AMOP Lectures 5.0-5.5
Tue 2.11 & Thur 2.13 2014

Relativity of lightwaves and Lorentz-Minkowski coordinates V.

(Ch. 0-4 of Unit 8)

Review of space-time (x,ct) and per-space-time (ω,ck) geometry

Space-time (x,ct) and per-space-time (ω,ck) geometry and its physics

- *All of those contraction and expansion coefficients*
  - *Detailed views Einstein time dilation*
    - *The old "smoke and mirrors" trick*
  - *Detailed views Lorentz contraction*
    - ➡ *Heighway's paradox 1 and 2*

Phase invariance used to derive (x,ct)↔(x′,ct′) Einstein Lorentz Transformations (ELT)

Introducing the stellar aberration angle σ vs. rapidity ρ

Trigonometry: From circular to hyperbolic and back

- *Finish "Sin-Tan" blackboard construction*
- *Group vs. phase velocity and tangent contacts*

Epstein's[†] space-proper-time (x,cτ) plots ("c-tau" plots)

[†]Lewis Carroll Epstein, *Relativity Visualized*
Insight Press, San Francisco, CA 94107

See also: L. C. Epstein, *Thinking Physics Press, Insight Press, San Francisco, CA 94107*

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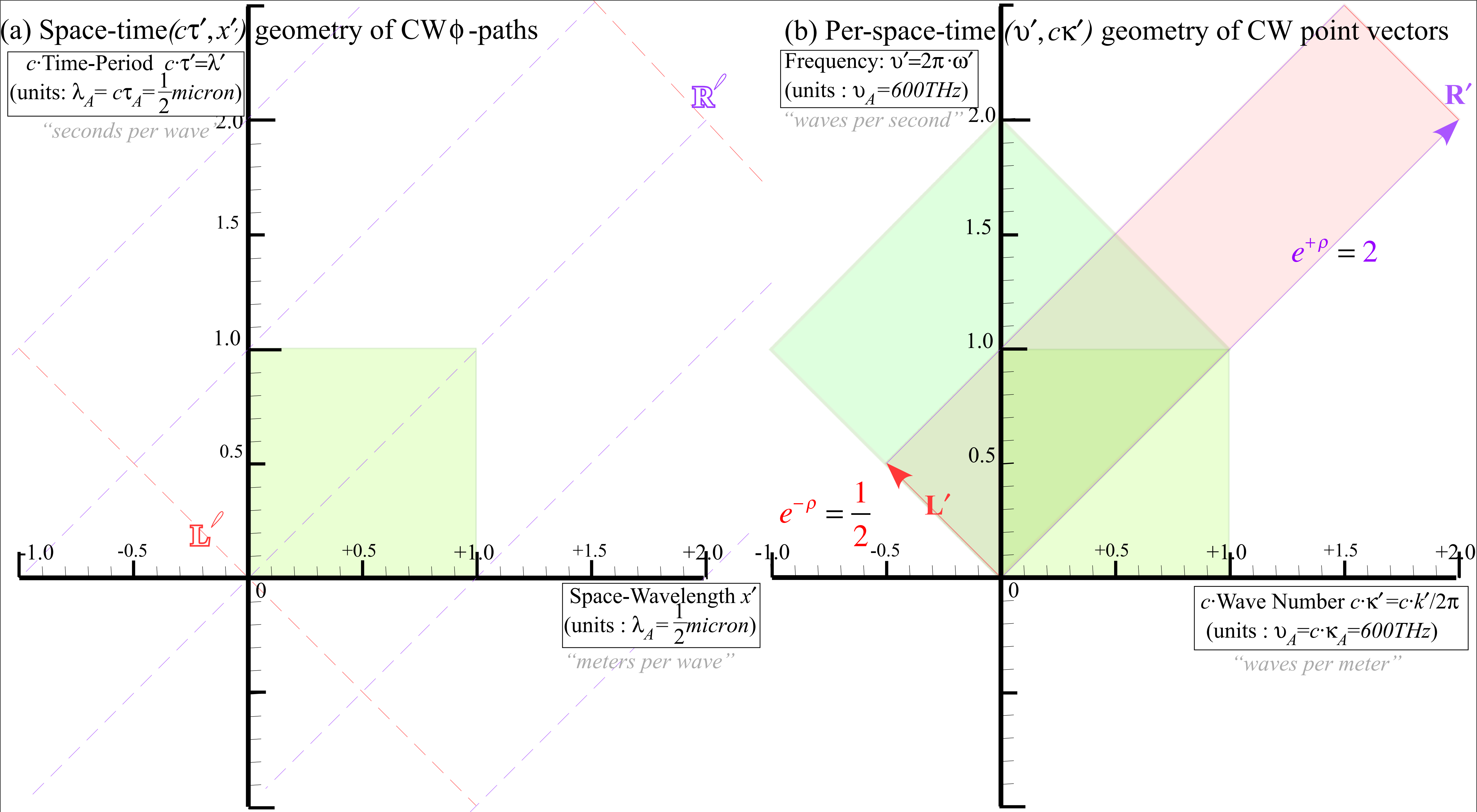
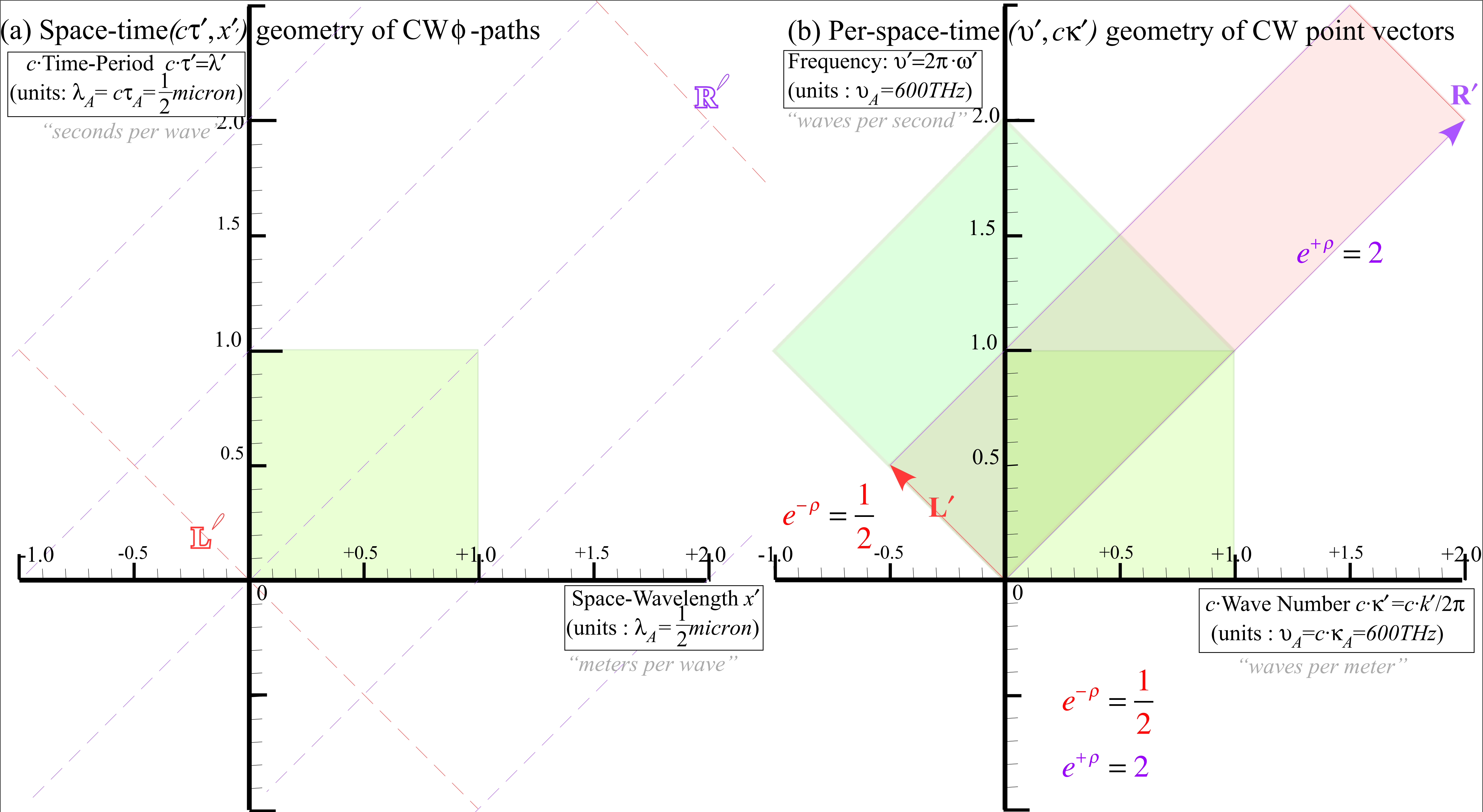


Fig. 7 SRQMbyR&C



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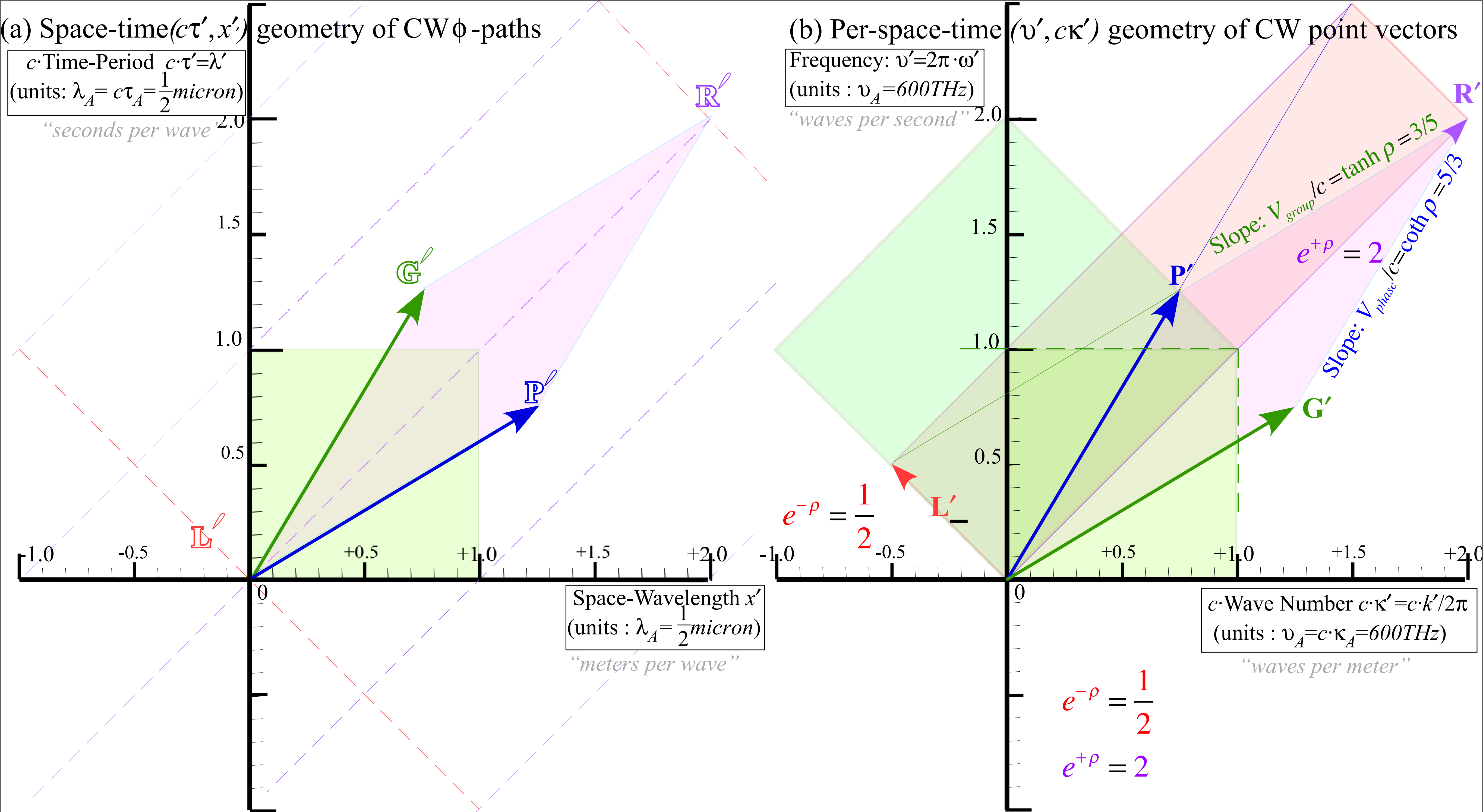

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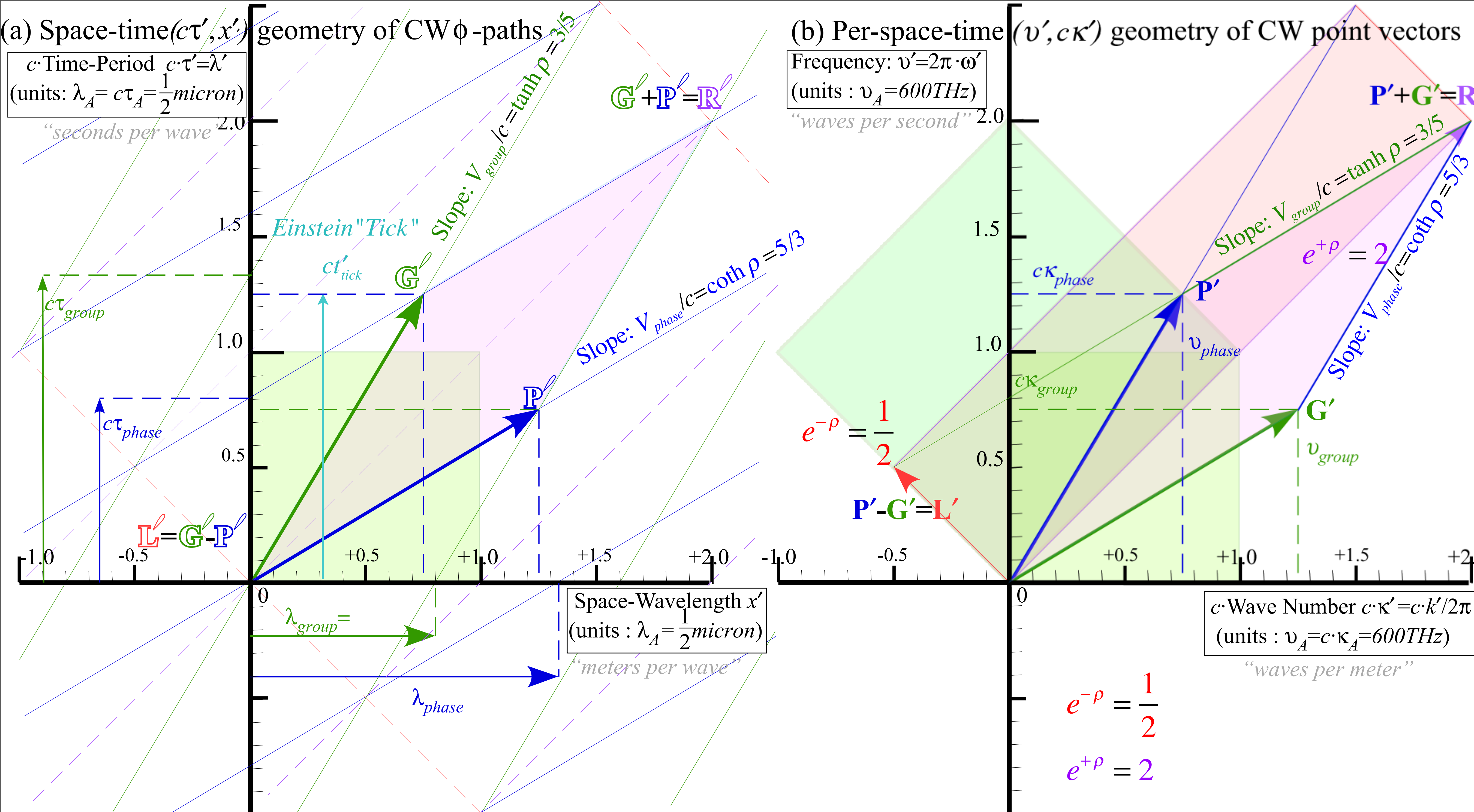

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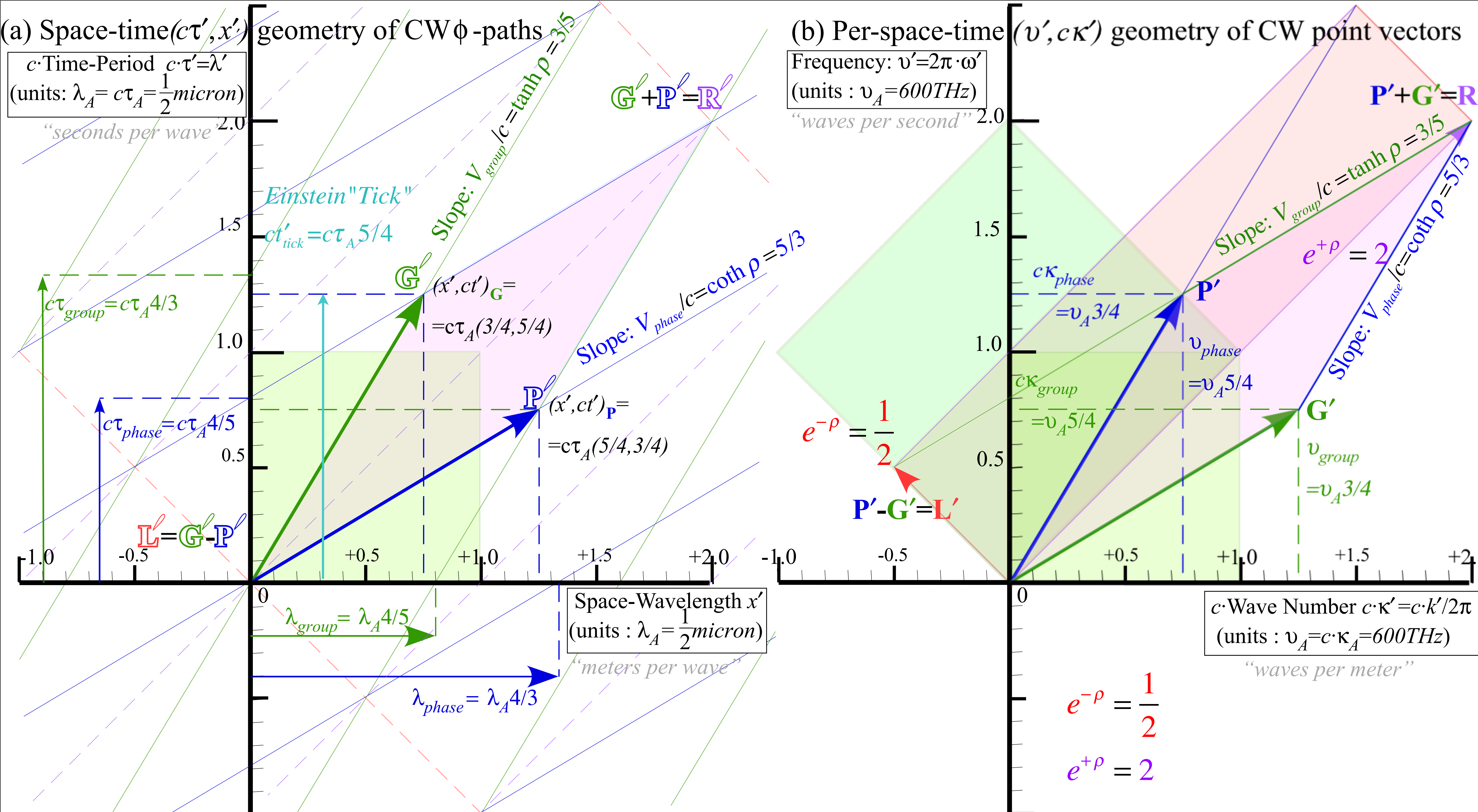

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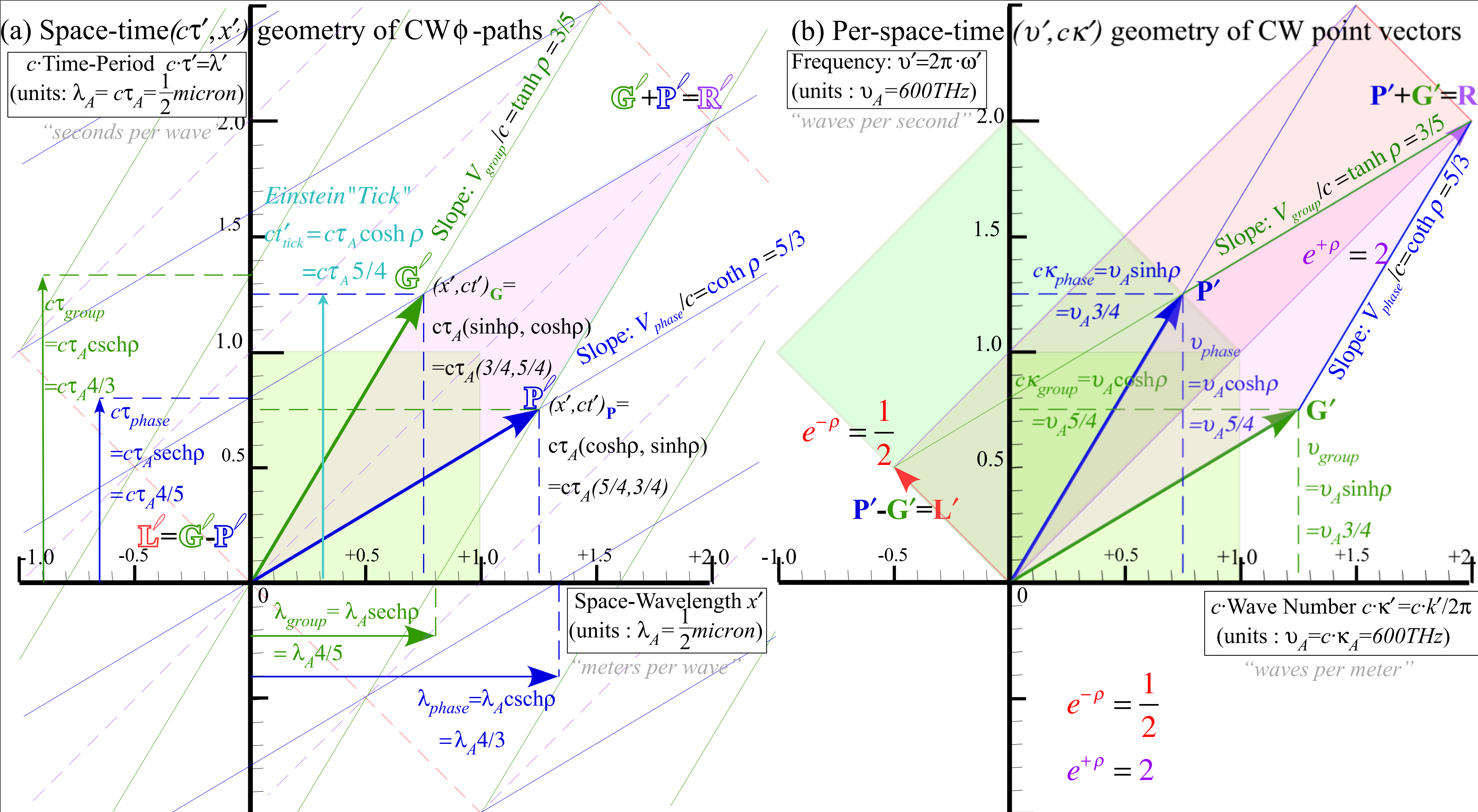

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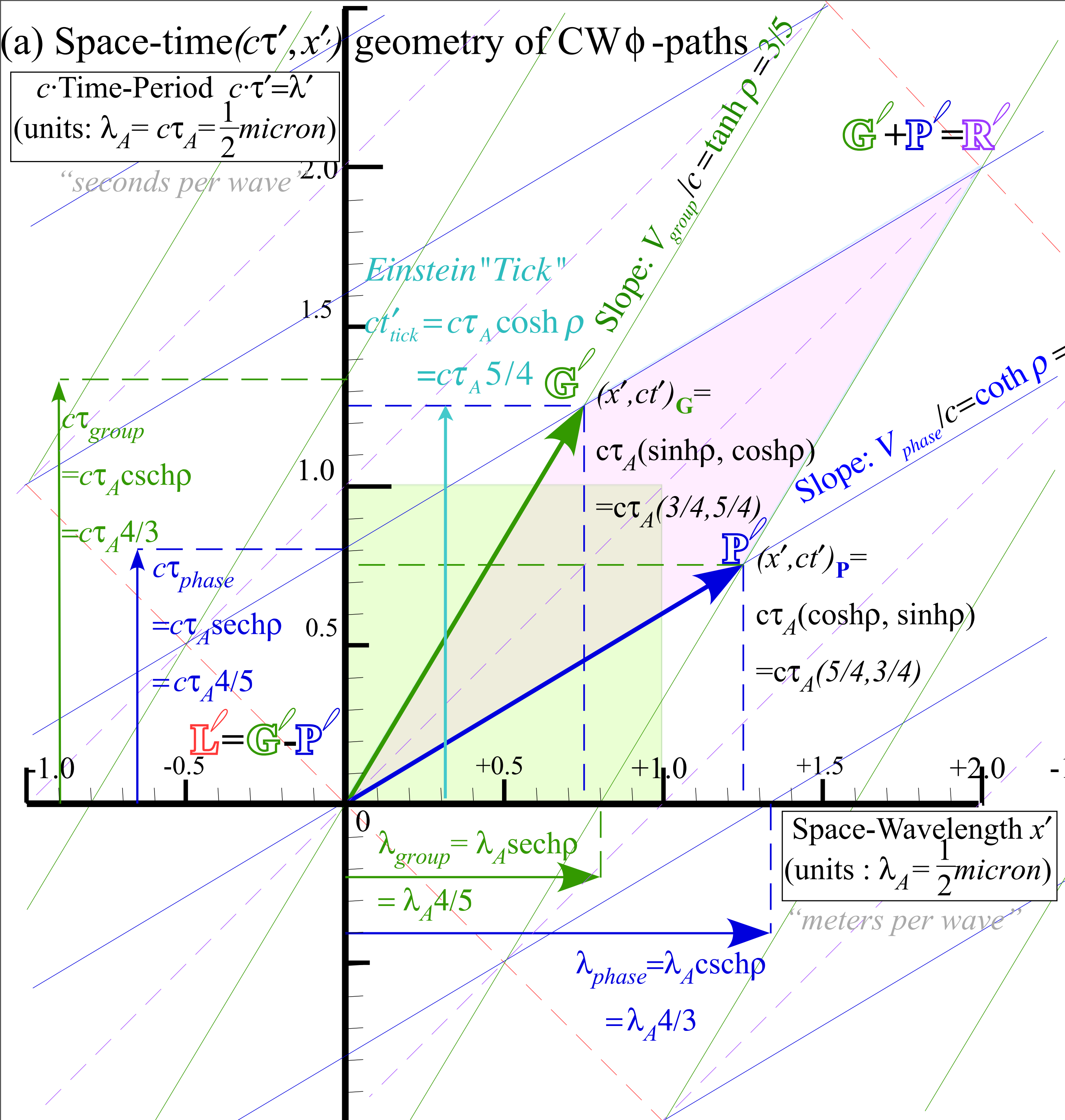

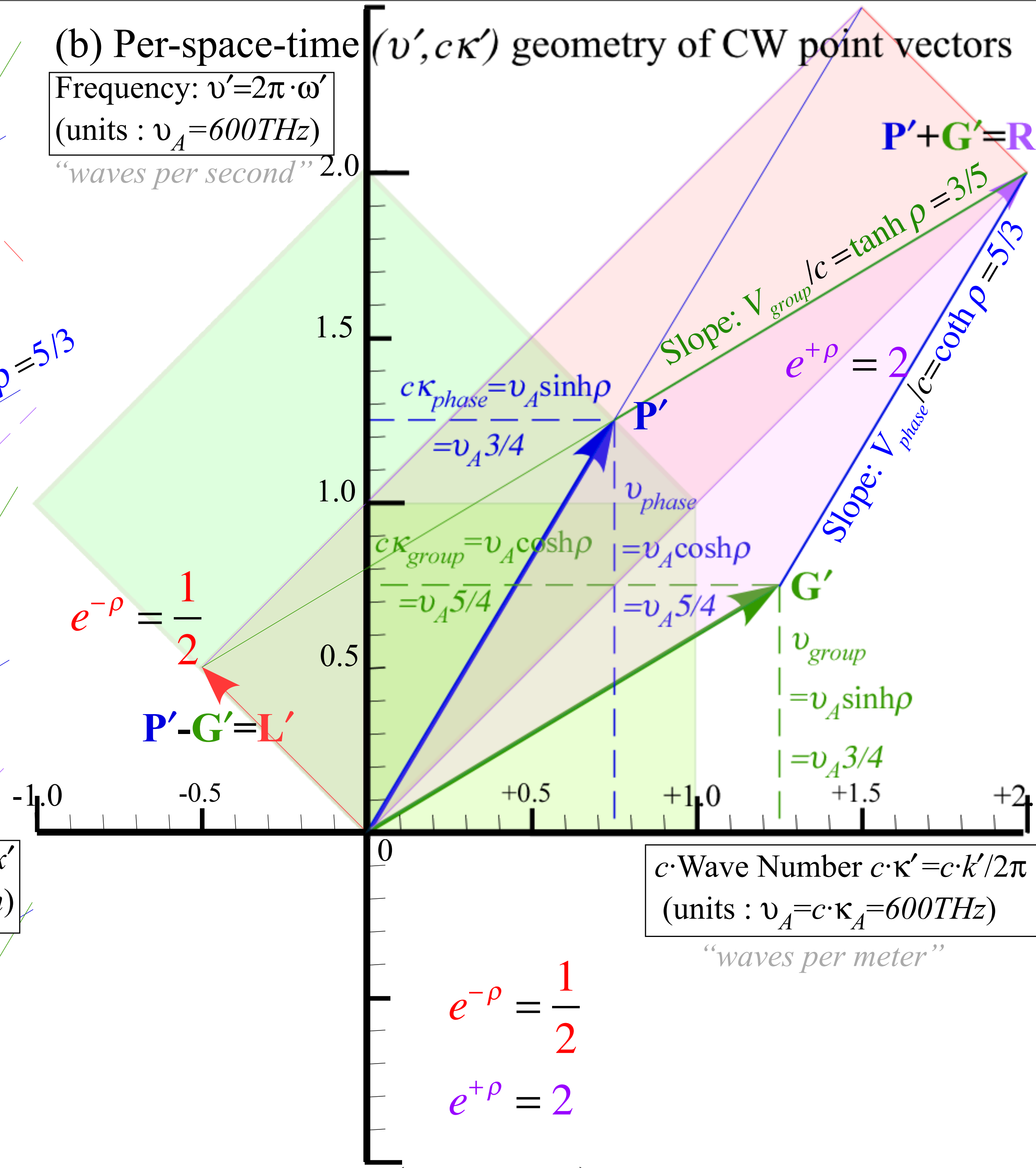

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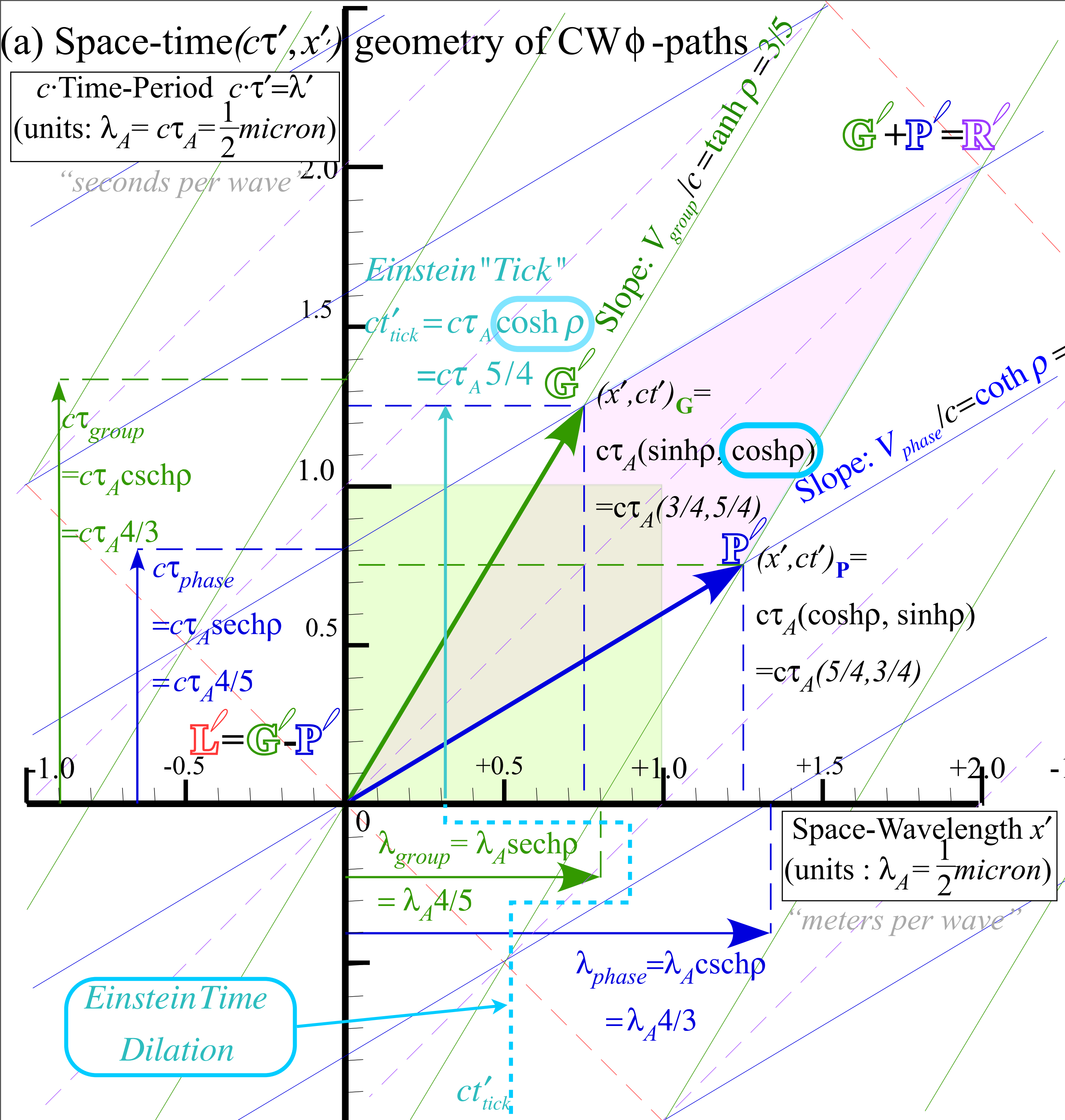

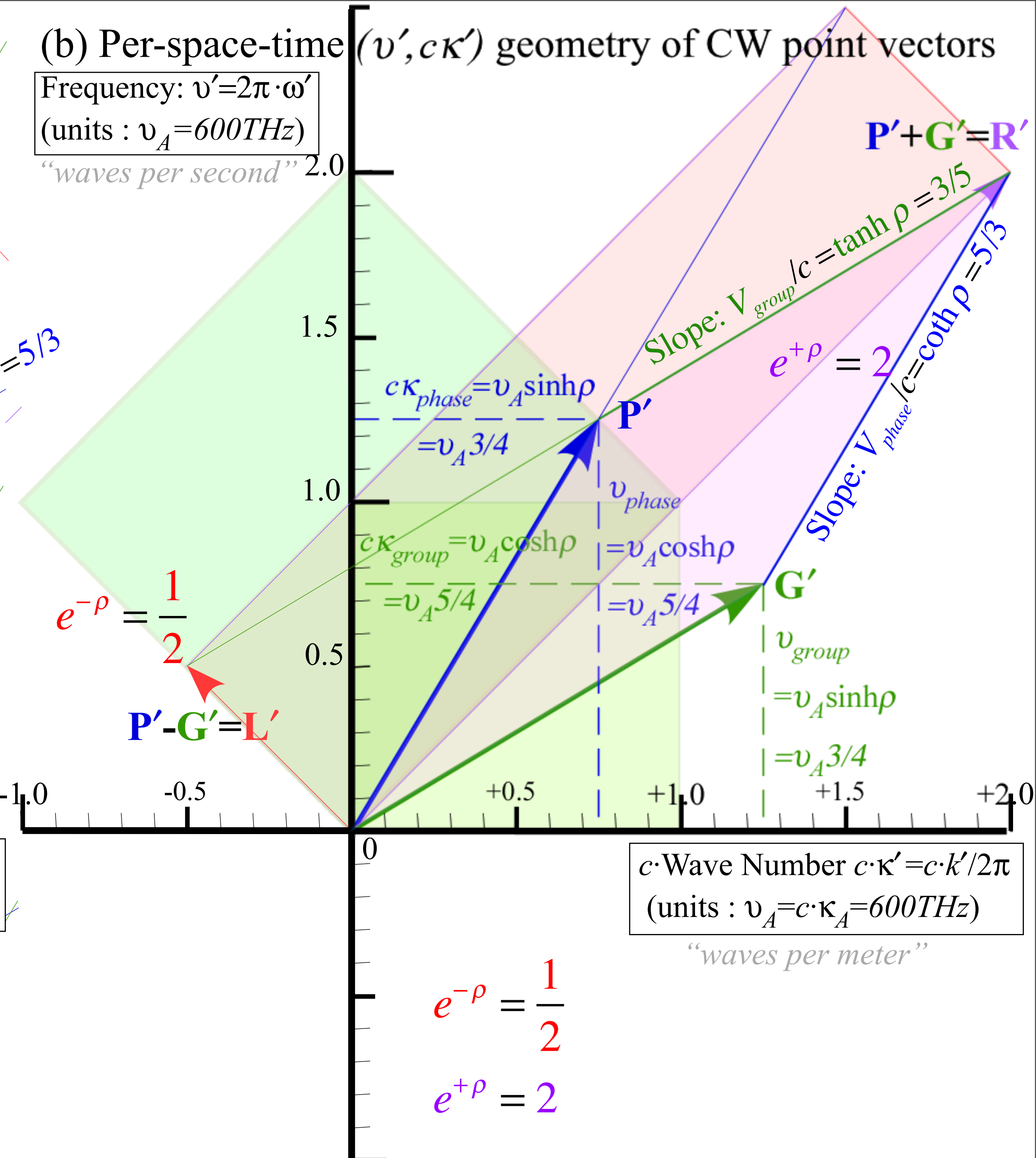

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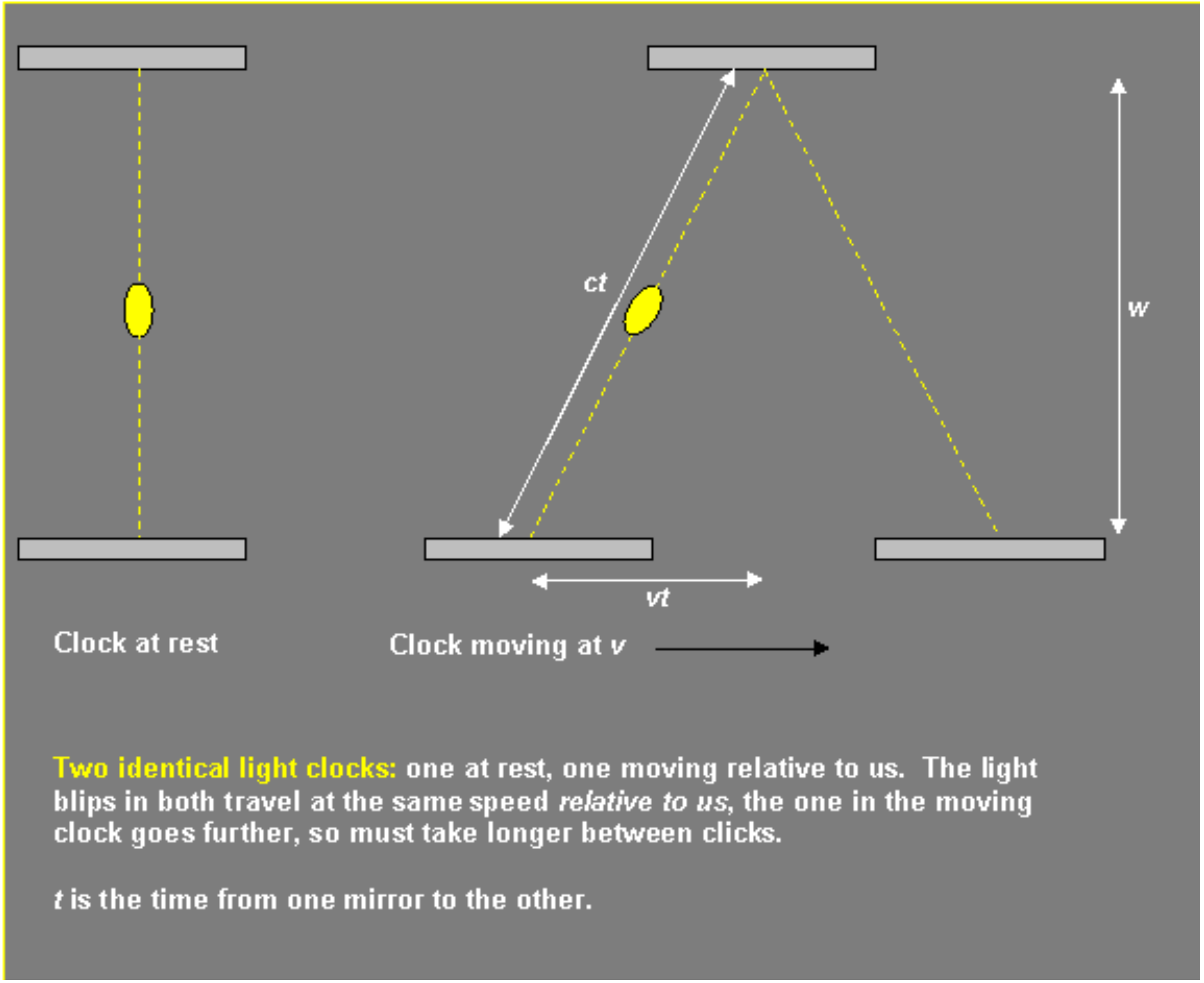

$c^2t^2 = v^2t^2 + w^2$

$t^2(c^2 - v^2) = w^2$

time between clicks for Jill's clock to be:

$t^2(1 - v^2/c^2) = w^2/c^2$

$$\text{time between clicks for moving clock} = \frac{2w}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$

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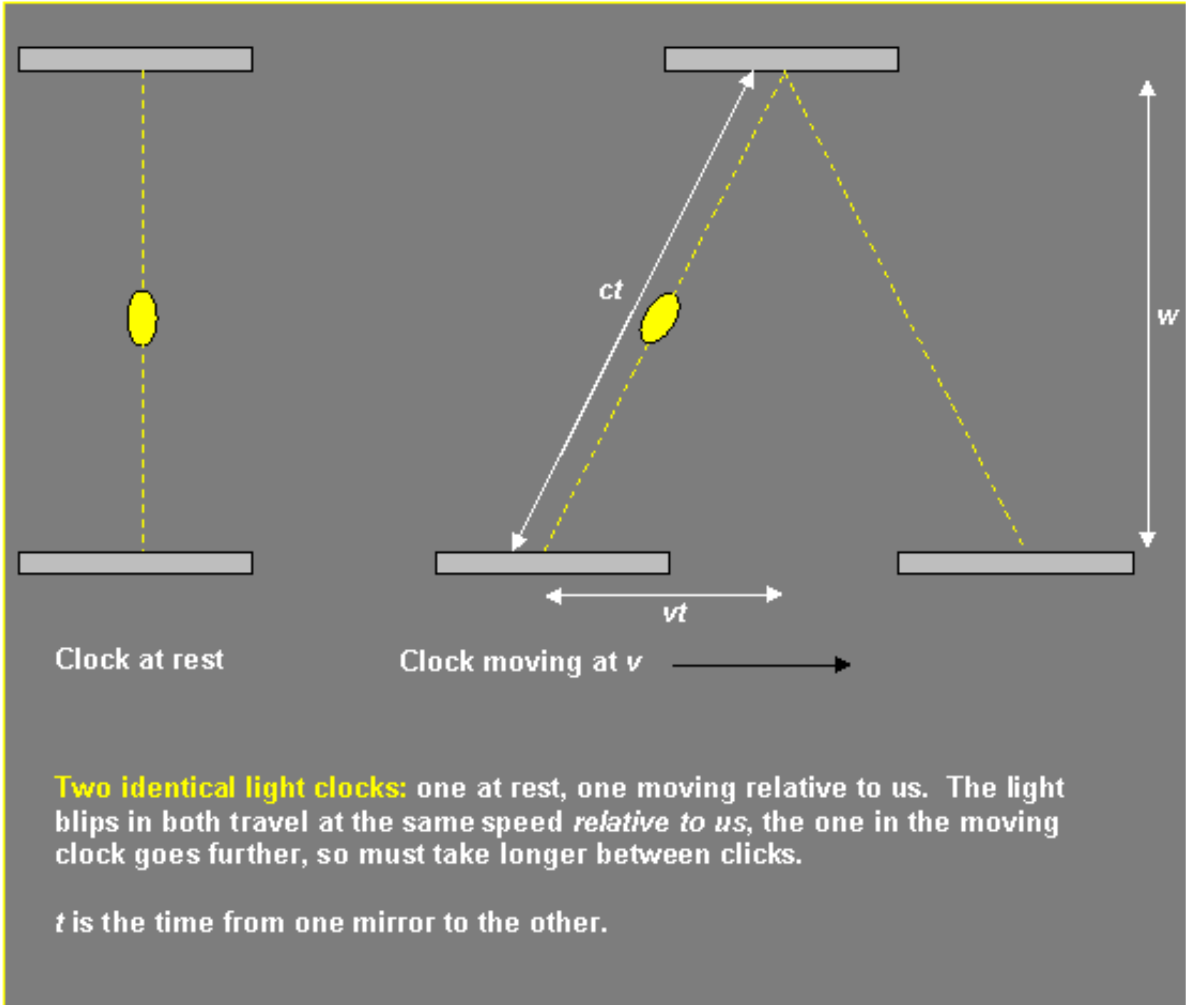
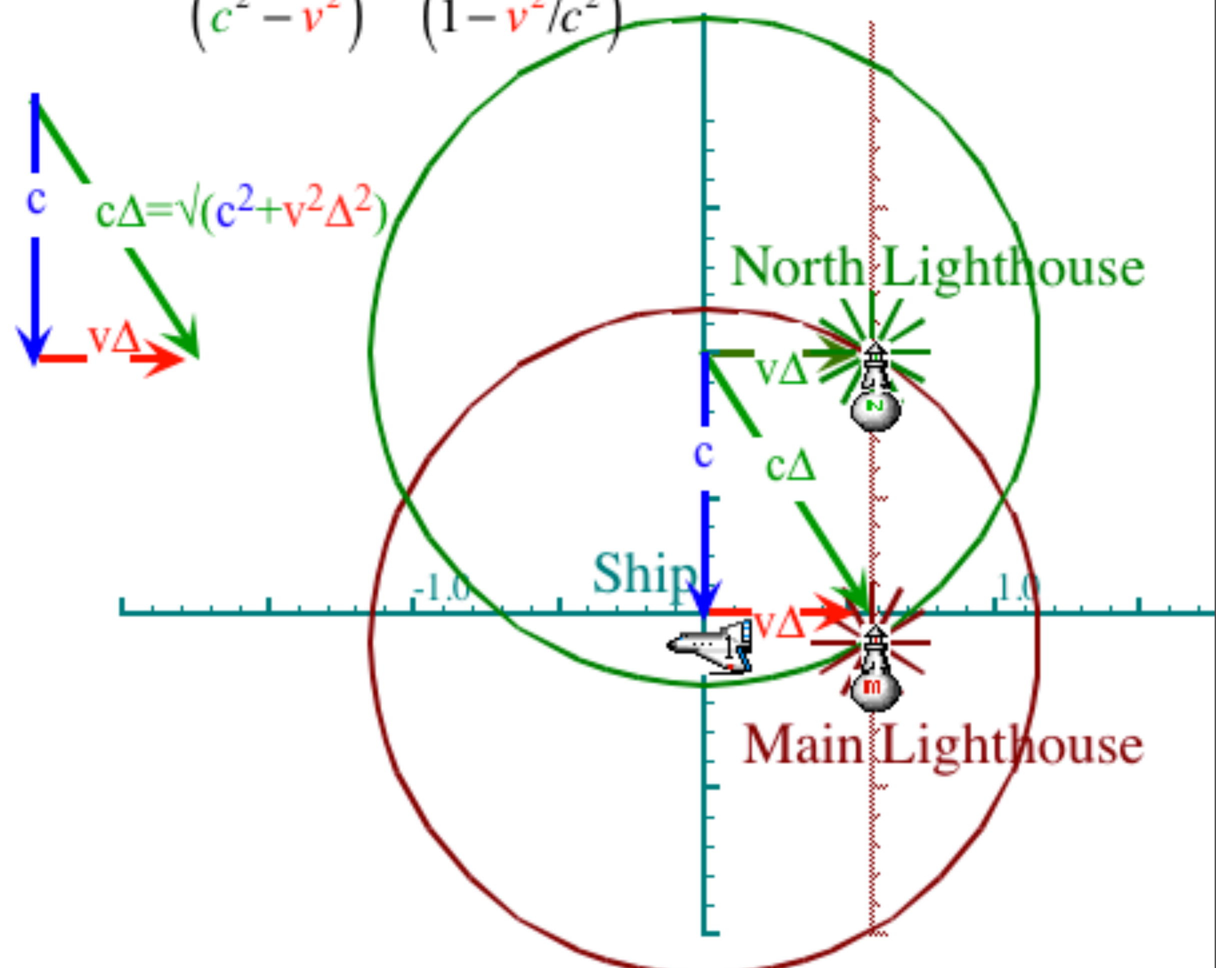


Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho = 1.15$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$

$$\Delta^2 = \frac{c^2}{\left(c^2 - v^2\right)} = \frac{1}{\left(1 - v^2/c^2\right)}$$



For $u/c = 1/2$

$\Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15$

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s

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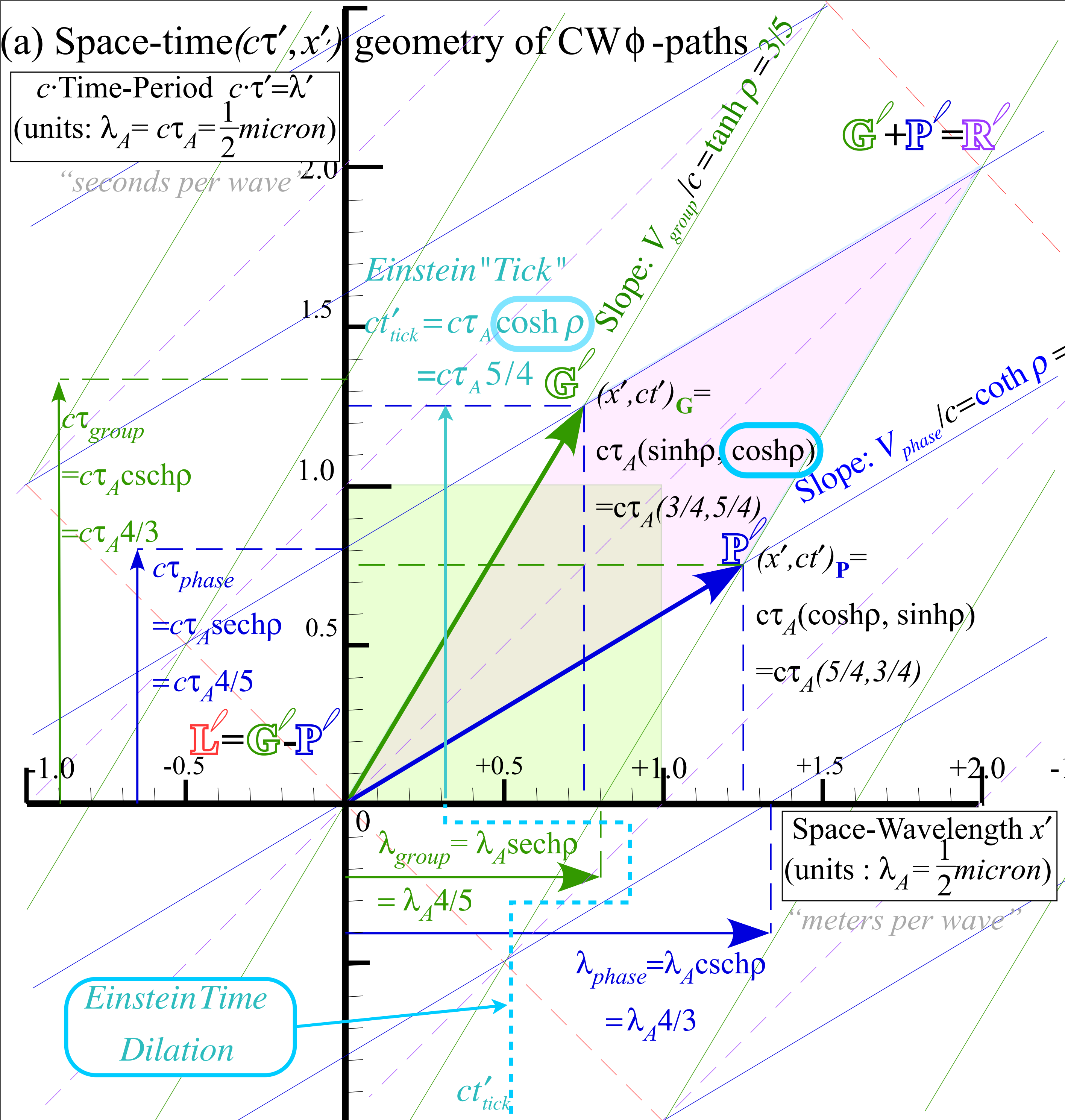
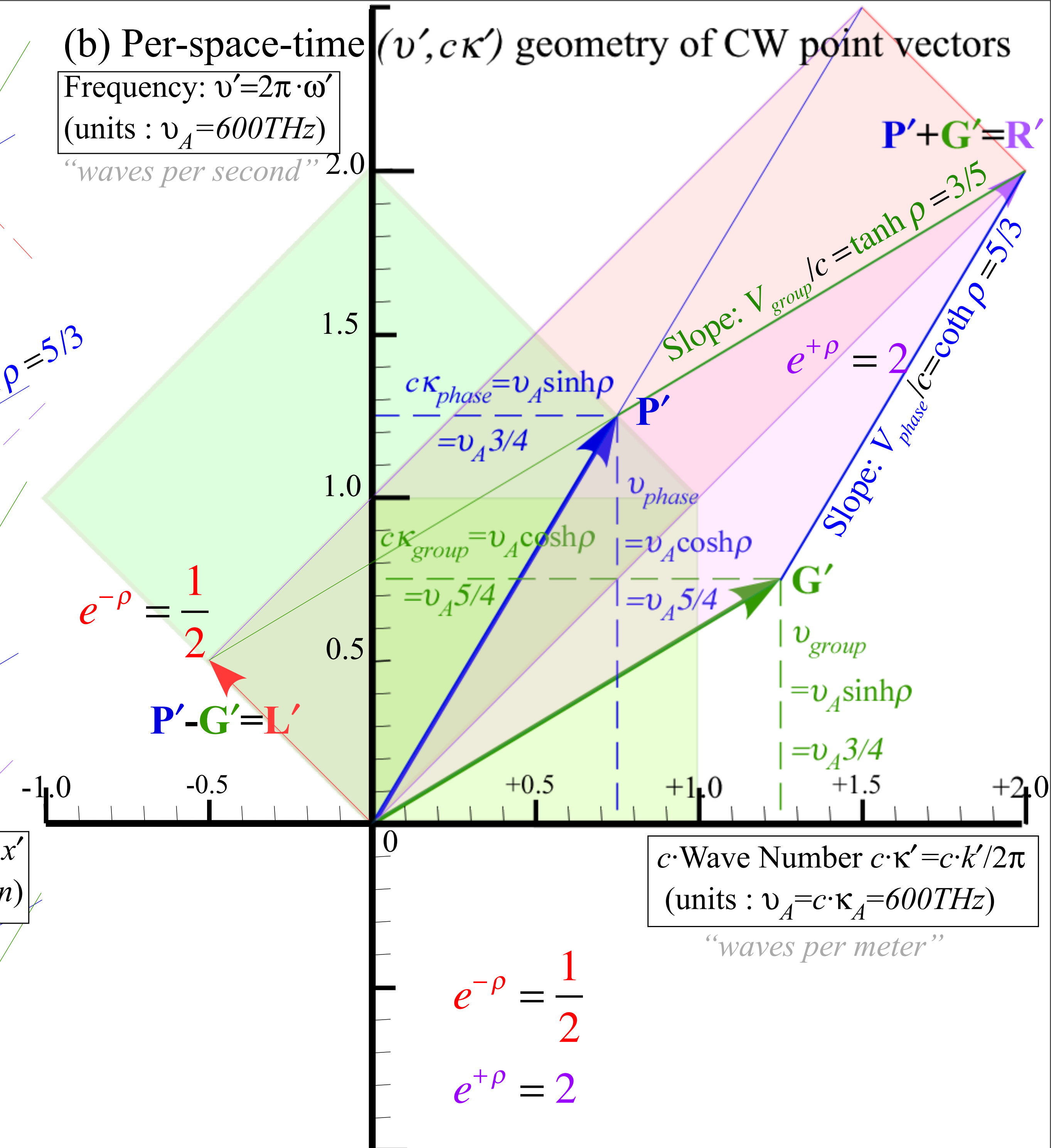
Now really All





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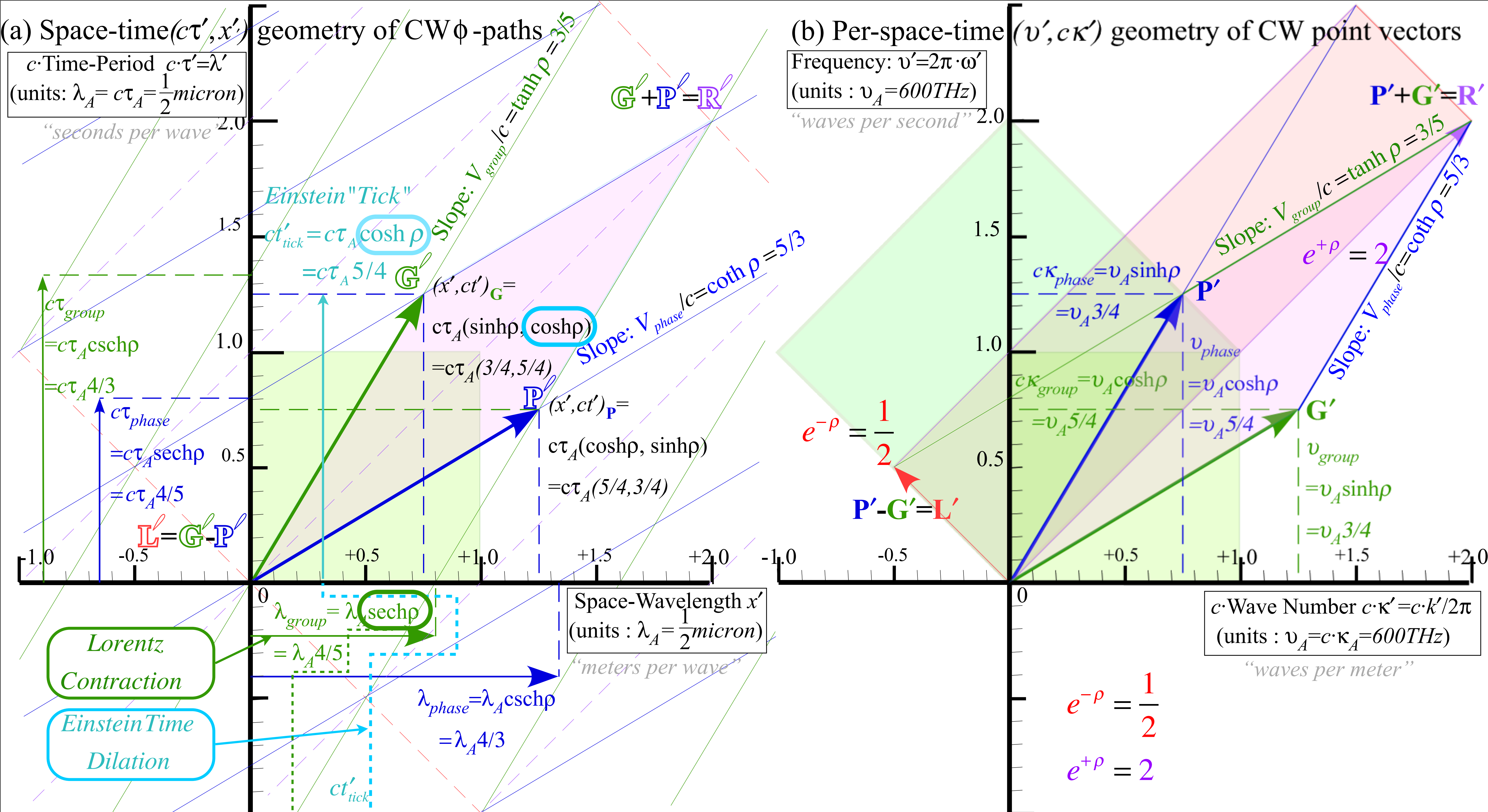

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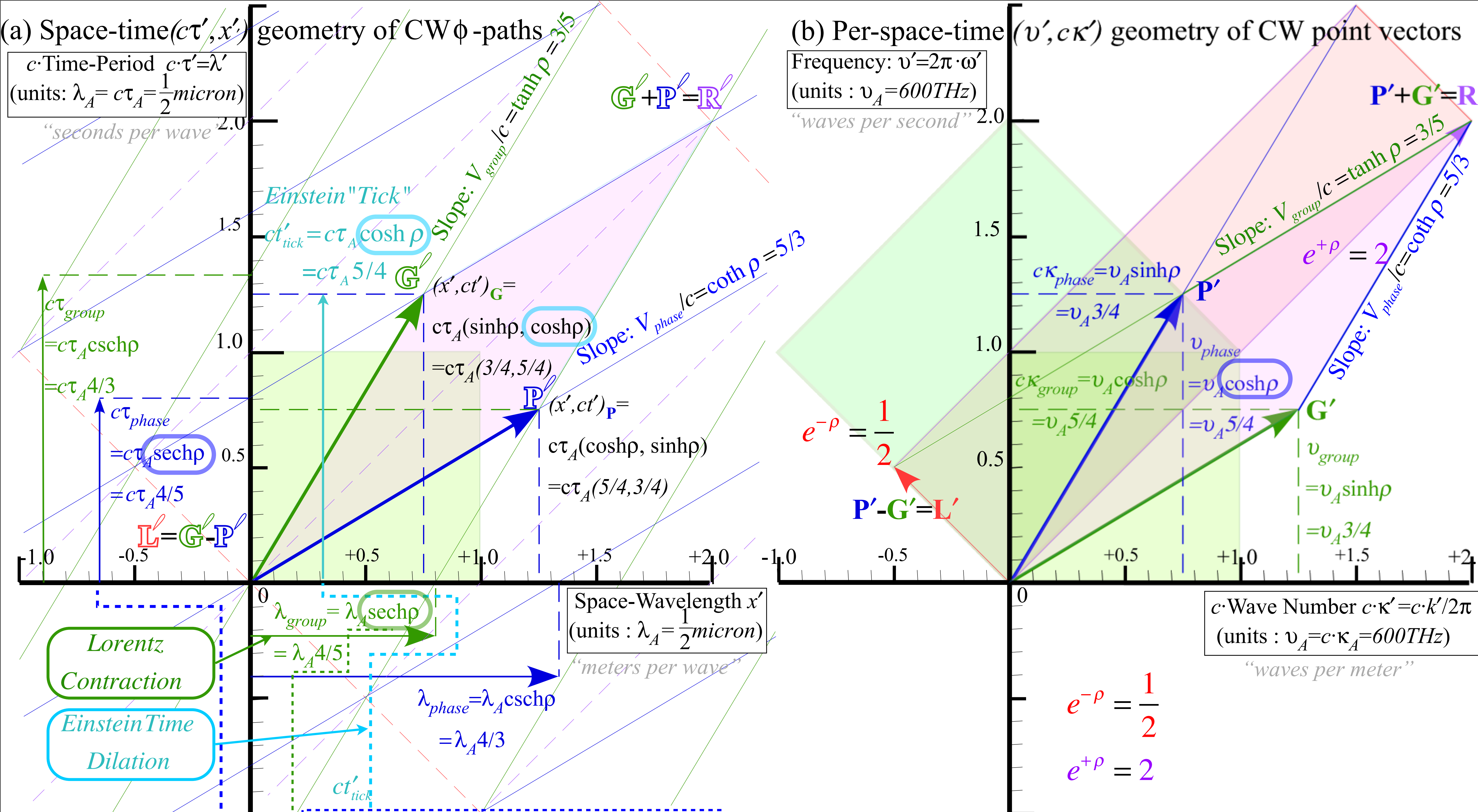

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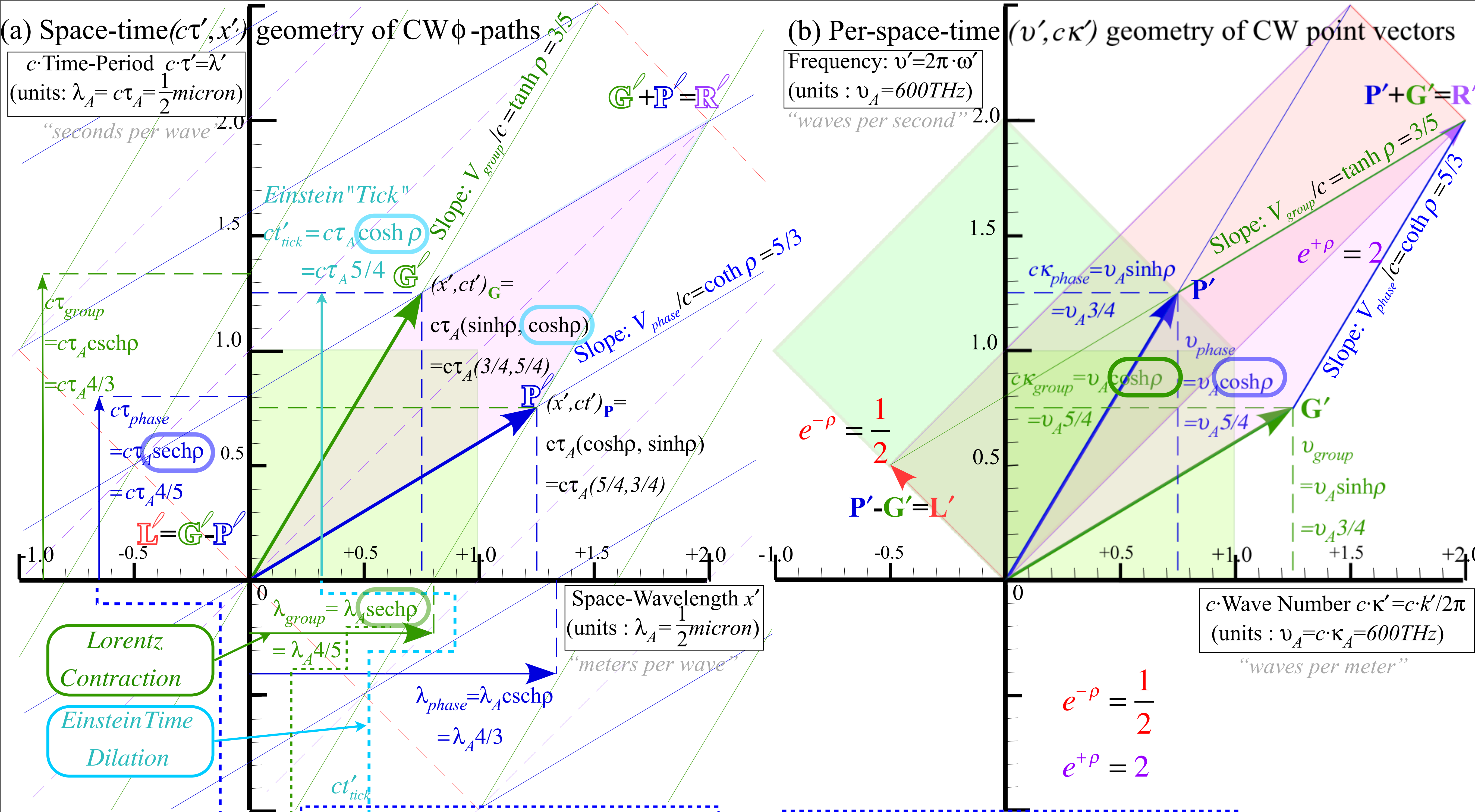

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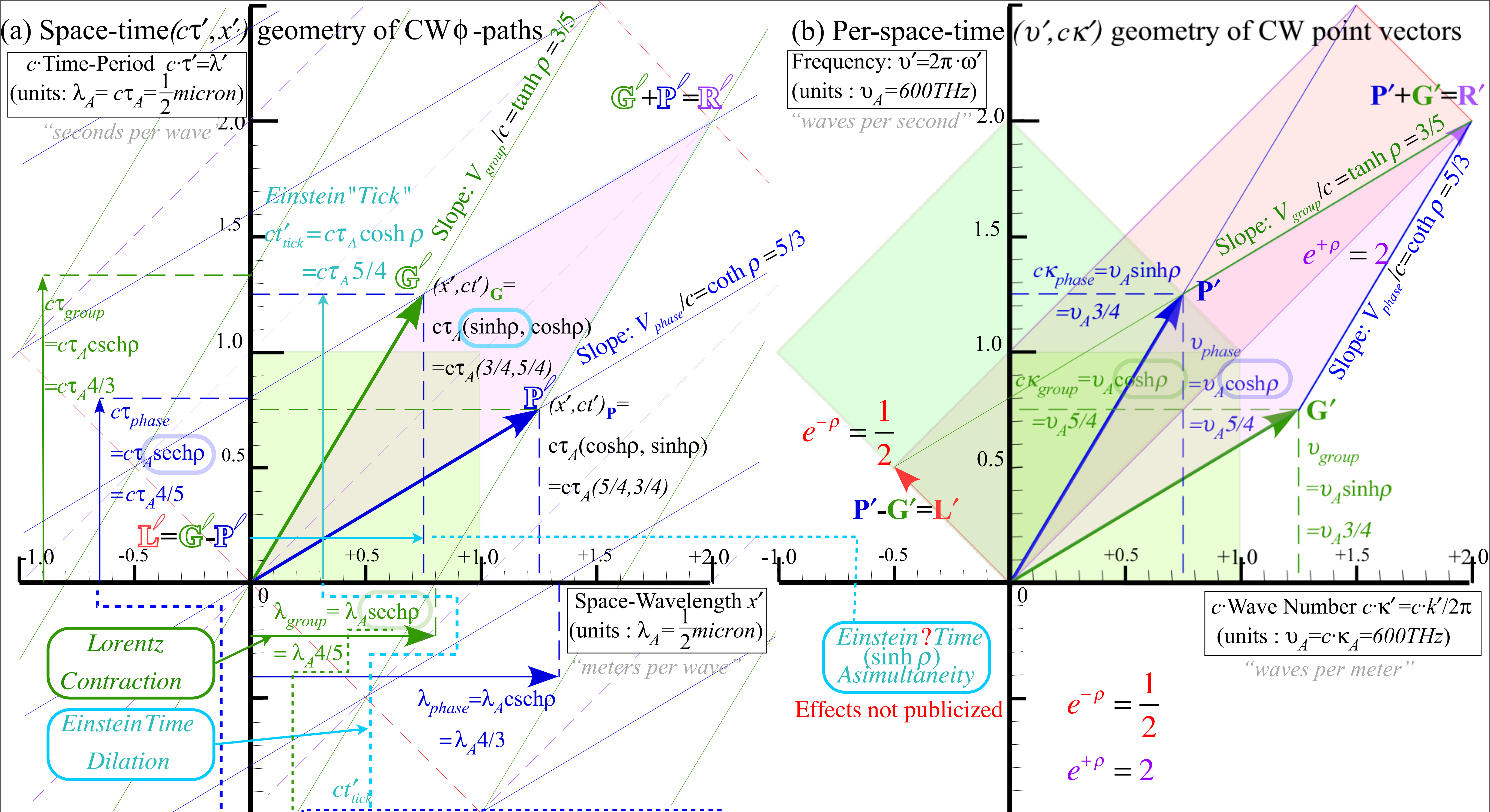

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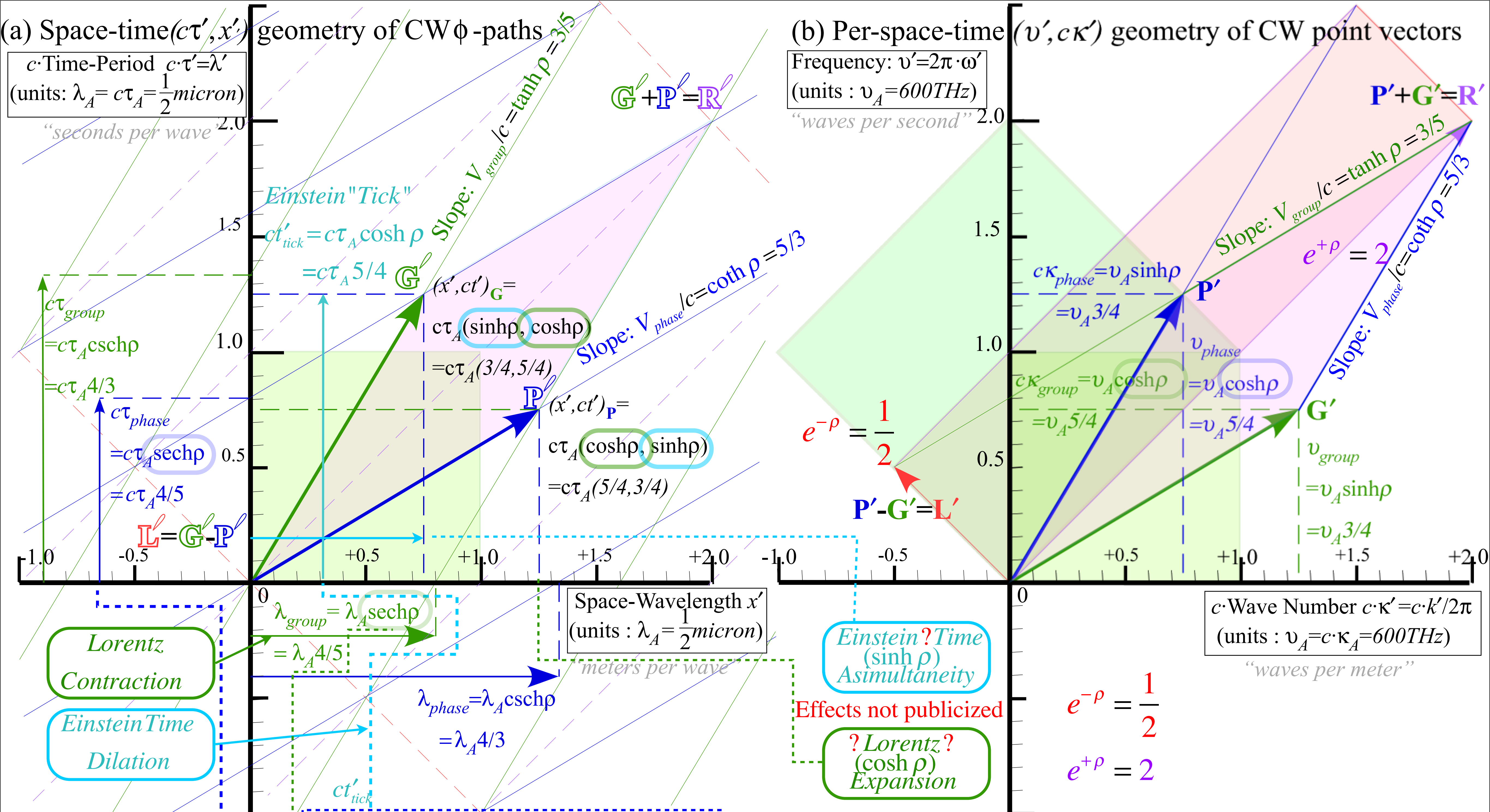

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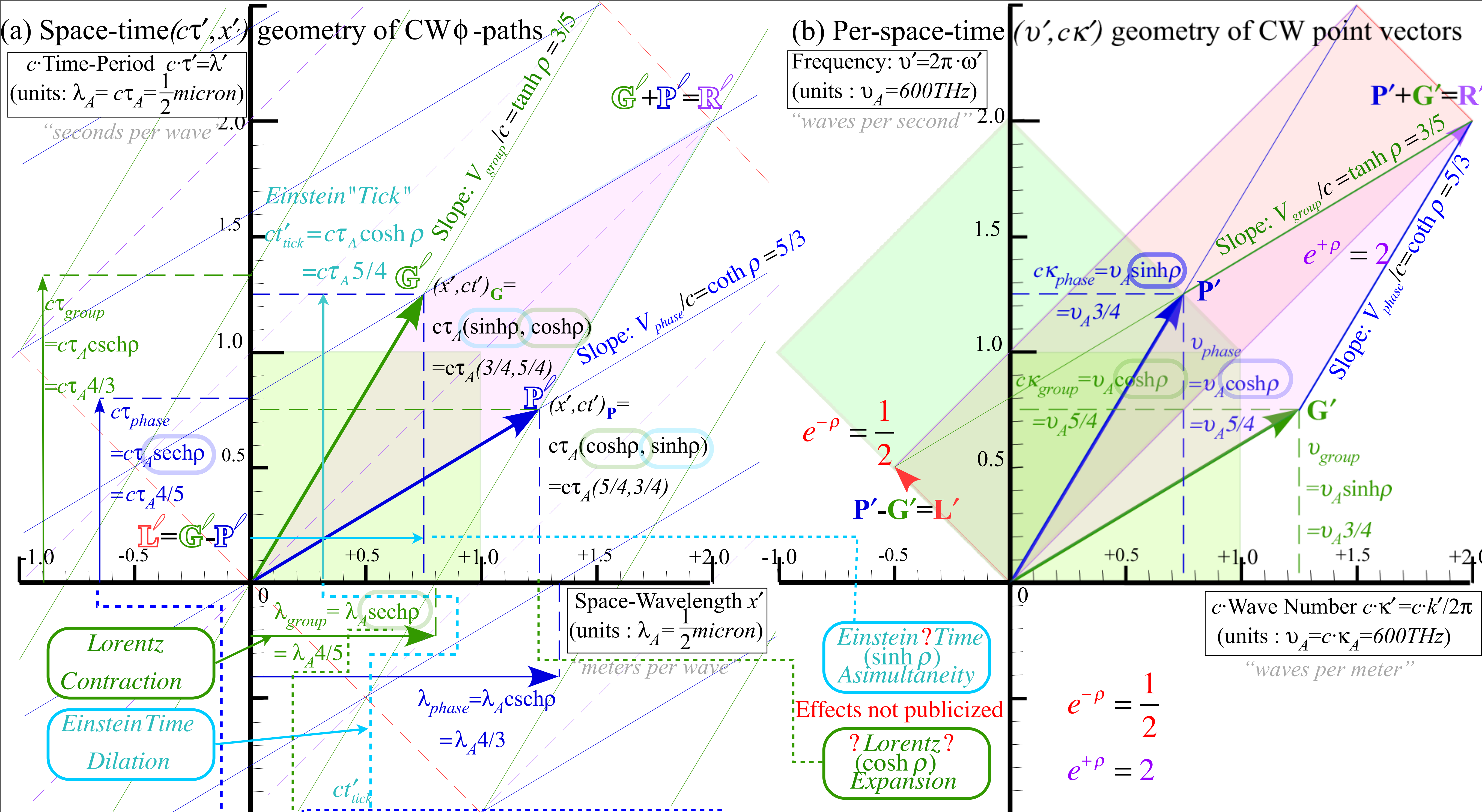
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$

$$\begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix} = \mathbf{P'} = \frac{\mathbf{R'}+\mathbf{L'}}{2} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}+e^{-\rho}}{2} \\ \frac{e^{+\rho}-e^{-\rho}}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix} = \mathbf{G'} = \frac{\mathbf{R'}-\mathbf{L'}}{2} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$

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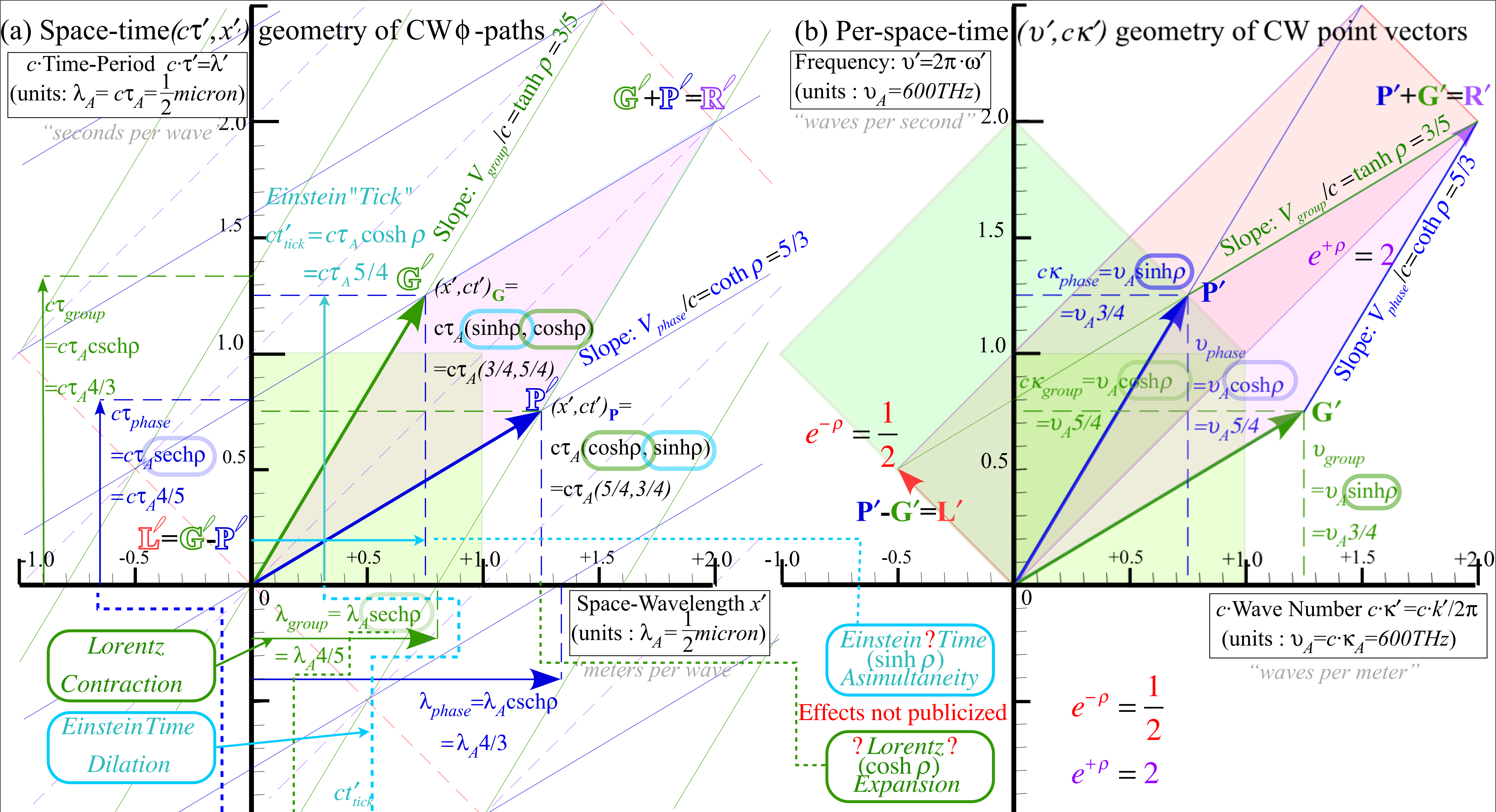
$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
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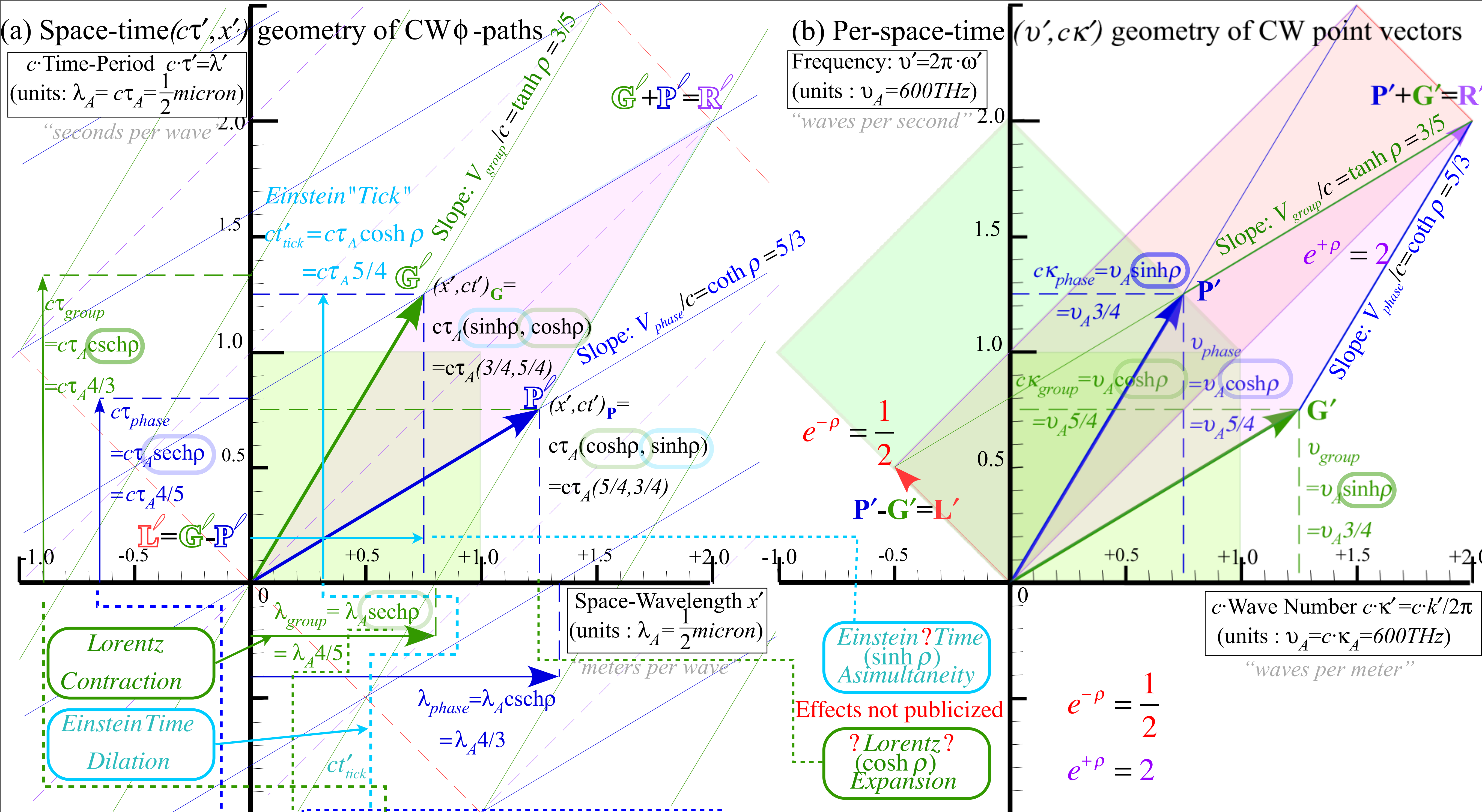

time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
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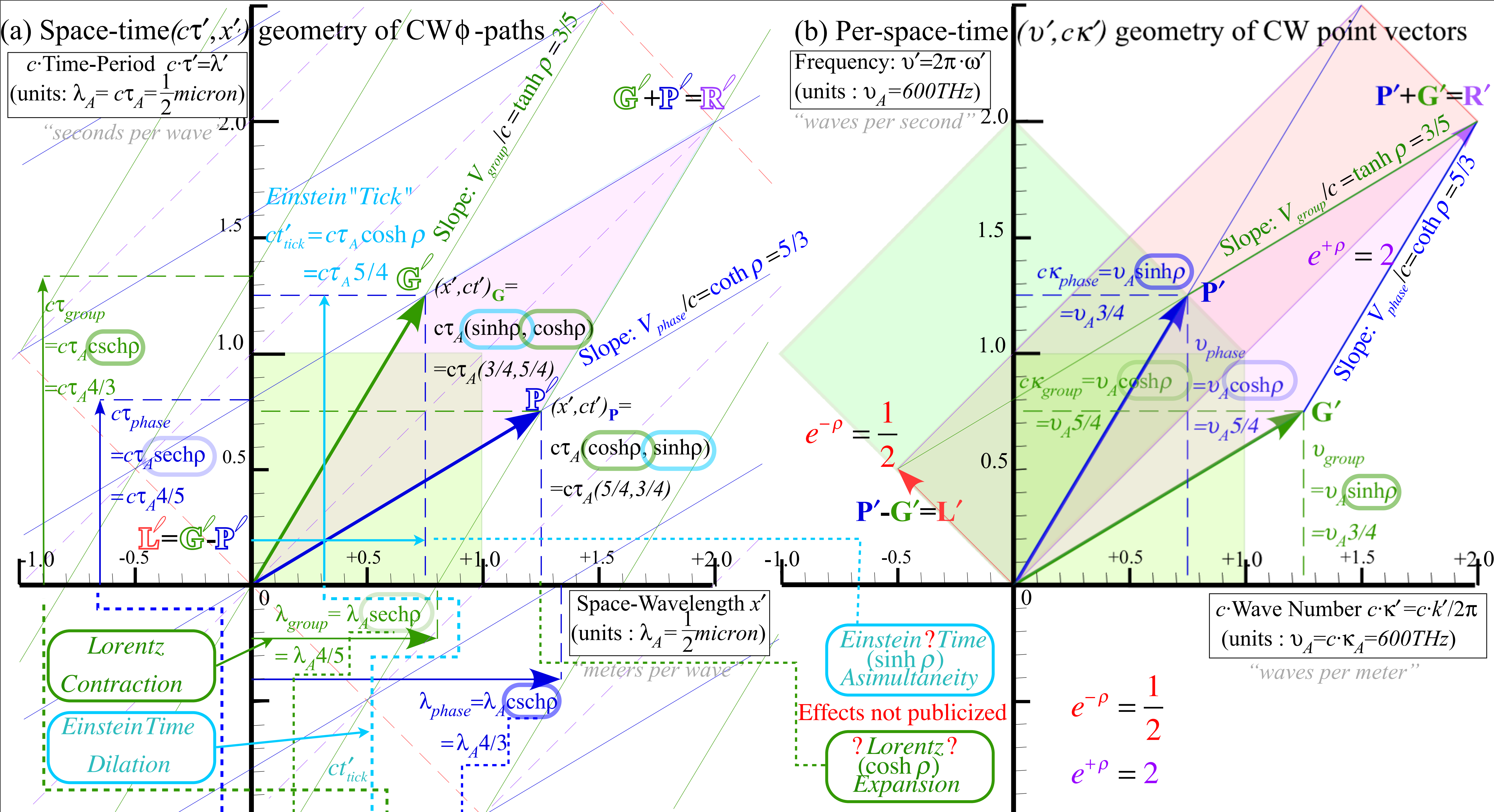

time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
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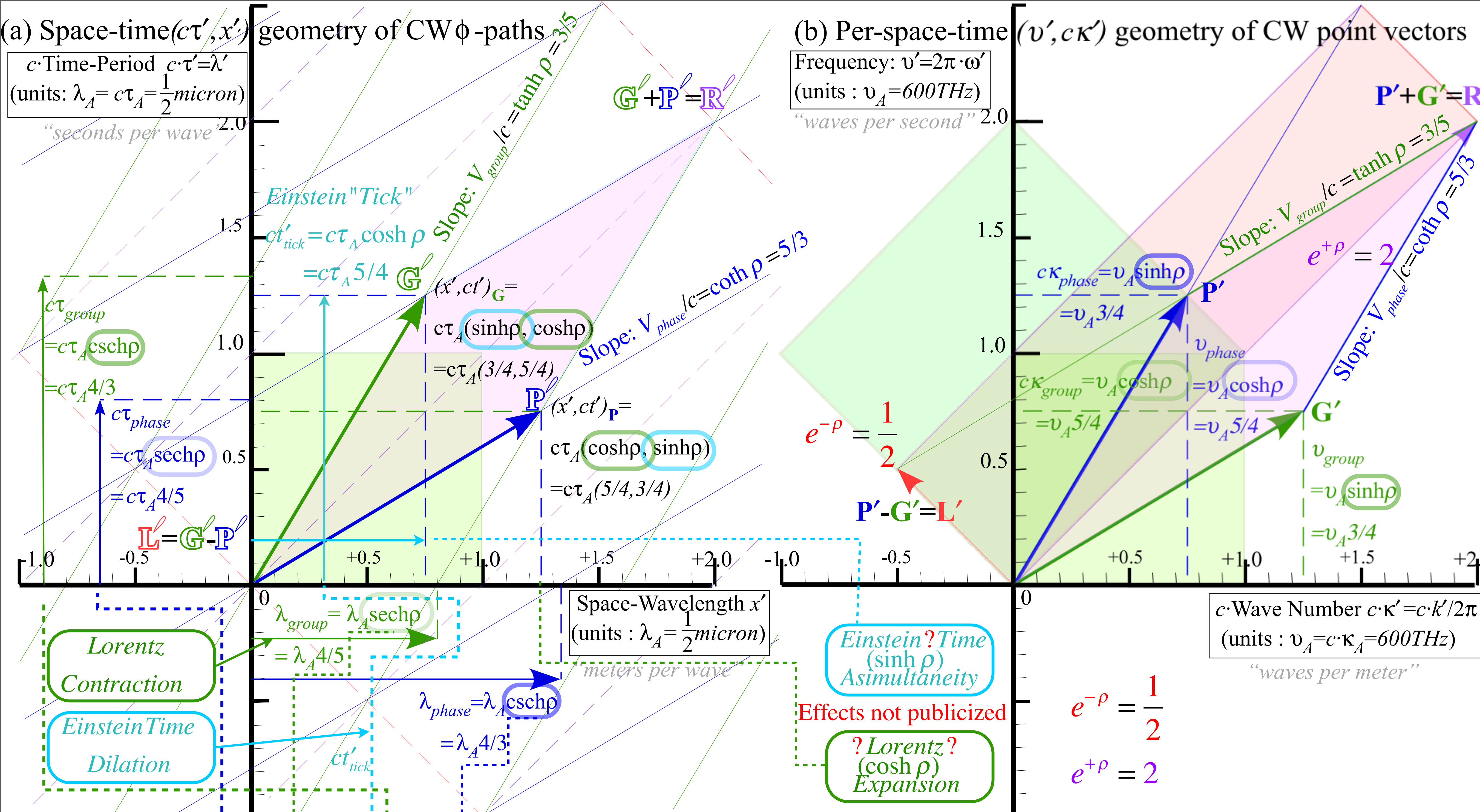

time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		Coming soon in theatre near you!						

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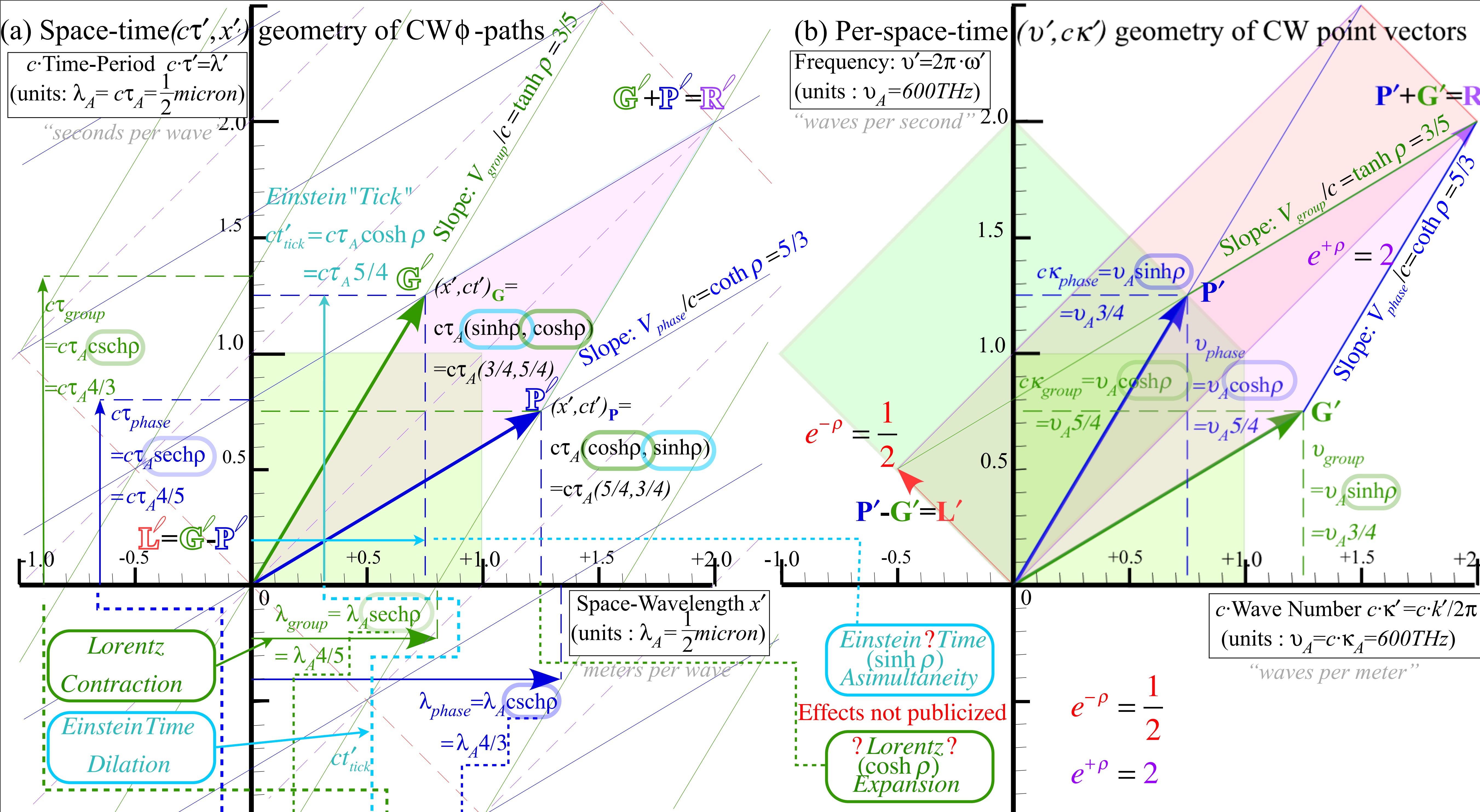


time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho=$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			

Coming soon in theatre near you!

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$

Space-time (x,ct) and per-space-time (ω,ck) geometry and its physics

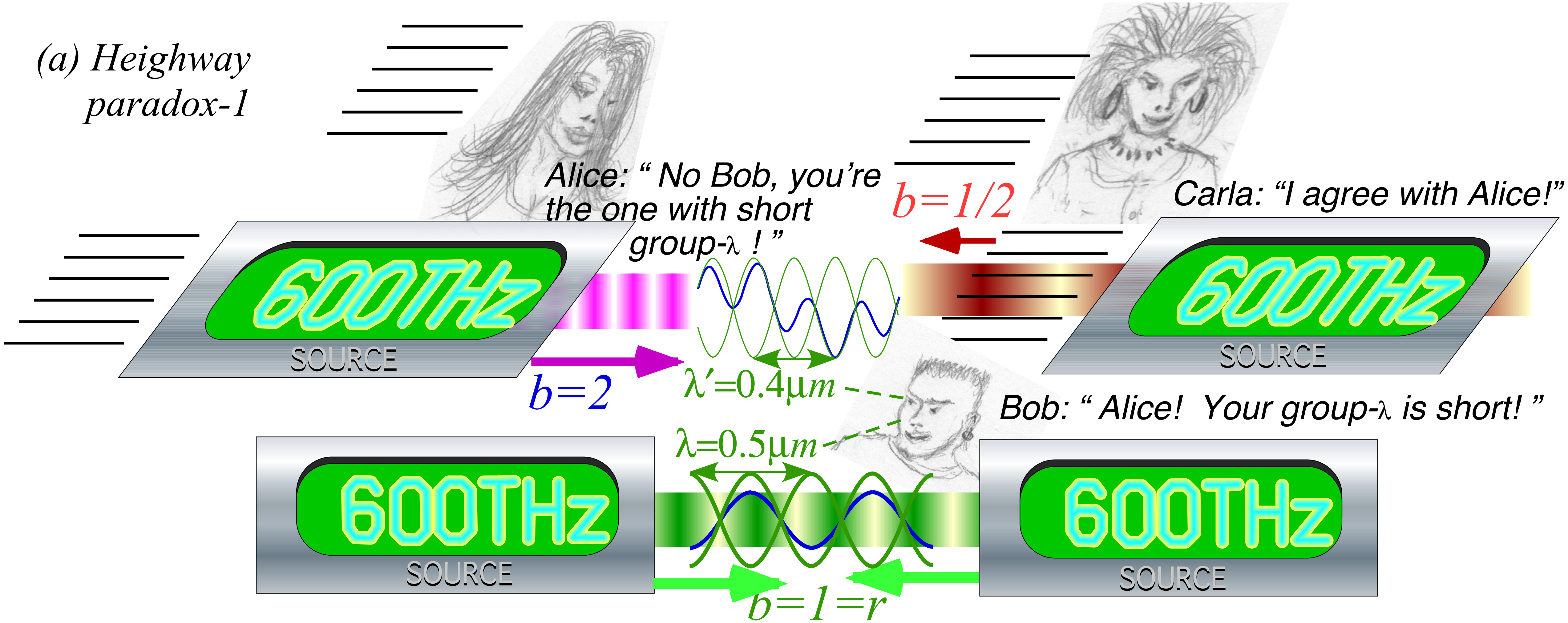
<u>All</u> of those contraction and expansion coefficients

Detailed views Einstein time dilation

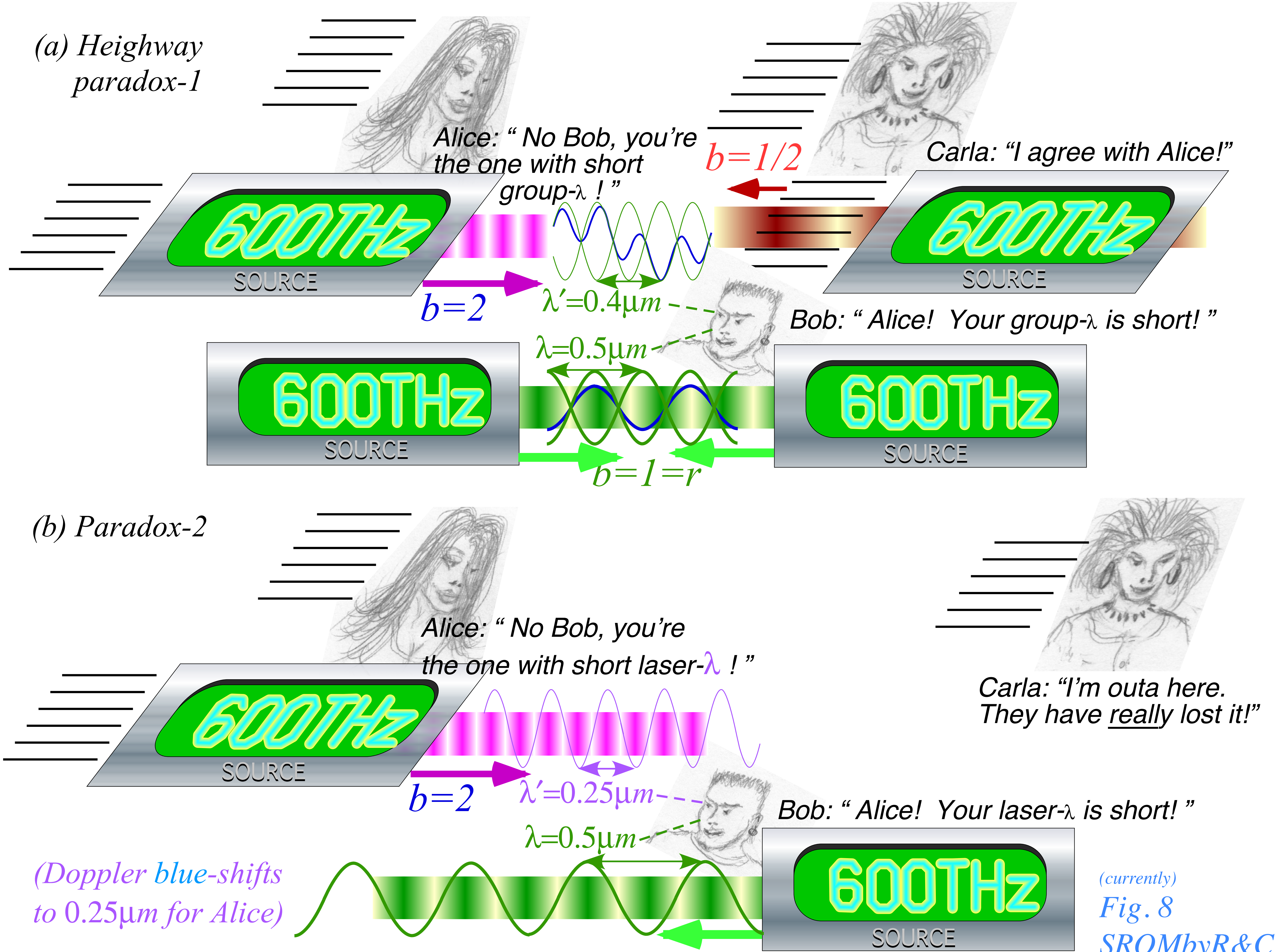
The old "smoke and mirrors" trick

Detailed views Lorentz contraction

Heighway's paradox 1 and 2



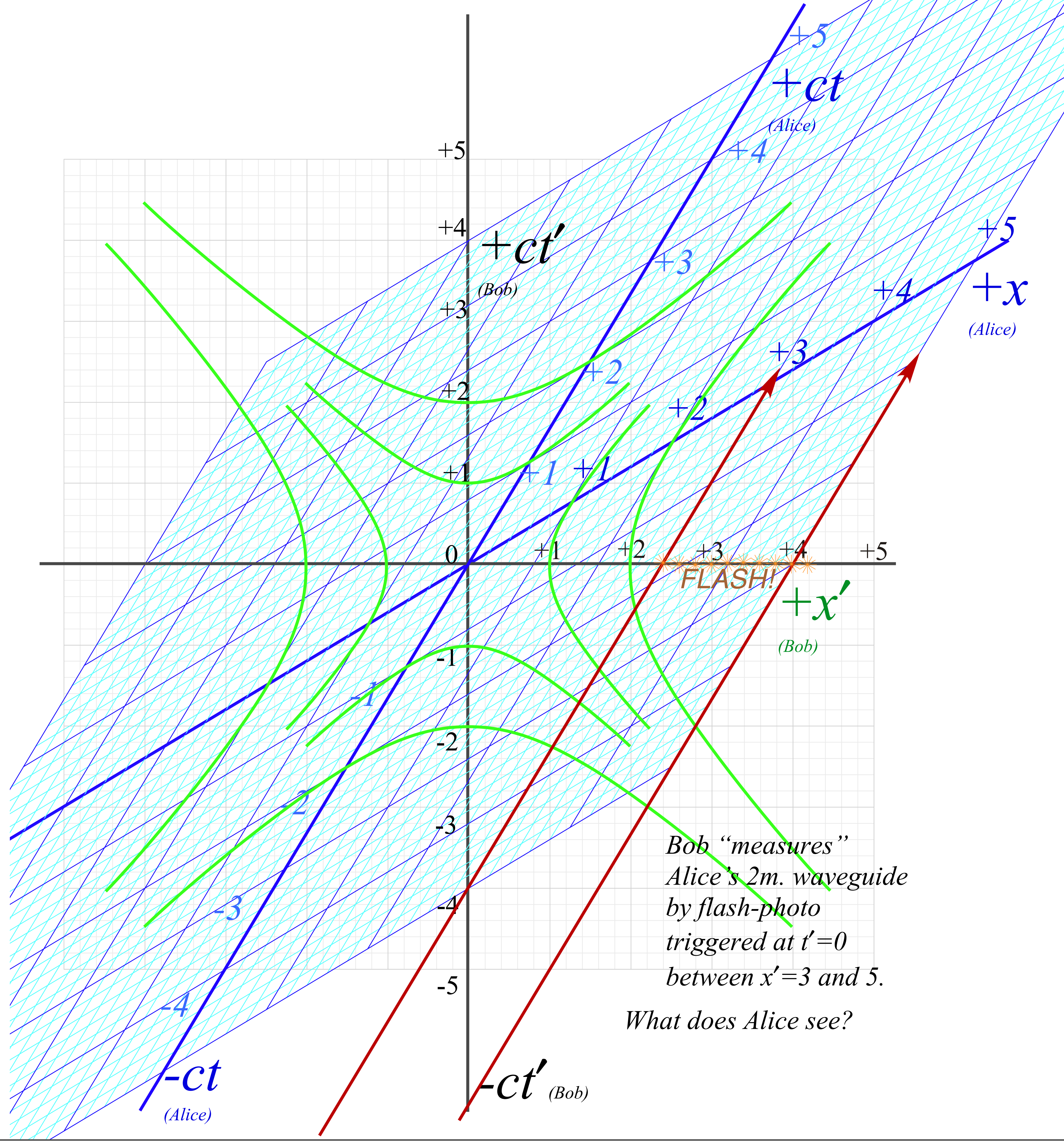
(currently part of)
Fig. 8
SRQMbyR&C



(currently) *Fig. 8* SRQMbyR&C

Detailed view of
Lorentz contraction
on space-time plot
Bob's axes:(x', ct')

Alice's 2m laser mirrors
lie at x=3 and x=5
These points trace time lines
shown at right of graph.



$$(c\tau)^2 = (+2)^2 = (ct)^2 - (x)^2 = (ct')^2 - (x')^2$$

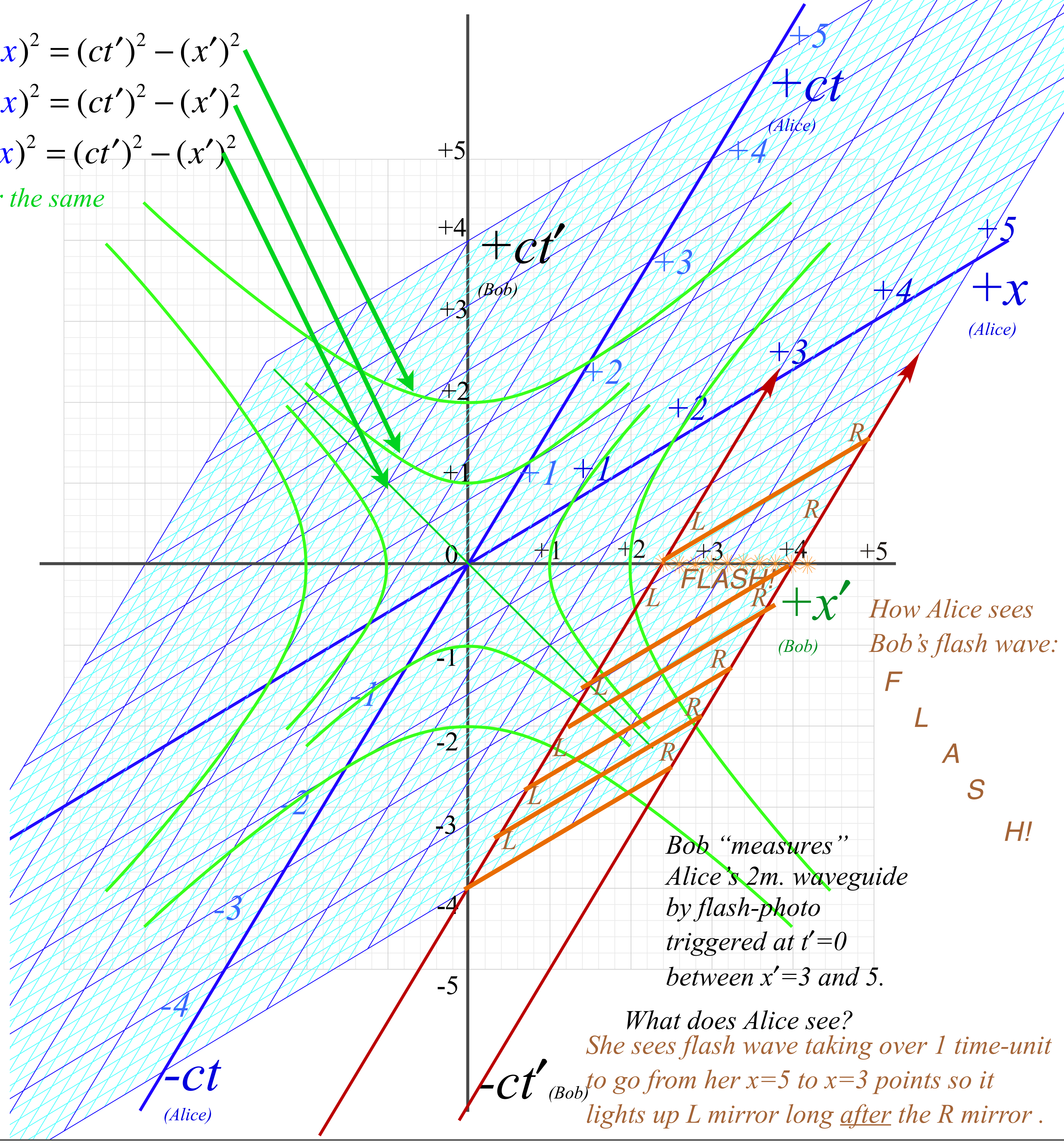
$$(c\tau)^2 = (+1)^2 = (ct)^2 - (x)^2 = (ct')^2 - (x')^2$$

$$(c\tau)^2 = (\ 0)^2 = (ct)^2 - (x)^2 = (ct')^2 - (x')^2$$

Invariant hyperbolas appear the same to both Bob and Alice.

Detailed view of Lorentz contraction on space-time plot Bob's axes: (x', ct')

Alice's 2m laser mirrors lie at x=3 and x=5
These points trace time lines shown at right of graph.



Phase invariance used to derive $(x,ct)\leftrightarrow(x',ct')$ *Einstein Lorentz Transformations (ELT)*

A. Transformations and phase invariance

Key points in SRQMbyR&C

A laser phasor sketched in Fig.4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ) in Fig.7a.

A. Transformations and phase invariance

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A laser phasor sketched in Fig.4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ) in Fig.7a. A time-stamp reading of phase ϕ at a particular space-time point should be equal for Alice and Bob in spite of having unequal readings (x, t) and (x', t') for that point and unequal frequency-wavevector readings (ω, k) and (ω', k') for a laser group-wave or its phase-wave.

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$$\phi'_{phase} = k'_{phase} x' - \omega'_{phase} t' = k_{phase} x - \omega_{phase} t \equiv \phi_{phase}$$

*Key point holds for **Any** phase*

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Key point holds for Any phase

Bob's (ω', k') components are in (14) and (15). Alice's (ω, k) are the same with $\rho = 0$. An Einstein-Lorentz Transformation (ELT) of Bob's (x', t') to Alice's (x, t) follows.

$$\phi'_{phase} = x'\frac{\omega_A}{c}\sinh\rho - t'\omega_A\cosh\rho = 0\cdot x - \omega_A t \quad \Rightarrow \quad ct = ct'\cosh\rho - x'\sinh\rho$$

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$$\phi'_{group} = x' \frac{\omega_A}{c} \cosh\rho - t'\omega_A \sinh\rho = \frac{\omega_A}{c} x - 0 \cdot t \quad \Rightarrow \quad x = -ct' \sinh\rho + x' \cosh\rho$$

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$$\phi'_{group} = x'\frac{\omega_A}{c}\cosh\rho - t'\omega_A\sinh\rho = \frac{\omega_A}{c}x - 0\cdot t \quad\Rightarrow\quad x = -ct'\sinh\rho + x'\cosh\rho$$

The ELT matrix form and its inverse complete the space-time side of Fig.7.

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} ct' \\ x' \end{pmatrix} \Rightarrow \begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\rho & +\sinh\rho \\ +\sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} ct \\ x \end{pmatrix} \qquad (22)$$

A. Transformations and phase invariance

Very key point in SRQMbyR&C

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Direct derivation of ELT uses base vectors $\mathbb{P}'$ and $\mathbb{G}'$ or $\mathbf{P}'$ and $\mathbf{G}'$ in (14) and (15).

$$\mathbf{P}' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \begin{pmatrix} \omega_A \\ 0 \end{pmatrix} \cosh\rho + \begin{pmatrix} 0 \\ \omega_A \end{pmatrix} \sinh\rho = \mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho \quad (23)$$

$$\mathbf{G}' = \begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \begin{pmatrix} \omega_A \\ 0 \end{pmatrix} \sinh\rho + \begin{pmatrix} 0 \\ \omega_A \end{pmatrix} \cosh\rho = \mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho \quad (24)$$

Introducing the stellar aberration angle σ vs. rapidity ρ

Trigonometry: From circular to hyperbolic and back

Finish "Sin-Tan" blackboard construction

Group vs. phase velocity and tangent contacts

Epstein's[†] space-proper-time (x,cτ) plots ("c-tau" plots)

[†]Lewis Carroll Epstein, *Relativity Visualized*
Insight Press, San Francisco, CA 94107

See also: L. C. Epstein, *Thinking Physics Press,*
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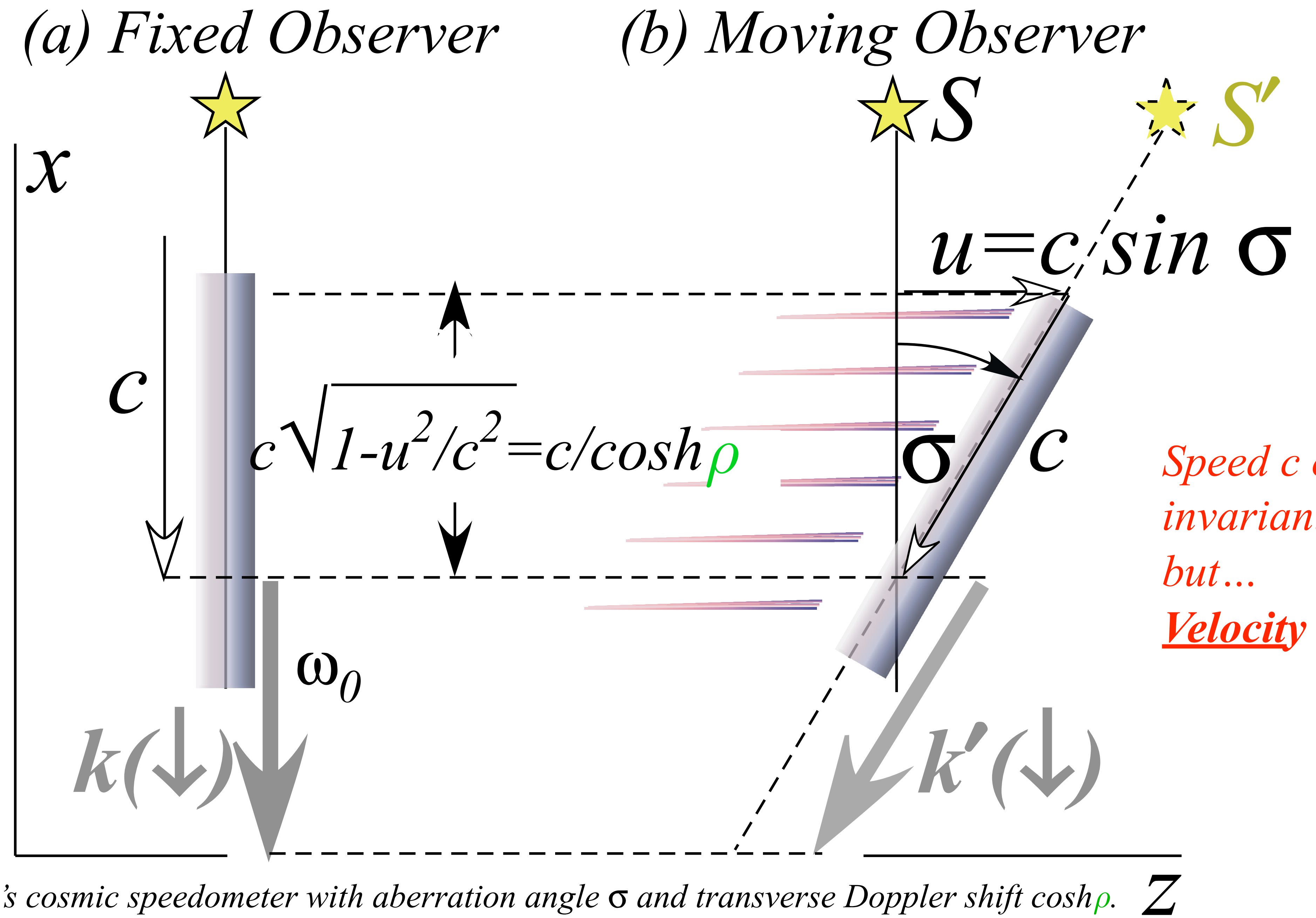
Introducing the stellar aberration angle σ vs. rapidity ρ

Together, rapidity ρ=ln b and stellar aberration angle σ are parameters of relative velocity

The rapidity ρ=ln b is based on longitudinal wave Doppler shift $b=e^{\rho}$ defined by $u/c=\tanh(\rho)$.
At low speed: $u/c\sim\rho$.

The stellar aberration angle σ is based on the transverse wave rotation $R=e^{i\sigma}$ defined by $u/c=\sin(\sigma)$.
At low speed: $u/c\sim\sigma$.



(Star appears to "race ahead")

Speed c of light is invariant to observer u but…
Velocity **c** *of light is not!*

Fig. 5.6 Epstein's cosmic speedometer with aberration angle σ and transverse Doppler shift coshρ.

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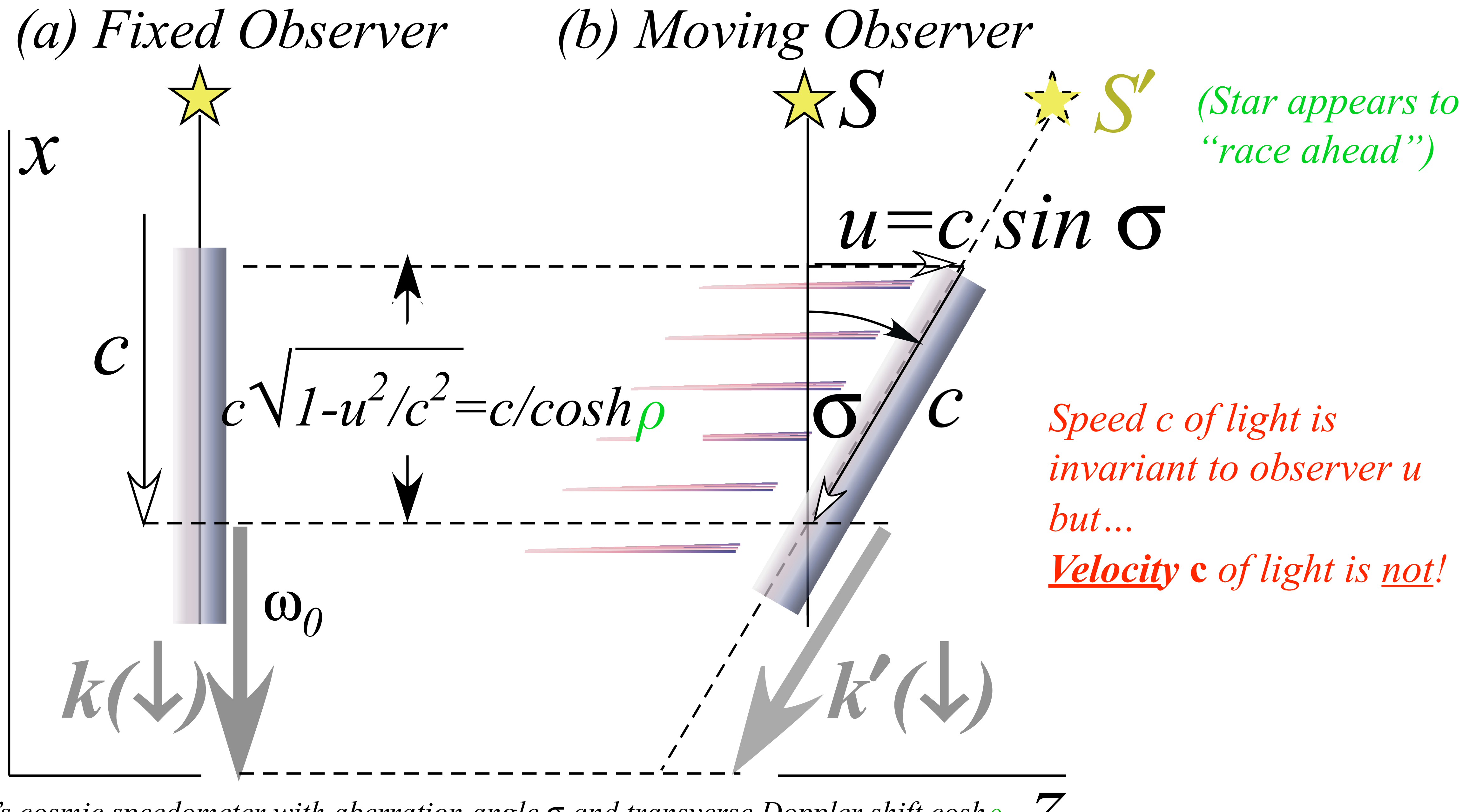
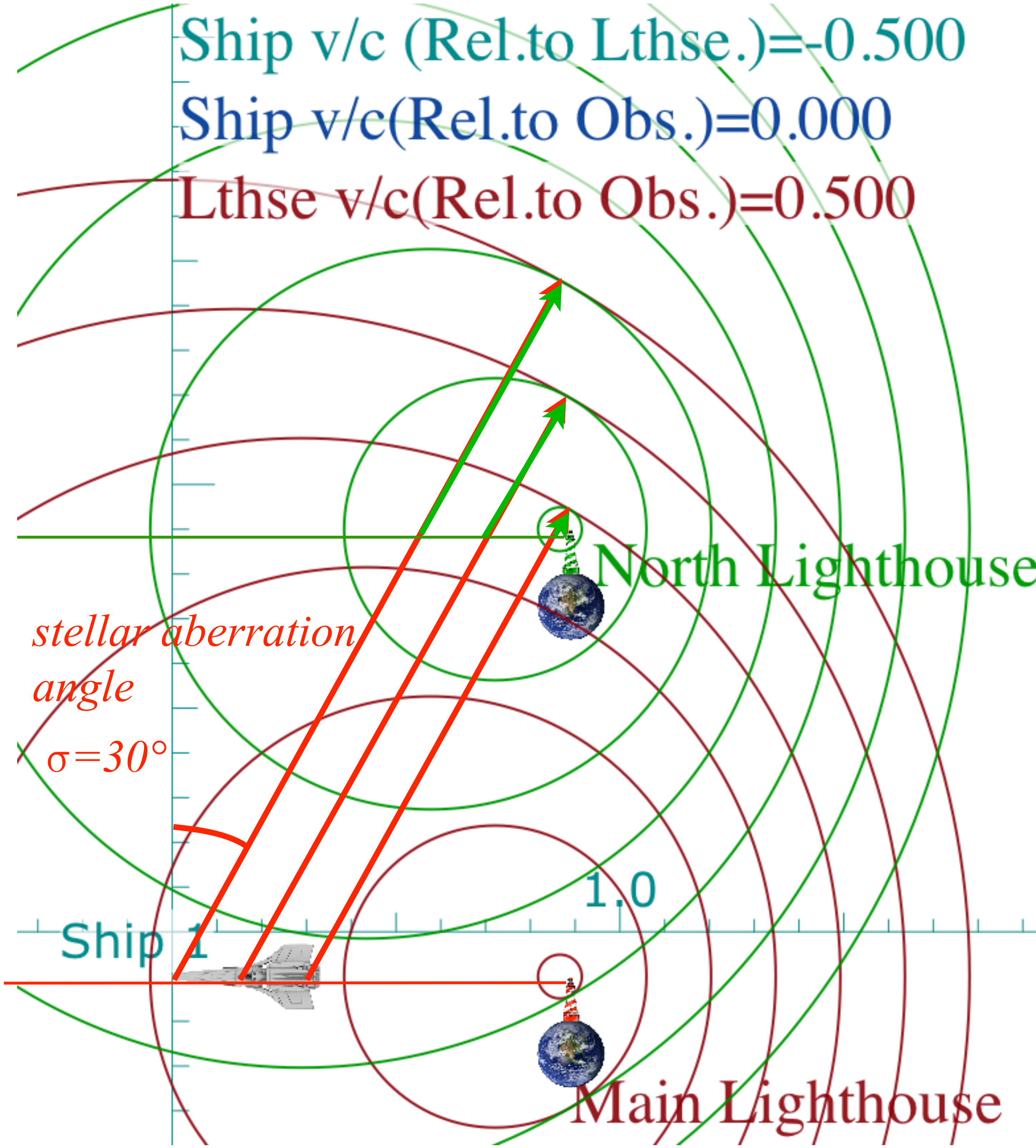
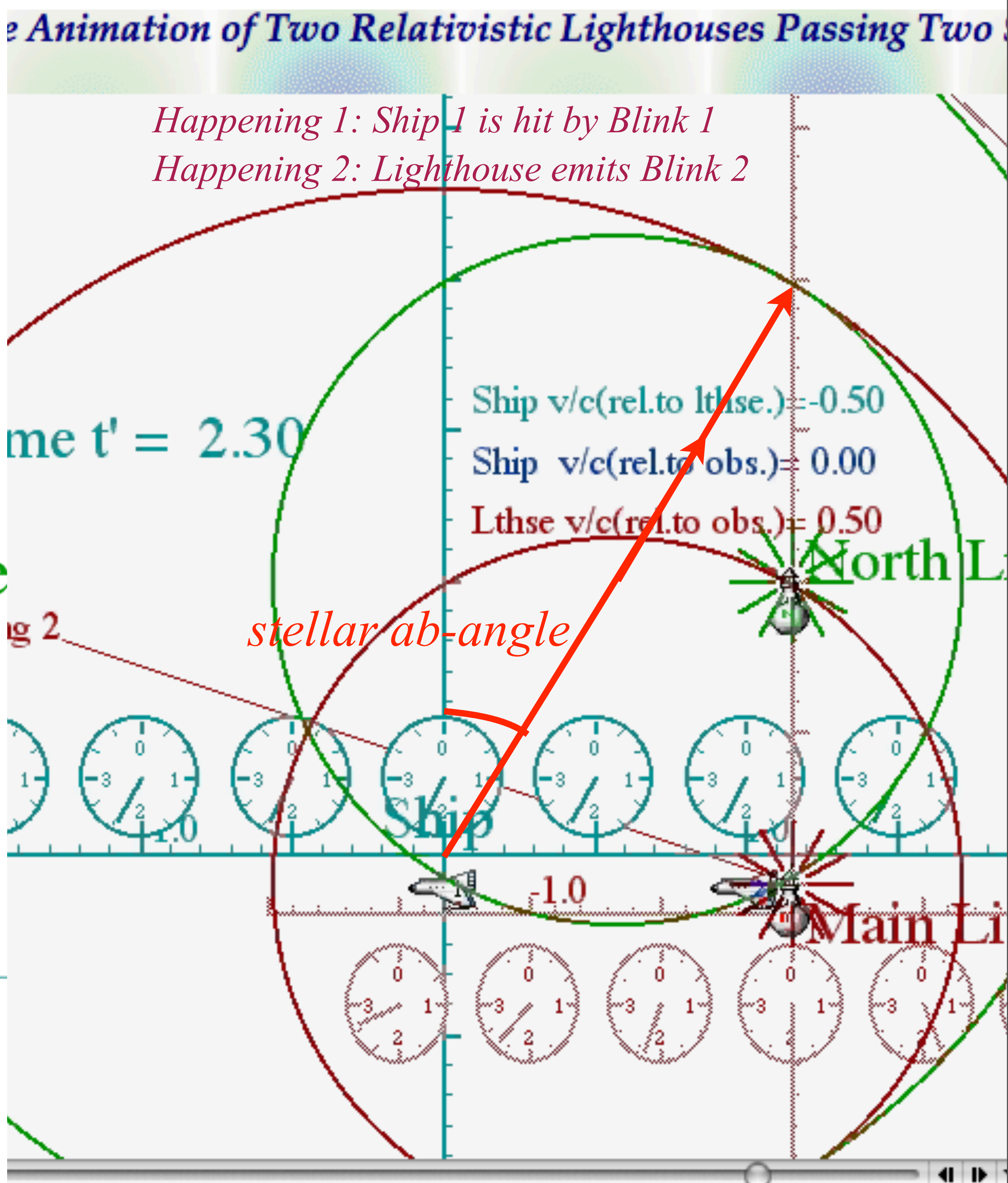


Fig. 5.6 Epstein's cosmic speedometer with aberration angle σ and transverse Doppler shift cosh ρ.

Lighthouse ship example of stellar aberration
(Here: ρ=atanh(1/2)=0.549)





(Here: ρ=atanh(1/2)=0.549...
and: σ=asin(1/2)=0.52 or 30°)

http://www.uark.edu/ua/modphys/testing/markup/RelativItWeb.html

www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php

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Trigonometry: From circular to hyperbolic and back

Finish "Sin-Tan" blackboard construction

Group vs. phase velocity and tangent contacts

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See also: L. C. Epstein, *Thinking Physics Press,*
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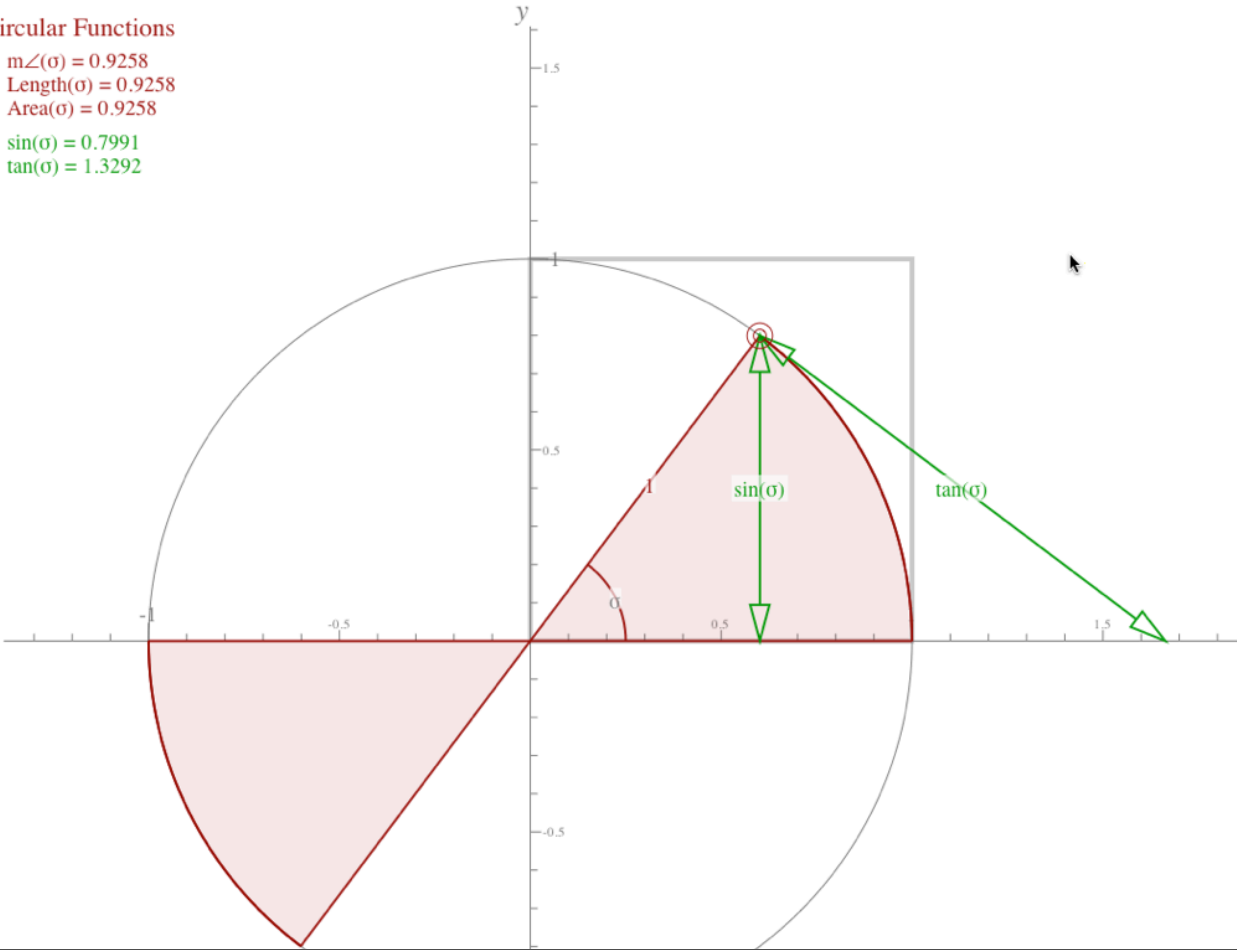
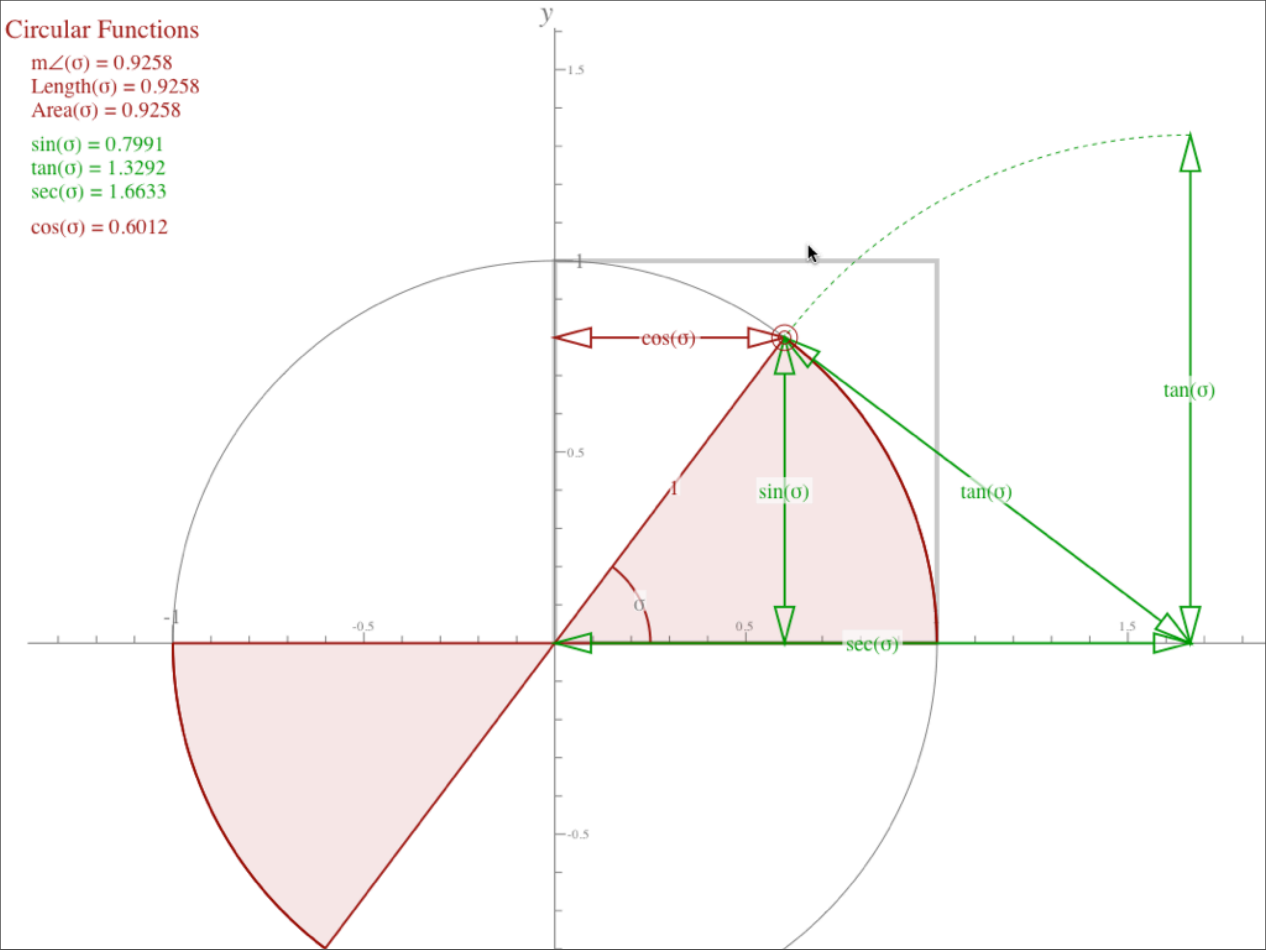
Circular Functions

m∠(σ) = 0.9258
Length(σ) = 0.9258
Area(σ) = 0.9258

sin(σ) = 0.7991
tan(σ) = 1.3292

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Area(σ) = 0.9258
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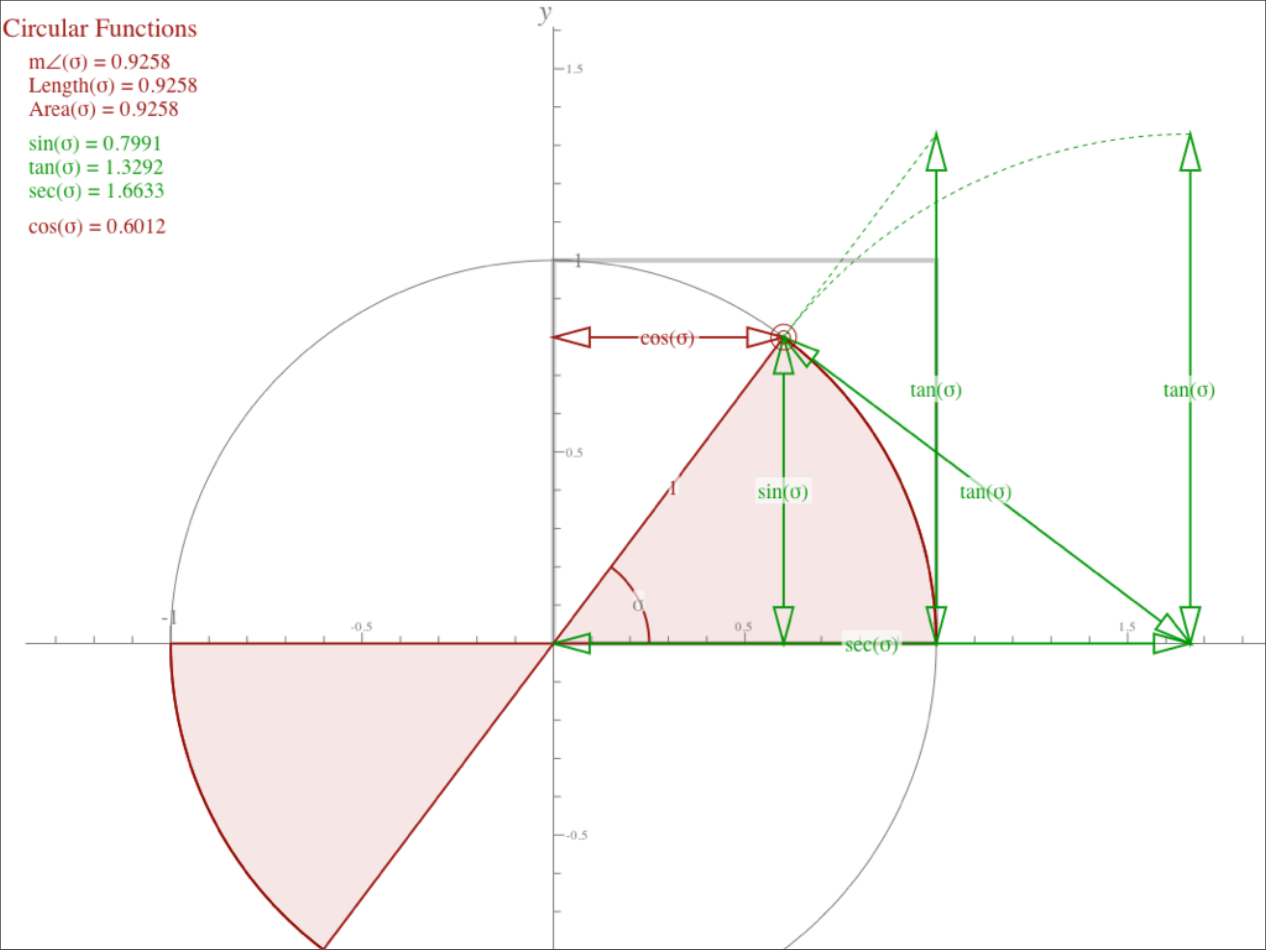
tan(σ) = 1.3292
sec(σ) = 1.6633
cos(σ) = 0.6012
y
1.5
1
0.5
-0.5
-1
-0.5
0.5
1.5
cos(σ)
sin(σ)
tan(σ)
tan(σ)
sec(σ)
σ
1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1.5
cos(σ)
tan(σ)
tan(σ)
sin(σ)
tan(σ)
1
σ
sec(σ)

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Area(σ) = 0.9258

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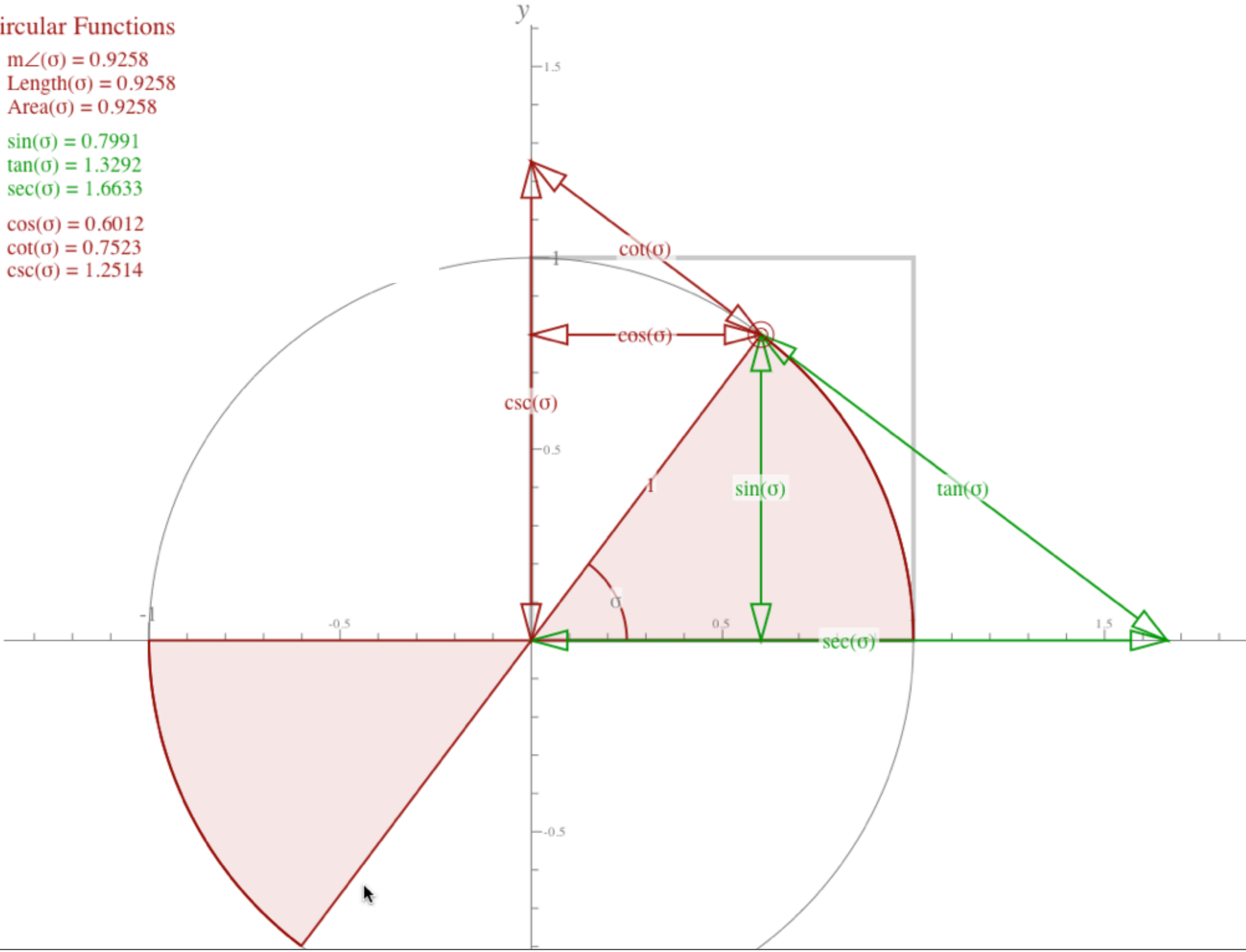
tan(σ) = 1.3292
sec(σ) = 1.6633

cos(σ) = 0.6012
cot(σ) = 0.7523
csc(σ) = 1.2514



Circular Functions

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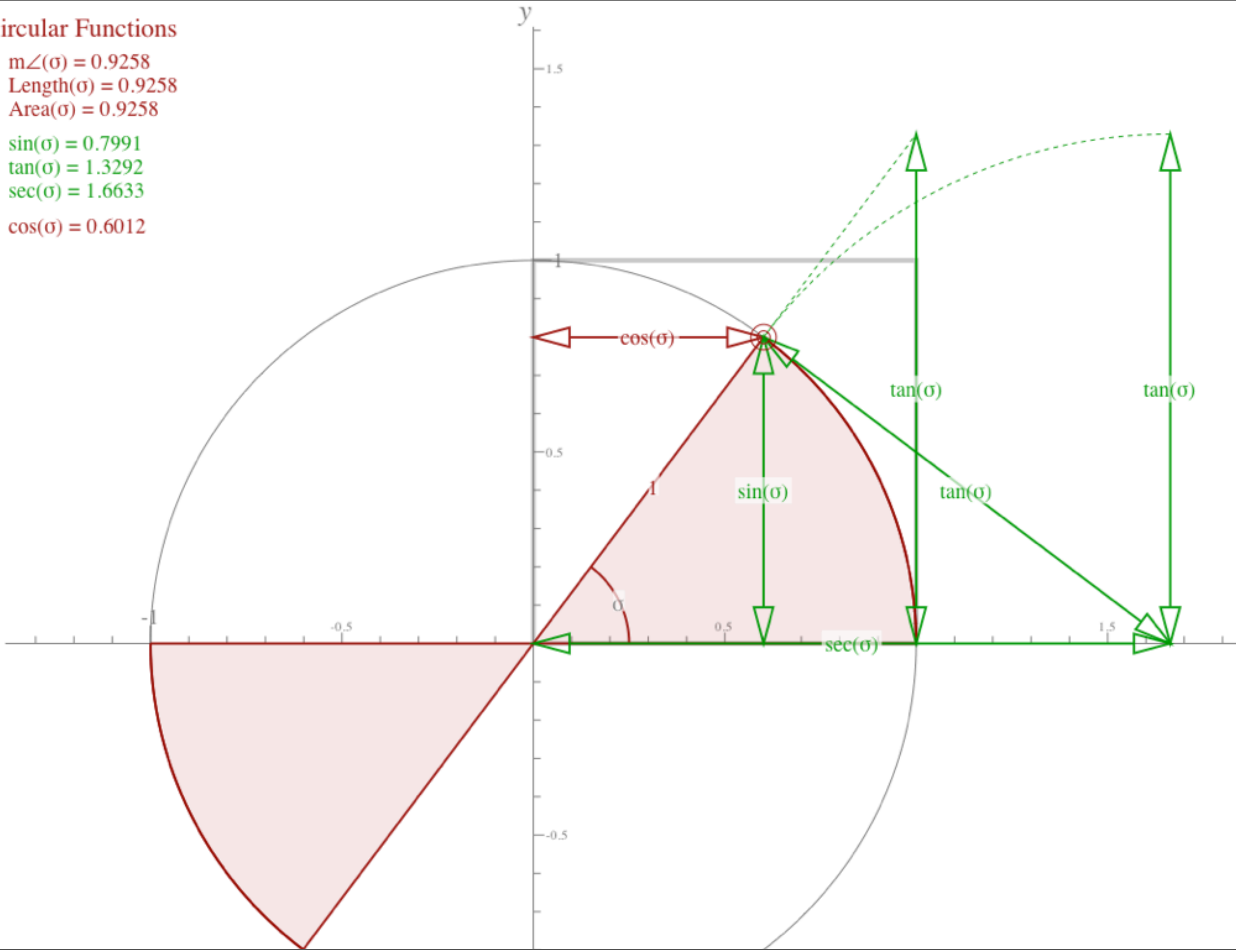
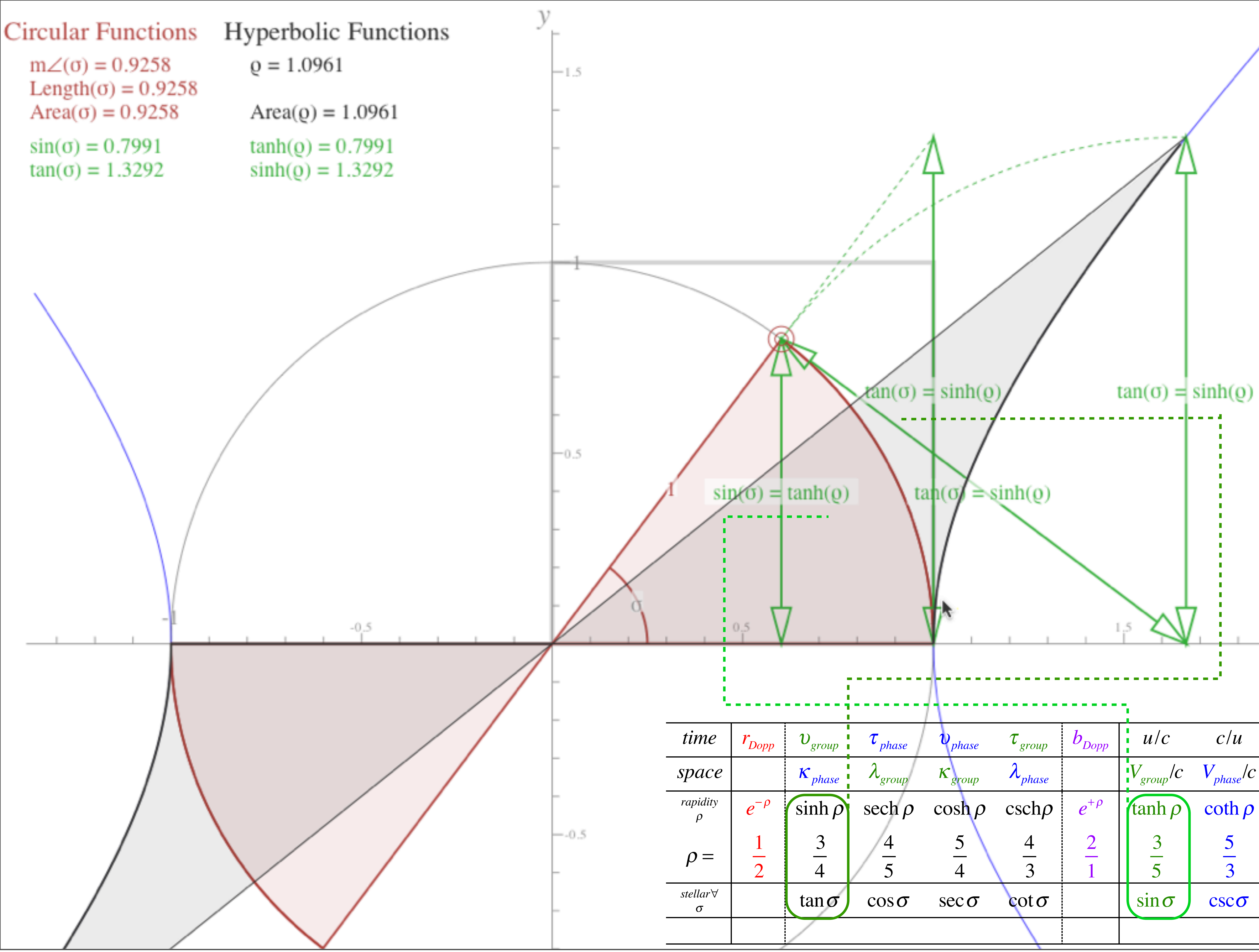
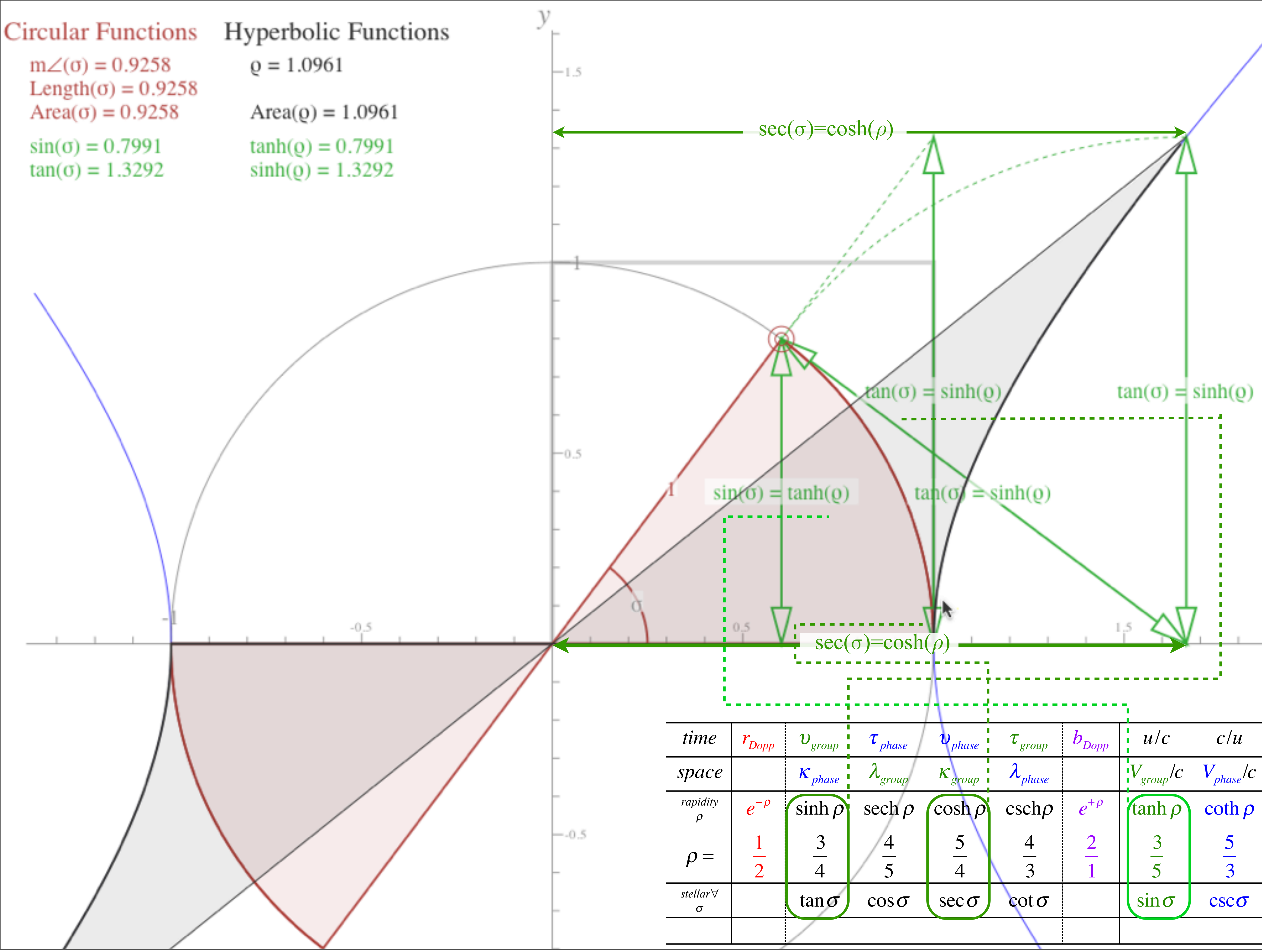




time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$



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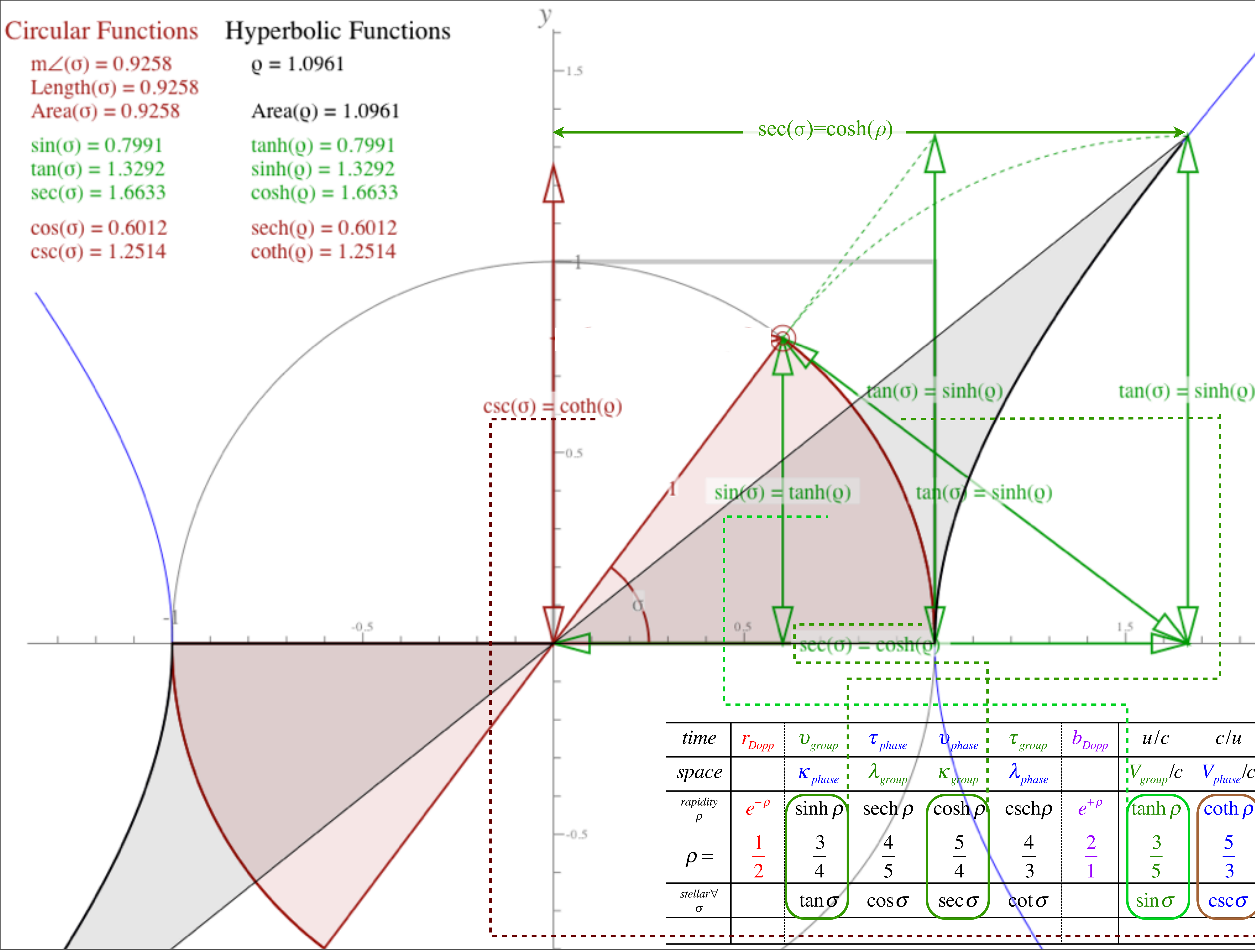

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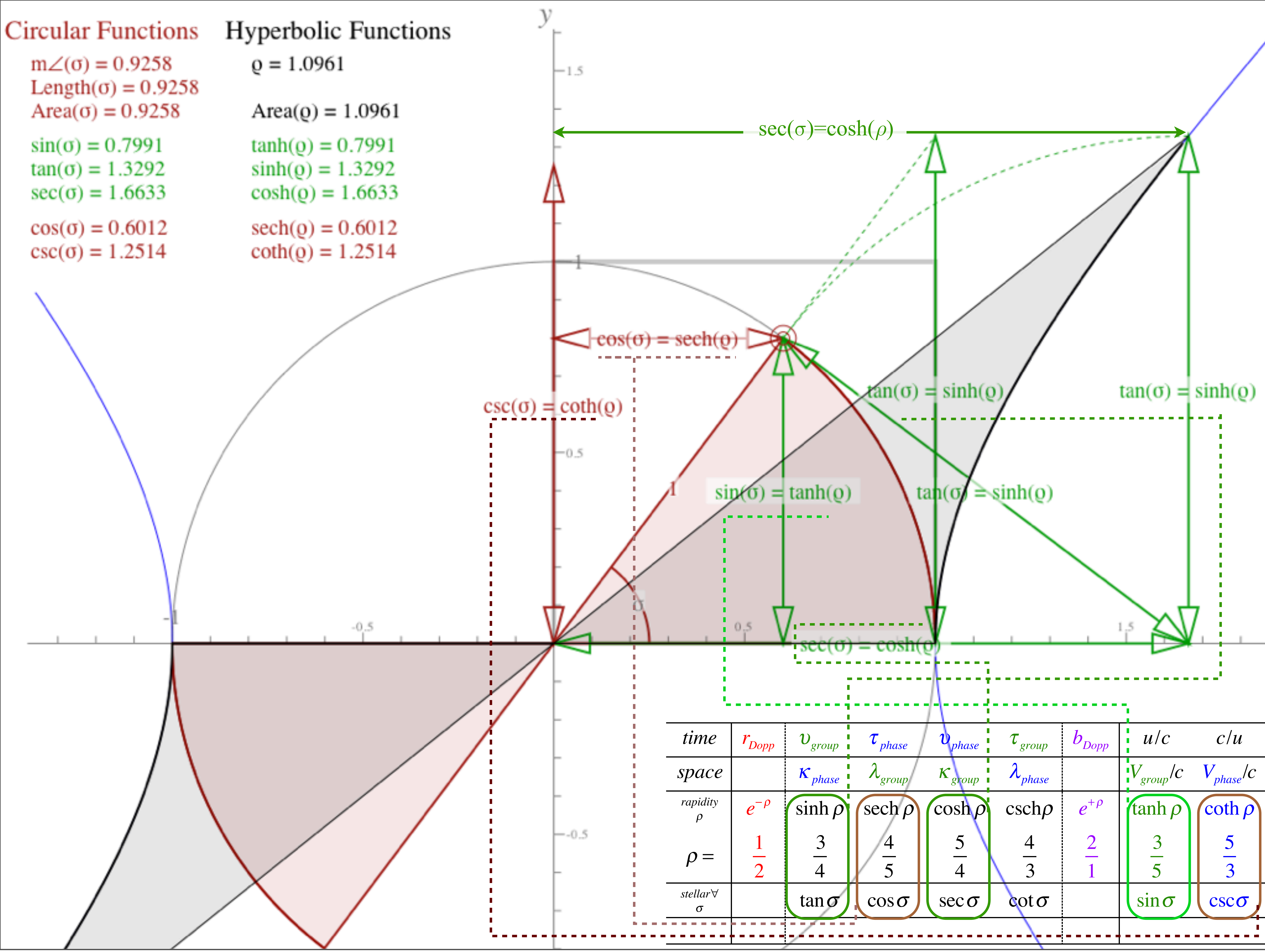

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sec(σ) = 1.6633

cos(σ) = 0.6012
cot(σ) = 0.7523

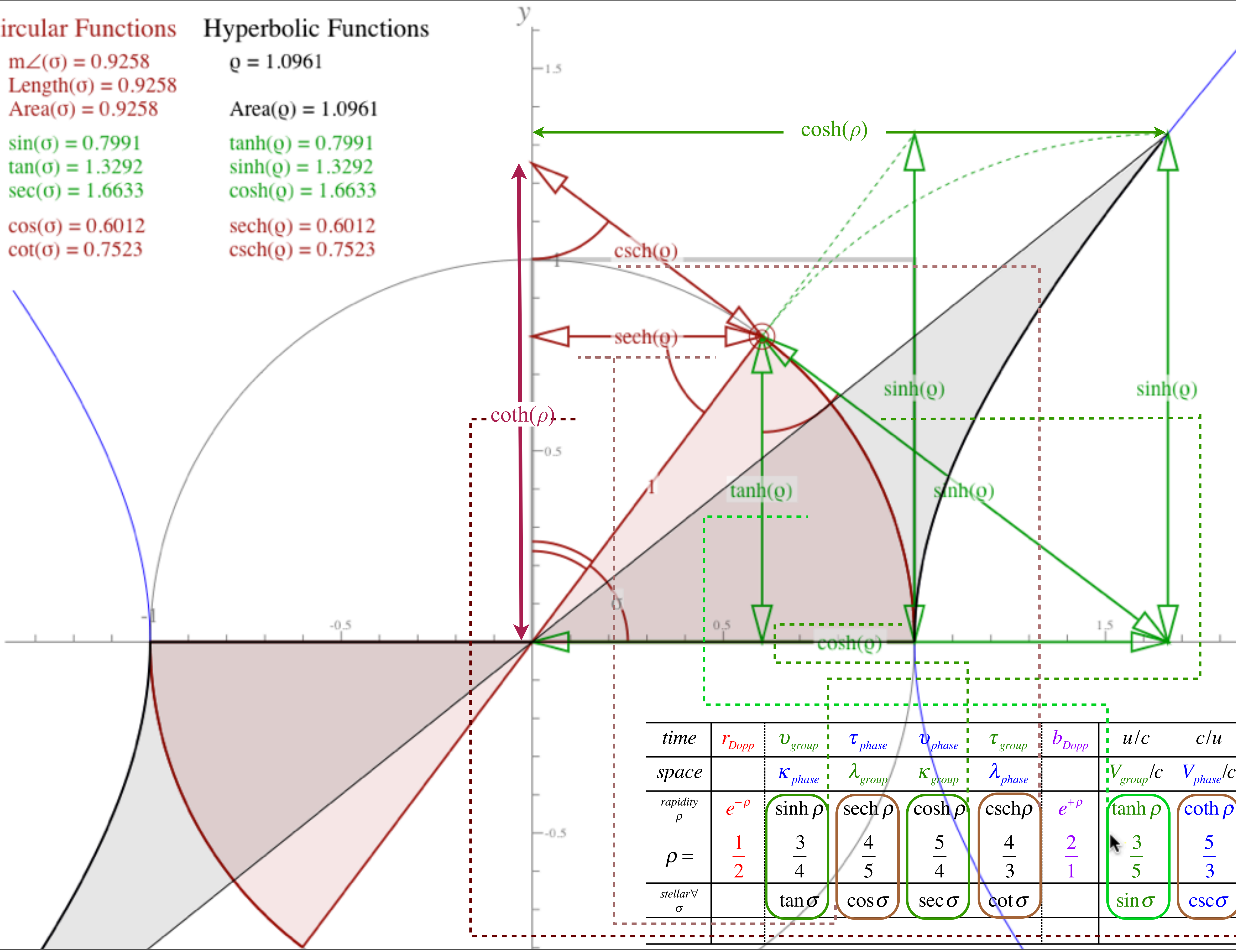
Hyperbolic Functions

ϱ = 1.0961

Area(ϱ) = 1.0961

tanh(ϱ) = 0.7991
sinh(ϱ) = 1.3292
cosh(ϱ) = 1.6633

sech(ϱ) = 0.6012
csch(ϱ) = 0.7523



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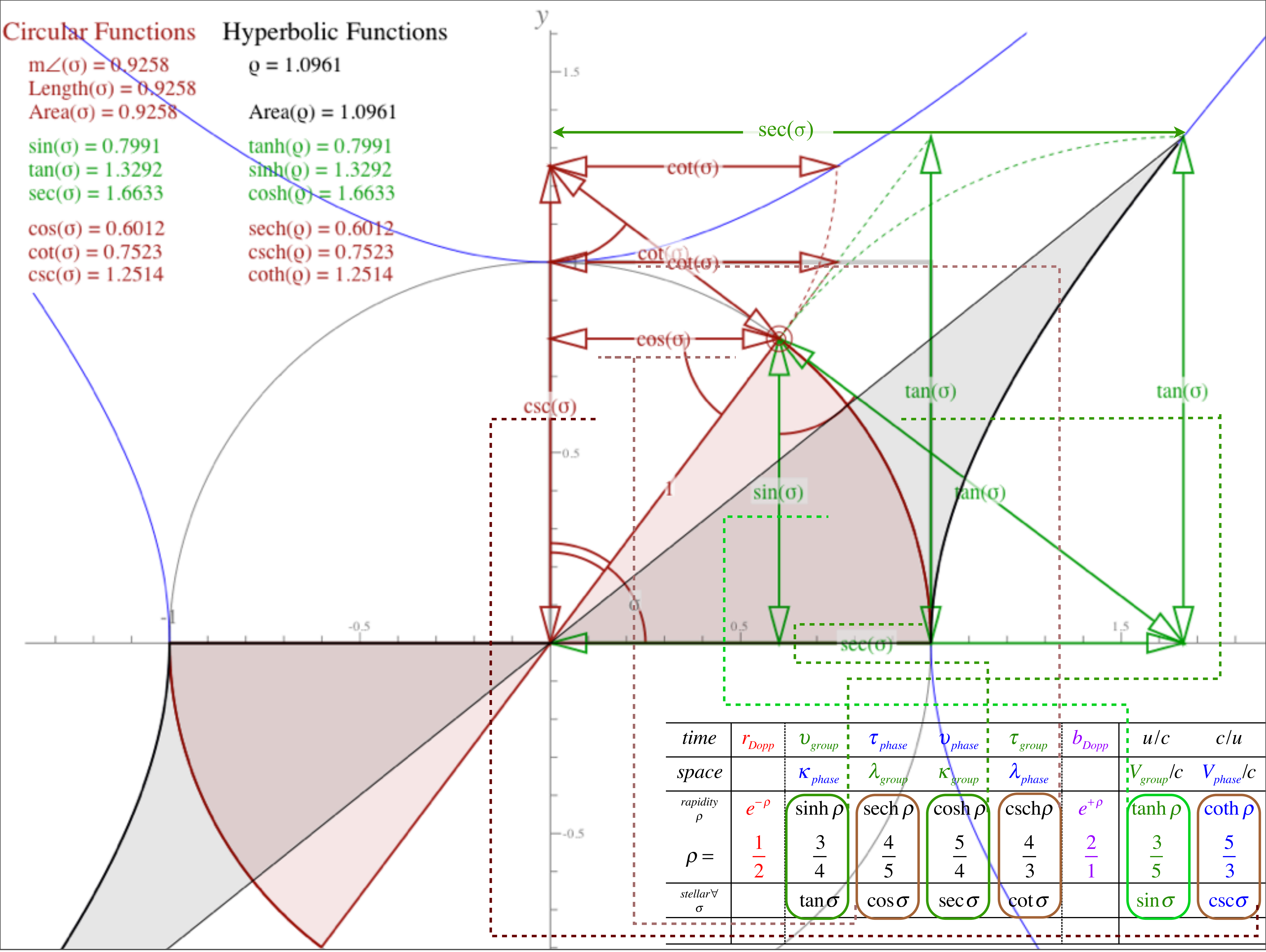
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$



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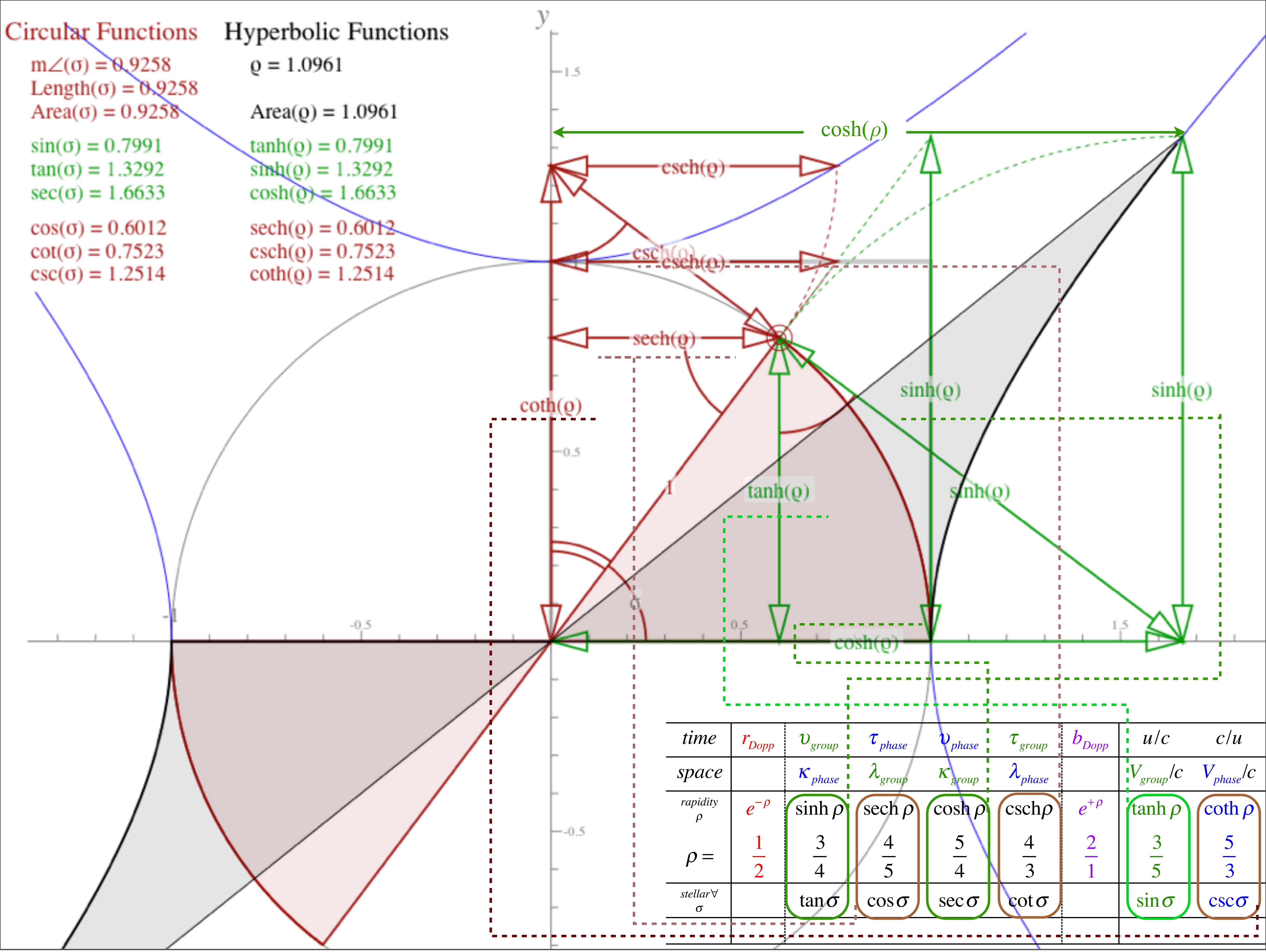

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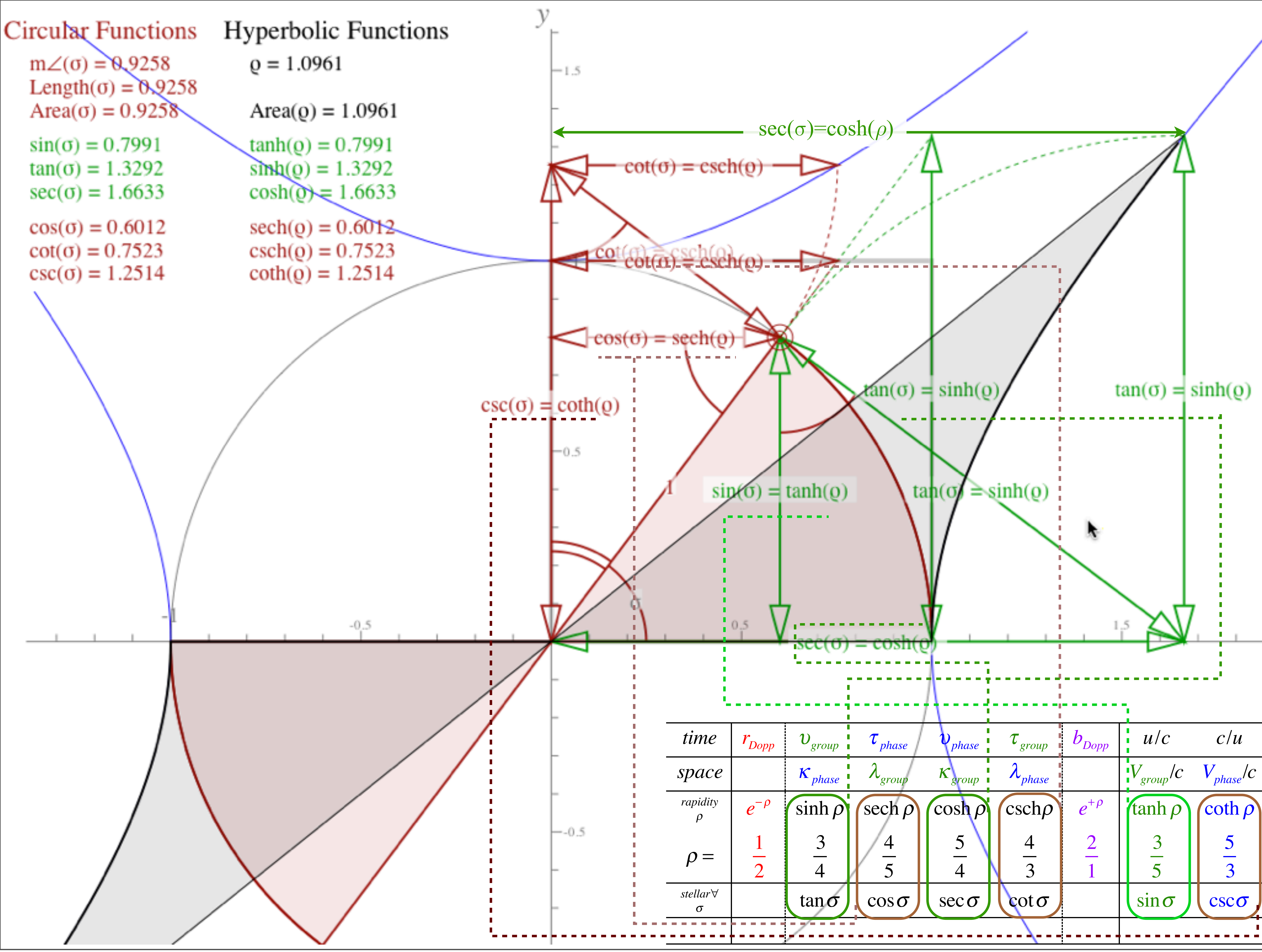

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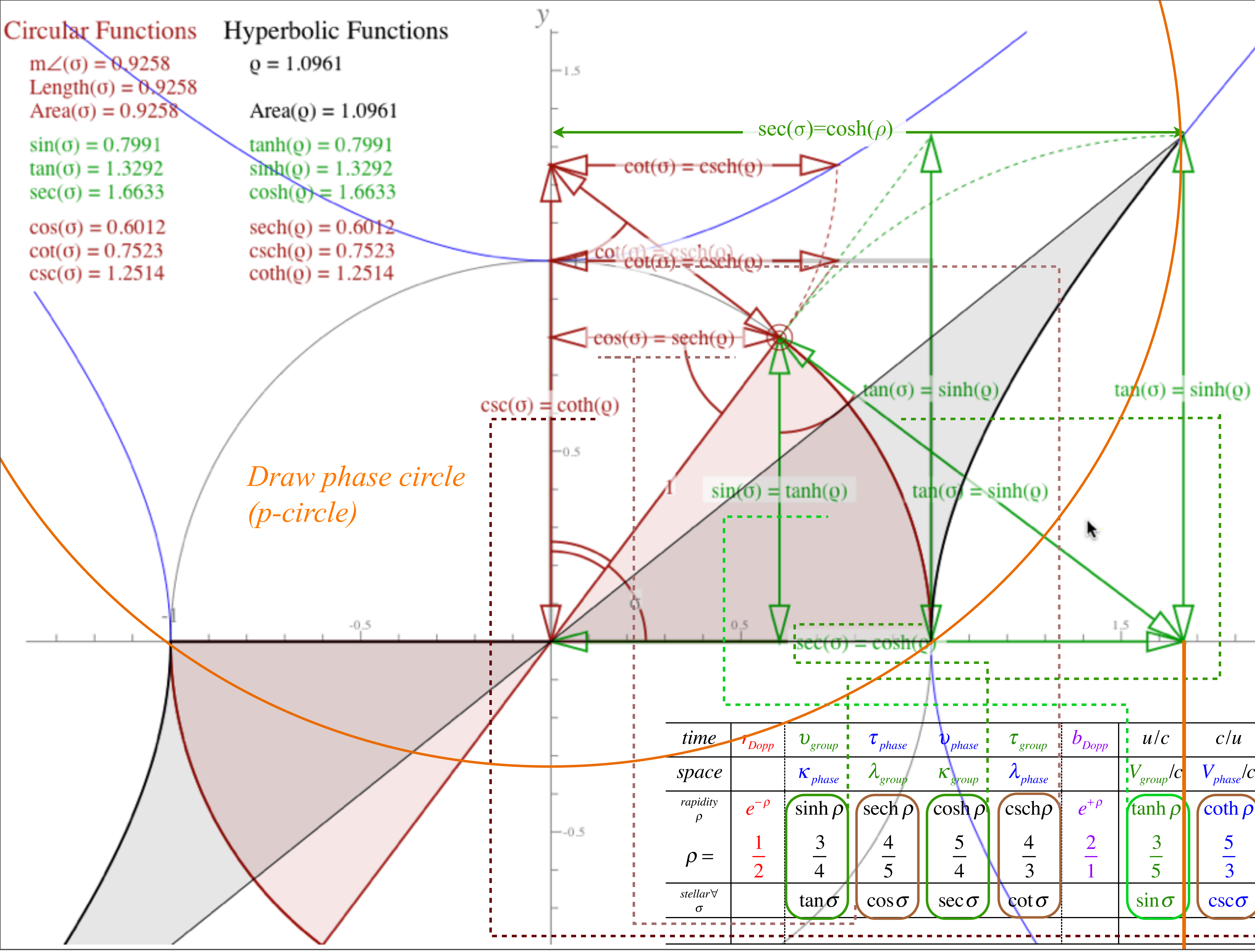

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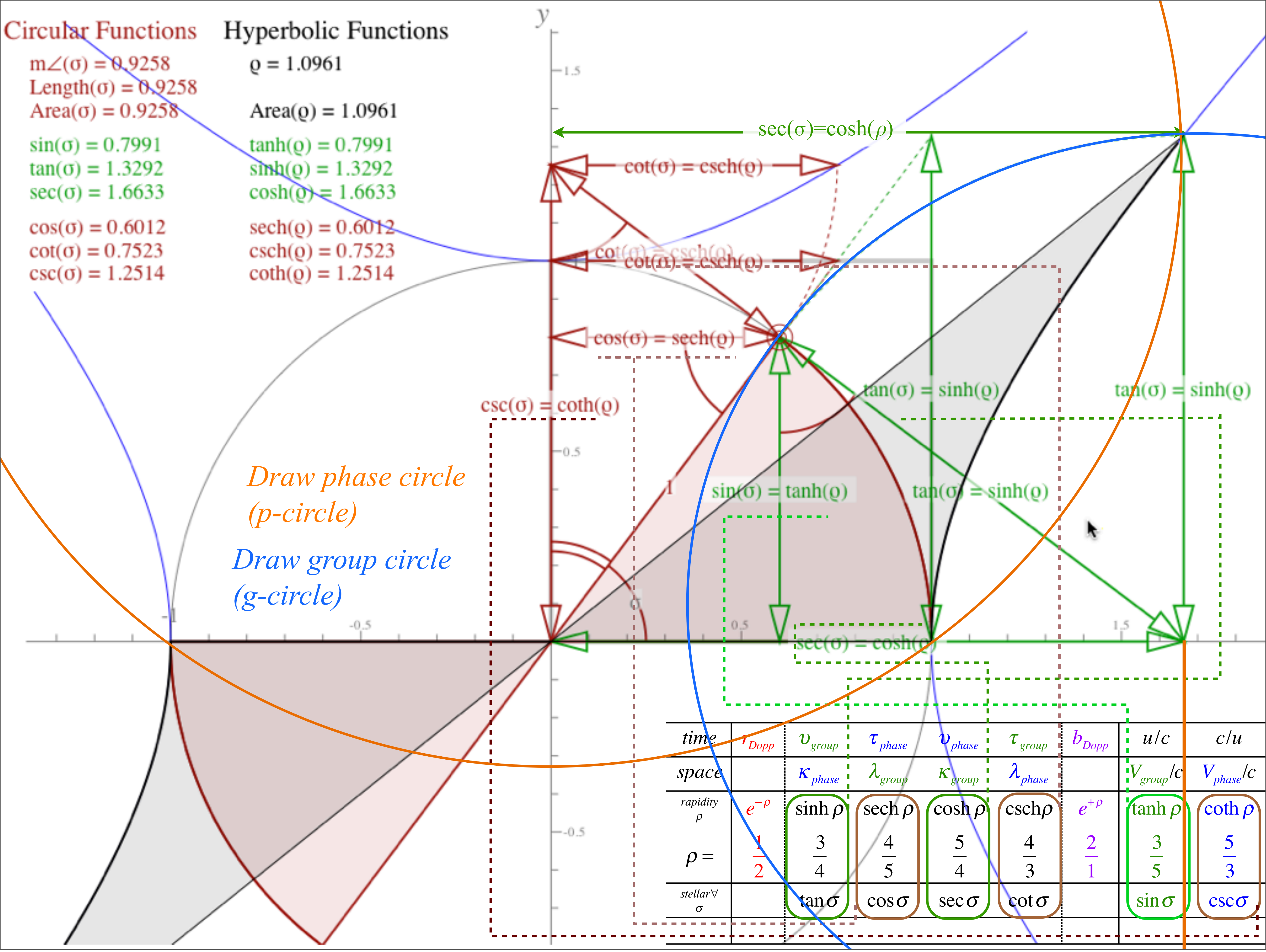

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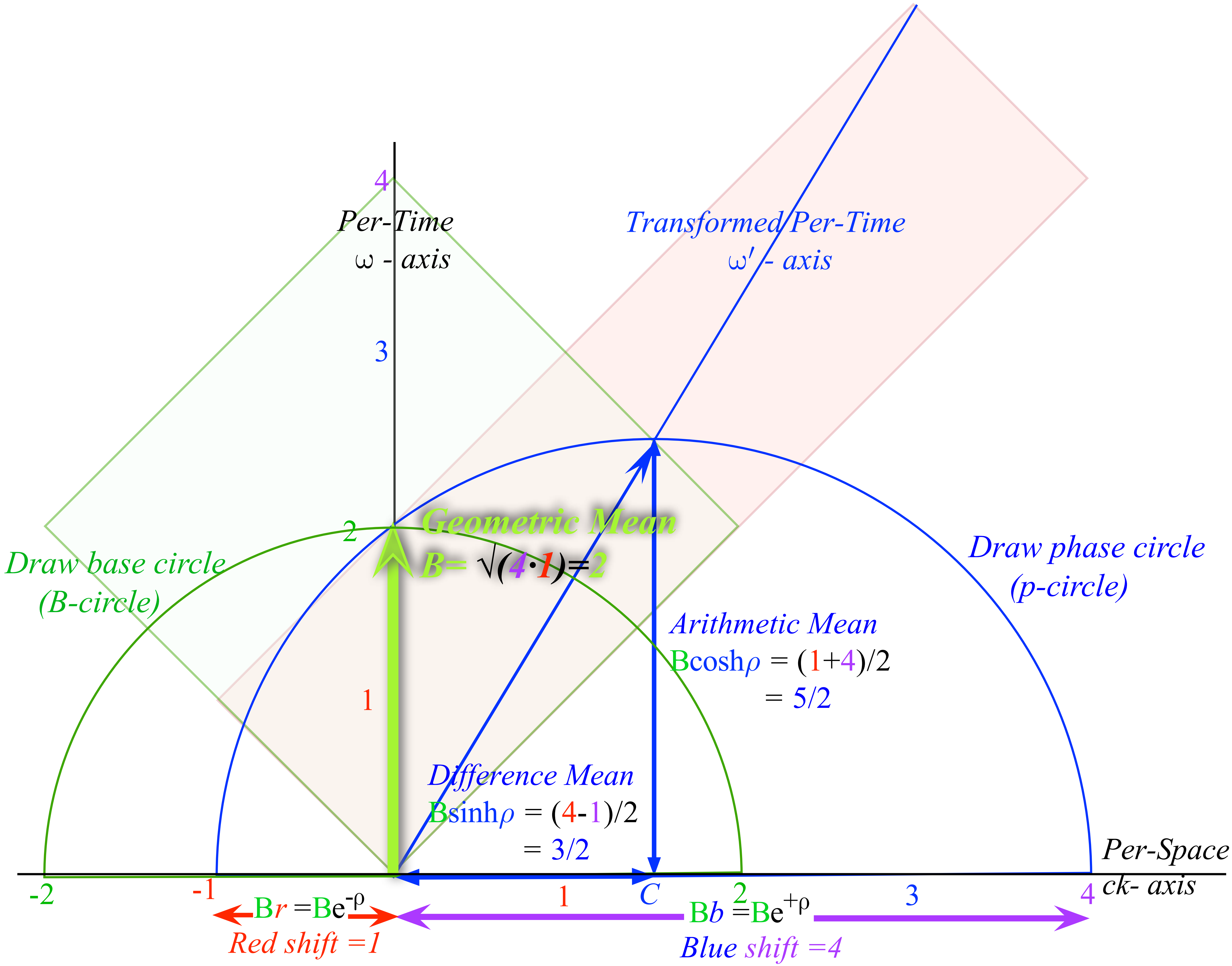
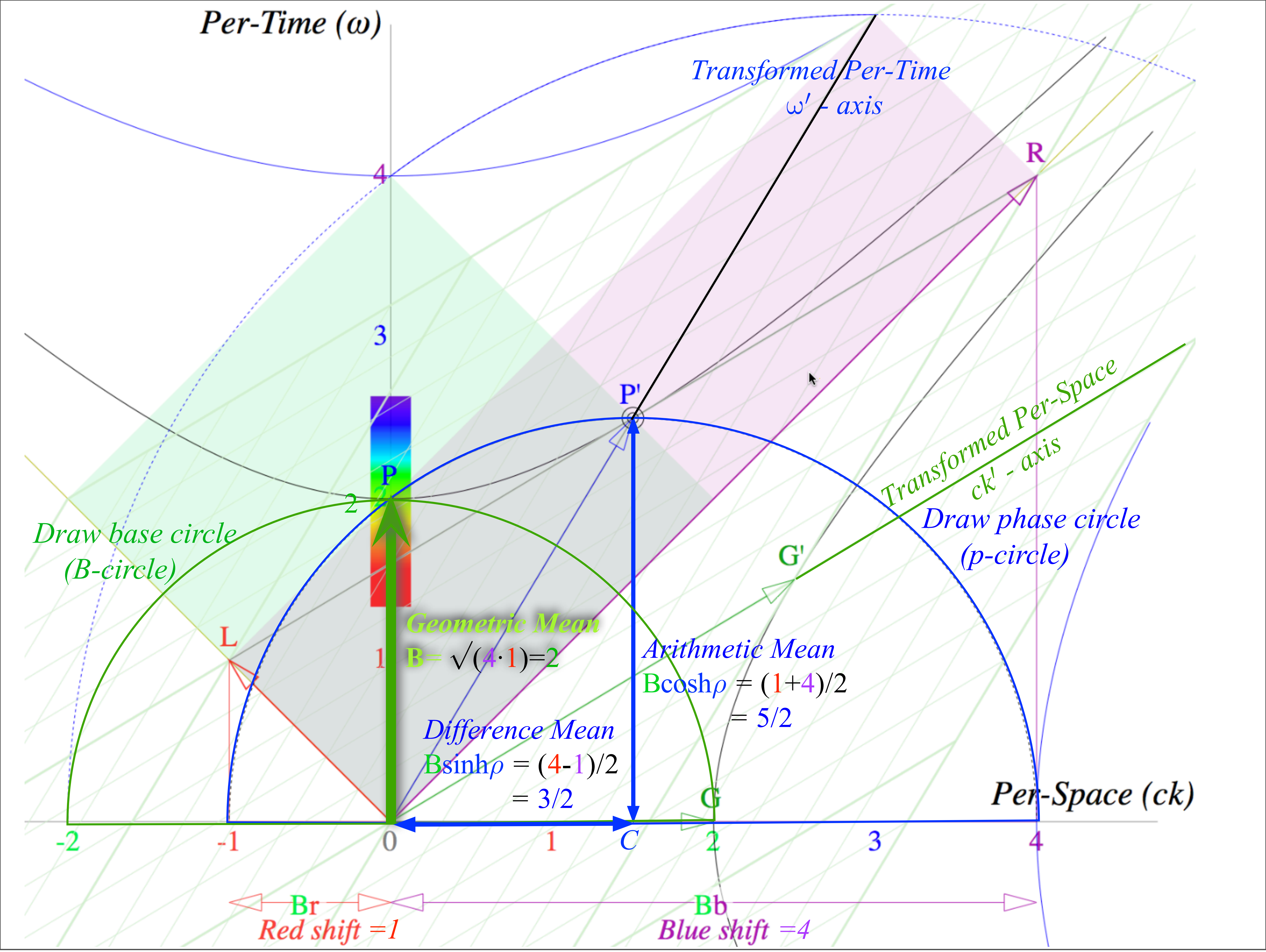

Per-Time
ω - axis
Transformed Per-Time
ω′ - axis
4
3
2
1
Geometric Mean
B= √(4·1)=2
Draw base circle
(B-circle)
Draw phase circle
(p-circle)
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
Per-Space
ck- axis
-2
-1
1
C
2
3
4
Br =Be^-ρ
Red shift =1
Bb =Be^+ρ
Blue shift =4



Per-Time (ω)
Transformed Per-Time
ω' - axis
R
4
3
P'
P
2
Transformed Per-Space
ck' - axis
Draw phase circle
(p-circle)
Draw base circle
(B-circle)
G'
Geometric Mean
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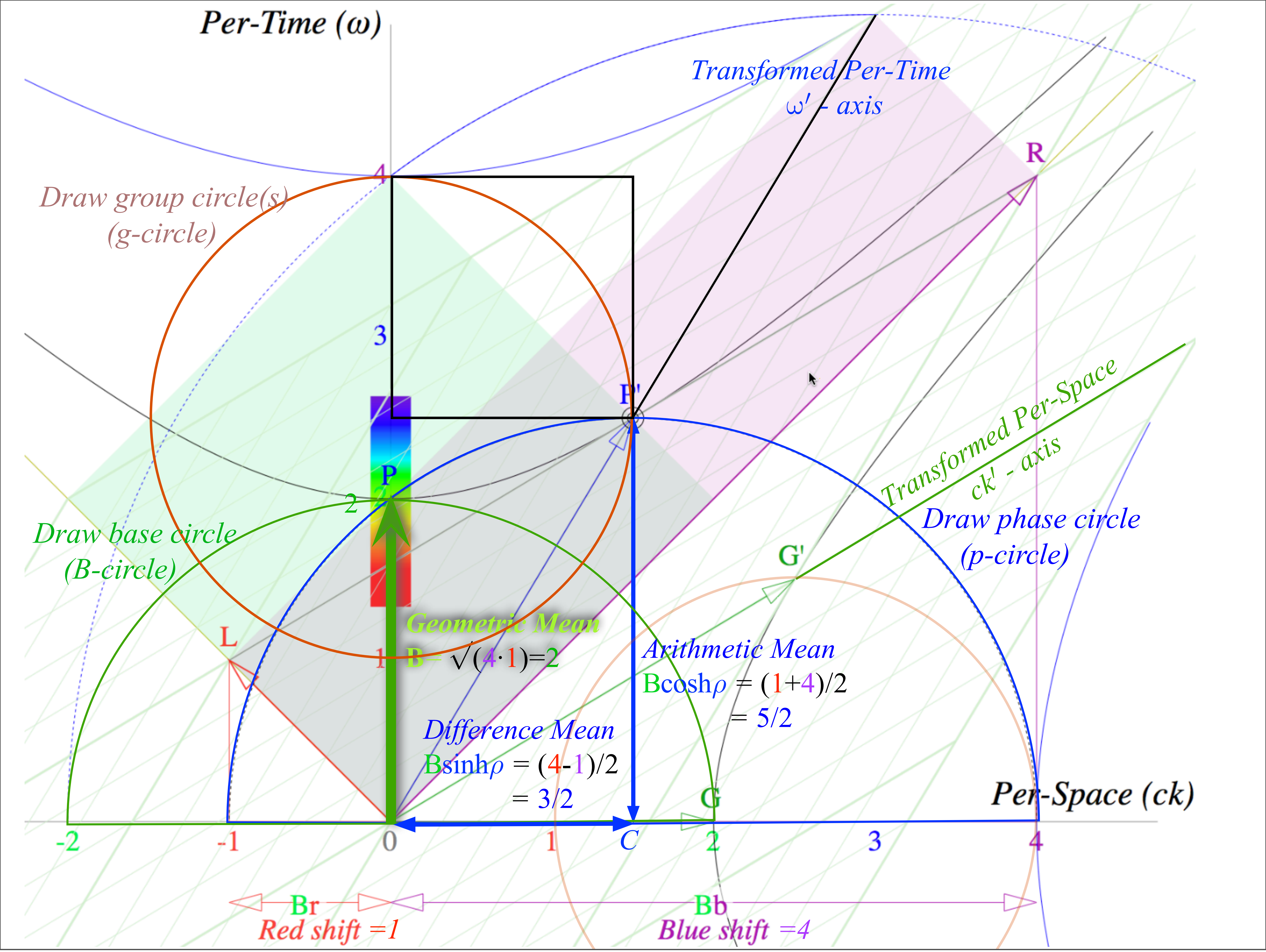
= 3/2
G
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Br
Bb
Red shift =1
Blue shift =4


Per-Time (ω)
Transformed Per-Time
ω′ - axis
R
Draw group circle(s)
(g-circle)
4
3
P′
P
2
Transformed Per-Space
ck′ - axis
Draw phase circle
(p-circle)
Draw base circle
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G
Per-Space (ck)
-2
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0
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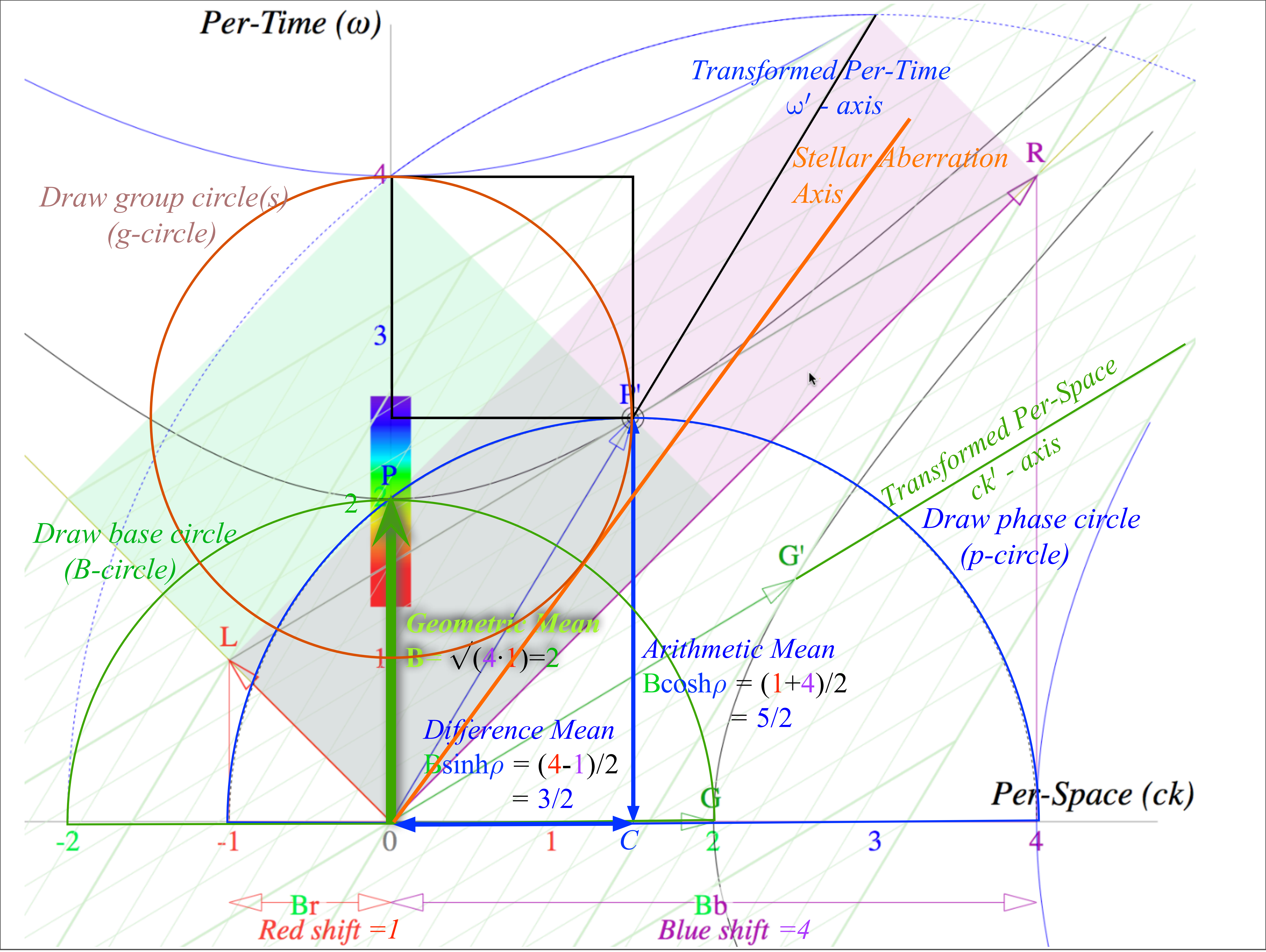
C
2
3
4
Br
Bb
Red shift =1
Blue shift =4


Per-Time (ω)
Transformed Per-Time
ω′ - axis
Stellar Aberration
Axis
R
Draw group circle(s)
(g-circle)
4
3
P′
P
2
Transformed Per-Space
ck′ - axis
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Draw base circle
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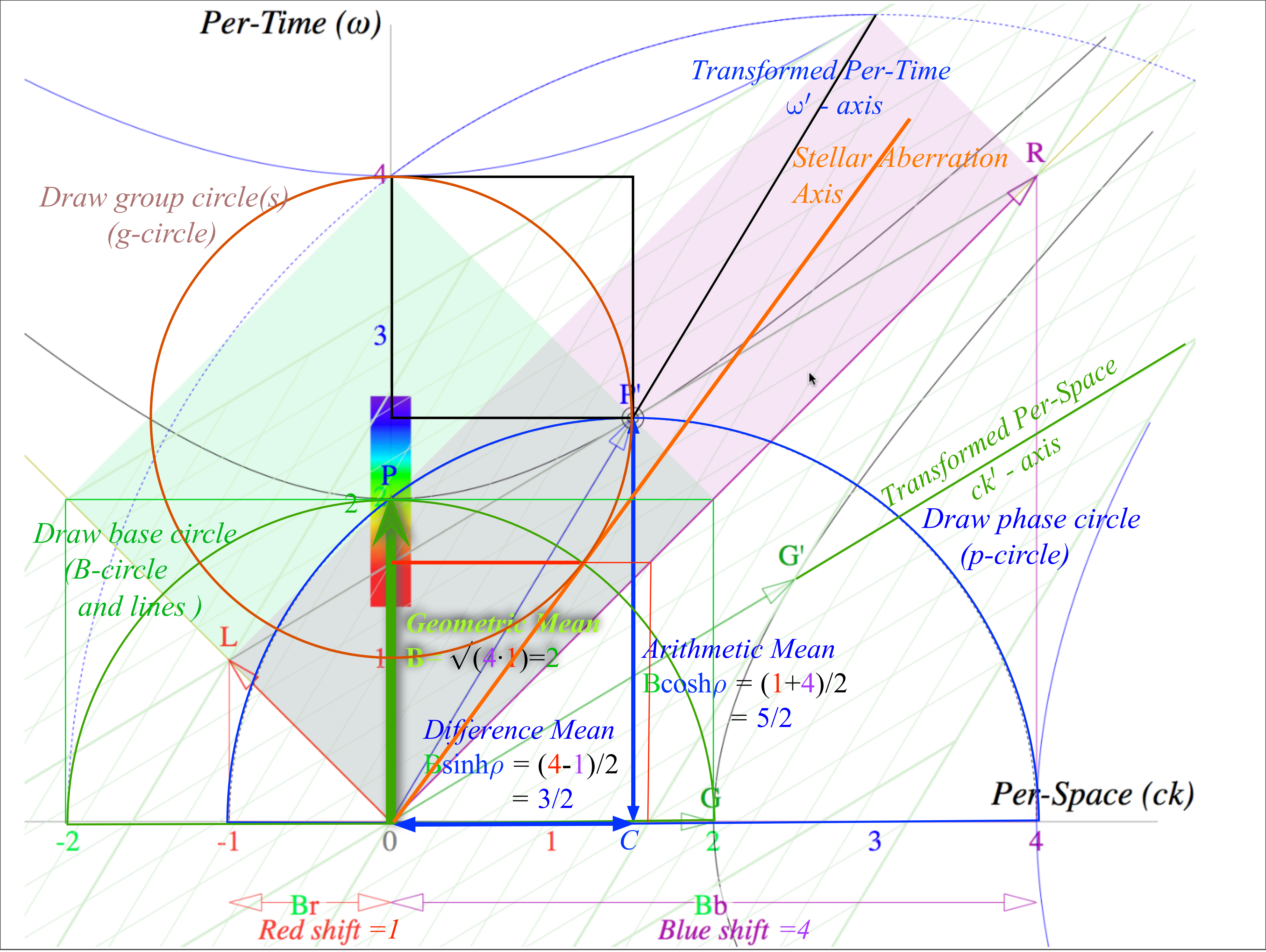
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Br
Red shift =1
Bb
Blue shift =4



Per-Time (ω)
Transformed Per-Time
ω' - axis
Stellar Aberration
Axis
R
Draw group circle(s)
(g-circle)
3
P'
P
2
Transformed Per-Space
ck' - axis
Draw phase circle
(p-circle)
Draw base circle
(B-circle
and lines)
G'
Geometric Mean
L
1
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Arithmetic Mean
Bcoshρ = (1+4)/2
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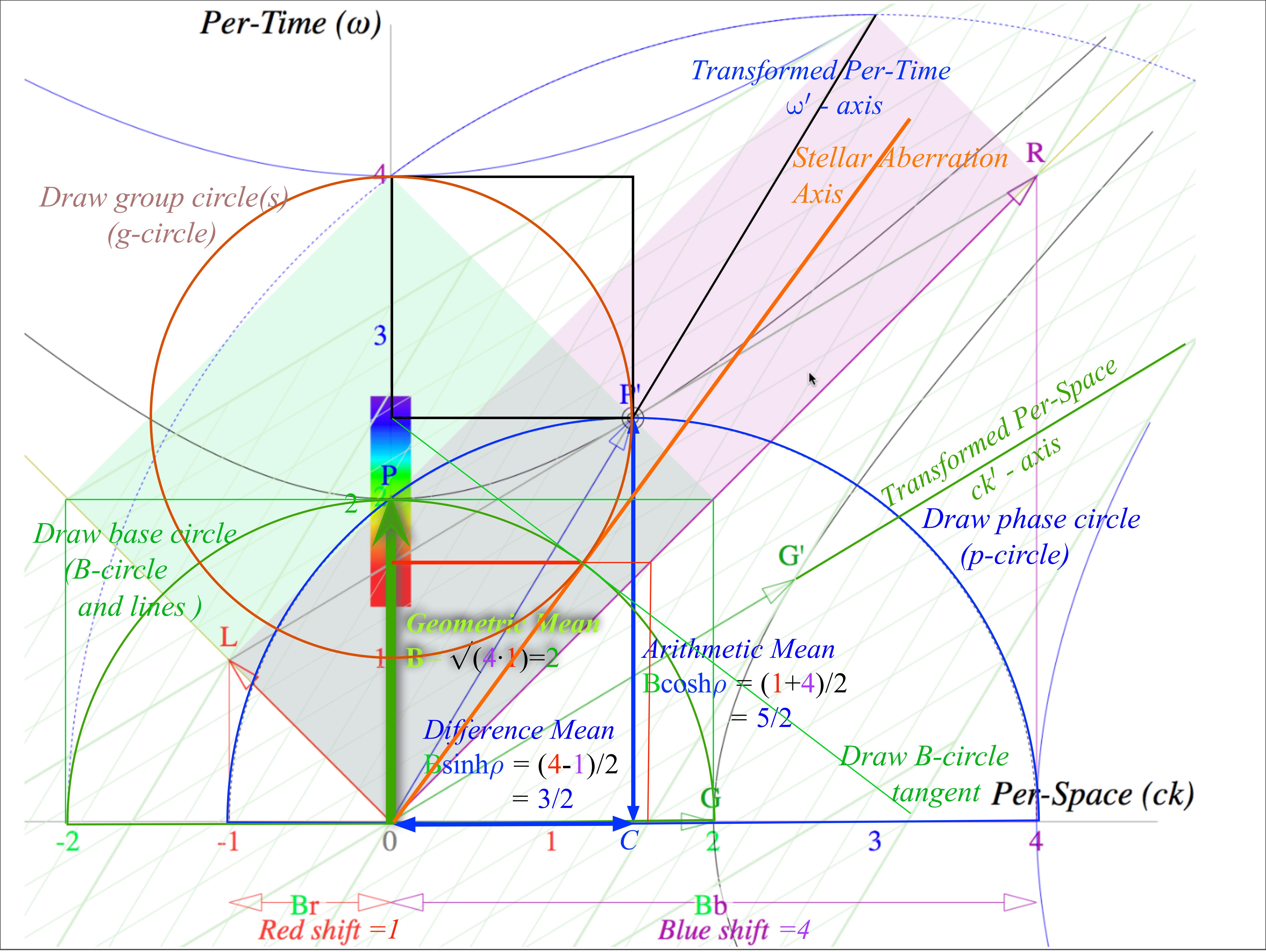
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
G
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Br
Bb
Red shift =1
Blue shift =4


Per-Time (ω)
Transformed Per-Time
ω' - axis
Stellar Aberration
Axis
R
Draw group circle(s)
(g-circle)
3
P'
P
2
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ck' - axis
Draw phase circle
(p-circle)
Draw base circle
(B-circle
and lines)
G'
Geometric Mean
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1
Arithmetic Mean
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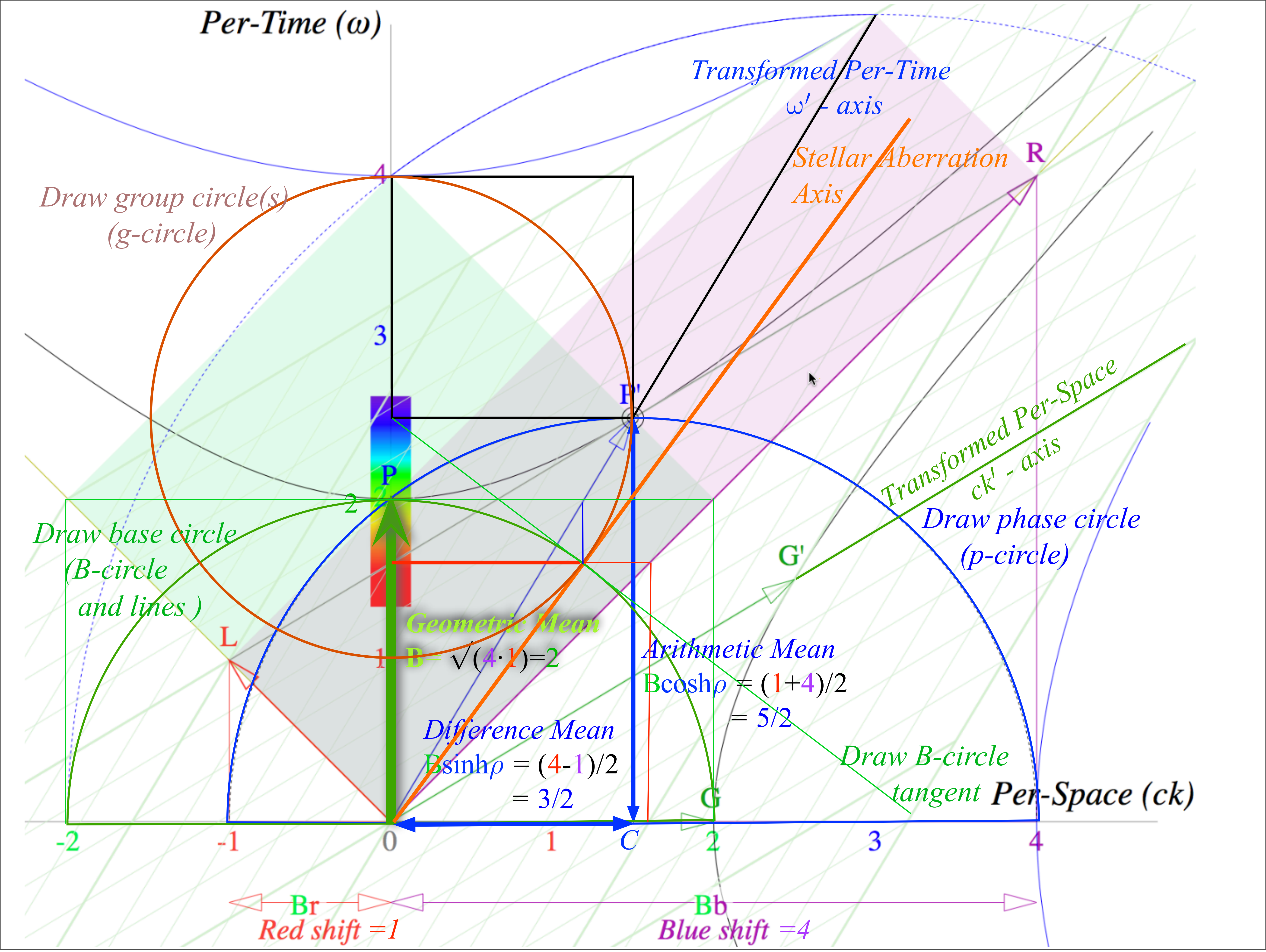
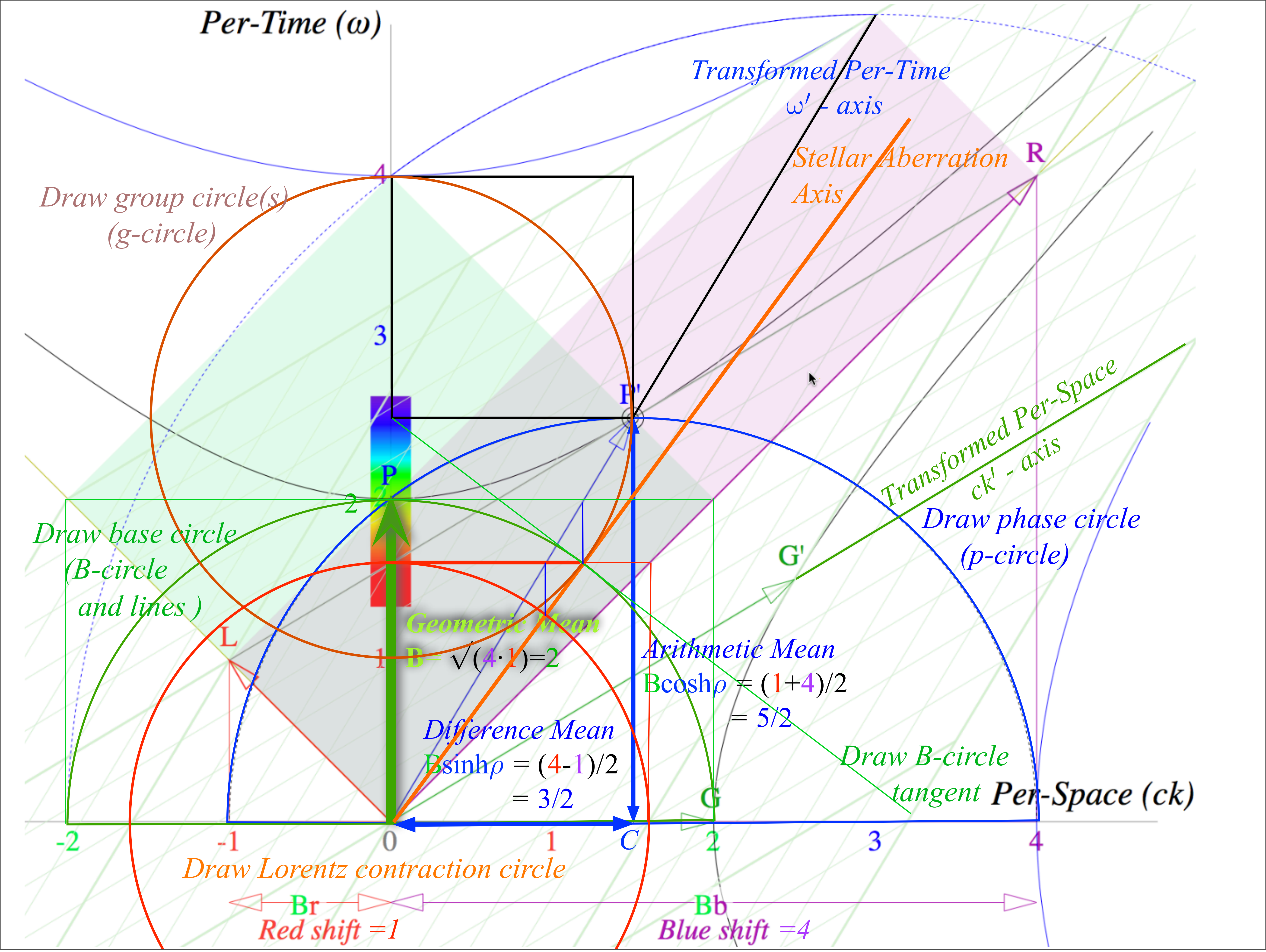
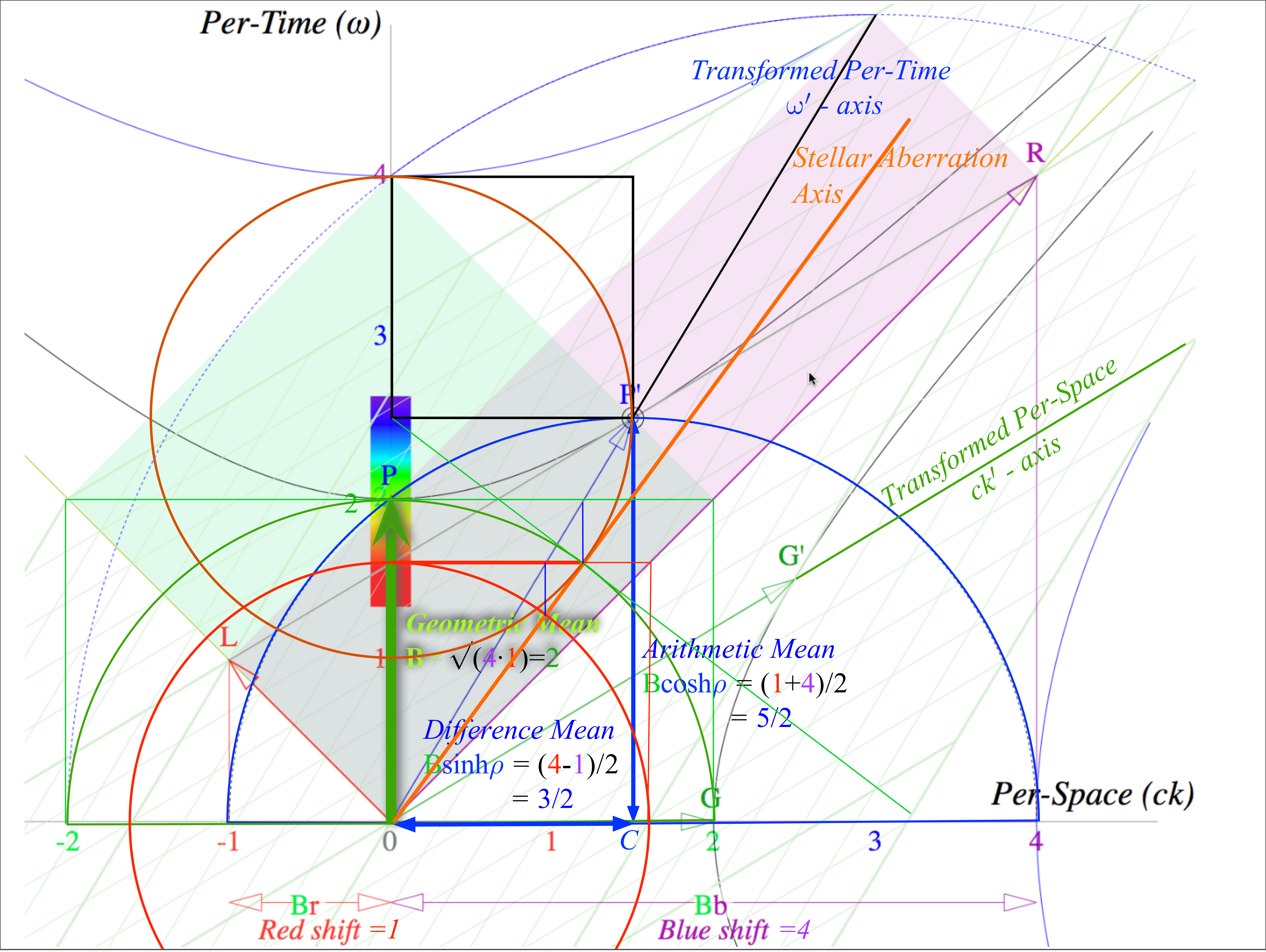
Draw B-circle
tangent
Per-Space (ck)
G
-2
-1
0
1
C
2
3
4
Br
Red shift =1
Bb
Blue shift =4

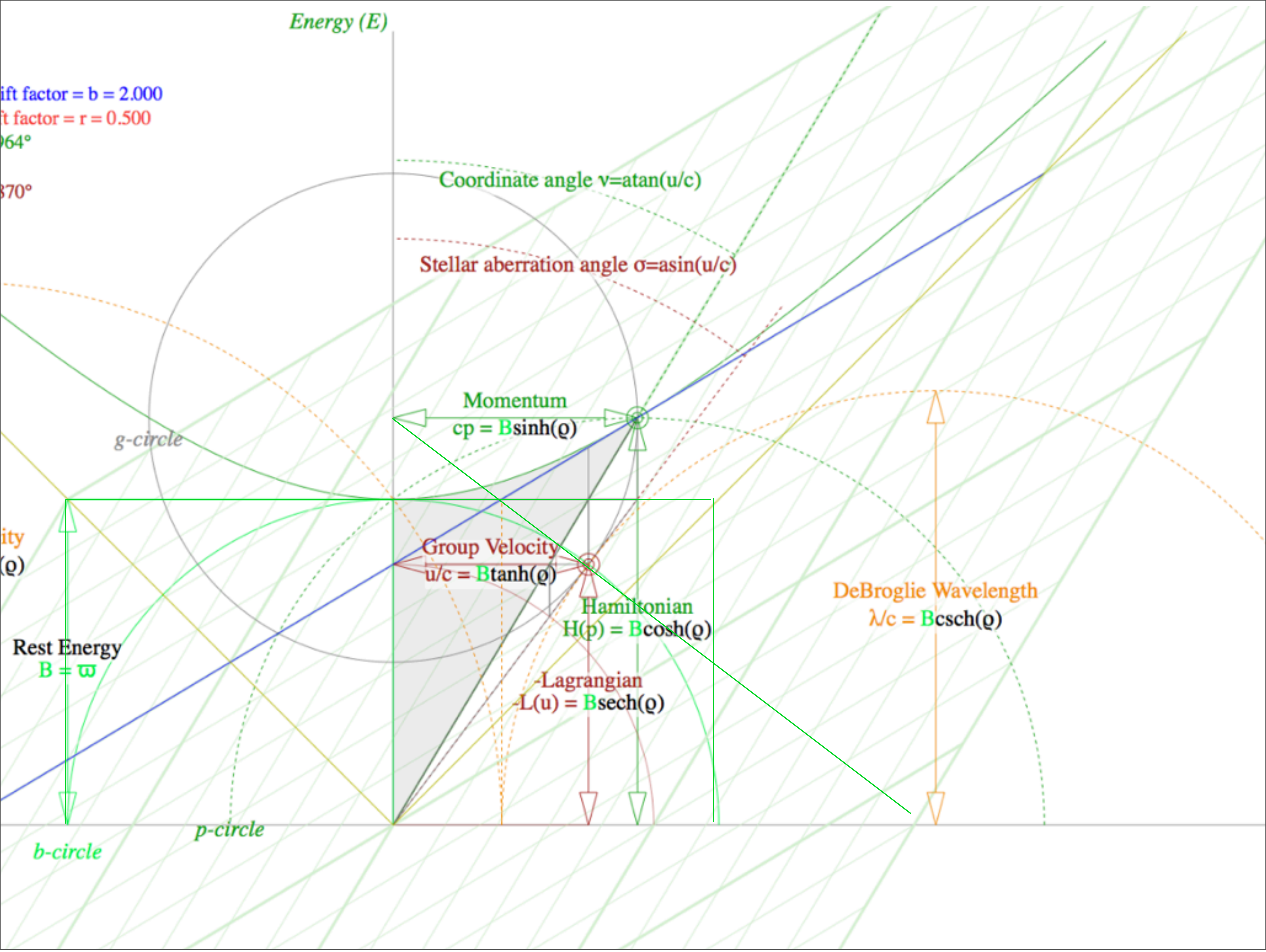


Per-Time (ω)
Transformed Per-Time
ω′ - axis
Stellar Aberration
Axis
R
Draw group circle(s)
(g-circle)
4
3
P′
P
2
Transformed Per-Space
ck′ - axis
Draw phase circle
(p-circle)
Draw base circle
(B-circle
and lines)
G′
Geometric Mean
B=√(4·1)=2
L
1
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
Draw B-circle
tangent
Per-Space (ck)
G
-2
-1
0
1
C
2
3
4
Br
Bb
Red shift =1
Blue shift =4

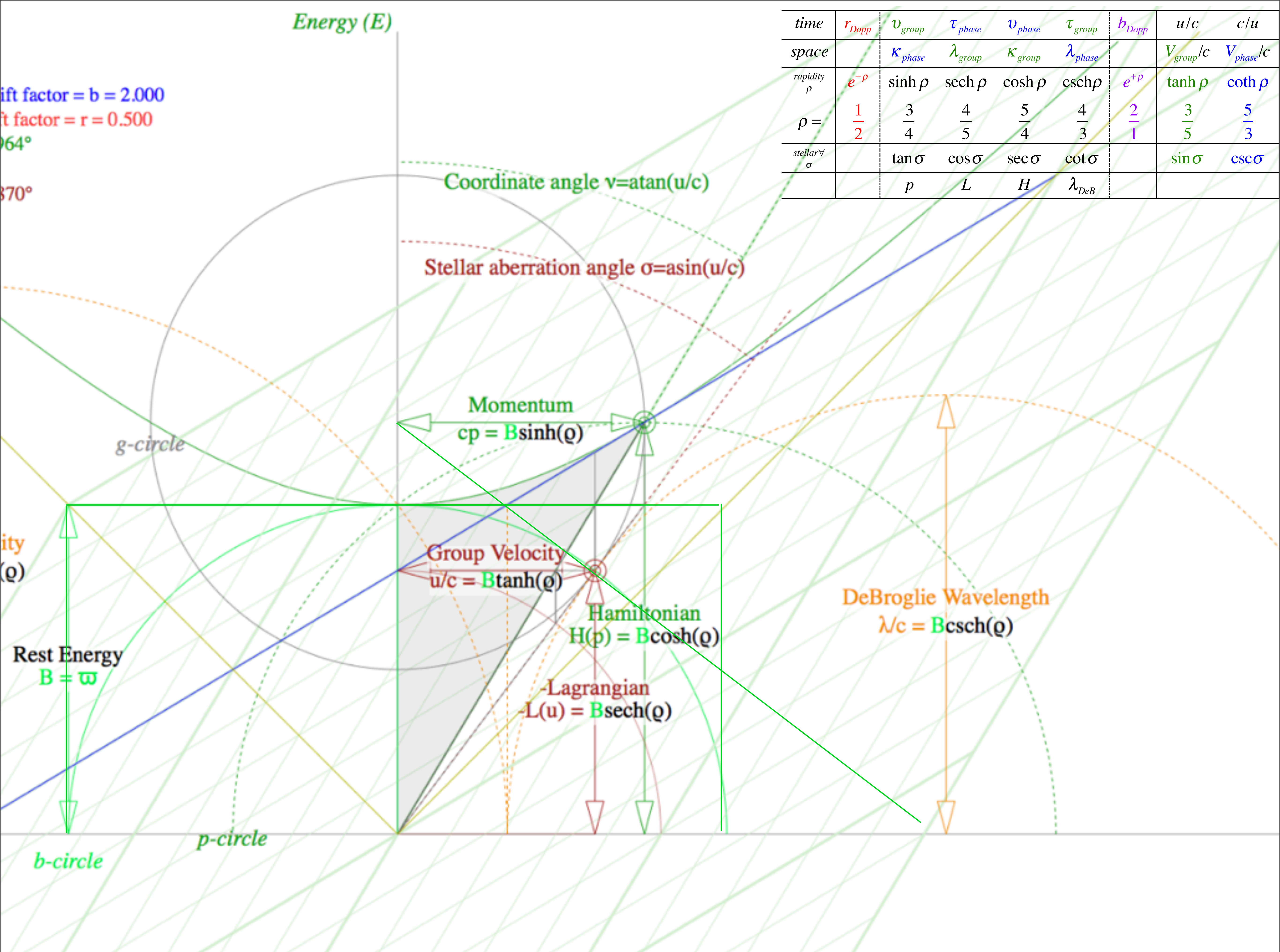

Per-Time (ω)
Transformed Per-Time
ω′ - axis
Stellar Aberration
Axis
R
Draw group circle(s)
(g-circle)
4
3
P′
P
2
Transformed Per-Space
ck′ - axis
Draw phase circle
(p-circle)
Draw base circle
(B-circle
and lines)
G′
L
Geometric Mean
B=√(4·1)=2
1
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
Draw B-circle
tangent
Per-Space (ck)
G
-2
-1
0
1
C
2
3
4
Draw Lorentz contraction circle
Br
Bb
Red shift =1
Blue shift =4



Per-Time (ω)
Transformed Per-Time
ω′ - axis
Stellar Aberration
Axis
R
4
3
P′
P
2
Transformed Per-Space
ck′ - axis
G′
Geometric Mean
B=√(4·1)=2
L
1
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
G
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Br
Red shift =1
Bb
Blue shift =4


Energy (E)
ift factor = b = 2.000
ft factor = r = 0.500
964°
370°
Coordinate angle ν=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Momentum
cp = Bsinh(ρ)
g-circle
ity
(ρ)
Group Velocity
u/c = Btanh(ρ)
Hamiltonian
H(ρ) = Bcosh(ρ)
DeBroglie Wavelength
λ/c = Bcsch(ρ)
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ρ)
p-circle
b-circle

time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			



Introducing the stellar aberration angle σ vs. rapidity ρ

Trigonometry: From circular to hyperbolic and back

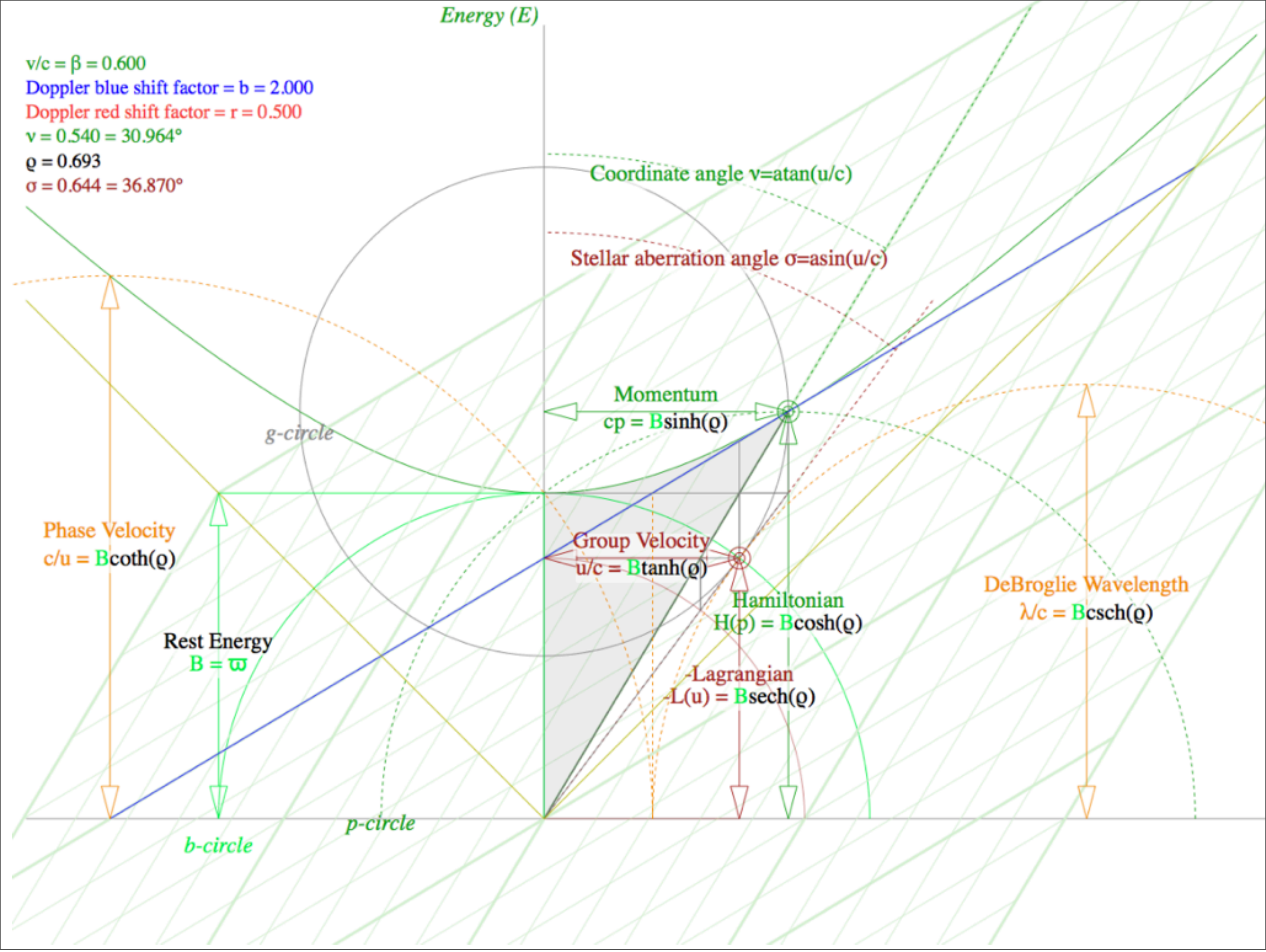
Finish "Sin-Tan" blackboard construction

Group vs. phase velocity and tangent contacts

Epstein's[†] space-proper-time (x,cτ) plots ("c-tau" plots)

[†]Lewis Carroll Epstein, *Relativity Visualized*
Insight Press, San Francisco, CA 94107

See also: L. C. Epstein, *Thinking Physics Press,*
Insight Press, San Francisco, CA 94107



Energy (E)
v/c = β = 0.600
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
ν = 0.540 = 30.964°
ϱ = 0.693
σ = 0.644 = 36.870°
Coordinate angle ν=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Momentum
cp = Bsinh(ϱ)
g-circle
Phase Velocity
c/u = Bcoth(ϱ)
Group Velocity
u/c = Btanh(ϱ)
Hamiltonian
H(ϱ) = Bcosh(ϱ)
DeBroglie Wavelength
λ/c = Bcsch(ϱ)
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ϱ)
p-circle
b-circle

Laser frequency = B = 2 = 600THz
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
$\varrho = 0.693$

CW Light Axioms

All colors go c: $\omega/k = c$ or L&R on diagonals

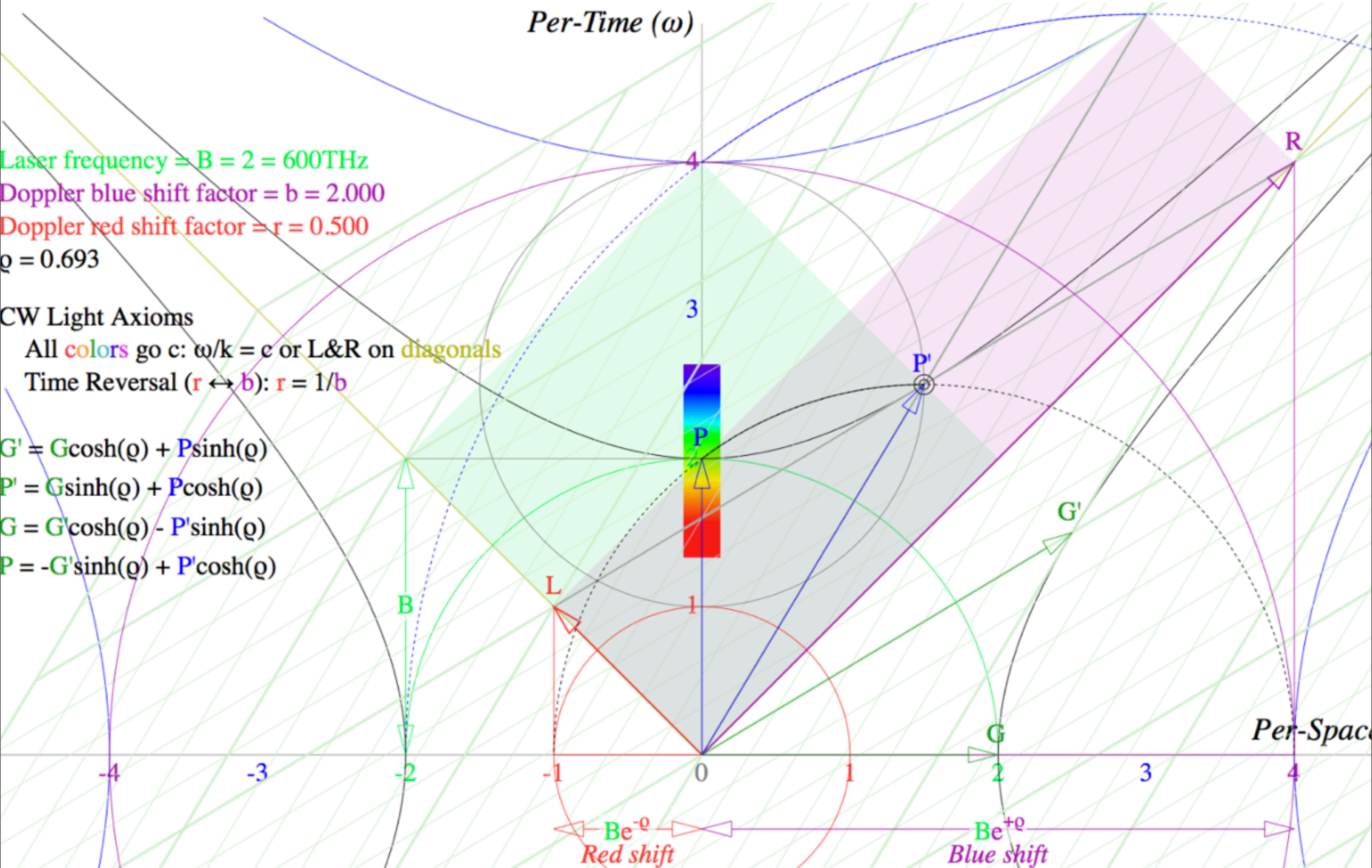
Time Reversal ($r \leftrightarrow b$): $r = 1/b$

$G' = G\cosh(\varrho) + P\sinh(\varrho)$

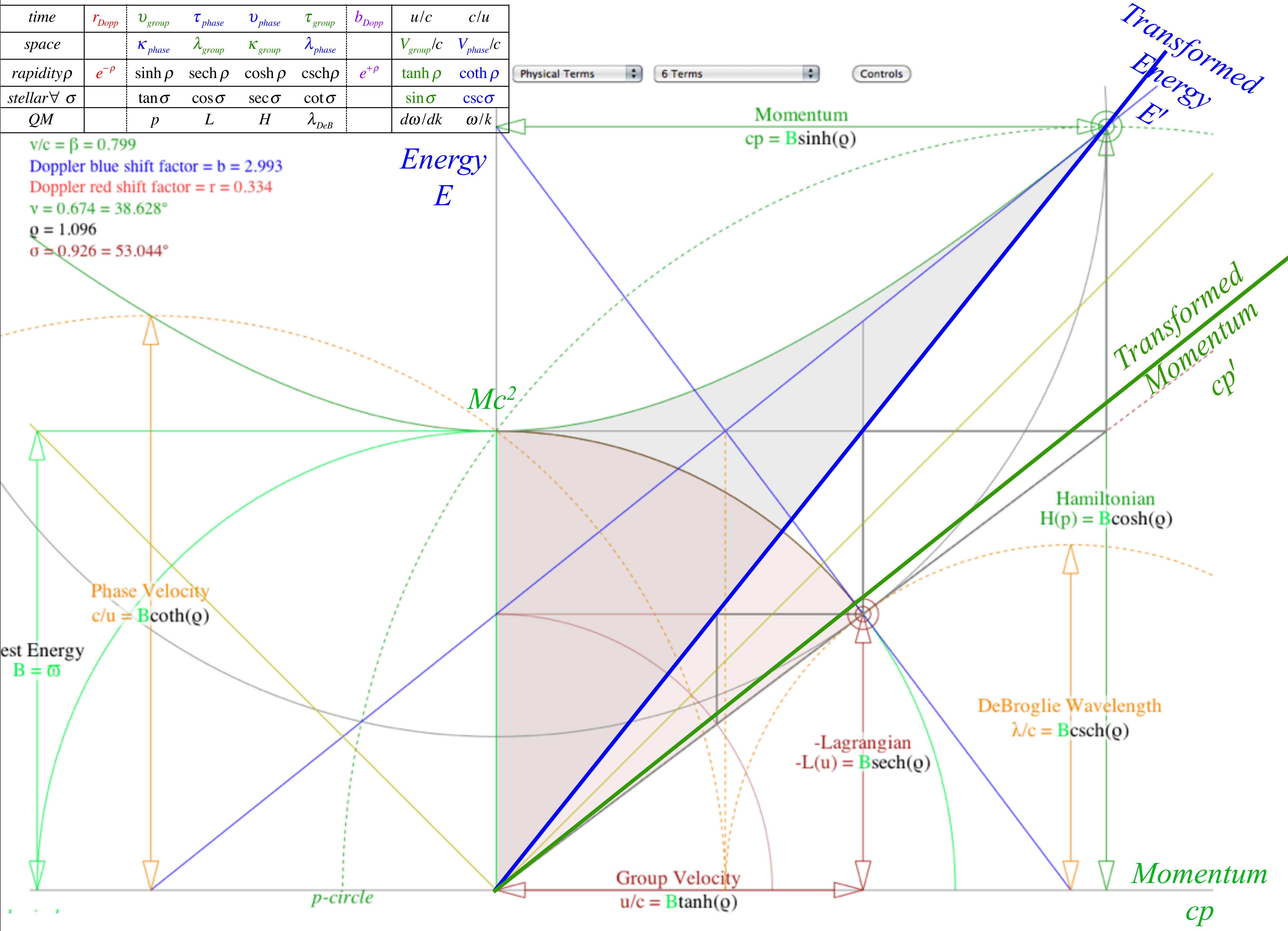
$P' = G\sinh(\varrho) + P\cosh(\varrho)$

$G = G'\cosh(\varrho) - P'\sinh(\varrho)$

$P = -G'\sinh(\varrho) + P'\cosh(\varrho)$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
QM		p	L	H	λ_{DeB}		$d\omega/dk$	ω/k



Introducing the stellar aberration angle σ vs. rapidity ρ

Trigonometry: From circular to hyperbolic and back

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Epstein's[†] space-proper-time (x,cτ) plots ("c-tau" plots)

[†] Lewis Carroll Epstein, *Relativity Visualized* *Insight Press, San Francisco, CA 94107*

See also: L. C. Epstein, *Thinking Physics Press, Insight Press, San Francisco, CA 94107*

Epstein's space-proper-time $(x,c\tau)$ plots ("c-tau" plots)

Time contraction-dilation revisited

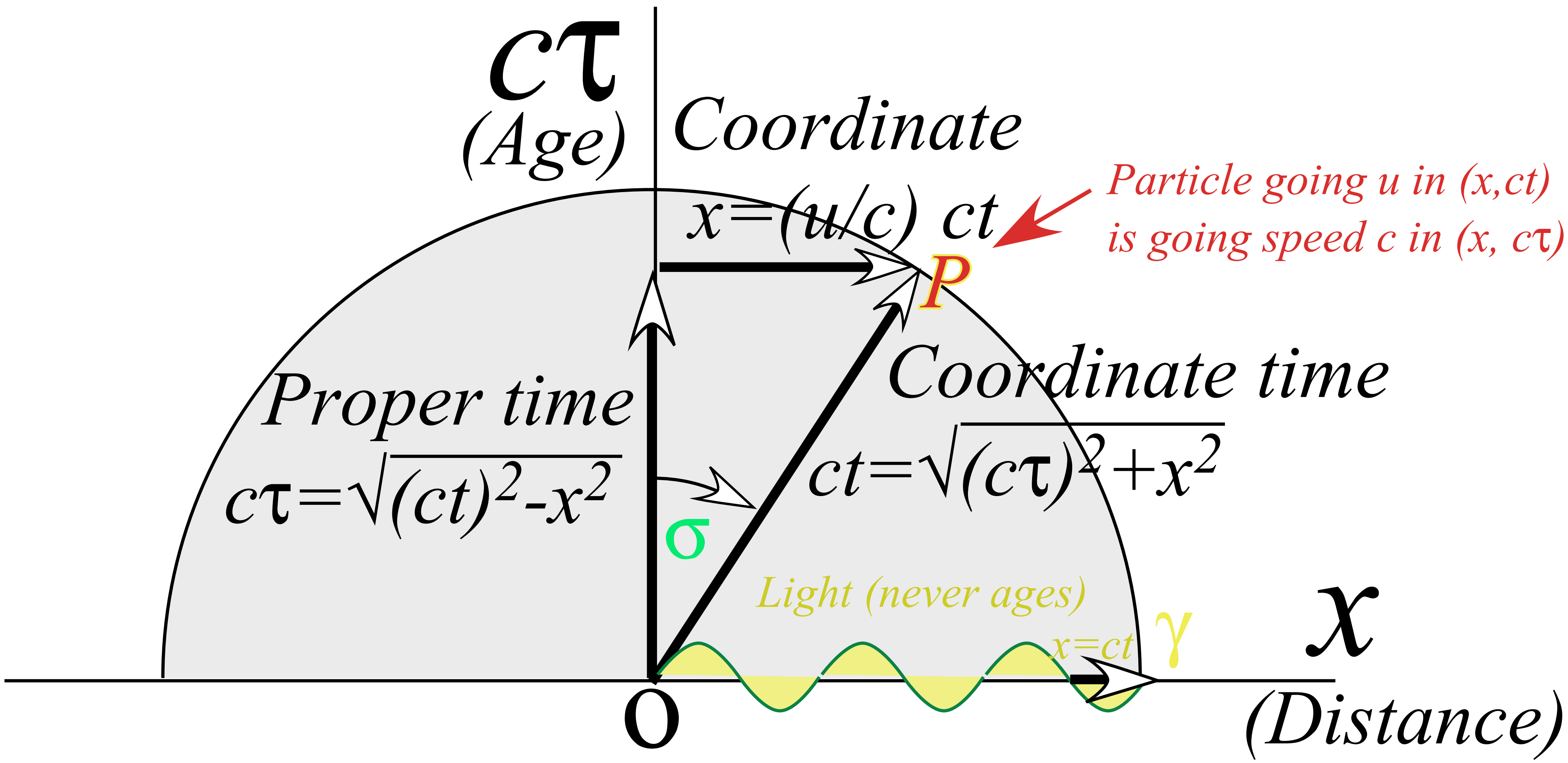


Fig. 5.8 Space-proper-time plot makes all objects move at speed c in their "cosmic speedometer.†"

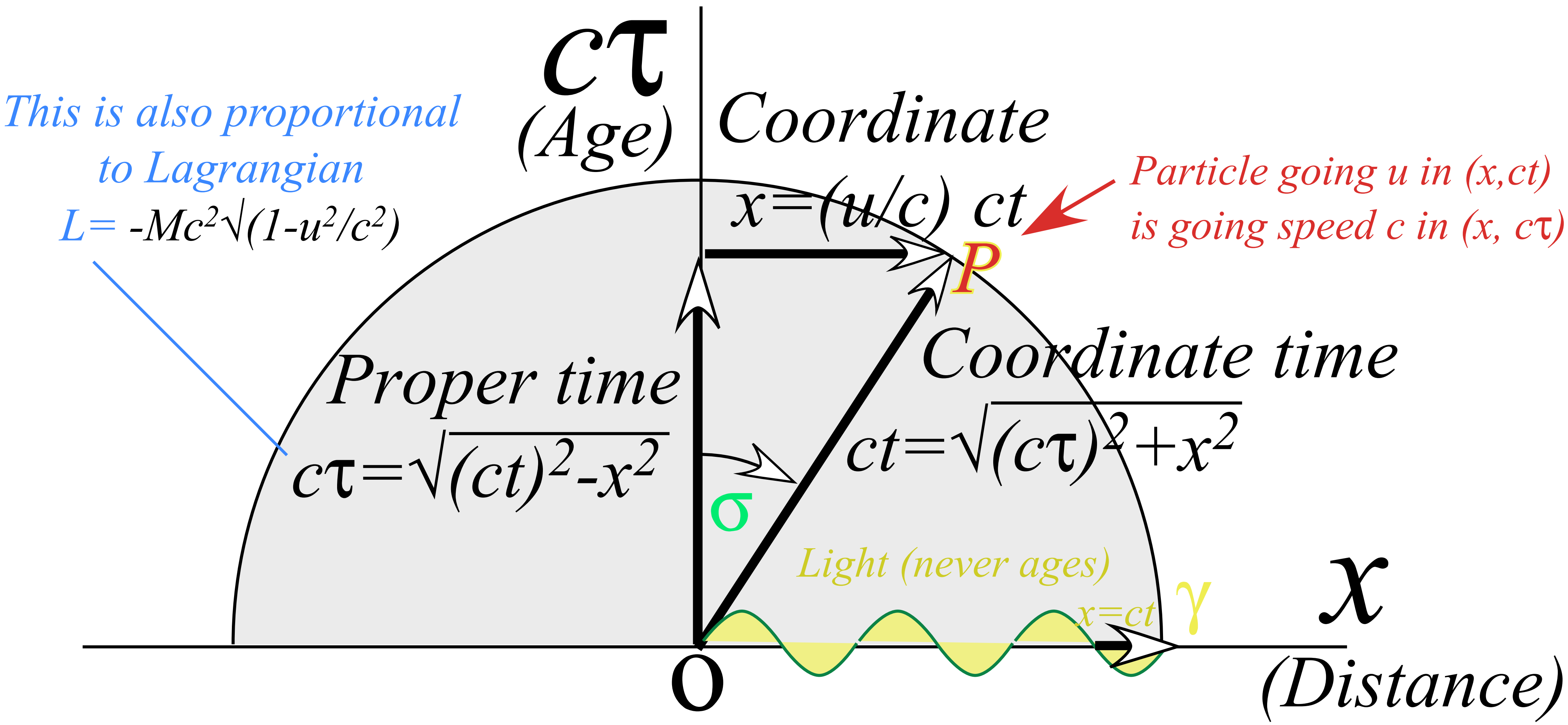
Fig. 5.8 from CMwBang! Ch. 5 of Unit 8.

†Lewis Carroll Epstein, *Relativity Visualized Insight Press, San Francisco, CA 94107*

Epstein, views stellar aberration angle σ as speedometer reading.

Epstein's space-proper-time $(x,c\tau)$ plots ("c-tau" plots)

Time contraction-dilation revisited



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Epstein, views stellar aberration angle σ as speedometer reading.

Fig. 5.8 from CMwBang! Ch. 5 of Unit 8.

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Time contraction-dilation revisited

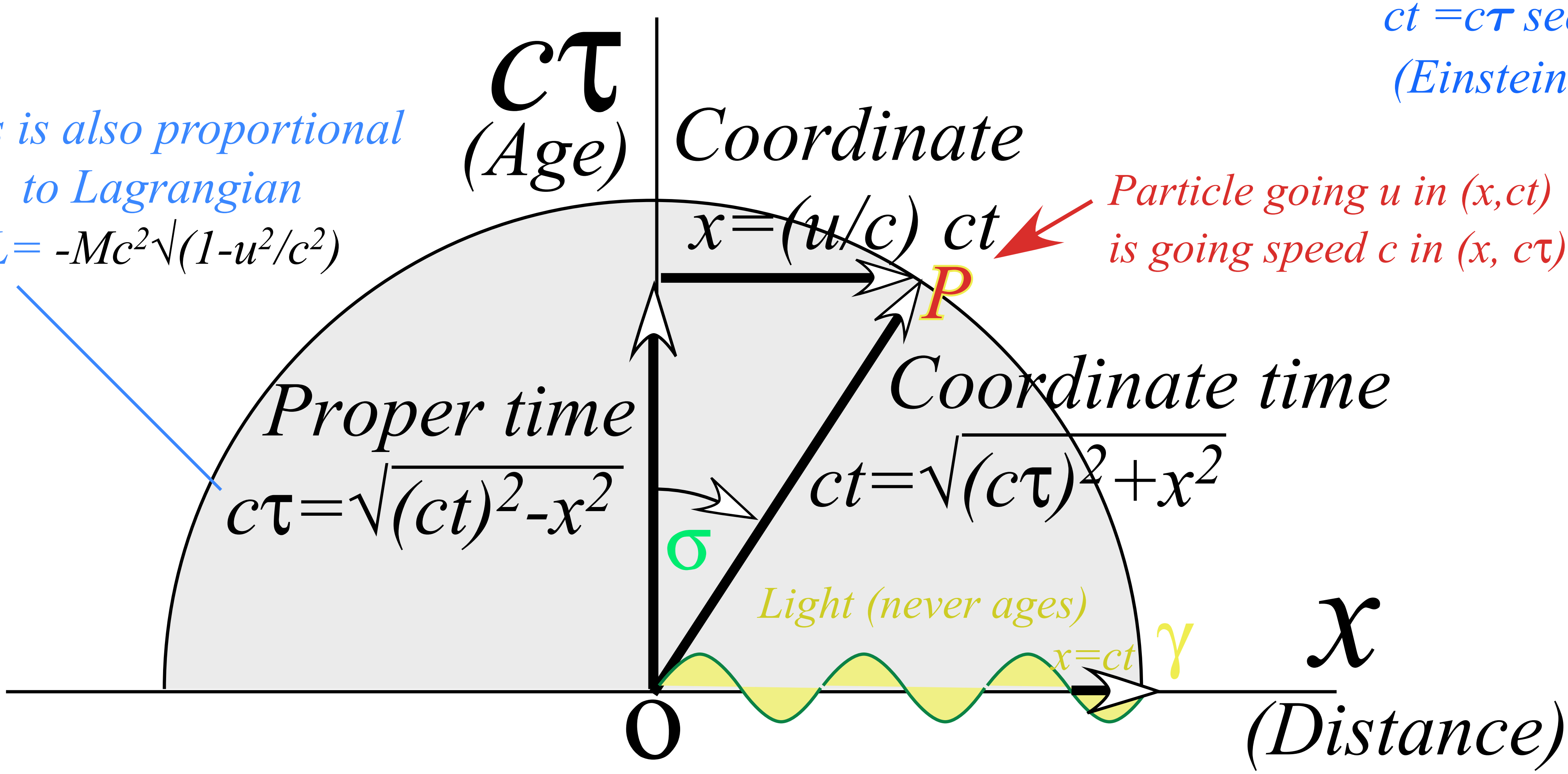
Observed coordinate length:

$ct = c\tau \sec\sigma = c\tau \cosh\rho$

(Einstein time dilation)

$ct = \frac{c\tau}{\sqrt{1-u^2/c^2}}$

This is also proportional to Lagrangian $L = -Mc^2\sqrt{(1-u^2/c^2)}$



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Epstein, views stellar aberration angle σ as speedometer reading.

Fig. 5.8 from CMwBang! Ch. 5 of Unit 8.

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Length contraction-dilation revisited

A cute Epstein feature is that Lorentz-Fitzgerald contraction of a proper length L to $L'=L\sqrt{1-u^2/c^2}$ is simply rotational projection onto the x-axis of a length L rotated by σ.

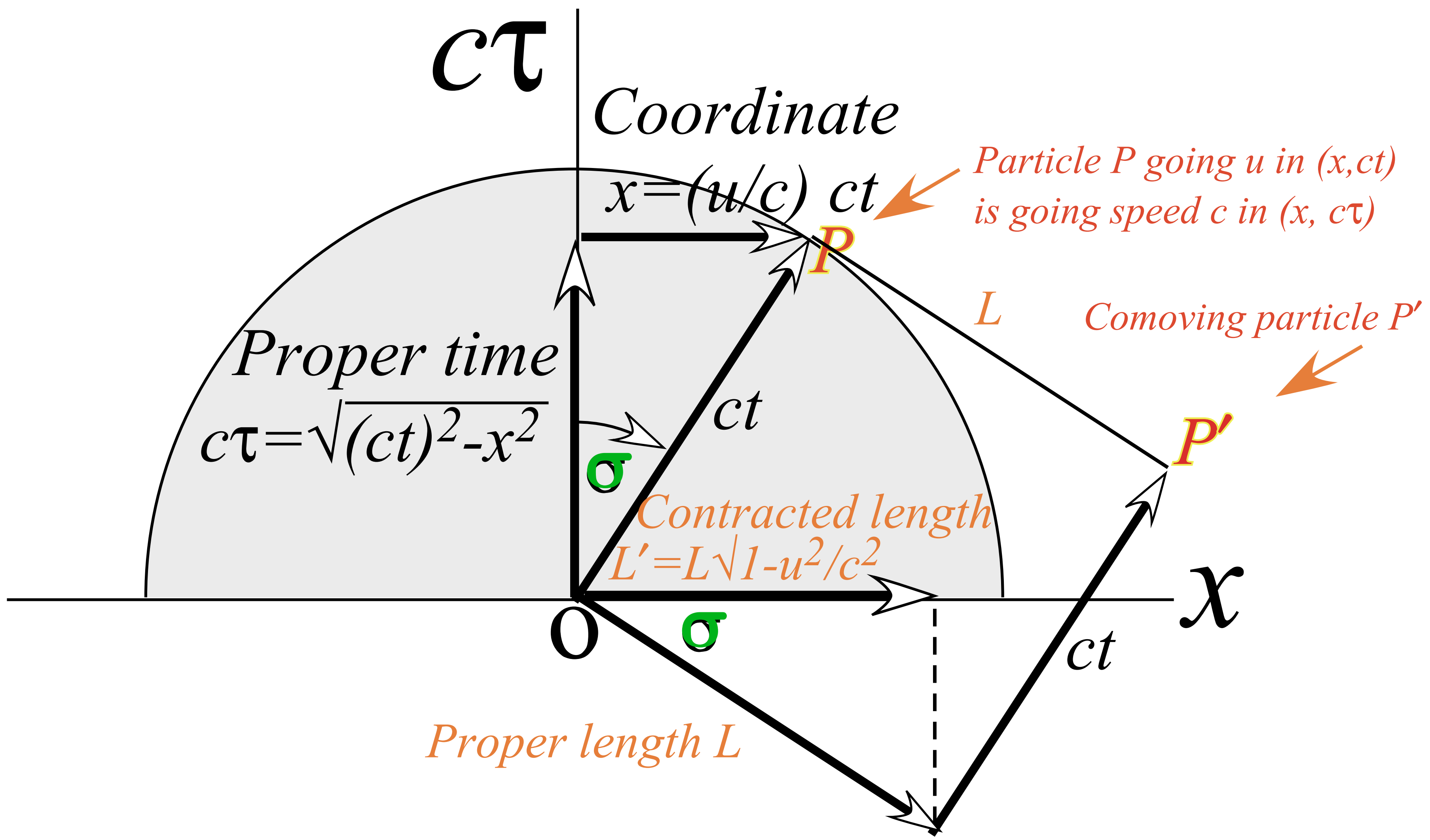
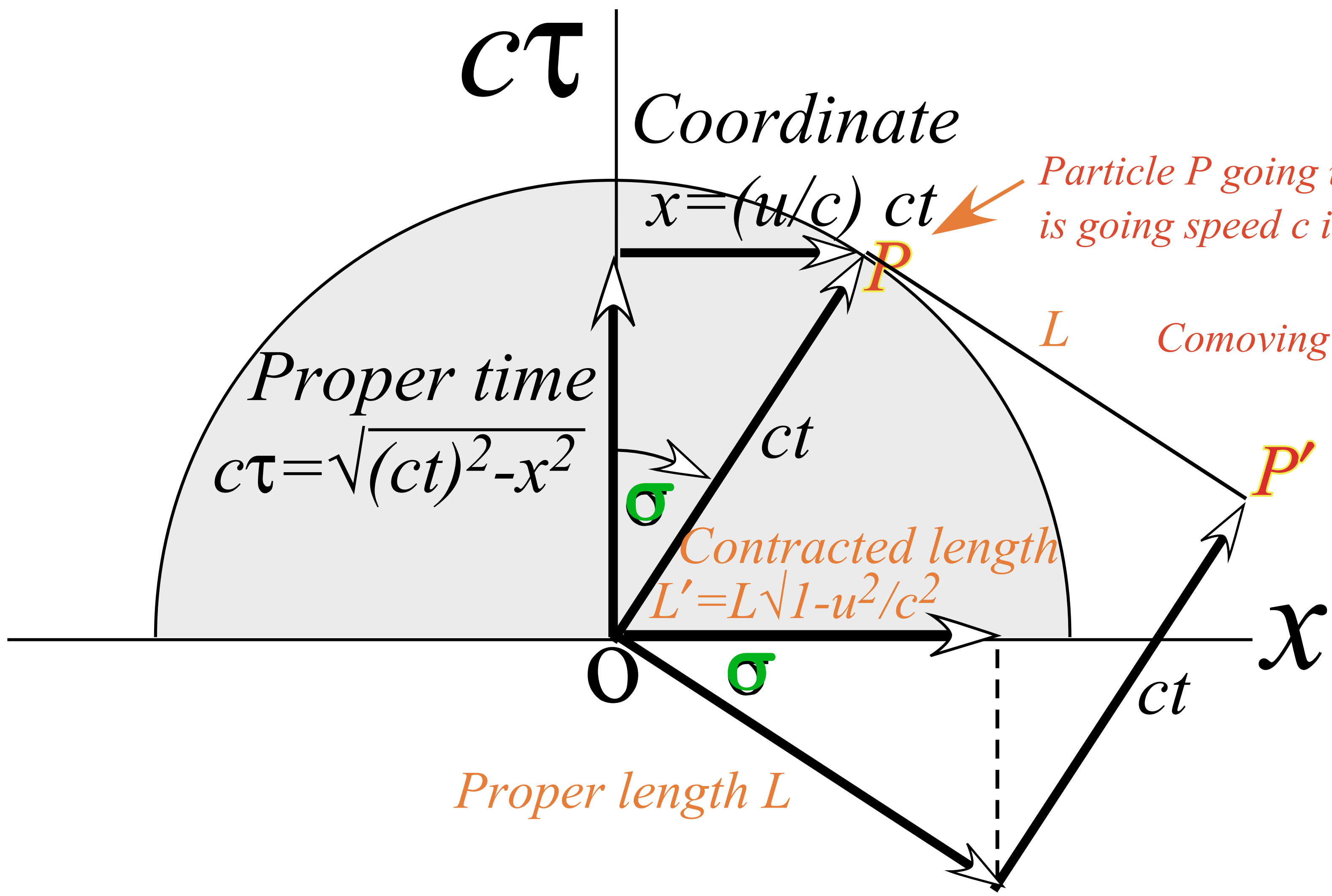


Fig. 5.9 from CMwBang! Ch. 5 of Unit 8.

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Length contraction-dilation revisited

A cute Epstein feature is that Lorentz-Fitzgerald contraction of a proper length L to $L'=L\sqrt{1-u^2/c^2}$ is simply rotational projection onto the x-axis of a length L rotated by σ.



Observed coordinate length: $L'=L\cos\sigma = L\,\mathrm{sech}\,\rho$

(Lorentz contraction)

Fig. 5.9 from CMwBang! Ch. 5 of Unit 8.

Epstein's space-proper-time $(x,c\tau)$ plots ("c-tau" plots)

Length contraction-dilation revisited

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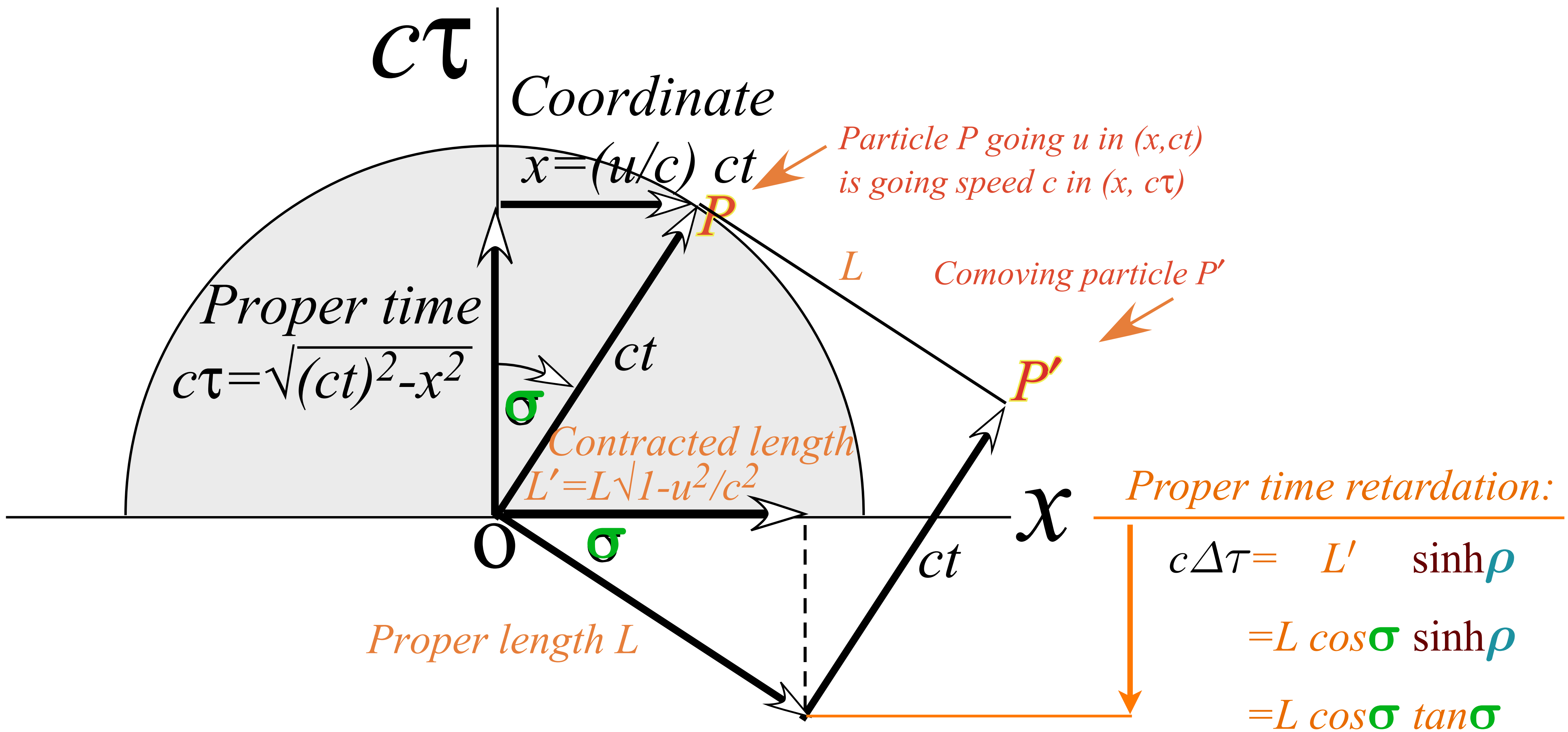


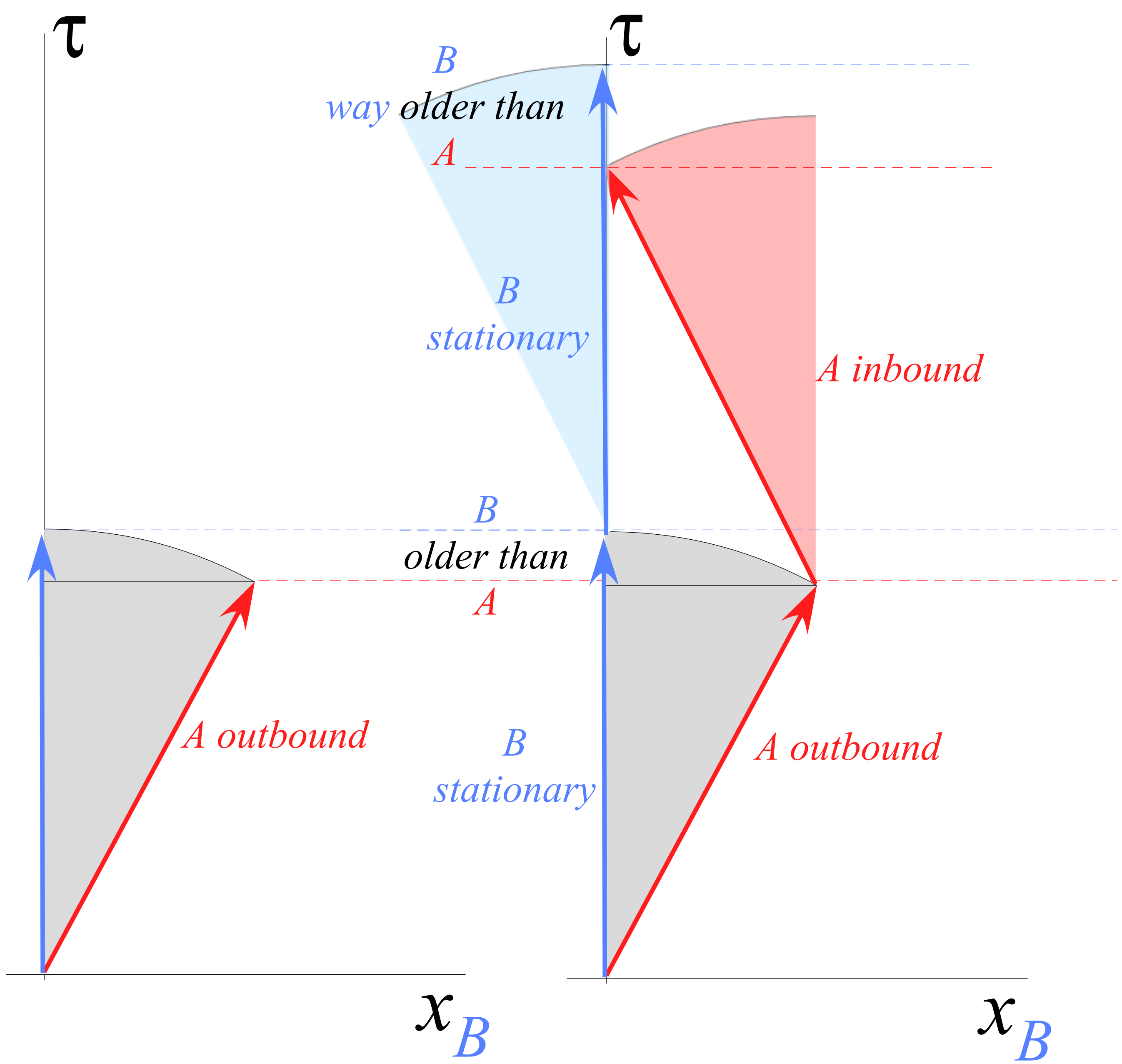
Observed coordinate length: $L'=L\cos\sigma=L\,\mathrm{sech}\rho$

(Lorentz contraction)

Fig. 5.9 from CMwBang! Ch. 5 of Unit 8.

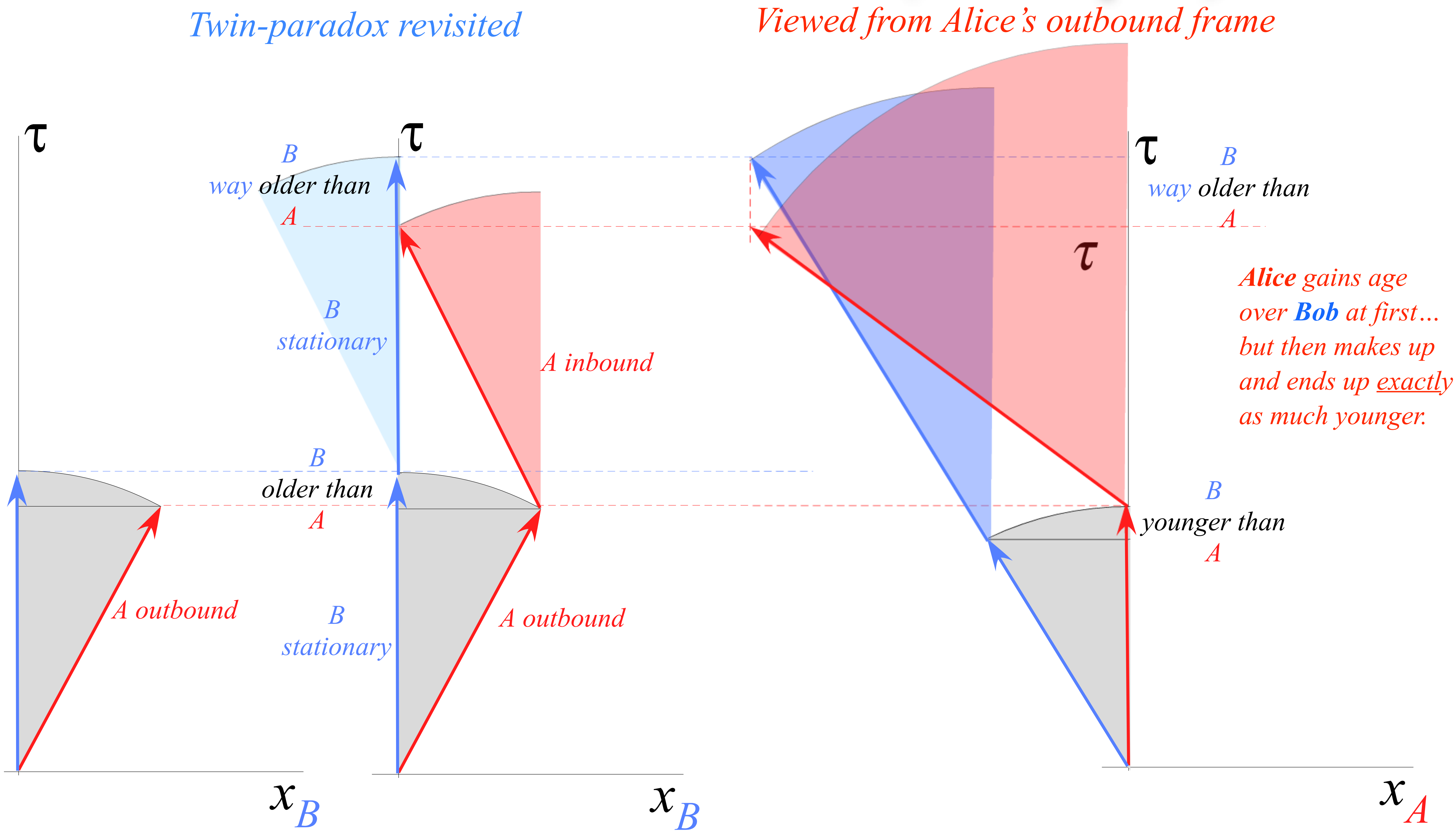
Epstein's space-proper-time $(x,c\tau)$ plots ("c-tau" plots)

Twin-paradox revisited



Anthropomorphic Analog:
"Active-Alice"
does not age as
much as
"Be-at-home Bob"

Epstein's space-proper-time $(x, c\tau)$ plots ("c-tau" plots)



Bob stationary viewpoint

Alice outbound to right

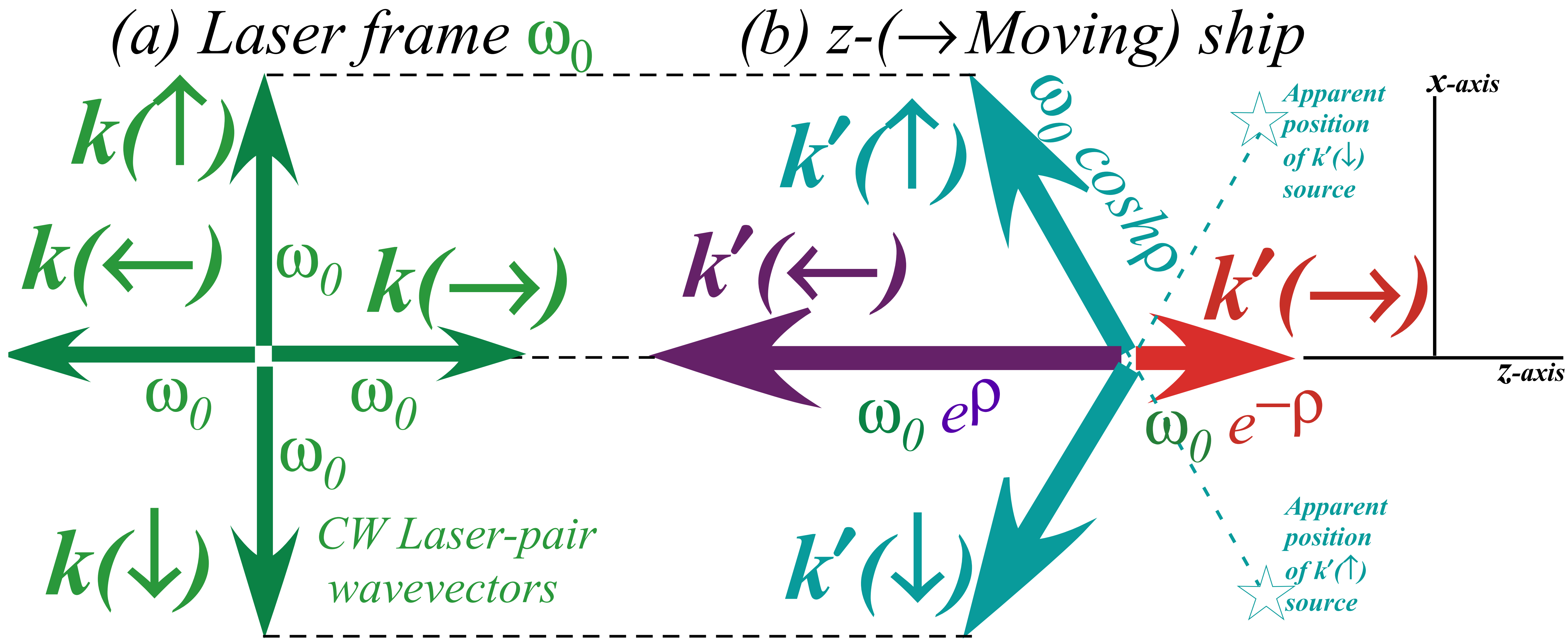


Fig. 5.7 from CMwBang! Ch. 5 of Unit 8.

Geometry of Lorentz boost of counter-propagating waves.

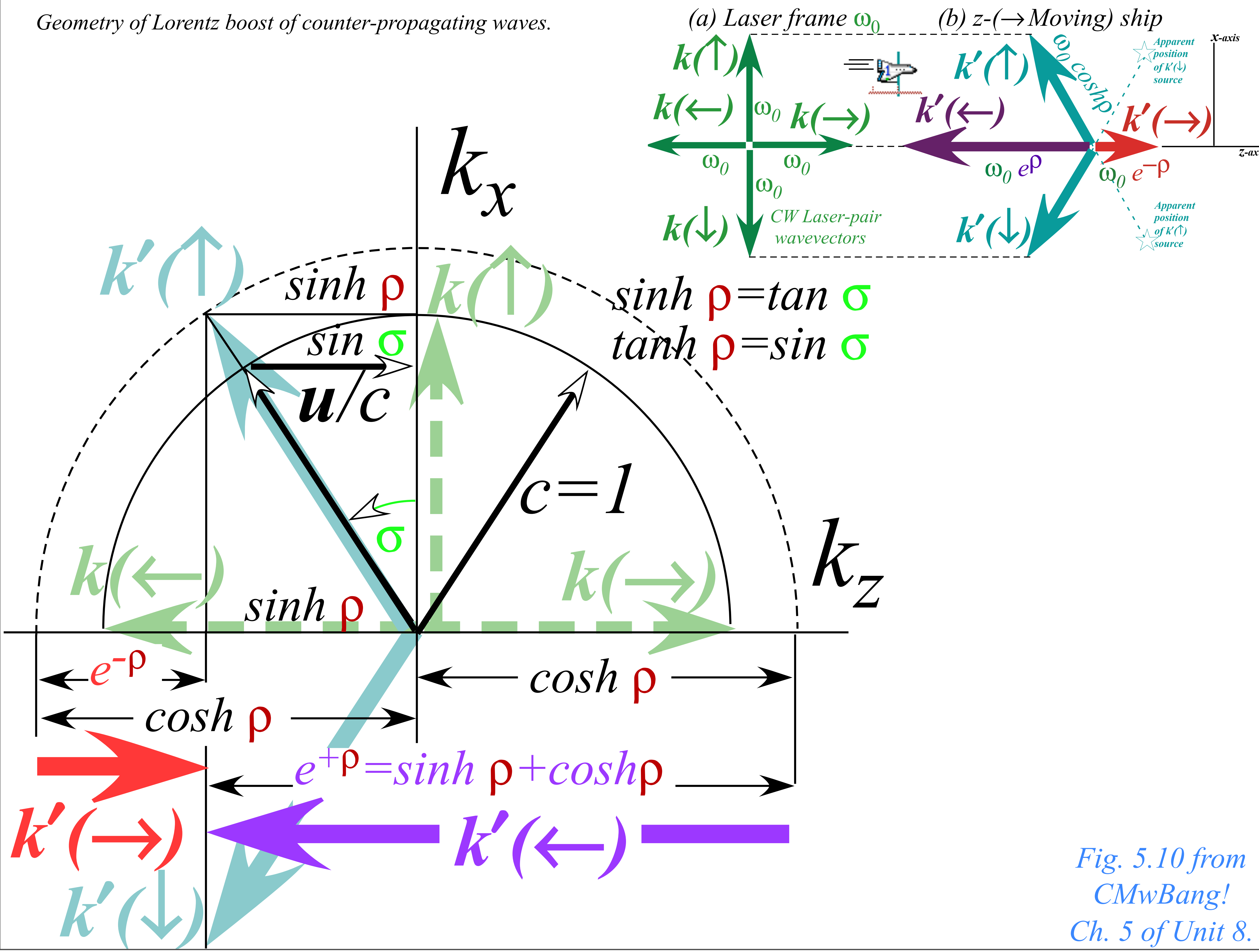
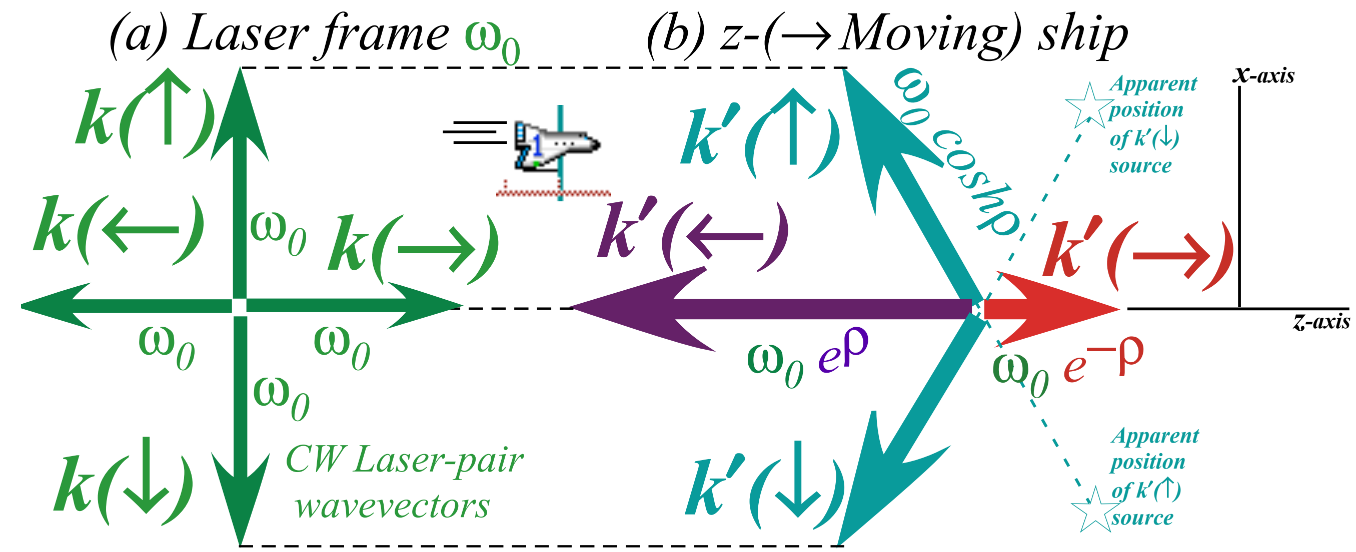


Fig. 5.10 from CMwBang! Ch. 5 of Unit 8.

Geometry of Lorentz boost of counter-propagating waves.



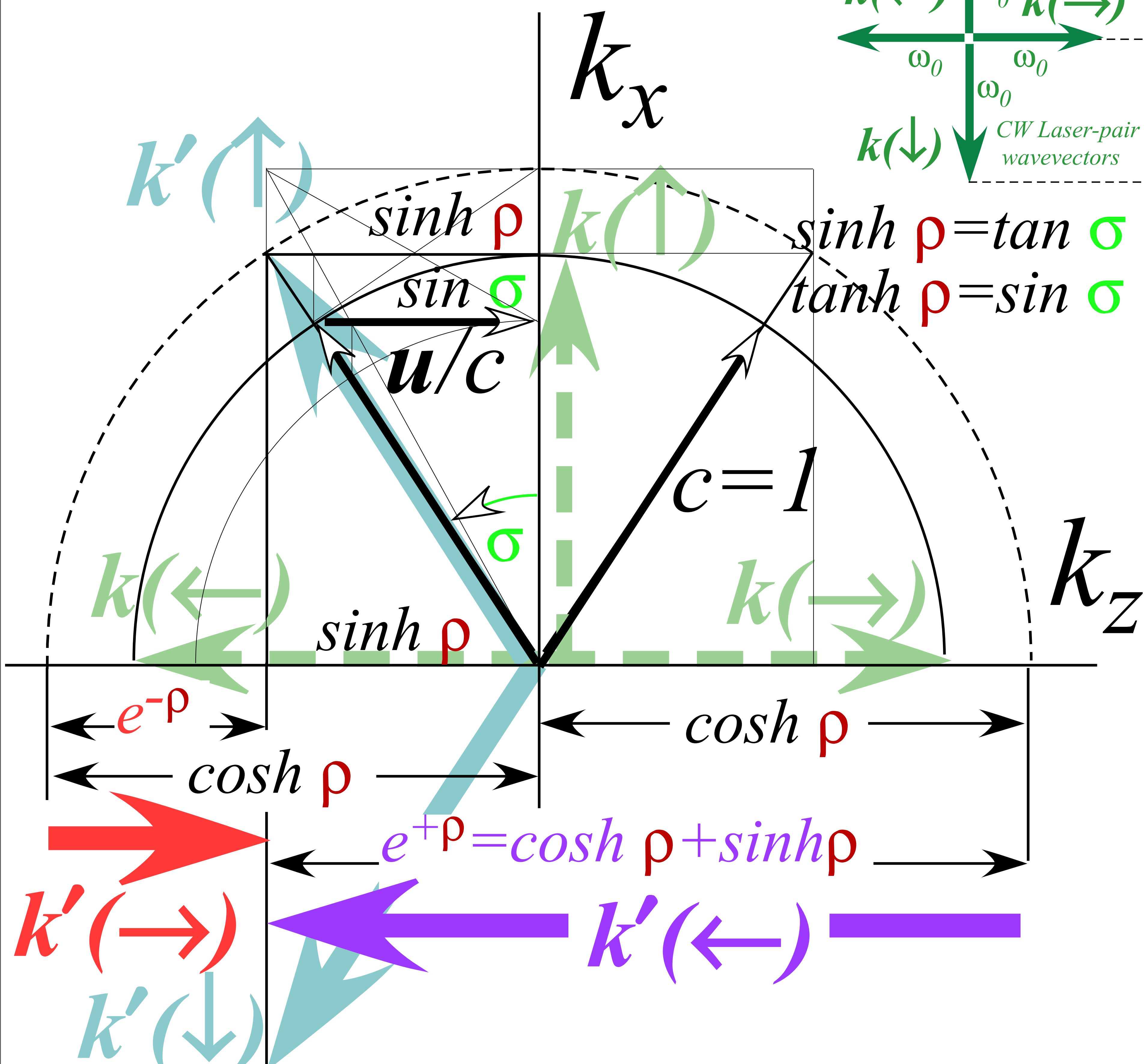


Fig. 5.10 from CMwBang! Ch. 5 of Unit 8.

Introducing the stellar aberration angle σ vs. rapidity ρ

Trigonometry: From circular to hyperbolic and back

Finish "Sin-Tan" blackboard construction

Group vs. phase velocity and tangent contacts

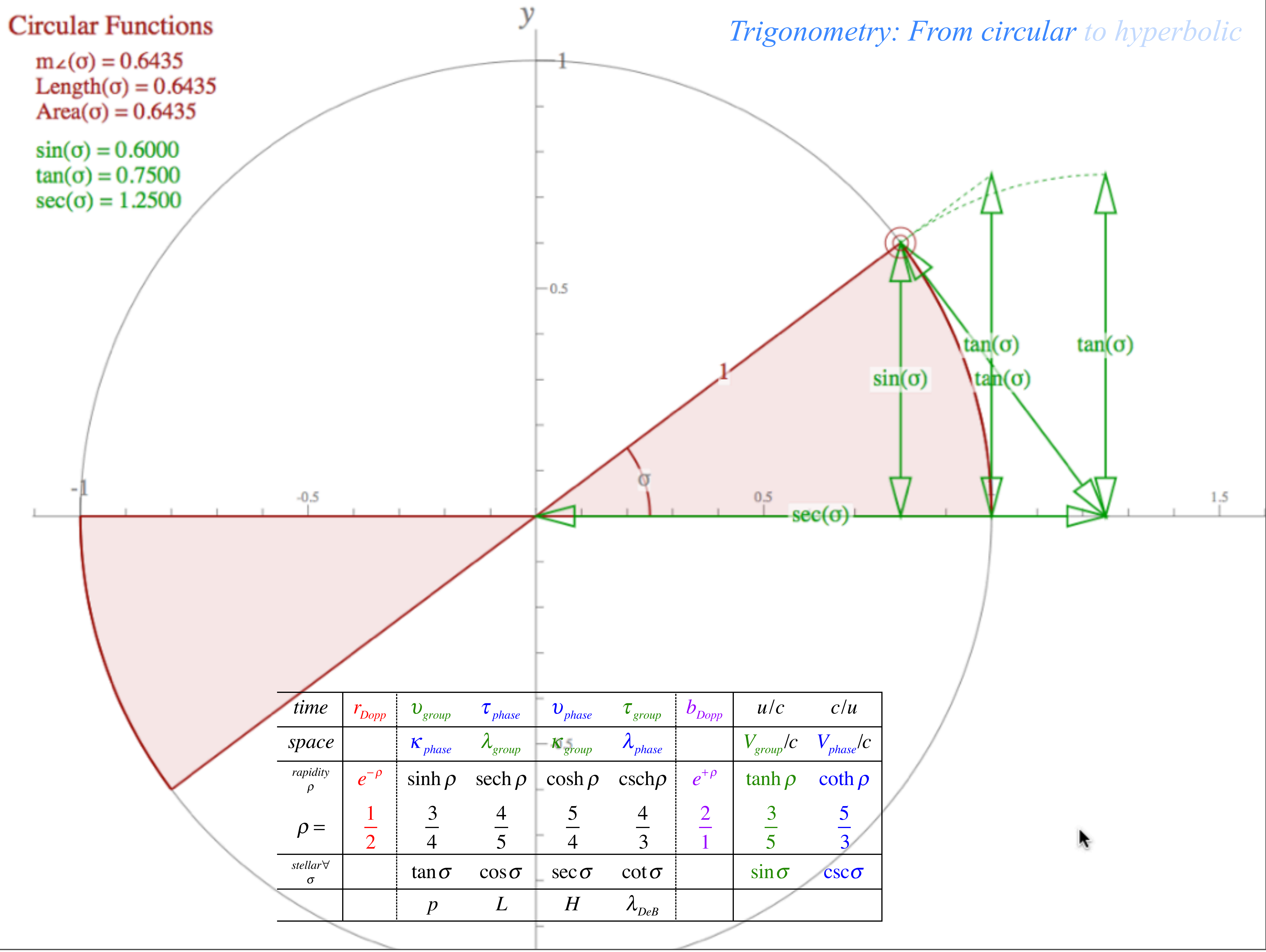
Epstein's† space-proper-time (x,cτ) plots ("c-tau" plots)

Extrastuff

†Lewis Carroll Epstein, *Relativity Visualized*
Insight Press, San Francisco, CA 94107

See also: L. C. Epstein, *Thinking Physics Press,*
Insight Press, San Francisco, CA 94107

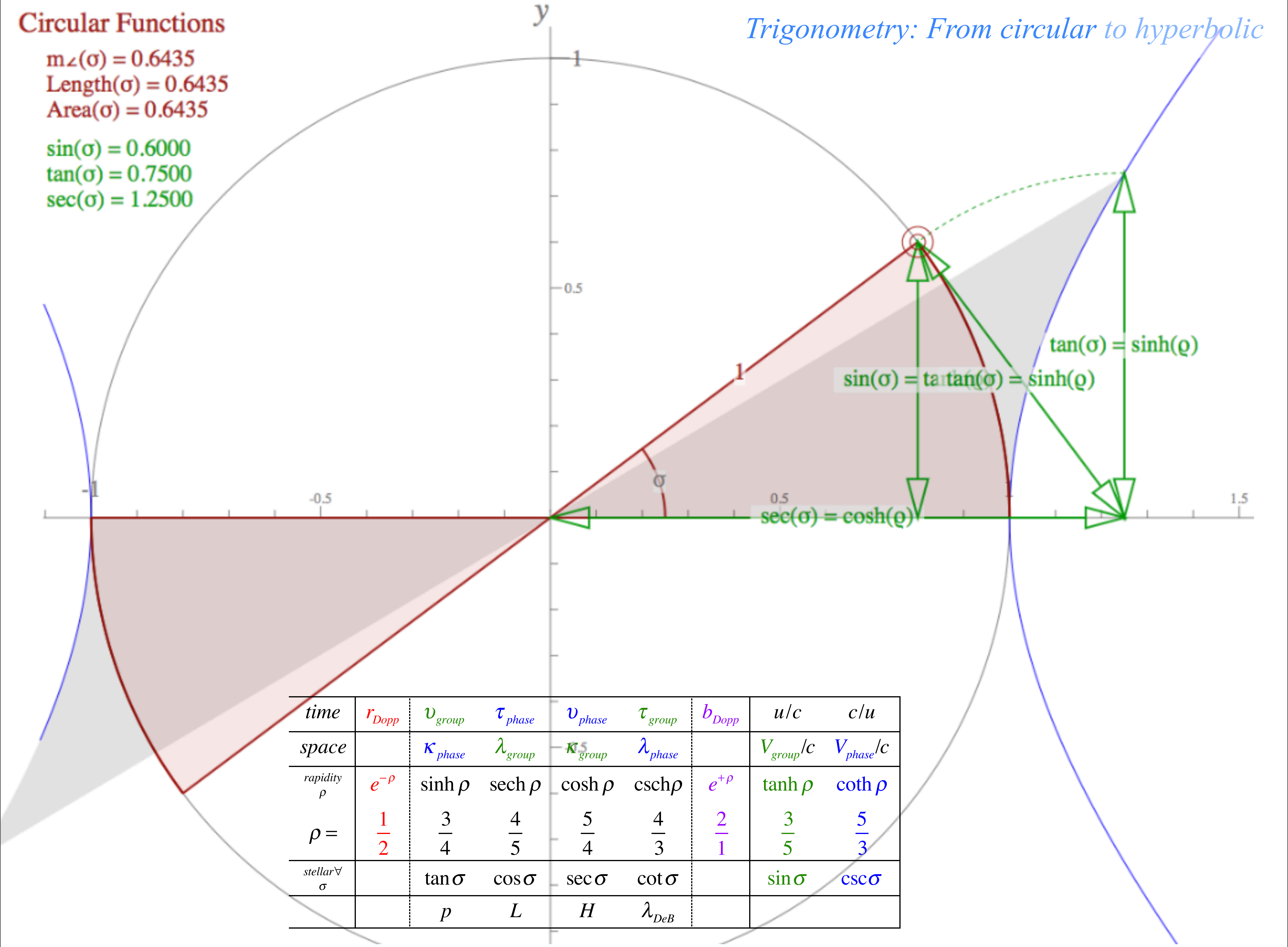
Trigonometry: From circular to hyperbolic



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			

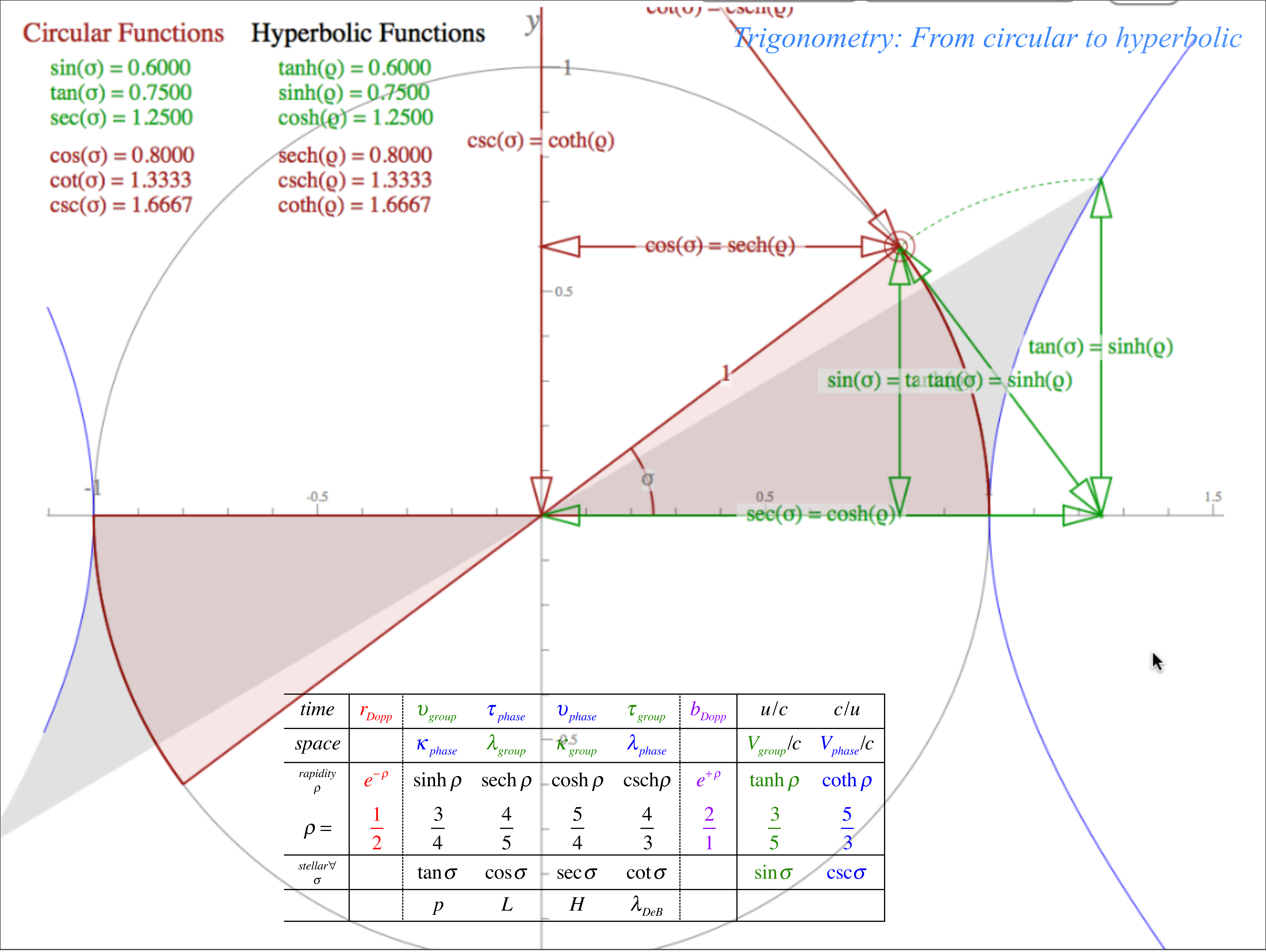
Circular Functions

Trigonometry: From circular to hyperbolic



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			

Trigonometry: From circular to hyperbolic



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			

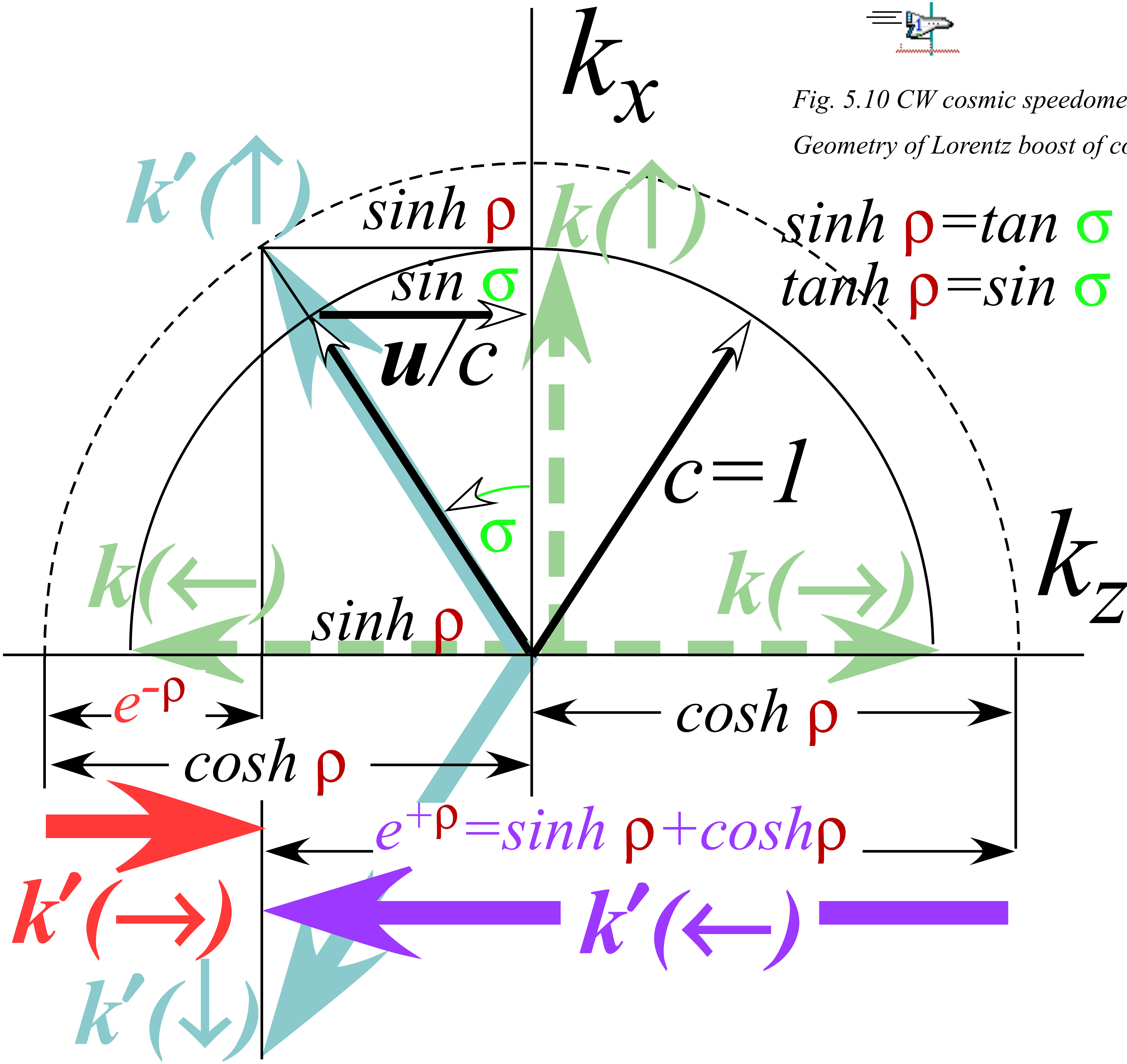
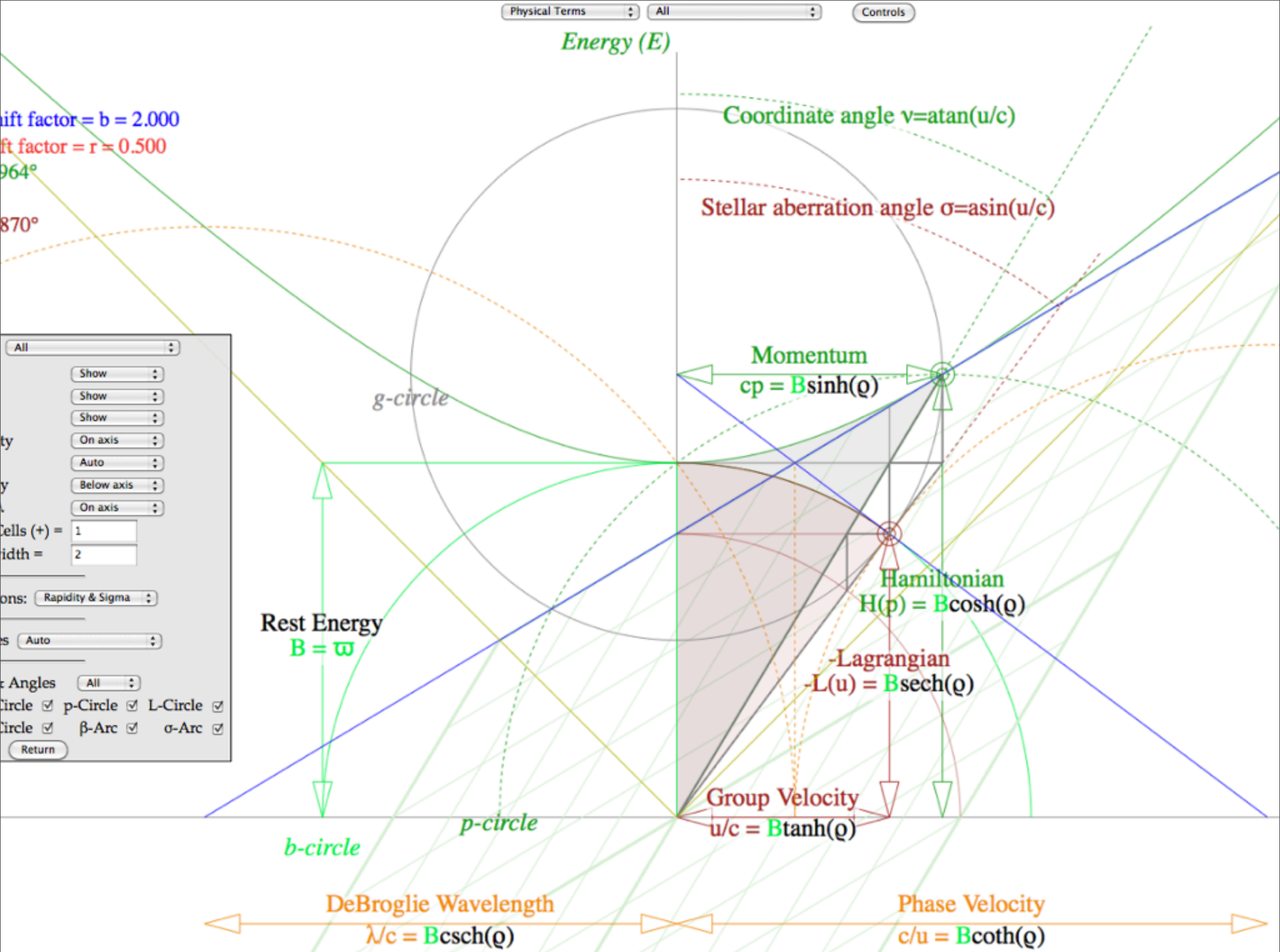


Fig. 5.10 CW cosmic speedometer.

Geometry of Lorentz boost of counter-propagating waves.


Physical Terms
All
Controls
Energy (E)
ift factor = b = 2.000
ft factor = r = 0.500
964°
870°
Coordinate angle ν=atan(u/c)
Stellar aberration angle σ=asin(u/c)
All
Show
Show
Show
ty
On axis
Auto
y
Below axis
On axis
Cells (+) =
1
idth =
2
ons:
Rapidity & Sigma
es
Auto
Angles
All
Circle
p-Circle
L-Circle
Circle
β-Arc
σ-Arc
Return
g-circle
Momentum
cp = Bsinh(ϱ)
Hamiltonian
H(p) = Bcosh(ϱ)
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ϱ)
Group Velocity
u/c = Btanh(ϱ)
p-circle
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DeBroglie Wavelength
λ/c = Bcsch(ϱ)
Phase Velocity
c/u = Bcoth(ϱ)