AMOP Lecture 5
Tue 2.11.2014

Relativity of lightwaves and Lorentz-Minkowski coordinates V.

(Ch. 0-4 of Unit 8)

Review of space-time (x,ct) and per-space-time (ω,ck) geometry

Space-time (x,ct) and per-space-time (ω,ck) geometry and its physics

- *__All__ of those contraction and expansion coefficients*
  - *Detailed views Einstein time dilation*
    - *The old "smoke and mirrors" trick*
  - *Detailed views Lorentz contraction*
    - *Heighway's paradox 1 and 2*

Phase invariance used to derive (x,ct)↔(x′,ct′) Einstein Lorentz Transformations (ELT)

Introducing the stellar aberration angle σ vs. rapidity ρ

Epstein's† space-proper-time (x,cτ) plots ("c-tau" plots)

- *Trigonometry: From circular to hyperbolic and back*
  - *Group vs. phase velocity and tangent contacts*

†Lewis Carroll Epstein, *Relativity Visualized*
Insight Press, San Francisco, CA 94107

See also: L. C. Epstein, *Thinking Physics Press,*
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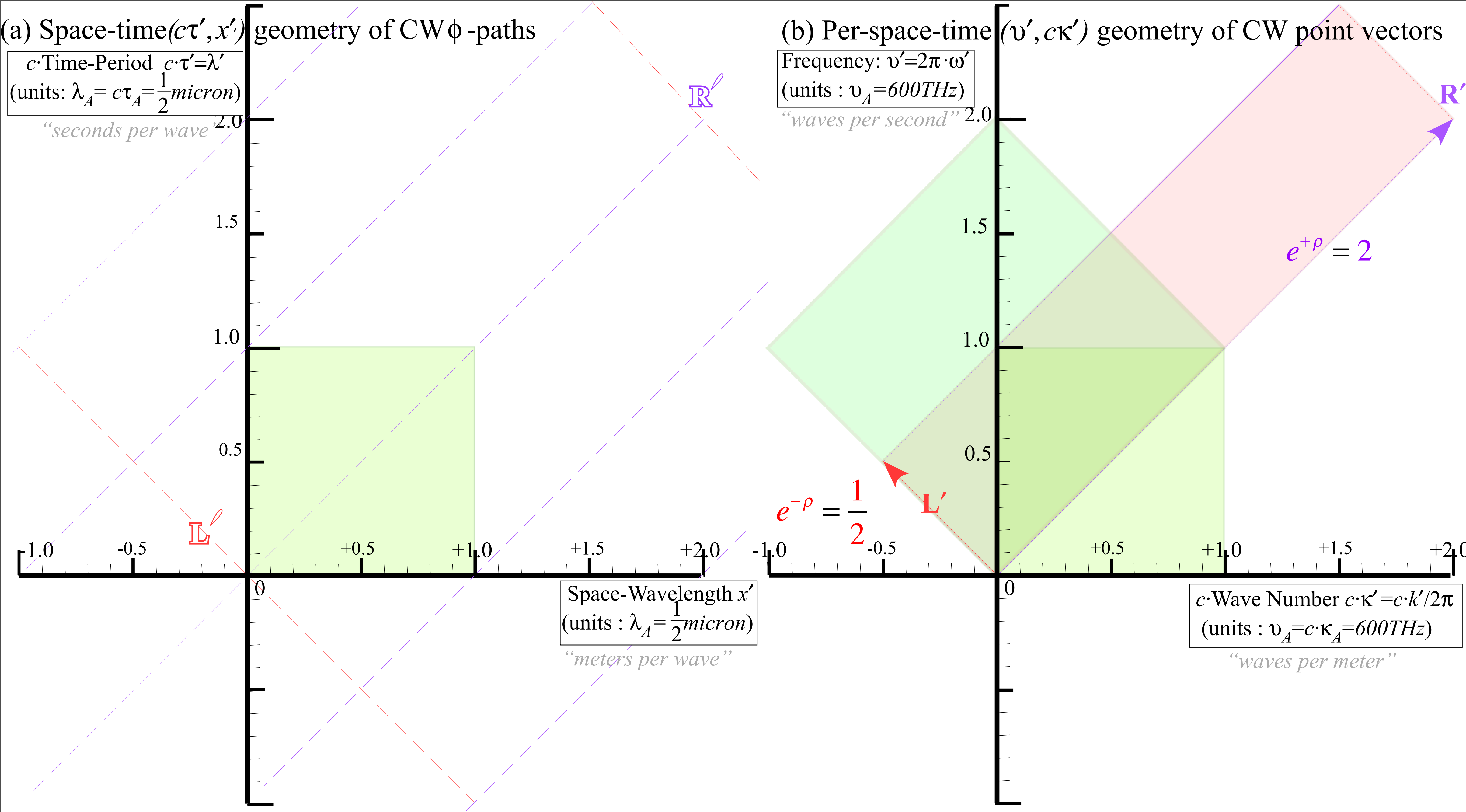
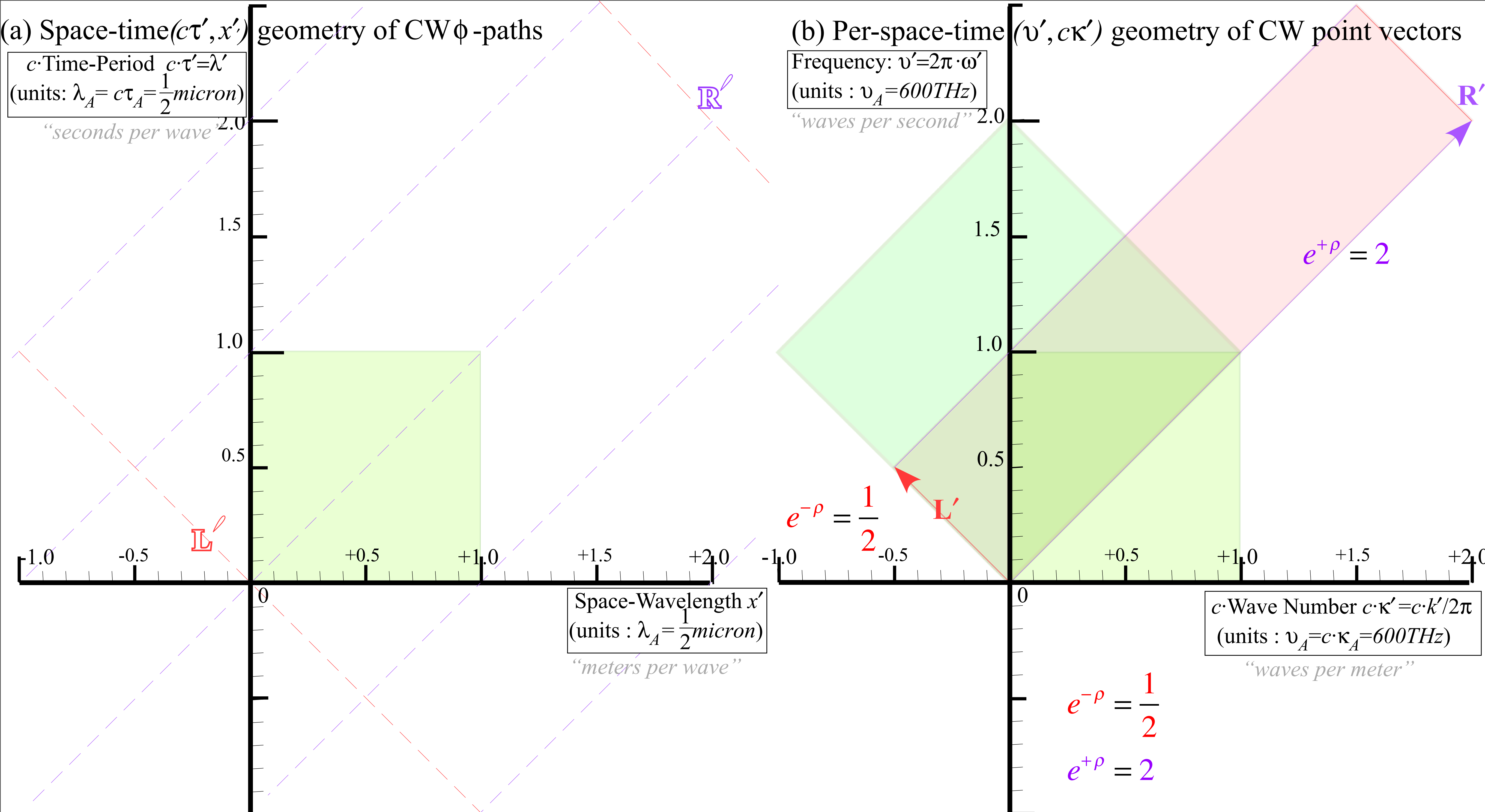


Fig. 7 SRQMbyR&C



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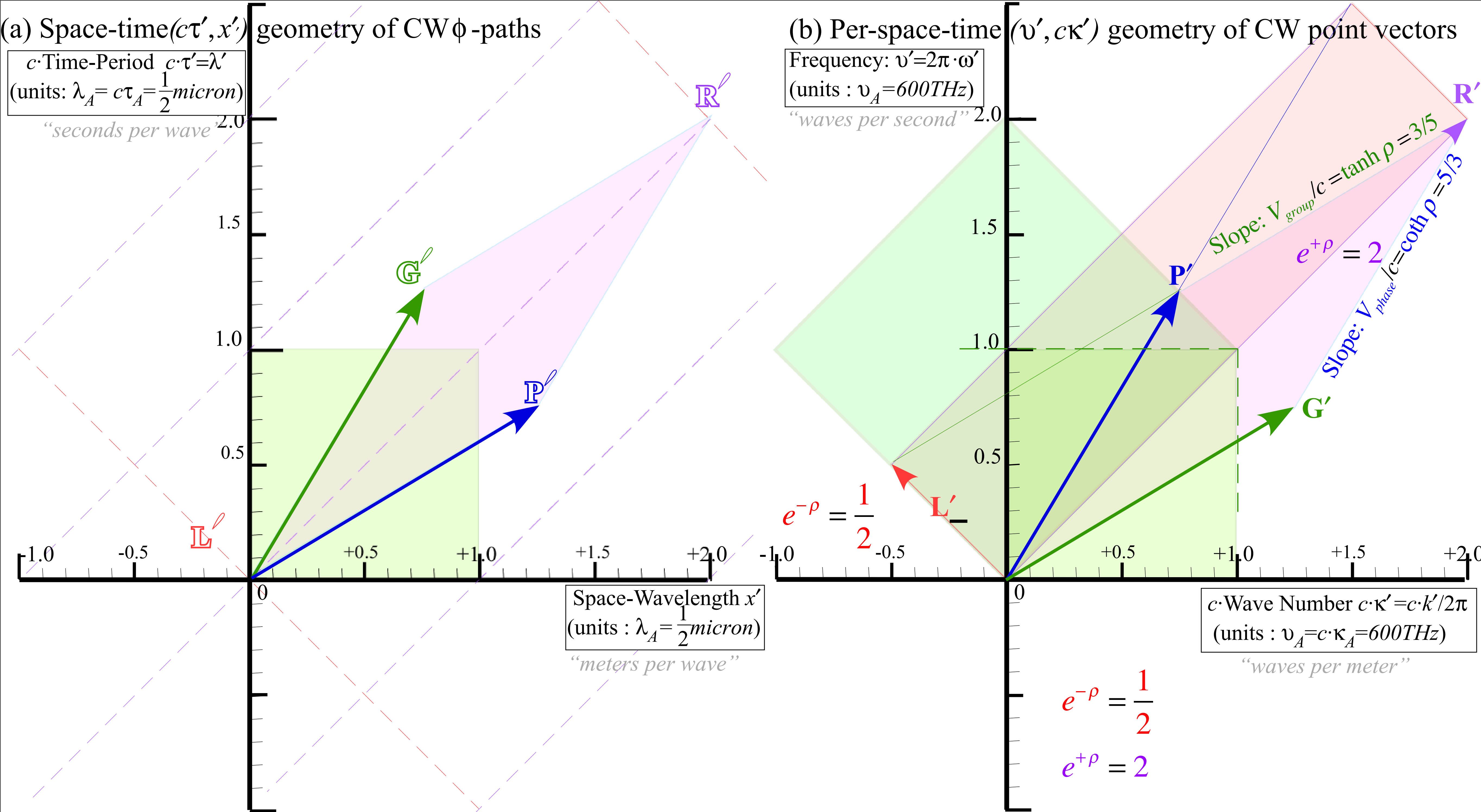

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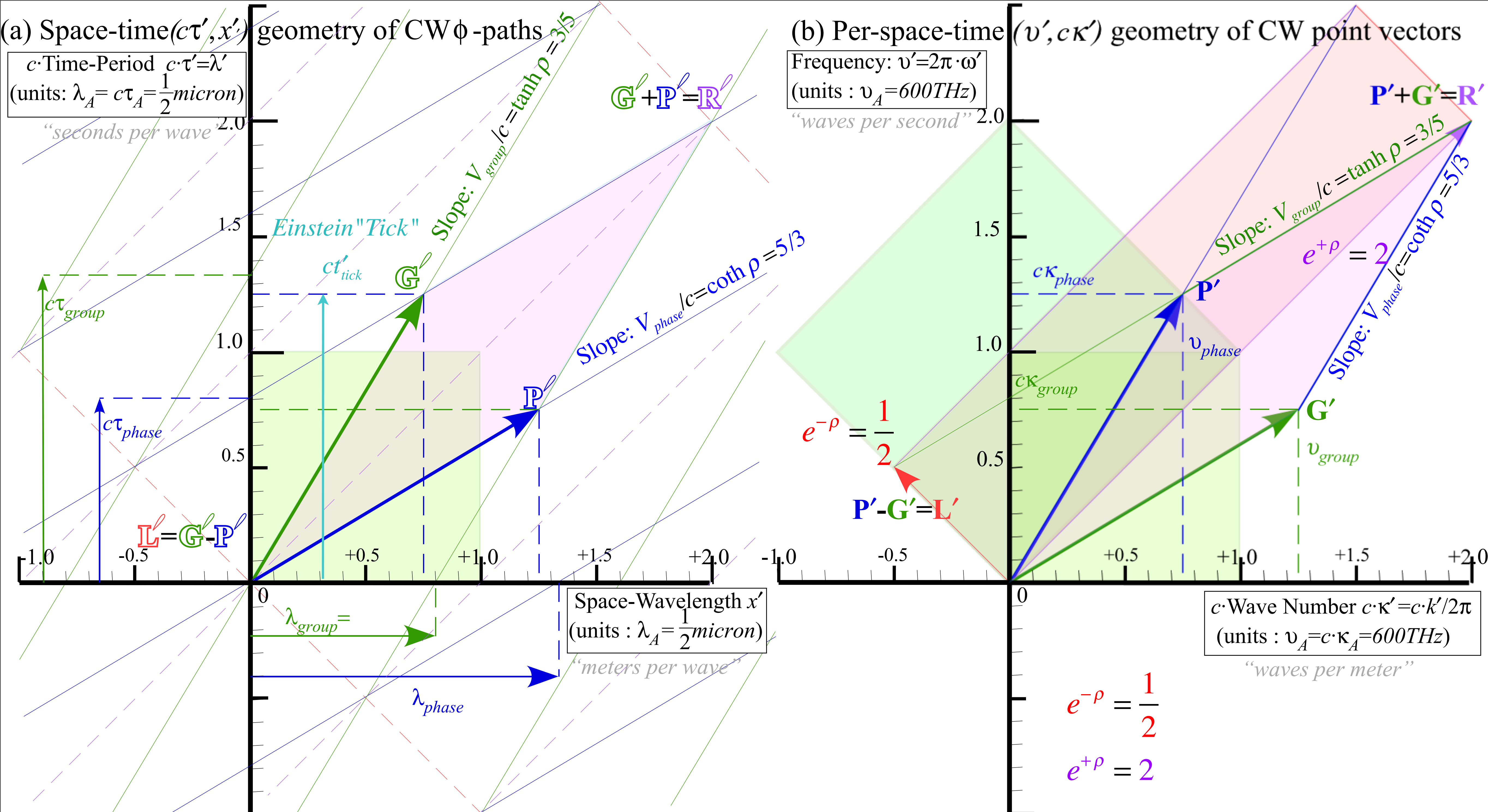

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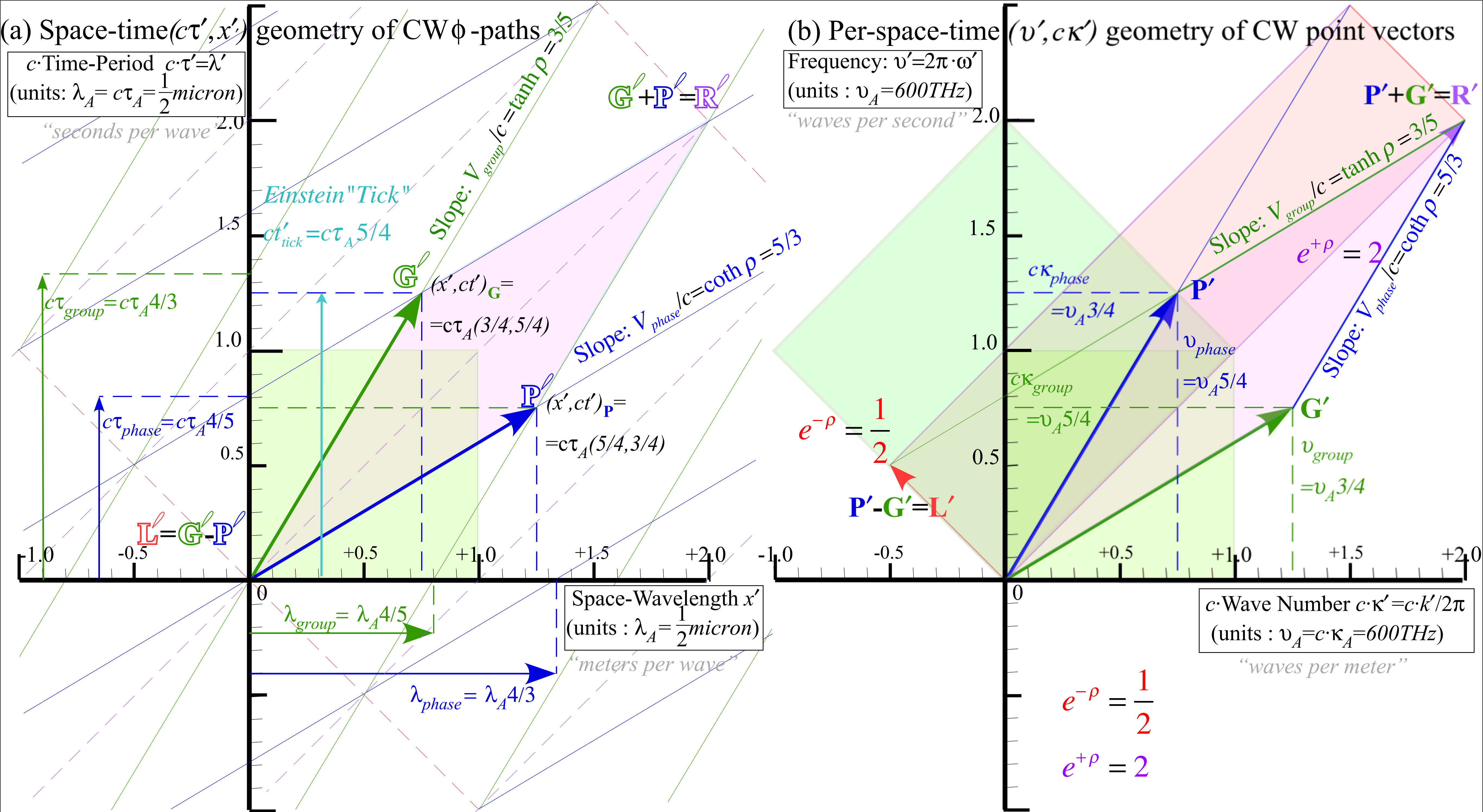

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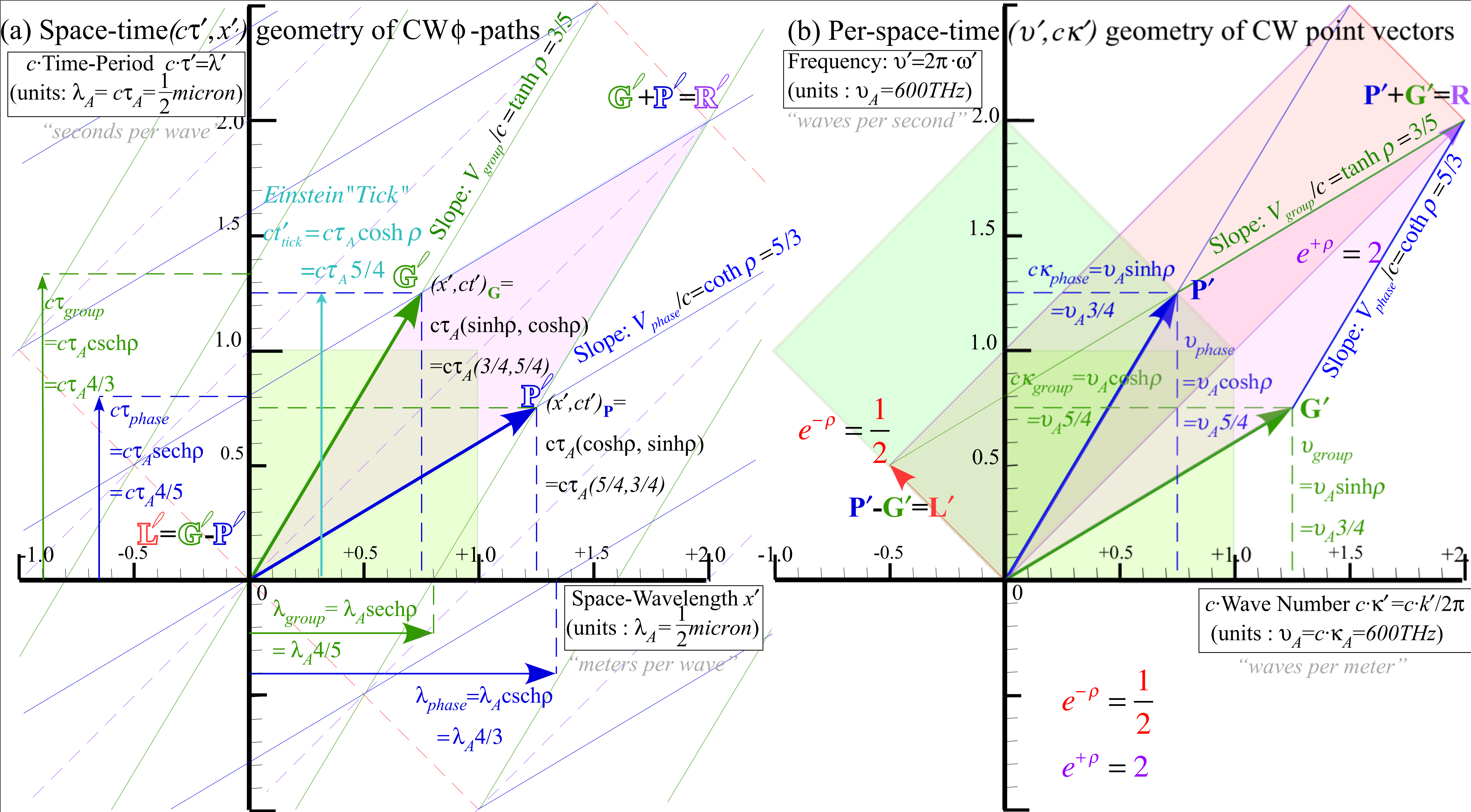

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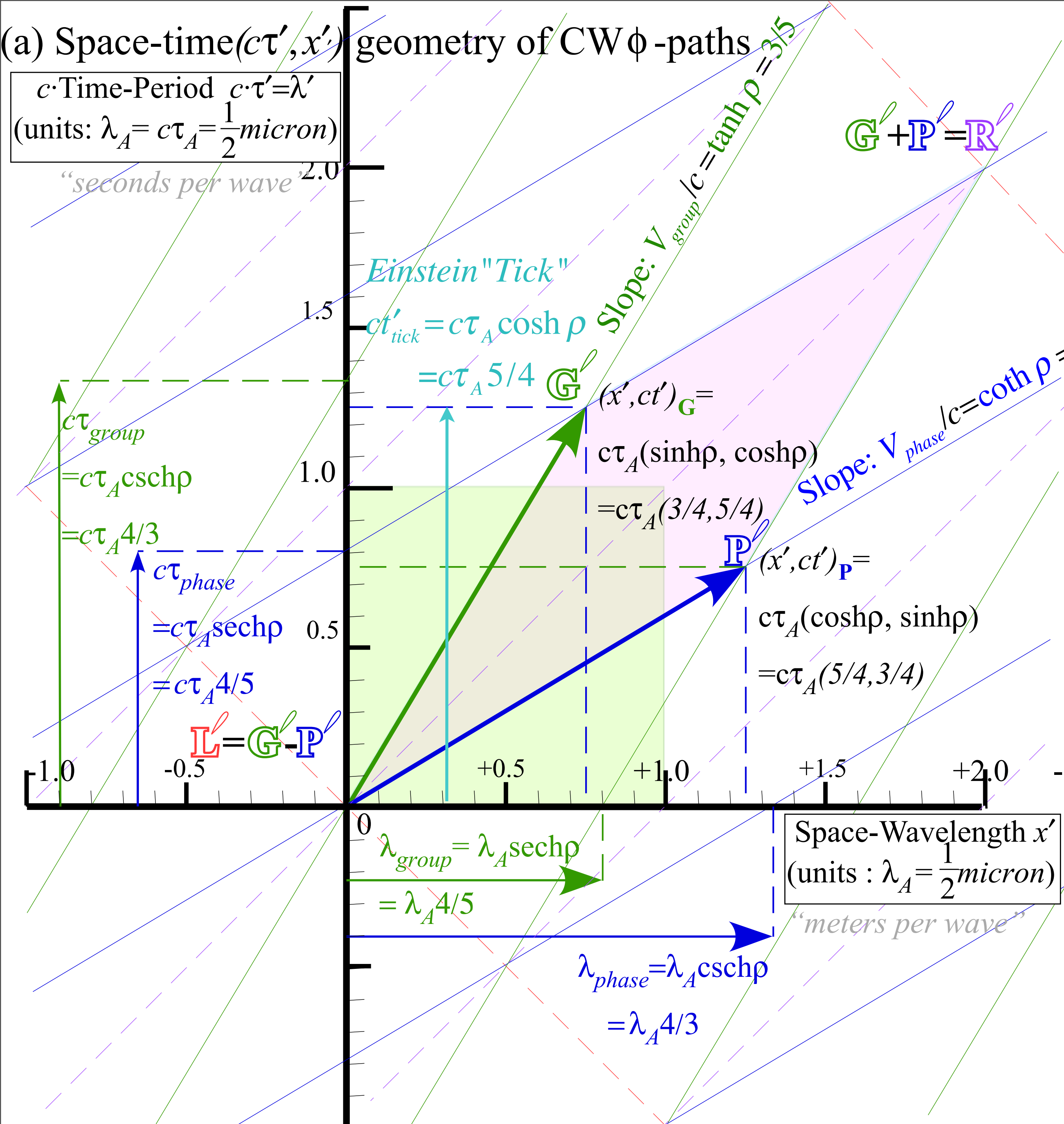

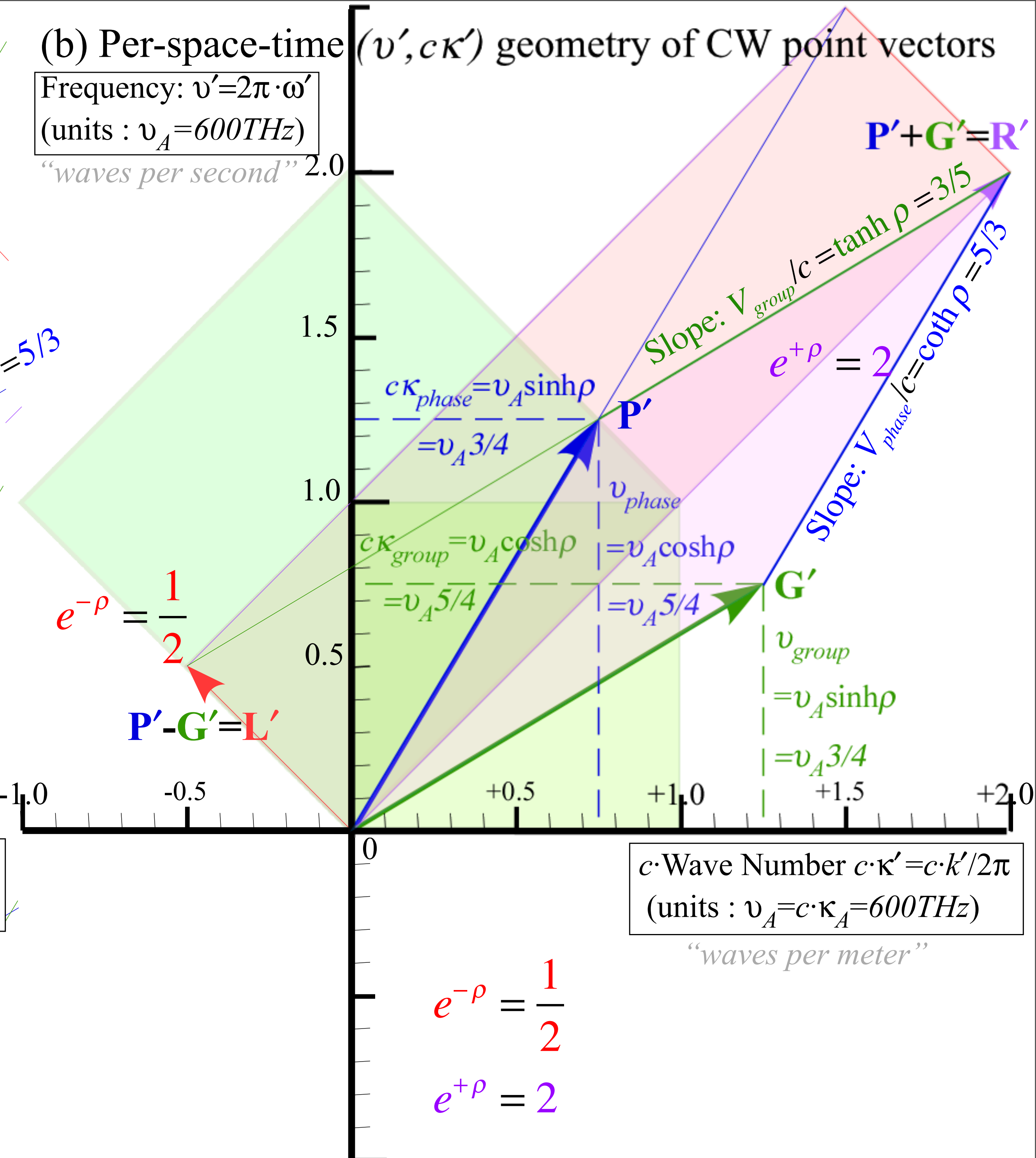

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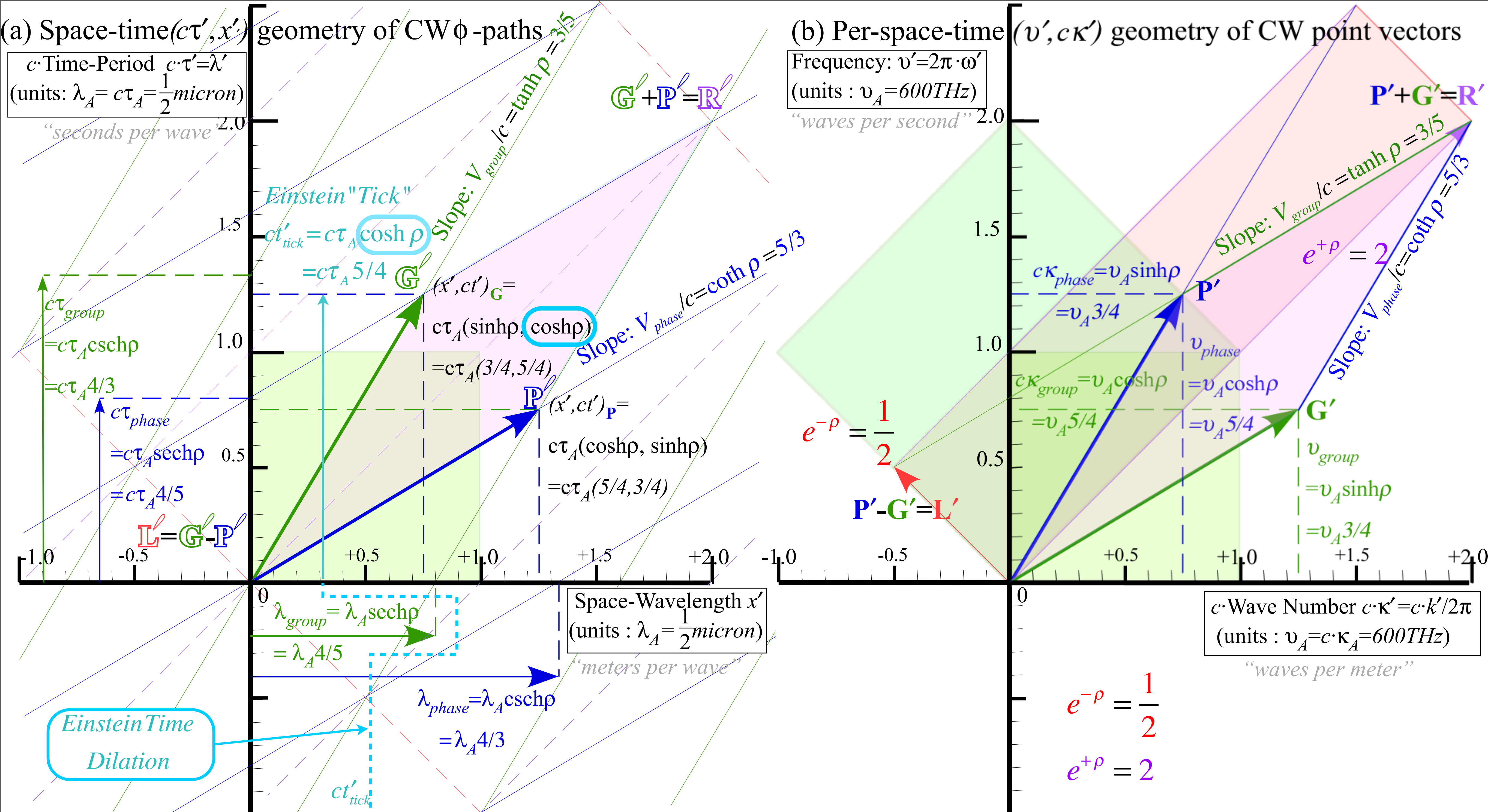

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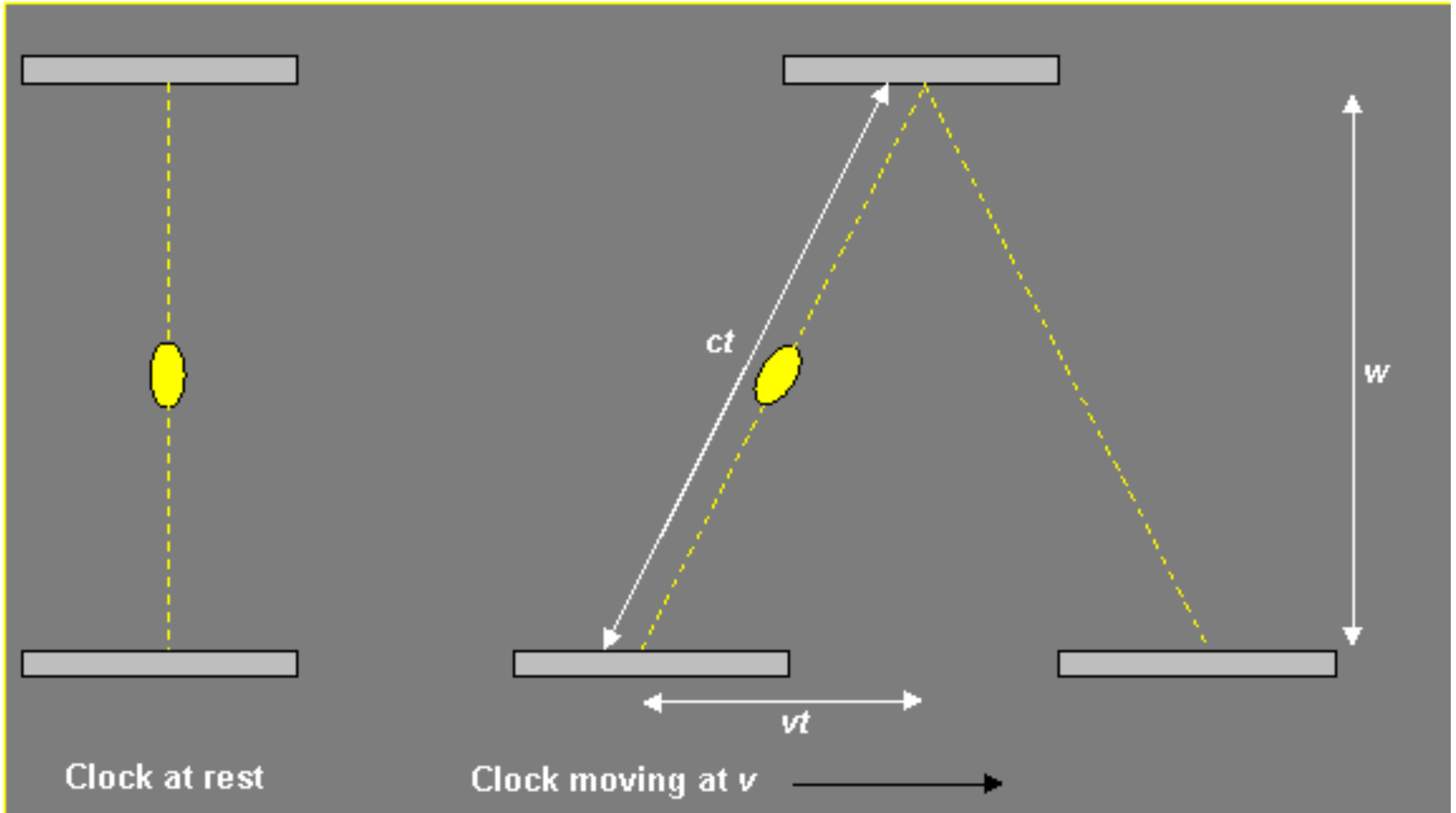
http://galileoandeinstein.physics.virginia.edu/lectures/srelwhat.html



Two identical light clocks: one at rest, one moving relative to us. The light blips in both travel at the same speed *relative to us*, the one in the moving clock goes further, so must take longer between clicks.

t is the time from one mirror to the other.

$$c^2t^2 = v^2t^2 + w^2$$

$$t^2(c^2 - v^2) = w^2$$

time between clicks for Jill's clock to be:

$$t^2(1 - v^2/c^2) = w^2/c^2$$

$$\text{time between clicks for moving clock} = \frac{2w}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$

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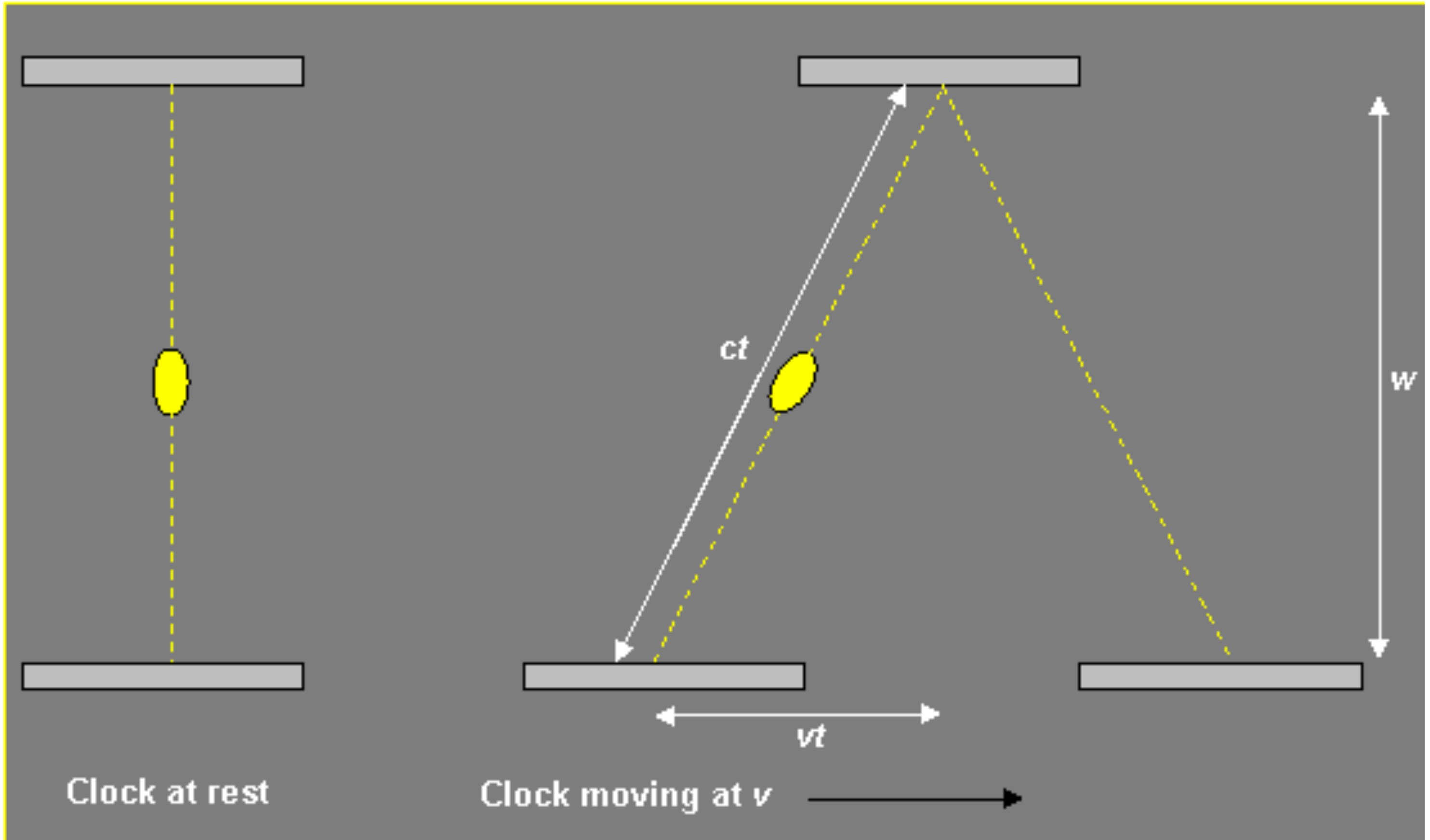

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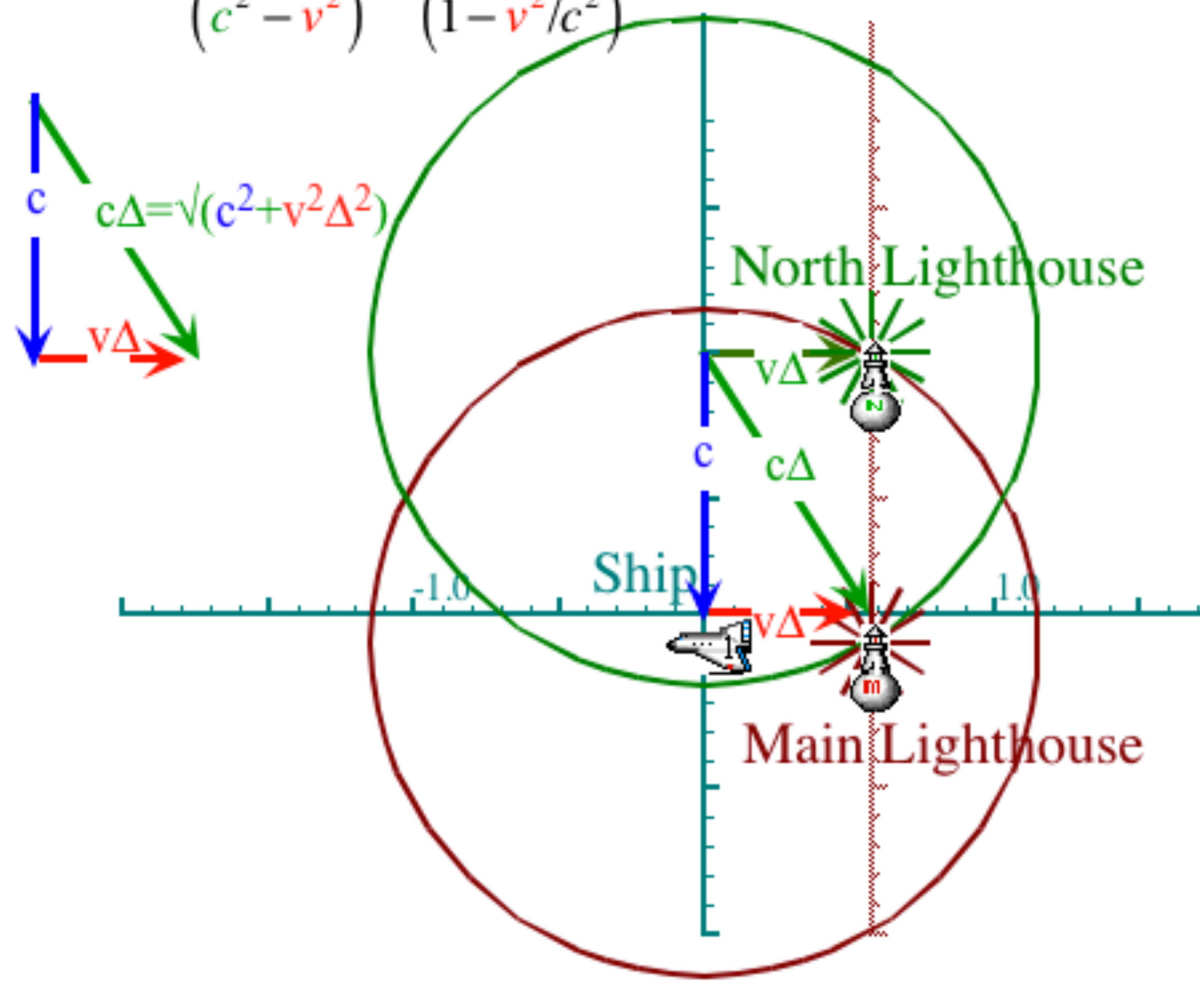
Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho = 1.15$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$

$$\Delta^2 = \frac{c^2}{\left(c^2 - v^2\right)} = \frac{1}{\left(1 - v^2/c^2\right)}$$



For u/c=1/2

$\Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15$

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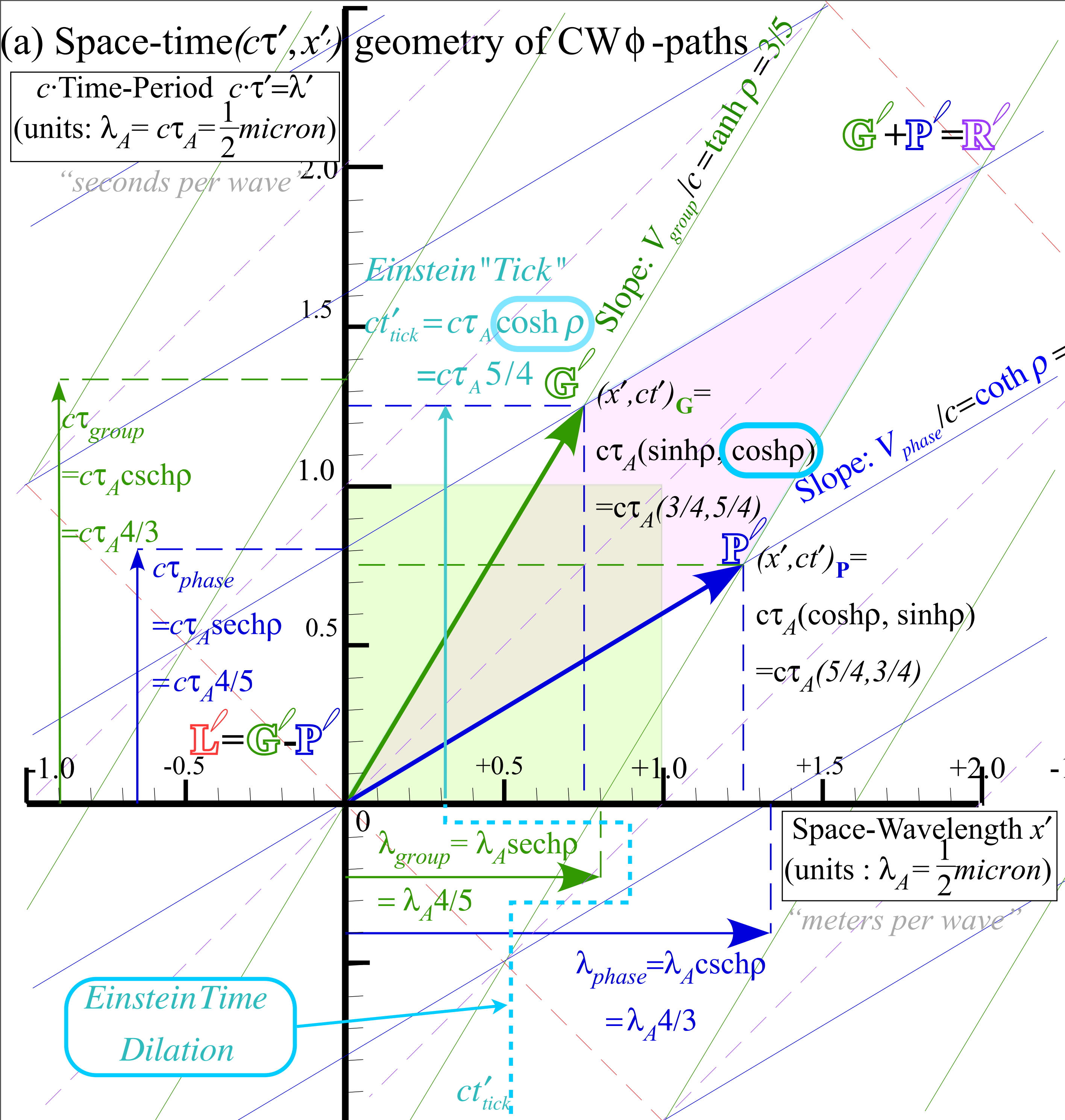

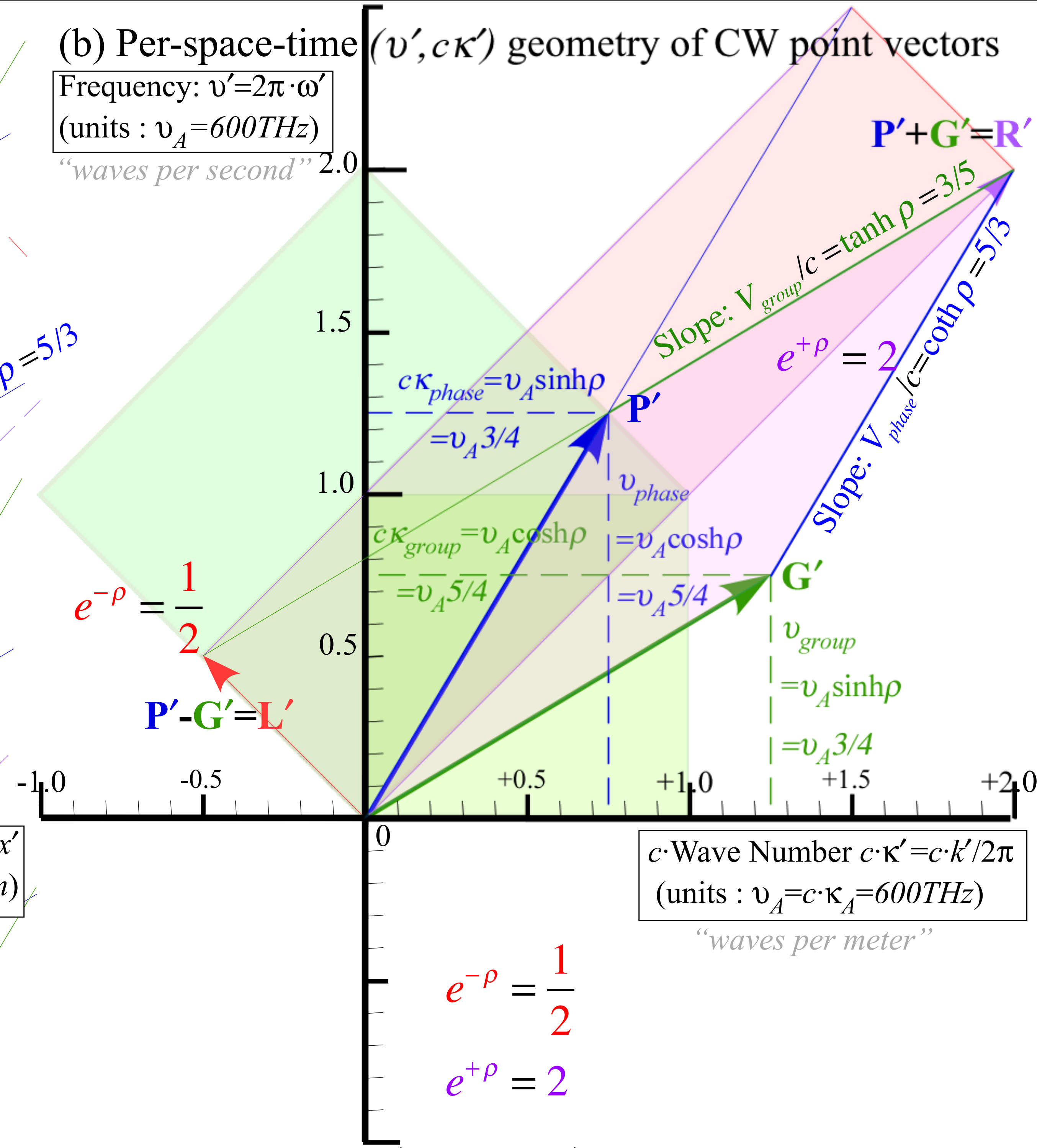

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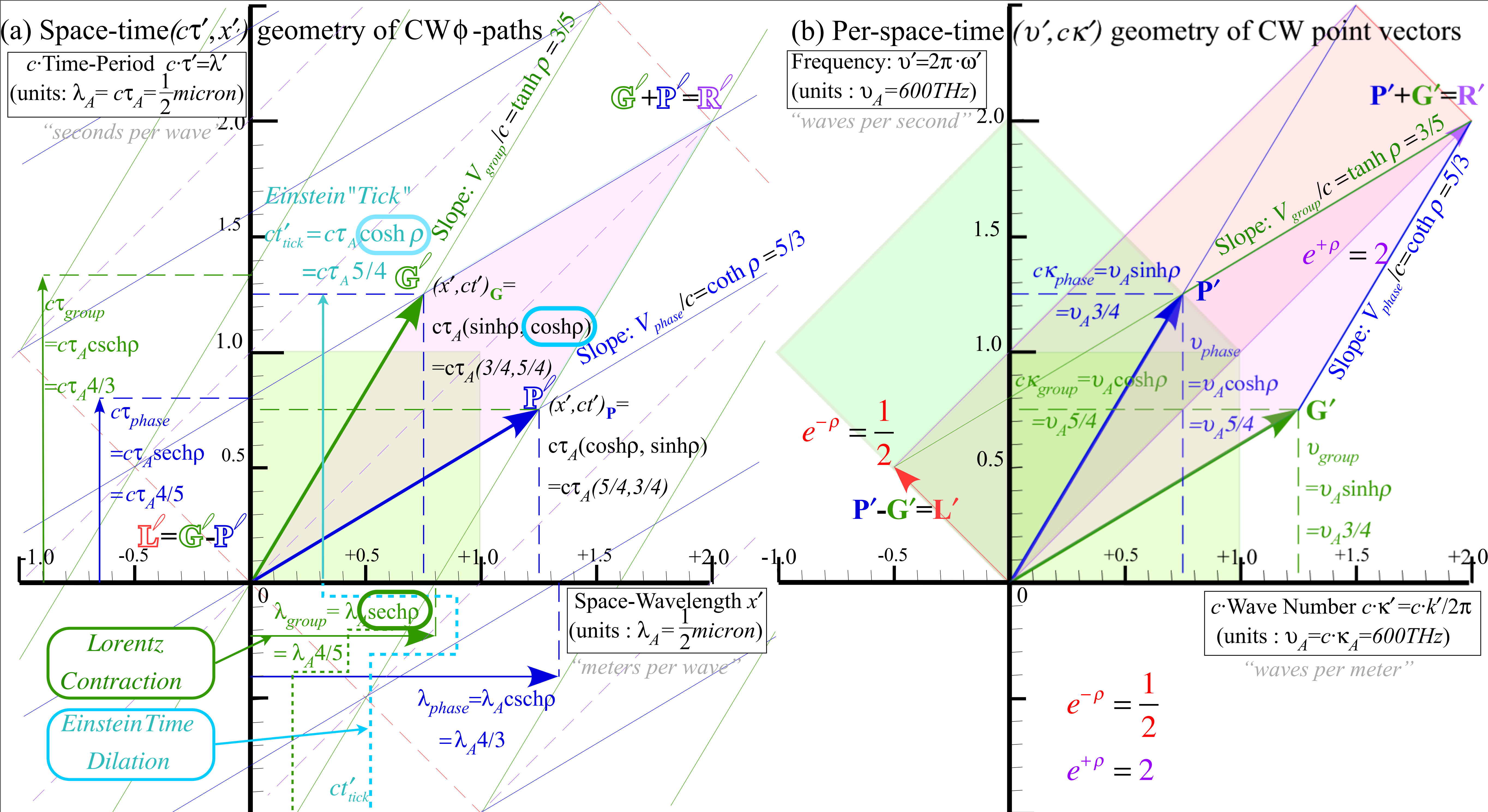

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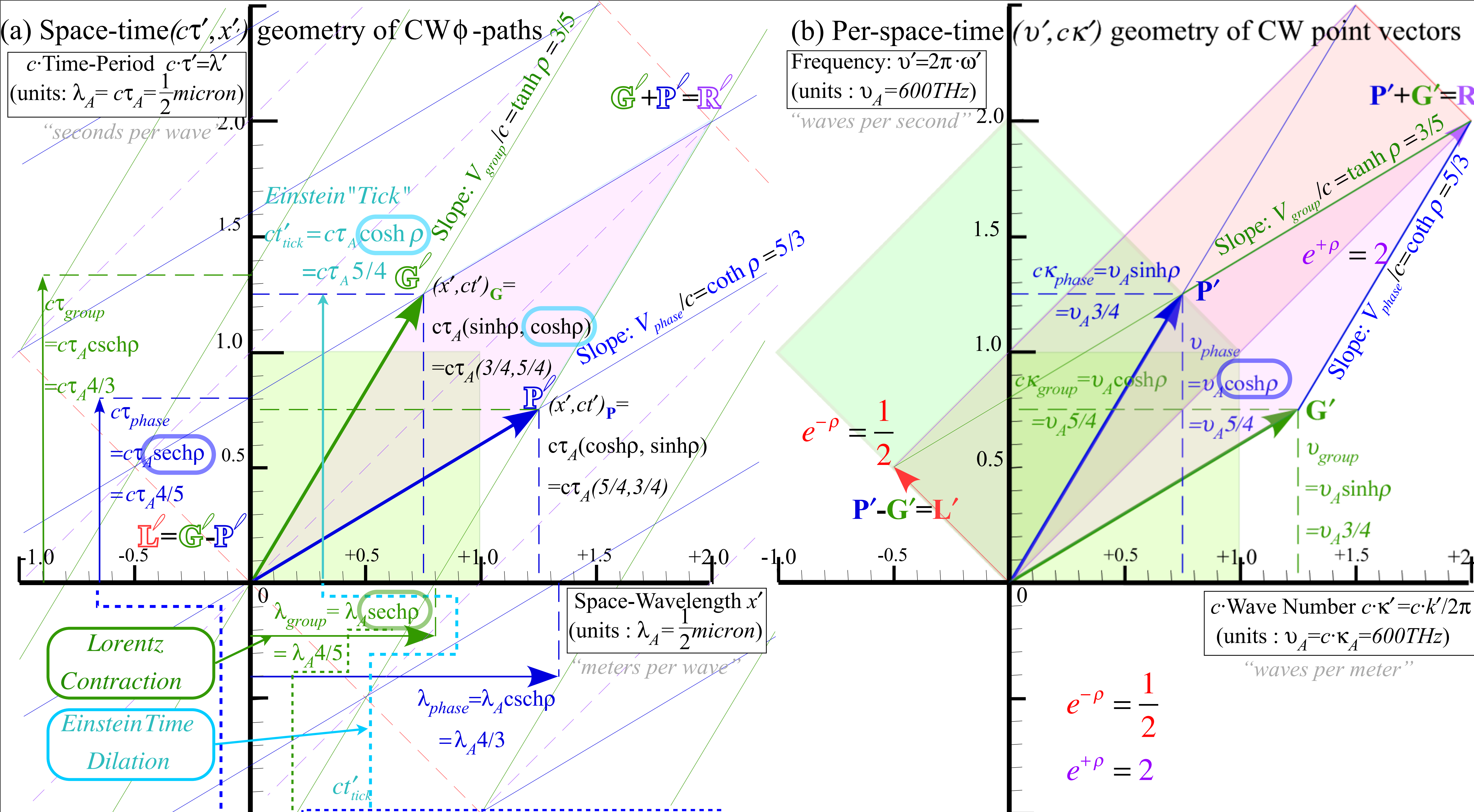

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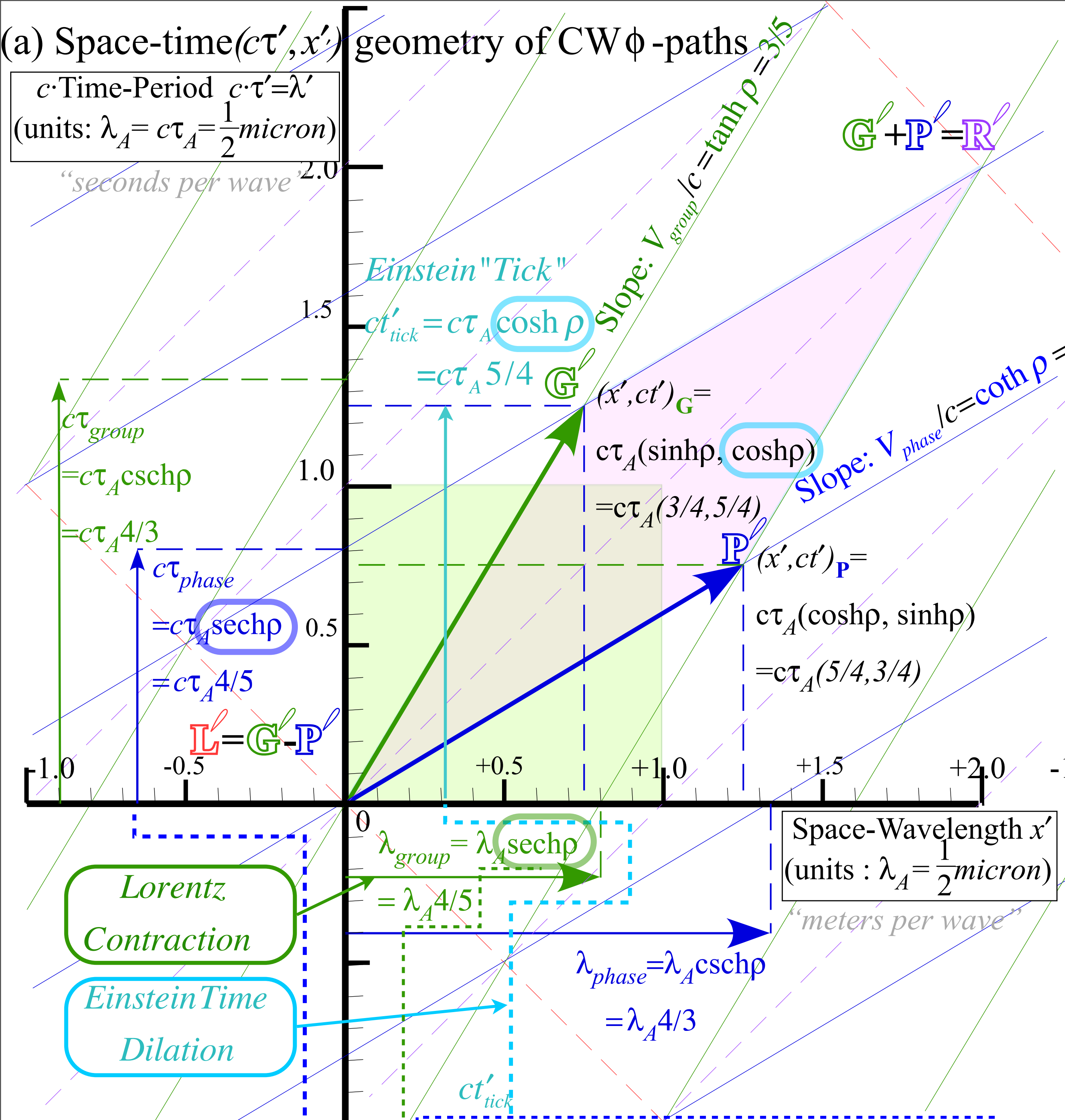

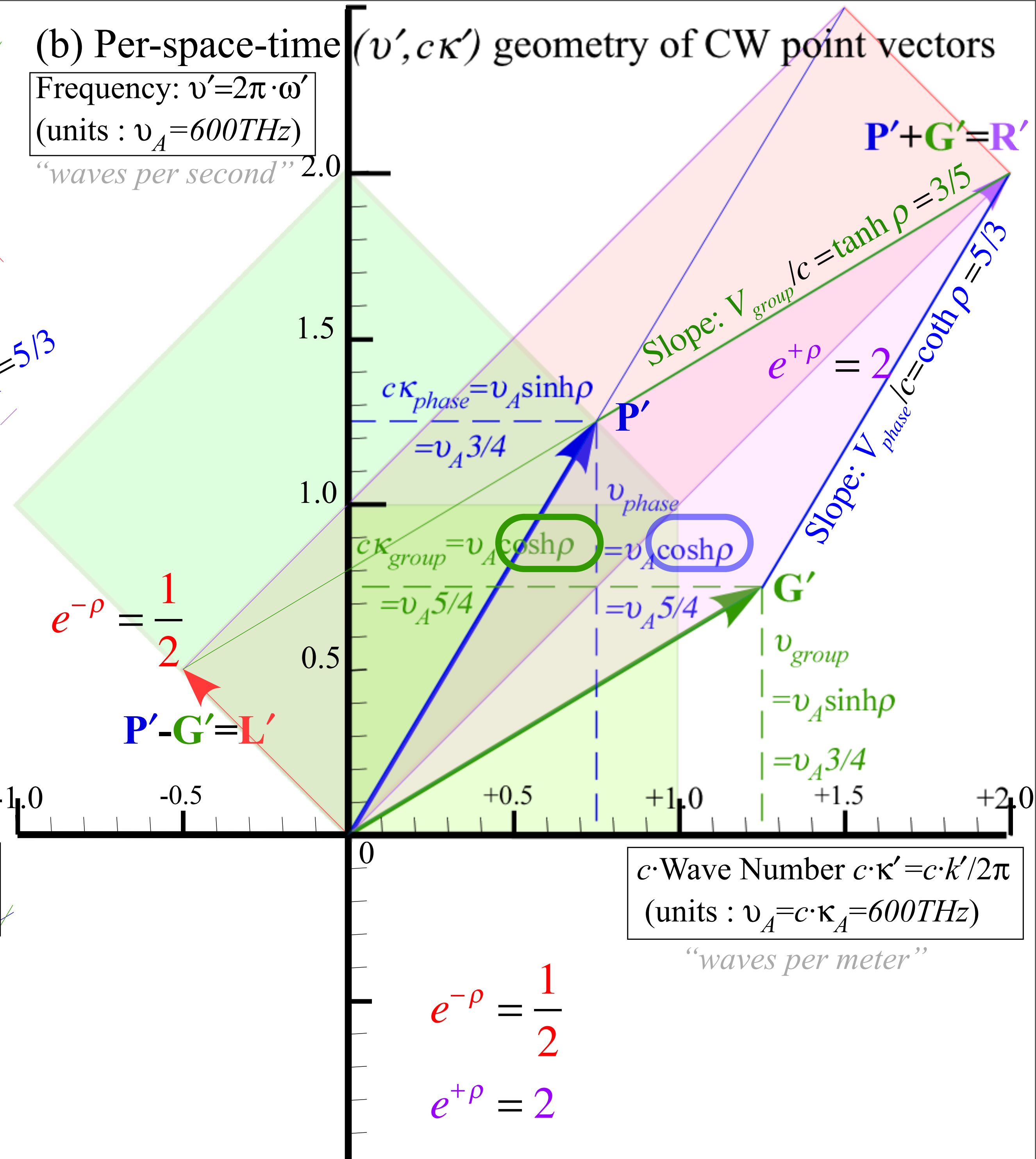

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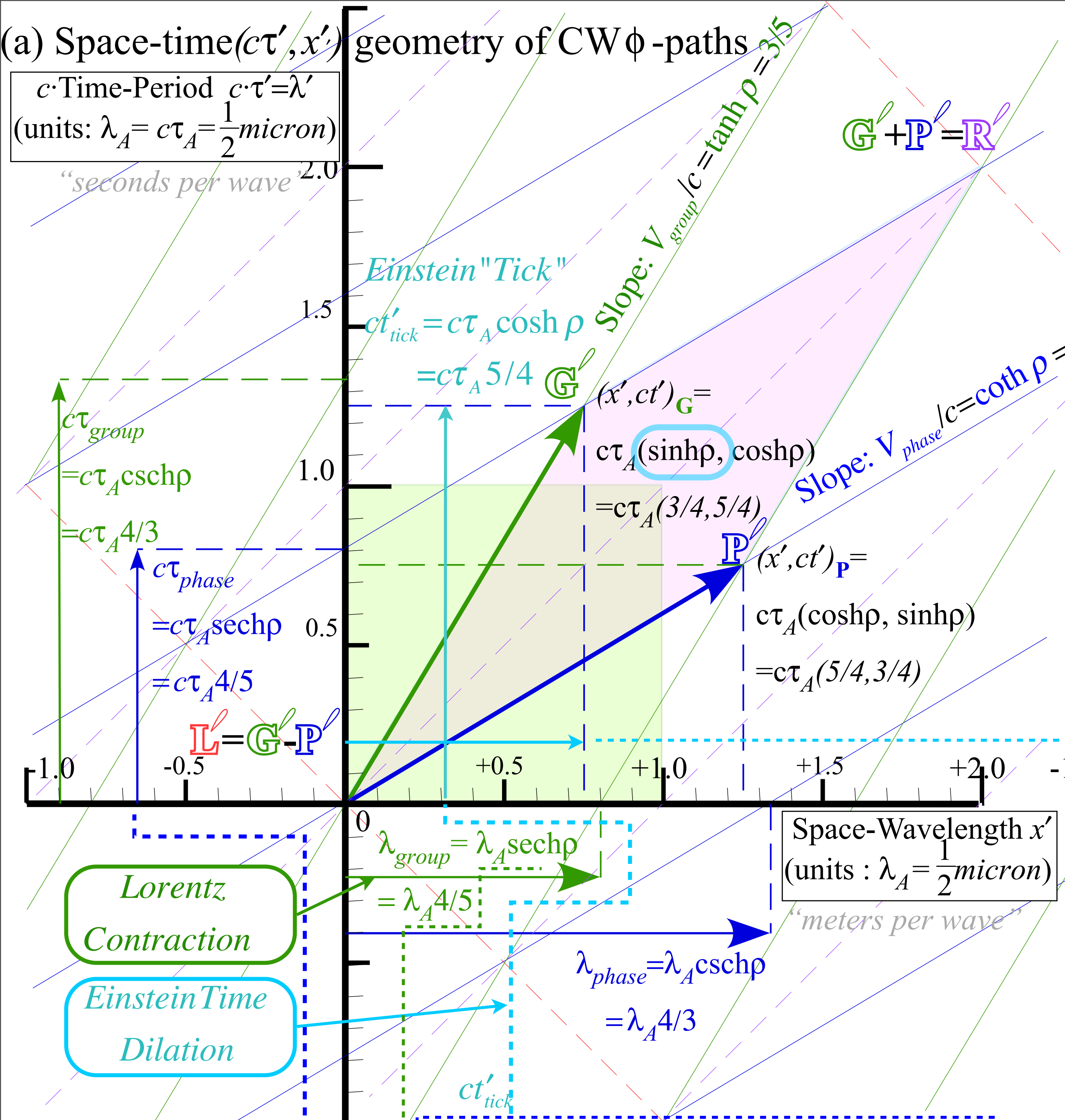

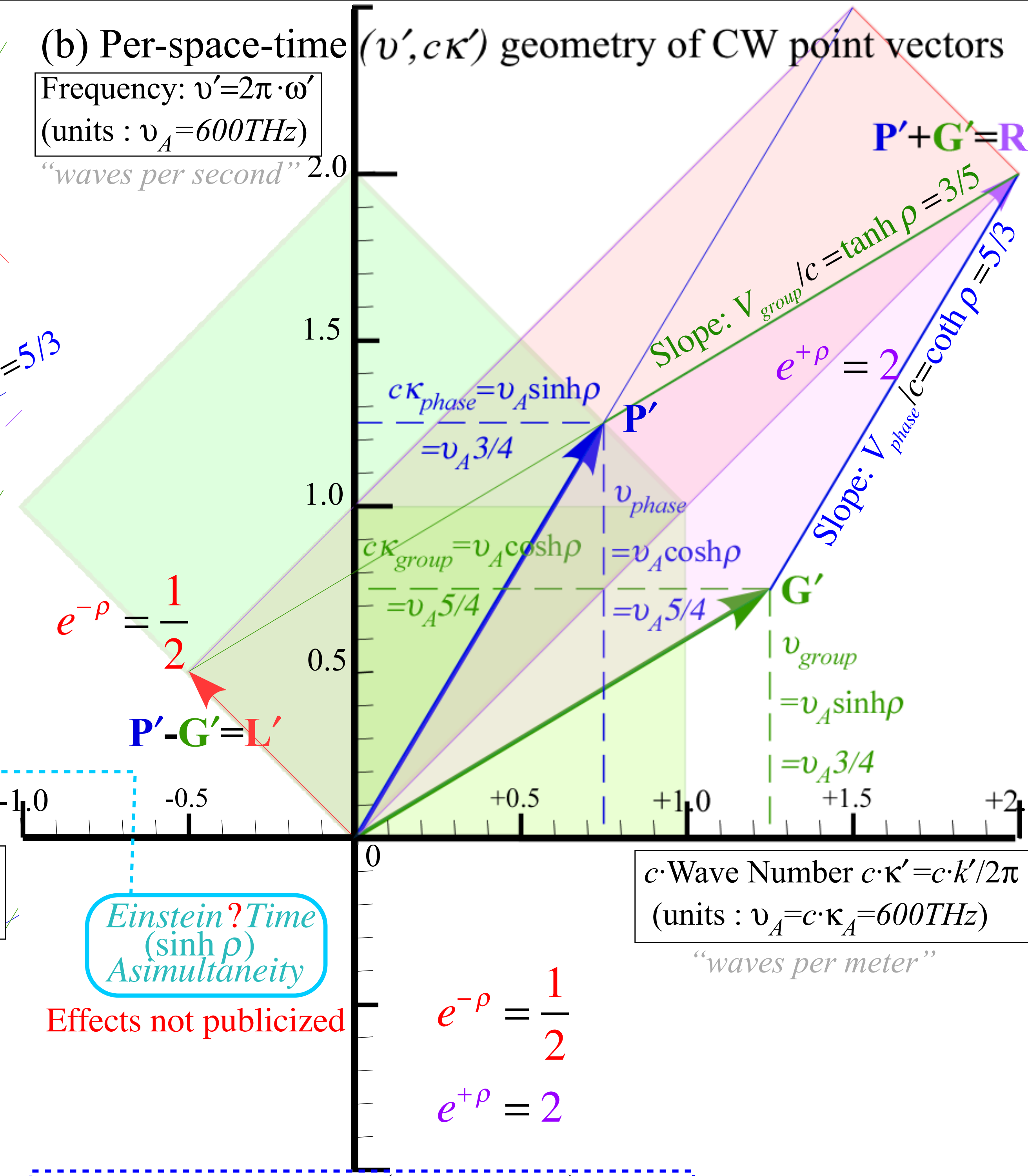

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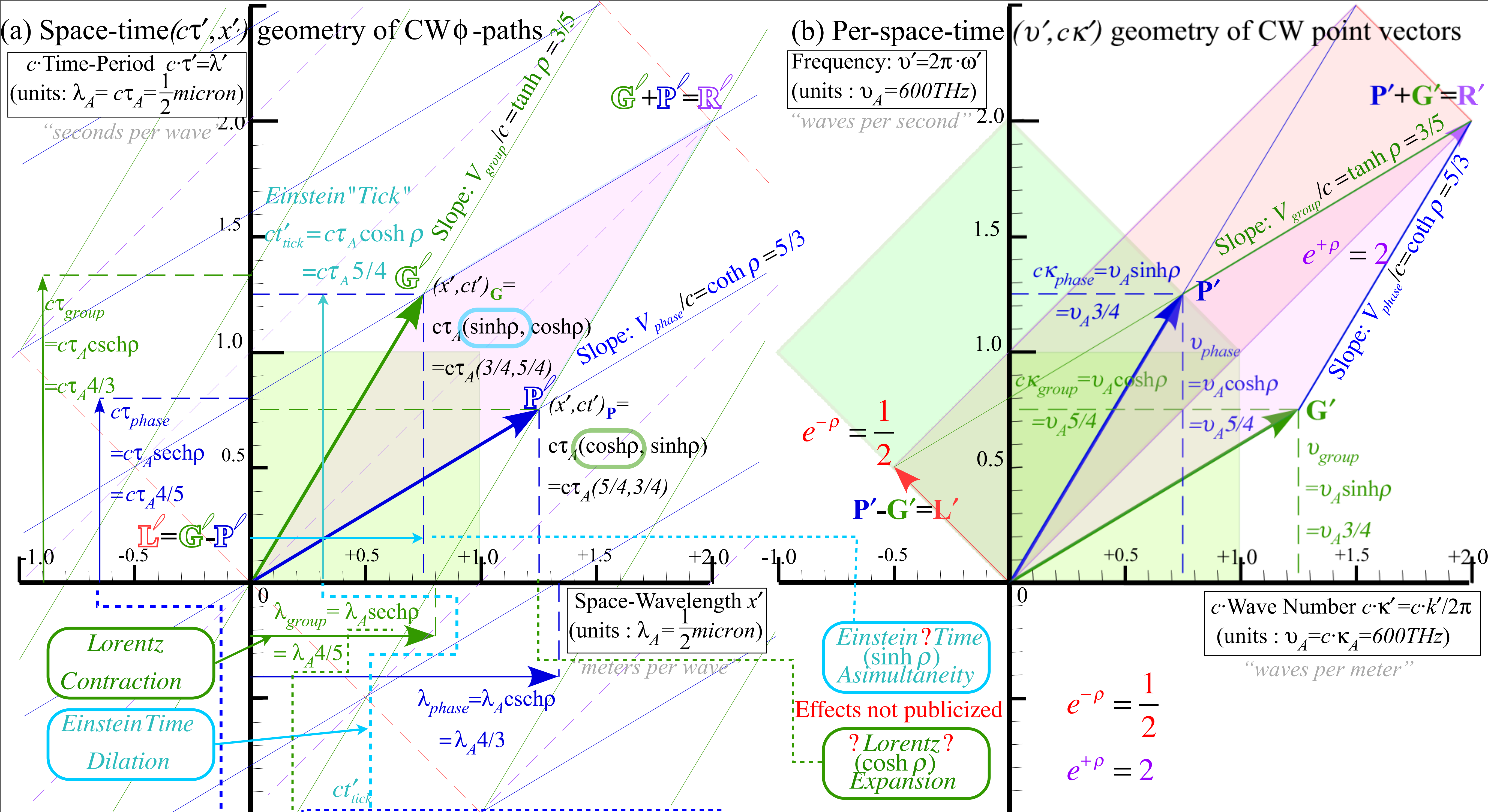
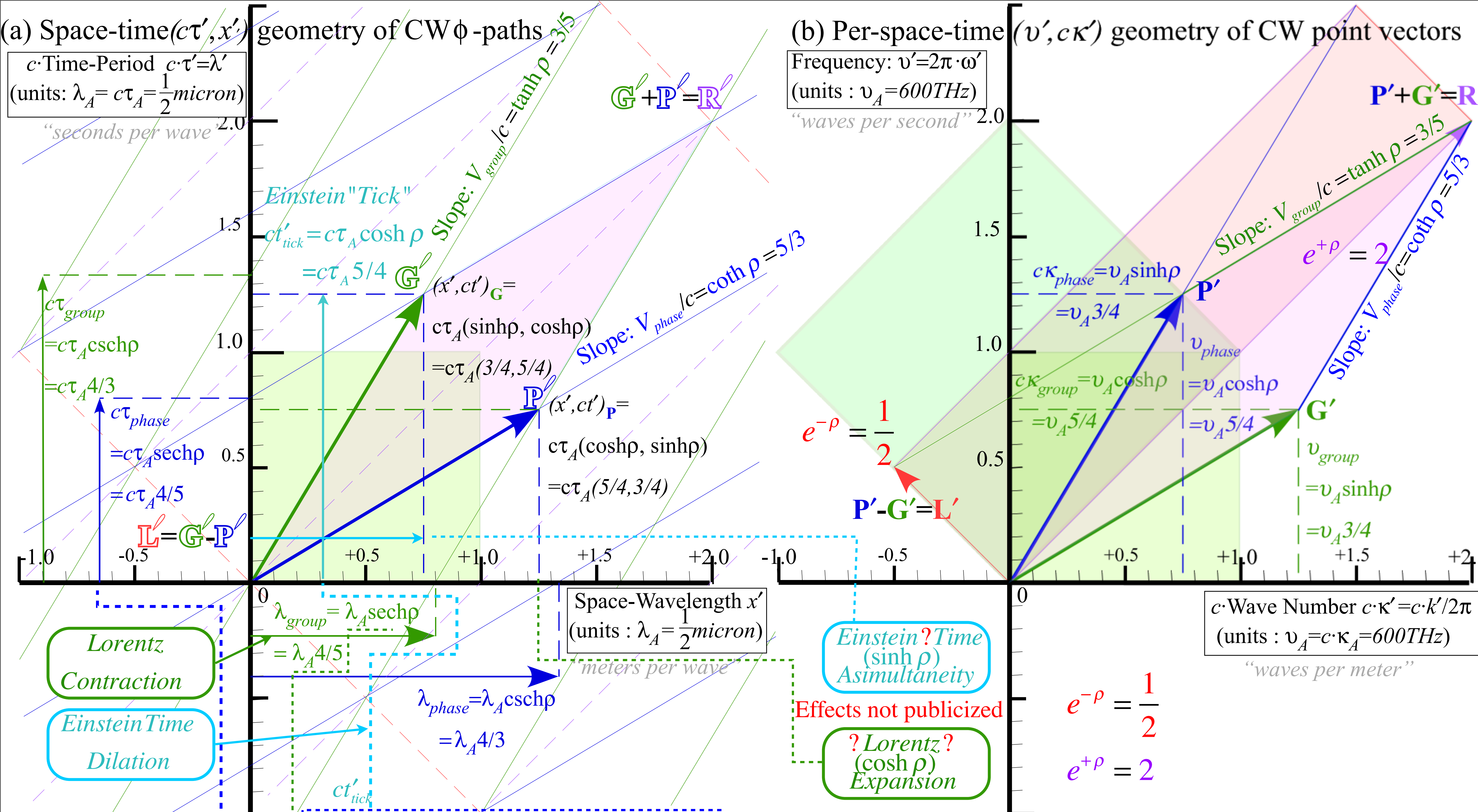


time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

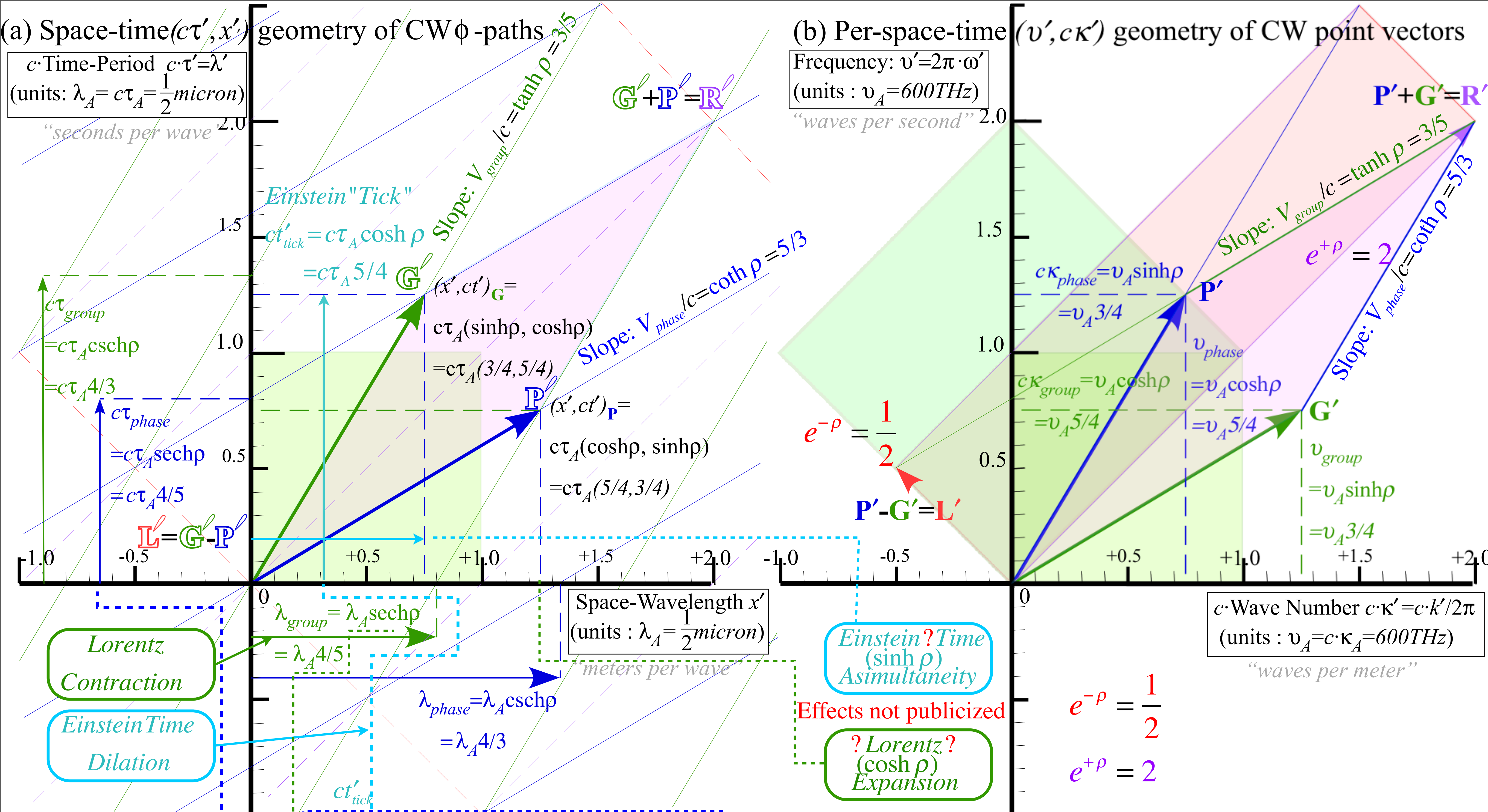
$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

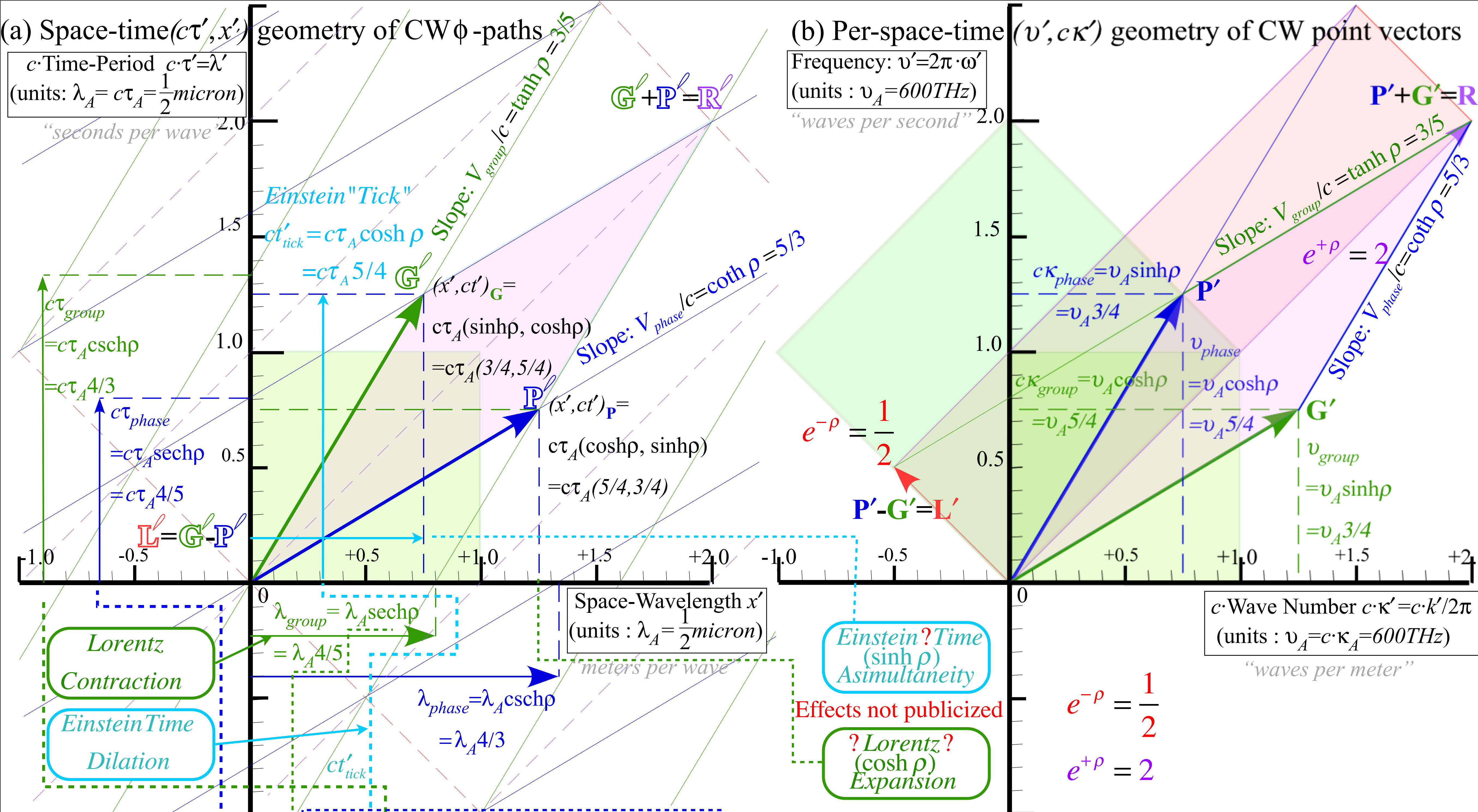
$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A\begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

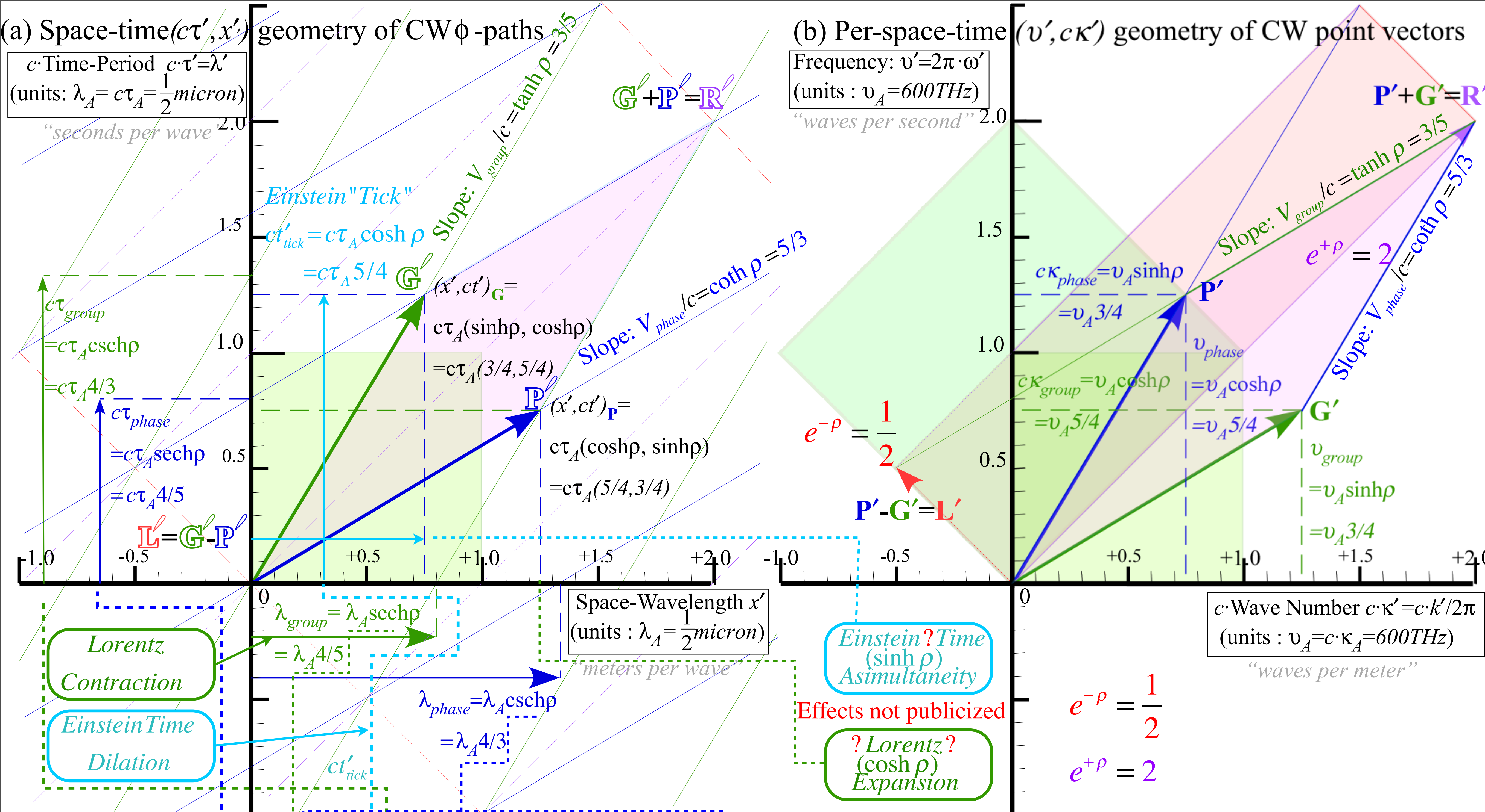
$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix}=\mathbf{G}'=\frac{\mathbf{R}'-\mathbf{L}'}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$

$$\begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}+e^{-\rho}}{2} \\ \frac{e^{+\rho}-e^{-\rho}}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}$$

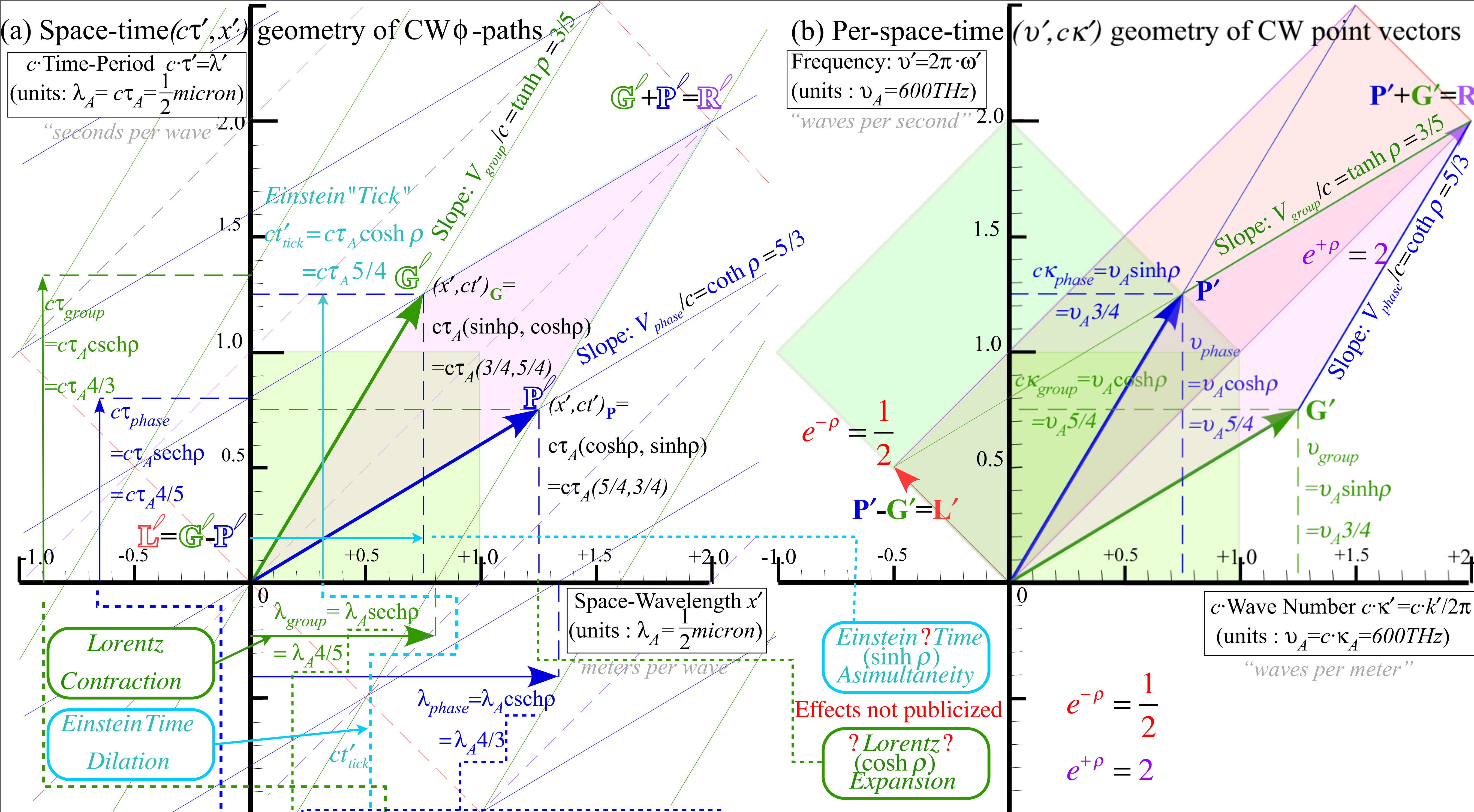
$$\begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
	$e^{-\rho}$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix}=\mathbf{P'}=\frac{\mathbf{R'}+\mathbf{L'}}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

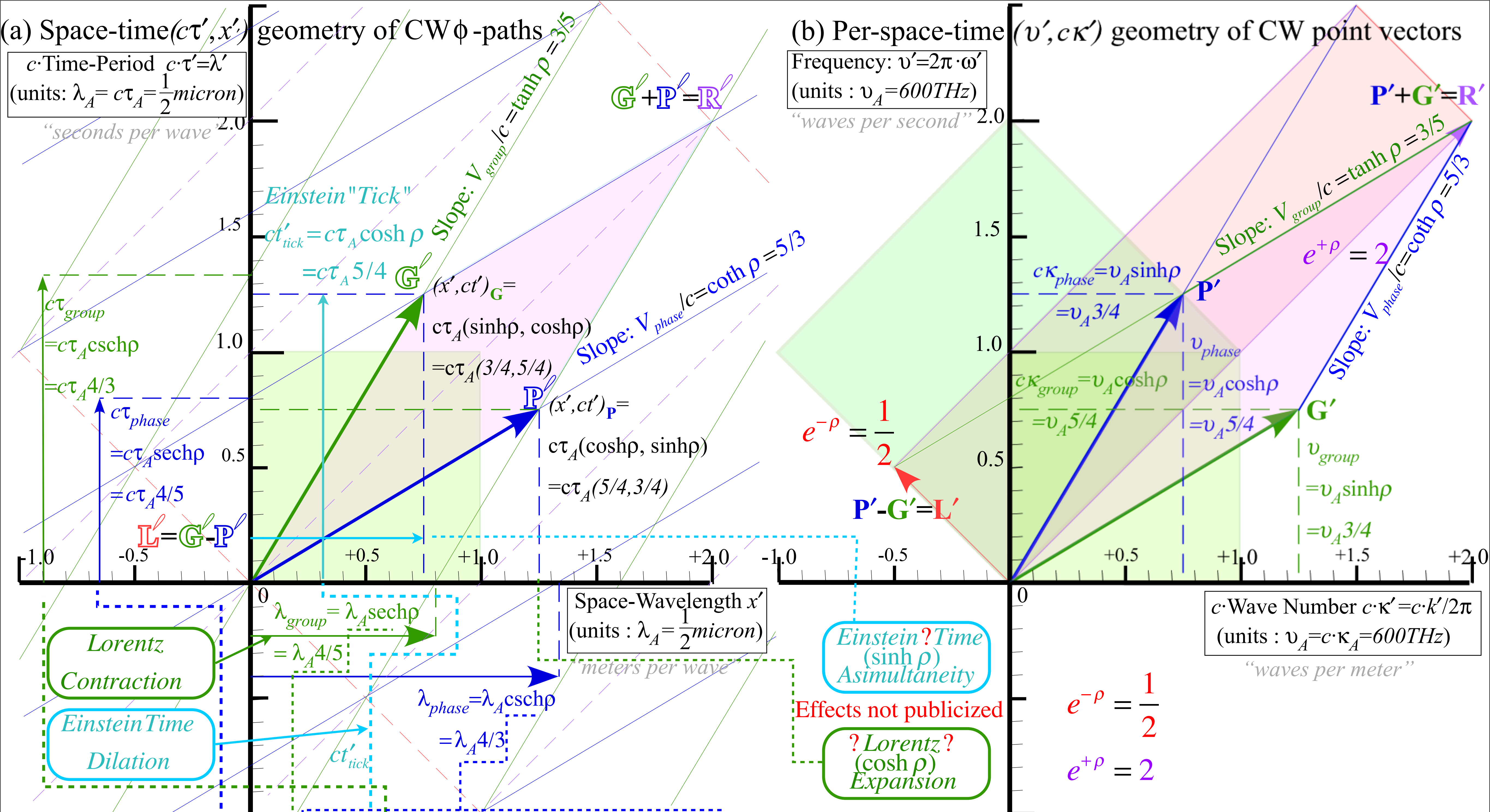
$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix}=\mathbf{G'}=\frac{\mathbf{R'}-\mathbf{L'}}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho=$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix}=\mathbf{G}'=\frac{\mathbf{R}'-\mathbf{L}'}{2}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			

$$\begin{pmatrix}\upsilon'_{phase}\\ c\kappa'_{phase}\end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

$$\begin{pmatrix}\upsilon'_{group}\\ c\kappa'_{group}\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A\begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix} = \upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$

Space-time (x,ct) and per-space-time (ω,ck) geometry and its physics

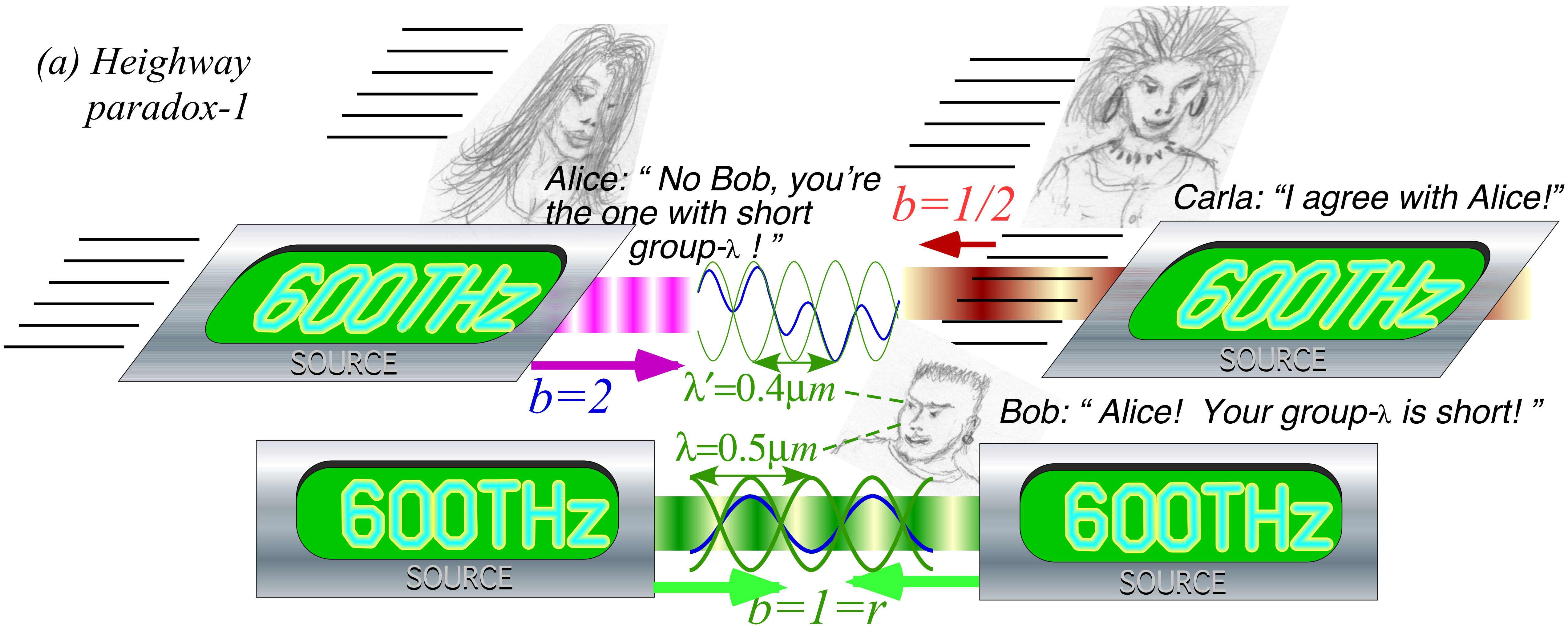
<u>All</u> of those contraction and expansion coefficients

Detailed views Einstein time dilation

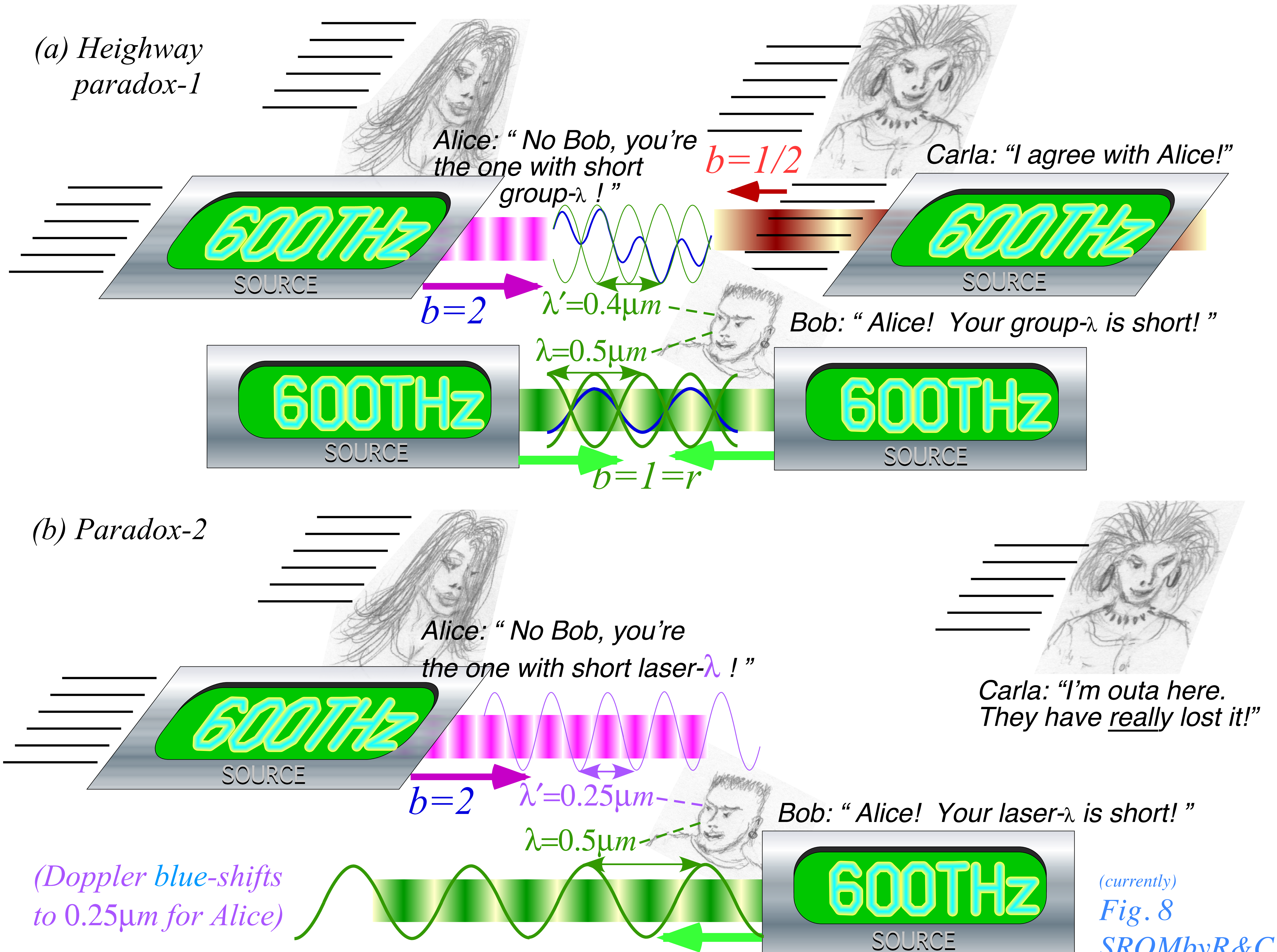
The old "smoke and mirrors" trick

Detailed views Lorentz contraction

Heighway's paradox 1 and 2



(currently part of)
Fig. 8
SRQMbyR&C



(currently) *Fig. 8* *SRQMbyR&C*

Phase invariance used to derive $(x,ct) \leftrightarrow (x',ct')$ Einstein Lorentz Transformations (ELT)

A. Transformations and phase invariance

Key points in SRQMbyR&C

A laser phasor sketched in Fig.4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ) in Fig.7a.

A. Transformations and phase invariance

Very key point in SRQMbyR&C

A laser phasor sketched in Fig.4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ) in Fig.7a. A time-stamp reading of phase ϕ at a particular space-time point should be equal for Alice and Bob in spite of having unequal readings (x, t) and (x', t') for that point and unequal frequency-wavevector readings (ω, k) and (ω', k') for a laser group-wave or its phase-wave.

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$$\phi'_{phase} = k'_{phase}x' - \omega'_{phase}t' = k_{phase}x - \omega_{phase}t \equiv \phi_{phase}$$

Key point holds for Any phase

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$$\phi'_{phase} = k'_{phase} x' - \omega'_{phase} t' = k_{phase} x - \omega_{phase} t \equiv \phi_{phase}$$

$$\phi'_{group} = k'_{group} x' - \omega'_{group} t' = k_{group} x - \omega_{group} t \equiv \phi_{group}$$

*Key point holds for **Any** phase*

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$$\phi'_{group} = k'_{group}x' - \omega'_{group}t' = k_{group}x - \omega_{group}t \equiv \phi_{group}$$

Key point holds for __Any__ phase

Bob's (ω', k') components are in (14) and (15). Alice's (ω, k) are the same with $\rho = 0$. An Einstein-Lorentz Transformation (ELT) of Bob's (x', t') to Alice's (x, t) follows.

$$\phi'_{phase} = x'\frac{\omega_A}{c}\sinh\rho - t'\omega_A\cosh\rho = 0\cdot x - \omega_A t \quad \Rightarrow \quad ct = ct'\cosh\rho - x'\sinh\rho$$

$$\begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^{+\rho} + e^{-\rho}}{2} \\ \frac{e^{+\rho} - e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix}$$

A. Transformations and phase invariance

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$$\phi'_{group} = x'\frac{\omega_A}{c}\cosh\rho - t'\omega_A\sinh\rho = \frac{\omega_A}{c}x - 0\cdot t \quad \Rightarrow \quad x = -ct'\sinh\rho + x'\cosh\rho$$

$$\begin{pmatrix}\omega'_{phase}\\ ck'_{phase}\end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \omega_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix} = \omega_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}$$

$$\begin{pmatrix}\omega'_{group}\\ ck'_{group}\end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \omega_A\begin{pmatrix}\frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix} = \omega_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}$$

A. Transformations and phase invariance

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Key point holds for Any phase

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$$\phi'_{group} = x'\frac{\omega_A}{c}\cosh\rho - t'\omega_A\sinh\rho = \frac{\omega_A}{c}x - 0\cdot t \quad\Rightarrow\quad x = -ct'\sinh\rho + x'\cosh\rho$$

The ELT matrix form and its inverse complete the space-time side of Fig.7.

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} ct' \\ x' \end{pmatrix} \Rightarrow \begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\rho & +\sinh\rho \\ +\sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} ct \\ x \end{pmatrix} \qquad (22)$$

A. Transformations and phase invariance

Very key point in SRQMbyR&C

A laser phasor sketched in Fig.4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ) in Fig.7a.

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Key point holds for Any phase

The ELT matrix form and its inverse complete the space-time side of Fig.7.

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix} \Rightarrow \begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\rho & +\sinh\rho \\ +\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \quad (22)$$

Direct derivation of ELT uses base vectors $\mathbb{P}'$ and $\mathbb{G}'$ or $\mathbf{P}'$ and $\mathbf{G}'$ in (14) and (15).

$$\mathbf{P}' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \begin{pmatrix} \omega_A \\ 0 \end{pmatrix} \cosh\rho + \begin{pmatrix} 0 \\ \omega_A \end{pmatrix} \sinh\rho = \mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho \quad (23)$$

$$\mathbf{G}' = \begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \begin{pmatrix} \omega_A \\ 0 \end{pmatrix} \sinh\rho + \begin{pmatrix} 0 \\ \omega_A \end{pmatrix} \cosh\rho = \mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho \quad (24)$$

Introducing the stellar aberration angle σ vs. rapidity ρ

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Trigonometry: From circular to hyperbolic and back

Group vs. phase velocity and tangent contacts

Introducing the stellar aberration angle σ vs. rapidity ρ

Together, rapidity $\rho = \ln b$ and stellar aberration angle σ are parameters of relative velocity

The rapidity $\rho = \ln b$ is based on longitudinal wave Doppler shift $b = e^{\rho}$ defined by $u/c = \tanh(\rho)$.
At low speed: $u/c \sim \rho$.

The stellar aberration angle σ is based on the transverse wave rotation $R = e^{i\sigma}$ defined by $u/c = \sin(\sigma)$.
At low speed: $u/c \sim \sigma$.

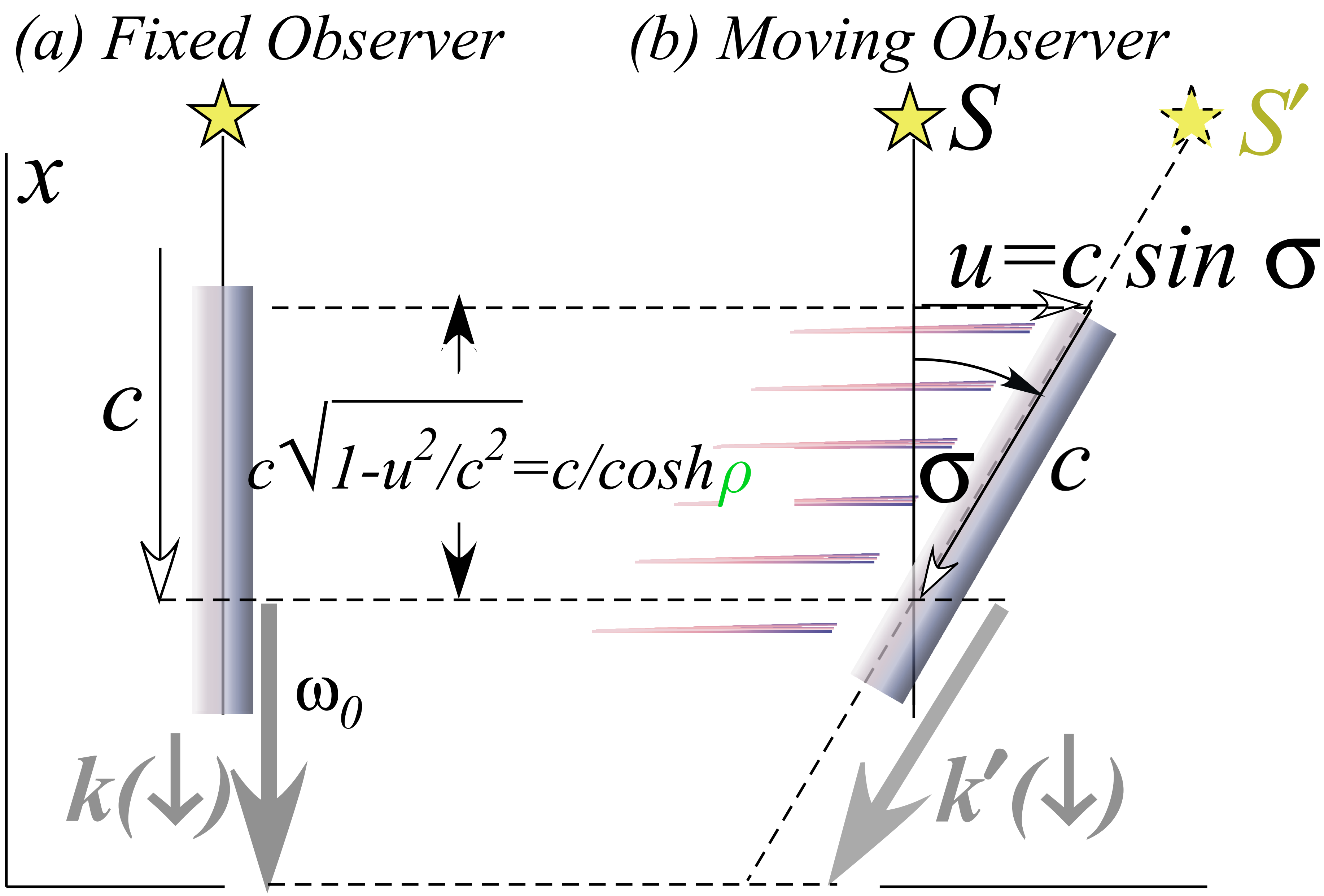
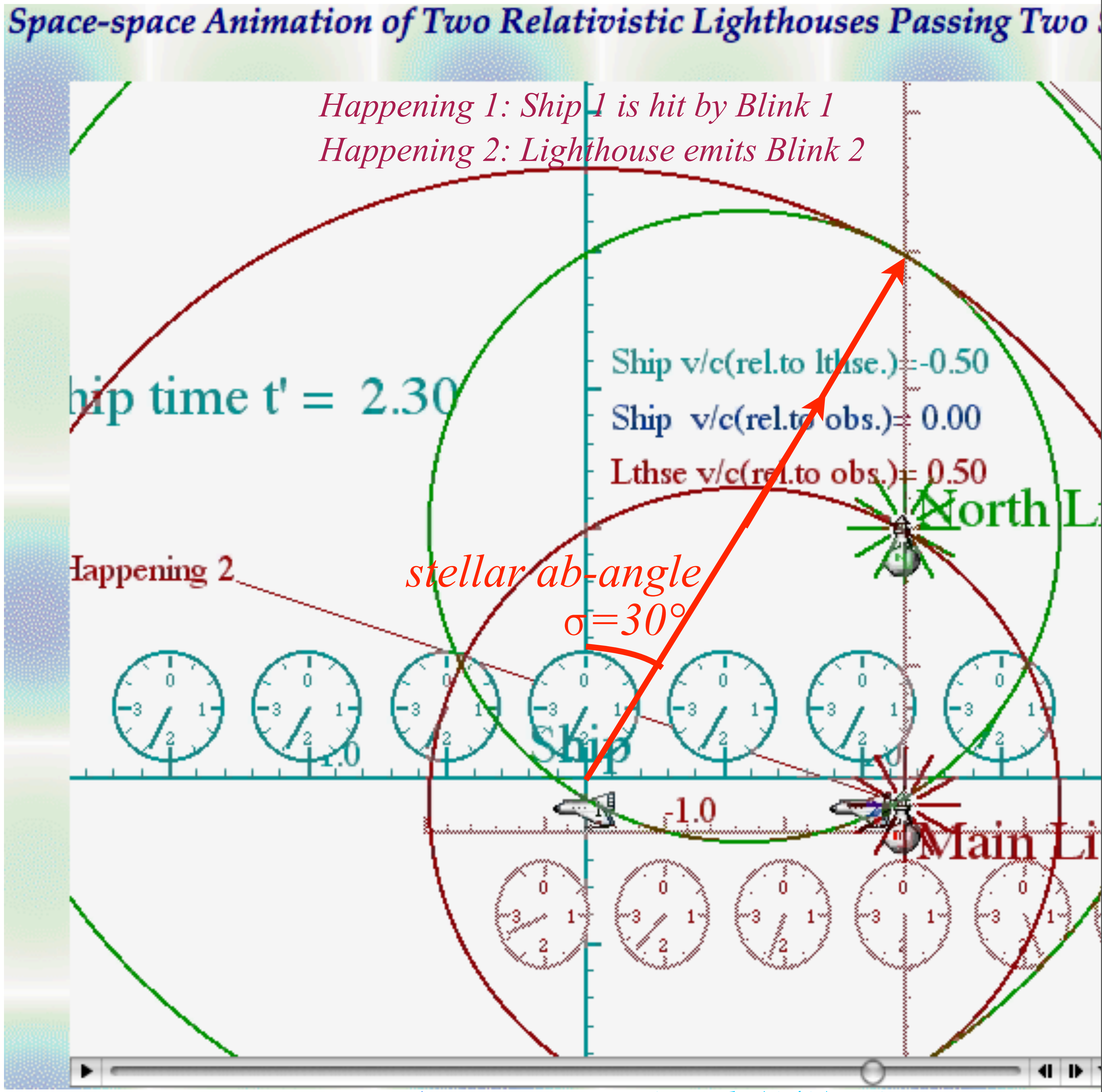


Fig. 5.6 Epstein's cosmic speedometer with aberration angle σ and transverse Doppler shift $\cosh\upsilon_Z$.

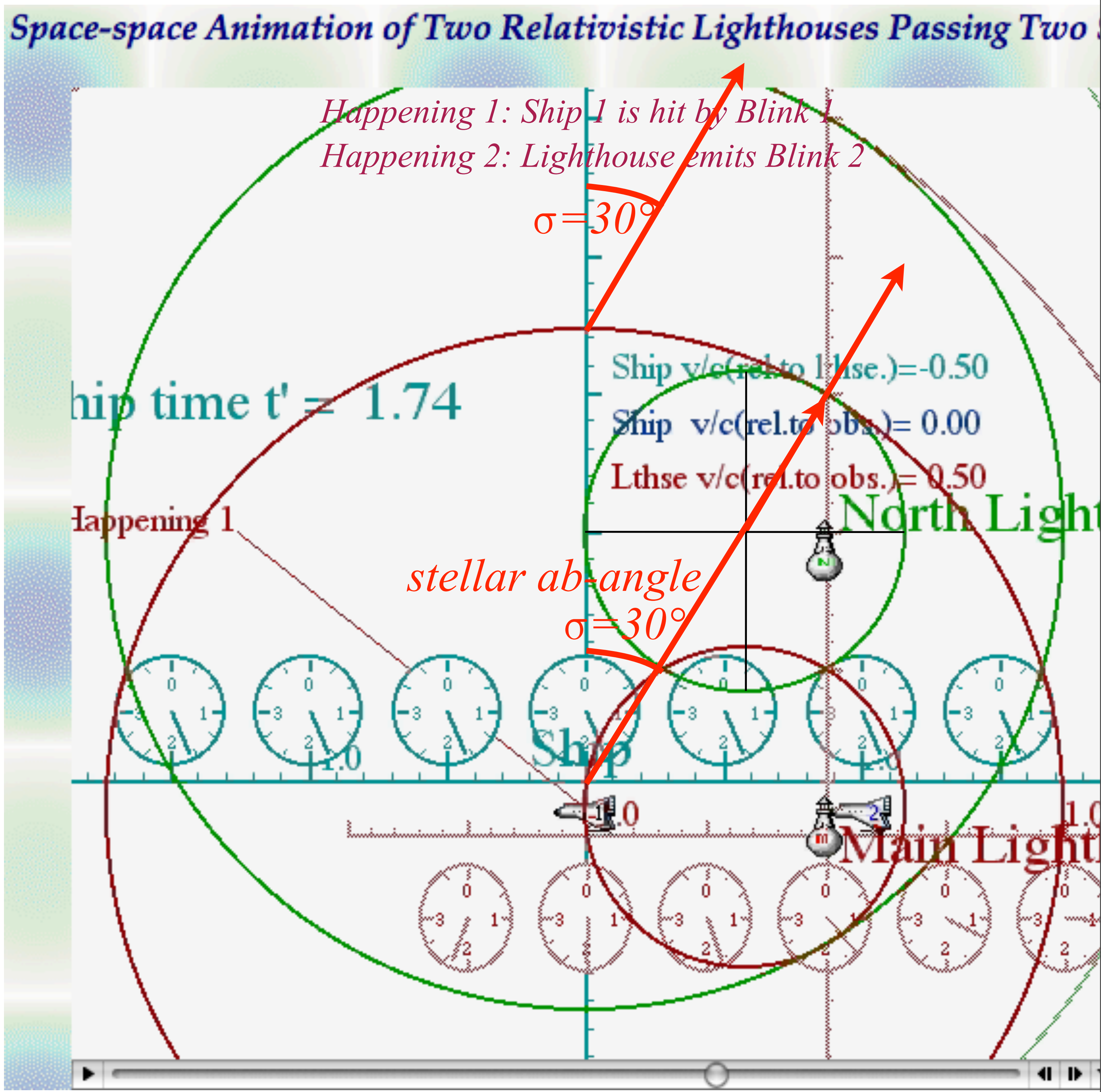
Lighthouse ship example of stellar aberration
(Here: ρ=atanh(1/2)=0.549)



(Here: ρ=Atanh(1/2)=0.55,
and: σ=Asin(1/2)=0.52 or 30°)

www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php

Lighthouse ship example of stellar aberration
(Here: ρ=atanh(1/2)=0.549)



(Here: ρ=Atanh(1/2)=0.55,
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www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php

Introducing the stellar aberration angle σ vs. rapidity ρ

Epstein's[†] space-proper-time (x,cτ) plots ("c-tau" plots)

Trigonometry: From circular to hyperbolic and back

Group vs. phase velocity and tangent contacts

[†] Lewis Carroll Epstein, *Relativity Visualized*
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See also: L. C. Epstein, *Thinking Physics Press,*
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Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Time contraction-dilation revisited

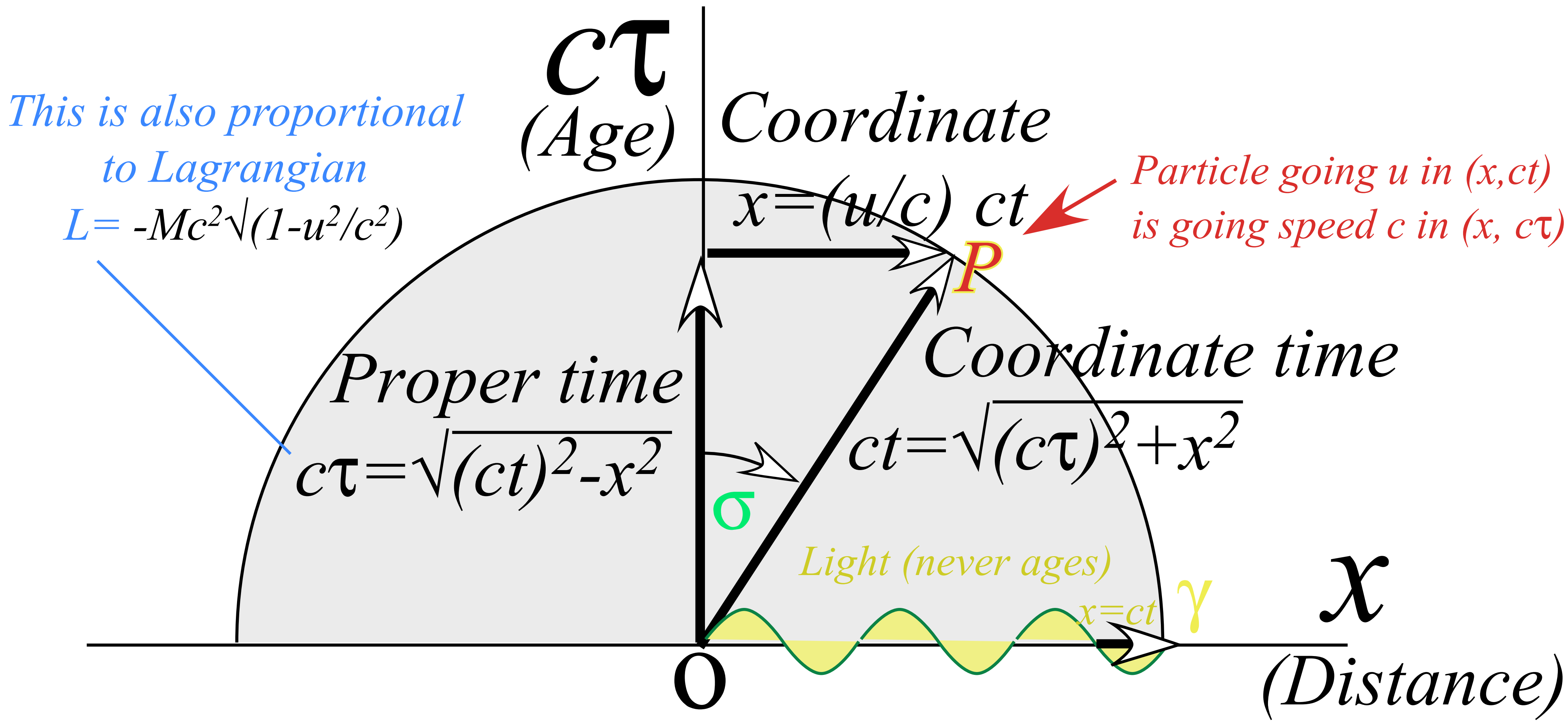
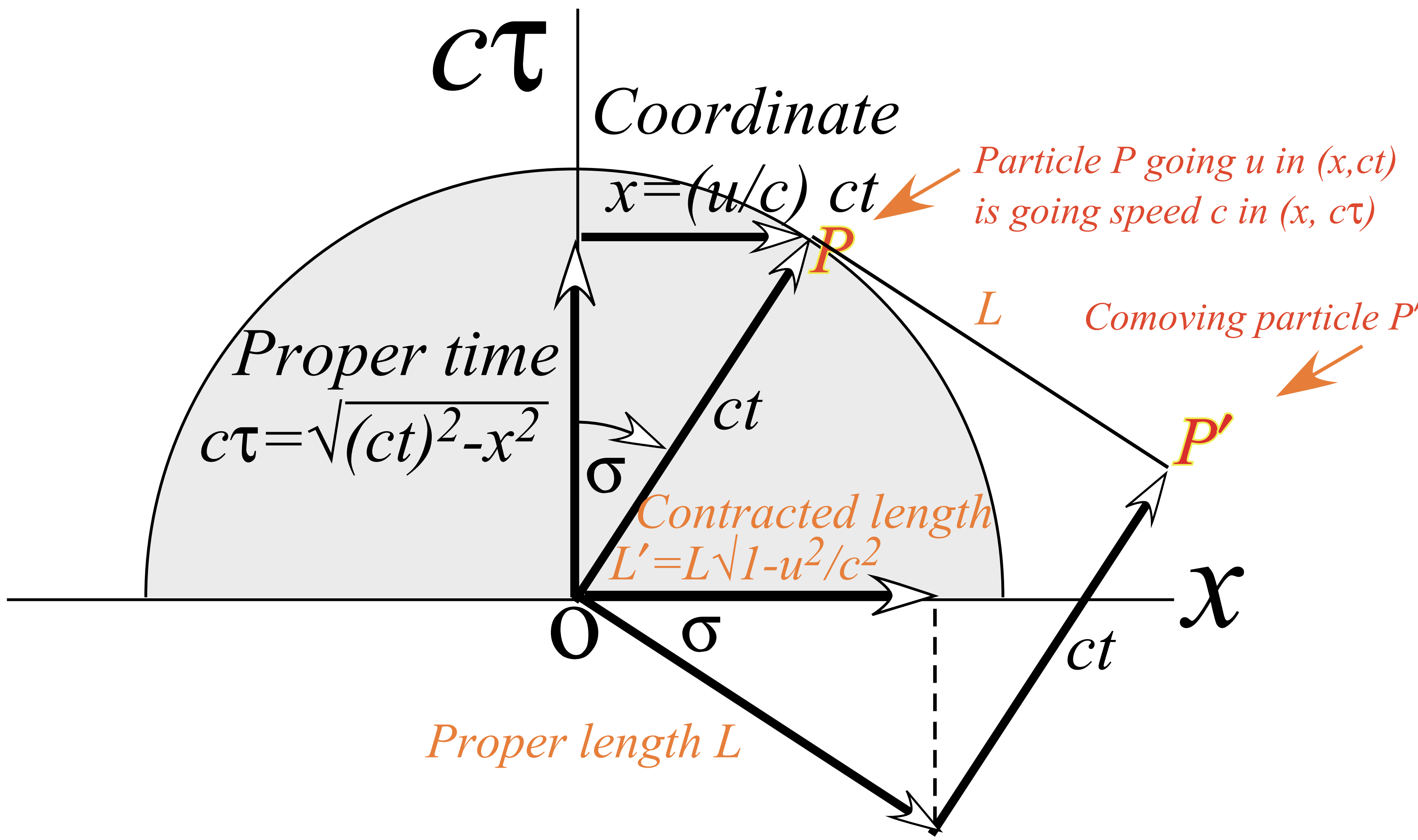


Fig. 5.8 Space-proper-time plot makes all objects move at speed c along their cosmic speedometer.

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

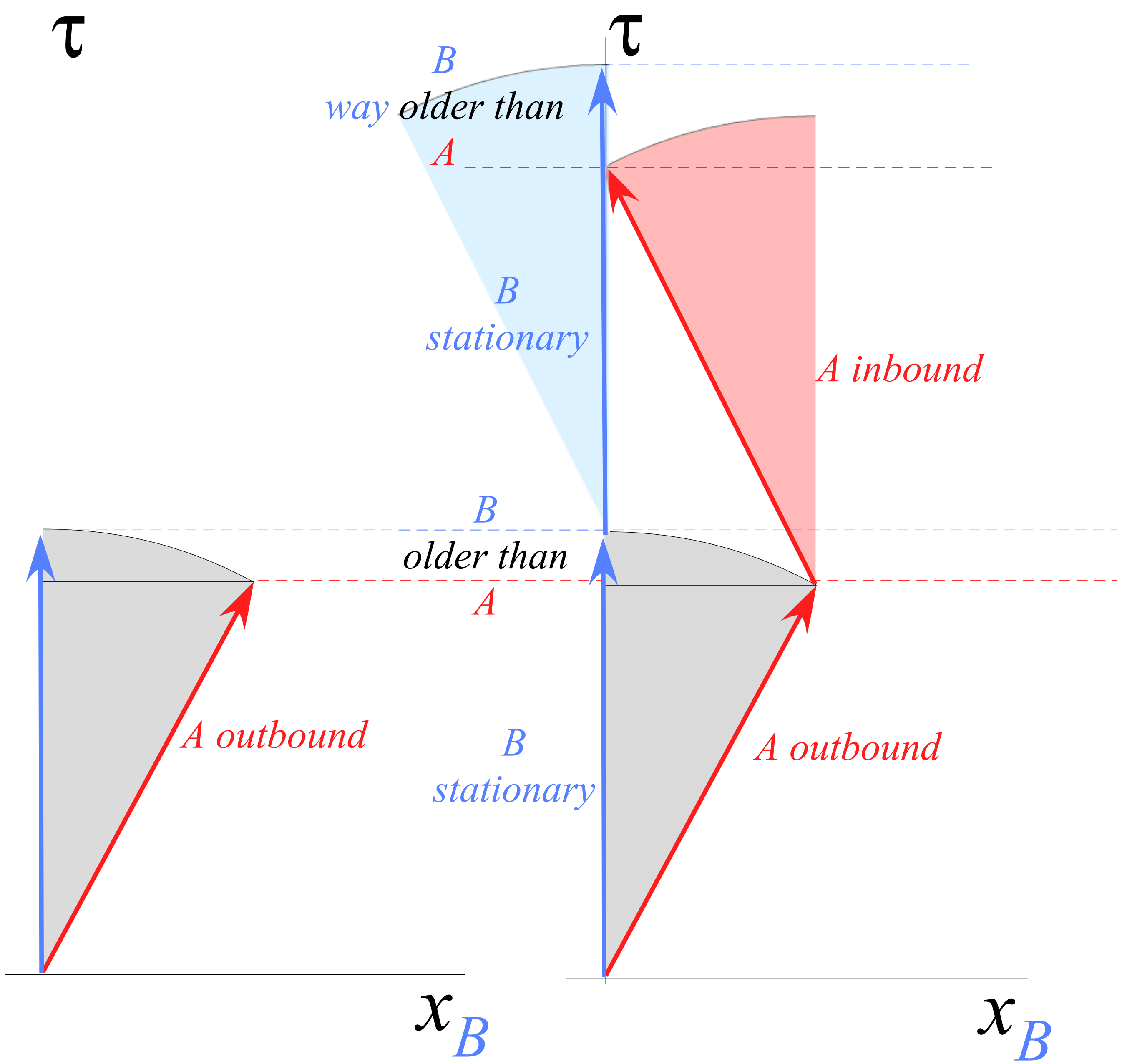
Length contraction-dilation revisited

A cute Epstein feature is that Lorentz-Fitzgerald contraction of a proper length L to $L'=L\sqrt{1-u^2/c^2}$ is simply rotational projection onto the x-axis of a length L rotated by σ.



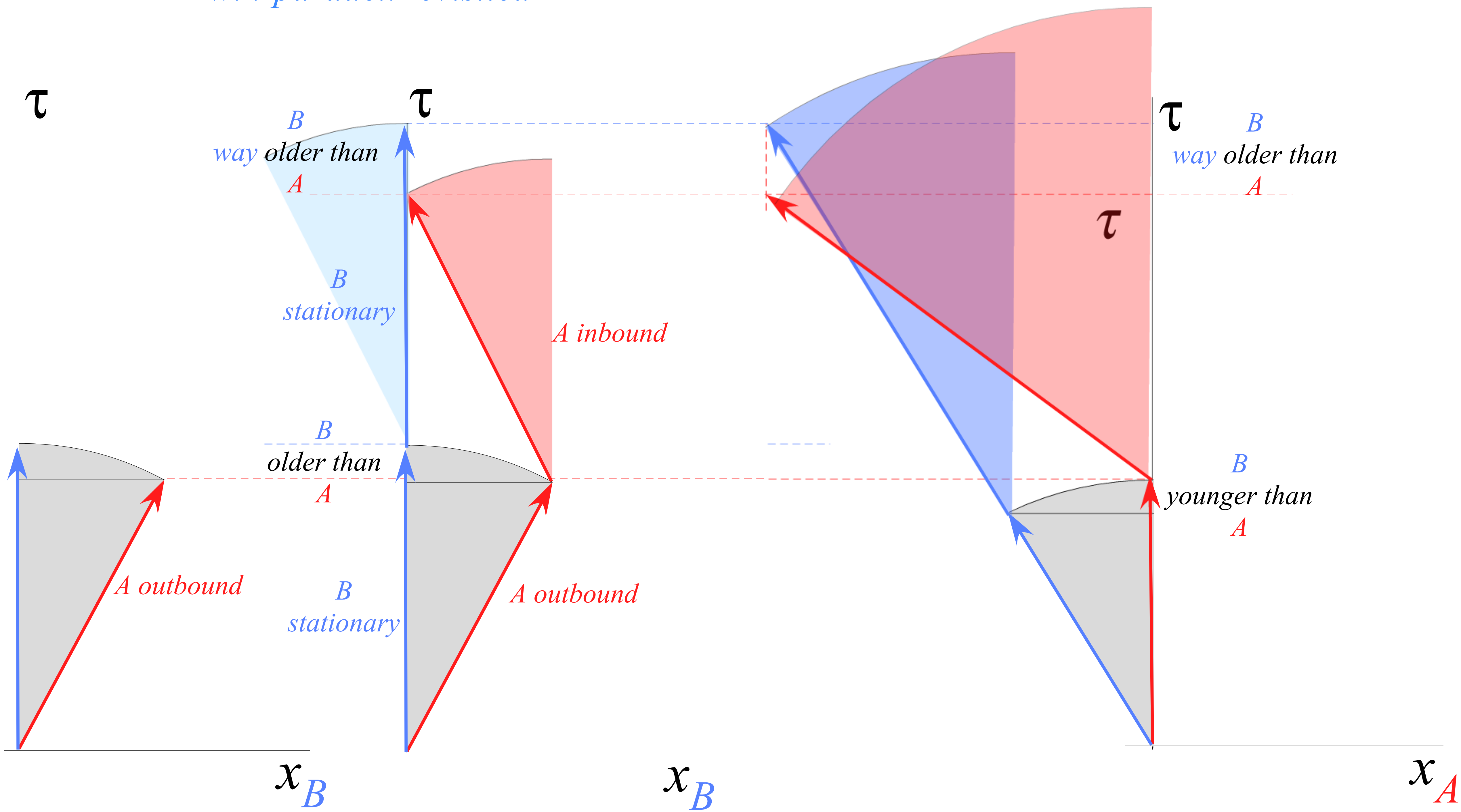
Epstein's space-proper-time $(x,c\tau)$ plots ("c-tau" plots)

Twin-paradox revisited



Epstein's space-proper-time $(x,c\tau)$ plots ("c-tau" plots)

Twin-paradox revisited



B stationary viewpoint

A outbound to right

A outbound appears stationary viewpoint

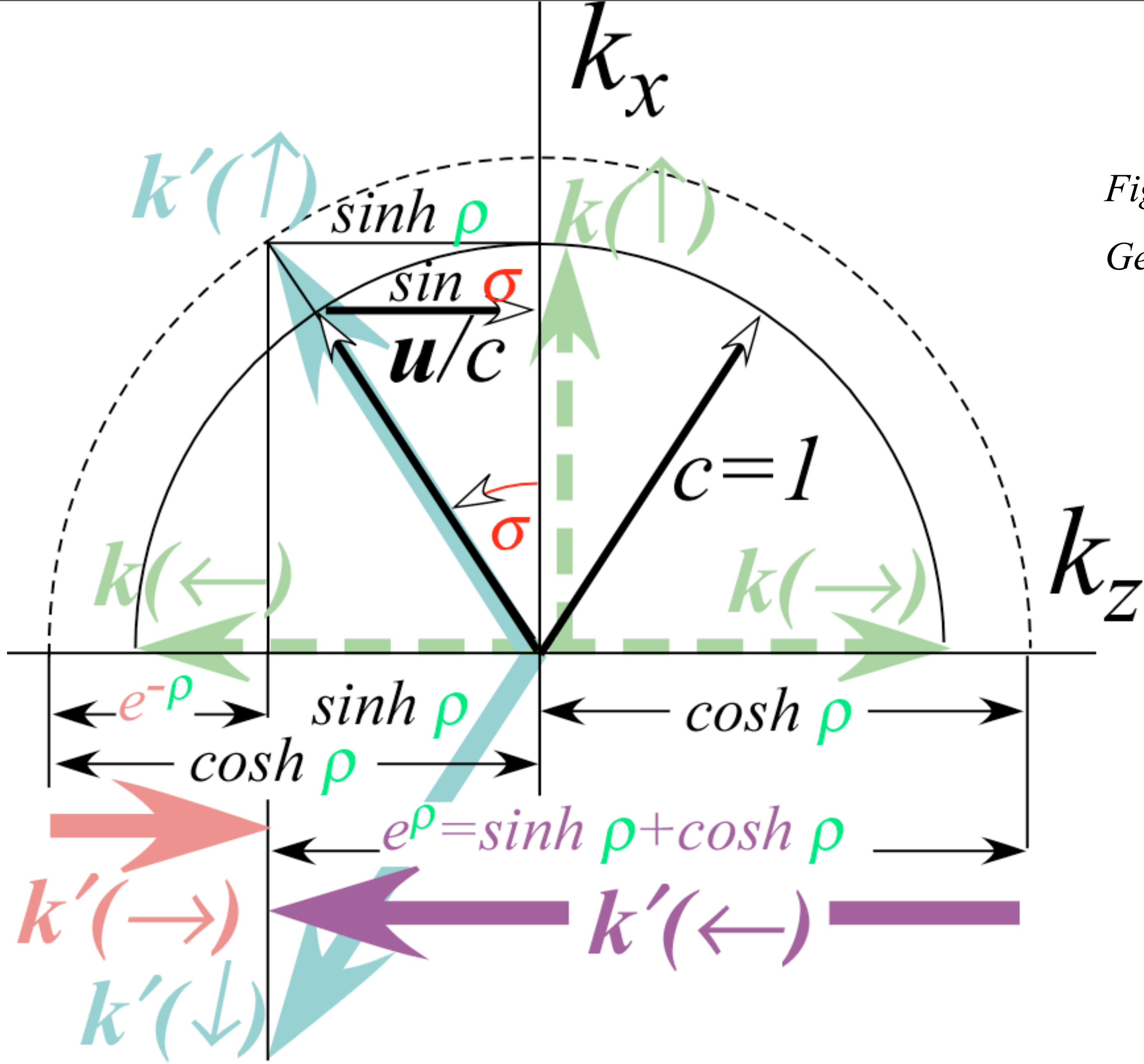


Fig. 5.10 CW cosmic speedometer.

Geometry of Lorentz boost of counter-propagating waves.

Introducing the stellar aberration angle σ vs. rapidity ρ

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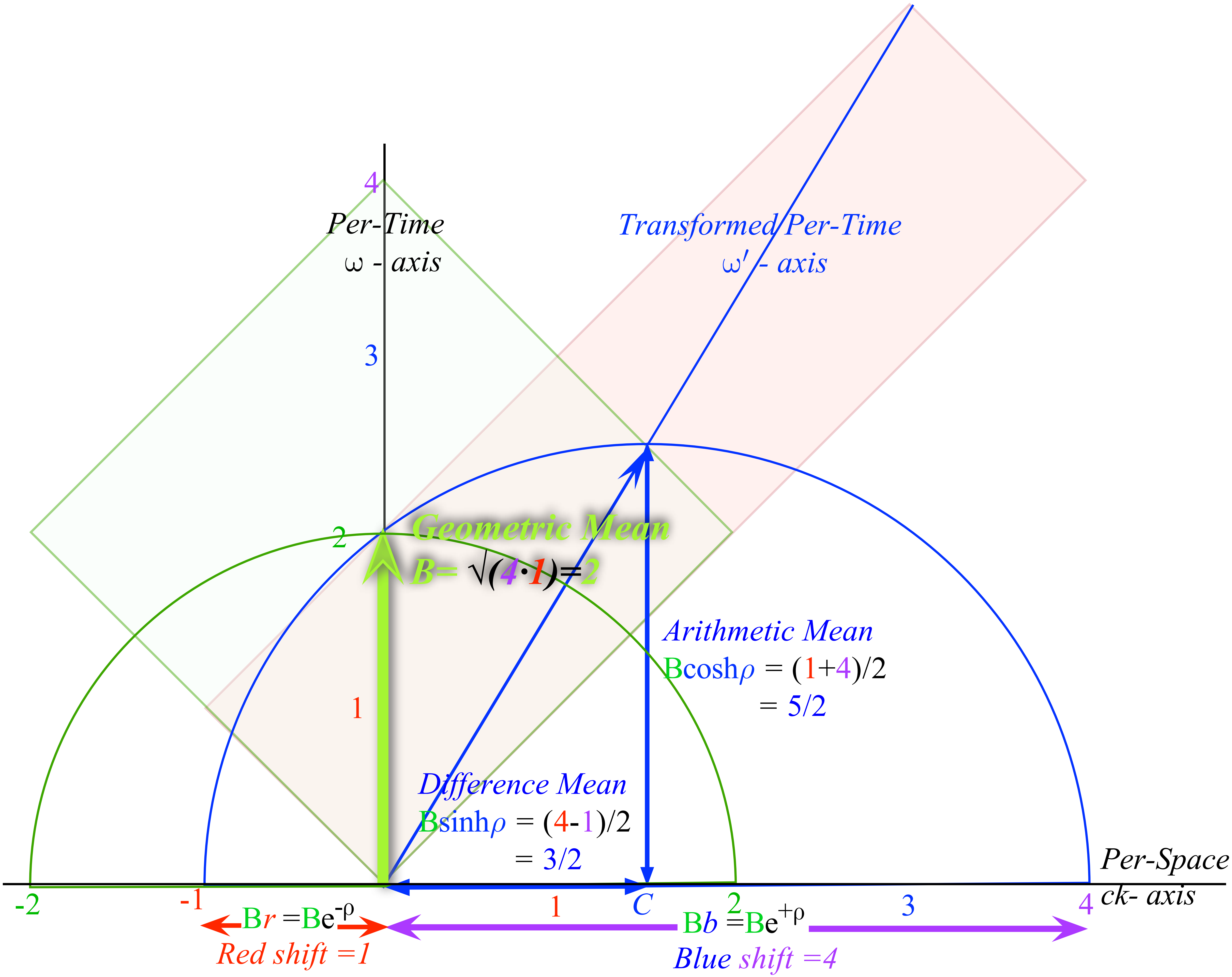
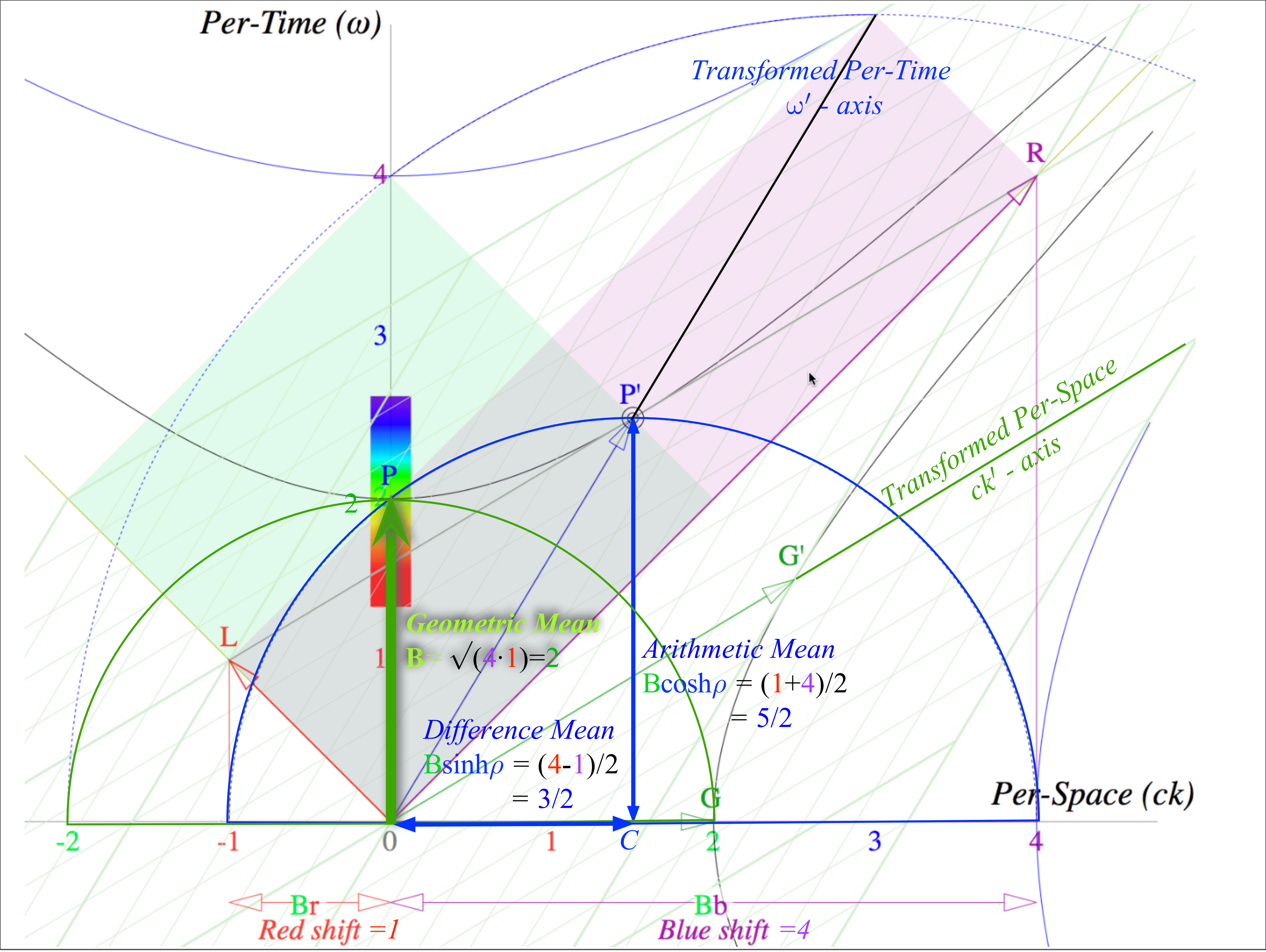
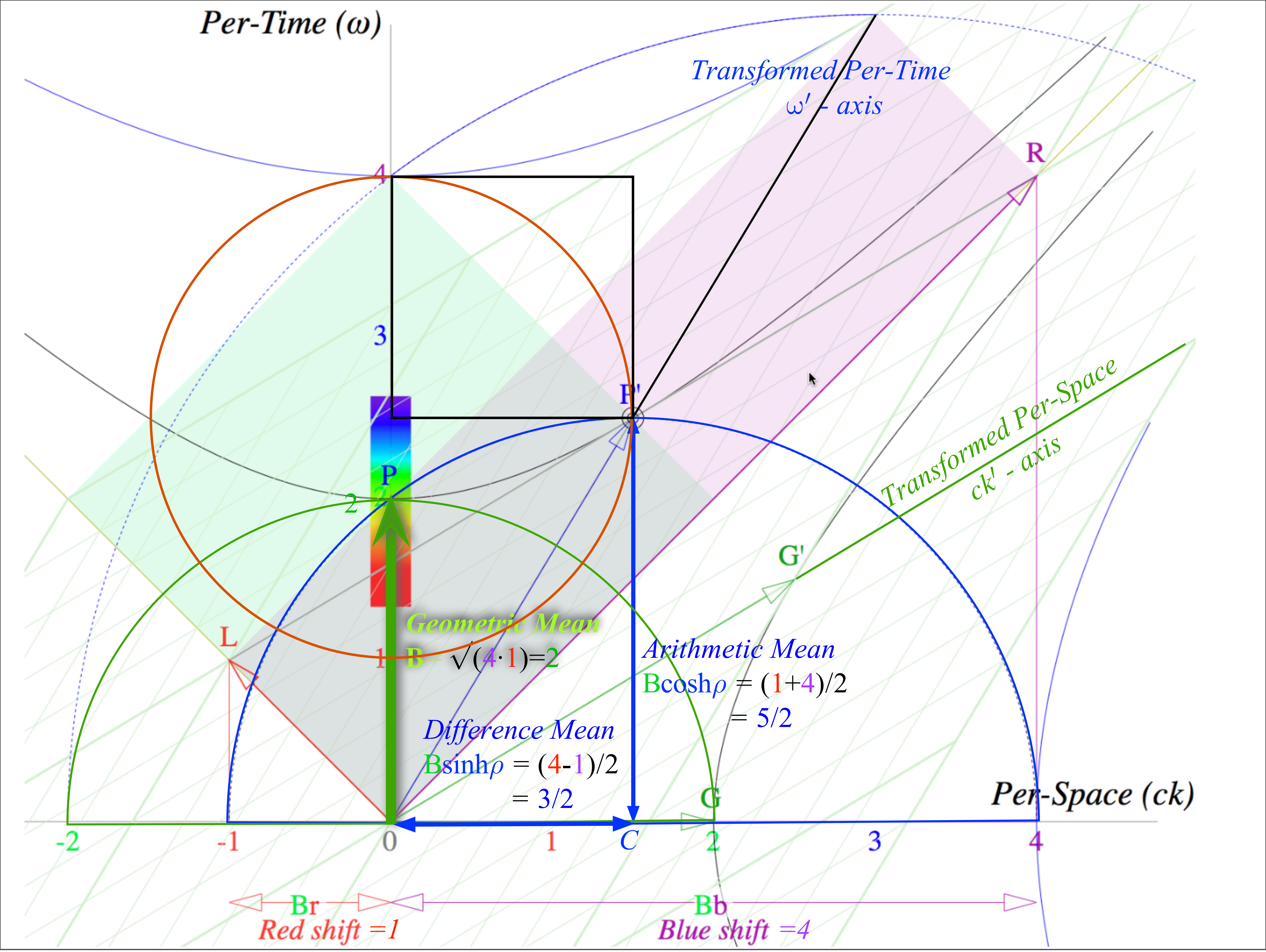
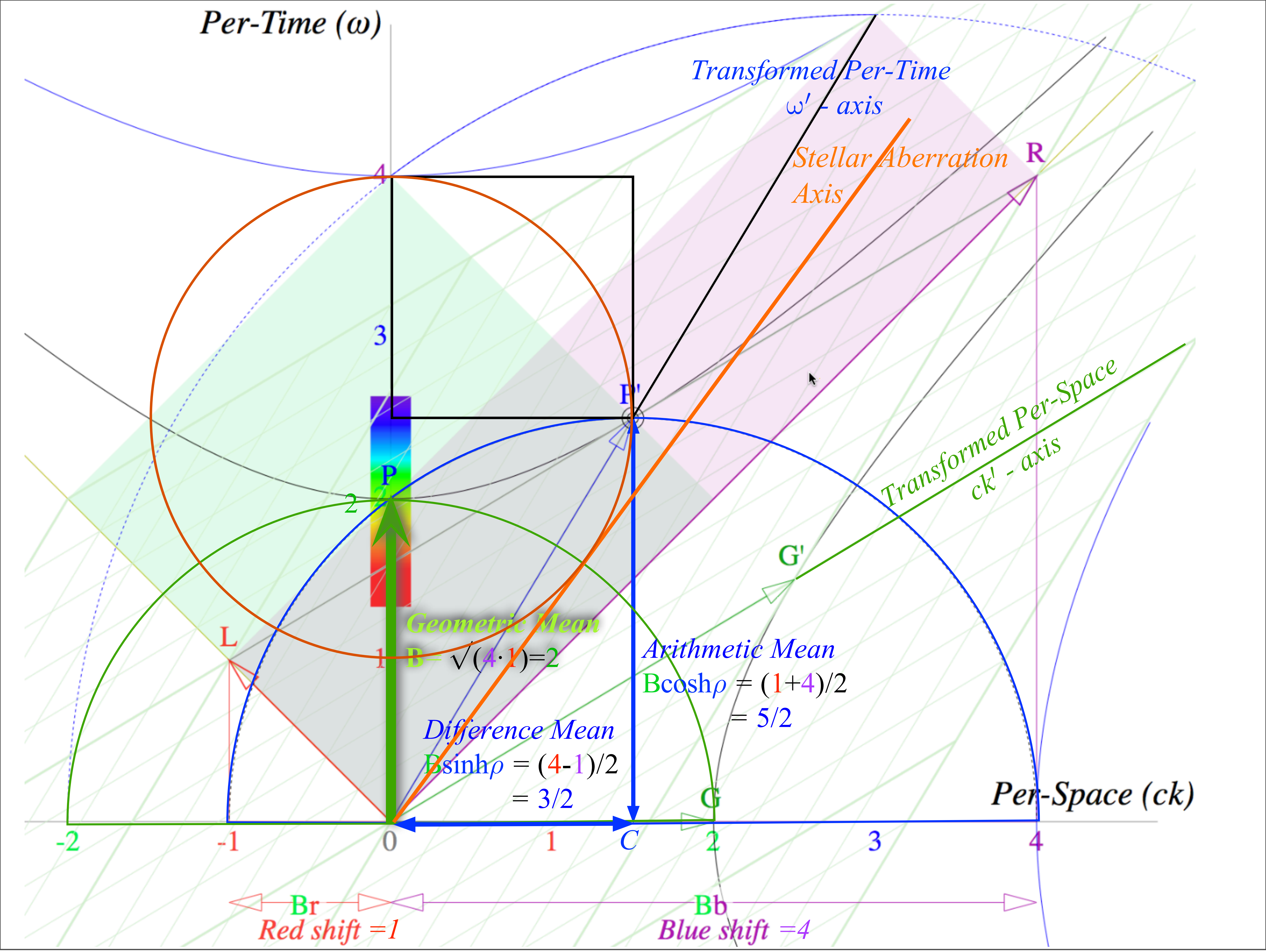
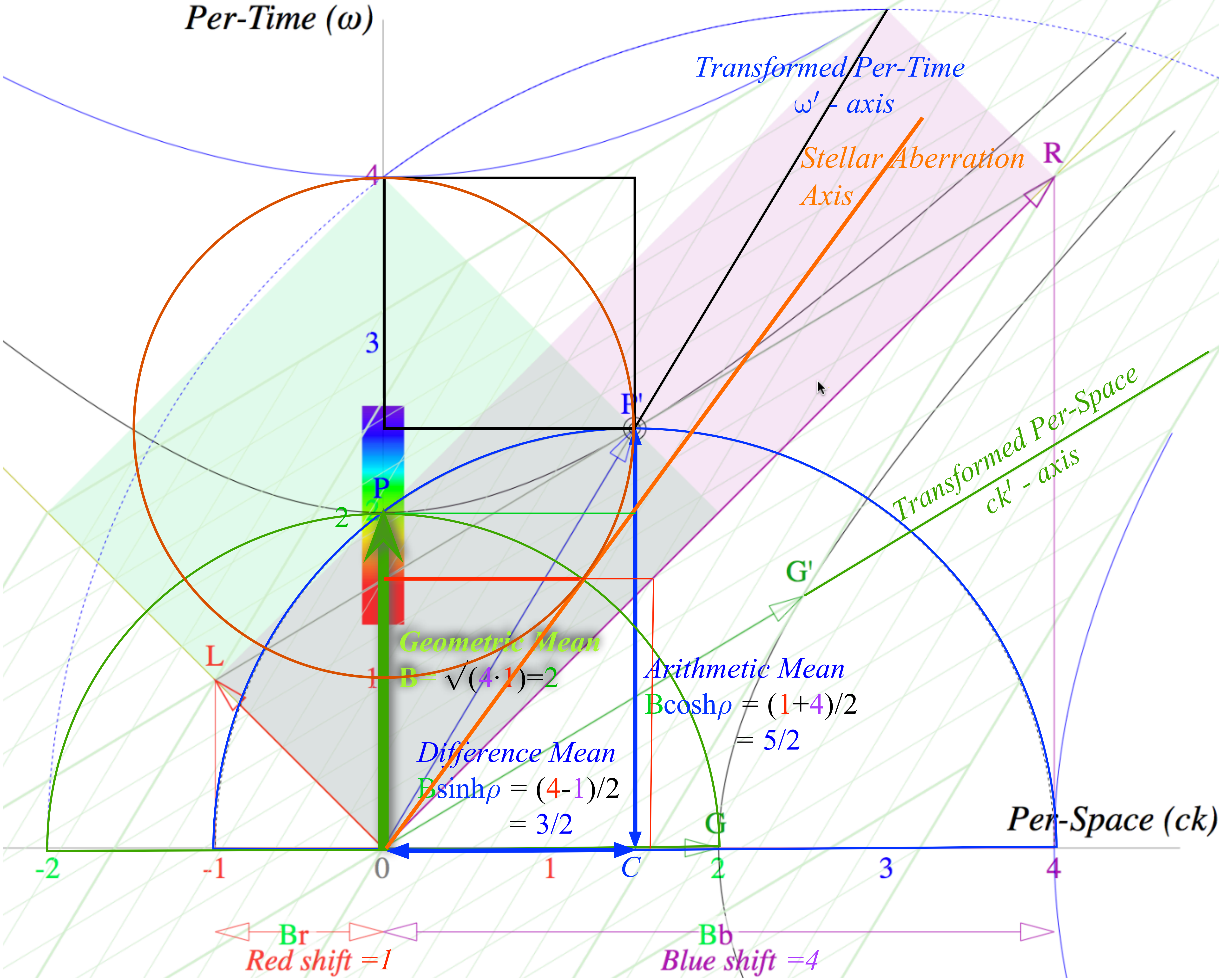
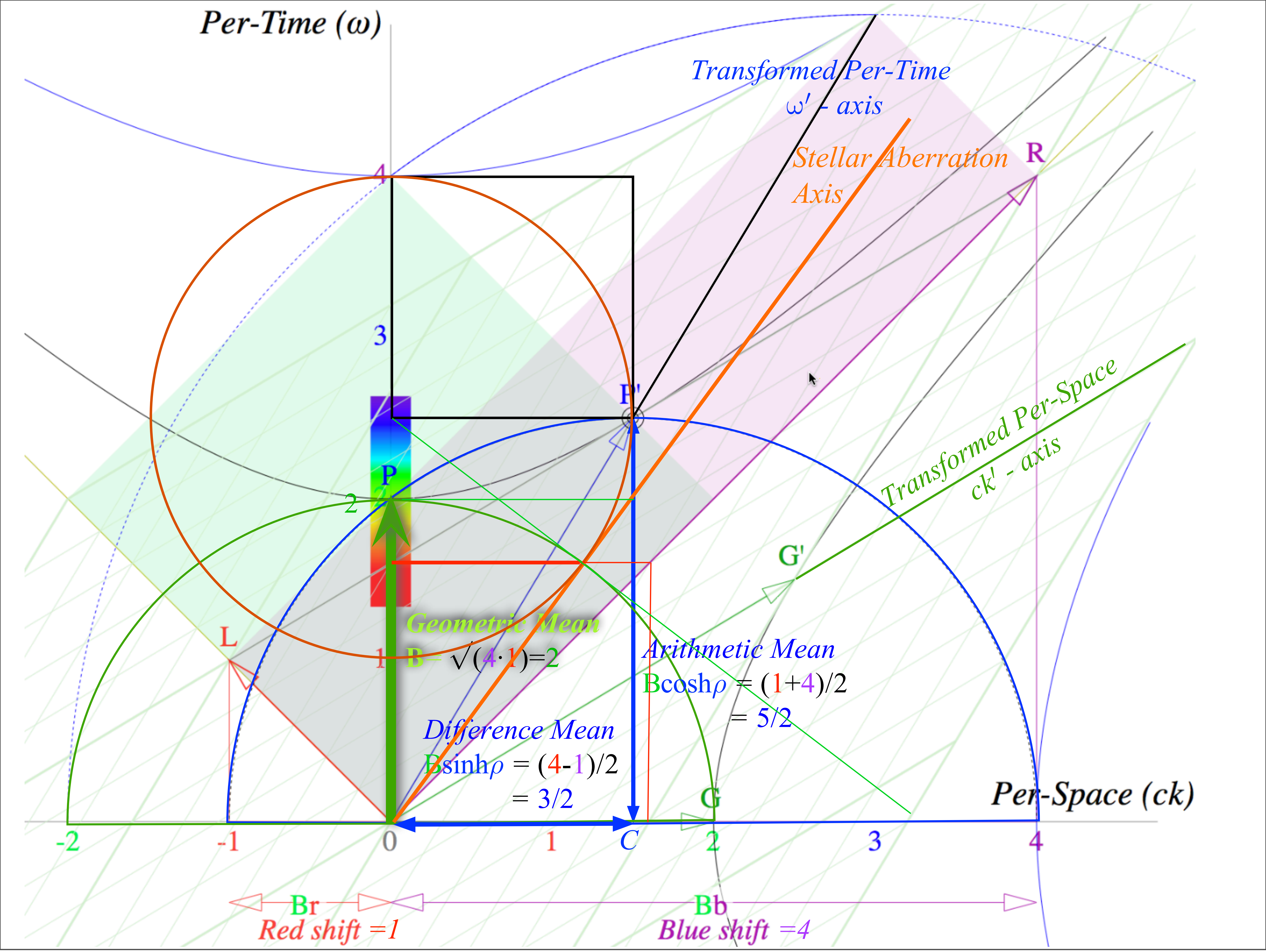

4
Per-Time
ω - axis
Transformed Per-Time
ω′ - axis
3
2
Geometric Mean
B= √(4·1)=2
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
1
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
Per-Space
ck- axis
-2
-1
1
C
2
3
4
Br =Be^-ρ
Bb =Be^+ρ
Red shift =1
Blue shift =4

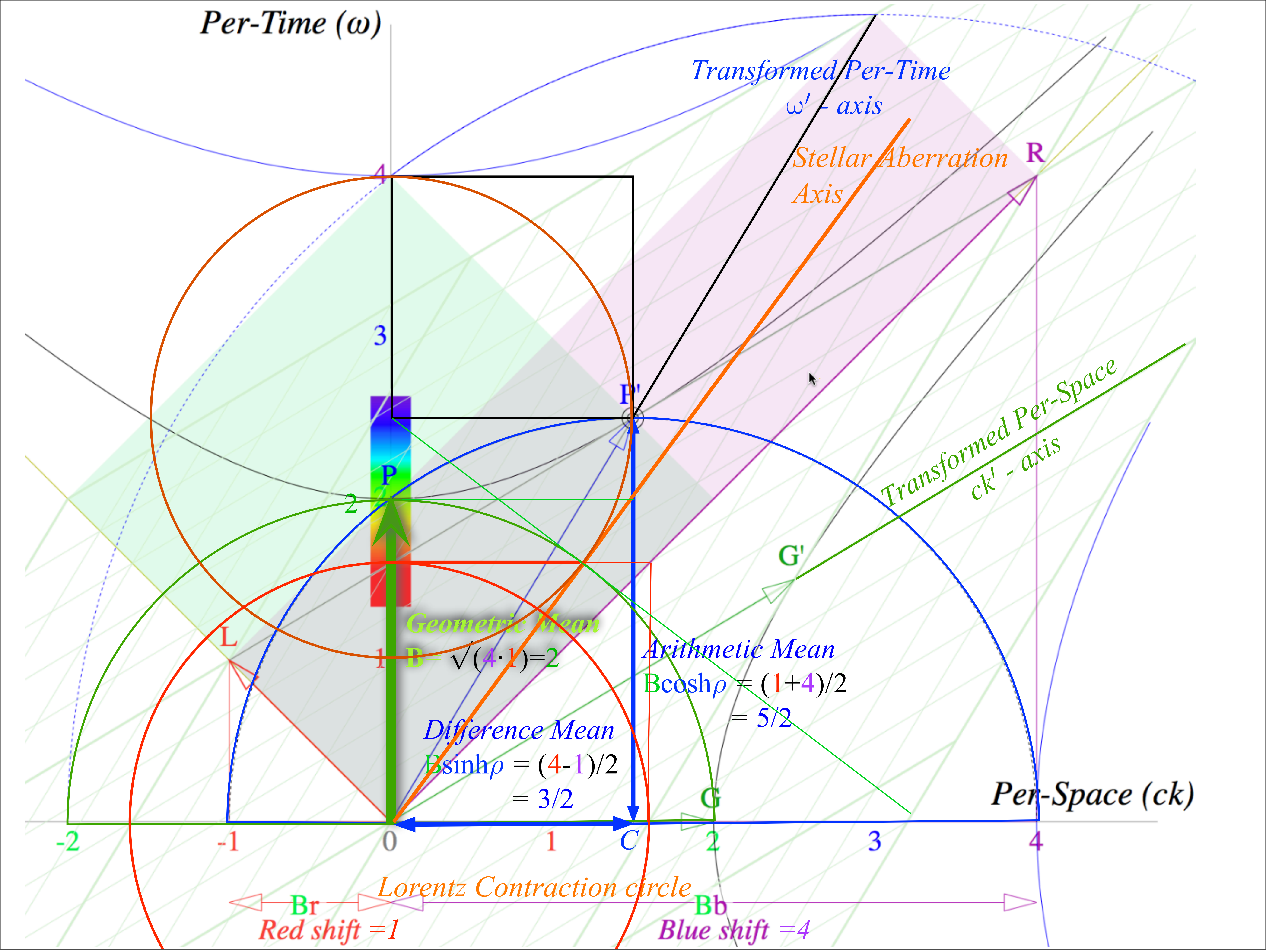

Per-Time (ω)
Transformed Per-Time
ω′ - axis
R
4
3
P′
P
2
Transformed Per-Space
ck′ - axis
G′
Geometric Mean
B= √(4·1)=2
L
1
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
Per-Space (ck)
G
-2
-1
0
1
C
2
3
4
Br
Bb
Red shift =1
Blue shift =4


Per-Time (ω)
Transformed Per-Time
ω' - axis
R
4
3
P'
P
2
Transformed Per-Space
ck' - axis
G'
Geometric Mean
B=√(4·1)=2
L
1
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
G
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Br
Bb
Red shift =1
Blue shift =4

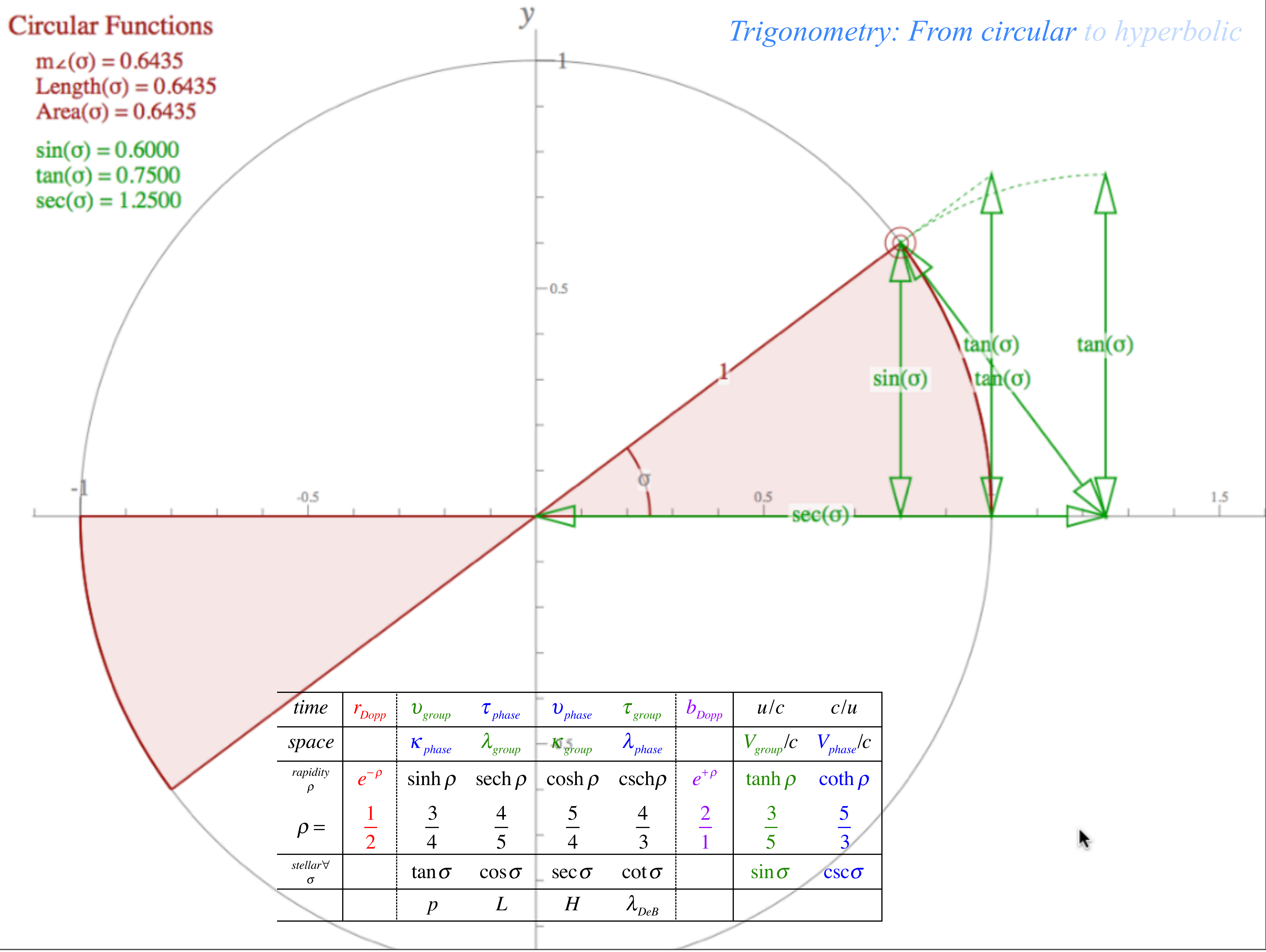

Per-Time (ω)
Transformed Per-Time
ω′ - axis
Stellar Aberration
Axis
R
4
3
P′
P
2
Transformed Per-Space
ck′ - axis
G′
Geometric Mean
B=√(4·1)=2
L
1
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
G
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Br
Bb
Red shift =1
Blue shift =4


Per-Time (ω)
Transformed Per-Time
ω' - axis
Stellar Aberration
Axis
R
3
P'
P
2
Transformed Per-Space
ck' - axis
G'
Geometric Mean
L
1
B=√(4·1)=2
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
G
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Br
Red shift =1
Bb
Blue shift =4


Per-Time (ω)
Transformed Per-Time
ω′ - axis
Stellar Aberration
Axis
R
3
P′
P
2
G′
Transformed Per-Space
ck′ - axis
Geometric Mean
B=√(4·1)=2
L
1
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
G
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Br
Red shift =1
Bb
Blue shift =4

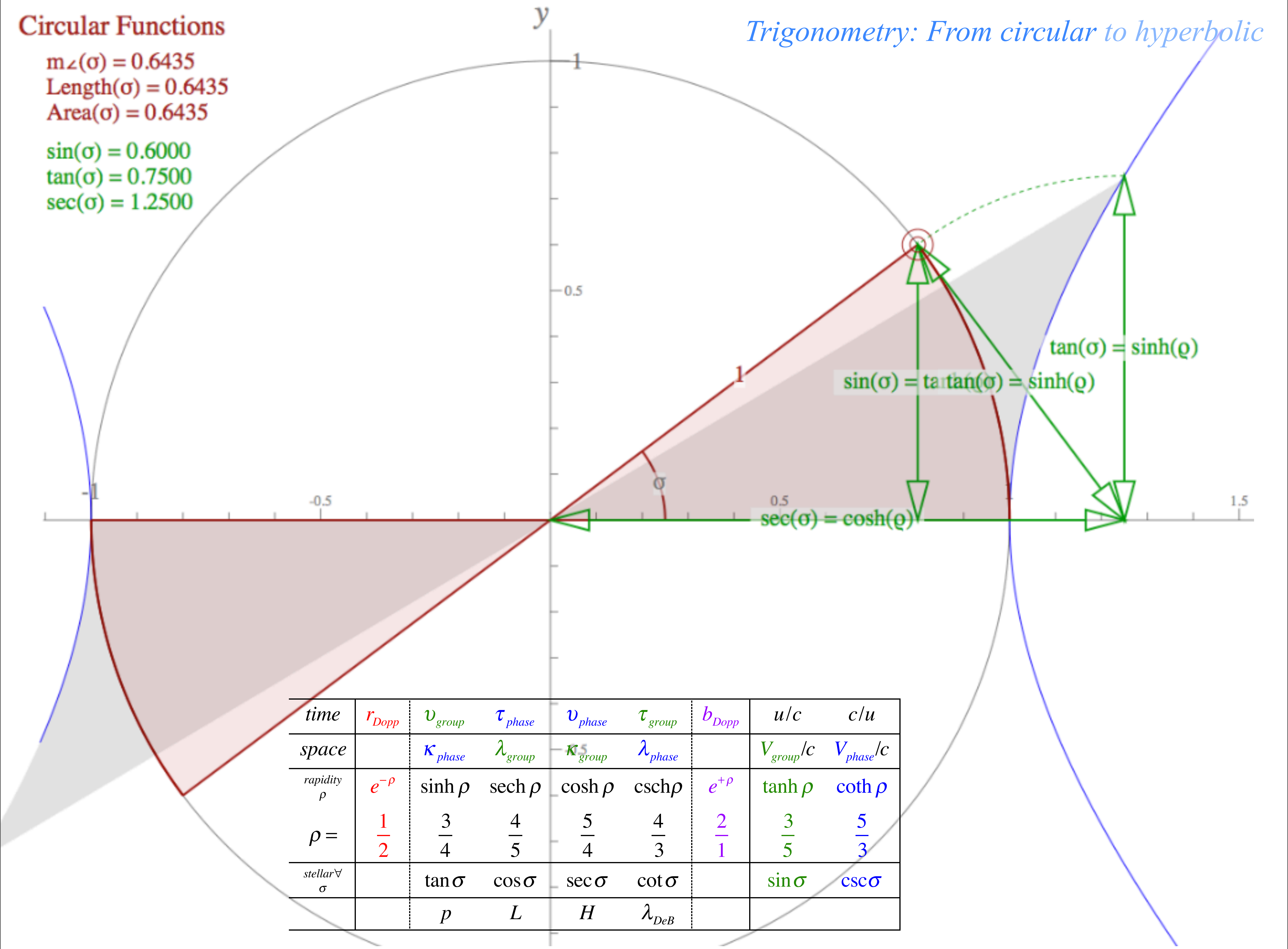


Per-Time (ω)
Transformed Per-Time
ω′ - axis
Stellar Aberration
Axis
R
4
3
P'
P
2
G'
Transformed Per-Space
ck′ - axis
Geometric Mean
B=√(4·1)=2
L
1
Arithmetic Mean
Bcoshρ = (1+4)/2
= 5/2
Difference Mean
Bsinhρ = (4-1)/2
= 3/2
G
Per-Space (ck)
-2
-1
0
1
C
2
3
4
Lorentz Contraction circle
Br
Bb
Red shift =1
Blue shift =4



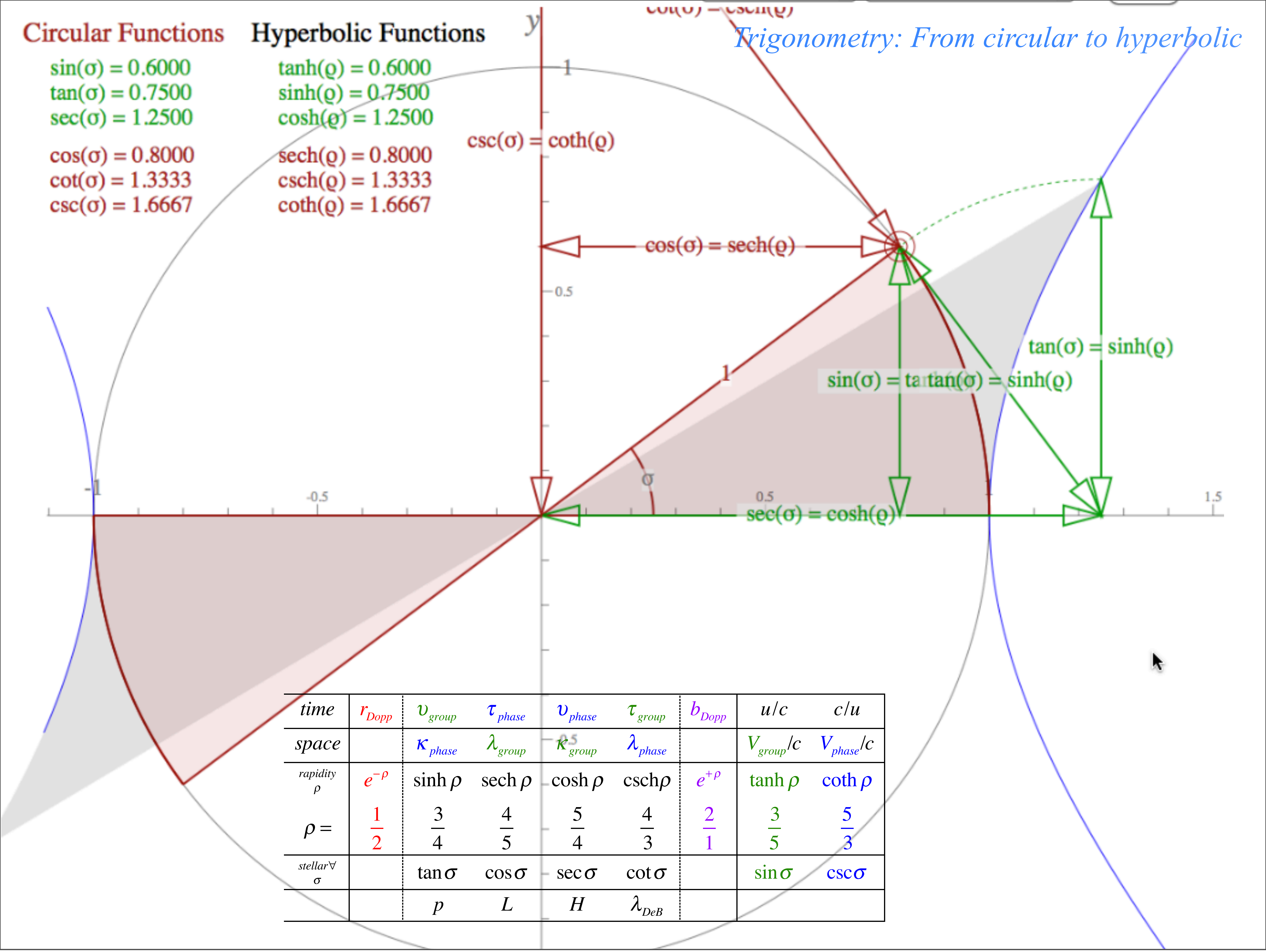
time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			

Trigonometry: From circular to hyperbolic

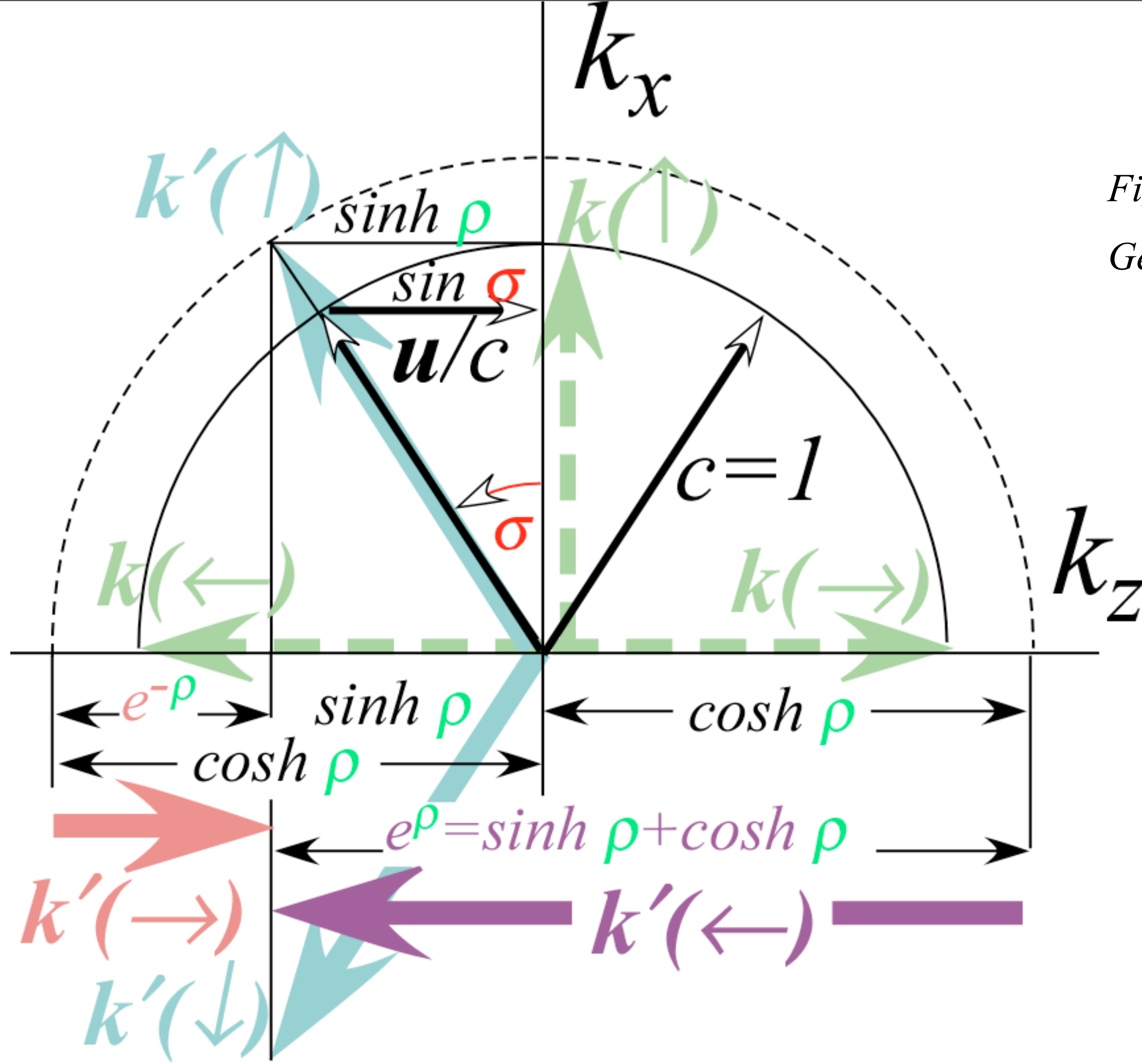


time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			

Trigonometry: From circular to hyperbolic



time	r_{Dopp}	υ_{group}	τ_{phase}	υ_{phase}	τ_{group}	b_{Dopp}	u/c	c/u
space		κ_{phase}	λ_{group}	κ_{group}	λ_{phase}		V_{group}/c	V_{phase}/c
rapidity ρ	$e^{-\rho}$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$e^{+\rho}$	$\tanh\rho$	$\coth\rho$
$\rho =$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{2}{1}$	$\frac{3}{5}$	$\frac{5}{3}$
stellar∀ σ		$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$		$\sin\sigma$	$\csc\sigma$
		p	L	H	λ_{DeB}			



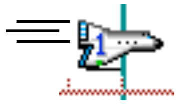


Fig. 5.10 CW cosmic speedometer.

Geometry of Lorentz boost of counter-propagating waves.

Introducing the stellar aberration angle σ vs. rapidity ρ

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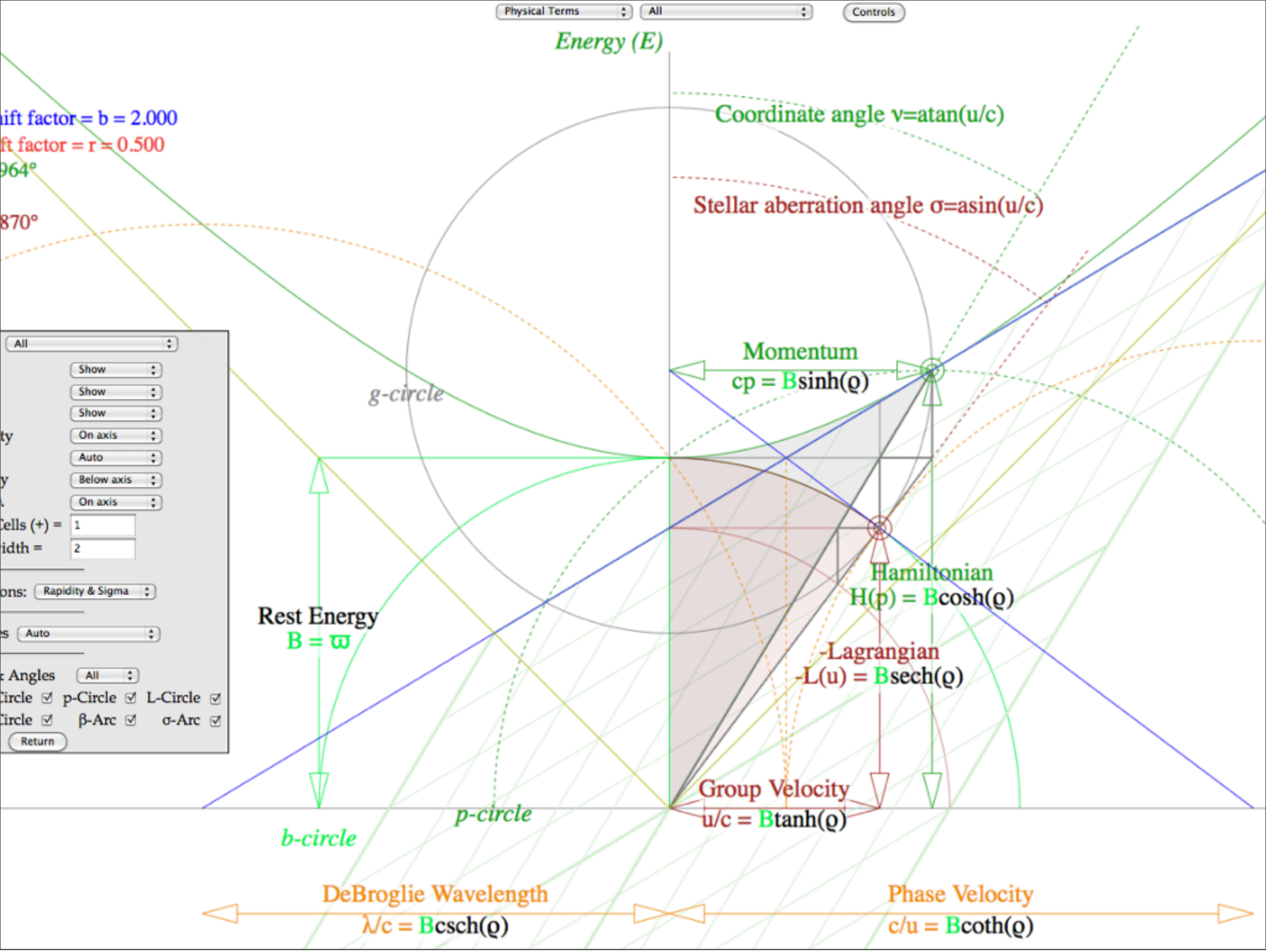
Physical Terms
All
Controls
Energy (E)
ift factor = b = 2.000
ft factor = r = 0.500
964°
870°
Coordinate angle ν=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Momentum
cp = Bsinh(ϱ)
g-circle
All
Show
Show
Show
ty
On axis
Auto
y
Below axis
On axis
Cells (+) =
1
idth =
2
ons:
Rapidity & Sigma
es
Auto
Angles
All
Circle
p-Circle
L-Circle
Circle
β-Arc
σ-Arc
Return
Rest Energy
B = ϖ
Hamiltonian
H(p) = Bcosh(ϱ)
-Lagrangian
-L(u) = Bsech(ϱ)
Group Velocity
u/c = Btanh(ϱ)
p-circle
b-circle
DeBroglie Wavelength
λ/c = Bcsch(ϱ)
Phase Velocity
c/u = Bcoth(ϱ)