# AMOP Lectures 9.0
# Tue. 3.4-Thur. 3.6 2014

## *Relativity of interfering and galloping waves: Amplitude and SWR.*

*(Ch. 2-5 of CMwBang-Unit 8 Ch. 6 of QTforCA Unit 2 )*

*Unmatched amplitudes giving galloping waves*
- *Standing Wave Ratio (SWR) and Standing Wave Quotient (SWQ)*
  - *Analogy with group and phase*
- *Galloping waves*
- *Analogy between wave galloping, Keplarian IHO orbits, and optical polarization*
  - *Galloping dynamics algebra*
- *Waves that go back in time - The Feynman-Wheeler Switchback*
  - *The Ship-Barn-and-Butler saga of confused causality*

*1st Quantization: Quantizing phase variables $\omega$ and k*
- *Understanding how quantum transitions require "mixed-up" states*
  - *Closed cavity vs ring cavity*

*Relativistic effects on charge, current, and Maxwell Fields*
- *Current density changes by Lorentz asynchrony*
  - *Magnetic B-field is relativistic $\sinh\rho$ $1^{st}$ order-effect*

*Unmatched amplitudes giving galloping waves*

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# *Unmatched amplitudes giving galloping waves*

2-CW dynamics has two 1-CW amplitudes $A_{\rightarrow}$ and $A_{\leftarrow}$ that we now allow to be *un*matched. $(A_{\rightarrow} \neq A_{\leftarrow})$

$$A_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)} + A_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)} = e^{i(k_{\Sigma}x-\omega_{\Sigma}t)}[A_{\rightarrow}e^{i(k_{\Delta}x-\omega_{\Delta}t)} + A_{\leftarrow}e^{-i(k_{\Delta}x-\omega_{\Delta}t)}]$$

Waves have half-sum mean-phase rates $(k_{\Sigma},\omega_{\Sigma})$ and half-difference group rates $(k_{\Delta},\omega_{\Delta})$ .

$$k_{\Sigma} = (k_{\rightarrow} + k_{\leftarrow})/2 \qquad k_{\Delta} = (k_{\rightarrow} - k_{\leftarrow})/2$$

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# *Galloping waves due to unmatched amplitudes*

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Also important is amplitude mean $A_{\Sigma} = (A_{\rightarrow} + A_{\leftarrow})/2$ and amplitude half-difference $A_{\Delta} = (A_{\rightarrow} - A_{\leftarrow})/2$.

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Detailed wave motion depends on standing-wave-ratio *SWR* or the inverse standing-wave-quotient *SWQ*.

$$SWR = \frac{(A_{\rightarrow} - A_{\leftarrow})}{(A_{\rightarrow} + A_{\leftarrow})} \qquad SWQ = \frac{(A_{\rightarrow} + A_{\leftarrow})}{(A_{\rightarrow} - A_{\leftarrow})} = \frac{1}{SWR}$$

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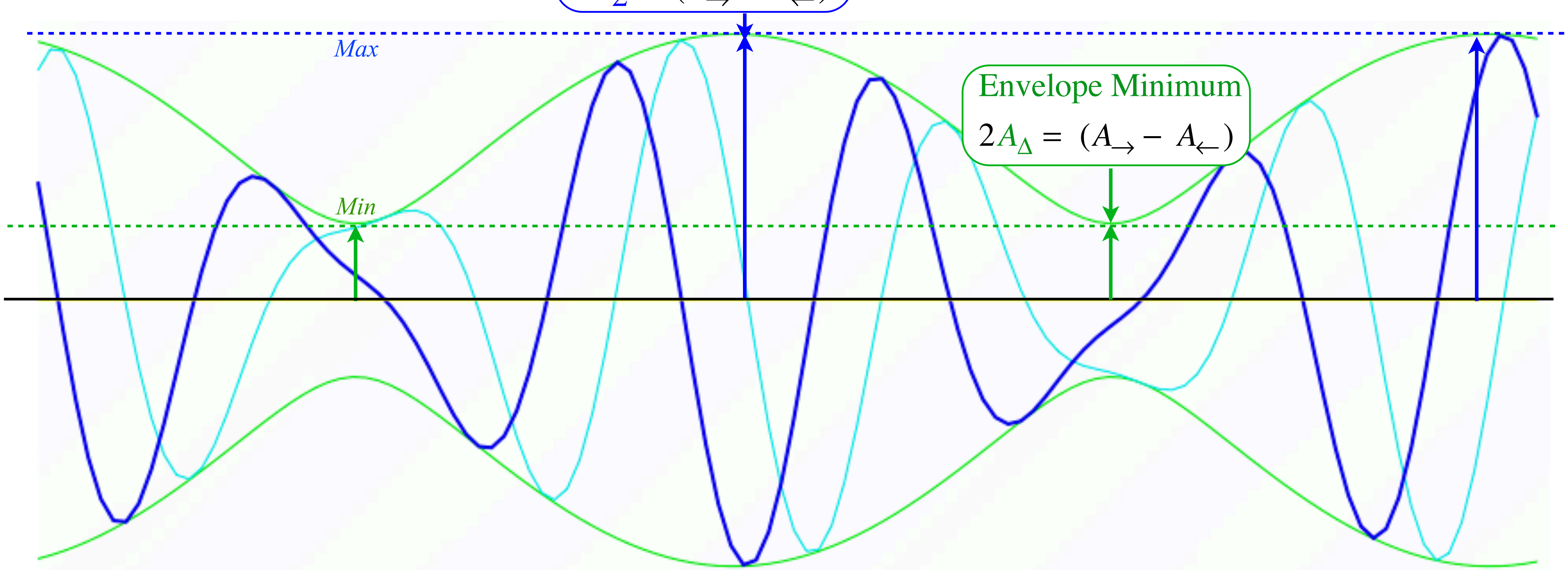
$$\frac{Envelope-Min.}{Envelope-Max.} = SWR = \frac{(A_{\rightarrow} - A_{\leftarrow})}{(A_{\rightarrow} + A_{\leftarrow})}$$

Envelope Maximum
$2A_{\Sigma} = (A_{\rightarrow} + A_{\leftarrow})$

$$SWQ = \frac{(A_{\rightarrow} + A_{\leftarrow})}{(A_{\rightarrow} - A_{\leftarrow})} = \frac{1}{SWR}$$



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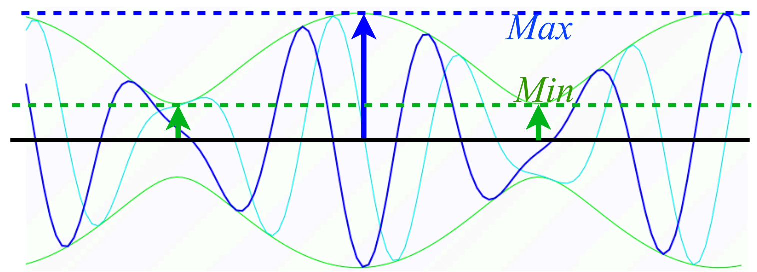


$$SWQ = \frac{(A_{\rightarrow} + A_{\leftarrow})}{(A_{\rightarrow} - A_{\leftarrow})} = \frac{1}{SWR}$$

These are analogous to frequency ratios for group velocity $V_{group}<c$ and its inverse that is phase velocity $V_{phase}>c$.

$$V_{group} = \frac{\omega_{\Delta}}{k_{\Delta}} = \frac{(\omega_{\rightarrow} - \omega_{\leftarrow})}{(k_{\rightarrow} - k_{\leftarrow})} = c\frac{(\omega_{\rightarrow} - \omega_{\leftarrow})}{(\omega_{\rightarrow} + \omega_{\leftarrow})}$$

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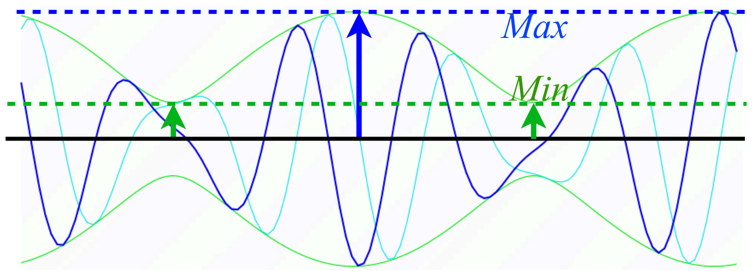


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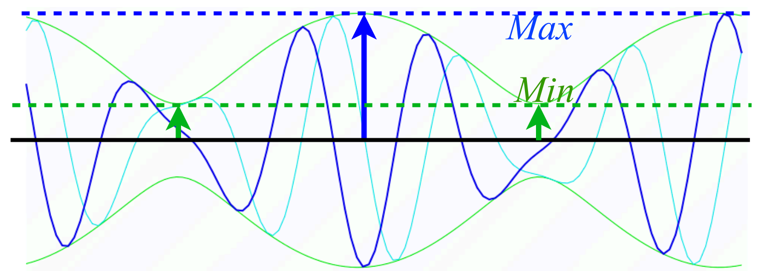


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$$\frac{V_{group}}{c} = \frac{c}{V_{phase}}$$ is analogous to: $$SWR = \frac{1}{SWQ}$$

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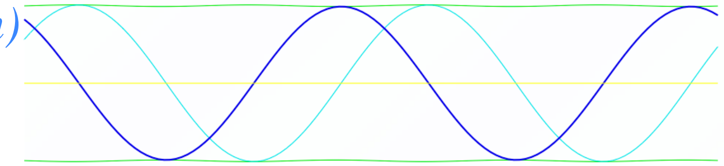
(not shown)

SWR=1



(not shown in (x,ct) plots)

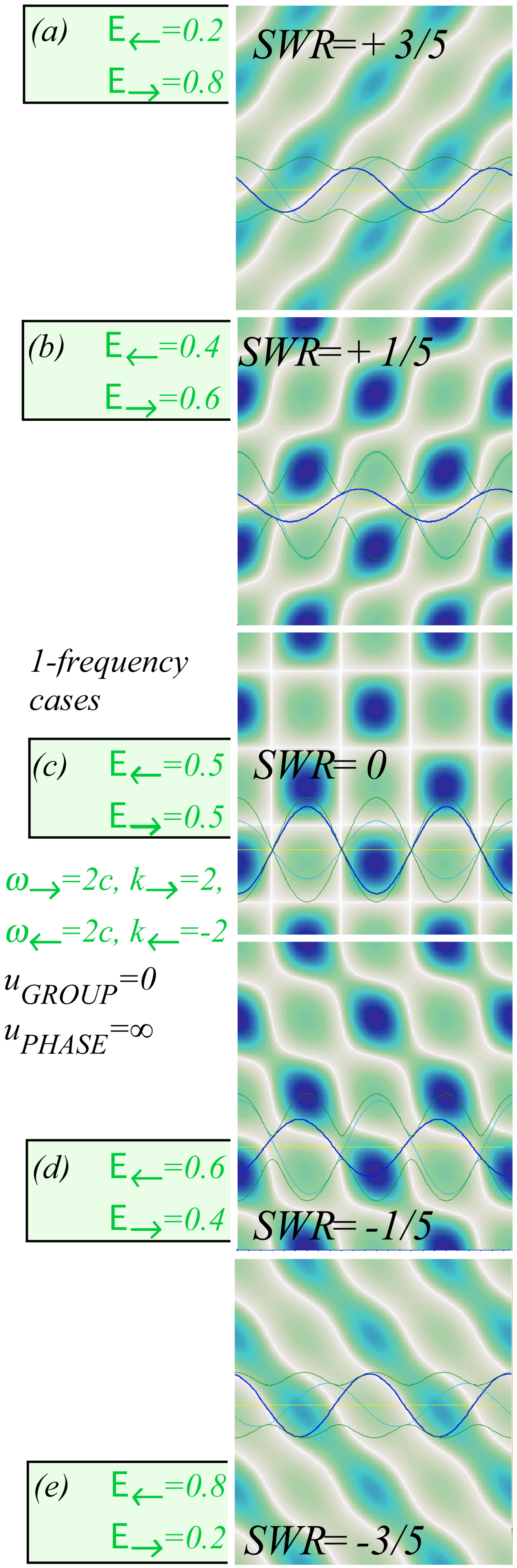



Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.

SWR=+3/5

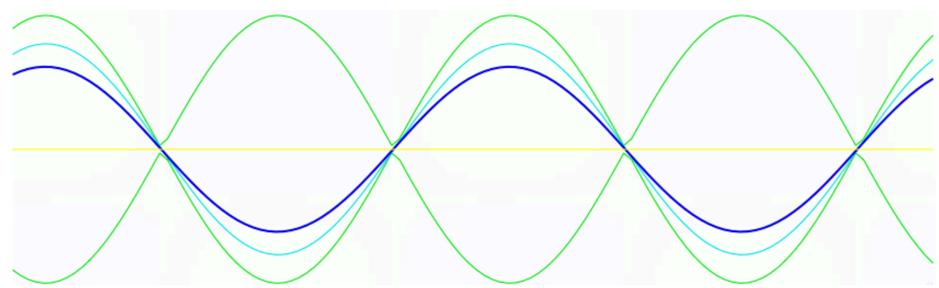
SWR=+1/5

SWR=1

Two extremes for Standing Wave Ratio

SWR=0

SWR=0



SWR=-1/5

SWR=-3/5

from:Fig. 4.5.2
QTforCA
Unit 2 Ch.4

from:Fig. 8.6.3
CMwBang!
Unit 8 Ch.6

(a) $E_{\leftarrow}=0.2$ $E_{\rightarrow}=0.8$

(b) $E_{\leftarrow}=0.4$ $E_{\rightarrow}=0.6$

*1-frequency cases*

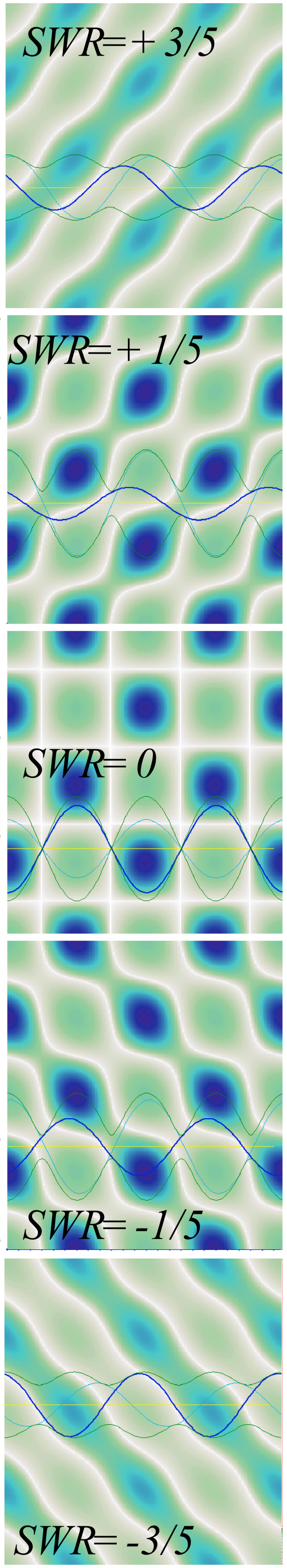
(c) $E_{\leftarrow}=0.5$ $E_{\rightarrow}=0.5$

$\omega_{\rightarrow}=2c$, $k_{\rightarrow}=2$,
$\omega_{\leftarrow}=2c$, $k_{\leftarrow}=-2$

$u_{GROUP}=0$
$u_{PHASE}=\infty$

(d) $E_{\leftarrow}=0.6$ $E_{\rightarrow}=0.4$

(e) $E_{\leftarrow}=0.8$ $E_{\rightarrow}=0.2$

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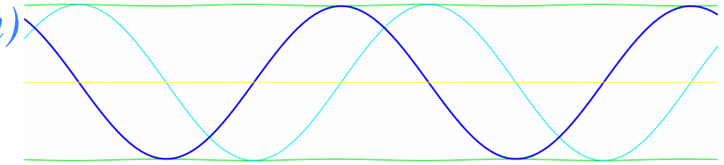
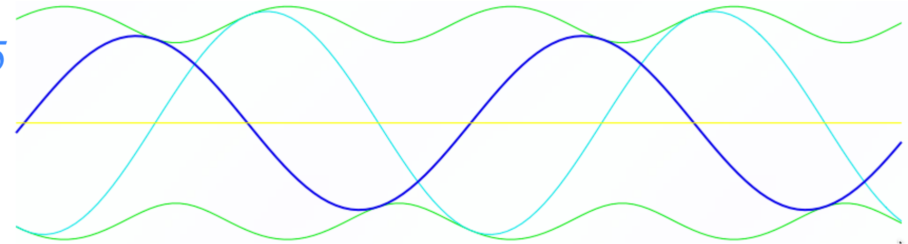
*(not shown)*
*SWR=1*

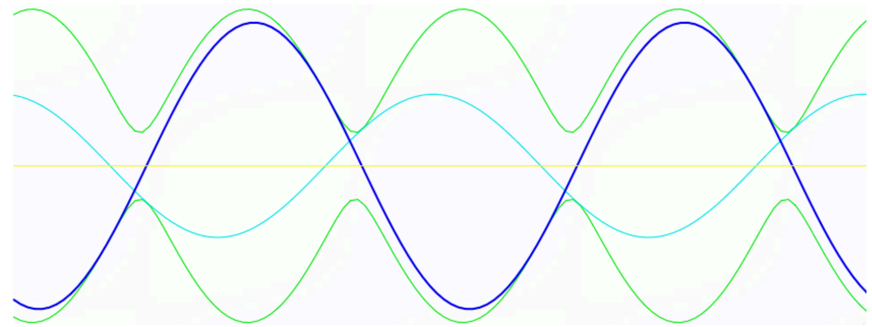


*(not shown in (x,ct) plots)*

*SWR=+3/5*

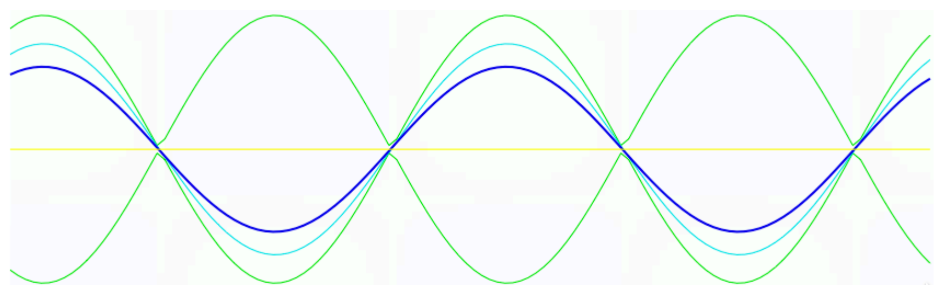


*SWR=+1/5*



*SWR=1*
*Two extremes for Standing Wave Ratio*
*SWR=0*

*SWR=0*



*SWR=-1/5*

*SWR=-3/5*

*from:Fig. 4.5.2*
*QTforCA*
*Unit 2 Ch.4*

*from:Fig. 8.6.3*
*CMwBang!*
*Unit 8 Ch.6*

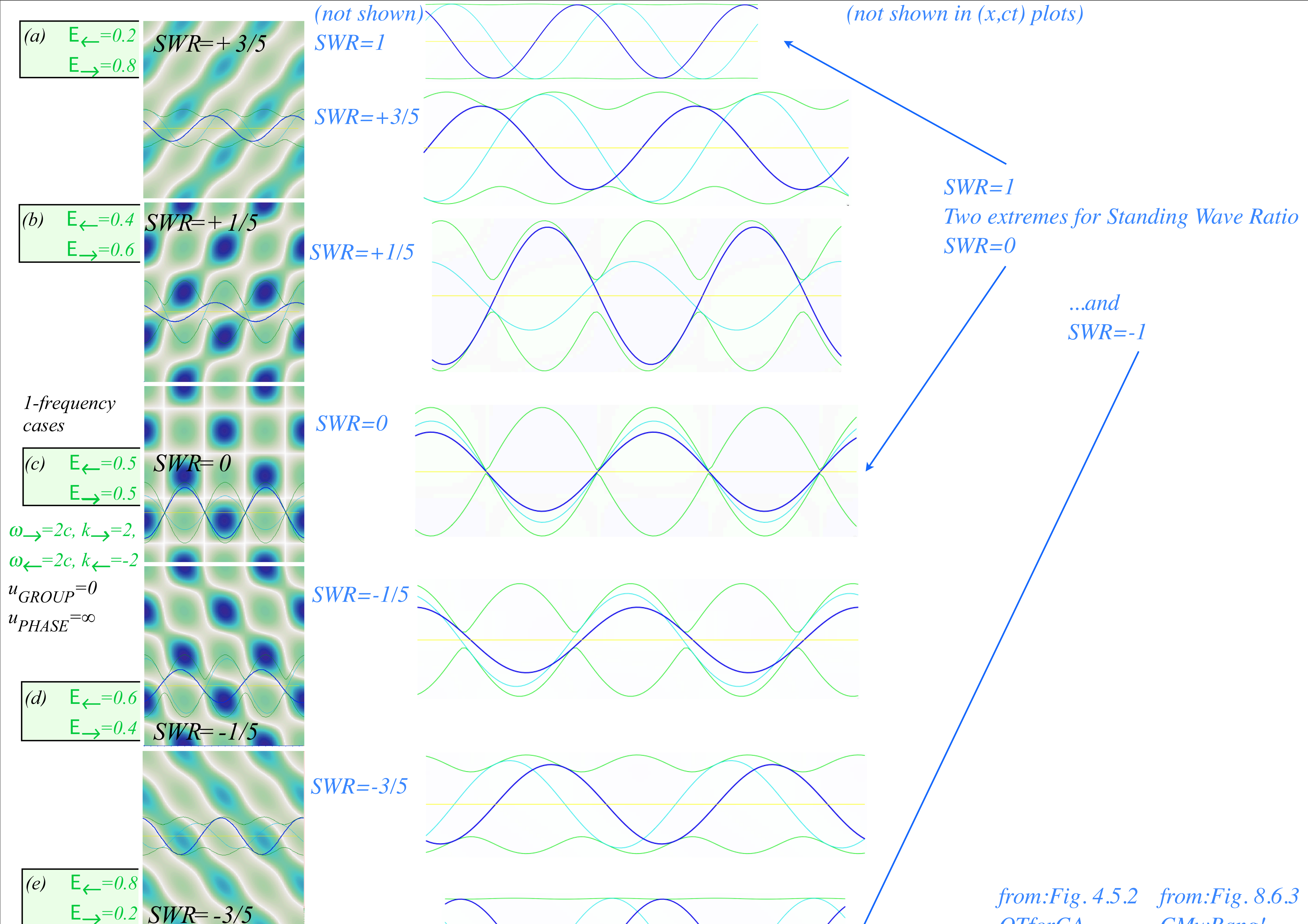
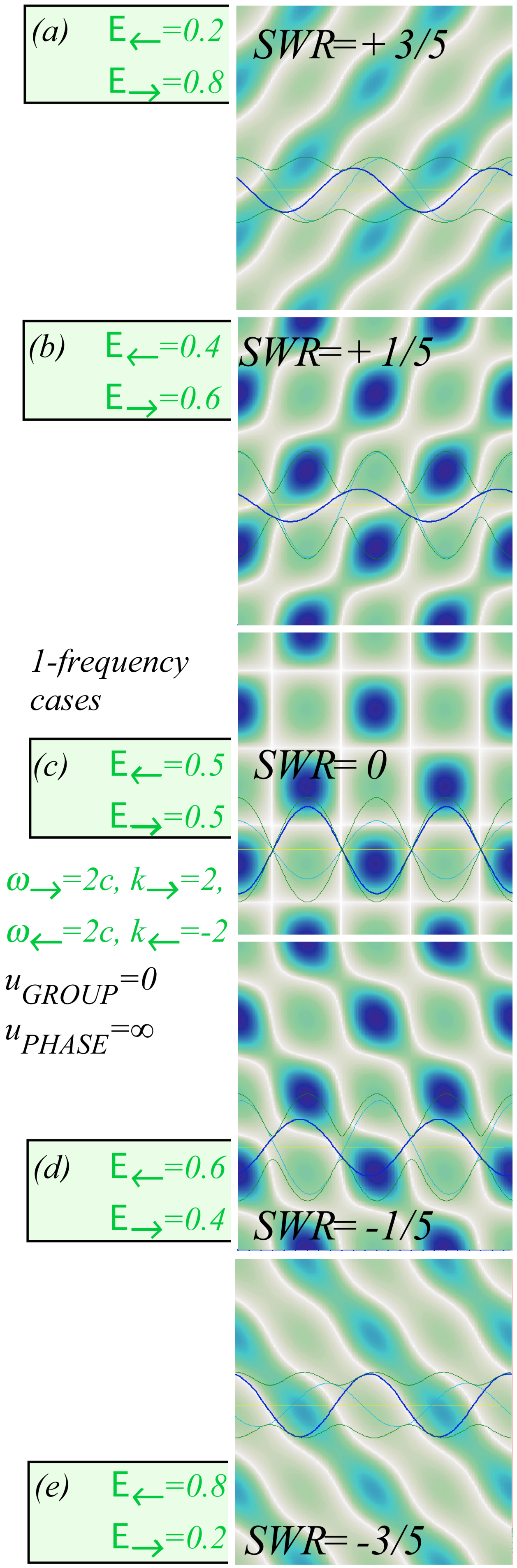



Fig. 6.1 Monochromatic (1-frequency) 2-CW wave space-time patterns.

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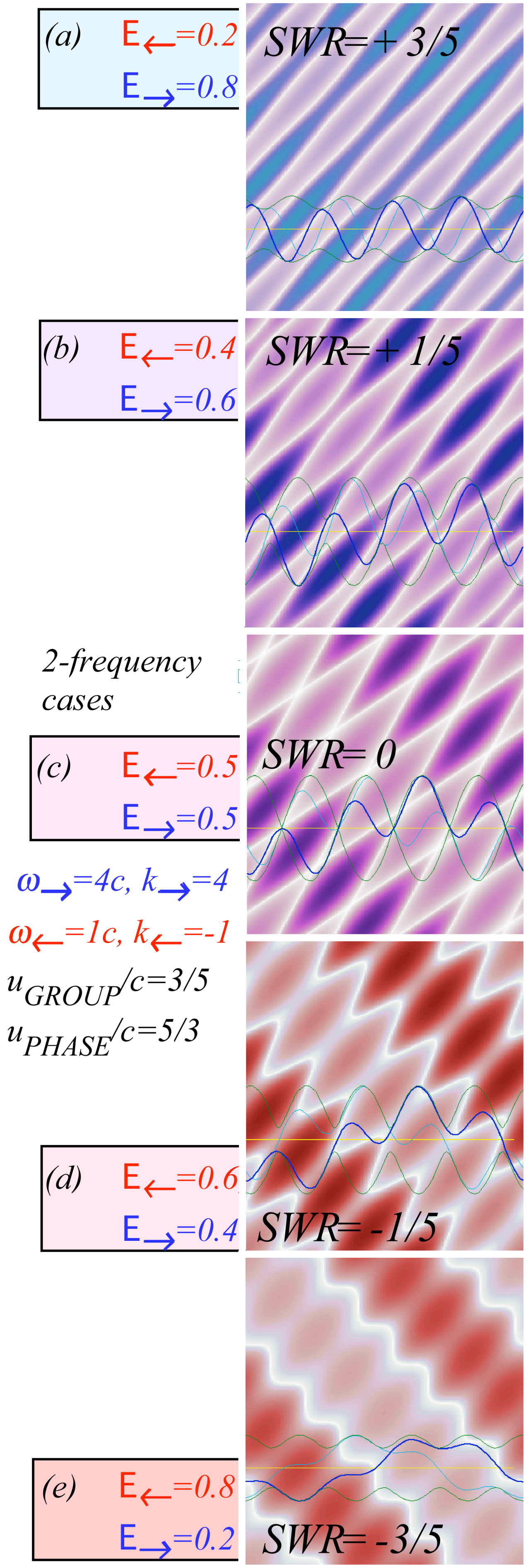


*Fig. 6.2 Dichromatic (2-frequency) 2-CW wave space-time patterns.*

SWR=1 (not shown)

SWR=+3/5

SWR=+1/5

SWR=0

SWR=-1/5

SWR=-3/5

SWR=-1 (not shown)

Same SWR cases viewed at u/c = 3/5

SWR=1
Two extremes for Standing Wave Ratio
SWR=0

...and
SWR=-1

www.uark.edu/ua/pirelli/php/amplitude_probability_2.php

from: Fig. 4.5.2 QTforCA Unit 2 Ch.4

from: Fig. 8.6.3 CMwBang! Unit 8 Ch.6

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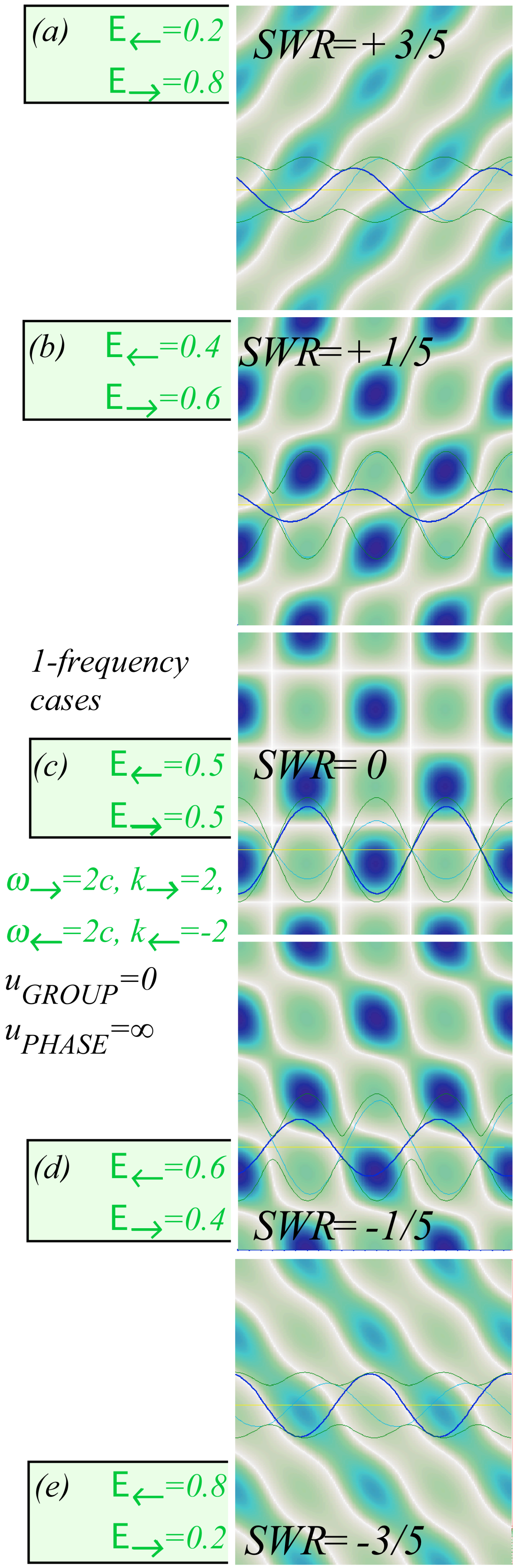


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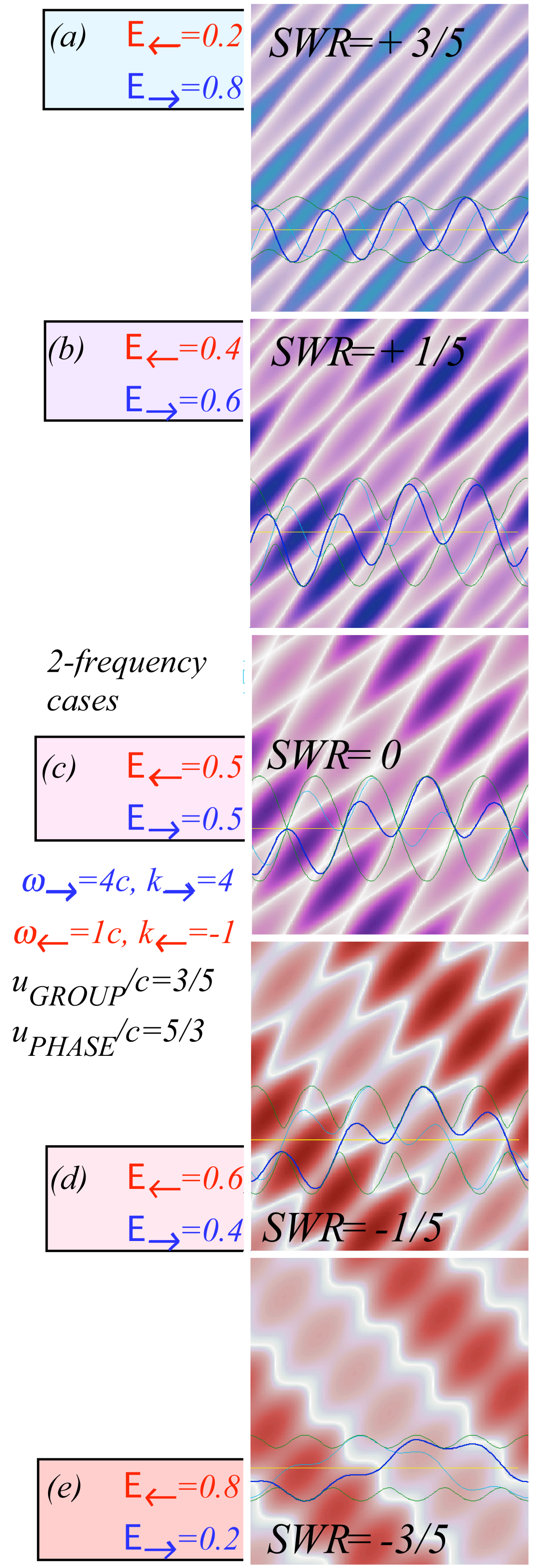



Fig. 6.2 Dichromatic (2-frequency) 2-CW wave space-time patterns.

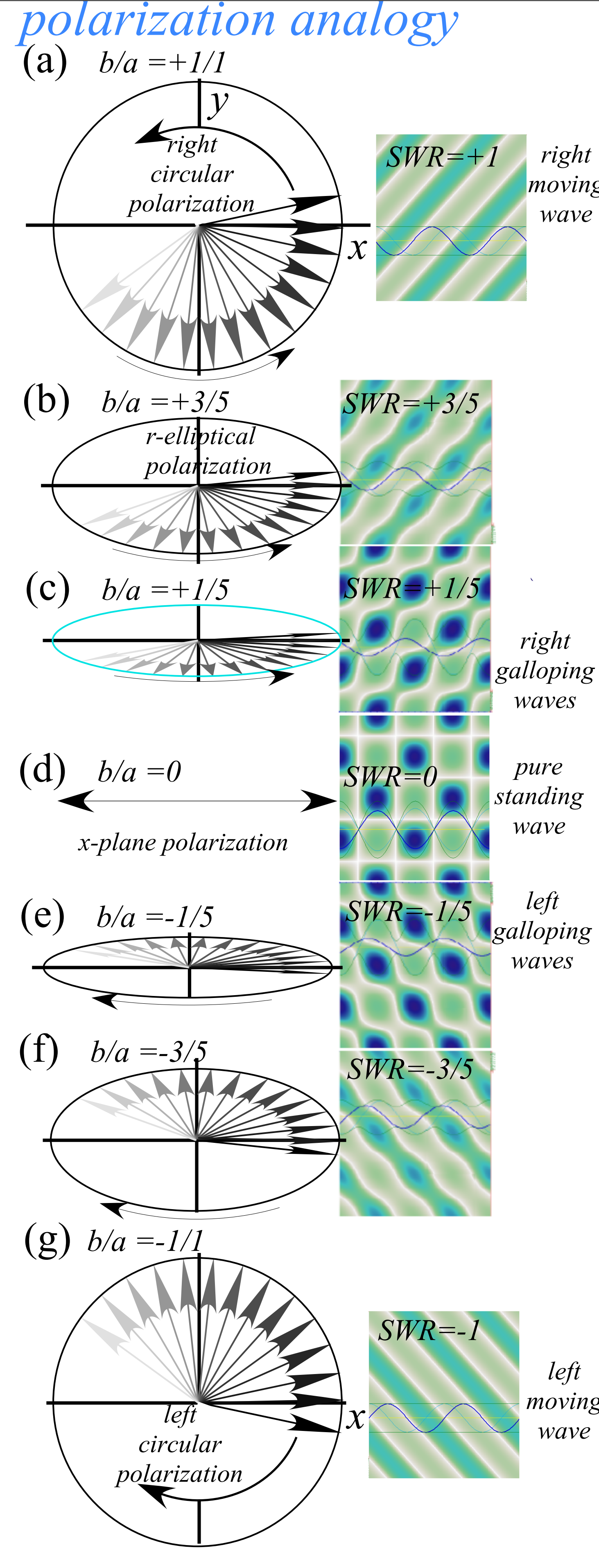
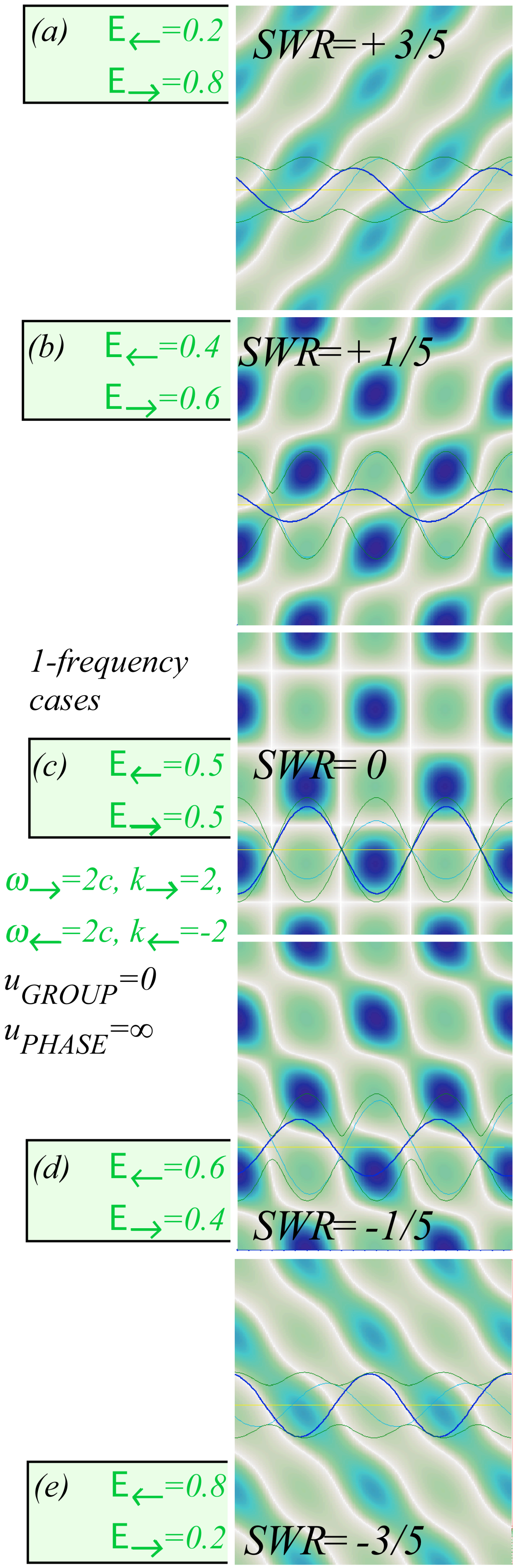



Fig. 6.3 (a-g) Elliptic polarization ellipses relate to galloping waves in Fig. 6.1. (h-i) Kepler anomalies.

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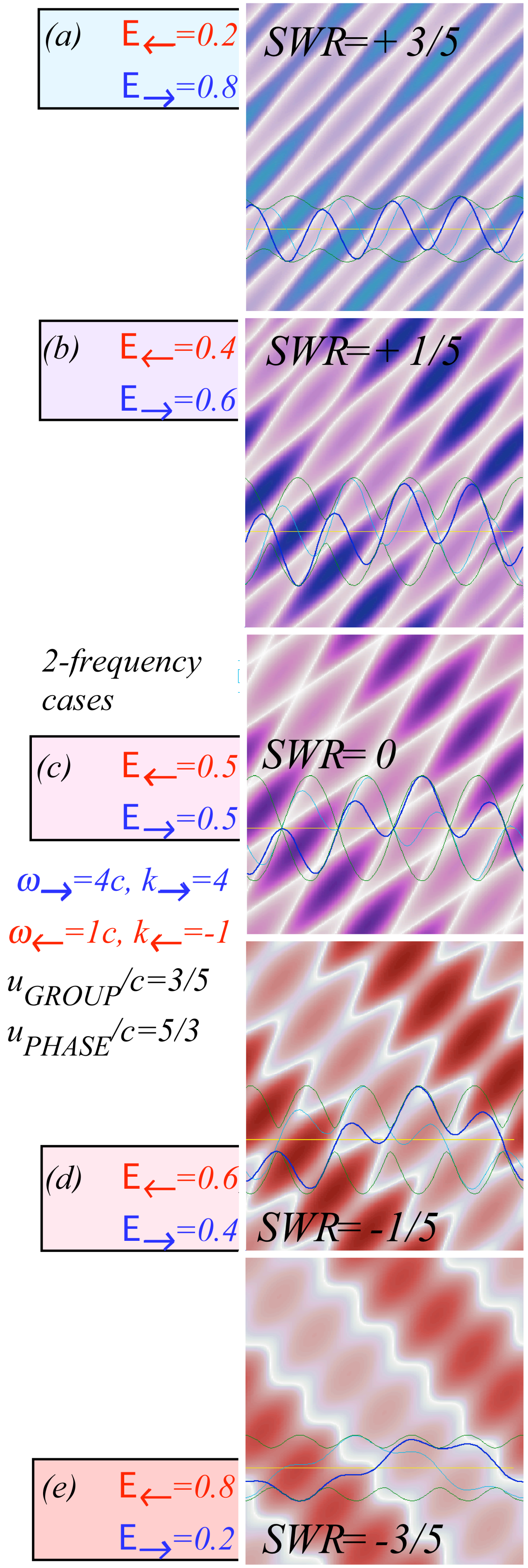


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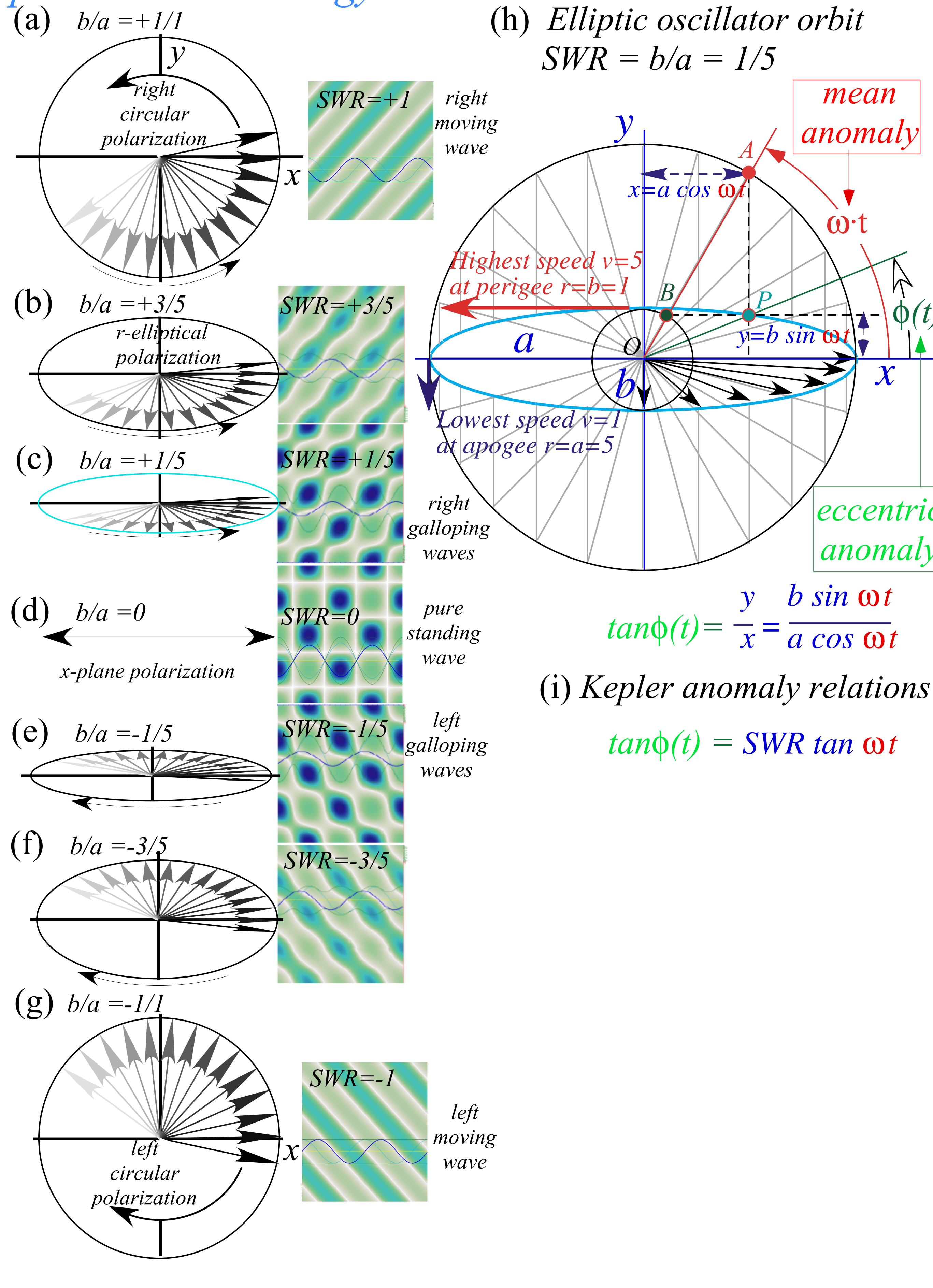
## polarization analogy

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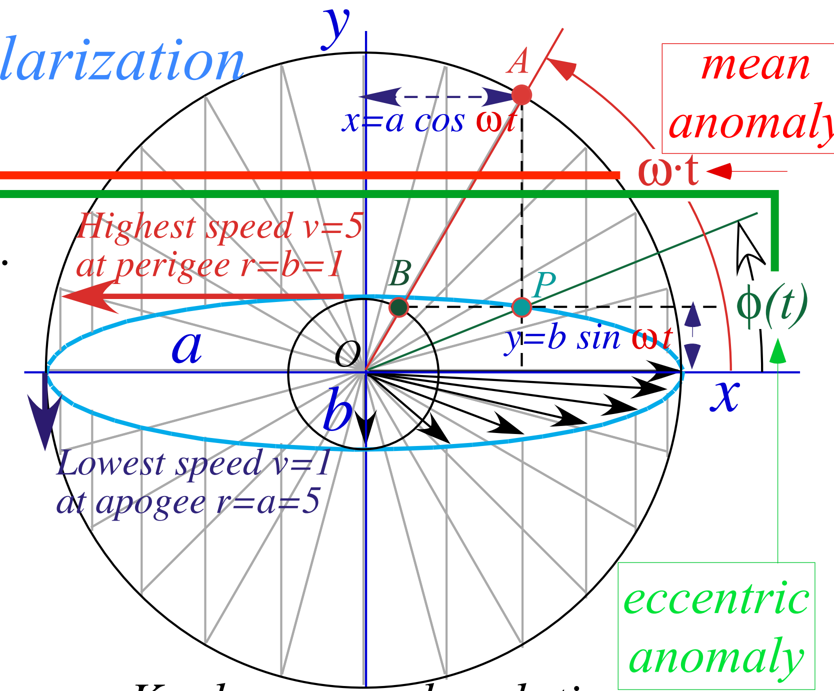
→ *Galloping dynamics algebra*

*Waves that go back in time - The Feynman-Wheeler Switchback*

# *Analogy between wave galloping, Keplarian IHO orbits, and optical polarization*

We'll show wave galloping is analogous to Keplarian orbital motion of angles $\omega \cdot t$ and $\phi$ of orbits.

$$\tan\phi(t) = \frac{b}{a}\tan\omega\cdot t$$



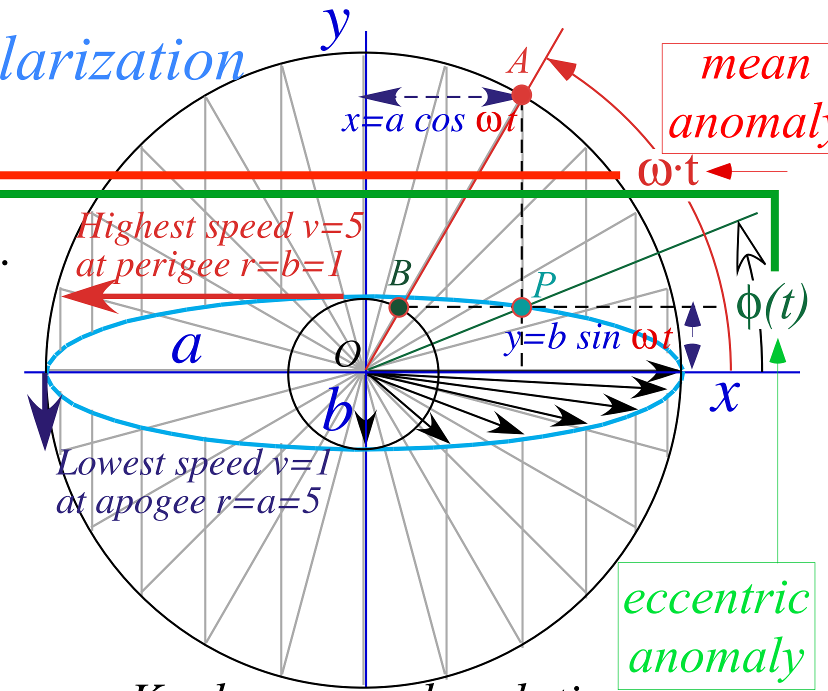

*Kepler anomaly relations*

$$\tan\phi(t) = \frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t} = SWR\cdot \tan\omega t$$

*from:Fig. 4.5.2 QTforCA Unit 2 Ch.4*

*from:Fig. 8.6.3 CMwBang! Unit 8 Ch.6*

# Analogy between wave galloping, Keplarian IHO orbits, and optical polarization

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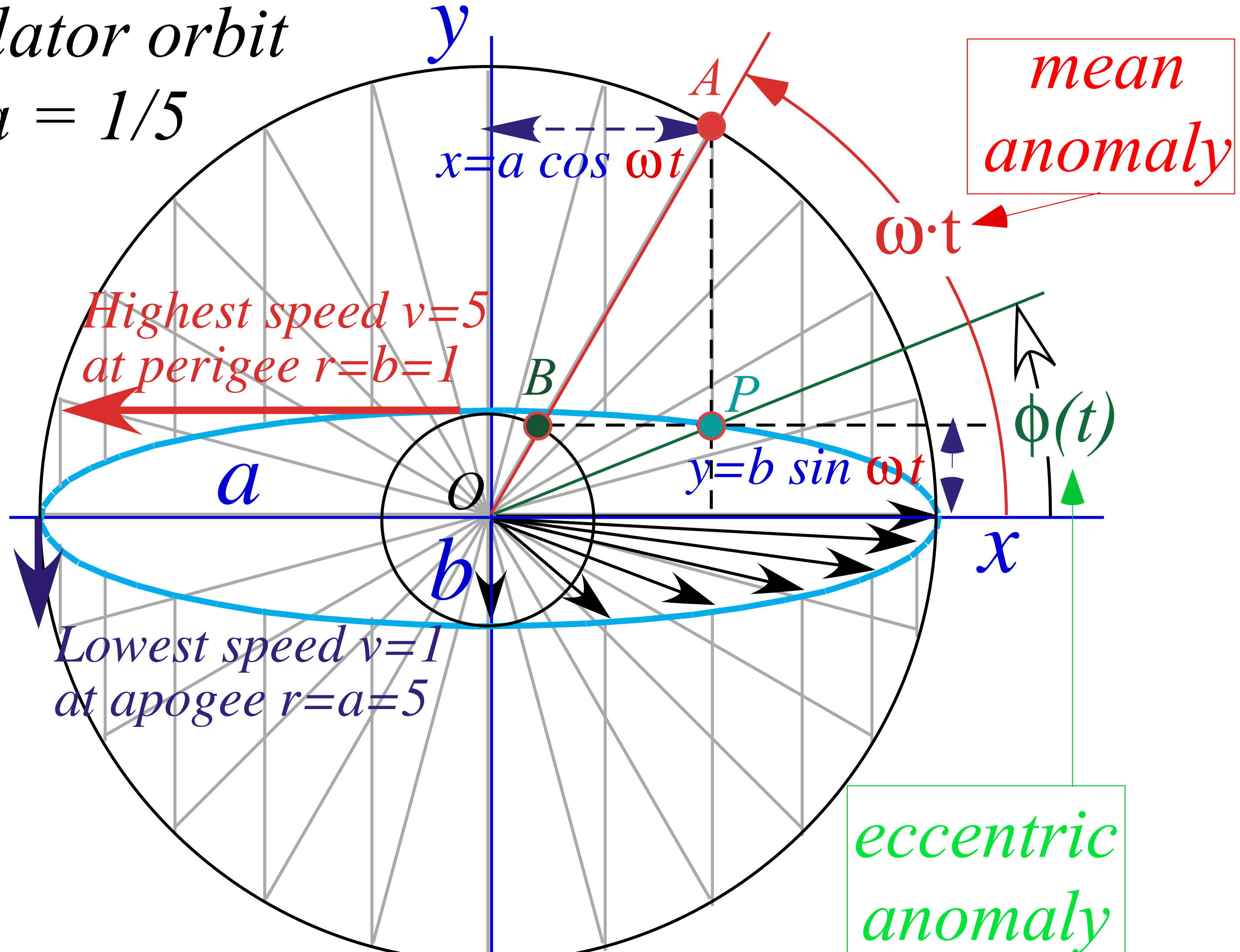
## Elliptic oscillator orbit

$SWR = b/a = 1/5$



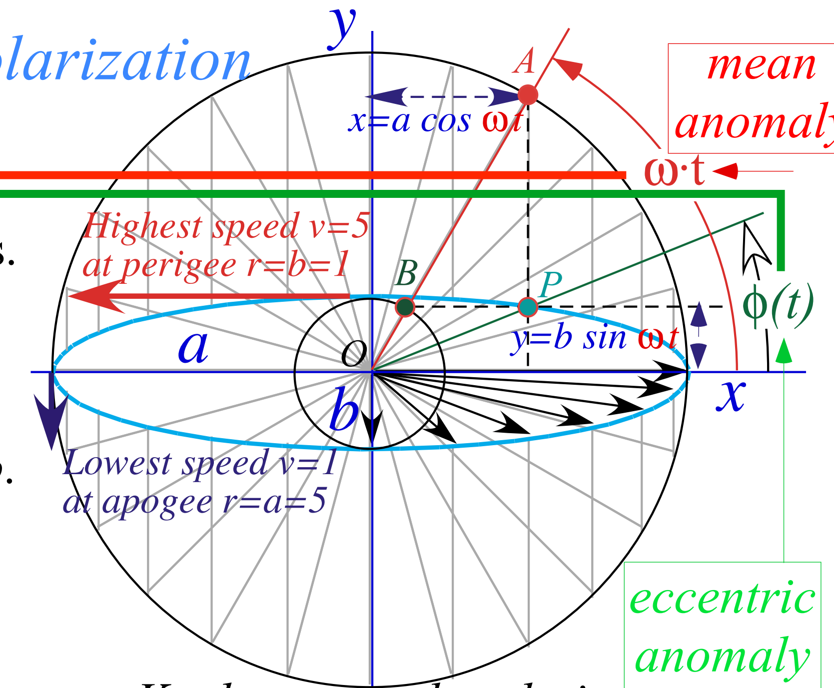

Kepler anomaly relations

$$\tan\phi(t) = SWR\cdot\tan\omega t \qquad \tan\phi(t) = \frac{y}{x} = \frac{b \sin \omega t}{a \cos \omega t}$$

from:Fig. 4.5.2 QTforCA Unit 2 Ch.4

from:Fig. 8.6.3 CMwBang! Unit 8 Ch.6

## *Analogy between wave galloping, Keplarian IHO orbits, and optical polarization*




We'll show wave galloping is analogous to Keplarian orbital motion of angles $\omega\cdot t$ and $\phi$ of orbits.

$$\tan\phi(t) = \frac{b}{a}\tan\omega\cdot t$$

The eccentric anomaly time derivative of $\phi$ (angular velocity) gallops between $\omega\cdot b/a$ and $\omega\cdot a/b$.

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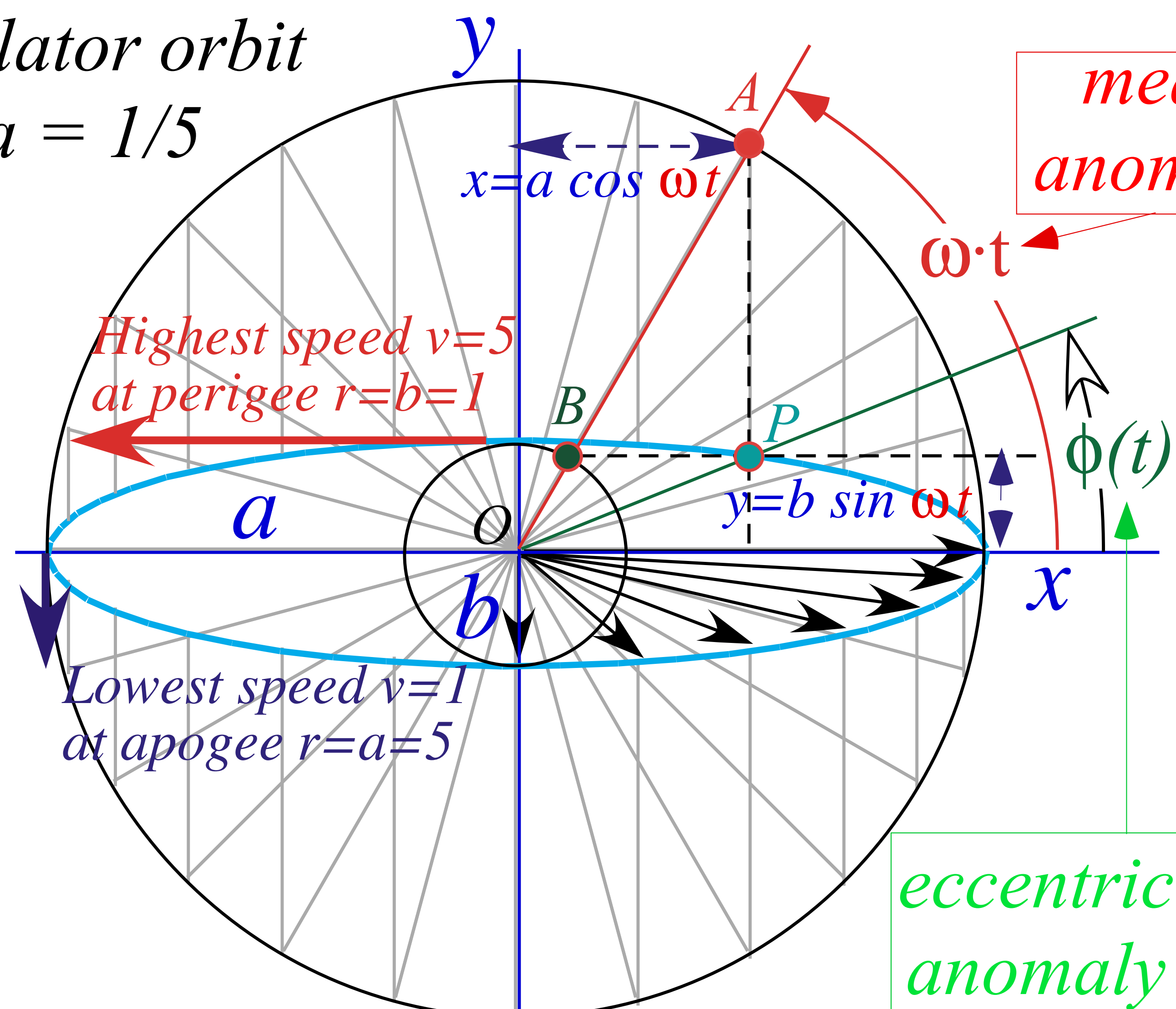
*Kepler anomaly relations*

$$\tan\phi(t) = \frac{y}{x} = \frac{b\sin\omega t}{a\cos\omega t} = SWR\cdot\tan\omega t$$

*Elliptic oscillator orbit*

*SWR = b/a = 1/5*




*Kepler anomaly relations*

$$\tan\phi(t) = SWR\cdot\tan\omega t$$

$$\tan\phi(t) = \frac{y}{x} = \frac{b\sin\omega t}{a\cos\omega t}$$

*from: Fig. 4.5.2 QTforCA Unit 2 Ch.4*

*from: Fig. 8.6.3 CMwBang! Unit 8 Ch.6*

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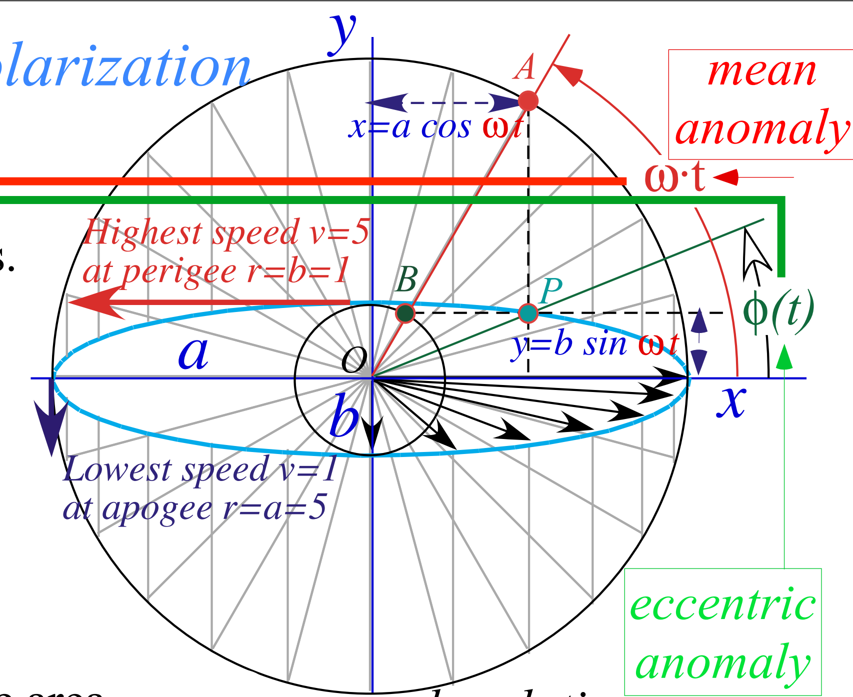

anomaly relations

$$tan\phi(t) = \frac{y}{x} = \frac{b\ sin\ \omega t}{a\ cos\ \omega t} = SWR\cdot tan\ \omega t$$

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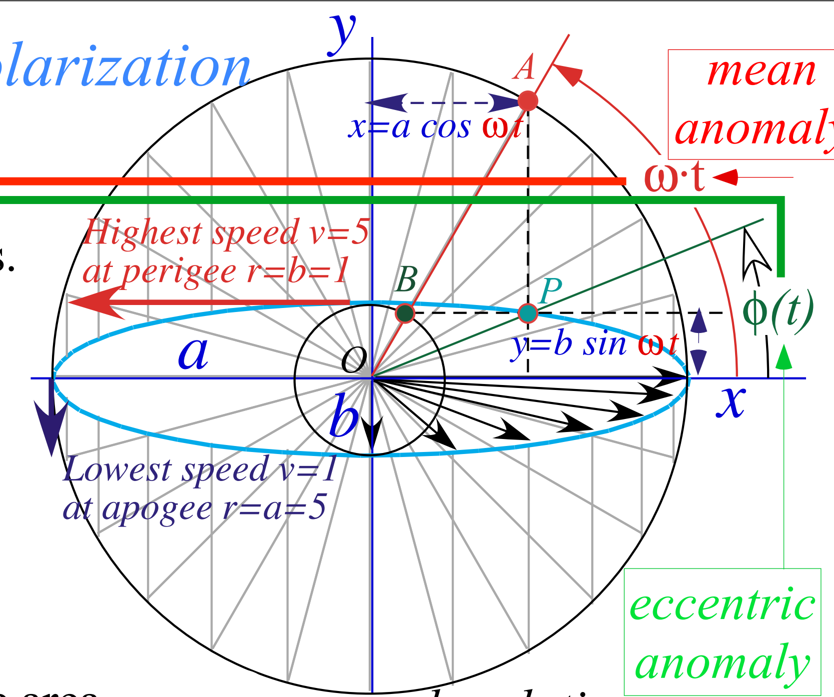


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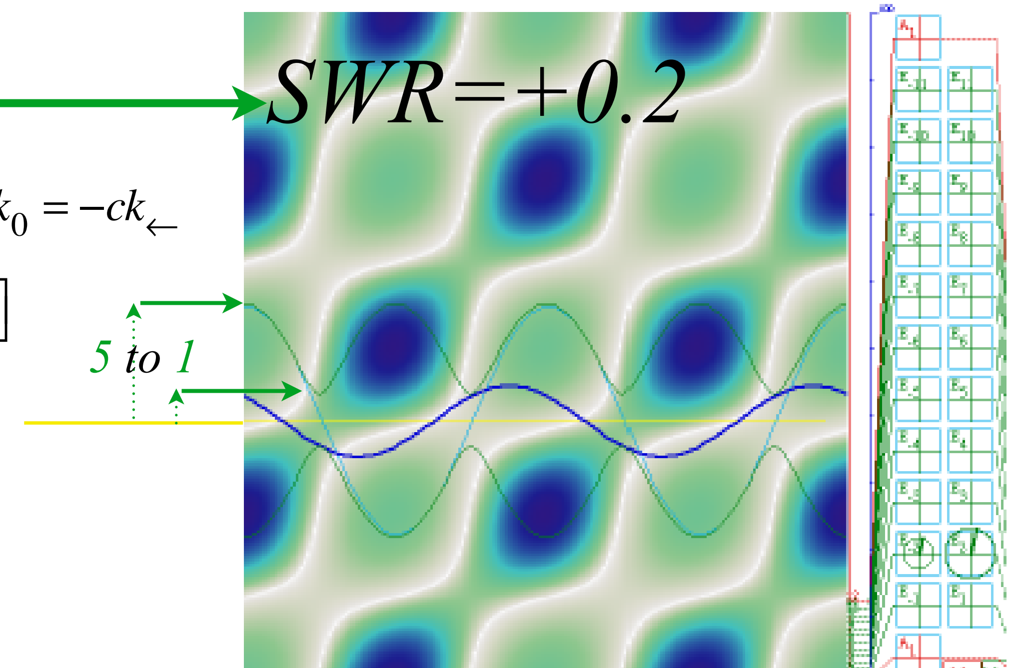
Consider galloping wave zeros of a monochromatic wave having *SWR =1/5*.

$$0 = \mathrm{Re}\,\Psi(x,t) = \mathrm{Re}\left[A_{\rightarrow}e^{i(k_0x-\omega_0t)} + A_{\leftarrow}e^{i(-k_0x-\omega_0t)}\right] \quad \text{where: } \omega_{\rightarrow} = \omega_0 = \omega_{\leftarrow} = ck_0 = -ck_{\leftarrow}$$

$$0 = A_{\rightarrow}\left[\cos k_0x\ \cos\omega_0t + \sin k_0x\ \sin\omega_0t\right] + A_{\leftarrow}\left[\cos k_0x\ \cos\omega_0t - \sin k_0x\ \sin\omega_0t\right]$$

$$\left(A_{\rightarrow} + A_{\leftarrow}\right)\left[\cos k_0x\ \cos\omega_0t\right] = -\left(A_{\rightarrow} - A_{\leftarrow}\right)\left[\sin k_0x\ \sin\omega_0t\right]$$




$E_{\leftarrow}=0.4,\quad E_{\rightarrow}=0.6$

# Analogy between wave galloping, Keplarian IHO orbits, and optical polarization

We'll show wave galloping is analogous to Keplarian orbital motion of angles $\omega\cdot t$ and $\phi$ of orbits.

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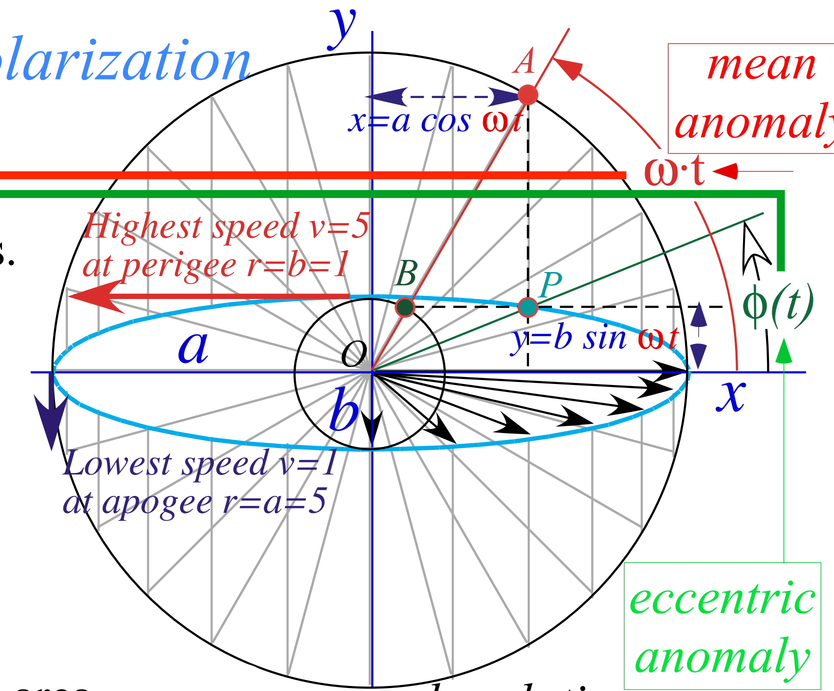


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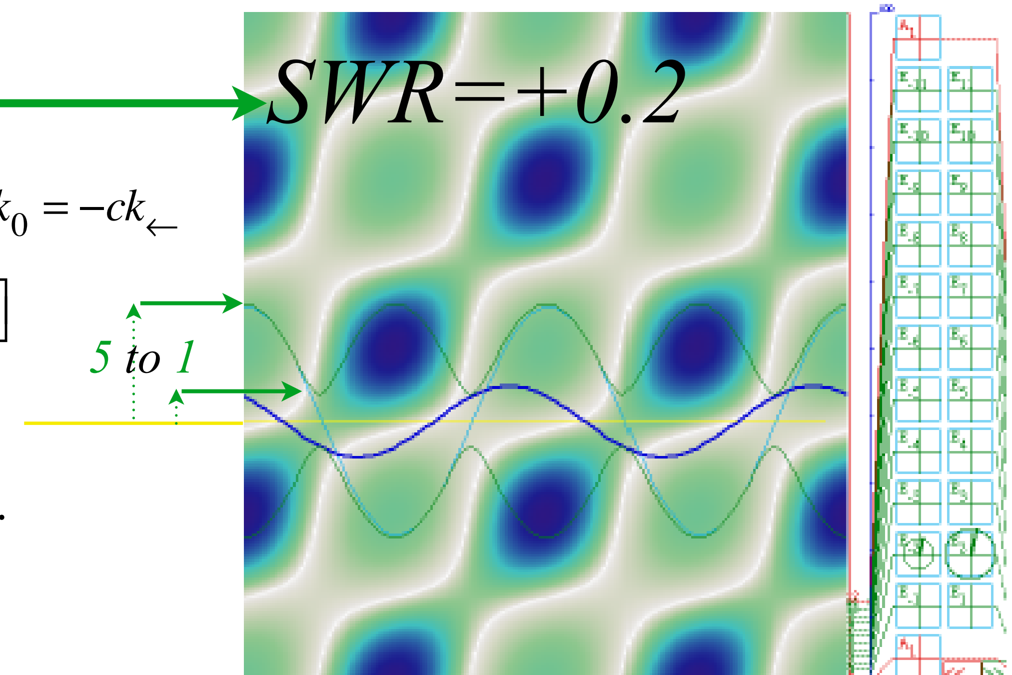
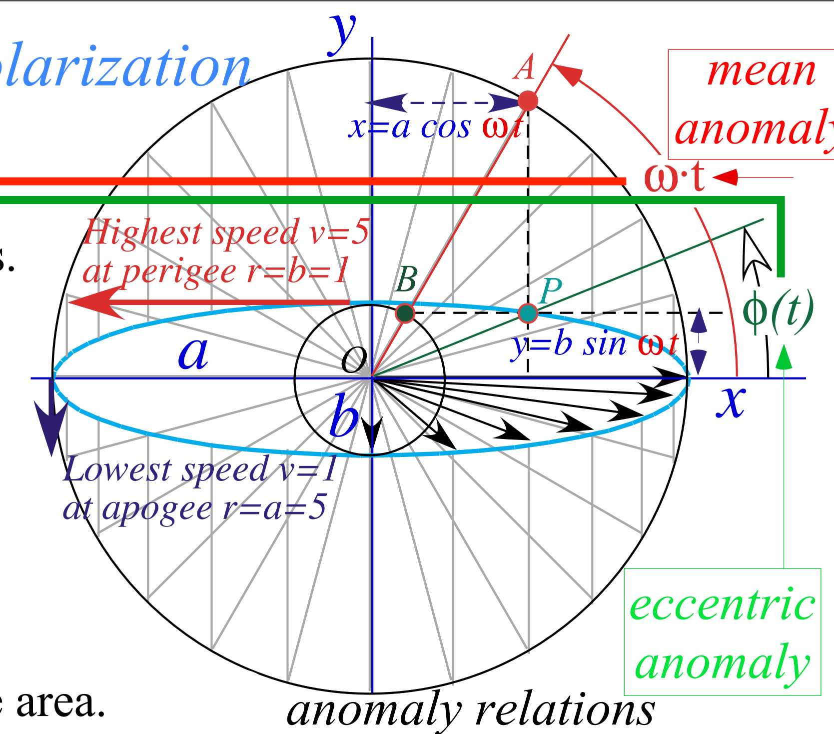



$E_{\leftarrow}=0.4,\quad E_{\rightarrow}=0.6$

Space $k_0x$ varies with time $\omega_0t$ in the same way that eccentric anomaly $\phi$ varies with $\omega\cdot t$ .

$$\tan k_0x = -SWR\cdot\cot\omega_0t = SWR\cdot\tan\omega_0\bar{t} \quad \text{where: } \omega_0\bar{t} = \omega_0t - \pi/2$$

# Analogy between wave galloping, Keplarian IHO orbits, and optical polarization

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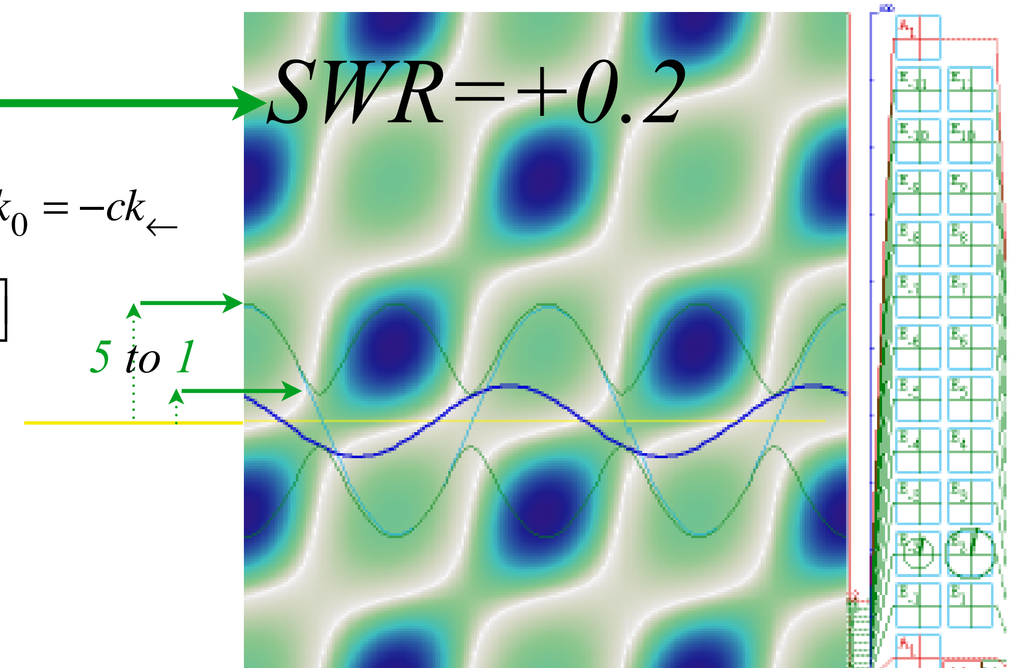



$$0=\mathrm{Re}\,\Psi(x,t)=\mathrm{Re}\left[A_{\rightarrow}e^{i(k_0x-\omega_0t)}+A_{\leftarrow}e^{i(-k_0x-\omega_0t)}\right]\quad\text{where: }\omega_{\rightarrow}=\omega_0=\omega_{\leftarrow}=ck_0=-ck_{\leftarrow}$$

$$0=A_{\rightarrow}\left[\cos k_0x\ \cos\omega_0t+\sin k_0x\sin\omega_0t\right]+A_{\leftarrow}\left[\cos k_0x\ \cos\omega_0t-\sin k_0x\sin\omega_0t\right]$$

$$(A_{\rightarrow}+A_{\leftarrow})\left[\cos k_0x\ \cos\omega_0t\right]=-(A_{\rightarrow}-A_{\leftarrow})\left[\sin k_0x\sin\omega_0t\right]$$

Space $k_0x$ varies with time $\omega_0t$ in the same way that eccentric anomaly $\phi$ varies with $\omega\cdot t$.

$$\tan k_0x=-SWR\cdot\cot\omega_0t=SWR\cdot\tan\omega_0\bar{t}\quad\text{where: }\omega_0\bar{t}=\omega_0t-\pi/2$$

$E_{\leftarrow}=0.4,\quad E_{\rightarrow}=0.6$

**Speed of galloping wave zeros is the time derivative of root location *x* in units of light velocity *c*.**

$$\frac{dx}{dt}=c\cdot SWR\frac{\sec^2\omega_0\bar{t}}{\sec^2k_0x}=\frac{c\cdot SWR}{\cos^2\omega_0\bar{t}+SWR^2\cdot\sin^2\omega_0\bar{t}}=\begin{cases}c\cdot SWR & \text{for: }\bar{t}=0,\ \pi,\ 2\pi...\\ c\cdot SWQ & \bar{t}=\pi/2,\ 3\pi/2,...\end{cases}$$

# Wave-Zero Speed-Limits

## *Standing Wave Ratio SWR and Quotient SWQ*

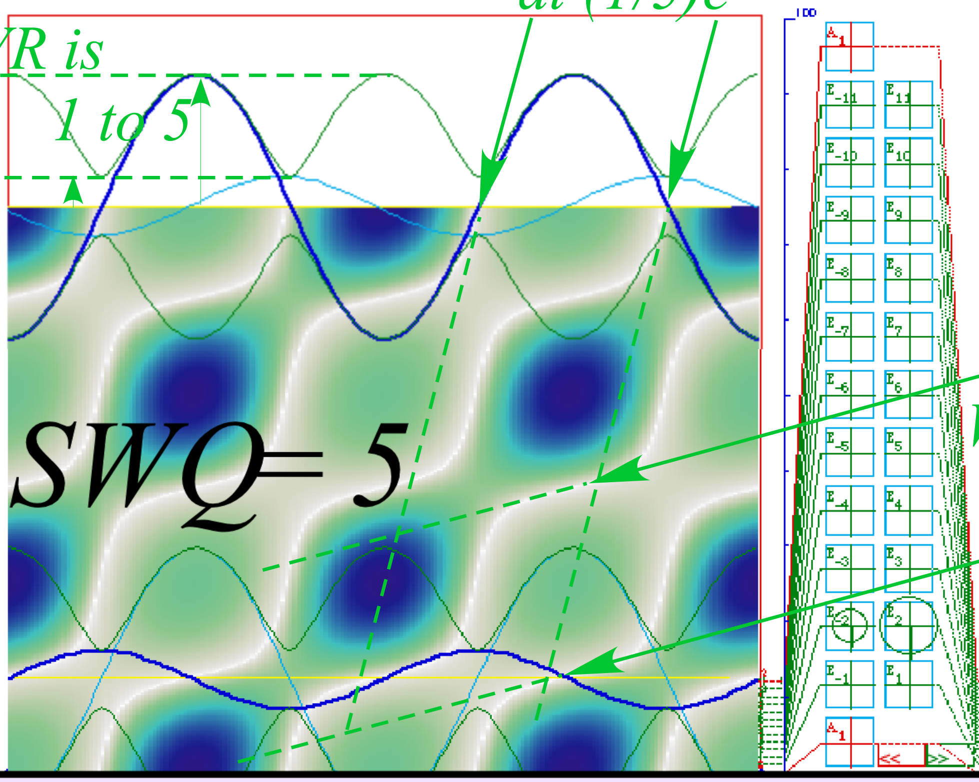
$$SWR = (E_{\rightarrow} - E_{\leftarrow})/(E_{\rightarrow} + E_{\leftarrow}) = 1/SWQ$$

Wave zeros "galloping" at ∞-speed

SWR=1/5

Wave zeros "resting" at (1/5)c

SWR is 1 to 5




Wave zeros "galloping" at 5c

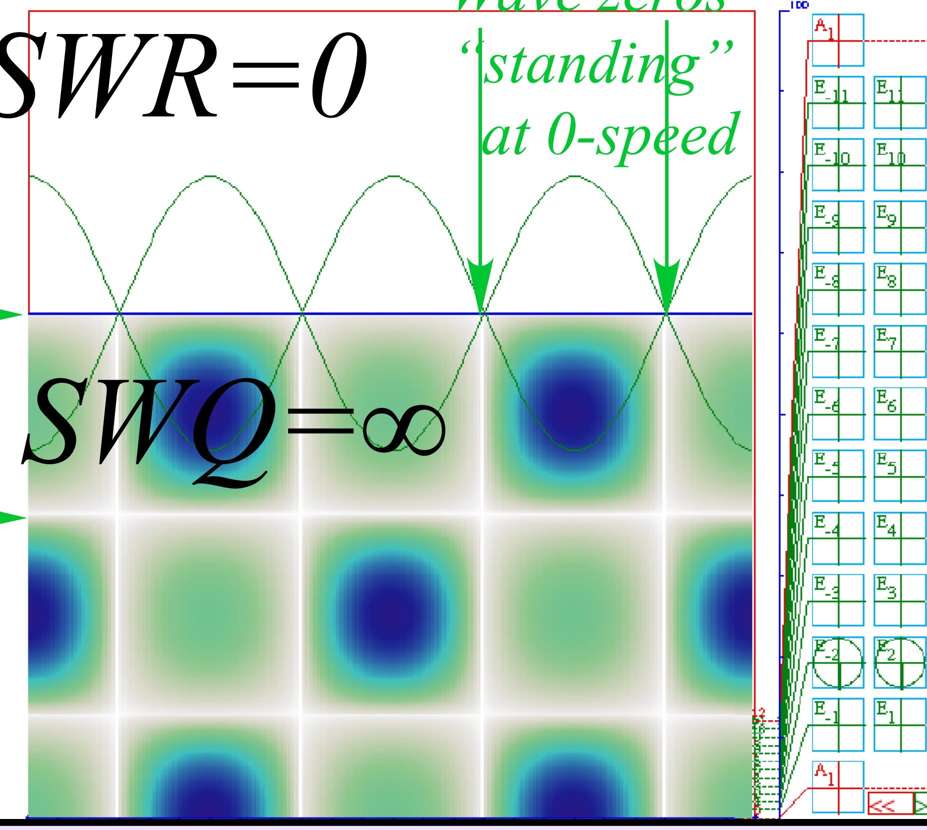
$\omega_{\rightarrow}=2c \quad \omega_{\leftarrow}=2c$

$k_{\rightarrow}=2 \quad k_{\leftarrow}=-2$

$u_{GROUP}=0 \ , \ u_{PHASE}=\infty$

SWR=0

Wave zeros "standing" at 0-speed




$E_{\leftarrow}=0.5, \ E_{\rightarrow}=0.5$

E = 0.4, E = 0.6

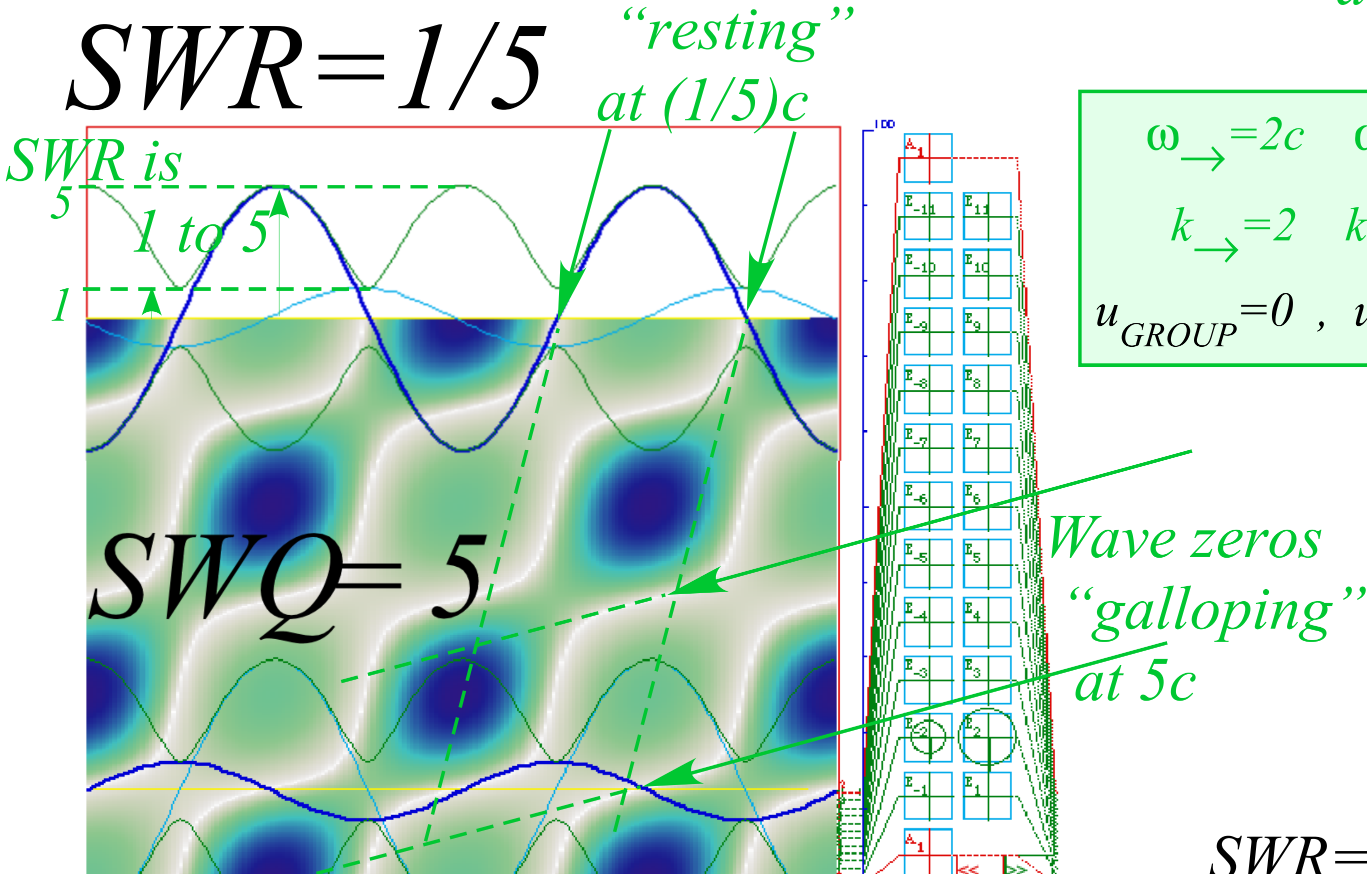
**Speed of galloping wave zeros is the time derivative of root location *x* in units of light velocity *c*.**

$$\frac{dx}{dt} = c \cdot SWR \frac{\sec^2 \omega_0 \bar{t}}{\sec^2 k_0 x} = \frac{c \cdot SWR}{\cos^2 \omega_0 \bar{t} + SWR^2 \cdot \sin^2 \omega_0 \bar{t}} = \begin{cases} c \cdot SWR & \text{for: } \bar{t} = 0, \pi, 2\pi ... \\ c \cdot SWQ & \bar{t} = \pi/2, 3\pi/2, ... \end{cases}$$

# Wave-Zero Speed-Limits

## *Standing Wave Ratio SWR and Quotient SWQ*

$$SWR=(E_{\rightarrow}-E_{\leftarrow})/(E_{\rightarrow}+E_{\leftarrow})=1/SWQ$$

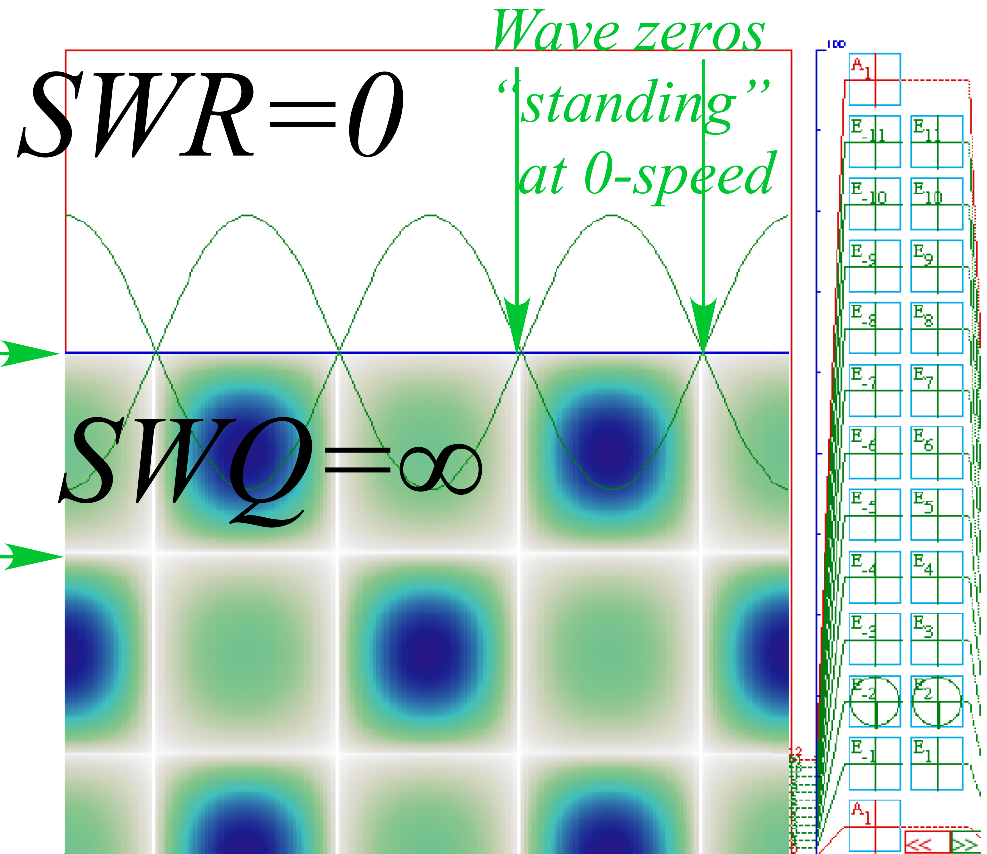



$E \; =0.4, E \; =0.6$

$\omega_{\rightarrow}=2c \quad \omega_{\leftarrow}=2c$

$k_{\rightarrow}=2 \quad k_{\leftarrow}=-2$

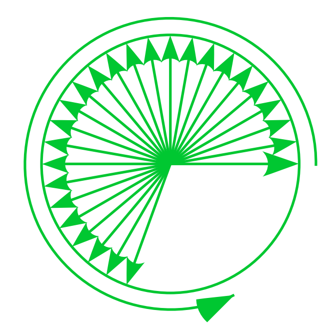
$u_{GROUP}=0 \;, \; u_{PHASE}=\infty$



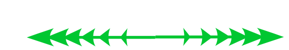

$E_{\leftarrow}=0.5, E_{\rightarrow}=0.5$

*SWR=1/5 is analogous to (5-to-1)*
*Right Elliptic Polarization*

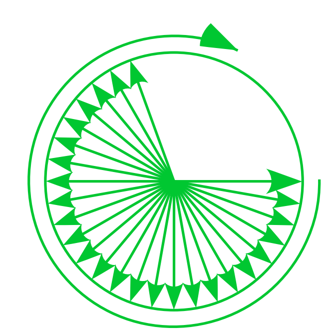

*SWR=1 is analogous to (1,i)*
*Right Circular Polarization*

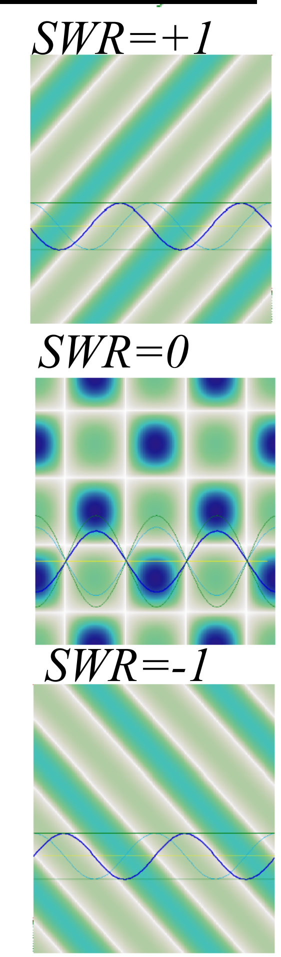


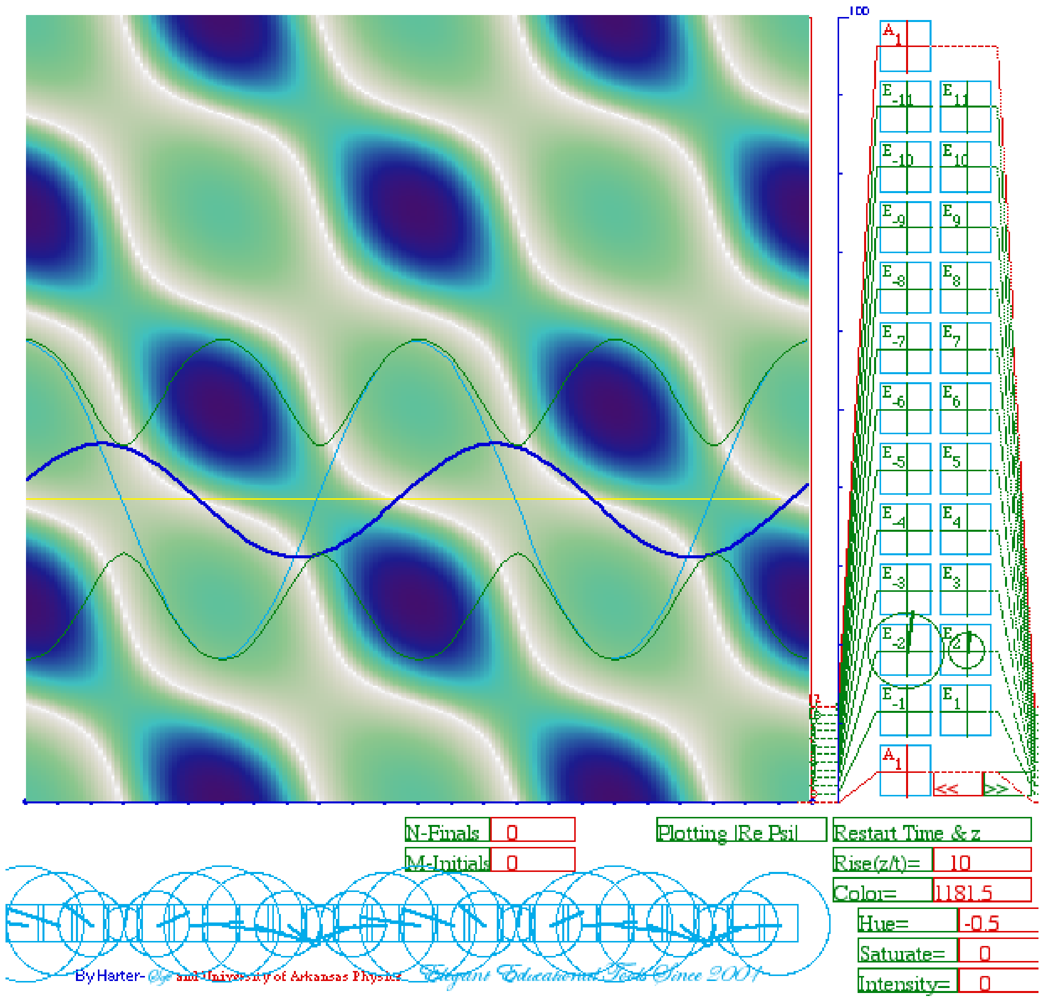
*SWR=0 is analogous to (1,0)*
*x-Plane Linear Polarization*

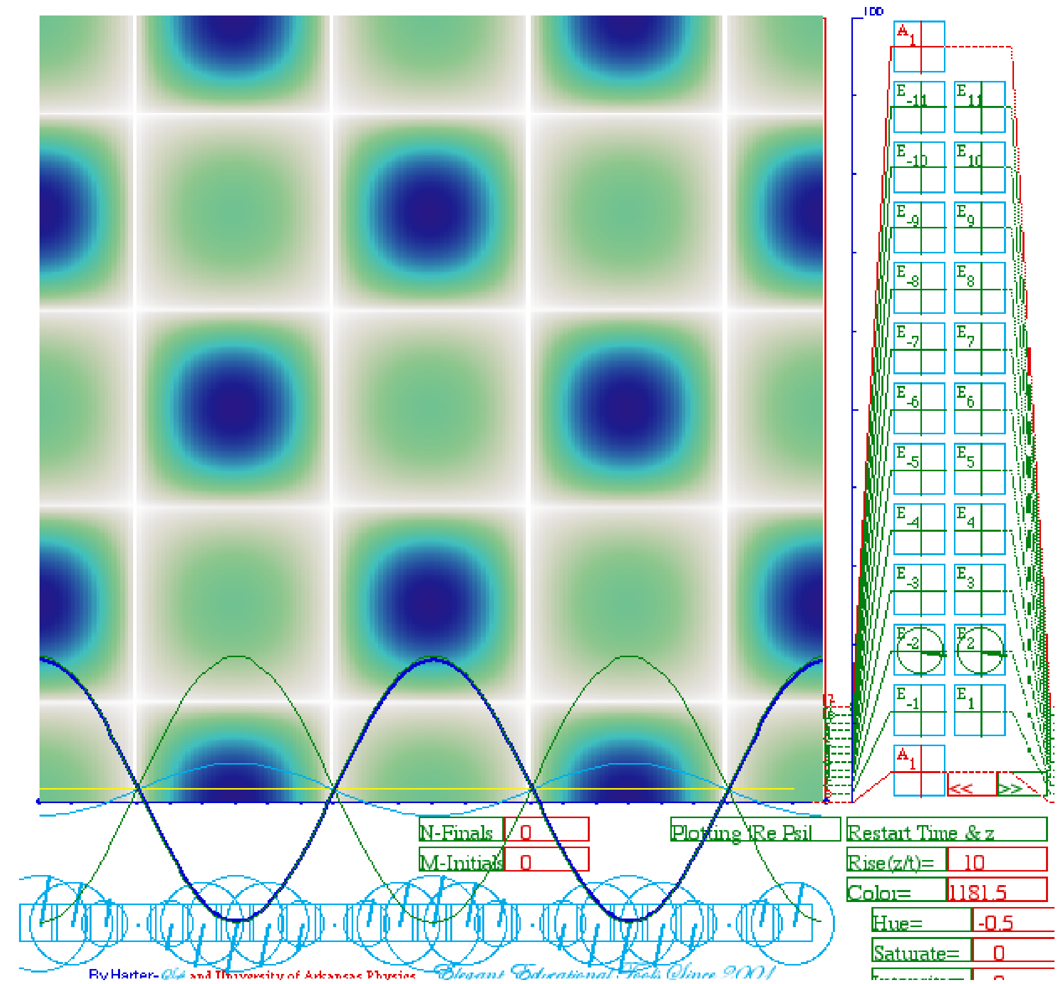


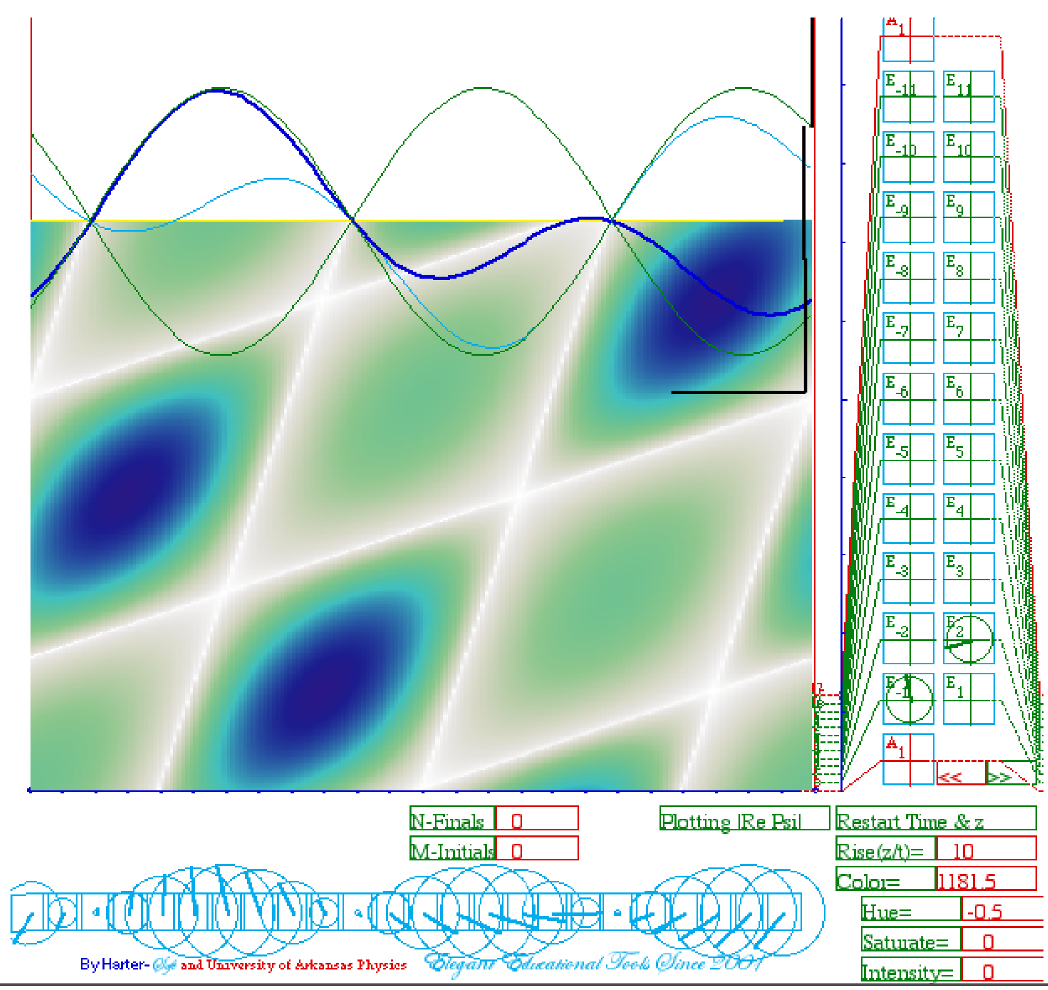
*SWR=-1 is analogous to (1,-i)*
*Left Circular Polarization*

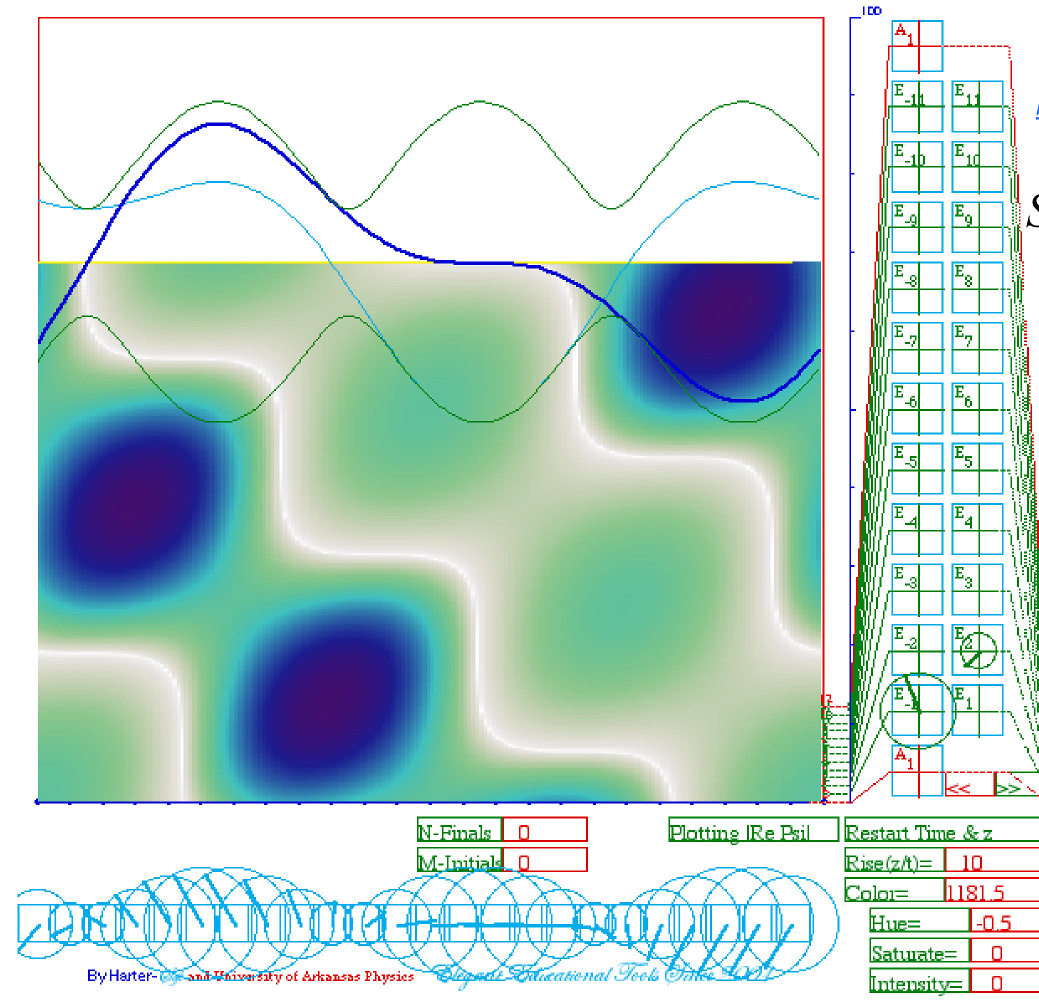







*Staircase Galloping*
Speed of galloping
*SWR*=+*1/2* cancelled
by group velocity
$u_{GROUP}/c = -1/2$.

*Unmatched amplitudes giving galloping waves*

*Standing Wave Ratio (SWR) and Standing Wave Quotient (SWQ)*

*Analogy with group and phase*

*Galloping waves*

*Analogy between wave galloping, Keplarian IHO orbits, and optical polarization*

*Galloping dynamics algebra*

*Waves that go back in time - The Feynman-Wheeler Switchback*

*The Ship-Barn-and-Butler saga of confused causality*

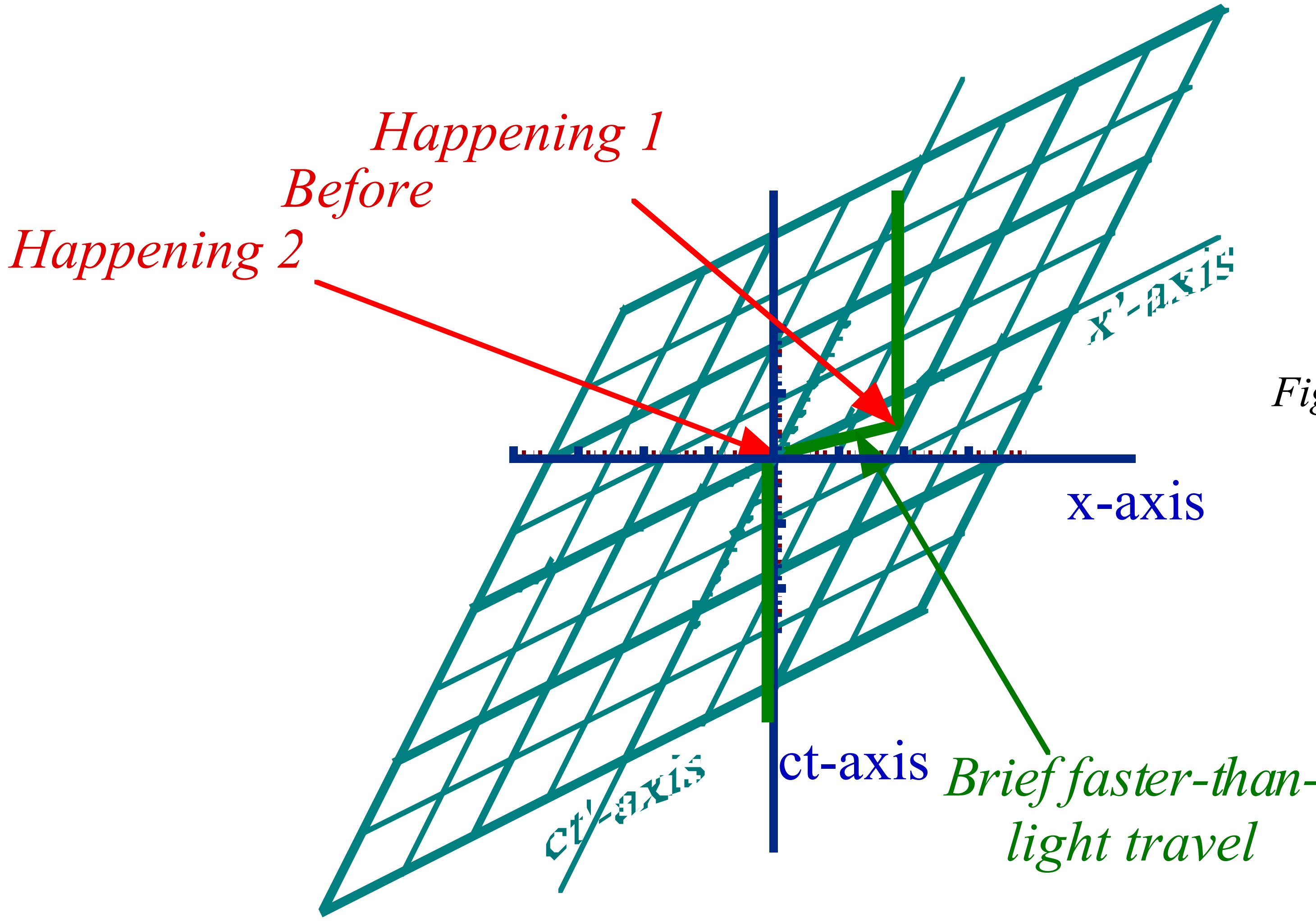
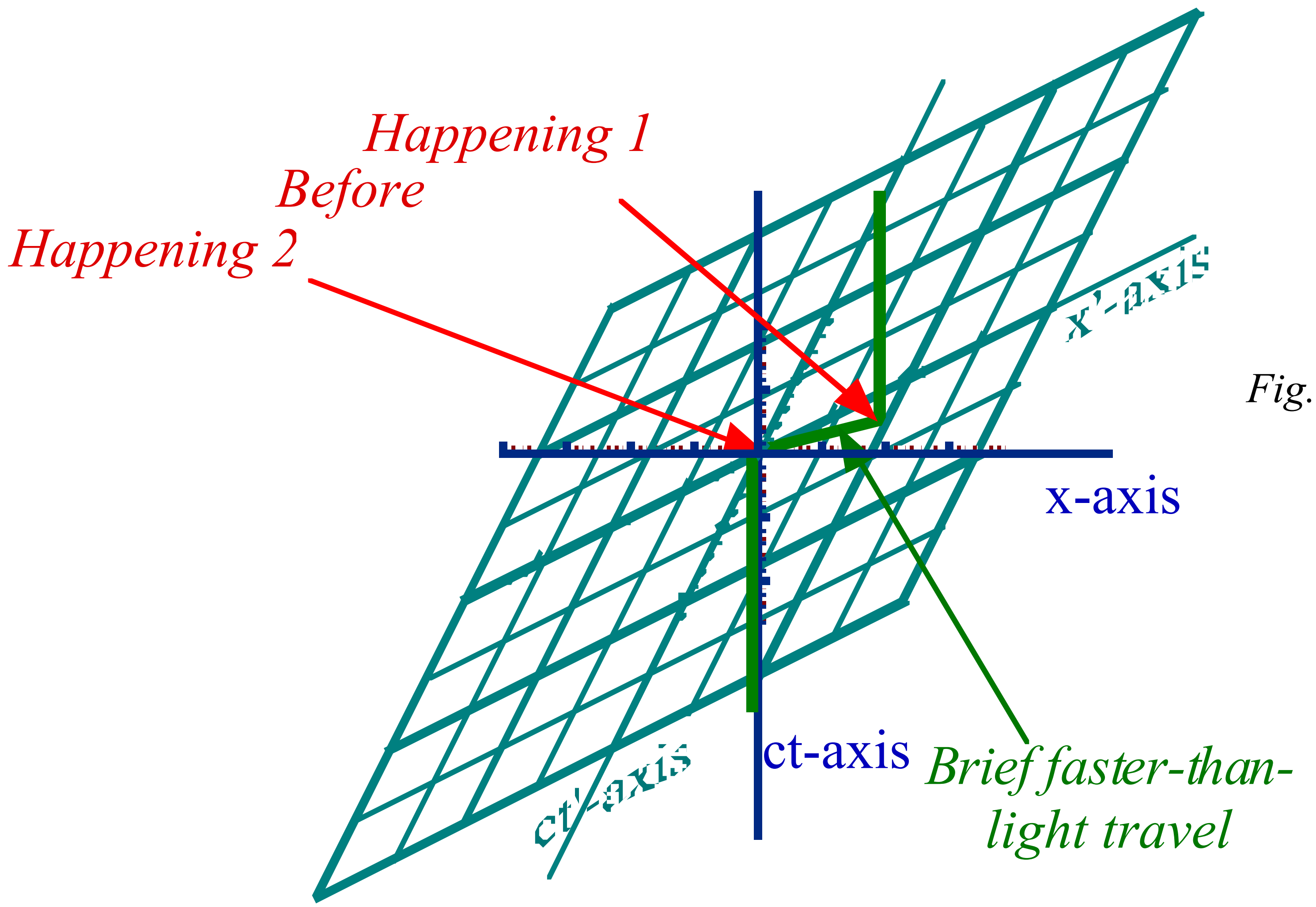



*Fig. 2.B.10 Lighthouse plot of two Happenings*

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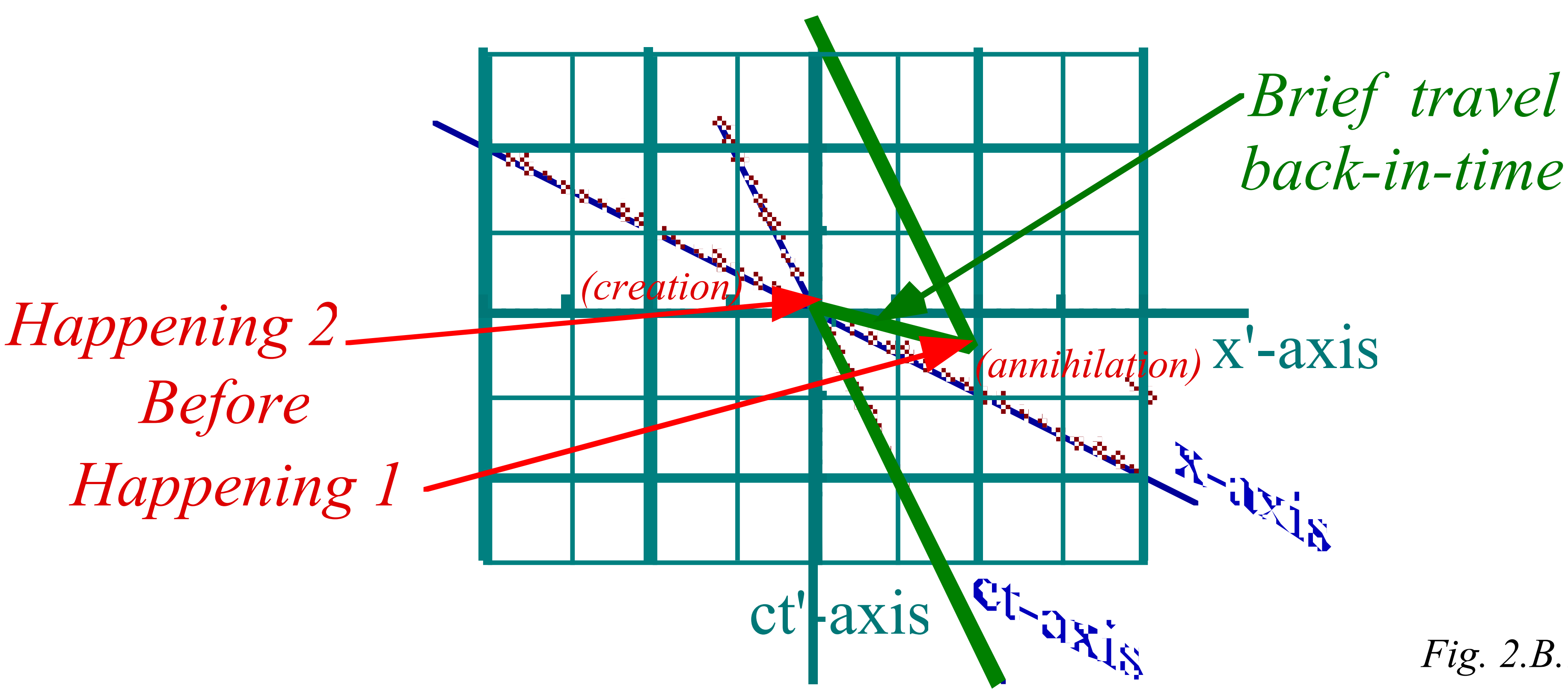



*Fig. 2.B.11 Ship plot of two Happenings*

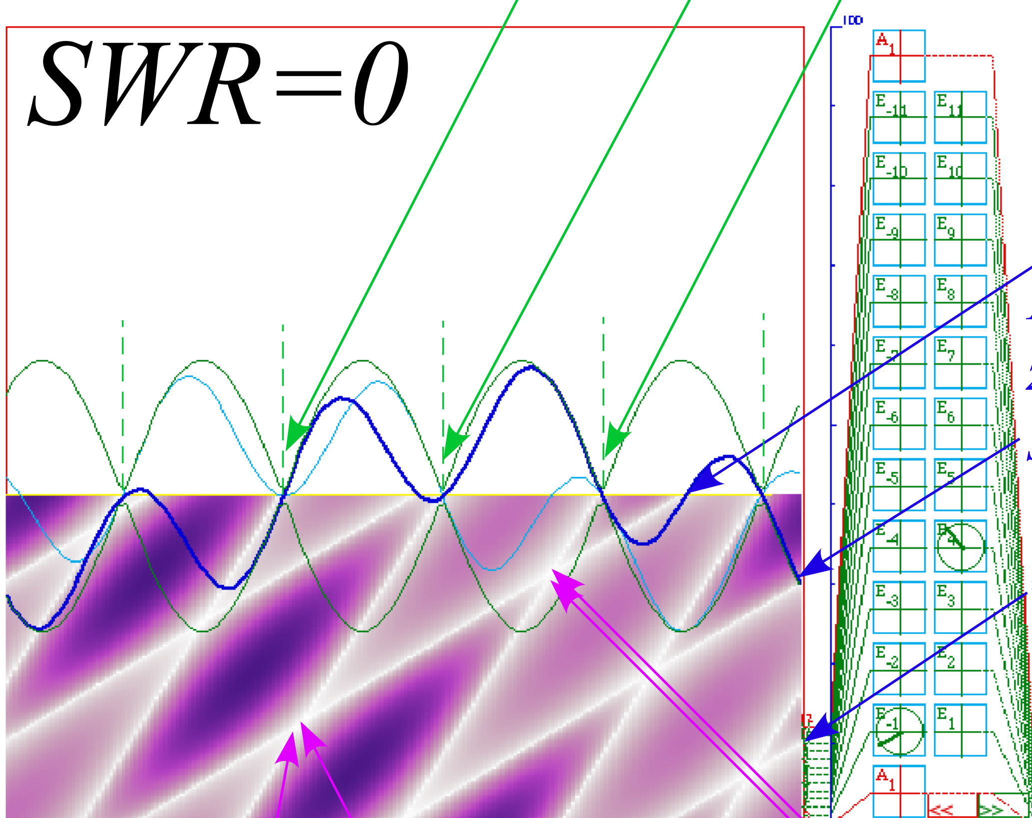
# Minkowski Zero-Grids are Spacetime *Switchbacks* for $-u_{GROUP}<SWR<0$

| | |
|---|---|
| $\omega_{\rightarrow}=4c$ | $\omega_{\leftarrow}=1c$ |
| $k_{\rightarrow}=4,$ | $k_{\leftarrow}=-1$ |
| $u_{GROUP}=c3/5$ | $u_{PHASE}=c5/3$ |

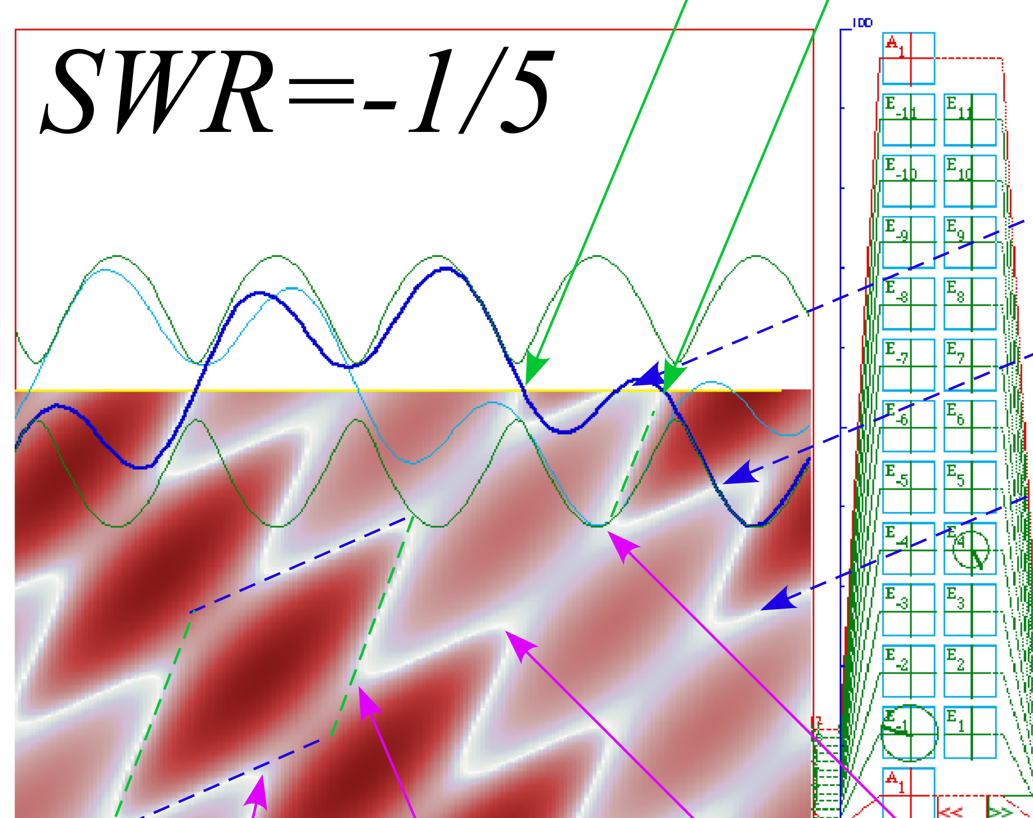
*Group-zero speed* $u_{GROUP}=c3/5$

*Group zero speed limit* $\dfrac{u_{GROUP}+SWR}{1+u_{GROUP}\cdot\dfrac{SWR}{c^2}}=5c/11$

$$\frac{\frac{3}{5}+\frac{-1}{5}}{1+\frac{3}{5}\frac{-1}{5}}=\frac{\frac{2}{5}}{\frac{22}{25}}=\frac{5}{11}$$




*Phase zero speed* $u_{PHASE}=c5/3$




*Phase "anti-zero" going "back-in-time"*

*Phase zero speed limit* $\dfrac{u_{PHASE}+SWR}{1+u_{PHASE}\cdot\dfrac{SWR}{c^2}}=11c/5$

$$\frac{\frac{5}{3}+\frac{-1}{5}}{1+\frac{5}{3}\frac{-1}{5}}=\frac{\frac{22}{15}}{\frac{10}{25}}=\frac{11}{5}$$

$E_{\leftarrow}=0.5$, $E_{\rightarrow}=0.5$

$E_{\leftarrow}=0.6$, $E_{\rightarrow}=0.4$

Wave zero-anti-zero annihilation *and* creation occur *together* at *the same* spacetime point for $SWR=0$

Wave zero-anti-zero annihilation and creation occur *separately* at *different* spacetime points for $-u_{GROUP}<SWR<0$

# *At High Speed 2-CW Modes Look More Like 1-CW Beams*

$\psi = E\sqrt{\frac{\varepsilon_0}{h\upsilon}}$

Various combinations of *opposite-k 1-CW* beams occur with open boundaries.

E-*wave:* $E = E_{\rightarrow} e^{i(k_{\rightarrow}x - \omega_{\rightarrow}t)} + E_{\leftarrow} e^{i(k_{\leftarrow}x - \omega_{\leftarrow}t)}$ *is related to* Ψ-*wave:* $\Psi = \psi_{\rightarrow} e^{i(k_{\rightarrow}x - \omega_{\rightarrow}t)} + \psi_{\leftarrow} e^{i(k_{\leftarrow}x - \omega_{\leftarrow}t)}$

*Standing Wave Ratio (or Quotient)*

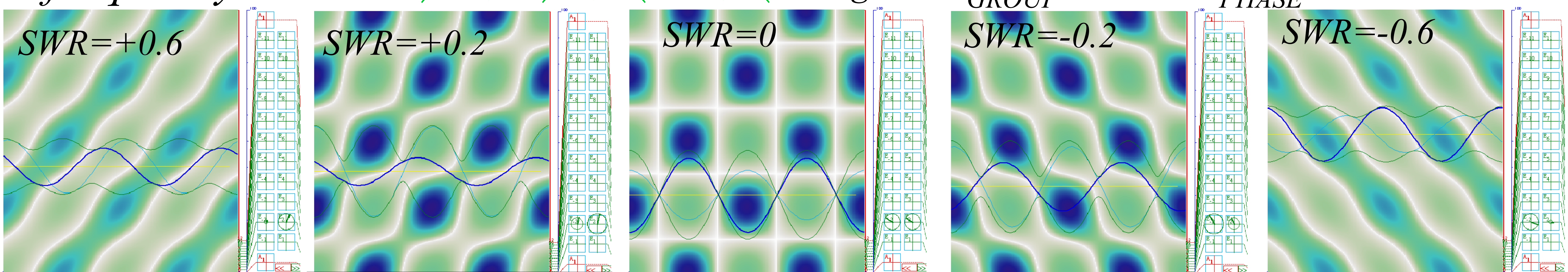
$SWR = (E_{\rightarrow} - E_{\leftarrow})/(E_{\rightarrow} + E_{\leftarrow}) = 1/SWQ$

*key numbers*

*Wave Group (or Phase) Velocity*

$u_{GROUP}/c = (\omega_{\rightarrow} - \omega_{\leftarrow})/(\omega_{\rightarrow} + \omega_{\leftarrow}) = c/u_{PHASE}$

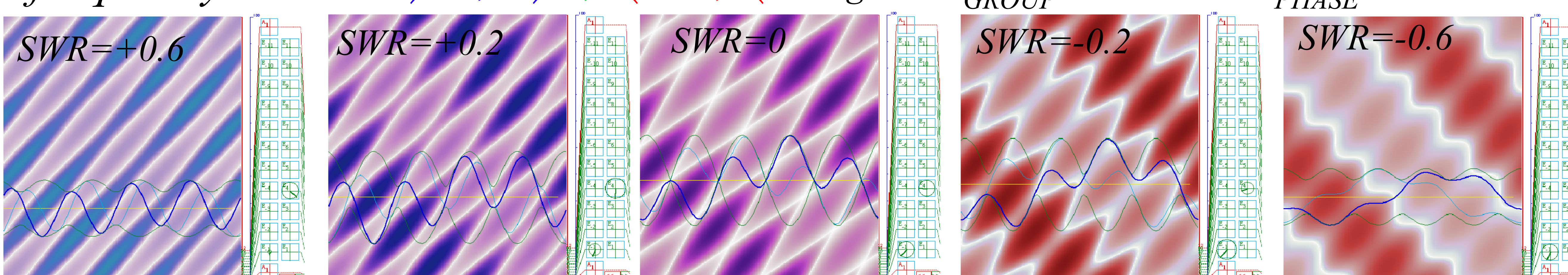
*1-frequency case :* $\omega_{\rightarrow} = 2c,\ k_{\rightarrow} = 2,\ \omega_{\leftarrow} = 2c,\ k_{\leftarrow} = -2$ gives: $u_{GROUP} = 0$ and $u_{PHASE} = \infty$




| | | | | |
|---|---|---|---|---|
| $E_{\leftarrow} = 0.4,\ E_{\rightarrow} = 1.0$ | $E_{\leftarrow} = 0.4,\ E_{\rightarrow} = 0.6$ | $E_{\leftarrow} = 0.5,\ E_{\rightarrow} = 0.5$ | $E_{\leftarrow} = 0.6,\ E_{\rightarrow} = 0.4$ | $E_{\leftarrow} = 1.0,\ E_{\rightarrow} = 0.4$ |

*Staircase Galloping*

*2-frequency case :* $\omega_{\rightarrow} = 4c,\ k_{\rightarrow} = 4,\ \omega_{\leftarrow} = 1c,\ k_{\leftarrow} = -1$ gives: $u_{GROUP}/c = 3/5$ and $u_{PHASE}/c = 5/3$




| | | | | |
|---|---|---|---|---|
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*Understanding how quantum transitions require "mixed-up" states*

*Closed cavity vs ring cavity*

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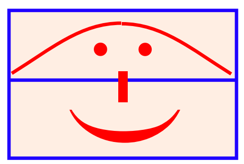
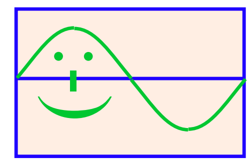
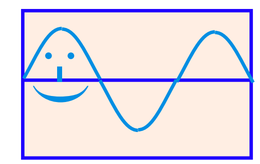
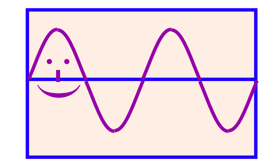

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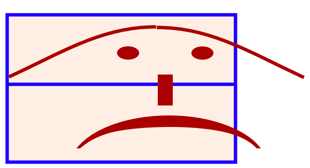
too fat!



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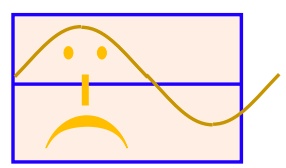
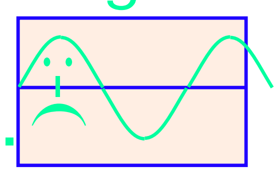


wrong color again!

misfits...



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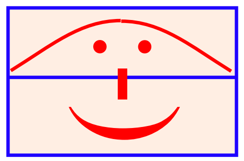
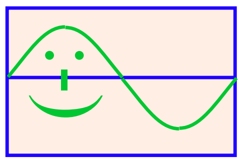
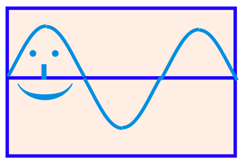
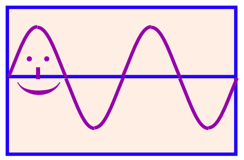

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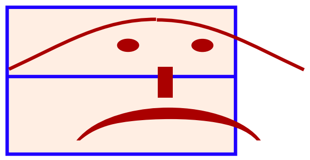

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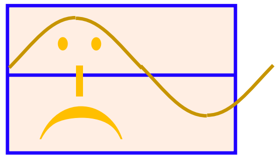

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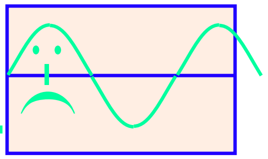

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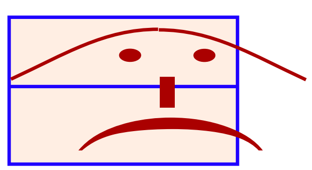

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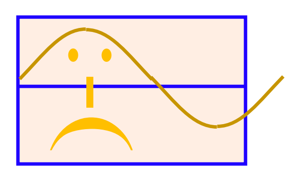

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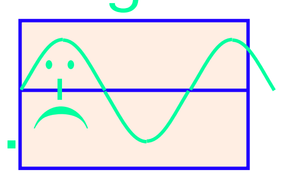

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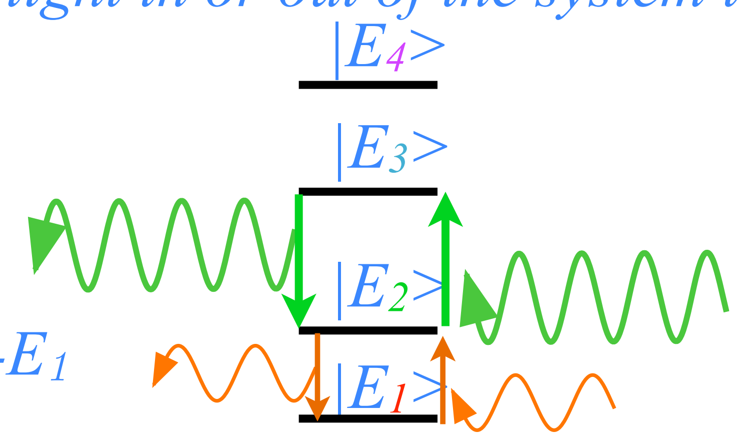
*frequency* $\hbar\omega_{32} = E_3 - E_2$

*frequency* $\hbar\omega_{21} = E_2 - E_1$



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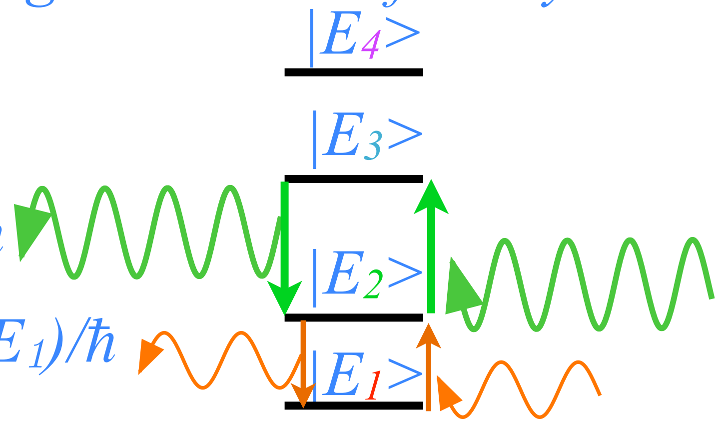
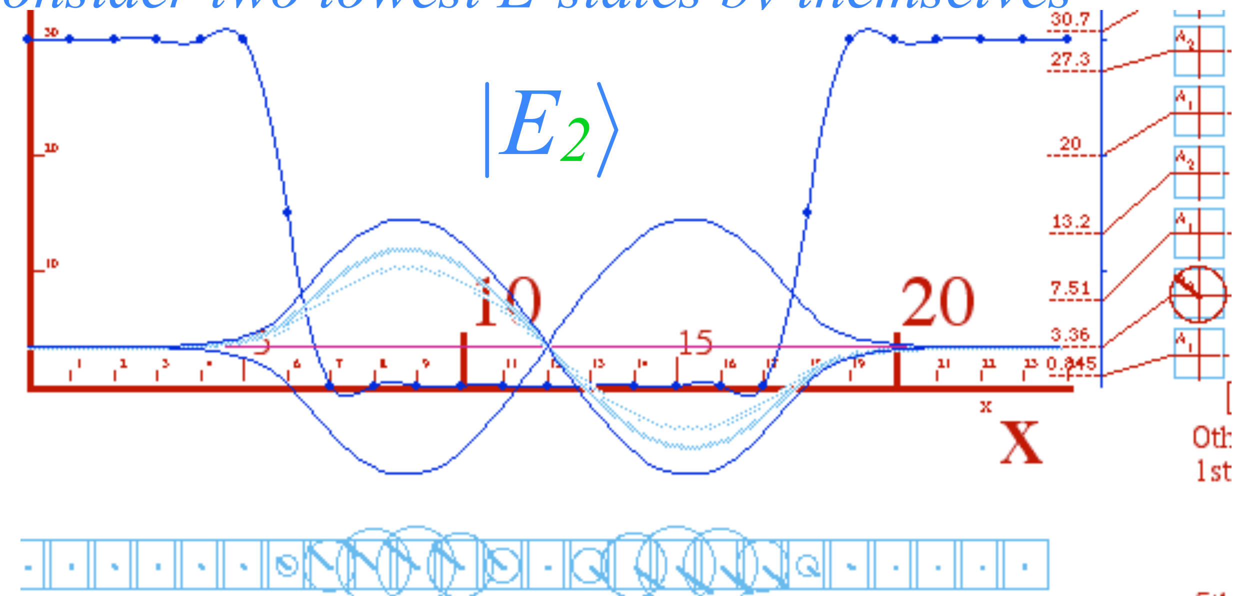
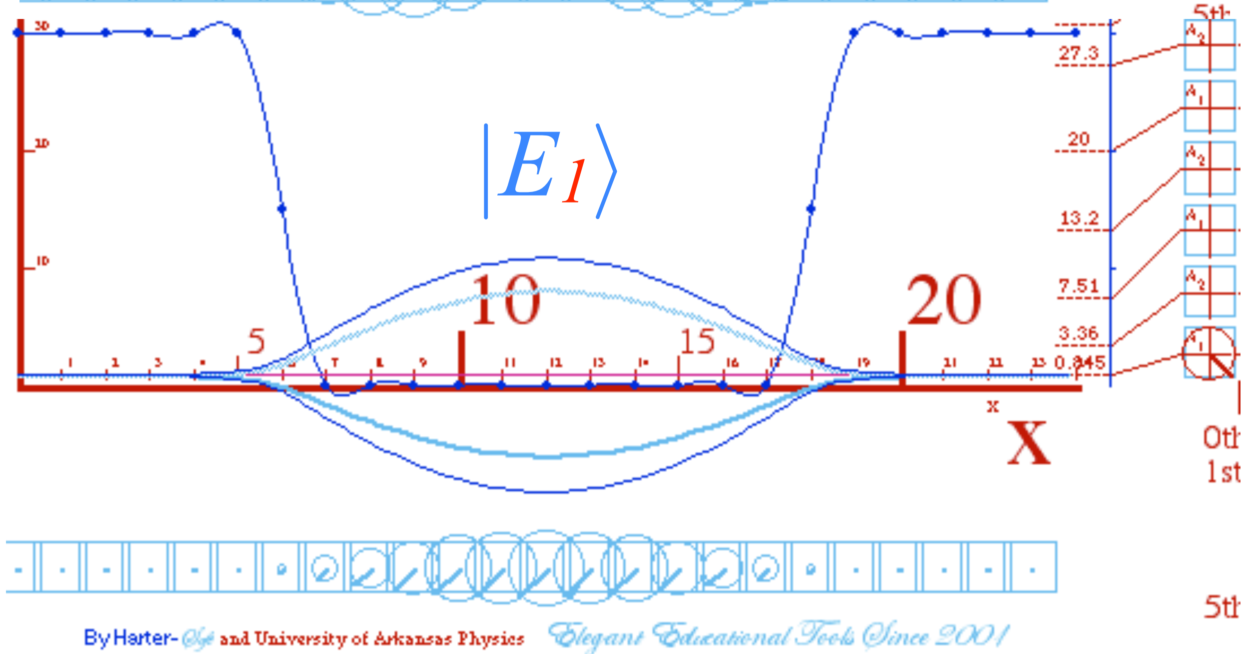


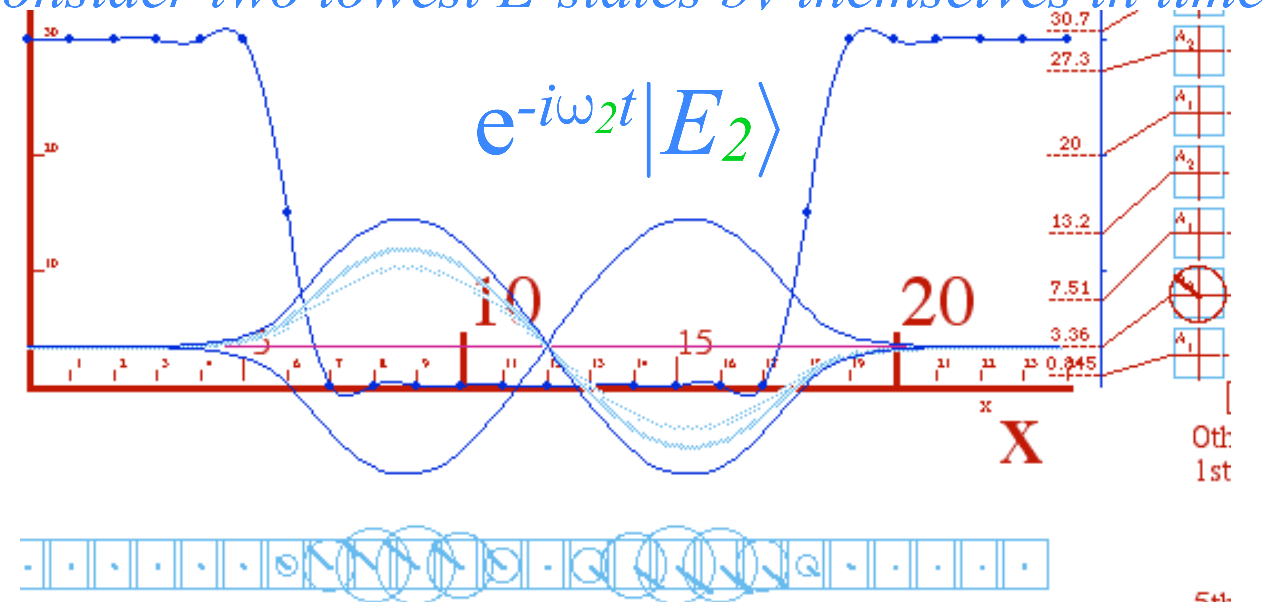

*These eigenstates are the only ways the system can "play dead"... ... " sleep with the fishes"...*

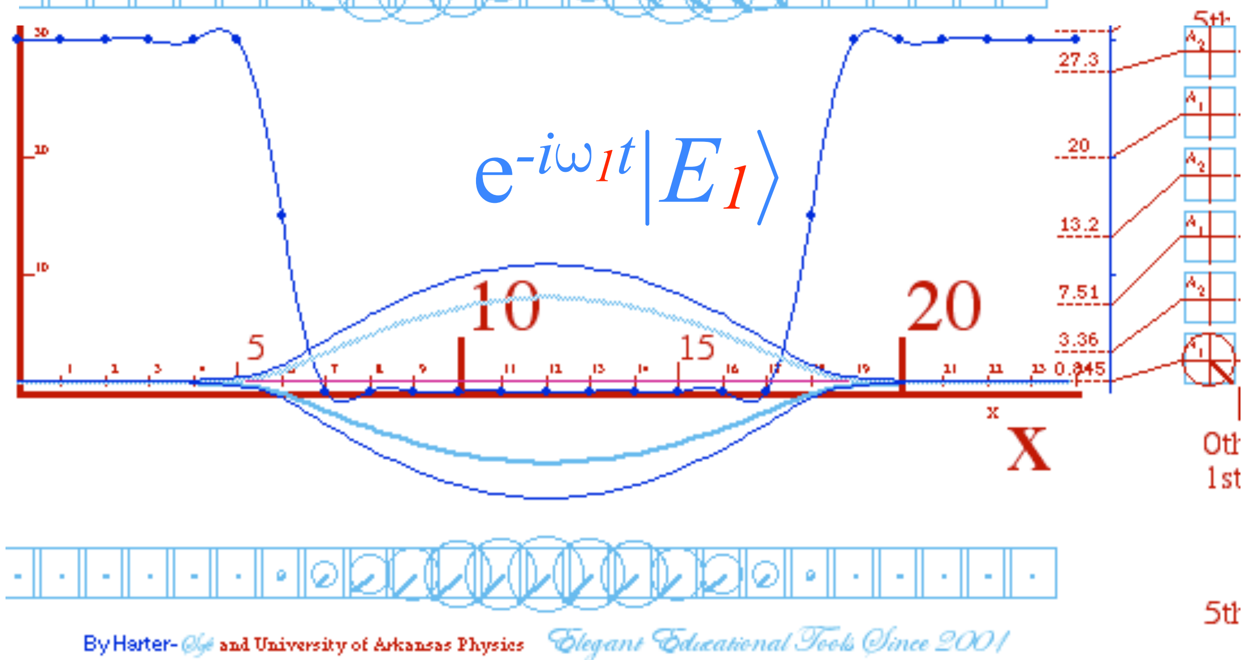
*Consider two lowest E-states by themselves*



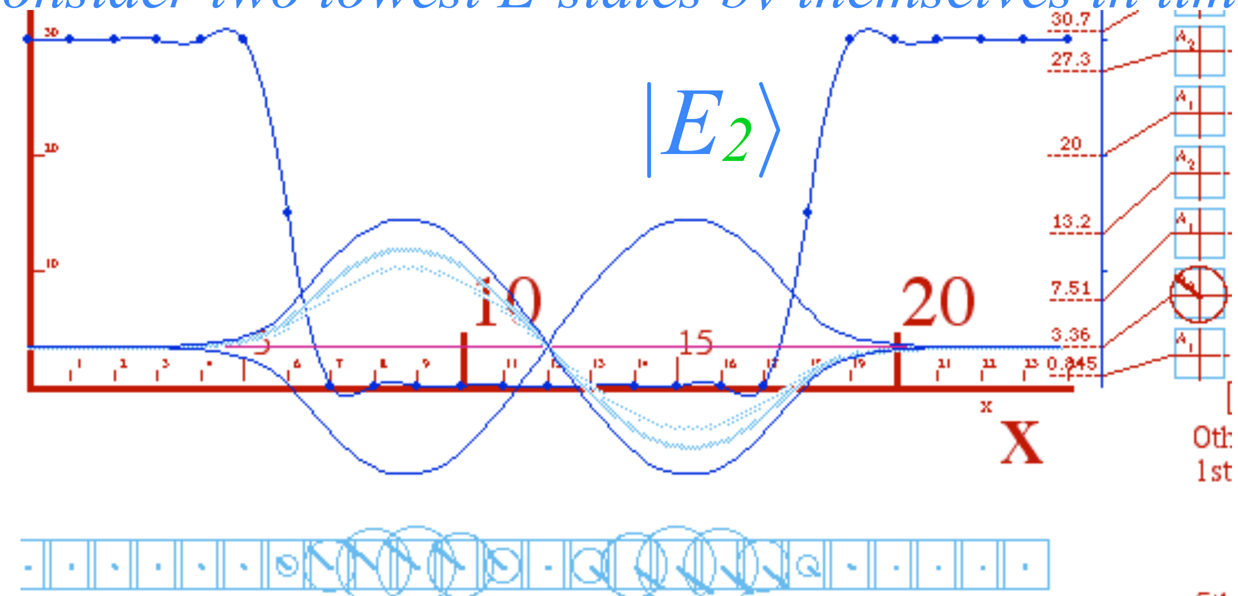
*Consider two lowest E-states by themselves in time*

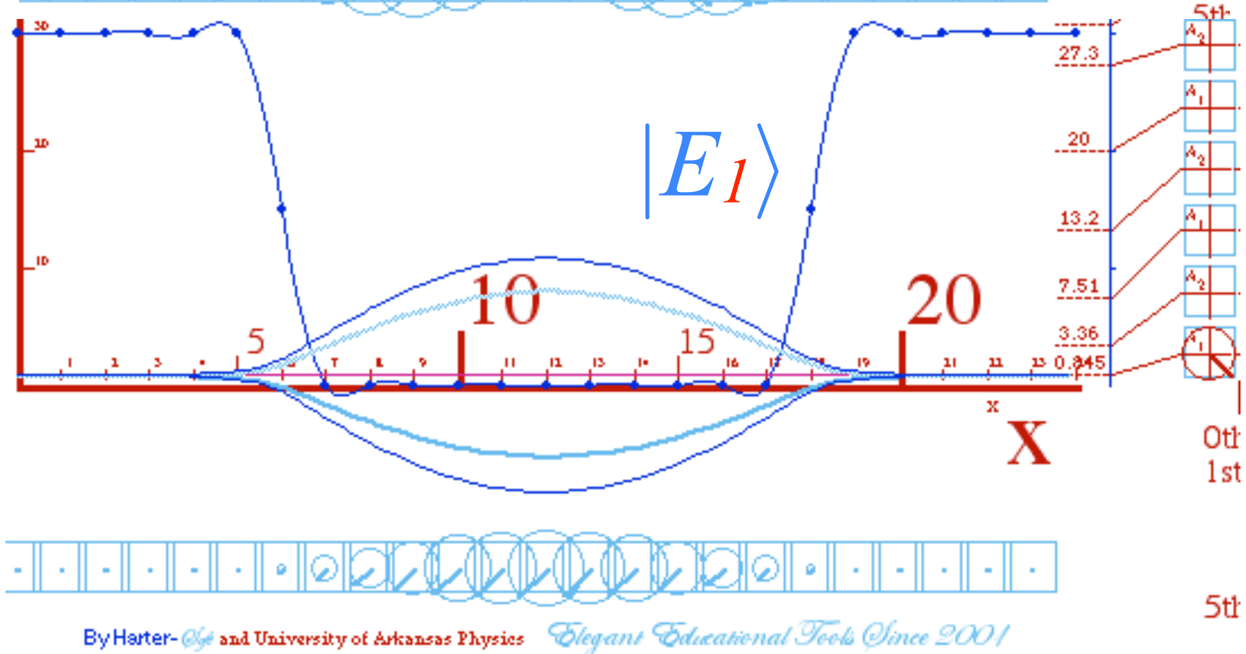


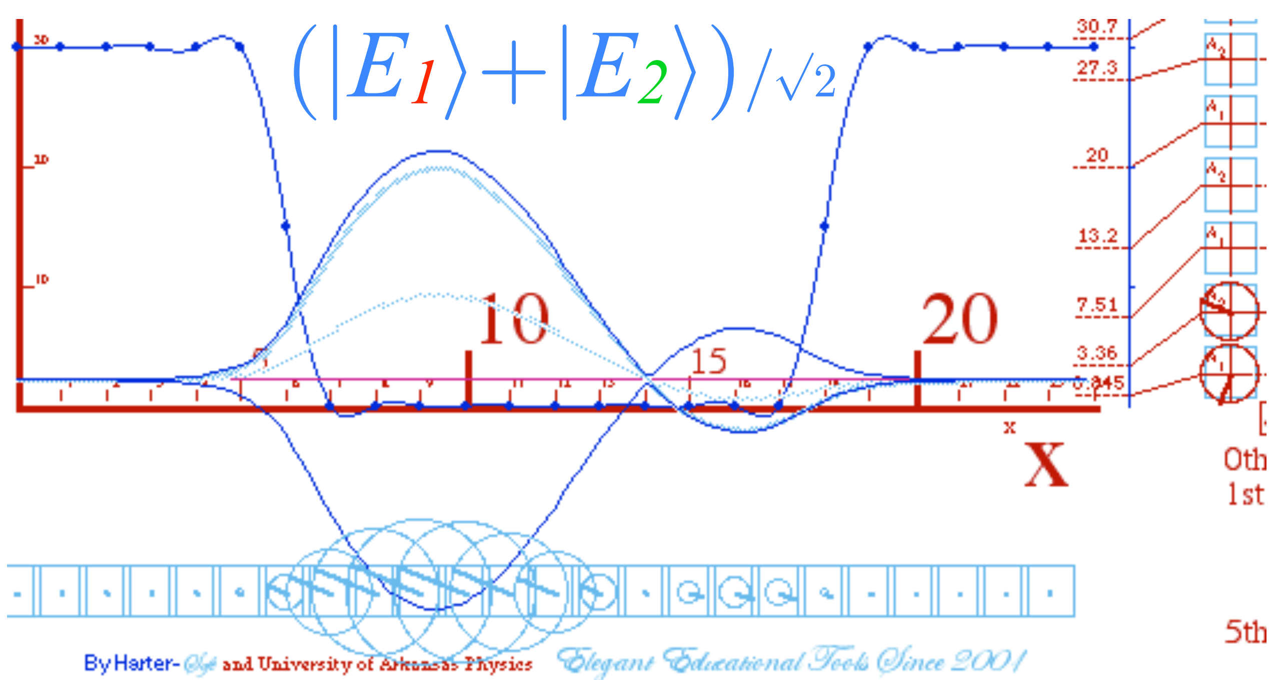



*Consider two lowest E-states by themselves in time*

*Now combine (add) them*



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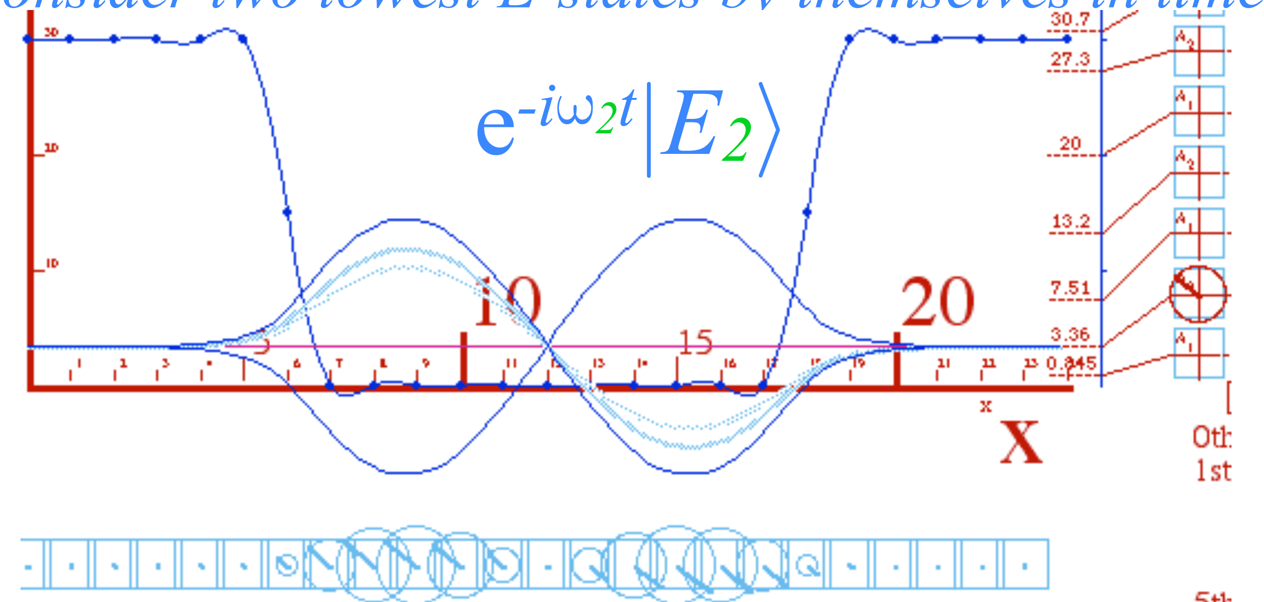
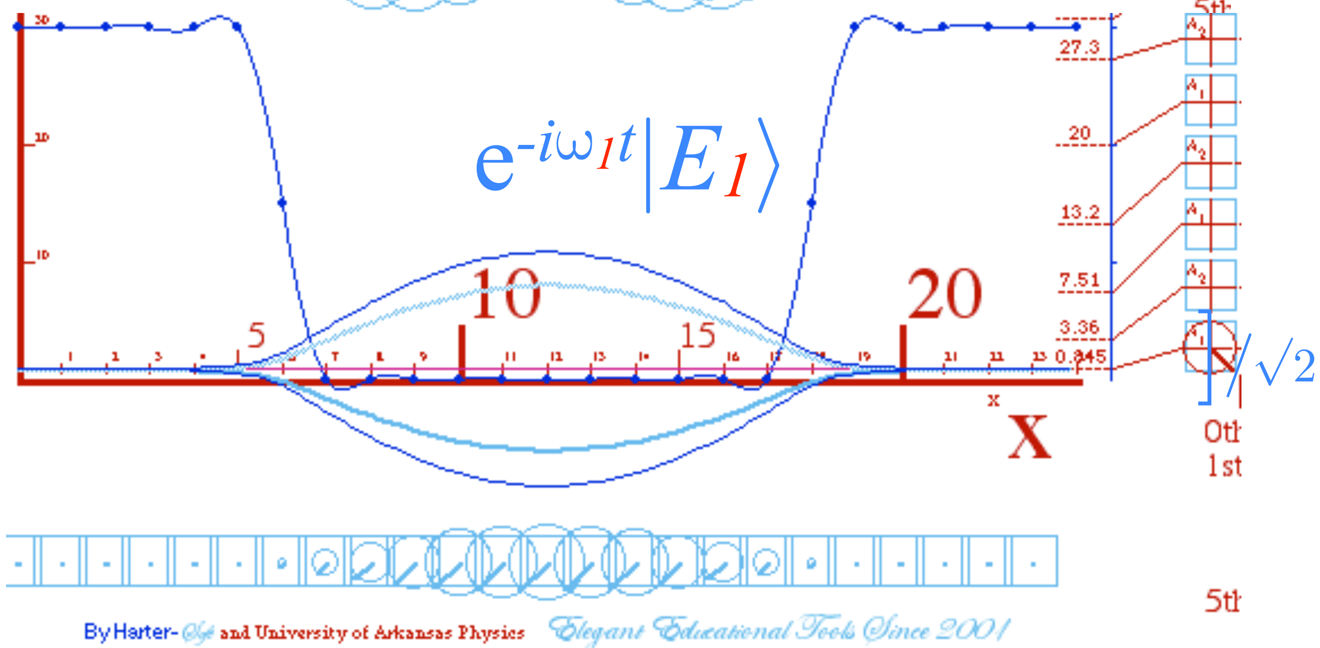
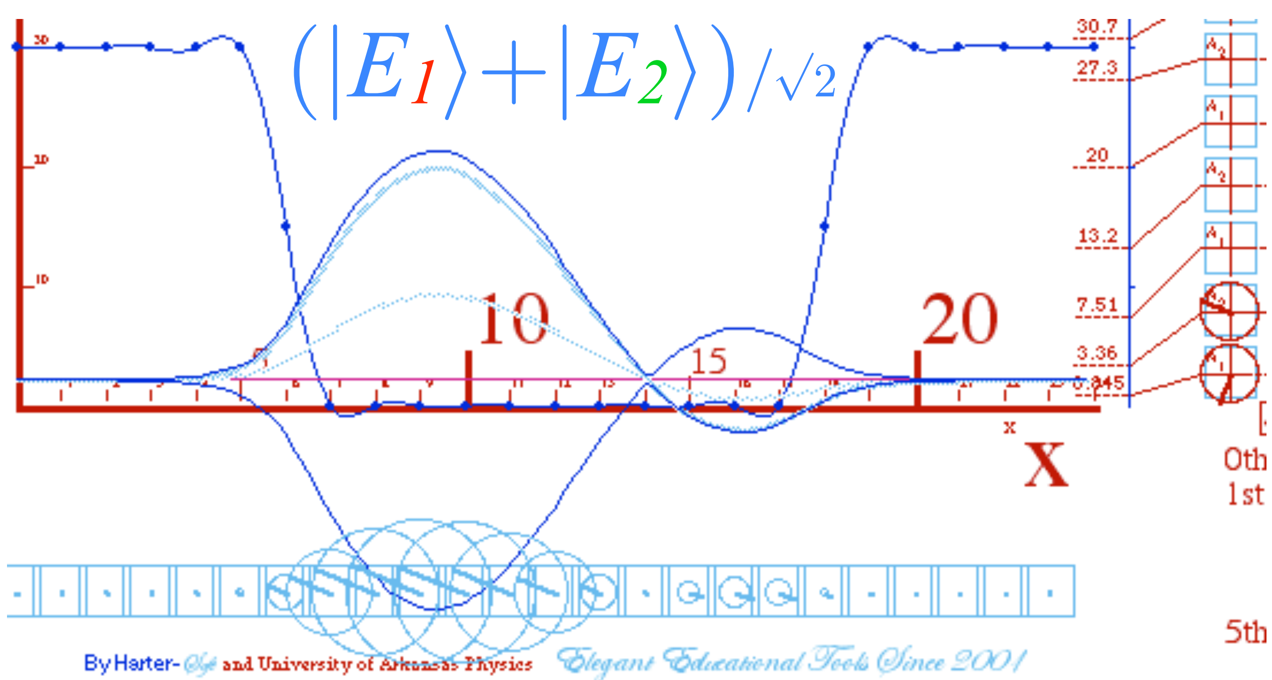
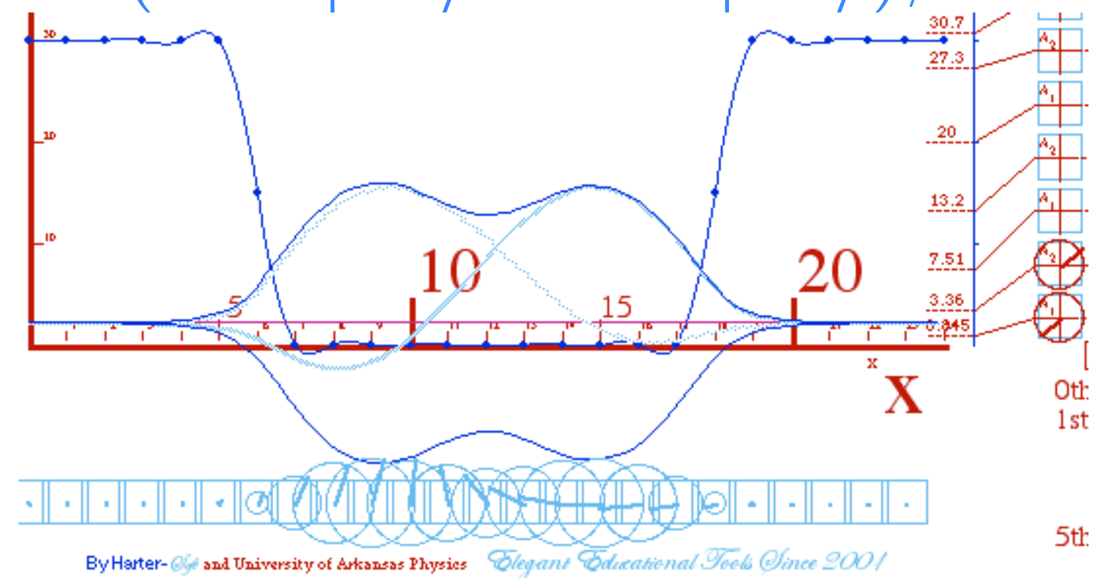



*Now combine (add) them and let time roll!*

$$\left(e^{-i\omega_1 t}|E_1\rangle + e^{-i\omega_2 t}|E_2\rangle\right)/\sqrt{2}$$



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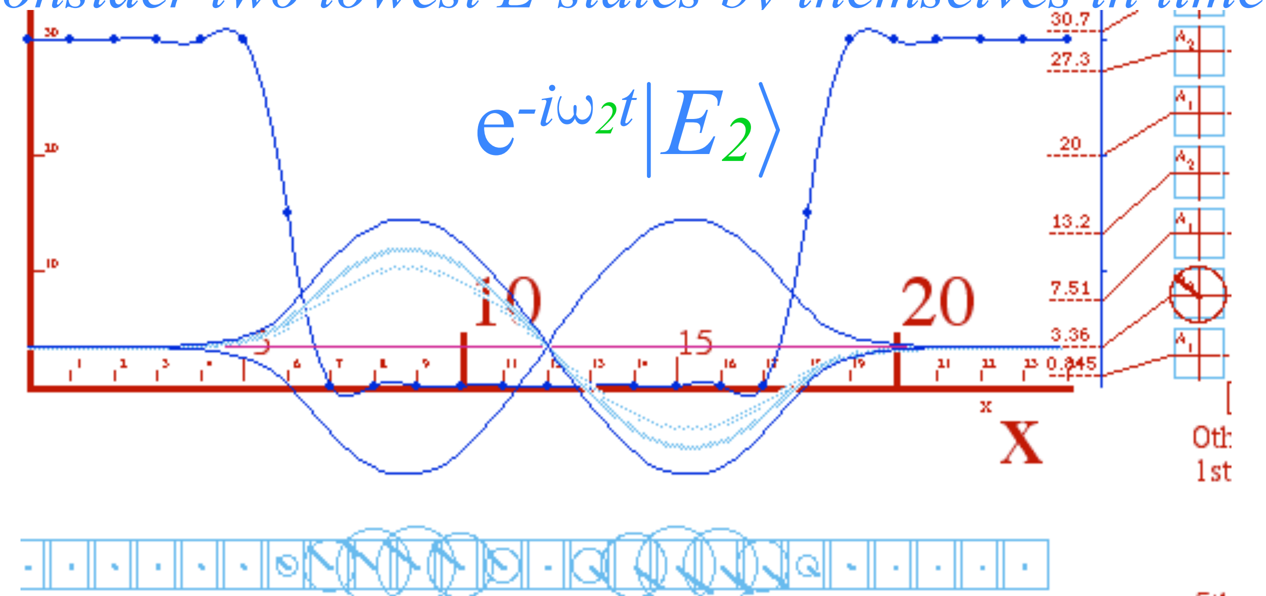


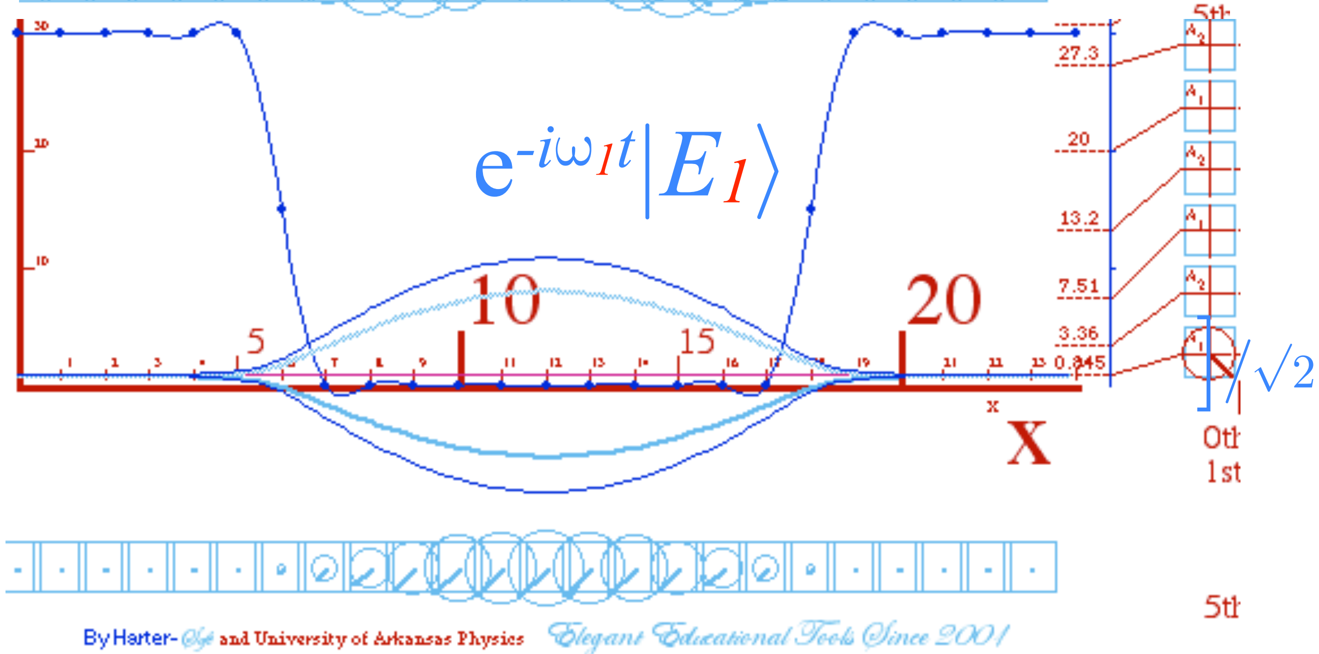


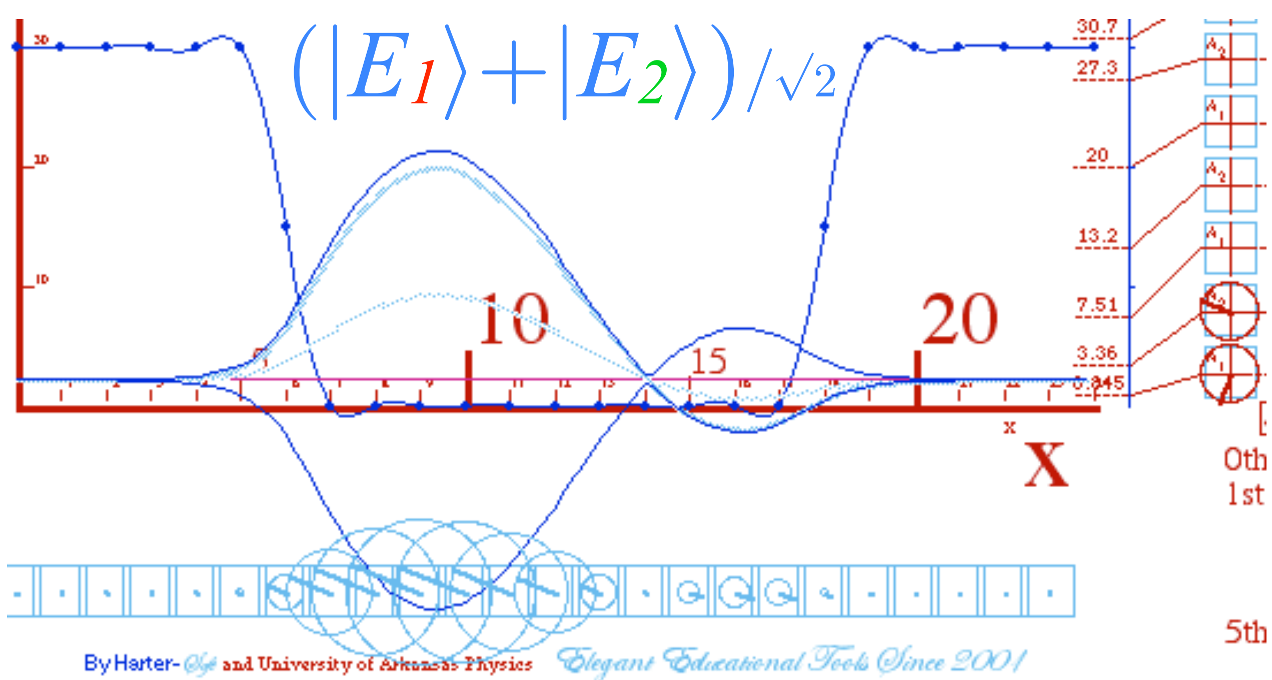


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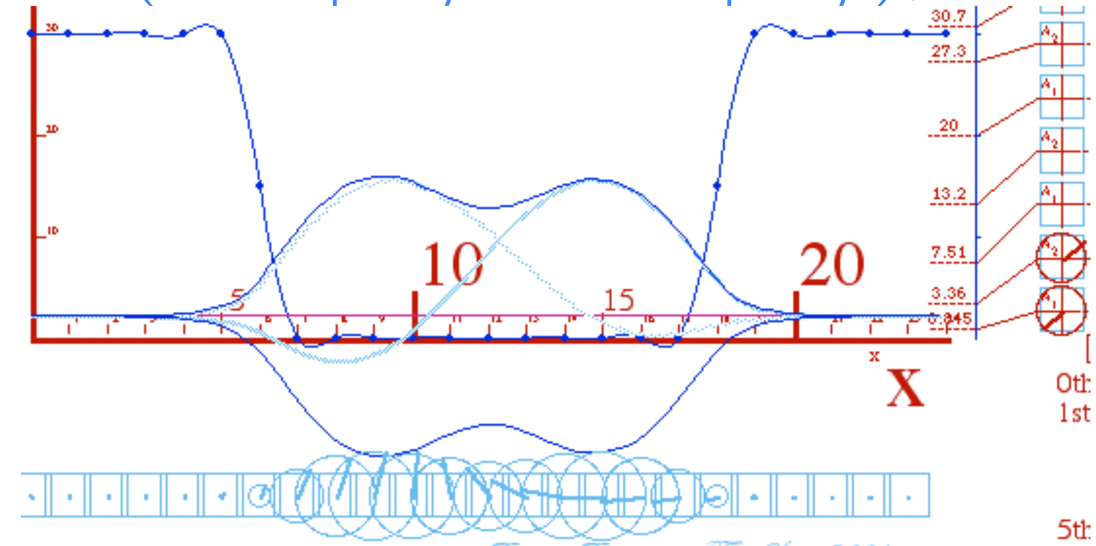


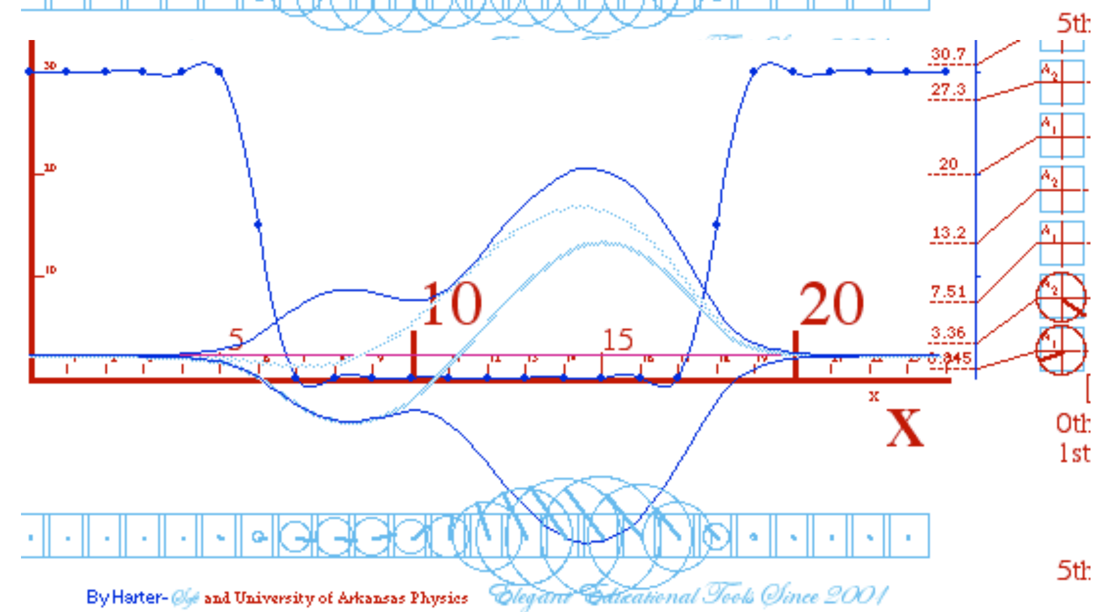

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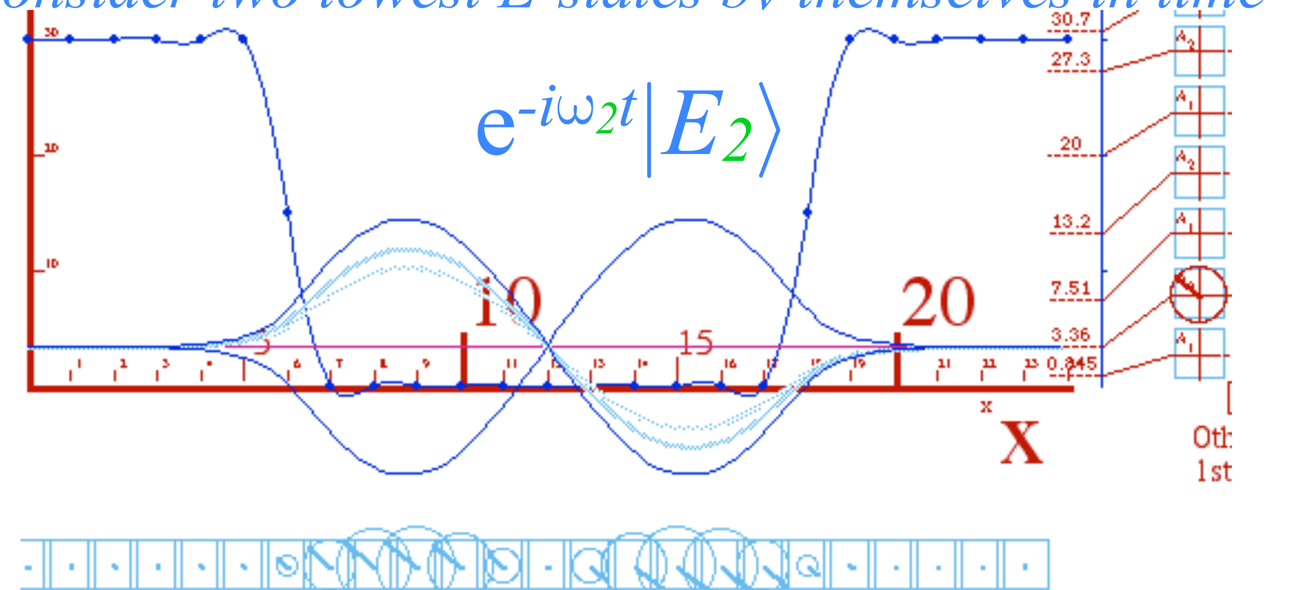
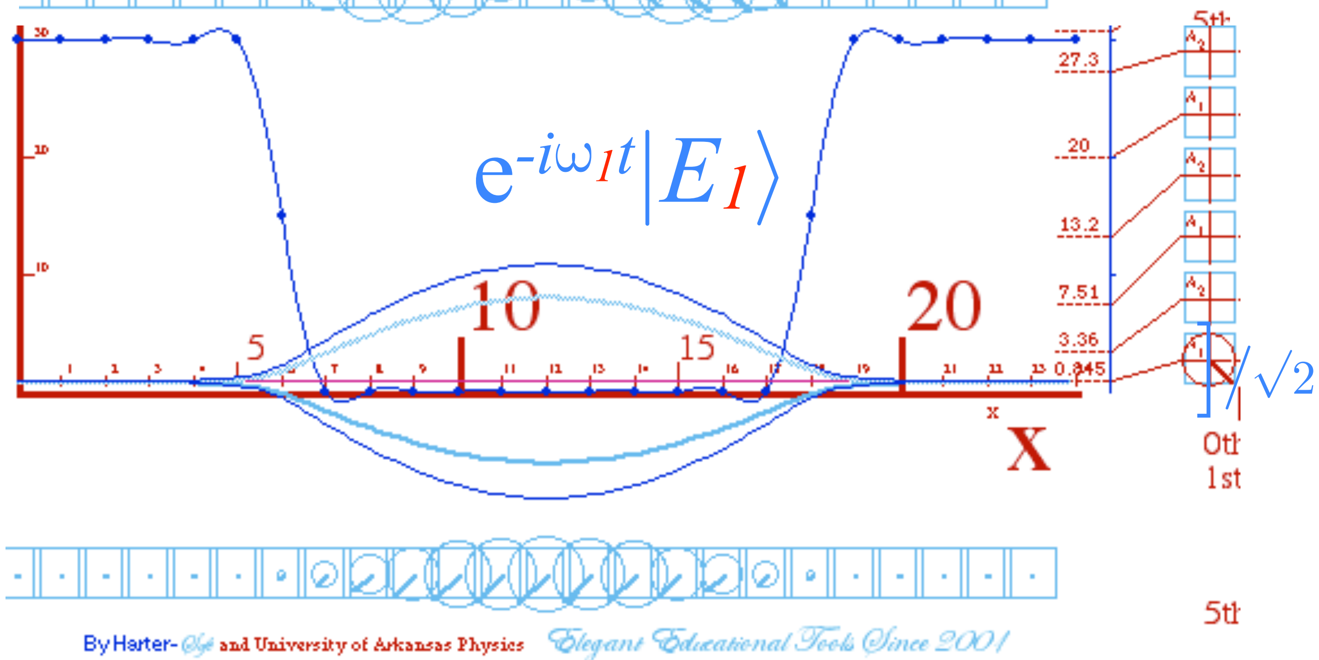
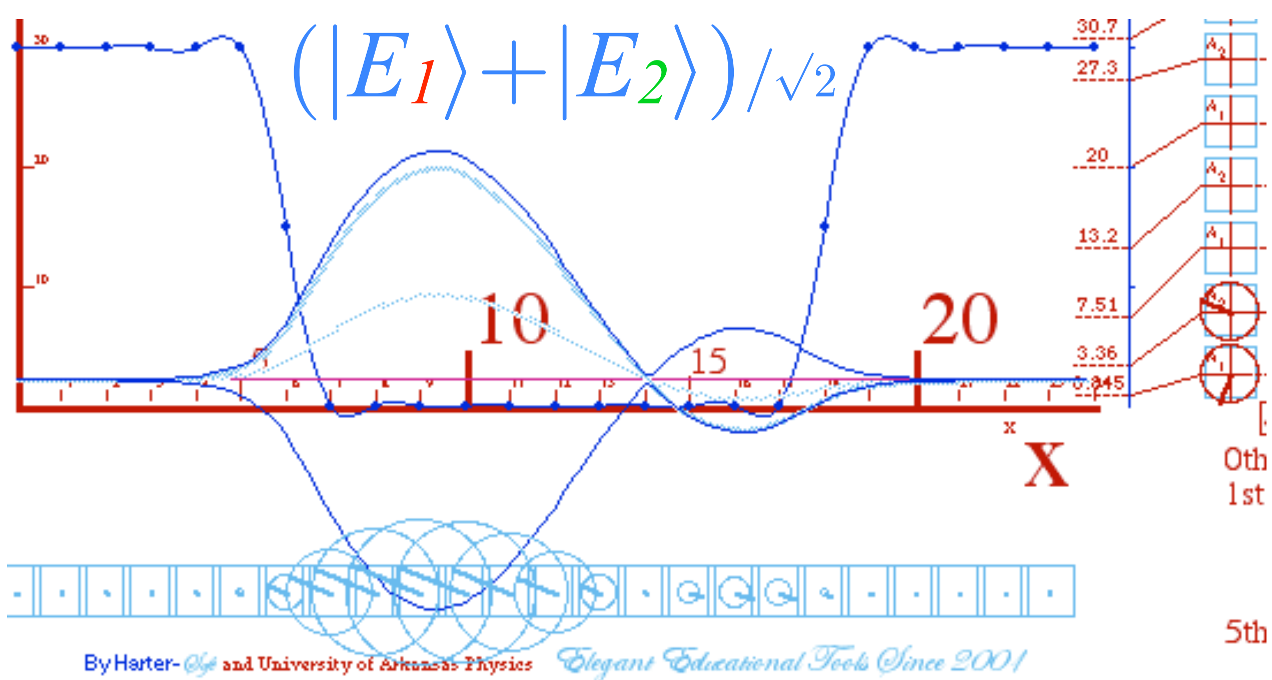
$e^{-i\omega_2 t}|E_2\rangle$




$e^{-i\omega_1 t}|E_1\rangle$




$(|E_1\rangle + |E_2\rangle)/\sqrt{2}$

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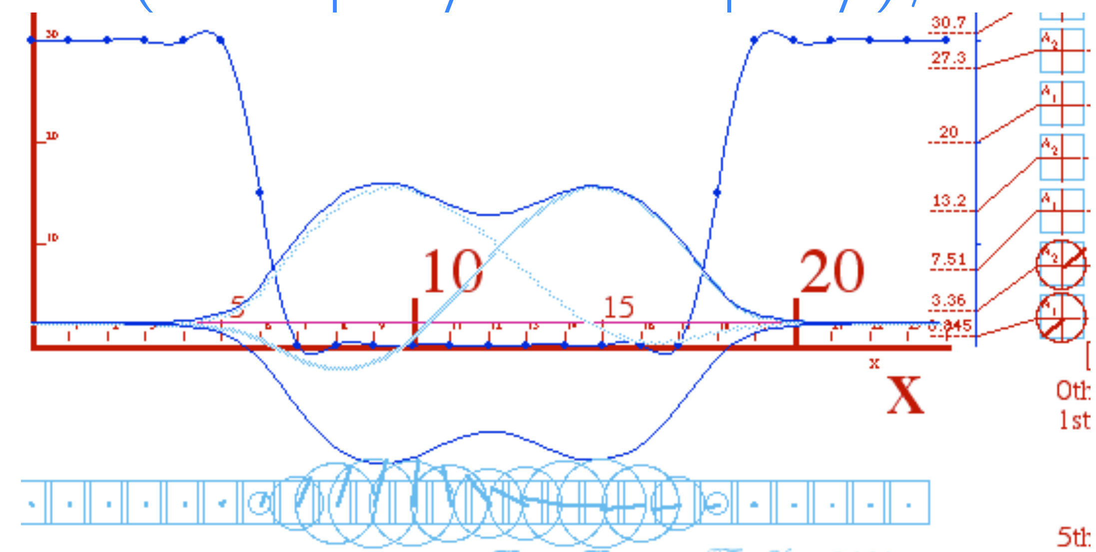


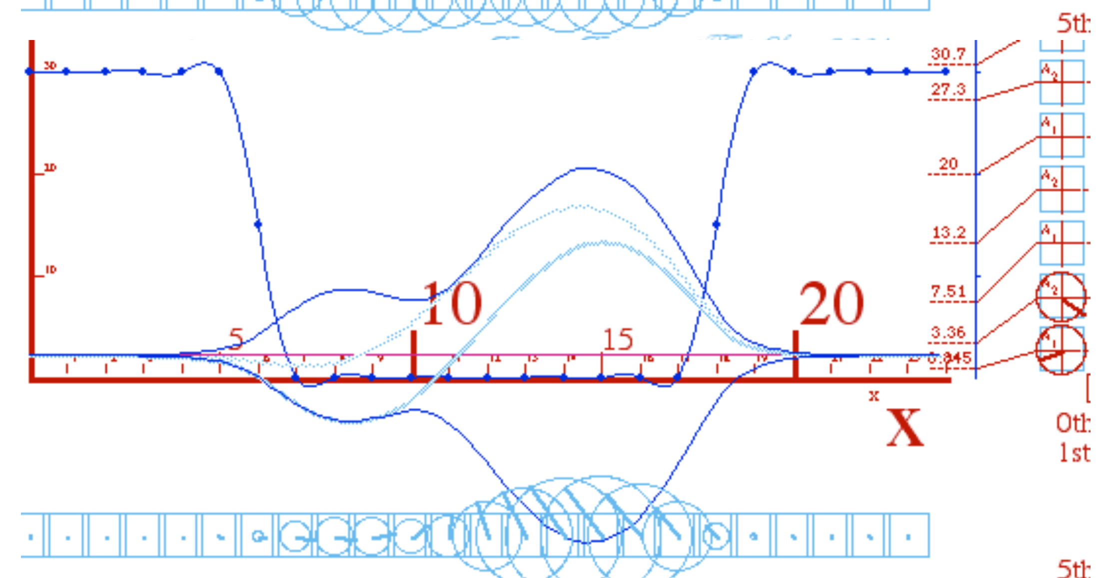


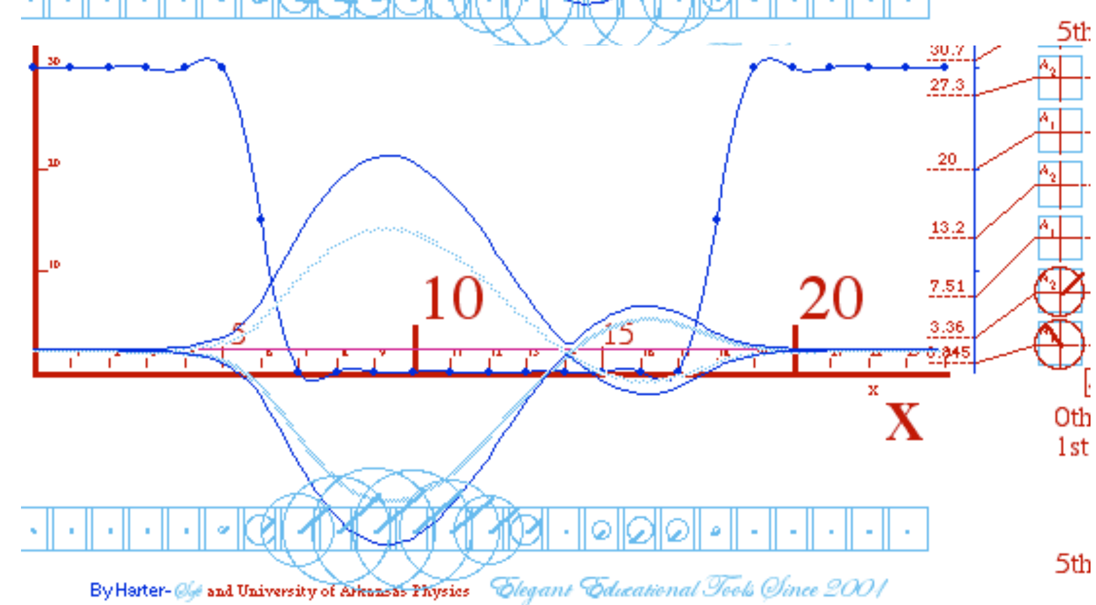

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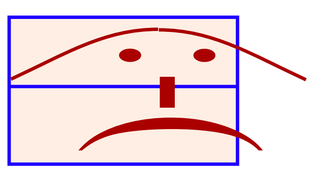

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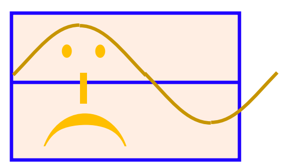

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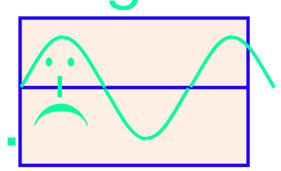

...not tolerated !

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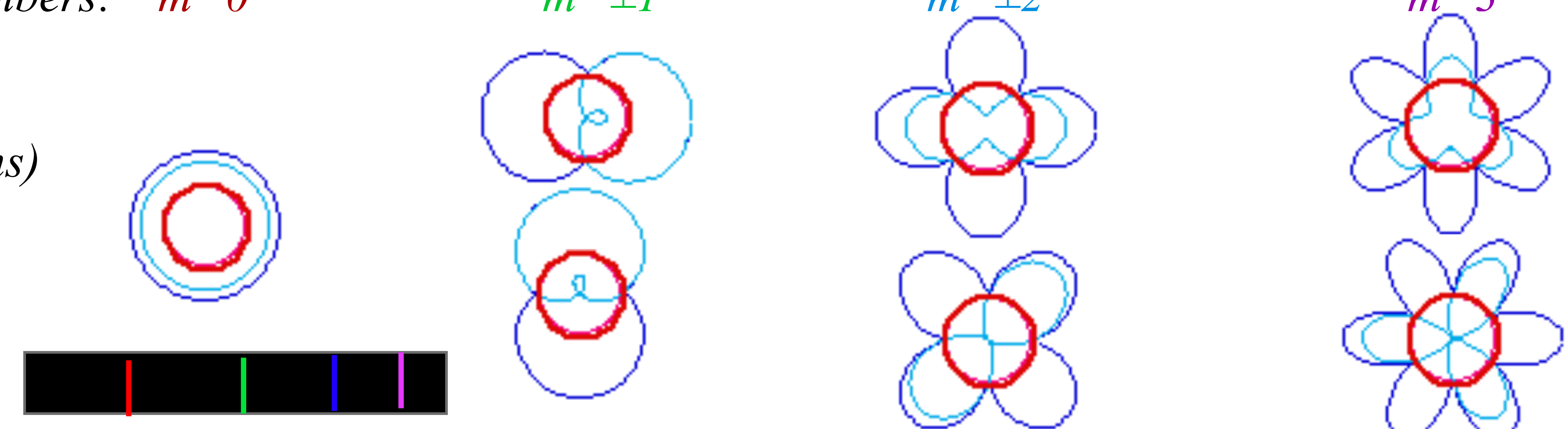
Rings tolerate a *zero* (kinkless) quantum wave but require ±*integral wave number.*

*OK ring quantum numbers:* *m=0* *m=±1* *m=±2* *m=3*

*(± integral number of wavelengths)*



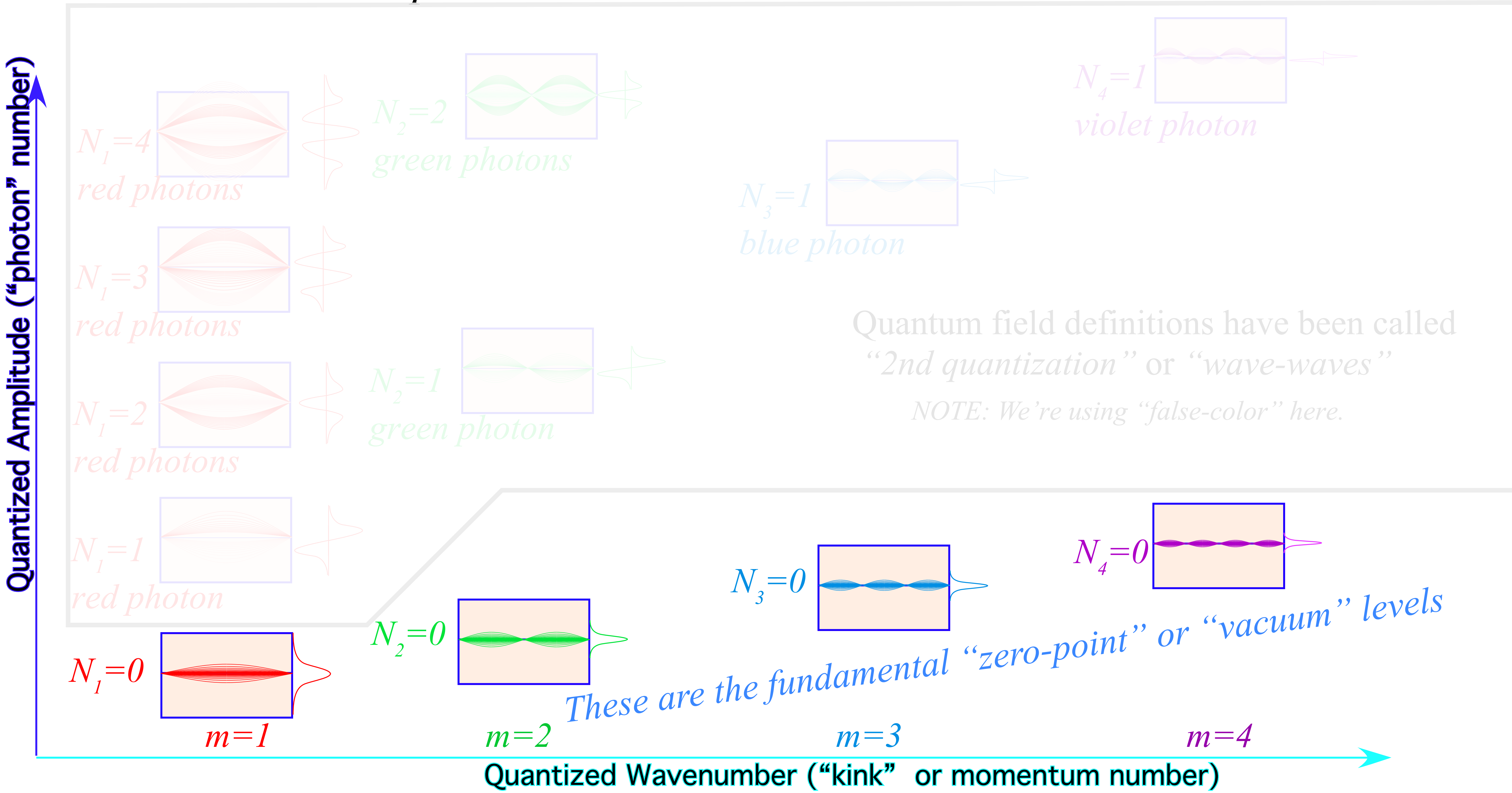
Bohr's models of *atomic spectra (1913-1923)* are beginnings of *quantum wave mechanics* built on *Planck-Einstein (1900-1905)* relation *E=hυ*. *DeBroglie* relation *p=h/λ* comes around *1923*.

- *2nd Quantization: Quantizing amplitudes ("photons","vibrons", and "what-ever-ons")*
  - *Introducing coherent states (What lasers use)*
    - *Analogy with (ω,k) wave packets*
    - *Wave coordinates need coherence*

# Quantized *Amplitude* *Counting "photon" number*

Planck's relation $E=Nh\upsilon$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it.

A current view is that it represents the *quantization* of optical field *amplitude*. We picture this below as *N-photon* wave states for each box-mode of *m* wave kinks.



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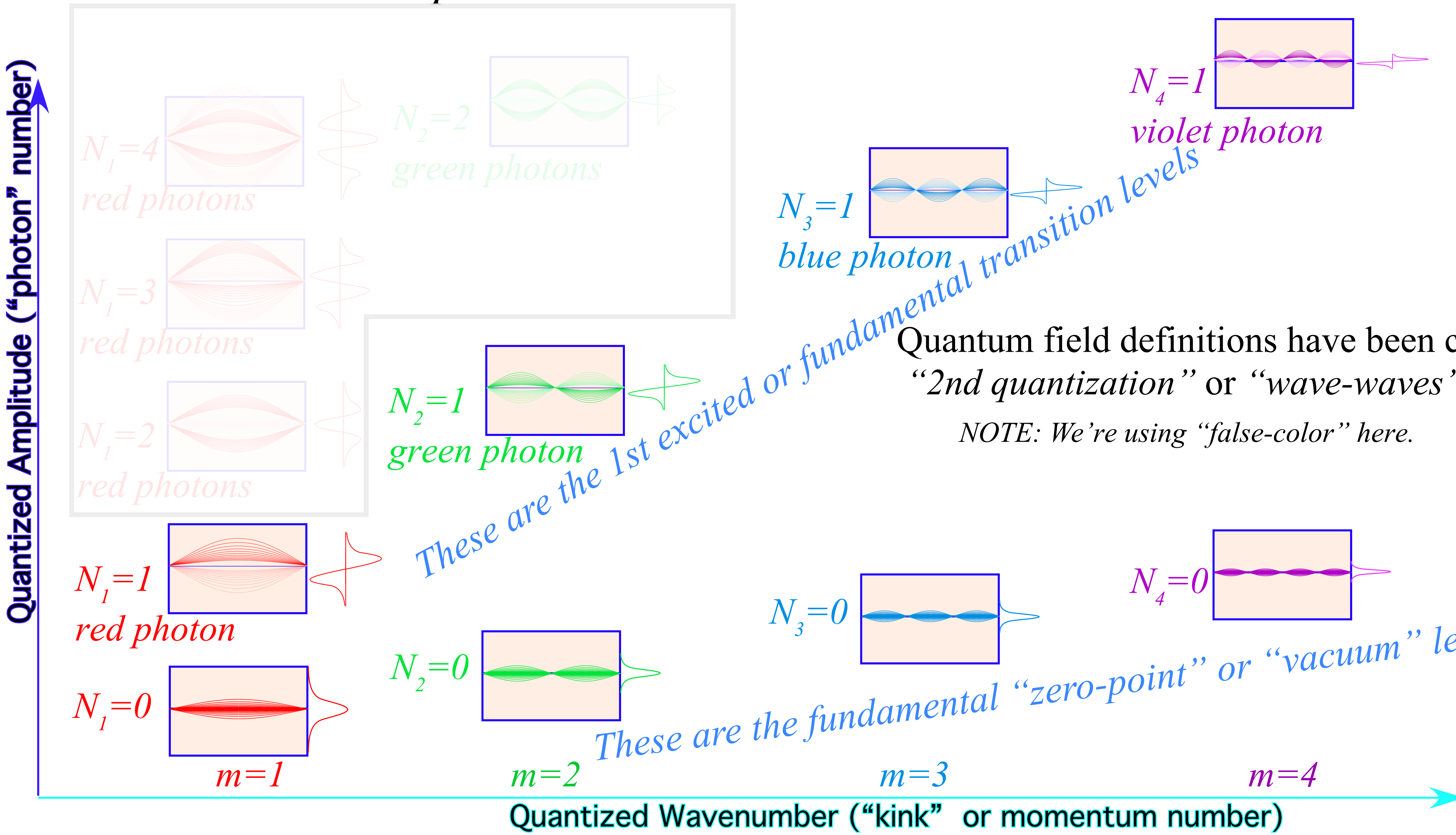


Quantum field definitions have been called "*2nd quantization*" or "*wave-waves*"

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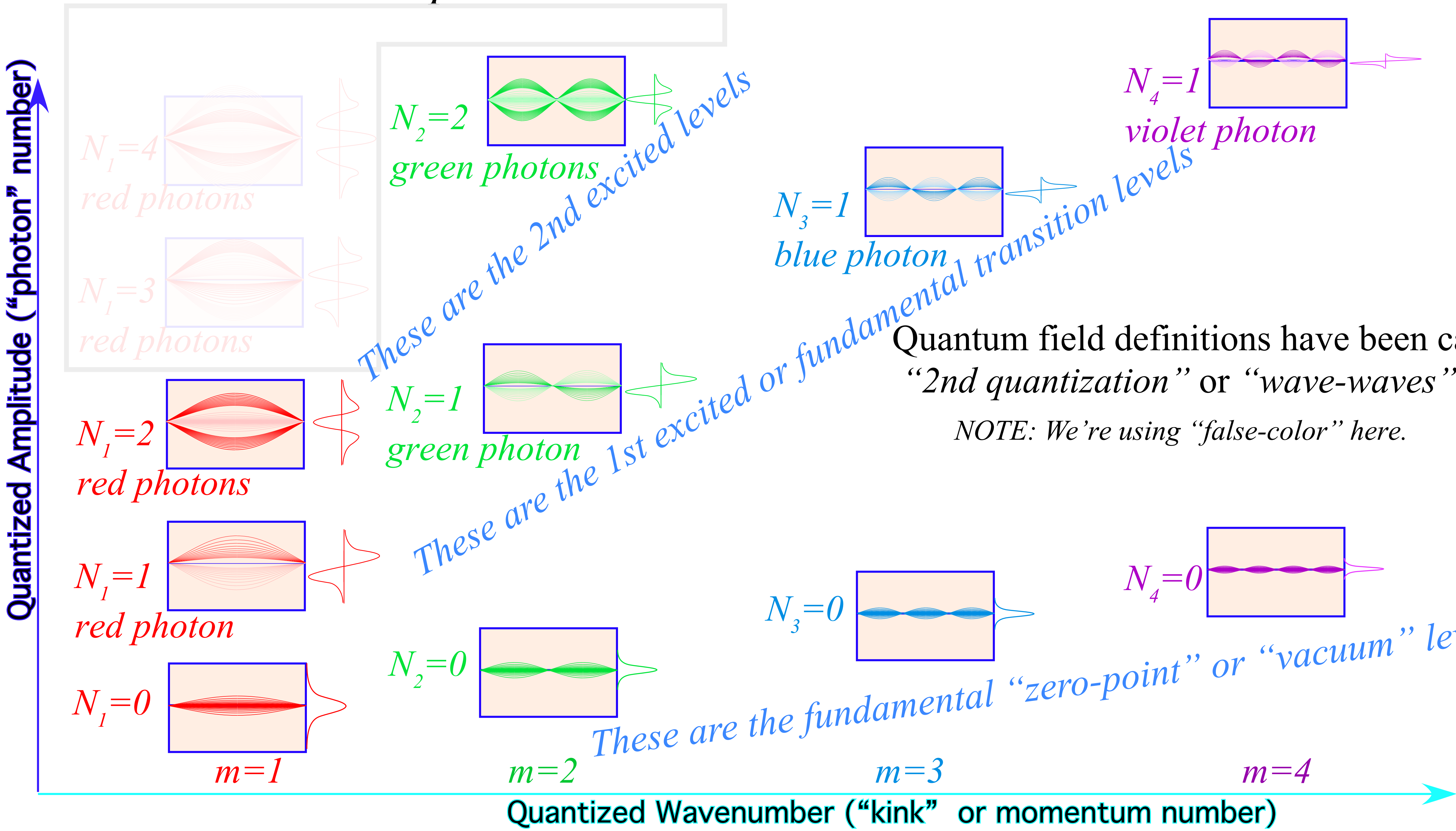

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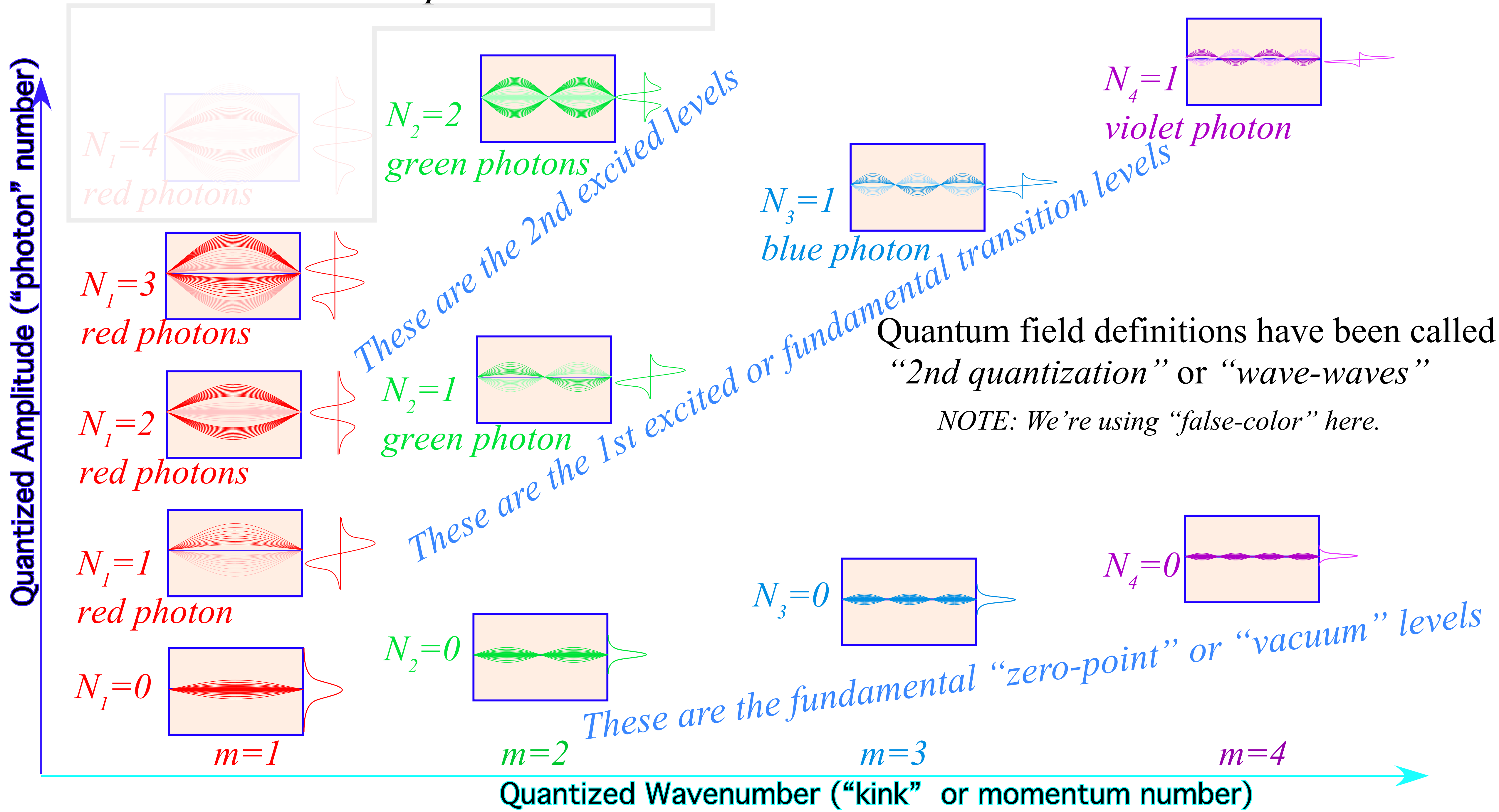
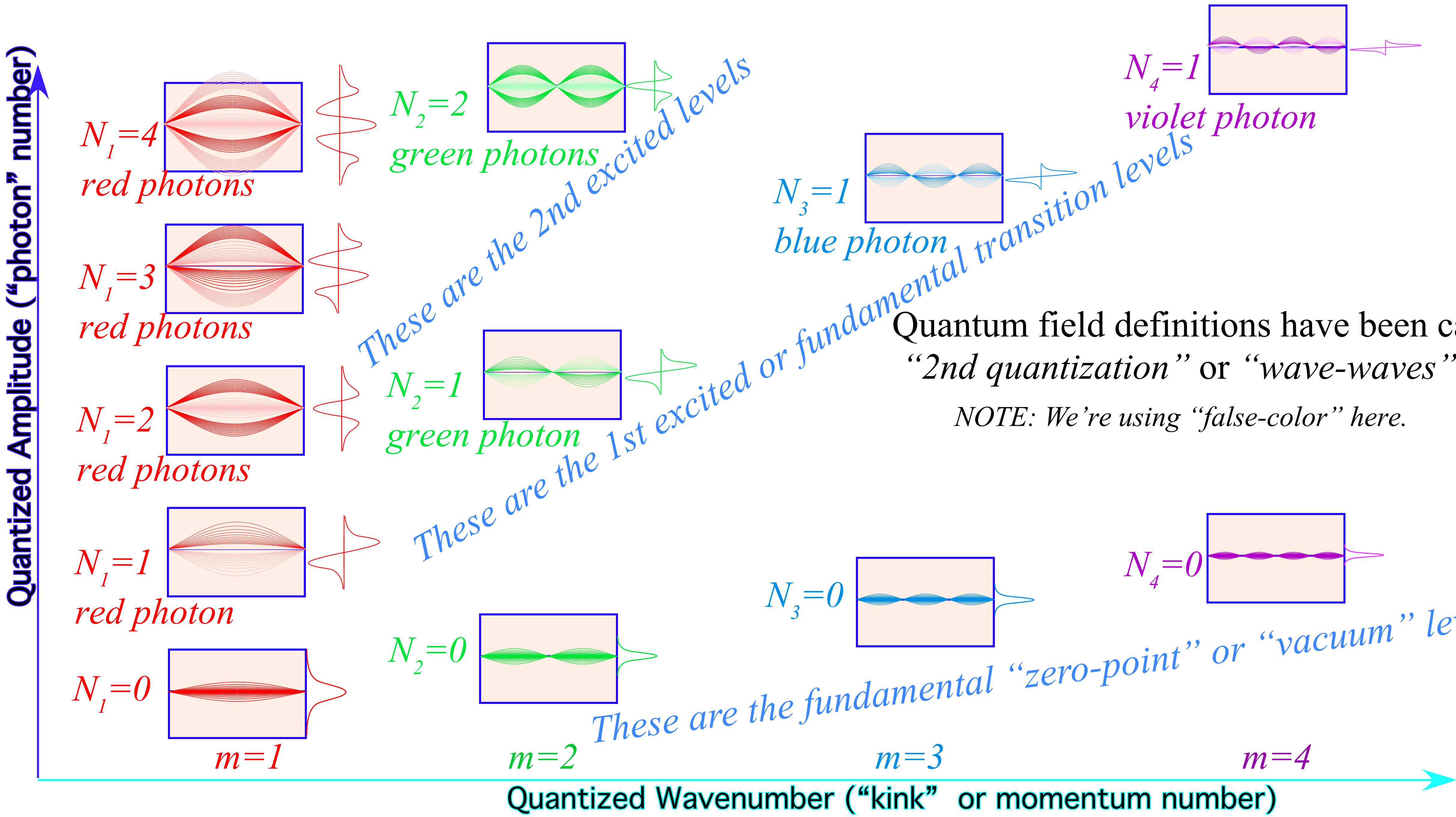


# Quantized *Amplitude* *Counting "photon" number*

Planck's relation $E=Nh\upsilon$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it.

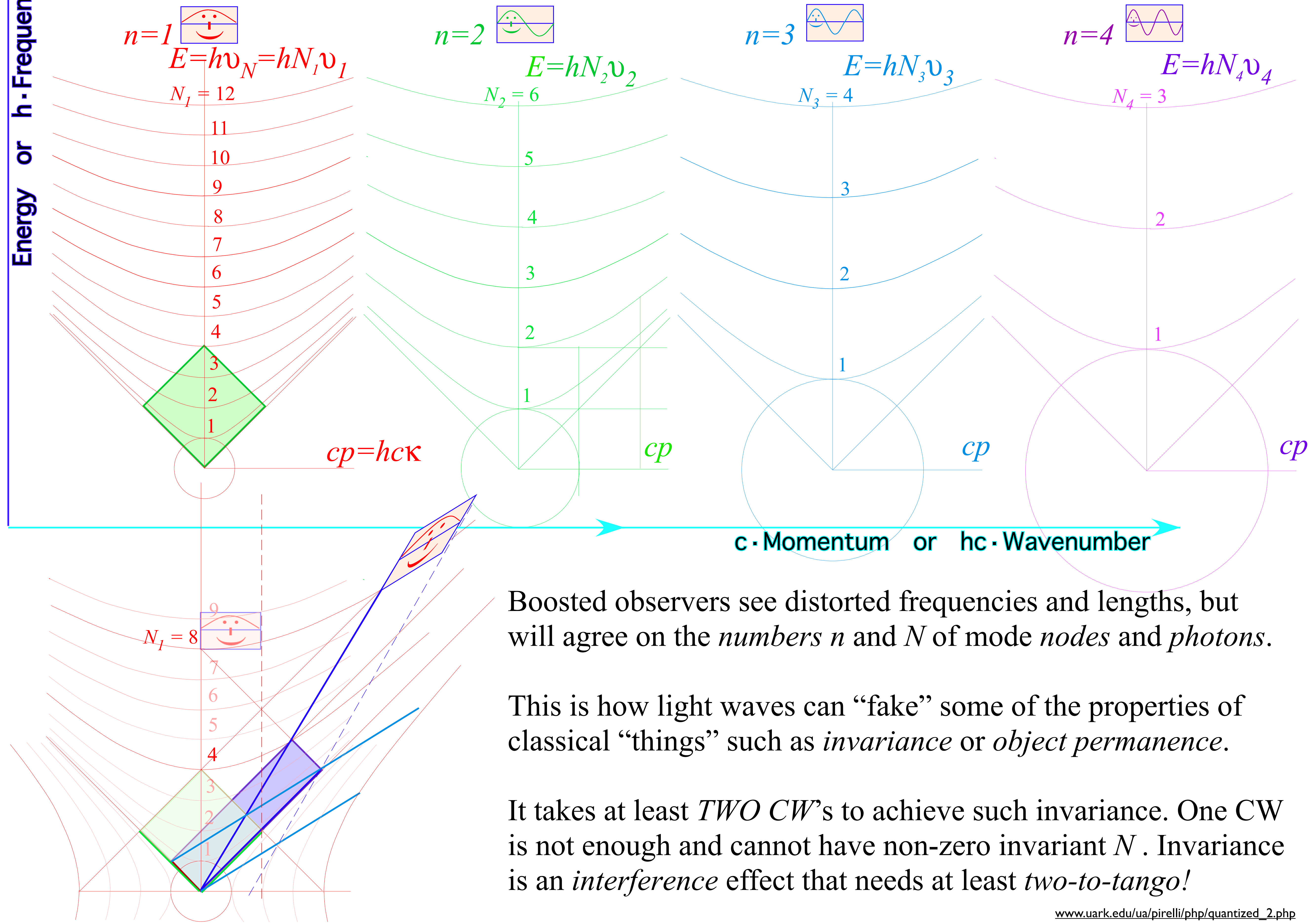
A current view is that it represents the *quantization* of optical field *amplitude*. We picture this below as *N-photon* wave states for each box-mode of *m* wave kinks.




Quantum field definitions have been called "2nd quantization" or "wave-waves"

*NOTE: We're using "false-color" here.*

Quantum numbers $N$ of field or $n, m,..$ of modes are *invariants* and *not* changed by boosting velocity.
Each mode fundamental frequency $\upsilon_n = n\upsilon_1$ and its $N$-photon multiples belong to invariant hyperbolas.




Boosted observers see distorted frequencies and lengths, but will agree on the *numbers n* and $N$ of mode *nodes* and *photons*.

This is how light waves can "fake" some of the properties of classical "things" such as *invariance* or *object permanence*.

It takes at least *TWO CW*'s to achieve such invariance. One CW is not enough and cannot have non-zero invariant $N$ . Invariance is an *interference* effect that needs at least *two-to-tango!*

www.uark.edu/ua/pirelli/php/quantized_2.php

*Lecture 30 ended here*

*2nd Quantization: Quantizing amplitudes ("photons", "vibrons", and "what-ever-ons")*

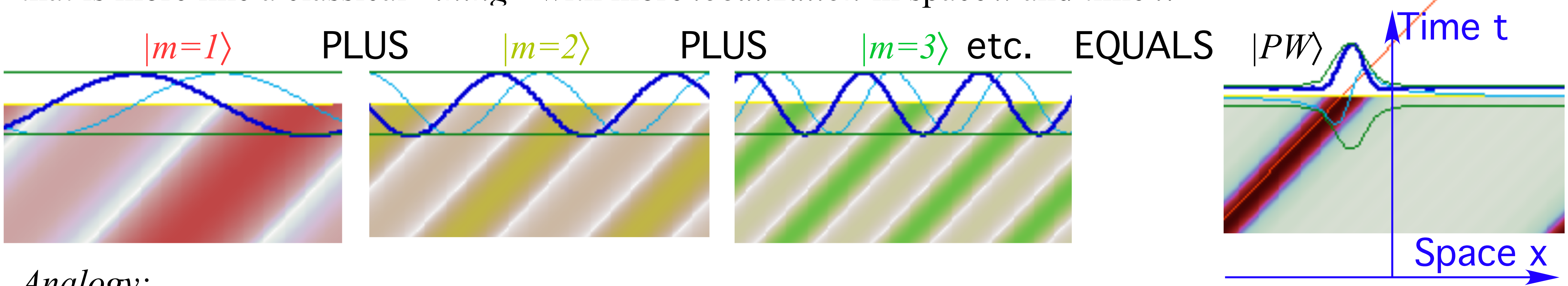
*Introducing coherent states (What lasers use)*

➡ *Analogy with (ω,k) wave packets*

*Wave coordinates need coherence*

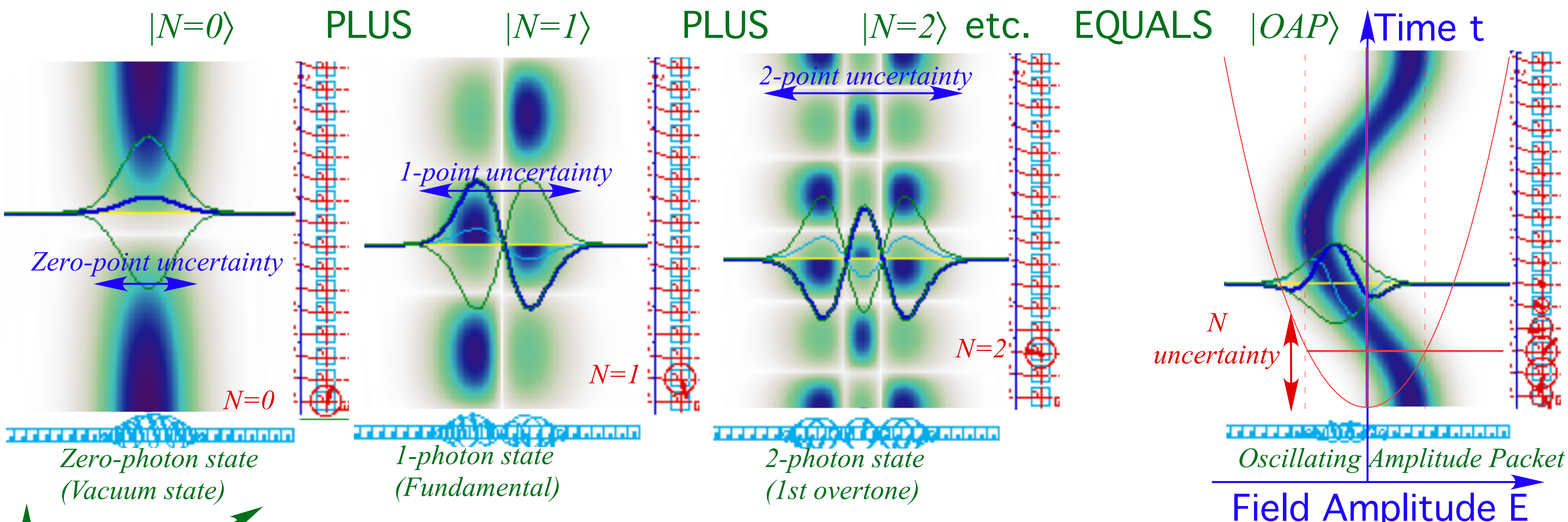
# *Coherent States: Oscillator Amplitude Packets analogous to Wave Packets*

We saw how adding *CW*'s *(Continuous Waves m=1,2,3...)* can make *PW (Pulse Wave)* or *WP (Wave Packet)* that is more like a classical *"thing"* with more *localization* in space *x* and time *t*.




*Analogy:*

Adding *photons (Quantized amplitude N=0,1,2...)* can make a *CS (Coherent State)* or *OAP (Oscillator Amplitude Packet)* that is more like a *classical wave oscillation* with more *localization* in field amplitude.

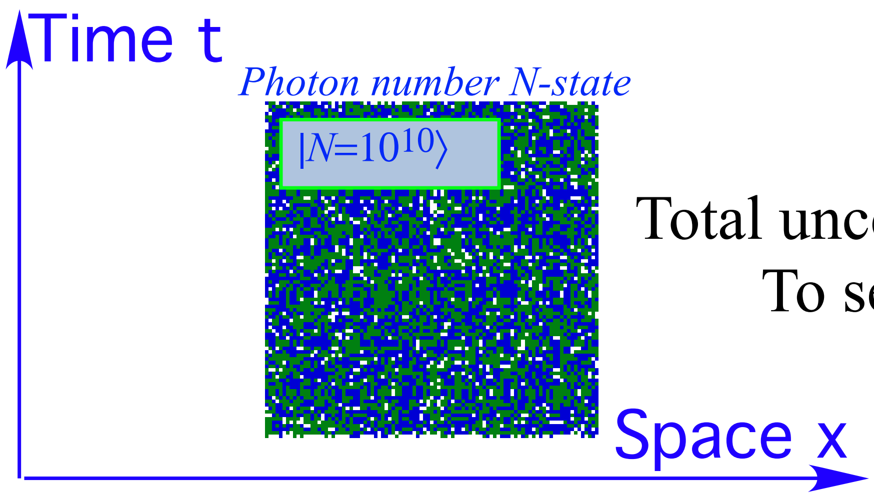



Pure photon states have localized (certain) *N* but *de*localized (*un*certain) amplitude and phase.

*OAP* states have *de*localized (*un*certain) *N* but more localized (certain) amplitude and phase.

## *Coherent States(contd.)* *Spacetime wave grid is impossible without coherent states*

Pure photon number *N*-states would make useless spacetime coordinates

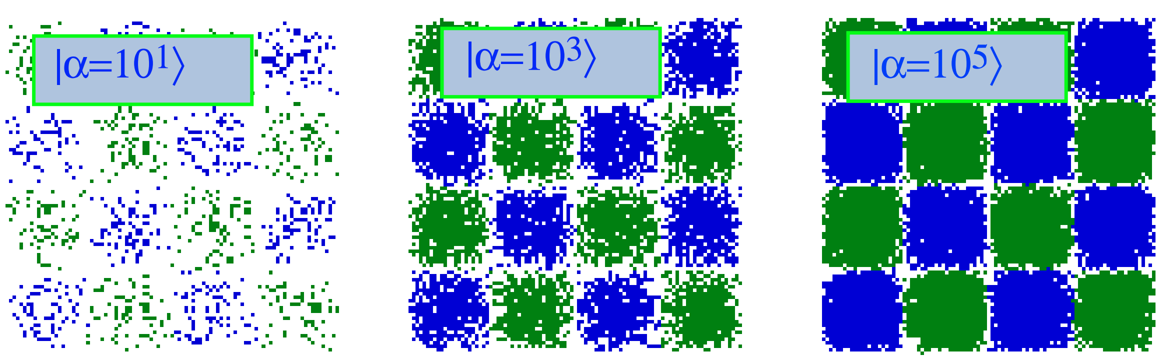



Total uncertainty of amplitude and phase makes the count pattern a wash.

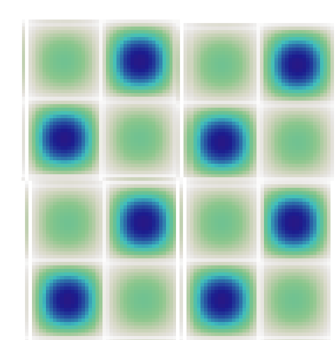
To see grids *some N-uncertainty is necessary!*

Coherent-α-states are defined by continuous amplitude-packet parameter α whose square is average photon number $\bar{N}=|\alpha|^2$. Coherent-states make better spacetime coordinates for larger $\bar{N}=|\alpha|^2$.

*Quantum field coherent α-states*




*Classical limit*



$\bar{N}=100$ $\Delta N=10$ | $\bar{N}=10^6$ $\Delta N=10^3$ | $\bar{N}=10^{10}$ $\Delta N=10^5$

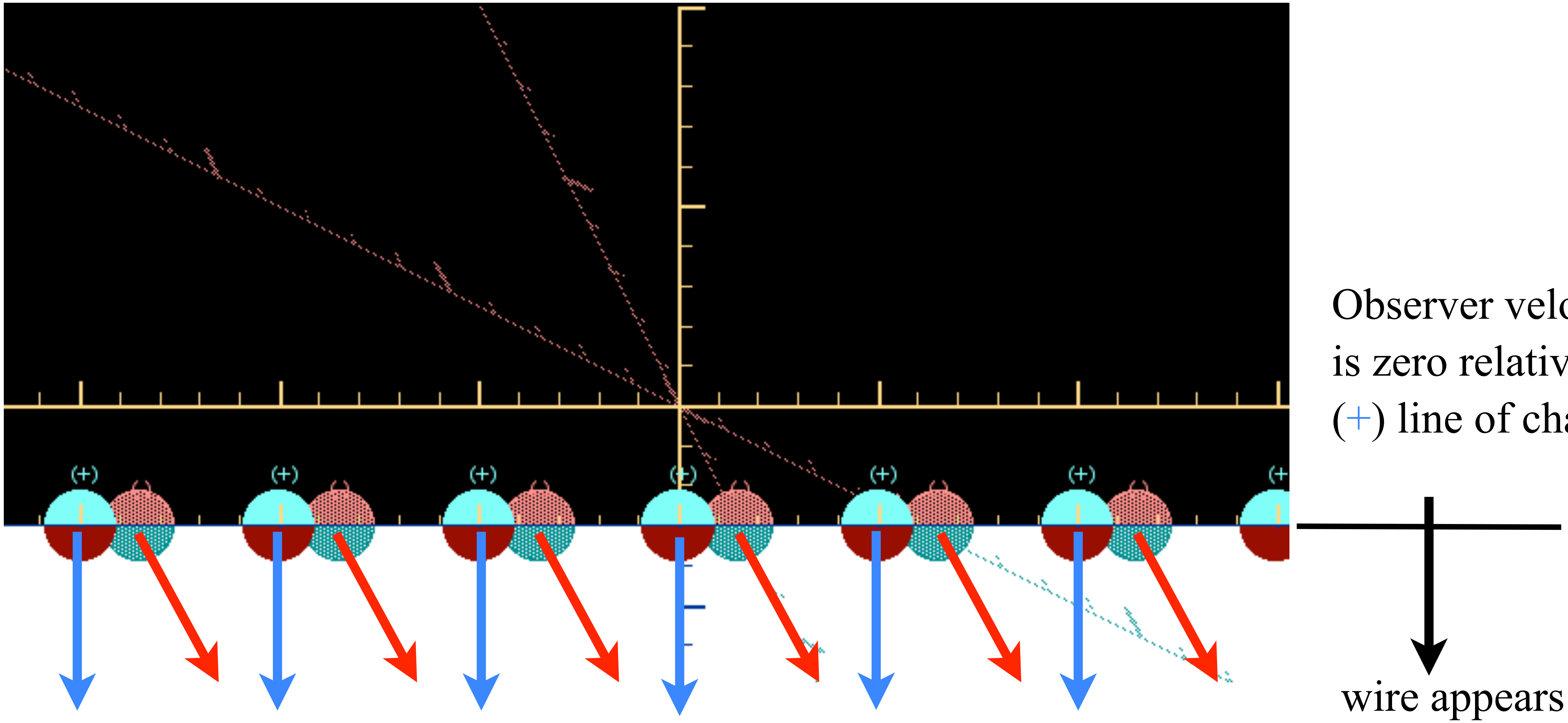
Coherent-state uncertainty in photon number (and mass) varies with amplitude parameter $\Delta N \sim \alpha \sim \sqrt{N}$ so a coherent state with $\bar{N}=|\alpha|^2=10^6$ only has a 1-in-1000 uncertainty $\Delta N \sim \alpha \sim \sqrt{N}=1000$.

## *Relativistic effects on charge, current, and Maxwell Fields*

➡ *Current density changes by Lorentz asynchrony*

*Magnetic B-field is relativistic* sinh$\rho$ *1st order-effect*

# *Relativistic effects on charge, current, and Maxwell Fields*



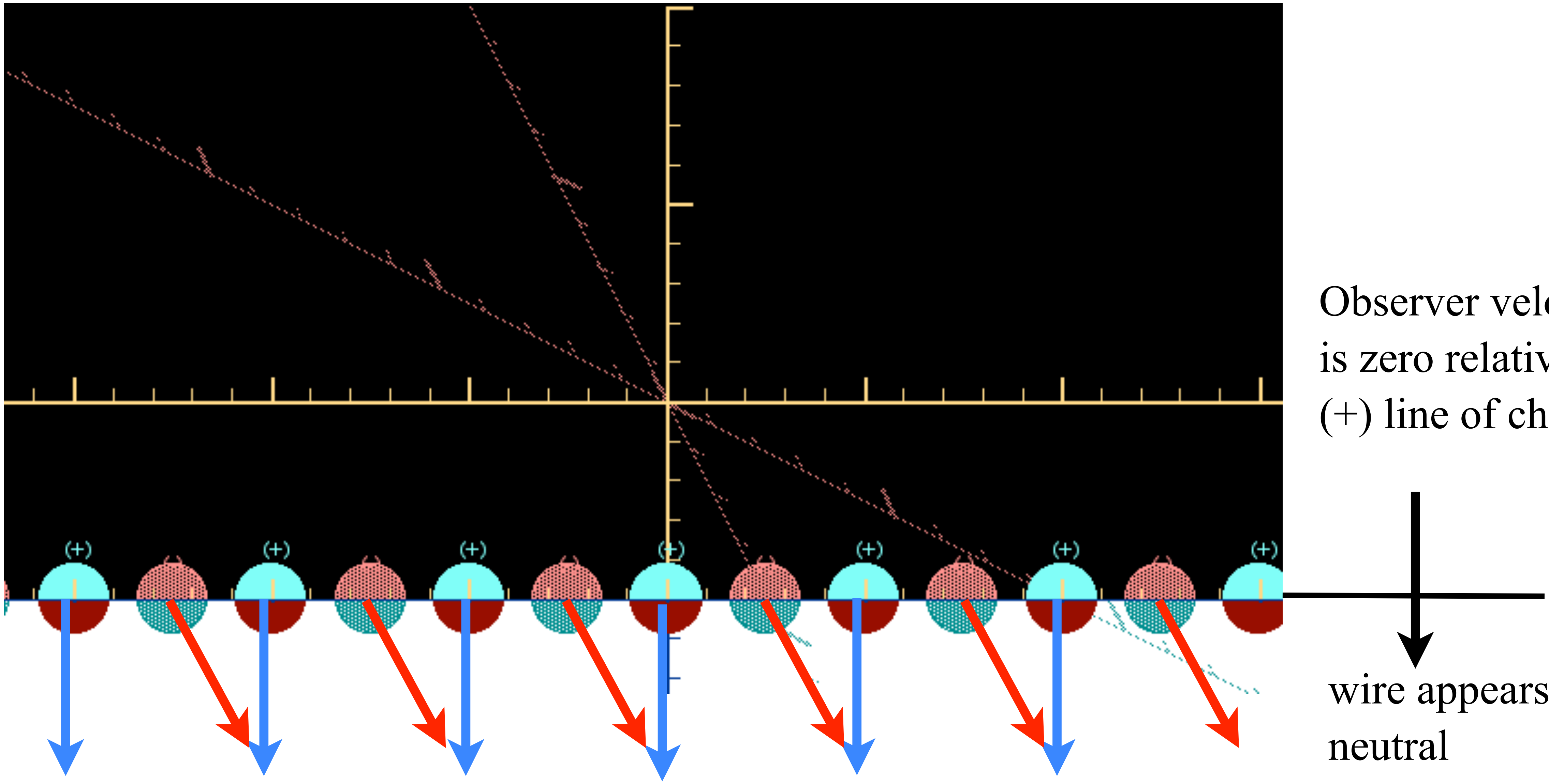

Observer velocity is zero relative to (+) line of charge

wire appears neutral

(+) Charge fixed (-) Charge moving to right (*Negative current density)*

(+) Charge density is Equal to the (-) Charge density

# *Relativistic effects on charge, current, and Maxwell Fields*

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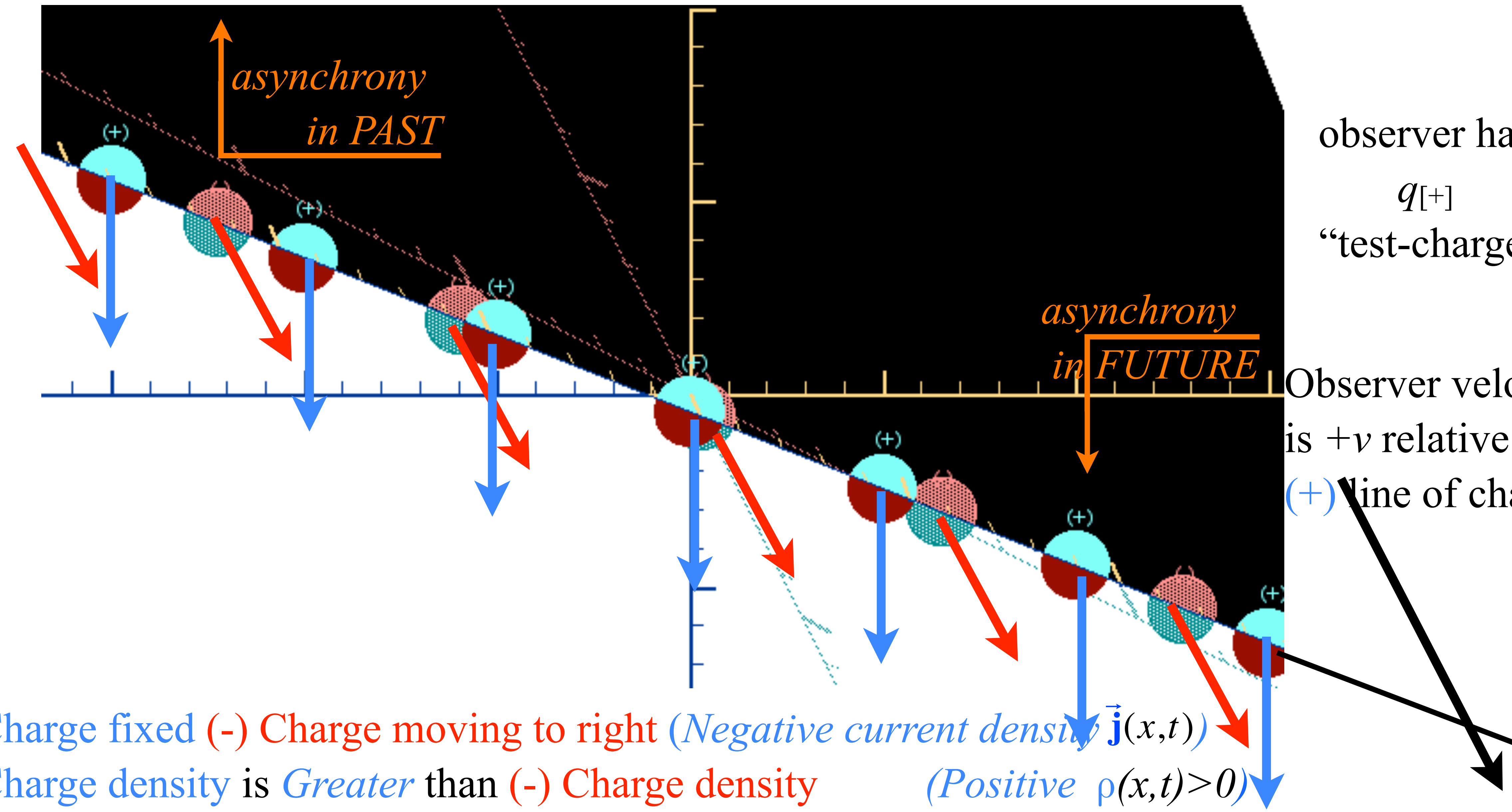
(+) Charge density is Equal to the (-) Charge density (*Zero* $\rho(x,t)=0$)

# *Relativistic effects on charge, current, and Maxwell Fields*

*Current density changes by Lorentz asynchrony*

*Asynchrony due to off-diagonal* $\sinh\rho$ (a 1st-order effect)

$$in\ Lorentz\ tranform: \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$$




observer has $q_{[+]}$ "test-charge"

Observer velocity is $+v$ relative to (+) line of charge

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

(+) Charge density is *Greater* than (-) Charge density (*Positive* $\rho(x,t)>0$)

wire appears postive (+) (repulsive to observer $q_{[+]}$)

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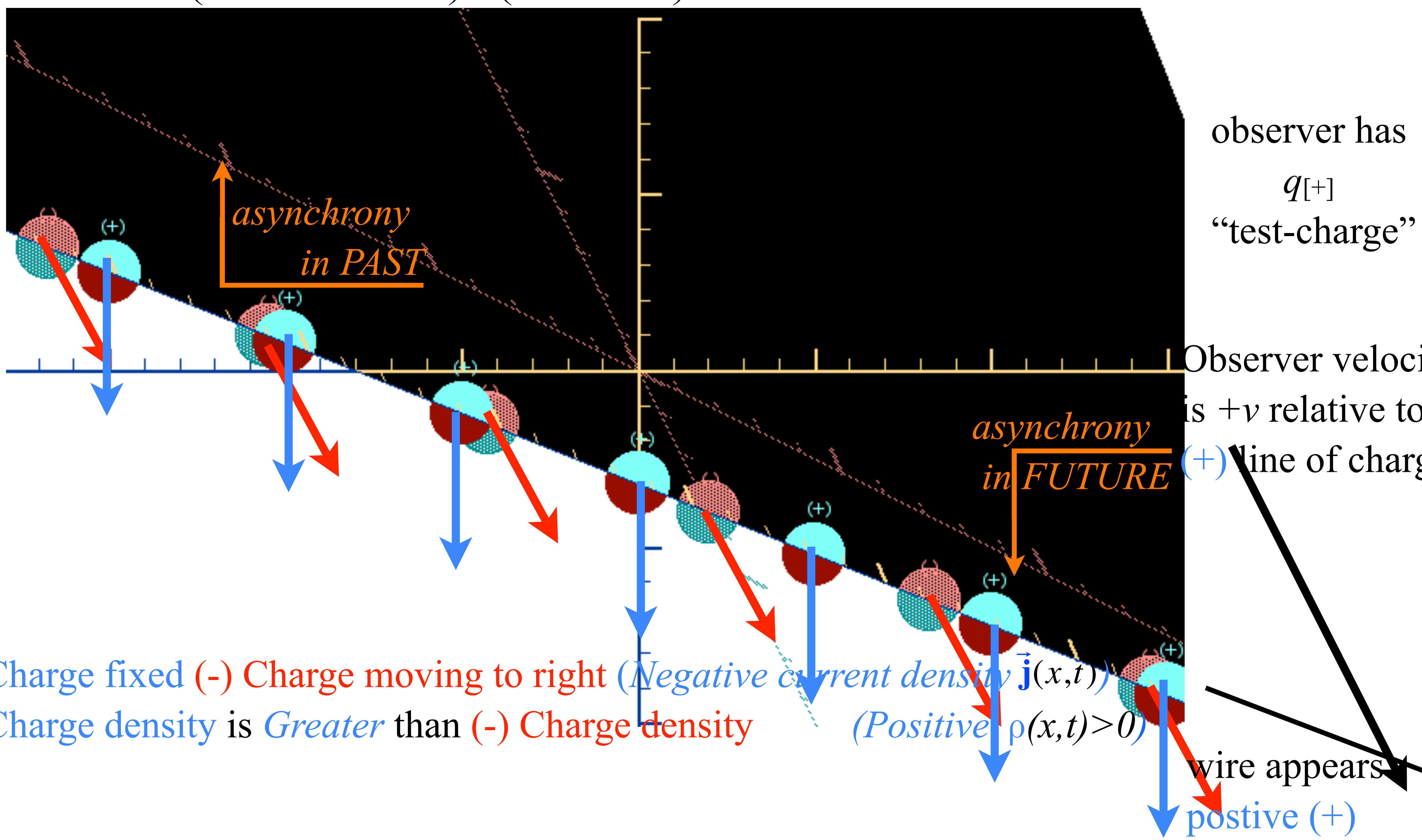


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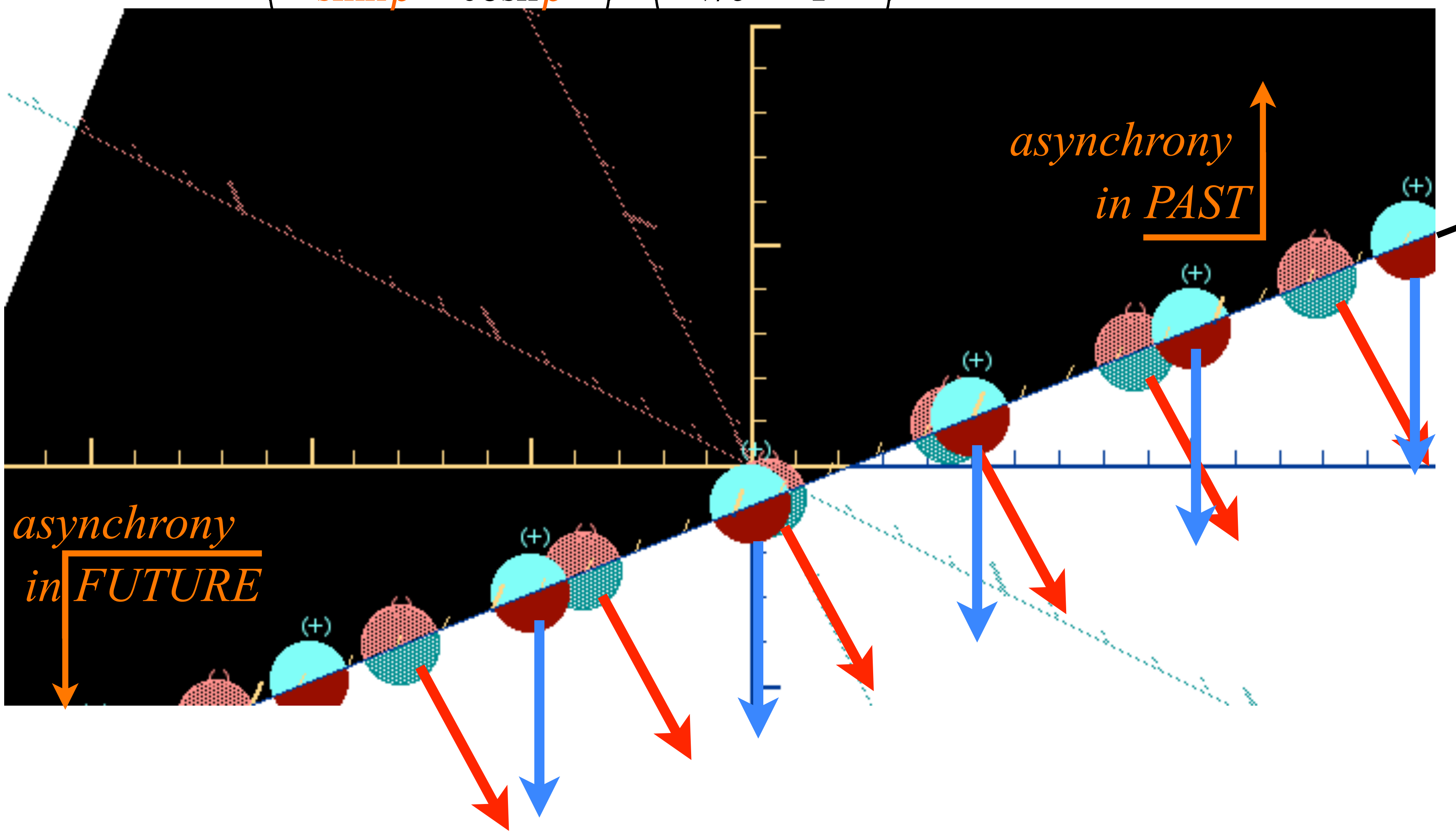
observer has $q_{[+]}$ "test-charge"

Observer velocity is -*v* relative to (+) line of charge




wire appears negative (-) (attractive to observer $q_{[+]}$)

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

(+) Charge density is *Less* than (-) Charge density (*Negative* $\rho(x,t)<0$)

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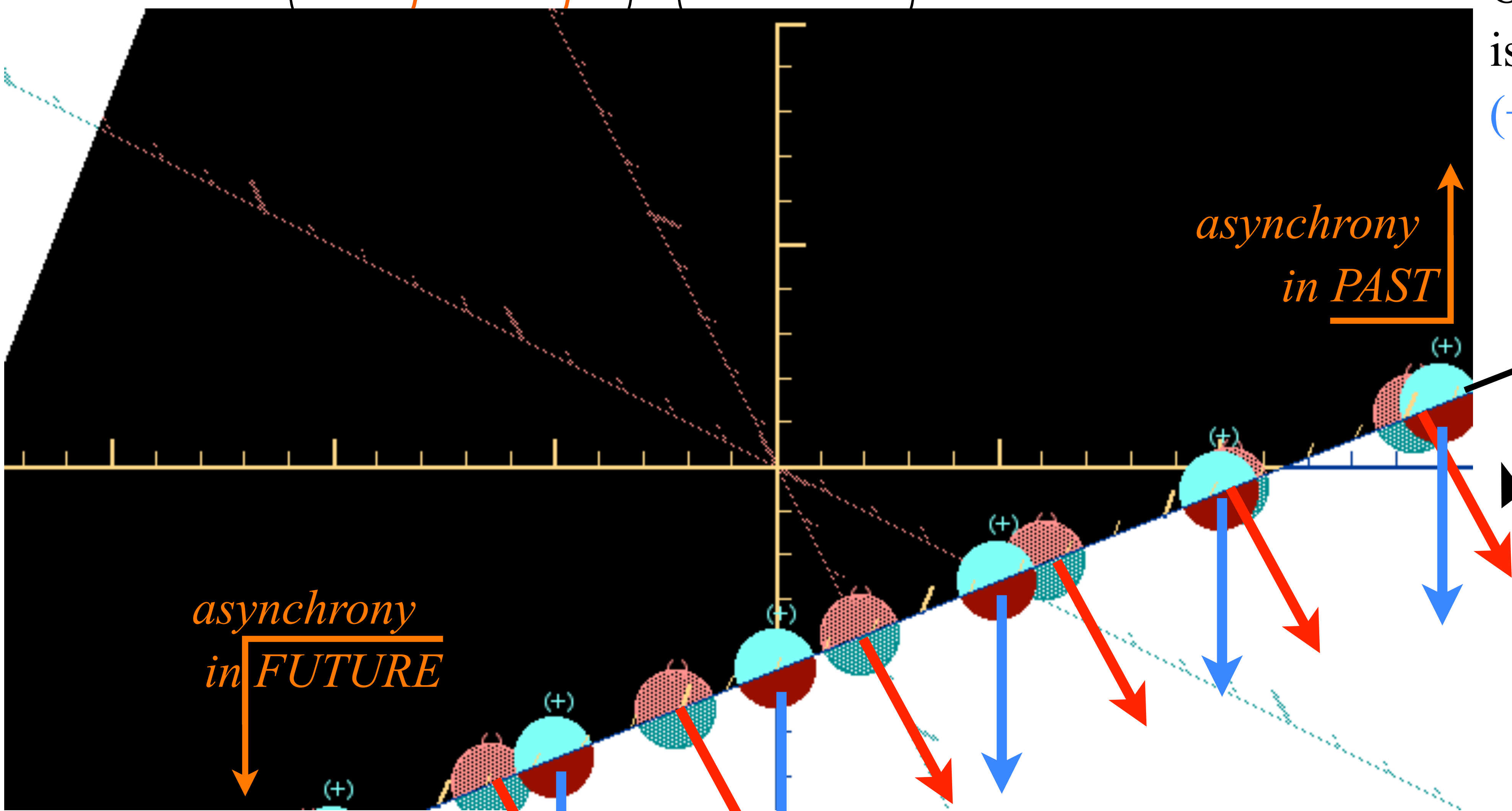
observer has $q_{[+]}$ "test-charge"

Observer velocity is $-v$ relative to (+) line of charge

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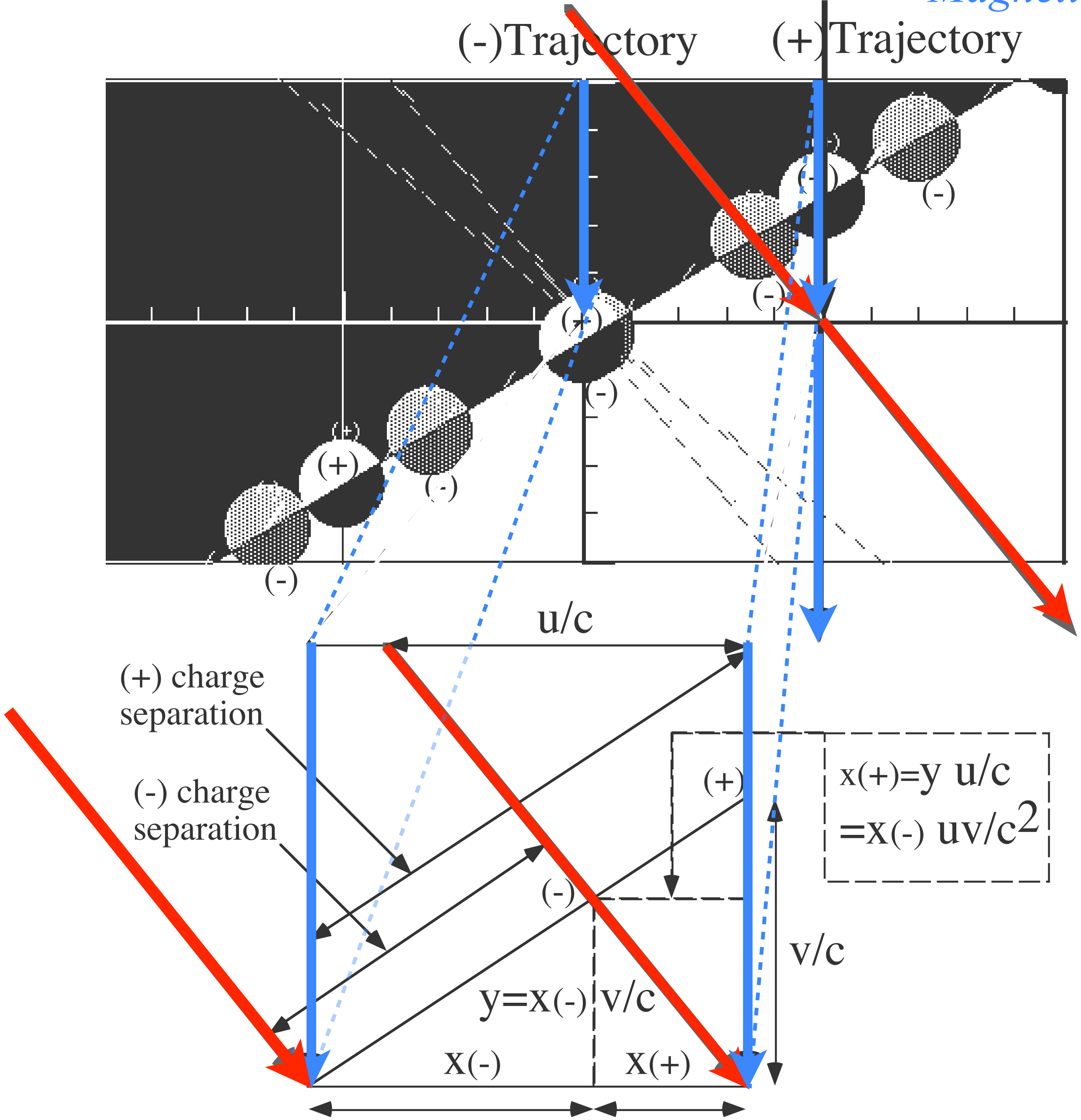



$$\frac{\rho(-)}{\rho(+)} = \frac{(+)\text{ charge separation}}{(-)\text{ charge separation}} = \frac{x(+)+x(-)}{x(-)}$$

$$\frac{\rho(-)}{\rho(+)} = \frac{x(+)}{x(-)} + 1 = \frac{uv}{c^2} + 1$$

$$\rho(+) - \rho(-) = \rho(+)\left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2}\rho(+)$$

Unit square: (u/c) /1 = x(+)/y

(v/c) /1 = y/x(-)

The electric force field **E** of a charged line varies inversely with radius. The Gauss formula for force in mks units :

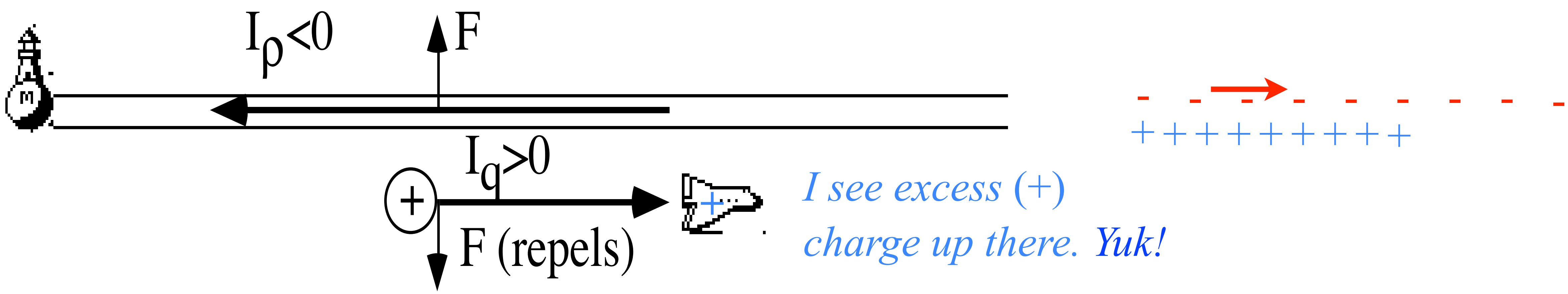
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2\rho}{r}\right] \quad , \quad \text{where:} \quad \frac{1}{4\pi\varepsilon_0} = 9\times 10^9 \frac{N\cdot m^2}{Coul.}$$

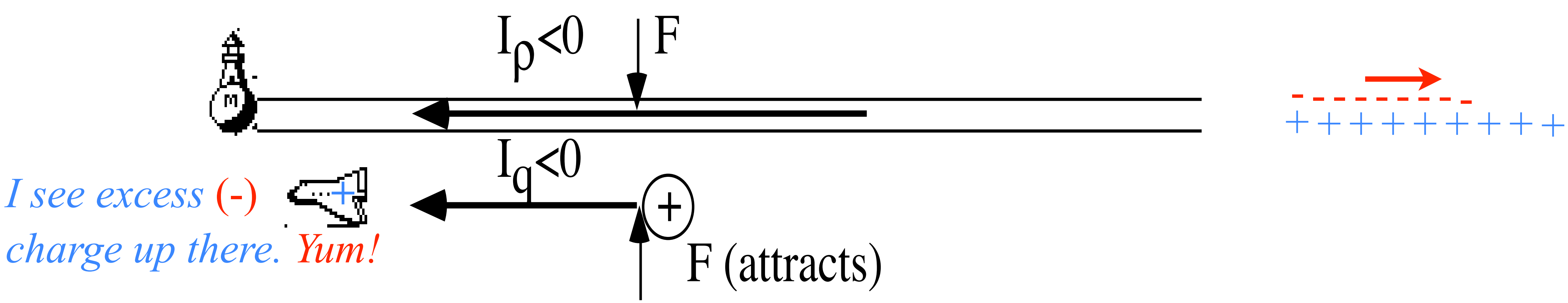
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2}{r}\left(-\frac{uv}{c^2}\rho(+)\right)\right] = -\frac{2\,qv\,\rho(+)u}{4\pi\varepsilon_0 c^2\, r} = -2\times 10^{-7}\frac{I_q\, I_\rho}{r}$$

$1/4\pi\varepsilon_0 = 9\cdot 10^9$

$c^2 = 9\cdot 10^{-16}$

$1/(4\pi\varepsilon_0 c^2) = 10^{-7}$



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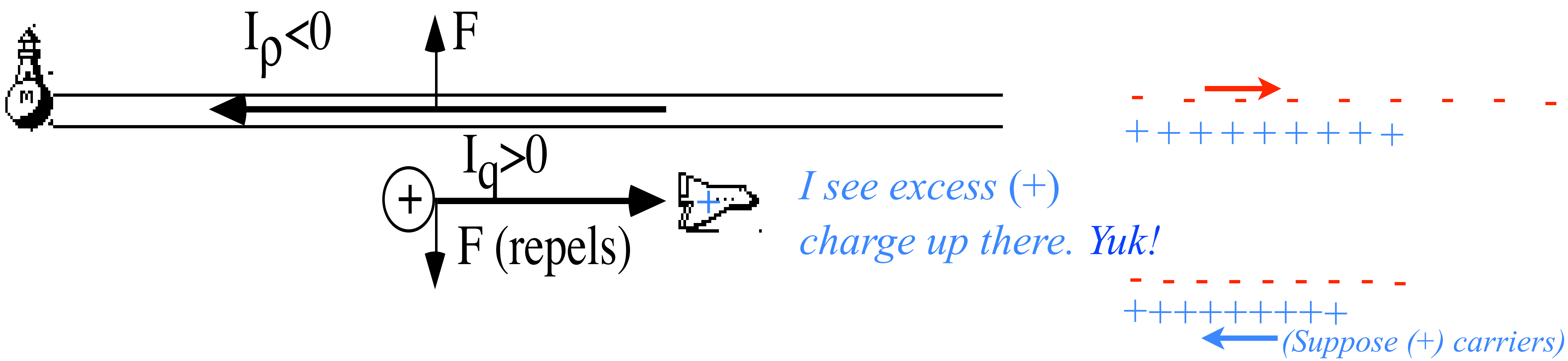


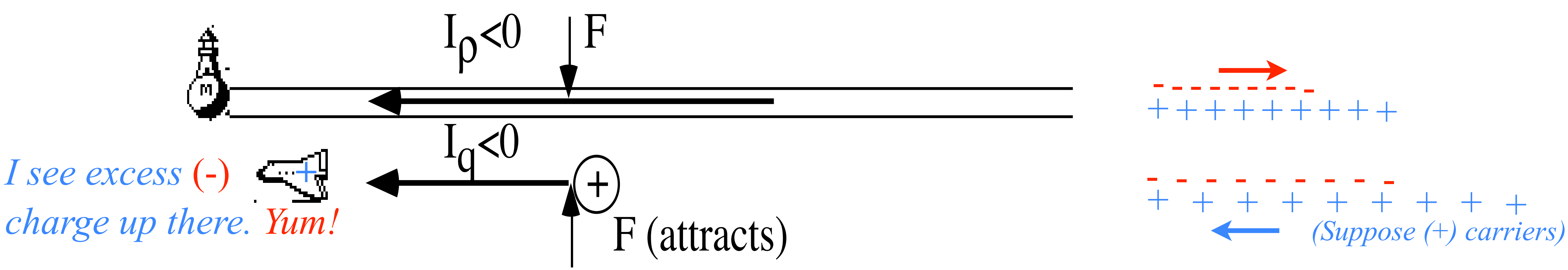

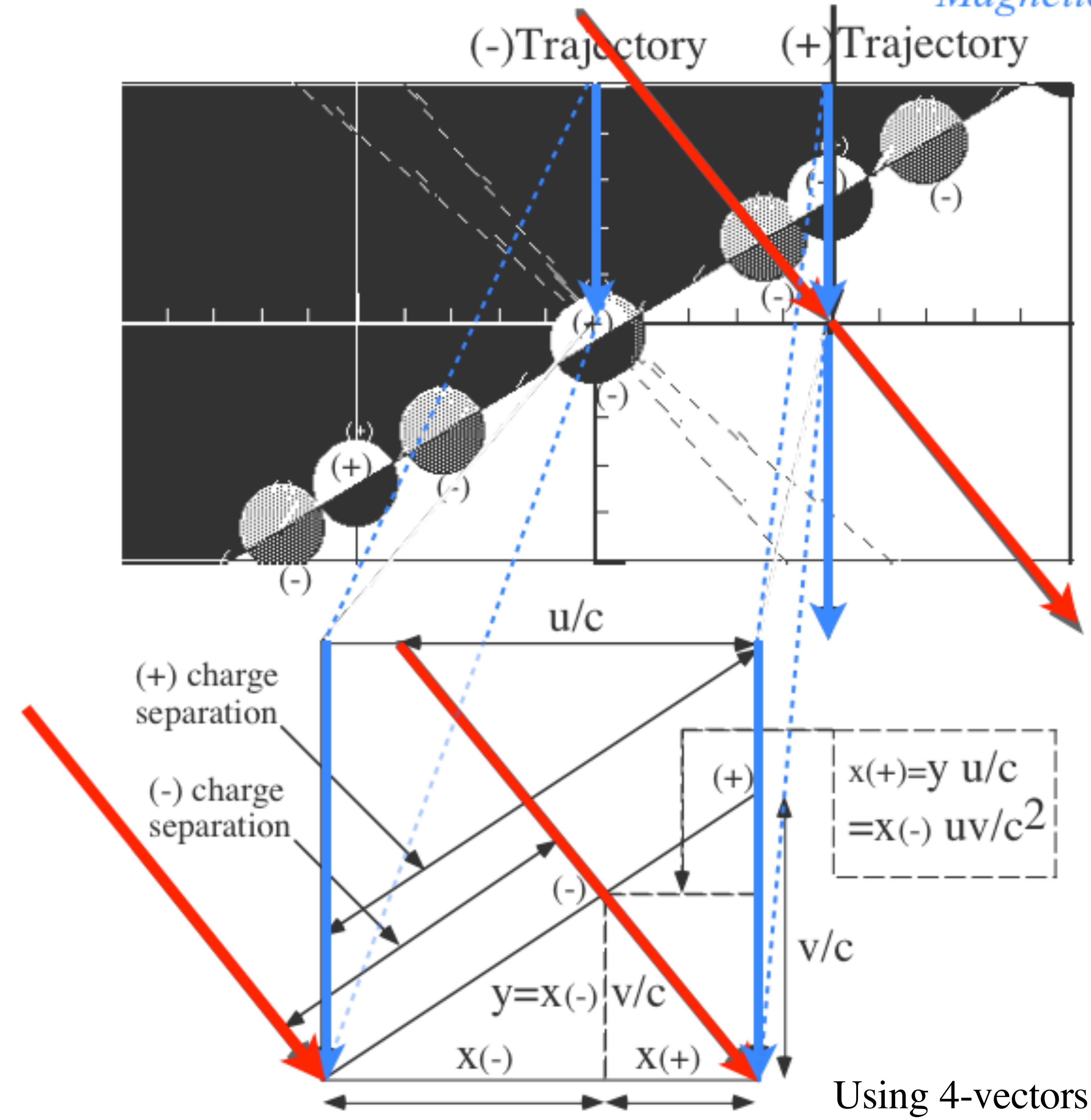


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Unit square: (u/c) /1 = x(+)/y

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Using 4-vectors to EL Transform (charge-current)=($c\rho$, $\mathbf{j}$)

$$\begin{pmatrix} c\rho' \\ j_{x'} \\ j_{y'} \\ j_{z'} \end{pmatrix}=\begin{pmatrix} \cosh\rho & \sinh\rho & \cdot & \cdot \\ \sinh\rho & \cosh\rho & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix}\begin{pmatrix} c\rho \\ j_x \\ j_y \\ j_z \end{pmatrix}$$