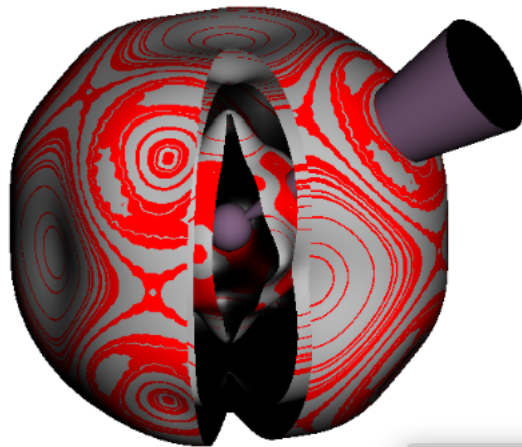


*ENERGY-LEVEL-CLUSTER RELATED
NUCLEAR-SPIN EFFECTS
AND
SUPER-HYPERFINE SPECTRAL PATTERNS:
HOW MOLECULES DO SELF-NMR:
internal-rotor molecules
and spin symmetry conversion effects*

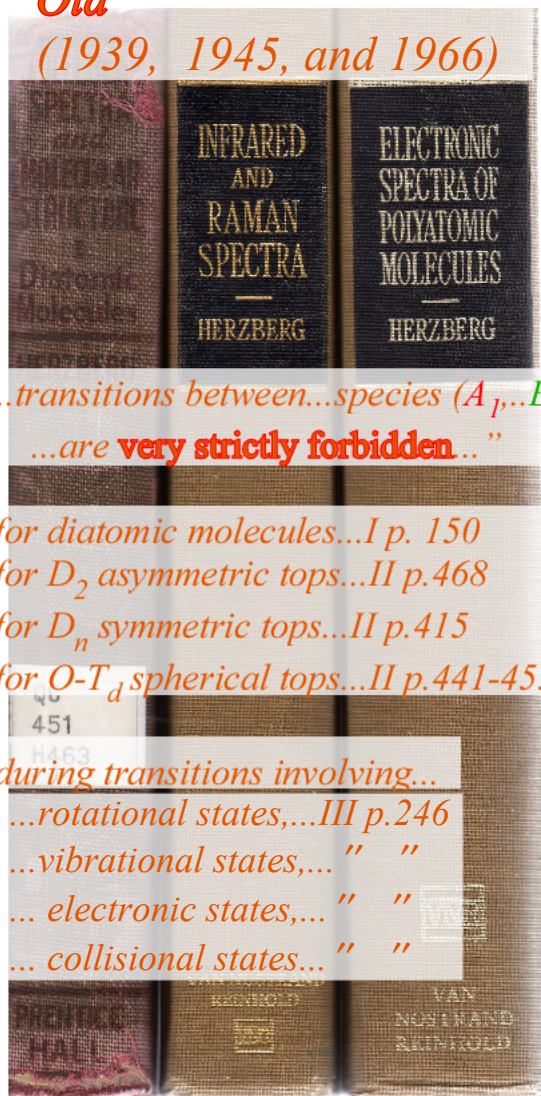
*William G. Harter and Justin C. Mitchell
Department of Physics, University of Arkansas
Fayetteville, AR 72701*



CONSERVATION OF ROVIBRONIC SPECIES - Two Views:

Old

(1939, 1945, and 1966)



versus

New (1978-present)

www.sciencemag.org SCIENCE VOL 310 23 DECEMBER 2005

CHEMISTRY

Nuclear Spin Conversion in Molecules

Jon T. Hougen and Takeshi Oka

Molecules with identical nuclei having nonzero spin can exist in different states called nuclear spin modifications by most researchers and nuclear spin isomers by some. Once prepared in

as initially shown by Bonhoeffer and Harteck in 1929 (3). Once prepared, a *para*- H_2 sample can be preserved for months.

[review of C_2H_4 study:
Sun, Takagi, Matsushima,
Science 310, 1938(2005)]

“...transitions between...species ($A_1, \dots, E, \dots, T_2, \dots$)
...are **very strictly forbidden**...”

...for diatomic molecules...I p. 150
...for D_2 asymmetric tops...II p.468
...for D_n symmetric tops...II p.415
...for $O-T_d$ spherical tops...II p.441-453

...during transitions involving...
...rotational states,...III p.246
...vibrational states,... " "
... electronic states,... " "
... collisional states... " "

Strictly versus **NOT!**
Conservation and
preservation?

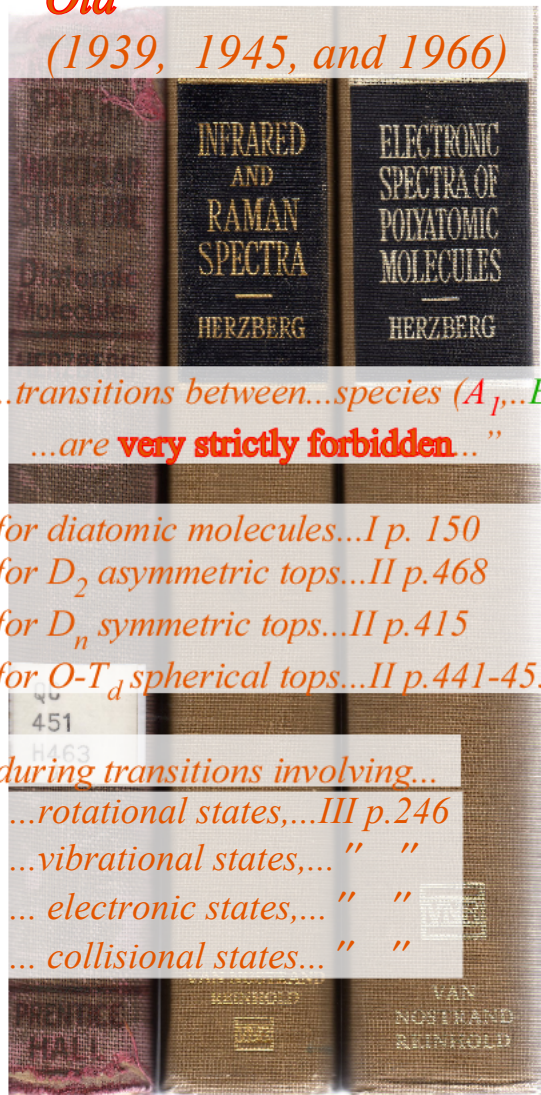
No Way! versus **WAY!**
Conversion, perversion
or transition?



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...during transitions involving...
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 ...vibrational states,... " "
 ... electronic states,... " "
 ... collisional states... " "

Strictly versus **NOT!**
 Conservation and preservation?

No Way! versus **WAY!**
 Conversion, perversion or transition?

To **conserve** vs. To **convert**
 To **preserve** vs. To **pervert**

perversion
 Widespread and extreme mixing of species reported in CF₄, SiF₄ and SF₆:

Ch. Borde, Phys. Rev. A20,254(1978)(expt.)
 Harter, Phys. Rev. A24,192 (1981)(theory)

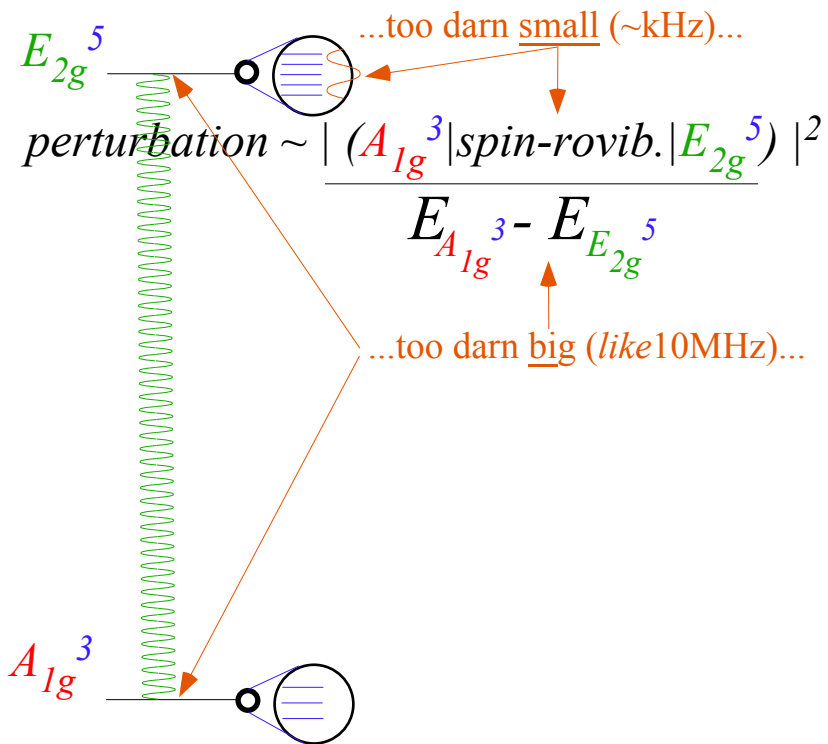
HOW **CONSERVED** IS ROVIBRONIC-SPIN SYMMETRY?

A_{2u}^1

What preserves it? versus **What messes it up?**

No Way!

...because nuclear moments...
...are so very slight..."



HOW **CONSERVED** IS ROVIBRONIC-SPIN SYMMETRY?

or *perverted?*

What preserves it? versus **What mixes it up?**

A_{2u}^1

No Way!

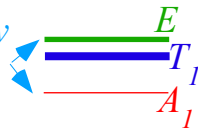
“...because nuclear moments...
...are so very slight...”

...too darn **small** (~kHz)...

$$\text{perturbation} \sim \frac{|(A_{1g}^3 | \text{spin-rovib.} | E_{2g}^5)|^2}{E_{A_{1g}^3} - E_{E_{2g}^5}}$$

...too darn **big** (like 10MHz)...

...exponentially
tiny!
(like 10^{-50} Hz)



RE Superhyperfine transitions

Hyperfine effects may rule! $A_1 T_1 E T_2 A_2$ get seriously mixed up.

Harter, Patterson, and daPaixao, Rev. Mod. Phys. 50, 37(1978)

Harter and Patterson, Phys. Rev. A19, 2277(1979) (CF₄)

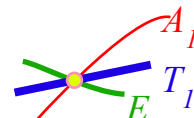
Harter, Phys. Rev. A24, 192-262(1981) (SF₆)

WAY!

...because levels of different species
are forced together by angular wave
localization or “level-clustering” or
(rarely) by “accidental” degeneracy.

“Accidental” degeneracy

Lea, Leask & Wolf JPCSol.23,1381(1962)



Level-clustering

Dorney and Watson JMS 42,135(1972)

Harter and Patterson PRL38,224(1977)

JCP 66,4872(1977)

RE Surface precession vs. tunneling

Harter and Patterson JMP 20,1453(1979)

JCP 80,4241(1984)

$v_3/2v_4$ See **RJ06 & RI09**

Symmetry-level-cluster effects in SF_6 , SiF_4 , CH_4 , CF_4 **Mitchell & Boudon**)

Graphical approach to rotation-vibration-spin Hamiltonian

$$\langle H \rangle \sim v_{\text{vib}} + BJ(J+1) + \langle H^{\text{Scalar Coriolis}} \rangle + \langle H^{\text{Tensor Centrifugal}} \rangle + \langle H^{\text{Nuclear Spin}} \rangle + \langle H^{\text{Tensor Coriolis}} \rangle + \dots$$

to help understand complex rotational spectra and dynamics.

OUTLINE

Introductory review

Example(s)

- *Rovibronic nomograms and PQR structure* v_3 and v_4 SF_6
- *Rotational Energy Surfaces (RES) and Θ_K^J -cones* v_4 P(88) SF_6
- *Spin symmetry correlation tunneling and entanglement* SF_6

Recent developments

- *Analogy between PE surface and RES dynamics*
- *Rotational Energy Eigenvalue Surfaces (REES)* v_3 SF_6

Graphical approach to rotation-vibration-spin Hamiltonian

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OUTLINE

Introductory review

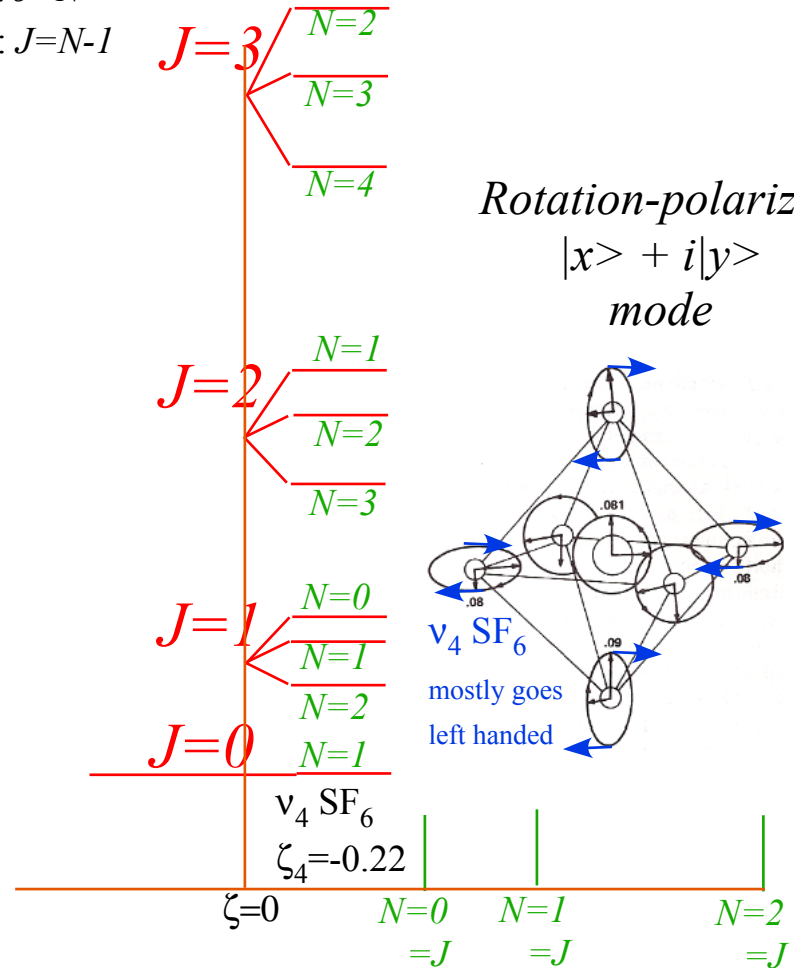
Example(s)

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- *Spin symmetry correlation tunneling and entanglement* SF₆
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- *Rotational Energy Eigenvalue Surfaces (REES)* v_3 SF₆

$$\langle H \rangle \sim v_{\text{vib}} + BJ(J+1) + \langle H^{\text{Scalar Coriolis}} \rangle + \langle H^{\text{Tensor Centrifugal}} \rangle + \langle H^{\text{Tensor Coriolis}} \rangle + \langle H^{\text{Nuclear Spin}} \rangle + \dots$$

$$\langle H \rangle \sim v_{\text{vib}} + BN(N+1) + 2B(1-\zeta) \cdot \begin{cases} N+1 & \text{for } J=N+1 \\ 0 & \text{for } J=N \\ N & \text{for } J=N-1 \end{cases}$$

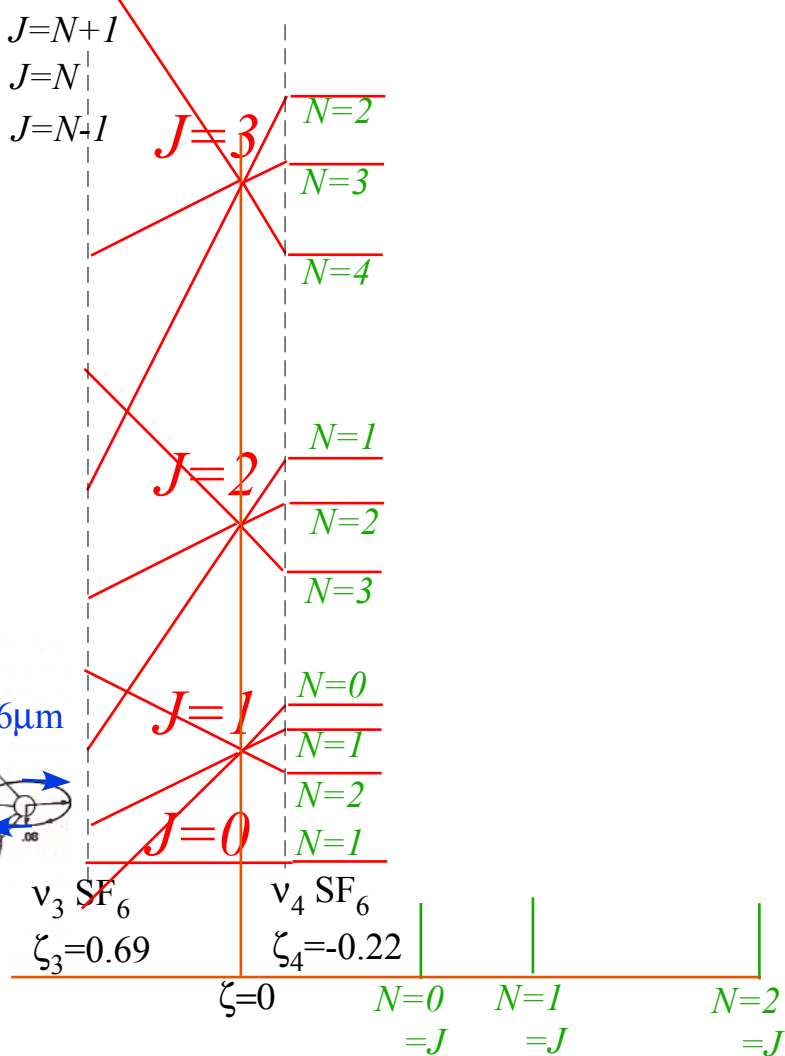
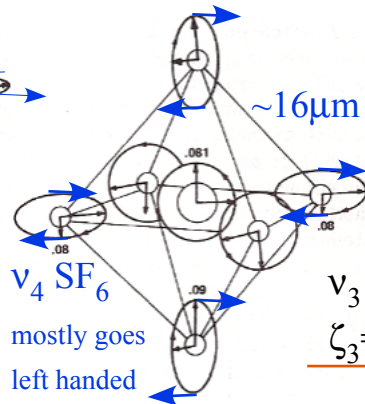
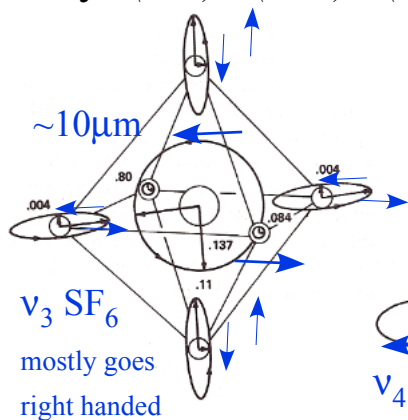
$$\begin{aligned} H^{\text{Scalar Coriolis}} &= -B\zeta 2\mathbf{J}^{\text{Total}} \cdot \boldsymbol{\ell}^{\text{vibe}} \\ &= -B\zeta [\mathbf{J}^2 - (\mathbf{J}^2 - \boldsymbol{\ell}^2) + \boldsymbol{\ell}^2] \\ &= -B\zeta [\mathbf{J}^2 - N^2 + \boldsymbol{\ell}^2] \\ &= -B\zeta [J(J+1) - N(N+1) + \ell(\ell+1)] \end{aligned}$$



$$\langle H \rangle \sim v_{\text{vib}} + BJ(J+1) + \langle H^{\text{Scalar Coriolis}} \rangle + \langle H^{\text{Tensor Centrifugal}} \rangle + \langle H^{\text{Tensor Coriolis}} \rangle + \langle H^{\text{Nuclear Spin}} \rangle + \dots$$

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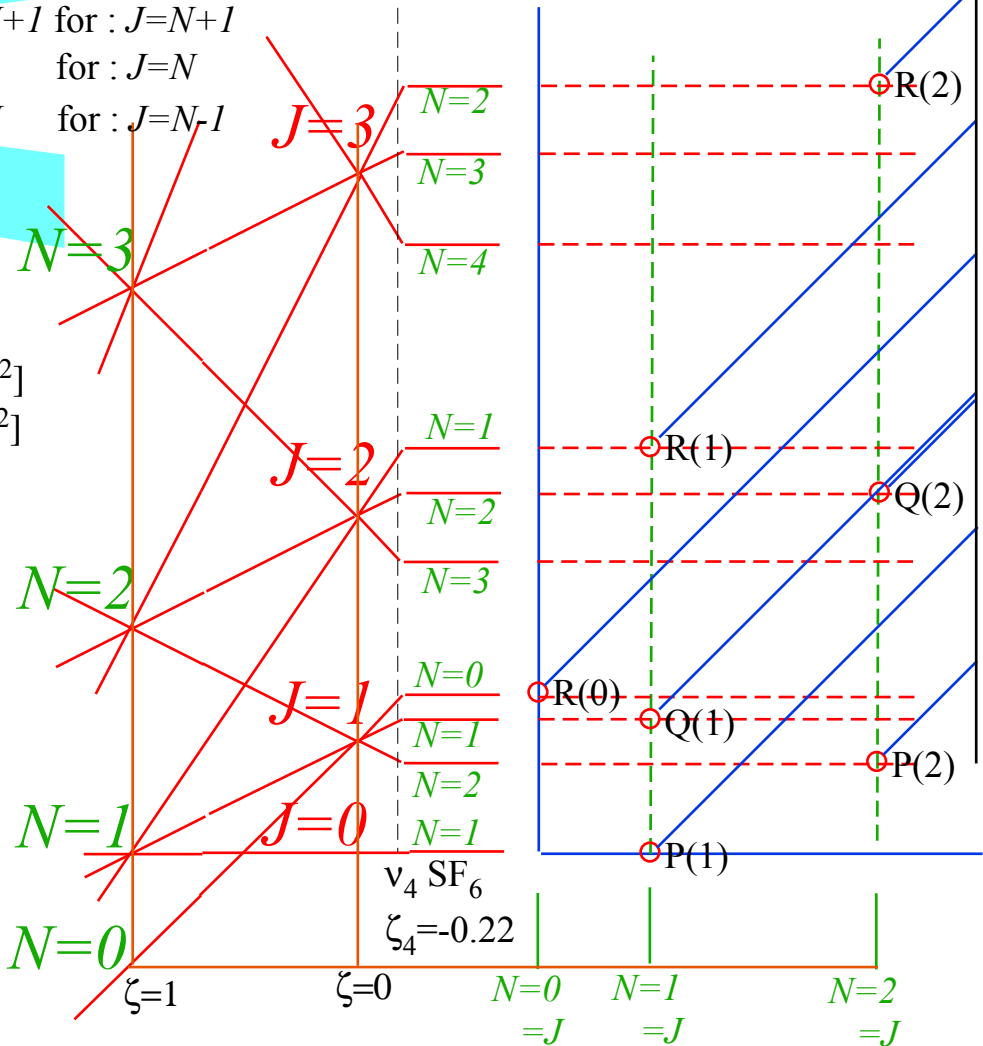
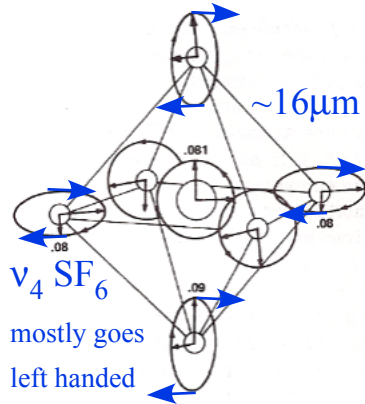
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$$\langle H \rangle \sim v_{\text{vib}} + BJ(J+1) + \langle H^{\text{Scalar Coriolis}} \rangle + \langle H^{\text{Tensor Centrifugal}} \rangle + \langle H^{\text{Tensor Coriolis}} \rangle + \langle H^{\text{Nuclear Spin}} \rangle + \dots$$

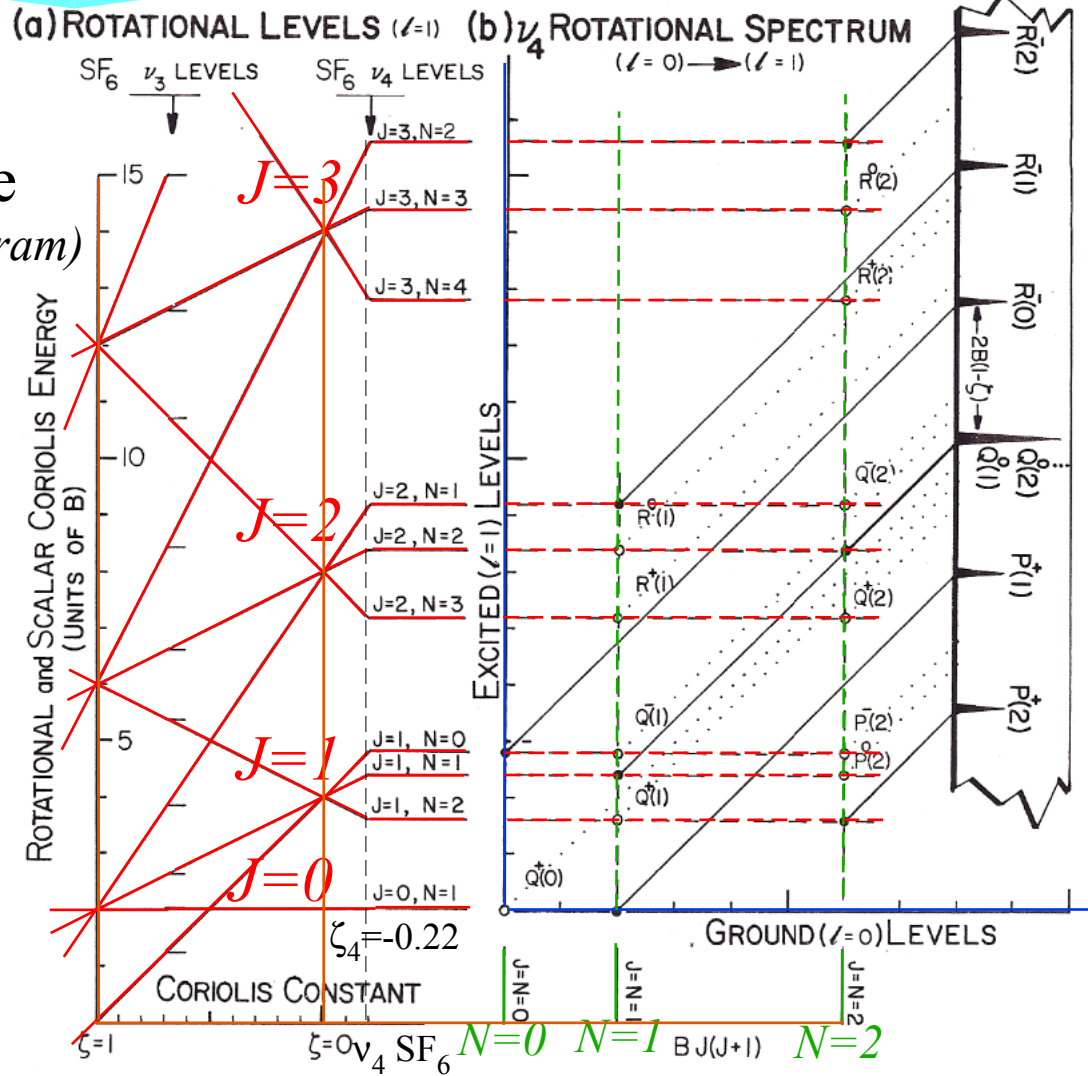
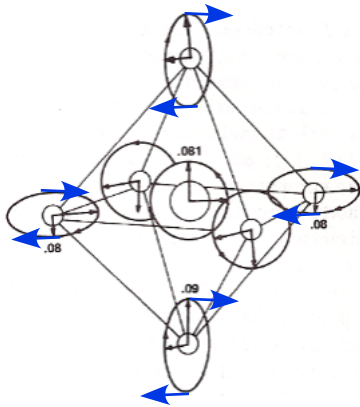
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Summary of
low-J (PQR)
ro-vibe structure
(Using ro vib. nomogram)



Graphical approach to rotation-vibration-spin Hamiltonian

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Introductory review

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v_3 and v_4 SF₆

• *Rotational Energy Surfaces (RES) and Θ_K^J -cones*

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Recent developments

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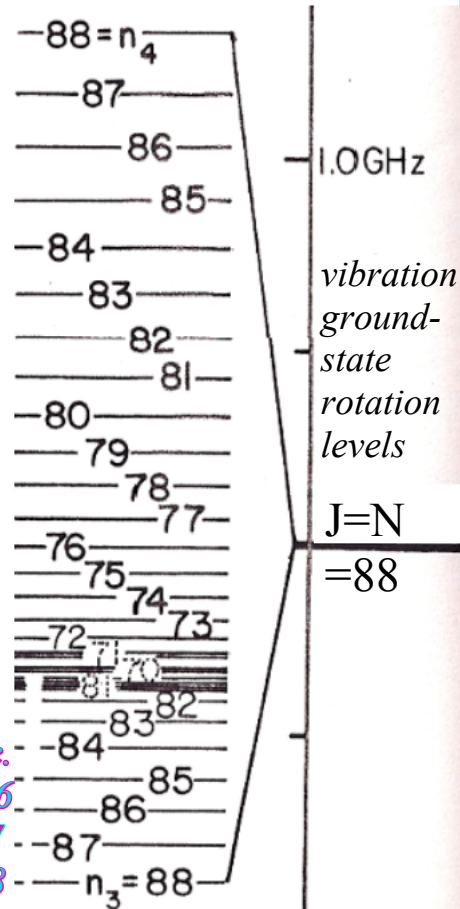
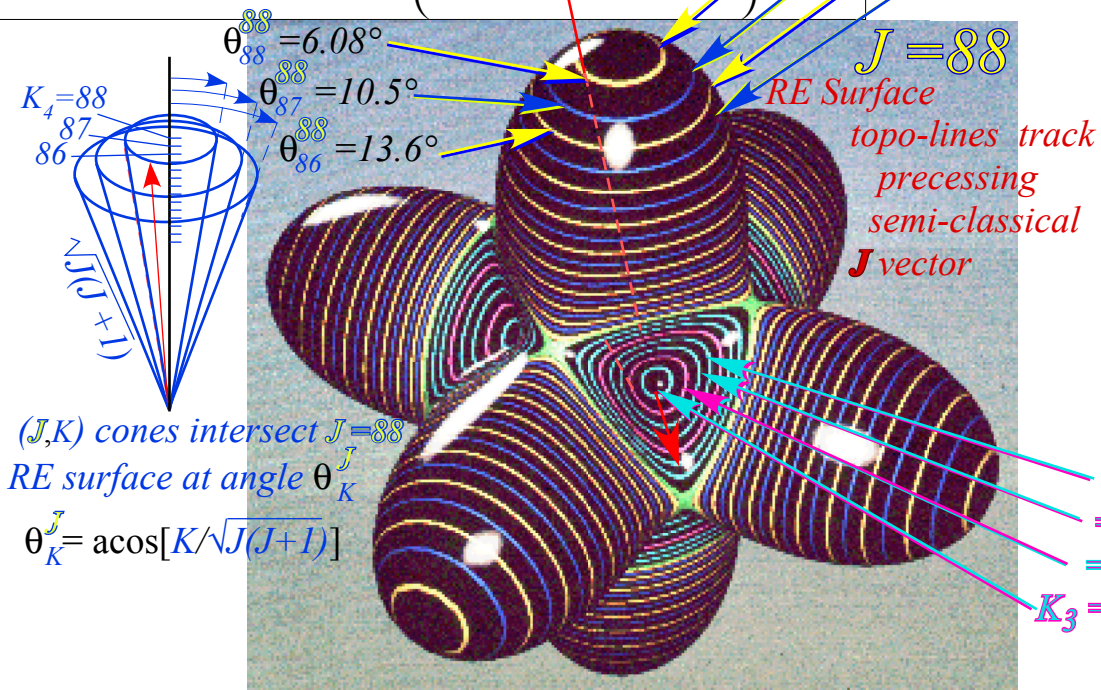
v_3 SF₆

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O_h or T_d Spherical Top: (Hecht CH_4 Hamiltonian 1960)

$$H = B(\mathbf{J}_x^2 + \mathbf{J}_y^2 + \mathbf{J}_z^2) + t_{440} \left(\mathbf{J}_x^4 + \mathbf{J}_y^4 + \mathbf{J}_z^4 - \frac{3}{5} J^4 \right) + \dots$$

$$= B J^2 + t_{440} \left(\mathbf{T}_0^4 + \sqrt{\frac{5}{14}} [\mathbf{T}_4^4 + \mathbf{T}_{-4}^4] \right) + \dots$$



(J,K) cones intersect $J=88$
RE surface at angle θ_K^J

$$\theta_K^J = \arccos\left[\frac{K}{\sqrt{J(J+1)}}\right]$$

(next page shows slice)

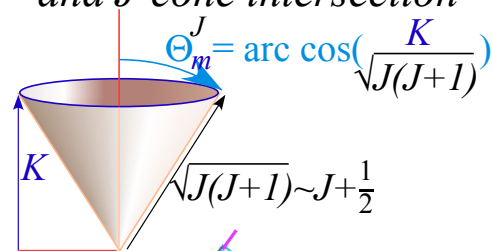


SF₆ Spectra of O_h Ro-vibronic Hamiltonian described by RE Tensor Topography

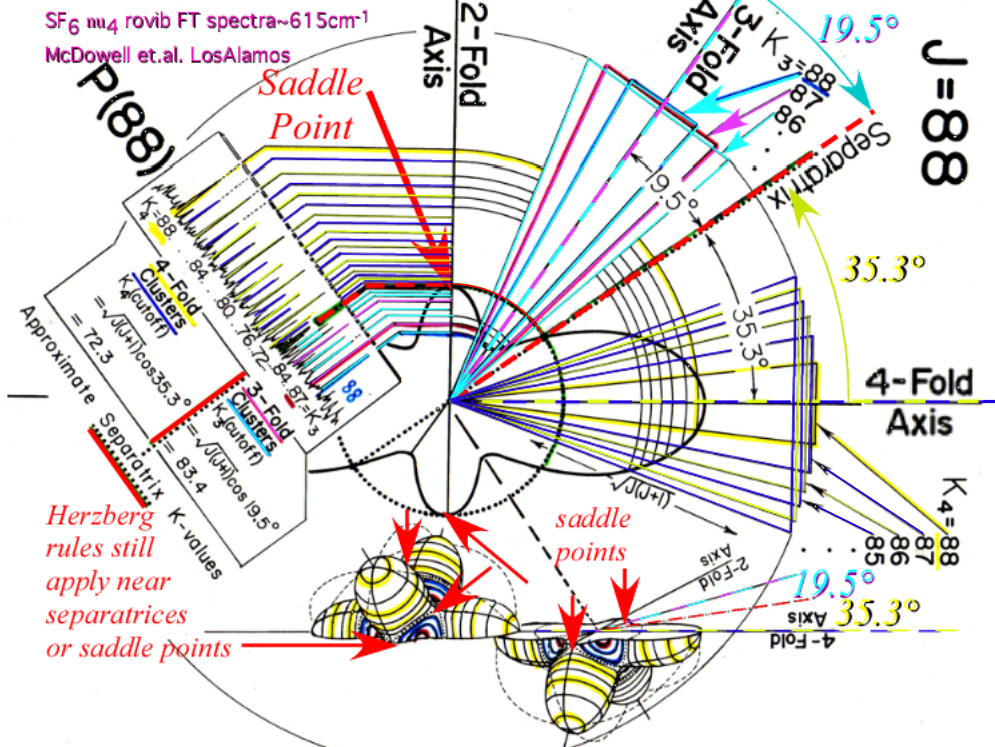
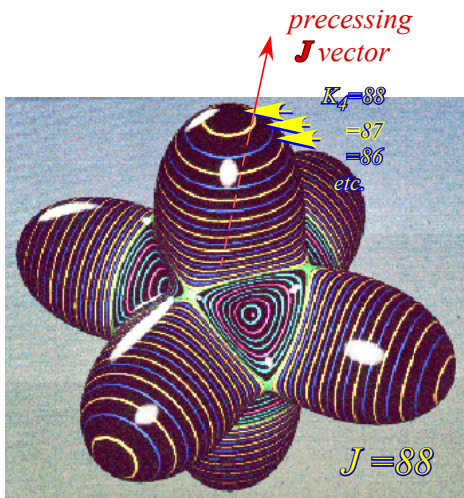
$$\mathbf{H} = B(\mathbf{J}_x^2 + \mathbf{J}_y^2 + \mathbf{J}_z^2) + t_{440} \left(\mathbf{J}_x^4 + \mathbf{J}_y^4 + \mathbf{J}_z^4 - \frac{3}{5} J^4 \right) + \dots$$

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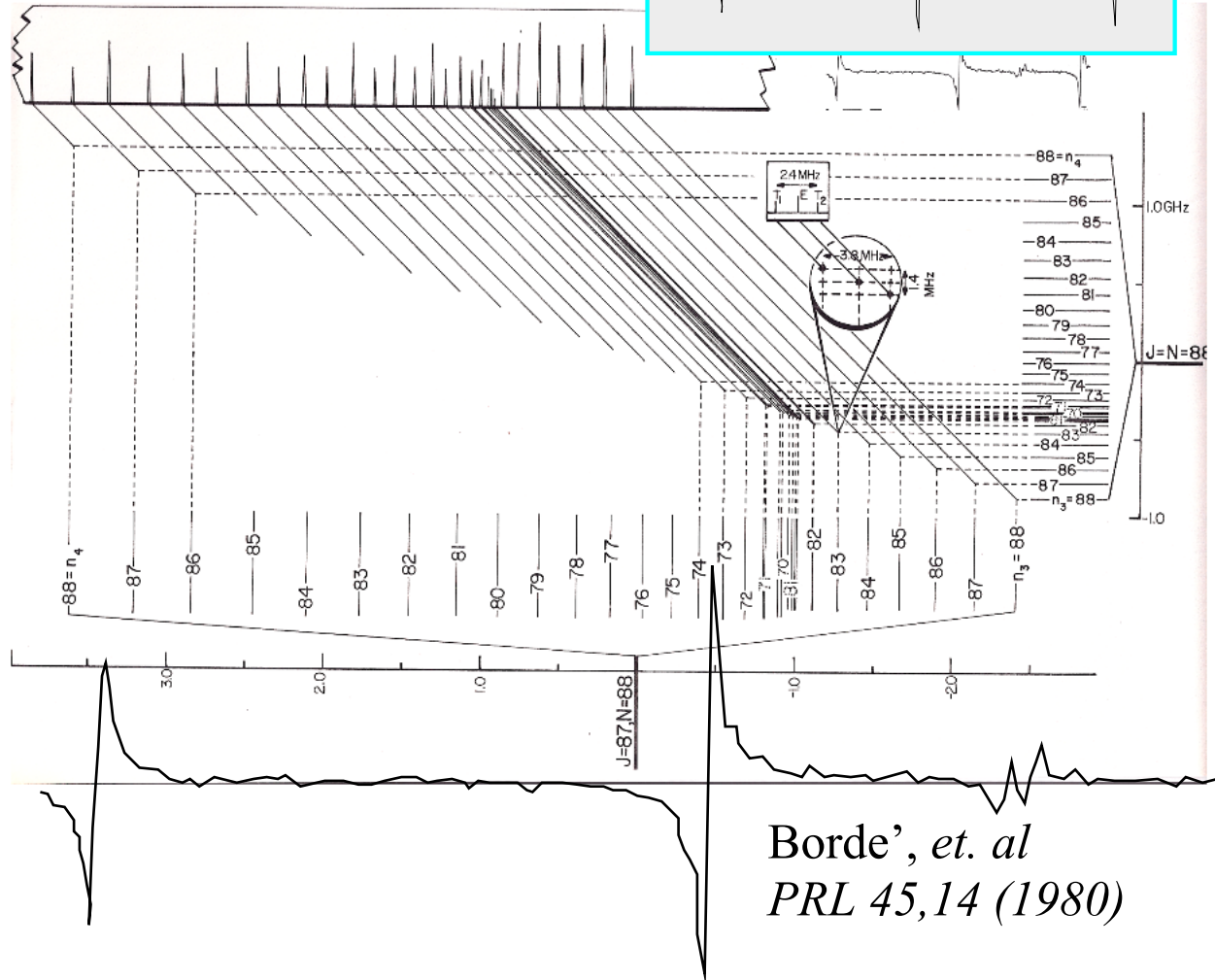
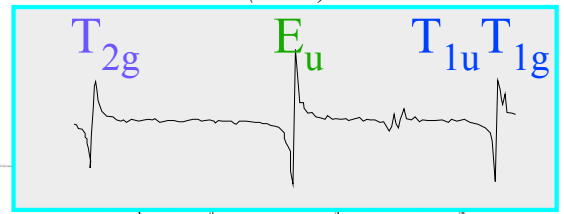
and J-cone intersection



Rovibronic Energy (RE) Tensor Surface



Borde', et. al
PRL 45,14 (1980)



Borde', et. al
PRL 45,14 (1980)

Graphical approach to rotation-vibration-spin Hamiltonian

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OUTLINE

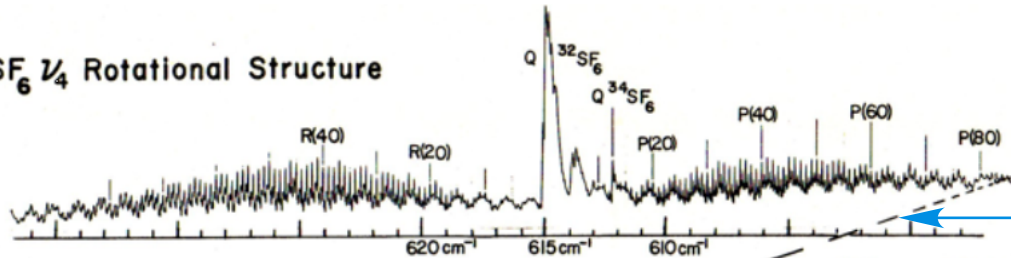
Introductory review

- | | <u>Example(s)</u> |
|---|---------------------------------|
| • <i>Rovibronic nomograms and PQR structure</i> | v_3 and v_4 SF ₆ |
| • <i>Rotational Energy Surfaces (RES) and Θ_K^J-cones</i> | v_4 P(88) SF ₆ |
| • <i>Spin symmetry correlation tunneling and entanglement</i> | SF ₆ |

Recent developments

- | | |
|---|-----------------------|
| • <i>Analogy between PE surface and RES dynamics</i> | |
| • <i>Rotational Energy Eigenvalue Surfaces (REES)</i> | v_3 SF ₆ |

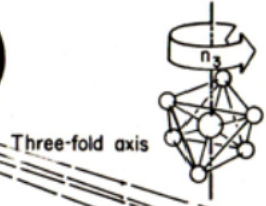
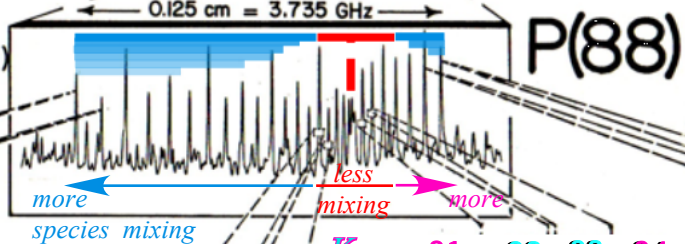
(a) SF₆ 1/4 Rotational Structure



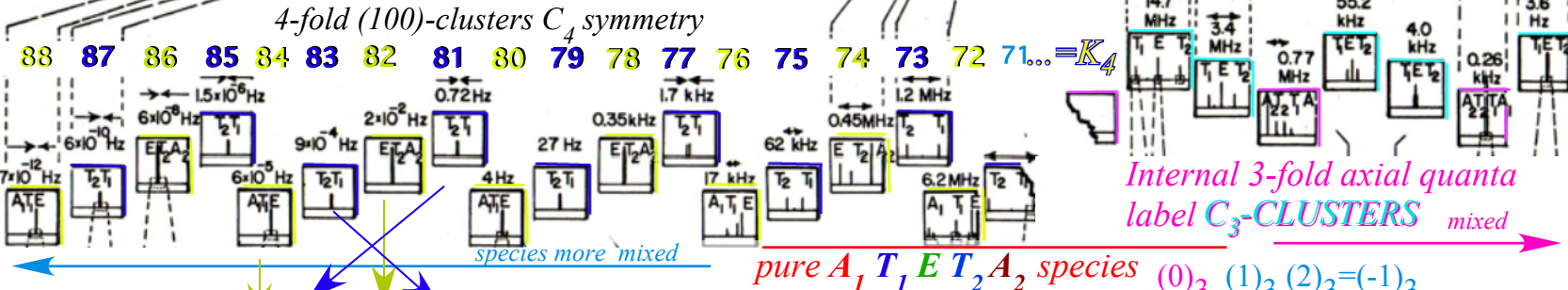
FT IR and Laser Diode Spectra
K.C. Kim, W.B. Person, D. Seitz, and B.J. Krohn
J. Mol. Spectrosc. 76, 322 (1979).

Primary AET species mixing increases with distance from "separatrix"

(b) P(88) Fine Structure (Rotational anisotropy effects)



(c) Superfine Structure (Rotational axis tunneling)



Cubic Octahedral symmetry O

A ₁	1	•	•	•
A ₂	•	•	1	•
E	1	•	1	•
T ₁	1	1	•	1
T ₂	•	1	1	1

3 modulo 4 equals -1 modulo 4 (and 83 mod 4) 83=84-1

4-fold (100) C₄ symmetry clusters

3-fold (111) C₃ symmetry clusters

A ₁	1	•	•
A ₂	1	•	•
E	•	1	1
T ₁	1	1	1
T ₂	1	1	1

(2 modulo 3 equals -1 modulo 3 and 86 mod 3) 86=88-1



Duality: The "Flip Side" of Symmetry Analysis.

OUTSIDE or LAB
Symmetry reduction
results in
Level or Spectral
SPLITTING
External B-field
does Zeeman splitting

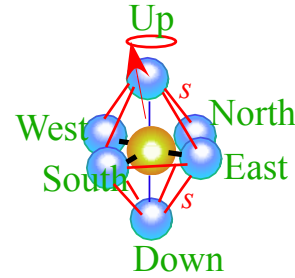
LAB versus BODY, **STATE versus PARTICLE,**
boils down to :
OUTSIDE versus INSIDE

INSIDE or BODY
Symmetry reduction
results in
Level or Spectral
UN-SPLITTING
("clustering")

Example:
Cubic-Octahedral O
reduced to
Tetragonal C_4

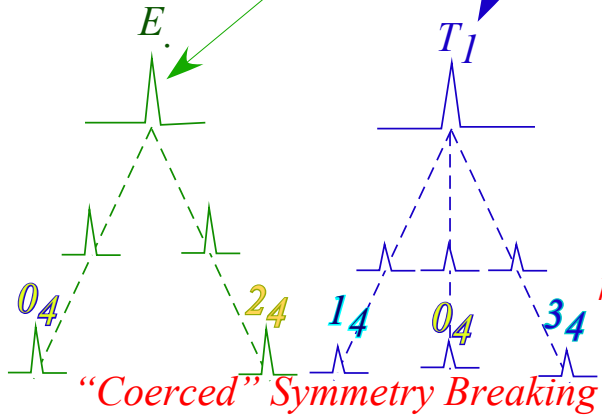
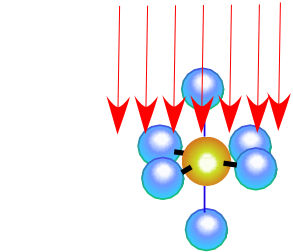
C_4	0_4	1_4	2_4	3_4
A_1	1	.	.	.
A_2	.	.	1	.
E	1.	.	1	.
T_1	1	1	.	1
T_2	.	1	1	1

Internal J gets "stuck" on RES axes
Must "tunnel" axis-to-axis at rate s



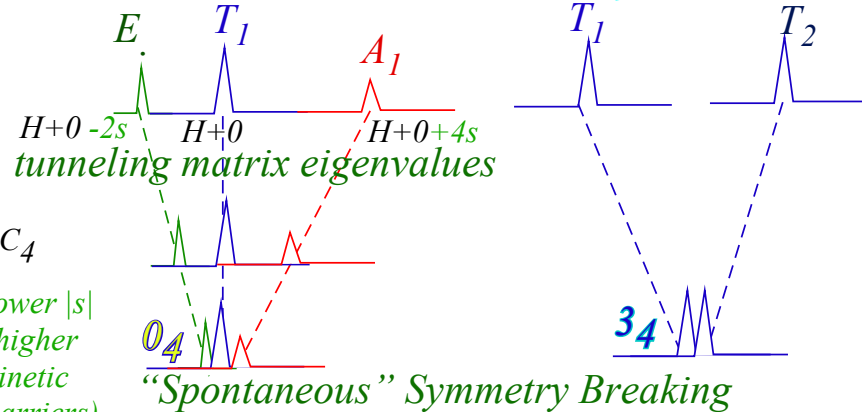
$|U\rangle|D\rangle|E\rangle|W\rangle|N\rangle|S\rangle$

H	0	s	s	s	s
0	H	s	s	s	s
s	s	H	0	s	s
s	s	0	H	s	s
s	s	s	s	H	0
s	s	s	s	0	H



Stronger C_4

higher $|B|$ \downarrow lower $|s|$
(higher kinetic barriers)



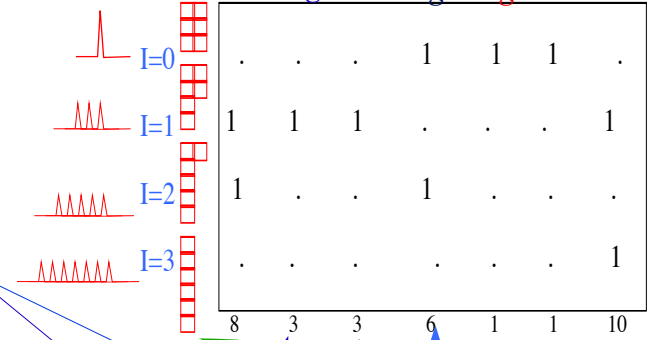
DISentanglement!

Spin-Permutation to Octahedral Correlations $S_6 \supset O_h$
 $E_u T_{1g} T_{1u} T_{2g} A_{1g} A_{1u} A_{2u}$

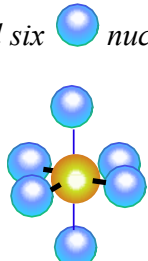
How F-nuclei become distinguished
 (but not distinguishable)
 in SF_6 .

C_4

	0 ₄	1 ₄	2 ₄	3 ₄
A_1	1	.	.	.
A_2	.	.	1	.
E	1	.	1	.
T_1	1	1	.	1
T_2	.	1	1	1

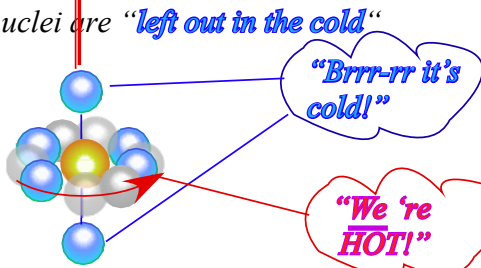


If rotation is not too stuck on C_4 axis
 all six nuclei are equivalent



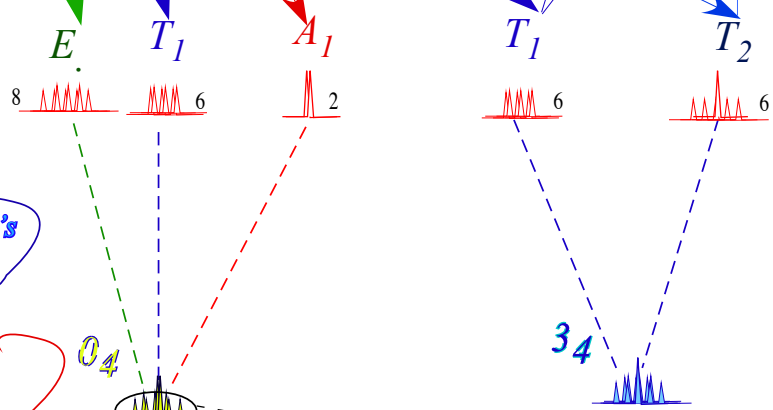
With rotation stuck on C_4 axis

polar nuclei are "left out in the cold"



"Brrr-rr it's cold!"

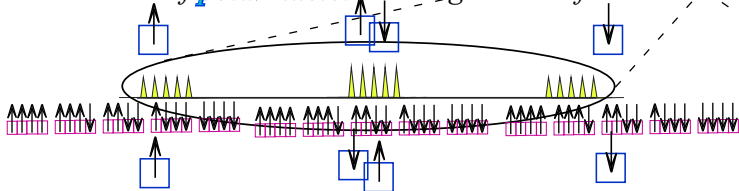
"We're HOT!"



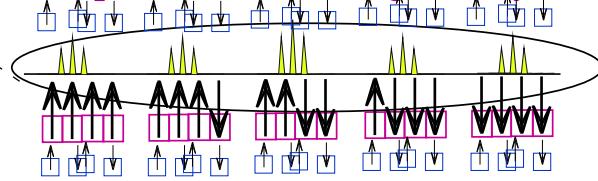
Species Spin Weights

Greatly simplified sketches of ultra high resolution IR SF_6 spectroscopy of Christian Borde', C. Saloman, and Oliver Pfister who did SiF_4 , too.

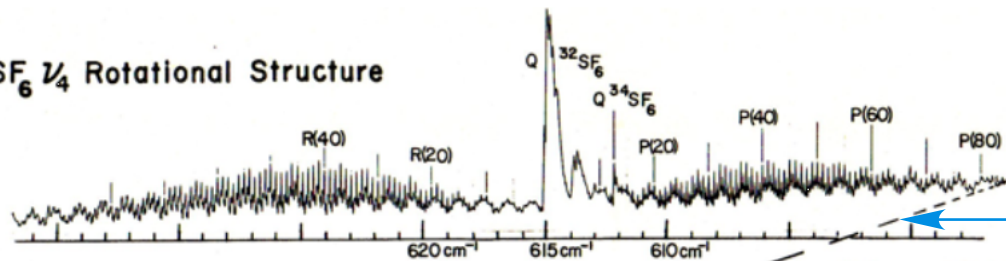
If polar nuclei have a greater B-field



If equatorial nuclei have a greater B-field



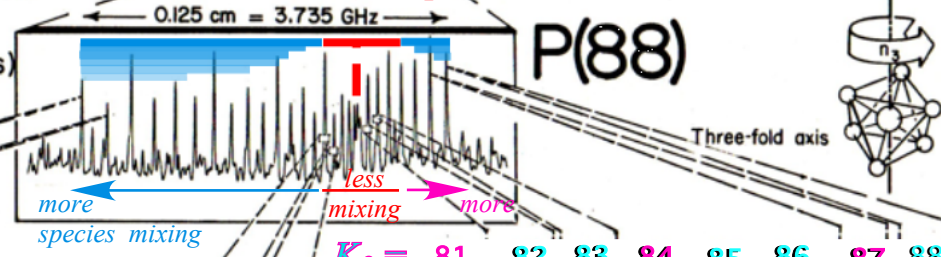
(a) SF₆ ¼ Rotational Structure



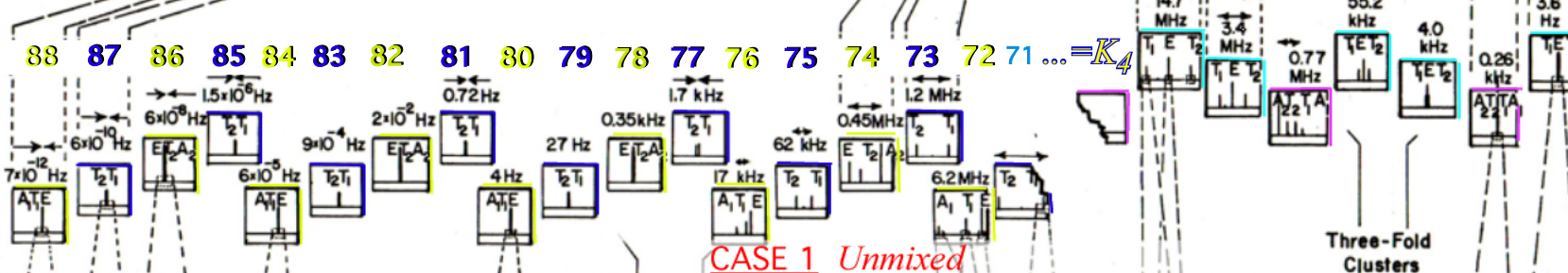
FT IR and Laser Diode Spectra
K.C. Kim, W.B. Person, D. Seitz, and B.J. Krohn
J. Mol. Spectrosc. 76, 322 (1979).

Primary *AET* species mixing
increases with distance from
"separatrix"

(b) P(88) Fine Structure (Rotational anisotropy effects)



(c) Superfine Structure (Rotational axis tunneling)



(d) Hyperfine Structure (Nuclear spin-rotation effects)



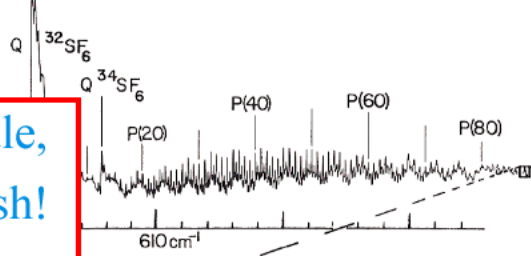
(e) Superhyperfine Structure (Spin frame correlation effects)



CASE 2₃-
Major mixing
in lowest two
C₃-CLUSTERS

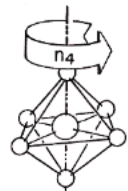
a) SF₆ 1/4 Rotational Structure

For a zero-spin X¹⁶O₆ molecule, hundreds of lines would vanish! Just eight A₁ singlets remain.

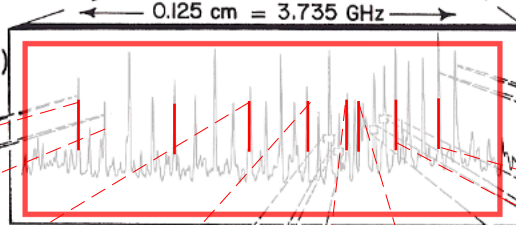


FT IR and Laser Diode Spectra
K.C. Kim, W.B. Person, D. Seitz, and B.J. Kroh
J. Mol. Spectrosc. 76, 322 (1979).

b) P(88) Fine Structure (Rotational anisotropy effects)

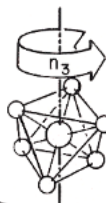


Four fold axis

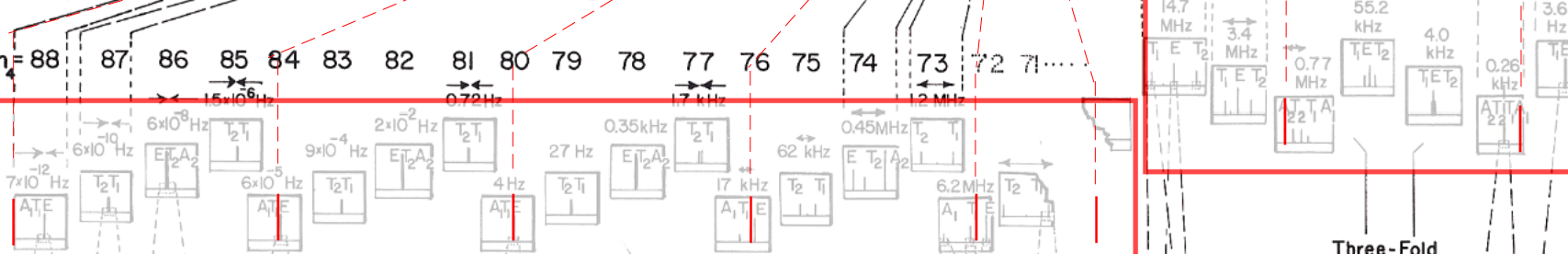


P(88)

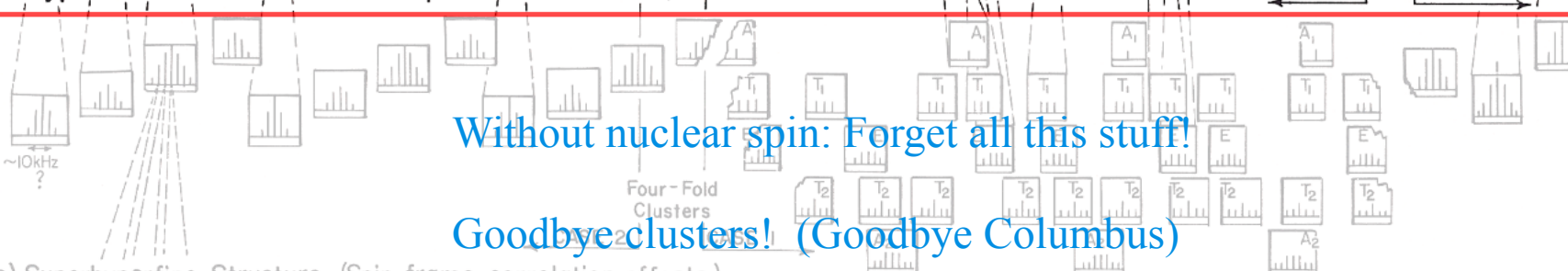
Three-fold axis



c) Superfine Structure (Rotational axis tunneling)



d) Hyperfine Structure (Nuclear spin-rotation effects)



Without nuclear spin: Forget all this stuff!

Goodbye clusters! (Goodbye Columbus)

e) Superhyperfine Structure (Spin frame correlation effects)



Graphical approach to rotation-vibration-spin Hamiltonian

$$\langle H \rangle \sim v_{\text{vib}} + B J(J+1) + \langle H^{\text{Scalar Coriolis}} \rangle + \langle H^{\text{Tensor Centrifugal}} \rangle + \langle H^{\text{Nuclear Spin}} \rangle + \langle H^{\text{Tensor Coriolis}} \rangle + \dots$$

OUTLINE

Introductory review

- | | <u>Example(s)</u> |
|---|---------------------------------|
| • <i>Rovibronic nomograms and PQR structure</i> | v_3 and v_4 SF ₆ |
| • <i>Rotational Energy Surfaces (RES) and Θ_K^J-cones</i> | v_4 P(88) SF ₆ |
| • <i>Spin symmetry correlation tunneling and entanglement</i> | SF ₆ |

Recent developments

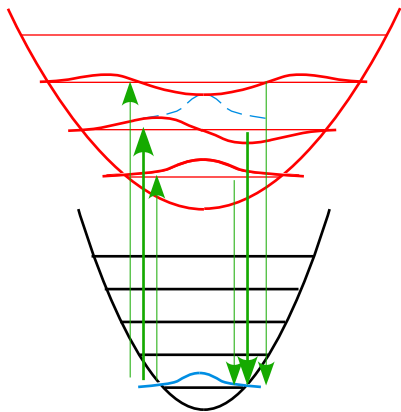
- | | |
|---|-----------------------|
| • <i>Analogy between PE surface and RES dynamics</i> | |
| • <i>Rotational Energy Eigenvalue Surfaces (REES)</i> | v_3 SF ₆ |

Potential Energy Surface (PES) Dynamics

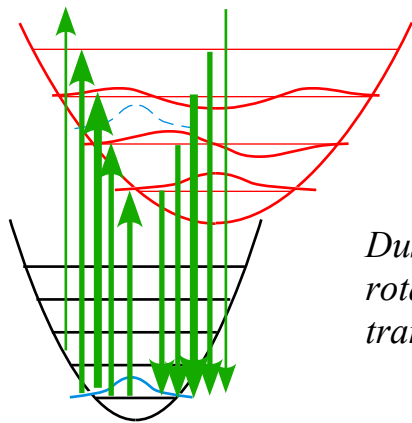
Inter-PES electronic transitions

Vibrational Franck-Condon effects

- Frequency mismatch of PES



- Shape or position mismatch of PES



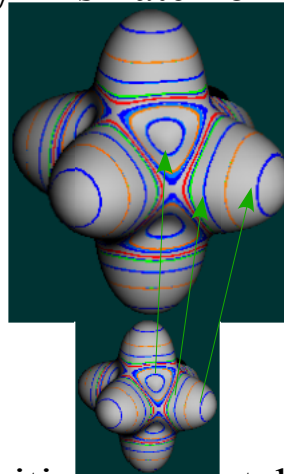
Duschinsky rotation or translation

Rotation Energy Surface (RES) Dynamics

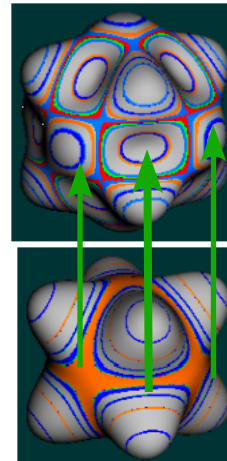
Inter-PES electronic transitions

Rotational “Franck-Condon” effects

- Frequency mismatch of RES



- Shape or position mismatch of RES



Analogy
between
Vibronic and **Rovibronic**

Non-Born-Oppenheimer Surfaces
Strong vibration-electronic mixing

Jahn-Teller-Renner effects

- Multiple and variable conformer minima

Rotation Energy Eigen-Surfaces (REES)

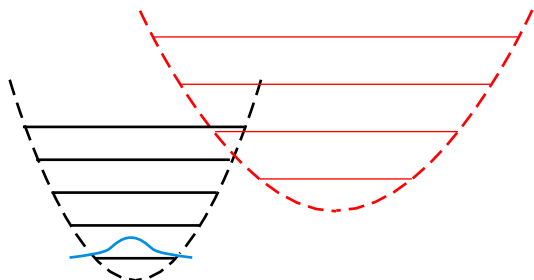
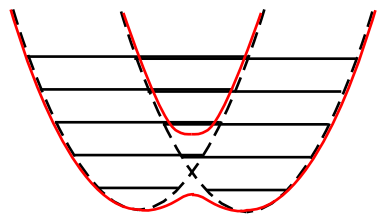
Inter-PES electronic transitions

Rotational JTR effects

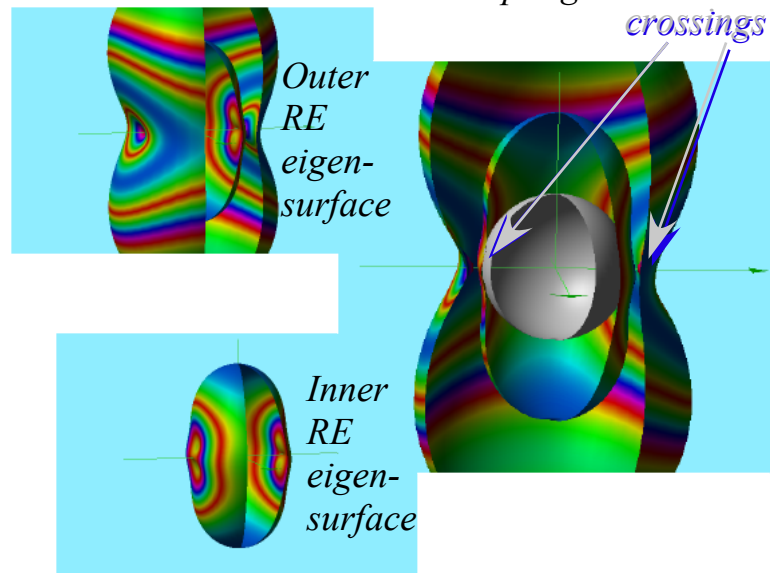
- Multiple and variable J-axes

Analogy
between

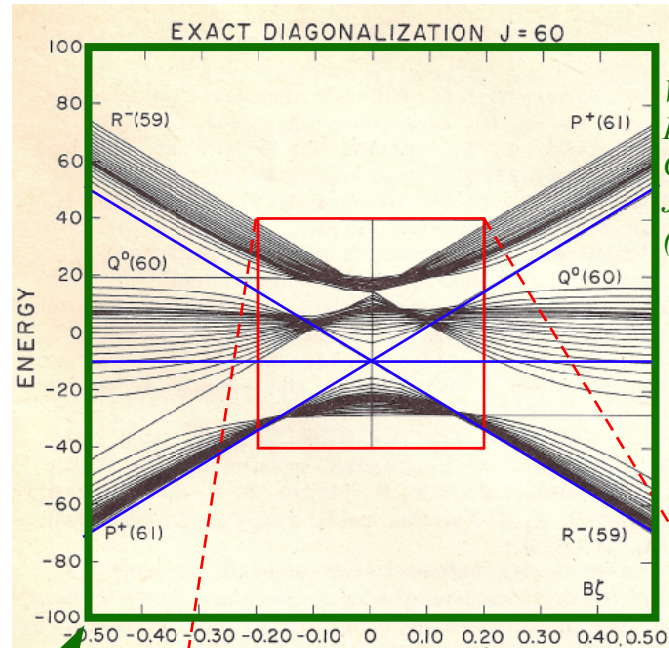
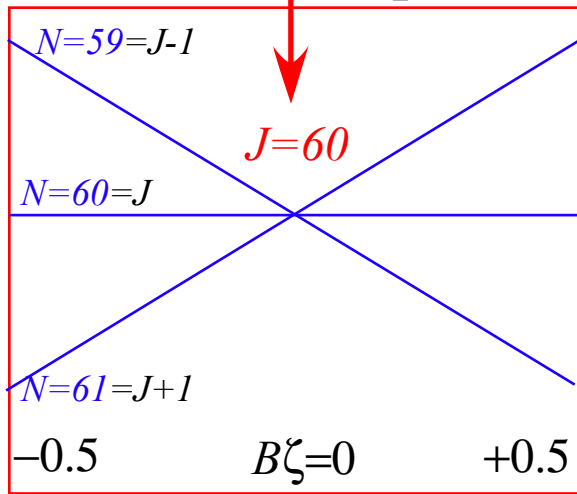
Vibronic and Rovibronic



Example for 2-state
vibronic-rotor coupling

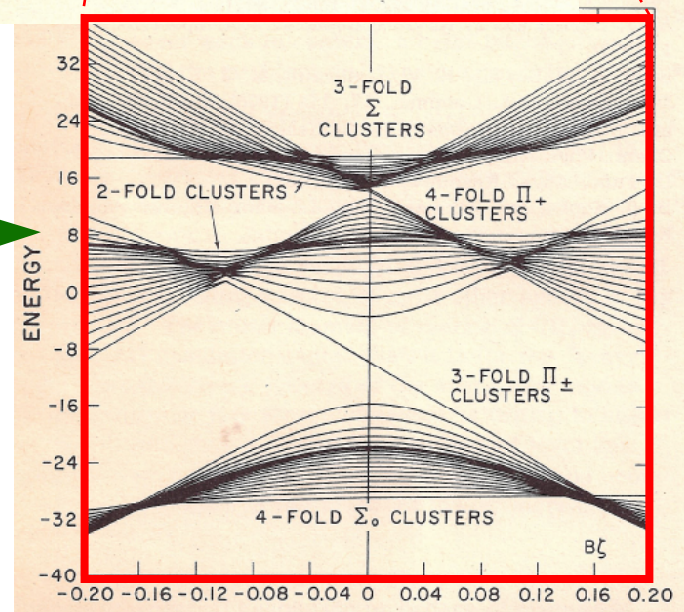


Recall scalar Coriolis
 PQR plots vs. $B\zeta$
 Here is a $J=60$ piece of it:



*WGH,
 Patterson,
 Galbraith
 JCP 69, 4906
 (1978)*

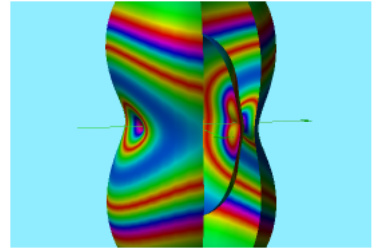
Now consider this plot
 with *tensor* Coriolis, too
 (Just 4th-rank $[2 \times 2]^4$ tensor here.
 See next talk **RJ06** and a 4PM talk **RI09**
 by *Mitchell et. al.* and *Boudon et. al.* who will
 pull much *higher rank!*)



How to display such monstrous avoided cluster crossings: REES: *Rotational Energy Eigenvalue Surfaces*

Vibration (or vibronic) momentum ℓ retains its quantum representation(s).

For $\ell=1$ that is the usual 3-by-3 matrices.



Rotational momentum J is treated semi-classically. $|J| = \sqrt{J(J+1)}$

Usually \mathbf{J} is written in Euler coordinates: $J_x = |J| \cos\gamma \sin\beta$, etc.

Plot resulting H-matrix eigenvalues vs. classical variables.

($\ell=1$) 3-by-3 H-matrix e-values are polar plotted vs. azimuth γ and polar β .

Body- $\Sigma\Pi\pm$ -Basis

$$\langle H \rangle = (v_3 + B|J|^2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 2B\zeta|J| \begin{pmatrix} \cos\beta & \frac{1}{\sqrt{2}}e^{-i\gamma}\sin\beta & 0 \\ \frac{1}{\sqrt{2}}e^{i\gamma}\sin\beta & 0 & \frac{1}{\sqrt{2}}e^{-i\gamma}\sin\beta \\ 0 & \frac{1}{\sqrt{2}}e^{i\gamma}\sin\beta & -\cos\beta \end{pmatrix}$$

$$+ 2t_{224}|J|^2 \begin{pmatrix} 3\cos^2\beta - 1 & -\sqrt{8}e^{-i\gamma}\sin\beta\cos\beta & \sin^2\beta(6\cos 2\gamma + i4\sin 2\gamma) \\ -\sqrt{8}e^{i\gamma}\sin\beta\cos\beta & 0 & -6\cos^2\beta + 2 & \sqrt{8}e^{-i\gamma}\sin\beta\cos\beta \\ \sin^2\beta(6\cos 2\gamma - i4\sin 2\gamma) & \sqrt{8}e^{i\gamma}\sin\beta\cos\beta & 3\cos^2\beta - 1 \end{pmatrix}$$

Lab-PQR-Basis

$$\langle H \rangle = (v_3 + B|J|^2) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 2B\zeta|J| \begin{pmatrix} |P\rangle & |Q\rangle & |R\rangle \\ +1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

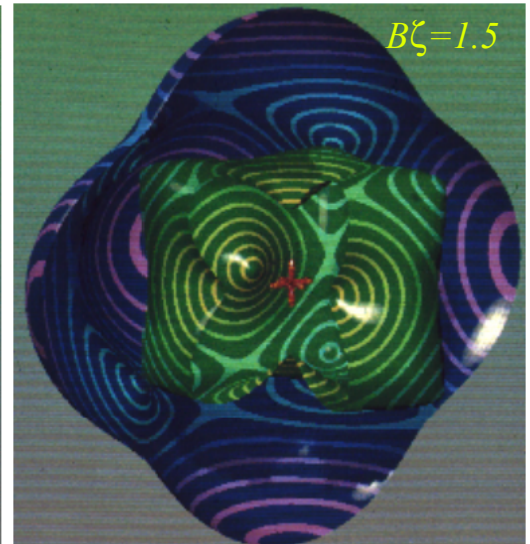
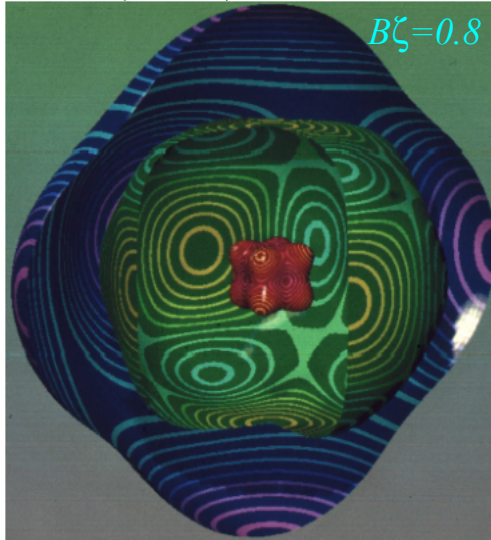
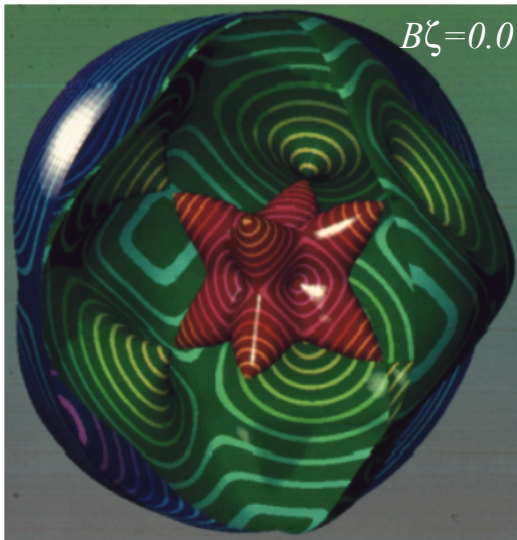
$$+ 2t_{224}|J|^2 \begin{pmatrix} H_{PP} & H_{PQ} & H_{PR} \\ H_{PQ}^* & H_{QQ} & H_{QR} \\ H_{RP}^* & H_{QR}^* & H_{RR} \end{pmatrix}$$

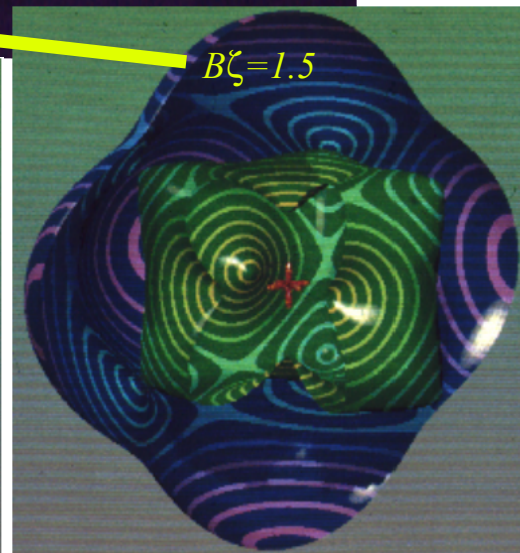
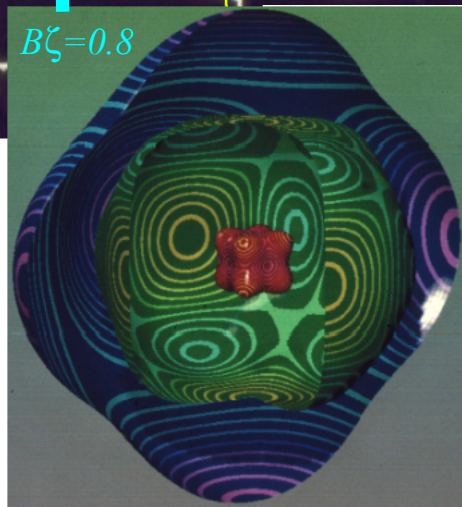
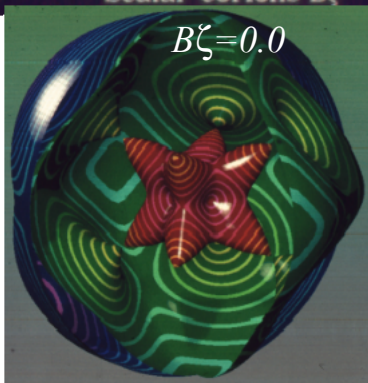
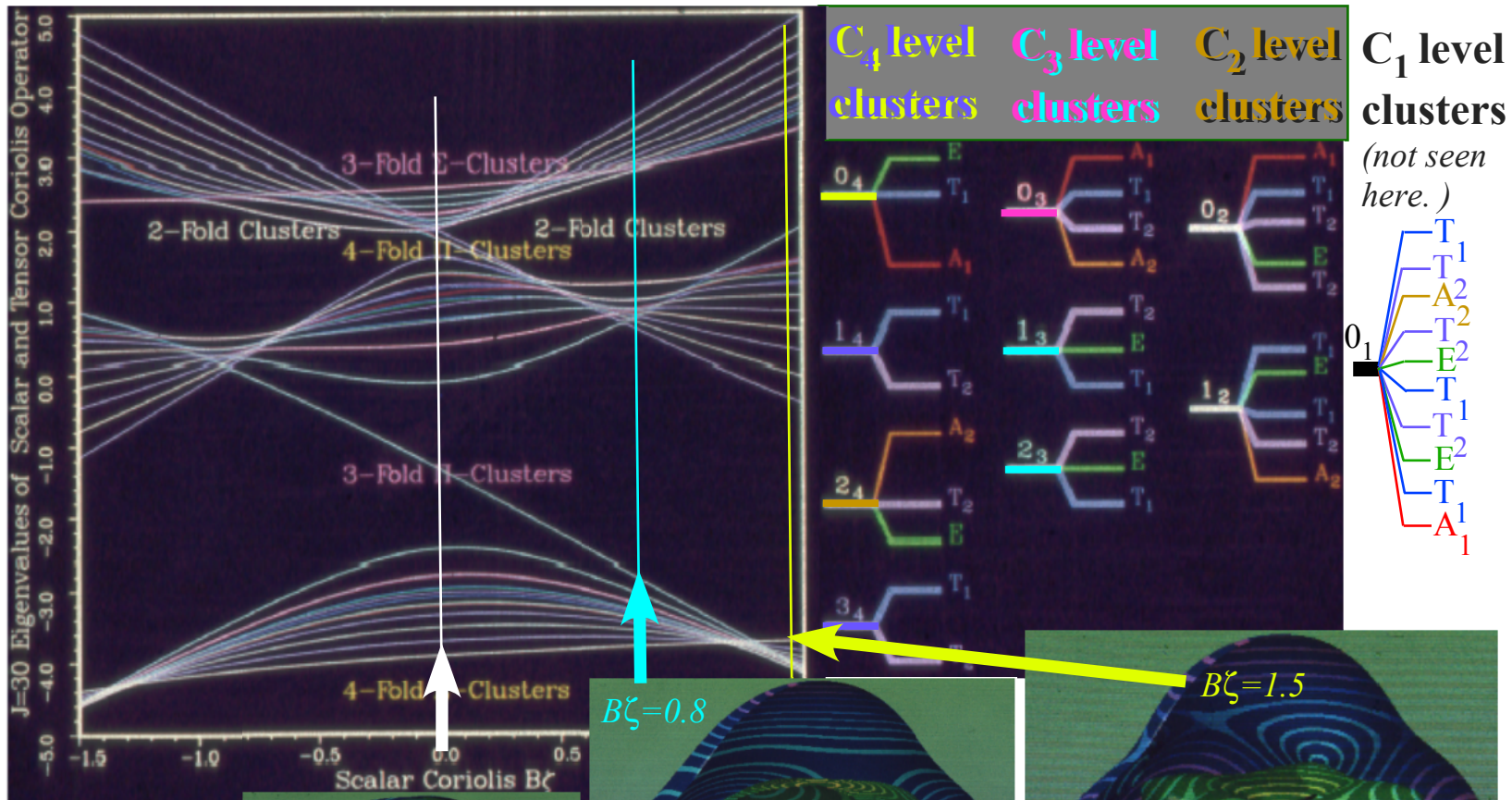
(Either basis should give same REES)

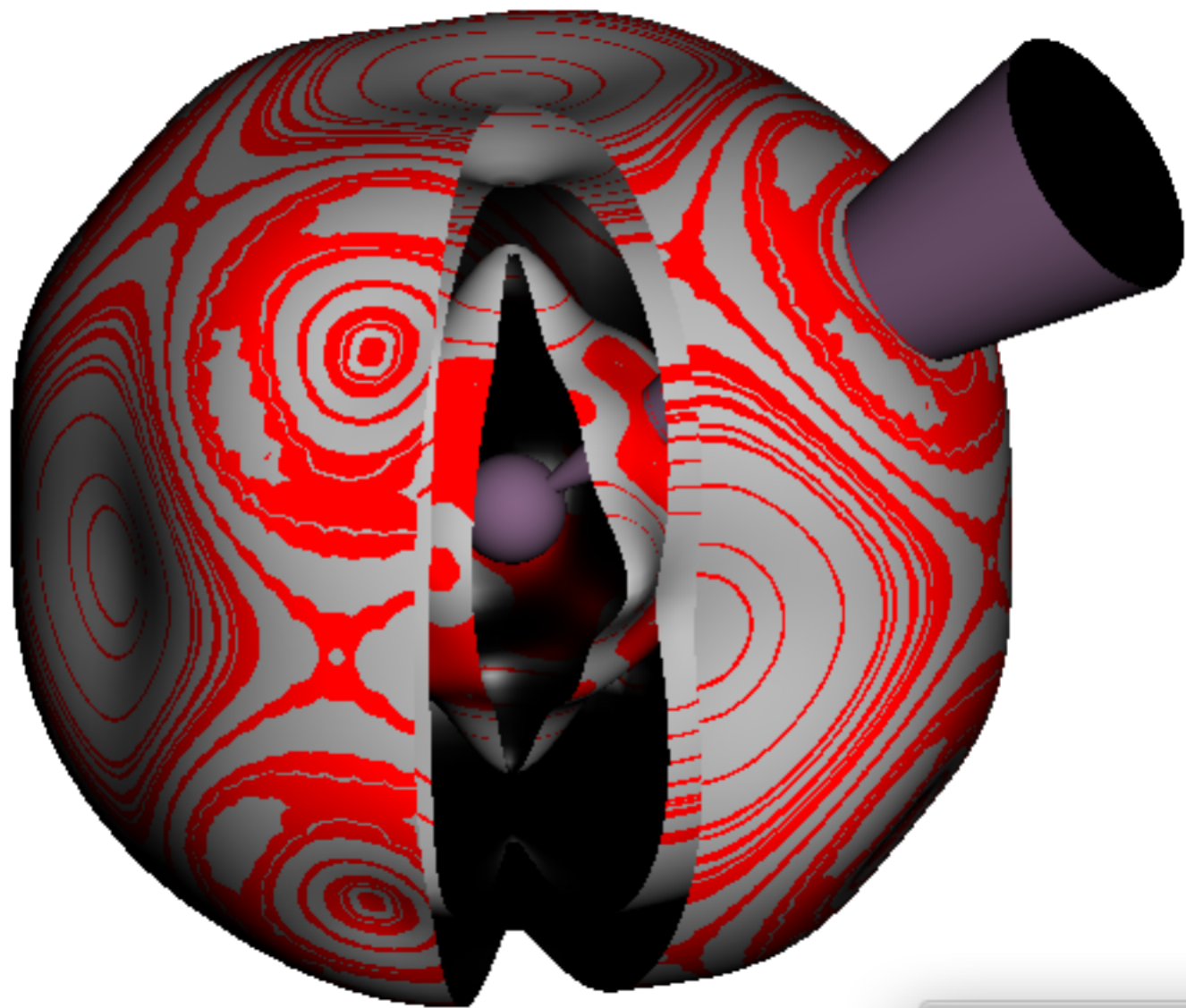
$$H_{PP} = (35\cos^4\beta - 30\cos^2\beta + 5\sin^2\beta\sin 4\gamma + 5)/4 = H_{RR}$$

$$H_{PQ} = 5\sin\beta(7\cos^2\beta - 3\cos\beta - \sin^2\beta(\cos\beta\cos 4\gamma + i\sin 4\gamma))/\sqrt{8} = H_{QR}$$

$$H_{PQ} = 5(-7\cos^4\beta + 8\cos^2\beta + (1 - \cos^4\beta)\cos 4\gamma + 2i\cos\beta\sin^2\beta\sin 4\gamma - 1)/4$$





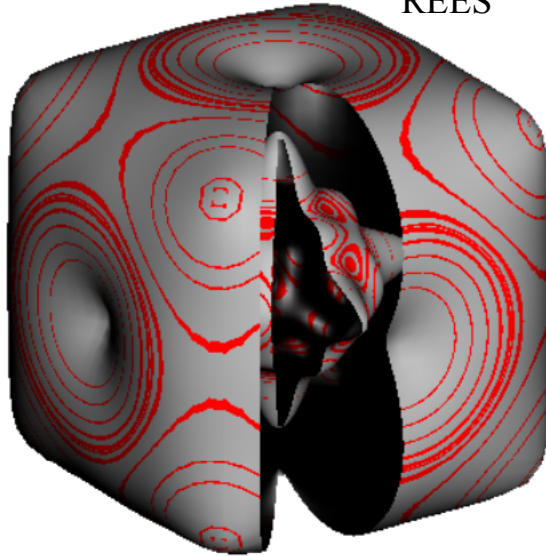


Lowest v_3
REES

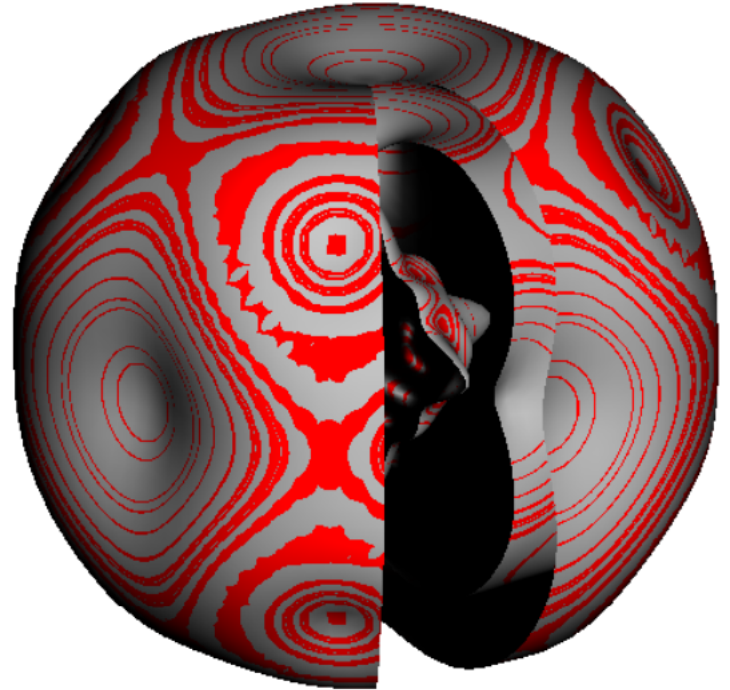


REES for v_3 with no scalar Coriolis ($B\zeta_3 = 0$)

Middle v_3
REES



Highest v_3
REES

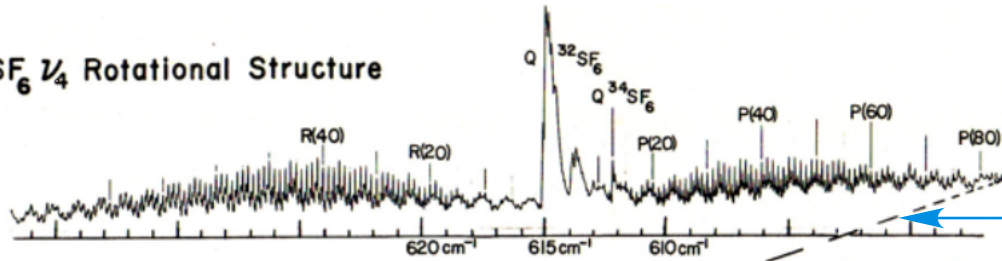


Summary

- *Spin symmetry, orientation, rotation, and permutation are underlying properties that molecules have before any excitation begins. Ignore them and you may miss some cool stuff!*
- *Graphical techniques help to expose symmetry properties. We discussed*
 - rovibronic nomograms*
 - rotational energy surfaces (RES)*
 - rotation energy eigenvalue surfaces (REES)*
 - effects that entangle and disentangle spin states*
- *REES effects have useful analogy with vibronic effects.*
- *Spin symmetry species are quite mutable.*

Perhaps, they may be optically controlled.

(a) SF₆ V₄ Rotational Structure



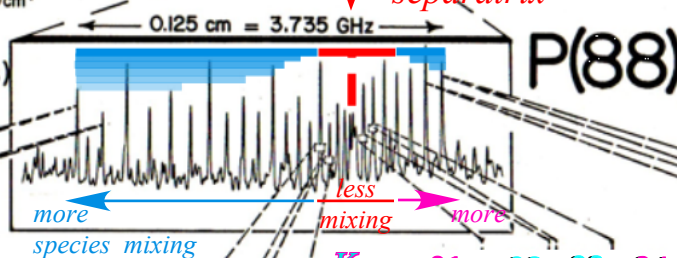
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Primary AET species mixing
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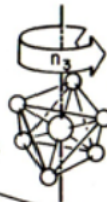
(b) P(88) Fine Structure (Rotational anisotropy effects)



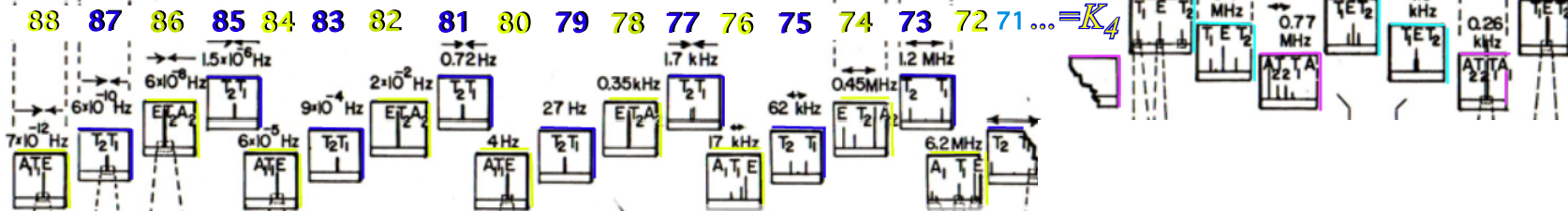
Four fold axis



Three-fold axis

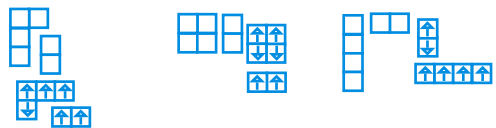


(c) Superfine Structure (Rotational axis tunneling)



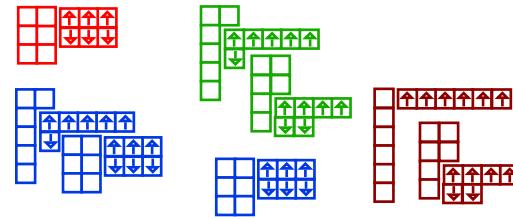
CASE 2₄

Broken 4 + 2 tableau state description



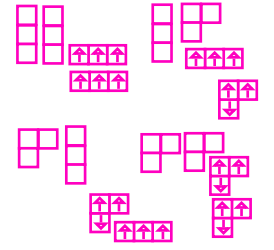
CASE 1 Unmixed

primary A₁ T₁ E T₂ A₂ species
(Whole 6-box tableaus)



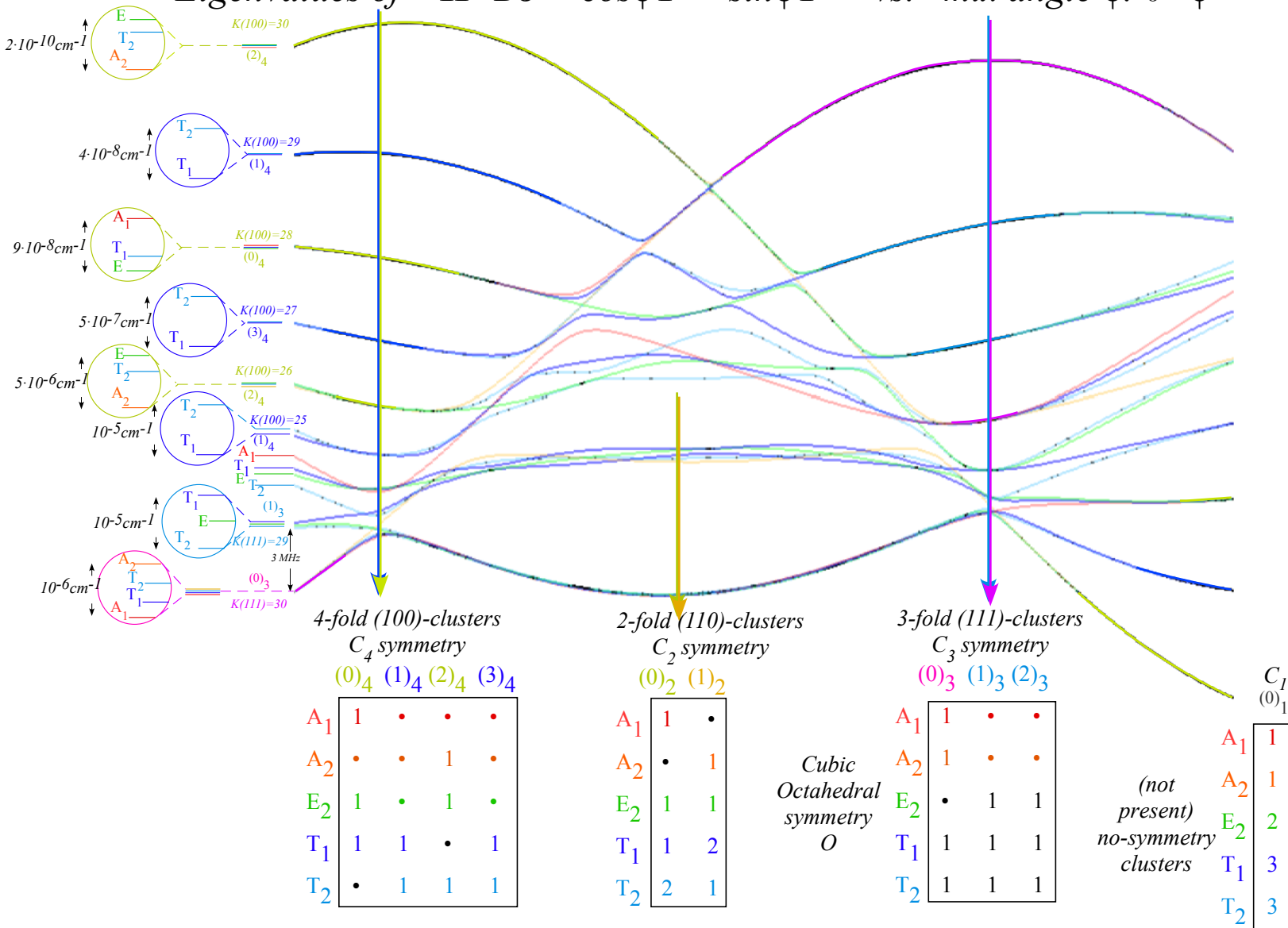
CASE 2₃

Broken 3 + 3 Tableaus



Spin-rovib ENTANGLEMENT symmetry
might be controllable!

Eigenvalues of $\mathbf{H} = \mathbf{B}\mathbf{J}^2 + \cos\phi\mathbf{T}^{[4]} + \sin\phi\mathbf{T}^{[6]}$ vs. mix angle $\phi: 0 < \phi < \pi$



Eigenvalues of $\mathbf{H} = B\mathbf{J}^2 + \cos\phi\mathbf{T}^{[4]} + \sin\phi\mathbf{T}^{[6]}$ vs. mix angle $\phi: 0 < \phi <$

