"*SIMPLEST MOLECULE*" CLARIFIES MODERN PHYSICS I. CW LASER SPACE-TIME FRAME DYNAMICS

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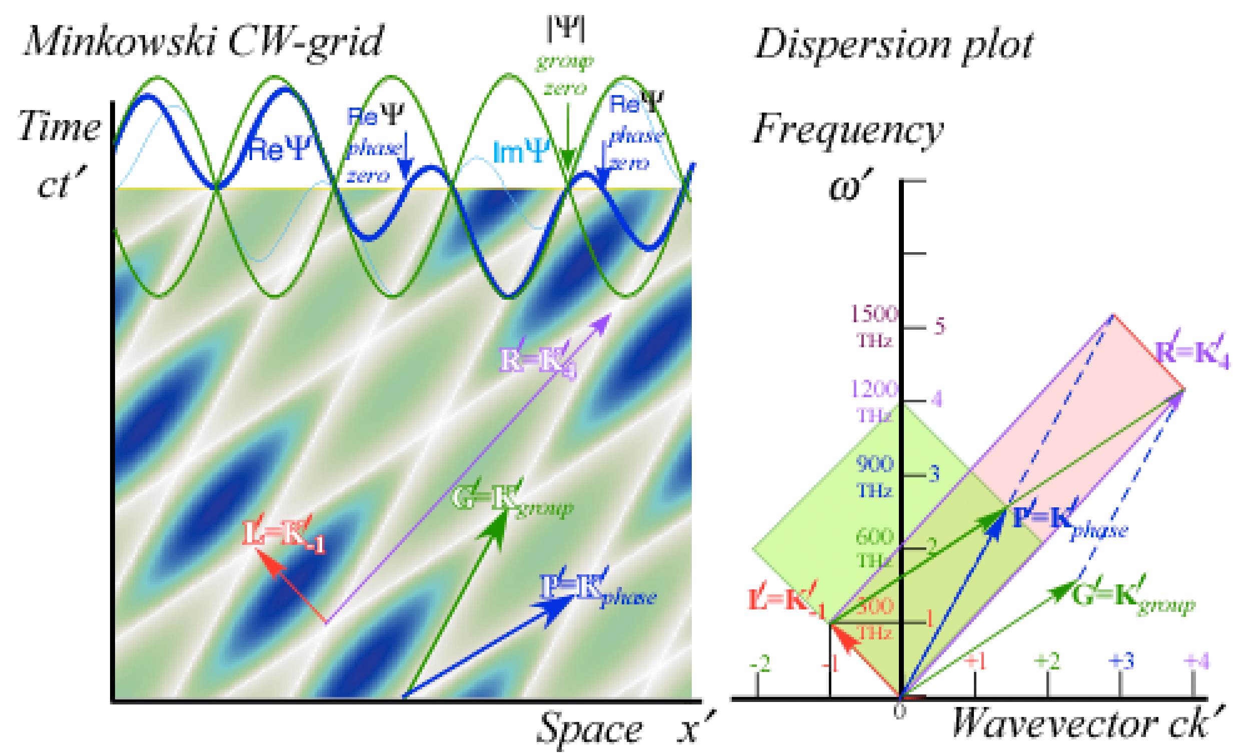
Molecular spectroscopy makes very precise applications of quantum theory including GPS, BEC, and laser clocks. Now it can return the favor by shedding some light on modern physics mysteries by further unifying quantum theory and relativity.

We first ask, "What is the simplest molecule?" Hydrogen H_2 is the simplest stable molecule. *Positronium* is an electron-positron (e^+e^-)-pair. An even simpler "molecule" or "radical" is a photon-pair (γ, γ) that under certain conditions can create an (e^+e^-)-pair.

To help unravel relativistic and quantum mysteries consider CW laser beam pairs or TE-waveguides. Remarkably, their wave interference immediately gives Minkowski space-time coordinates and clearly relates eight kinds of space-time wave dilations or contractions to shifts in Doppler frequency or wavenumber.



Modern physics students may find this approach significantly simplifies and clarifies relativistic physics in space-time *(x,ct)* and inverse time-space *(ω,ck)*. It resolves some mysteries surrounding super-constant *c=299,792,458m/s* by proving "Evenson's Axiom" named in honor of NIST metrologist Ken Evenson (1932-2002) whose spectroscopy established c to start a precision-renaissance in spectroscopy and GPS metrology.

The following Talk II applies this approach to relativistic quantum mechanics.

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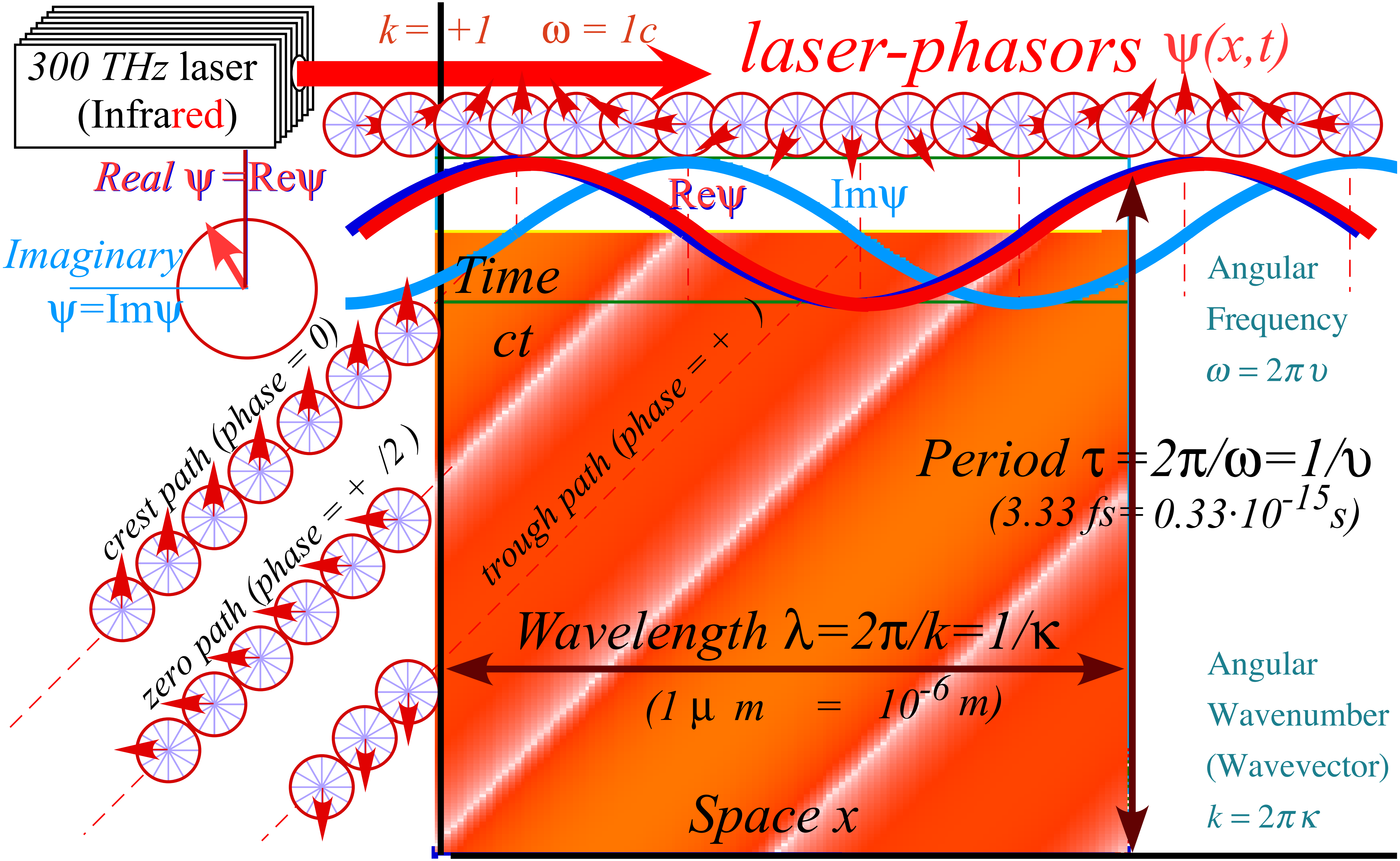
http://www.uark.edu/ua/modphys/testing/markup/Harter-SoftWebApps.html

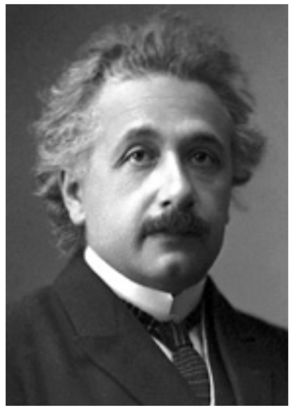
http://www.uark.edu/ua/modphys/pdfs/QTCA_Pdfs/QTCA_PapersNTalks/SRQM.pdf

Goal: Understand relativity *(and QM in next talk)* using *Laser-Phasor clocks*



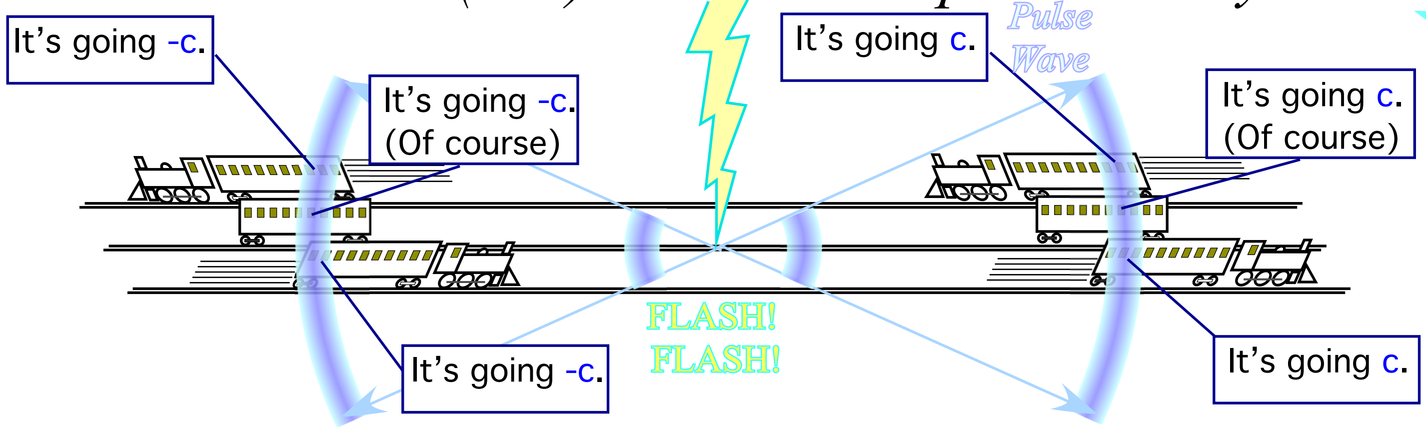
Improving on Einstein's PW axiom...

Albert Einstein



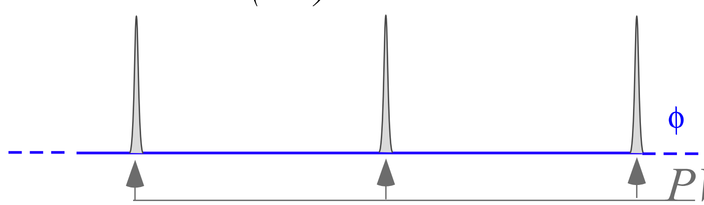
1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



A "road-runner" axiom
is a "show-stopper"

Pulse wave (PW) train



$A_1\cos\omega t + A_2\cos 2\omega t + A_3\cos 3\omega t + A_4\cos 4\omega t + \ldots$

Complicated

PW peaks precisely locate places where wave is.

Improving on Einstein's PW axiom…with Occam's Razors & Evenson's Lasers

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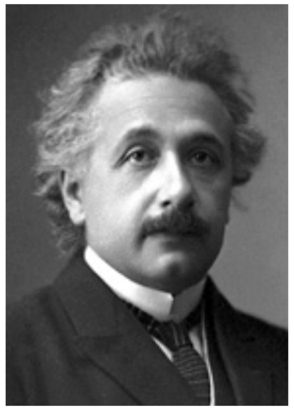

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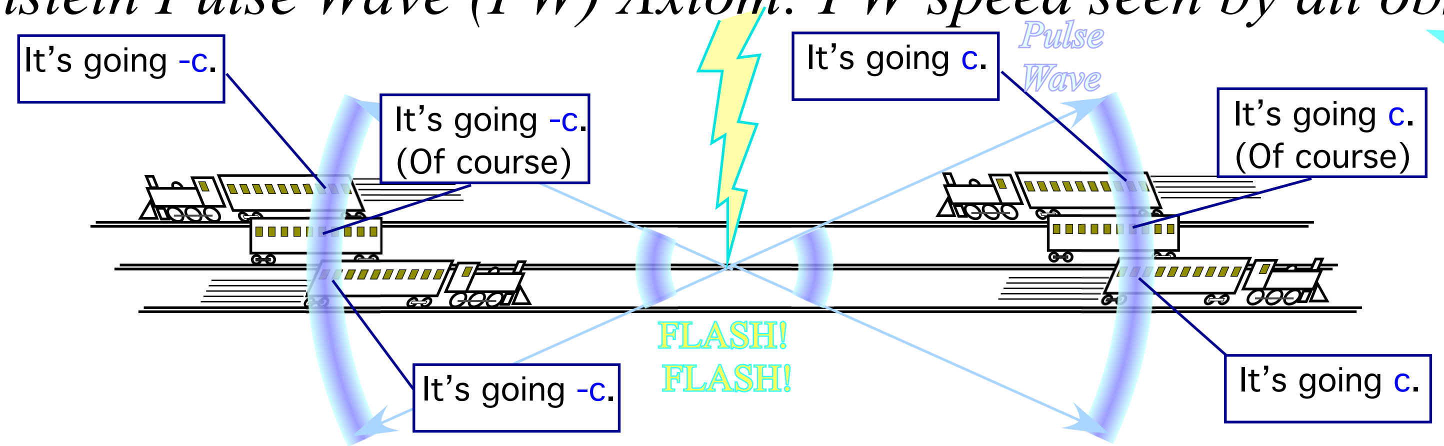

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William of Ockham

1285-1349

Using Occam's Razor

(and Evenson's lasers)

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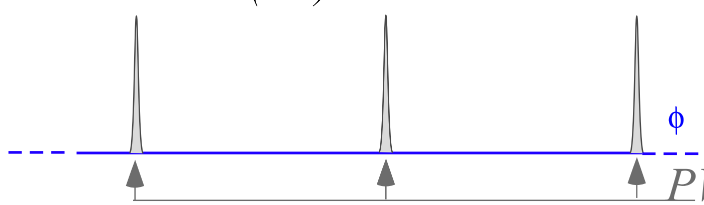

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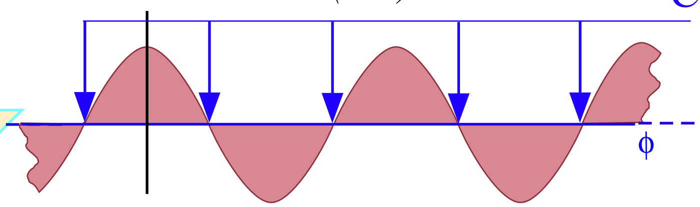
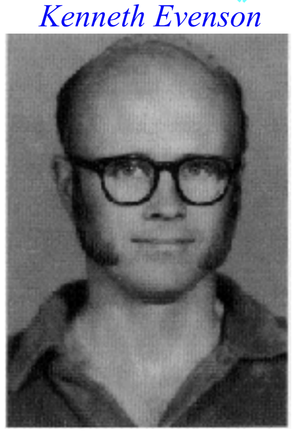
Continuous wave (CW) train



$A\cos \omega t$

Simpler

Kenneth Evenson



1929-2002
c=299,792,458 m/s

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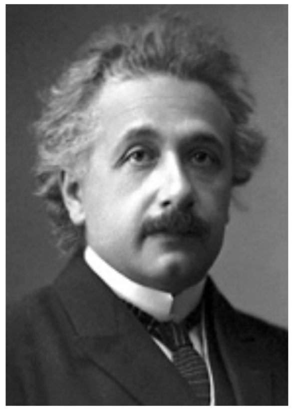

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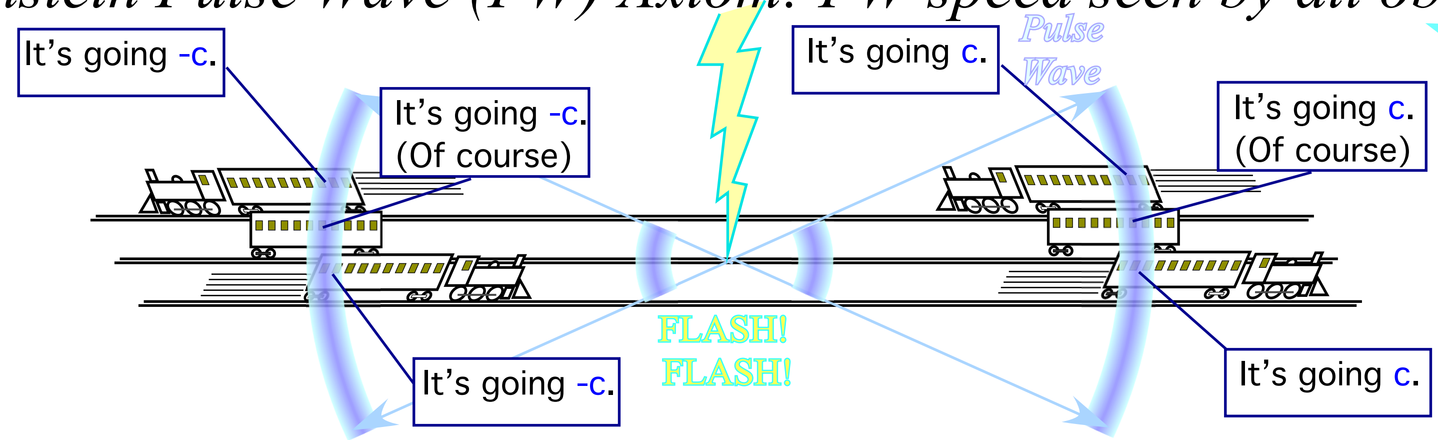

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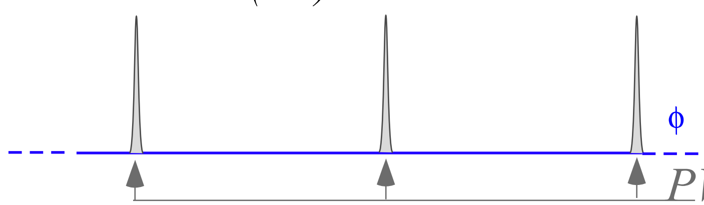

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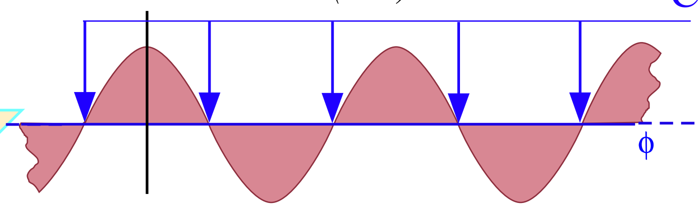

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Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

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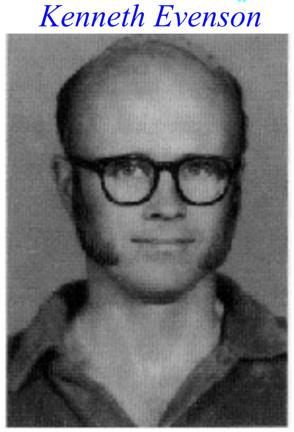

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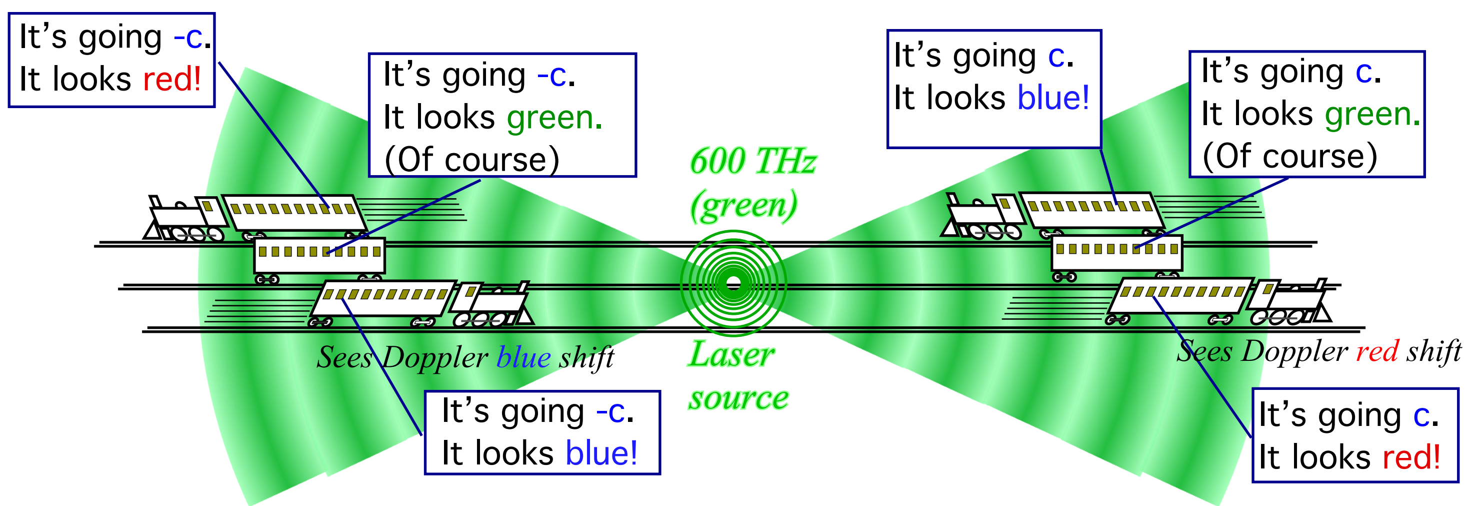
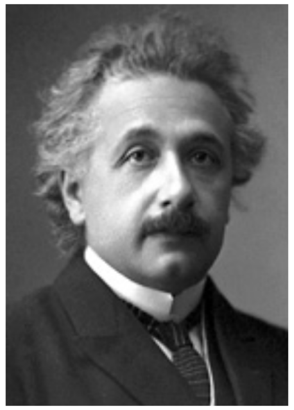


More self-evident "must-be" axiom

Improving on Einstein's PW axiom…with Occam's Razors & Evenson's Lasers
..and a little help from Alice, Bob, and Carla

Albert Einstein



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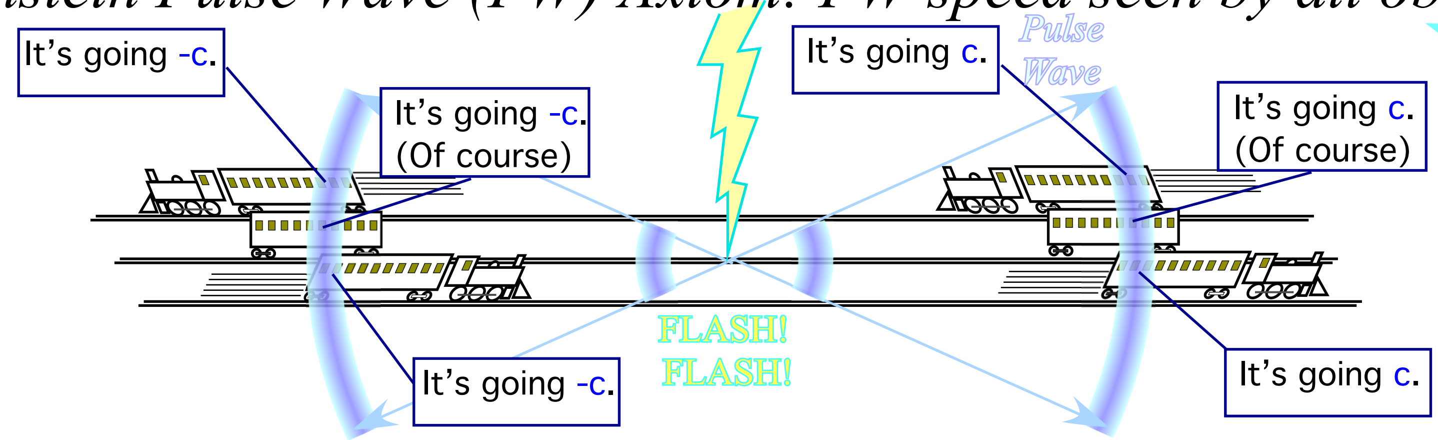

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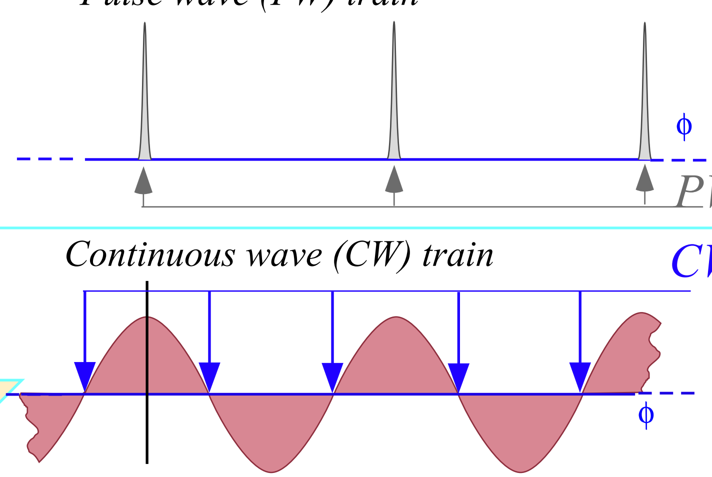

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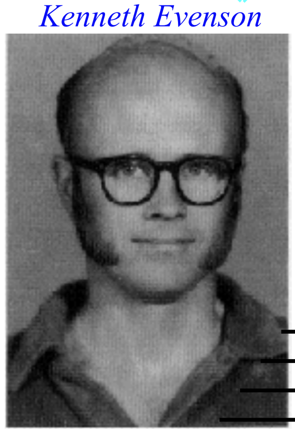

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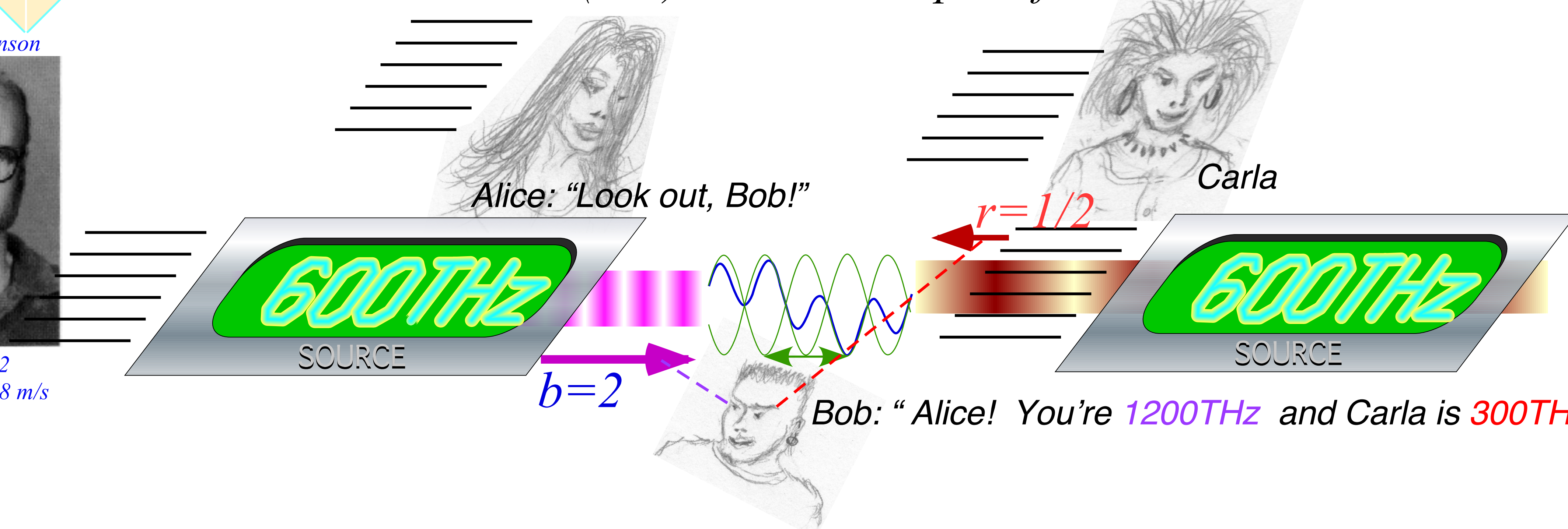

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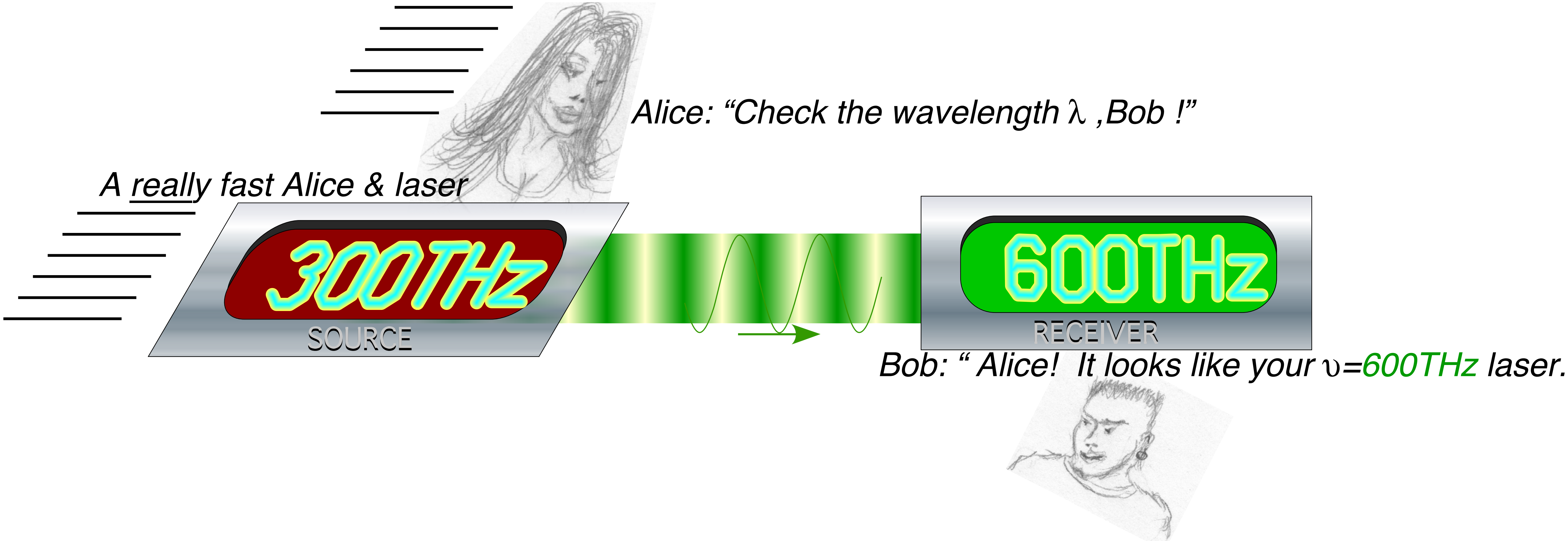

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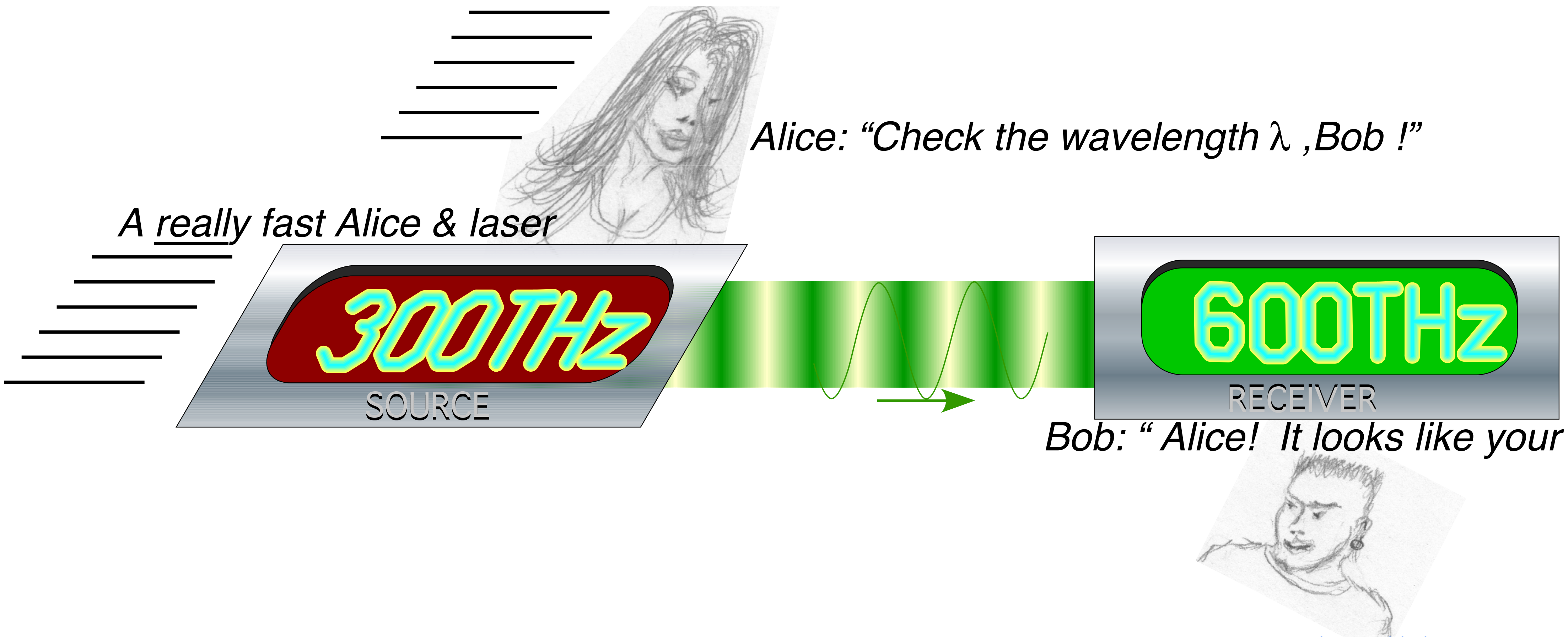

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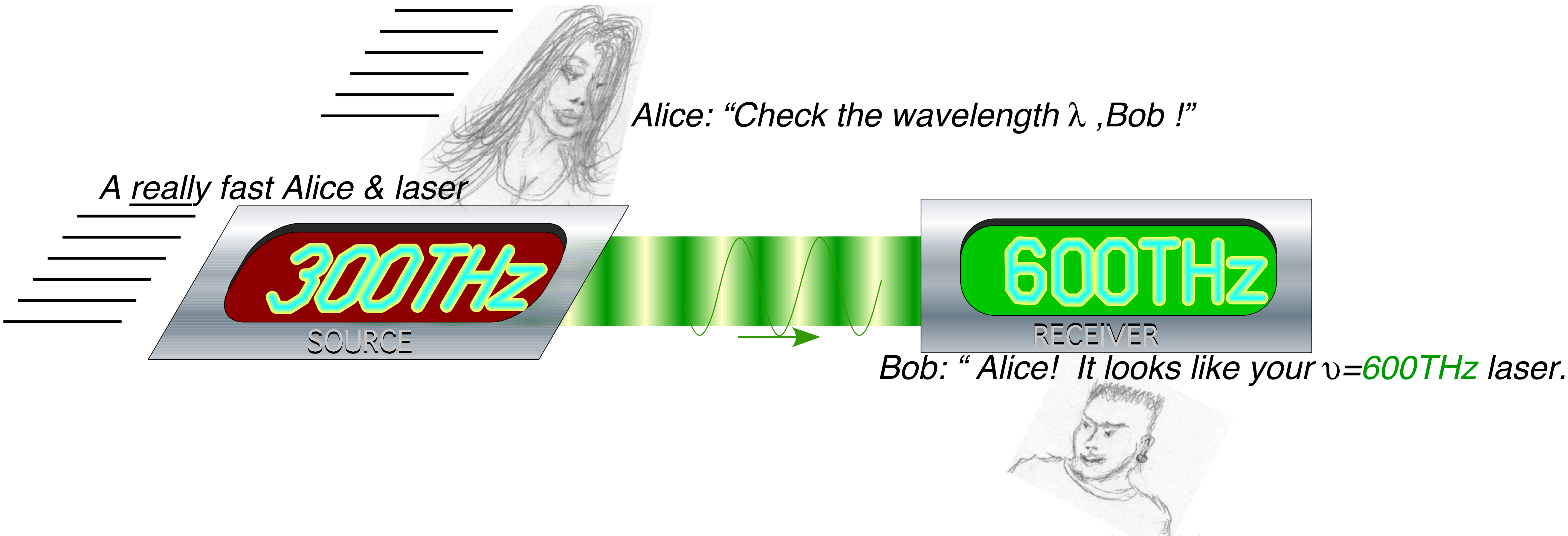

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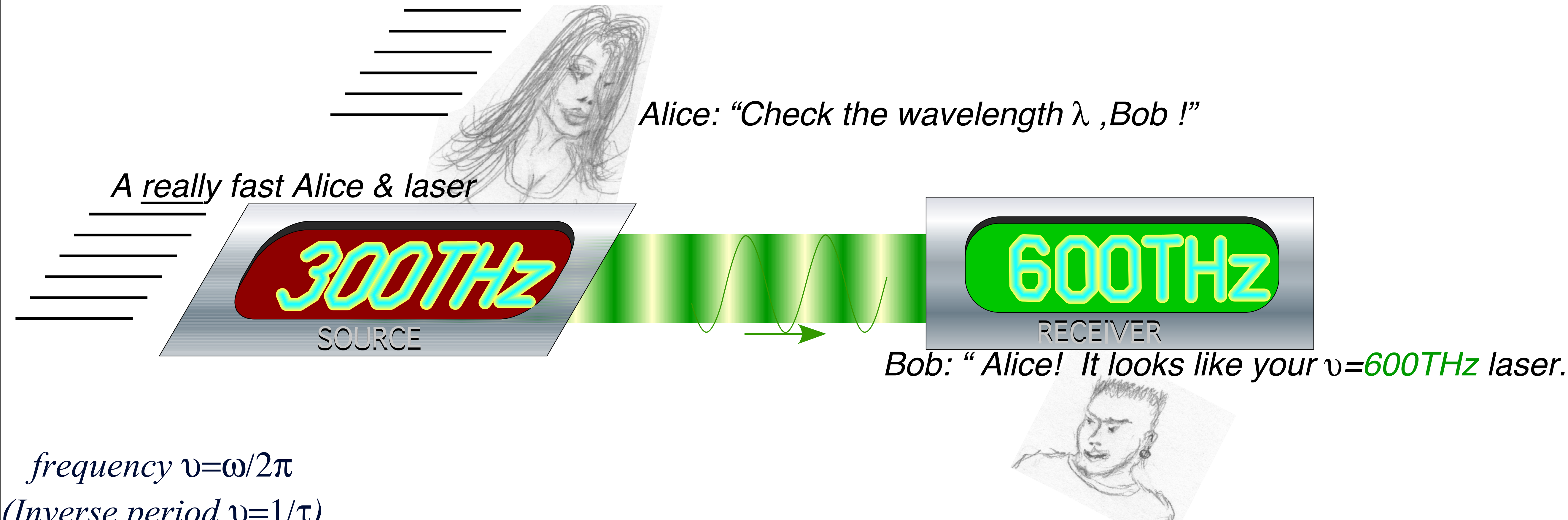

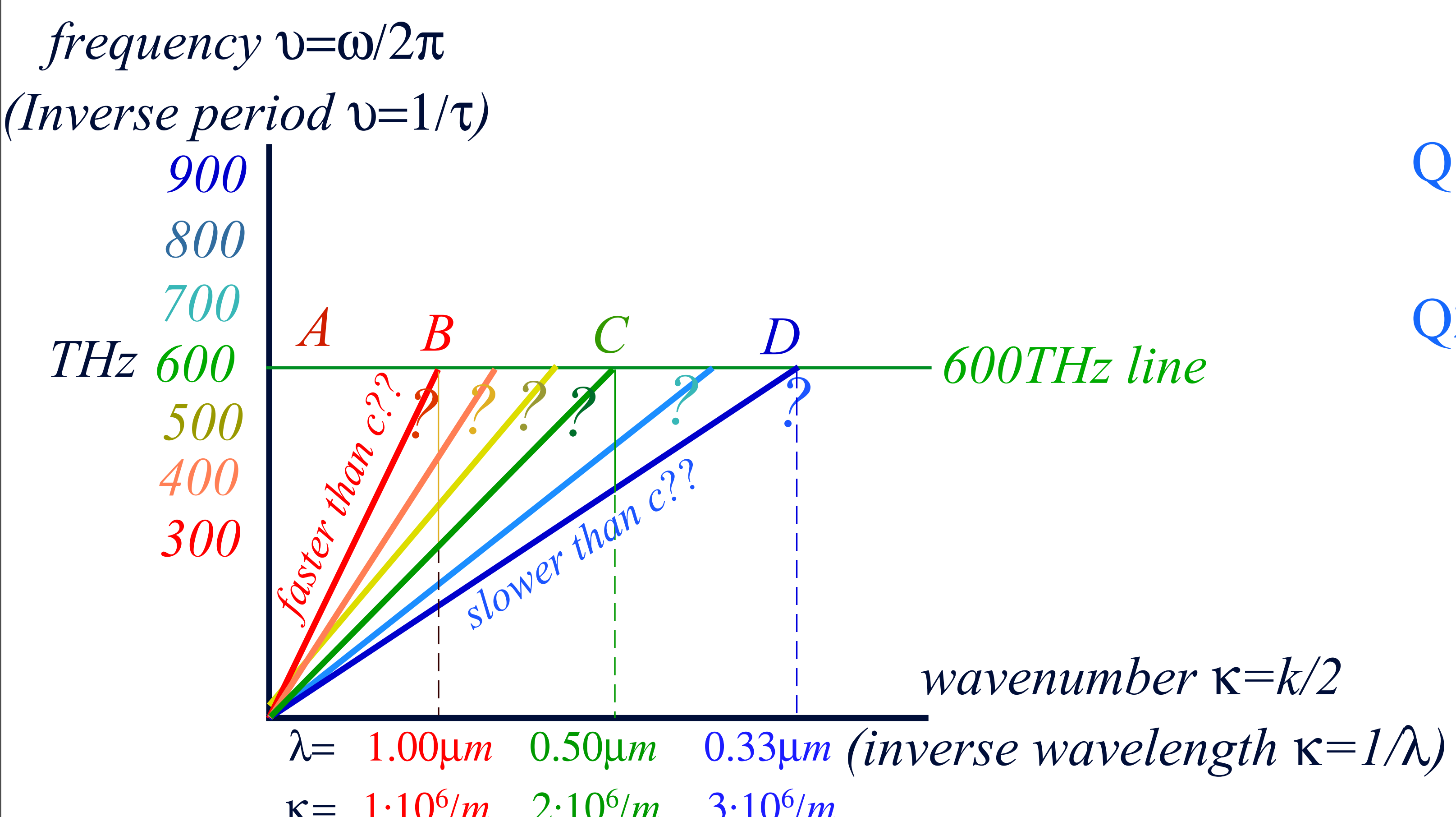

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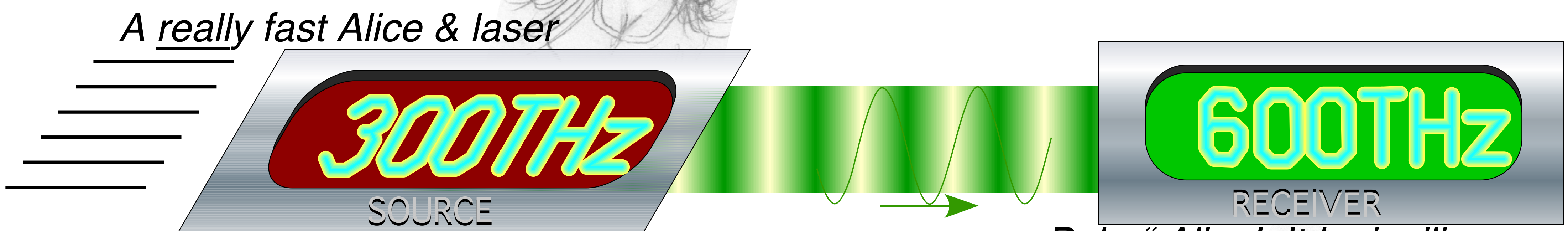

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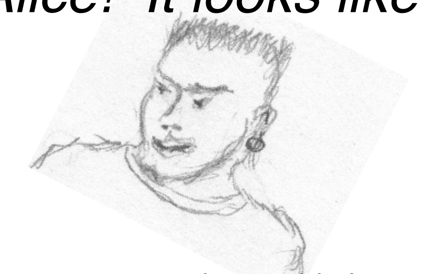

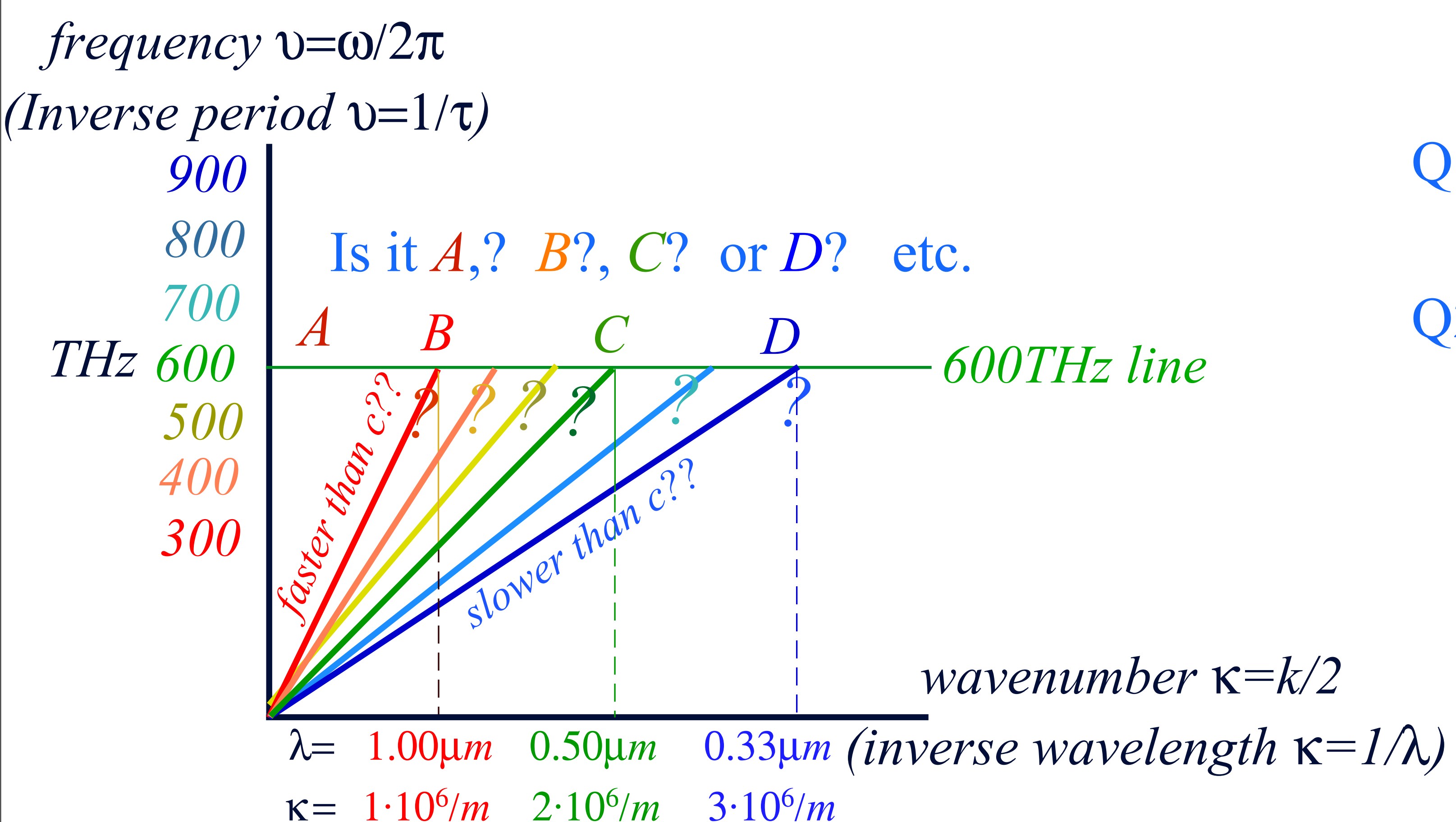

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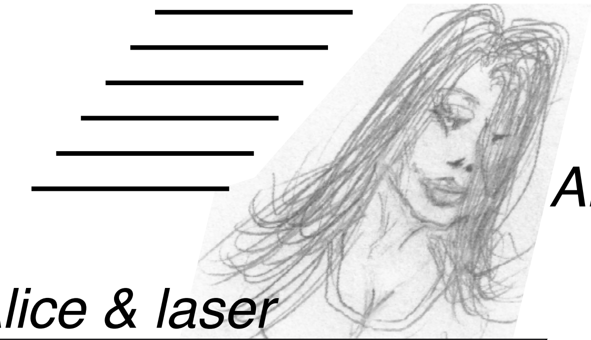

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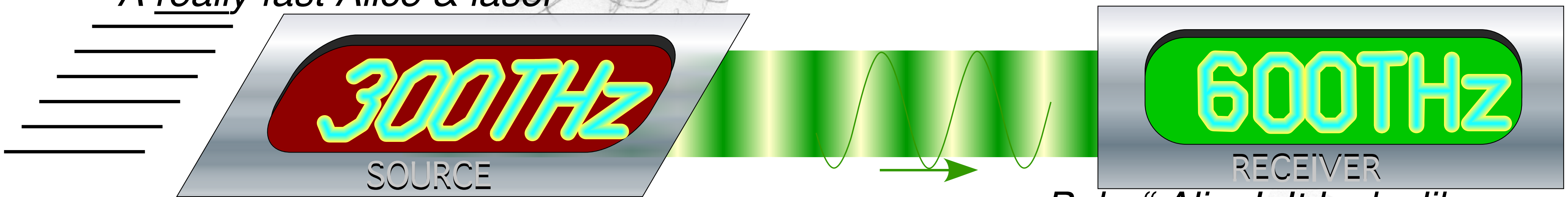

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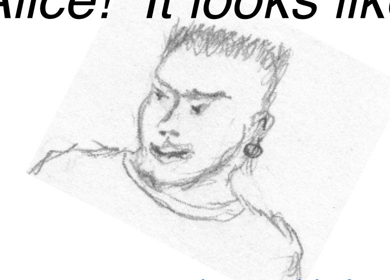

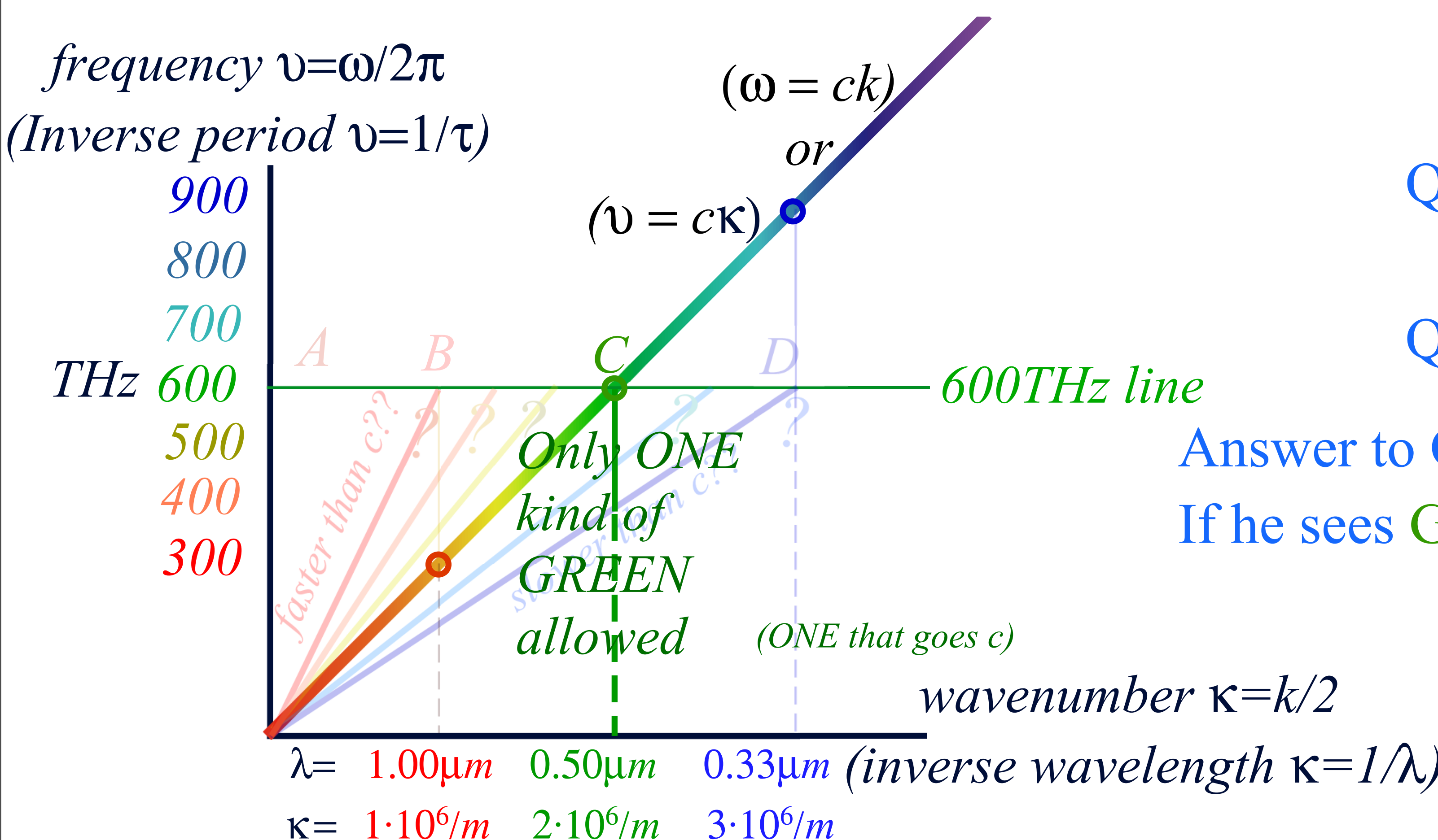

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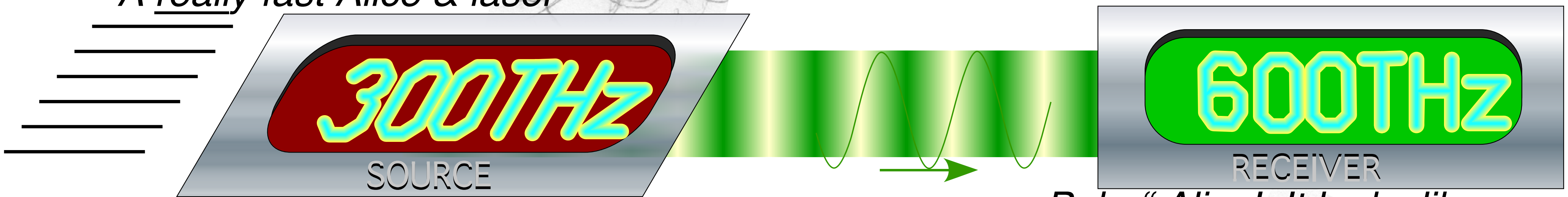

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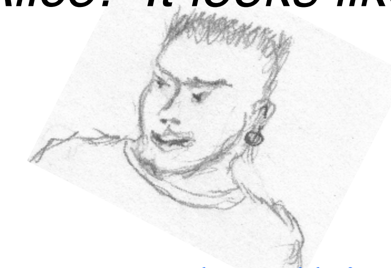

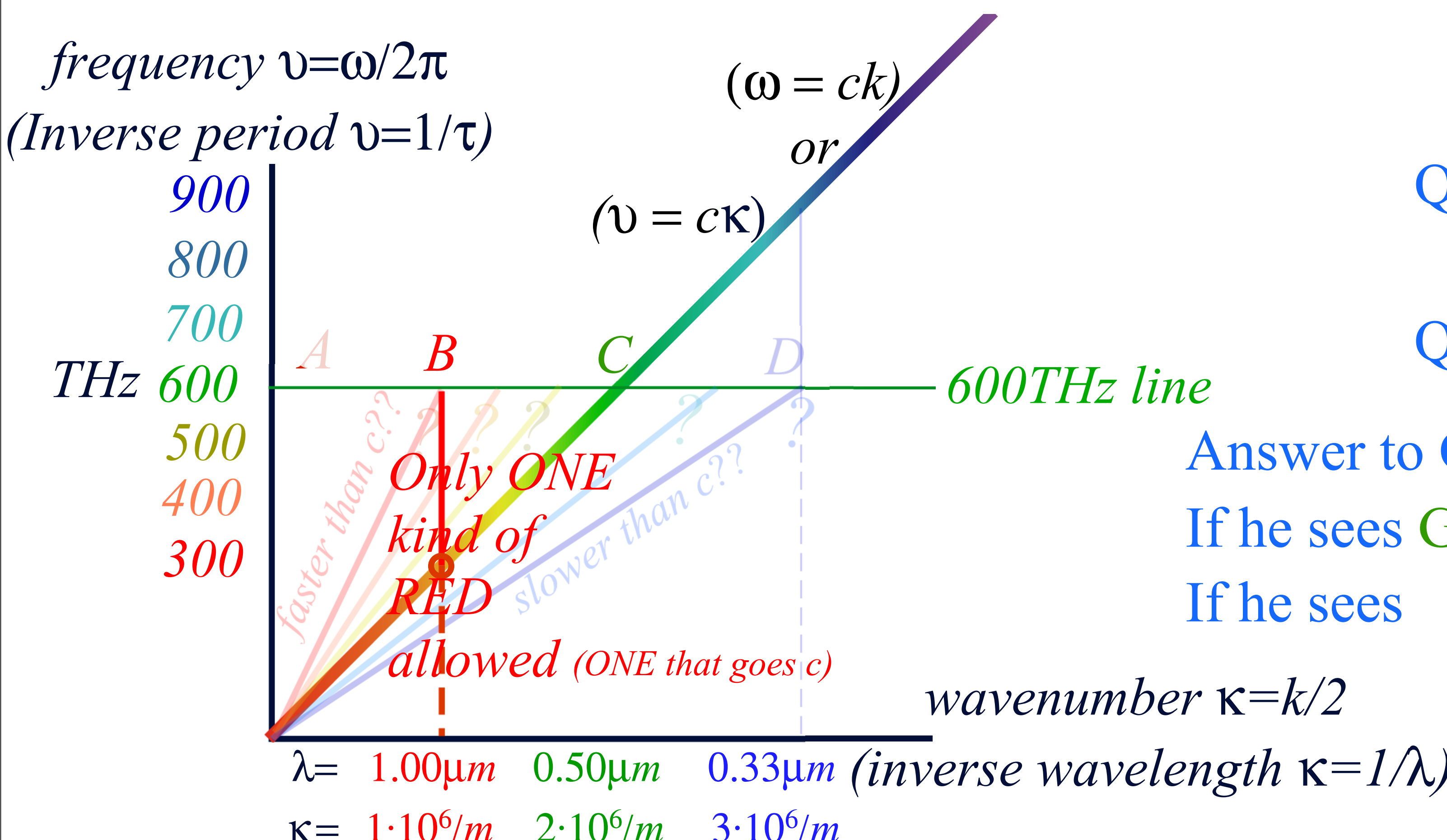

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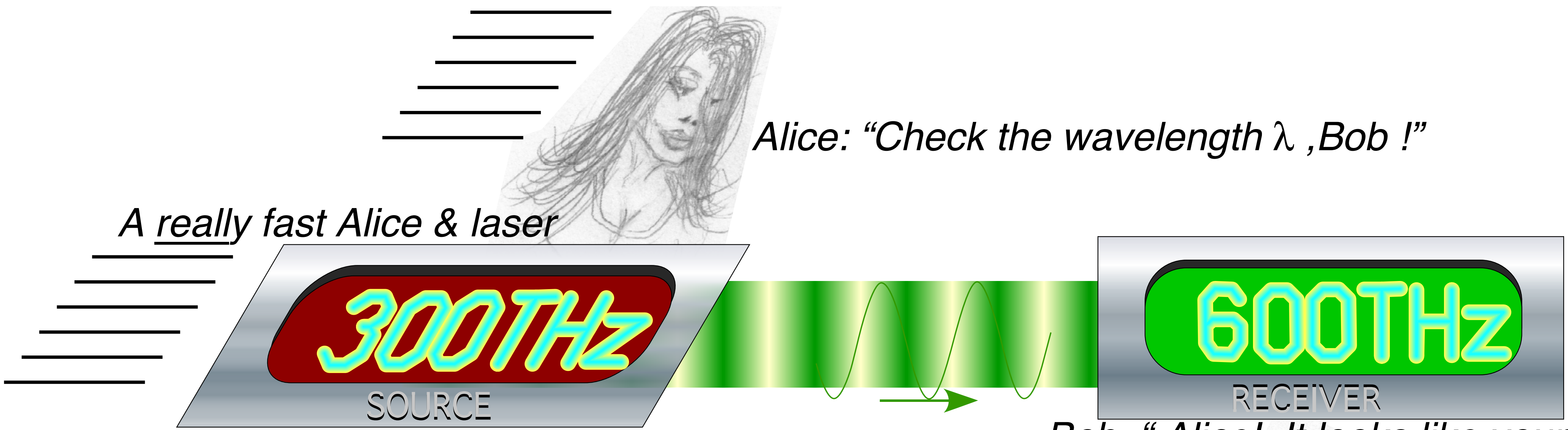

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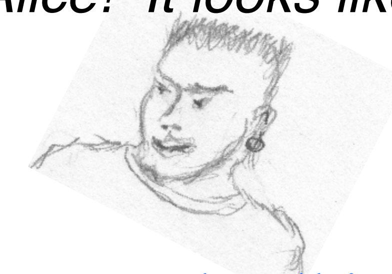

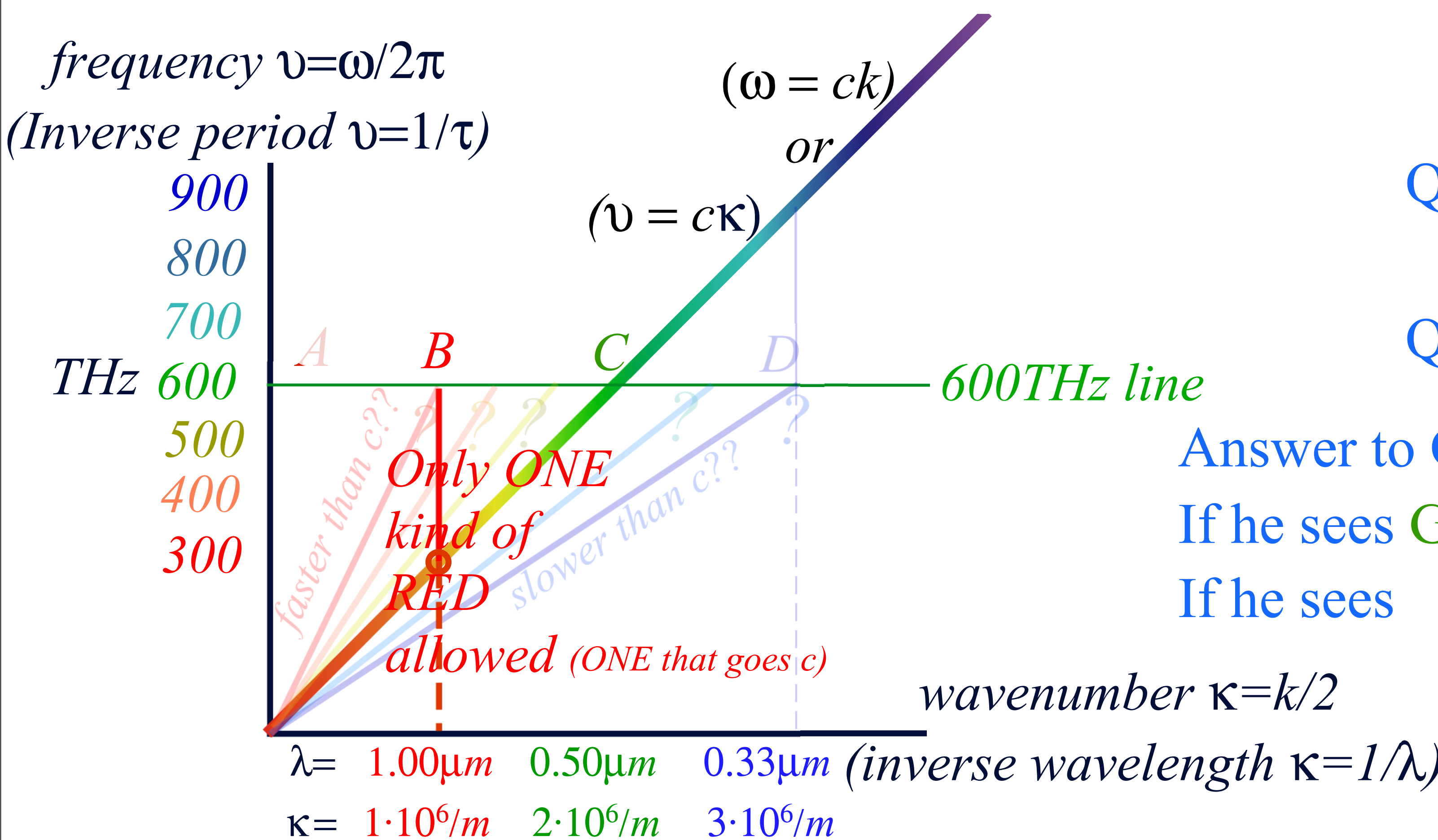

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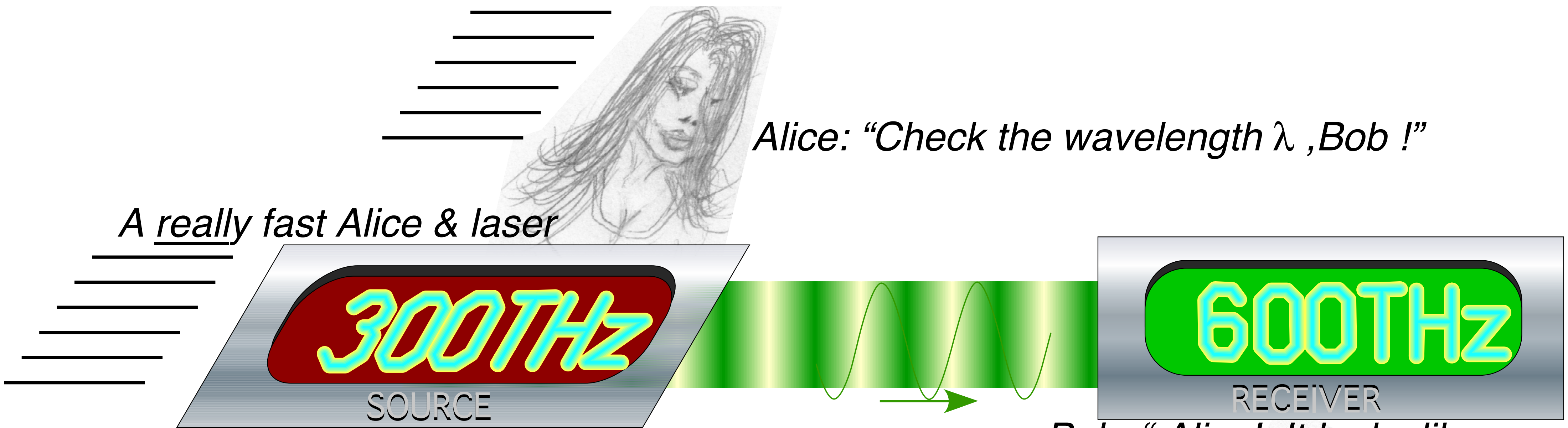

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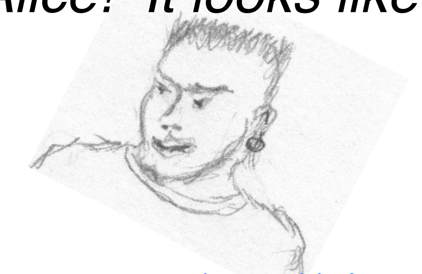

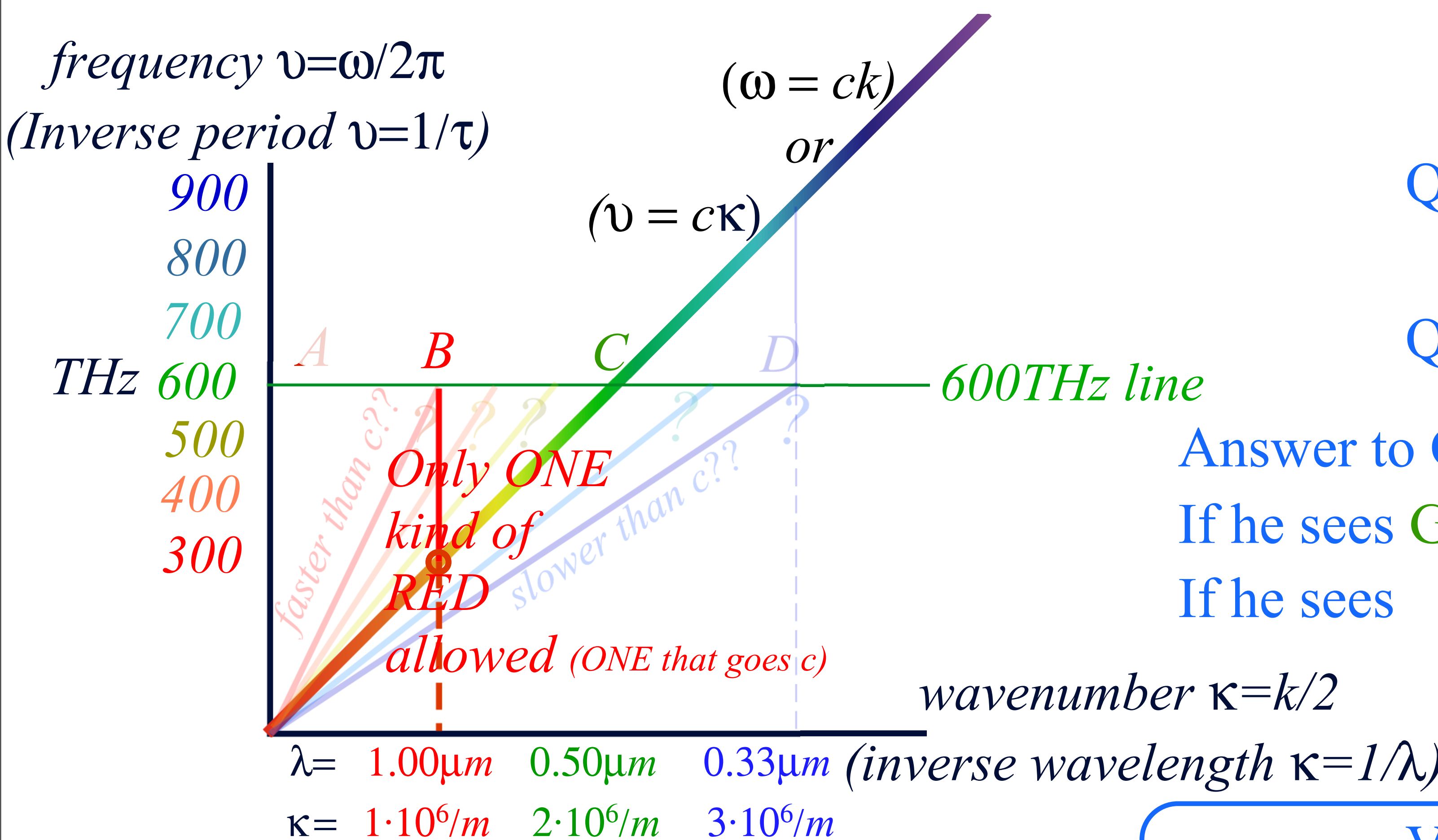

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Vacuum only makes one λ for each υ.*

"All colors go $c = \lambda\upsilon = \upsilon/\kappa = \omega/k$"

Then *Evenson's axiom* holds:

*for each beam and polarization orientation

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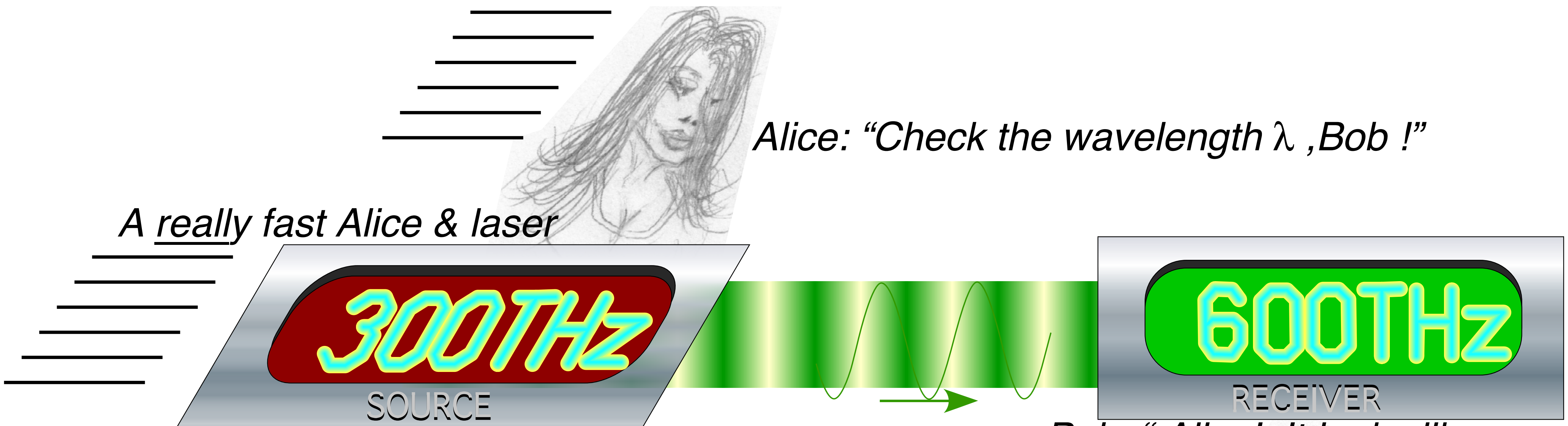

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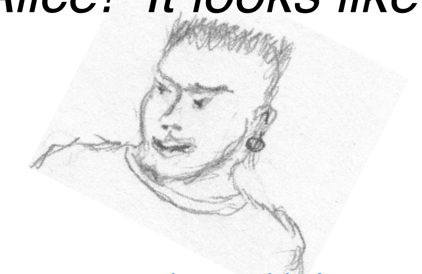

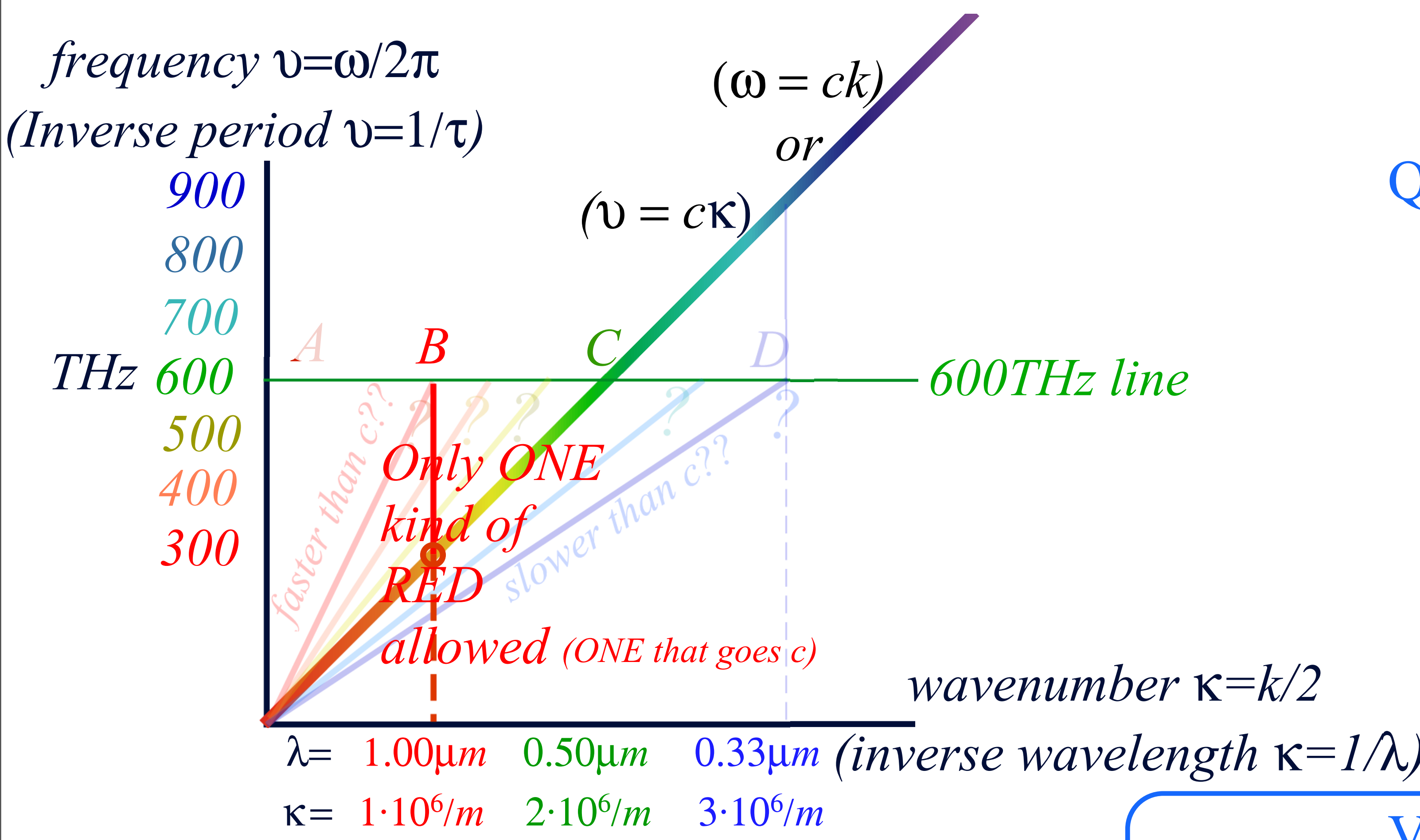

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Also could be labeled :

Linear dispersion axiom

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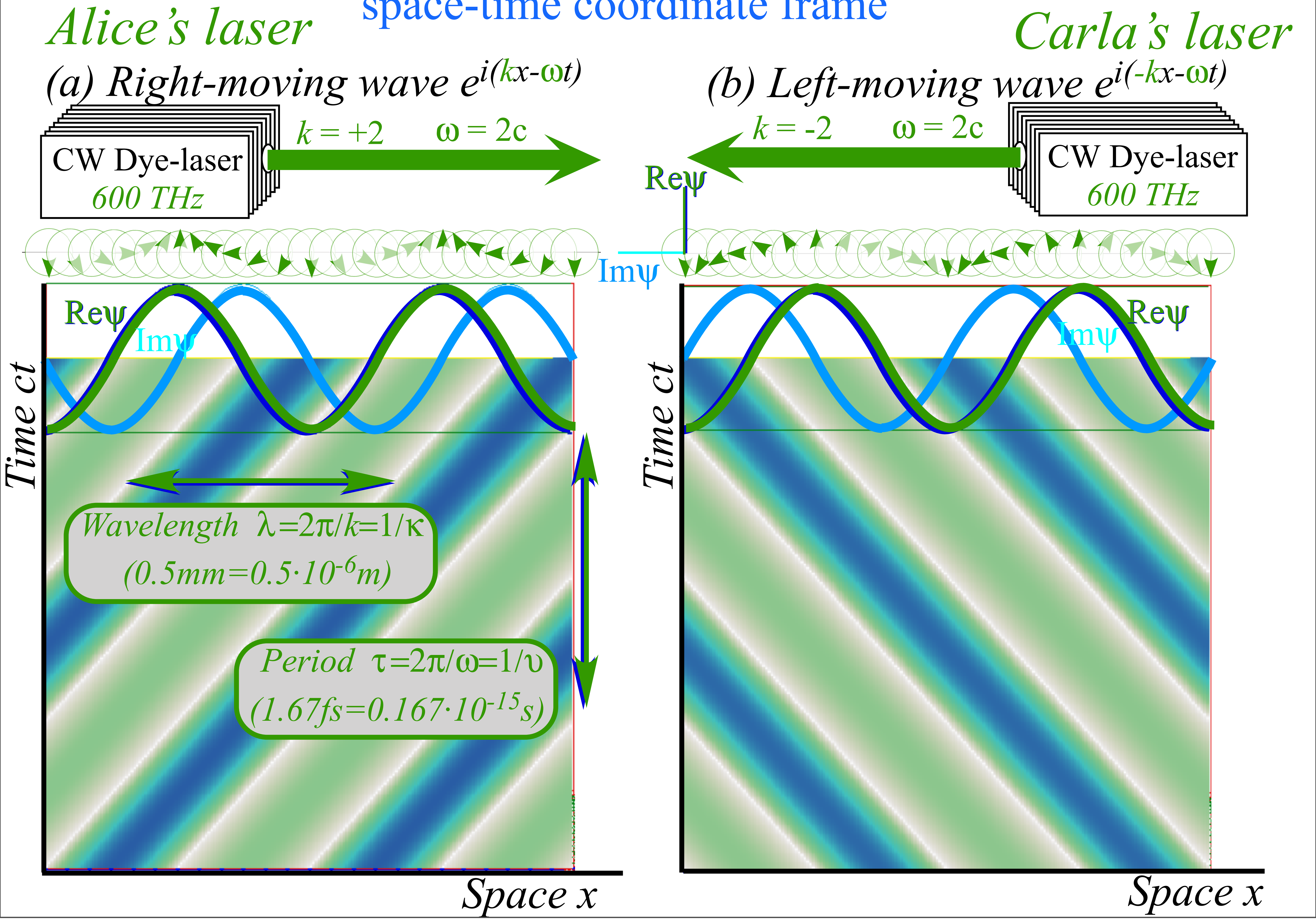
Then *Evenson's axiom* holds:

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_anim.php

http://www.uark.edu/ua/pirelli/php/waveit_1way_disp2_phasor.php

Colliding CW laser beams make space-time coordinate frame



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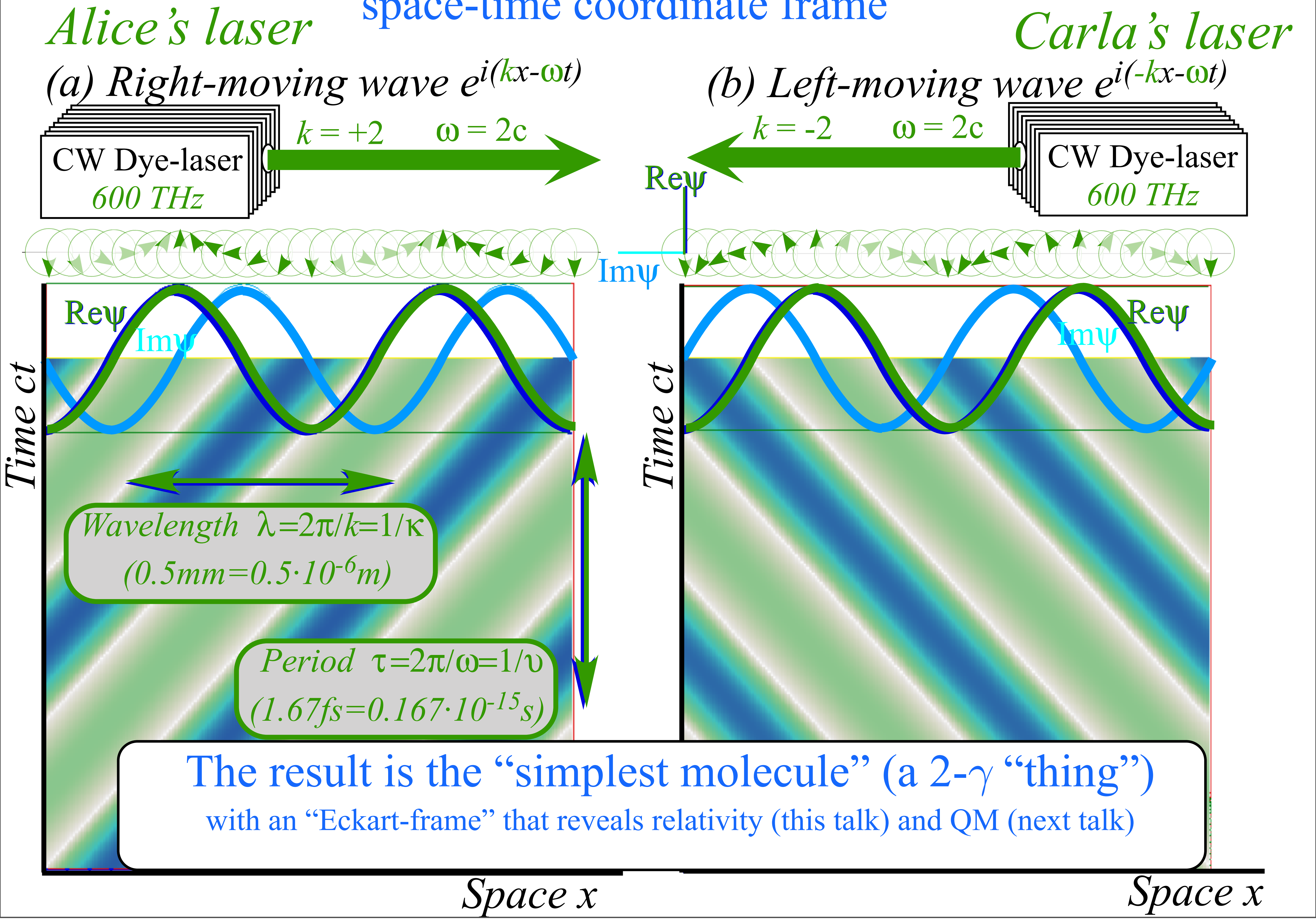

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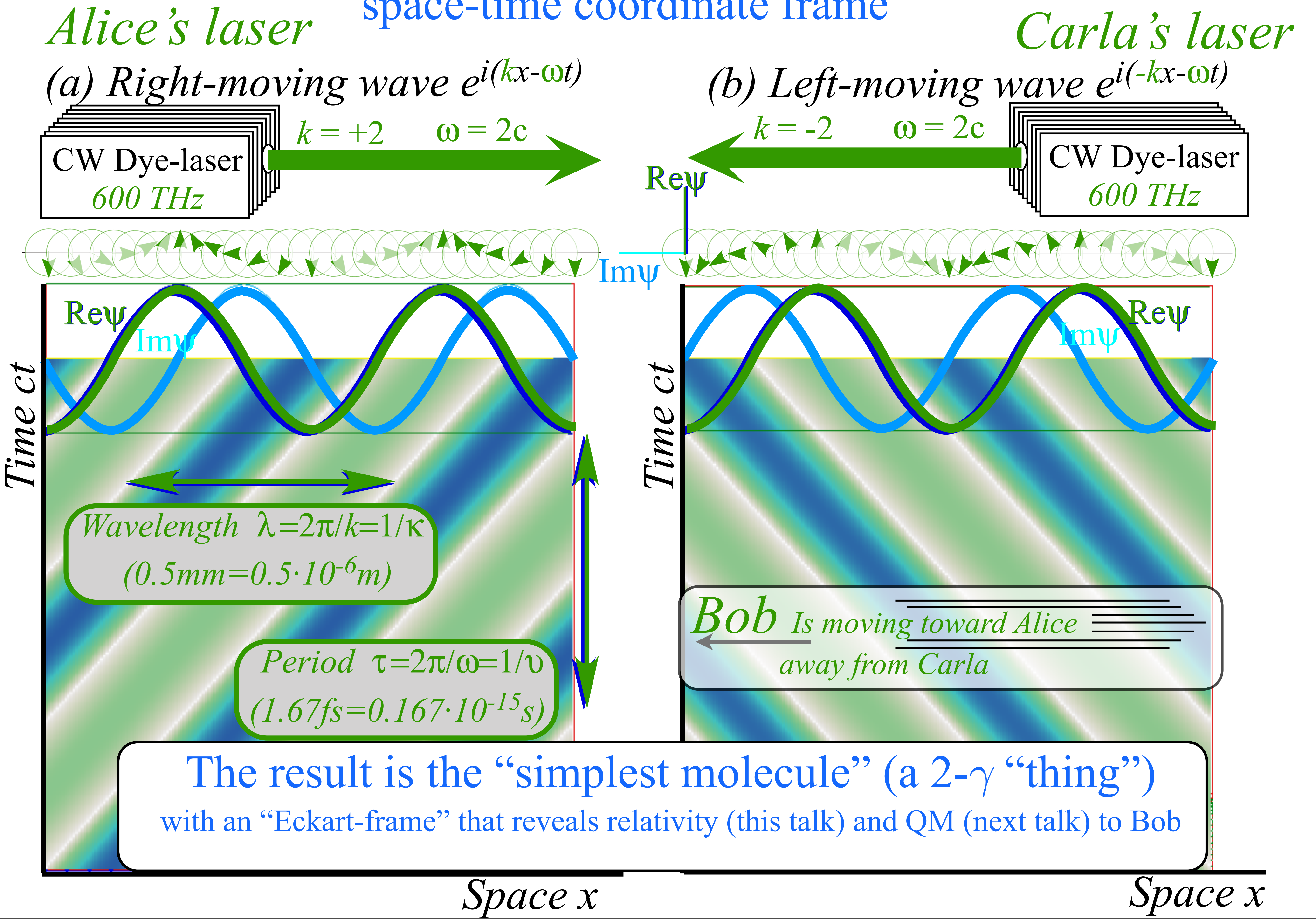
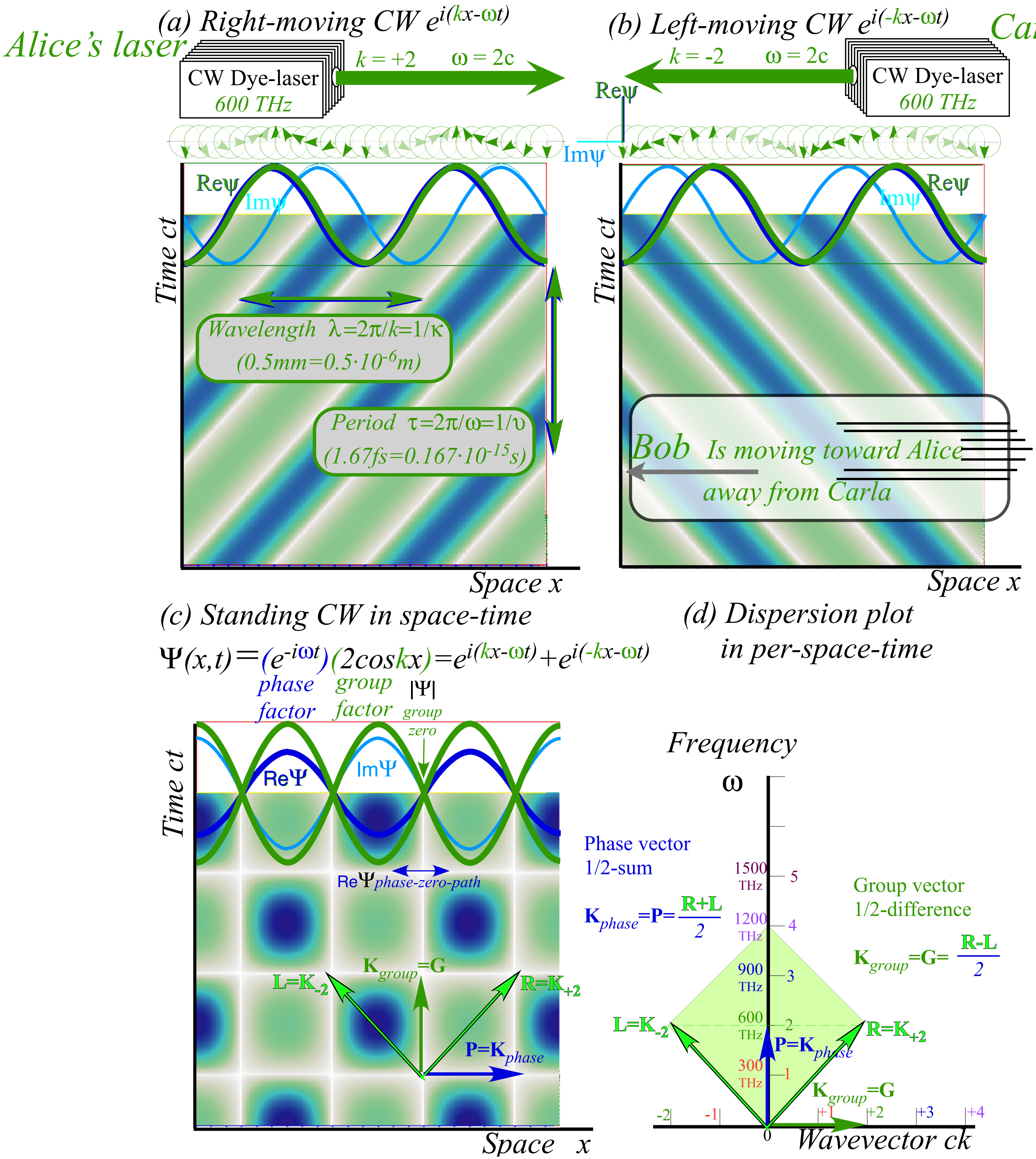




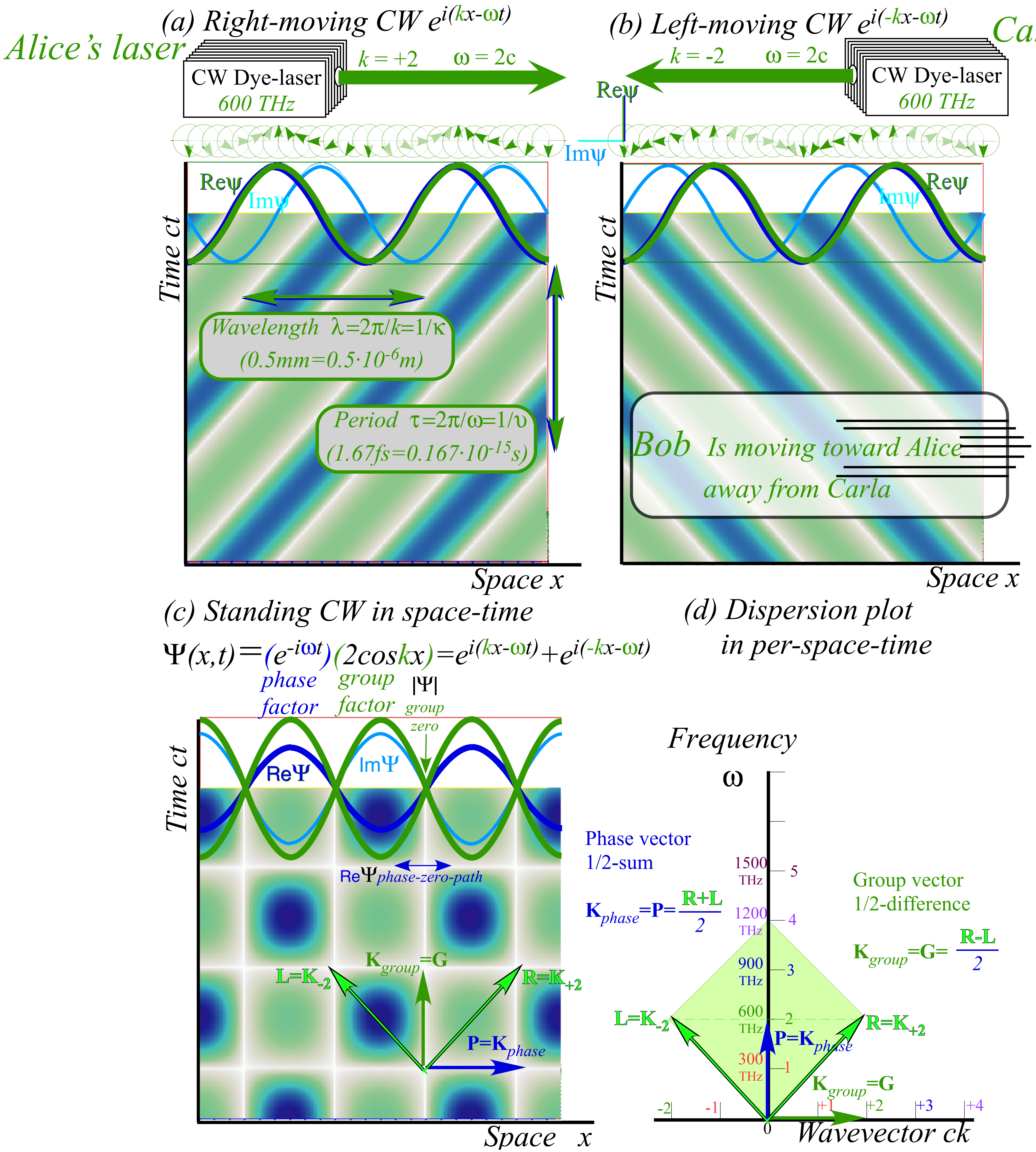
How do we find wave zeros?

Q : How is wave sum:

$$\Psi = e^{ia} + e^{ib}$$

factored into:

$$\Psi = (\psi_{phase}) \cdot (\psi_{group})?$$



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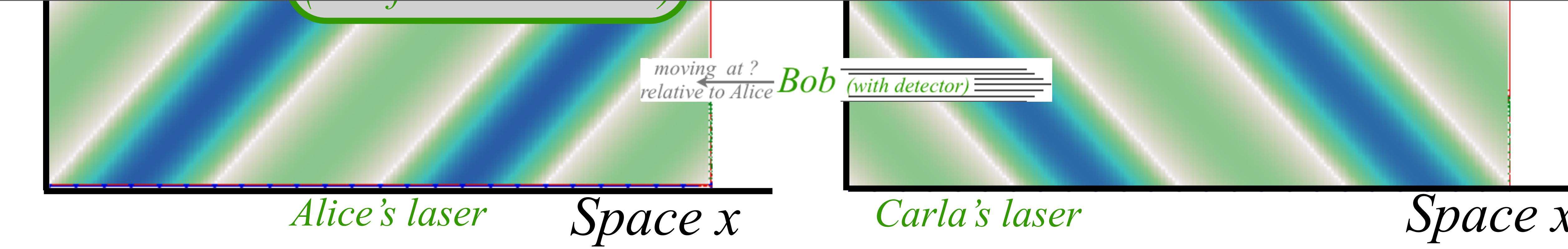
A: Easily! :

$$\Psi = e^{ia} + e^{ib} =$$

$$e^{i\frac{a+b}{2}}\left[e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right]$$

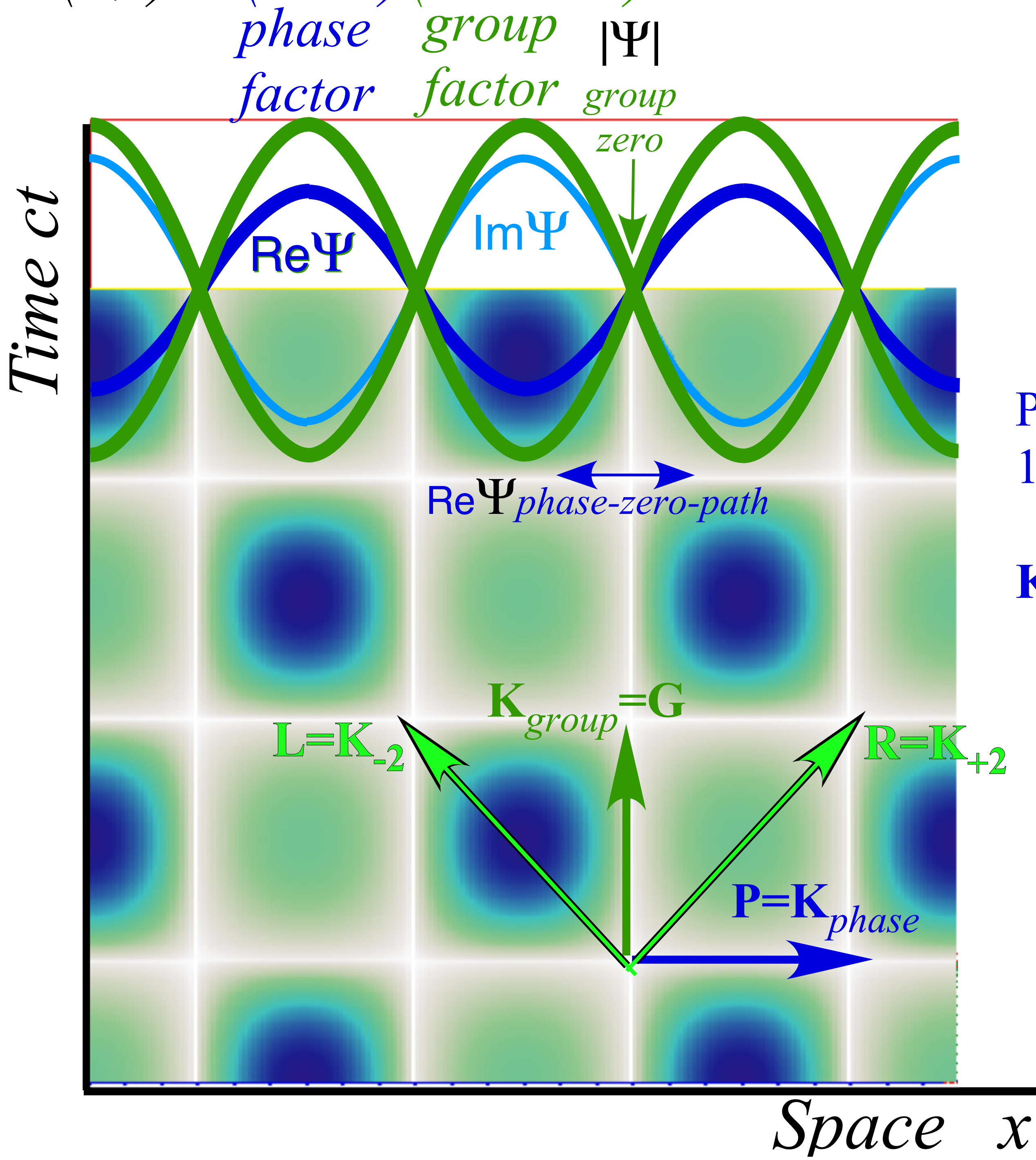
$$=(e^{i\frac{a+b}{2}})\left[2\cos\left(\frac{a-b}{2}\right)\right]$$

$$=(\psi_{phase})\cdot[\psi_{group}]?$$

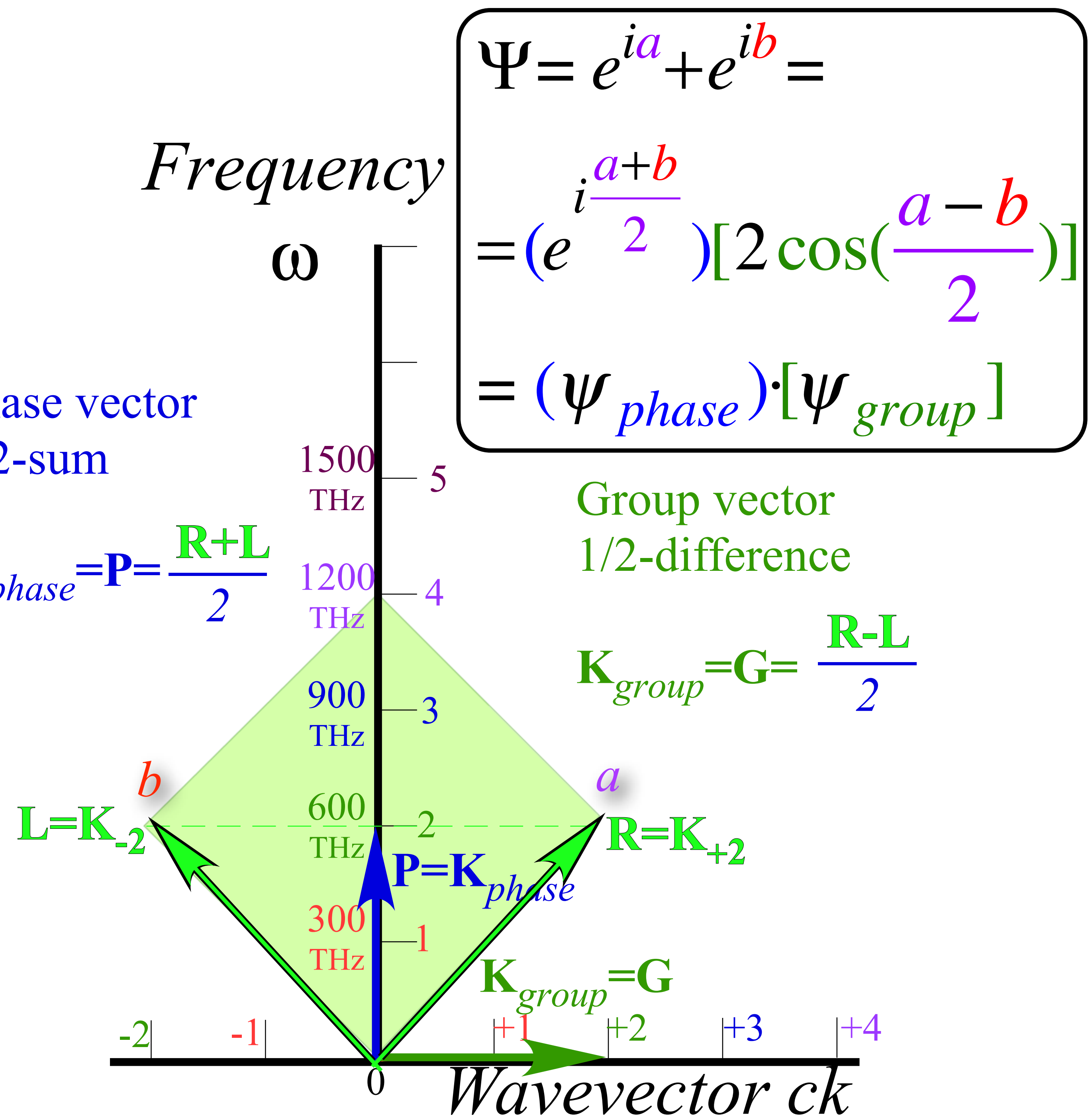


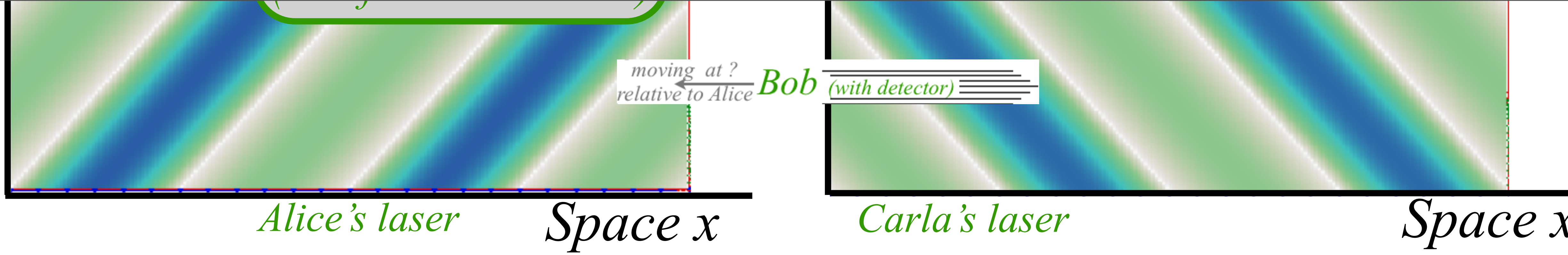
(c) Standing CW in space-time

$$\Psi(x,t) = (e^{-i\omega t})(2\cos kx) = e^{i(kx-\omega t)} + e^{i(-kx-\omega t)}$$



(d) Dispersion plot in per-space-time

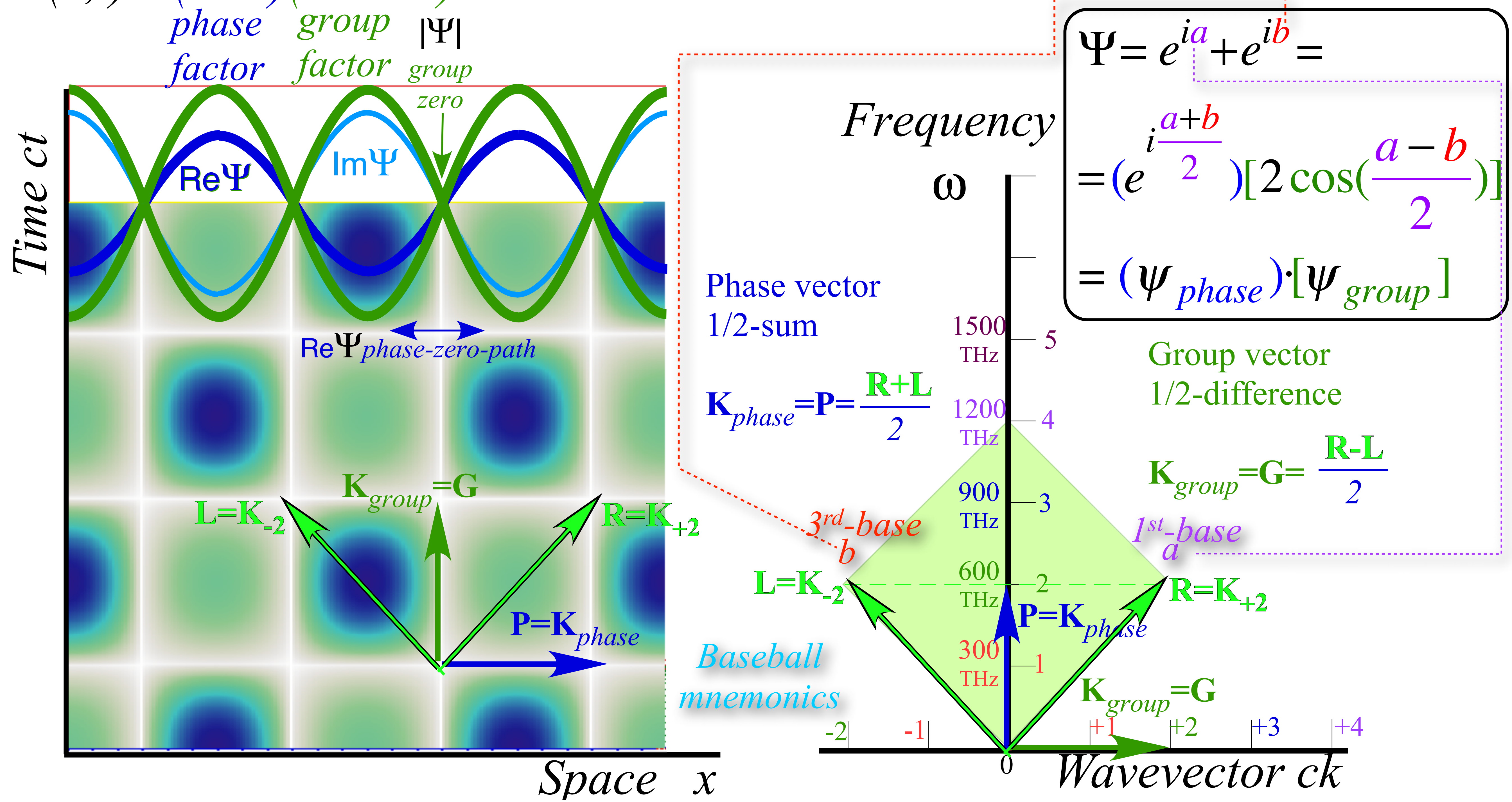


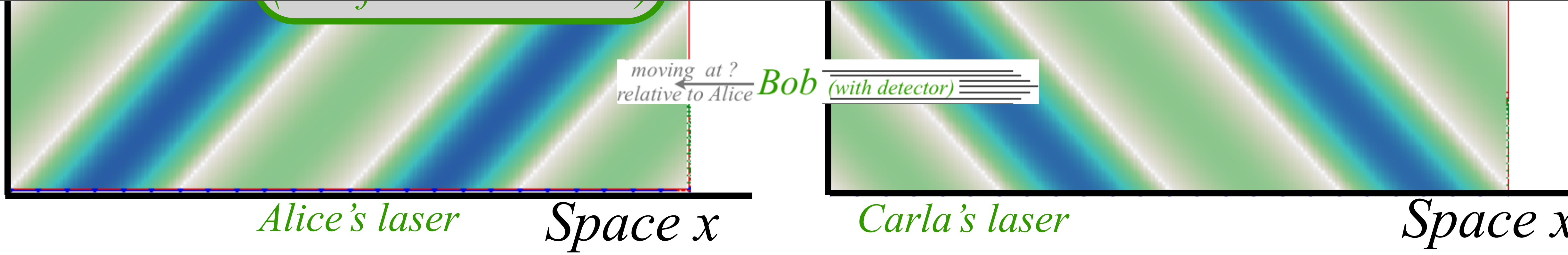


(c) Standing CW in space-time

(d) Dispersion plot in per-space-time

$$\Psi(x,t)=(e^{-i\omega t})(2\cos kx)=e^{i(kx-\omega t)}+e^{i(-kx-\omega t)}$$





(c) Standing CW in space-time

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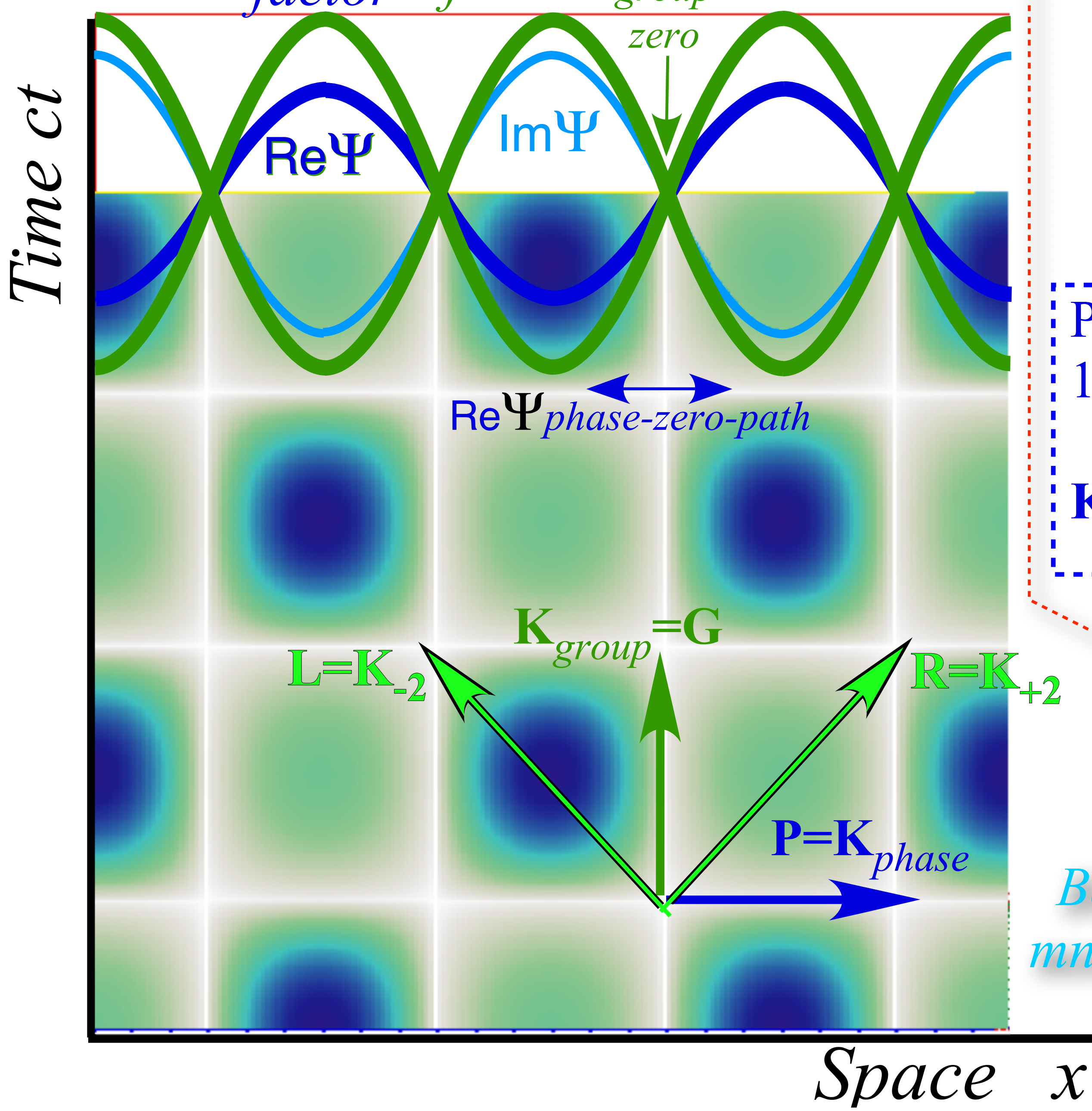
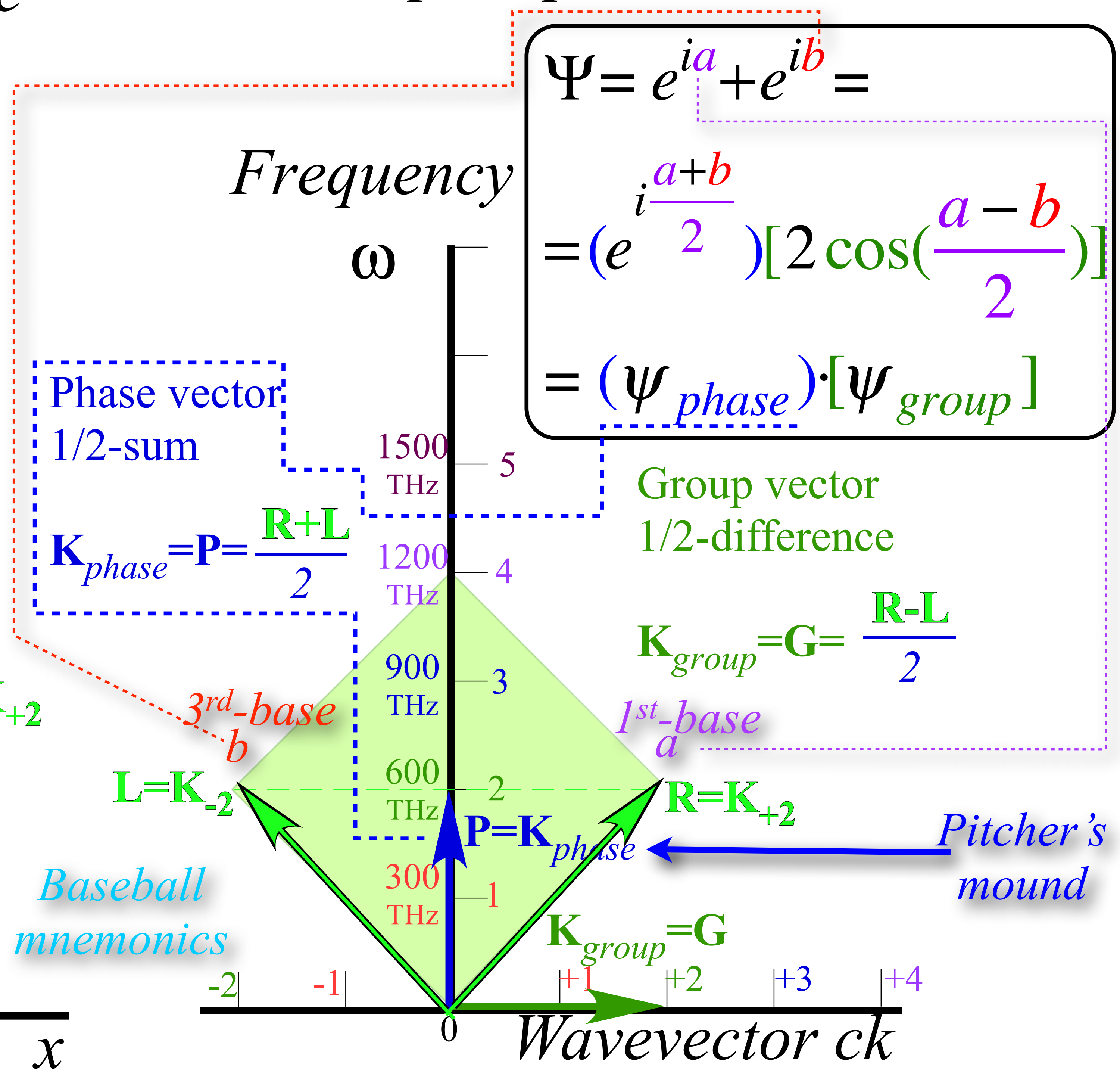
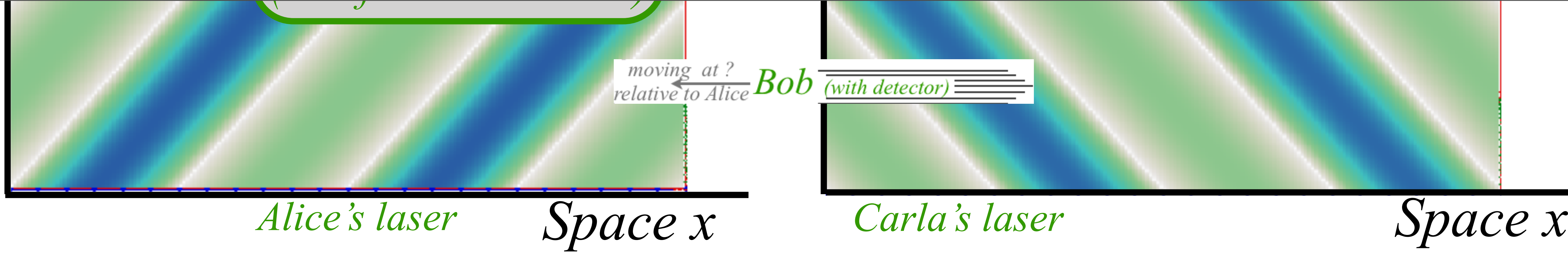


(d) Dispersion plot in per-space-time





(c) Standing CW in space-time

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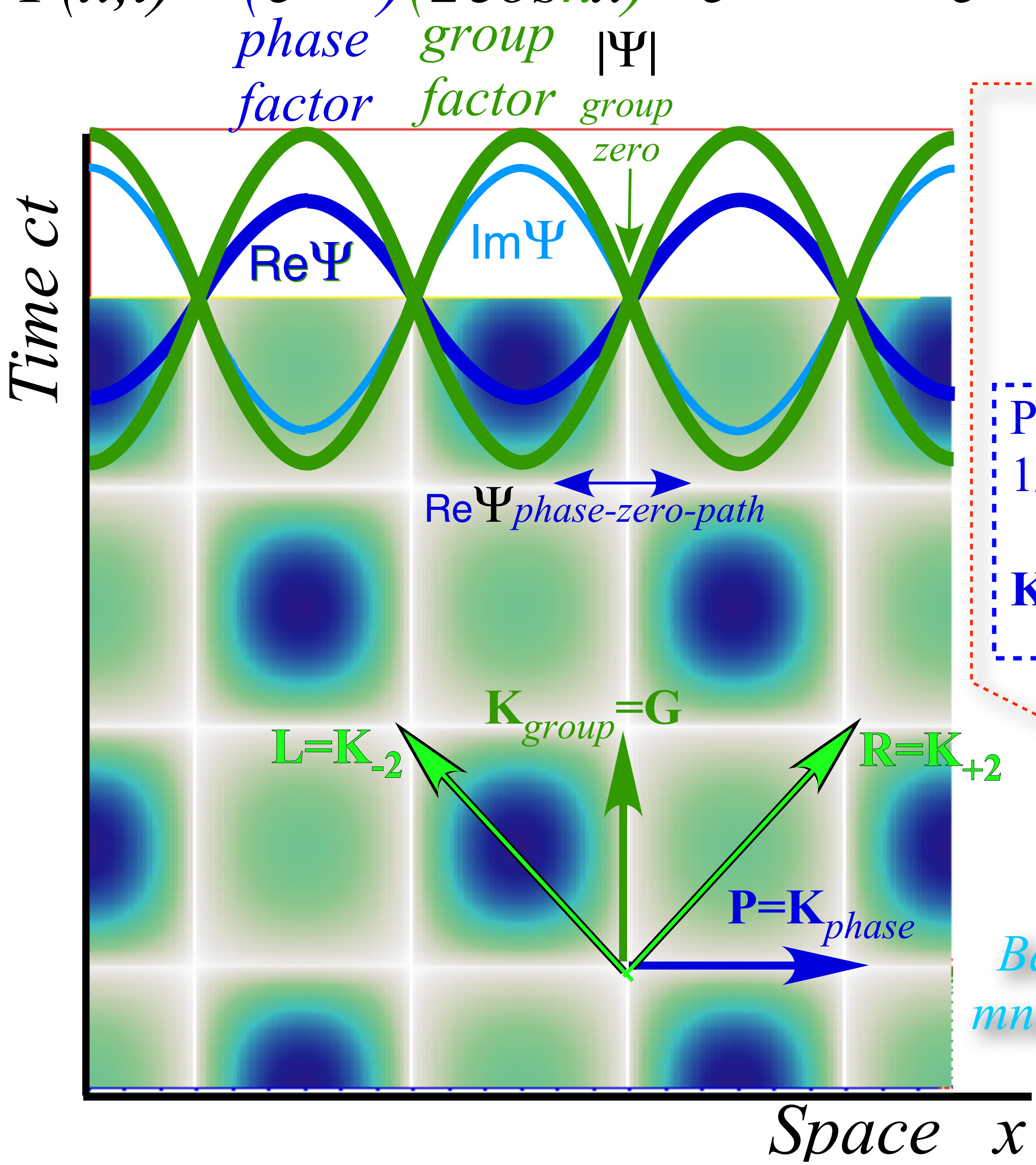
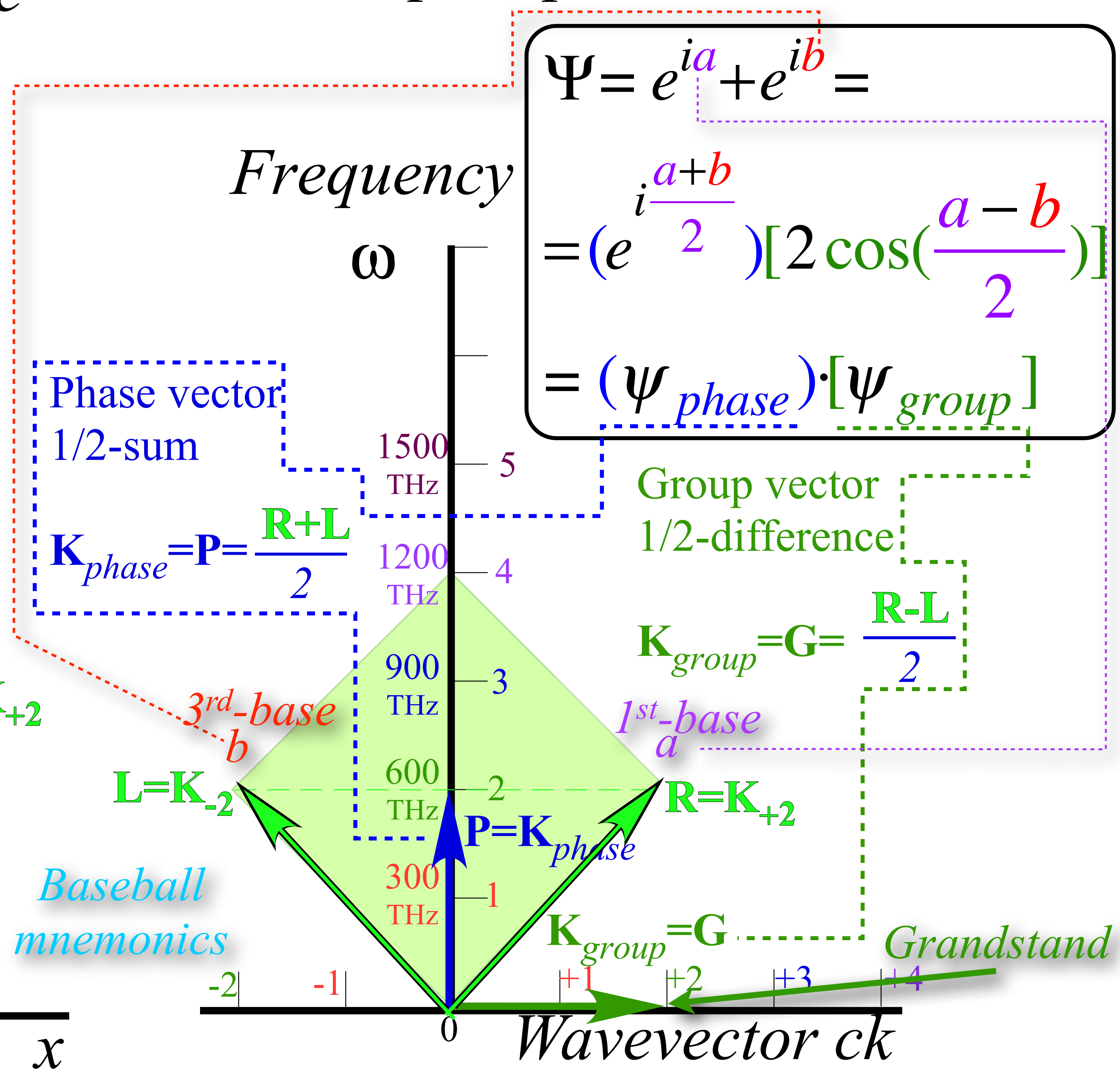
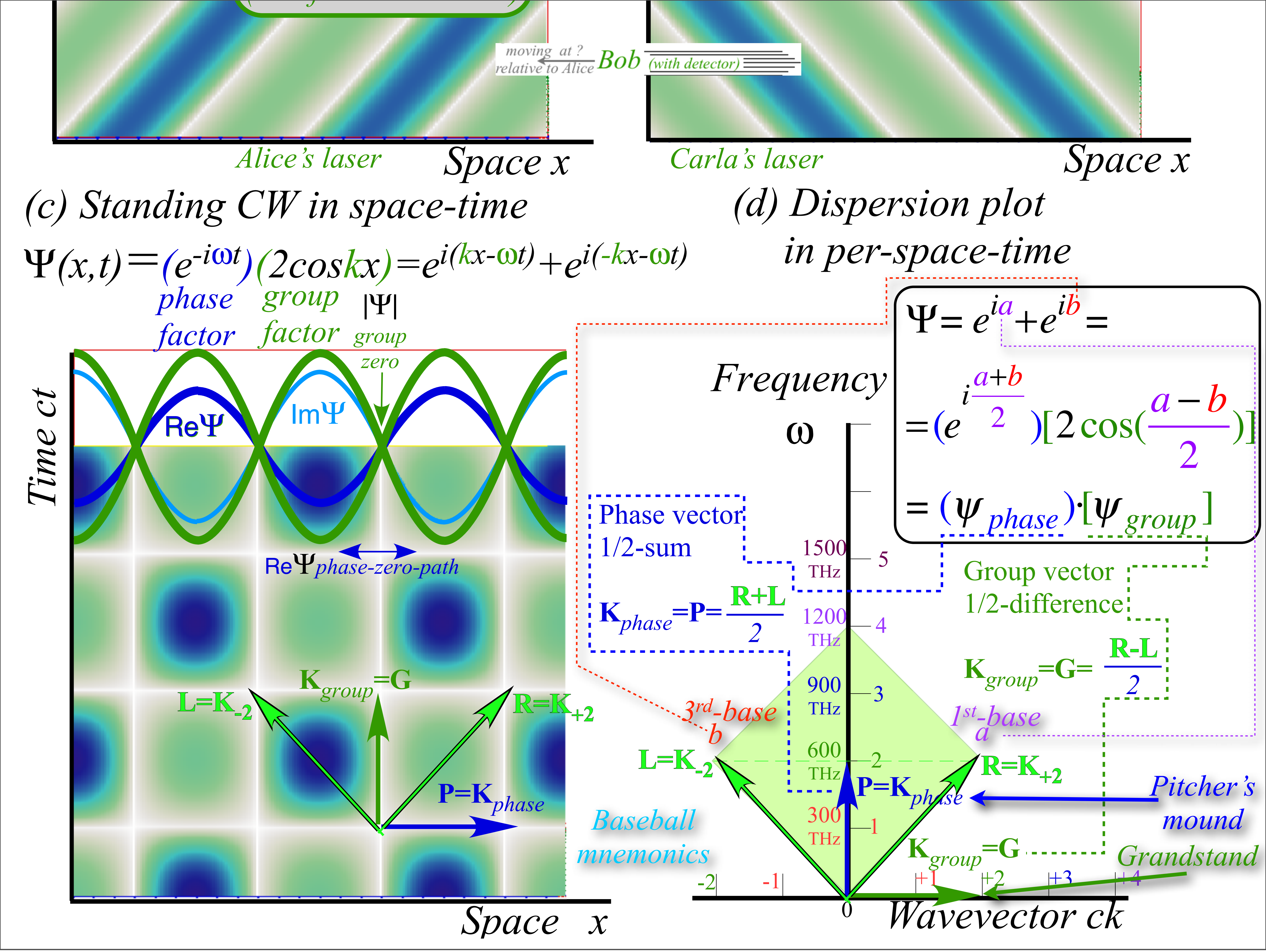
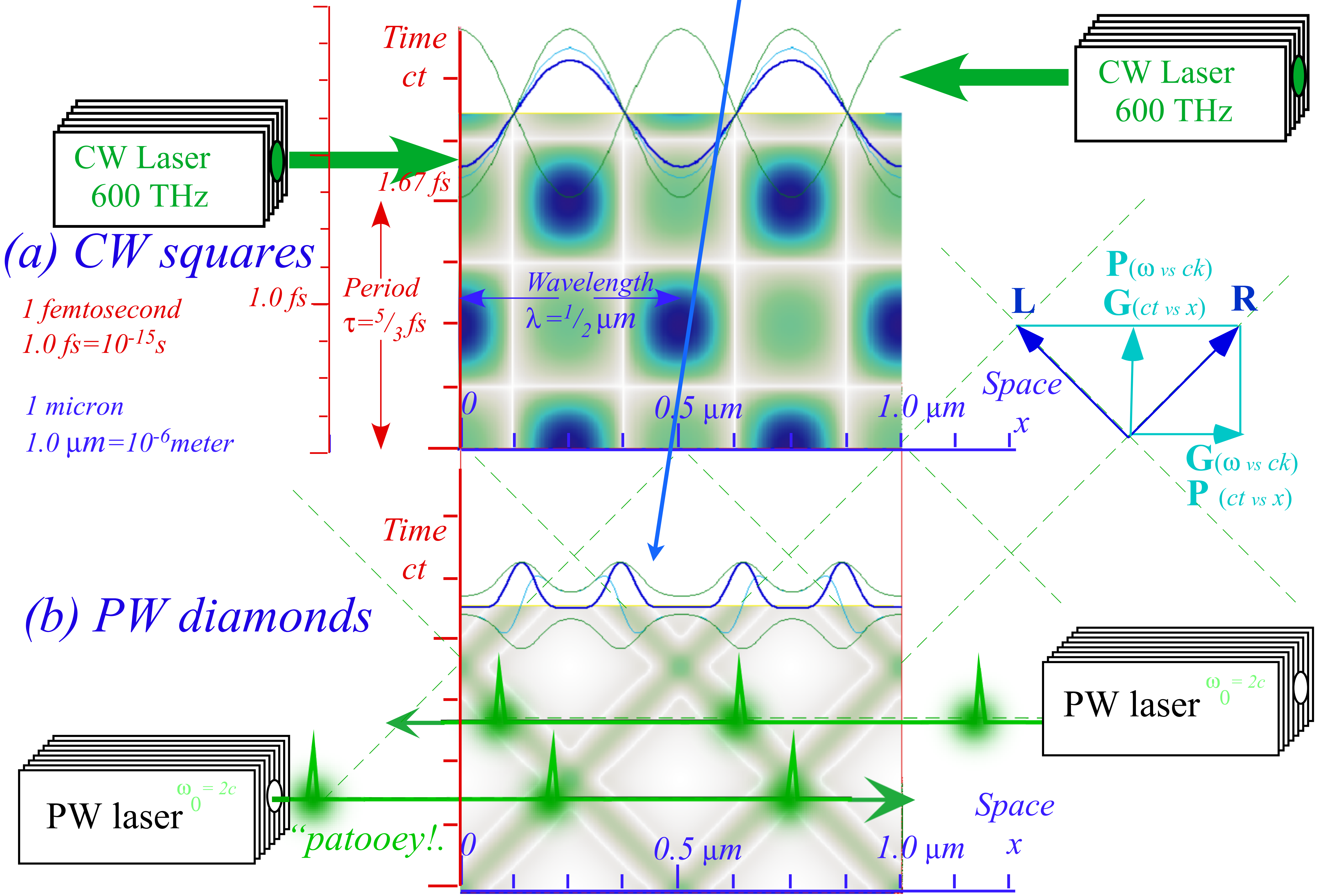


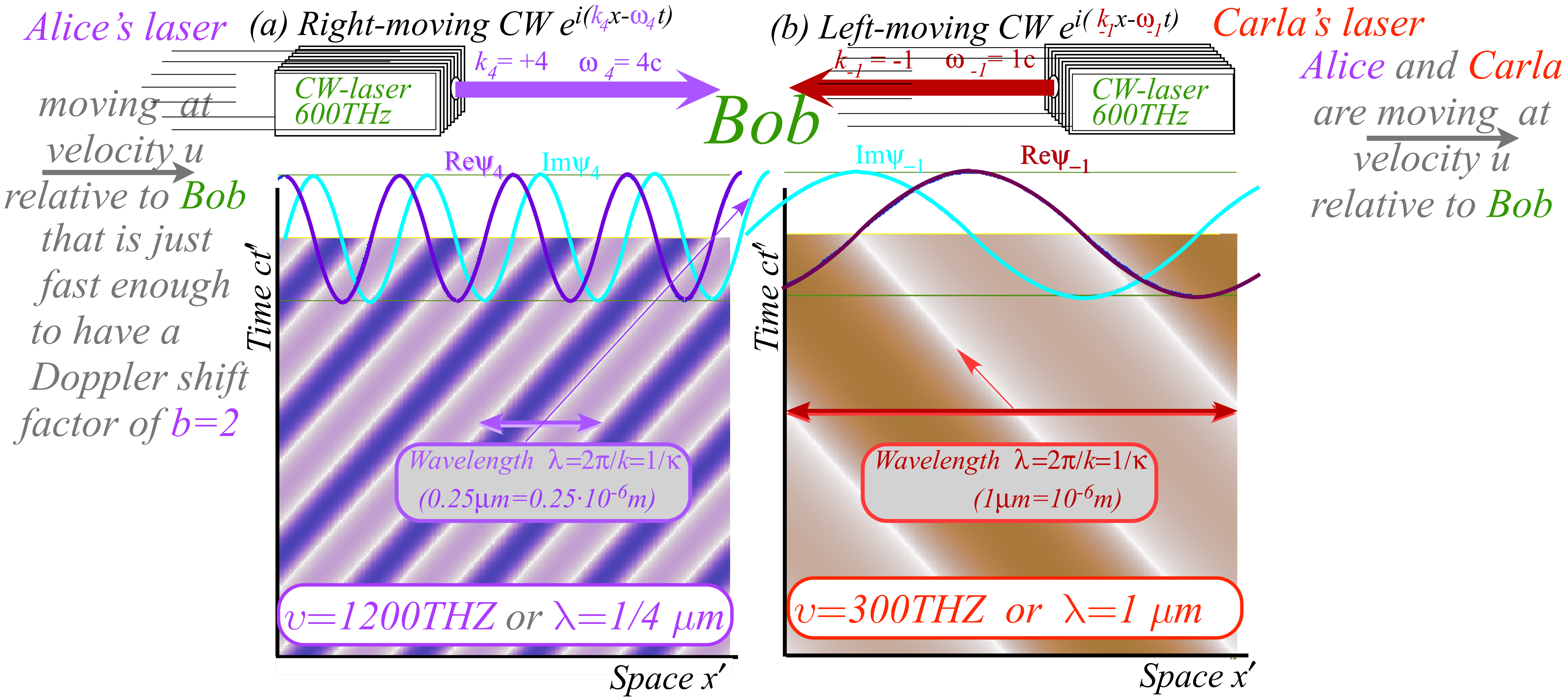
(d) Dispersion plot in per-space-time




moving at ? relative to Alice
Bob (with detector)
Alice's laser
Space x
Carla's laser
Space x
(c) Standing CW in space-time
(d) Dispersion plot in per-space-time
Ψ(x,t)=(e^{-iωt})(2cos kx)=e^{i(kx-ωt)}+e^{i(-kx-ωt)}
phase factor
group factor
|Ψ| group zero
Time ct
ReΨ
ImΨ
ReΨ phase-zero-path
K_group=G
L=K_-2
R=K_+2
P=K_phase
Space x
Baseball mnemonics
Frequency ω
Ψ= e^{ia}+e^{ib} =
=(e^{i(a+b)/2})[2 cos((a−b)/2)]
= (ψ_phase)·[ψ_group]
Phase vector 1/2-sum
K_phase=P=(R+L)/2
Group vector 1/2-difference
K_group=G=(R-L)/2
1500 THz
1200 THz
900 THz
600 THz
300 THz
5
4
3
2
1
3rd-base b
1st-base a
L=K_-2
R=K_+2
P=K_phase
K_group=G
Pitcher's mound
Grandstand
-2
-1
0
+1
+2
+3
+4
Wavevector ck

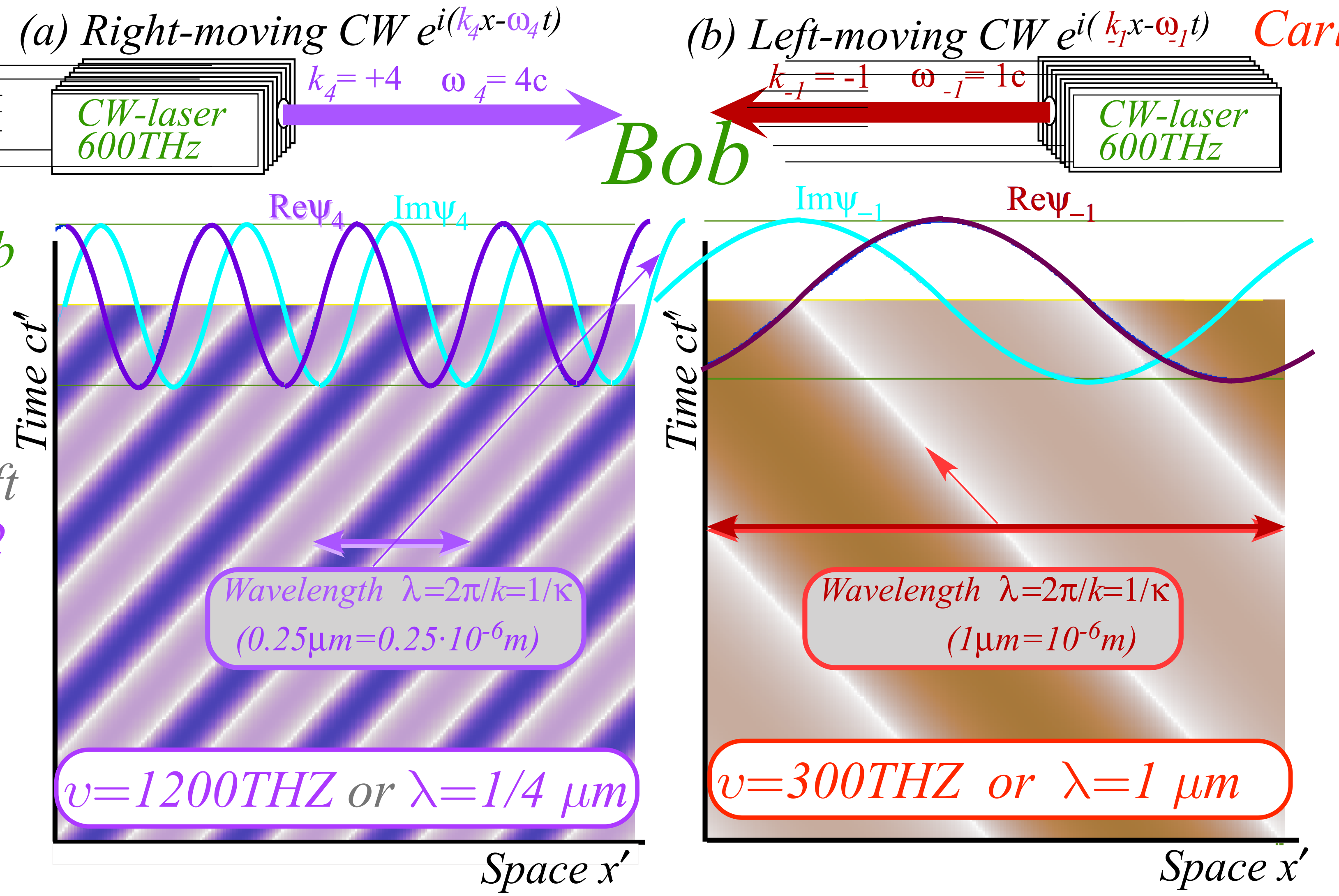
Pulse Waves (PW) make "baseball diamonds" in space-time




Alice's laser
(a) Right-moving CW $e^{i(k_4x-\omega_4t)}$
(b) Left-moving CW $e^{i(k_{-1}x-\omega_{-1}t)}$
Carla's laser
Alice and Carla are moving at velocity u relative to Bob
moving at velocity u relative to Bob that is just fast enough to have a Doppler shift factor of b=2
CW-laser 600THz
k_4= +4 ω $_4$= 4c
k_{-1} = -1 ω $_{-1}$= 1c
CW-laser 600THz
Bob
Reψ4 Imψ4
Imψ−1 Reψ−1
Time ct′
Wavelength λ=2π/k=1/κ (0.25μm=0.25·10-6m)
υ=1200THZ or λ=1/4 μm
Space x′
Time ct′
Wavelength λ=2π/k=1/κ (1μm=10-6m)
υ=300THZ or λ=1 μm
Space x′

Alice's laser

moving at velocity u relative to Bob that is just fast enough to have a Doppler shift factor of b=2



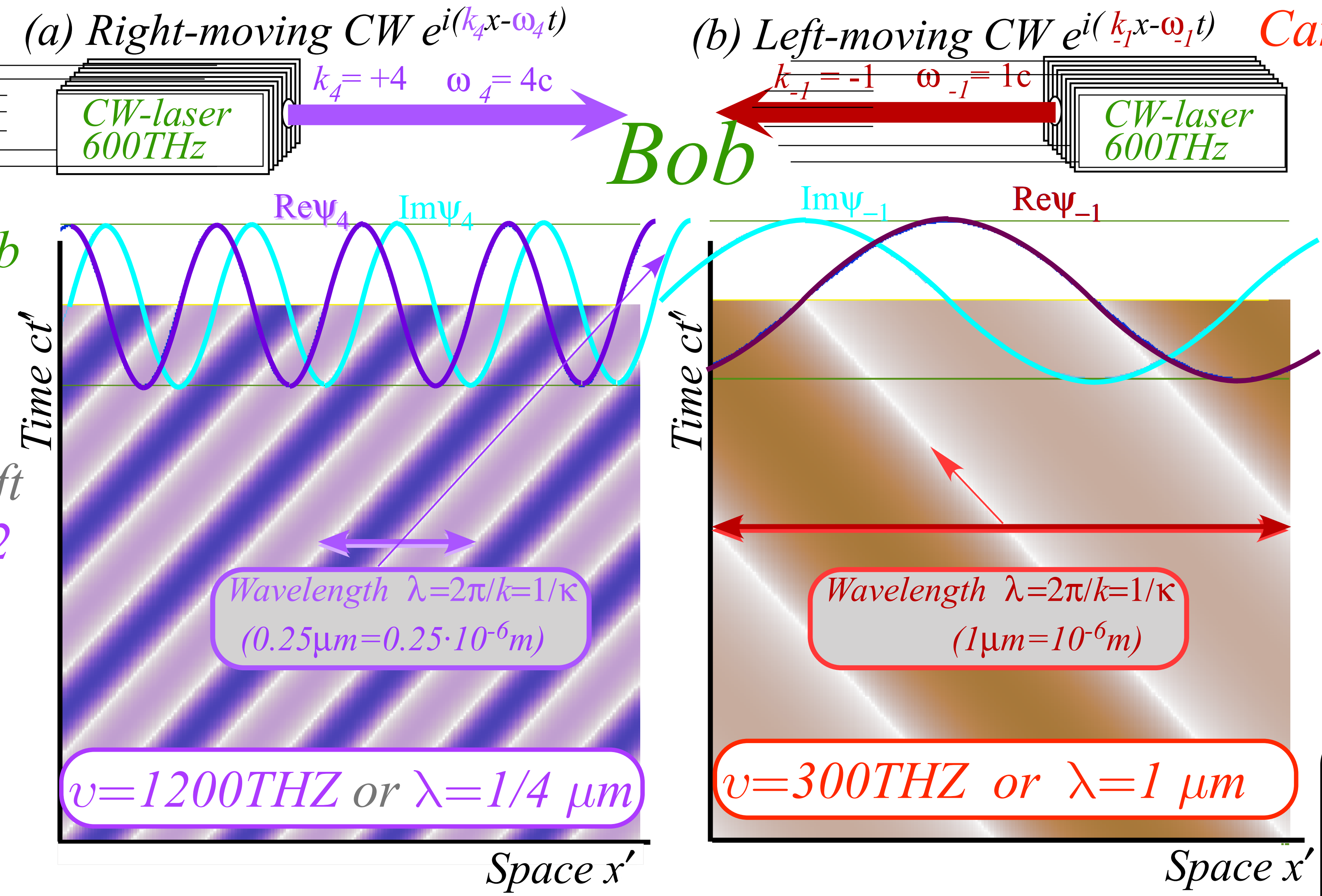
Carla's laser

Alice and Carla are moving at velocity u relative to Bob

What is velocity u?
and
Where are wave zeros in Bob's Frame?

Alice's laser

moving at velocity u relative to Bob that is just fast enough to have a Doppler shift factor of b=2



Carla's laser

Alice and Carla are moving at velocity u relative to Bob

What is velocity u?
and
Where are wave zeros in Bob's Frame?

Answer both questions by factoring wave sum:

$$\Psi = e^{ia} + e^{ib} =$$

$$= \left(e^{i\frac{a+b}{2}}\right)\left[2\cos\left(\frac{a-b}{2}\right)\right]$$

$$= (\psi_{phase}) \cdot [\psi_{group}]$$

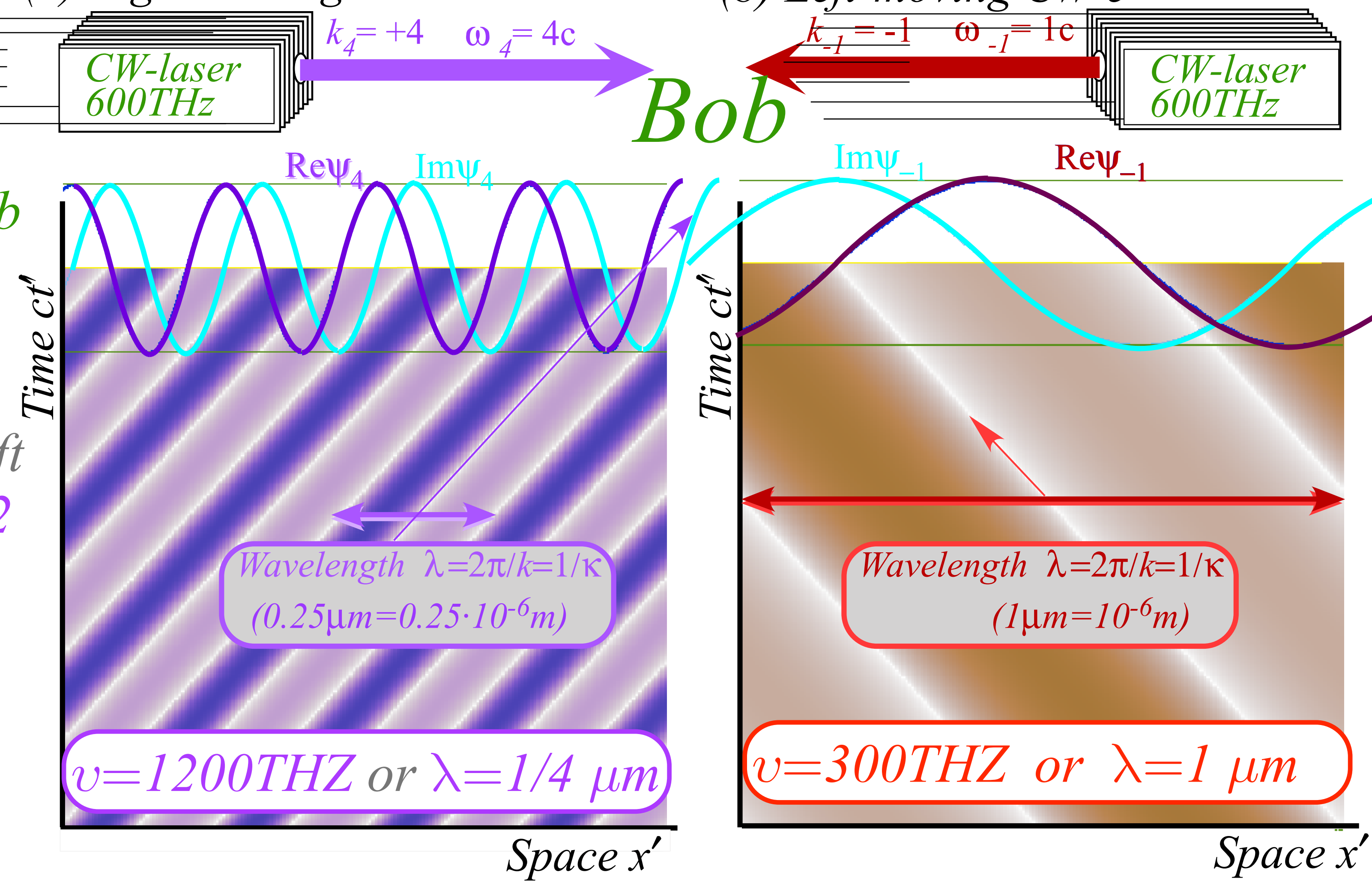
Alice's laser

moving at velocity u relative to Bob that is just fast enough to have a Doppler shift factor of b=2

(a) Right-moving CW $e^{i(k_4 x-\omega_4 t)}$

(b) Left-moving CW $e^{i(k_{-1} x-\omega_{-1} t)}$

Carla's laser



Alice and Carla are moving at velocity u relative to Bob

What is velocity u? and Where are wave zeros in Bob's Frame?

Answer both questions by factoring wave sum:

$$\Psi = e^{ia} + e^{ib} =$$

$$= \left(e^{i\frac{a+b}{2}}\right)\left[2\cos\left(\frac{a-b}{2}\right)\right]$$

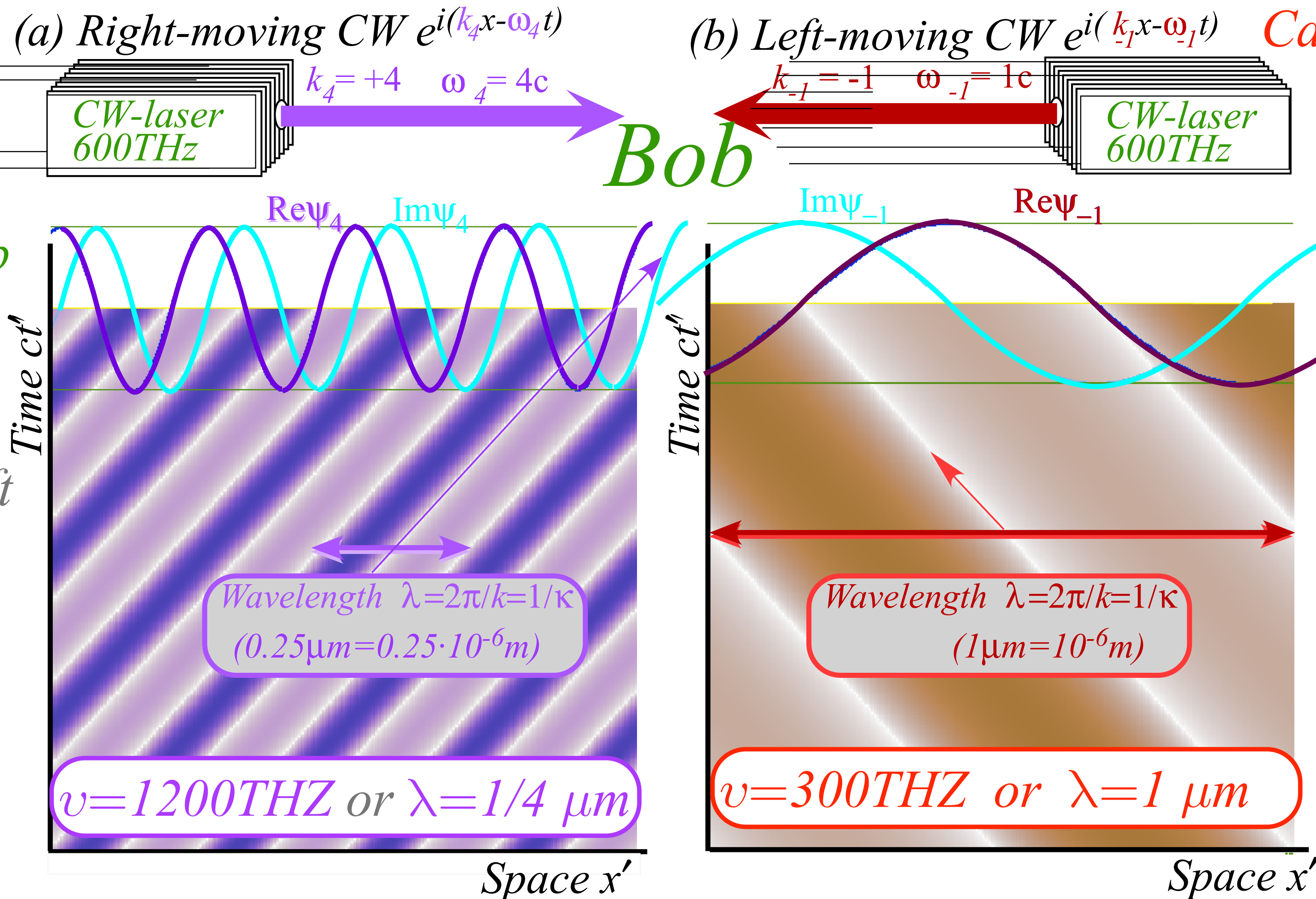
$$= (\psi_{phase}) \cdot [\psi_{group}]$$

Alice's 600THz laser moving at velocity u toward Bob is Doppler blue shifted by b=2 to 1200THZ for Bob

Carla's 600THz laser moving at velocity u away from Bob is Doppler red shifted by 1/b=1/2 to 300THZ for Bob

Alice's laser

moving at velocity u relative to Bob that is just fast enough to have a Doppler shift factor of $b=2$



Carla's laser

Alice and Carla are moving at velocity u relative to Bob

What is velocity u? and Where are wave zeros in Bob's Frame?

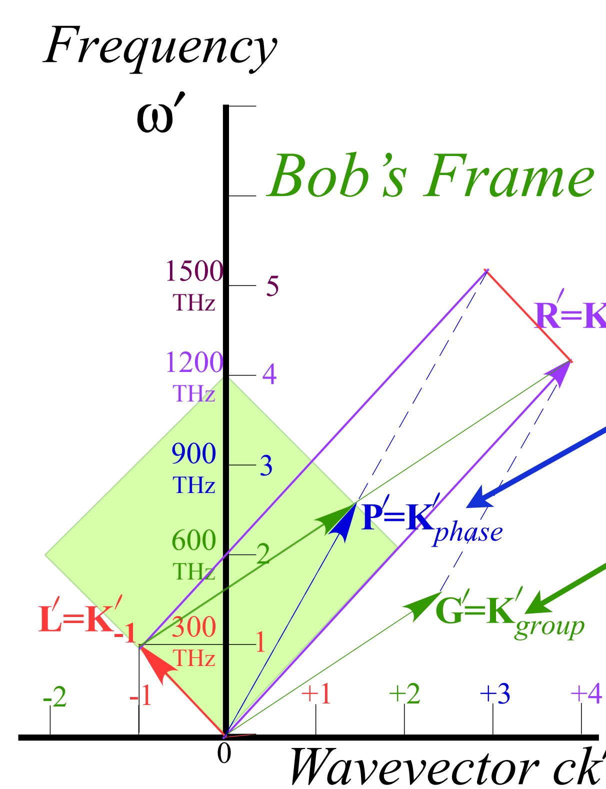
Answer both questions by factoring wave sum:

$$\Psi = e^{ia}+e^{ib} =$$

$$=(e^{i\frac{a+b}{2}})[2\cos(\frac{a-b}{2})]$$

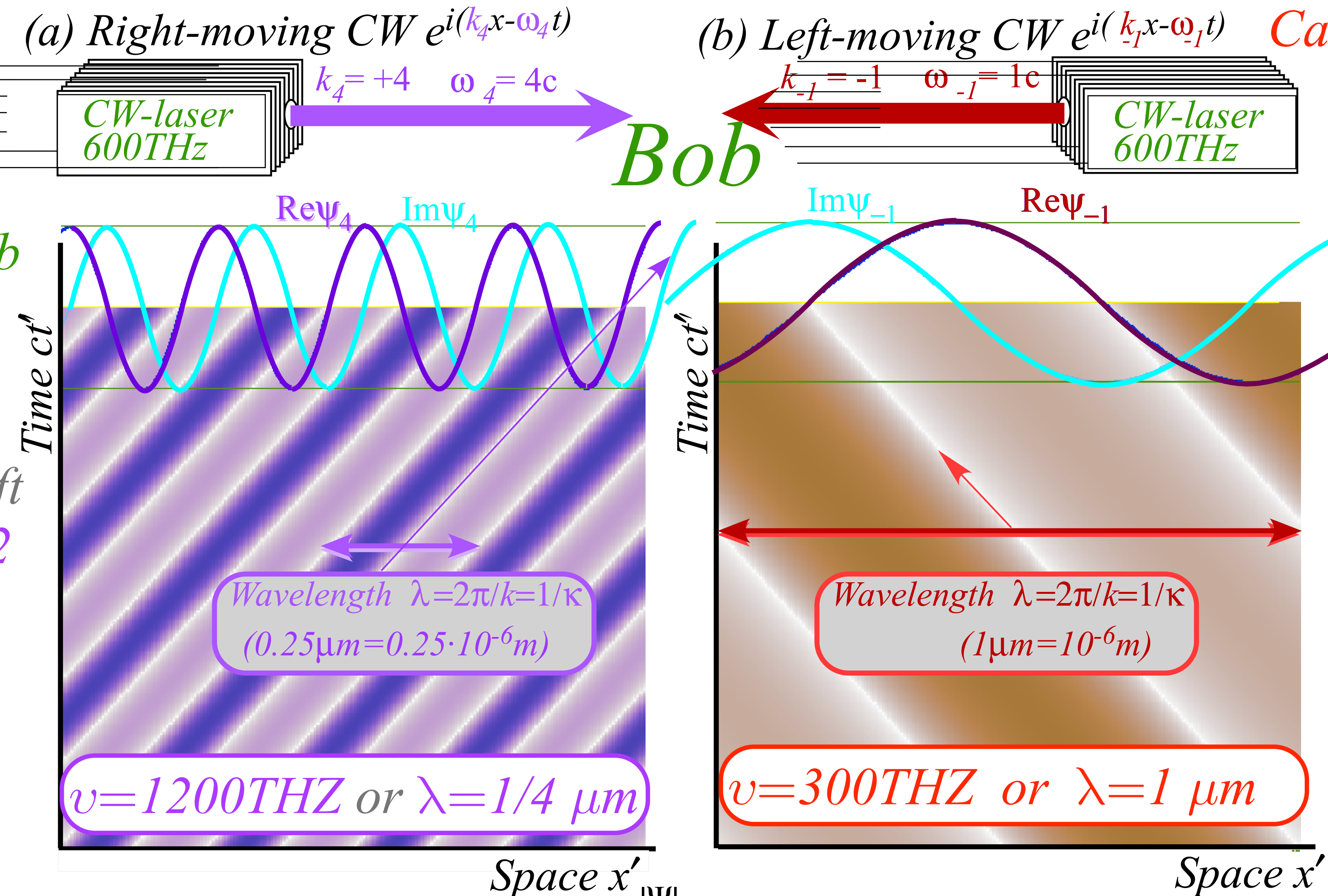
$$=(\psi_{phase})\cdot[\psi_{group}]$$

(d) Dispersion plot



This gives 1/2-sum Phase $\mathbf{P}'=(\mathbf{R}'+\mathbf{L}')/2$ vector and a 1/2-difference Group $\mathbf{G}'=(\mathbf{R}'-\mathbf{L}')/2$ vector in (frequency, wavevector) space (ω', k') for Bob.

Alice's laser moving at velocity u relative to Bob that is just fast enough to have a Doppler shift factor of b=2



Carla's laser

Alice and Carla are moving at velocity u relative to Bob

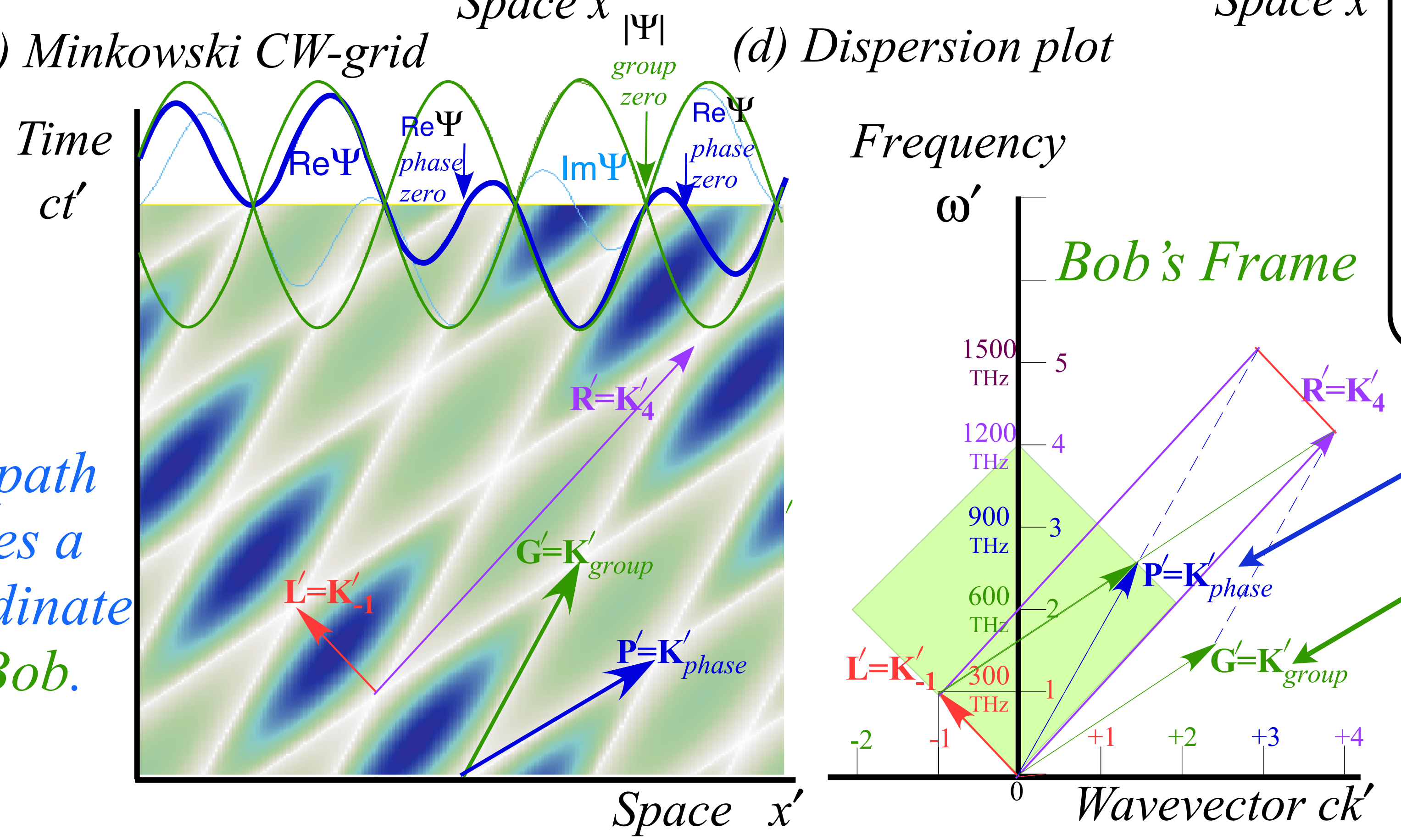
What is velocity u?
and
Where are wave zeros in Bob's Frame?

Answer both questions by factoring wave sum:

$$\Psi = e^{ia} + e^{ib} =$$

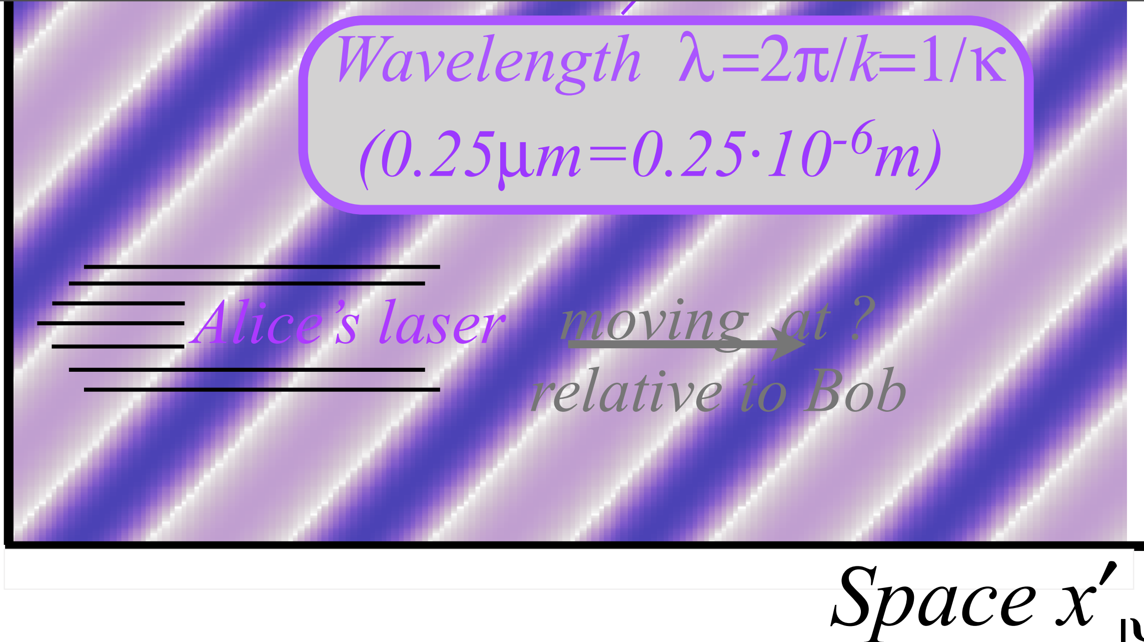
$$= \left(e^{i\frac{a+b}{2}}\right)\left[2\cos\left(\frac{a-b}{2}\right)\right]$$

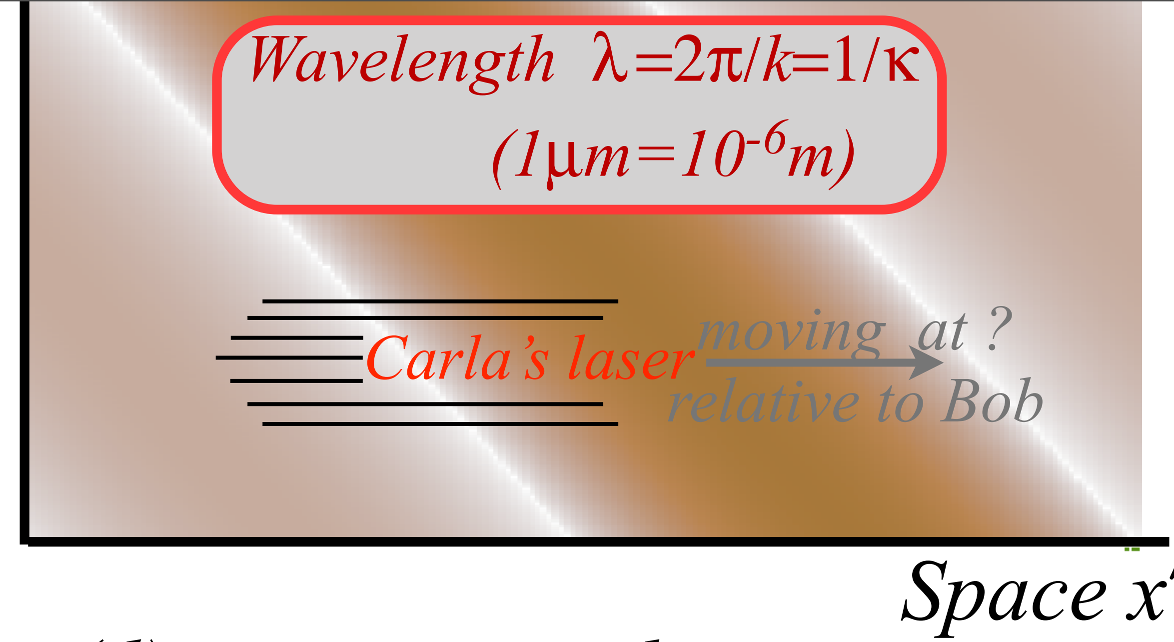
$$= (\psi_{phase})\cdot[\psi_{group}]$$



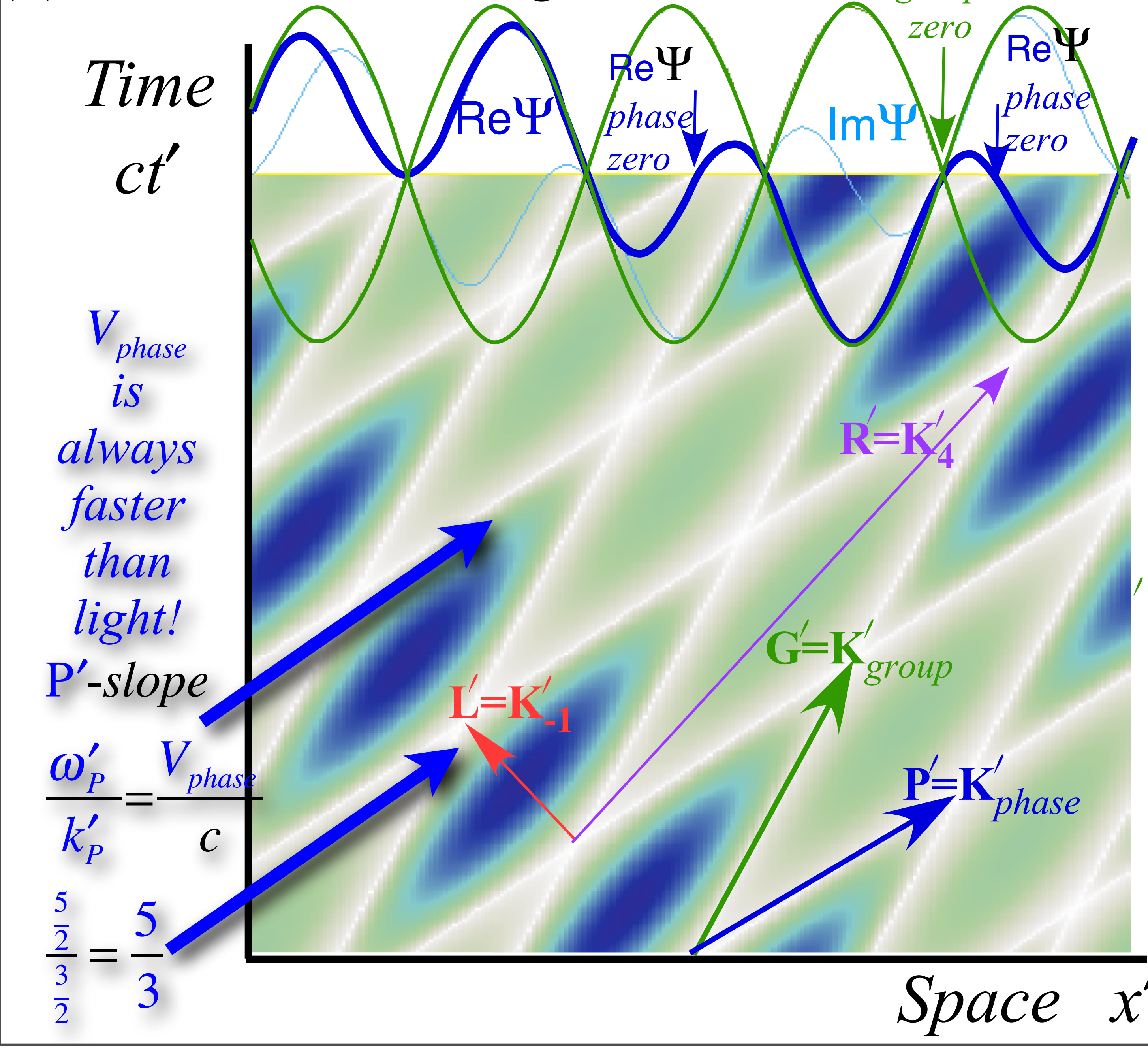
This determines space-time zero-path lattice that defines a Minkowski-coordinate grid as seen by Bob.

This gives 1/2-sum Phase **P′**=(**R′**+**L′**)/2 vector and a 1/2-difference Group **G′**=(**R′**-**L′**)/2 vector in (frequency,wavevector) space (ω′, k′) for Bob.

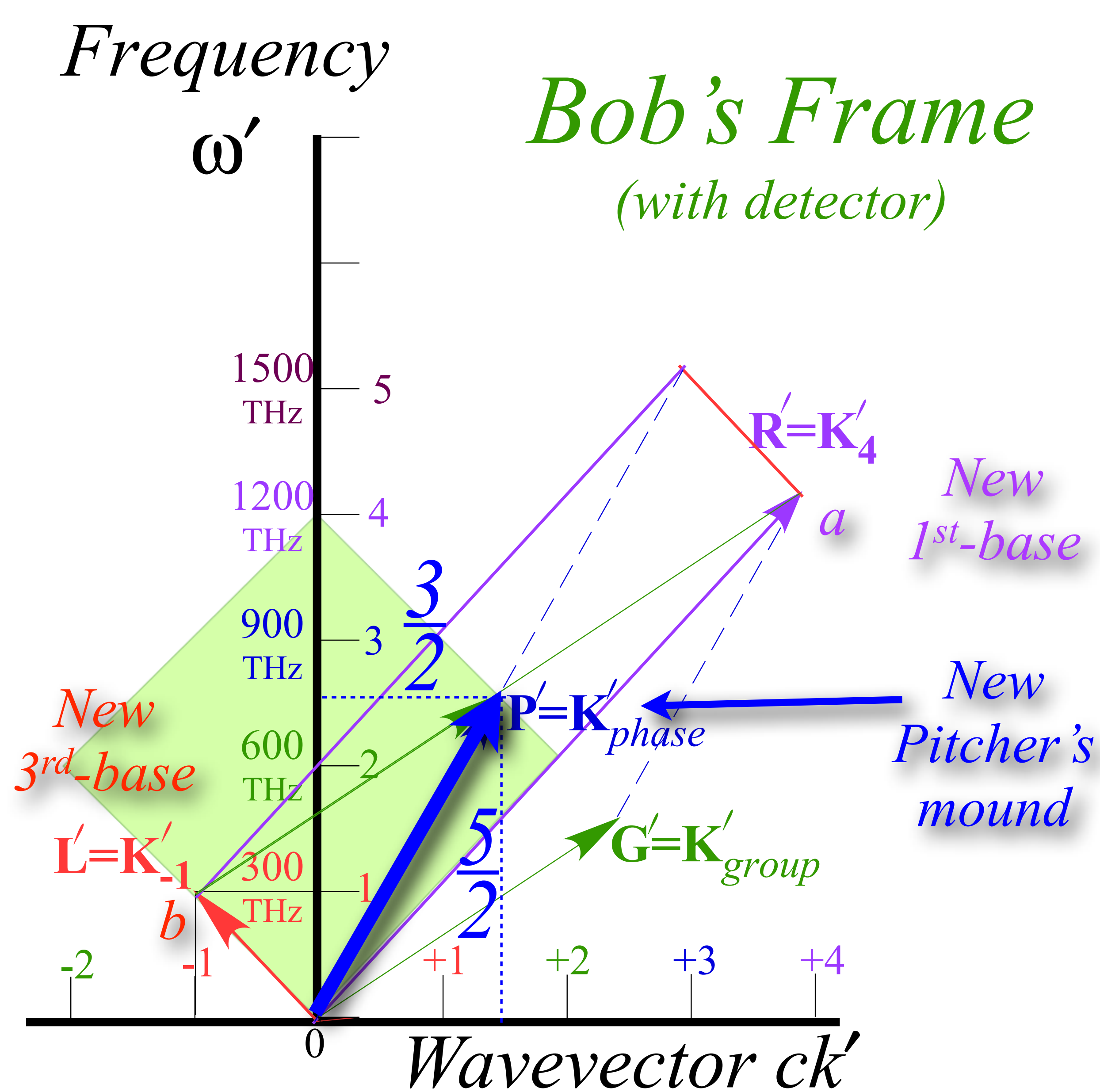


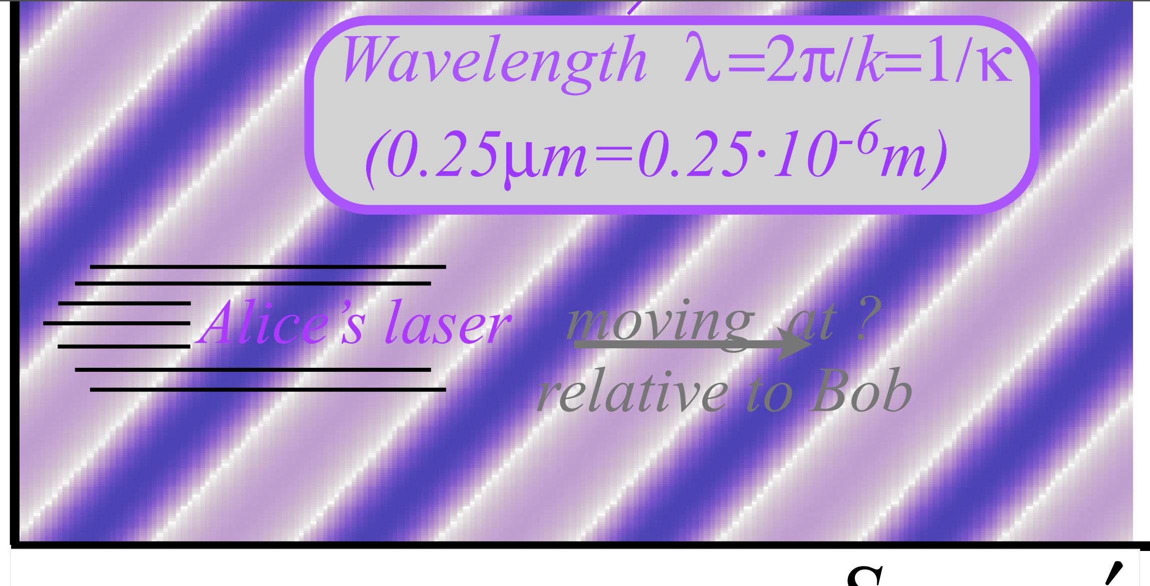


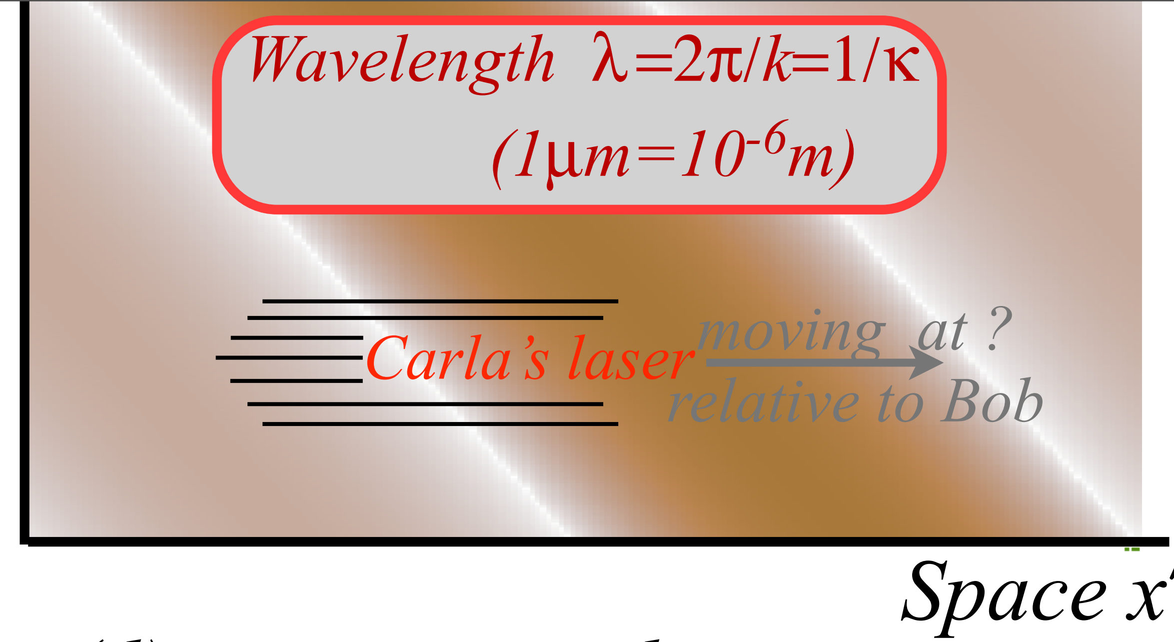
(c) Minkowski CW-grid



(d) Dispersion plot







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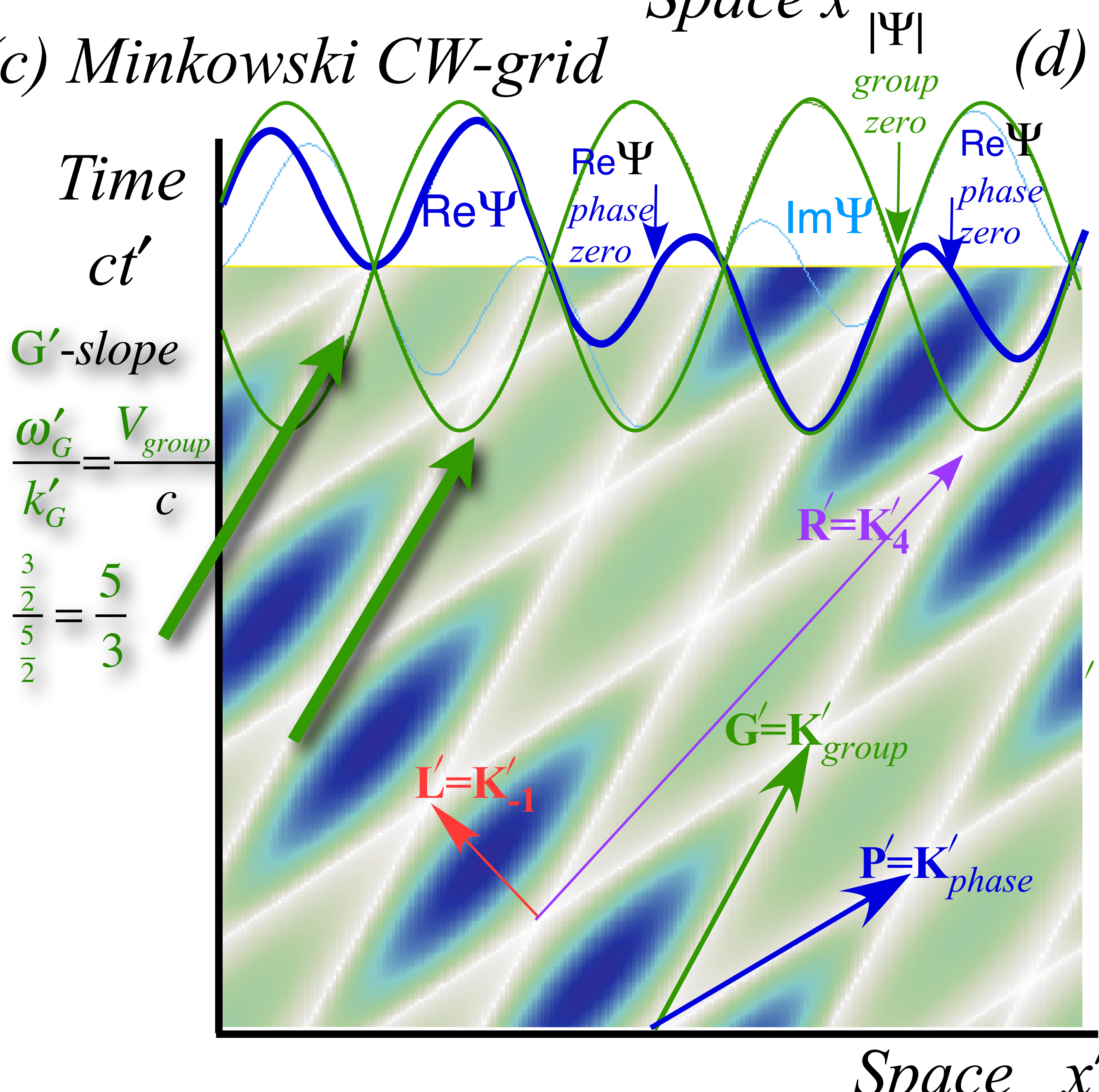
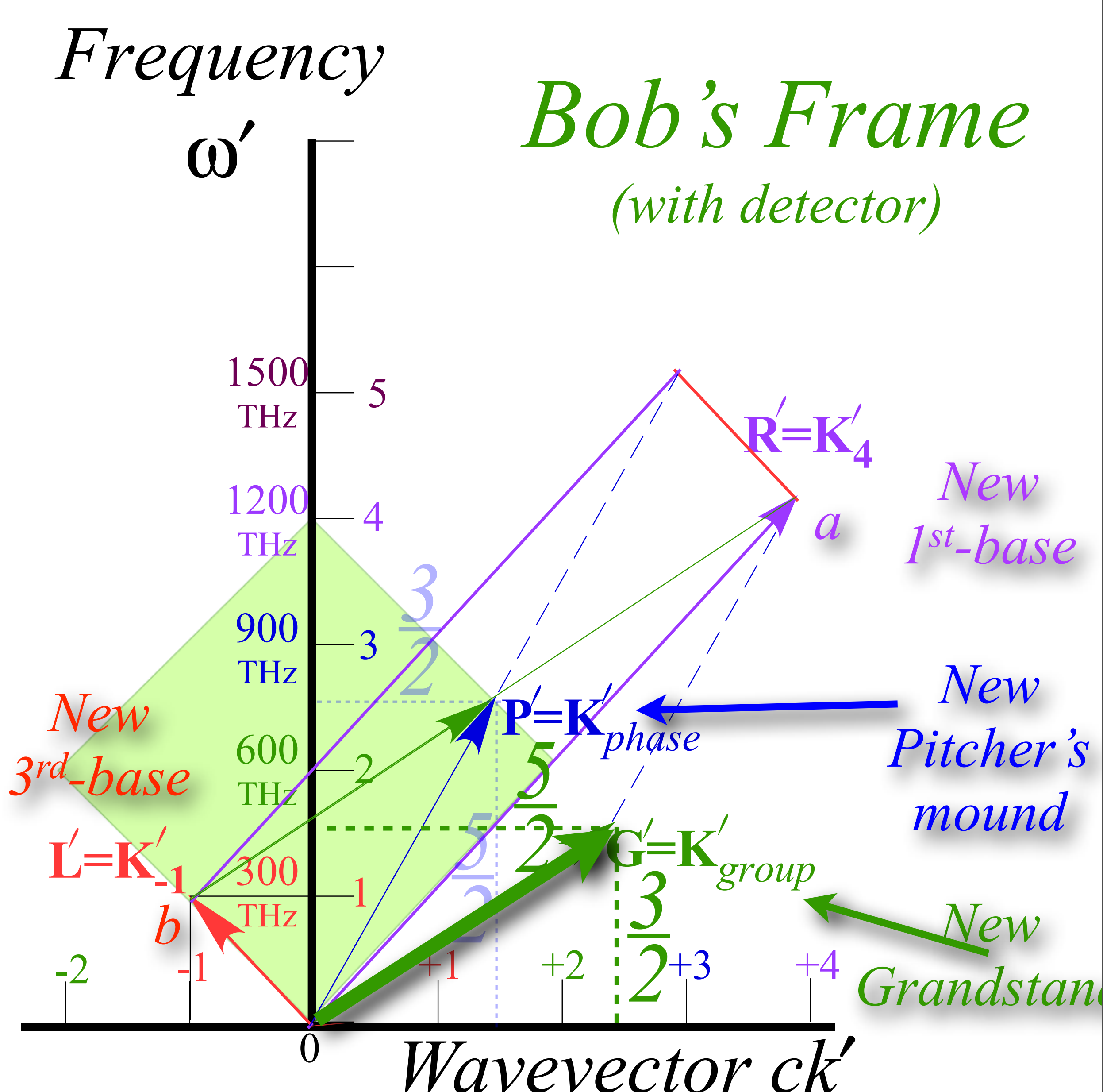
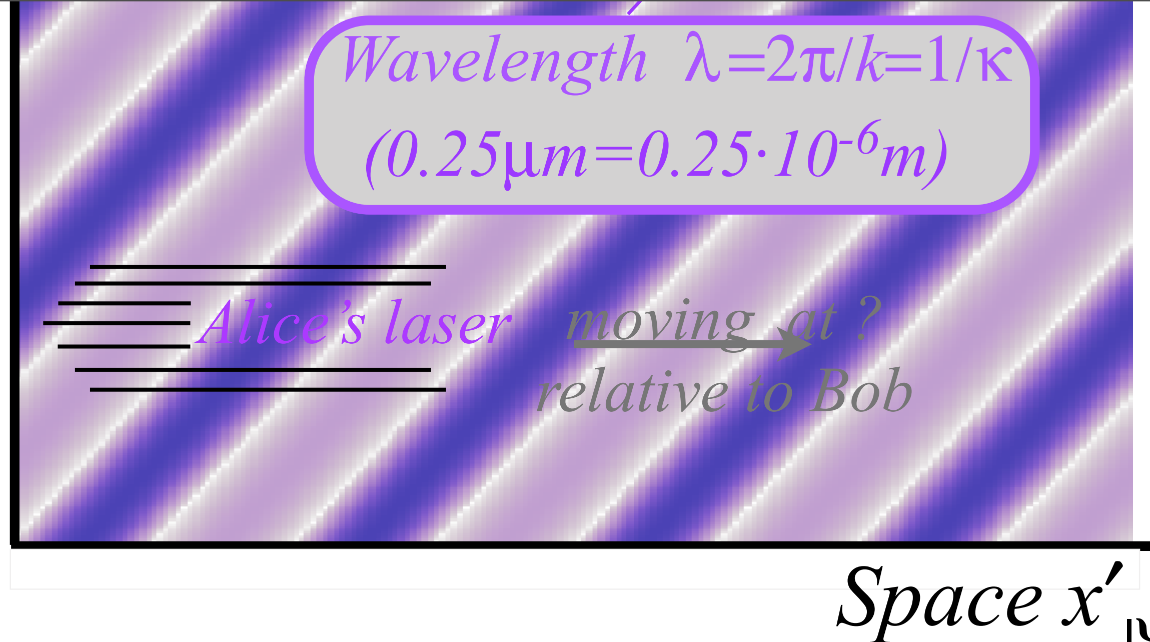
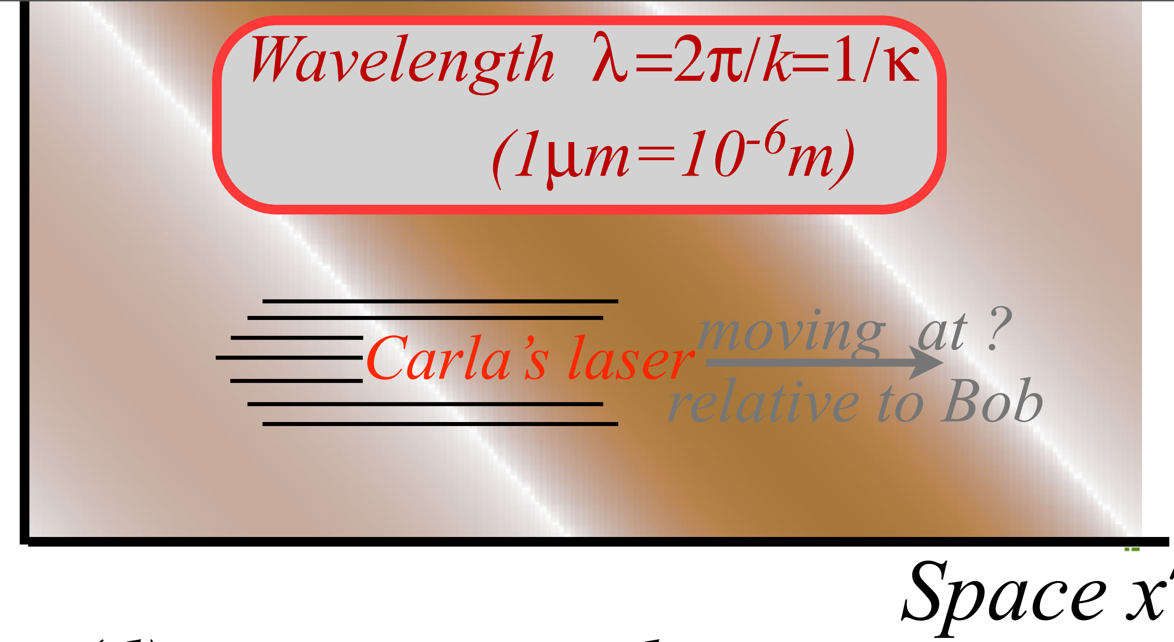


(d) Dispersion plot

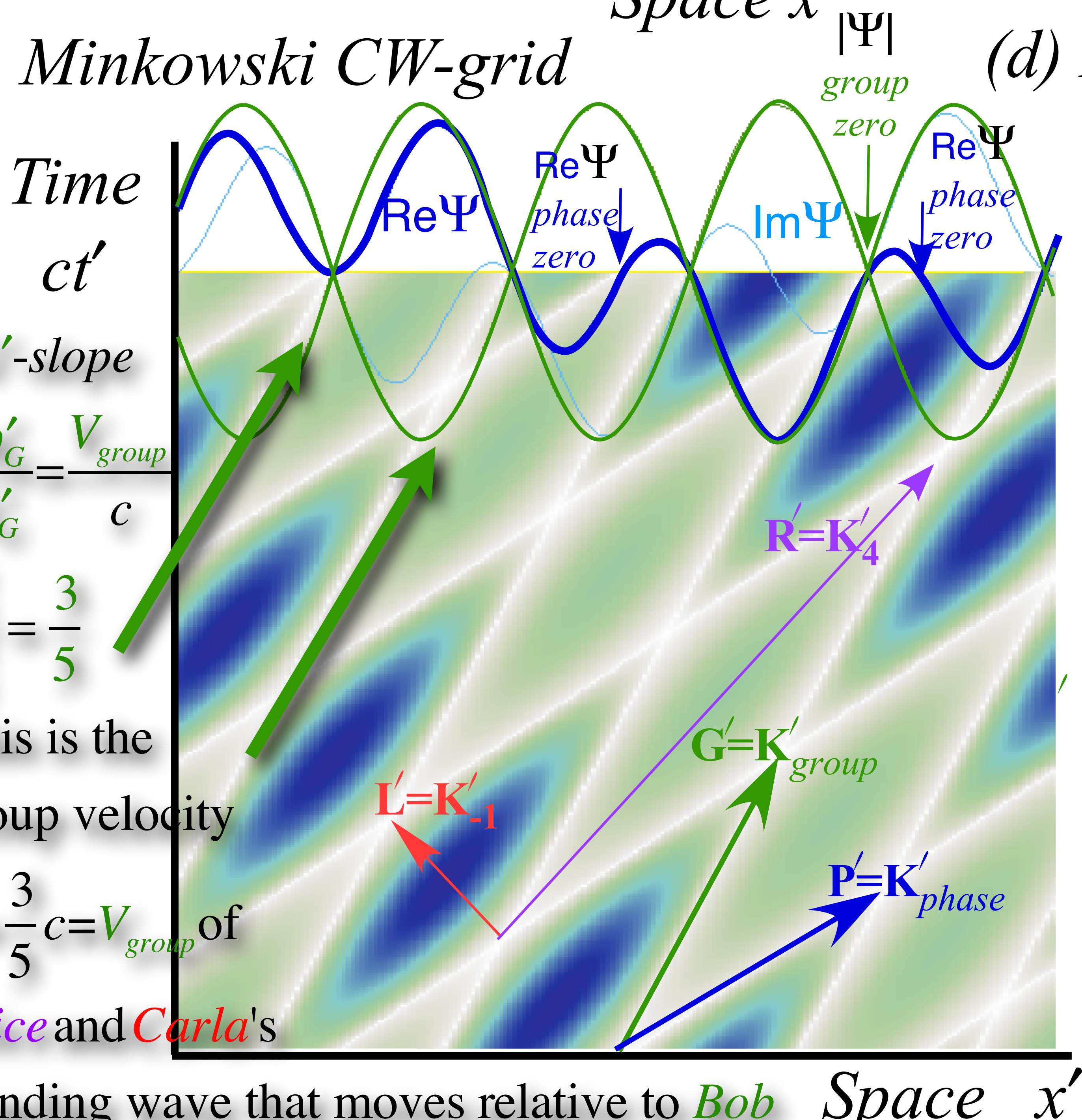






(c) Minkowski CW-grid

(d) Dispersion plot



G′-slope

$$\frac{\omega'_G}{k'_G} = \frac{V_{group}}{c}$$

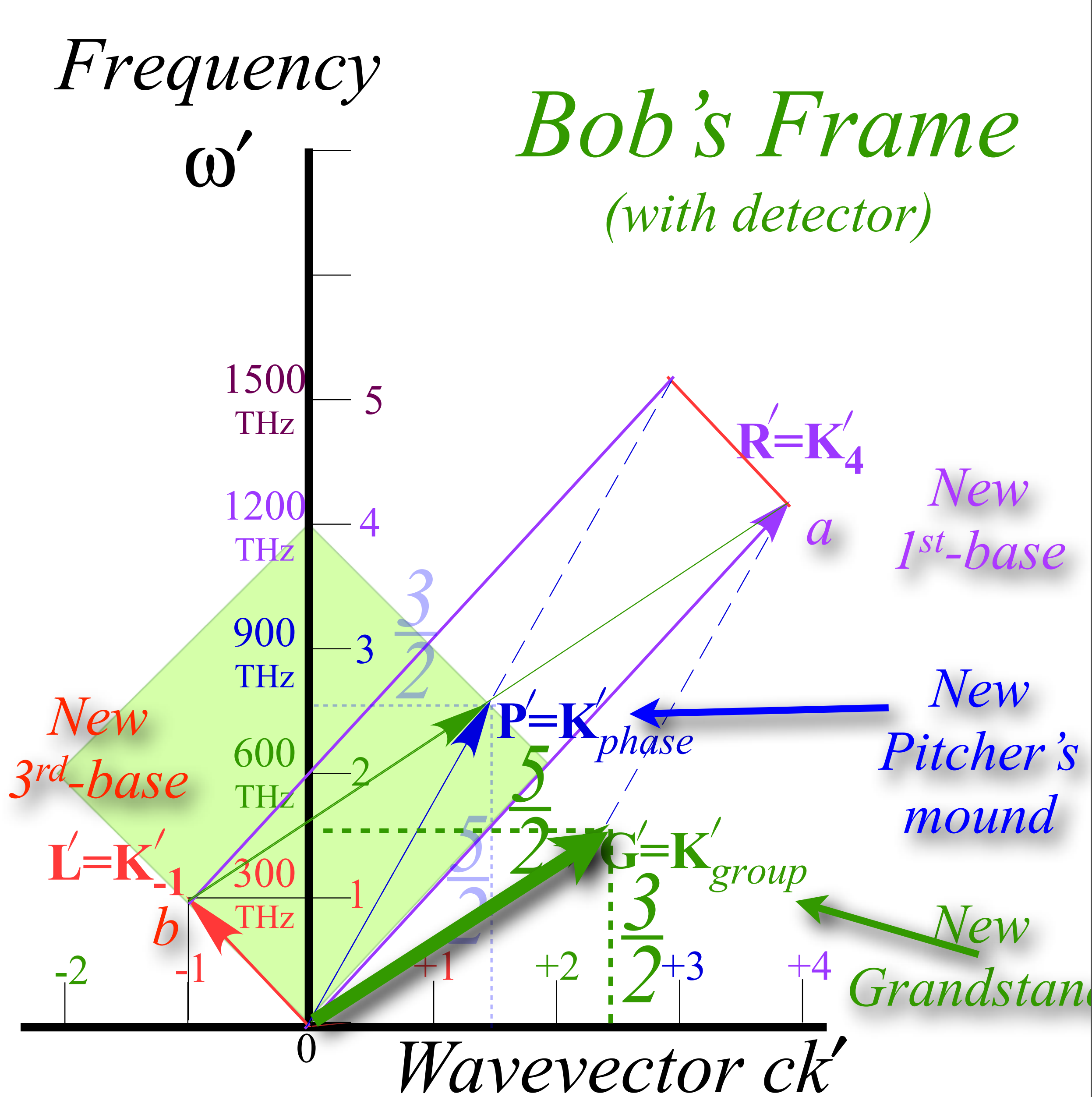
$$\frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5}$$

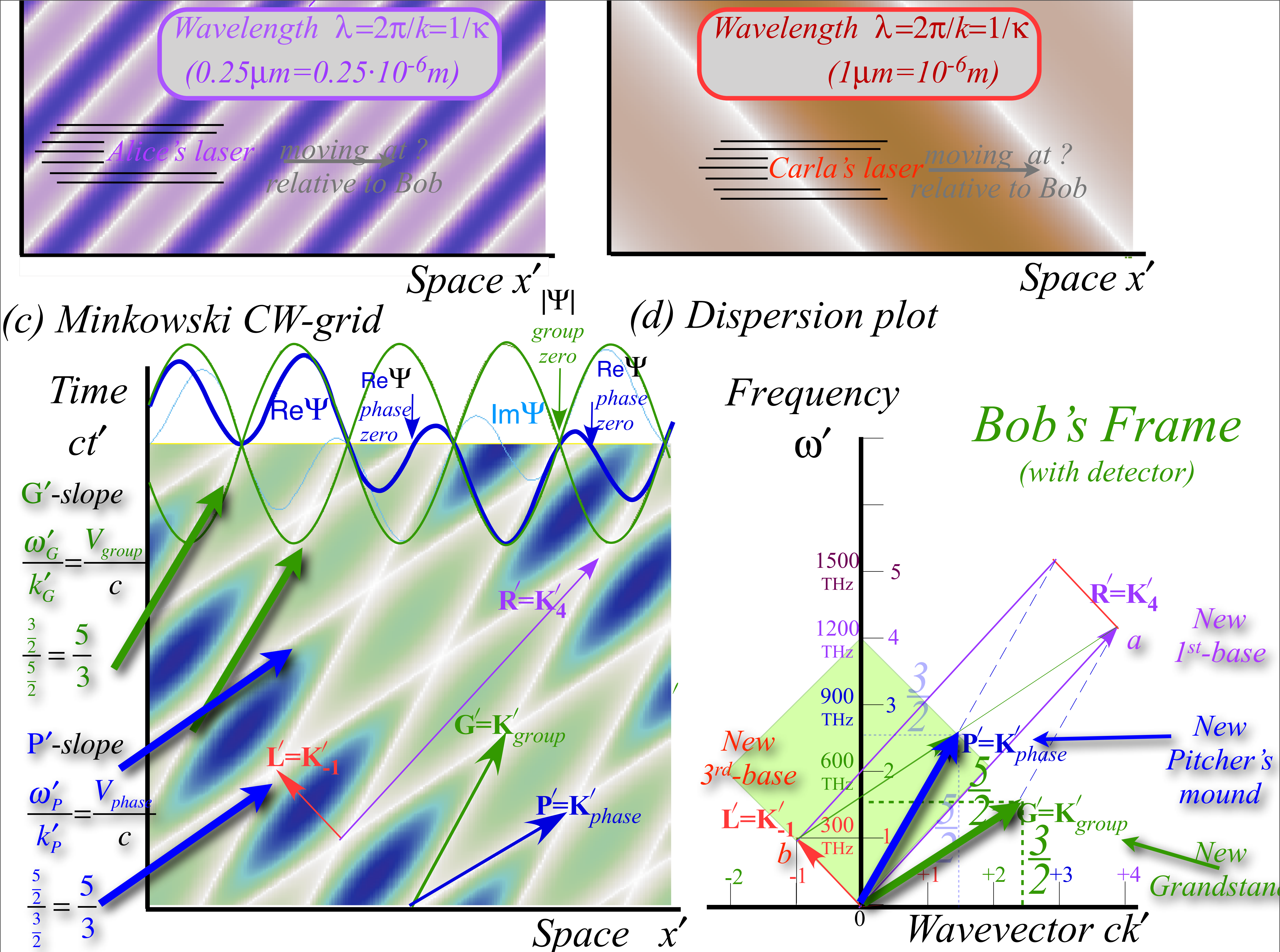
This is the group velocity

$u = \frac{3}{5}c = V_{group}$ of

Alice and *Carla*'s

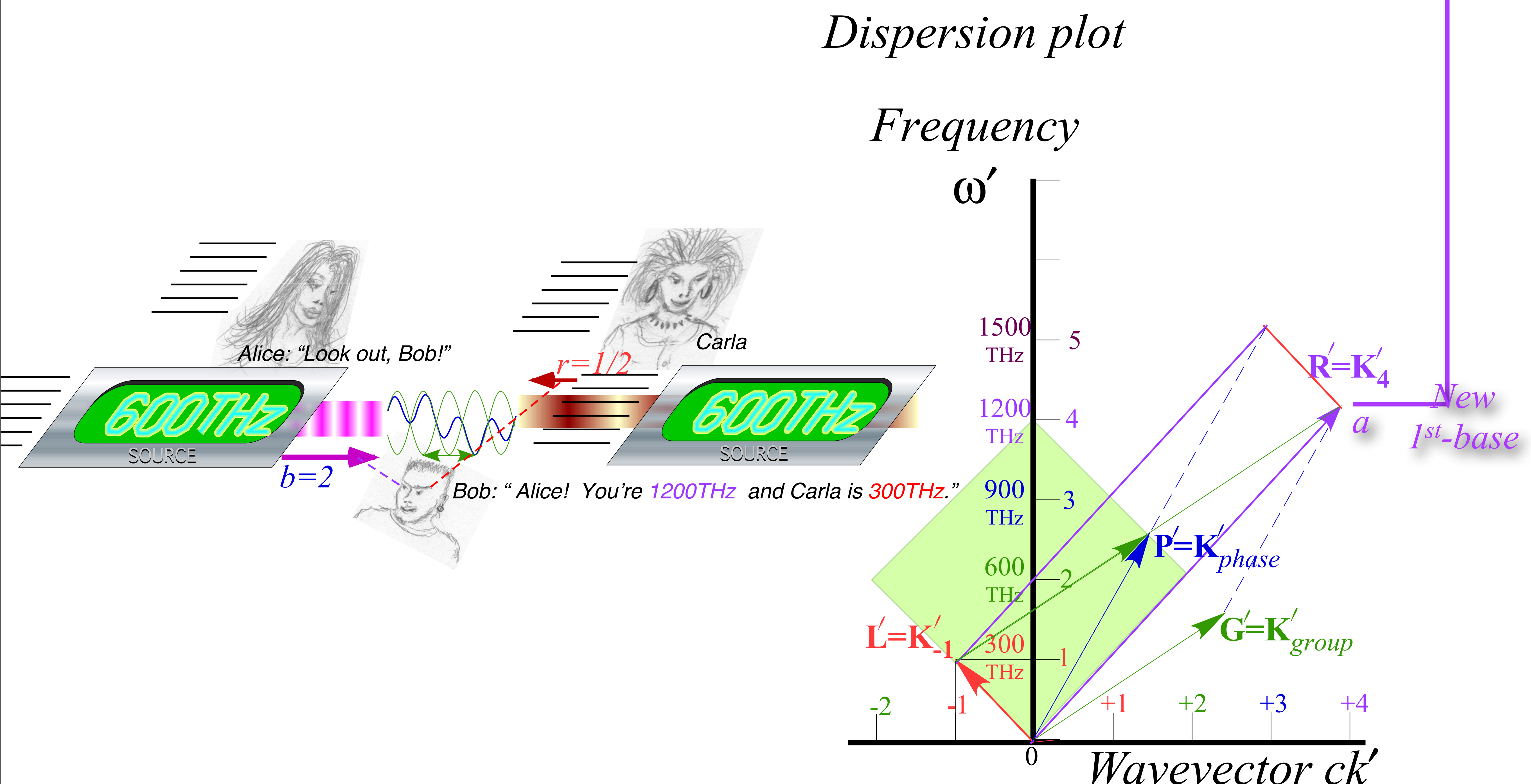
standing wave that moves relative to *Bob*





Wavelength λ=2π/k=1/κ
(0.25μm=0.25·10^-6m)
Alice's laser moving at ? relative to Bob
Space x′
Wavelength λ=2π/k=1/κ
(1μm=10^-6m)
Carla's laser moving at ? relative to Bob
Space x′
(c) Minkowski CW-grid
|Ψ|
group zero
ReΨ
ReΨ phase zero
ImΨ
ReΨ phase zero
Time ct′
G′-slope
ω′G/k′G = Vgroup/c
(3/2)/(5/2) = 5/3
P′-slope
ω′P/k′P = Vphase/c
(5/2)/(3/2) = 5/3
R=K′4
G′=K′group
L′=K′-1
P′=K′phase
Space x′
(d) Dispersion plot
Frequency ω′
Bob's Frame
(with detector)
1500 THz
1200 THz
900 THz
600 THz
300 THz
5
4
3
2
1
R′=K′4
a
New 1st-base
3/2
P′=K′phase
New Pitcher's mound
New 3rd-base
L′=K′-1
b
5/2
G′=K′group
3/2
New Grandstand
-2
-1
0
+1
+2
+3
+4
Wavevector ck′

According to Bob, Alice's right-moving *600THz* laser beam is blue-shifted by $b_{BA}=e^{\rho}=2$ as she approaches him. So vector $\mathbf{R}'$ that Bob ascribes to Alice is her vector $\mathbf{R}$ doubled in length to $\mathbf{R}'=\omega_A b_{BA}(1,+1)$. (It must stay on the 1st-baseline to obey Evenson's axiom.)



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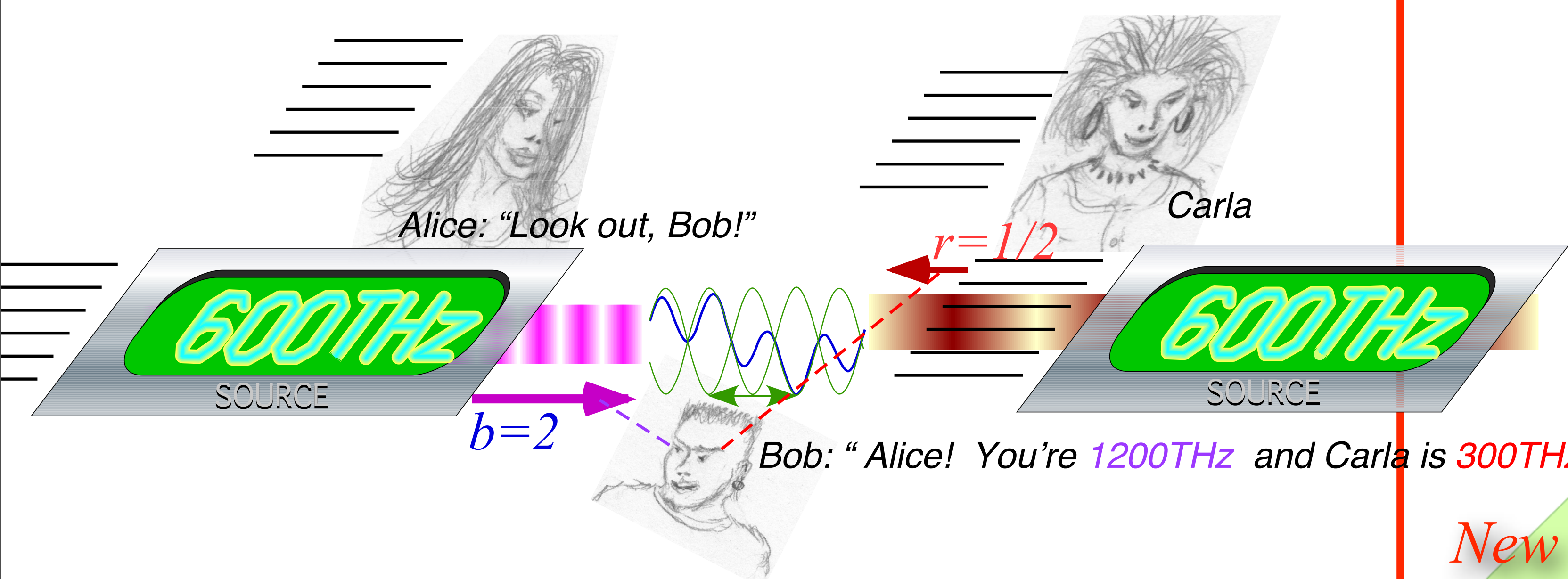
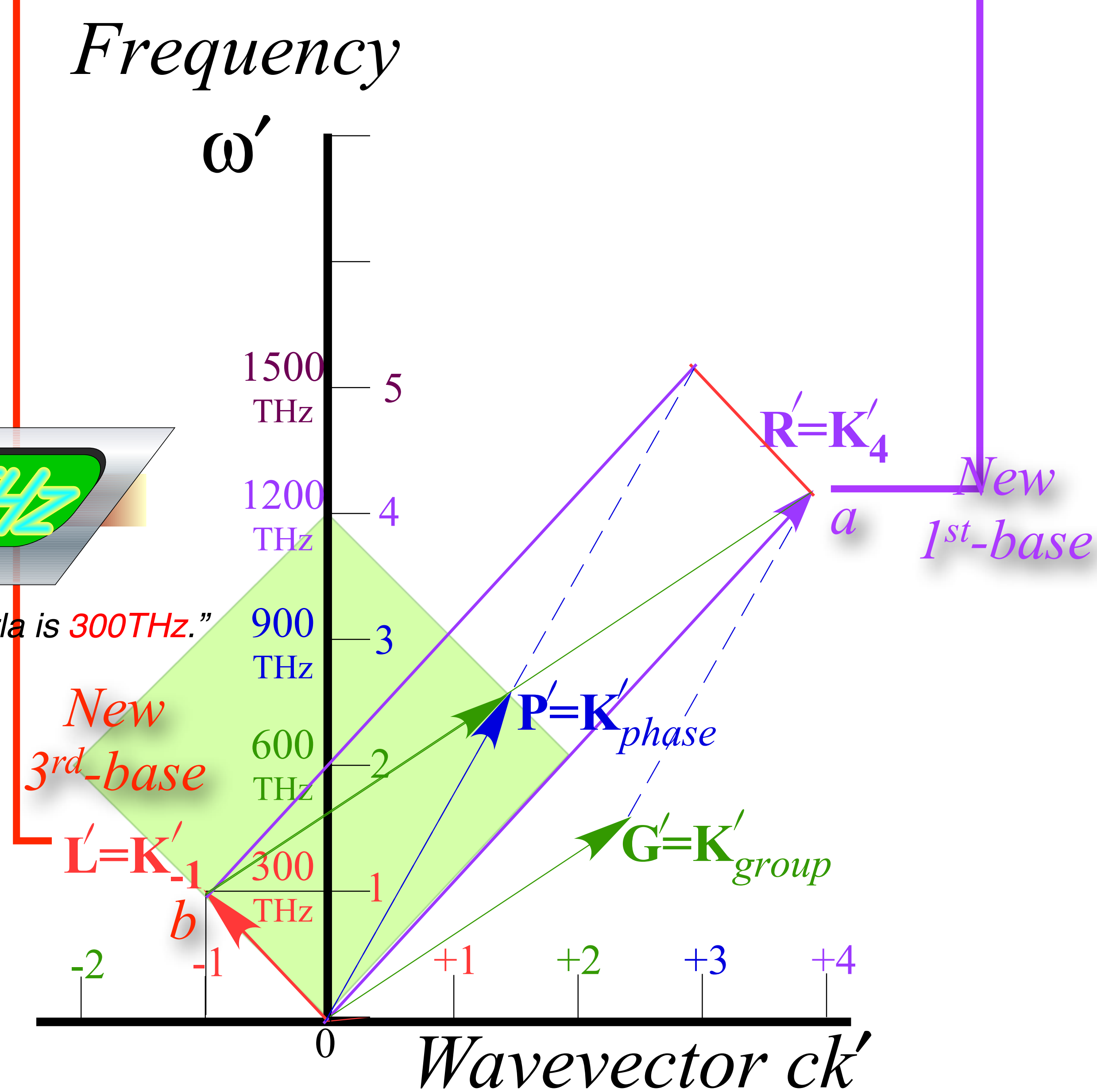
Meanwhile, Bob sees Carla's left-moving *600THz* laser beam red-shifted by $b_{BC}=e^{-\rho}=\frac{1}{2}$ as she recedes and her vector $\mathbf{L}$ halved in length to $\mathbf{L}'=\omega_A b_{BC}(1,-1)$ along the 3rd baseline.





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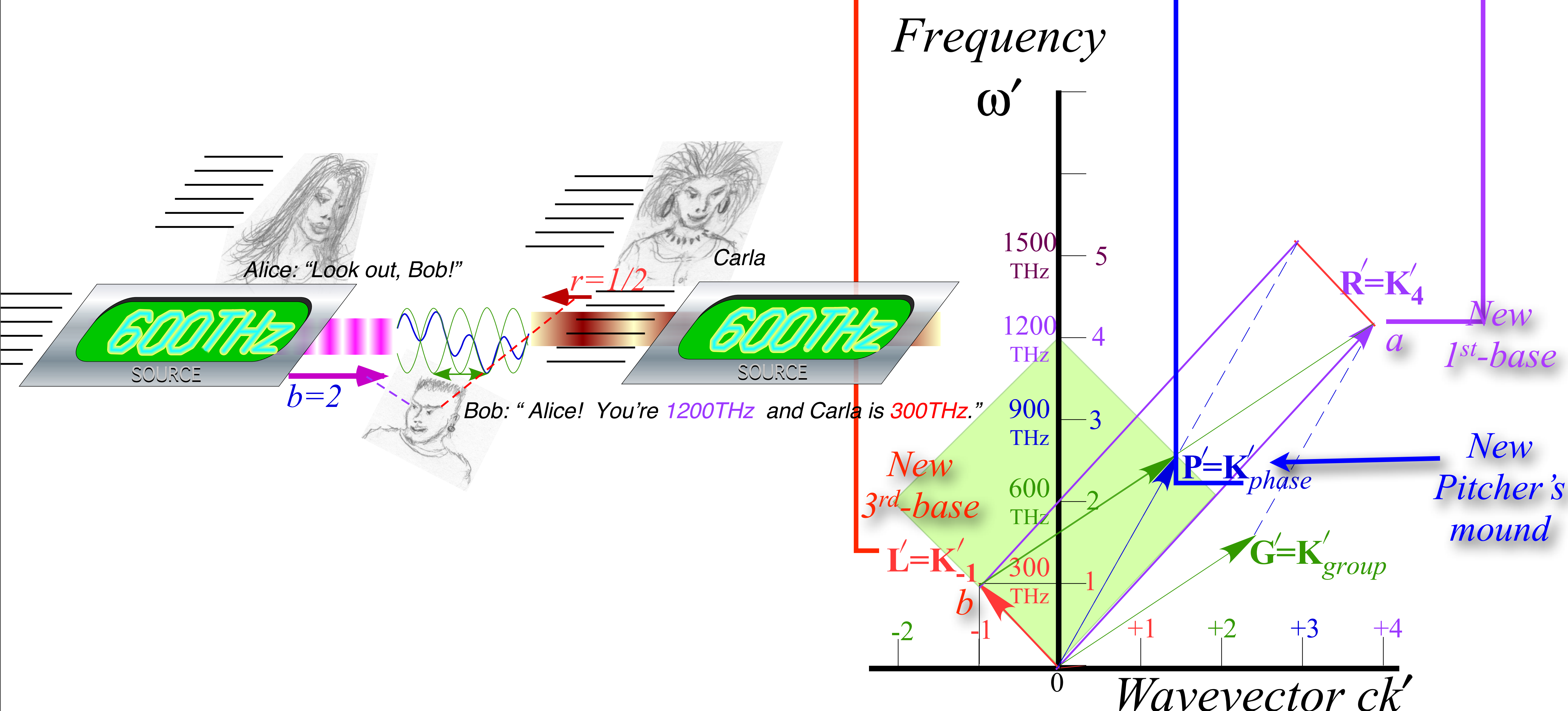
Bob's phase $\mathbf{P}'=(\mathbf{R}'+\mathbf{L}')/2$ and group $\mathbf{G}'=(\mathbf{R}'-\mathbf{L}')/2$ vectors define his warped view of Alice's baseball diamond



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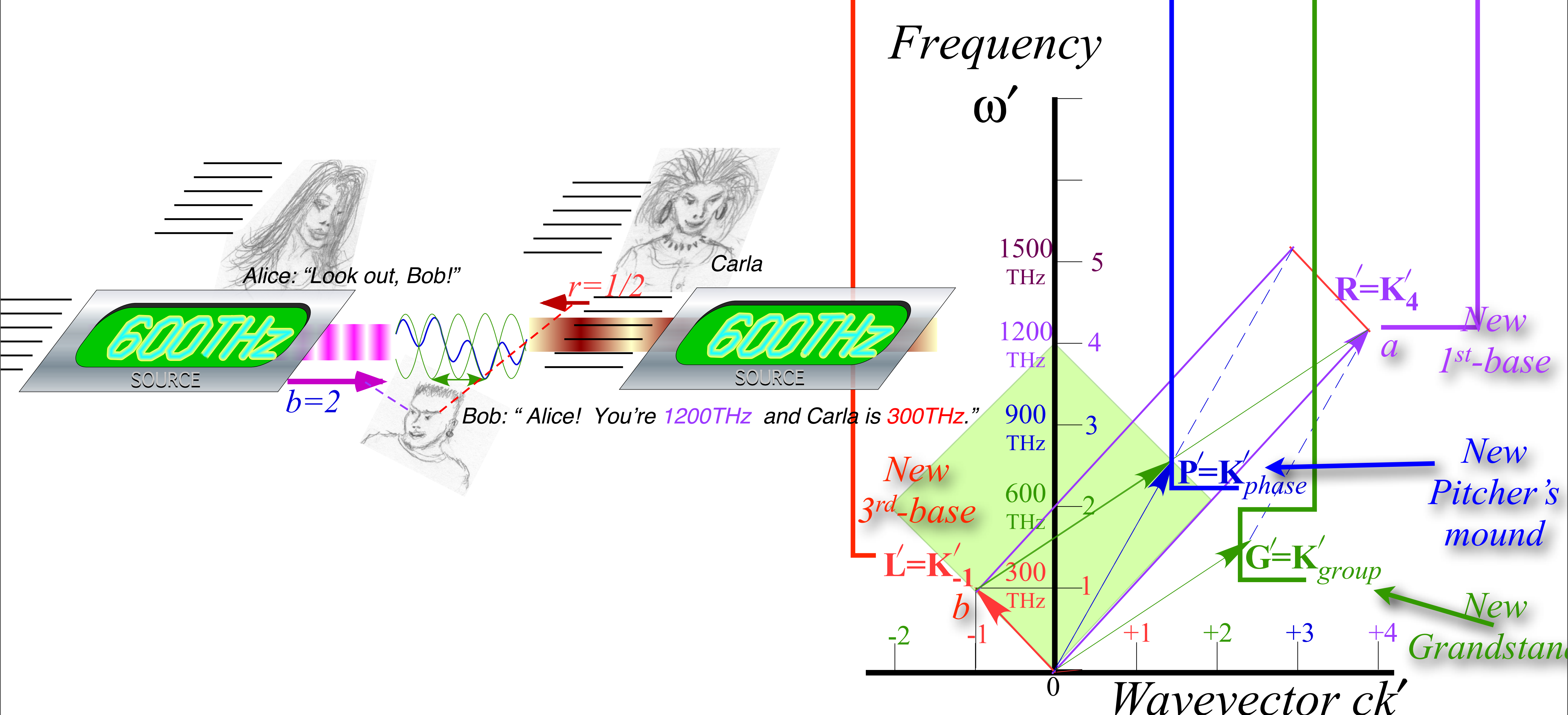

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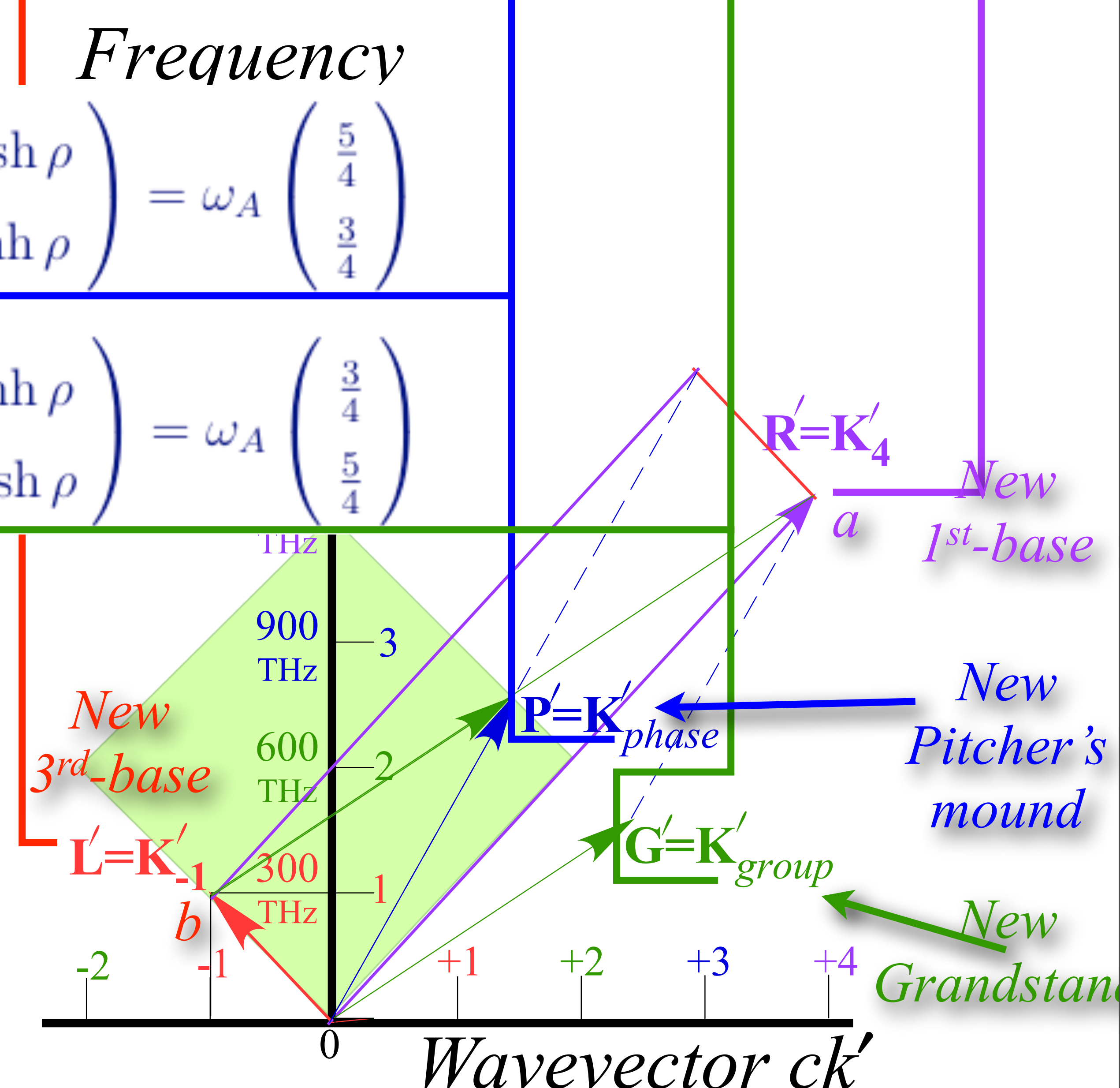
Bob's phase $\mathbf{P}'=(\mathbf{R}'+\mathbf{L}')/2$ and group $\mathbf{G}'=(\mathbf{R}'-\mathbf{L}')/2$ vectors define his warped view of Alice's baseball diamond

$$\begin{pmatrix}\omega'_{phase}\\ ck'_{phase}\end{pmatrix}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}=\omega_A\begin{pmatrix}\frac{e^{\rho}+e^{-\rho}}{2}\\ \frac{e^{\rho}-e^{-\rho}}{2}\end{pmatrix}=\omega_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\omega_A\begin{pmatrix}\frac{5}{4}\\ \frac{3}{4}\end{pmatrix}$$

$$\begin{pmatrix}\omega'_{group}\\ ck'_{group}\end{pmatrix}=\mathbf{G}'=\frac{\mathbf{R}'-\mathbf{L}'}{2}=\omega_A\begin{pmatrix}\frac{e^{\rho}-e^{-\rho}}{2}\\ \frac{e^{\rho}+e^{-\rho}}{2}\end{pmatrix}=\omega_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}=\omega_A\begin{pmatrix}\frac{3}{4}\\ \frac{5}{4}\end{pmatrix}$$

This derives Einstein-Lorentz transformation matrix

$$\begin{pmatrix}\cosh\rho & \sinh\rho\\ \sinh\rho & \cosh\rho\end{pmatrix}$$



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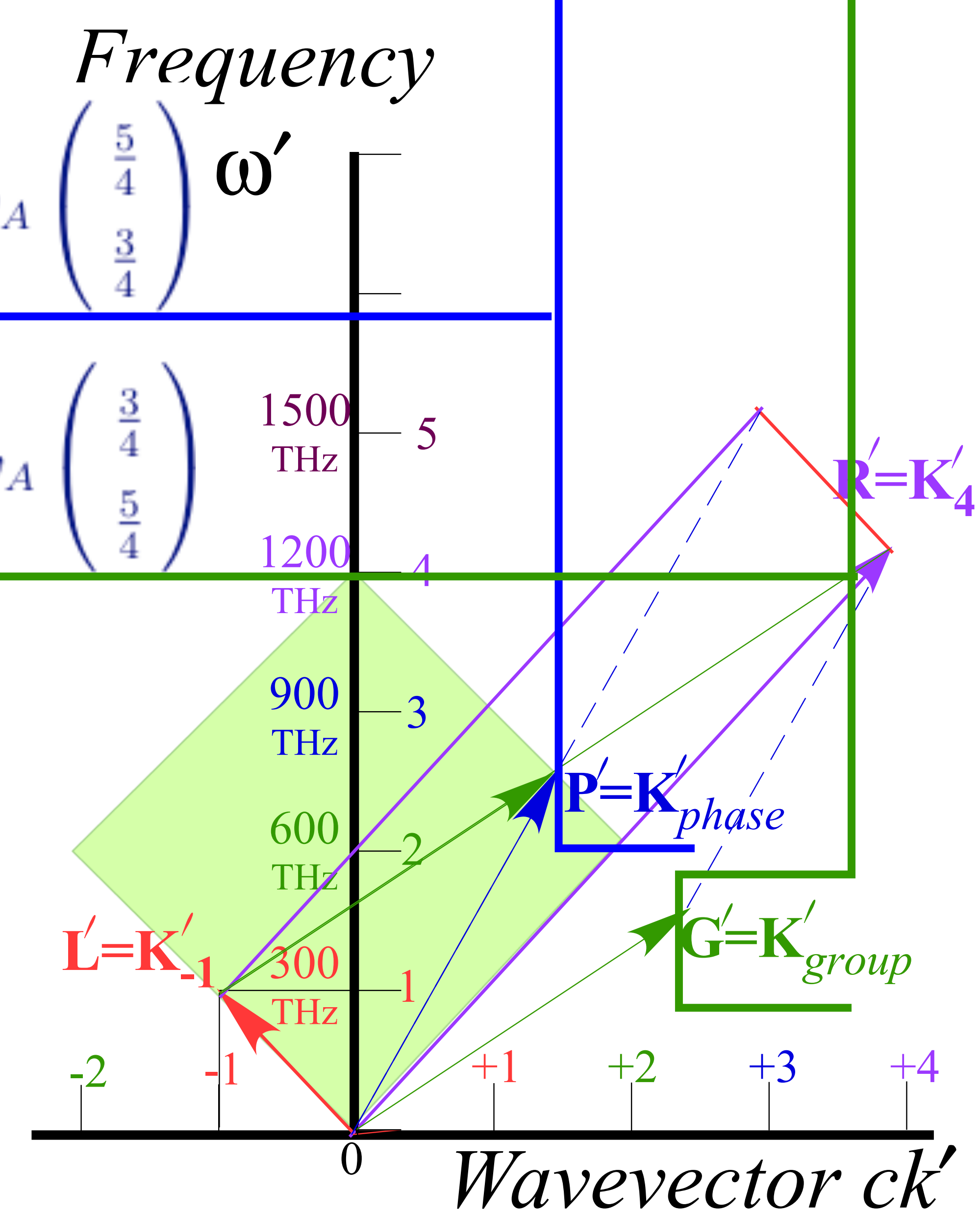

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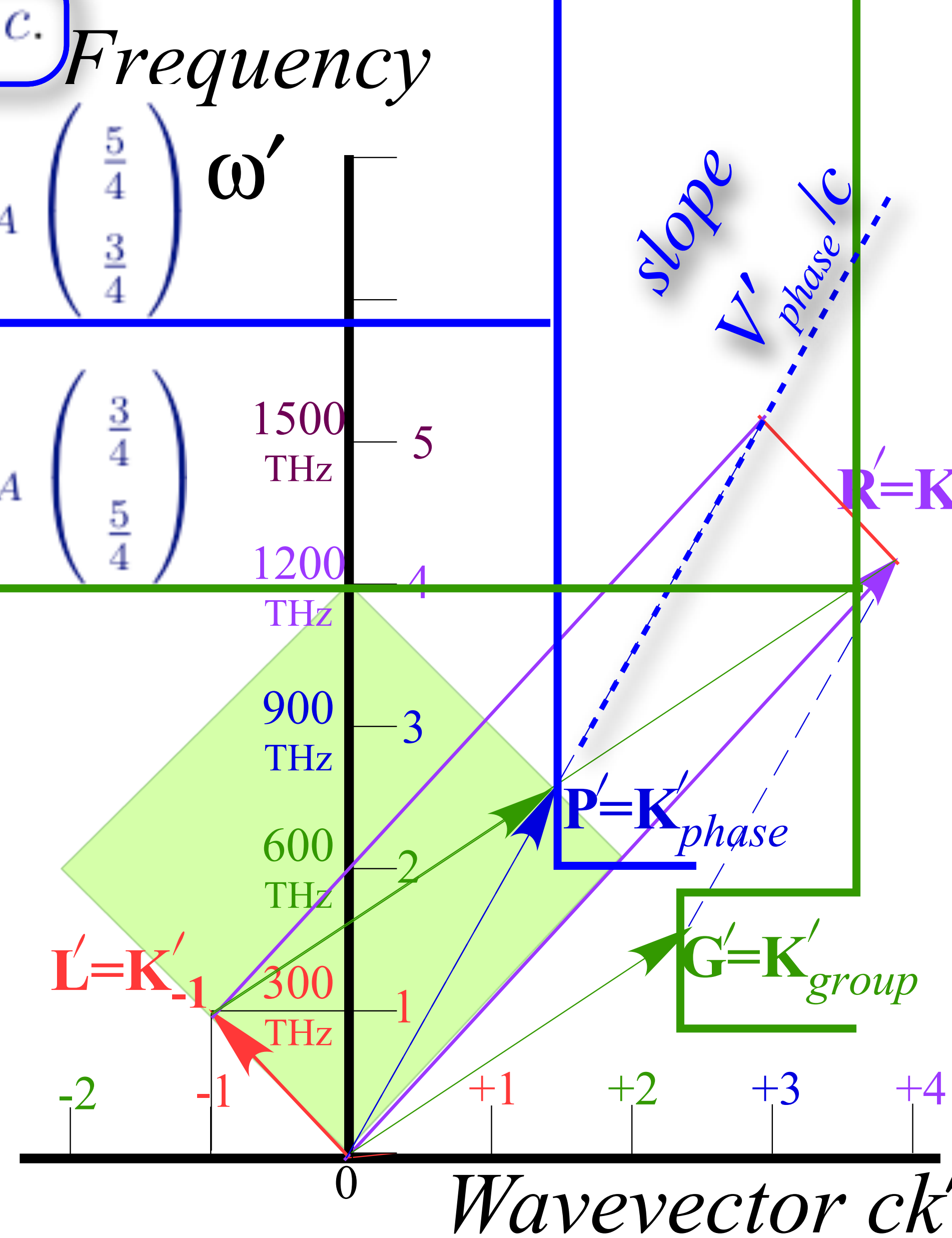

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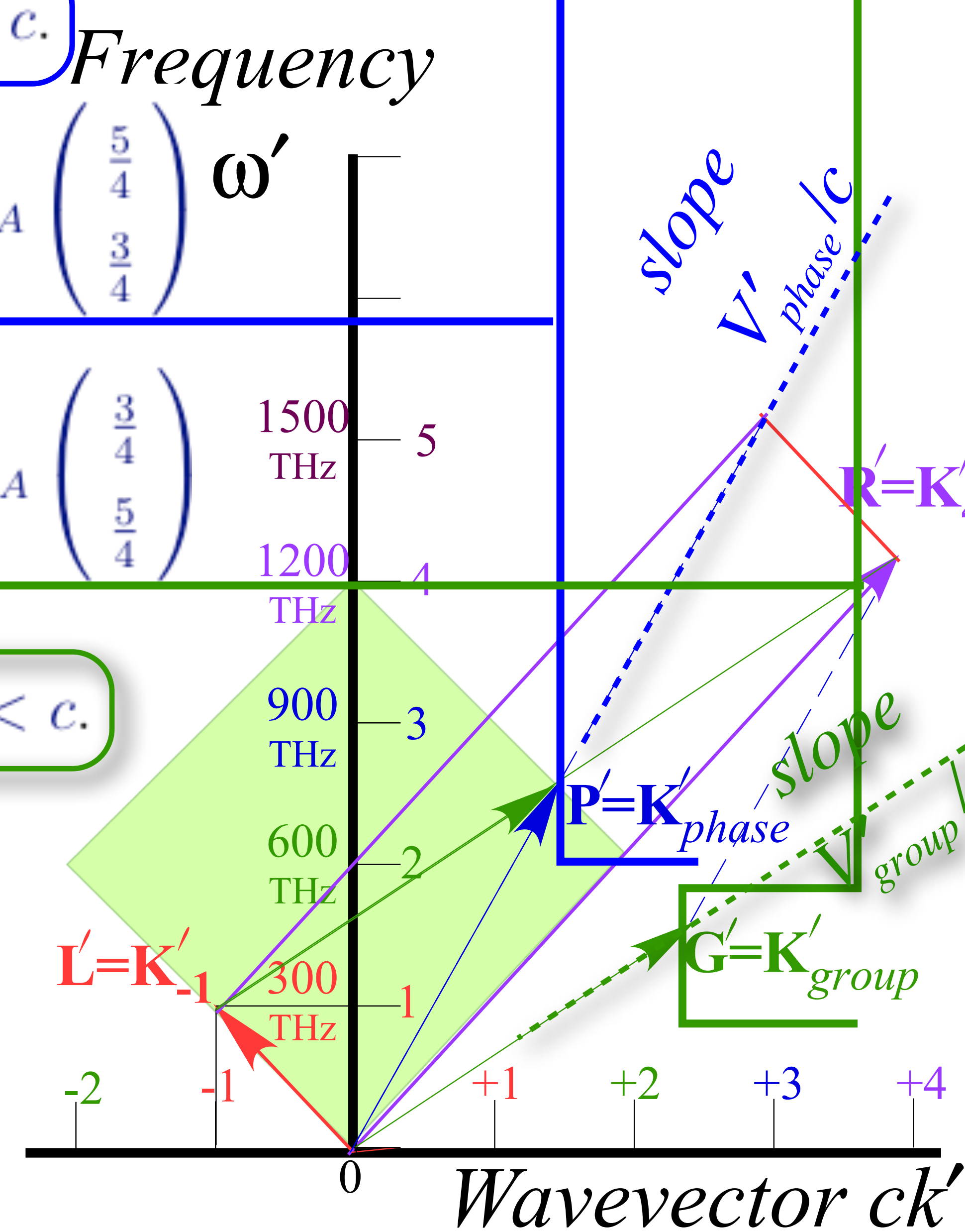

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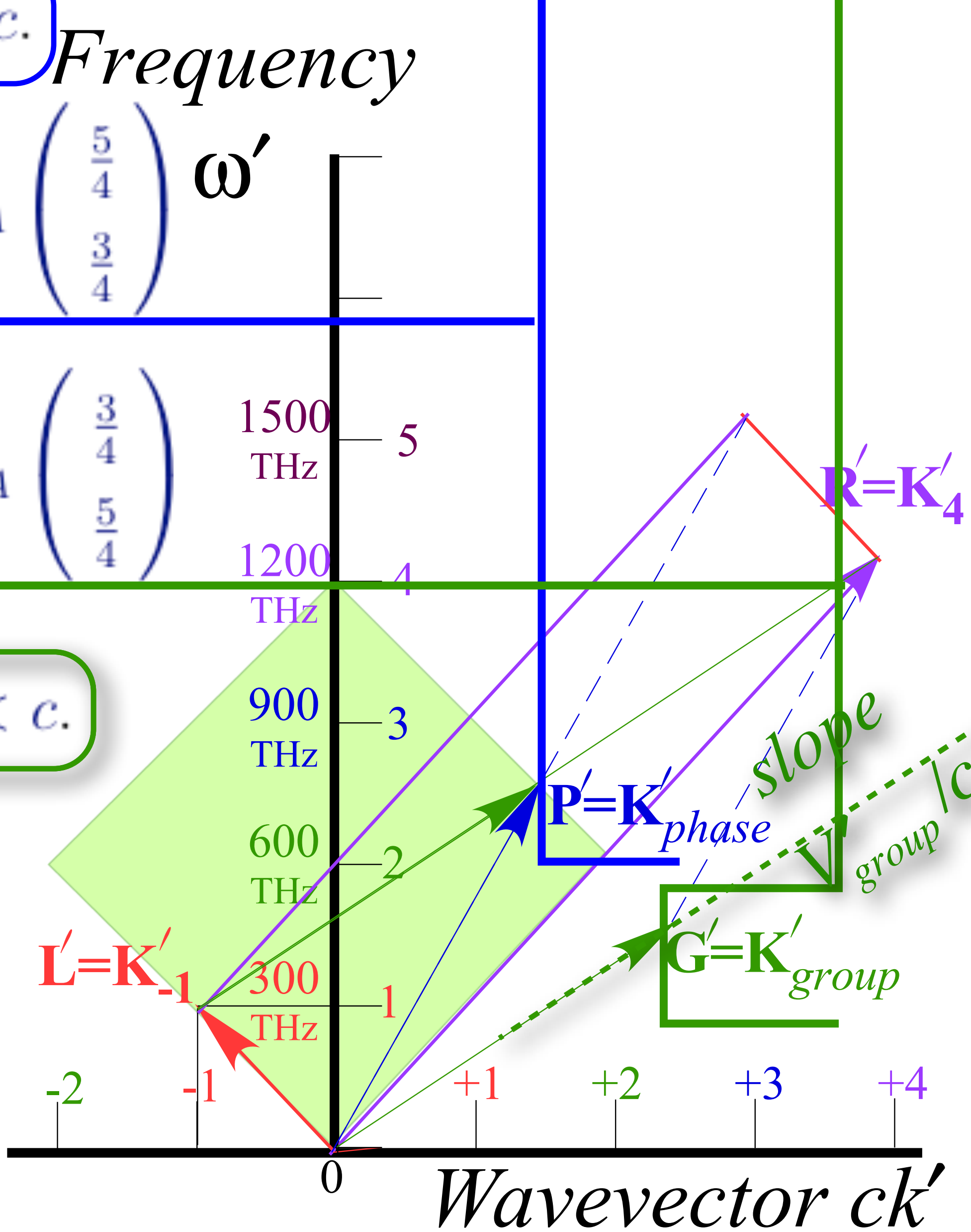
Alice and Carla see a *600THz* standing wave between them.



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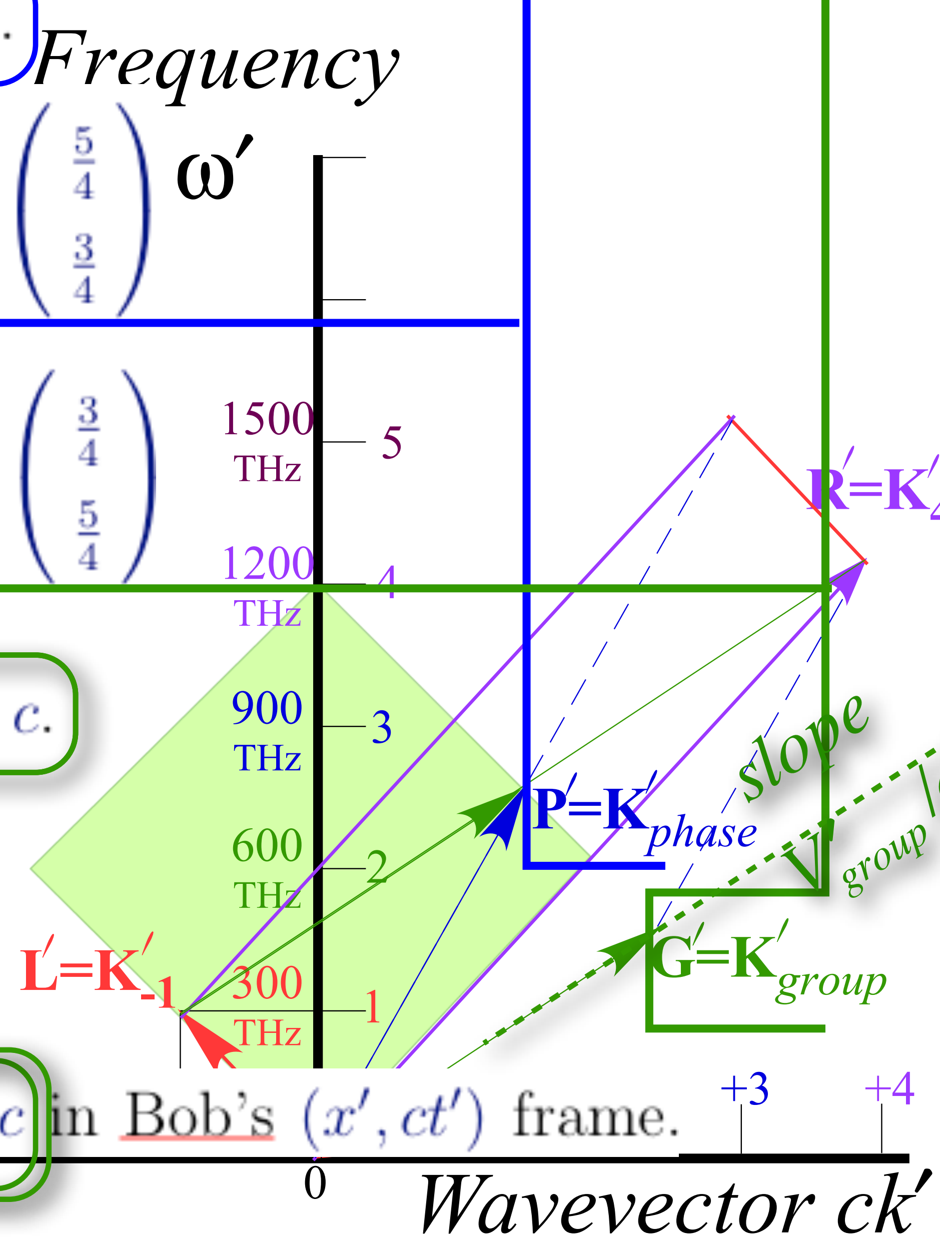
Alice and Carla see a *600THz* standing wave between them.

So velocity u of Alice, Carla, and standing wave is $V'_{group}=u=\frac{3}{5}c$ in Bob's (x', ct') frame.



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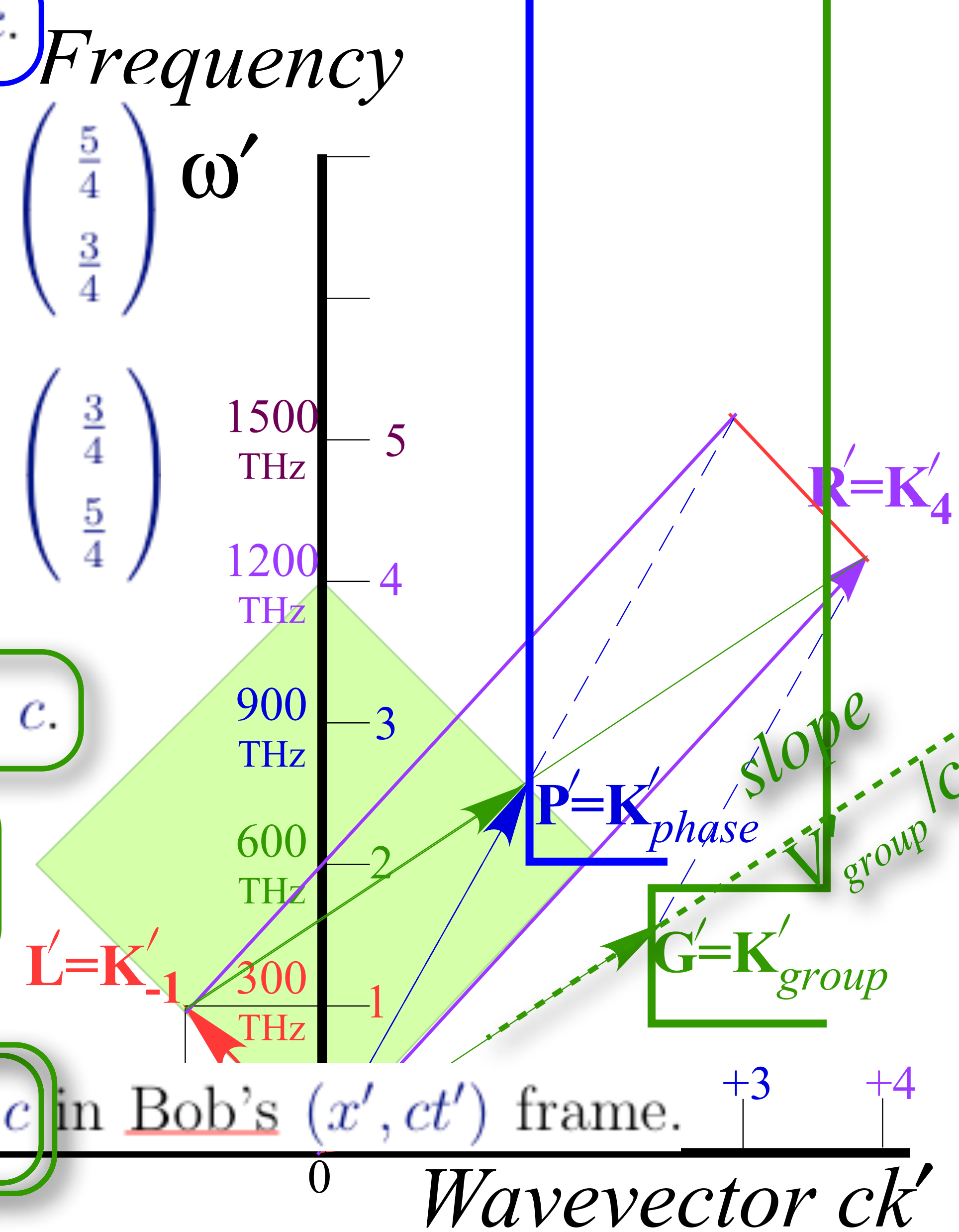

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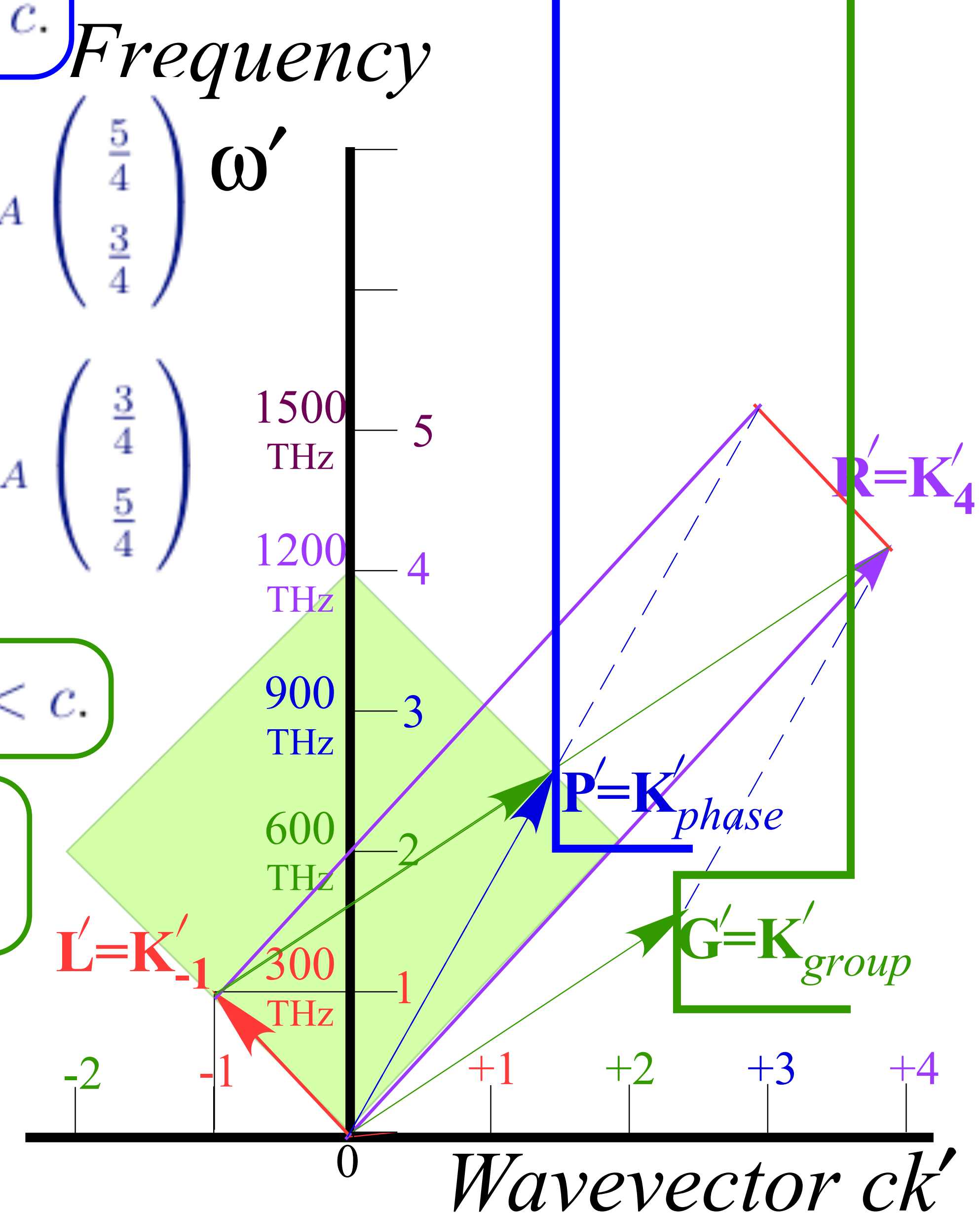
Inverse Doppler-blue

$$b=\sqrt{\frac{1+\beta}{1-\beta}}=\sqrt{\frac{1+u/c}{1-u/c}}$$



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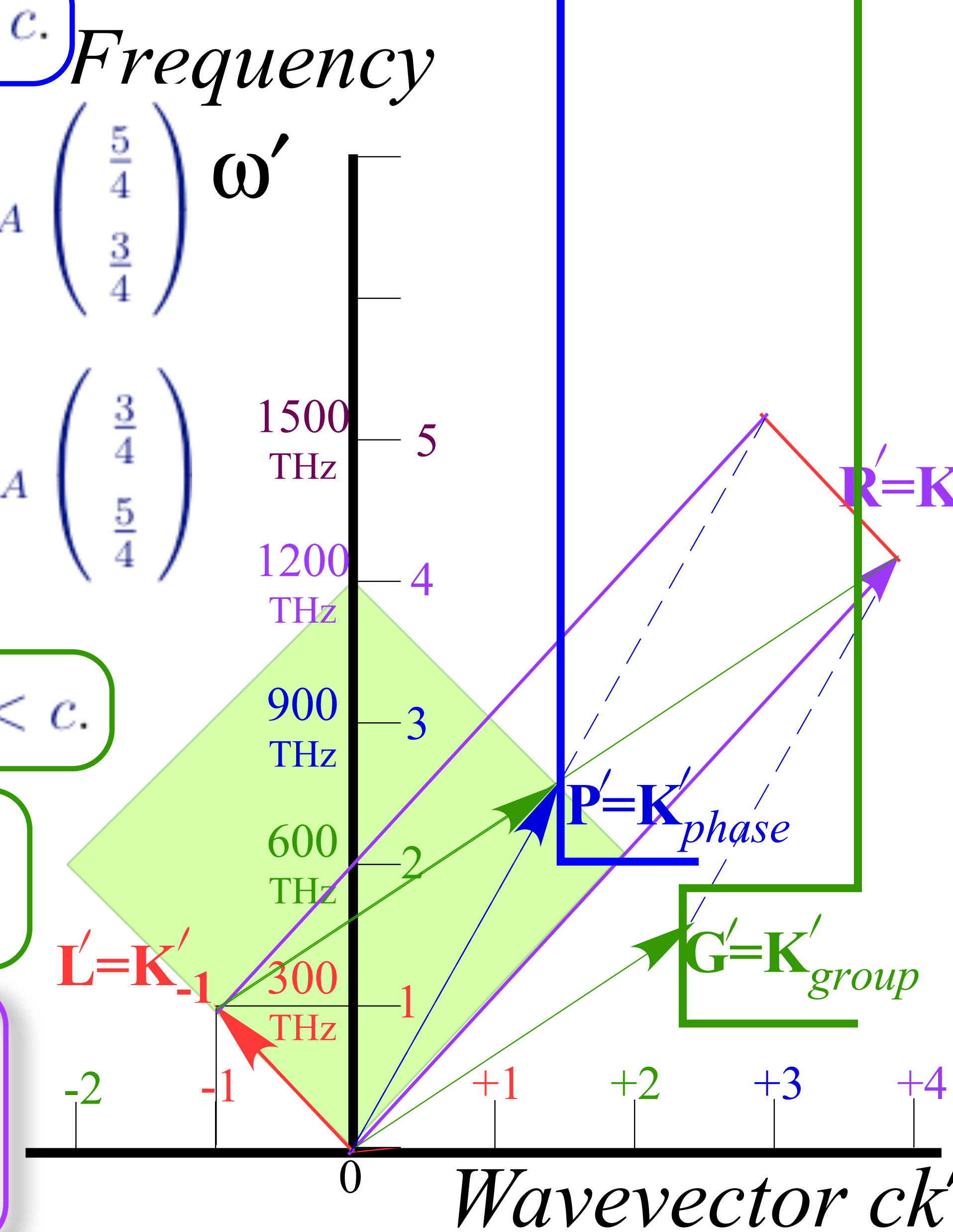
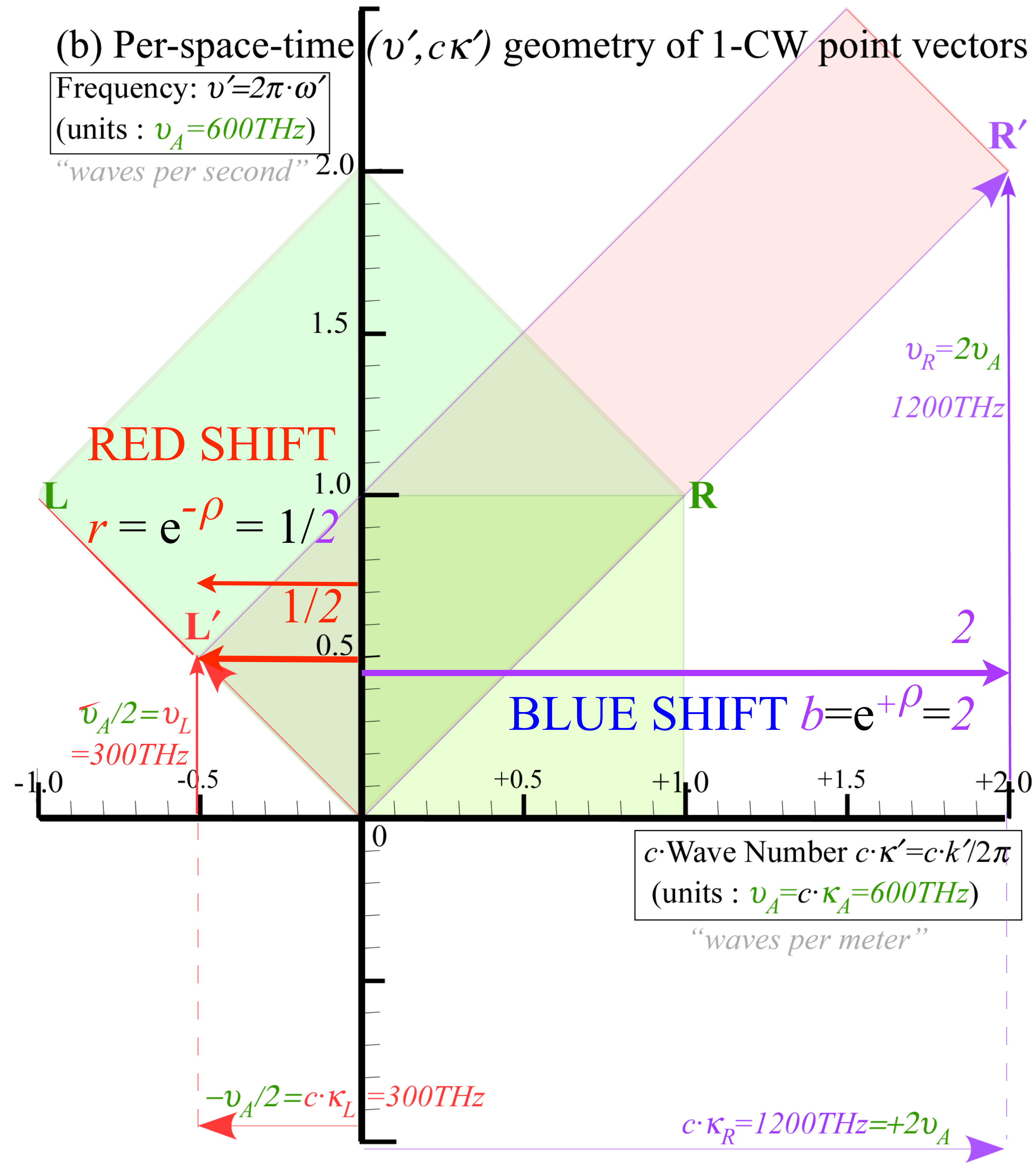
Inverse Doppler-blue includes *Lorentz coefficient* $\lambda=\sqrt{1-\beta^2}$

$$b=\sqrt{\frac{1+\beta}{1-\beta}}=\sqrt{\frac{1+u/c}{1-u/c}}=\frac{1+u/c}{\sqrt{1-u^2/c^2}}\equiv\frac{1+\beta}{\lambda}$$



Optical wave parameters for relativity

Doppler BLUE SHIFT $b = e^{+\rho}$ or RED SHIFT $r = e^{-\rho} = 1/b$ or RAPIDITY $\rho = \log_e b$



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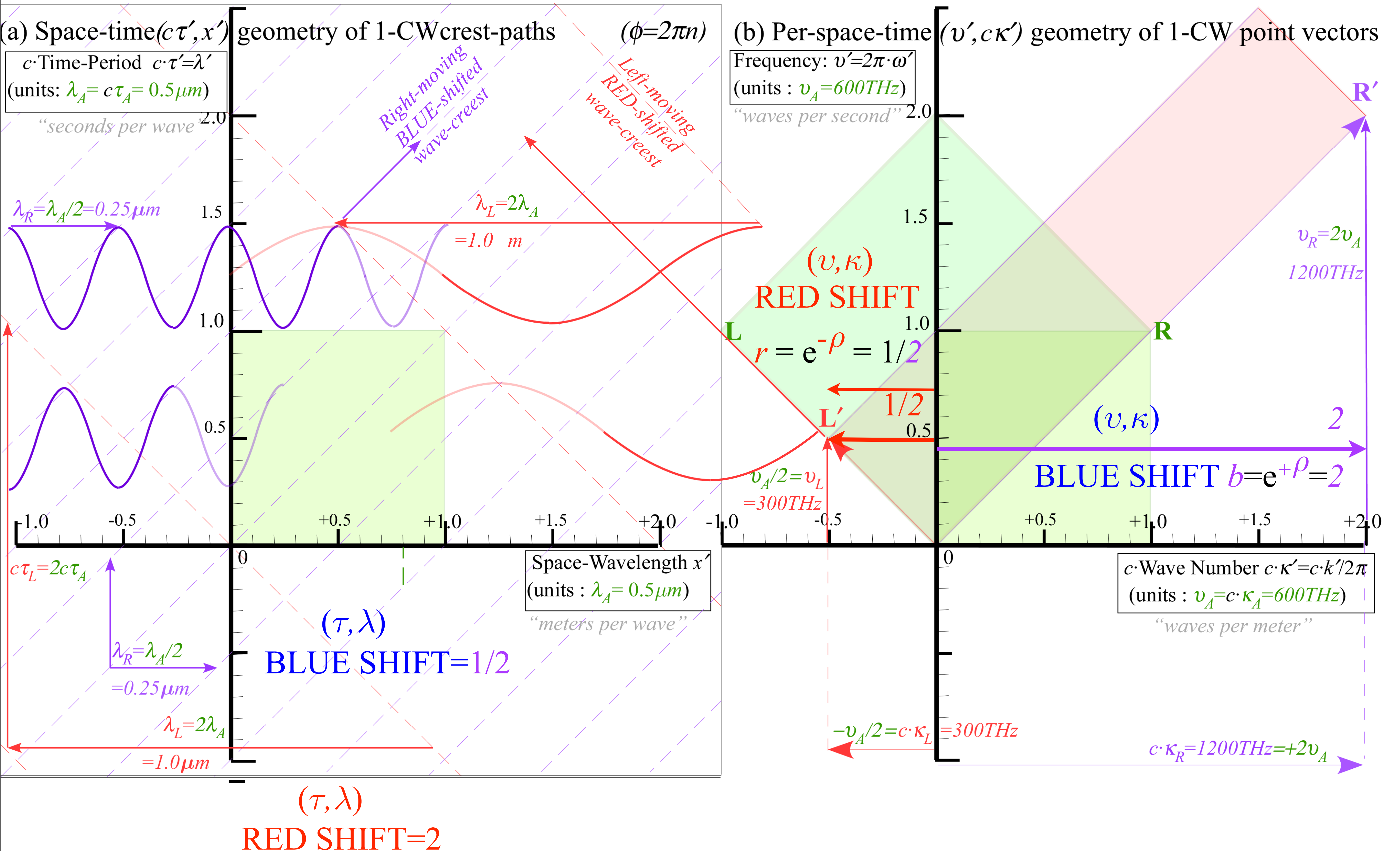
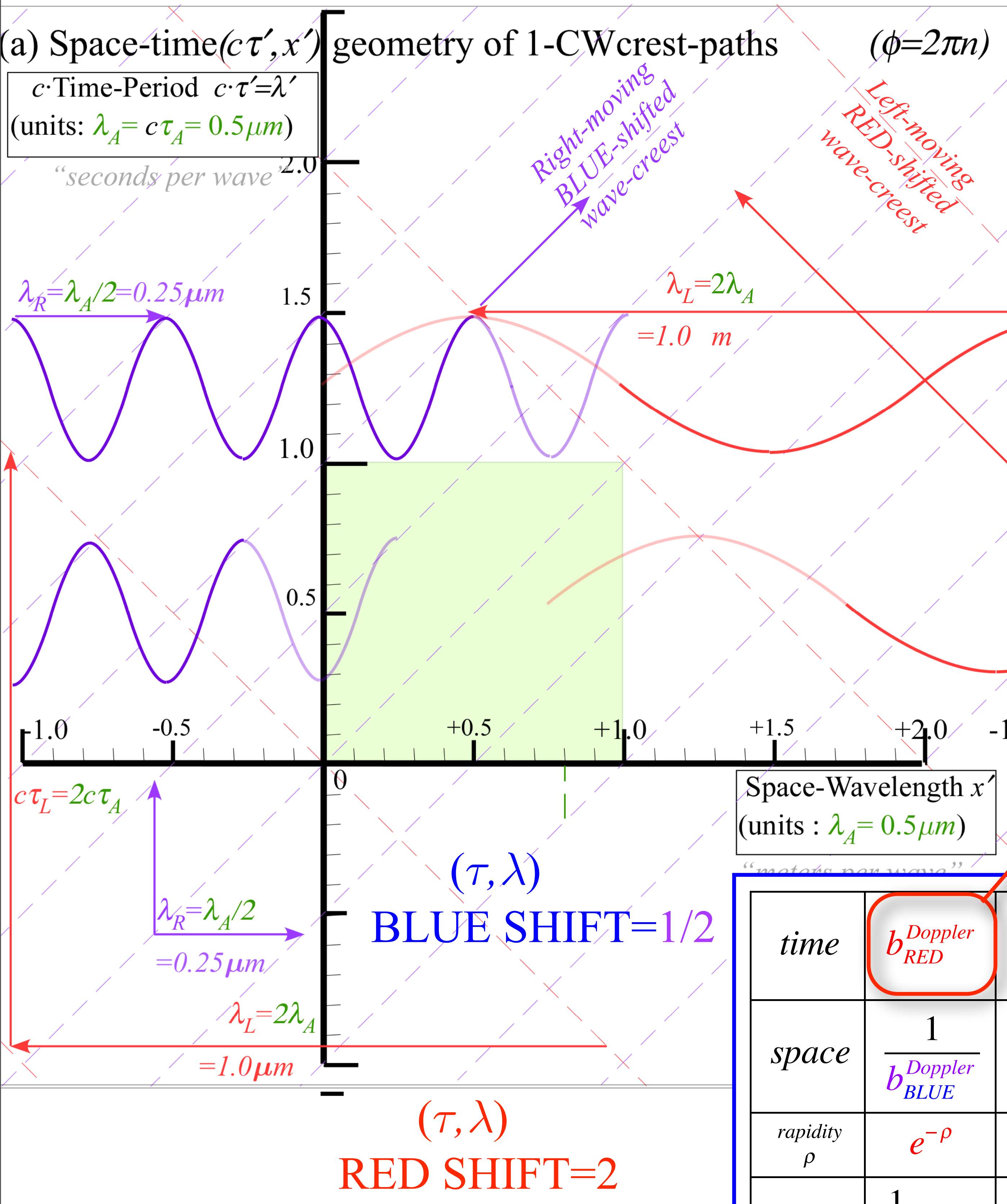
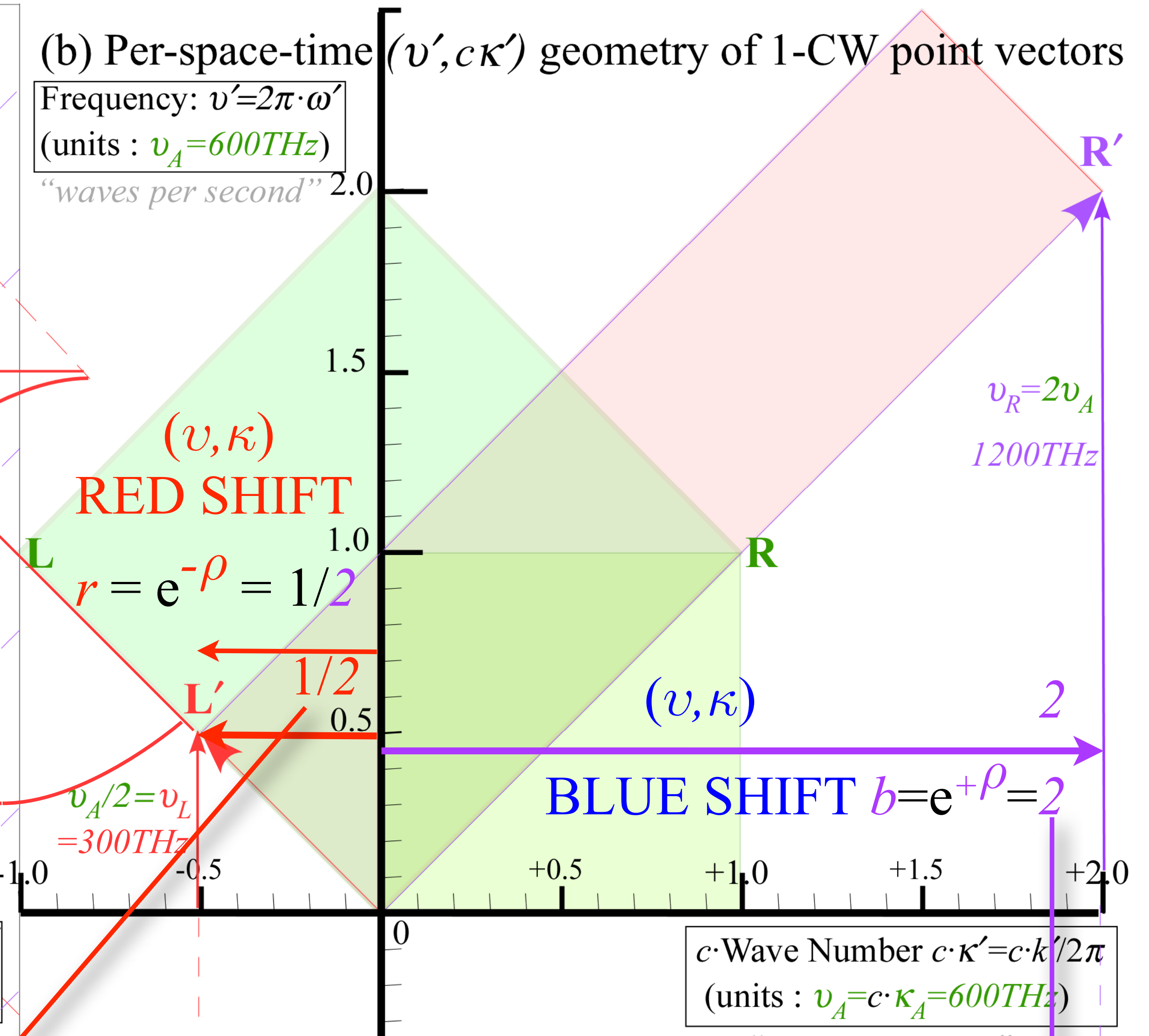


16 Optical wave parameters for relativity

based on Doppler BLUE SHIFT $b=e^{+\rho}$ or RED SHIFT $r=e^{-\rho}=1/b$ or RAPIDITY $\rho=\log_e b$





time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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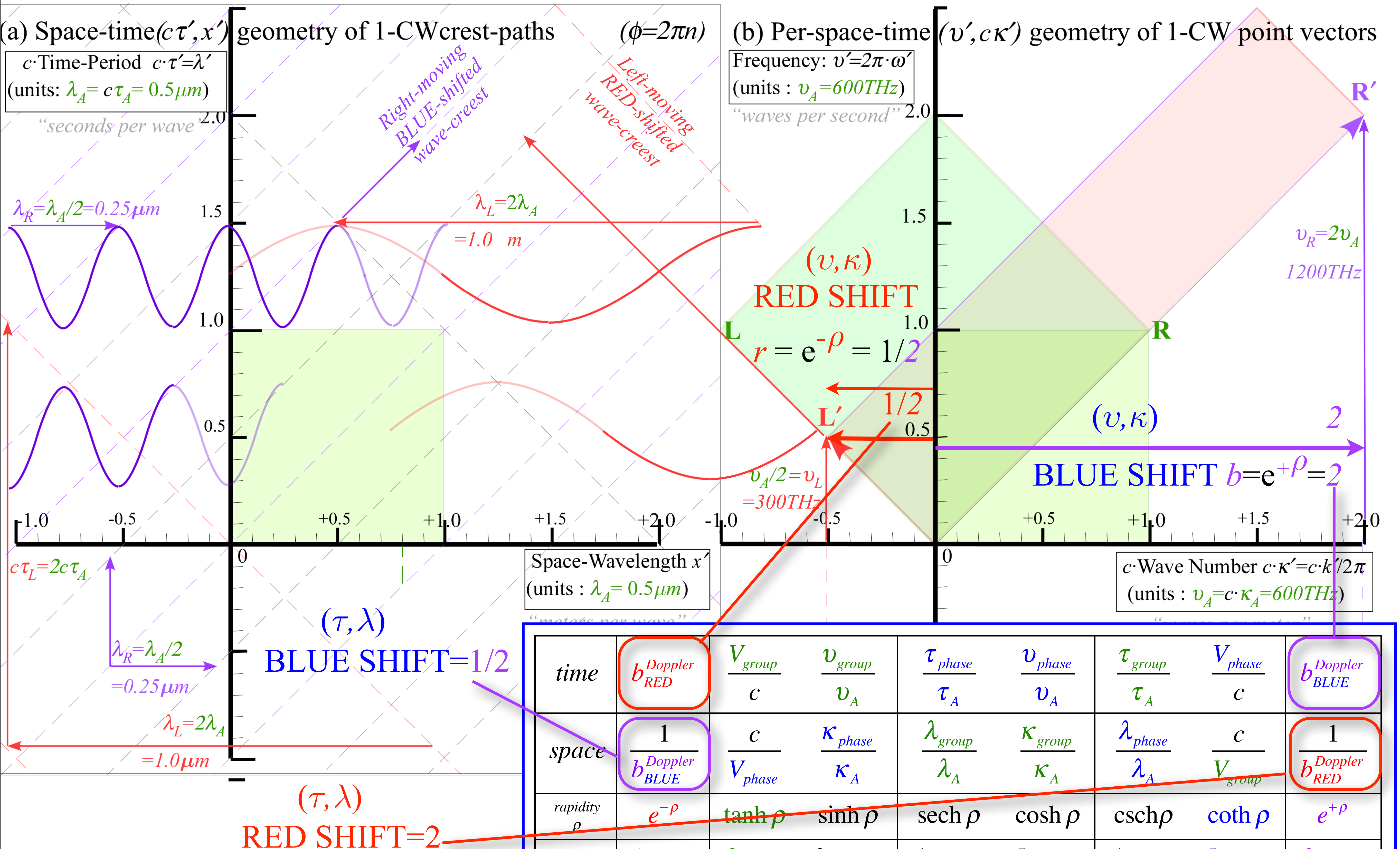

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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16 Optical wave parameters for relativity (including inverses and symmetry)

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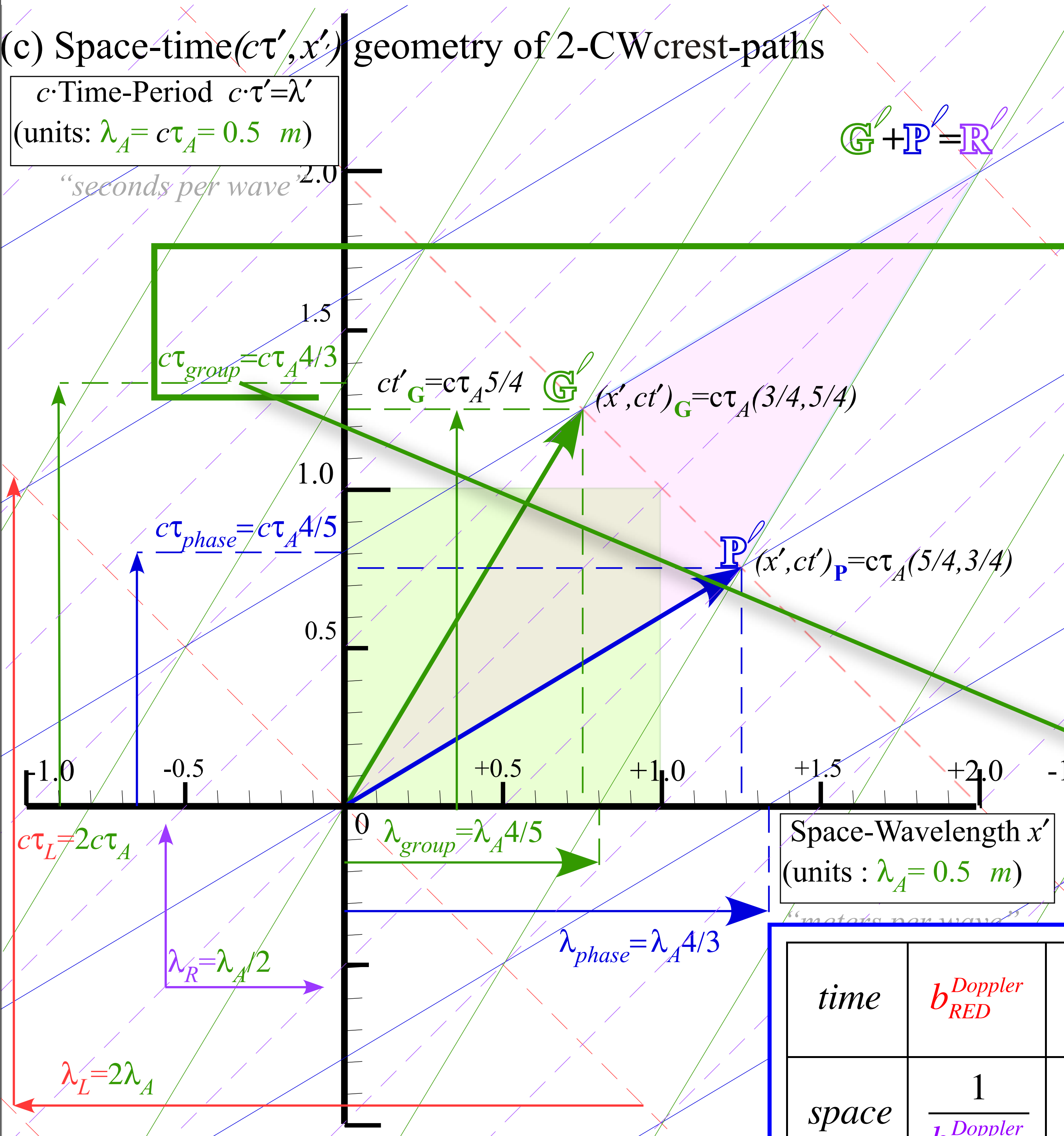

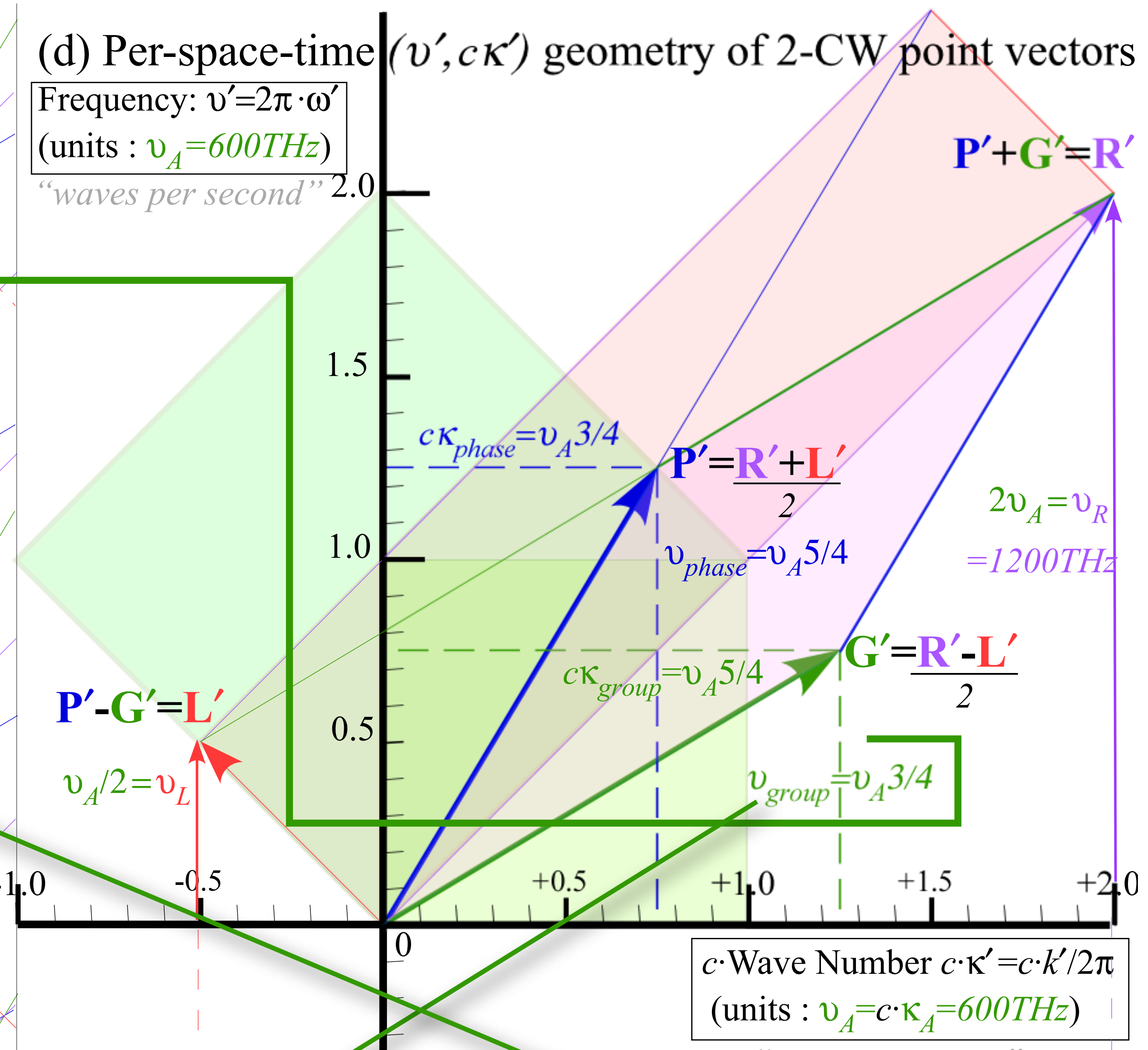

time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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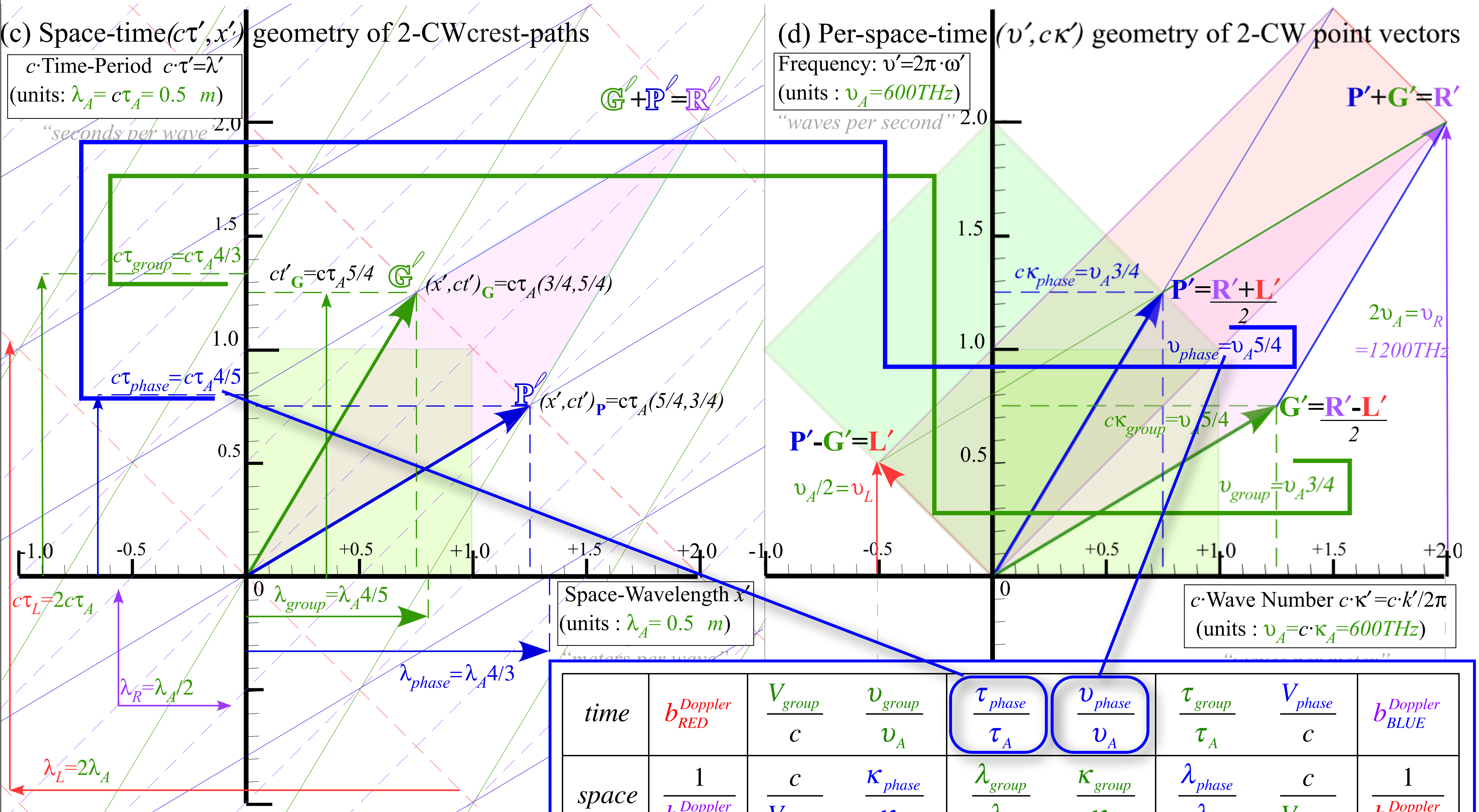
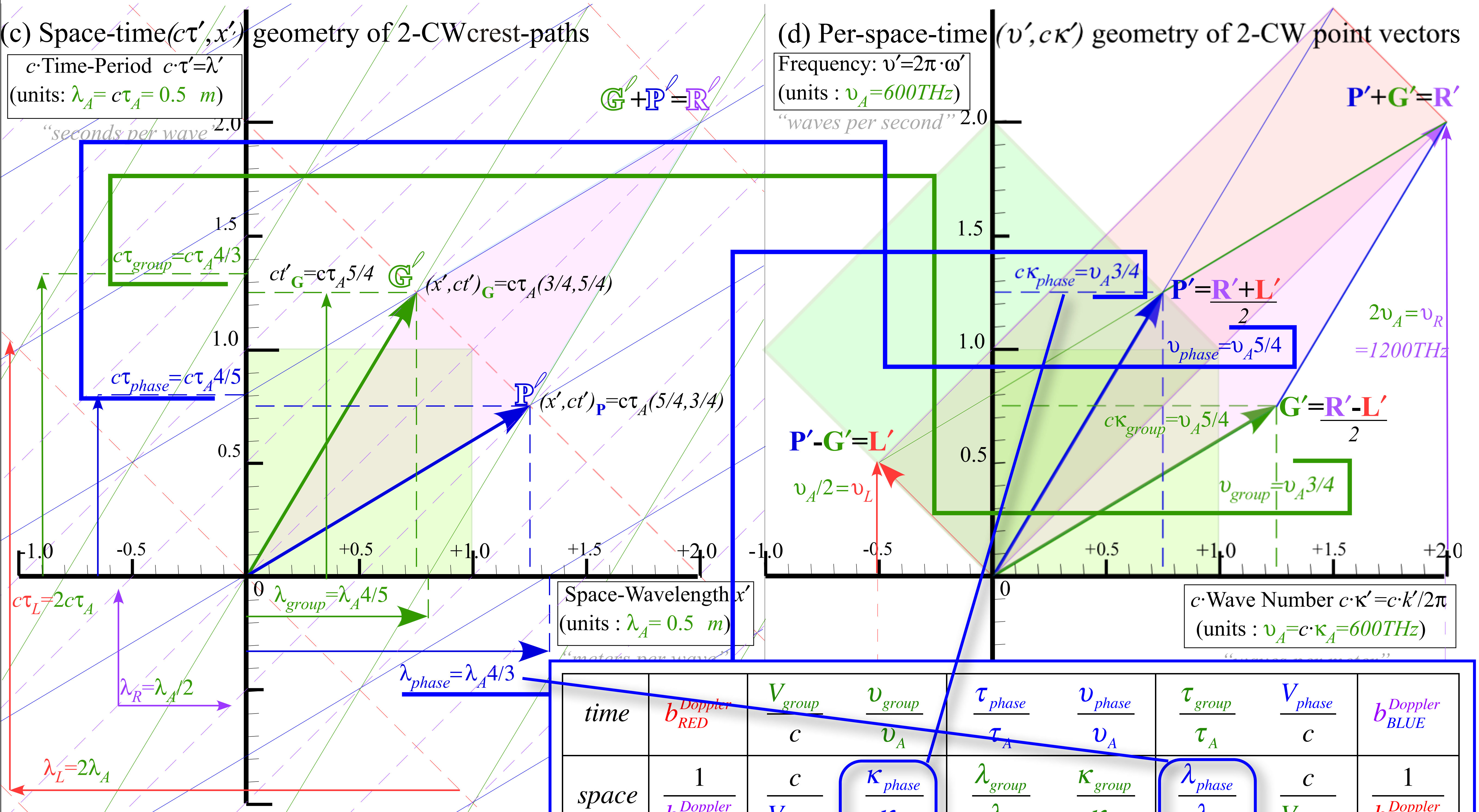


time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

16 Optical wave parameters for relativity (including inverses and symmetry)

based on Doppler BLUE SHIFT $b=e^{+\rho}$ or RED SHIFT $r=e^{-\rho}=1/b$ or RAPIDITY $\rho=\log_e b$



time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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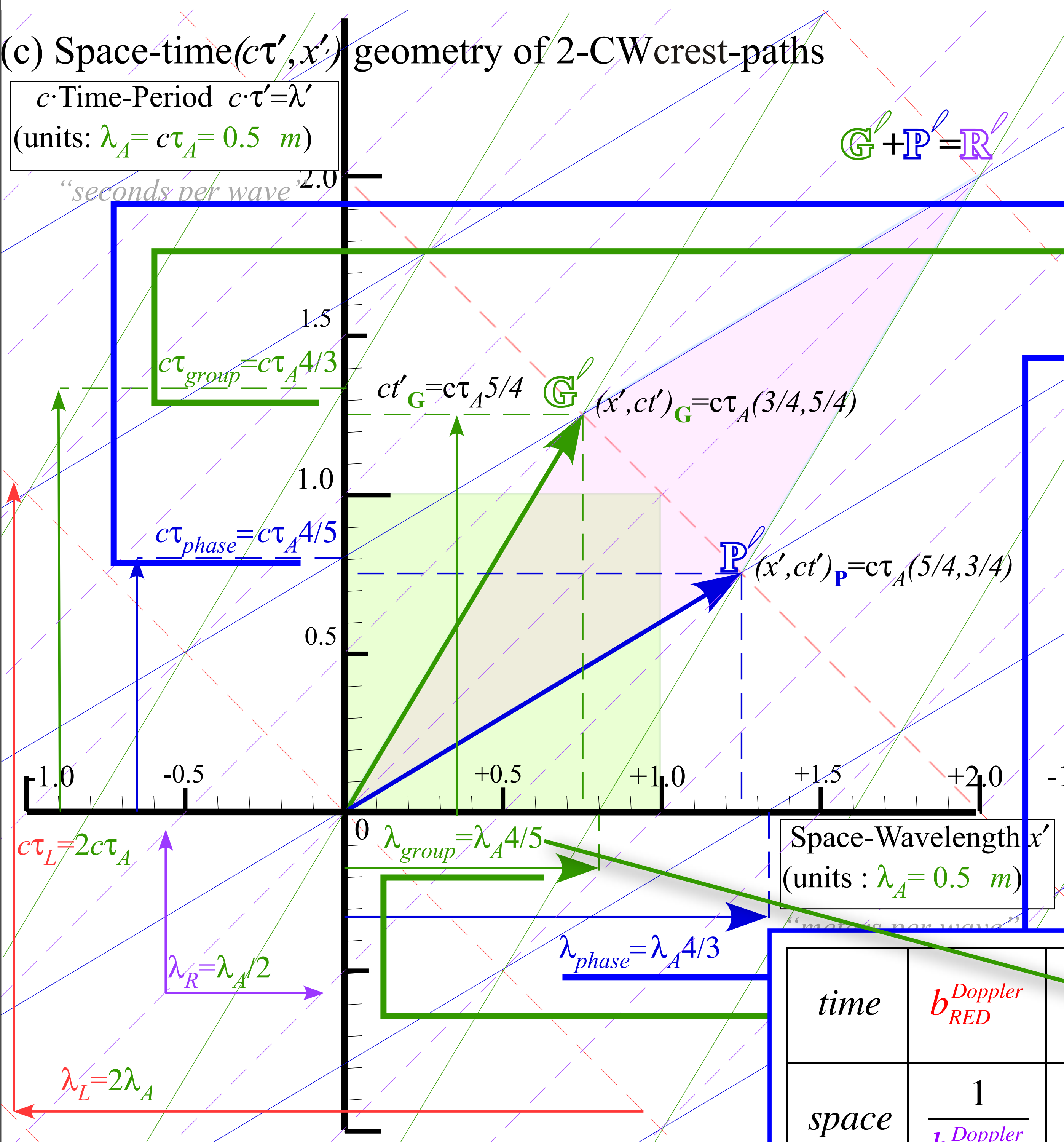
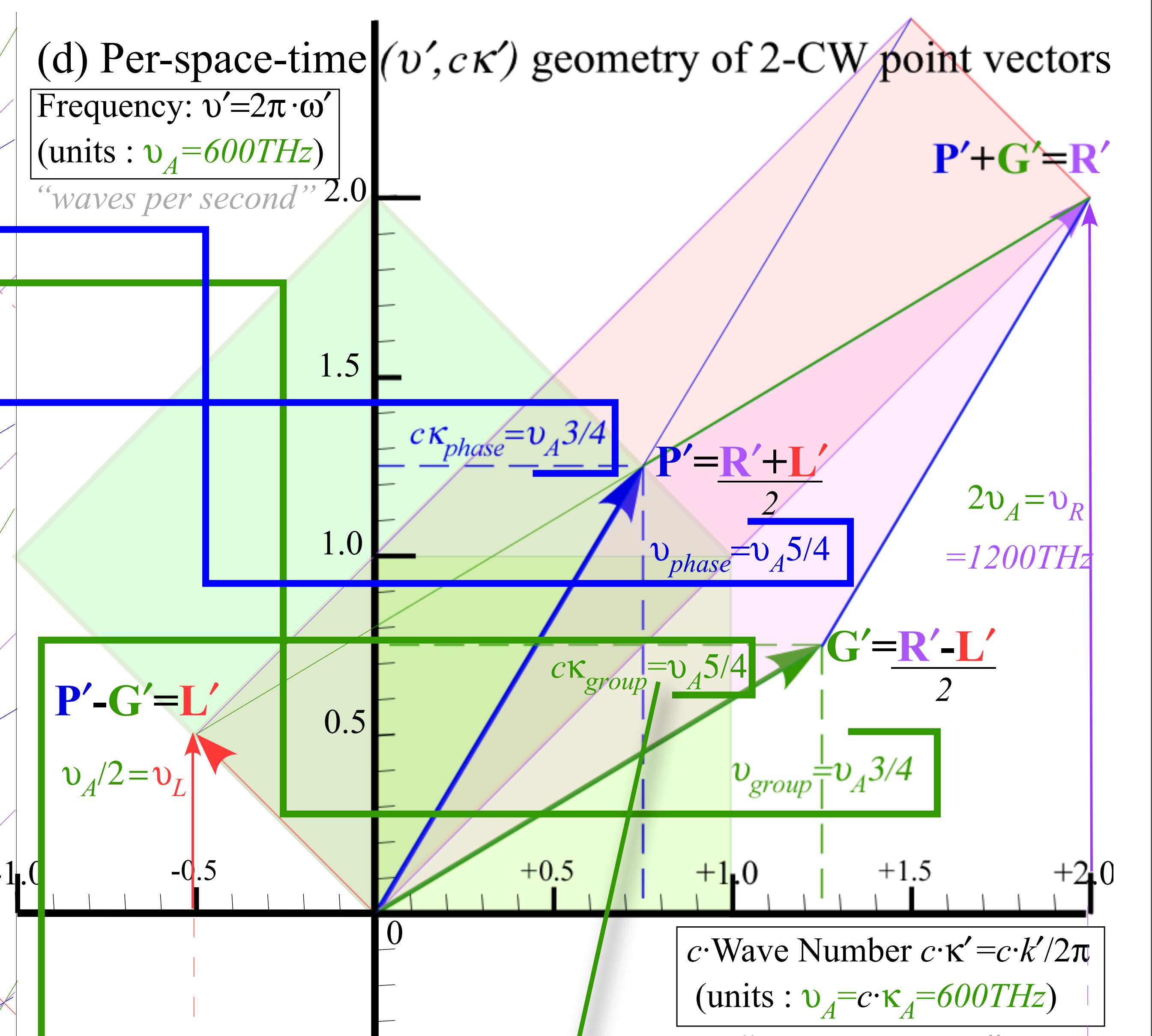
16 Optical wave parameters for relativity (including inverses and symmetry)

based on Doppler BLUE SHIFT $b = e^{+\rho}$ or RED SHIFT $r = e^{-\rho} = 1/b$ or RAPIDITY $\rho = \log_e b$





time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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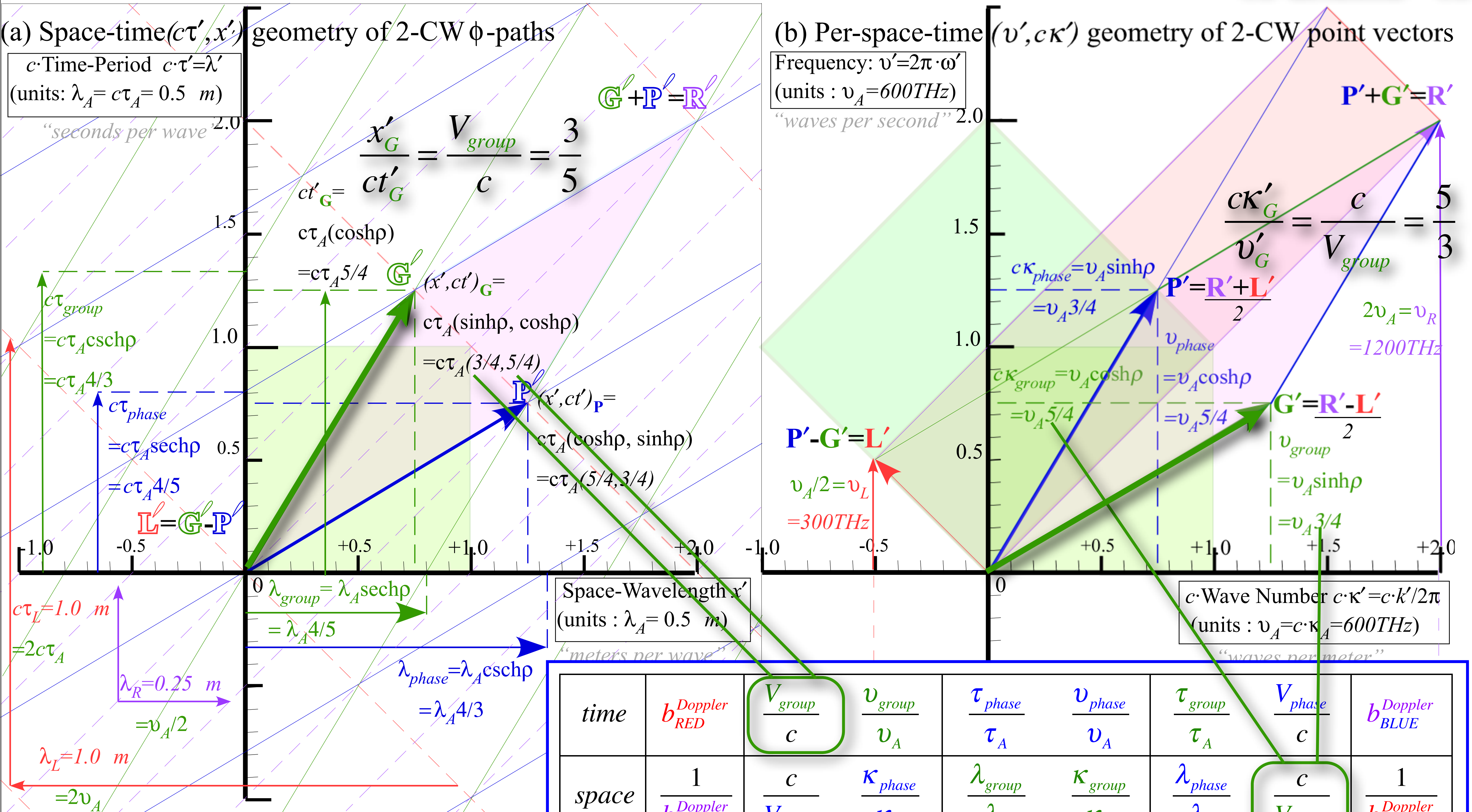
Space-time graph slope $\equiv \dfrac{t\text{-ordinate}}{x\text{-abscissa}} = \dfrac{ct'}{x'}$ Wavenumber-frequency graph slope $\equiv \dfrac{\upsilon\text{-ordinate}}{c\kappa\text{-abscissa}} = \dfrac{\upsilon'}{c\kappa'}$



time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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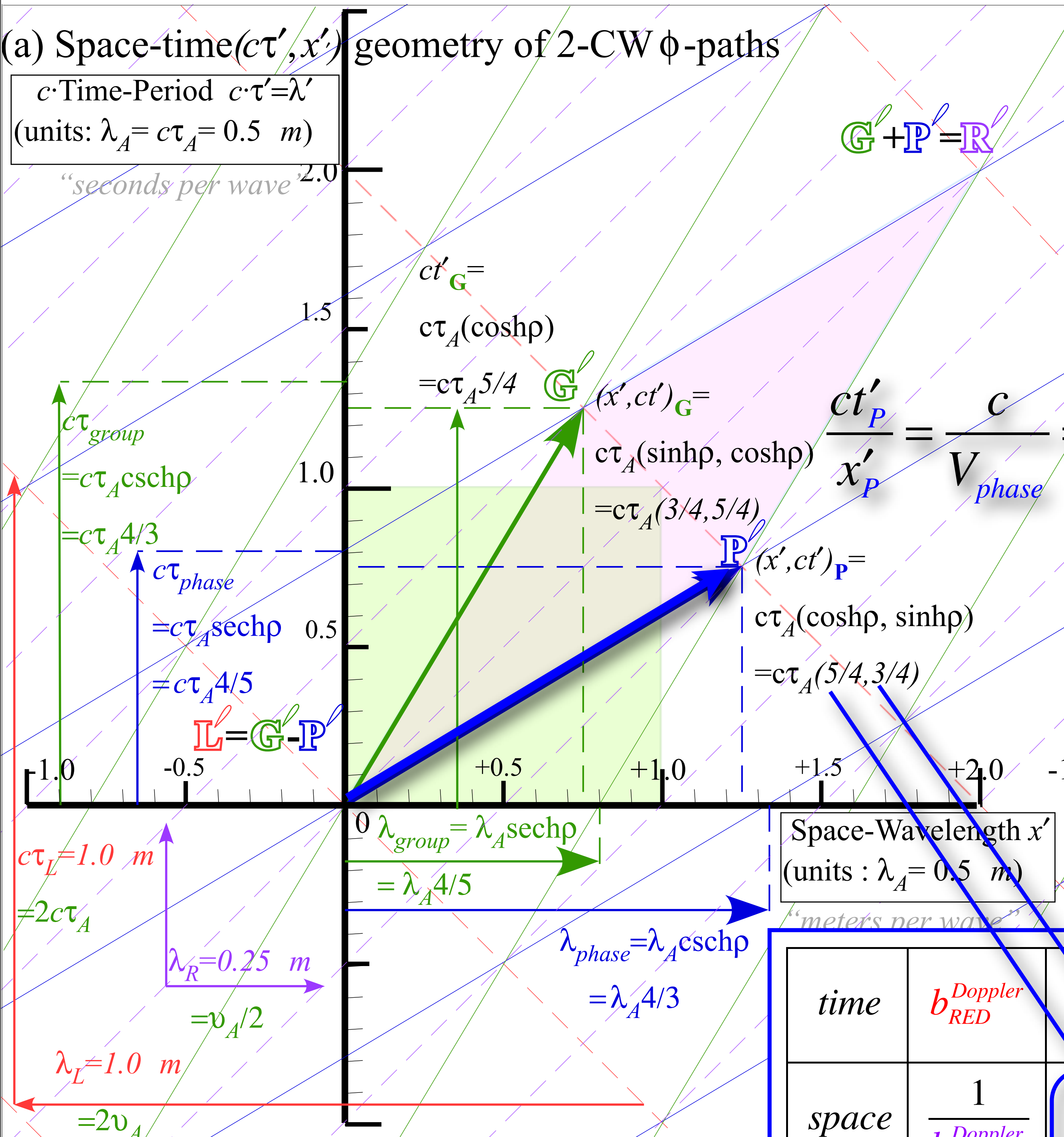
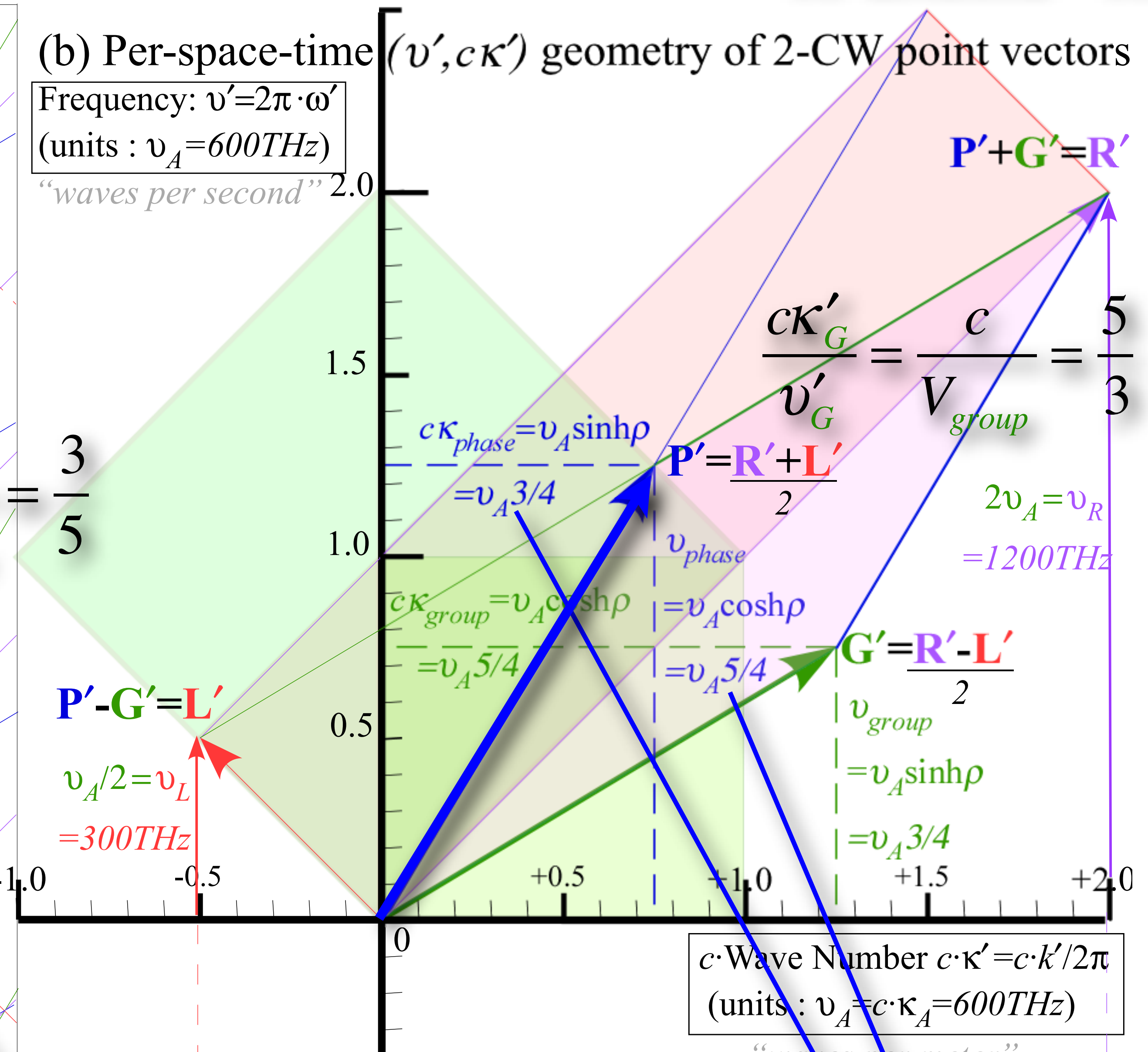
$$\text{Space-time graph slope} \equiv \frac{t\text{-ordinate}}{x\text{-abscissa}} = \frac{ct'}{x'} \qquad \text{Wavenumber-frequency graph slope} \equiv \frac{\upsilon\text{-ordinate}}{c\kappa\text{-abscissa}} = \frac{\upsilon'}{c\kappa'}$$





time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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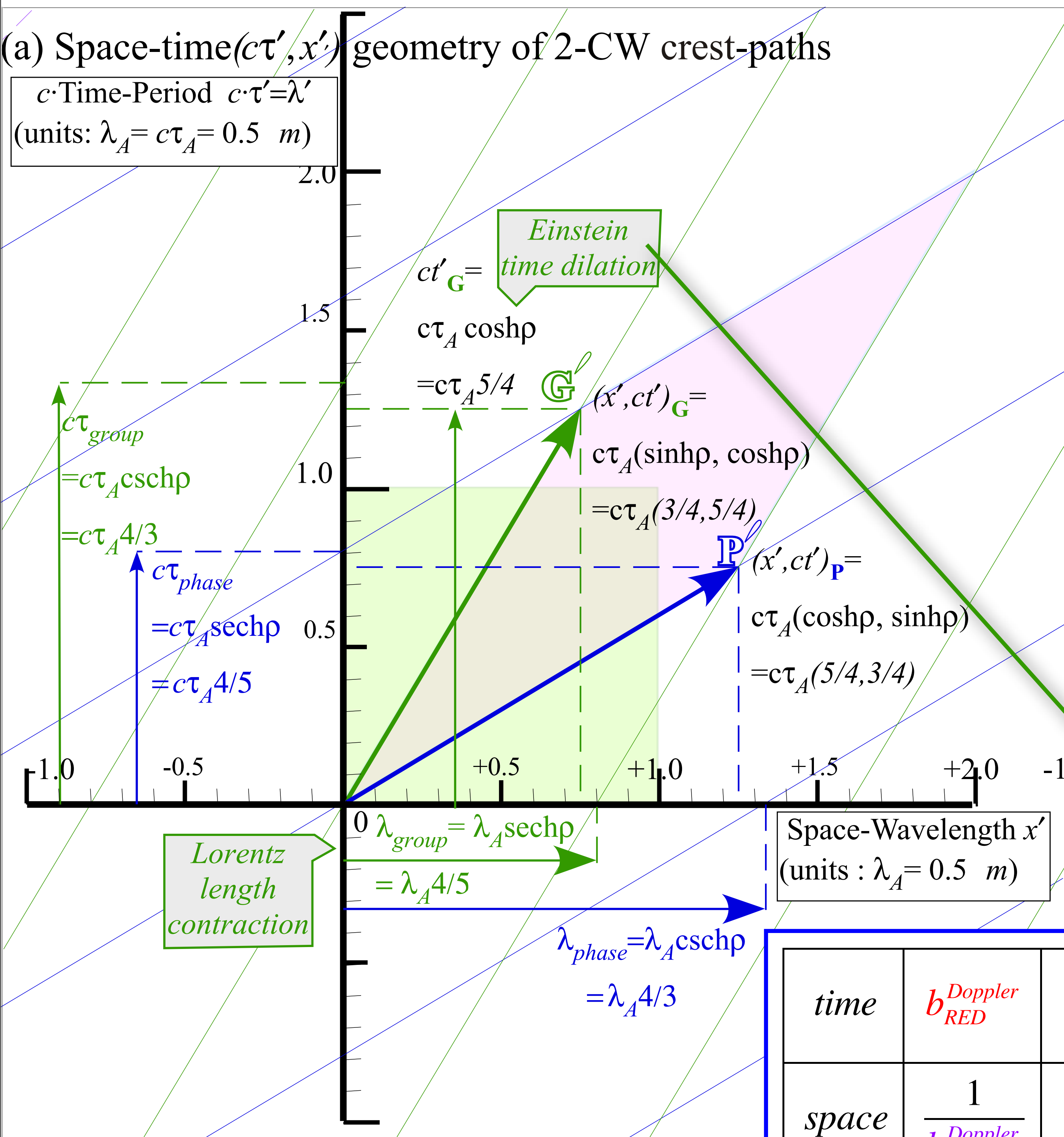
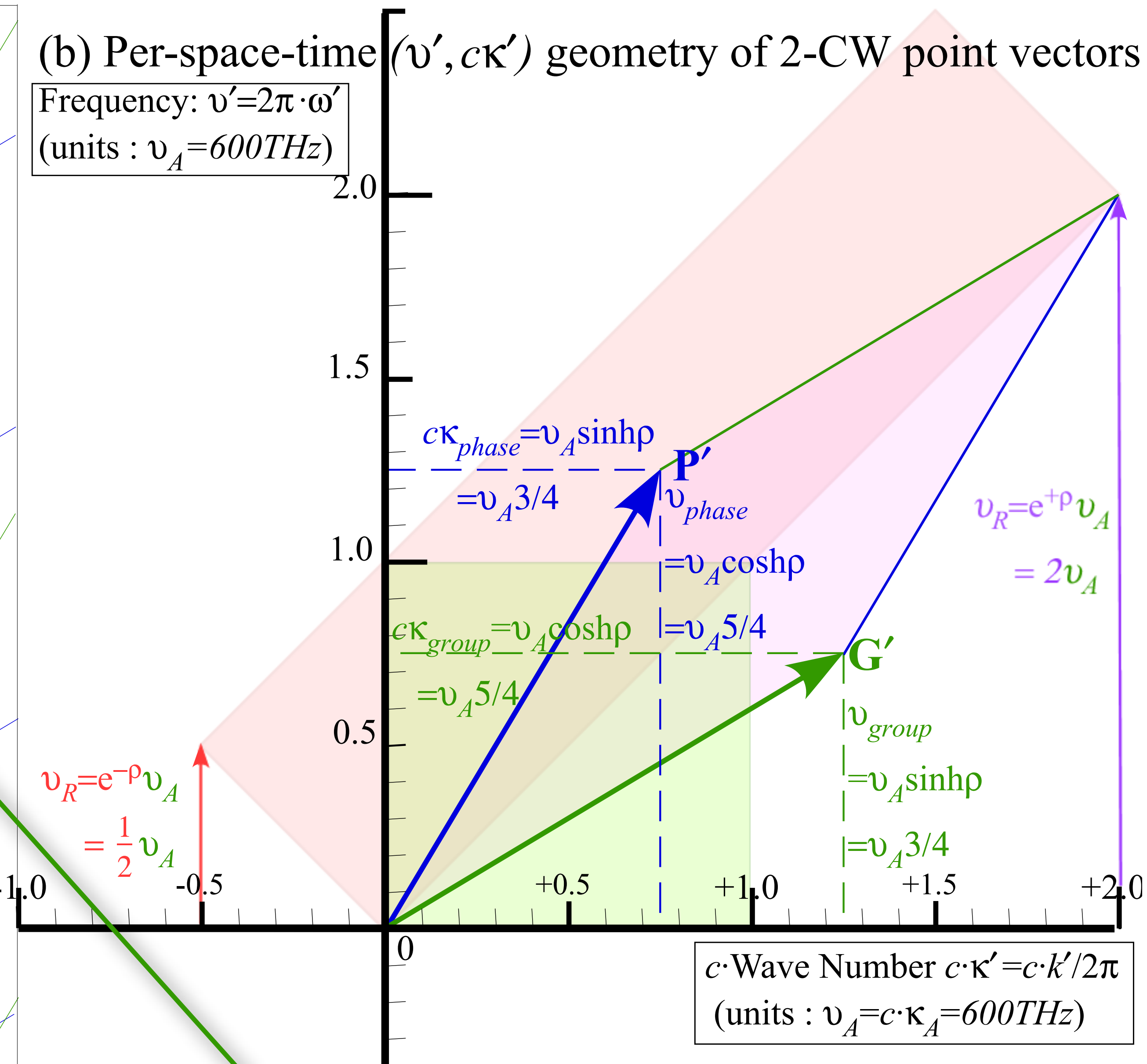
One MOST FAMOUS of 16 Optical wave parameters for relativity





time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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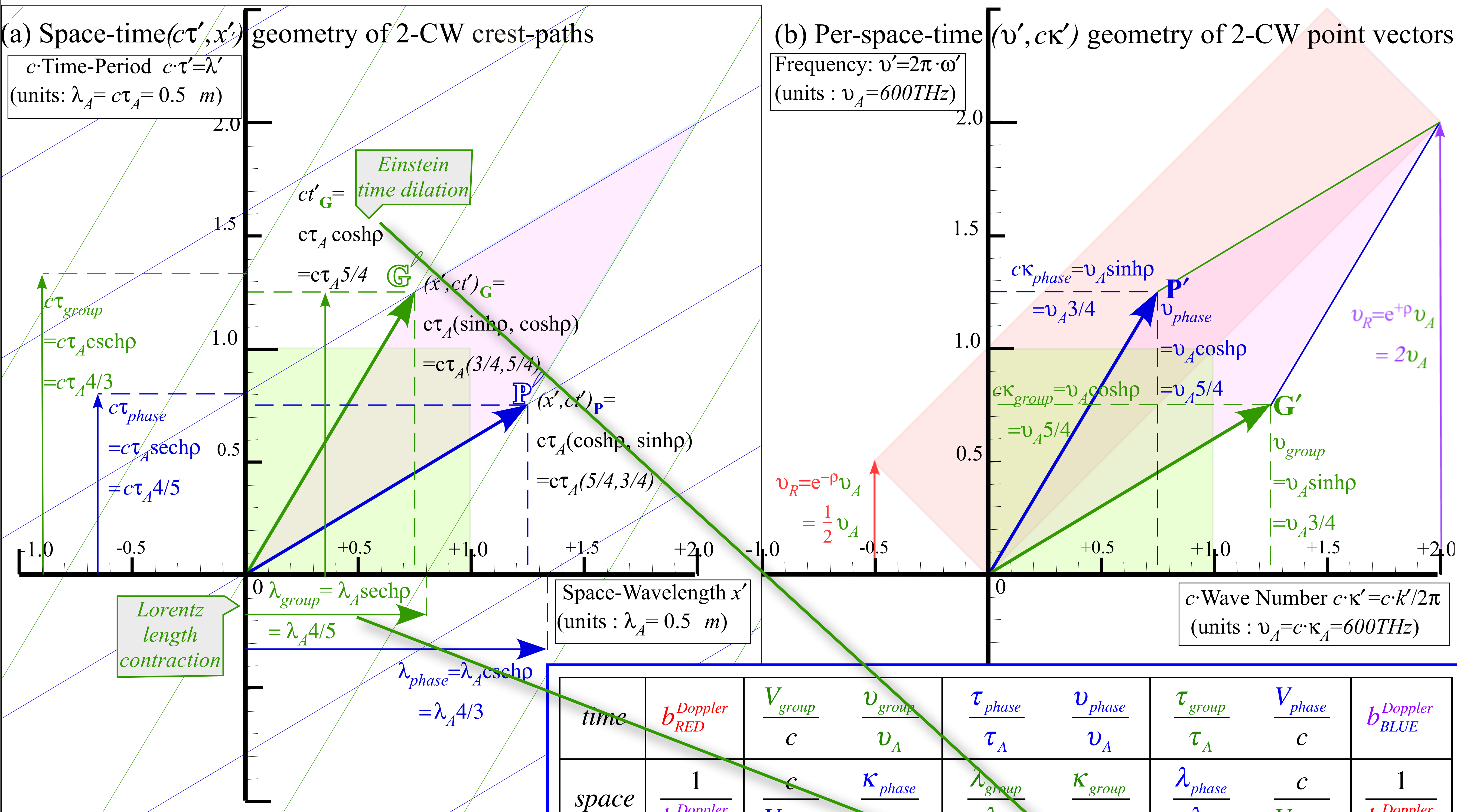
Two MOST FAMOUS of 16 Optical wave parameters for relativity



time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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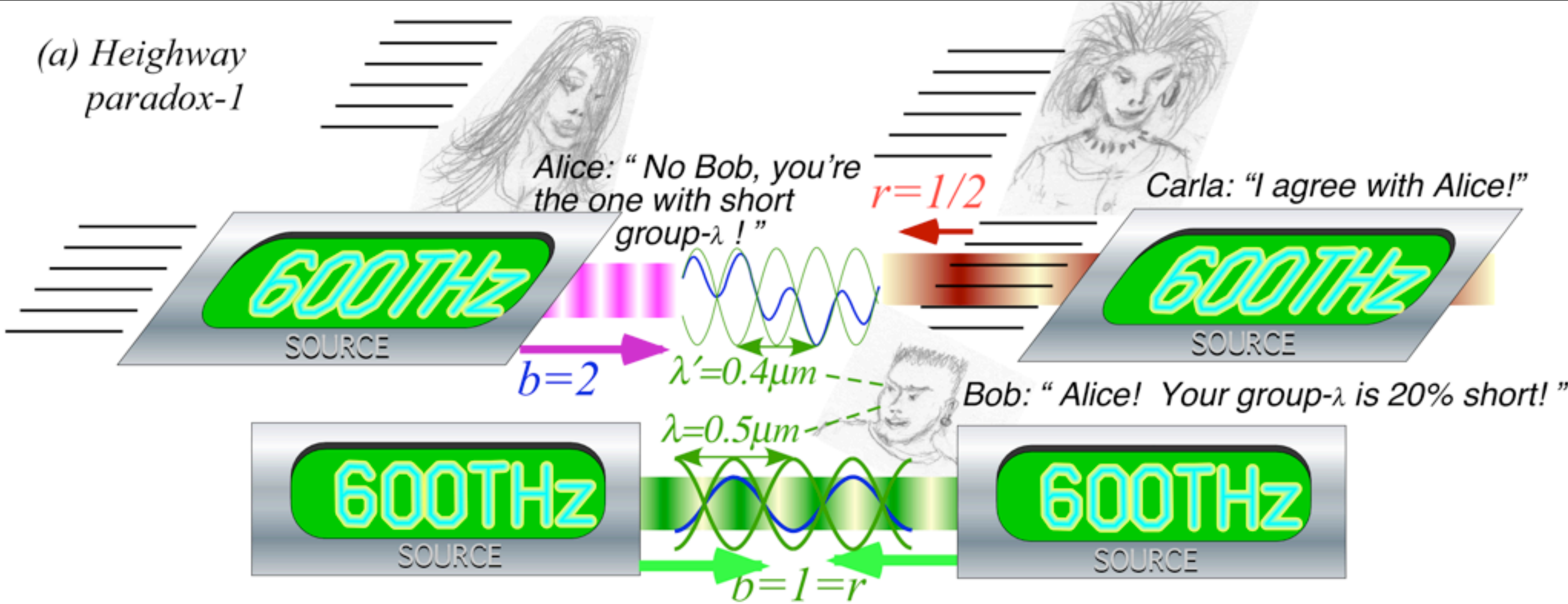
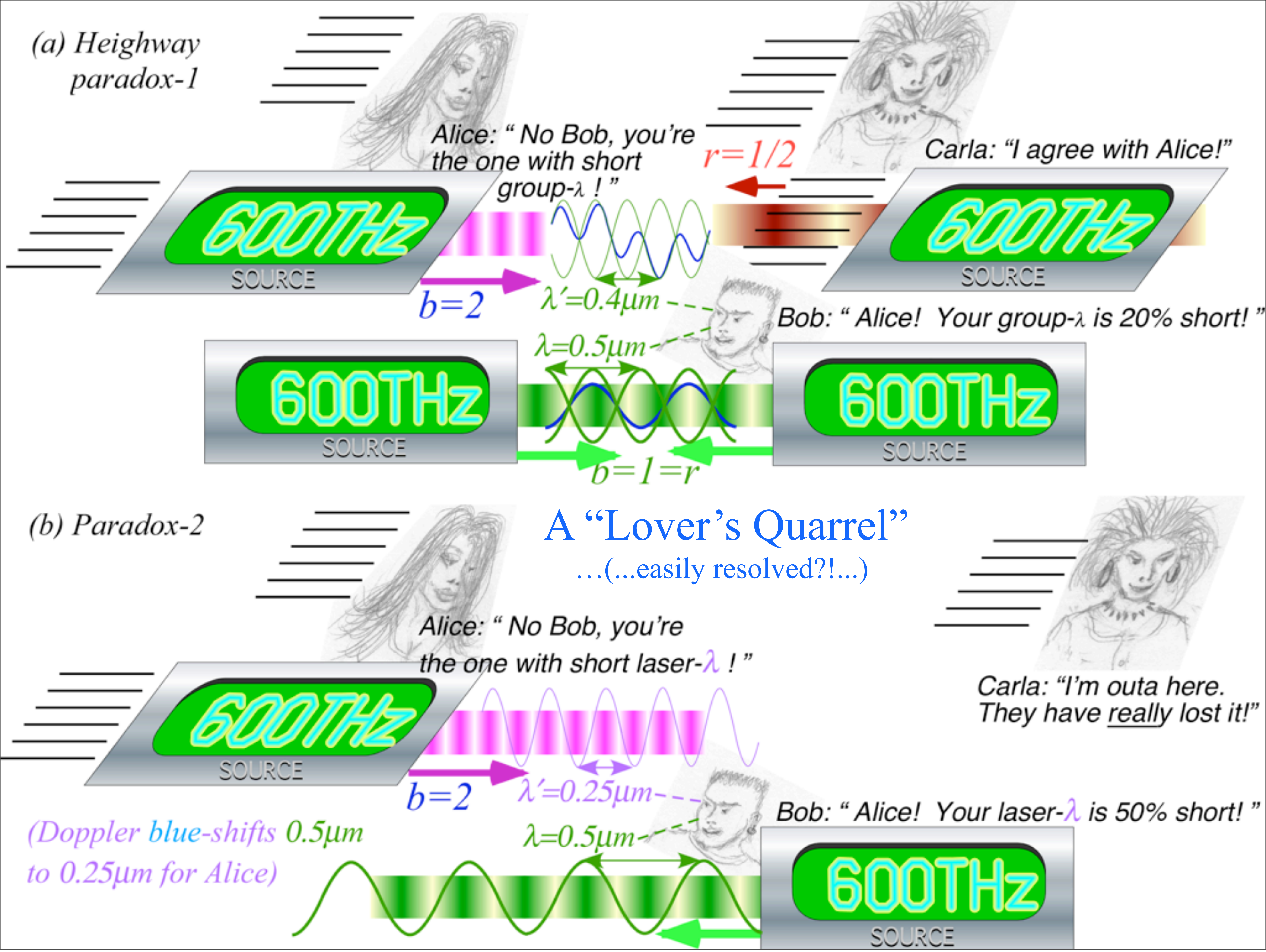
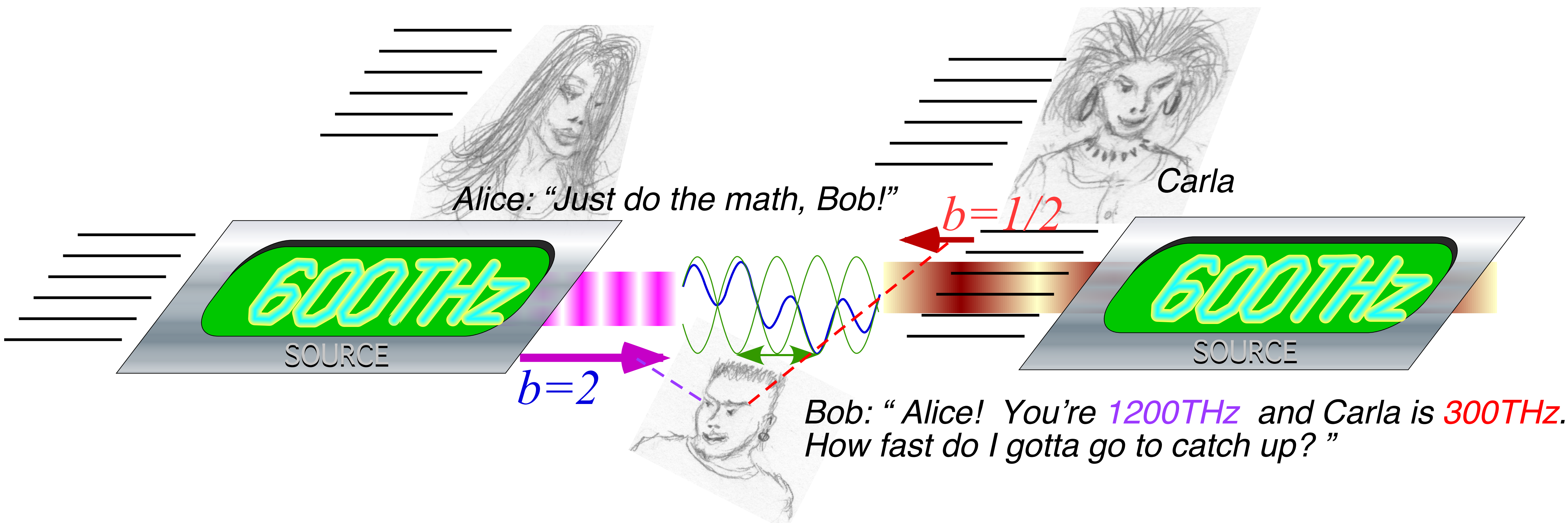


A "Lover's Quarrel"

…(The worst kind...when *both* are right *and* wrong)


(a) Heighway paradox-1
Alice: “ No Bob, you’re the one with short group-λ ! ”
r=1/2
Carla: “I agree with Alice!”
600THz
SOURCE
600THz
SOURCE
b=2
λ′=0.4μm
Bob: “ Alice! Your group-λ is 20% short! ”
λ=0.5μm
600THz
SOURCE
600THz
SOURCE
b=1=r
(b) Paradox-2
A “Lover’s Quarrel”
…(...easily resolved?!...)
Alice: “ No Bob, you’re the one with short laser-λ ! ”
Carla: “I’m outa here. They have really lost it!”
600THz
SOURCE
b=2
λ′=0.25μm
Bob: “ Alice! Your laser-λ is 50% short! ”
λ=0.5μm
(Doppler blue-shifts 0.5μm to 0.25μm for Alice)
600THz
SOURCE

Imagine as before, that Bob detects counter-propagating laser beams of frequency ω_R going left-to-right (previously Alice's laser) and ω_L going right-to-left (Carla's laser). We ask two questions: (1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E? And, (2.) What is that frequency ω_E?



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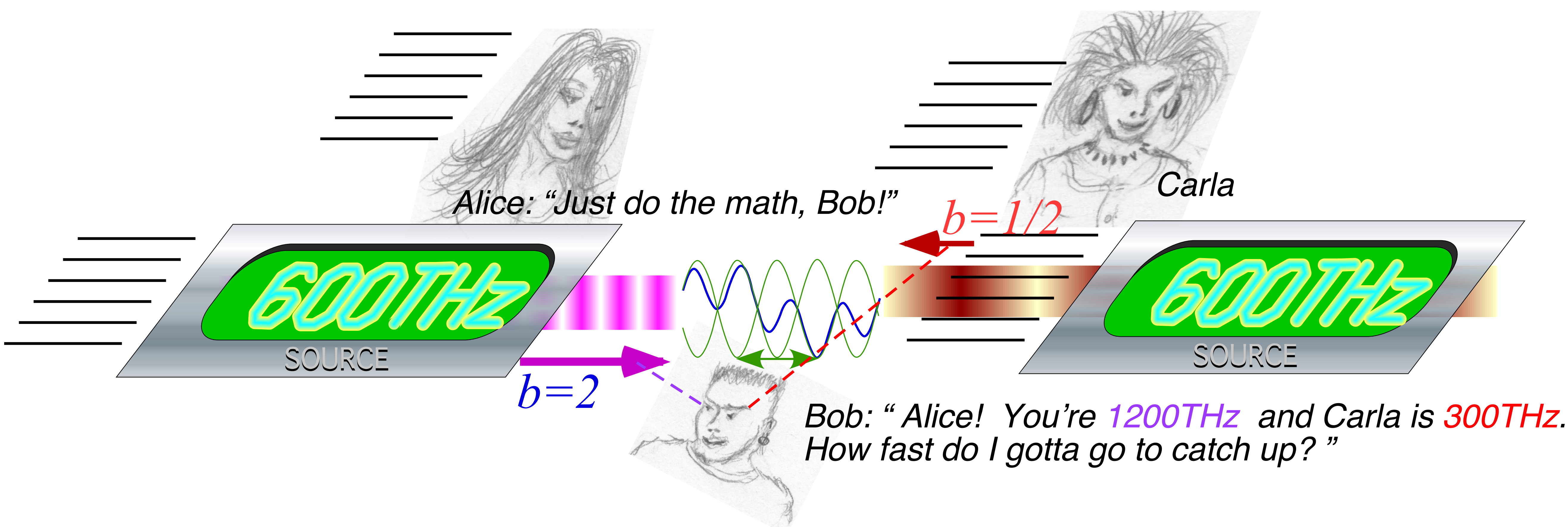
Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity? $\frac{\text{Difference Mean}}{\text{Arithmetic Mean}} =$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

$$\frac{1200 - 300}{1200 + 300}c = \frac{900}{1500}c = \frac{3}{5}c$$



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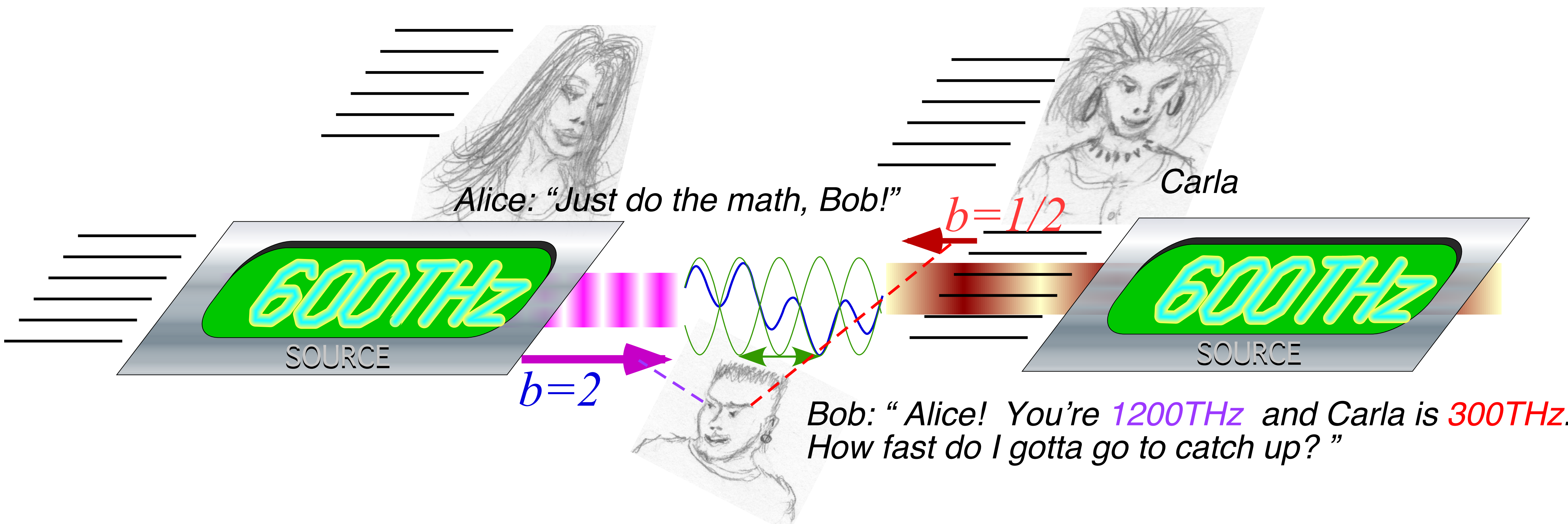
$$\frac{\text{Difference Mean}}{\text{Arithmetic Mean}} = \frac{1200-300}{1200+300}c = \frac{900}{1500}c = \frac{3}{5}c$$

Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R?

$$\omega_E = b\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R/\omega_L} \quad \Rightarrow \quad \boxed{\omega_E = \sqrt{\omega_R \cdot \omega_L}}$$

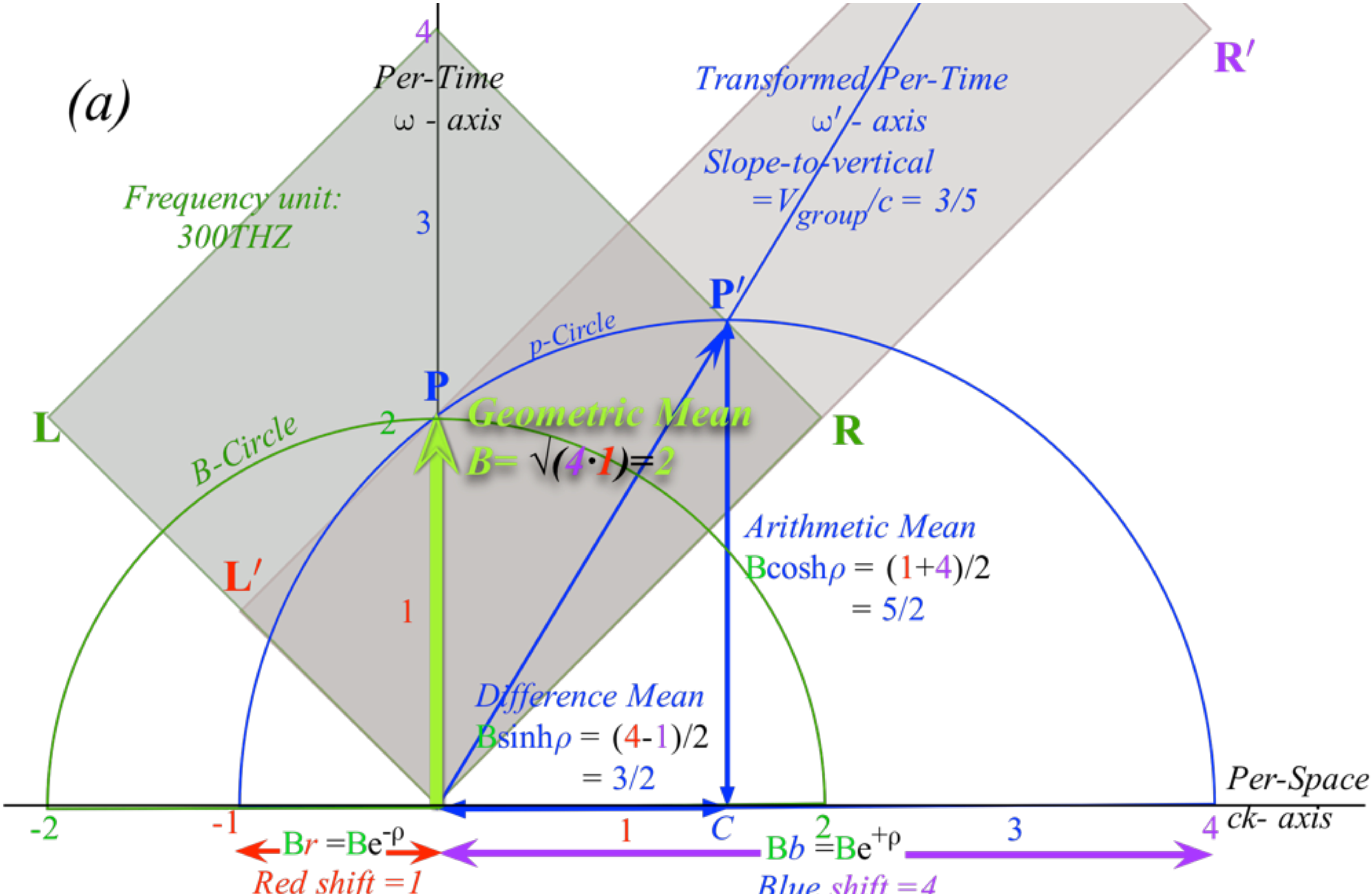
Geometric Mean

$$\sqrt{1200 \cdot 300} = 600$$



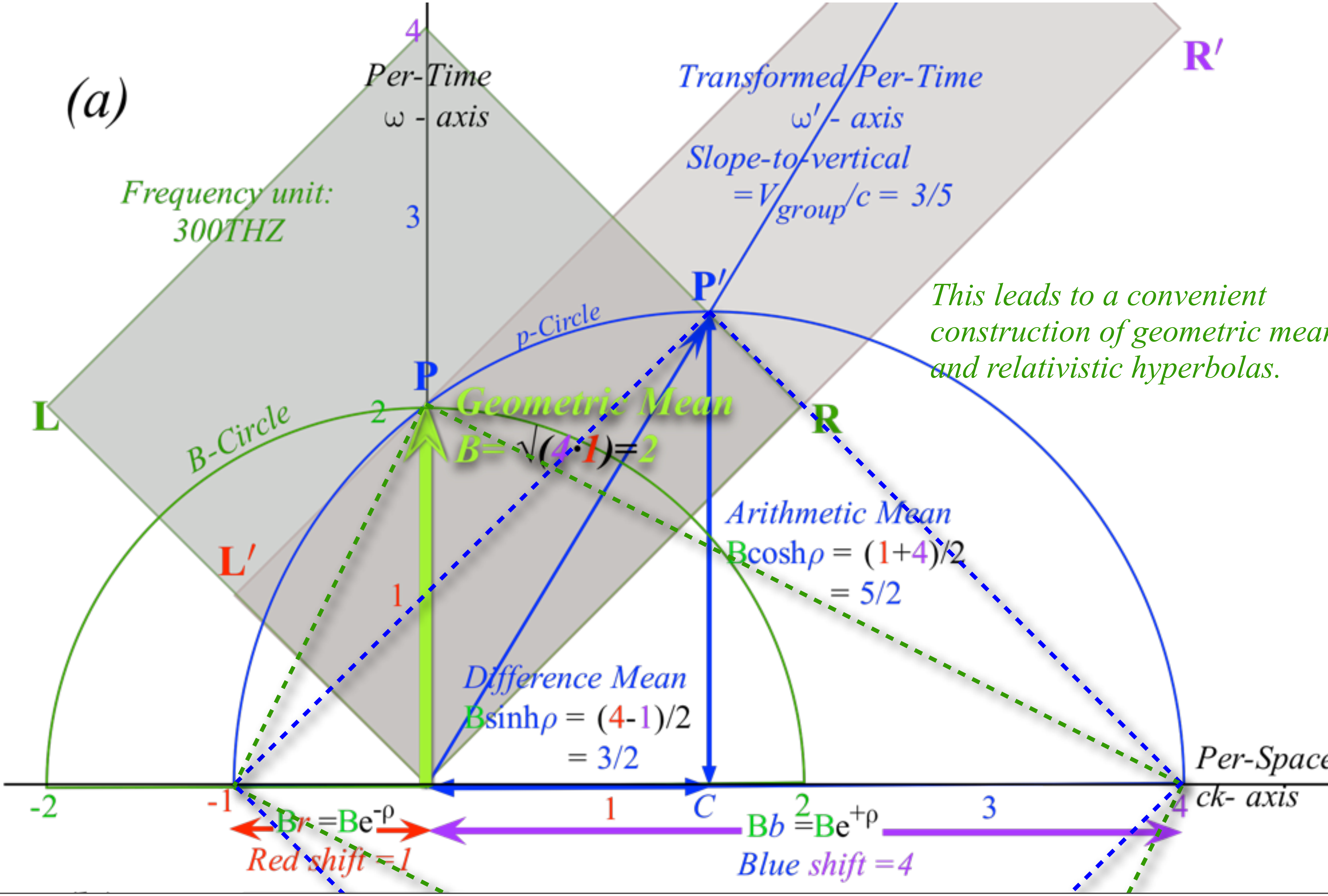
Thales Mean Geometry (600BCE)

helps “Relawavity”



Thales Mean Geometry (600BCE)

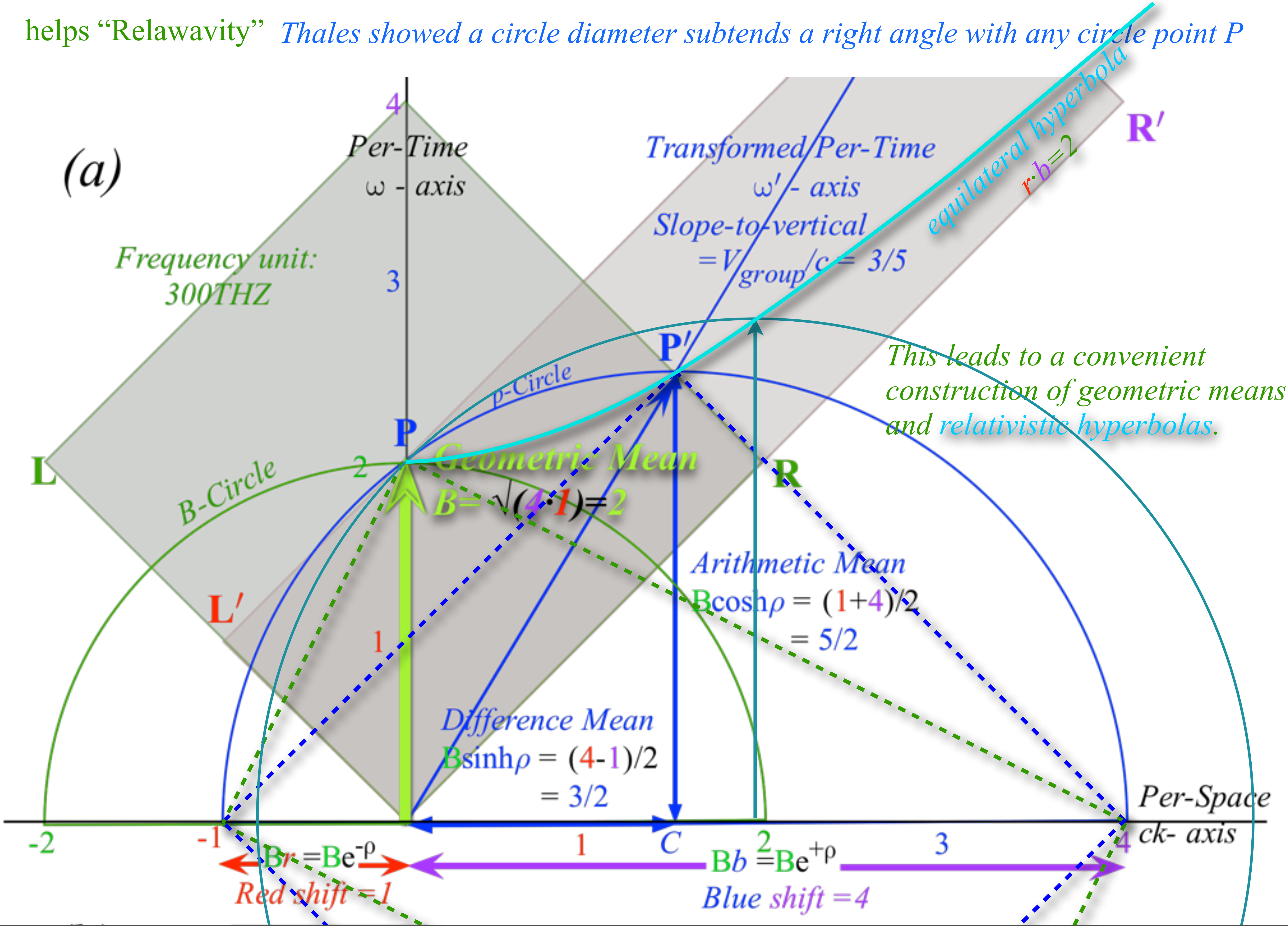
helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*



This leads to a convenient construction of geometric means and relativistic hyperbolas.

Thales Mean Geometry (600BCE)

helps “Relawavity” *Thales showed a circle diameter subtends a right angle with any circle point P*



Thales Mean Geometry (600BCE)

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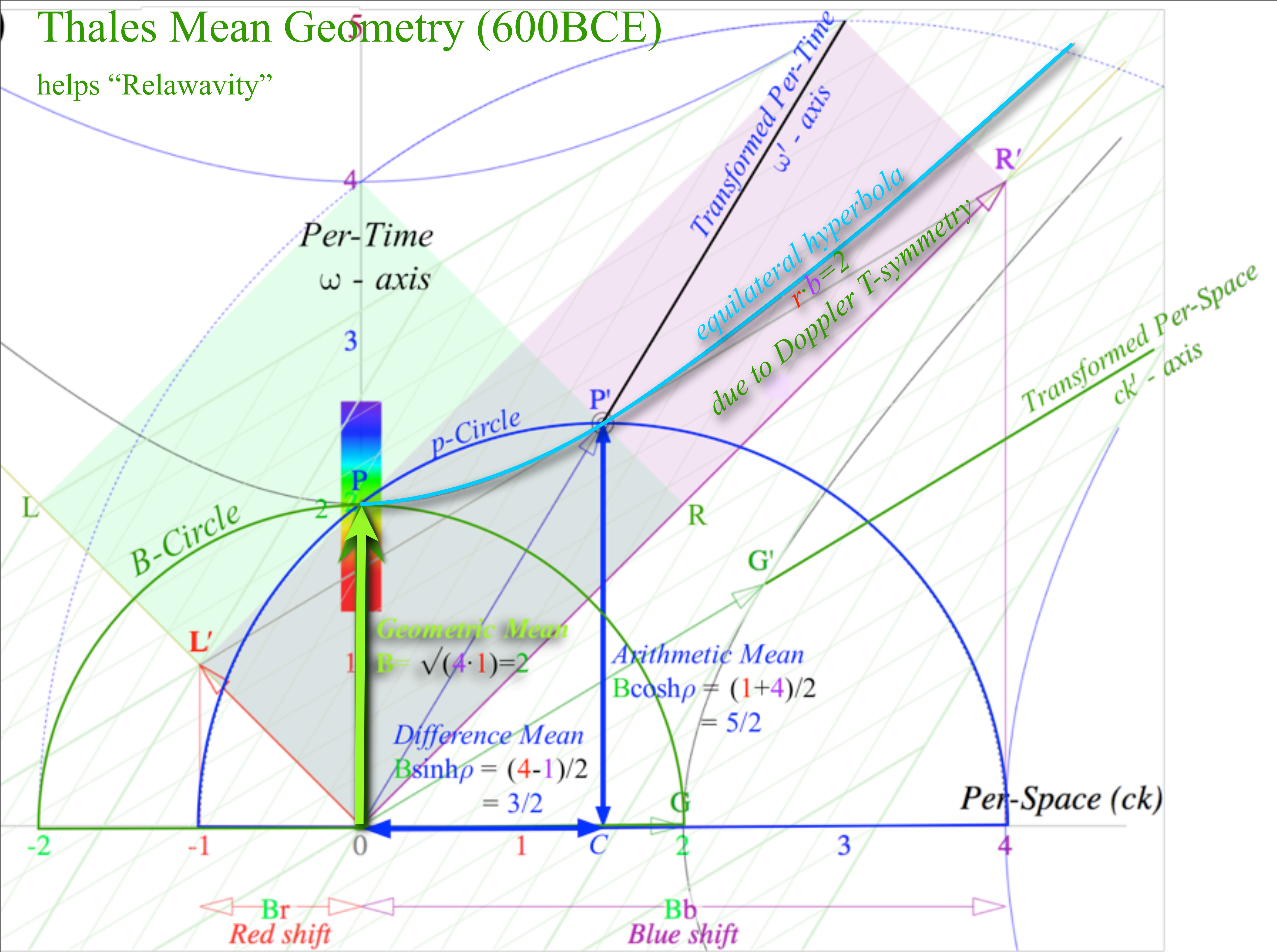


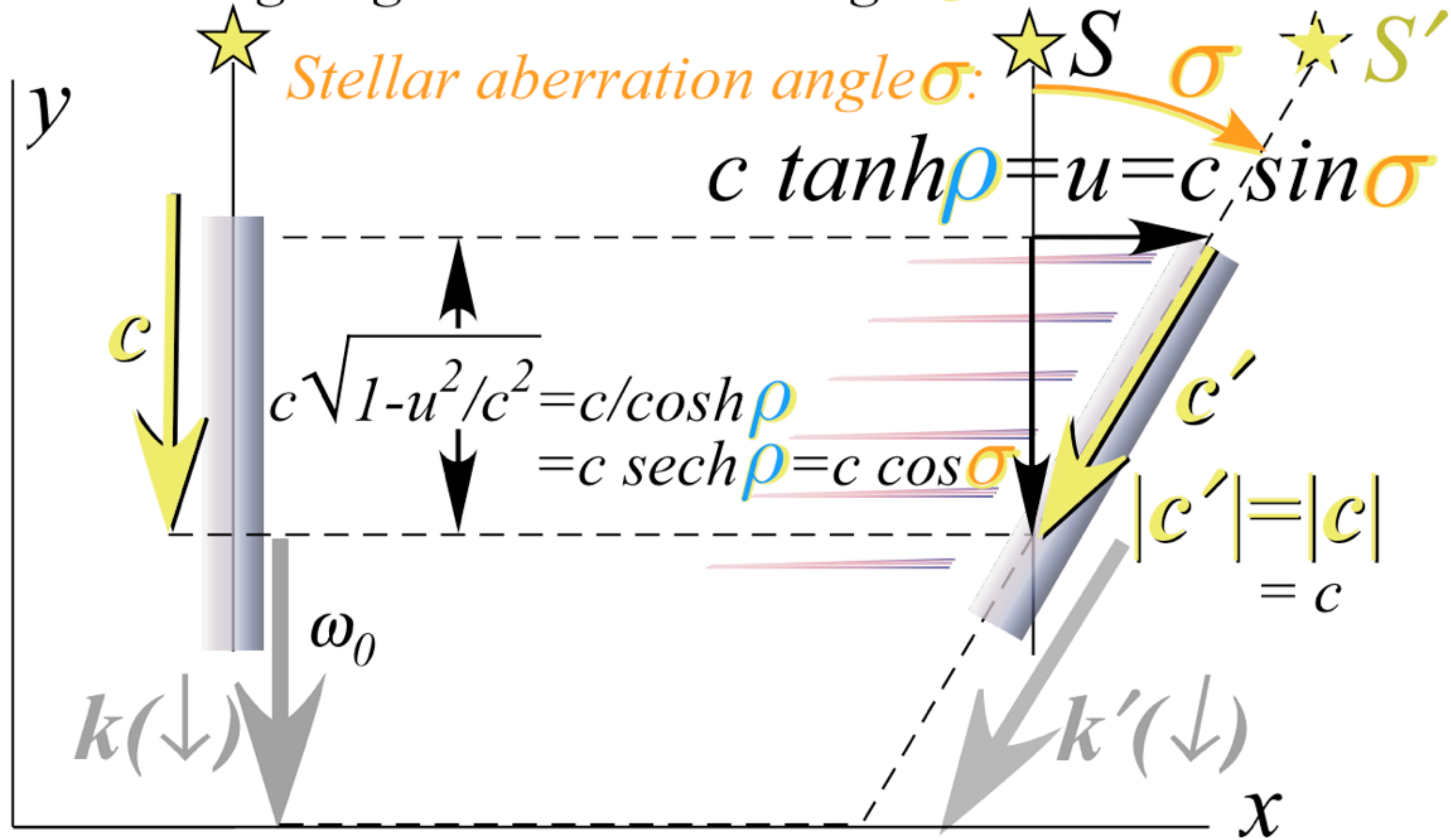
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

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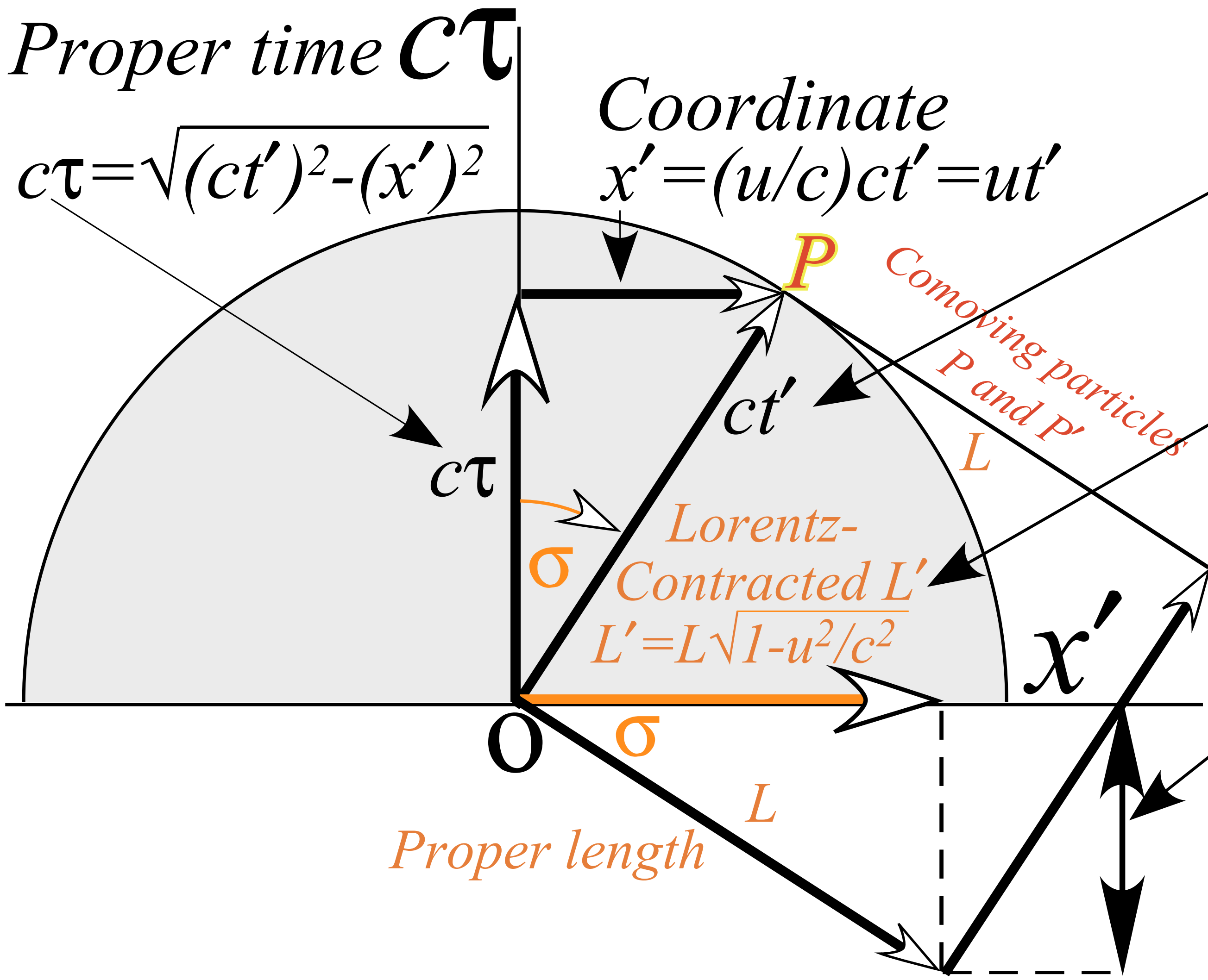
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

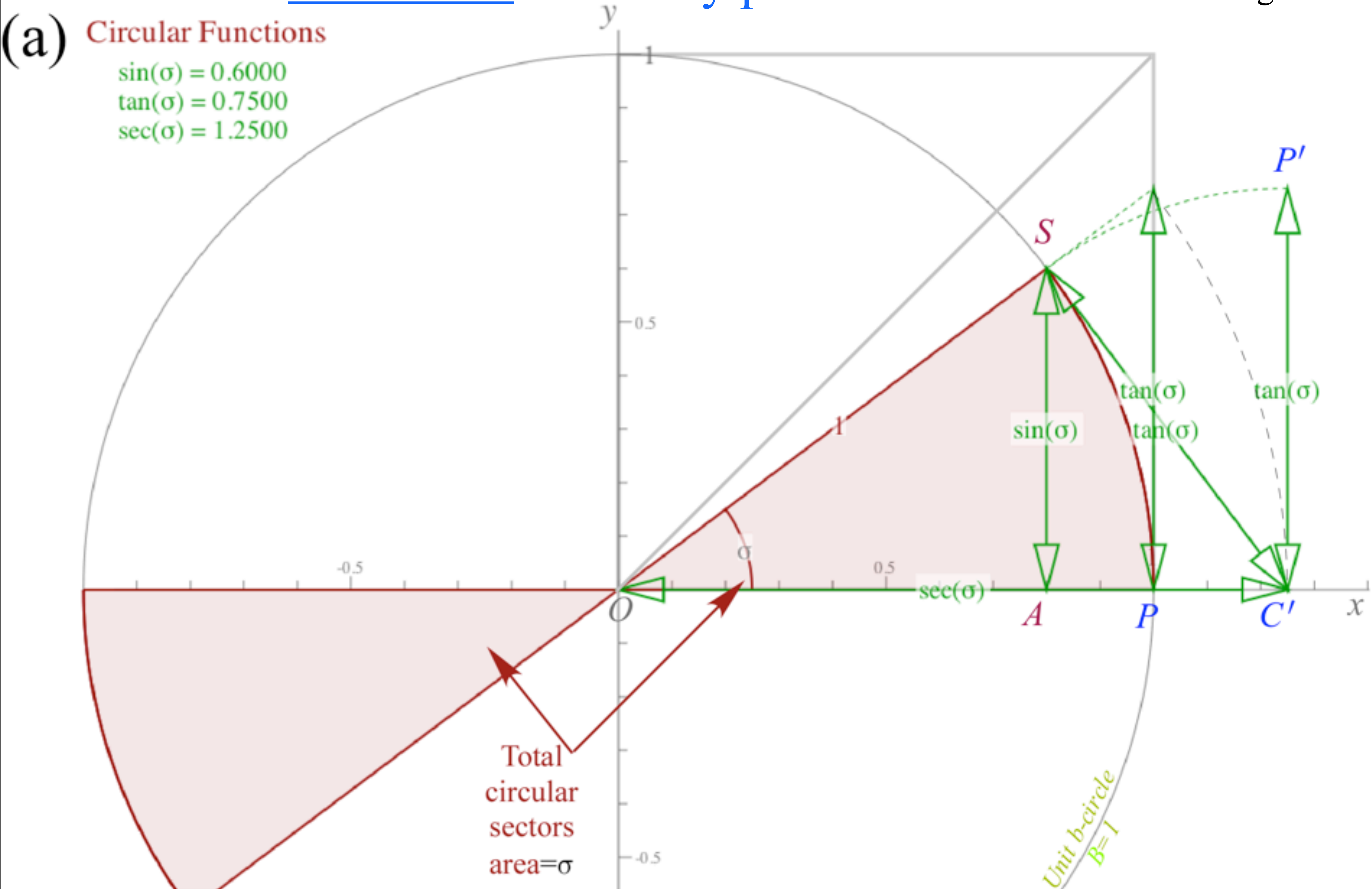
$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

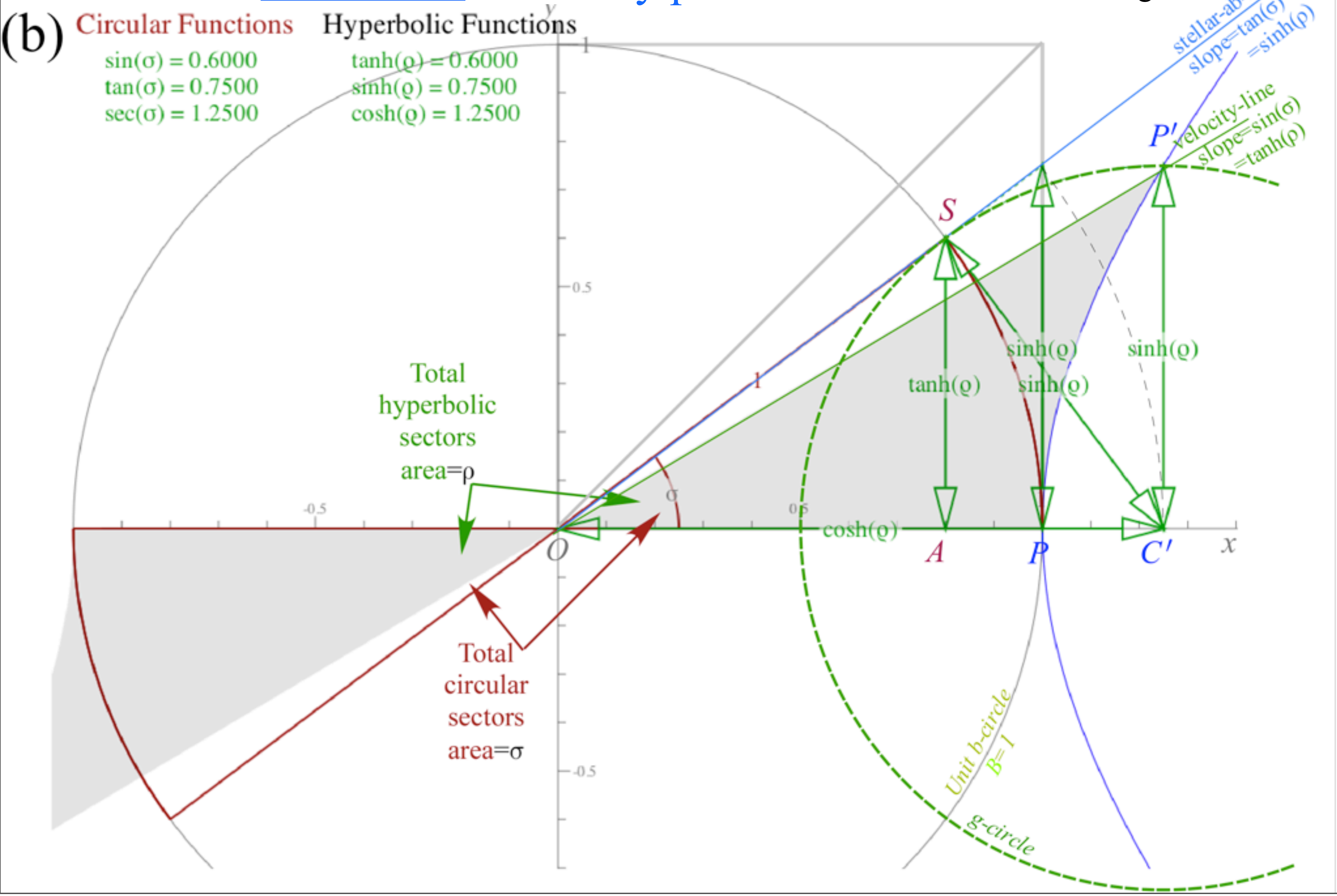
$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

to a Transverse relativity parameter: Stellar aberration angle σ

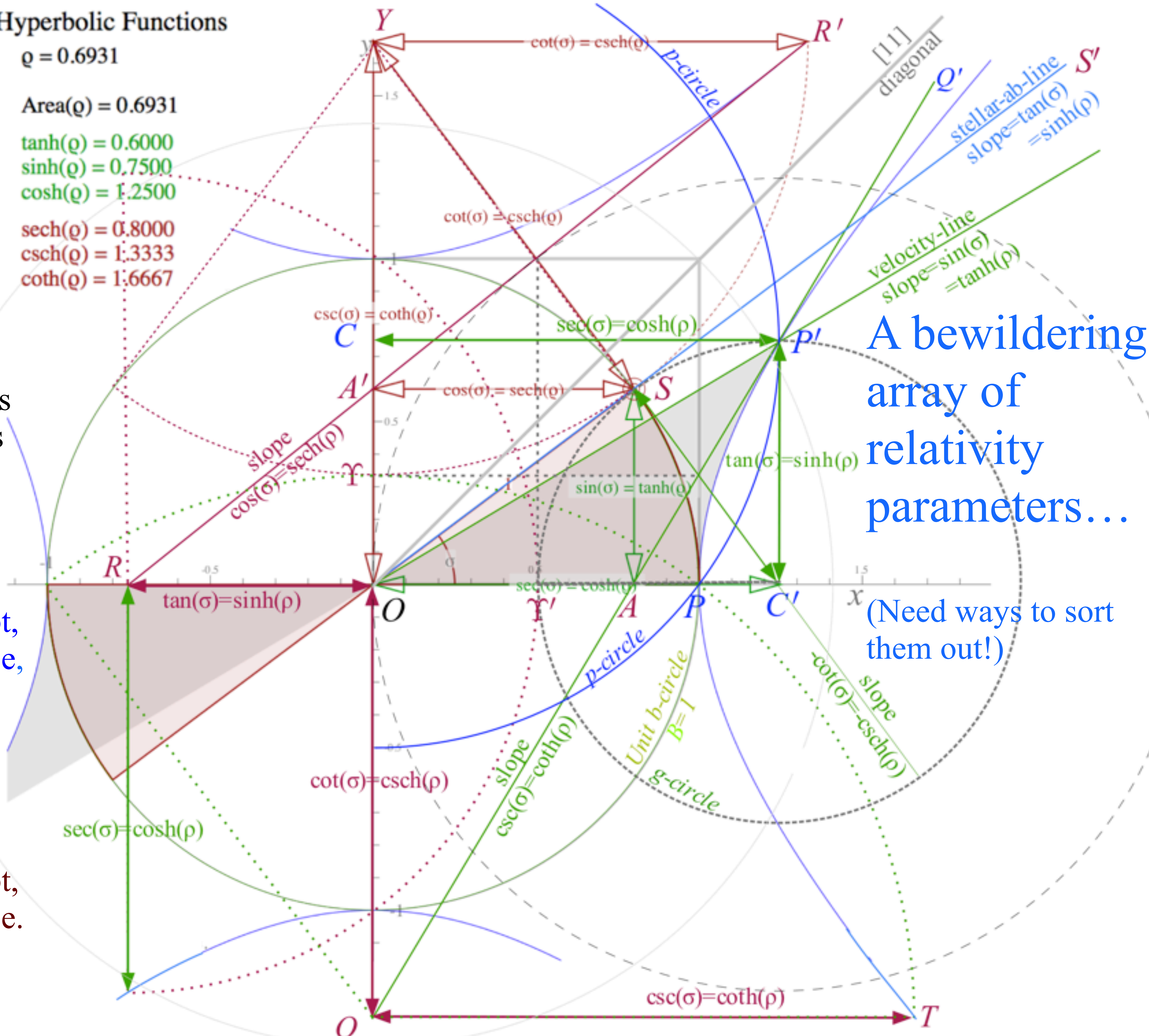


Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)
to a Transverse relativity parameter: Stellar aberration angle σ



Circular Functions	Hyperbolic Functions
m∠(σ) = 0.6435	ϱ = 0.6931
Length(σ) = 0.6435	
Area(σ) = 0.6435	Area(ϱ) = 0.6931
sin(σ) = 0.6000	tanh(ϱ) = 0.6000
tan(σ) = 0.7500	sinh(ϱ) = 0.7500
sec(σ) = 1.2500	cosh(ϱ) = 1.2500
cos(σ) = 0.8000	sech(ϱ) = 0.8000
cot(σ) = 1.3333	csch(ϱ) = 1.3333
csc(σ) = 1.6667	coth(ϱ) = 1.6667

Each of 6 trig (or trig*h*) functions serves at least once as a hyperbolic *x*,*y*, and *z* coordinate, *x*,*y*, and *z* tangent intercept, and tangent slope, and a circular *x*,*y*, and *z* coordinate, *x*,*y*, and *z* tangent intercept, and tangent slope.

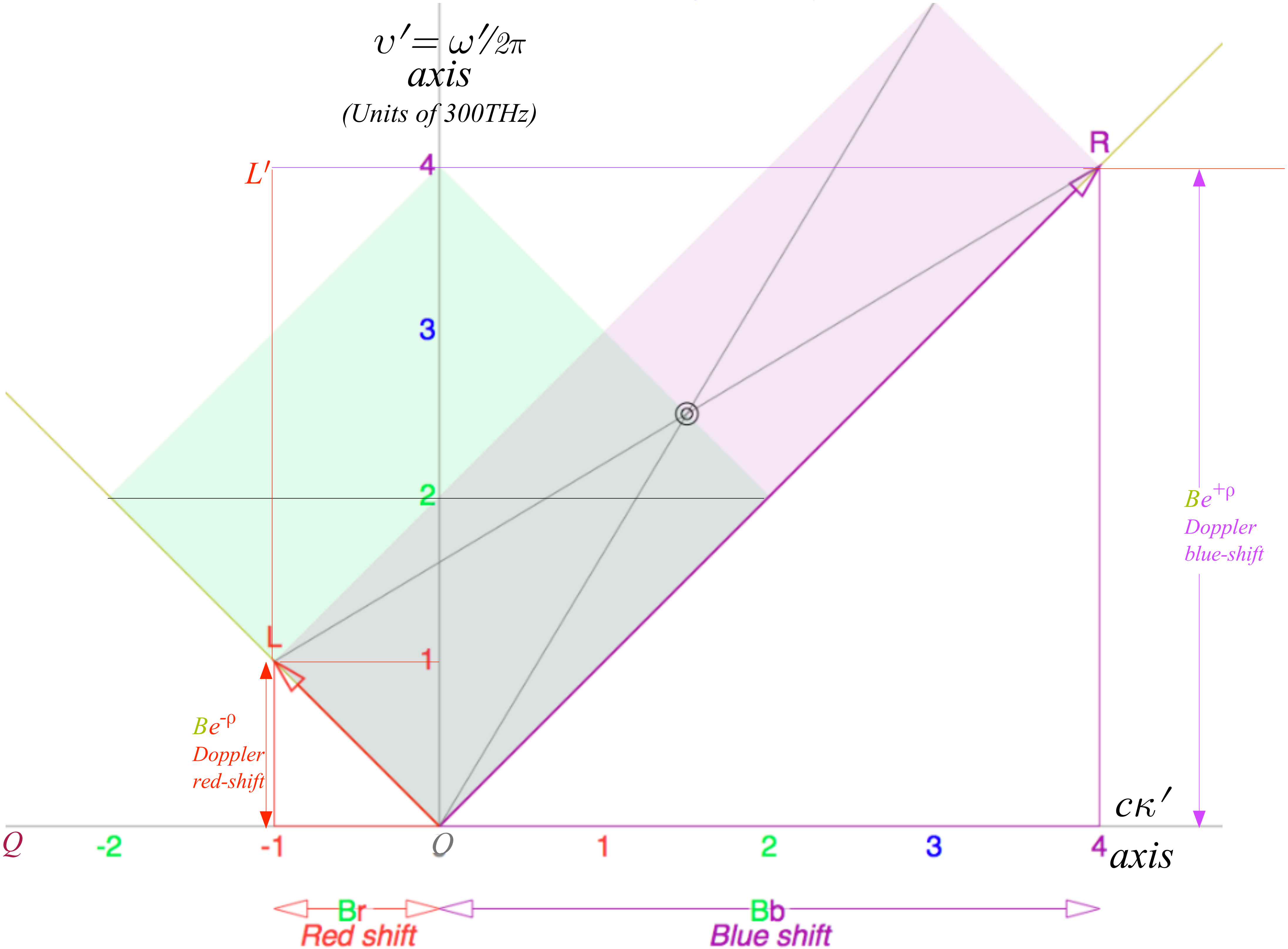


A bewildering array of relativity parameters…

(Need ways to sort them out!)

Optical wave parameters for relativity
...and their geometry

It's all based on Doppler shifts
RED r and BLUE b=1/r
RED $e^{-\rho}$ and BLUE $e^{+\rho}$

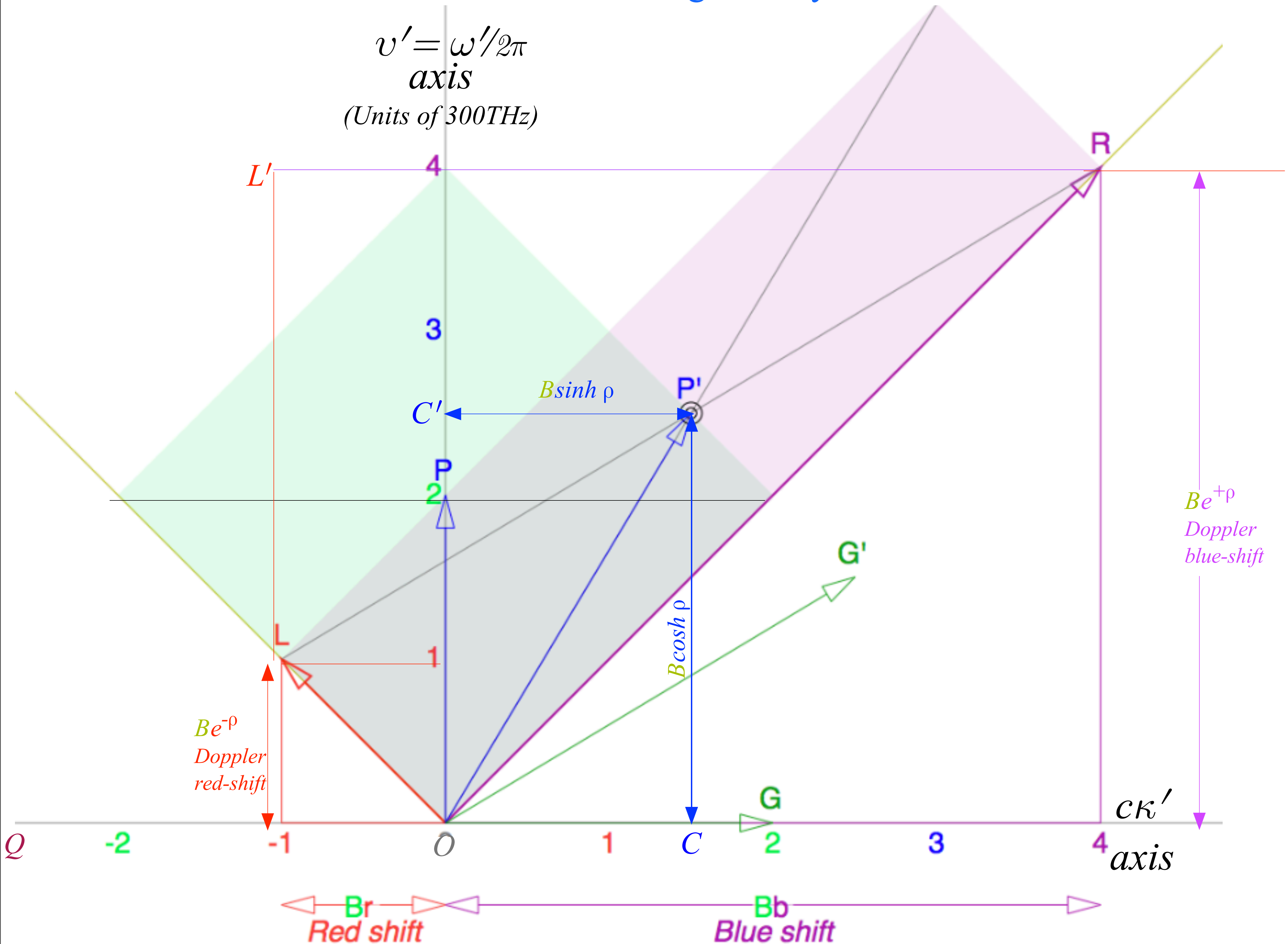


Optical wave parameters for relativity

...and their geometry

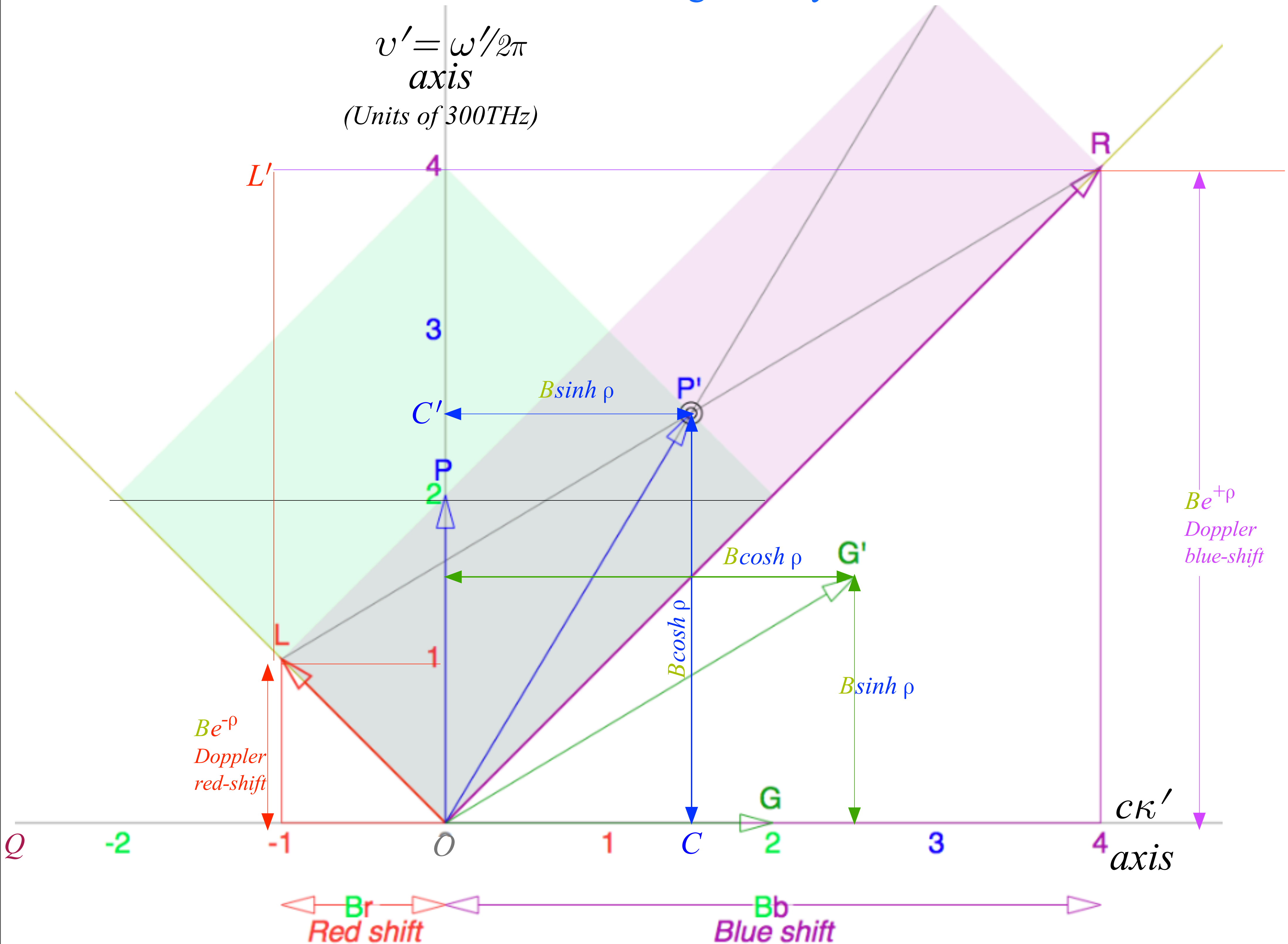
It's all based on Doppler shifts
RED r and BLUE b=1/r
RED $e^{-\rho}$ and BLUE $e^{+\rho}$

PHASE Freq is HALF-SUM
$B\cosh\rho$
PHASE k-vec is HALF-DIFF
$B\sinh\rho$



Optical wave parameters for relativity

...and their geometry



It's all based on Doppler shifts
RED r and BLUE b=1/r
RED $e^{-\rho}$ and BLUE $e^{+\rho}$

PHASE Freq is HALF-SUM
$B\cosh\rho$

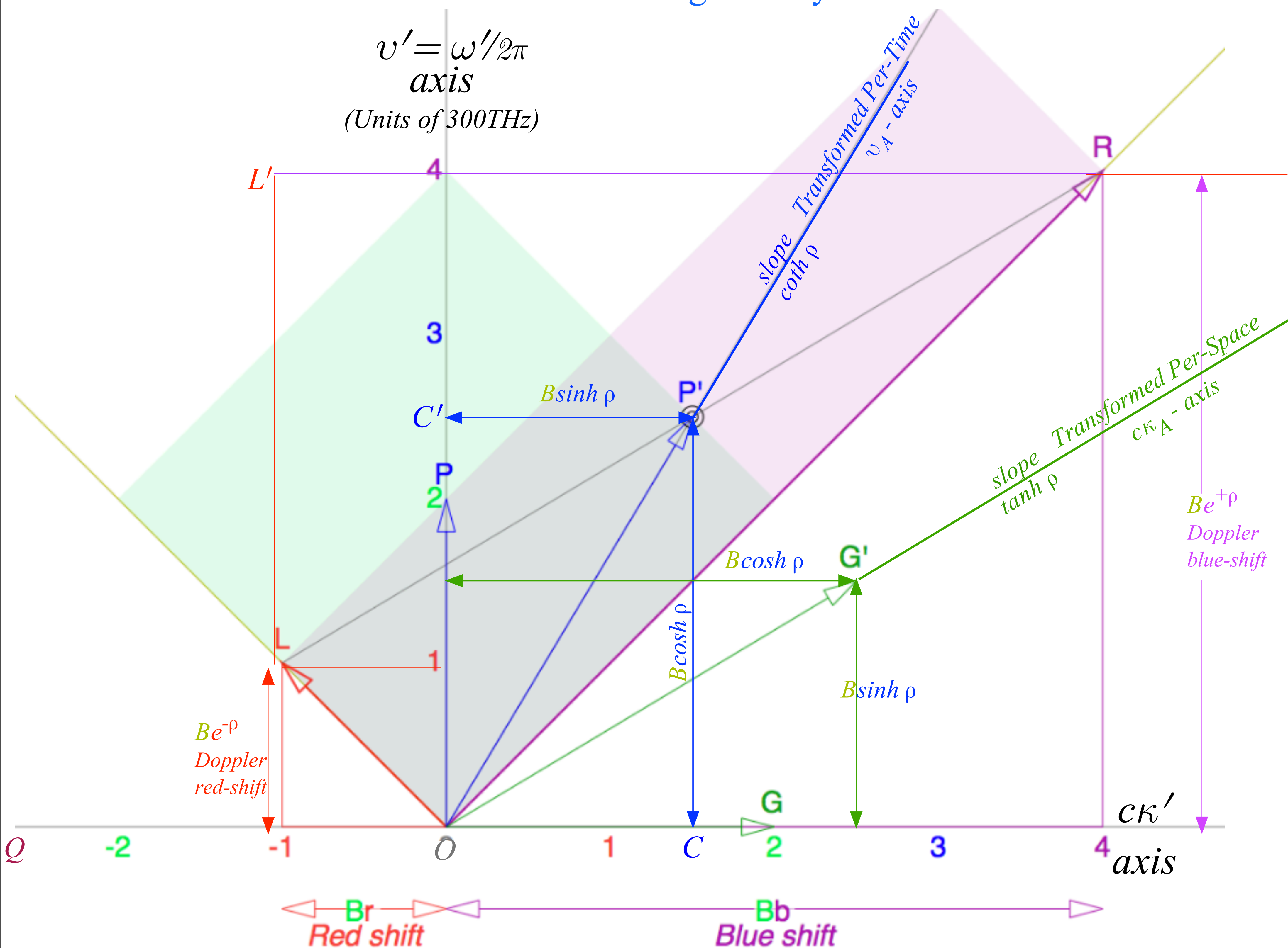
PHASE k-vec is HALF-DIFF
$B\sinh\rho$

GROUP Freq is HALF-DIFF
$B\sinh\rho$

GROUP k-vec is HALF-SUM
$B\cosh\rho$

Optical wave parameters for relativity

...and their geometry



It's all based on Doppler shifts
RED r and BLUE b=1/r
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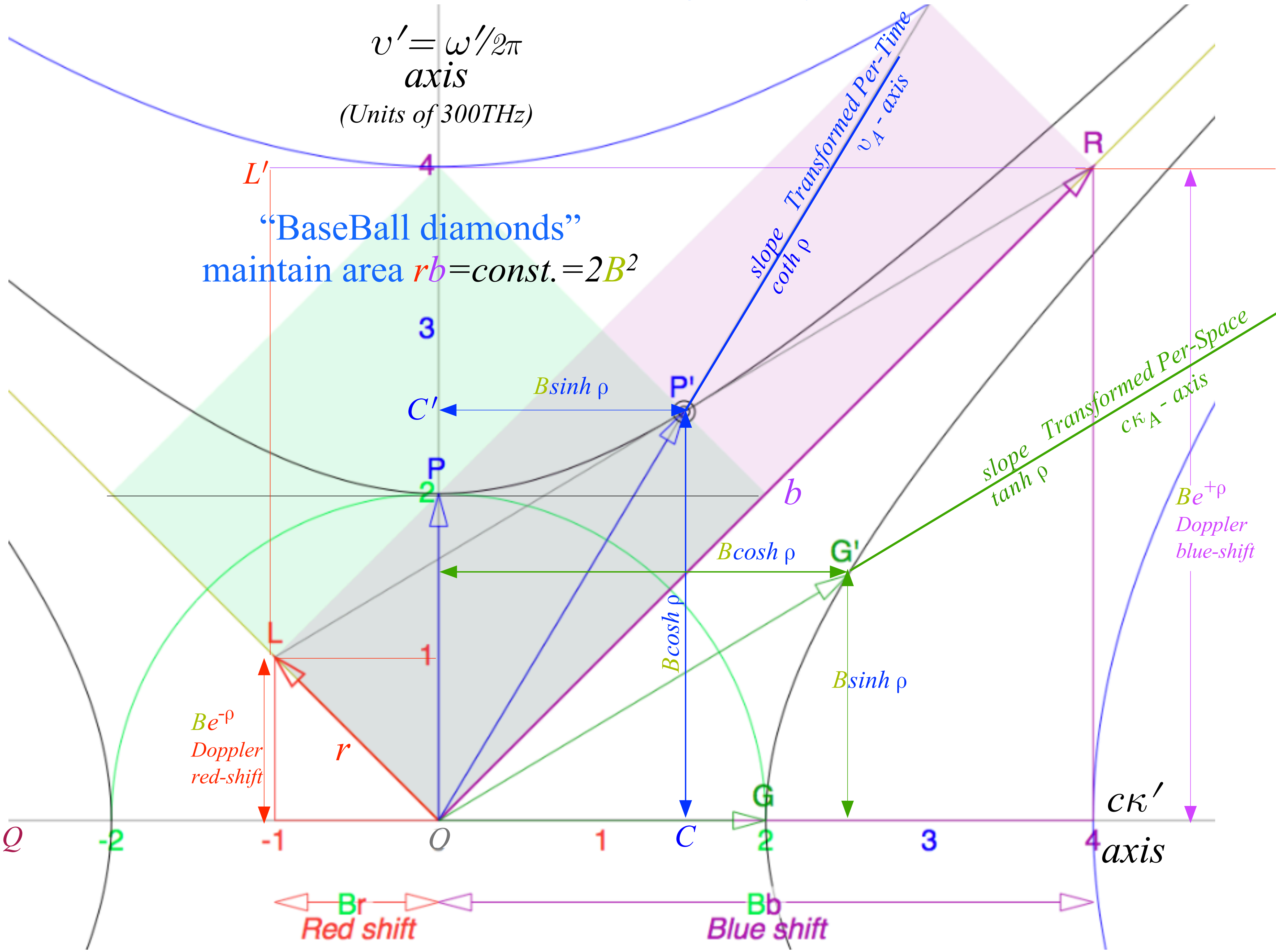
PHASE Freq is HALF-SUM
Bcoshρ
PHASE k-vec is HALF-DIFF
Bsinhρ

GROUP Freq is HALF-DIFF
Bsinhρ
GROUP k-vec is HALF-SUM
Bcoshρ

GROUP is per-Space axis or
$k_x = 2\pi\,\kappa_x$ = $Kappa_x$ dimension
PHASE is per-Time axis or
$\omega = 2\pi\,\upsilon$ = Nu dimension

Optical wave parameters for relativity

...and their geometry



It's all based on Doppler shifts
RED r and BLUE b=1/r
RED $e^{-\rho}$ and BLUE $e^{+\rho}$

PHASE Freq is HALF-SUM
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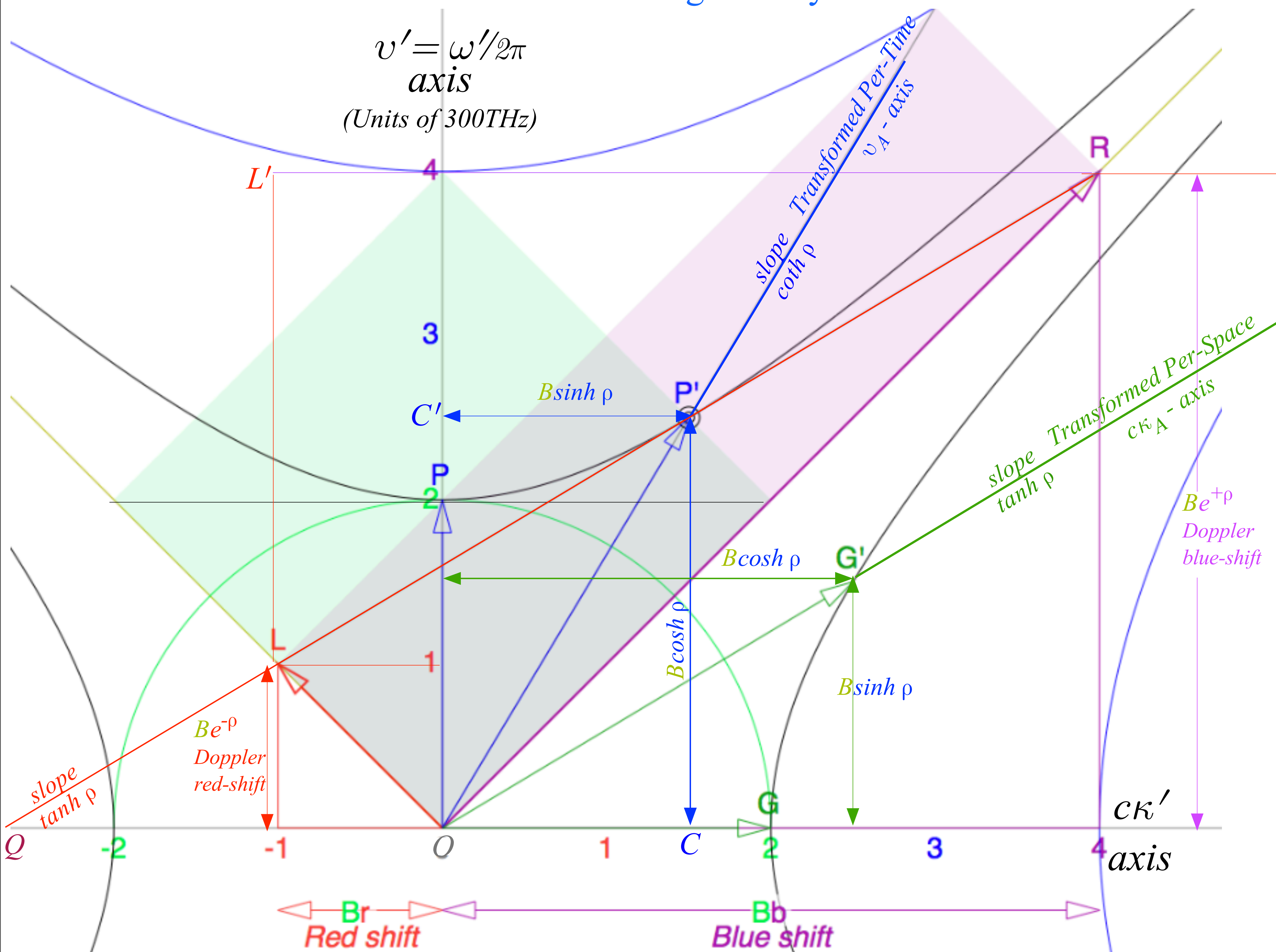

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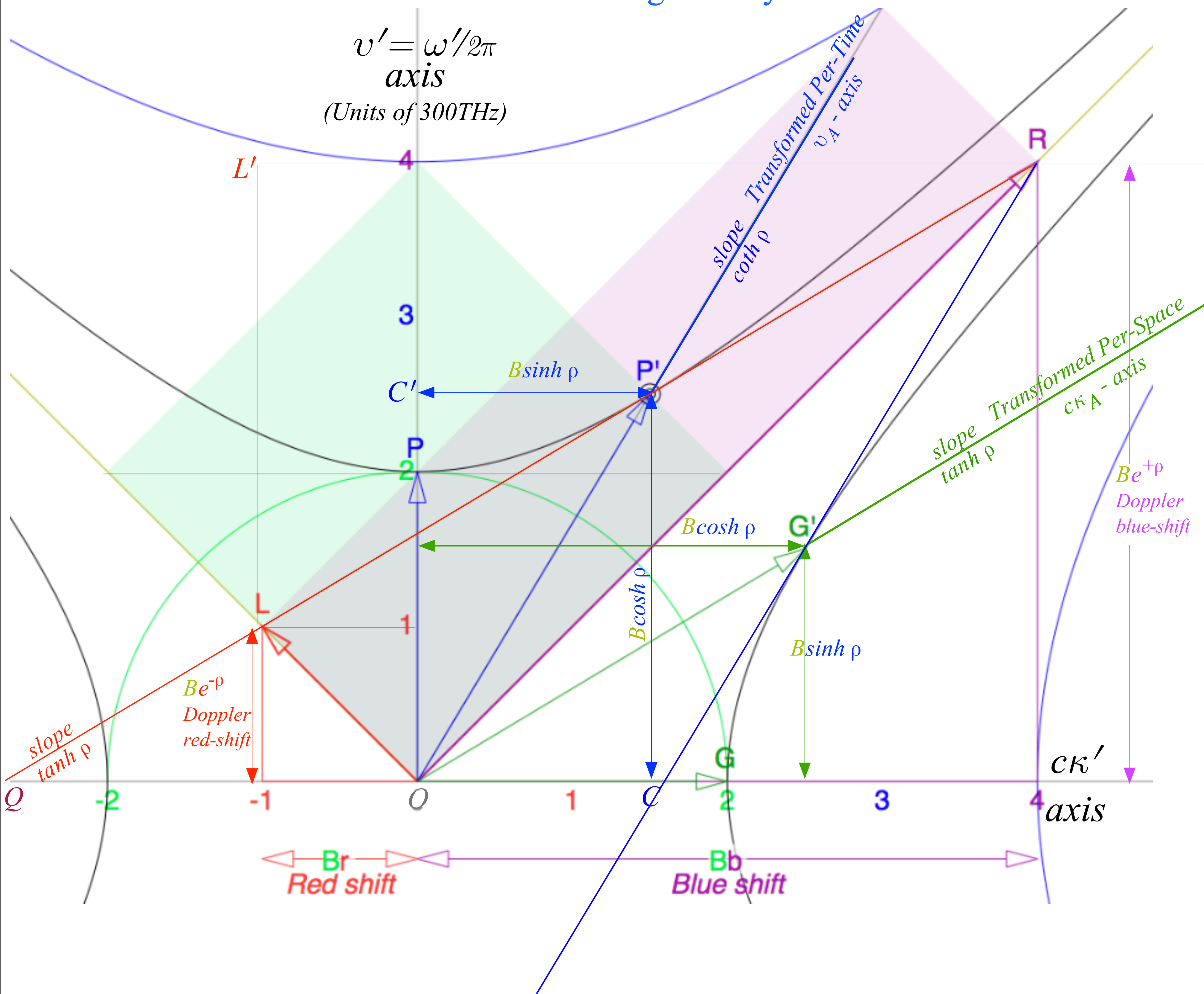

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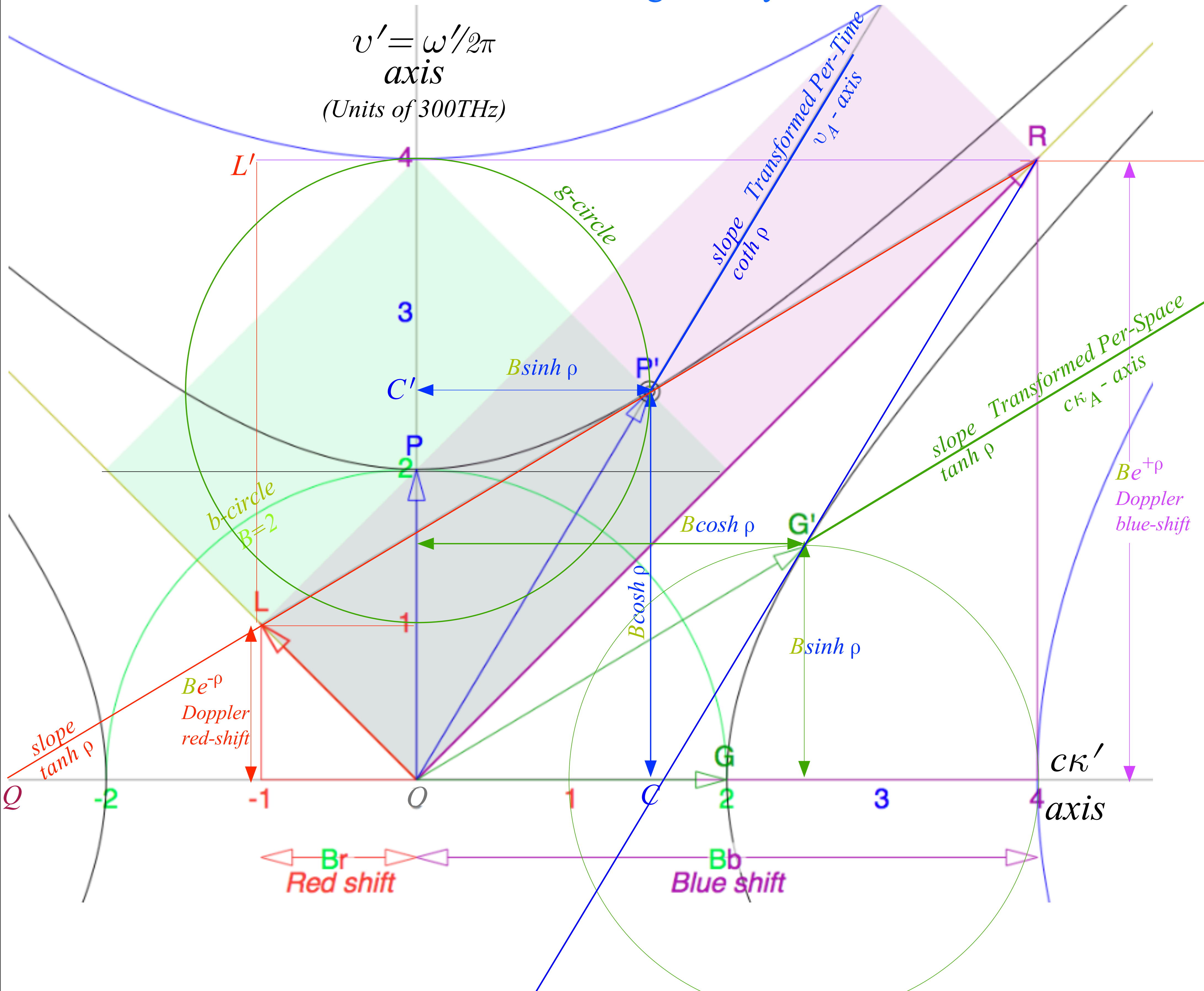

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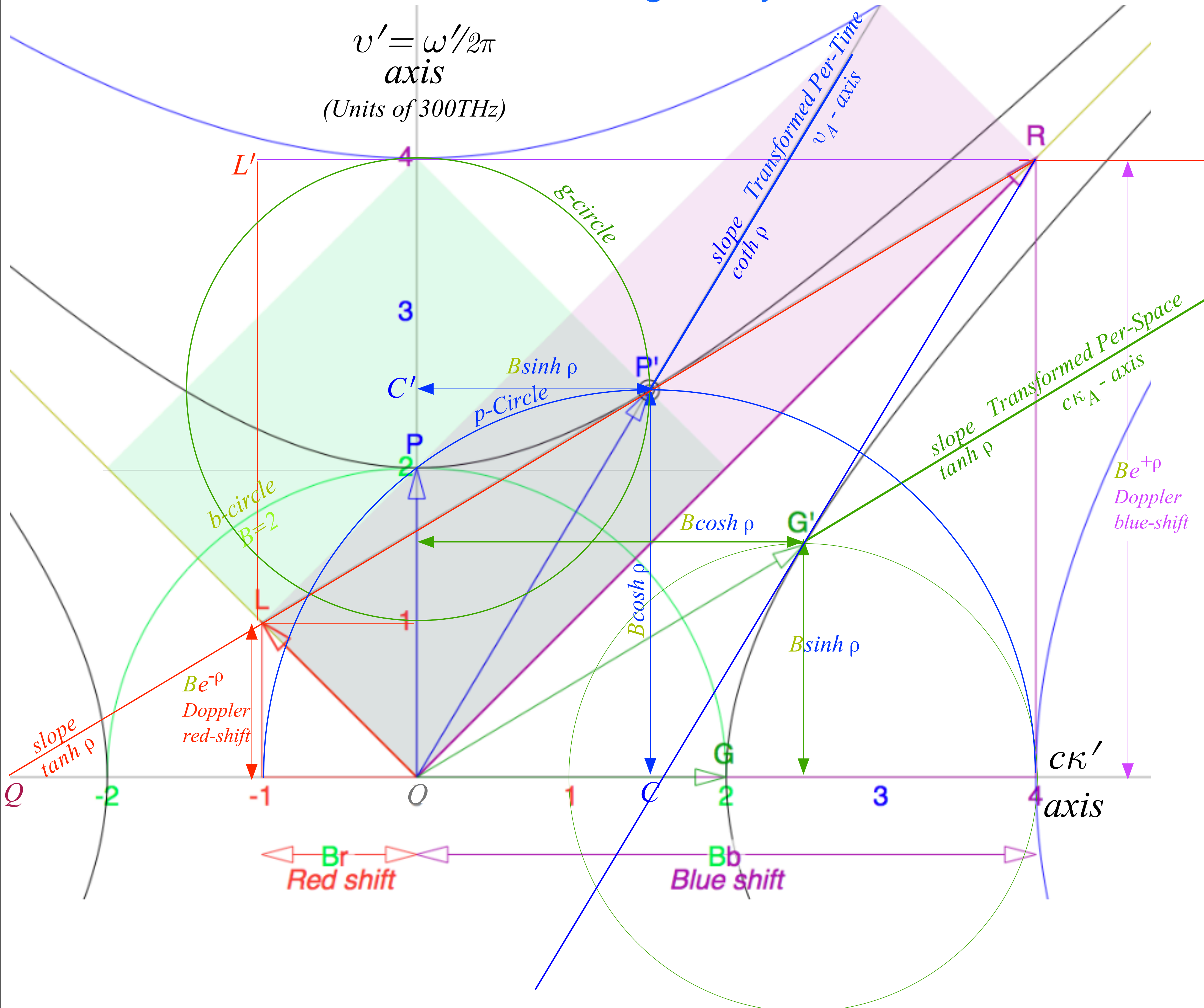

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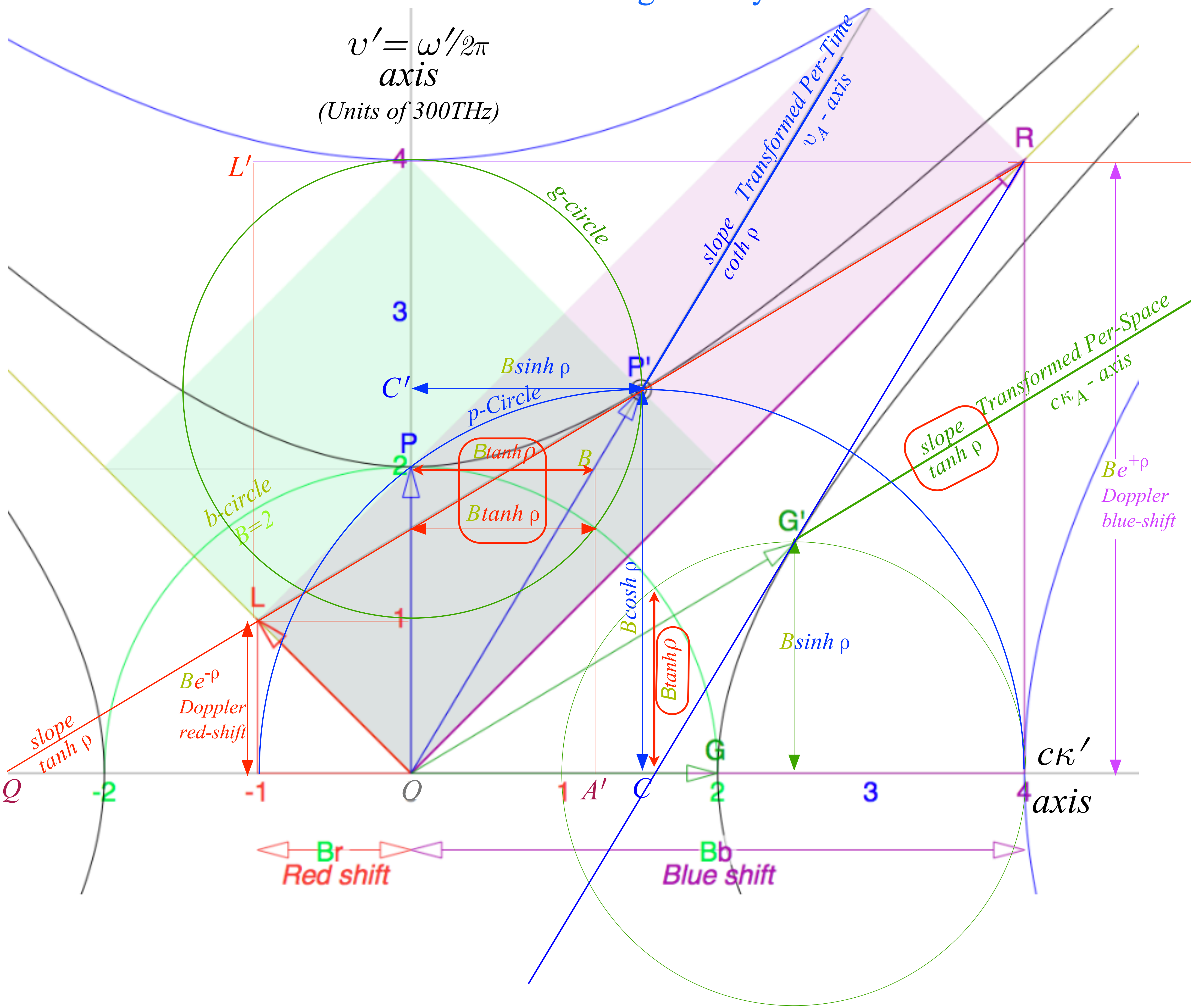

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$V_{group}/c = \tanh\rho$

Optical wave parameters for relativity

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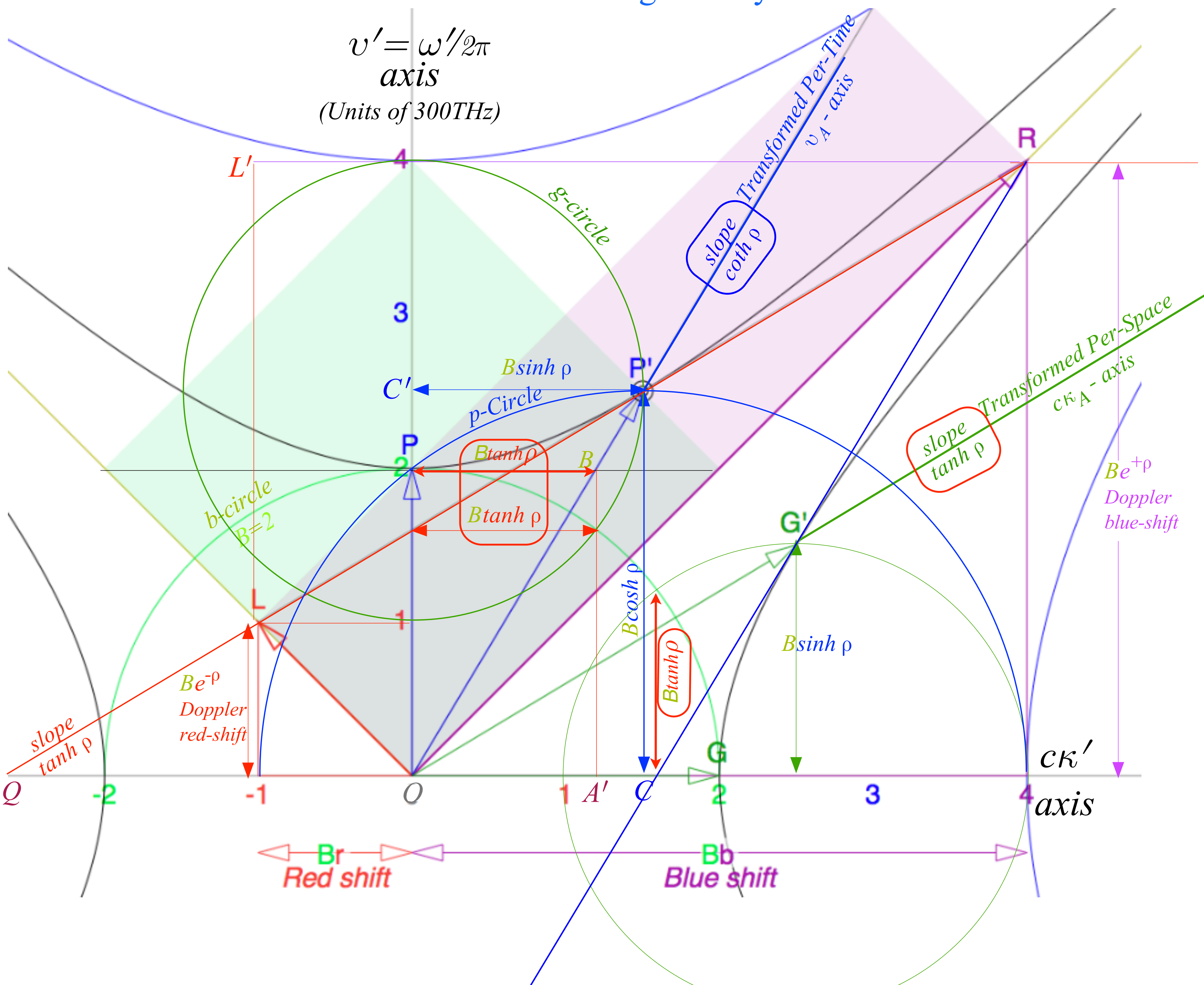

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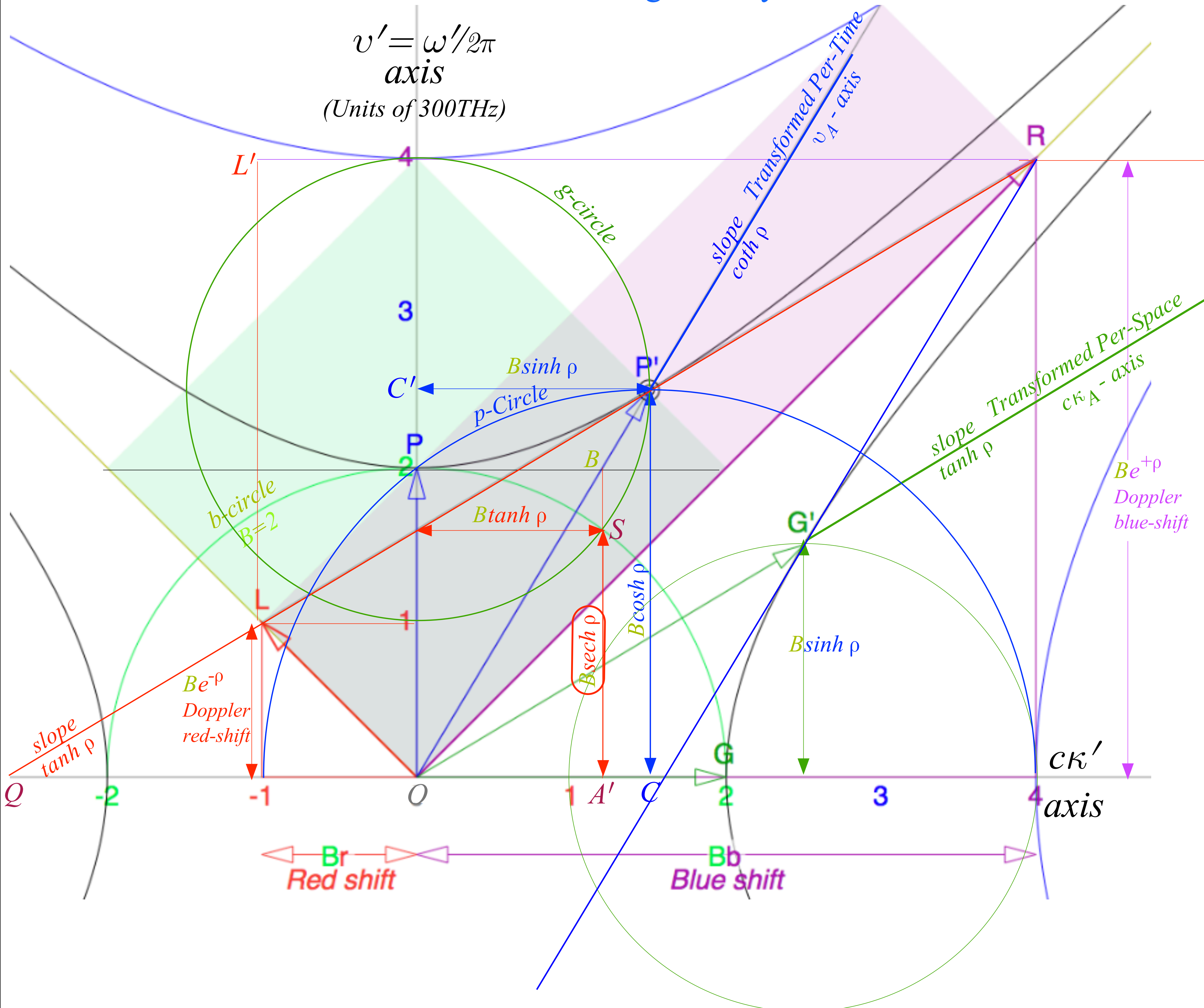

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is compliment coord to
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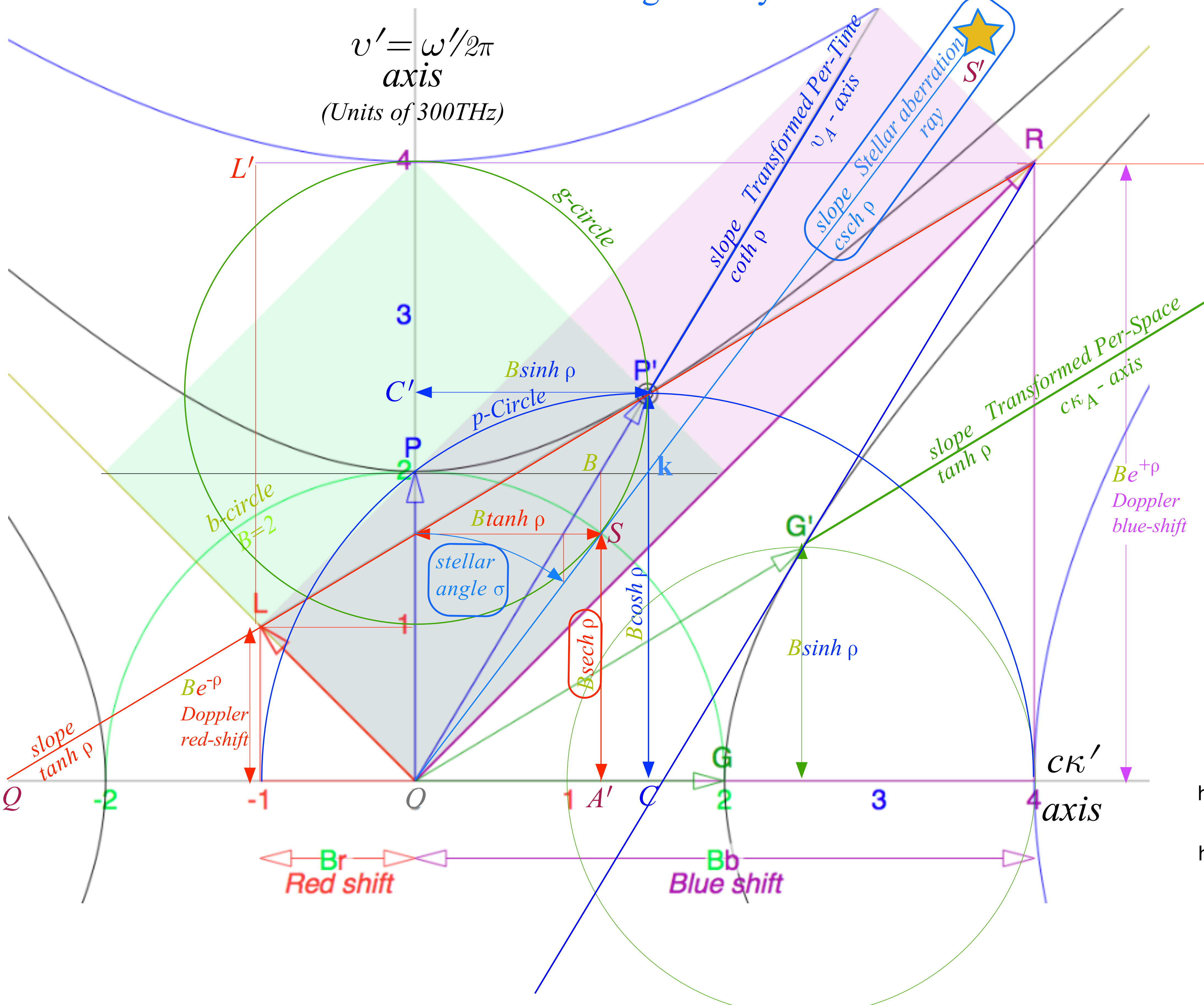
Optical wave parameters for relativity

...and their geometry



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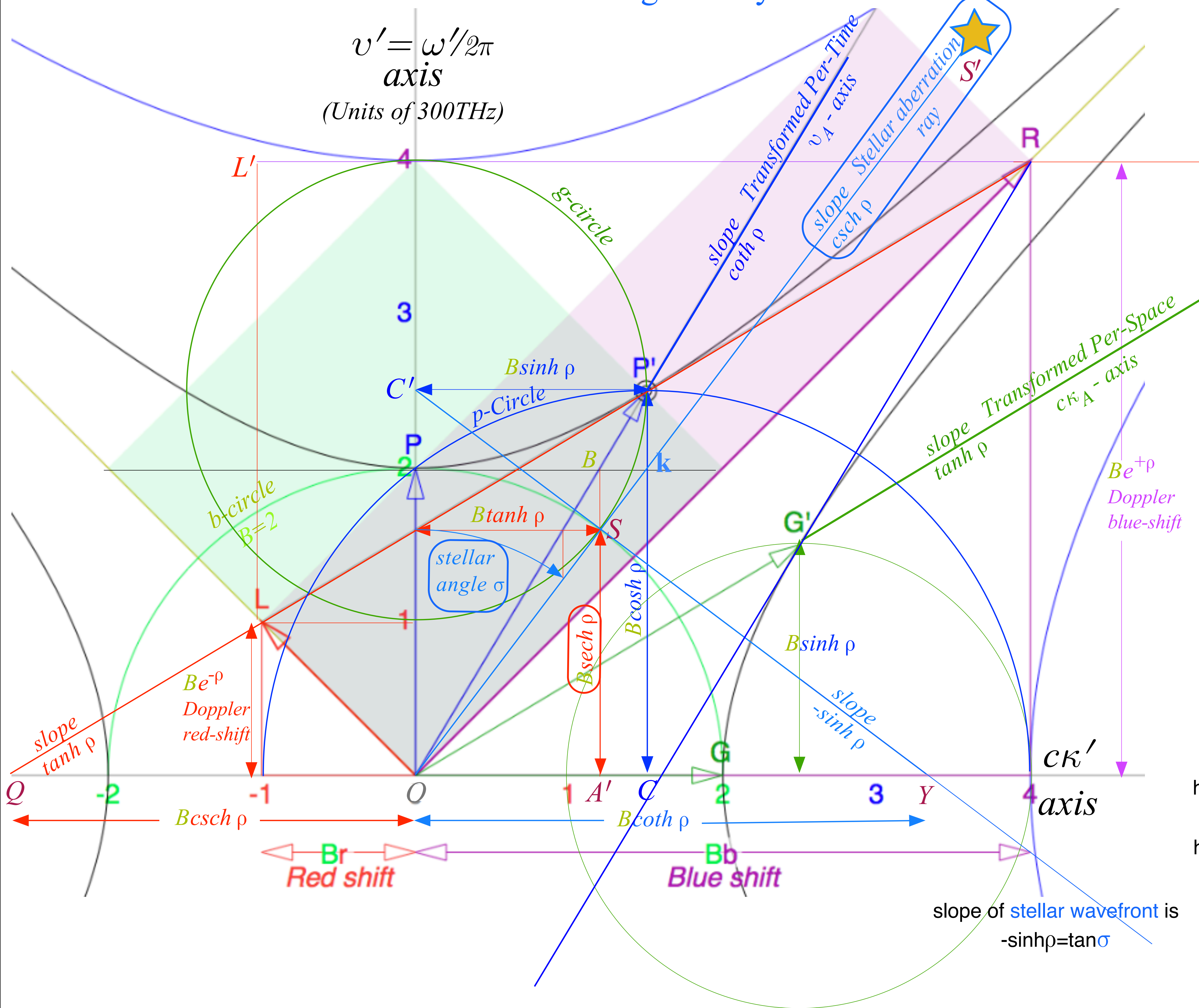
hyper-secant Bsechρ = Bcosσ is compliment coord to hyper-tangent Btanhρ = Bsinσ for stellar aberration angleσ slope of stellar k-vector is cschρ=cotσ

Optical wave parameters for relativity

...and their geometry



slope of stellar wavefront is -sinhρ=tanσ

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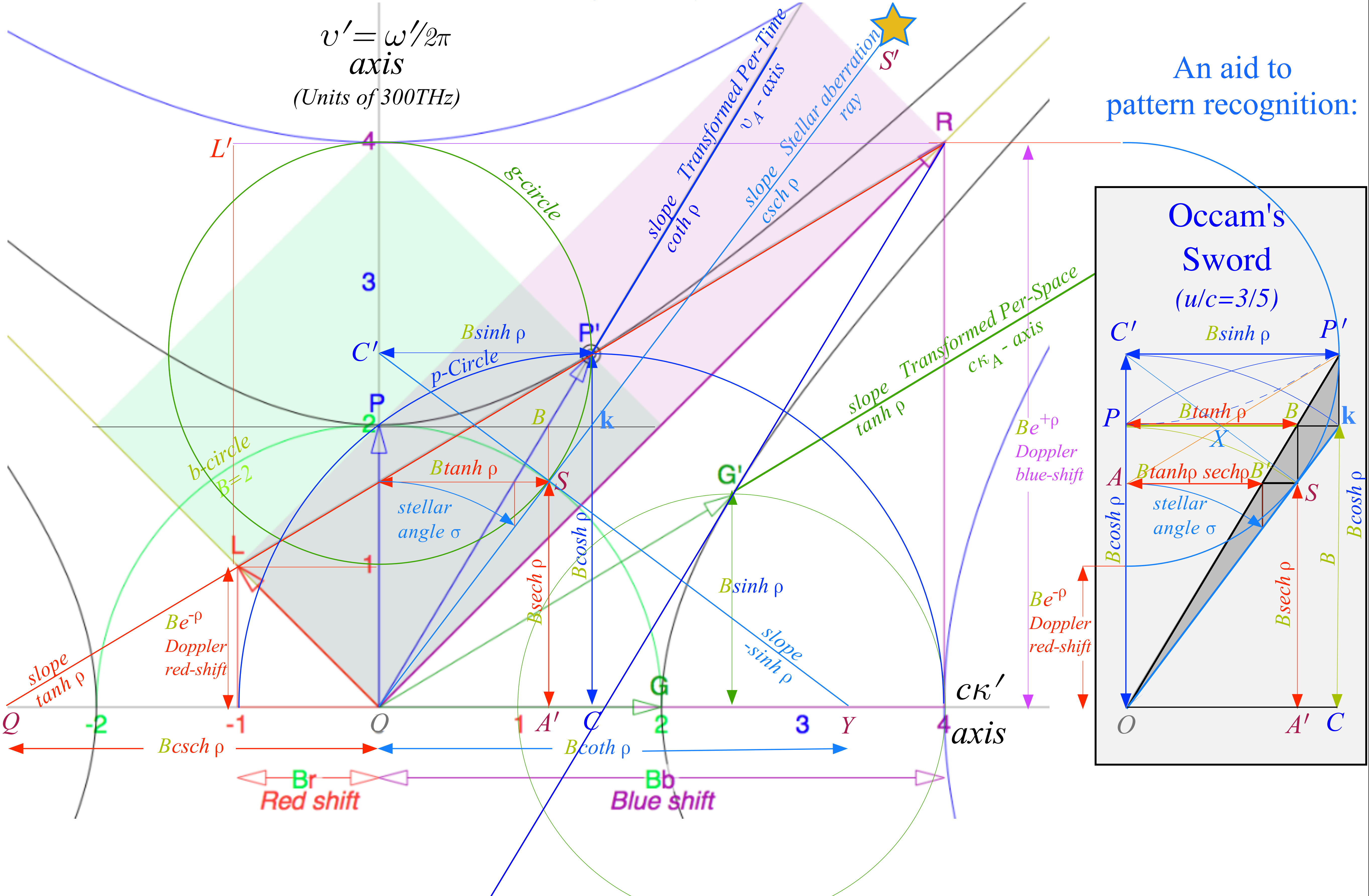
Optical wave parameters for relativity

...and their geometry

An aid to pattern recognition:



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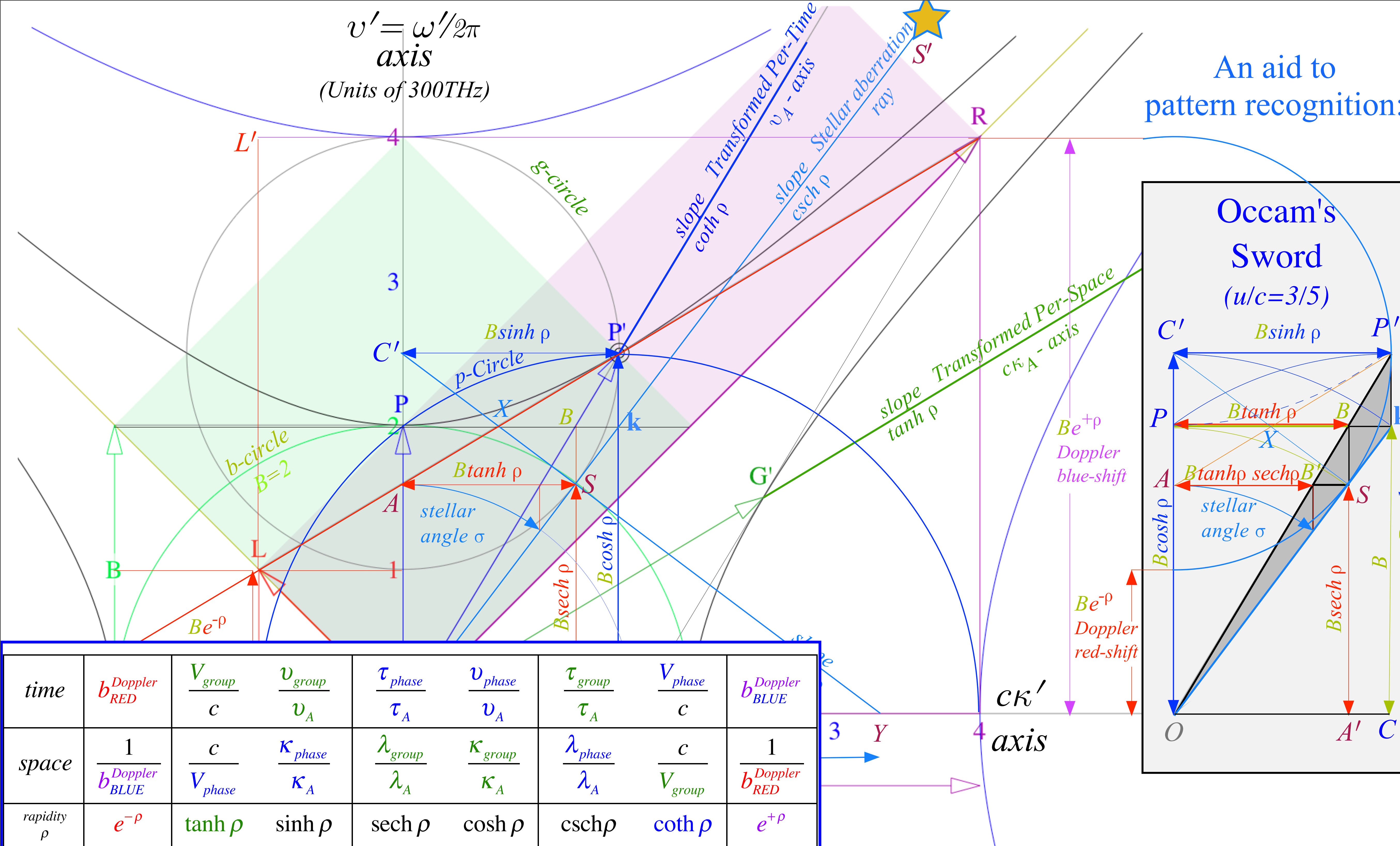


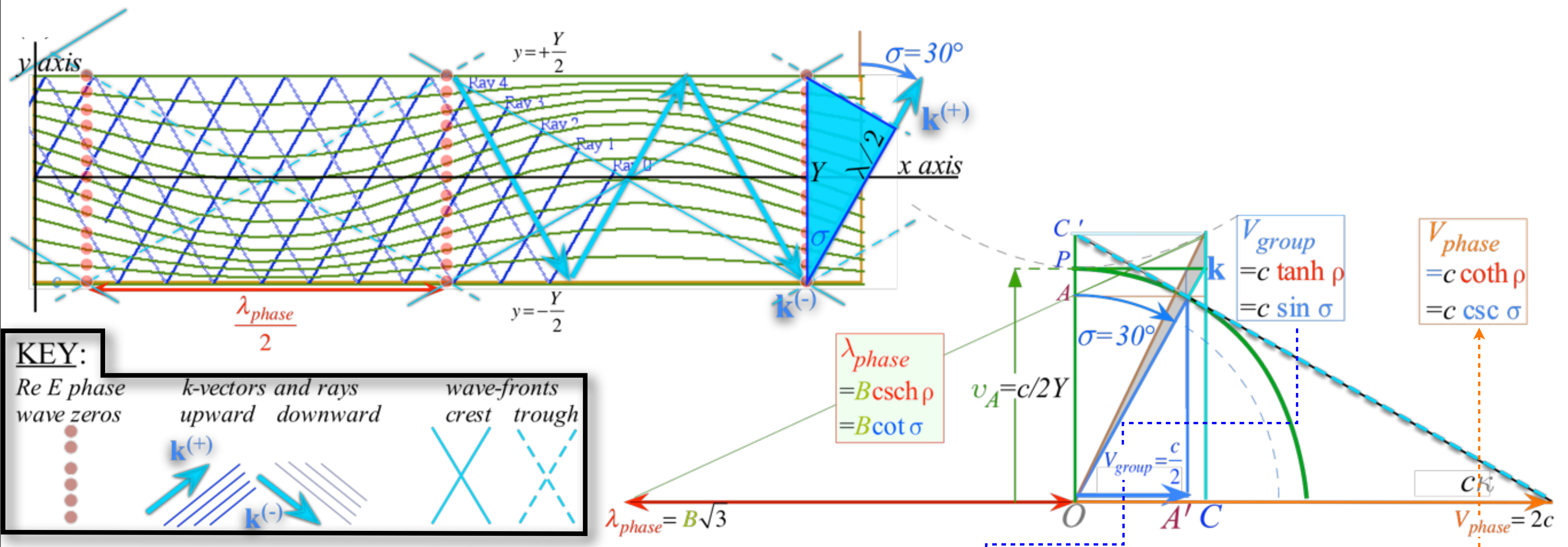
time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{\beta^2-1}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Table of 16 wave parameters (includes inverses) for relativity

...and values for $u/c=3/5$

Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time



Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

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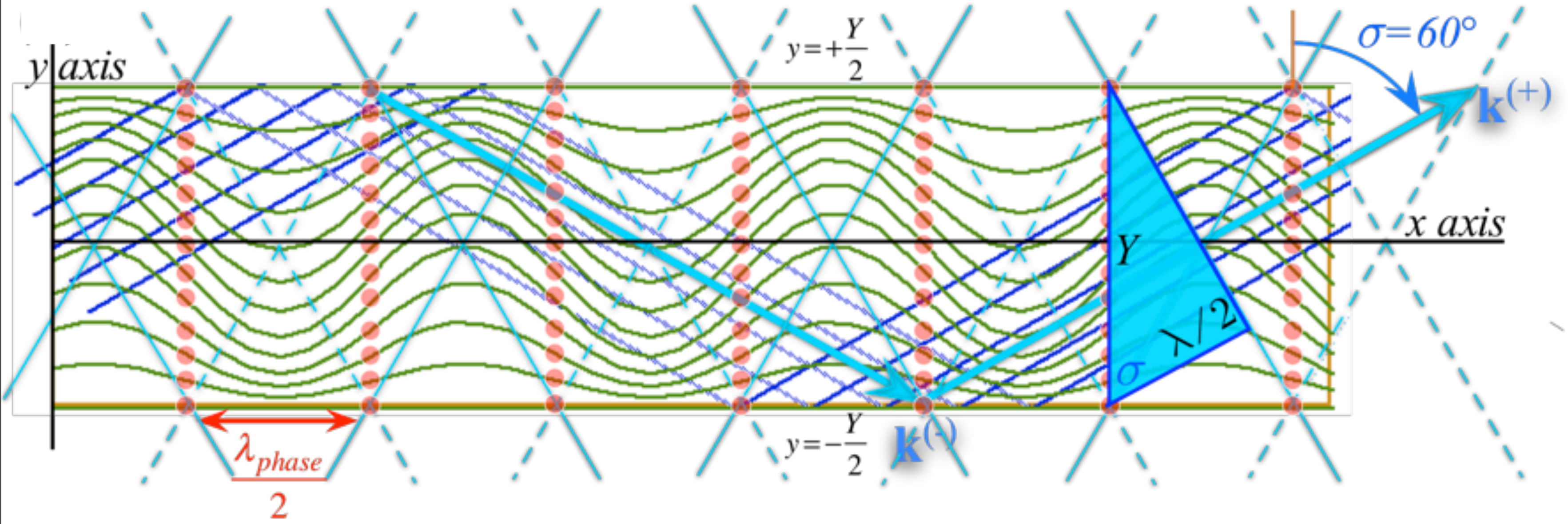

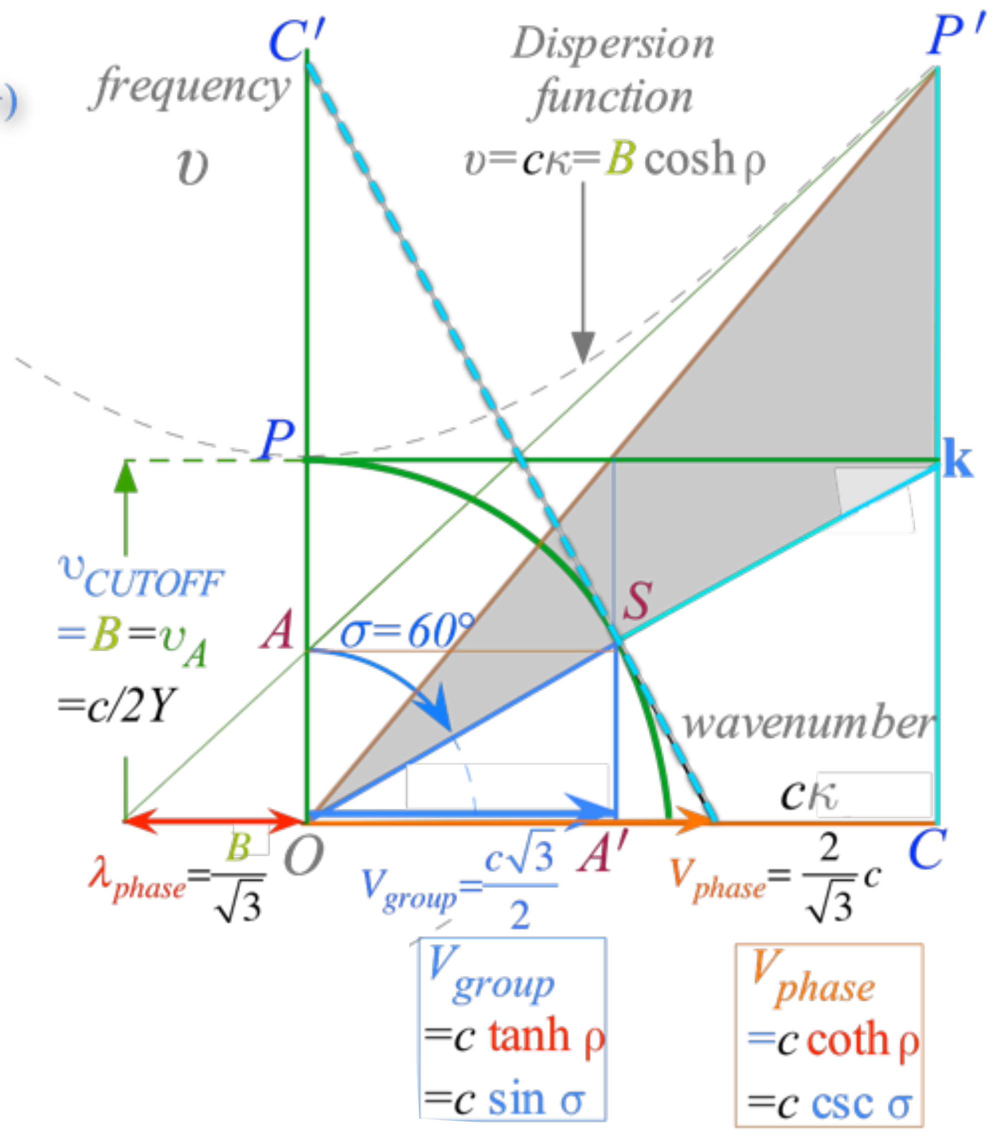

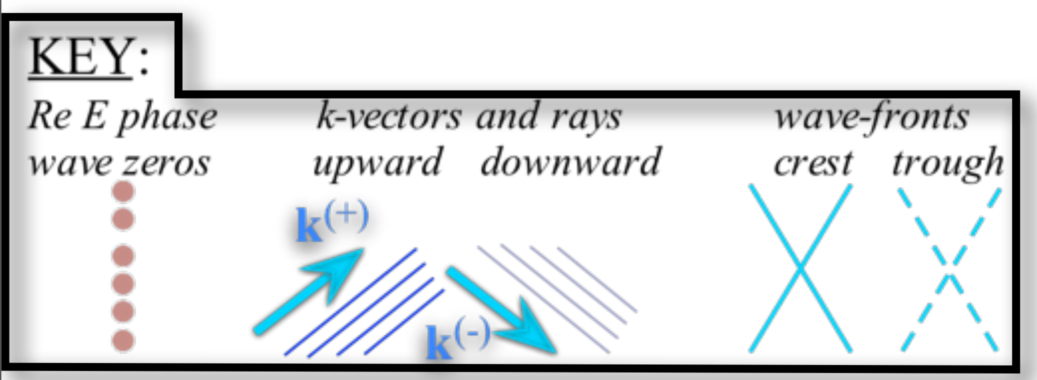

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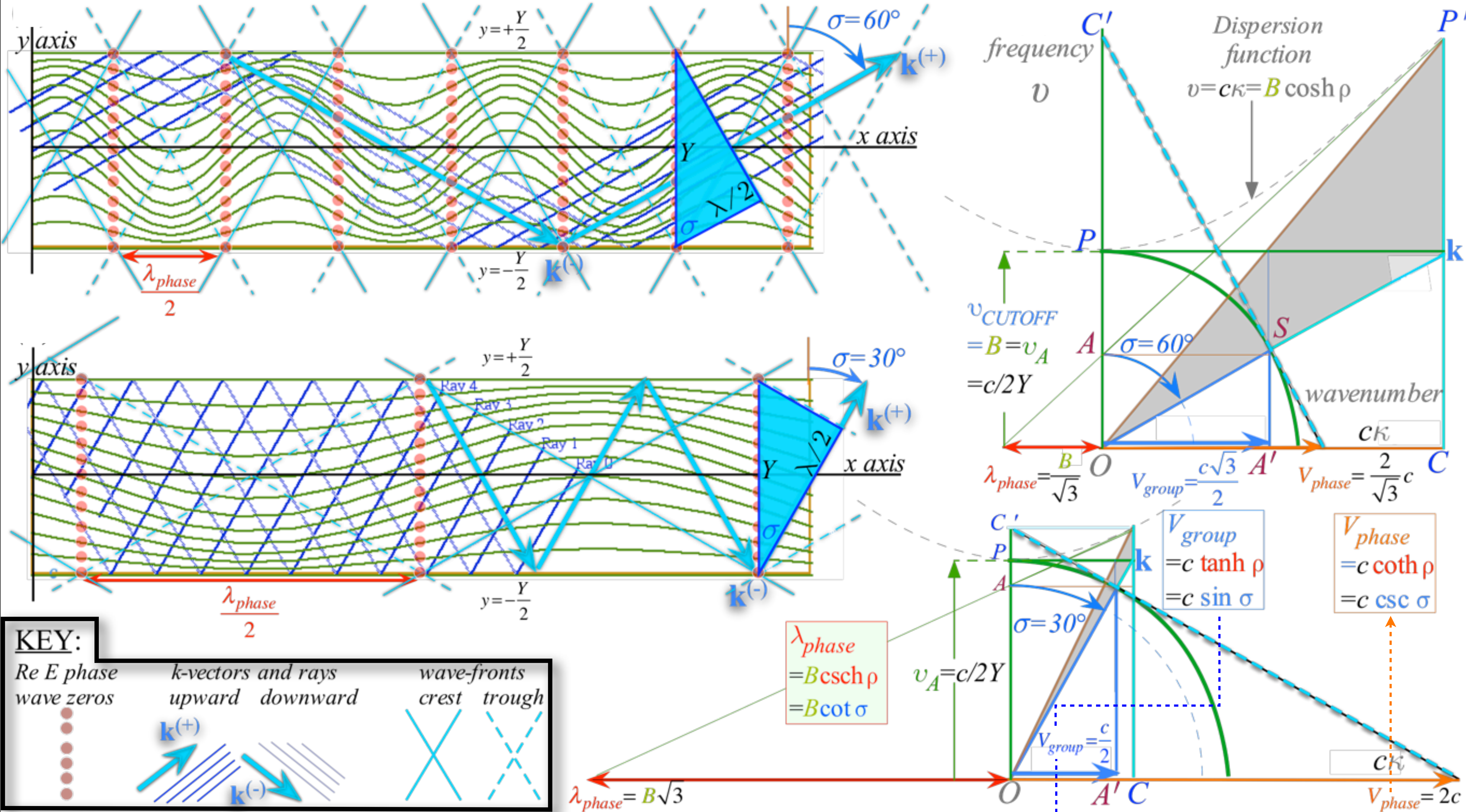
Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)



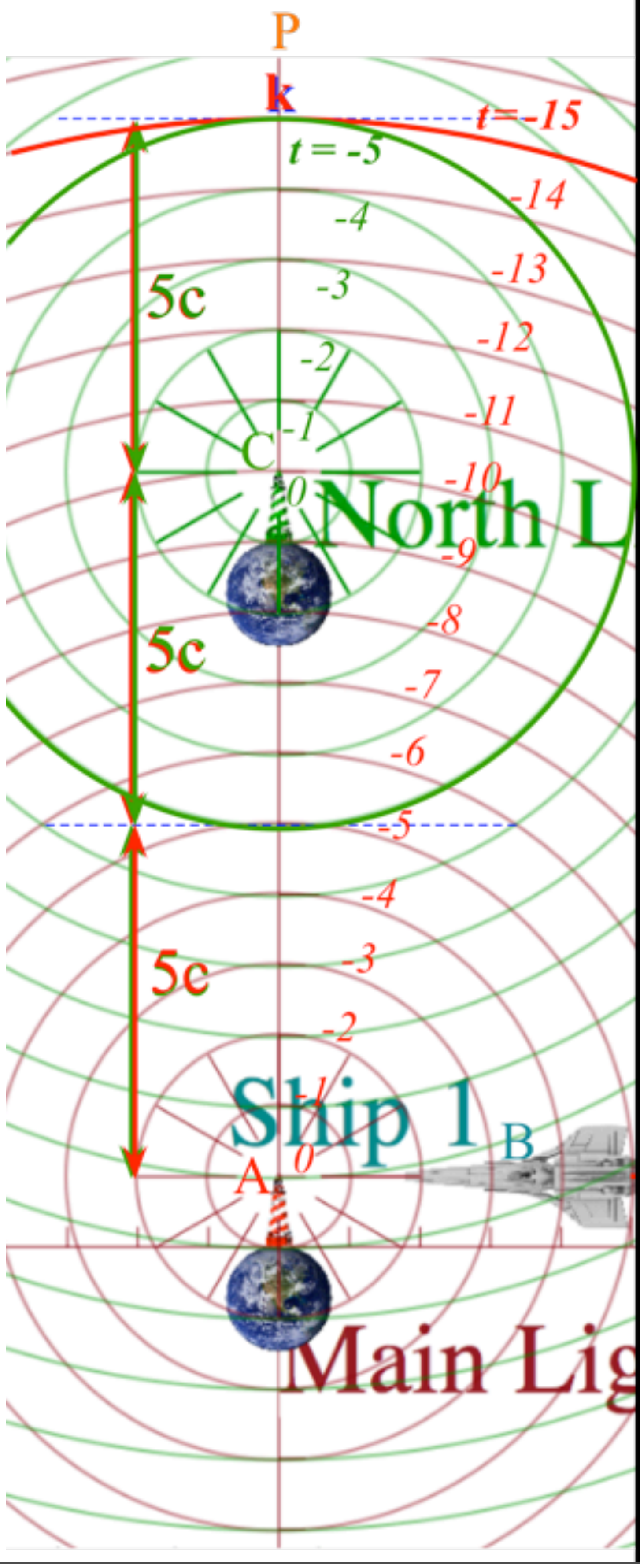
Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

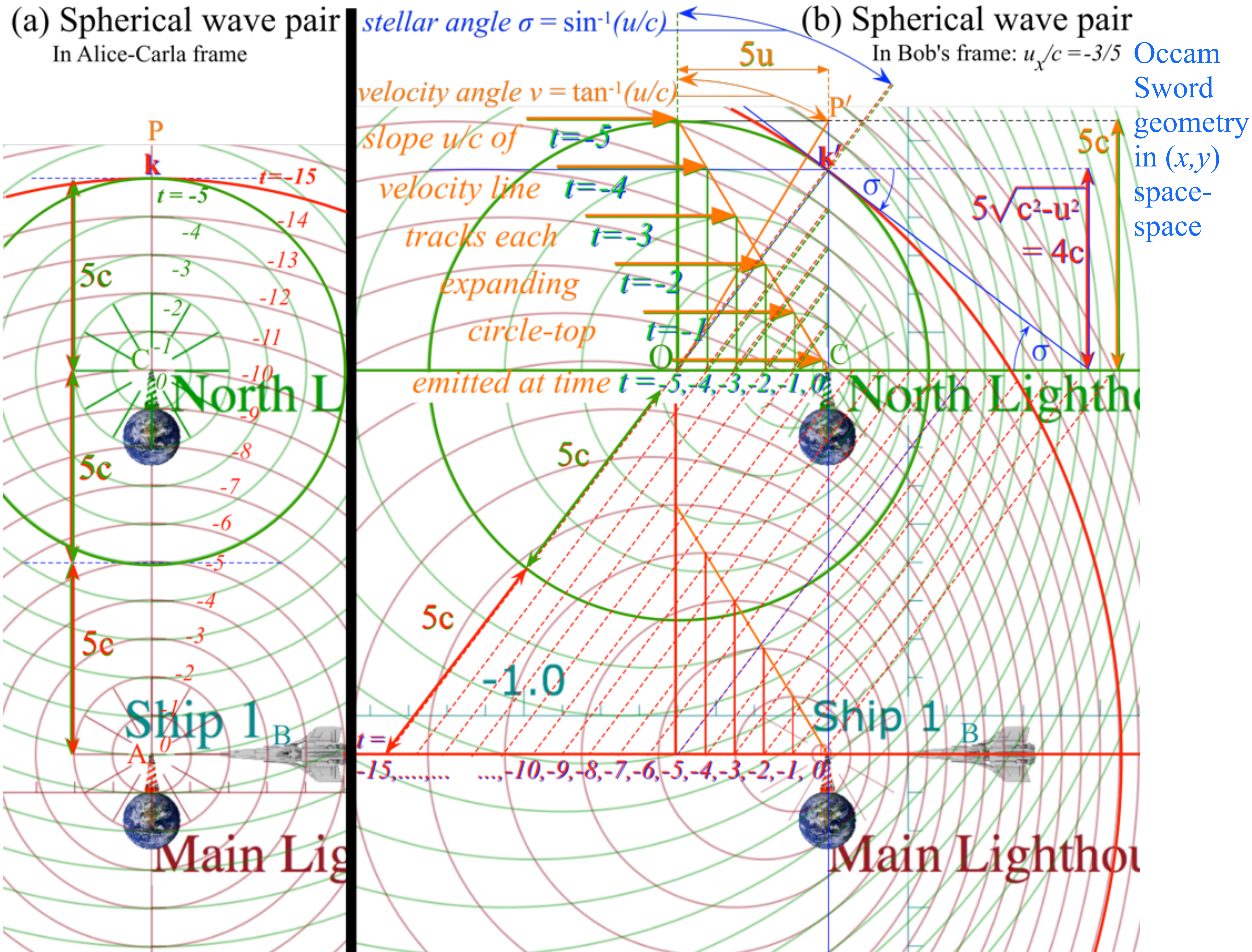
Spherical wave relativistic geometry

Also, aided by Occam's Sword

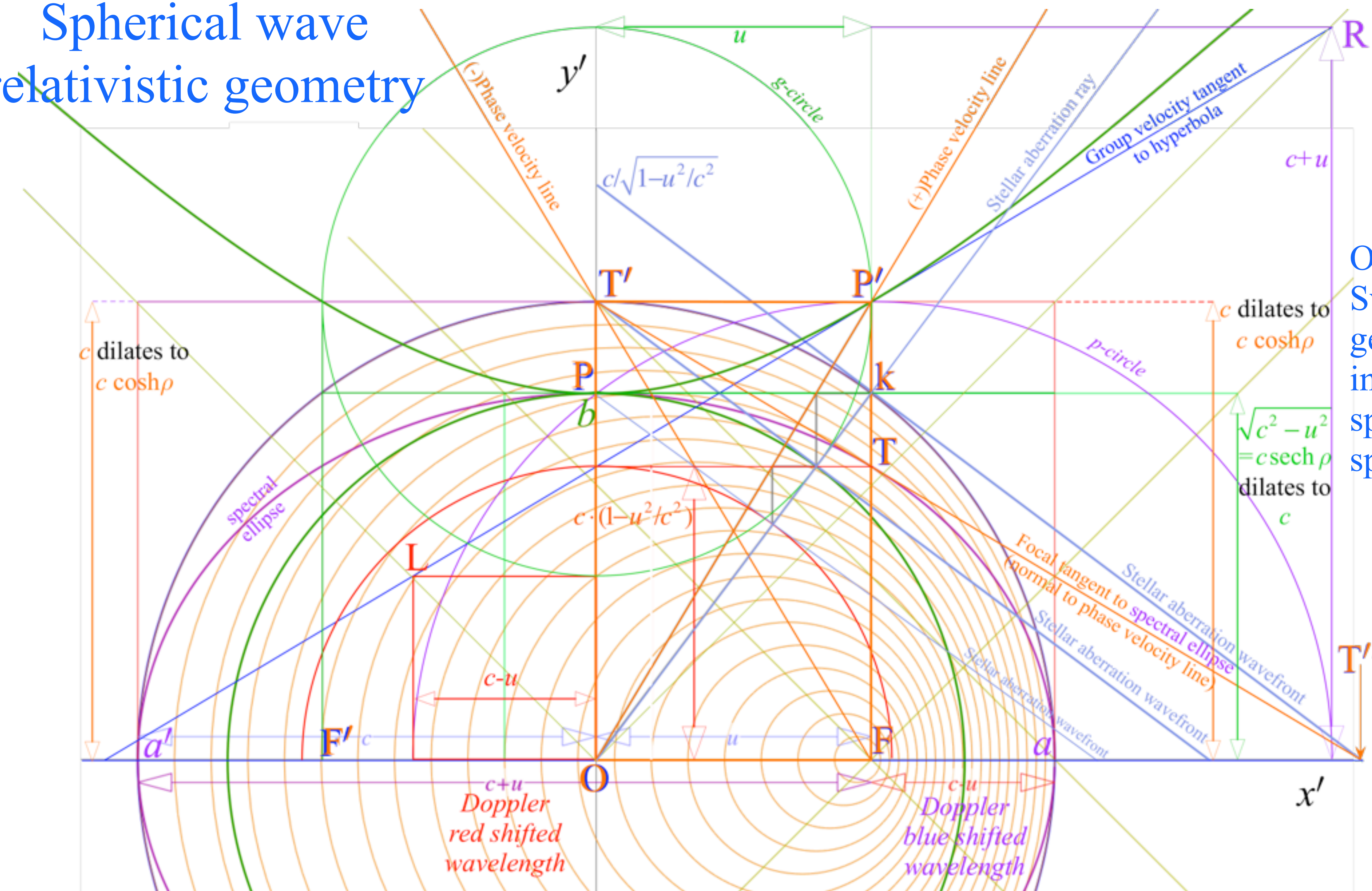
(a) Spherical wave pair

In Alice-Carla frame




(a) Spherical wave pair
In Alice-Carla frame
(b) Spherical wave pair
In Bob's frame: $u_x/c = -3/5$
Occam Sword geometry in (x,y) space-space
stellar angle $\sigma = \sin^{-1}(u/c)$
velocity angle $\nu = \tan^{-1}(u/c)$
slope u/c of velocity line tracks each expanding circle-top emitted at time t = -5,-4,-3,-2,-1,0
5u
5c
$5\sqrt{c^2-u^2} = 4c$
P
P′
k
k′
O
C
A
B
North Lighthouse
Main Lighthouse
Ship 1
-1.0
t = -15,.......,... ...,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1, 0

Spherical wave relativistic geometry



Occam Sword geometry in (x,y) space-space

Doppler Red λ=c+u
dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse major radius a=**OF**a=c
dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor:
$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse focal length **FO**= $u = c\tanh\rho$
dilates to: $u\cosh\rho = c\sinh\rho$

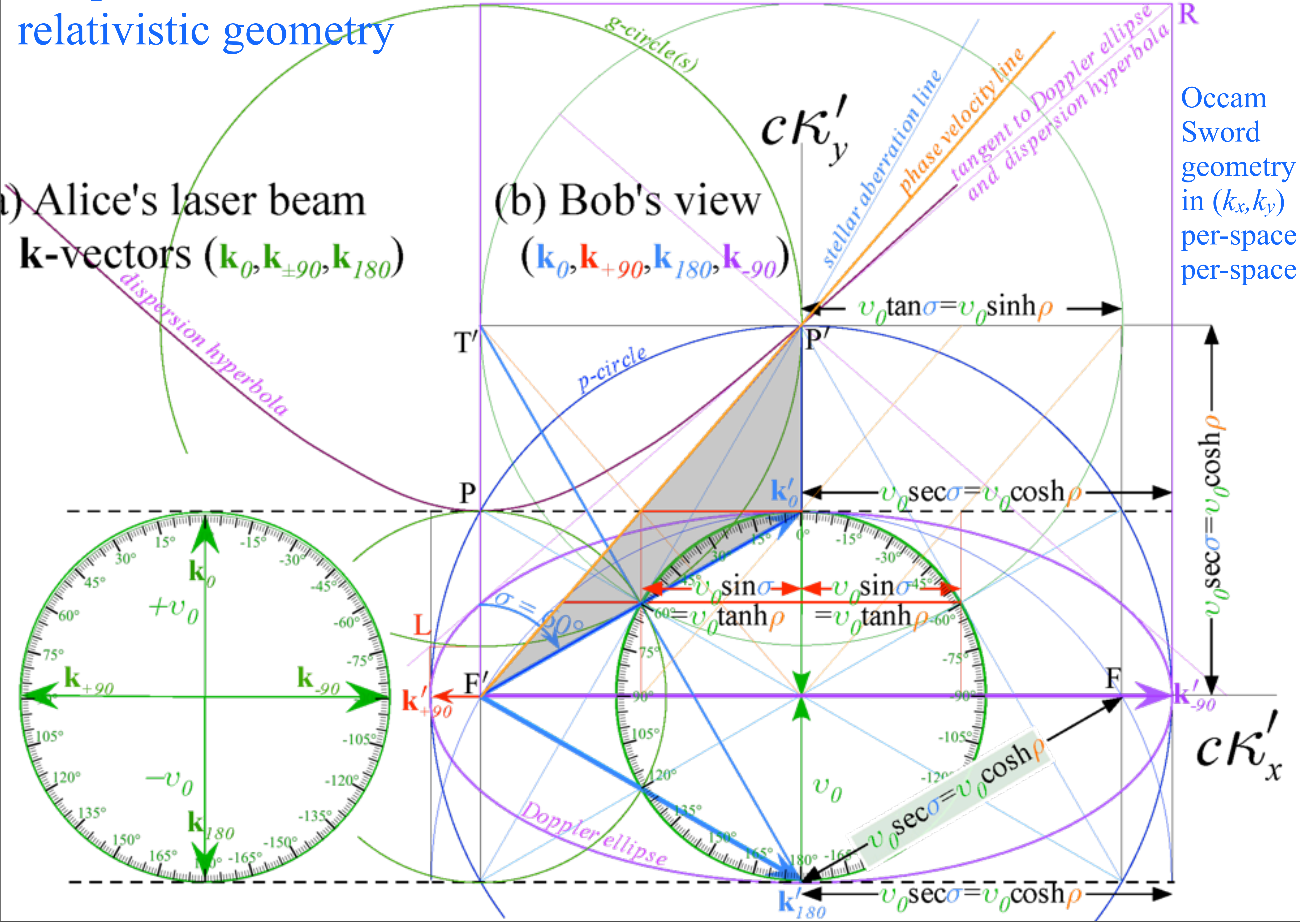
ellipse latus radius **FT**=$c(1-u^2/c^2)$
dilates to: $c(1-u^2/c^2)\cosh\rho$
$= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

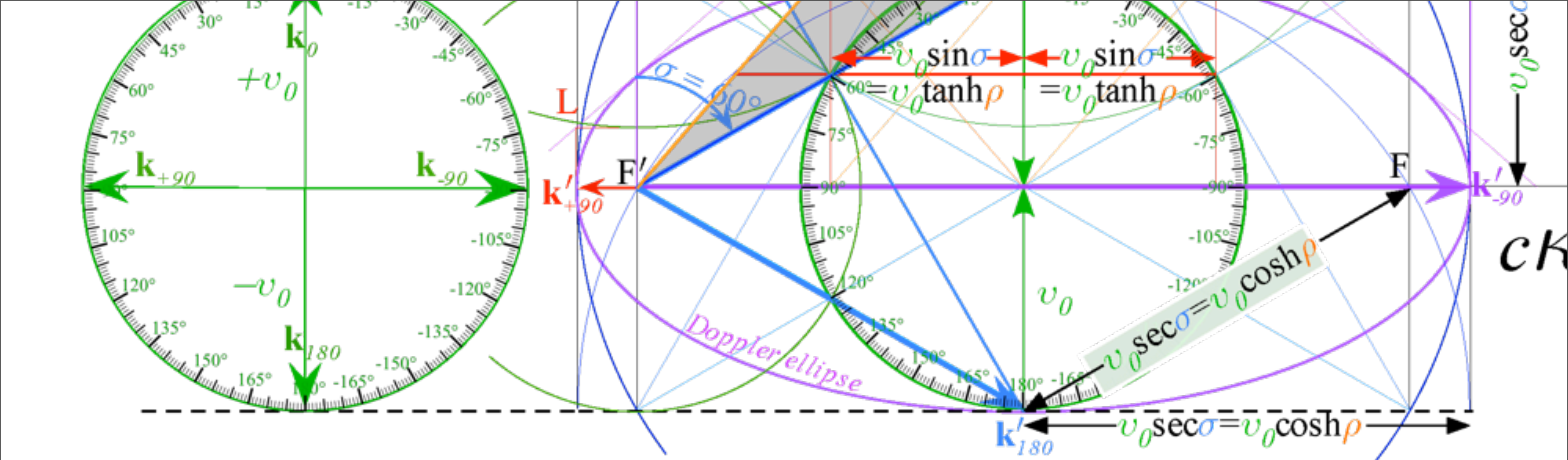
Doppler Blue λ=$c-u$
dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

Base height **FTk**=$\sqrt{c^2-u^2}$
dilates to: $\sqrt{c^2-u^2}\cosh\rho = c$
(equal to ellipse minor radius b)

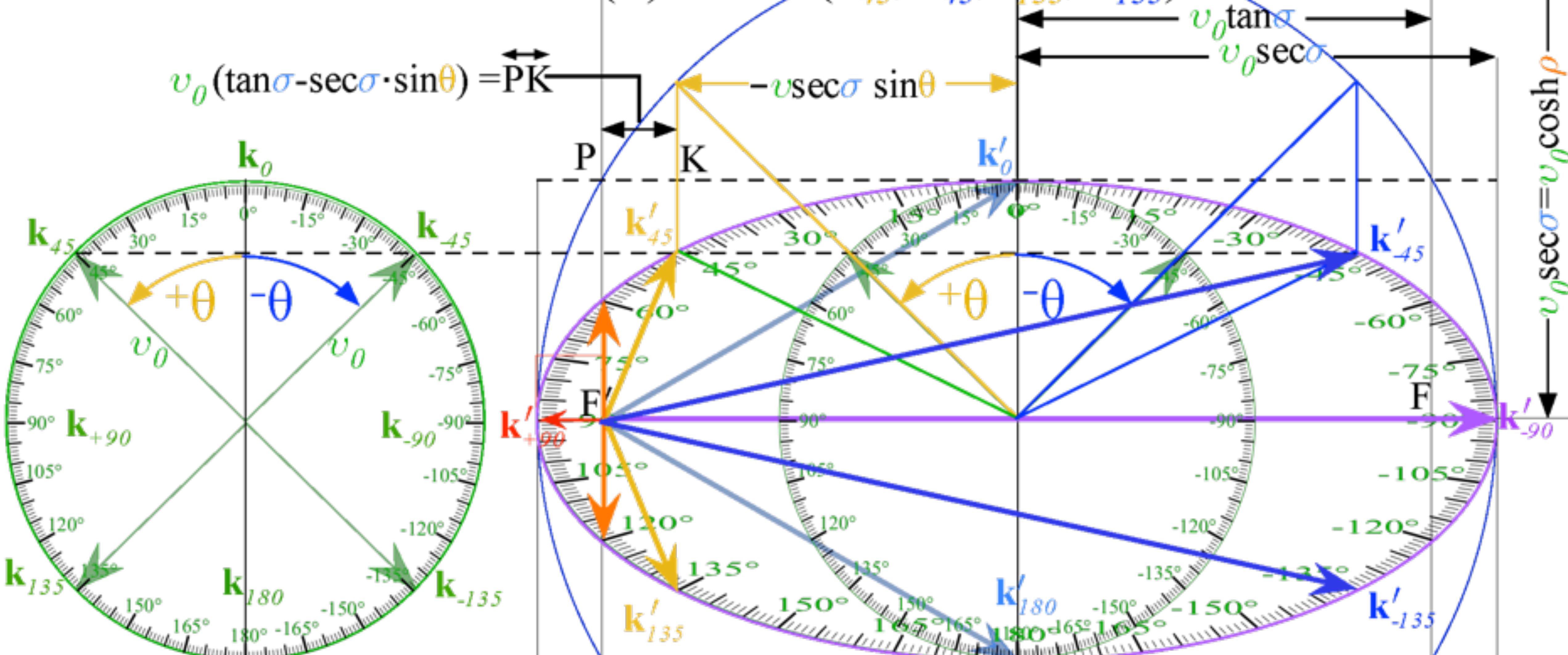
Spherical wave relativistic geometry

Occam Sword geometry in (k_x,k_y) per-space per-space



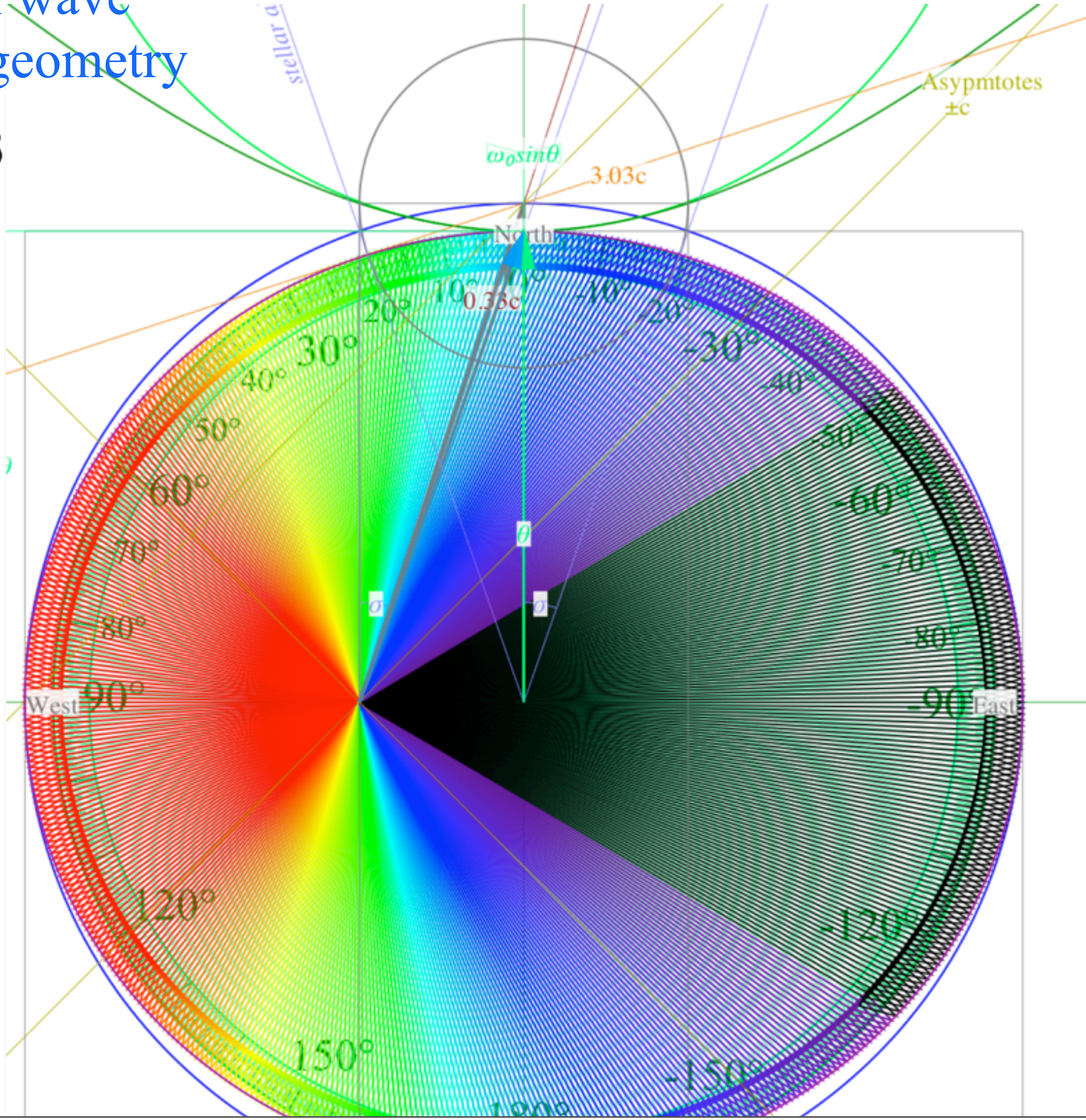


(c)Alice's ($\mathbf{k}_{45}$,$\mathbf{k}_{-45}$,$\mathbf{k}_{135}$,$\mathbf{k}_{-135}$) (d) Bob's($\mathbf{k}'_{45}$,$\mathbf{k}'_{-45}$,$\mathbf{k}'_{135}$,$\mathbf{k}'_{-135}$)

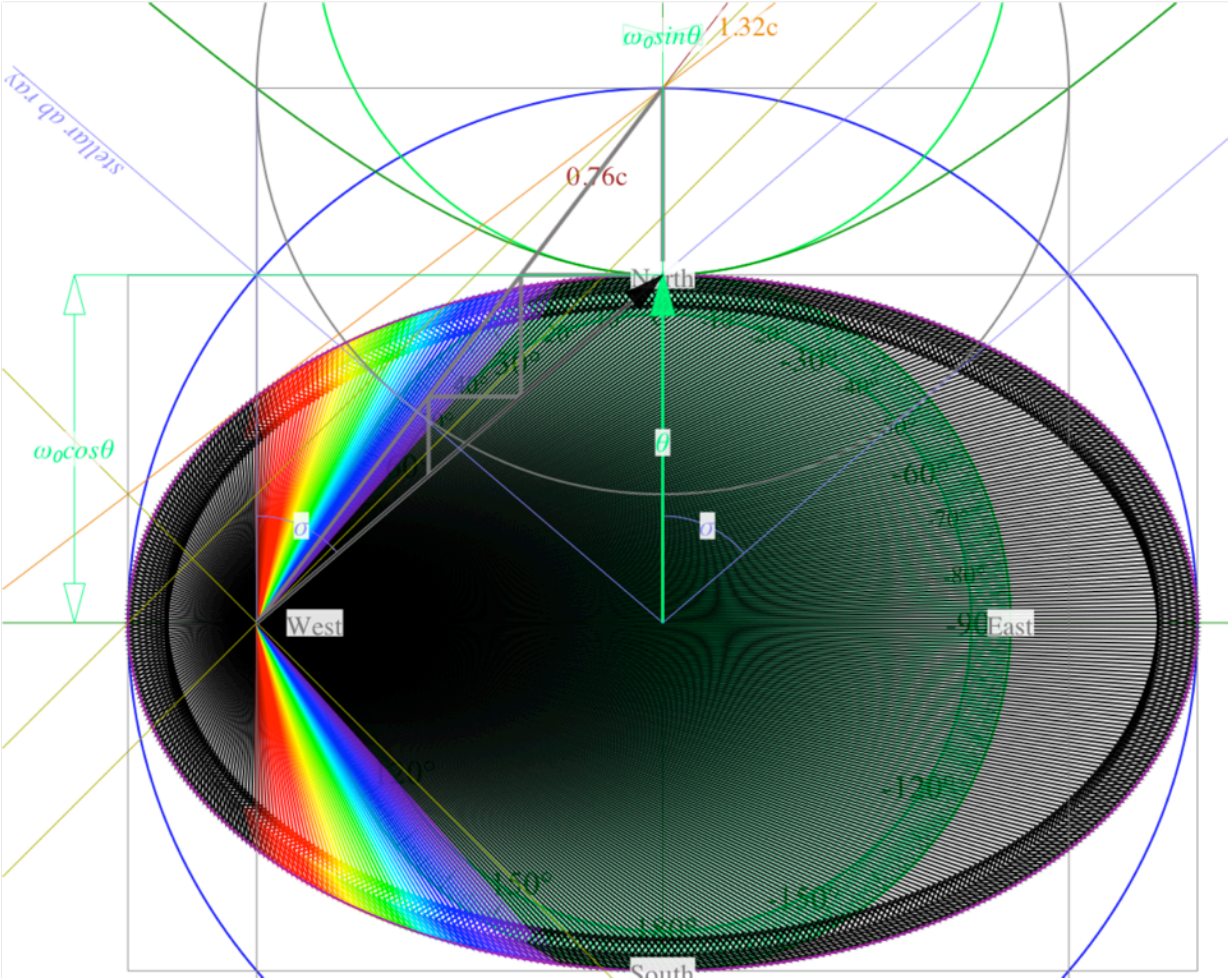


Spherical wave relativistic geometry

(a) u/c=1/3



(b) u/c=3/4



stellar ab ray
$\omega_0 sin\theta$
1.32c
0.76c
North
$\omega_0 cos\theta$
θ
σ
σ
West
East
-30°
-60°
-90
-120°
-150°
120°
150°
South