Lecture 7
Thur. 9.17.2013

Geometry and Motion of Isotropic Harmonic Oscillators

(Ch. 9 and Ch. 11 of Unit 1)

Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*
Contact-geometry of potential curve(s)
"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"
Earth matter vs nuclear matter:
*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

Sinusoidal space-time dynamics derived by geometry
Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)
Constructing 2D Isotropic harmonic oscillator orbits using phasor plots
Examples with x-y phase lag : $\alpha_{x\text{-}y}=\alpha_x-\alpha_y$ *=15°, 30°, and ±75°*

Geometry of idealized "Sophomore-physics Earth"

→ *Coulomb field <u>outside</u>* *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

Contact-geometry of potential curve(s)

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Gravity at r
due to shell mass elements

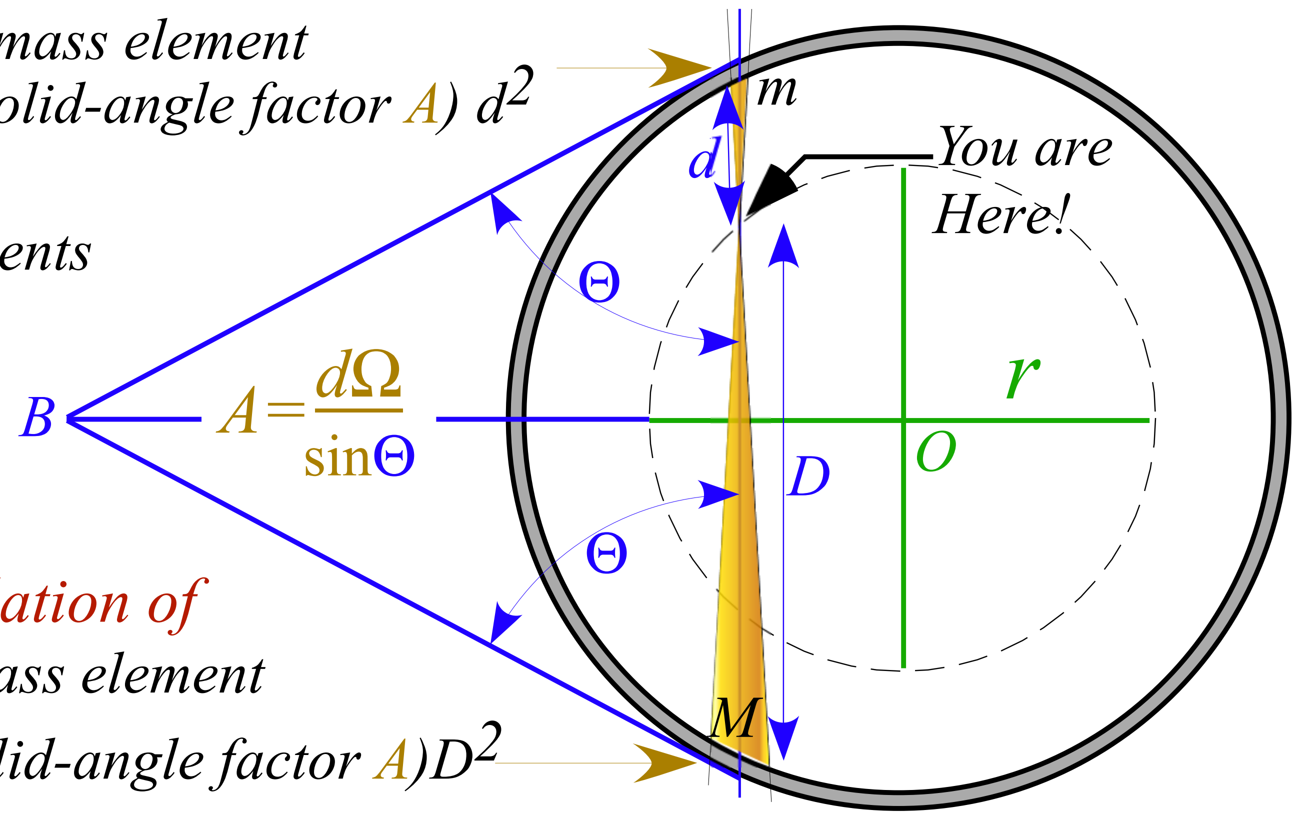
$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

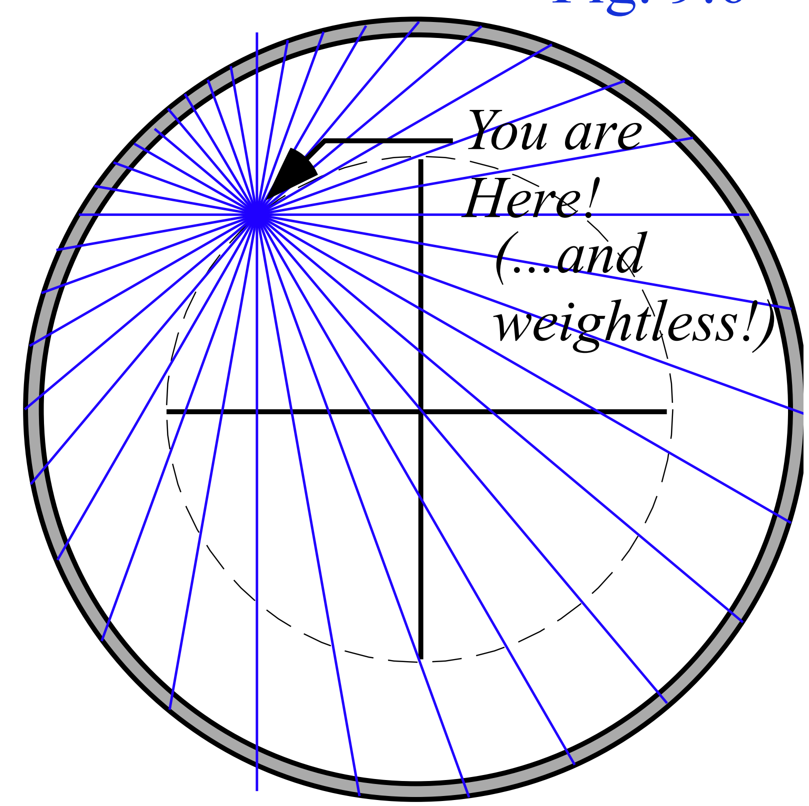
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

Cancellation of

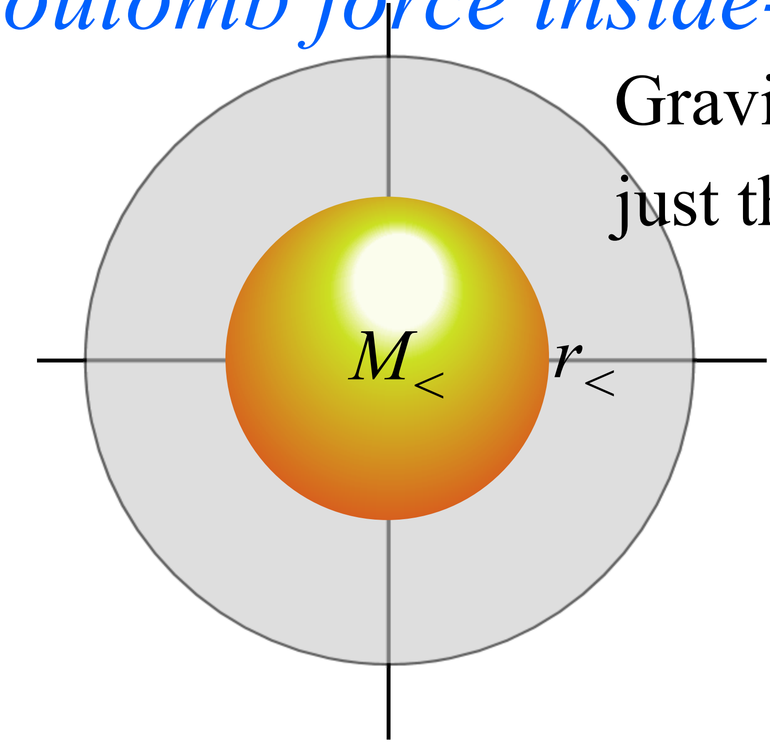
Shell mass element $m =$ (solid-angle factor A) d^2

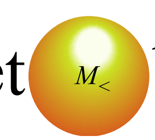
Shell mass element $M =$ (solid-angle factor A)D^2





Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet  below $r_<$

Coulomb force vanishes inside-spherical shell *(Gauss-law)*

Unit 1
Fig. 9.6

Shell mass element
m =(solid-angle factor A) d^2

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Shell mass element
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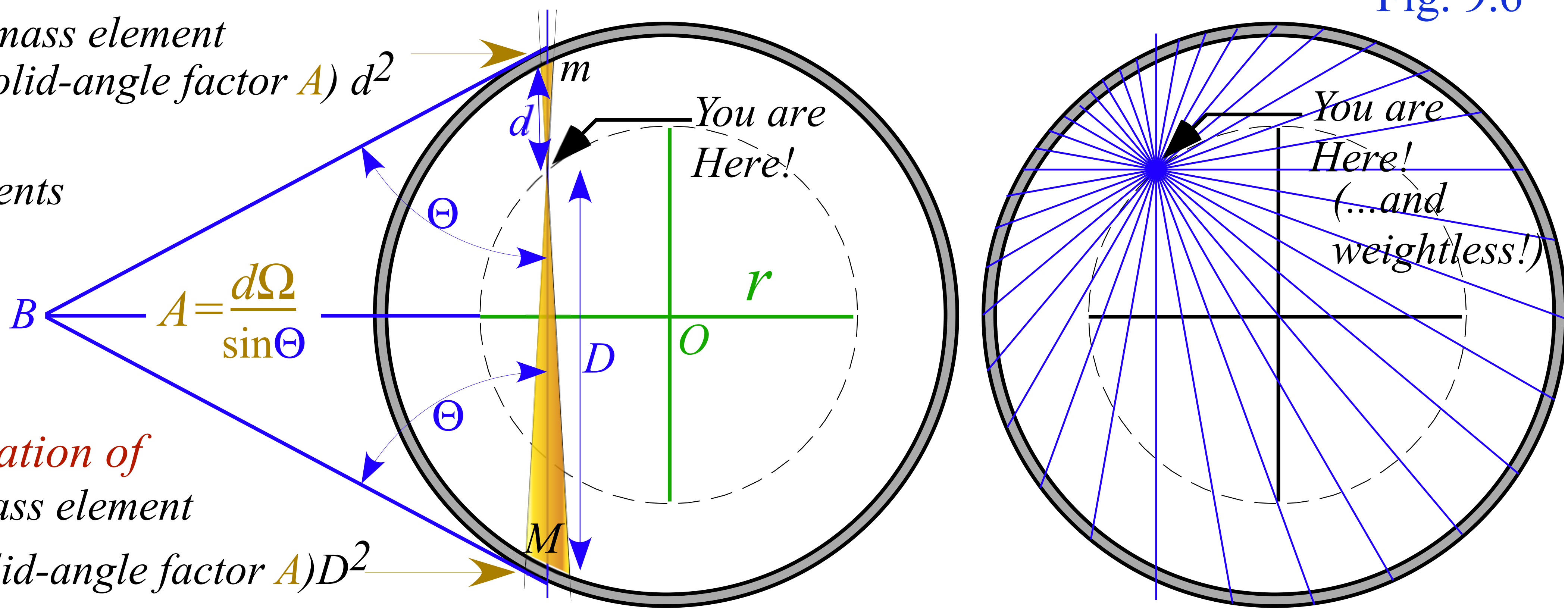

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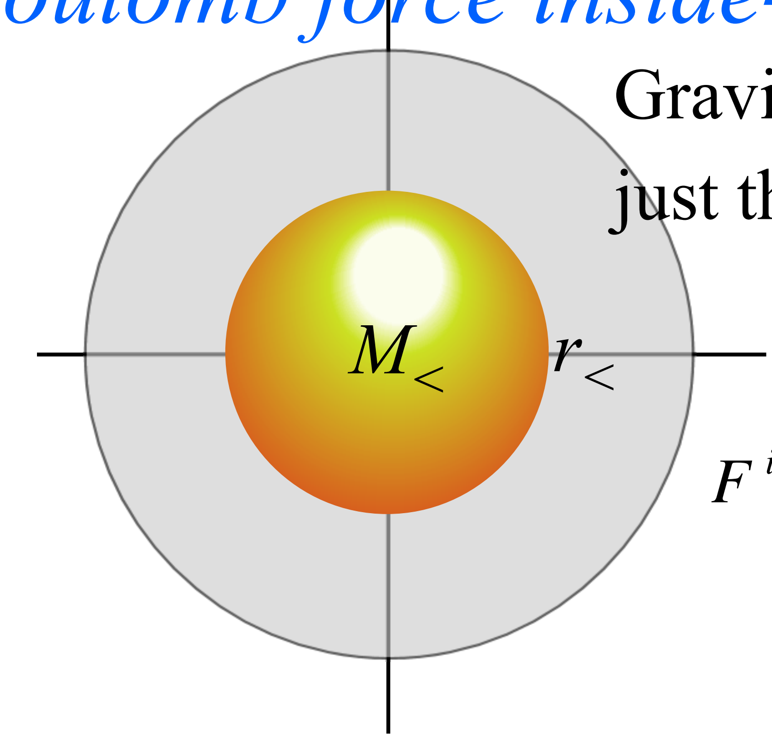

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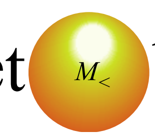
just that of planet  below $r_<$

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^{\,2}} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^{\,3}}r_<$$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

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m =(solid-angle factor A) d^2

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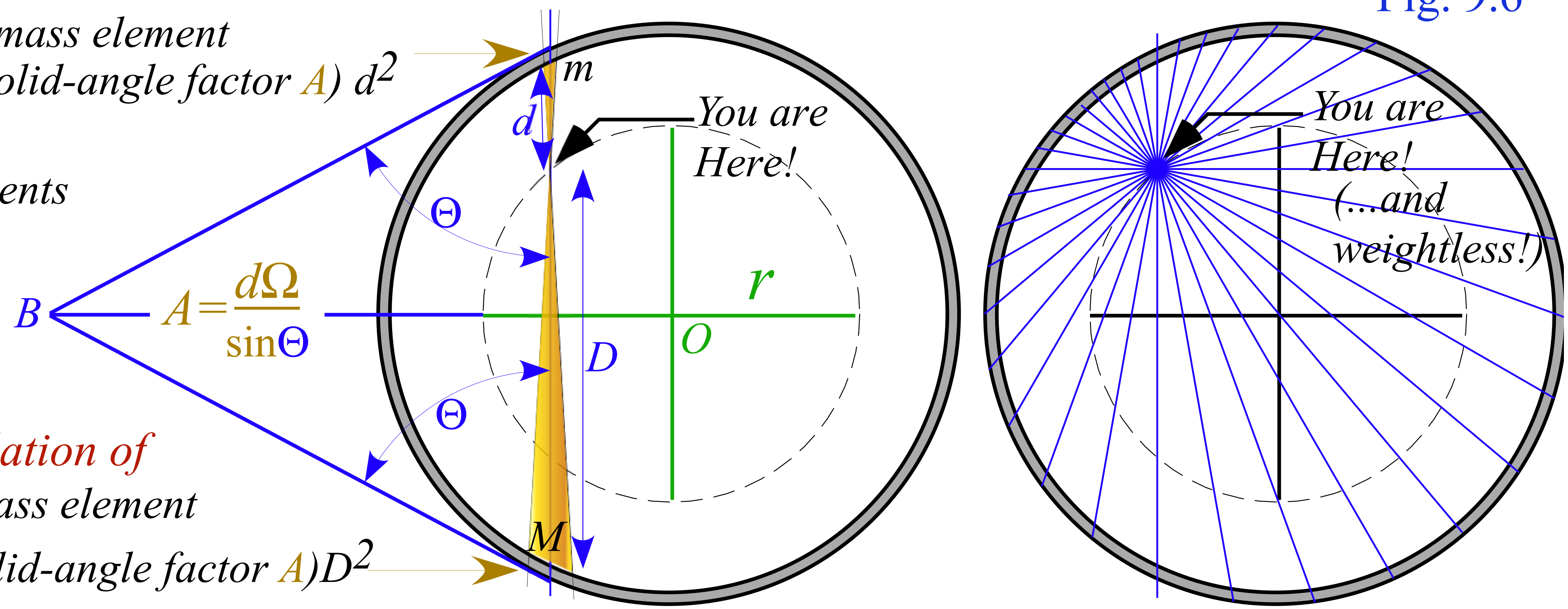
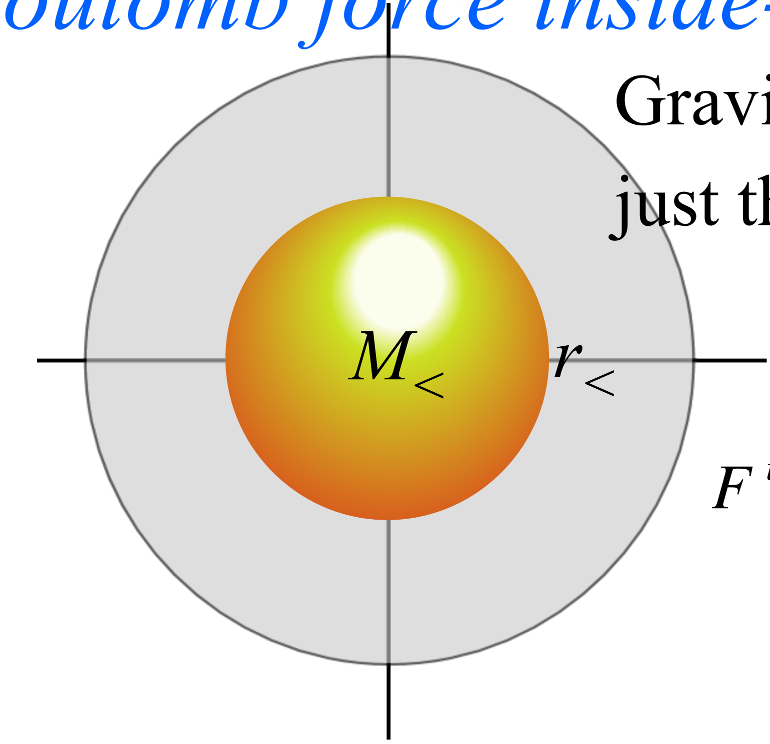


Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet below $r_<$

Note:
Hooke's (linear) force law
for $r_<$ inside uniform body

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg\cdot x$$

Coulomb force vanishes inside-spherical shell *(Gauss-law)*

Gravity at r due to shell mass elements

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Shell mass element
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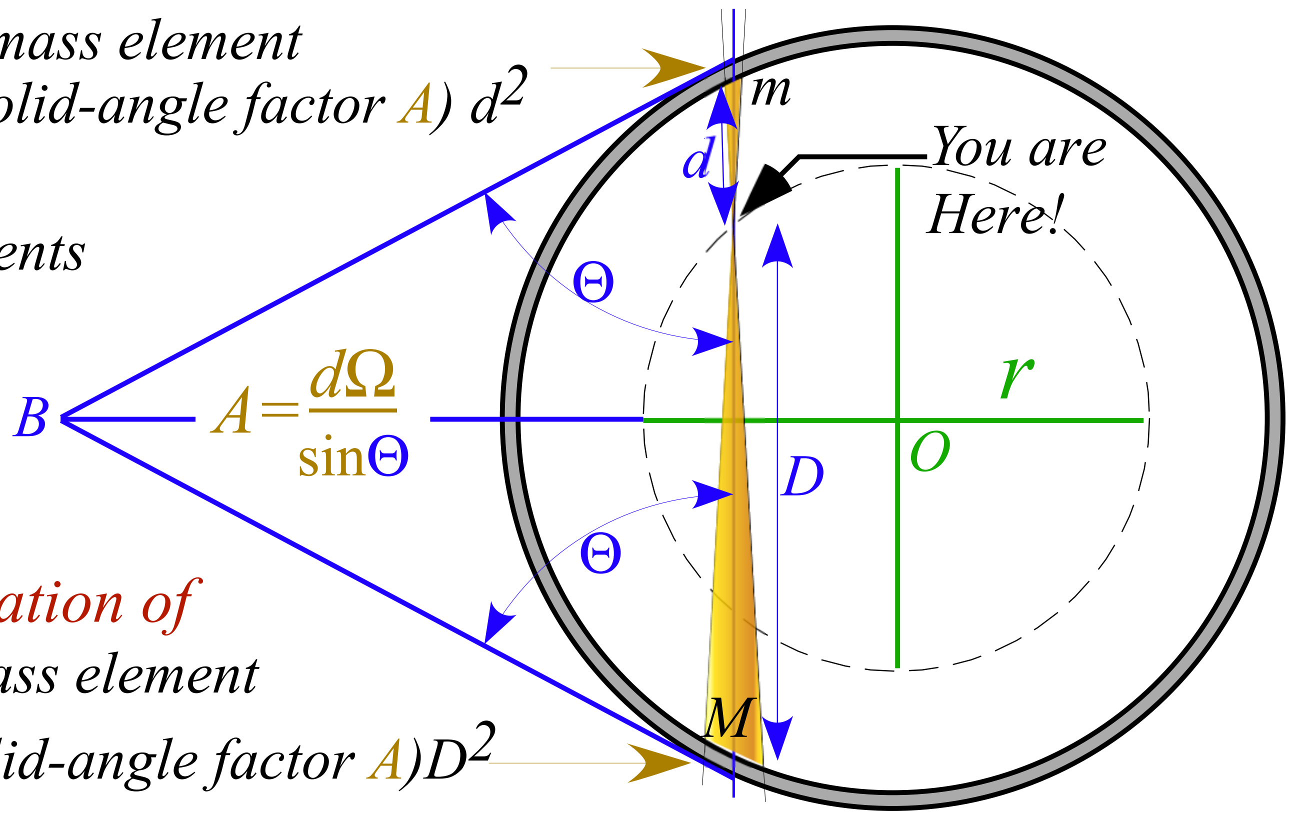

Cancellation of
Shell mass element
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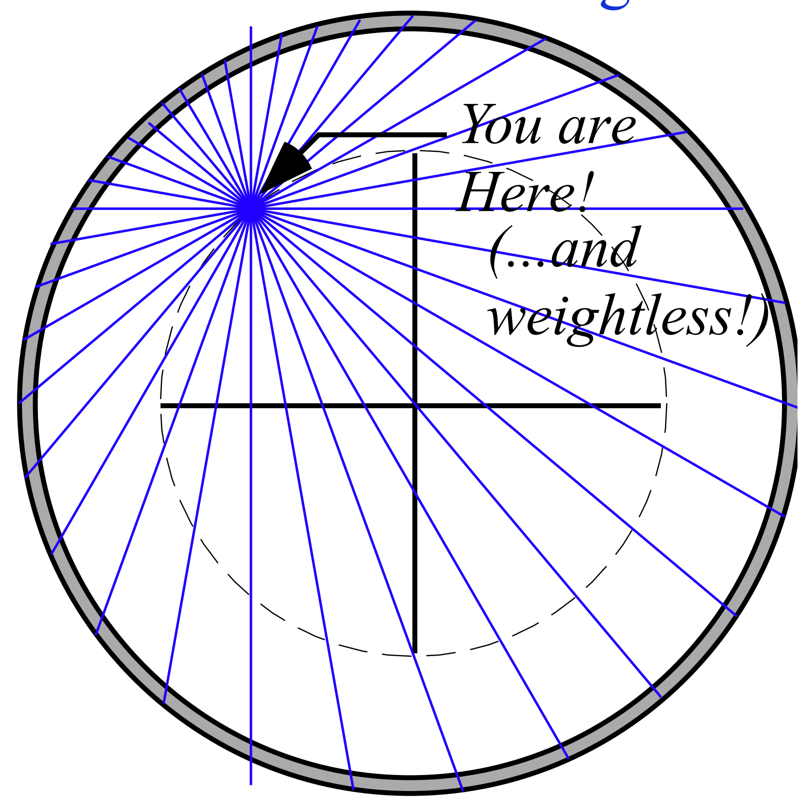
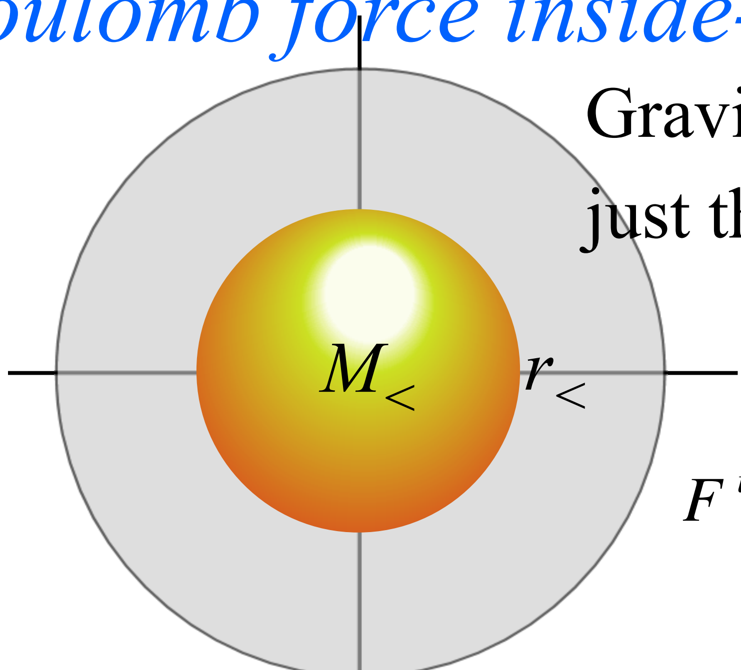
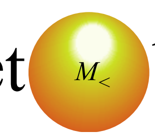


Coulomb force inside-spherical body due to stuff <u>below</u> you, <u>only</u>.



Gravitational force at $r_<$ is
just that of planet  below $r_<$

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Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s$

$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

Coulomb force vanishes inside-spherical shell *(Gauss-law)*

Unit 1
Fig. 9.6

Shell mass element
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due to shell mass elements

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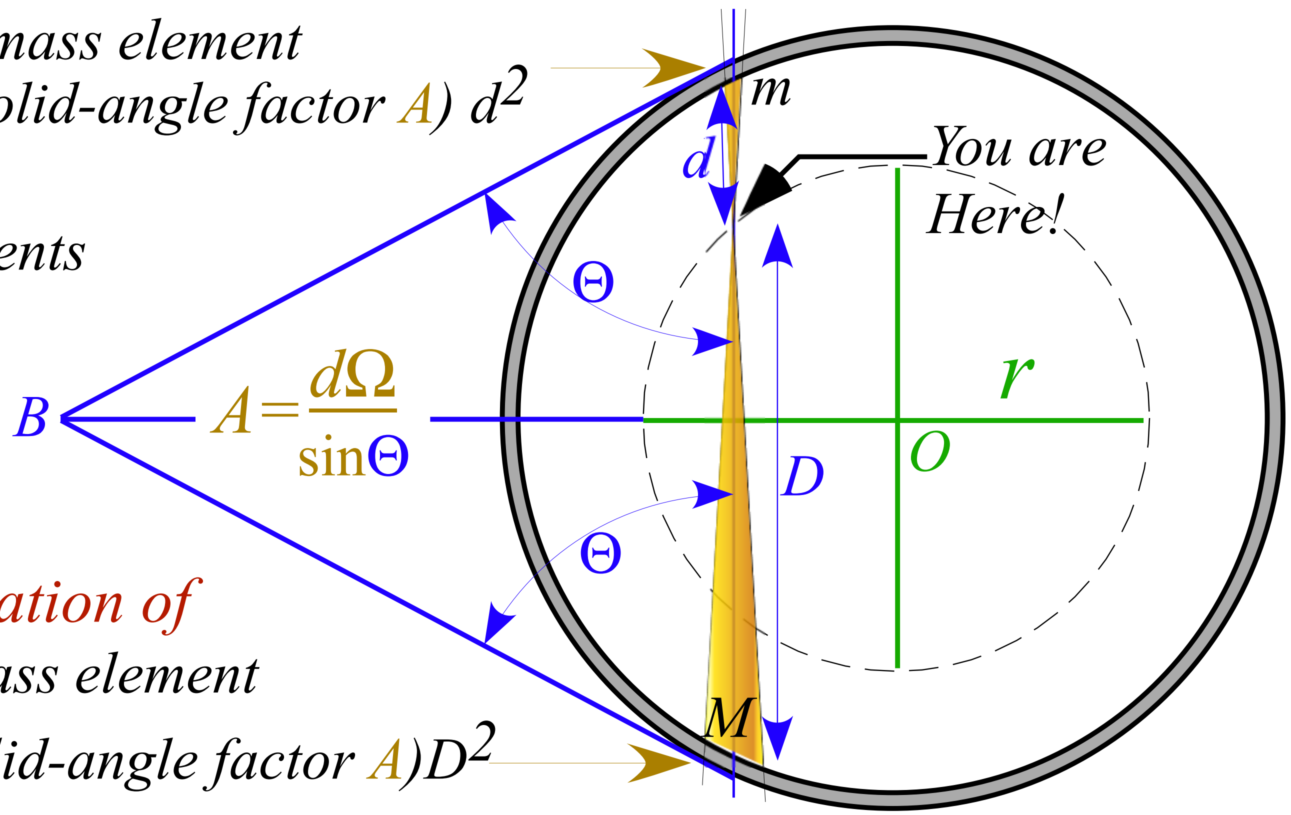

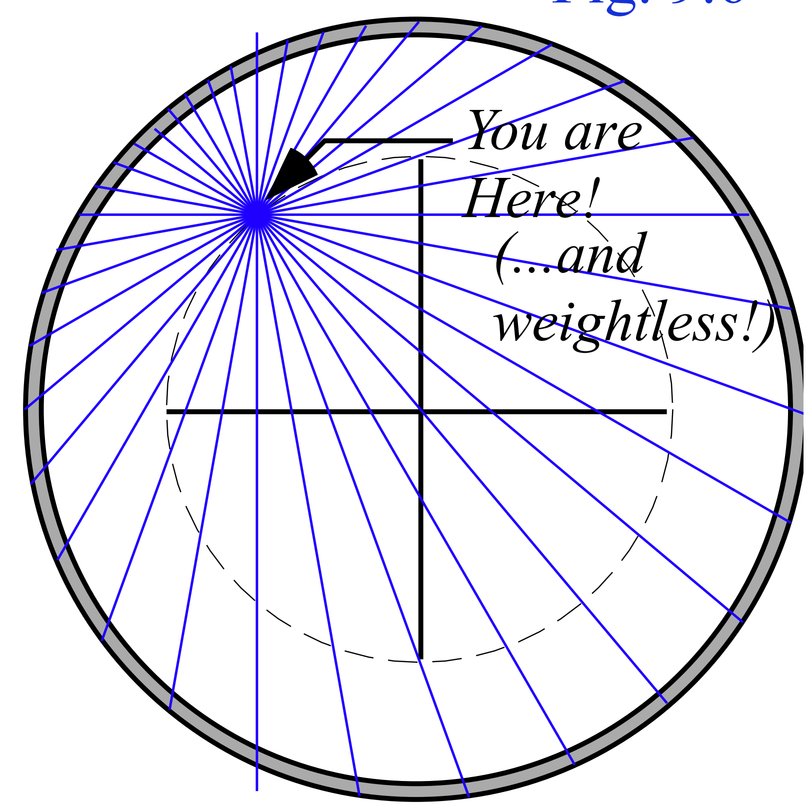

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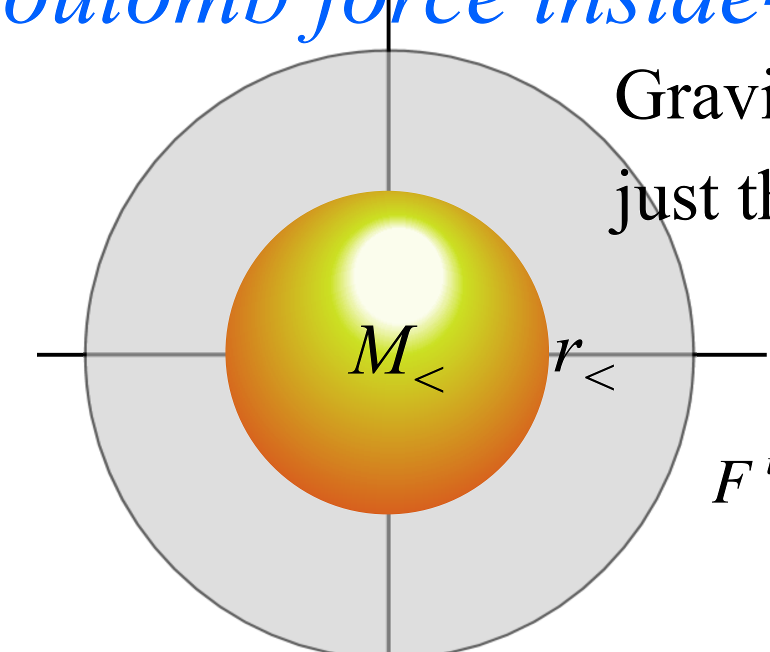

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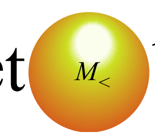
just that of planet  below $r_<$

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G=6.67384(80)·10^{-11}Nm^2/C^2 ~(2/3)10^{-10}

Earth radius : $R_\oplus = 6.371\cdot 10^6 m \simeq 6.4\cdot 10^6 m$

Earth mass : $M_\oplus = 5.9722 \times 10^{24} kg. \simeq 6.0\cdot 10^{24} kg.$

Solar radius : $R_\odot = 6.955 \times 10^8 m. \simeq 7.0\cdot 10^8 m.$

Solar mass : $M_\odot = 1.9889 \times 10^{30} kg. \simeq 2.0\cdot 10^{30} kg.$

Geometry of idealized "Sophomore-physics Earth"

Coulomb field <u>outside</u> *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

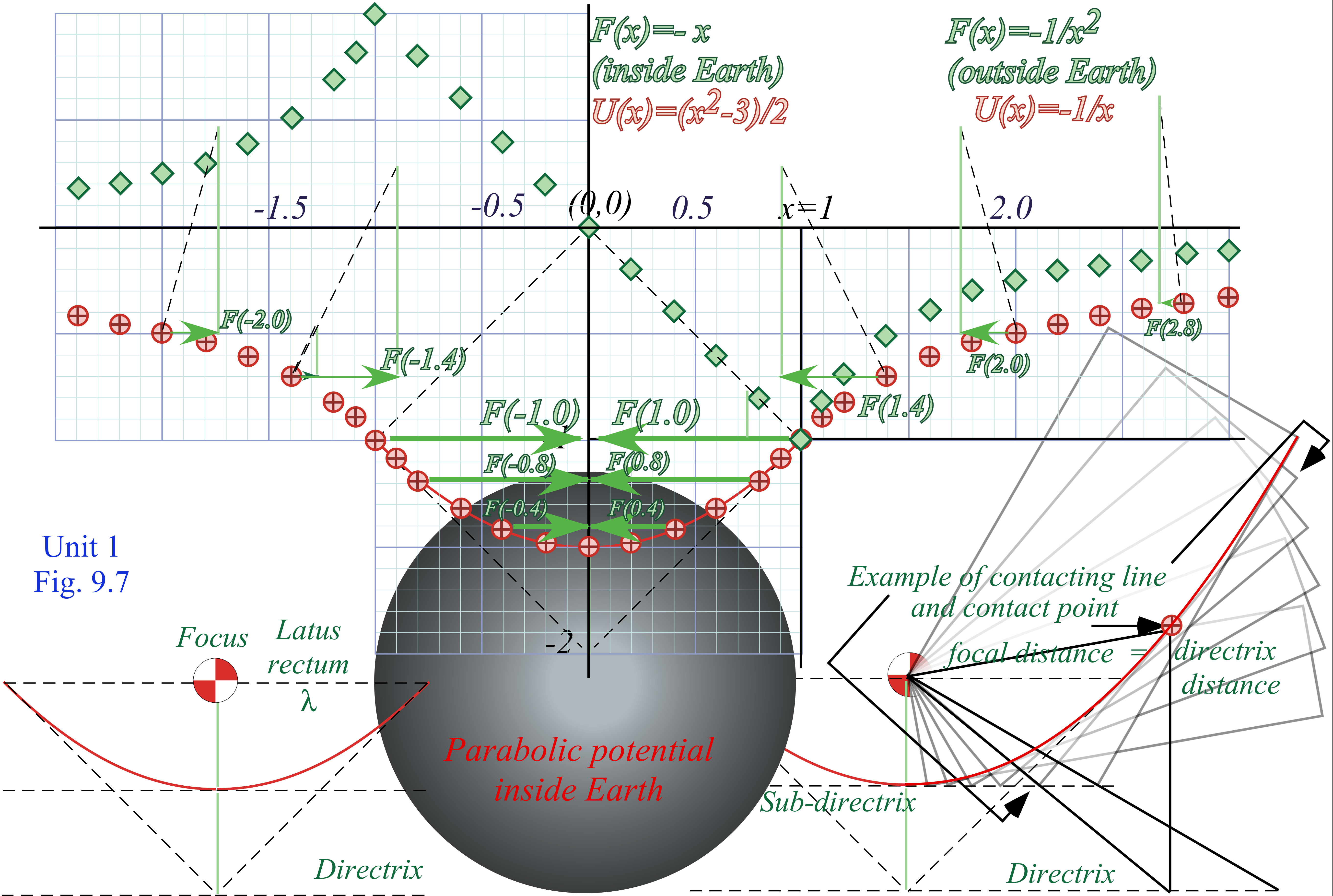
→ *Contact-geometry of potential curve(s)*

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

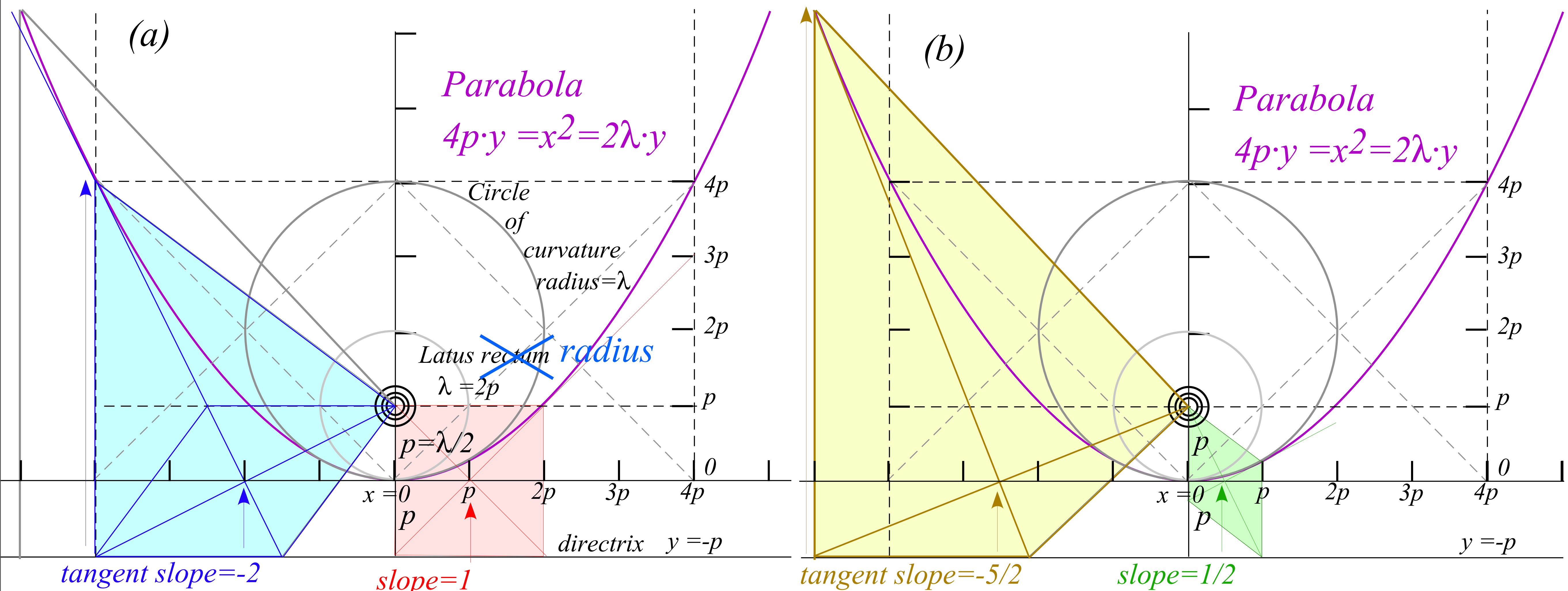
*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

The ideal "Sophomore-Physics-Earth" model of geo-gravity



...conventional parabolic geometry...carried to extremes...

(Review of Lect. 6 p.29)



Unit 1
Fig. 9.4

Geometry of idealized "Sophomore-physics Earth"

Coulomb field <u>*outside*</u>   *Isotropic Harmonic Oscillator (IHO) field* <u>*inside*</u>

Contact-geometry of potential curve(s)

→ *"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

Earth matter vs nuclear matter:

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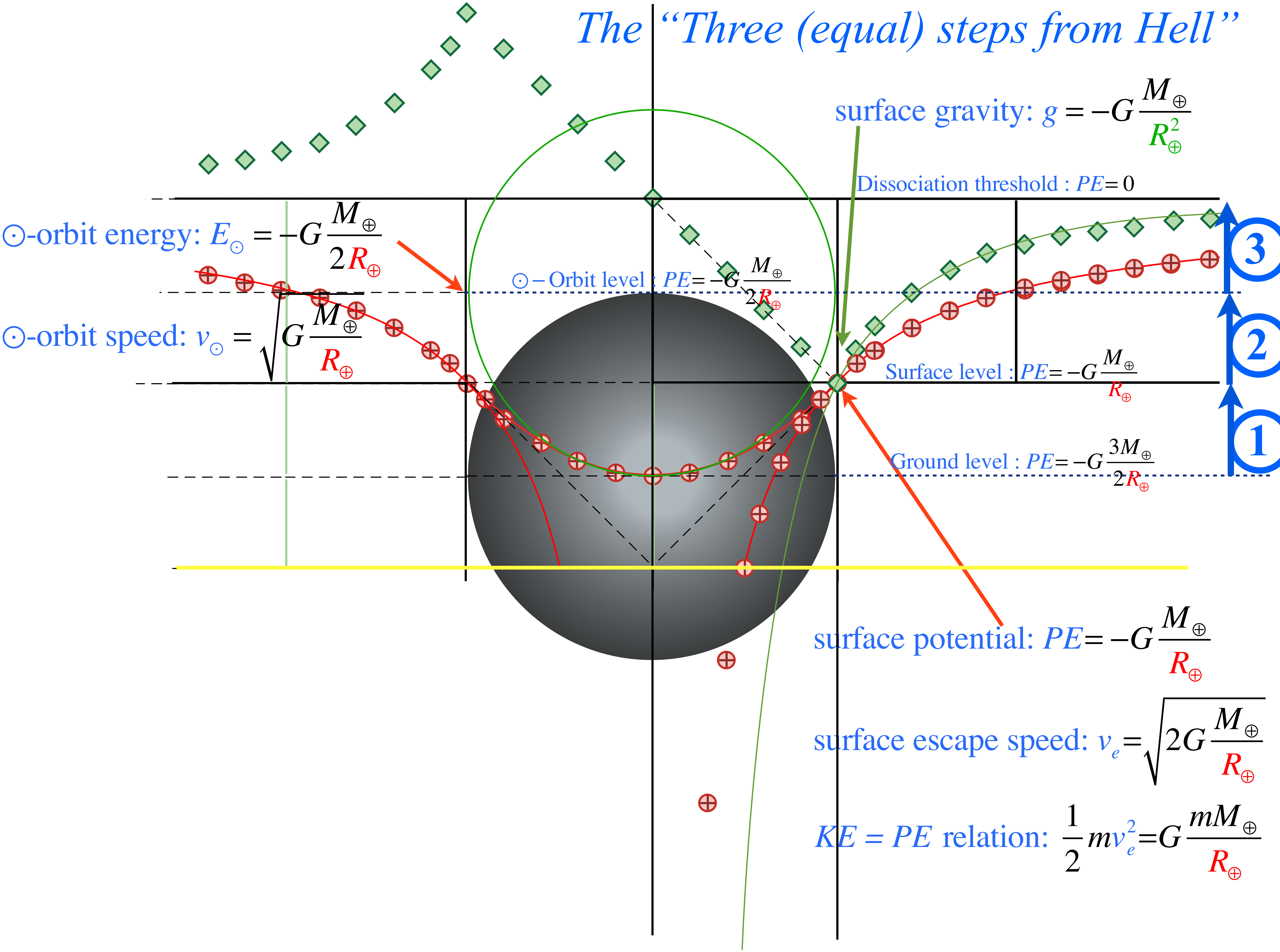
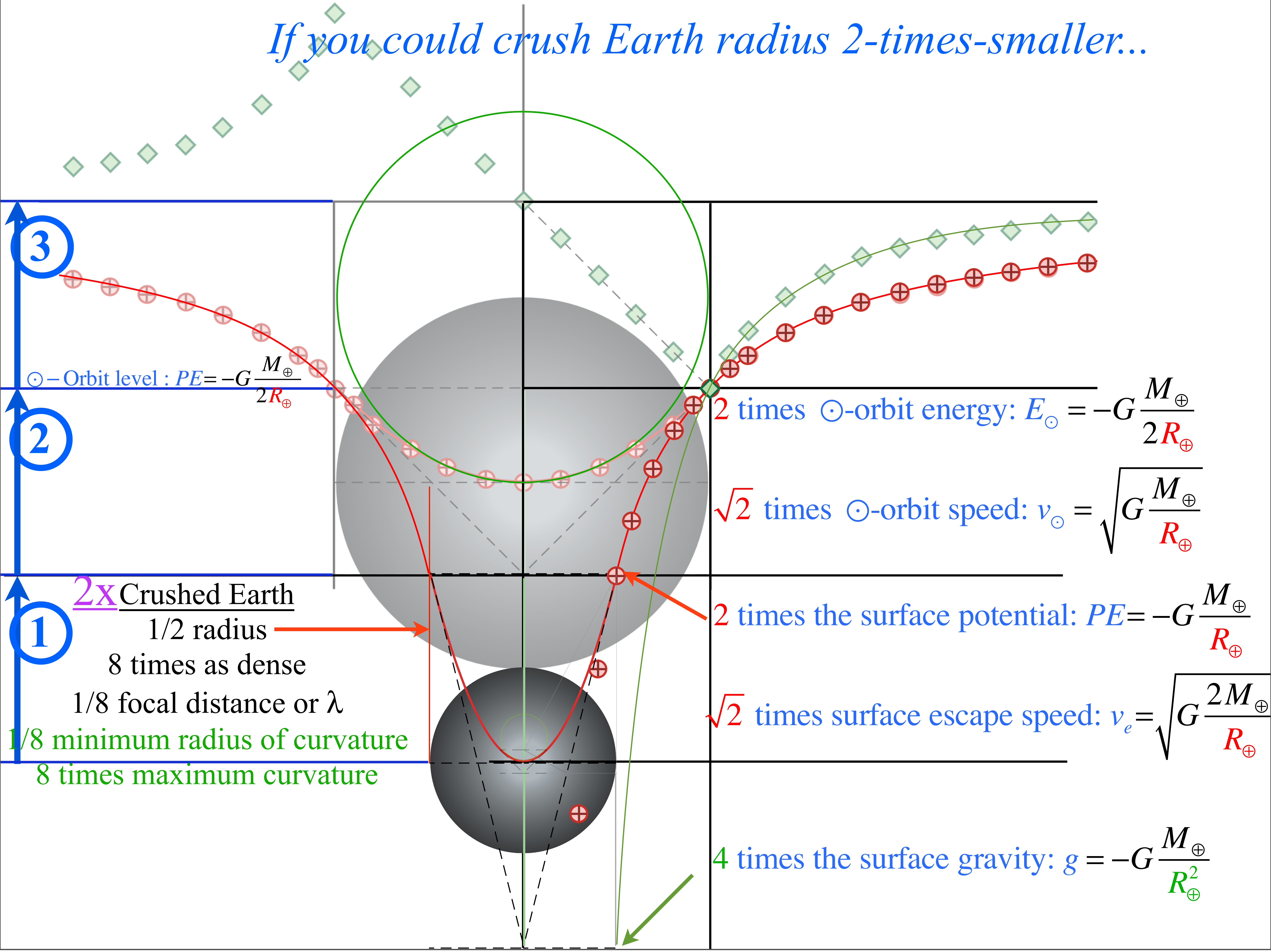
The "Three (equal) steps from Hell"



If you could crush Earth radius 2-times-smaller...



Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*

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Earth matter vs nuclear matter:

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Examples of "crushed" matter

Earth matter

$Earth\,mass: M_{\oplus} = 5.9722\ \times\ 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ Density $\rho_{\oplus}$??

$Earth\,radius: R_{\oplus} = 6.371 \cdot 10^{6}\,m \simeq 6.4 \cdot 10^{6}\,m \quad Earth\,volume: (4\pi / 3)R_{\oplus}{}^{3} \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^{3}$

$(6.4)^3 \sim 262$ and $(4\pi/3)260 = 1098 \sim 10^3$

Examples of "crushed" matter

Earth matter

$Earth\,mass: M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ Density $\rho_\oplus \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3 kg/m^3$

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Density of solid Fe $= 7.9 \cdot 10^{3}$ kg/m^{3}

Density of liquid Fe $= 6.9 \cdot 10^{3}$ kg/m^{3}

Examples of "crushed" matter

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Nuclear matter

Nucleon mass $= 1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

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That's $100 \cdot 10^{-27} = 10^{-25}\ kg$ packed into a volume of $\frac{4\pi}{3} r^{3} = \frac{4\pi}{3} (3 \cdot 10^{-15})^{3}\ m^{3}$ or about $10^{-43}\ m^{3}$.

$36\pi = 113 \sim 10^{2}$

Examples of "crushed" matter

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Nuclear density is $10^{-25+43} = 10^{18} kg/m^{3}$ or a *trillion* (10^{12}) kilograms in the size of a fingertip (1cc).

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300\,m$ would approach neutron-star density.

Examples of "crushed" matter

Earth matter

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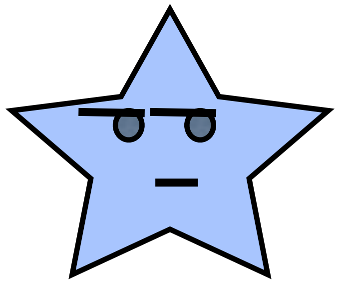
Introducing the "Neutron starlet"

1 cm³ of nuclear matter: mass= 10^{12} *kg.*



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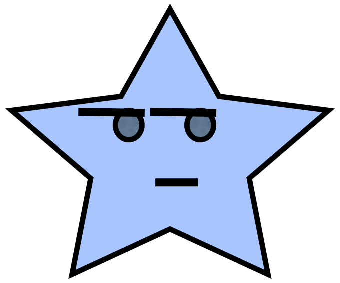
Introducing the "Neutron starlet"

1 cm³ of nuclear matter: mass= 10¹² kg.



Introducing the "**Black Hole Earth**"

Suppose Earth is crushed so that its surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^8 m/s$.

$c \equiv 299{,}792{,}458 m/s$ (EXACTLY)

$$c = \sqrt{(2GM/R_\otimes)}$$

$$R_\otimes = 2GM/c^2 = 8.9mm \sim 1cm$$ *(fingertip size!)*

$G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

→ *Sinusoidal space-time dynamics derived by geometry*

Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)

Constructing 2D Isotropic harmonic oscillator orbits using phasor plots

Examples with x-y phase lag : $\alpha_{x\text{-}y}=\alpha_x-\alpha_y=15°, 30°,$ *and* $\pm 75°$

[BoxIt simulation of U(2) orbits](http://www.uark.edu/ua/modphys/markup/BoxItWeb.html)
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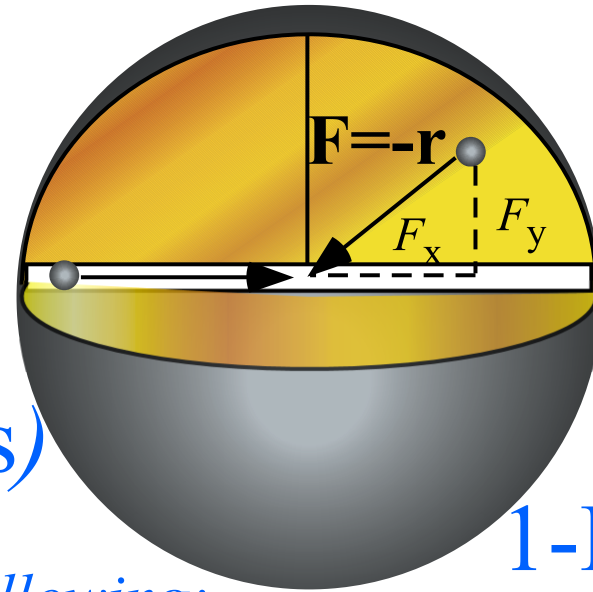
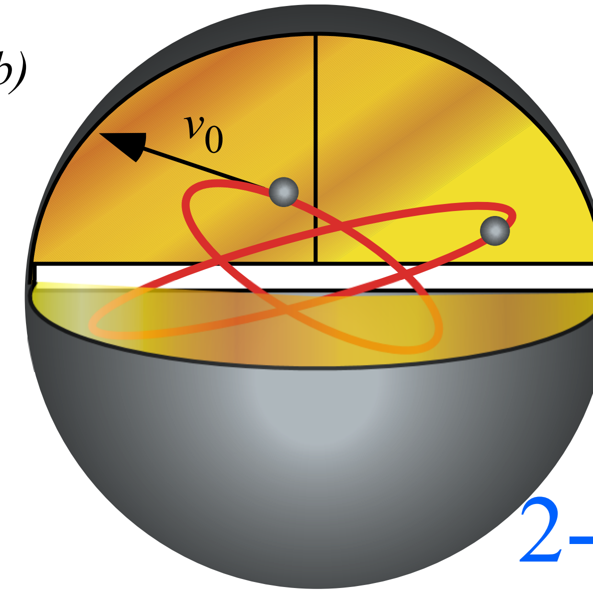
Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



1-D

(b)



2-D

(Paths are *always* 2-D ellipses if viewed right!)

Unit 1
Fig. 9.10

I.H.O. Force law

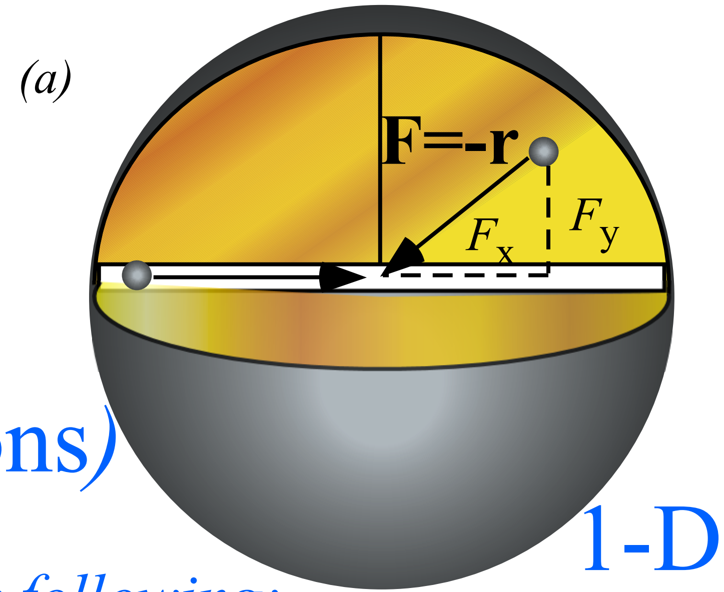
F =-x (1-Dimension)

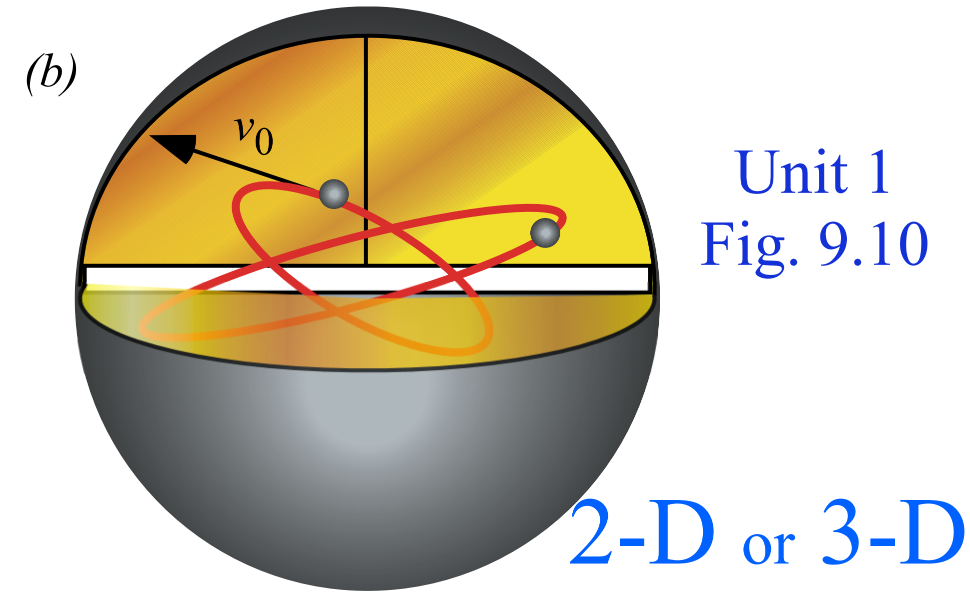
F =-r (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body





Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

F =-*x* (1-Dimension)

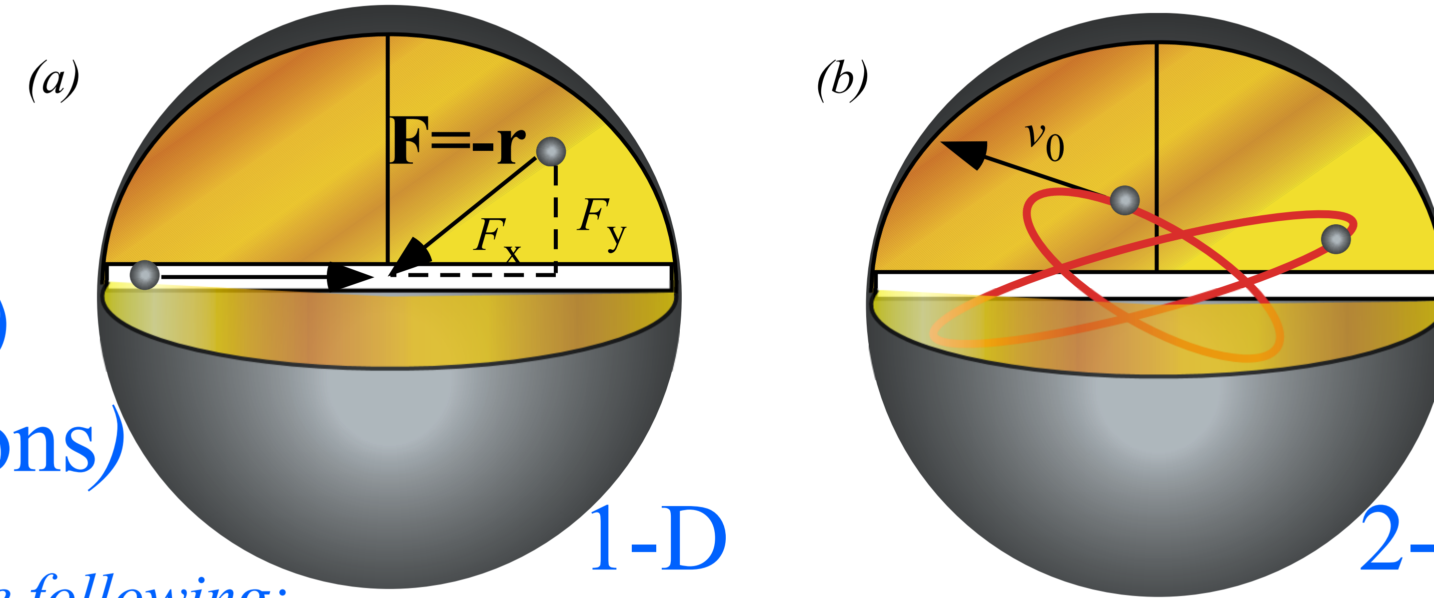
F =-**r** (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal *__isotropic__* *case.*

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

I.H.O. Force law

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$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

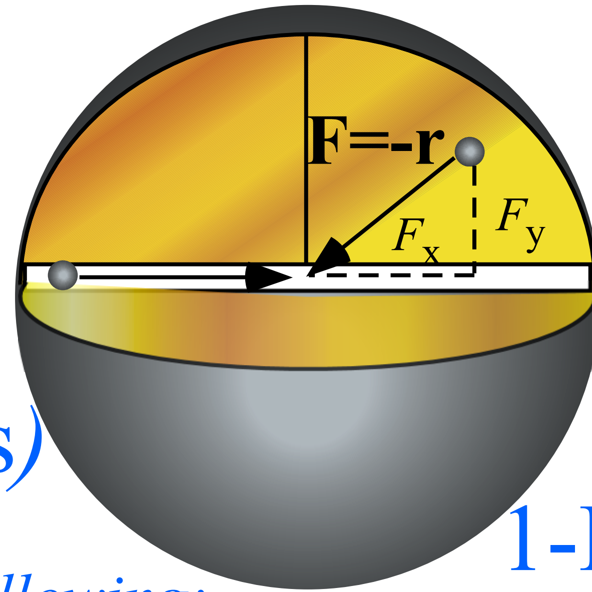
$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let:\ (1)\ v = \sqrt{2E/m}\cos\theta, \quad and:\ (2)\ x = \sqrt{2E/k}\sin\theta$$

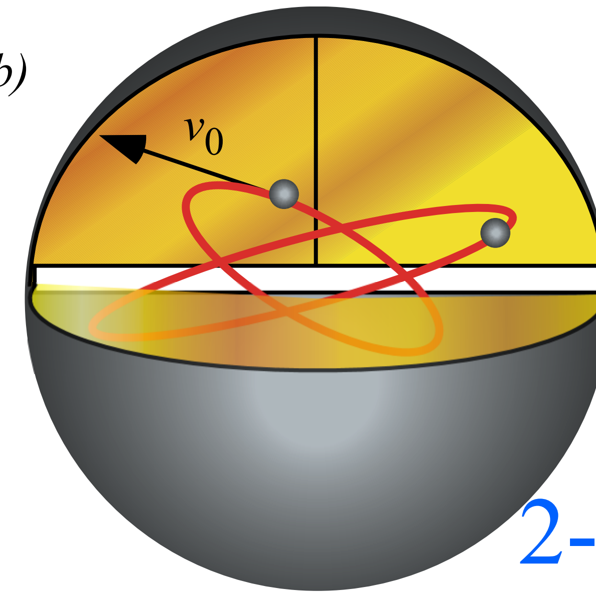
Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



1-D

(b)



Unit 1
Fig. 9.10

2-D or 3-D
(Paths are *always* 2-D ellipses if viewed right!)

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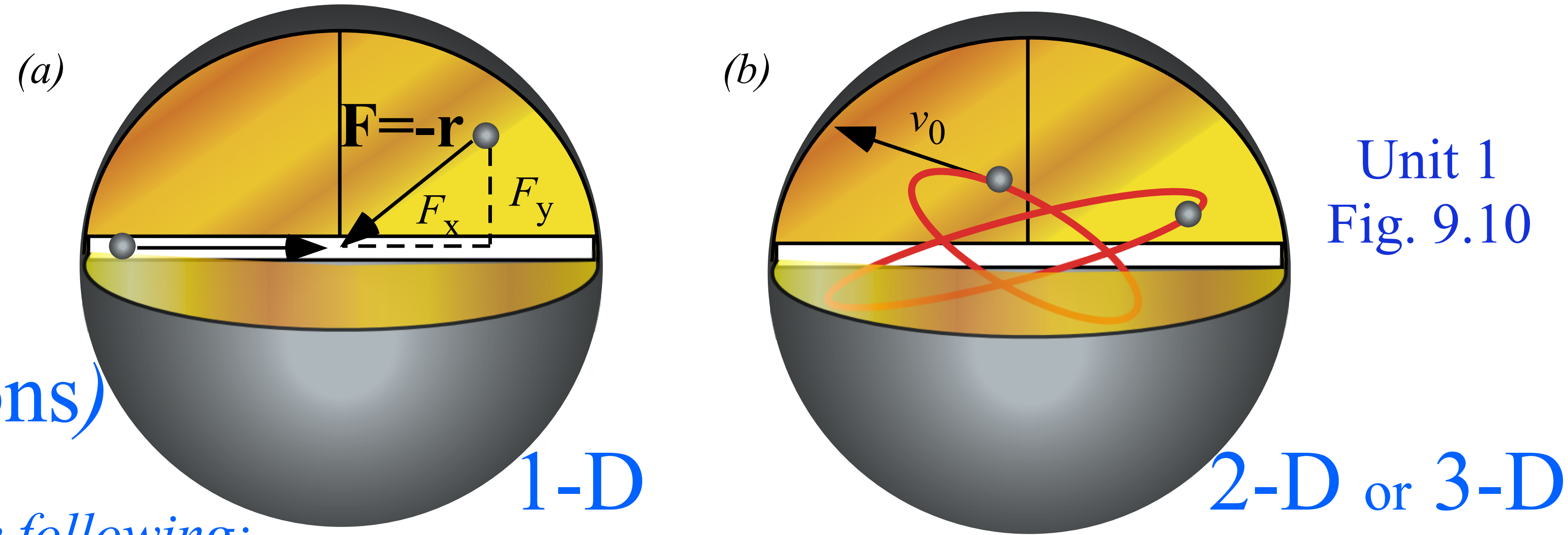
def. **(3)** $\omega = \frac{d\theta}{dt}$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta}$$

by **(1)**   *by def.* **(3)**

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

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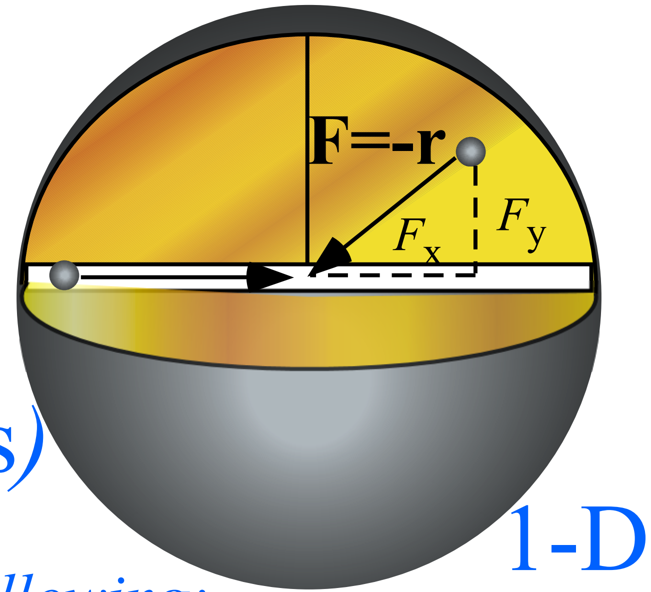
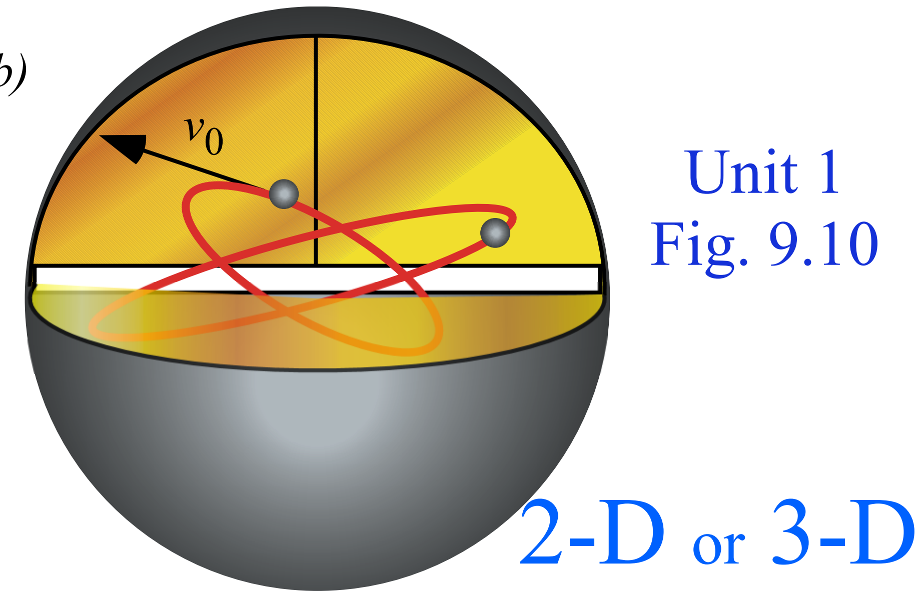
by (1) by def. (3) by (2)

Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



(b)



Unit 1
Fig. 9.10

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by (1) by def. (3) by (2)

by def. (3)

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

divide this by (1)

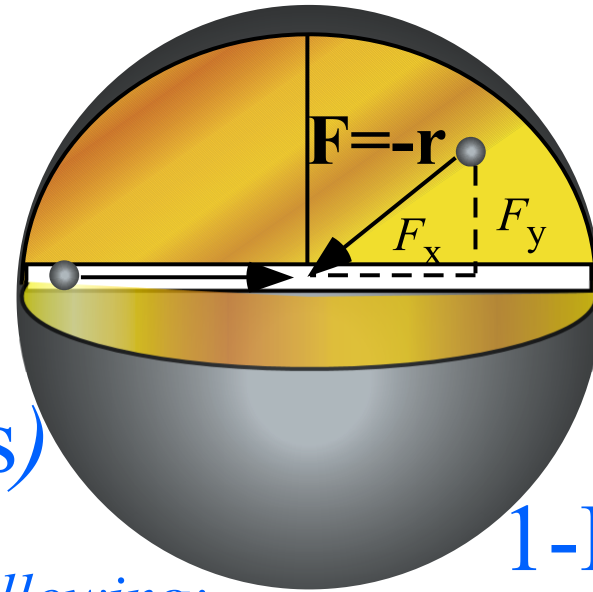
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

$F = -x$ (1-Dimension)

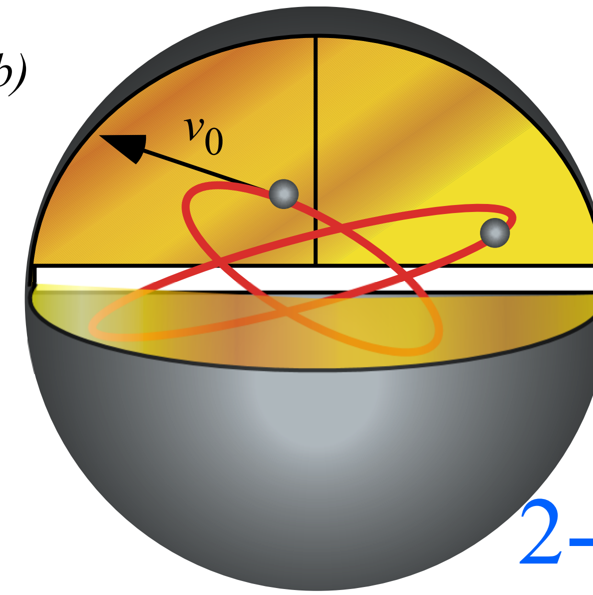
$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

(a)



1-D

(b)



Unit 1
Fig. 9.10

2-D or 3-D

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by (1) — *by def.* (3) — *by* (2)

by def. (3)

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

divide this by (1)

by integration given constant ω:

$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

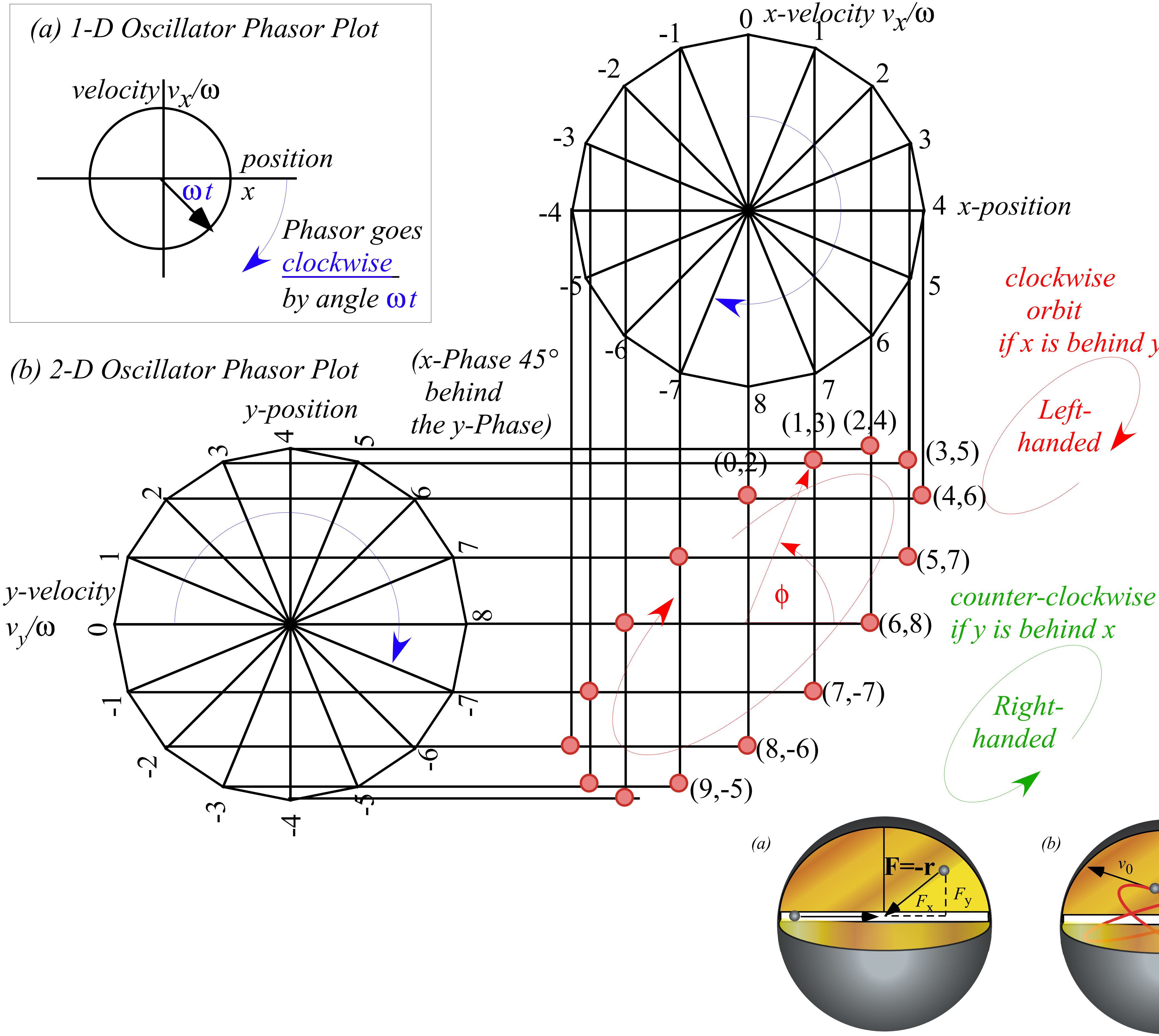
Sinusoidal space-time dynamics derived by geometry

Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)

→ *Constructing 2D Isotropic harmonic oscillator orbits using phasor plots*

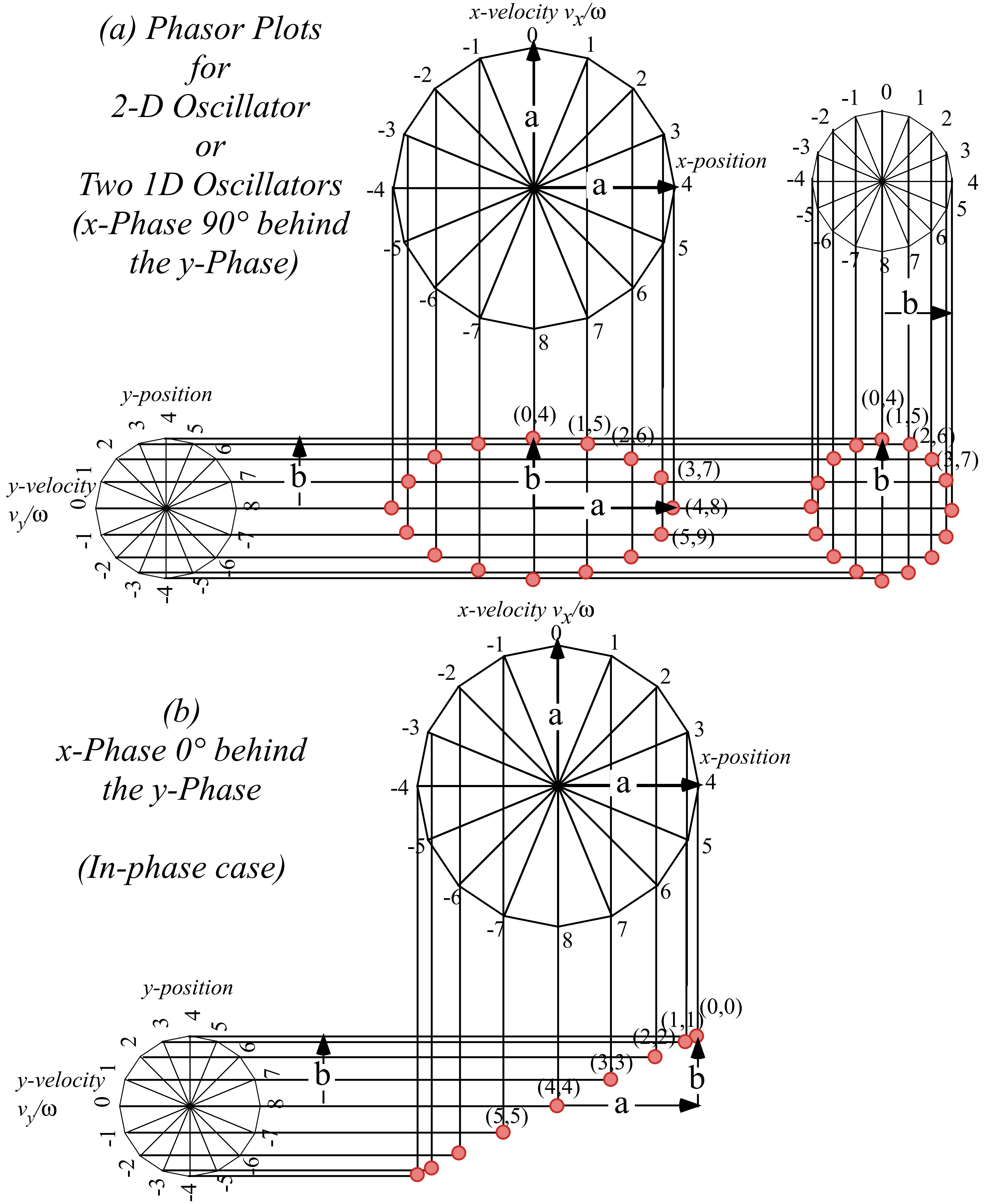
Examples with x-y phase lag : $\alpha_{x\text{-}y}=\alpha_x-\alpha_y=15°, 30°$, *and* $\pm 75°$

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

Unit 1
Fig. 9.12



These are more generic examples with radius of x-phasor differing from that of the y-phasor.

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

Sinusoidal space-time dynamics derived by geometry

Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)

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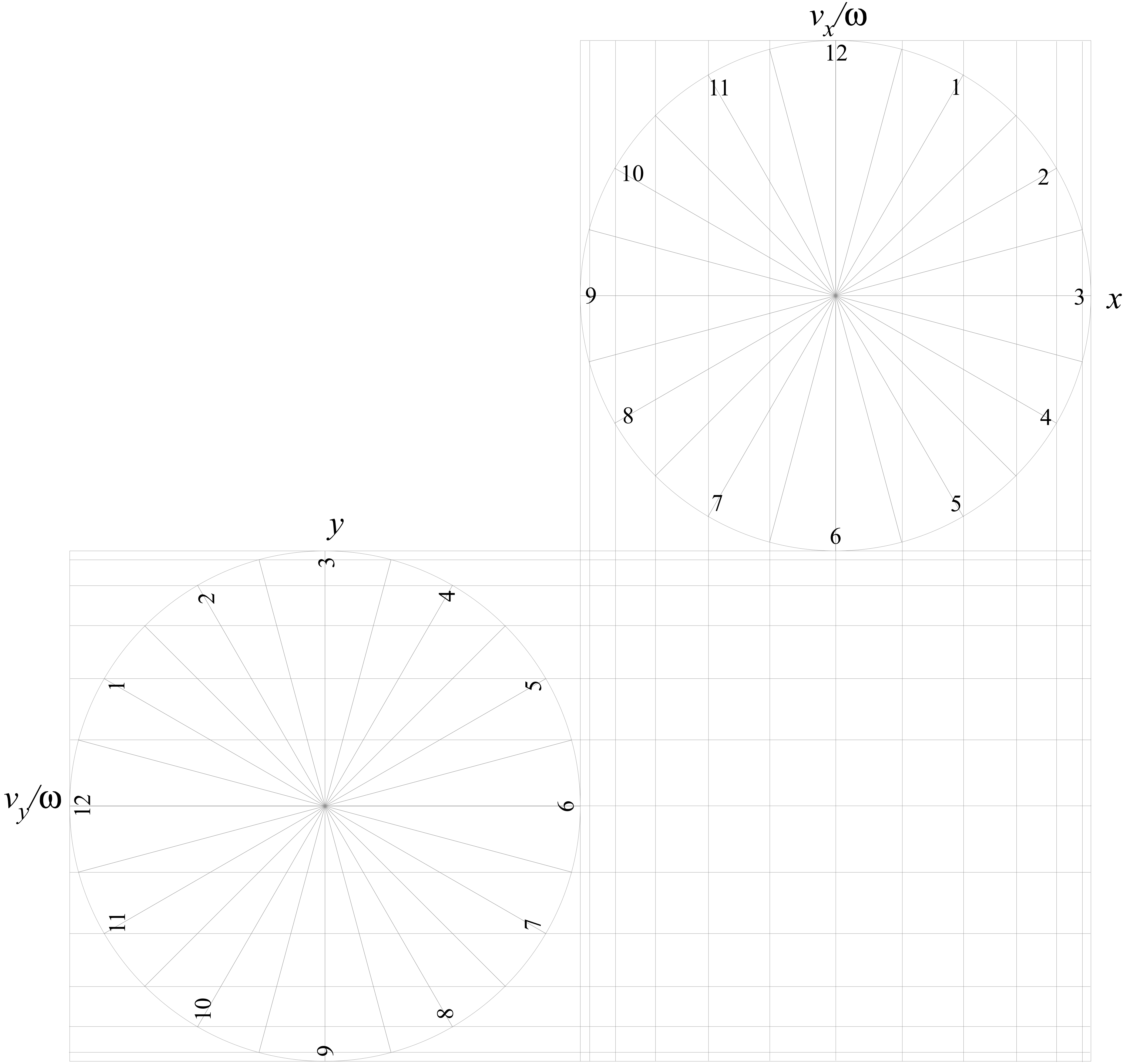
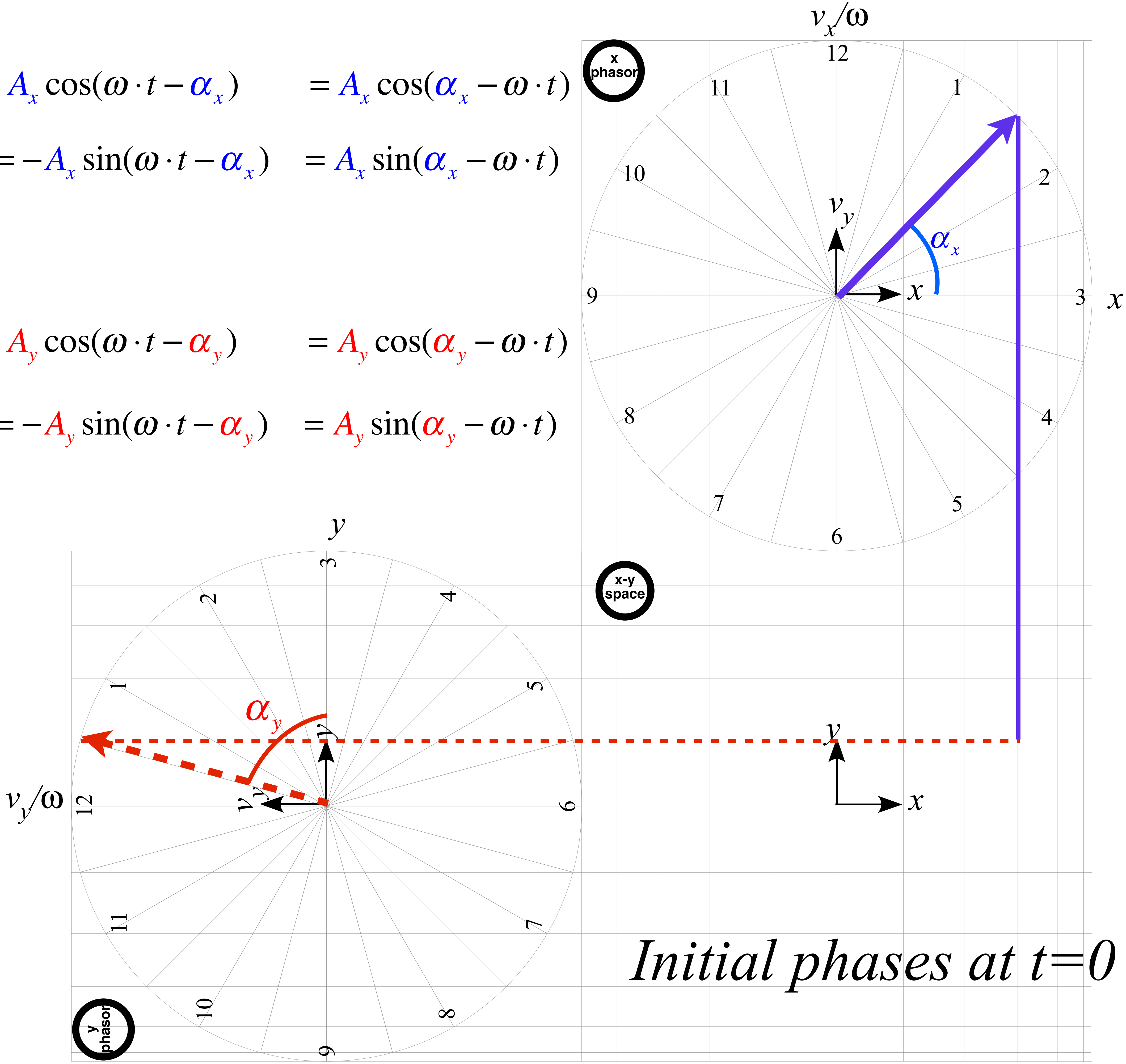

v_x/ω
x
12
1
2
3
4
5
6
7
8
9
10
11
y
v_y/ω
3
4
5
6
7
8
9
10
11
12
1
2

$$x(t) = A_x \cos(\omega \cdot t - \alpha_x) \qquad = A_x \cos(\alpha_x - \omega \cdot t)$$

$$\frac{v_x(t)}{\omega} = -A_x \sin(\omega \cdot t - \alpha_x) \quad = A_x \sin(\alpha_x - \omega \cdot t)$$

$$y(t) = A_y \cos(\omega \cdot t - \alpha_y) \qquad = A_y \cos(\alpha_y - \omega \cdot t)$$

$$\frac{v_y(t)}{\omega} = -A_y \sin(\omega \cdot t - \alpha_y) \quad = A_y \sin(\alpha_y - \omega \cdot t)$$



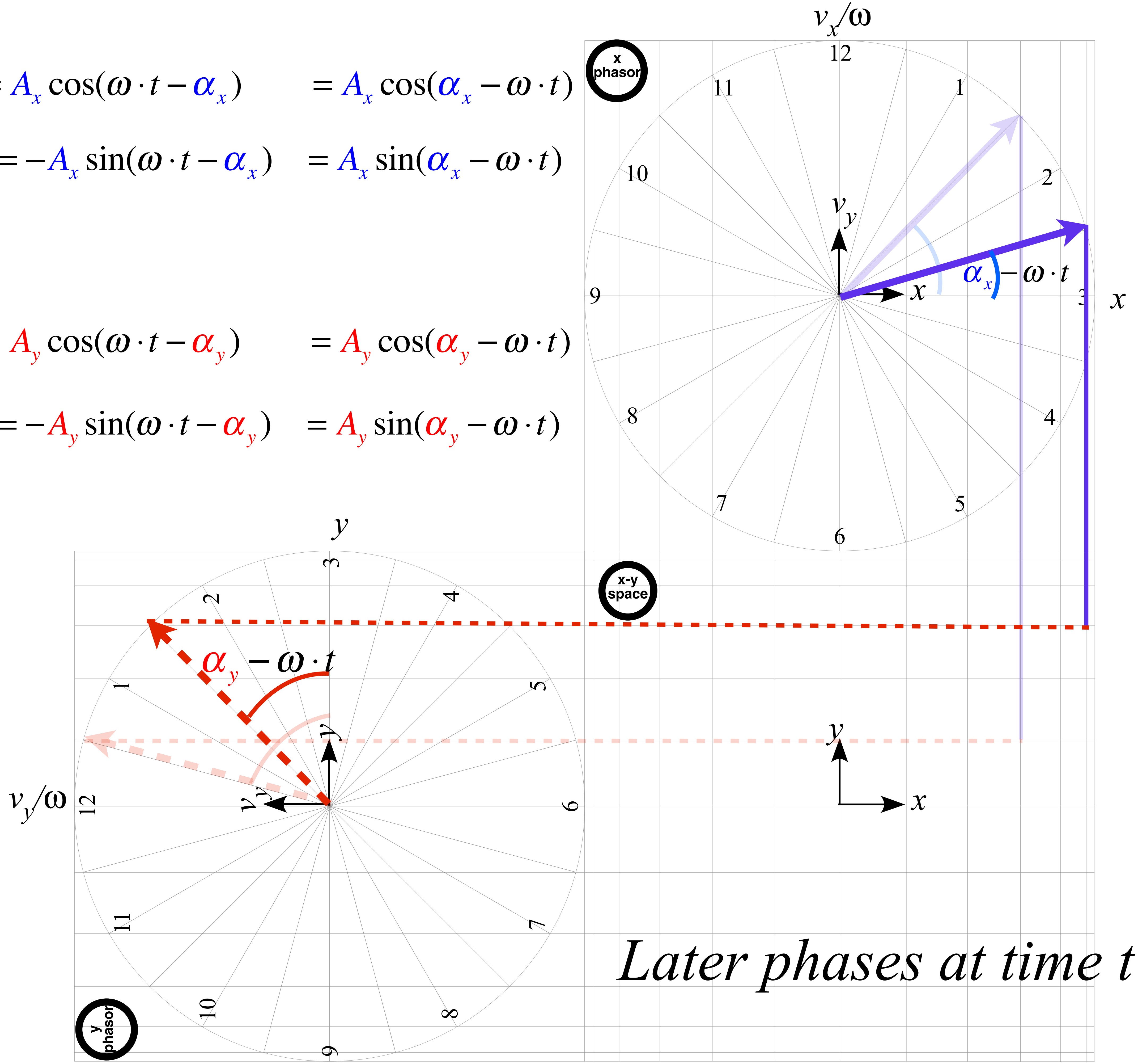
Initial phases at t=0

$$x(t) = A_x \cos(\omega \cdot t - \alpha_x) \qquad = A_x \cos(\alpha_x - \omega \cdot t)$$

$$\frac{v_x(t)}{\omega} = -A_x \sin(\omega \cdot t - \alpha_x) \quad = A_x \sin(\alpha_x - \omega \cdot t)$$

$$y(t) = A_y \cos(\omega \cdot t - \alpha_y) \qquad = A_y \cos(\alpha_y - \omega \cdot t)$$

$$\frac{v_y(t)}{\omega} = -A_y \sin(\omega \cdot t - \alpha_y) \quad = A_y \sin(\alpha_y - \omega \cdot t)$$

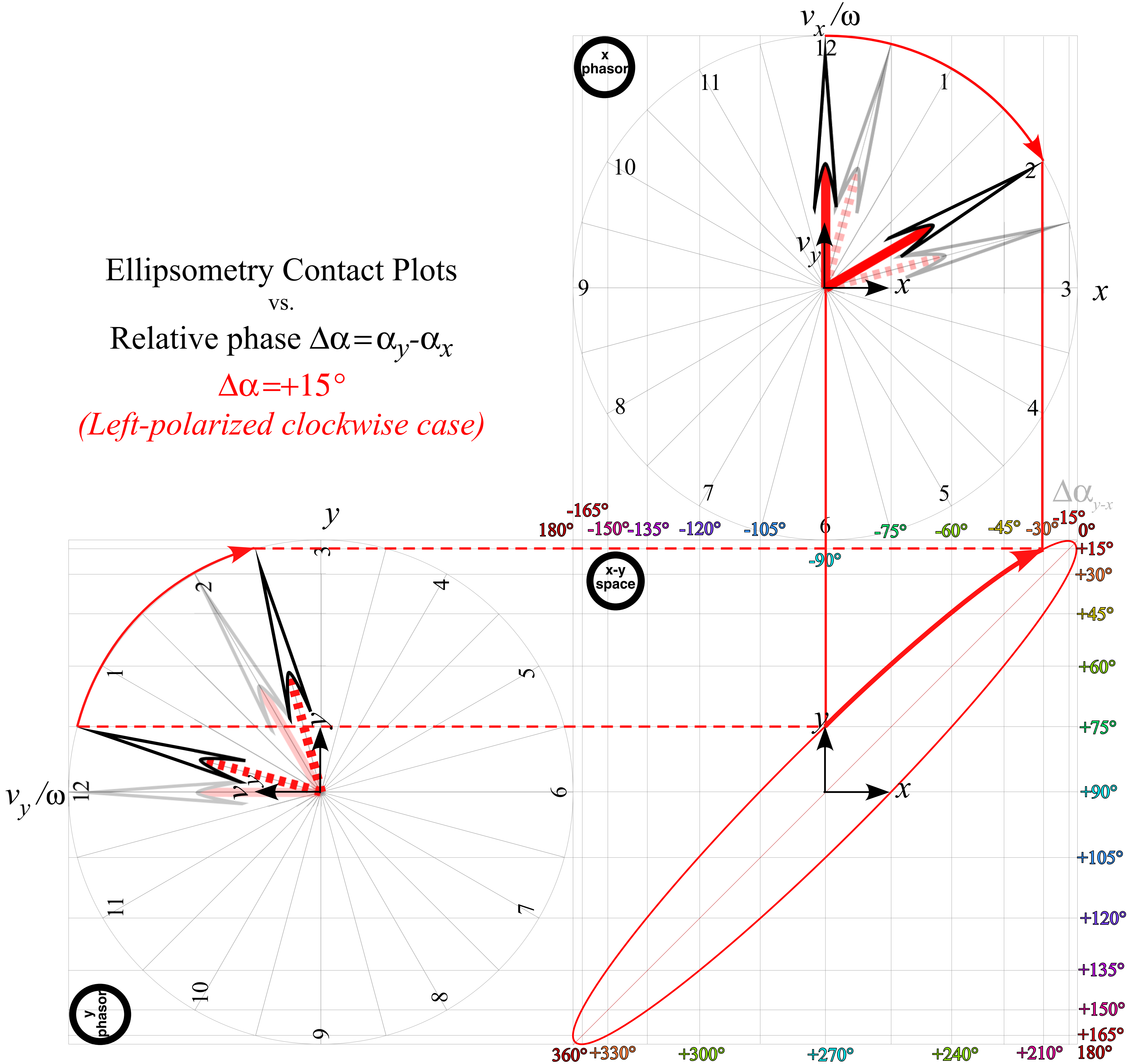


Later phases at time t

Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha = \alpha_y - \alpha_x$

$\Delta\alpha = +15°$

(Left-polarized clockwise case)

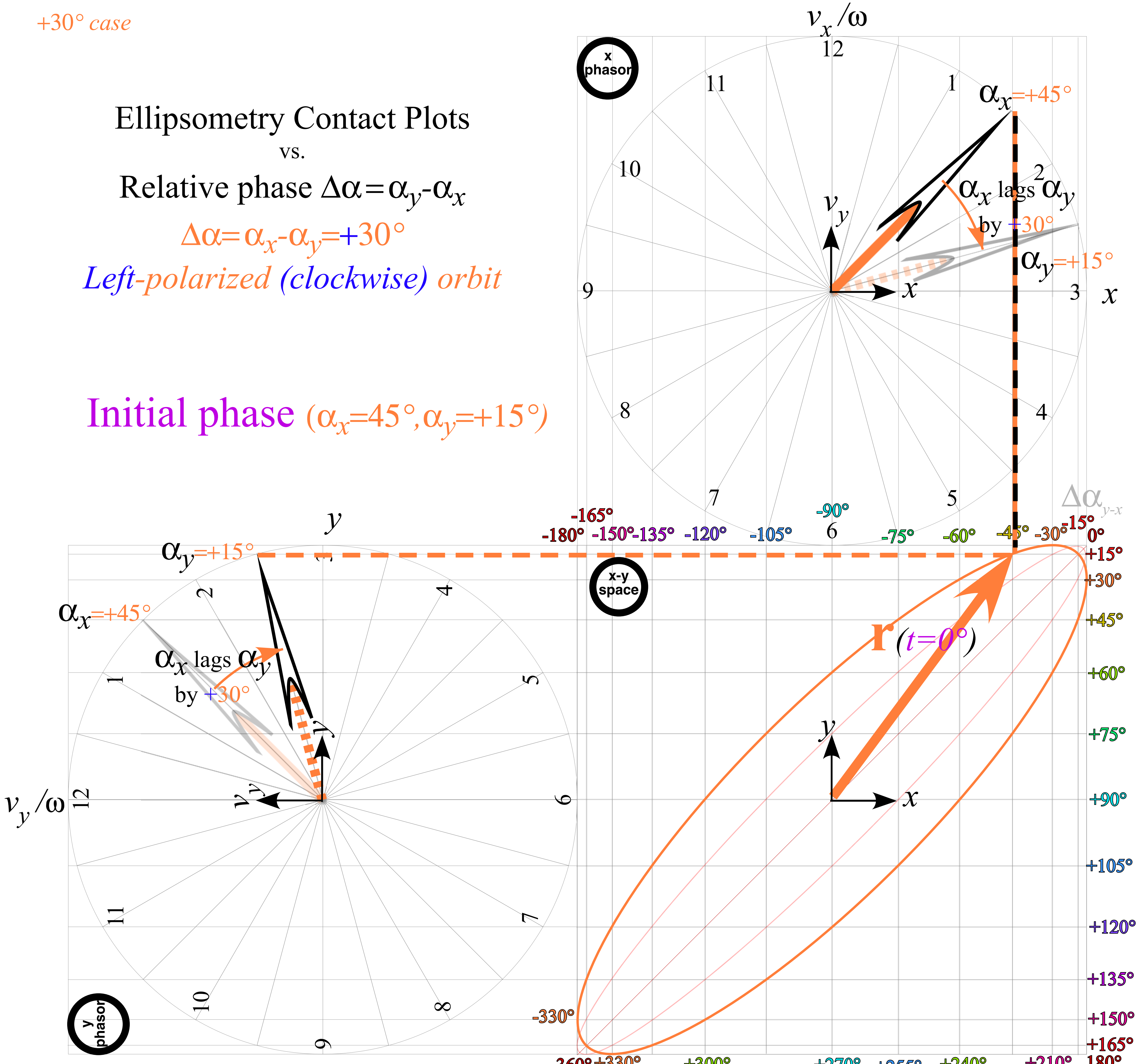


Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_y-\alpha_x$

$\Delta\alpha=\alpha_x-\alpha_y=+30°$

Left-polarized (clockwise) orbit

Initial phase (α_x=45°,α_y=+15°)



+30° case

Ellipsometry Contact Plots

vs.

Relative phase $\Delta\alpha=\alpha_y-\alpha_x$

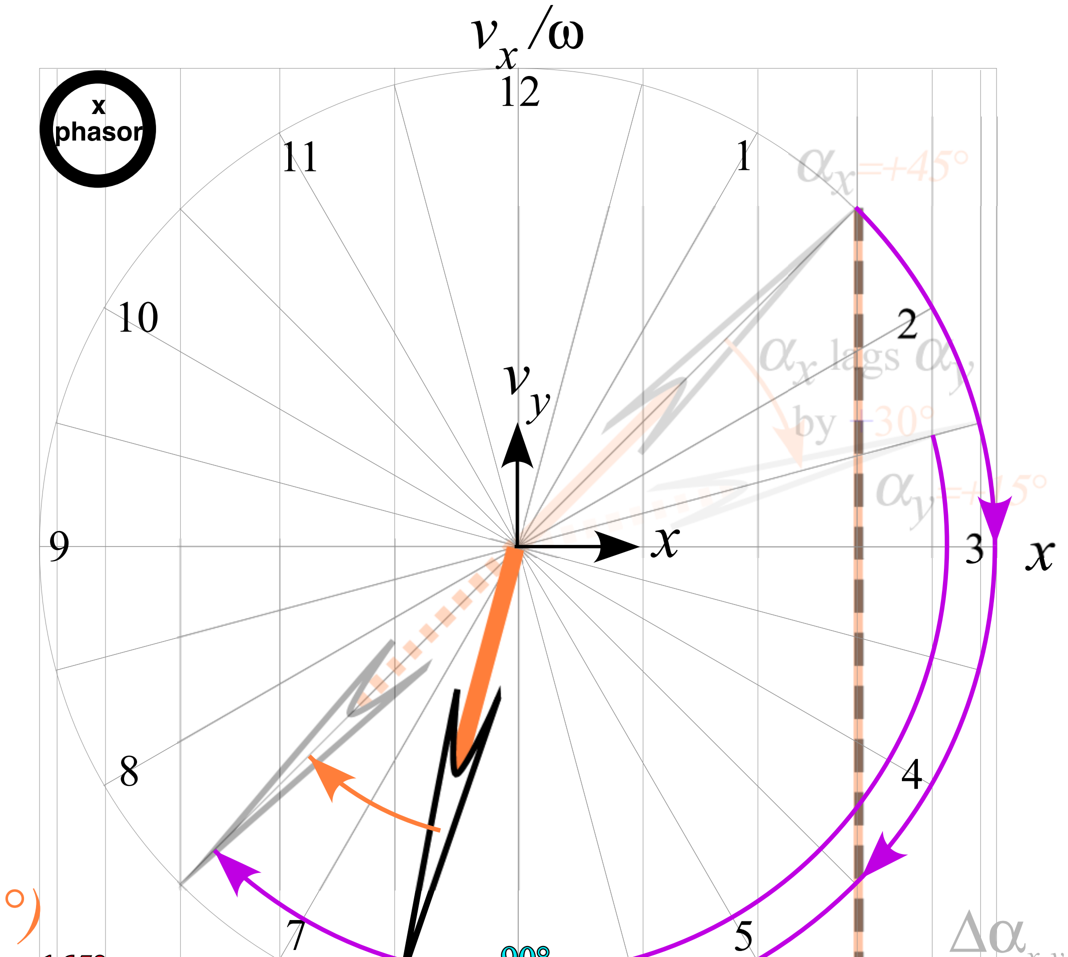
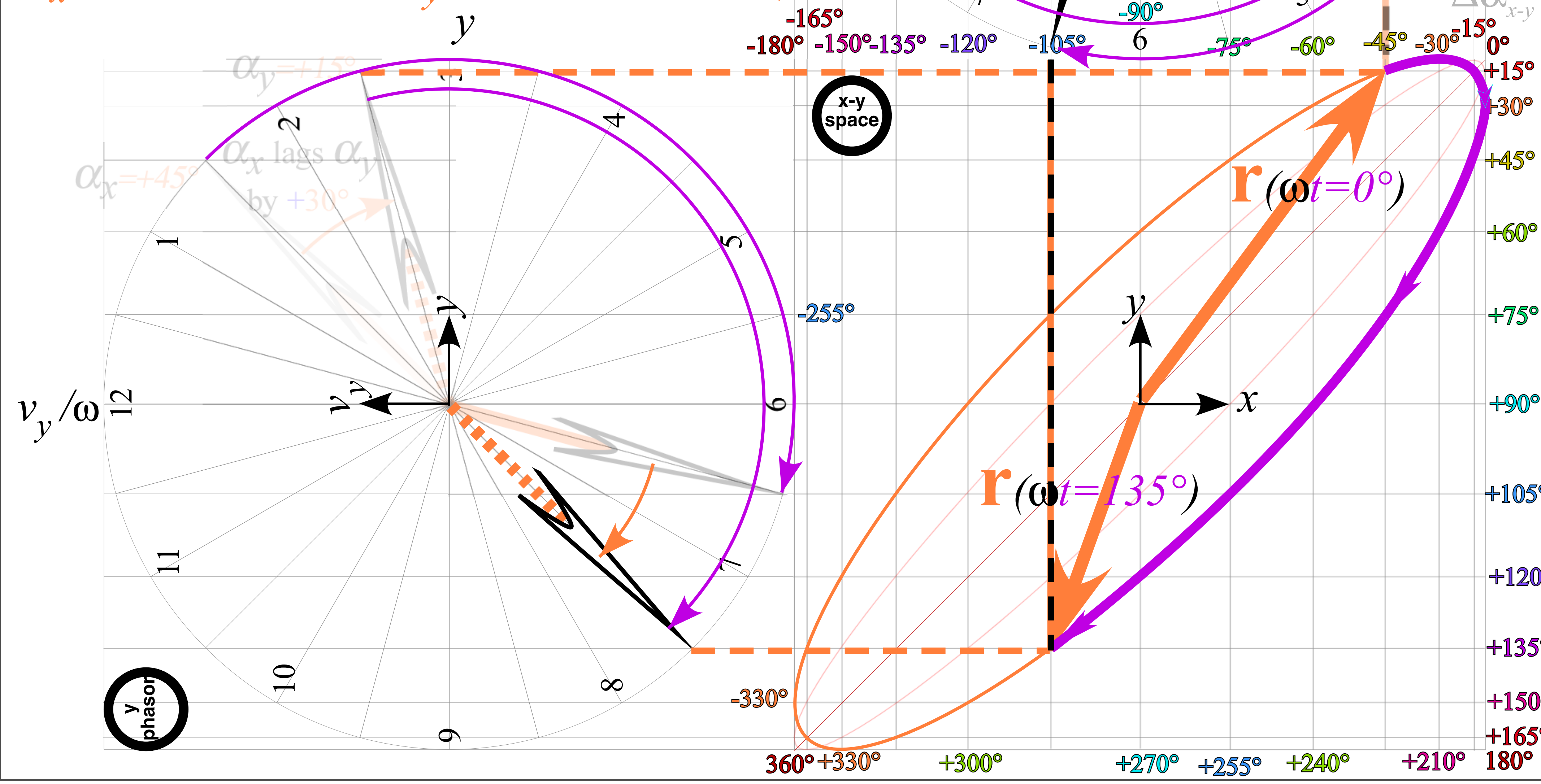
$\Delta\alpha=\alpha_x-\alpha_y=+30°$

Left-polarized (clockwise) orbit

Initial phase (α_x=45°,α_y=+15°)

...after time advances by *150°*:

(α_x=45°-*150°*=-*105°*,α_y=+15°-*150°*=-*135°*)

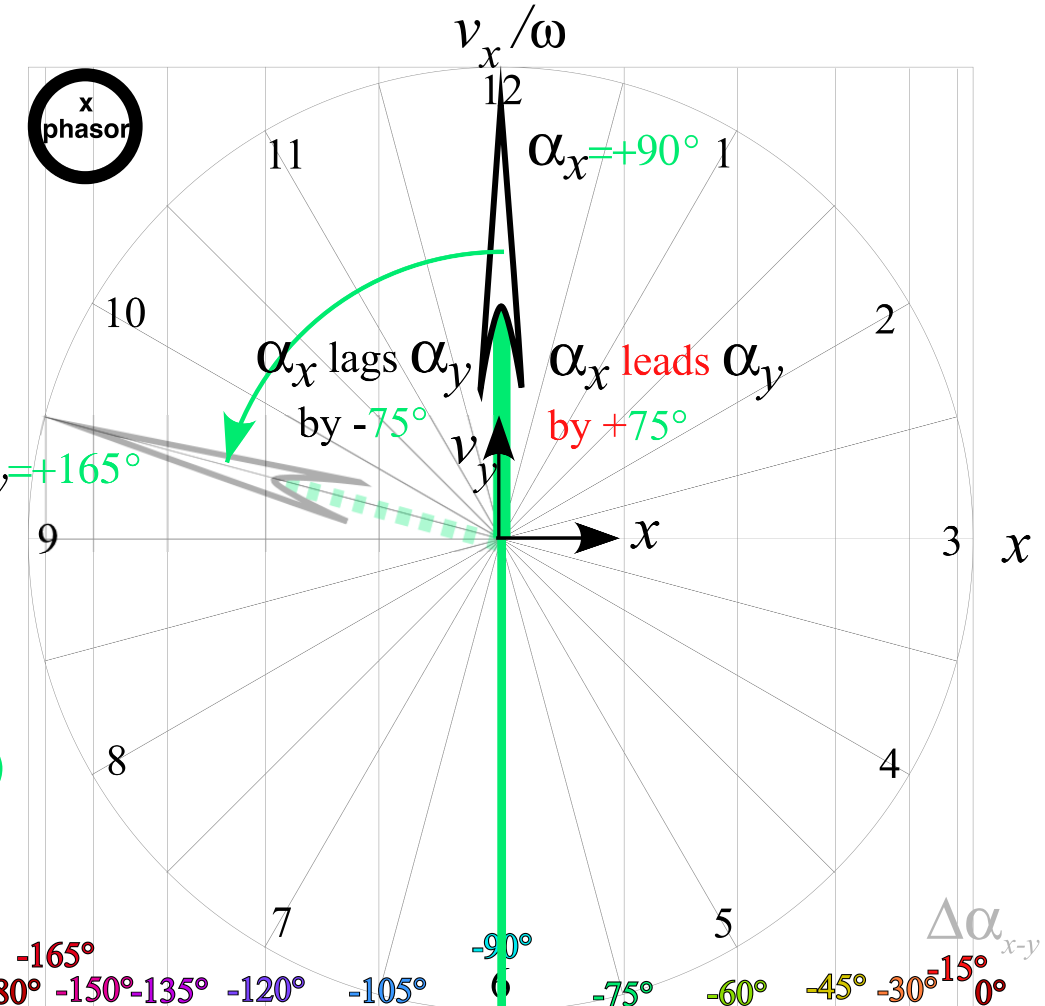



−75° case

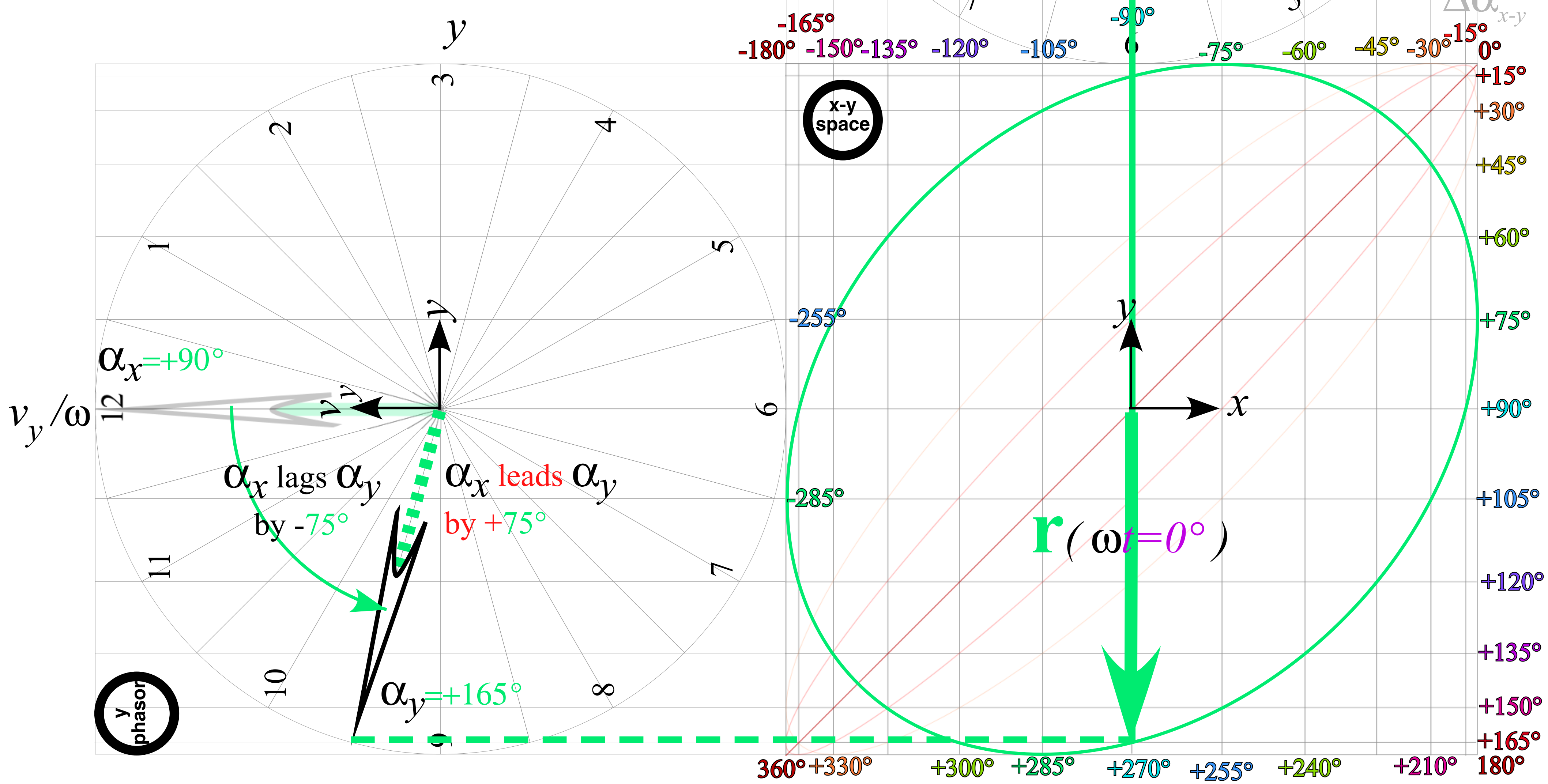
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_y-\alpha_x$

$\Delta\alpha=\alpha_x-\alpha_y=-75°$

Right-polarized (anti-clockwise) orbit



Initial phase (α_x=90°,α_y=+165°)



−75° case

Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_y-\alpha_x$

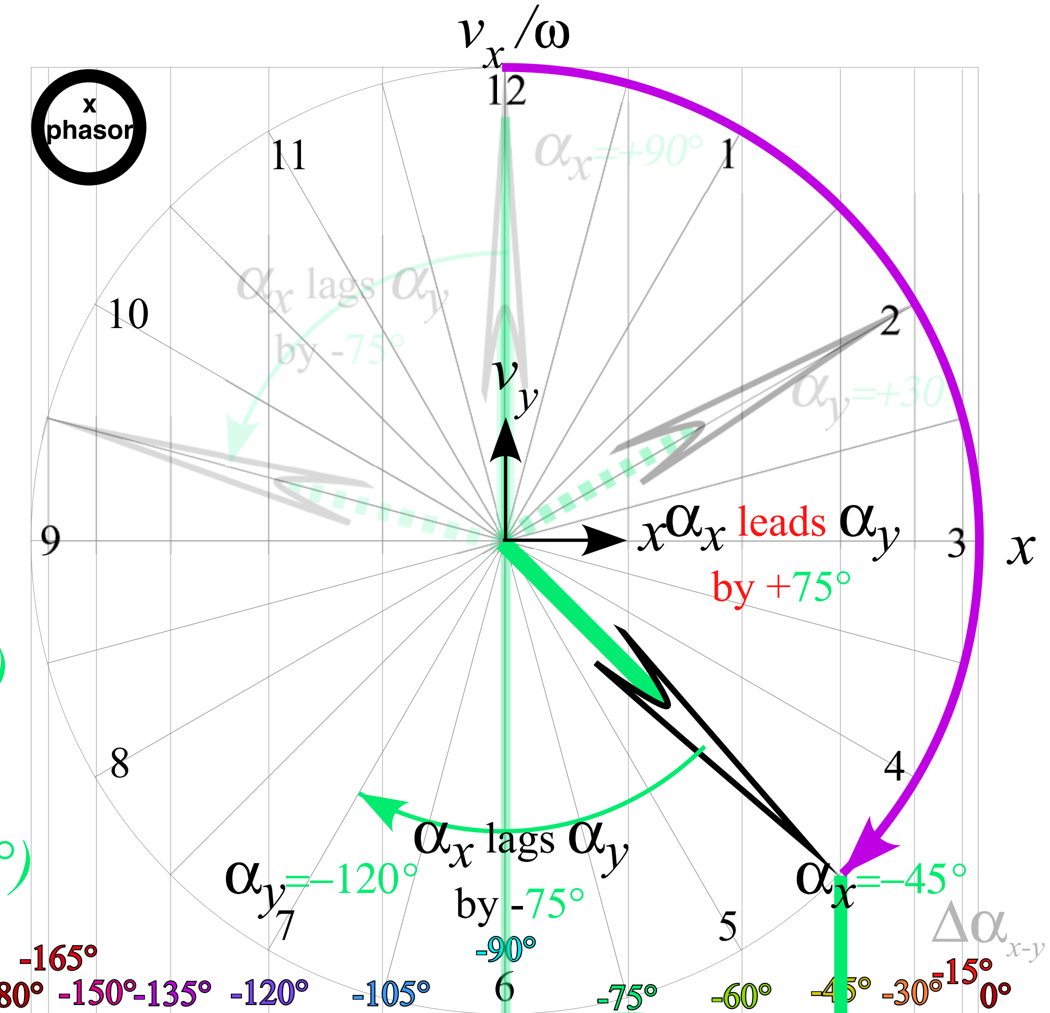
$\Delta\alpha=\alpha_x-\alpha_y=-75°$

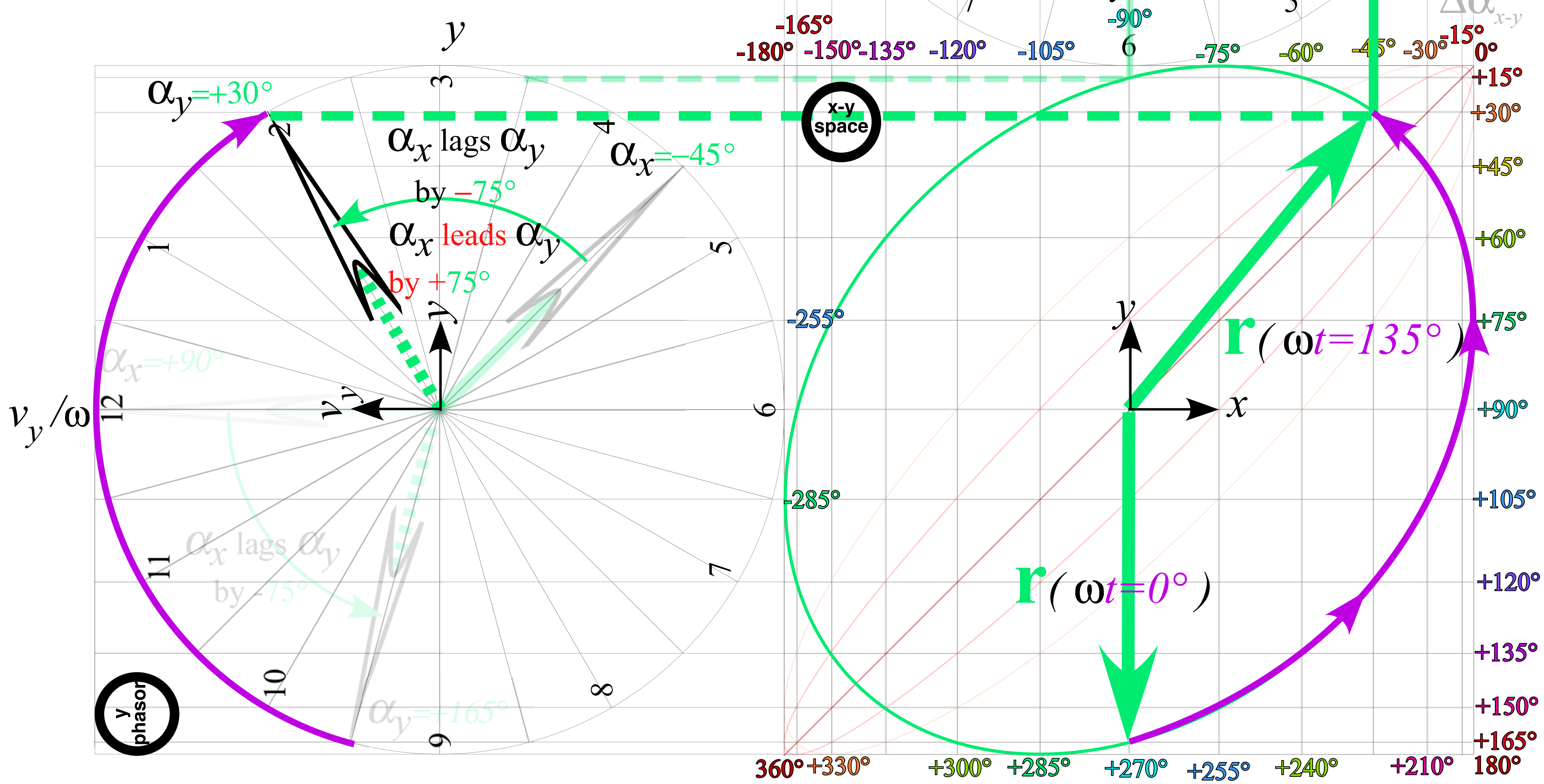
Right-polarized (anti-clockwise) orbit

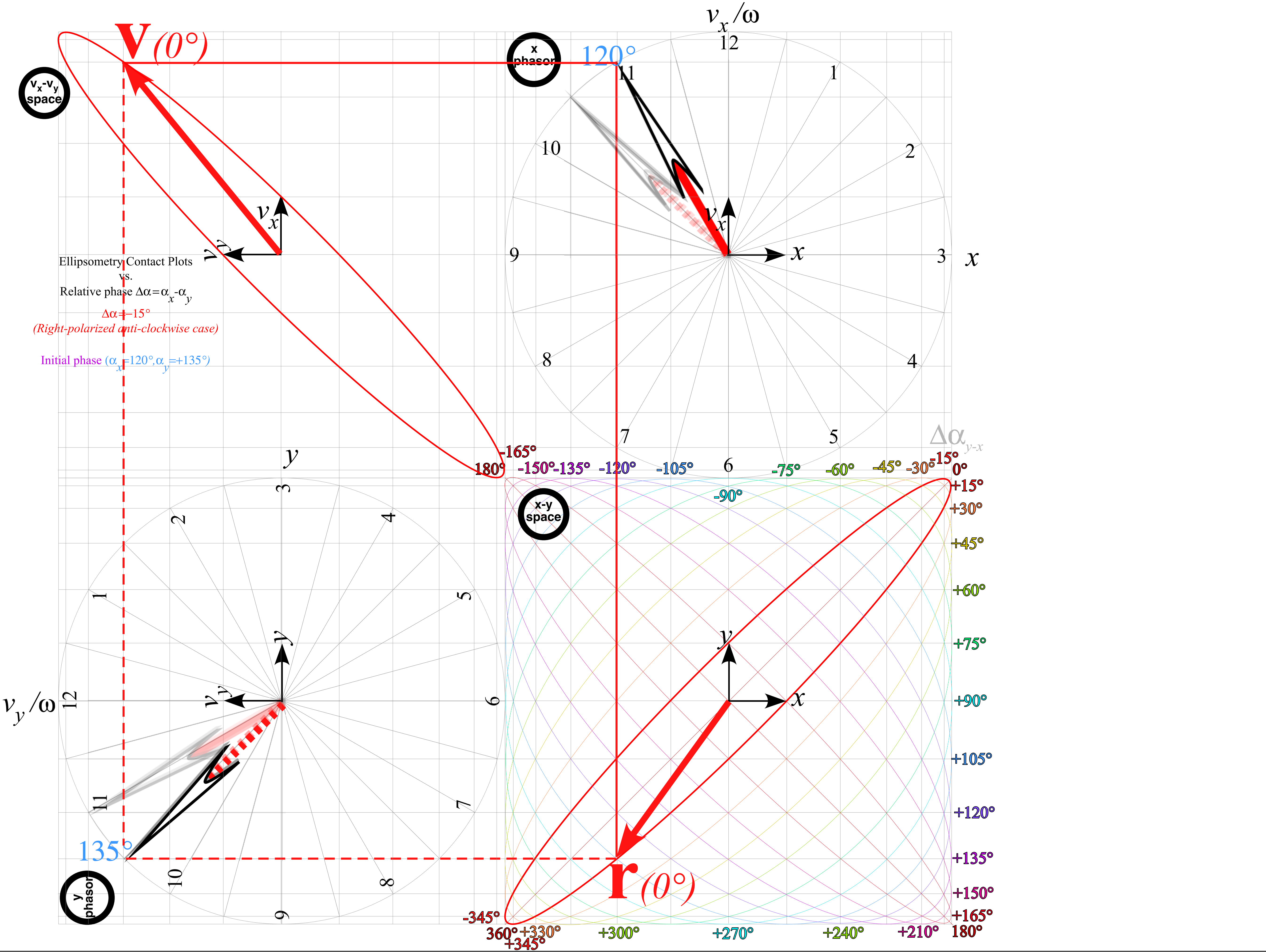
Initial phase (α_x=90°,α_y=+165°)

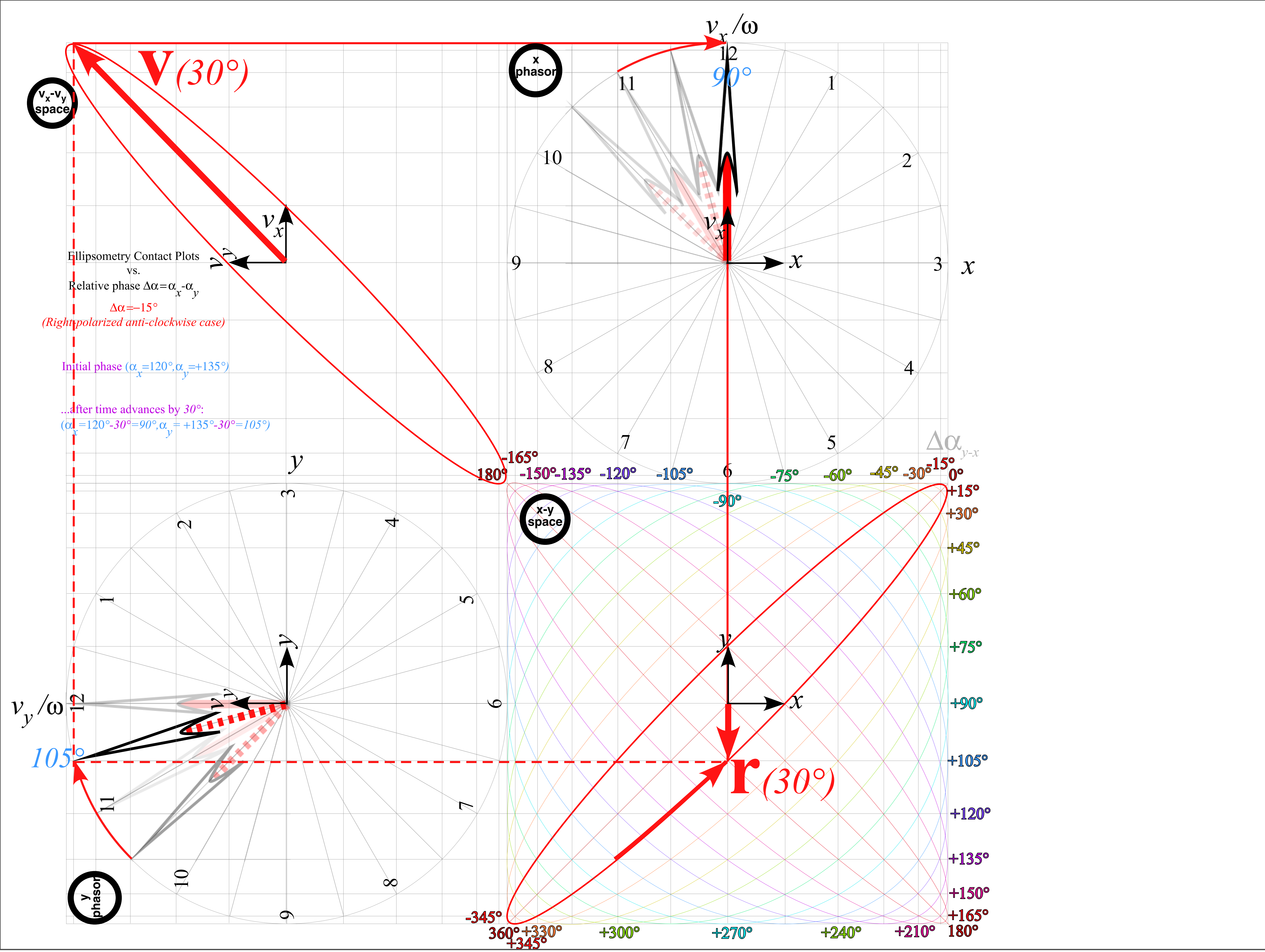
...after time advances by *135°*:

(α_x=90°-*135°*=-*45°*,α_y=+165°-*135°*=+*30°*)

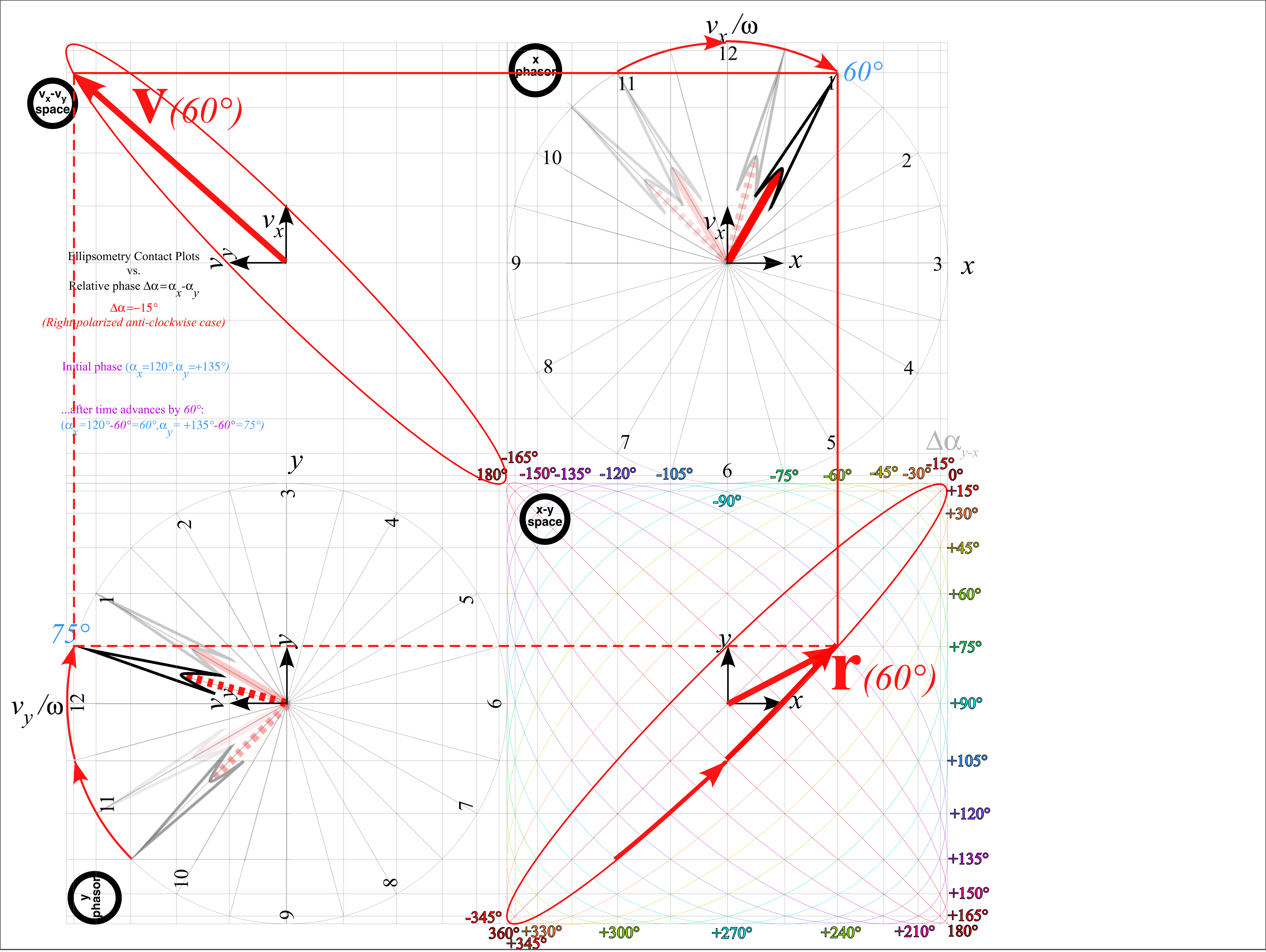




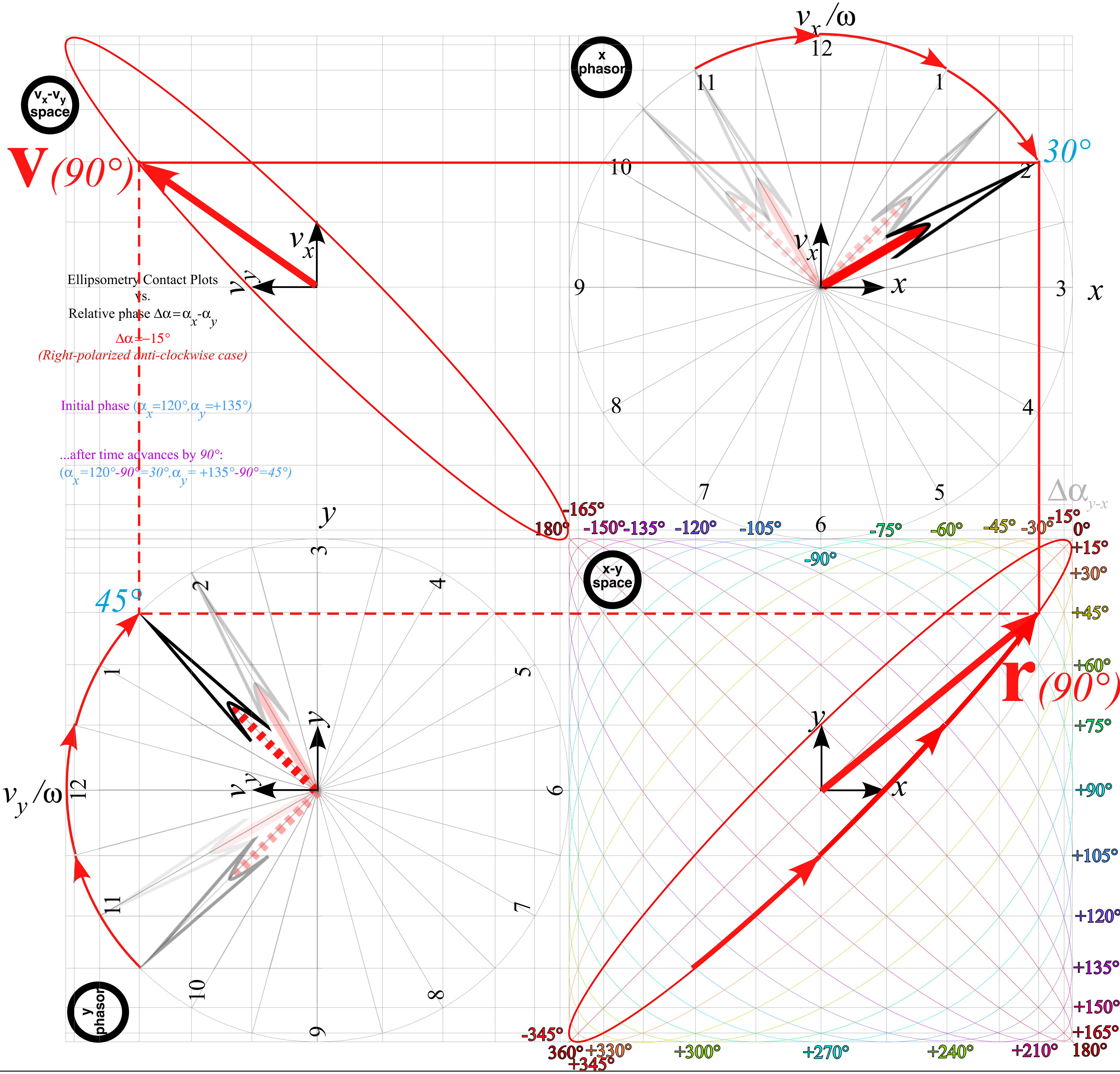

Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
V(0°)
r(0°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
120°
135°
Δα_y-x



V(30°)
v_x-v_y space
x phasor
v_x/ω
12
90°
11
1
10
2
9
3
8
4
7
5
6
x
v_x
v_y
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 30°:
(α_x=120°-30°=90°,α_y=+135°-30°=105°)
y
$\Delta\alpha_{y-x}$
-165°
180°
-150°
-135°
-120°
-105°
-90°
-75°
-60°
-45°
-30°
-15°
0°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
+135°
+150°
+165°
180°
x-y space
y phasor
v_y/ω
105°
r(30°)
-345°
360°
+345°
+330°
+300°
+270°
+240°
+210°



Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 60°:
(αx=120°-60°=60°,αy=+135°-60°=75°)
V(60°)
r(60°)
vx-vy space
x phasor
y phasor
x-y space
vx/ω
vy/ω
60°
75°
Δαy-x



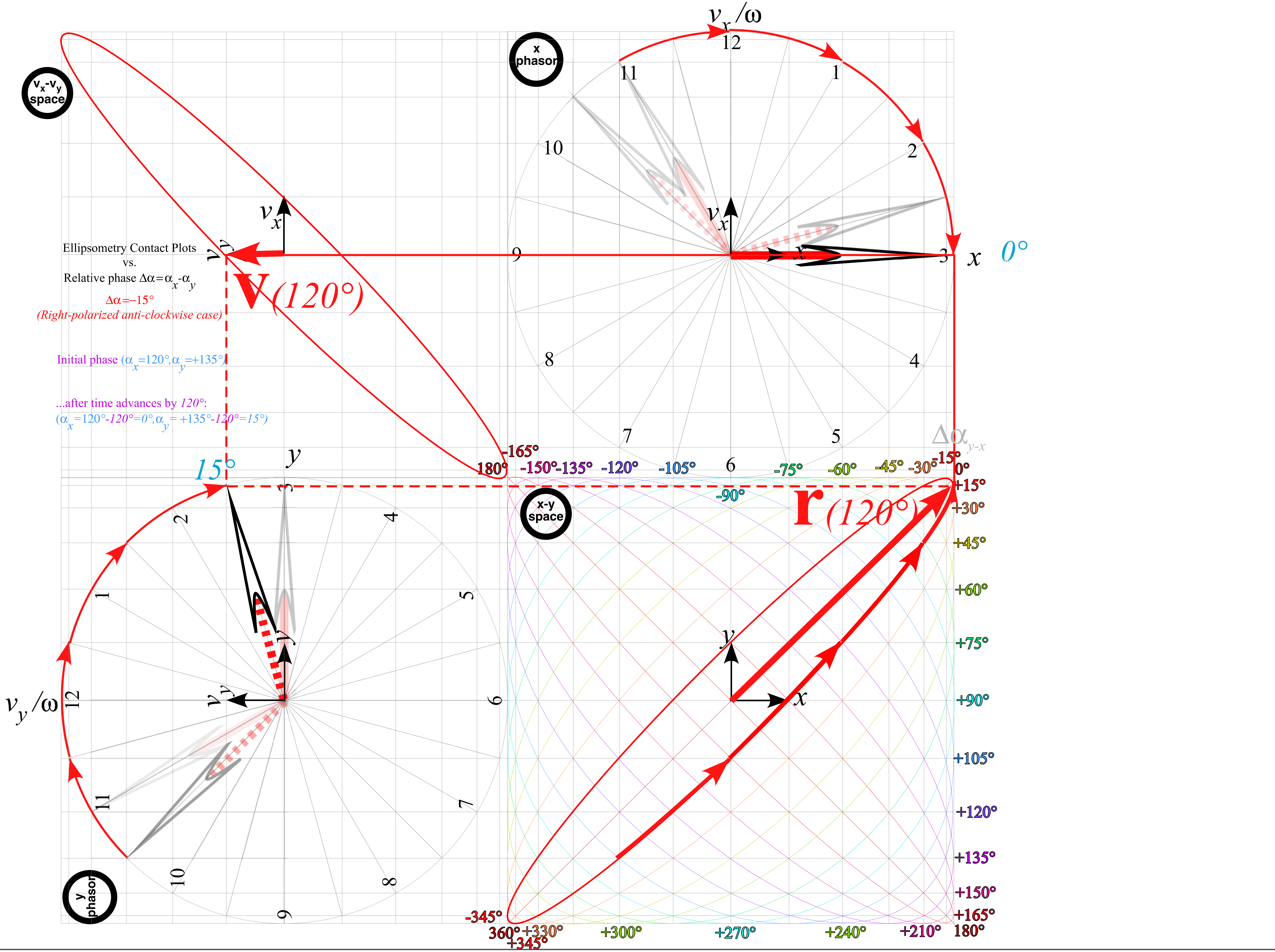
v_x/ω
x phasor
v_x-v_y space
V *(90°)*
30°
v_x
v_y
x
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by *90°*:
(α_x=120°-*90°*=*30°*,α_y = +135°-*90°*=*45°*)
y
45°
v_y/ω
y phasor
x-y space
r *(90°)*
$\Delta\alpha_{y-x}$
-165° 180° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°

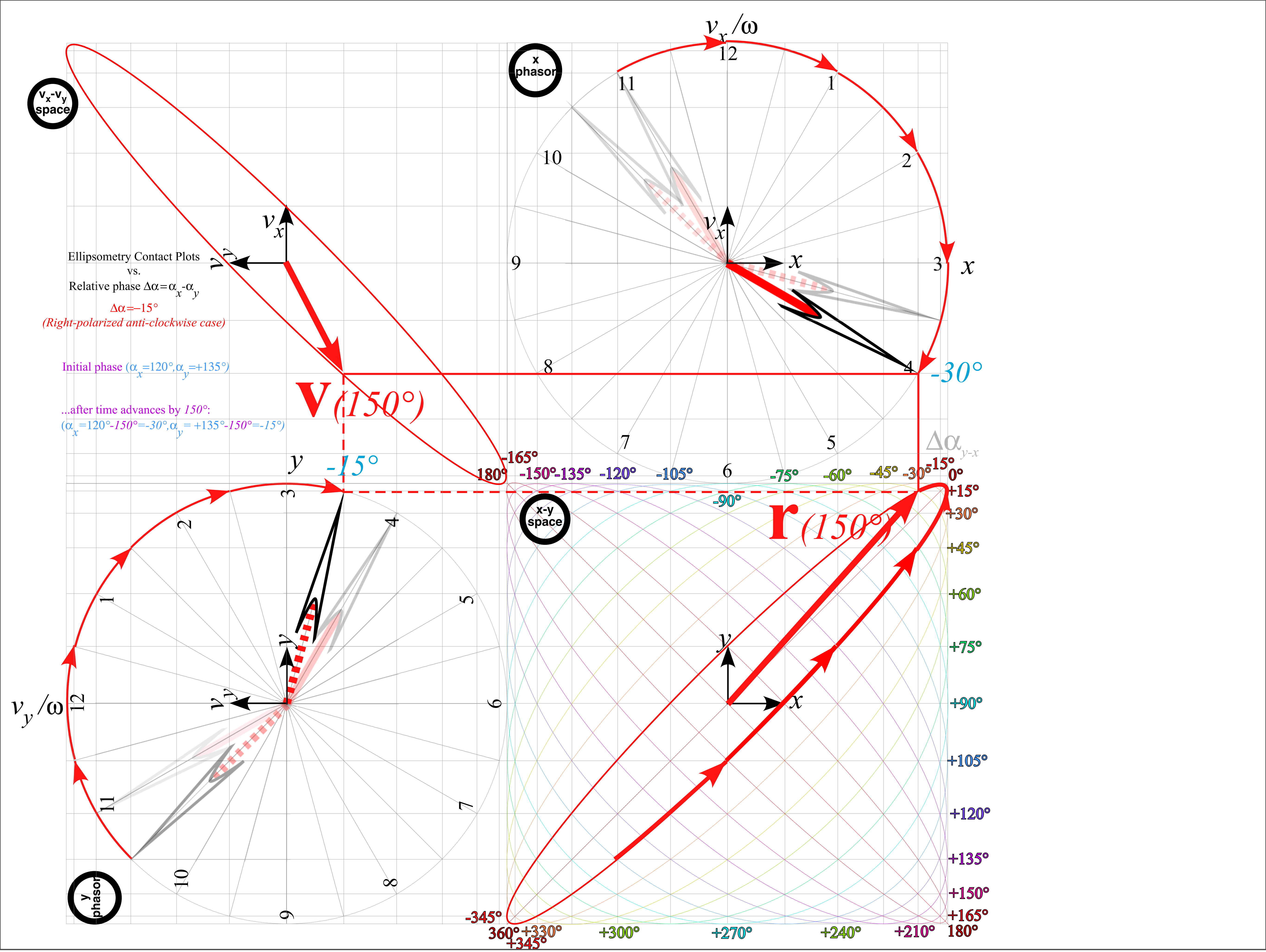
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$

$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)

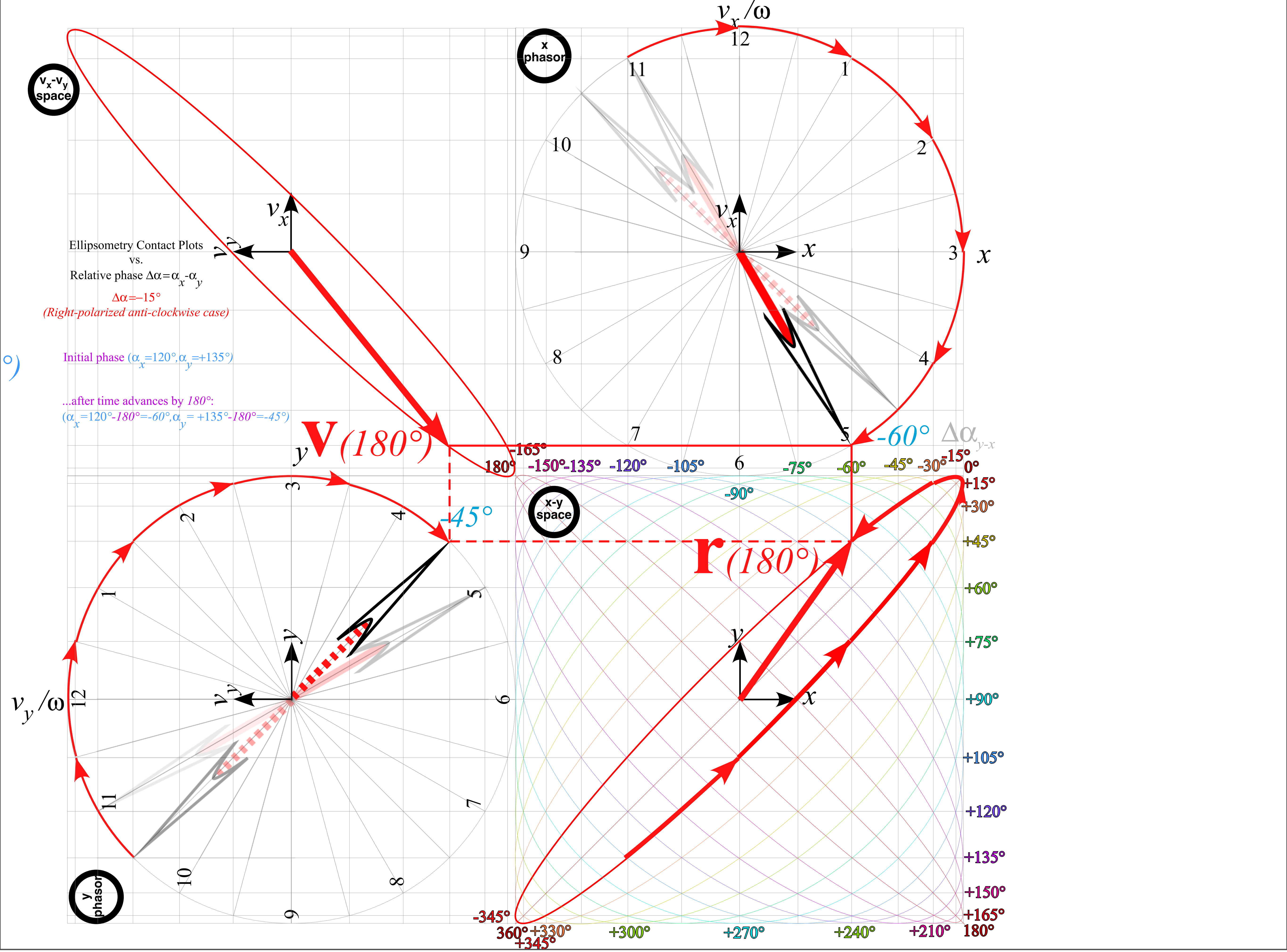
Initial phase ($\alpha_x=120°, \alpha_y=+135°$)

...after time advances by *120°*:
($\alpha_x=120°-120°=0°, \alpha_y=+135°-120°=15°$)

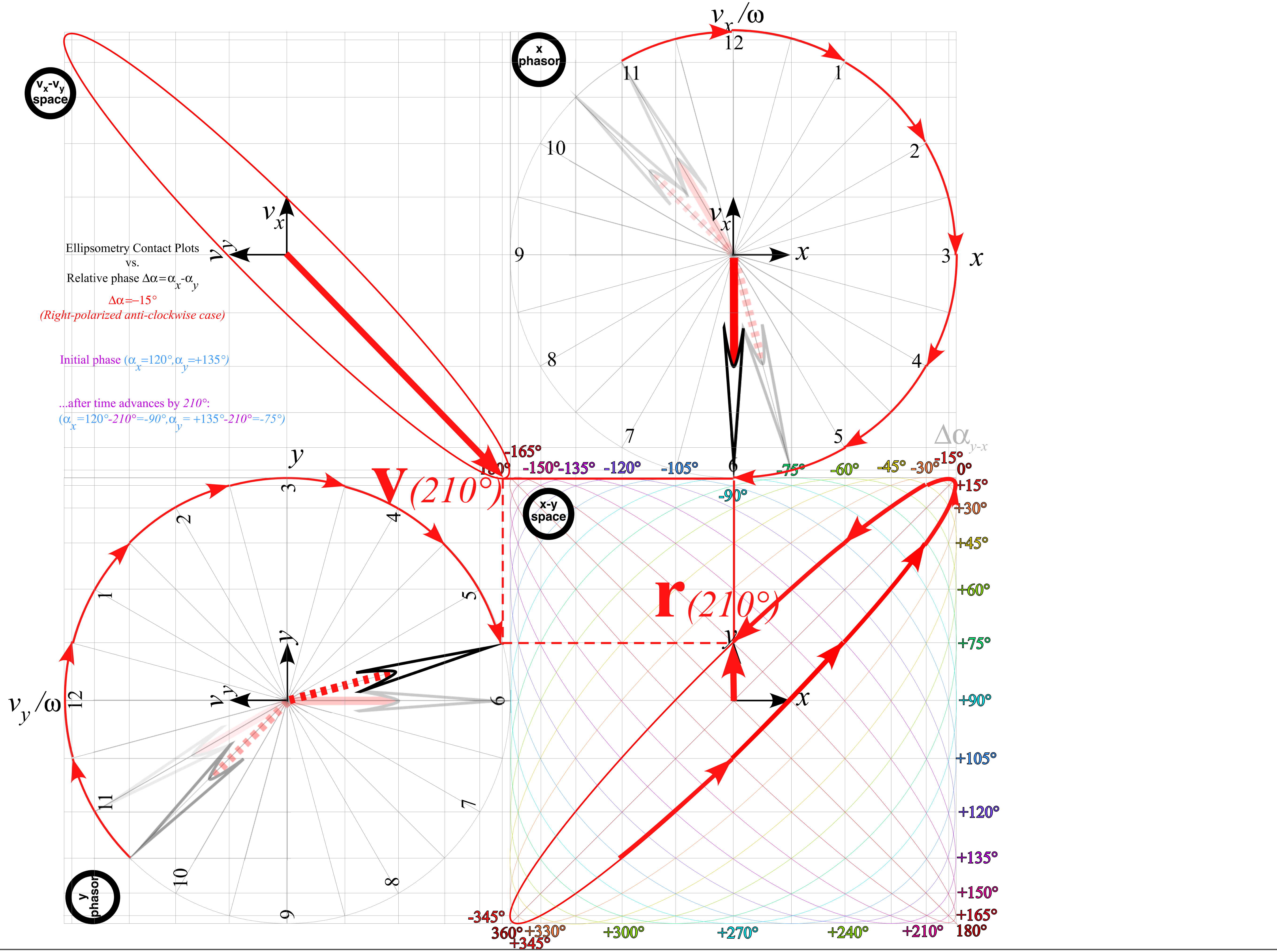




Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 150°:
(α_x=120°-150°=-30°,α_y= +135°-150°=-15°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(150°)
r(150°)
-30°
-15°
Δα_y-x
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 180°:
(α_x=120°-180°=-60°,α_y= +135°-180°=-45°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(180°)
r(180°)
-60°
-45°
$\Delta\alpha_{y-x}$


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 210°:
(α_x=120°-210°=-90°,α_y= +135°-210°=-75°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(210°)
r(210°)
Δα_y-x
-165°
-150°
-135°
-120°
-105°
-90°
-75°
-60°
-45°
-30°
-15°
0°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
+135°
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+165°
180°
+210°
+240°
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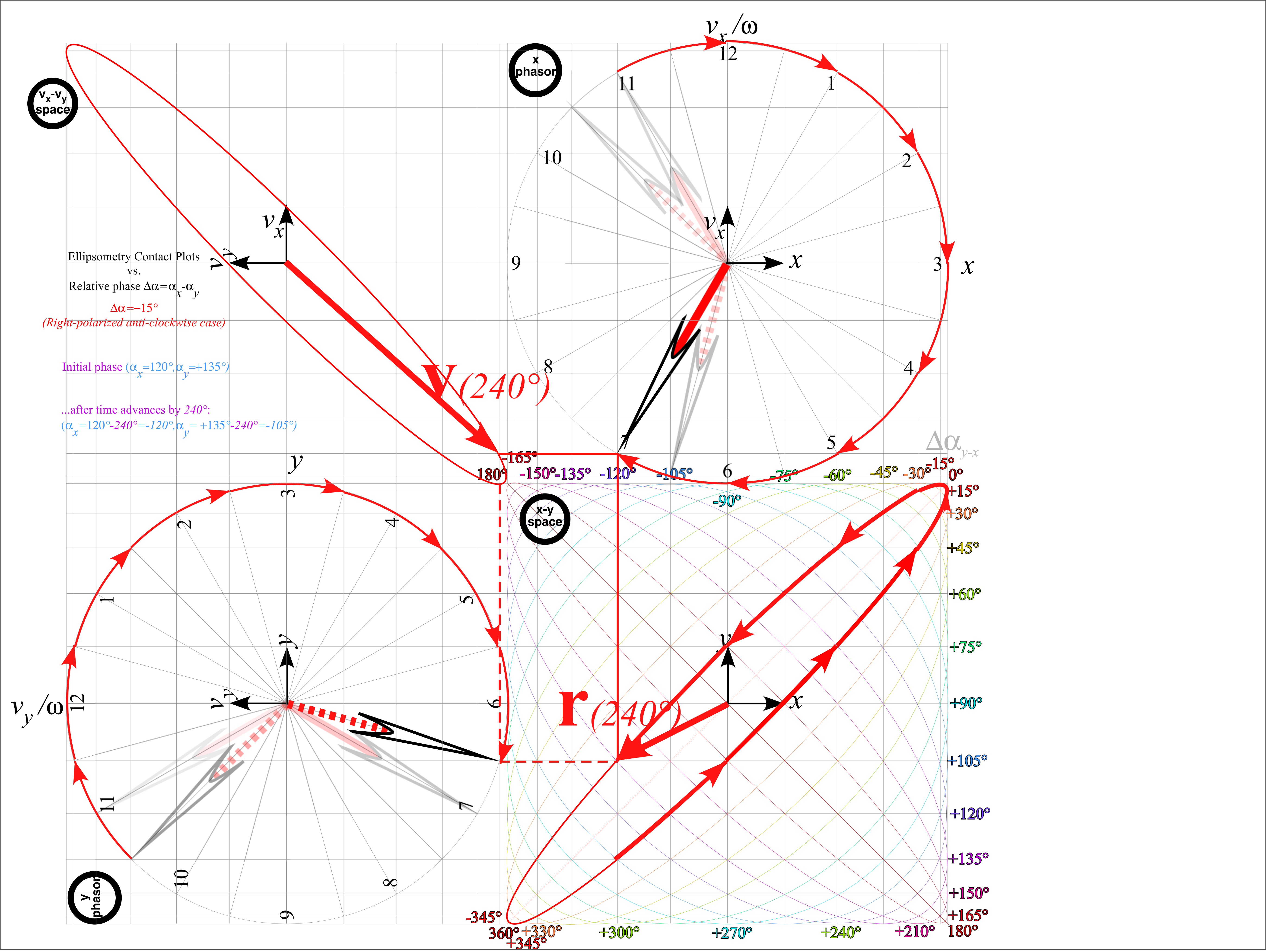
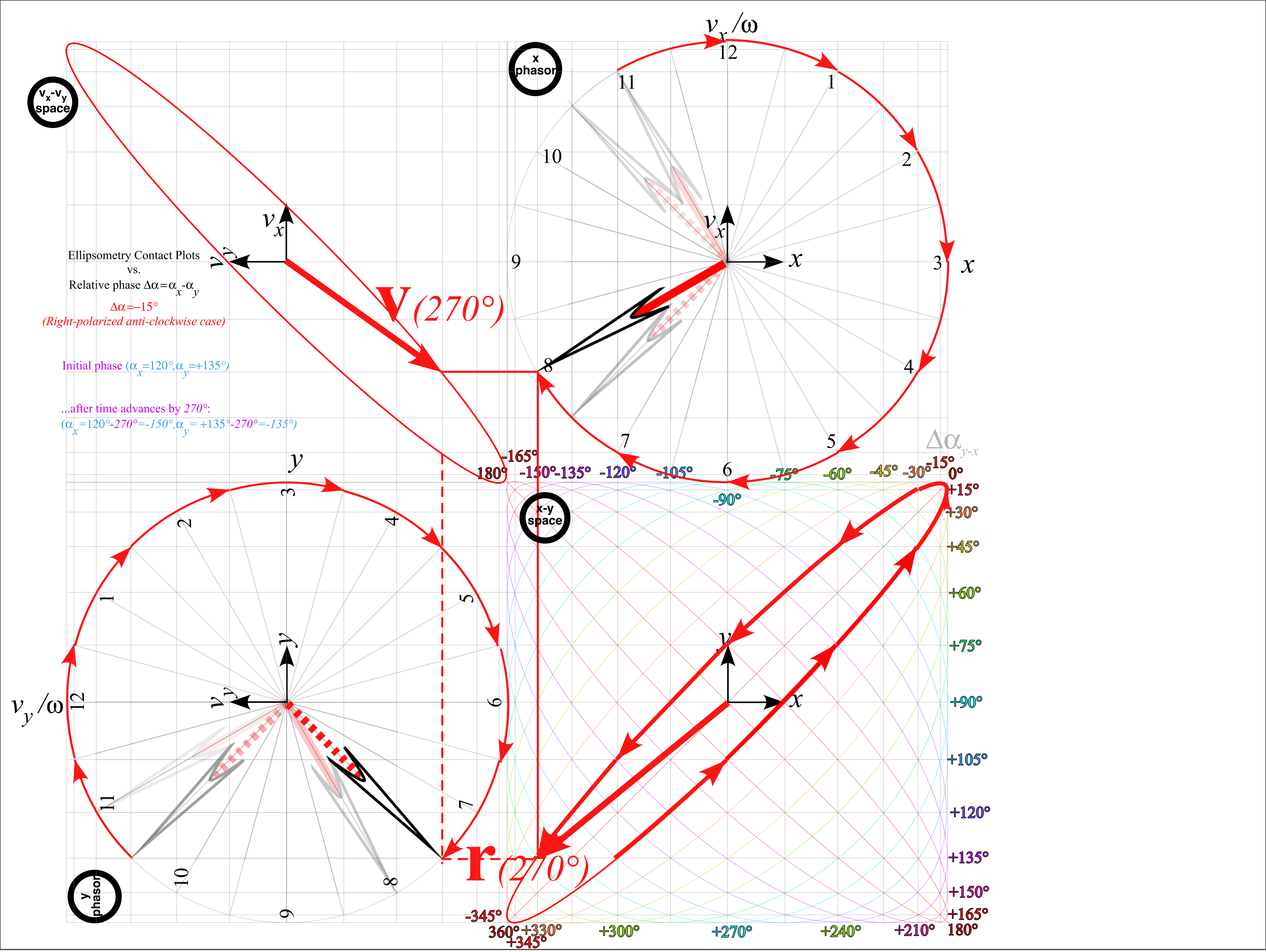
+300°
+330°
+345°
360°
-345°



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 240°:
(α_x=120°-240°=-120°,α_y=+135°-240°=-105°)
v_x-v_y space
x phasor
y phasor
x-y space
V(240°)
r(240°)
v_x/ω
v_y/ω
Δα_y-x



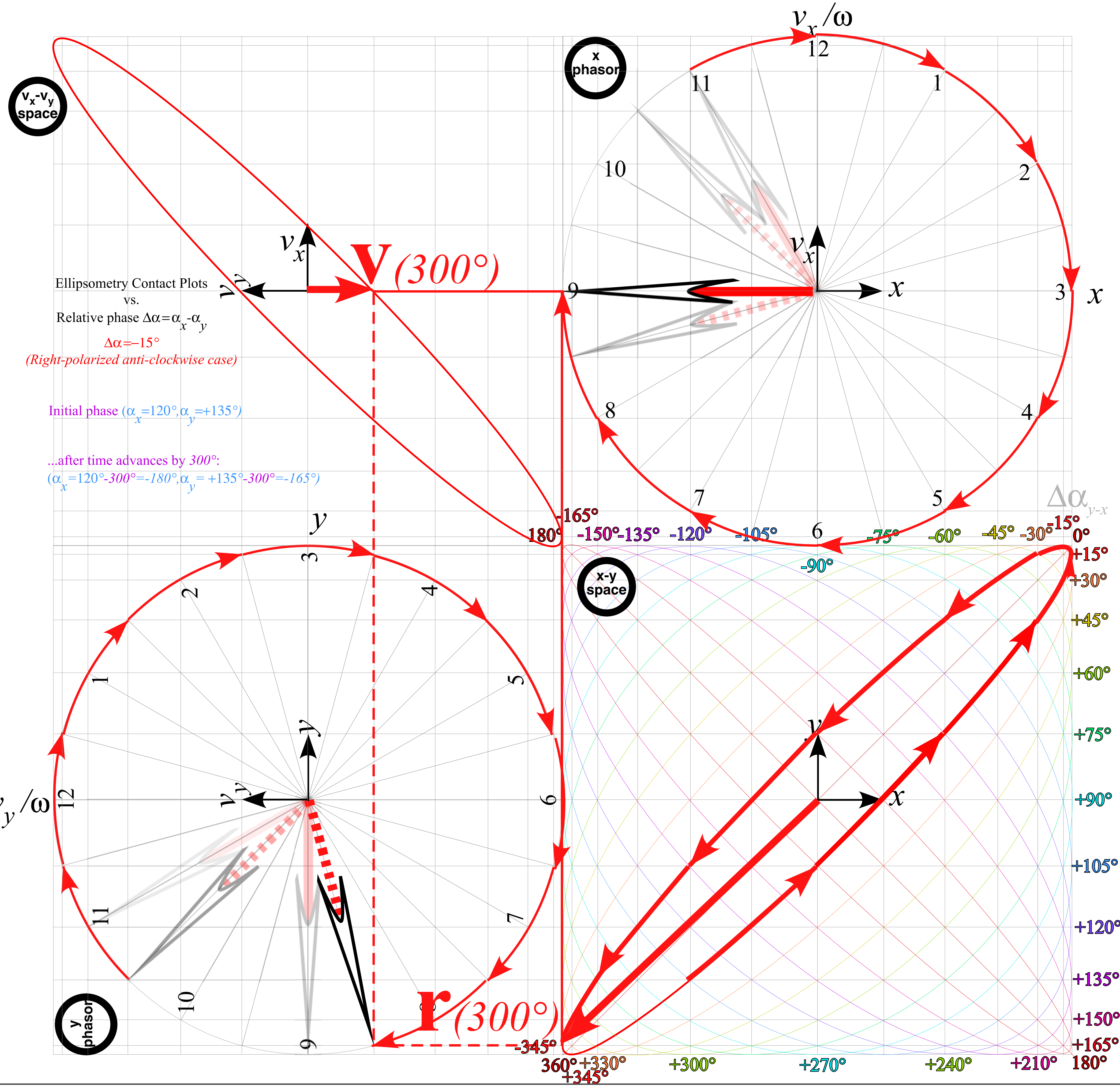
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 270°:
(α_x=120°-270°=-150°,α_y = +135°-270°=-135°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(270°)
r(270°)
v_x
v_y
x
y
Δα_y-x
0°
-15°
-30°
-45°
-60°
-75°
-90°
-105°
-120°
-135°
-150°
-165°
180°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
+135°
+150°
+165°
180°
+210°
+240°
+270°
+300°
+330°
+345°
360°
-345°

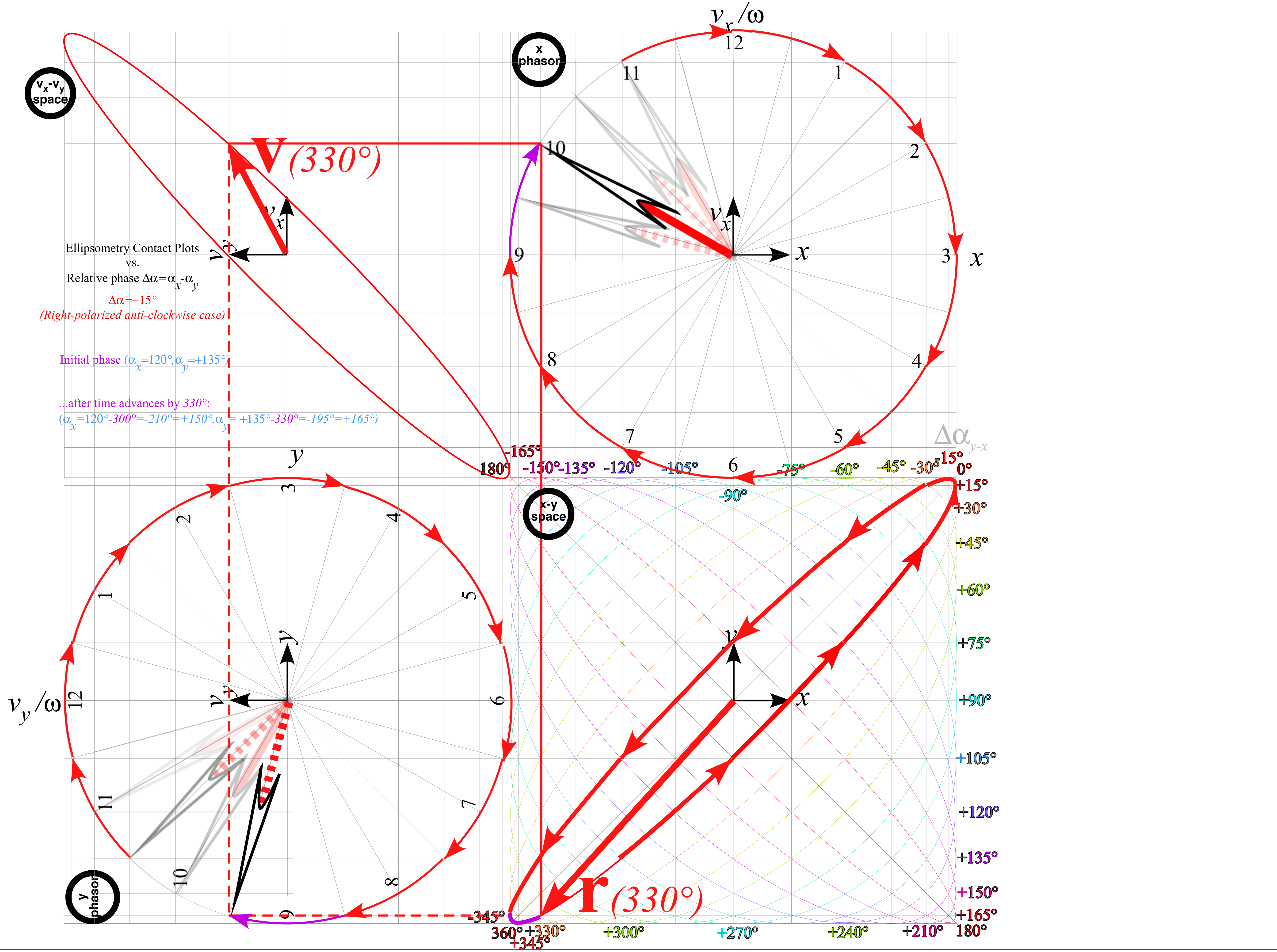
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$

$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)

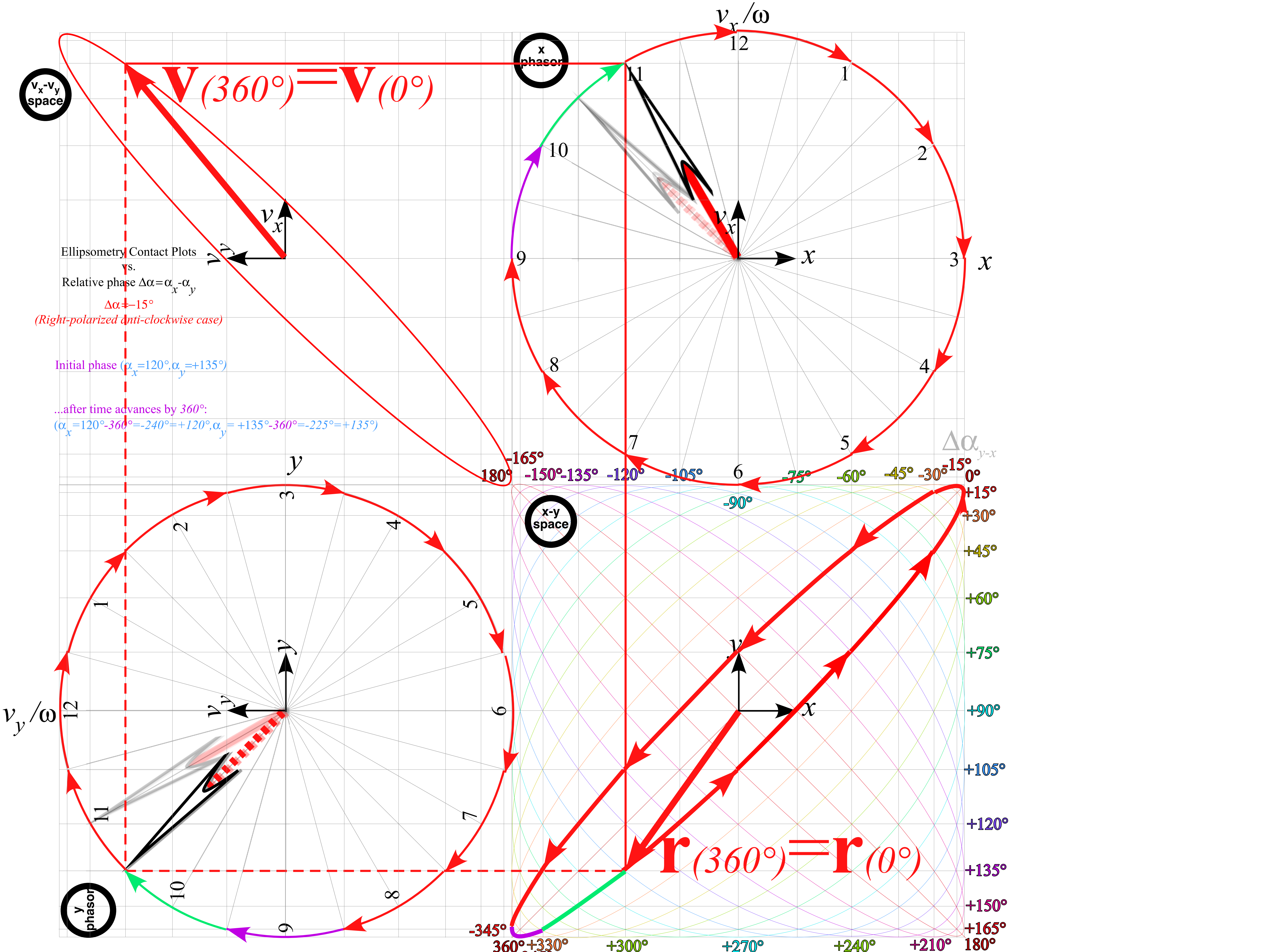
Initial phase ($\alpha_x=120°, \alpha_y=+135°$)

...after time advances by *300°*:
($\alpha_x=120°$-*300°*=*-180°*, α_y = +135°-*300°*=*-165°*)

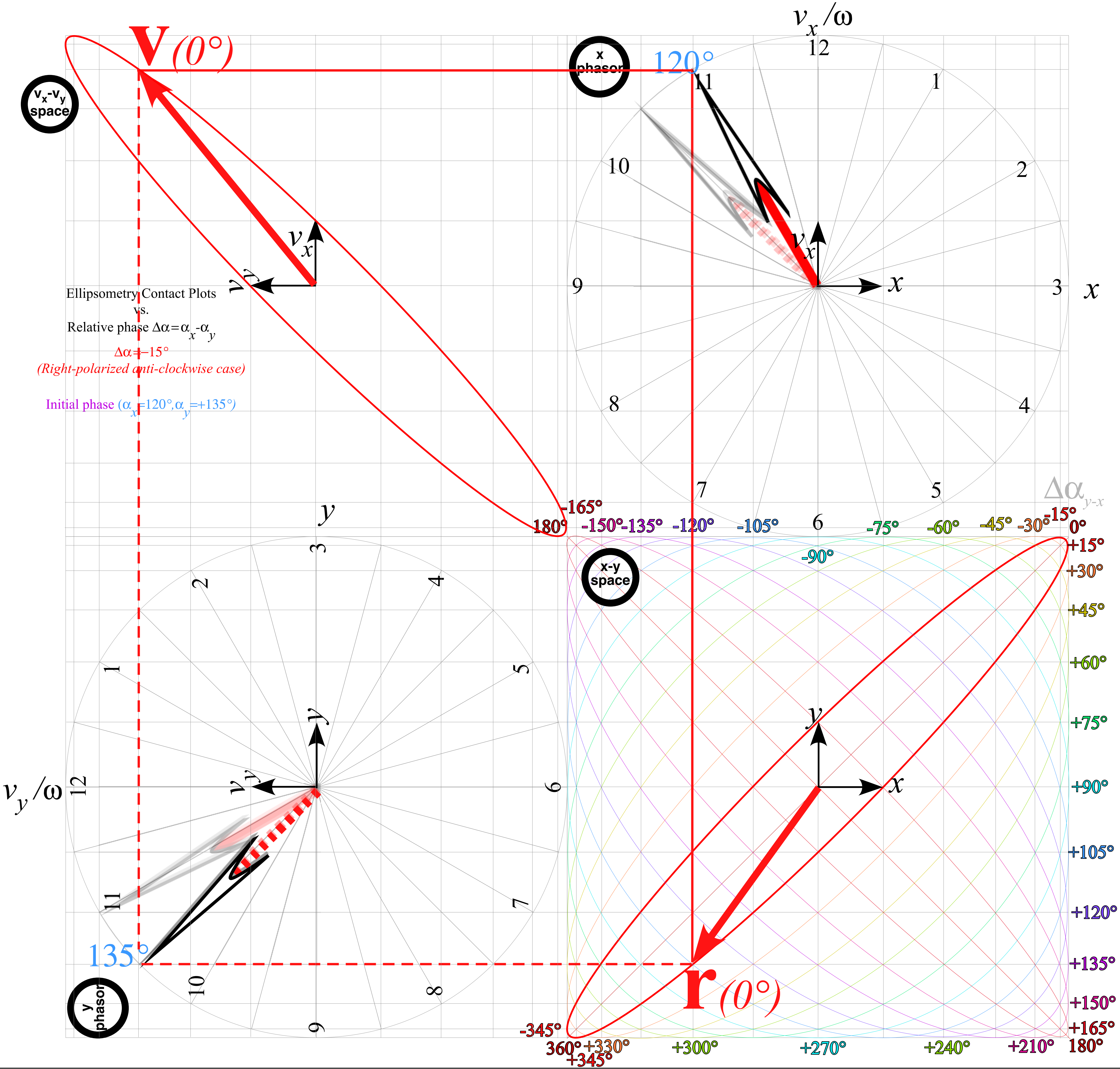




Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 330°:
(α_x=120°-300°=-210°=+150°,α_y = +135°-330°=-195°=+165°)
v_x-v_y space
x phasor
y phasor
x-y space
V (330°)
r (330°)
v_x/ω
v_y/ω
Δα_y-x



vx-vy space
x phasor
x-y space
y phasor
V(360°)=V(0°)
r(360°)=r(0°)
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 360°:
(αx=120°-360°=-240°=+120°,αy=+135°-360°=-225°=+135°)
vx/ω
vy/ω
Δαy-x


V (0°)
vx-vy space
x phasor
120°
vx/ω
12
1
2
3
4
5
6
7
8
9
10
11
x
vx
vy
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
y
vy/ω
y phasor
135°
x-y space
r (0°)
$\Delta\alpha_{y-x}$
-165°
180°
-150°
-135°
-120°
-105°
-90°
-75°
-60°
-45°
-30°
-15°
0°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
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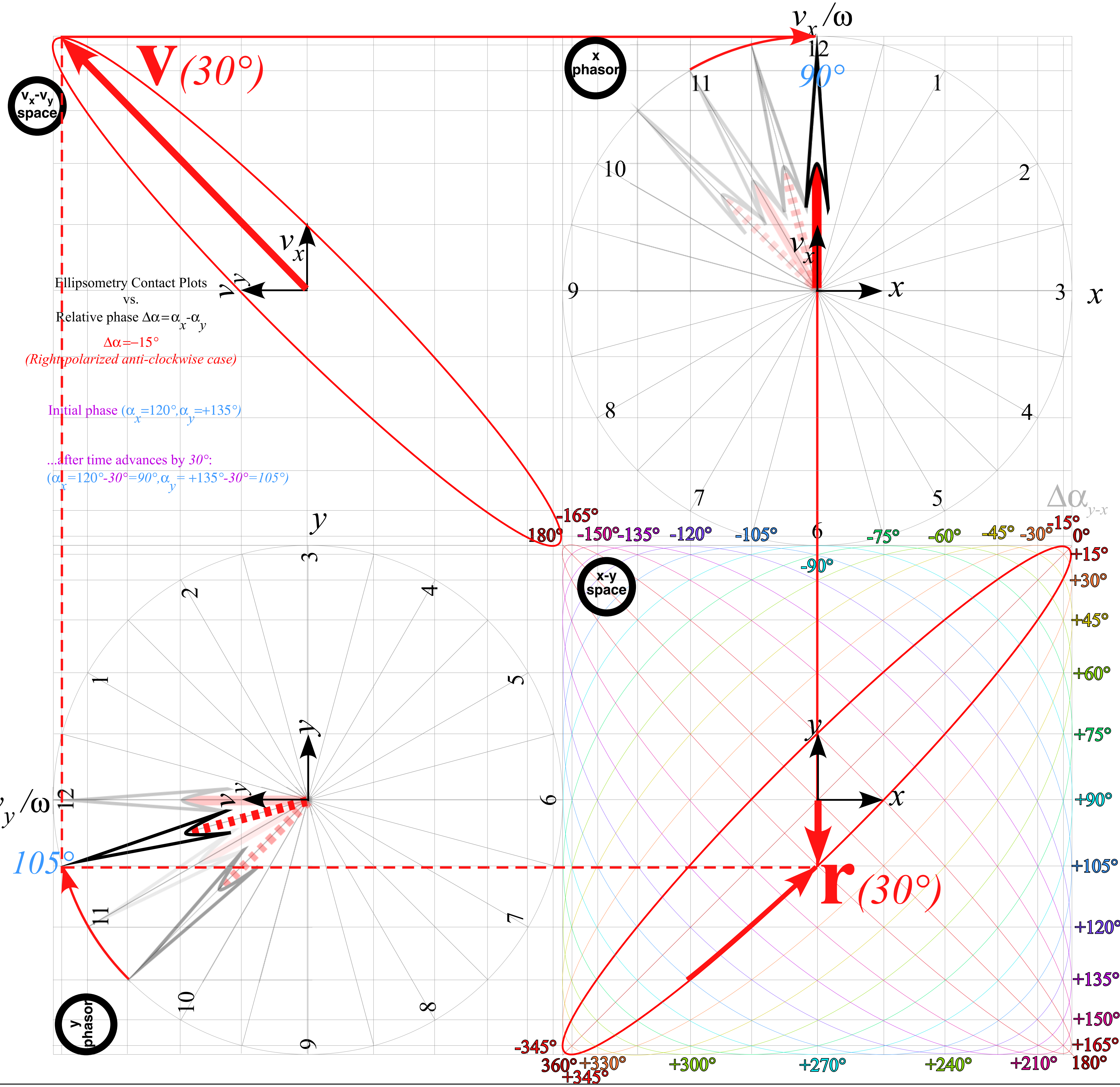
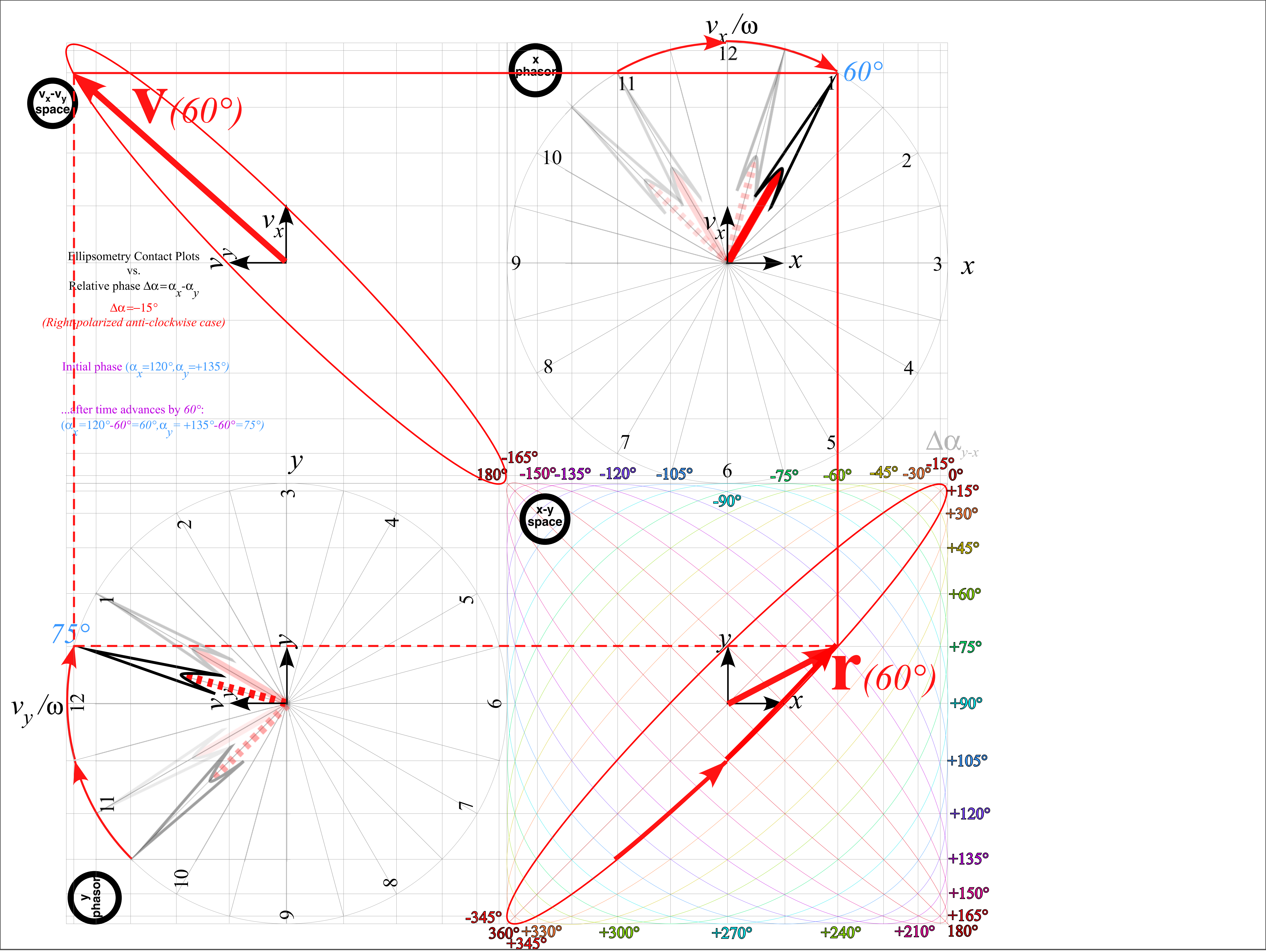
+300°
+330°
+345°
-345°
360°



V(30°)
vx-vy space
x phasor
vx/ω
90°
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 30°:
(αx=120°-30°=90°,αy= +135°-30°=105°)
x-y space
y phasor
vy/ω
105°
r(30°)
Δαy-x



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 60°:
(α_x=120°-60°=60°,α_y=+135°-60°=75°)
v_x-v_y space
V(60°)
x phasor
v_x/ω
60°
y phasor
v_y/ω
75°
x-y space
r(60°)
Δα_y-x
0°
-15°
-30°
-45°
-60°
-75°
-90°
-105°
-120°
-135°
-150°
-165°
180°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
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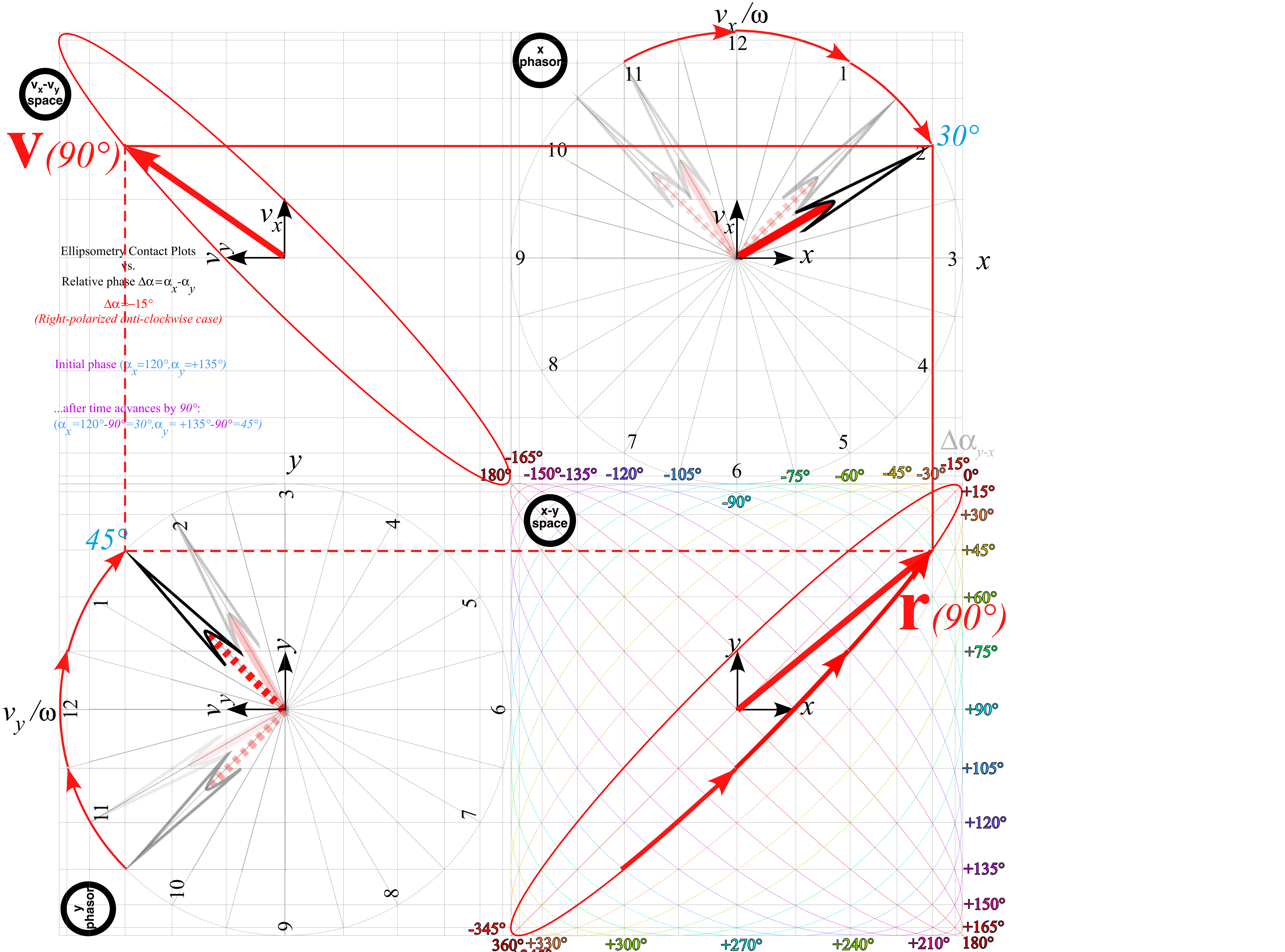
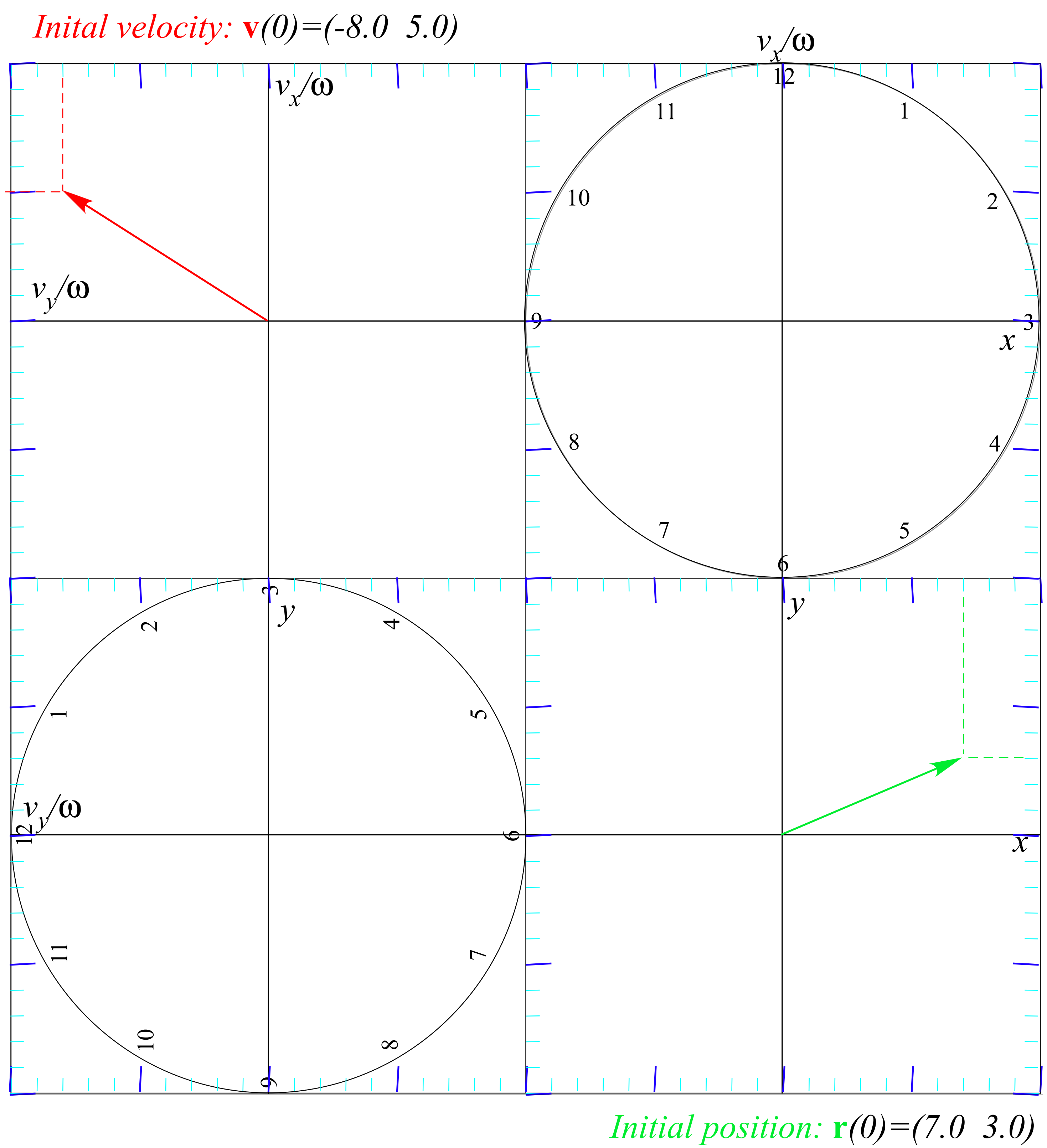
+300°
+330°
+345°
360°
-345°


Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 90°:
(αx=120°-90°=30°,αy= +135°-90°=45°)
vx-vy space
x phasor
y phasor
x-y space
V(90°)
r(90°)
30°
45°
vx/ω
vy/ω
Δαy-x

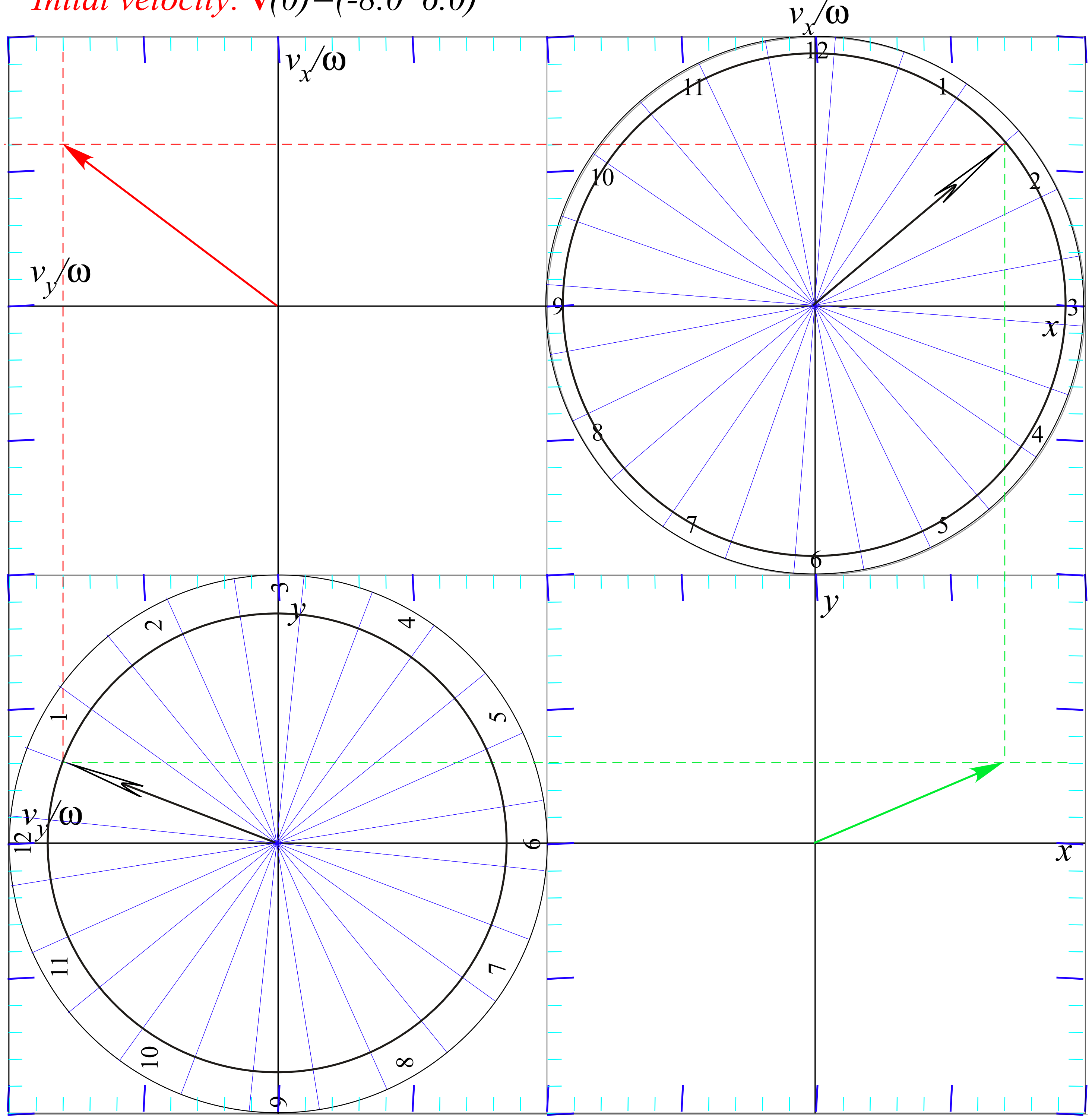
Constructing 2D IHO orbits by phasor plots

→ *Integrating IHO equations by phasor geometry (case of unequal x and y phasor area)*



[BoxIt simulation of U(2) orbits](http://www.uark.edu/ua/modphys/markup/BoxItWeb.html)
http://www.uark.edu/ua/modphys/markup/BoxItWeb.html

Inital velocity: **v***(0)=(-8.0 6.0)*



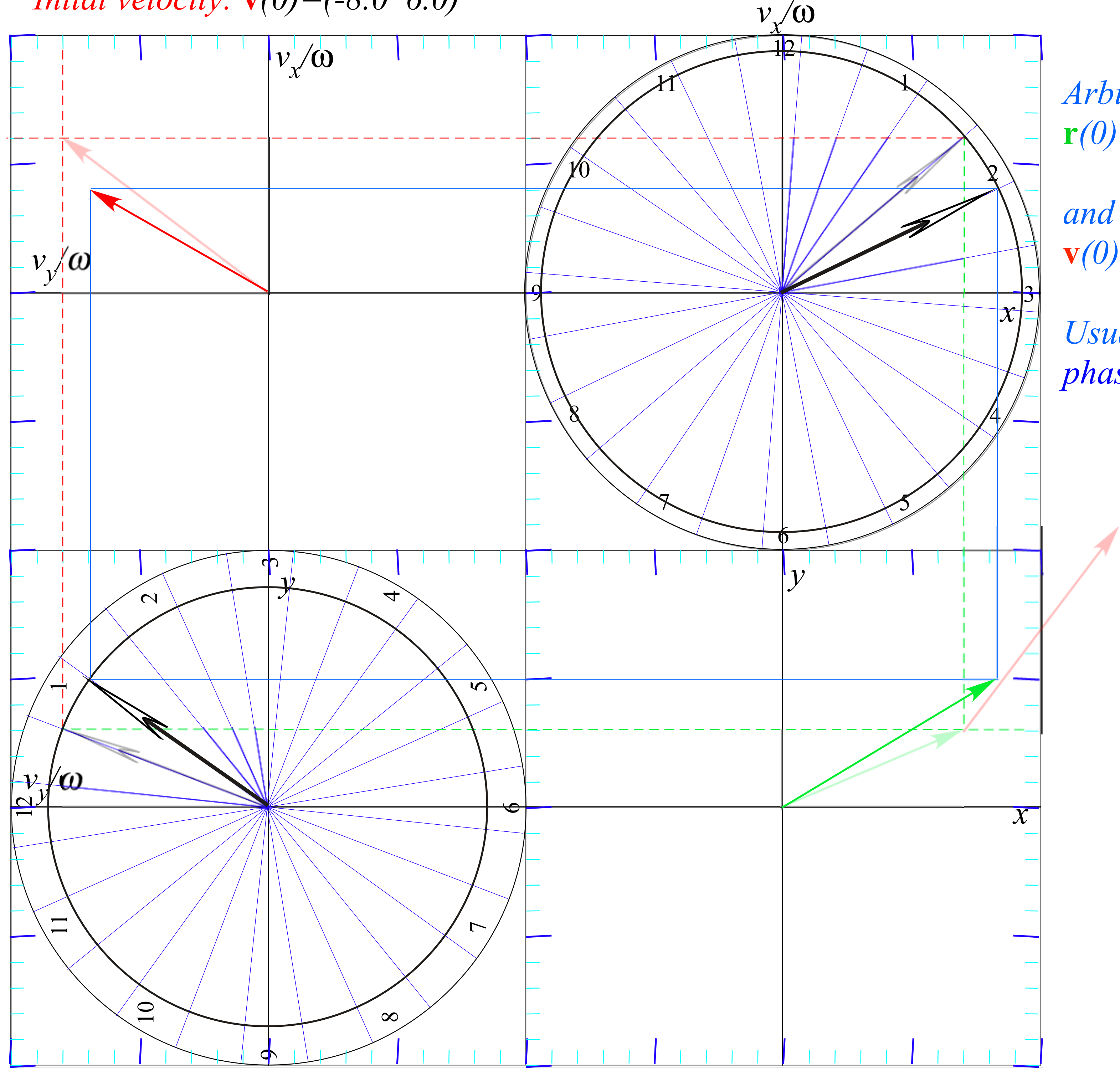
Arbitrary initial position
r*(0)=(x(0),y(0))*

and initial velocity
v*(0)=(v_x(0),v_y(0))*

Usually have x and y
phasor circles of unequal size

Initial position: **r***(0)=(7.0 3.0)*

Inital velocity: **v***(0)=(-8.0 6.0)*



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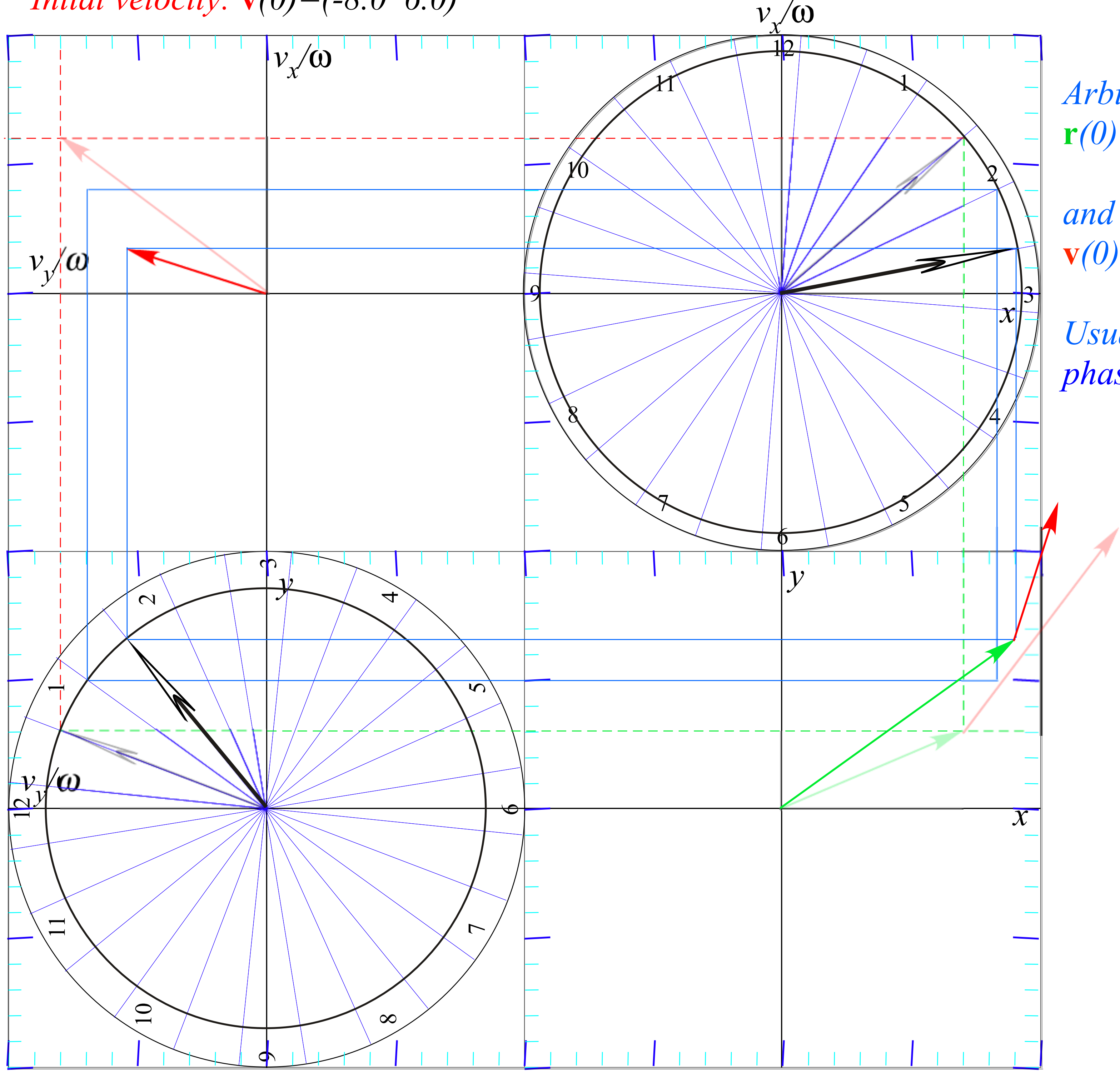

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v*(0)=(v$_x$(0),v$_y$(0)*

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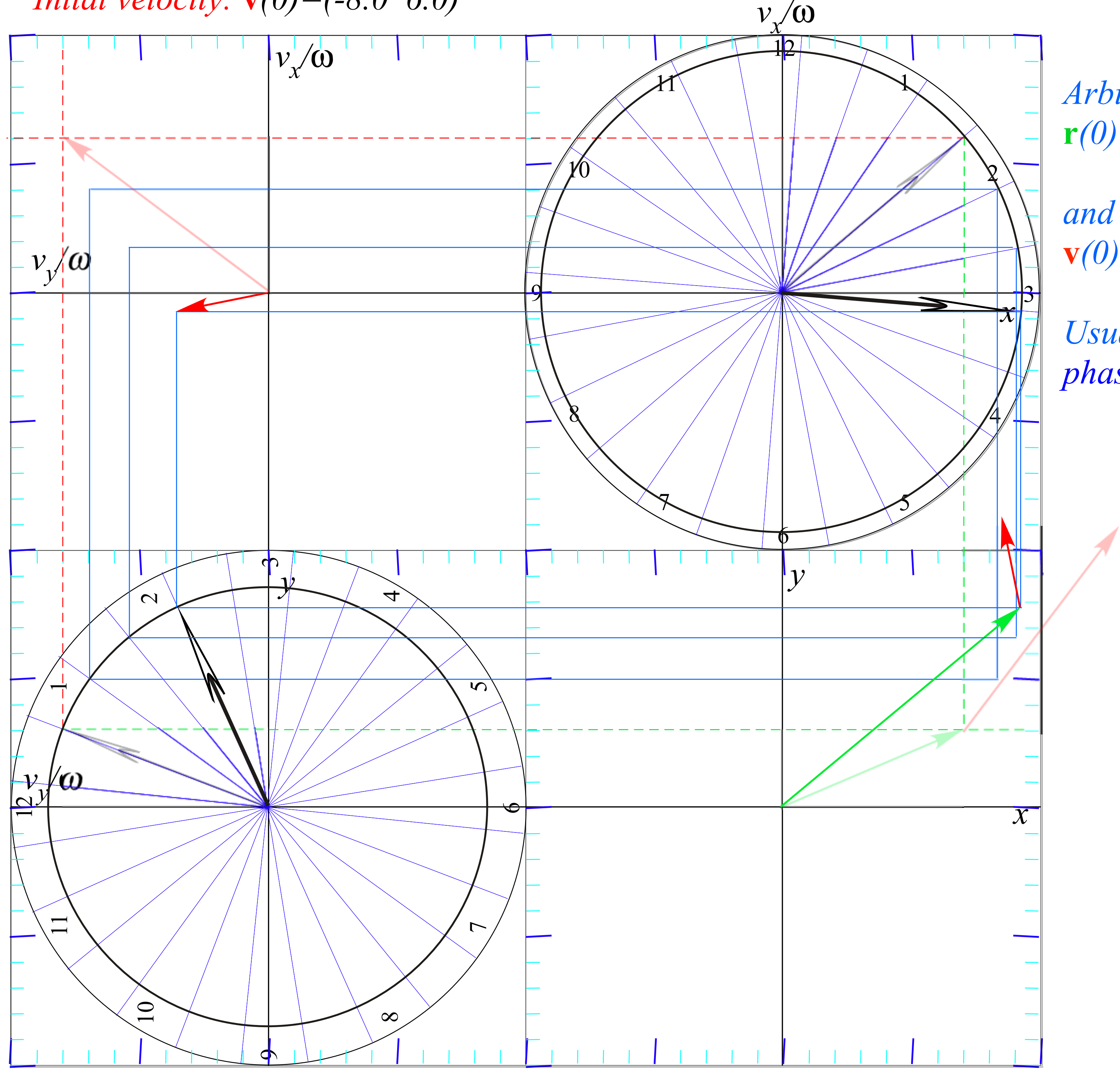
Inital velocity: **v**(0)=(-8.0 6.0)



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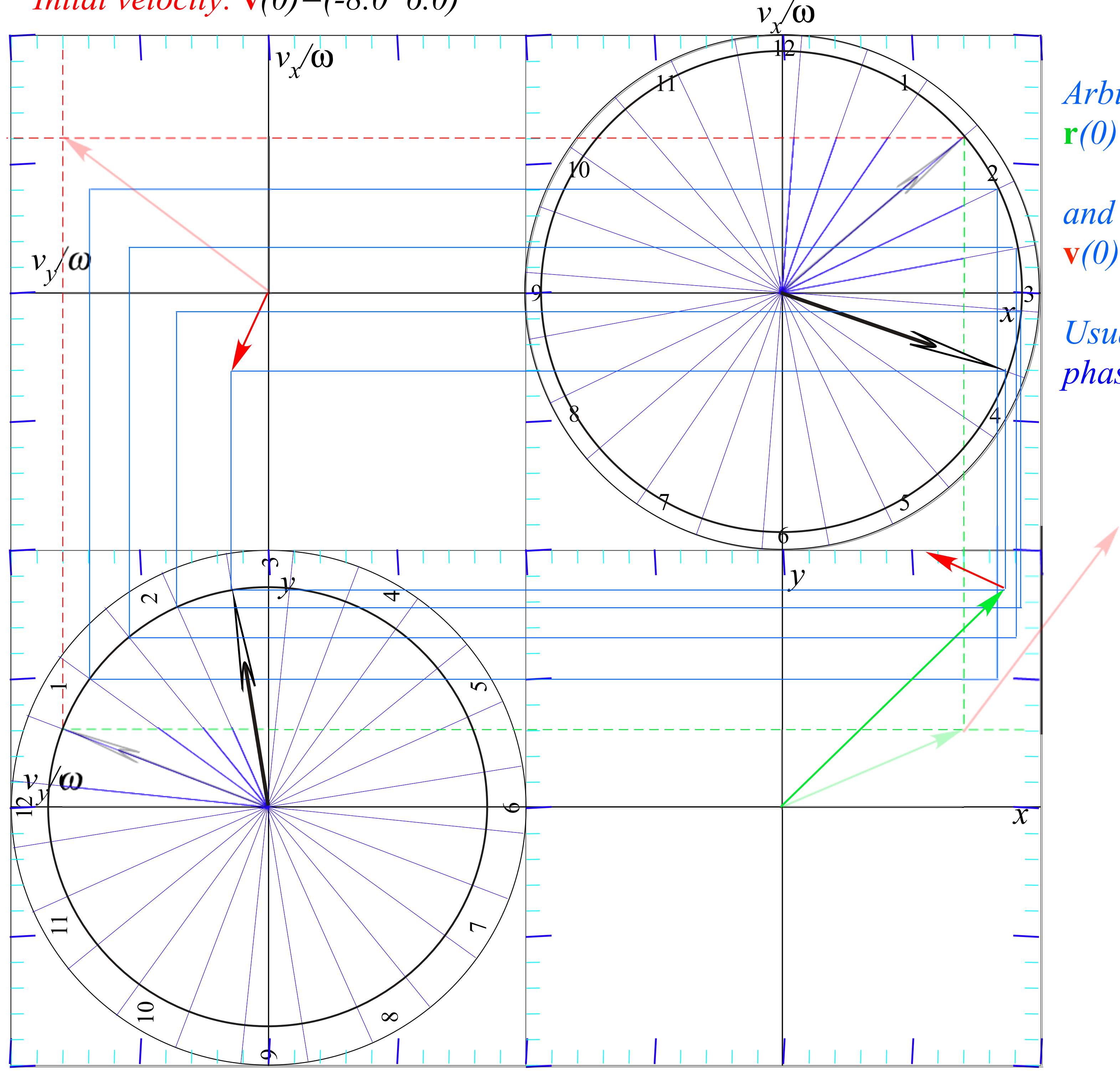

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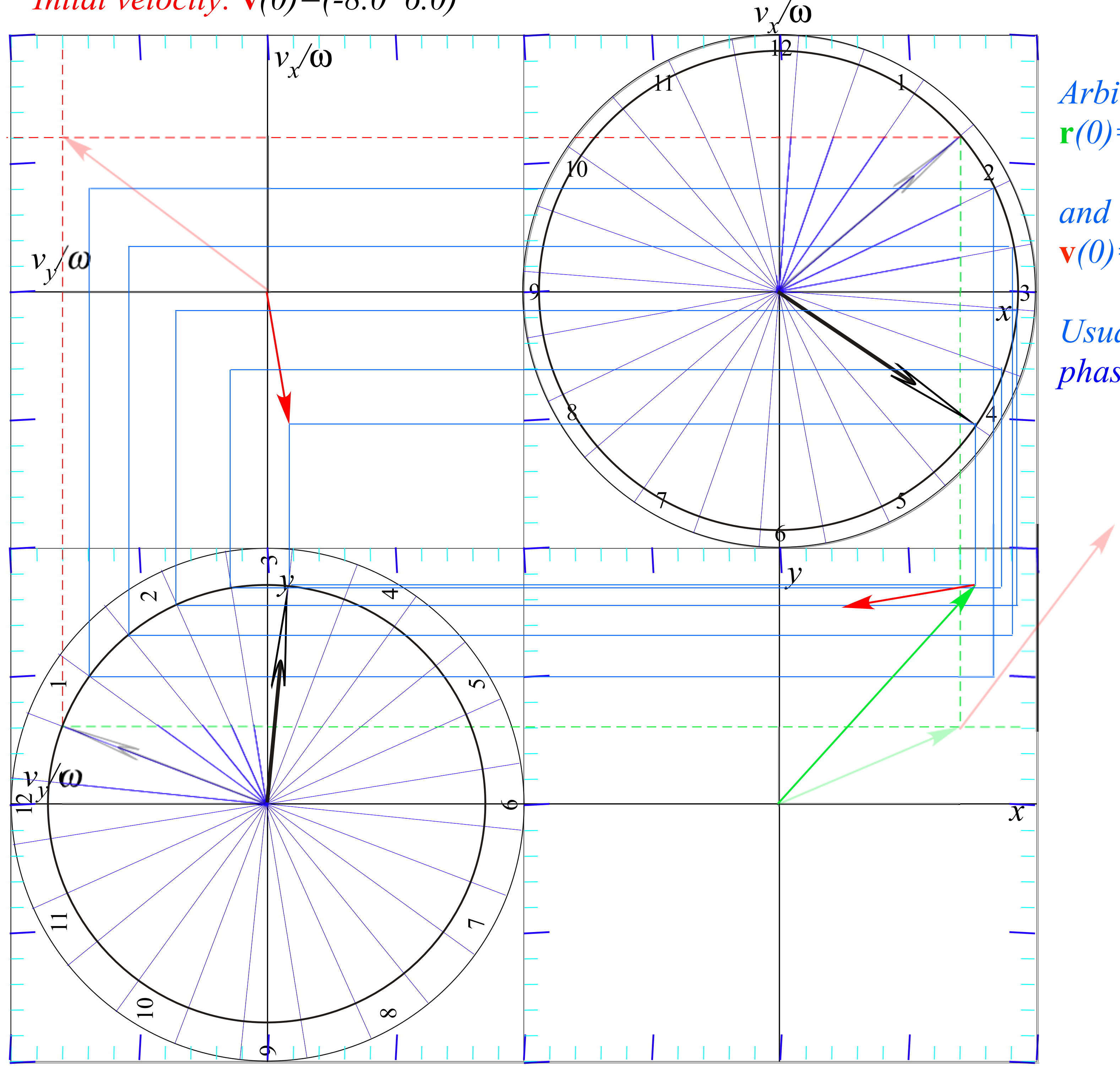

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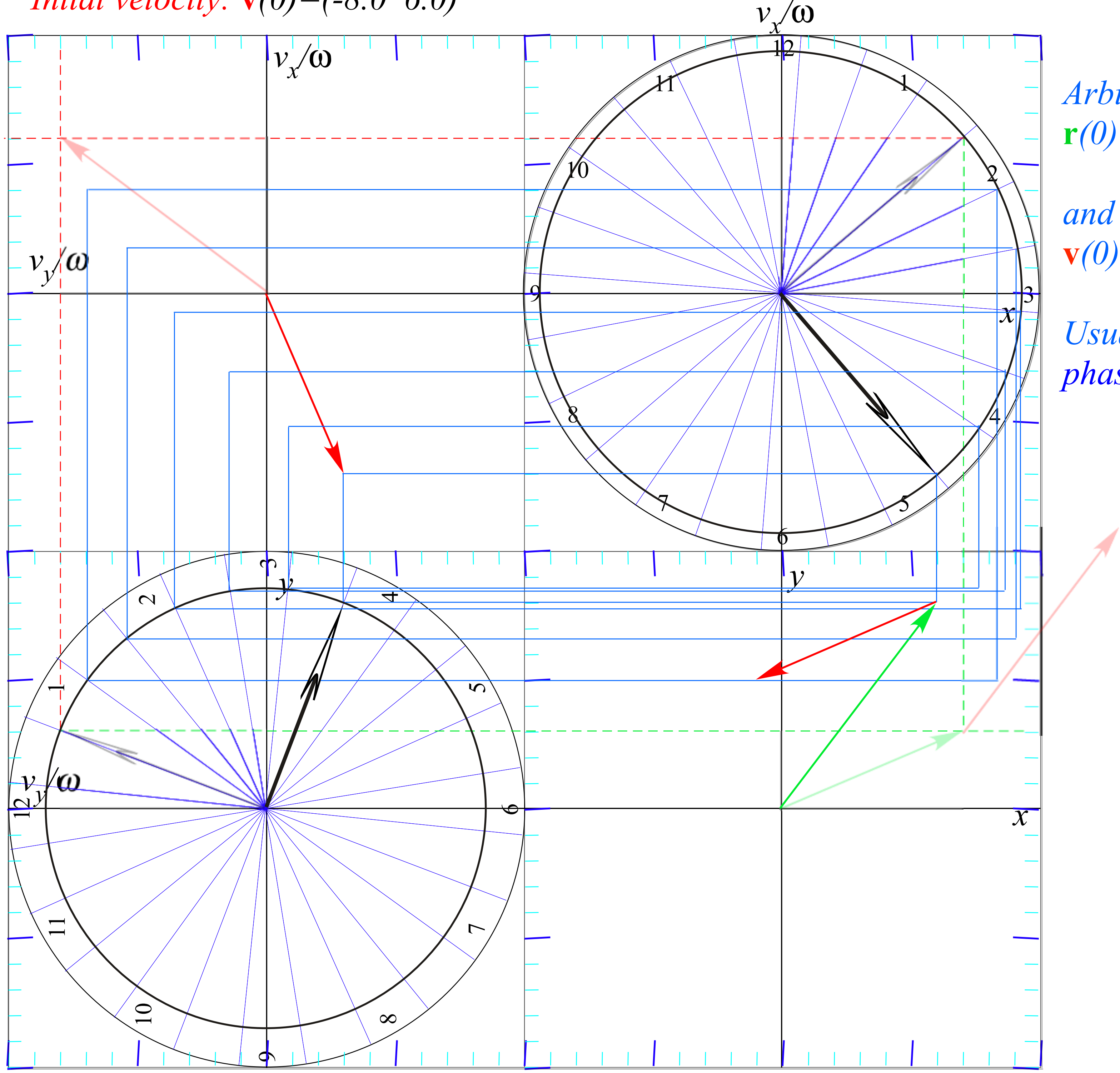

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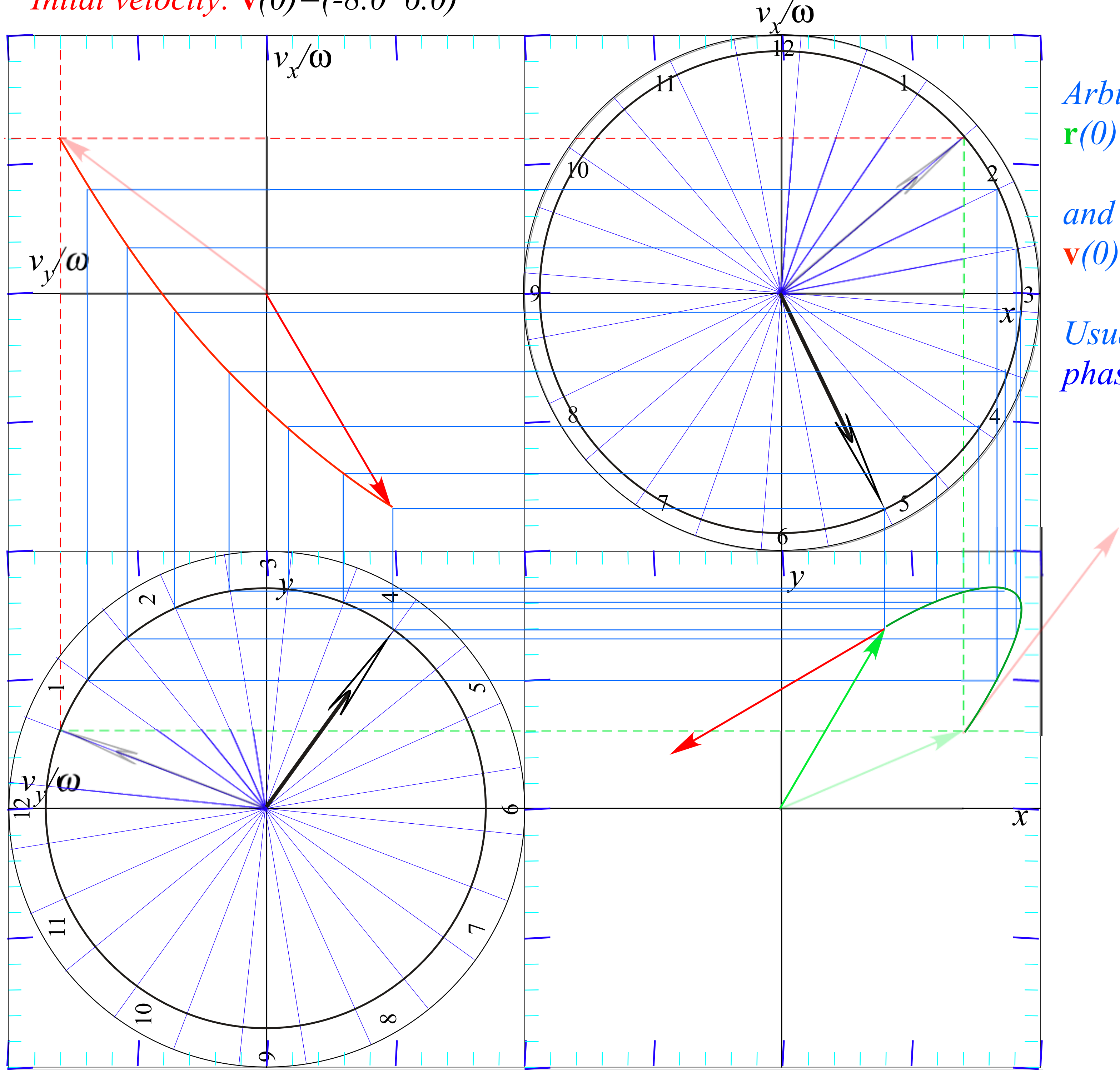

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and initial velocity
v*(0)=(v_x(0),v_y(0)*

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