Lecture 10
Wed. 9.24.2014

Geometry of Dual Quadratic Forms: Lagrange vs Hamilton

(Ch. 11 and Ch. 12 of Unit 1)

Review of dual IHO elliptic orbits (Lecture 9)
- *Construction by Phasor-pair projection (with example of dual-ellipses)*
- *Construction by Kepler anomaly projection*

Introduction to dual matrix operator geometry

Quadratic form ellipse $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$
- *Duality norm relations (* $\mathbf{r}\bullet\mathbf{p}=1$*)*
- *Q-Ellipse tangents* $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ *(* $\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$*)*

Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry
- *Vector calculus of tensor operation*

Introduction to Lagrangian-Hamiltonian duality

Review of partial differential relations
- *Chain rule and order* $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ *symmetry*

Duality relations of Lagrangian and Hamiltonian ellipse

Introducing the 1st (partial) differential equations of mechanics

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→ *Construction by Phasor-pair projection (with example of dual-ellipses)*

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Duality norm relations ($\mathbf{r}\bullet\mathbf{p}=1$*)*

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ *(* $\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$*)*

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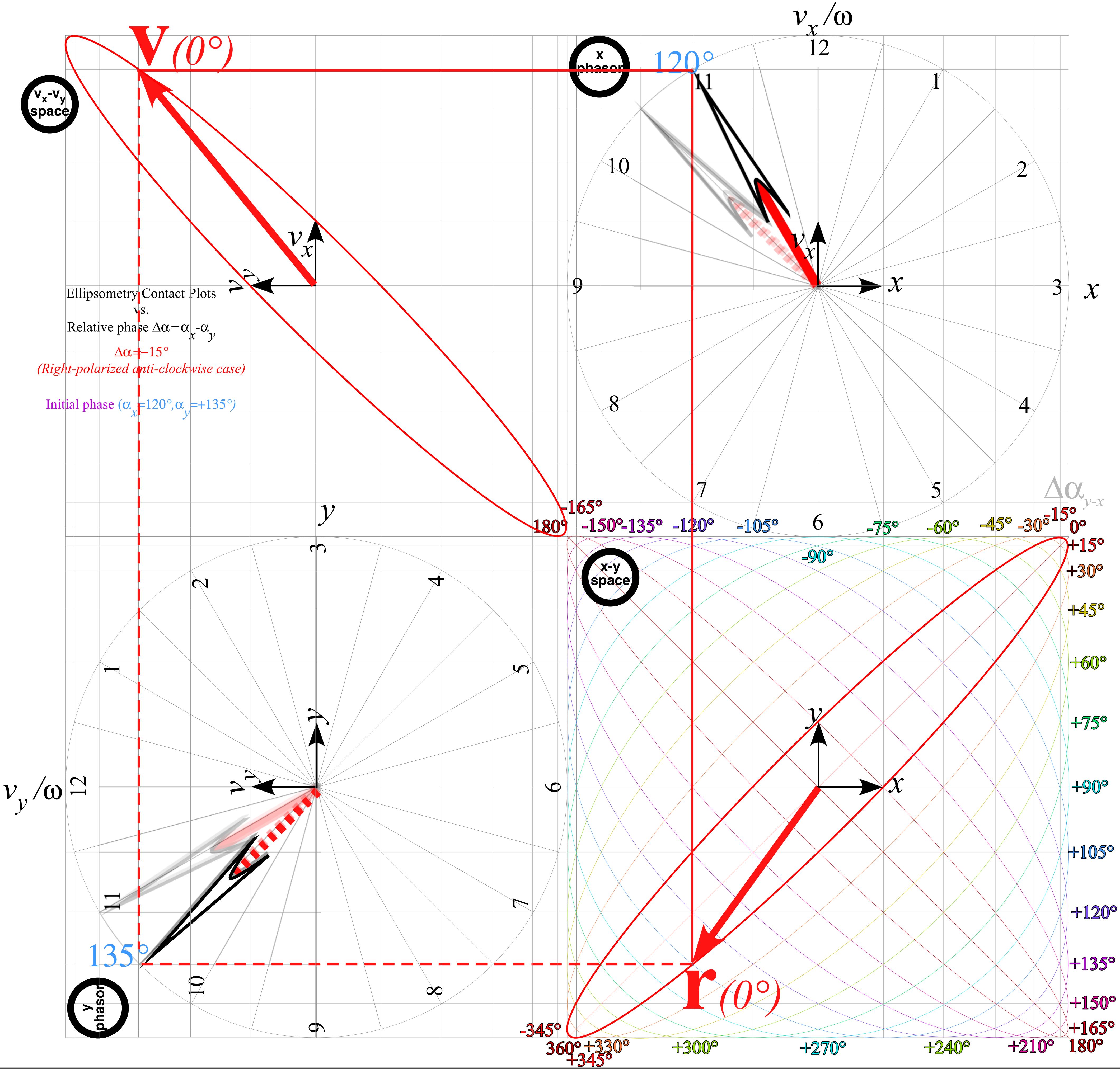
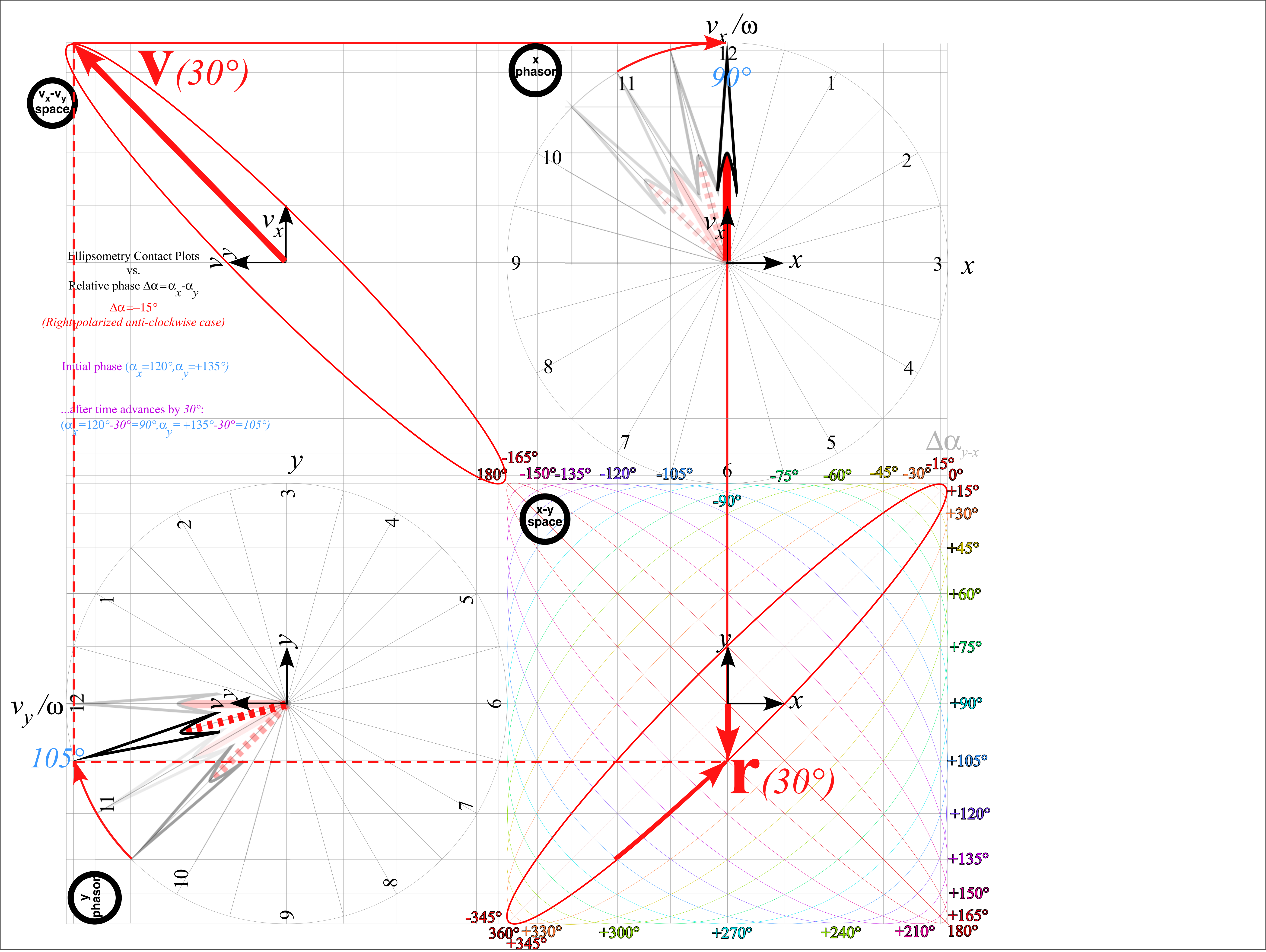
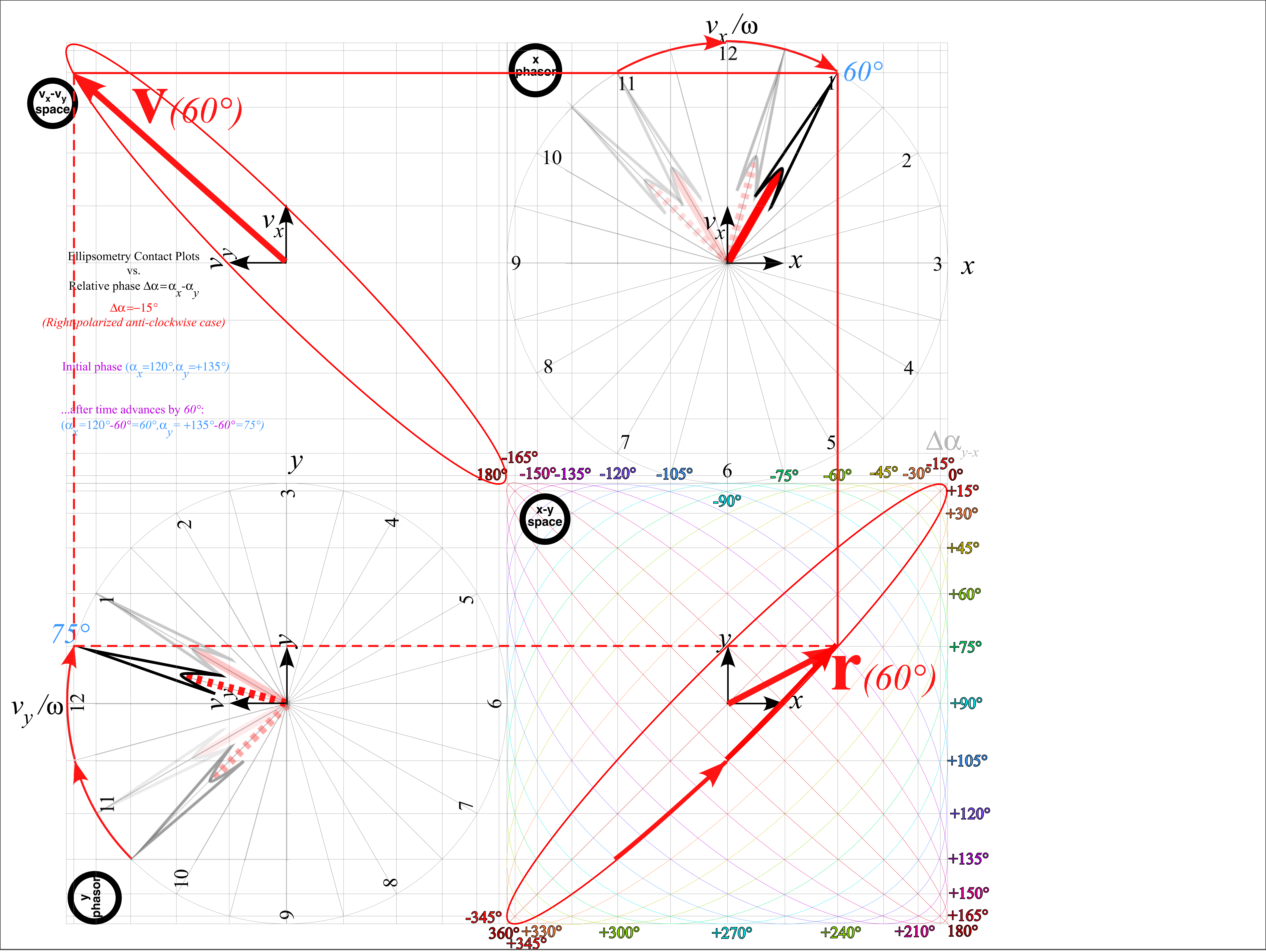
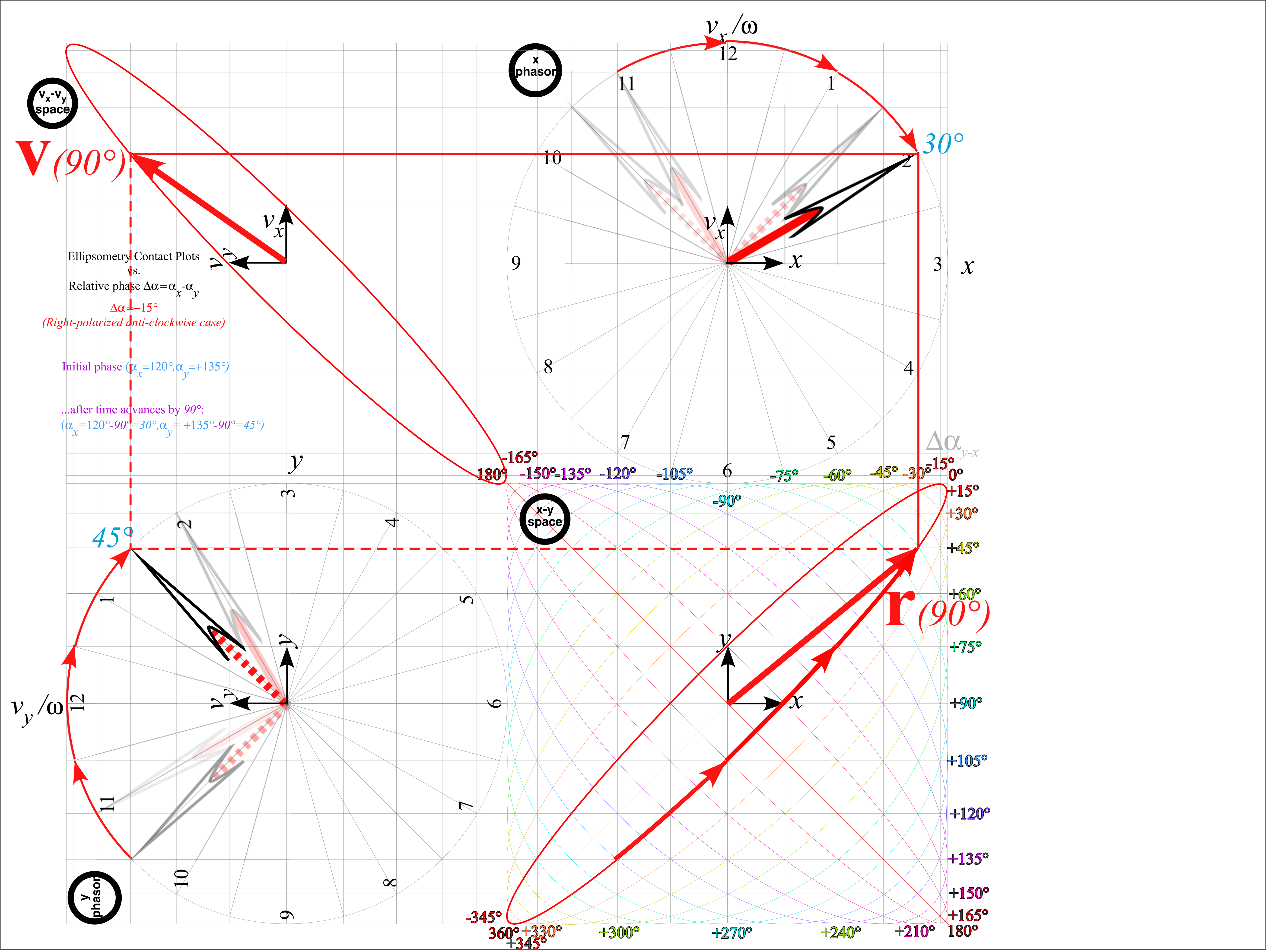


V (0°)
vx-vy space
x phasor
120°
vx/ω
12
1
2
3
4
5
6
7
8
9
10
11
x
vx
vy
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
y
x-y space
Δαy-x
-165° 180° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +330° +345° +300° +270° +240° +210°
vy/ω
135°
y phasor
r (0°)

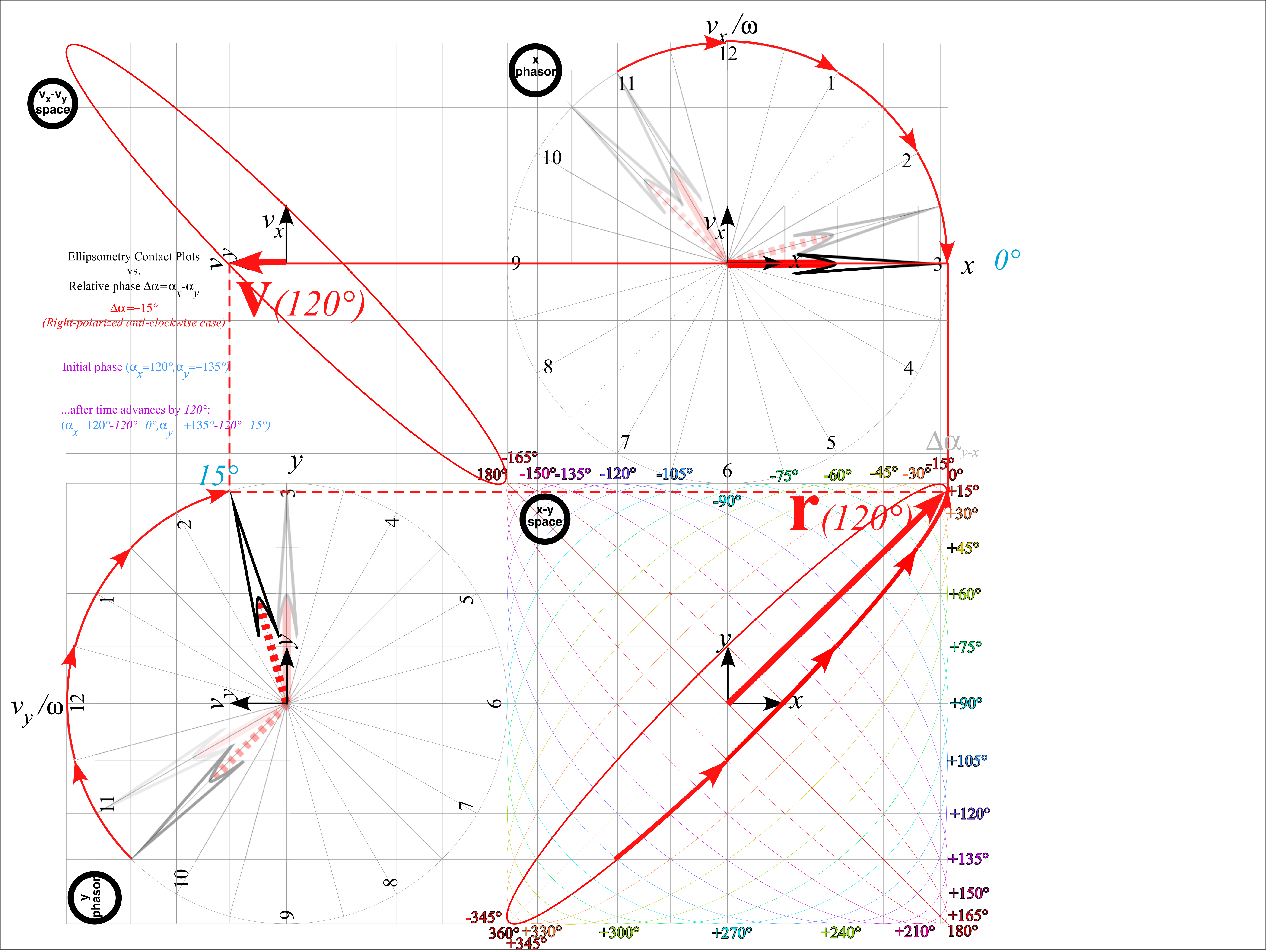


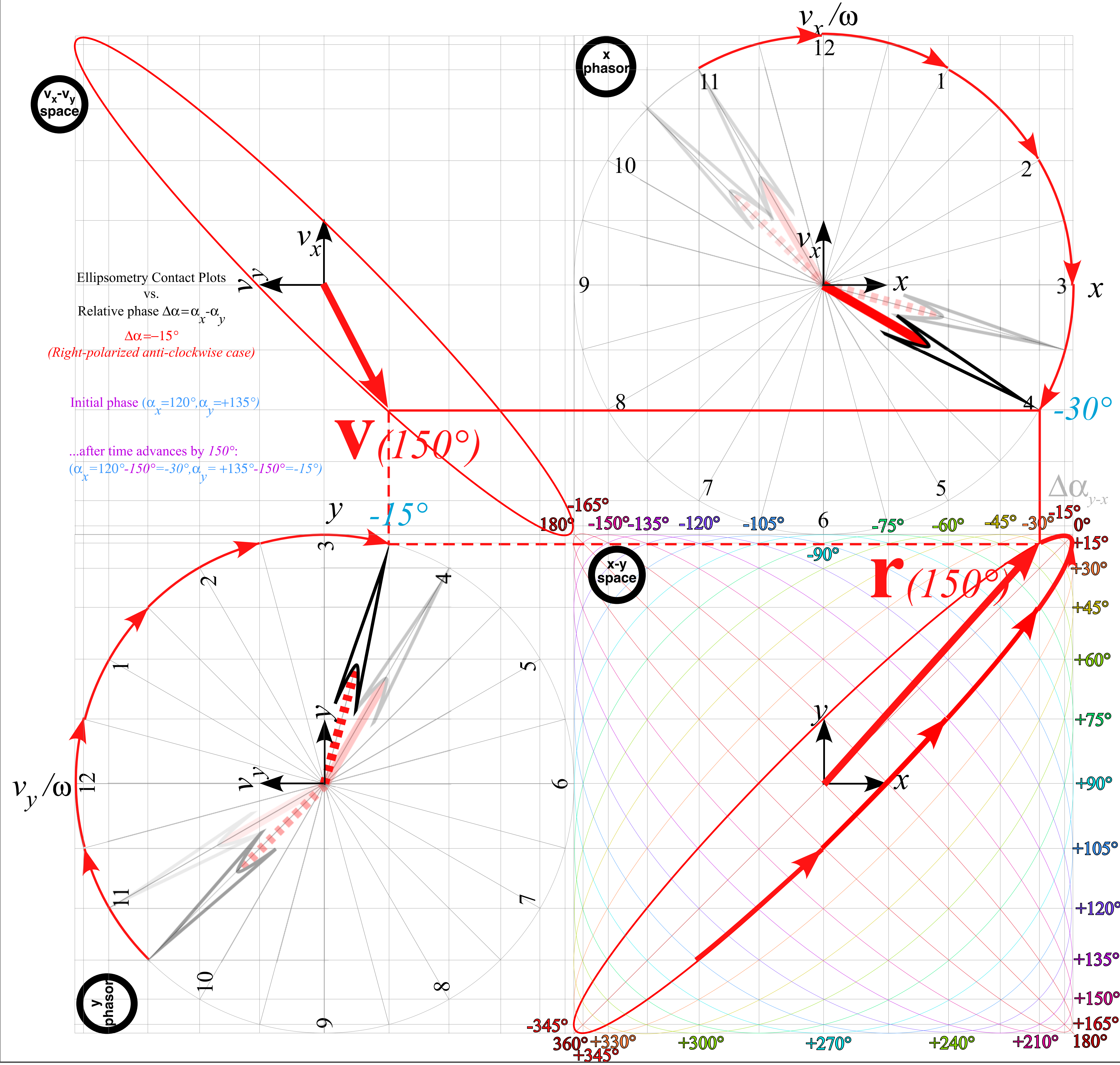
V(30°)
v_x-v_y space
x phasor
v_x/ω
12
90°
11
1
10
2
9
3
x
8
4
7
5
6
v_x
v_y
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 30°:
(α_x=120°-30°=90°,α_y= +135°-30°=105°)
y
$\Delta\alpha_{y-x}$
-165°
180°
-150°
-135°
-120°
-105°
-90°
-75°
-60°
-45°
-30°
-15°
0°
+15°
+30°
+45°
+60°
+75°
+90°
+105°
+120°
+135°
+150°
+165°
180°
x-y space
v_y/ω
105°
y phasor
r(30°)
-345°
360°
+330°
+345°
+300°
+270°
+240°
+210°


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 60°:
(α_x=120°-60°=60°,α_y = +135°-60°=75°)
V(60°)
r(60°)
60°
75°
v_x/ω
v_y/ω
x phasor
y phasor
v_x-v_y space
x-y space
Δα_y-x

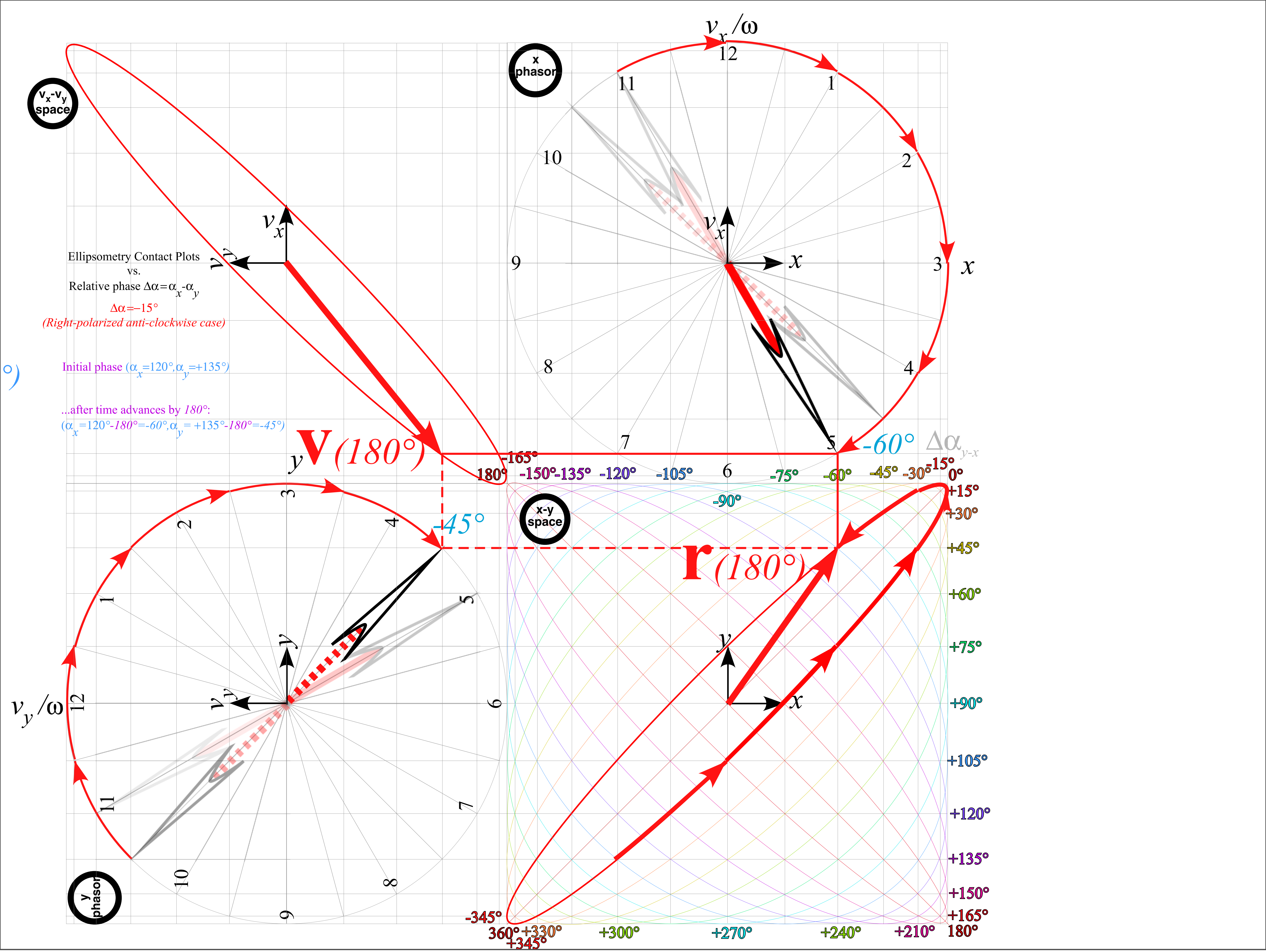


v_x/ω
x phasor
v_x-v_y space
V(90°)
30°
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 90°:
(α_x=120°-90°=30°,α_y = +135°-90°=45°)
$\Delta\alpha_{y-x}$
-165° 180° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +330° +345° +300° +270° +240° +210°
45°
x-y space
r(90°)
v_y/ω
y phasor

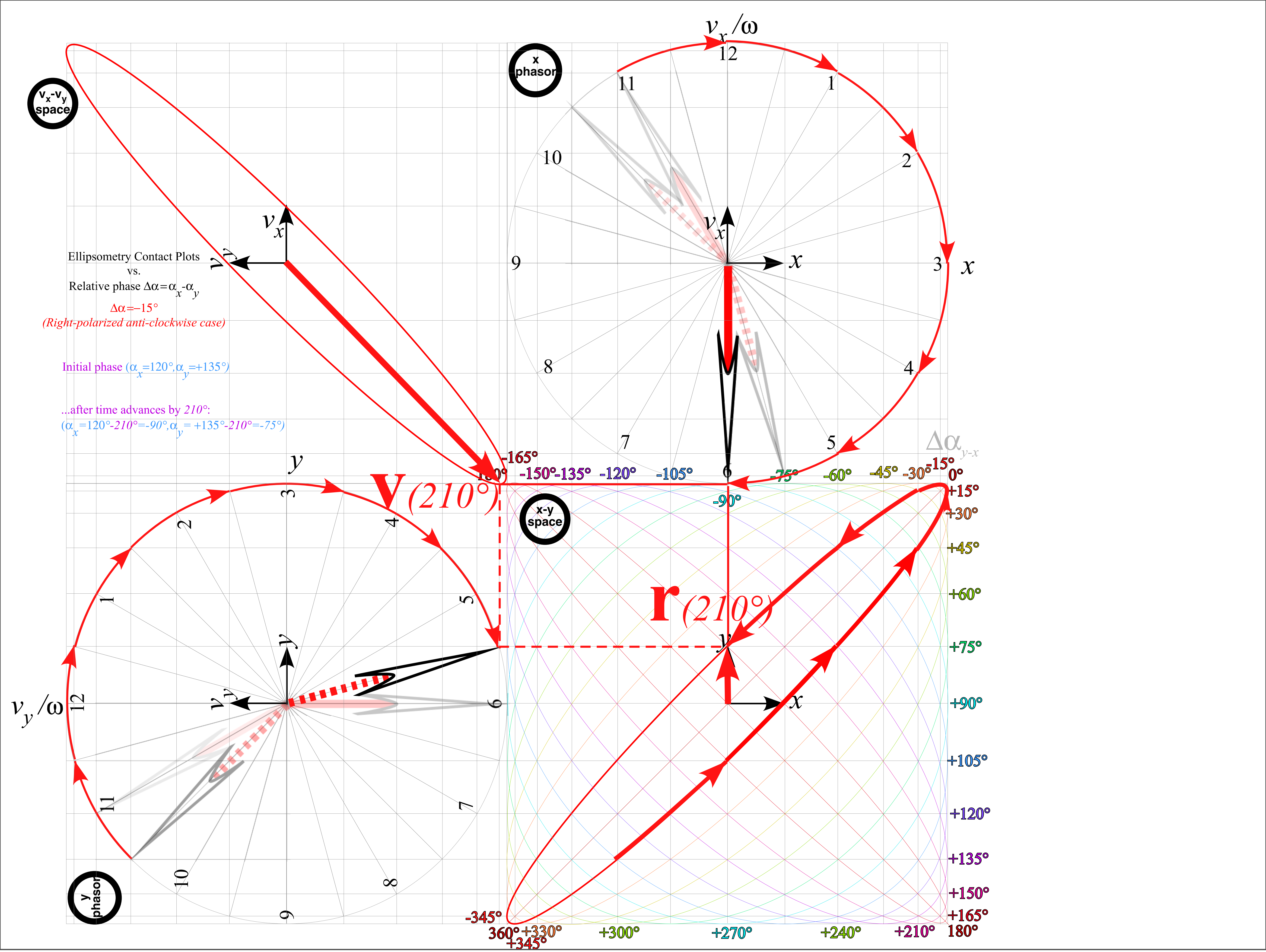

Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 120°:
(α_x=120°-120°=0°,α_y=+135°-120°=15°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(120°)
r(120°)
0°
15°
Δα_y-x
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°



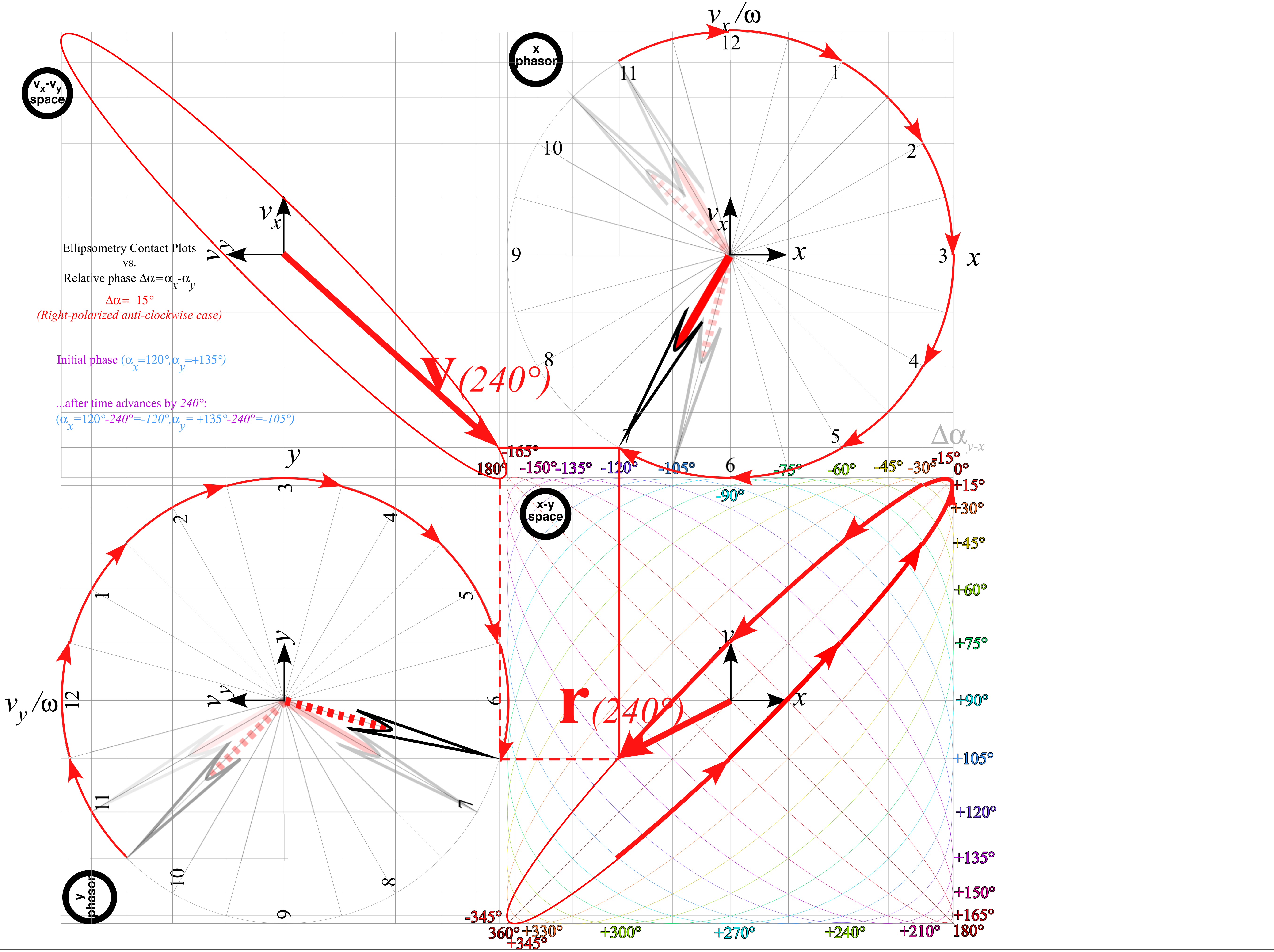
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 150°:
(α_x=120°-150°=-30°,α_y= +135°-150°=-15°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(150°)
r(150°)
-30°
-15°
Δα_y-x



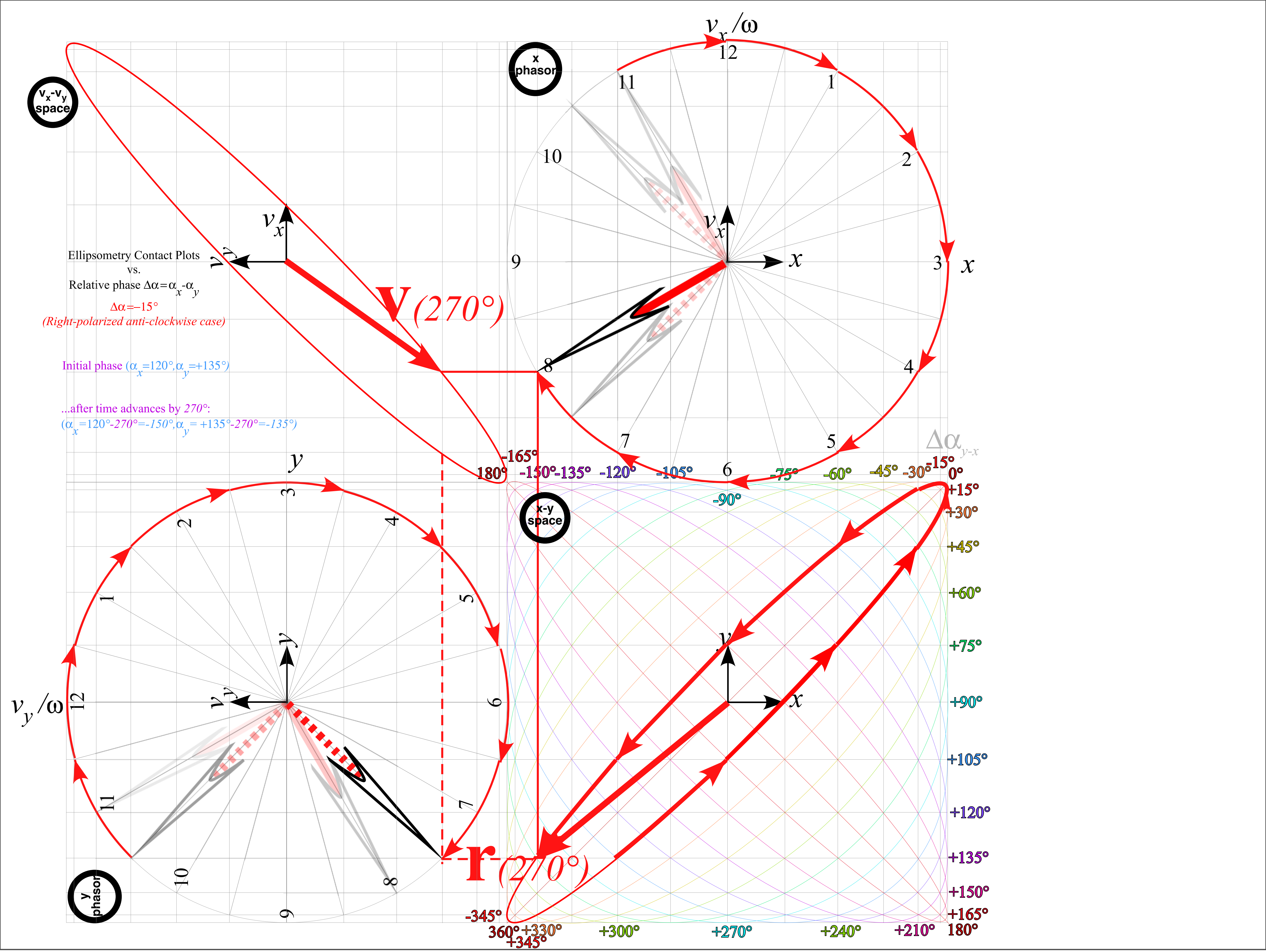
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 180°:
(α_x=120°-180°=-60°,α_y= +135°-180°=-45°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(180°)
r(180°)
-60°
-45°
Δα_y-x
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°



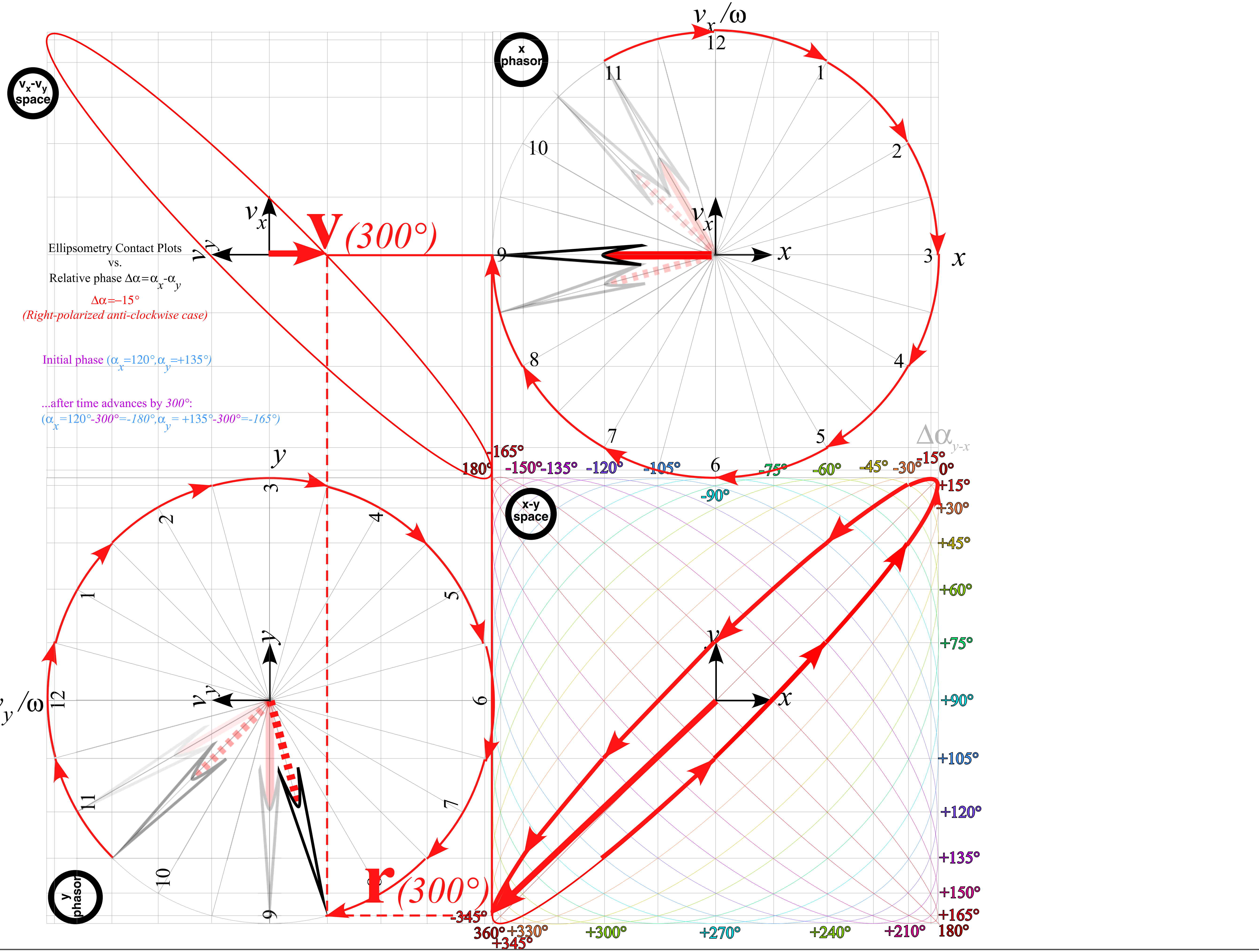
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 210°:
(α_x=120°-210°=-90°,α_y = +135°-210°=-75°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(210°)
r(210°)
Δα_y-x



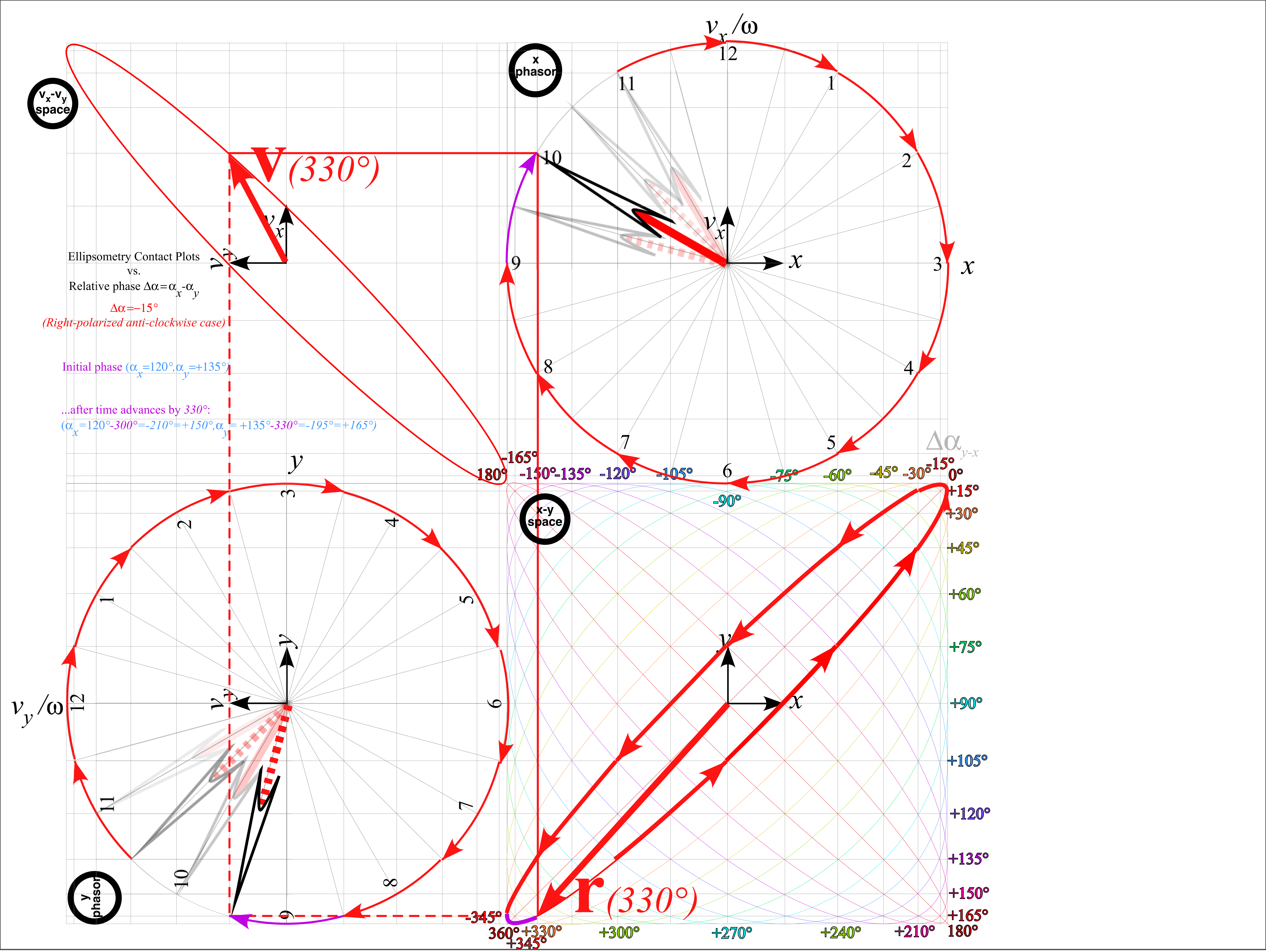
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 240°:
(α_x=120°-240°=-120°,α_y= +135°-240°=-105°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
v_x
v_y
x
y
V(240°)
r(240°)
$\Delta\alpha_{y-x}$
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°



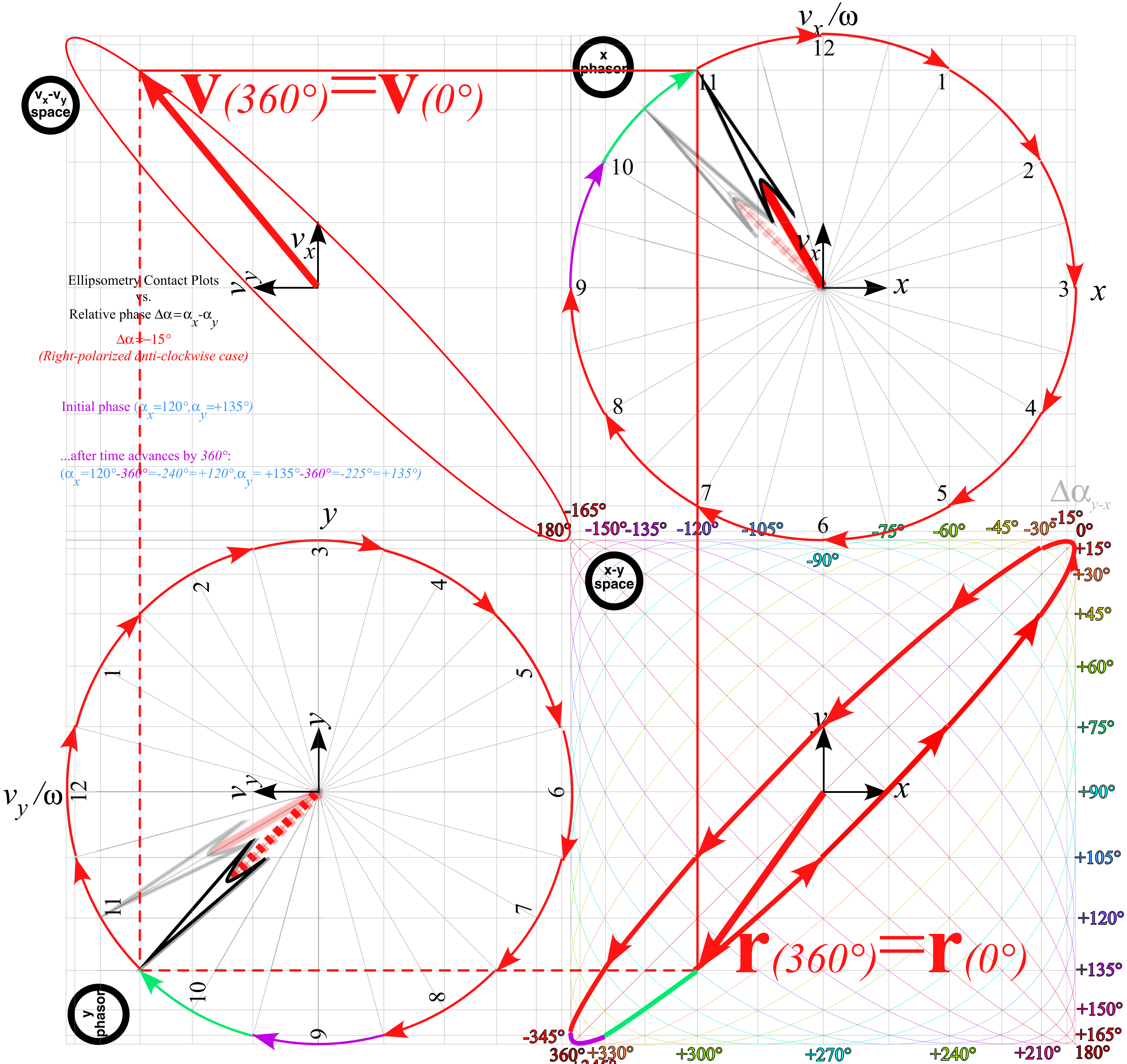
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 270°:
(α_x=120°-270°=-150°,α_y = +135°-270°=-135°)
v_x-v_y space
x phasor
y phasor
x-y space
V(270°)
r(270°)
v_x/ω
v_y/ω
Δα_y-x



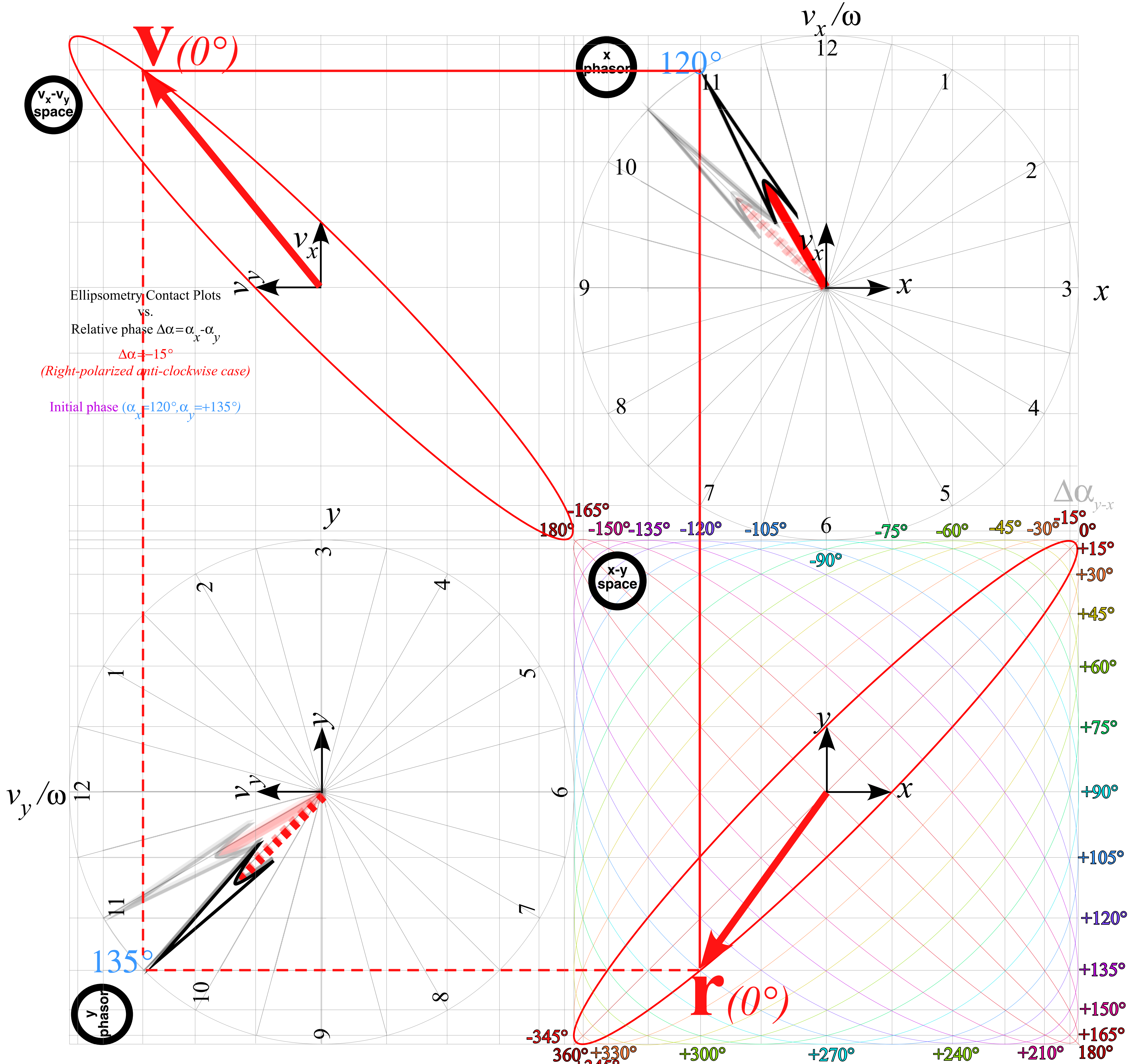
vx-vy space
x phasor
y phasor
x-y space
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase ($\alpha_x=120°,\alpha_y=+135°$)
...after time advances by *300°*:
($\alpha_x=120°-300°=-180°,\alpha_y=+135°-300°=-165°$)
v_x/ω
v_y/ω
V (300°)
r (300°)
$\Delta\alpha_{y-x}$



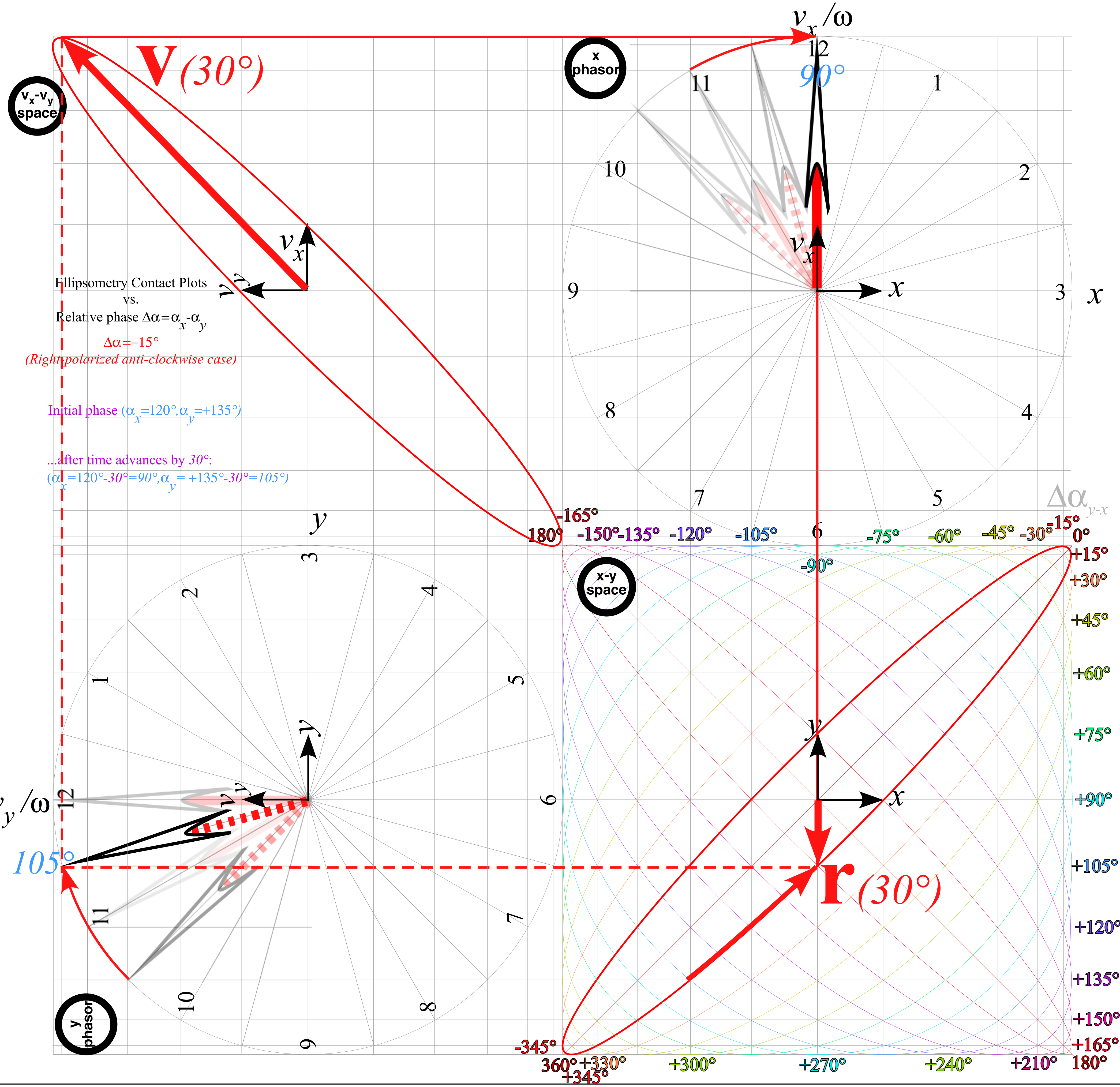
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 330°:
(α_x=120°-300°=-210°=+150°,α_y= +135°-330°=-195°=+165°)
V (330°)
r (330°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
Δα_y-x



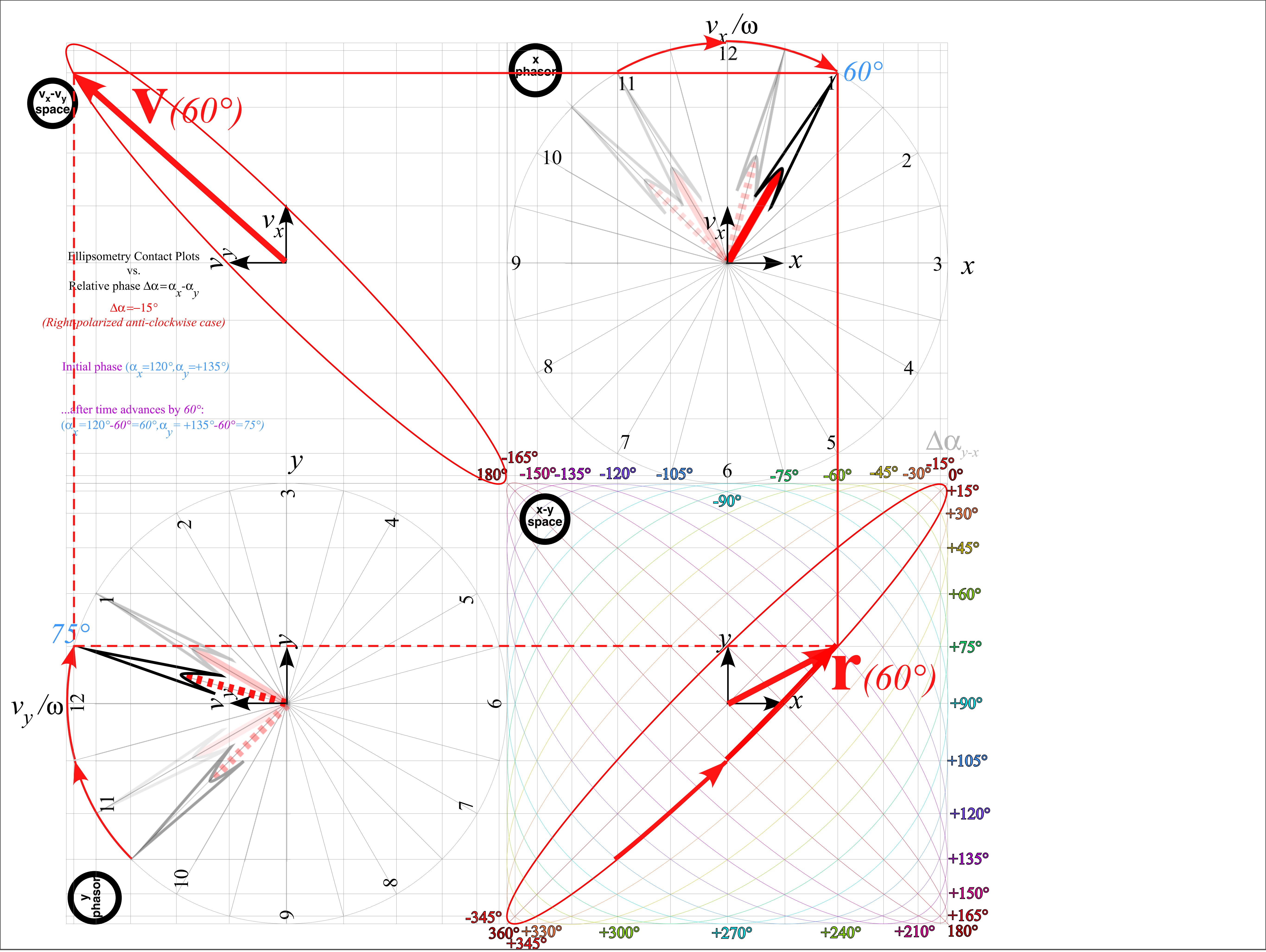
v_x/ω
x phasor
v_x-v_y space
V(360°)=V(0°)
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase ($\alpha_x=120°$, $\alpha_y=+135°$)
...after time advances by 360°:
($\alpha_x=120°-360°=-240°=+120°$, $\alpha_y=+135°-360°=-225°=+135°$)
$\Delta\alpha_{y-x}$
x-y space
y phasor
v_y/ω
r(360°)=r(0°)



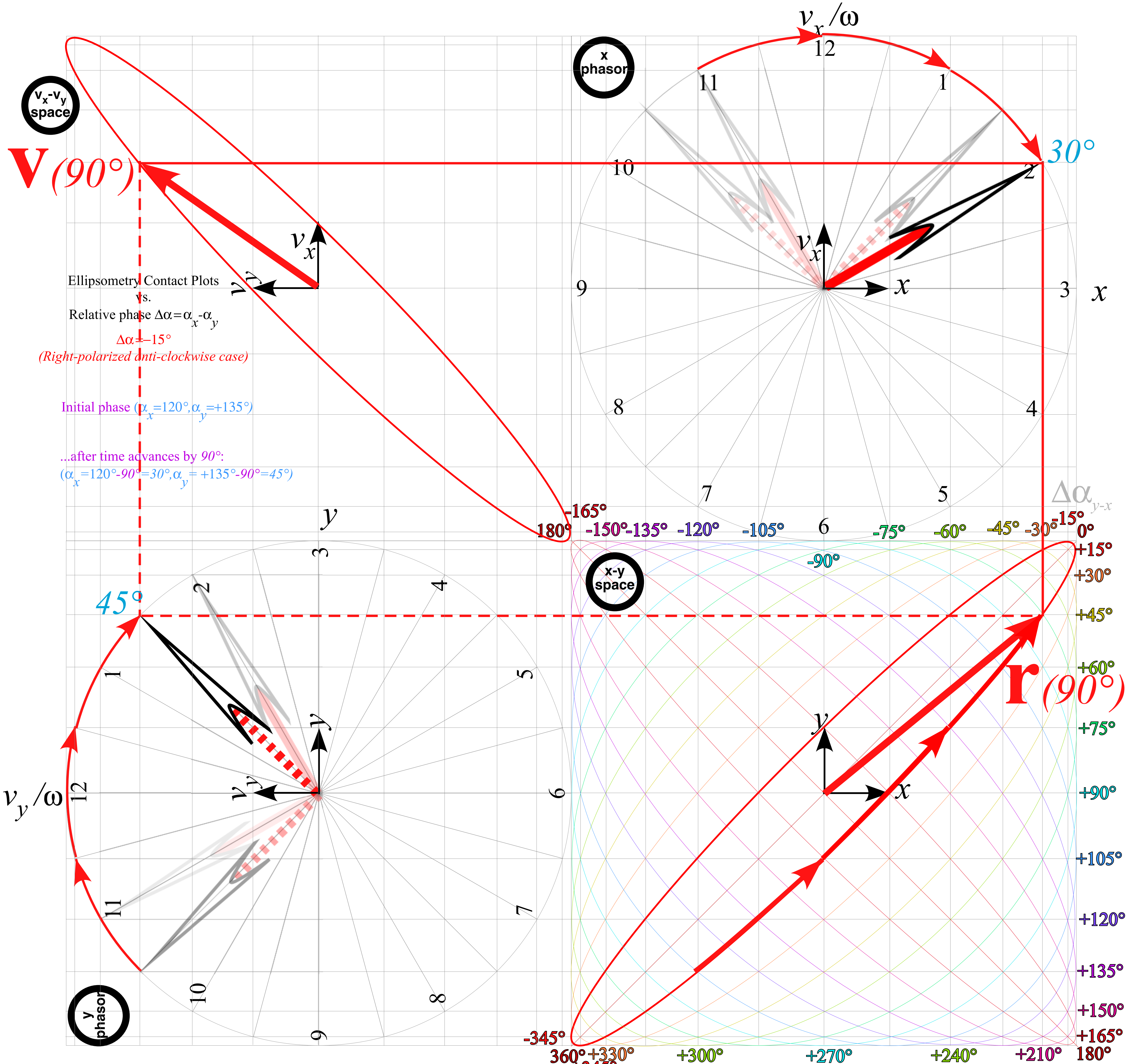
V(0°)
v_x-v_y space
x phasor
120°
v_x/ω
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
$\Delta\alpha_{y-x}$
x-y space
y phasor
135°
v_y/ω
r(0°)



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 30°:
(α_x=120°-30°=90°,α_y=+135°-30°=105°)
V(30°)
r(30°)
vx-vy space
x phasor
y phasor
x-y space
vx/ω
vy/ω
90°
105°
Δα_y-x



Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 60°:
(αx=120°-60°=60°,αy = +135°-60°=75°)
vx-vy space
V(60°)
x phasor
vx/ω
60°
x-y space
r(60°)
y phasor
vy/ω
75°
Δαy-x



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 90°:
(α_x=120°-90°=30°,α_y = +135°-90°=45°)
v_x-v_y space
x phasor
y phasor
x-y space
V(90°)
r(90°)
30°
45°
v_x/ω
v_y/ω
Δα_y-x

Review of dual IHO elliptic orbits (Lecture 9)

Construction by Phasor-pair projection (with example of dual-ellipses)

→ *Construction by Kepler anomaly projection*

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Duality norm relations ($\mathbf{r}\bullet\mathbf{p}=1$*)*

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ *(* $\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$*)*

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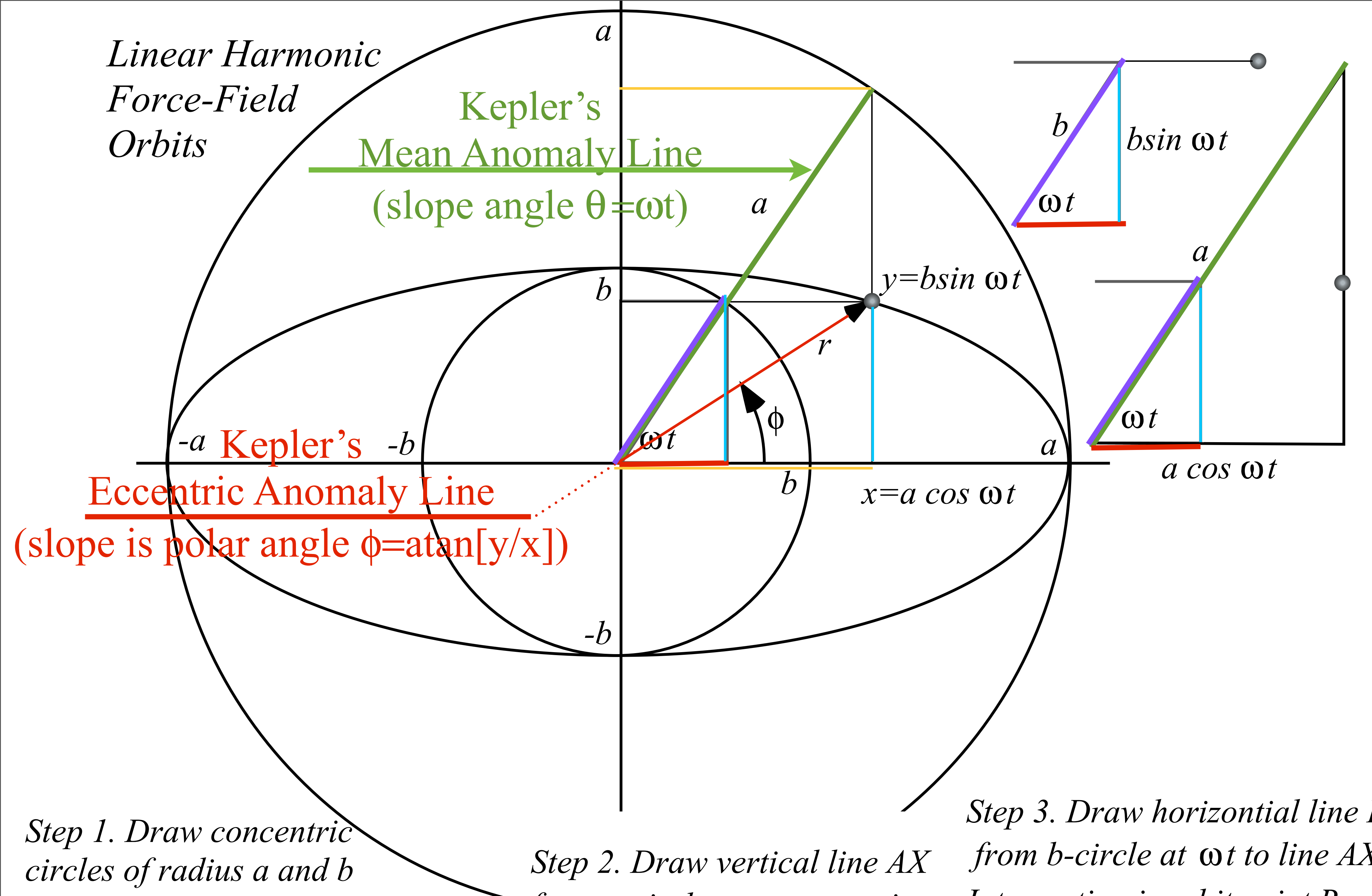
Duality relations of Lagrangian and Hamiltonian ellipse

Introducing the 1^{st} *(partial) differential equations of mechanics*

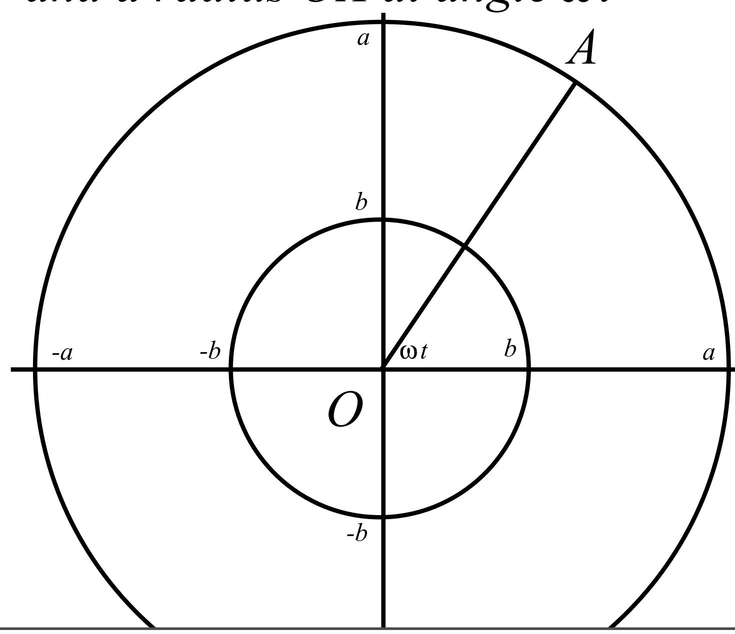
Linear Harmonic Force-Field Orbits



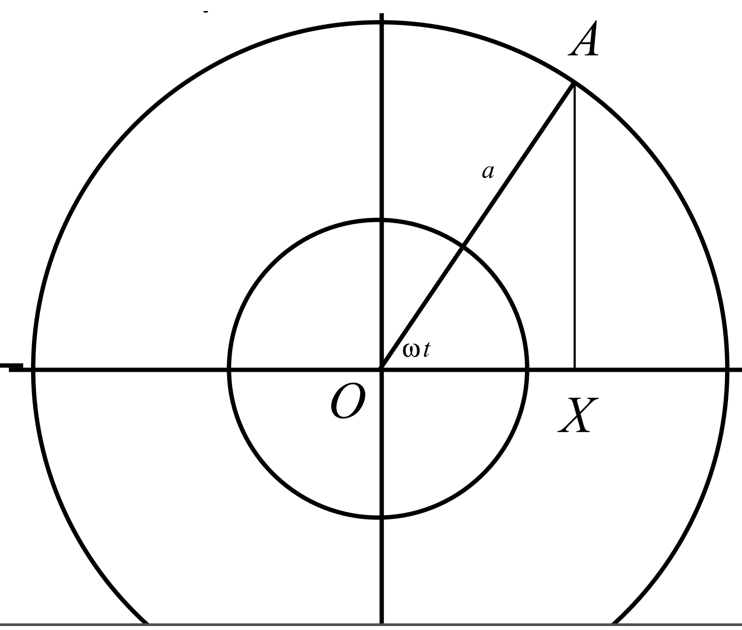
Unit 1
Fig. 11.1
(top 2/3's)

See Lecture 8 pages 17 to 25

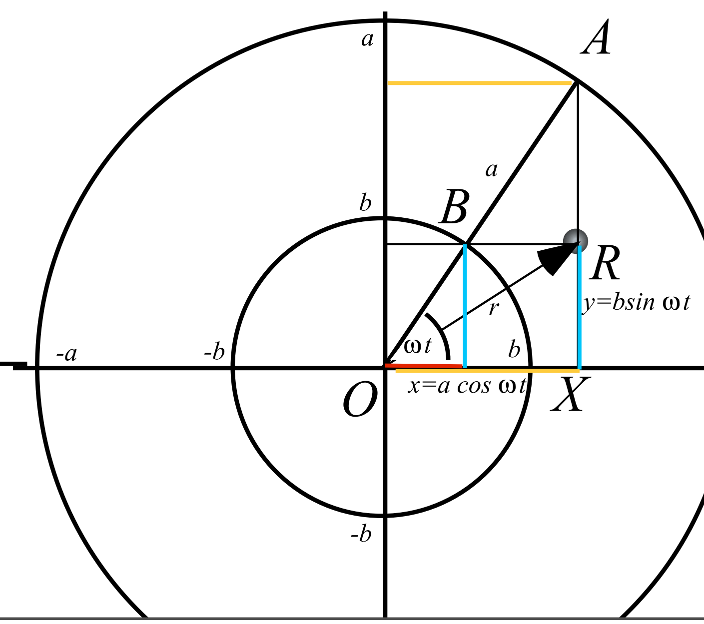
Step 1. Draw concentric circles of radius a and b and a radius OA at angle ωt



Step 2. Draw vertical line AX from a-circle at ωt to x-axis



Step 3. Draw horizontial line BR from b-circle at ωt to line AX. Intersection is orbit point R.



Review of dual IHO elliptic orbits (Lecture 9)

Construction by Phasor-pair projection

Construction by Kepler anomaly projection

Introduction to dual matrix operator geometry

→ *Quadratic form ellipse* $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$

Duality norm relations ($\mathbf{r}\bullet\mathbf{p}=1$)

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ ($\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$)

Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

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Chain rule and order $\partial^2\Psi/\partial x\partial y= \partial^2\Psi/\partial y\partial x$ *symmetry*

Duality relations of Lagrangian and Hamiltonian ellipse

Introducing the 1st (partial) differential equations of mechanics

Quadratic forms and tangent contact geometry of their ellipses

A matrix Q that generates an ellipse by **r**•*Q*•**r**=*1 is called* positive-definite (*if* **r**•*Q*•**r** *always >0*)

$$\overbrace{\begin{pmatrix} x & y \end{pmatrix} \bullet \overbrace{\begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix}}^{\mathbf{r}\bullet\mathbf{Q}\bullet\mathbf{r}} \bullet \begin{pmatrix} x \\ y \end{pmatrix}}^{=1} = 1 = \overbrace{\begin{pmatrix} x & y \end{pmatrix}}^{\mathbf{r}} \bullet \overbrace{\begin{pmatrix} \frac{x}{a^2} \\ \frac{y}{b^2} \end{pmatrix}}^{\mathbf{Q}\bullet\mathbf{r}} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

A inverse matrix Q^{-1} generates an ellipse by **p**•Q^{-1}•**p**=*1 called* inverse *or* dual *ellipse:*

$$\overbrace{\begin{pmatrix} p_x & p_y \end{pmatrix} \bullet \overbrace{\begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix}}^{\mathbf{p}\bullet\mathbf{Q}^{-1}\bullet\mathbf{p}} \bullet \begin{pmatrix} p_x \\ p_y \end{pmatrix}}^{=1} = 1 = \overbrace{\begin{pmatrix} p_x & p_y \end{pmatrix}}^{\mathbf{p}} \bullet \overbrace{\begin{pmatrix} a^2 p_x \\ b^2 p_y \end{pmatrix}}^{\mathbf{Q}^{-1}\bullet\mathbf{p}} = a^2 p_x^2 + b^2 p_y^2$$

Quadratic forms and tangent contact geometry of their ellipses

A matrix Q that generates an ellipse by **r**•*Q*•**r**=*1 is called* positive-definite (*if* **r**•*Q*•**r** *always >0*)

$$\overbrace{\begin{pmatrix} x & y \end{pmatrix} \bullet \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix}}^{\mathbf{r}\bullet\mathbf{Q}\bullet\mathbf{r}\ =1} = 1 = \overbrace{\begin{pmatrix} x & y \end{pmatrix}}^{\mathbf{r}} \bullet \overbrace{\begin{pmatrix} \frac{x}{a^2} \\ \frac{y}{b^2} \end{pmatrix}}^{\mathbf{Q}\bullet\mathbf{r}=\mathbf{p}} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

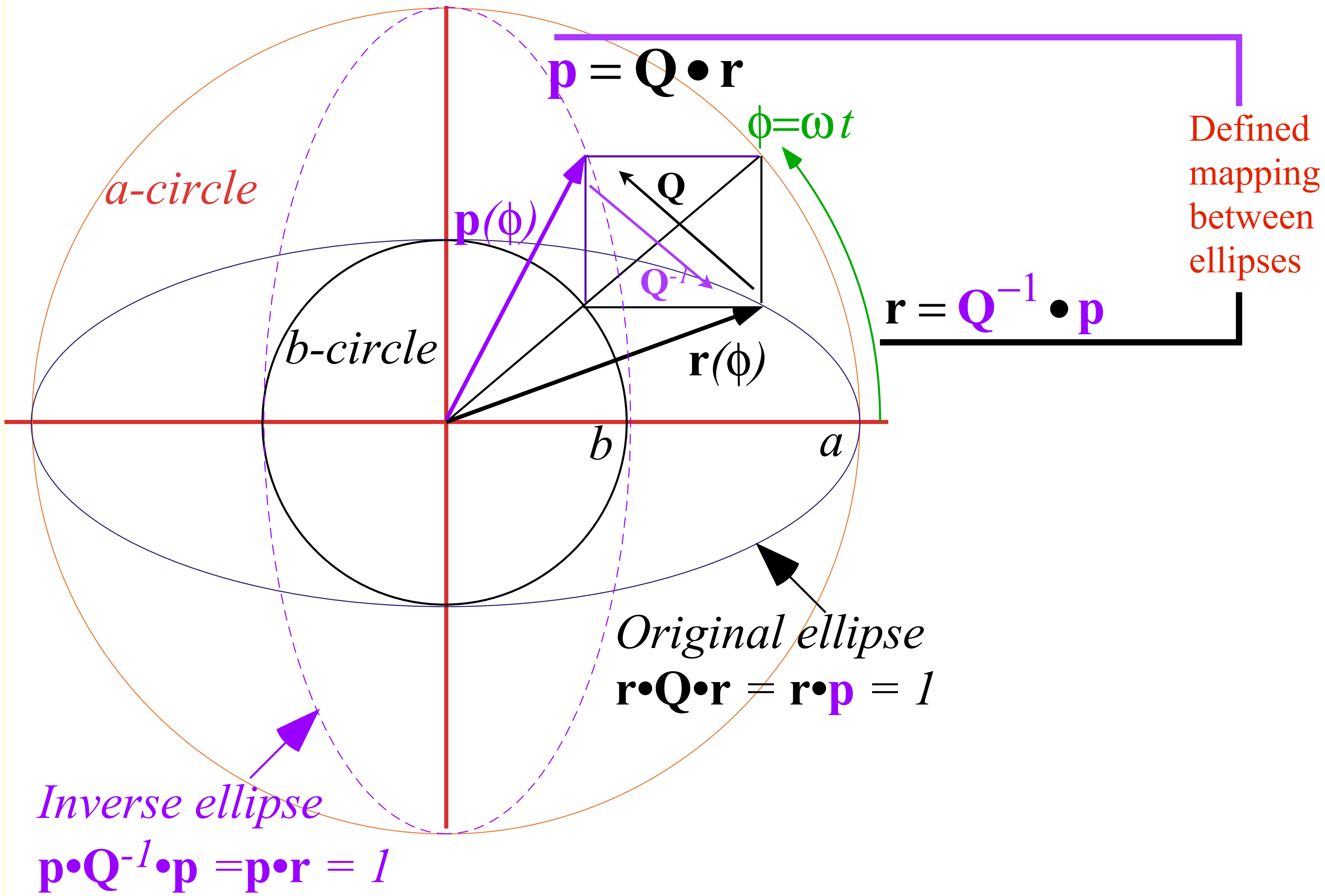
Defined mapping between ellipses

A inverse matrix Q^{-1} *generates an ellipse by* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$ *called* inverse *or* dual *ellipse:*

$$\overbrace{\begin{pmatrix} p_x & p_y \end{pmatrix} \bullet \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix} \bullet \begin{pmatrix} p_x \\ p_y \end{pmatrix}}^{\mathbf{p}\bullet\mathbf{Q}^{-1}\bullet\mathbf{p}\ =1} = 1 = \overbrace{\begin{pmatrix} p_x & p_y \end{pmatrix}}^{\mathbf{p}} \bullet \overbrace{\begin{pmatrix} a^2 p_x \\ b^2 p_y \end{pmatrix}}^{\mathbf{Q}^{-1}\bullet\mathbf{p}=\mathbf{r}} = a^2 p_x^2 + b^2 p_y^2$$

(a) Quadratic form ellipse and
Inverse quadratic form ellipse

based on
Unit 1
Fig. 11.6



Here plot of **p**-ellipse is re-scaled by scalefactor $S=a \cdot b$

p-ellipse x-radius=$1/a$ plotted at: $S(1/a)=b$ (=1 for a=2, b=1)

p-ellipse y-radius=$1/b$ plotted at: $S(1/b)=a$ (=2 for a=2, b=1)

Review of dual IHO elliptic orbits (Lecture 9)

Construction by Phasor-pair projection

Construction by Kepler anomaly projection

Introduction to dual matrix operator geometry

Quadratic form ellipse $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$

→ *Duality norm relations (* $\mathbf{r}\bullet\mathbf{p}=1$*)*

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ *(* $\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$*)*

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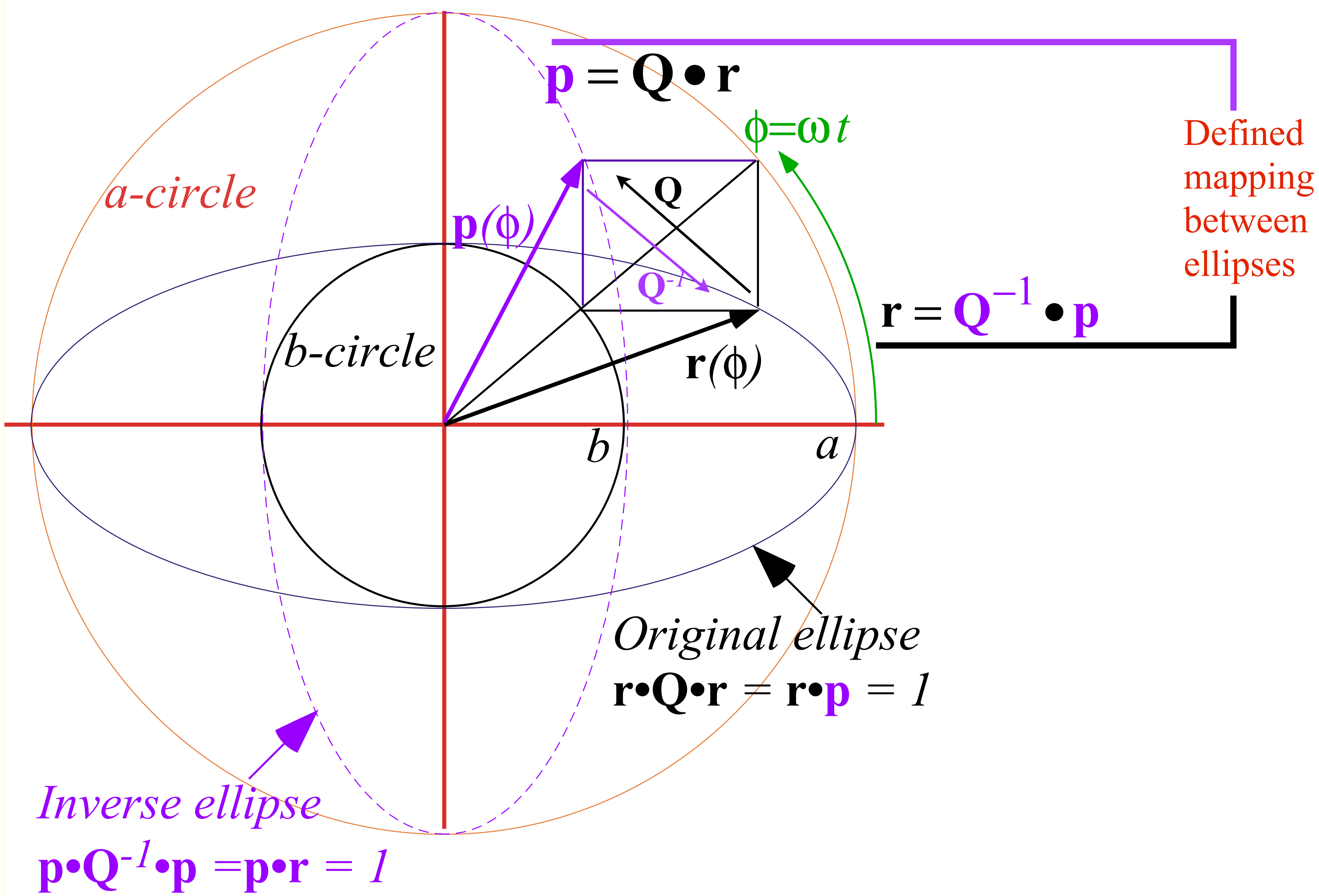
Chain rule and order $\partial^2\Psi/\partial x\partial y=\partial^2\Psi/\partial y\partial x$ *symmetry*

Duality relations of Lagrangian and Hamiltonian ellipse

Introducing the 1^{st} *(partial) differential equations of mechanics*

(a) Quadratic form ellipse and
Inverse quadratic form ellipse

based on
Unit 1
Fig. 11.6



Quadratic form **r•Q•r** $=1$ *has mutual duality relations with inverse form* **p•Q**$^{-1}$**•p** $=1=$ **p•r**

Here plot of **p**-ellipse is re-scaled by scalefactor $S=a \cdot b$

p-ellipse x-radius$=1/a$ plotted at: $S(1/a)=b$ (=1 for a=2, b=1)

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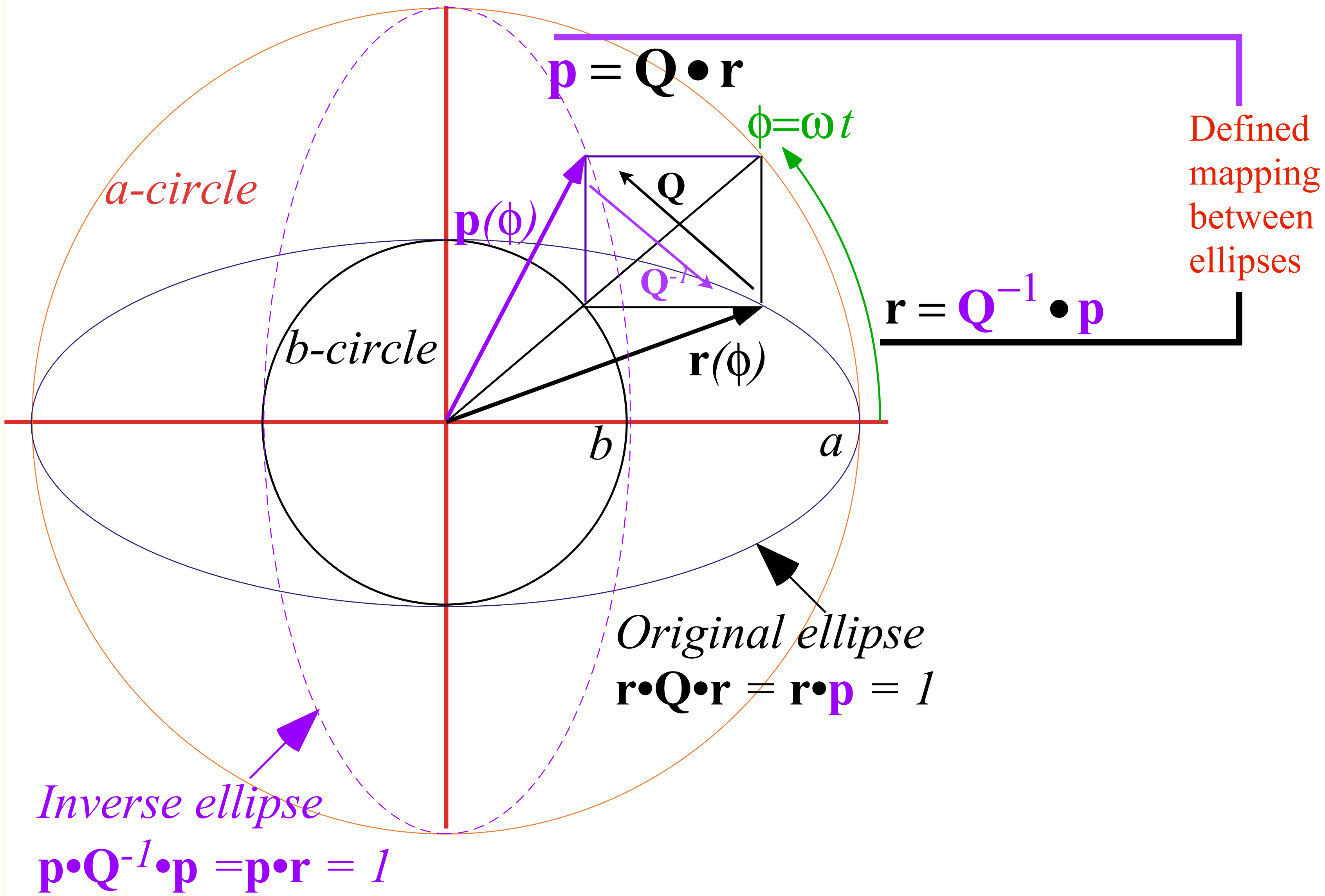

Quadratic form **r•Q•r** *=1 has mutual duality relations with inverse form* **p•Q**$^{-1}$**•p** *=1=* **p•r**

$$\mathbf{p} = \mathbf{Q}\bullet\mathbf{r} = \overbrace{\begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix}}^{\mathbf{Q}} \bullet \overbrace{\begin{pmatrix} x \\ y \end{pmatrix}}^{\mathbf{r}} = \overbrace{\begin{pmatrix} x/a^2 \\ y/b^2 \end{pmatrix}}^{\mathbf{p}} = \begin{pmatrix} (1/a)\cos\phi \\ (1/b)\sin\phi \end{pmatrix}$$

where: $x = r_x = a\cos\phi = a\cos\omega t$, $y = r_y = b\sin\phi = b\sin\omega t$ so: $\boxed{\mathbf{p}\bullet\mathbf{r} = 1}$

Here plot of **p**-ellipse is re-scaled by scalefactor $S=a\cdot b$

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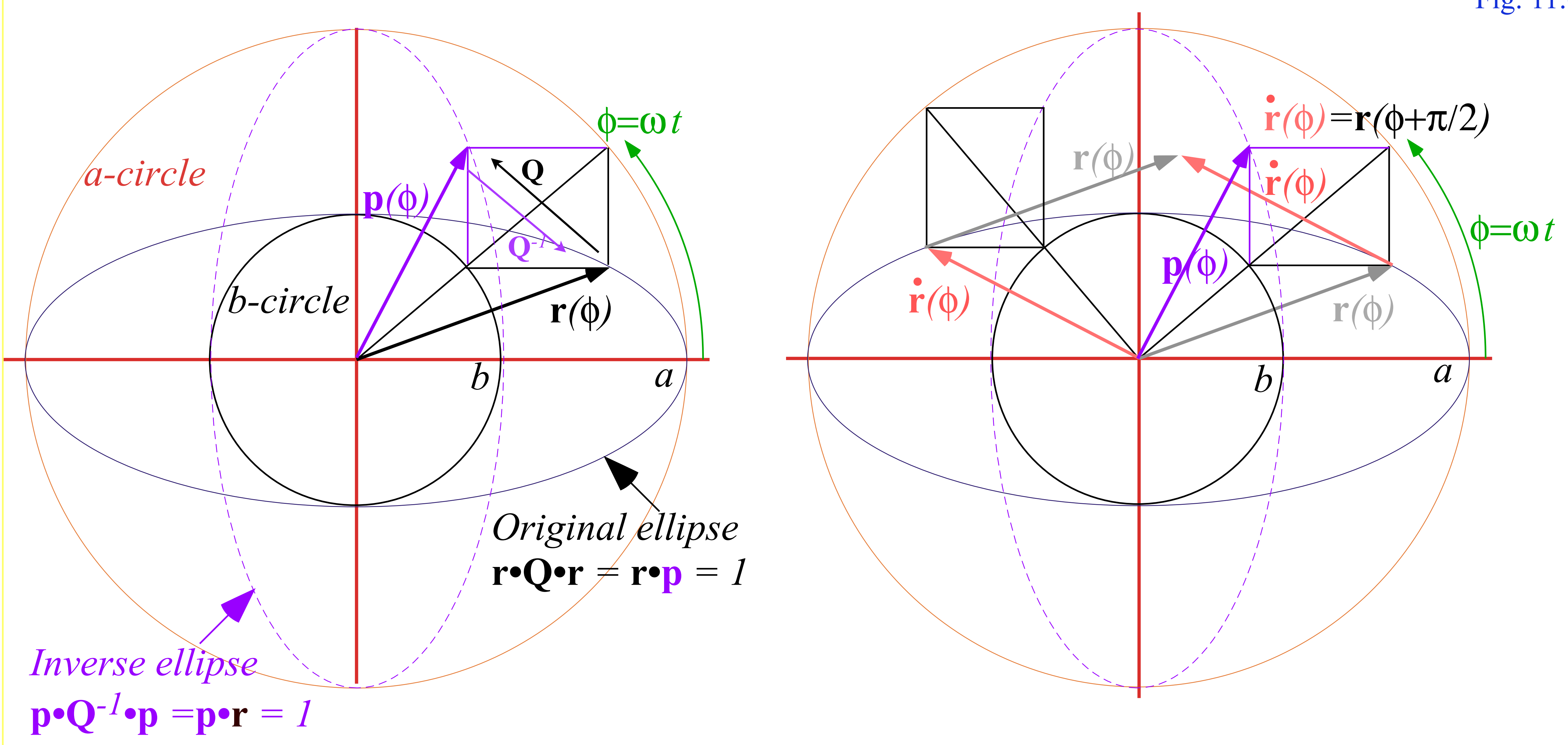
Introducing the 1st (partial) differential equations of mechanics

(a) Quadratic form ellipse and
Inverse quadratic form ellipse

(b) Ellipse tangents

based on
Unit 1
Fig. 11.6



Quadratic form **r•Q•r** *=1 has mutual duality relations with inverse form* **p•Q^{-1}•p** *=1=* **p•r**

$$\mathbf{p}=\mathbf{Q}\bullet\mathbf{r}=\begin{pmatrix}1/a^2 & 0\\ 0 & 1/b^2\end{pmatrix}\bullet\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x/a^2\\ y/b^2\end{pmatrix}=\begin{pmatrix}(1/a)\cos\phi\\ (1/b)\sin\phi\end{pmatrix}\quad \text{where: } \begin{matrix}x=r_x=a\cos\phi=a\cos\omega t\\ y=r_y=b\sin\phi=b\sin\omega t\end{matrix}\quad \text{so: } \boxed{\mathbf{p}\bullet\mathbf{r}=1}$$

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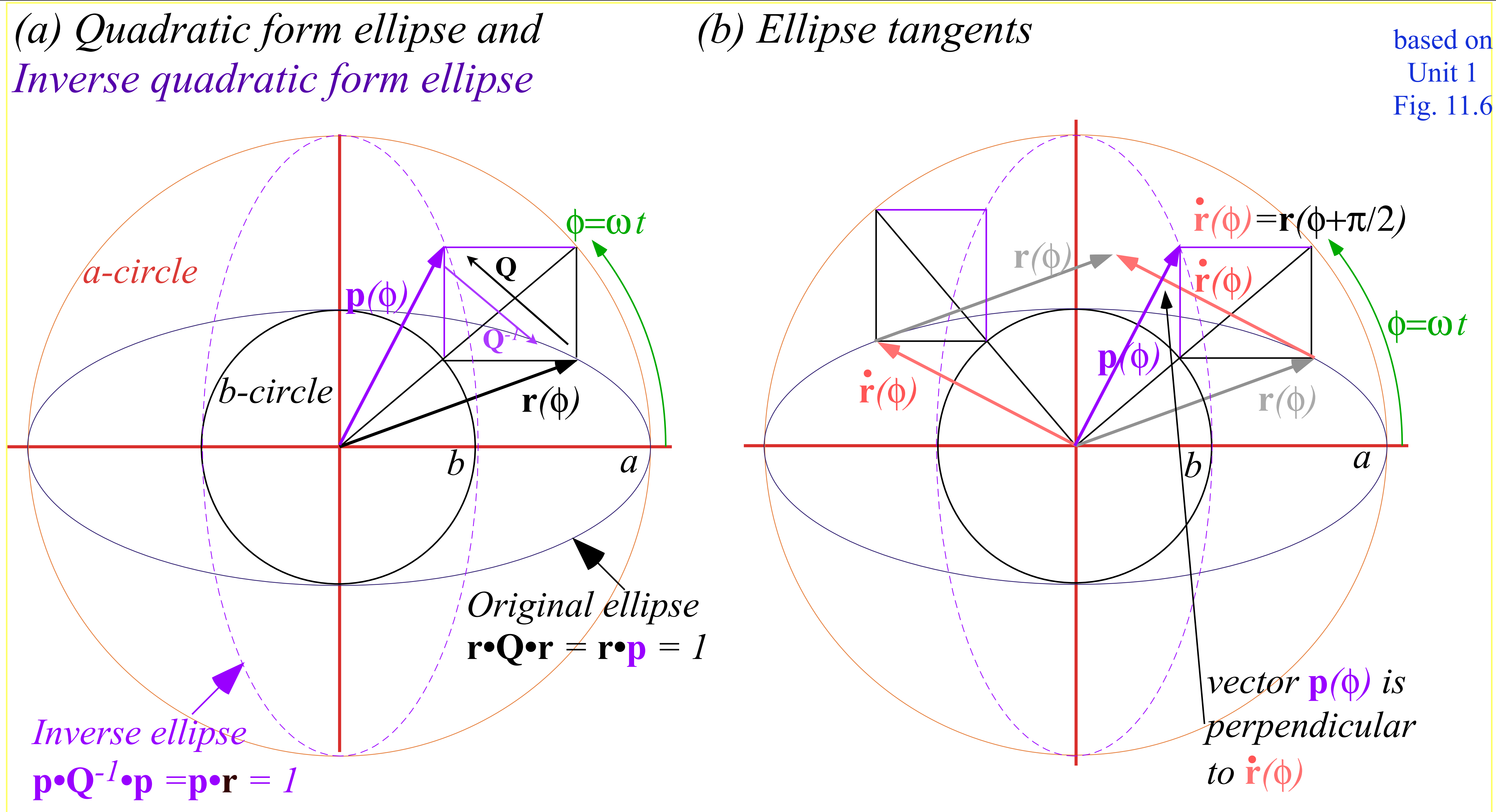

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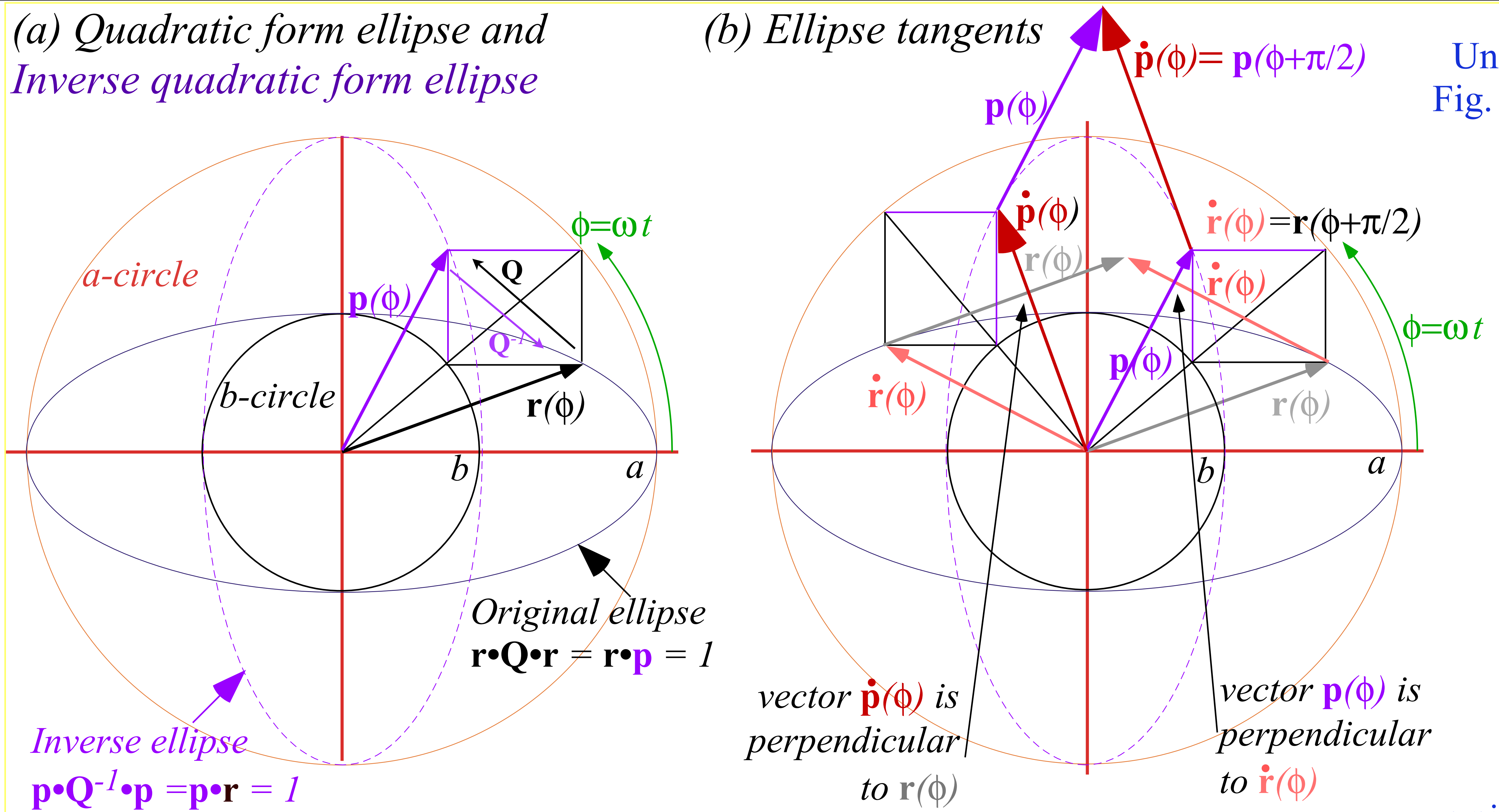
p *is perpendicular to velocity* **v**=**ṙ** *, a mutual orthogonality*

$$\boxed{\dot{\mathbf{r}}\bullet\mathbf{p}=0}=\begin{pmatrix}\dot{r}_x & \dot{r}_y\end{pmatrix}\bullet\begin{pmatrix}p_x\\ p_y\end{pmatrix}=\begin{pmatrix}-a\sin\phi & b\cos\phi\end{pmatrix}\bullet\begin{pmatrix}(1/a)\cos\phi\\ (1/b)\sin\phi\end{pmatrix}\quad \text{where: } \begin{matrix}\dot{r}_x=-a\sin\phi\\ \dot{r}_y=b\cos\phi\end{matrix}\quad \text{and: } \begin{matrix}p_x=(1/a)\cos\phi\\ p_y=(1/b)\sin\phi\end{matrix}$$



Quadratic form **r•Q•r** *=1 has mutual duality relations with inverse form* **p•Q**$^{-1}$**•p** *=1* — *unit mutual projection*

$$\mathbf{p}=\mathbf{Q}\bullet\mathbf{r}=\begin{pmatrix}1/a^2 & 0\\ 0 & 1/b^2\end{pmatrix}\bullet\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x/a^2\\ y/b^2\end{pmatrix}=\begin{pmatrix}(1/a)\cos\phi\\ (1/b)\sin\phi\end{pmatrix}\quad\text{where: }\begin{matrix}x=r_x=a\cos\phi=a\cos\omega t\\ y=r_y=b\sin\phi\ =b\sin\omega t\end{matrix}\quad\text{so: }\boxed{\mathbf{p}\bullet\mathbf{r}=1}$$

p *is perpendicular to velocity* **v**=**ṙ** *, a mutual orthogonality. So is* **r** *perpendicular to* **ṗ**: $\boxed{\dot{\mathbf{p}}\bullet\mathbf{r}=0}$

$$\boxed{\dot{\mathbf{r}}\bullet\mathbf{p}=0}=\begin{pmatrix}\dot r_x & \dot r_y\end{pmatrix}\bullet\begin{pmatrix}p_x\\ p_y\end{pmatrix}=\begin{pmatrix}-a\sin\phi & b\cos\phi\end{pmatrix}\bullet\begin{pmatrix}(1/a)\cos\phi\\ (1/b)\sin\phi\end{pmatrix}\quad\text{where: }\begin{matrix}\dot r_x=-a\sin\phi\\ \dot r_y=b\cos\phi\end{matrix}\quad\text{and: }\begin{matrix}p_x=(1/a)\cos\phi\\ p_y=(1/b)\sin\phi\end{matrix}$$

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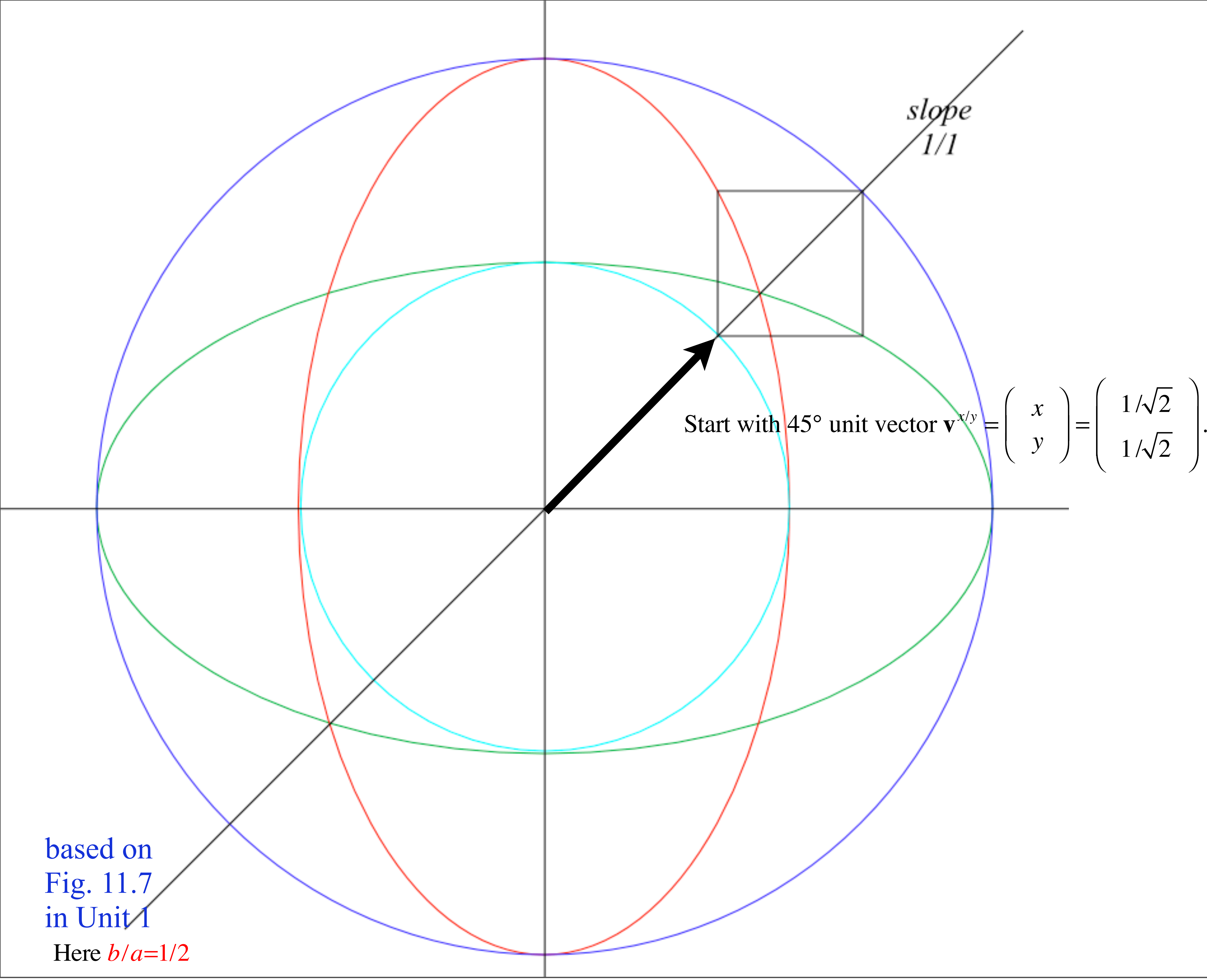

slope
1/1
Start with 45° unit vector $\mathbf{v}^{x/y} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$.
based on
Fig. 11.7
in Unit 1
Here b/a=1/2

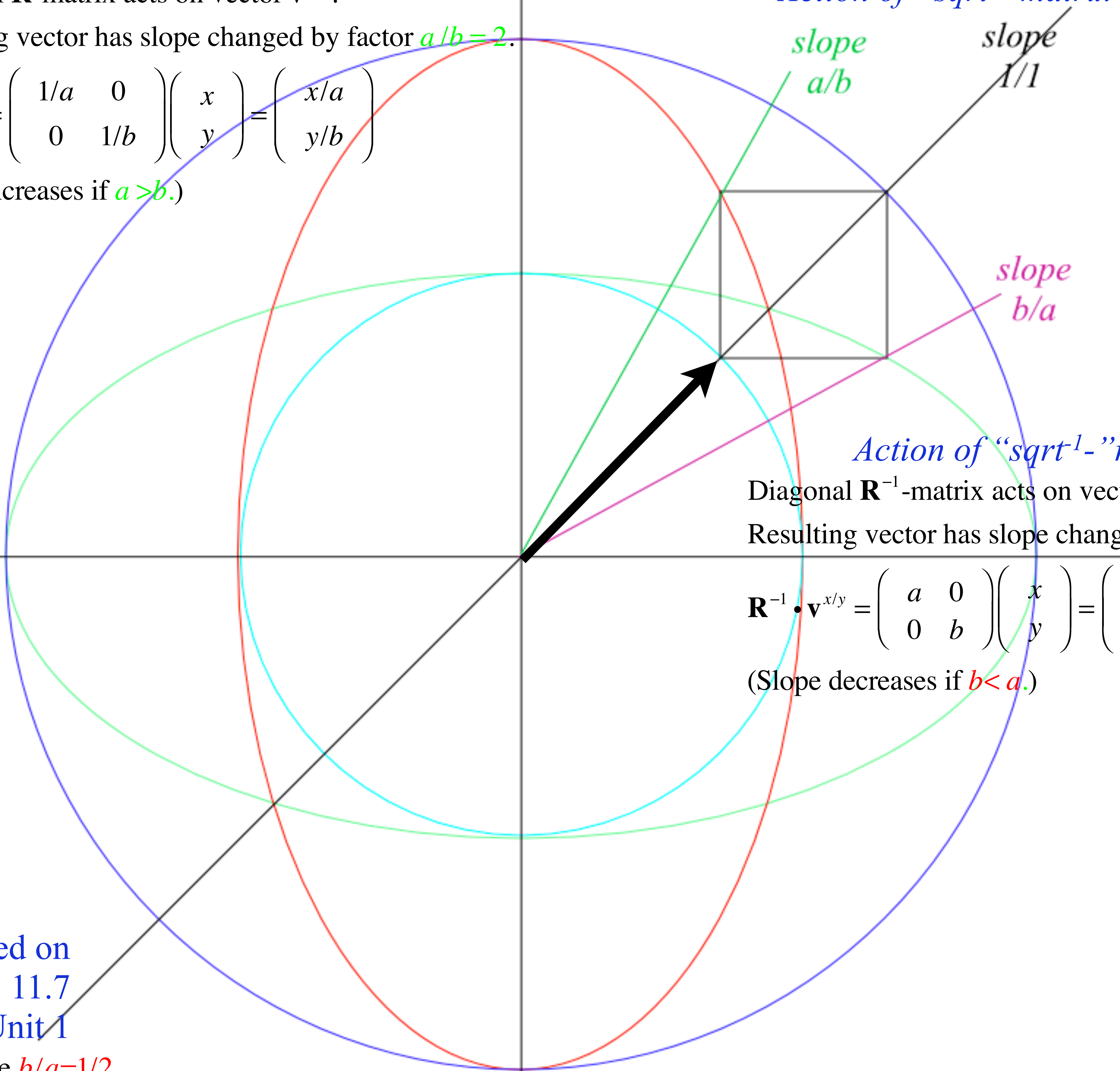
Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b = 2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a \\ y/b \end{pmatrix}$$

(Slope increases if $a > b$.)

Action of "sqrt-"matrix $R=\sqrt{Q}$



Action of "sqrt^{-1}-"matrix $R^{-1} = \sqrt{Q}^{-1}$

Diagonal $\mathbf{R}^{-1}$-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor b/a.

$$\mathbf{R}^{-1}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x\cdot a \\ y\cdot b \end{pmatrix}$$

(Slope decreases if $b < a$.)

based on
Fig. 11.7
in Unit 1

Here $b/a = 1/2$

Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b=2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}1/a & 0\\ 0 & 1/b\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x/a\\ y/b\end{pmatrix}$$

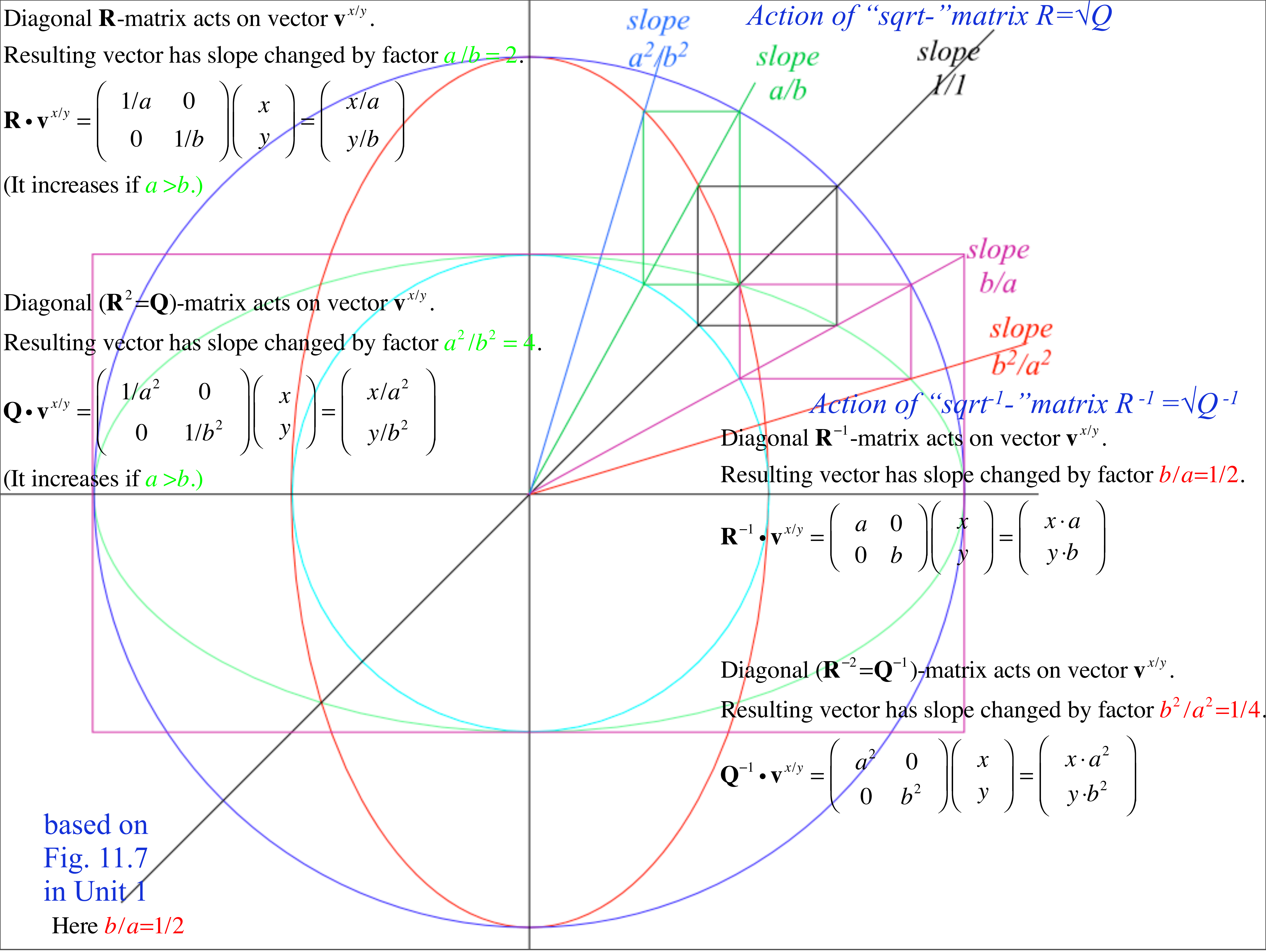
(It increases if $a>b$.)

Diagonal ($\mathbf{R}^2=\mathbf{Q}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^2/b^2=4$.

$$\mathbf{Q}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}1/a^2 & 0\\ 0 & 1/b^2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x/a^2\\ y/b^2\end{pmatrix}$$

(It increases if $a>b$.)



Diagonal $\mathbf{R}^{-1}$-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $b/a=1/2$.

$$\mathbf{R}^{-1}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\cdot a\\ y\cdot b\end{pmatrix}$$

Diagonal ($\mathbf{R}^{-2}=\mathbf{Q}^{-1}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $b^2/a^2=1/4$.

$$\mathbf{Q}^{-1}\bullet\mathbf{v}^{x/y}=\begin{pmatrix}a^2 & 0\\ 0 & b^2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\cdot a^2\\ y\cdot b^2\end{pmatrix}$$

based on Fig. 11.7 in Unit 1

Here $b/a=1/2$

Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b = 2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a \\ y/b \end{pmatrix}$$

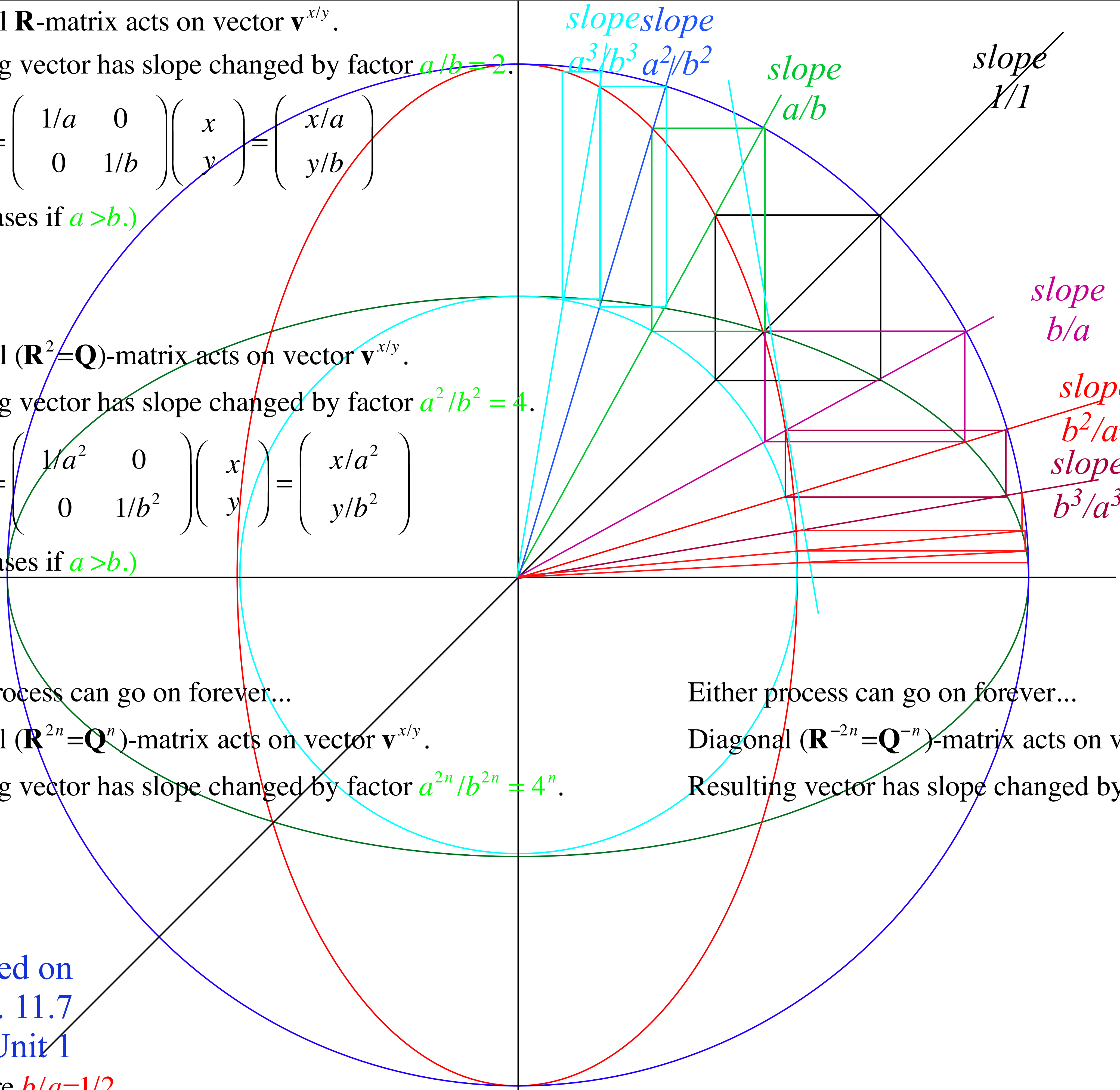
(It increases if $a > b$.)

Diagonal ($\mathbf{R}^2 = \mathbf{Q}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^2/b^2 = 4$.

$$\mathbf{Q}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a^2 \\ y/b^2 \end{pmatrix}$$

(It increases if $a > b$.)



Either process can go on forever...

Diagonal ($\mathbf{R}^{2n} = \mathbf{Q}^n$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^{2n}/b^{2n} = 4^n$.

Either process can go on forever...

Diagonal ($\mathbf{R}^{-2n} = \mathbf{Q}^{-n}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $b^{2n}/a^{2n} = 4^{-n}$.

based on
Fig. 11.7
in Unit 1

Here $b/a = 1/2$

Diagonal **R**-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a/b = 2$.

$$\mathbf{R}\bullet\mathbf{v}^{x/y} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/a \\ y/b \end{pmatrix}$$

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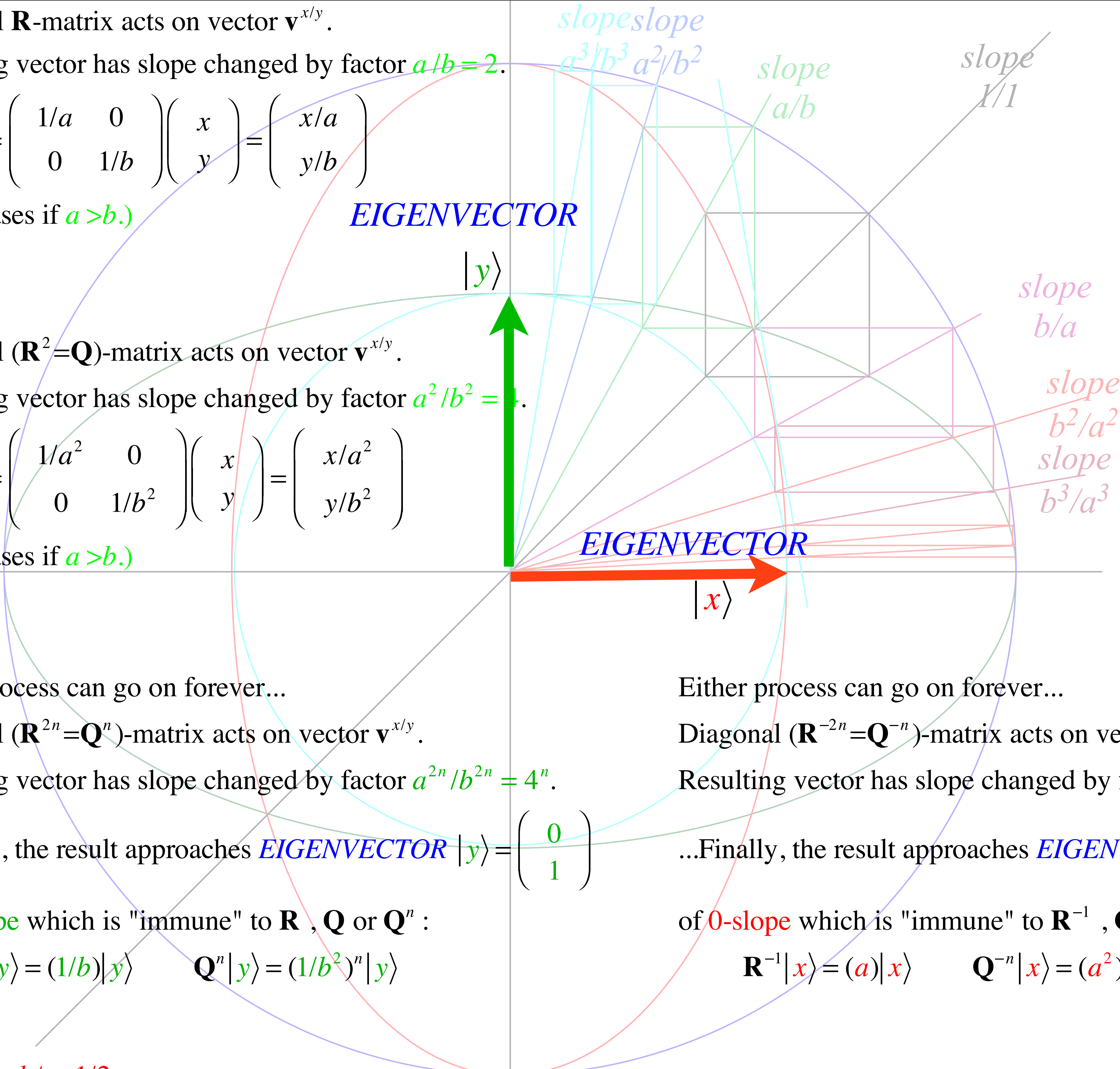

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Diagonal ($\mathbf{R}^{2n} = \mathbf{Q}^n$)-matrix acts on vector $\mathbf{v}^{x/y}$.

Resulting vector has slope changed by factor $a^{2n}/b^{2n} = 4^n$.

...Finally, the result approaches *EIGENVECTOR* $|y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

of ∞-slope which is "immune" to **R** , **Q** or $\mathbf{Q}^n$:

$$\mathbf{R}|y\rangle = (1/b)|y\rangle \qquad \mathbf{Q}^n|y\rangle = (1/b^2)^n|y\rangle$$

Either process can go on forever...

Diagonal ($\mathbf{R}^{-2n} = \mathbf{Q}^{-n}$)-matrix acts on vector $\mathbf{v}^{x/y}$.

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...Finally, the result approaches *EIGENVECTOR* $|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

of 0-slope which is "immune" to $\mathbf{R}^{-1}$, $\mathbf{Q}^{-1}$ or $\mathbf{Q}^{-n}$:

$$\mathbf{R}^{-1}|x\rangle = (a)|x\rangle \qquad \mathbf{Q}^{-n}|x\rangle = (a^2)^n|x\rangle$$

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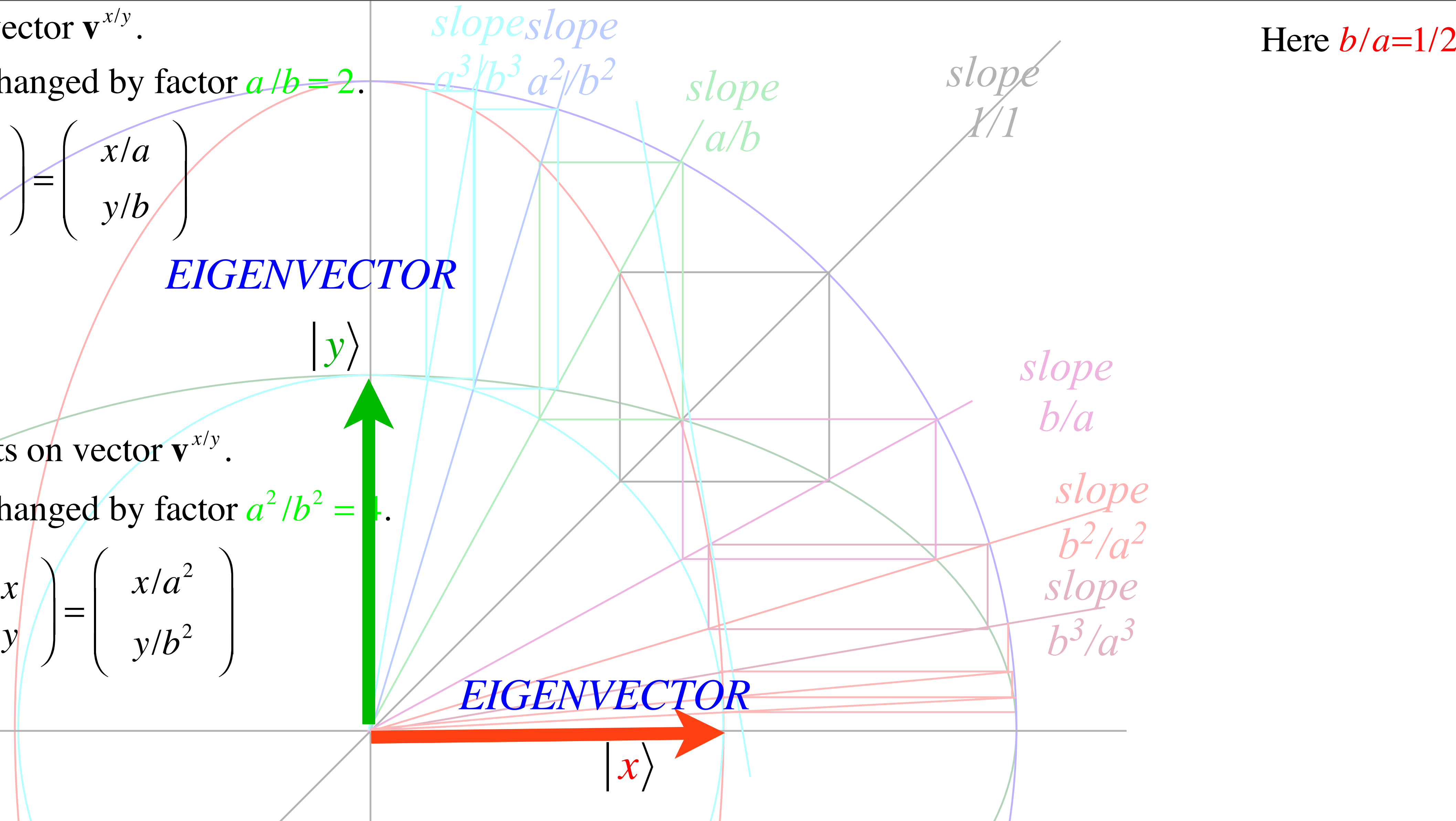

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of ∞-slope which is "immune" to **R** , **Q** or $\mathbf{Q}^n$:

Either process can go on forever...

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Resulting vector has slope changed by factor $b^{2n}/a^{2n} = 4^{-n}$.

...Finally, the result approaches *EIGENVECTOR* $|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

of 0-slope which is "immune" to $\mathbf{R}^{-1}$, $\mathbf{Q}^{-1}$ or $\mathbf{Q}^{-n}$:

Eigensolution Relations

$\mathbf{R}|y\rangle = (1/b)|y\rangle$ $\quad$ $\mathbf{Q}^n|y\rangle = (1/b^2)^n|y\rangle$

Eigenvalues

$\mathbf{R}^{-1}|x\rangle = (a)|x\rangle$ $\quad$ $\mathbf{Q}^{-n}|x\rangle = (a^2)^n|x\rangle$

Eigenvalues

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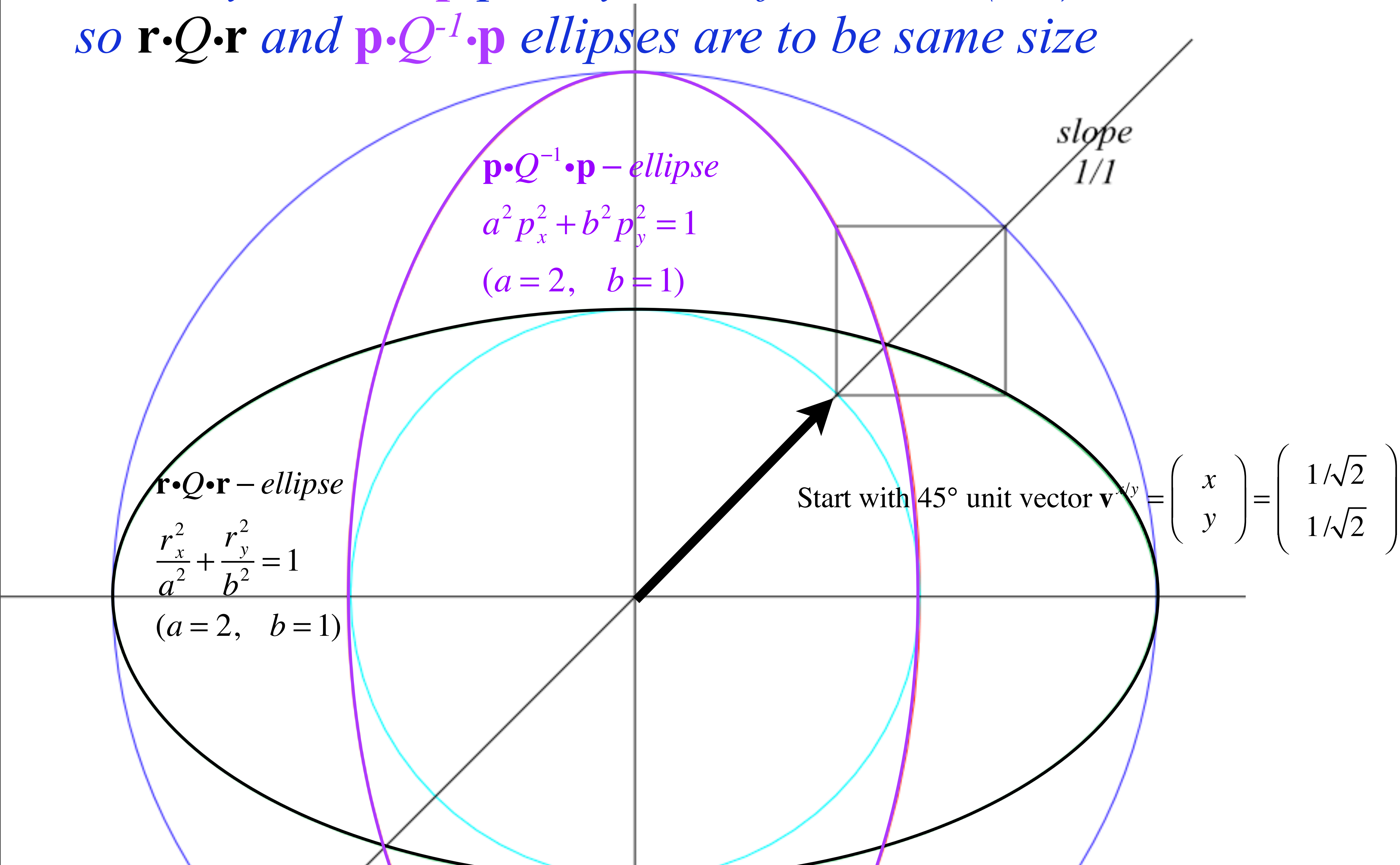
You may rescale **p***-plot by scale factor S=(a·b)*
so **r**·*Q*·**r** *and* **p**·Q^{-1}·**p** *ellipses are to be same size*

Here $b/a=1/2$



Here plot of **p**-ellipse is re-scaled by scalefactor $S=a{\cdot}b$

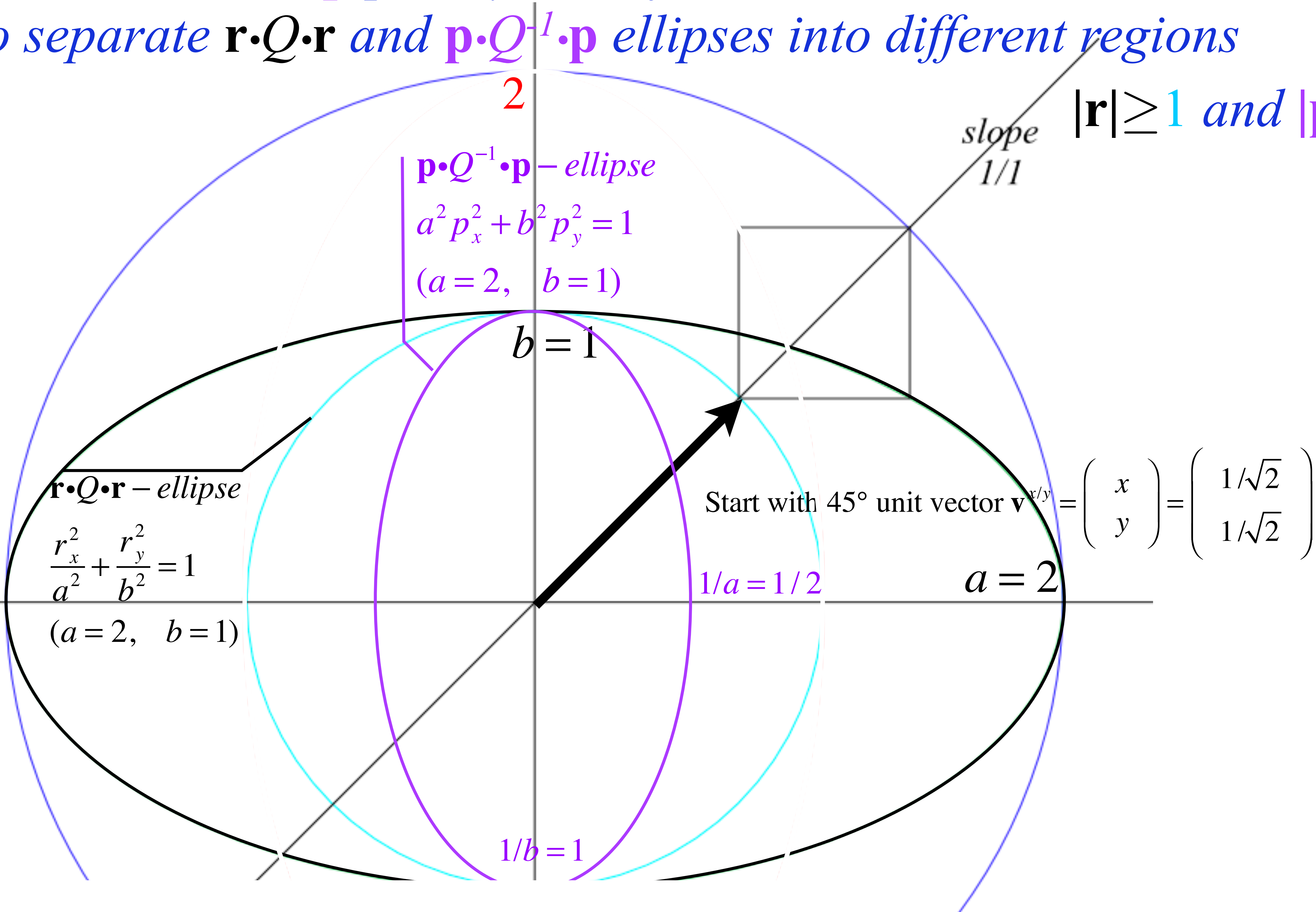
p-ellipse x-radius=$1/a$ plotted at: $S(1/a)=b$ (=1 for a=2, b=1)

p-ellipse y-radius=$1/b$ plotted at: $S(1/b)=a$ (=2 for a=2, b=1)

..or else rescale **p***-plot by scale factor* $S=b$
to separate $\mathbf{r}\cdot Q\cdot\mathbf{r}$ *and* $\mathbf{p}\cdot Q^{-1}\cdot\mathbf{p}$ *ellipses into different regions*

Here $b/a=1/2$

$|\mathbf{r}|\geq 1$ *and* $|\mathbf{p}|\leq 1$



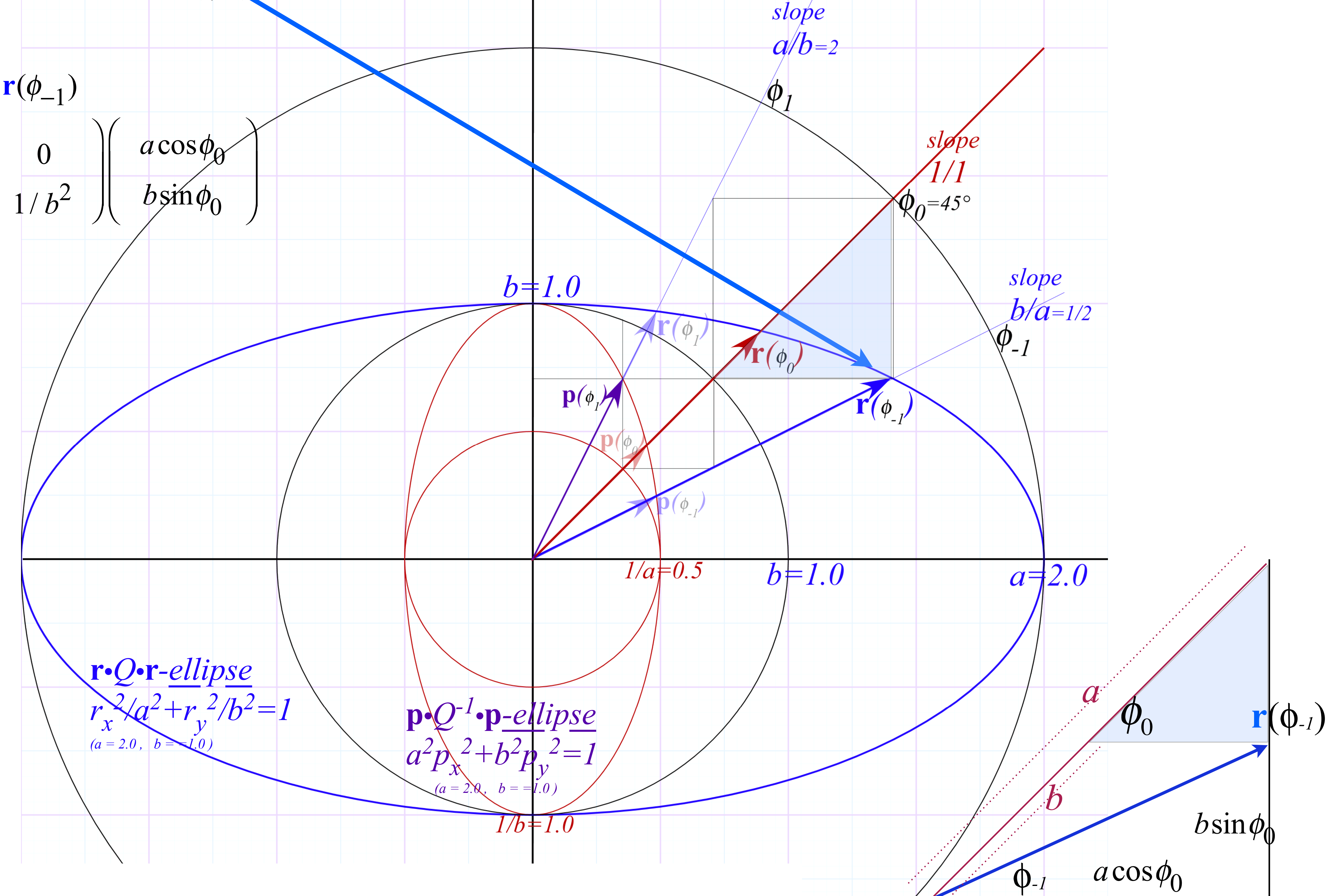
Here plot of **p**-ellipse is re-scaled by scalefactor $S=b$

p-ellipse x-radius=$1/a$ plotted at: $S(1/a)=b/a$ (=1/2 for $a=2$, $b=1$)

p-ellipse y-radius=$1/b$ plotted at: $S(1/b)=1$

Action of matrix Q that generates an **r***-ellipse* (**r**•*Q*• **r** *=1)* *on a single* **r***-vector* **r**(ϕ_{-1})...

$$\mathbf{p}(\phi_1) = \mathbf{Q} \bullet \mathbf{r}(\phi_{-1}) = \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \begin{pmatrix} a\cos\phi_0 \\ b\sin\phi_0 \end{pmatrix}$$



Variation of Fig. 11.7 in Unit 1

Here plot of **p**-ellipse is re-scaled by scalefactor $S=b$

p-ellipse x-radius=1/a plotted at: $S(1/a)=b/a$ (=1/2 for a=2, b=1)

p-ellipse y-radius=1/b plotted at: $S(1/b)$=1

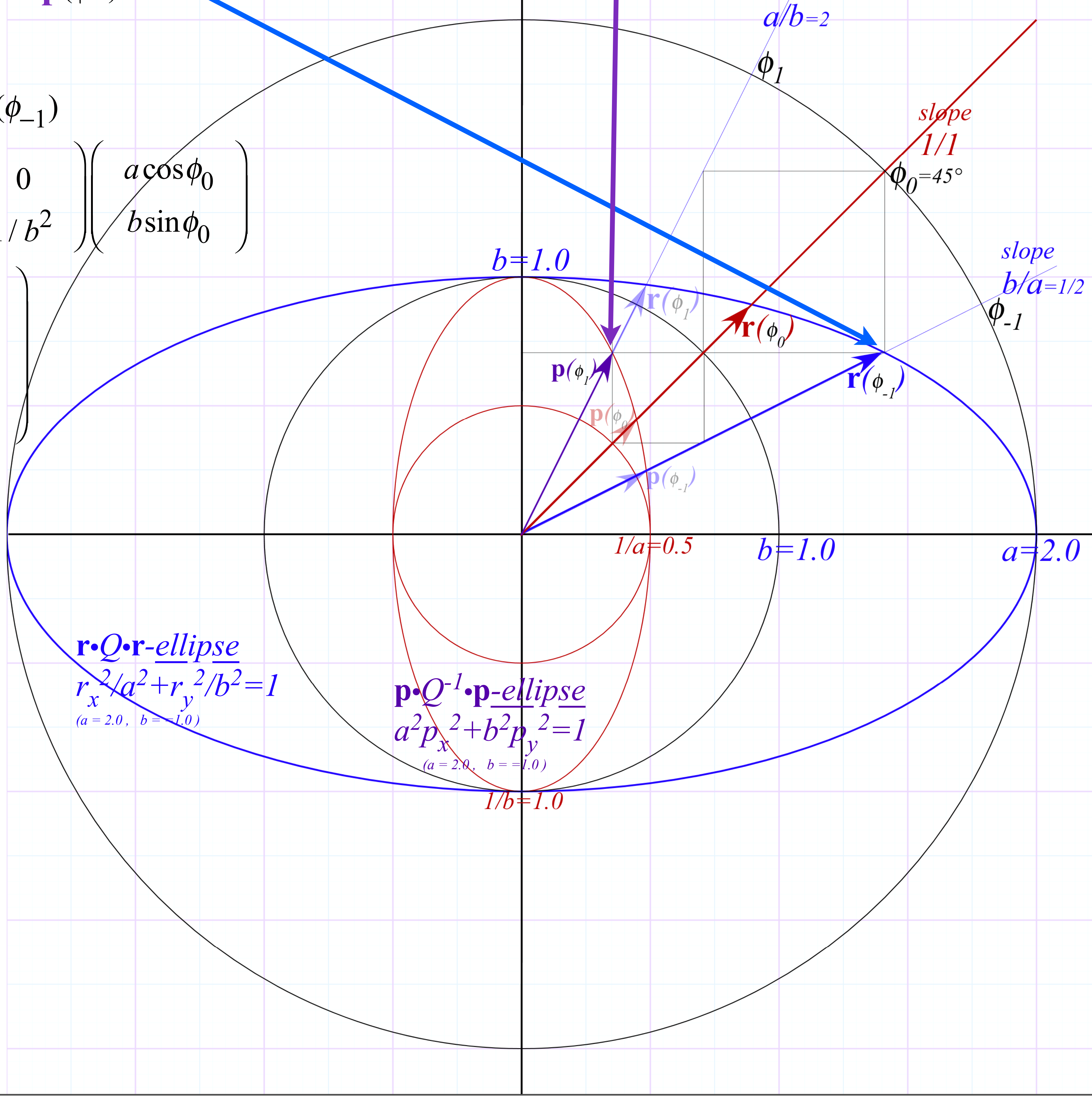
Action of matrix Q that generates an **r**-*ellipse* (**r**•*Q*•**r** =1) *on a single* **r**-*vector* **r**(ϕ₋₁)... *is to rotate it to a new vector* **p** *on the* **p**-*ellipse* (**p**•Q^{-1}• **p** =1), *that is,* Q•**r**(ϕ_{-1}) = **p**(ϕ_{+1})

$$\mathbf{p}(\phi_1) = \mathbf{Q} \bullet \mathbf{r}(\phi_{-1})$$

$$= \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix} \begin{pmatrix} a\cos\phi_0 \\ b\sin\phi_0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{a}\cos\phi_0 \\ \frac{1}{b}\sin\phi_0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}\frac{1}{\sqrt{2}} \\ \frac{1}{1}\frac{1}{\sqrt{2}} \end{pmatrix}$$



Variation of
Fig. 11.7
in Unit 1

Action of matrix Q that generates an **r***-ellipse (***r**•*Q*•**r** *=1)*
on a single **r***-vector* **r**(ϕ-1)*... is to rotate it to a new vector* **p** *on the* **p***-ellipse* (**p**•Q^{-1}• **p** =1),
that is, *Q*•**r**(ϕ-1) = **p**(ϕ+1)

$$\mathbf{p}(\phi_1) = \mathbf{Q}\bullet\mathbf{r}(\phi_{-1})$$
$$= \begin{pmatrix} 1/a^2 & 0 \\ 0 & 1/b^2 \end{pmatrix}\begin{pmatrix} a\cos\phi_0 \\ b\sin\phi_0 \end{pmatrix}$$
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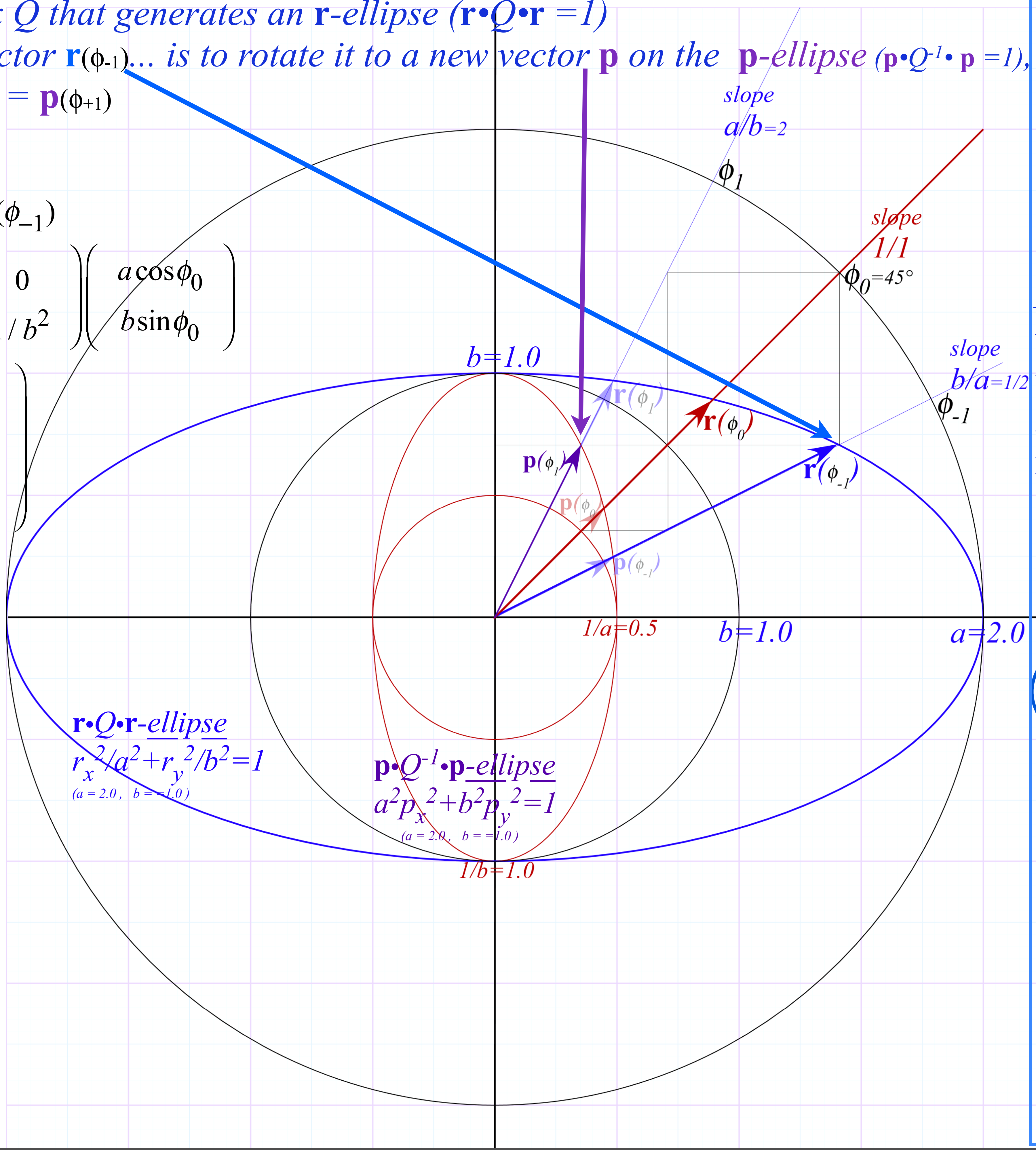
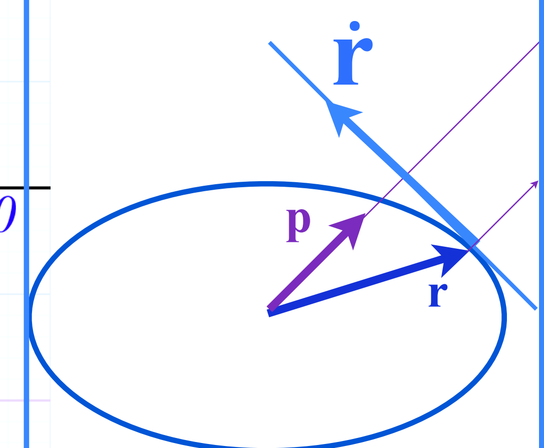


Key points of matrix geometry:

Matrix Q maps any vector **r** *to a new vector* **p** *normal to the tangent* $\dot{\mathbf{r}}$ *to its* **r**•*Q*•**r***-ellipse.*



Variation of Fig. 11.7 in Unit 1

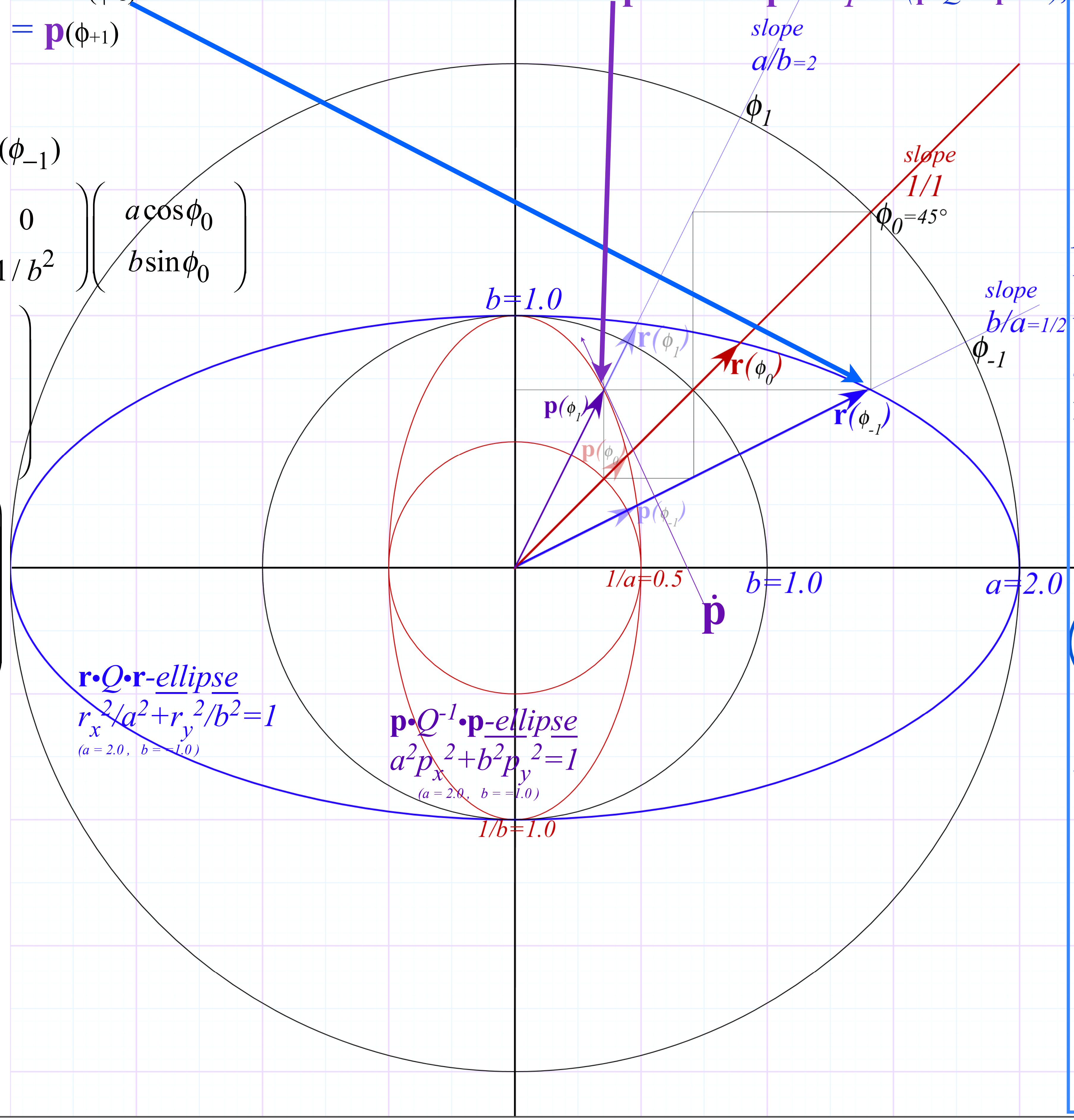
Action of matrix Q that generates an **r**-*ellipse* (**r**•*Q*•**r** =*1*)
on a single **r**-*vector* **r**(ϕ-1)... *is to rotate it to a new vector* **p** *on the* **p**-*ellipse* (**p**•Q^{-1}• **p** =*1*),
that is, *Q*•**r**(ϕ-1) = **p**(ϕ+1)

$$\mathbf{p}(\phi_1)=\mathbf{Q}\bullet\mathbf{r}(\phi_{-1})$$

$$=\begin{pmatrix}1/a^2 & 0\\ 0 & 1/b^2\end{pmatrix}\begin{pmatrix}a\cos\phi_0\\ b\sin\phi_0\end{pmatrix}$$

$$=\begin{pmatrix}\frac{1}{a}\cos\phi_0\\ \frac{1}{b}\sin\phi_0\end{pmatrix}$$

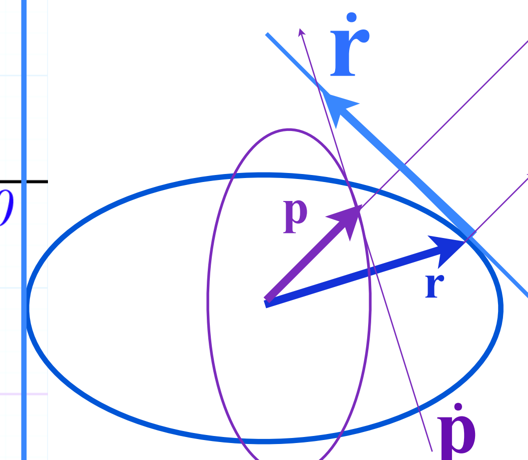
$$=\begin{pmatrix}\frac{1}{2}\frac{1}{\sqrt{2}}\\ \frac{1}{1}\frac{1}{\sqrt{2}}\end{pmatrix}$$



Variation of
Fig. 11.7
in Unit 1

Key points of matrix geometry:

Matrix Q maps any vector **r** *to a new vector* **p** *normal to the tangent* **ṙ** *to its* **r**•*Q*•**r**-*ellipse.*



Matrix Q^{-1} *maps* **p** *back to* **r** *that is normal to the tangent* **ṗ** *to its* **p**• Q^{-1}• **p**-*ellipse.*

Review of dual IHO elliptic orbits (Lecture 9)

Construction by Phasor-pair projection (with example of dual-ellipses)

Construction by Kepler anomaly projection

Introduction to dual matrix operator geometry

Quadratic form ellipse $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$

Duality norm relations ($\mathbf{r}\bullet\mathbf{p}=1$)

Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ ($\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}'$)

Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

→ *Vector calculus of tensor operation*

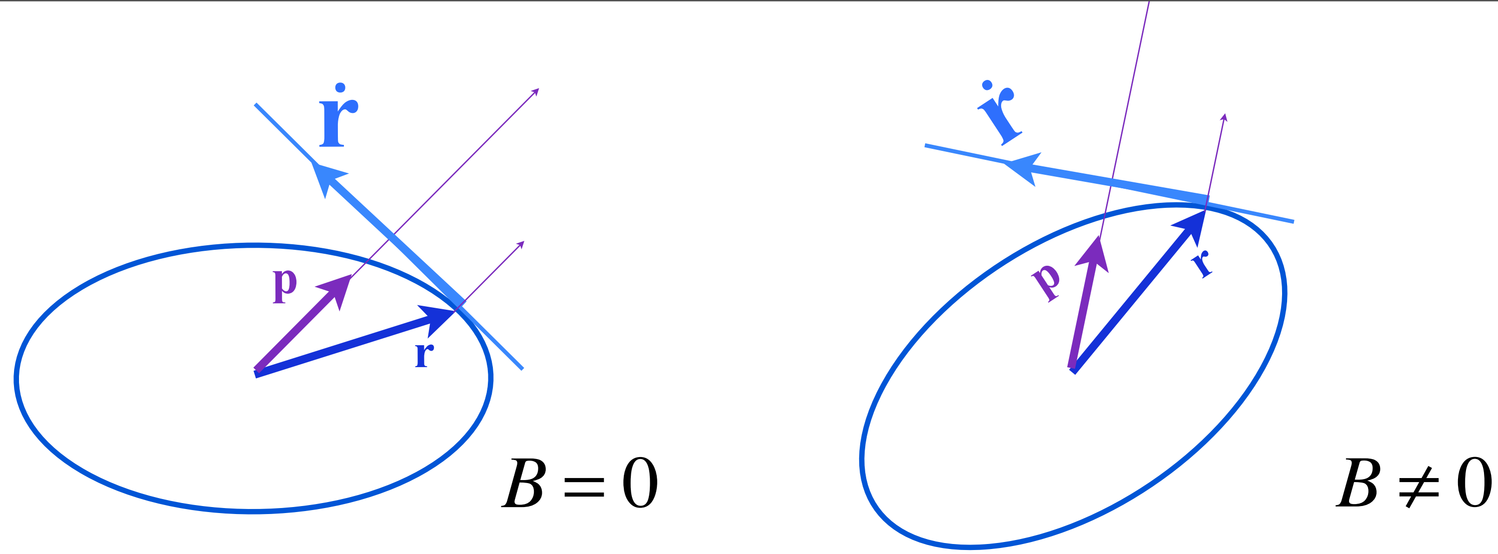
Introduction to Lagrangian-Hamiltonian duality

Review of partial differential relations

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ *symmetry*

Duality relations of Lagrangian and Hamiltonian ellipse

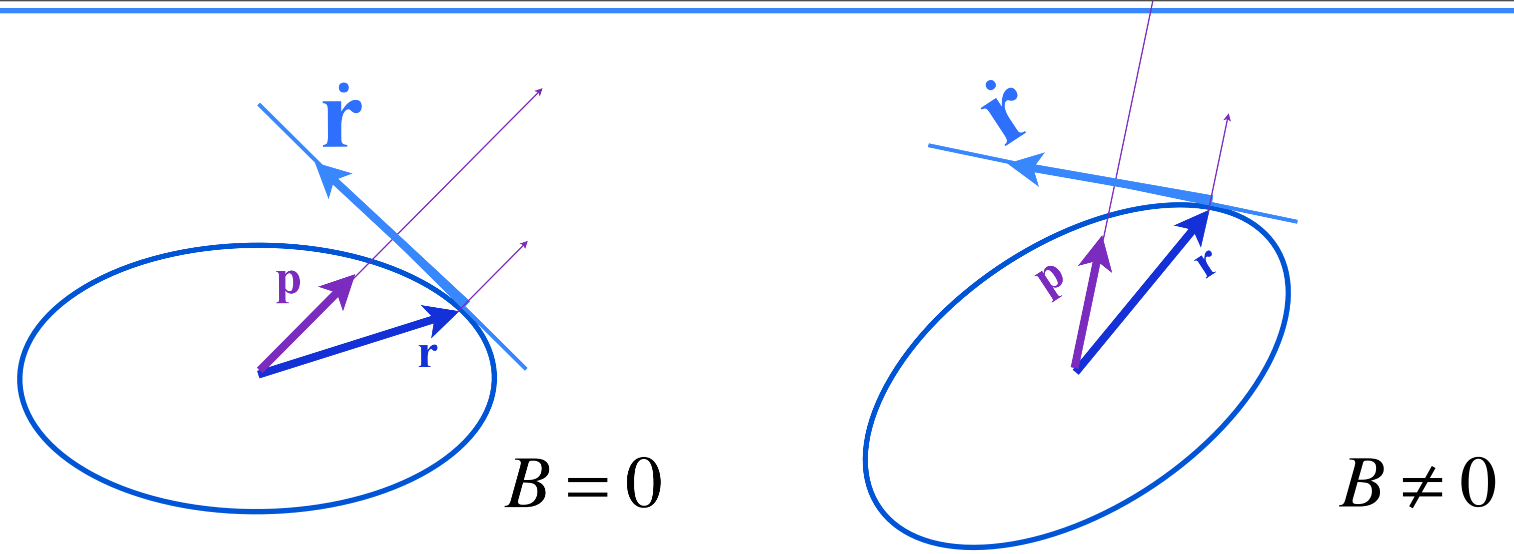
Introducing the 1^{st} *(partial) differential equations of mechanics*



Derive matrix "normal-to-ellipse"geometry by vector calculus:

Let matrix $Q = \begin{pmatrix} A & B \\ B & D \end{pmatrix}$

define the ellipse $1 = \mathbf{r}\bullet Q \bullet \mathbf{r} = \begin{pmatrix} x & y \end{pmatrix} \bullet \begin{pmatrix} A & B \\ B & D \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x & y \end{pmatrix} \bullet \begin{pmatrix} A\cdot x + B\cdot y \\ B\cdot x + D\cdot y \end{pmatrix} = A\cdot x^2 + 2B\cdot xy + D\cdot y^2 = 1$



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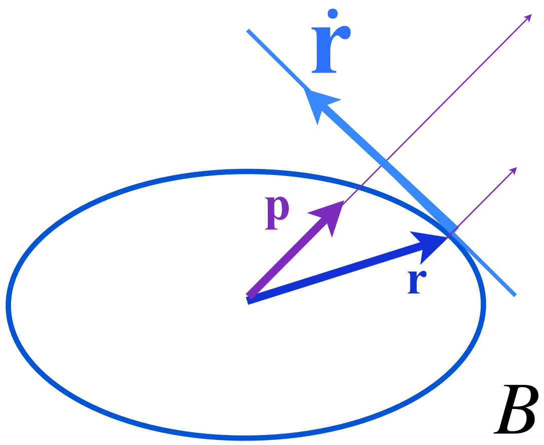
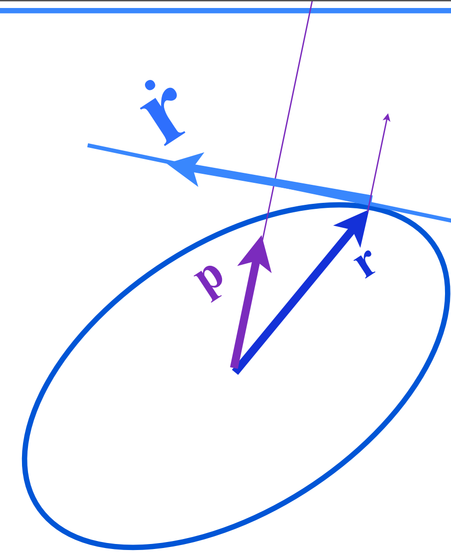
Compare operation by Q on vector **r** *with* *vector derivative or gradient of* $\mathbf{r}\bullet Q\bullet\mathbf{r}$

$$\frac{\partial}{\partial \mathbf{r}}(\mathbf{r}\bullet Q\bullet\mathbf{r}) = \nabla(\mathbf{r}\bullet Q\bullet\mathbf{r})$$

$$\begin{pmatrix} A & B \\ B & D \end{pmatrix}\bullet\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A\cdot x + B\cdot y \\ B\cdot x + D\cdot y \end{pmatrix}$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}\left(A\cdot x^2 + 2B\cdot xy + D\cdot y^2\right) = \begin{pmatrix} 2A\cdot x + 2B\cdot y \\ 2B\cdot x + 2D\cdot y \end{pmatrix}$$





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Let matrix $Q = \begin{pmatrix} A & B \\ B & D \end{pmatrix}$

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Compare operation by Q *on vector* $\mathbf{r}$ *with vector derivative or gradient of* $\mathbf{r}\bullet Q\bullet\mathbf{r}$

$$\begin{pmatrix} A & B \\ B & D \end{pmatrix}\bullet\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A\cdot x+B\cdot y \\ B\cdot x+D\cdot y \end{pmatrix}$$

$$\frac{\partial}{\partial \mathbf{r}}(\mathbf{r}\bullet Q\bullet\mathbf{r}) = \nabla(\mathbf{r}\bullet Q\bullet\mathbf{r})$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}\left(A\cdot x^2+2B\cdot xy+D\cdot y^2\right) = \begin{pmatrix} 2A\cdot x+2B\cdot y \\ 2B\cdot x+2D\cdot y \end{pmatrix}$$

Very simple result:

$$\frac{\partial}{\partial \mathbf{r}}\left(\frac{\mathbf{r}\bullet Q\bullet\mathbf{r}}{2}\right) = \nabla\left(\frac{\mathbf{r}\bullet Q\bullet\mathbf{r}}{2}\right) = Q\bullet\mathbf{r}$$

Review of dual IHO elliptic orbits (Lecture 9)

Construction by Phasor-pair projection

Construction by Kepler anomaly projection

Introduction to dual matrix operator geometry

Quadratic form ellipse $\mathbf{r}\bullet Q\bullet\mathbf{r}=1$ *vs.inverse form ellipse* $\mathbf{p}\bullet Q^{-1}\bullet\mathbf{p}=1$

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Q-Ellipse tangents $\mathbf{r}'$ *normal to dual* Q^{-1}*-ellipse position* $\mathbf{p}$ $(\mathbf{r}'\bullet\mathbf{p}=0=\mathbf{r}\bullet\mathbf{p}')$

(Still more)Operator geometric sequences and eigenvectors

Alternative scaling of matrix operator geometry

Vector calculus of tensor operation

Introduction to Lagrangian-Hamiltonian duality

Review of partial differential relations

Chain rule and order $\partial^2\Psi/\partial x\partial y=\partial^2\Psi/\partial y\partial x$ *symmetry*

Duality relations of Lagrangian and Hamiltonian ellipse

Introducing the 1^{st} *(partial) differential equations of mechanics*

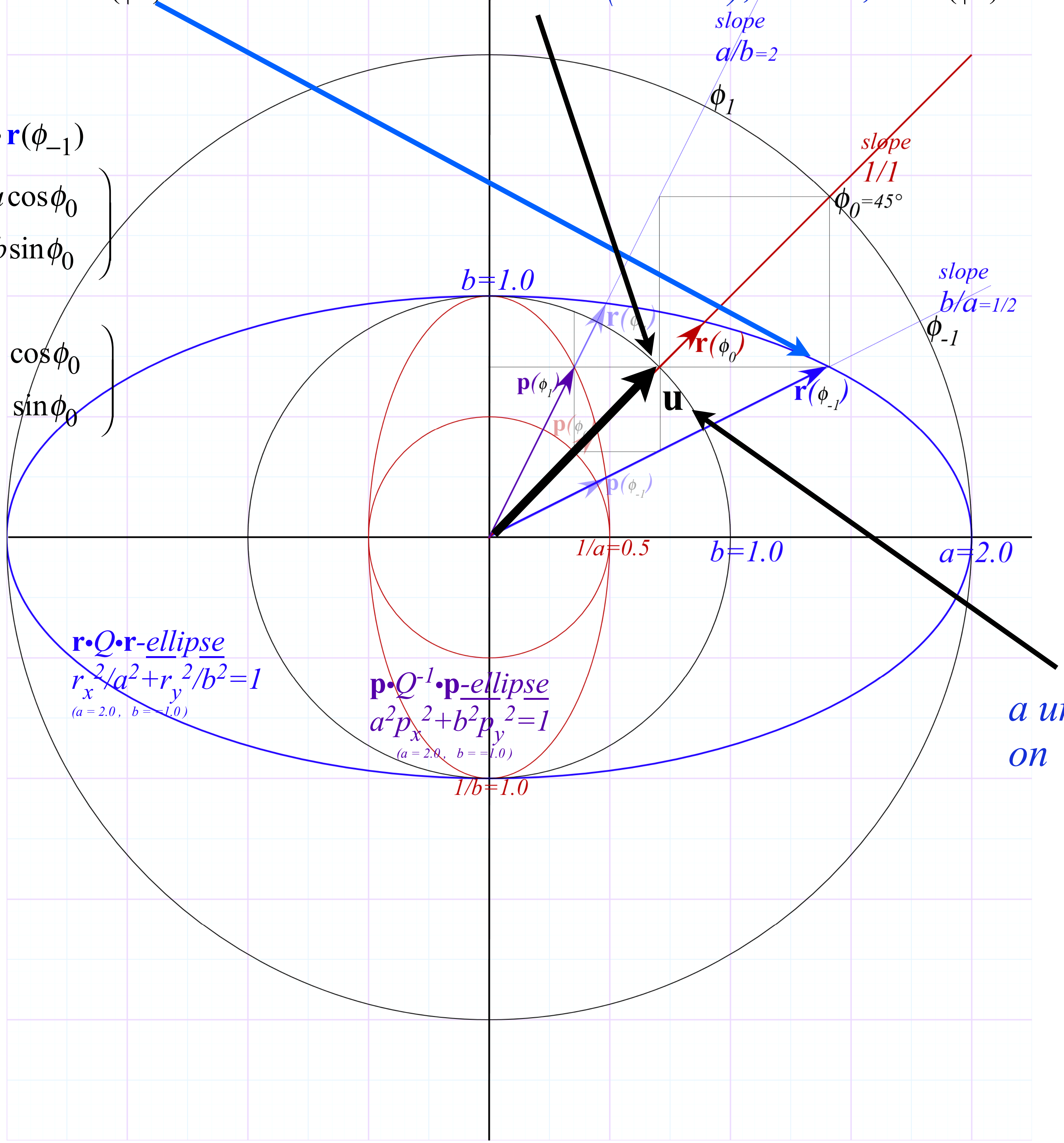
Action of "sqrt-"matrix R=√Q *(R generates another ellipse* **r**•*R*•**r** *=1 not shown)*
on a single **r**-*vector* **r**(ϕ-1)... *is to rotate it to* **u**-*circle* (**u**•**u** *=1), that is, R*•**r**(ϕ-1) = **u** =(*const.*)**r**(ϕ0)

$$\mathbf{u}=\sqrt{\mathbf{Q}}\bullet\mathbf{r}(\phi_{-1})=\mathbf{R}\bullet\mathbf{r}(\phi_{-1})$$

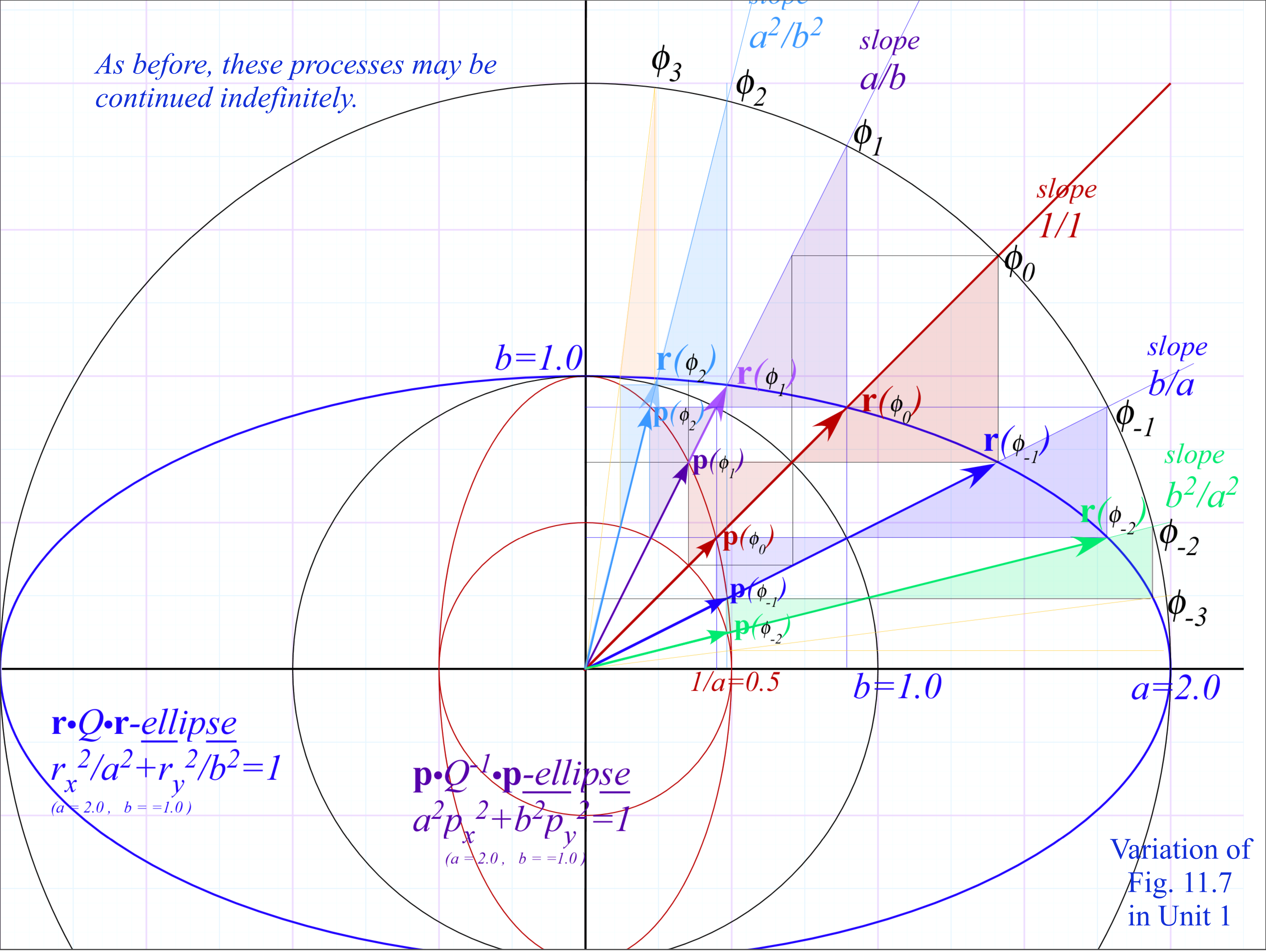
$$=\begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}\begin{pmatrix} a\cos\phi_0 \\ b\sin\phi_0 \end{pmatrix}$$

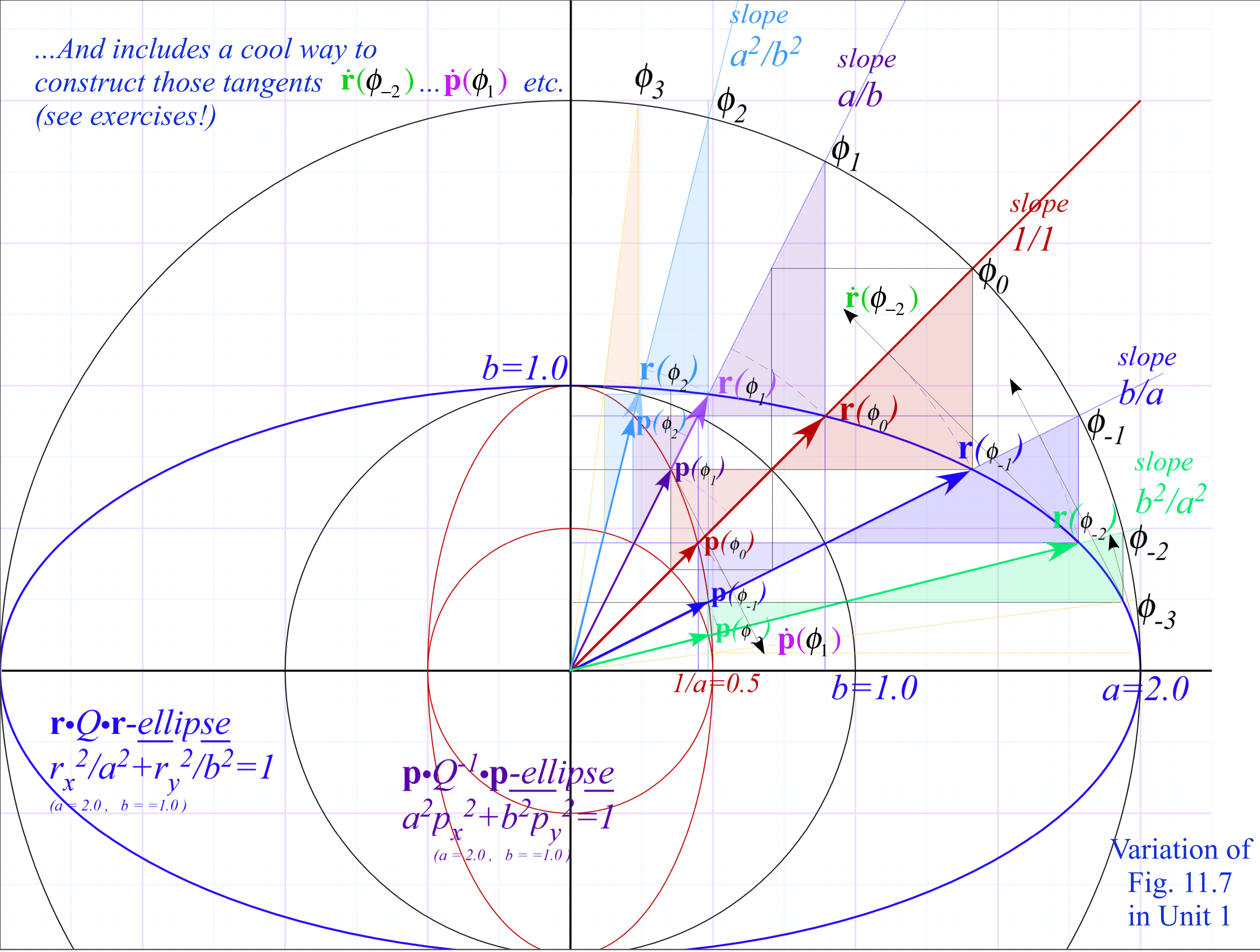
$$=\begin{pmatrix} \frac{1}{a}a\cos\phi_0 \\ \frac{1}{b}b\sin\phi_0 \end{pmatrix}=\begin{pmatrix} \cos\phi_0 \\ \sin\phi_0 \end{pmatrix}$$

$$=\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$



Variation of
Fig. 11.7
in Unit 1


As before, these processes may be continued indefinitely.
slope
a²/b²
slope
a/b
slope
1/1
slope
b/a
slope
b²/a²
φ3
φ2
φ1
φ0
φ-1
φ-2
φ-3
b=1.0
r(φ2)
r(φ1)
r(φ0)
r(φ-1)
r(φ-2)
p(φ2)
p(φ1)
p(φ0)
p(φ-1)
p(φ-2)
1/a=0.5
b=1.0
a=2.0
r•Q•r-ellipse
rx²/a²+ry²/b²=1
(a = 2.0 , b = =1.0)
p•Q⁻¹•p-ellipse
a²px²+b²py²=1
(a = 2.0 , b = =1.0)
Variation of Fig. 11.7 in Unit 1


...And includes a cool way to
construct those tangents ṙ(φ₋₂)...ṗ(φ₁) etc.
(see exercises!)
slope a²/b²
slope a/b
slope 1/1
slope b/a
slope b²/a²
φ3
φ2
φ1
φ0
φ-1
φ-2
φ-3
ṙ(φ₋₂)
b=1.0
r(φ2)
r(φ1)
r(φ0)
r(φ-1)
r(φ-2)
p(φ2)
p(φ1)
p(φ0)
p(φ-1)
p(φ-2)
ṗ(φ1)
1/a=0.5
b=1.0
a=2.0
r•Q•r-ellipse
rx²/a²+ry²/b²=1
(a = 2.0 , b = =1.0)
p•Q⁻¹•p-ellipse
a²px²+b²py²=1
(a = 2.0 , b = =1.0)
Variation of
Fig. 11.7
in Unit 1

Review of dual IHO elliptic orbits (Lecture 9)

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Introduction to dual matrix operator geometry

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Vector calculus of tensor operation

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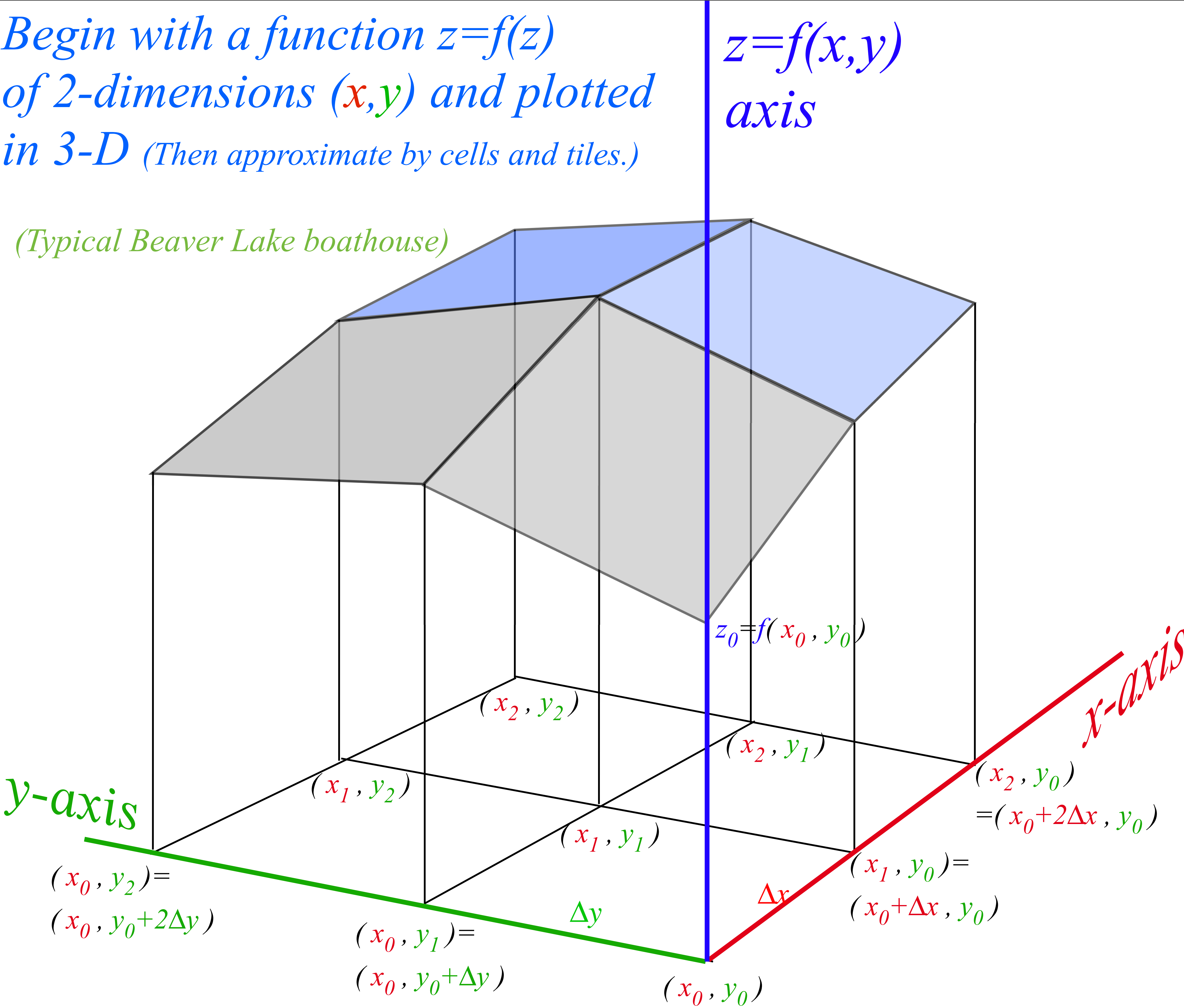
→ *Review of partial differential relations*

Chain rule and order $\partial^2\Psi/\partial x\partial y=\partial^2\Psi/\partial y\partial x$ *symmetry*

Duality relations of Lagrangian and Hamiltonian ellipse

Introducing the 1st (partial) differential equations of mechanics

Begin with a function z=f(z) of 2-dimensions (x,y) and plotted in 3-D *(Then approximate by cells and tiles.)*



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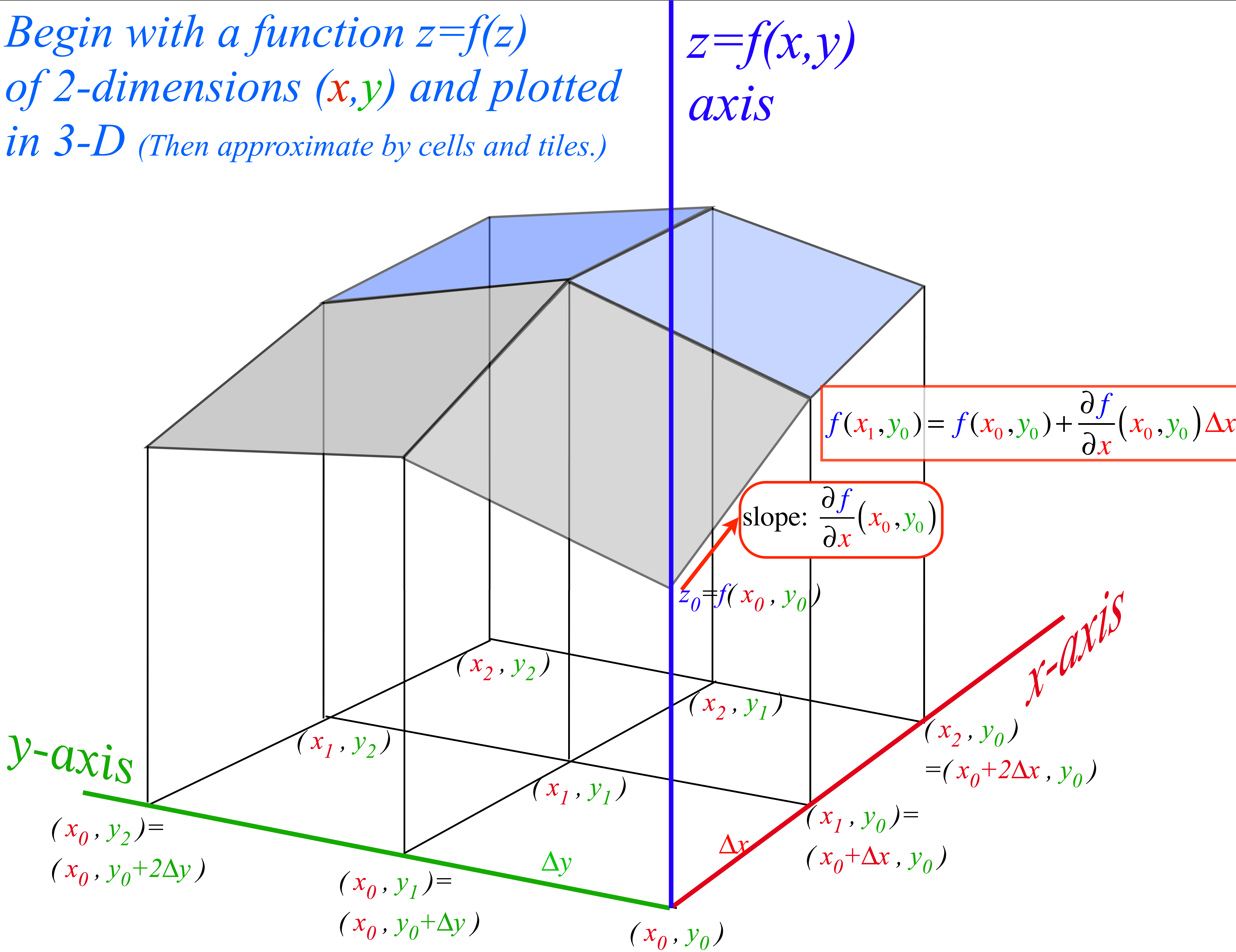

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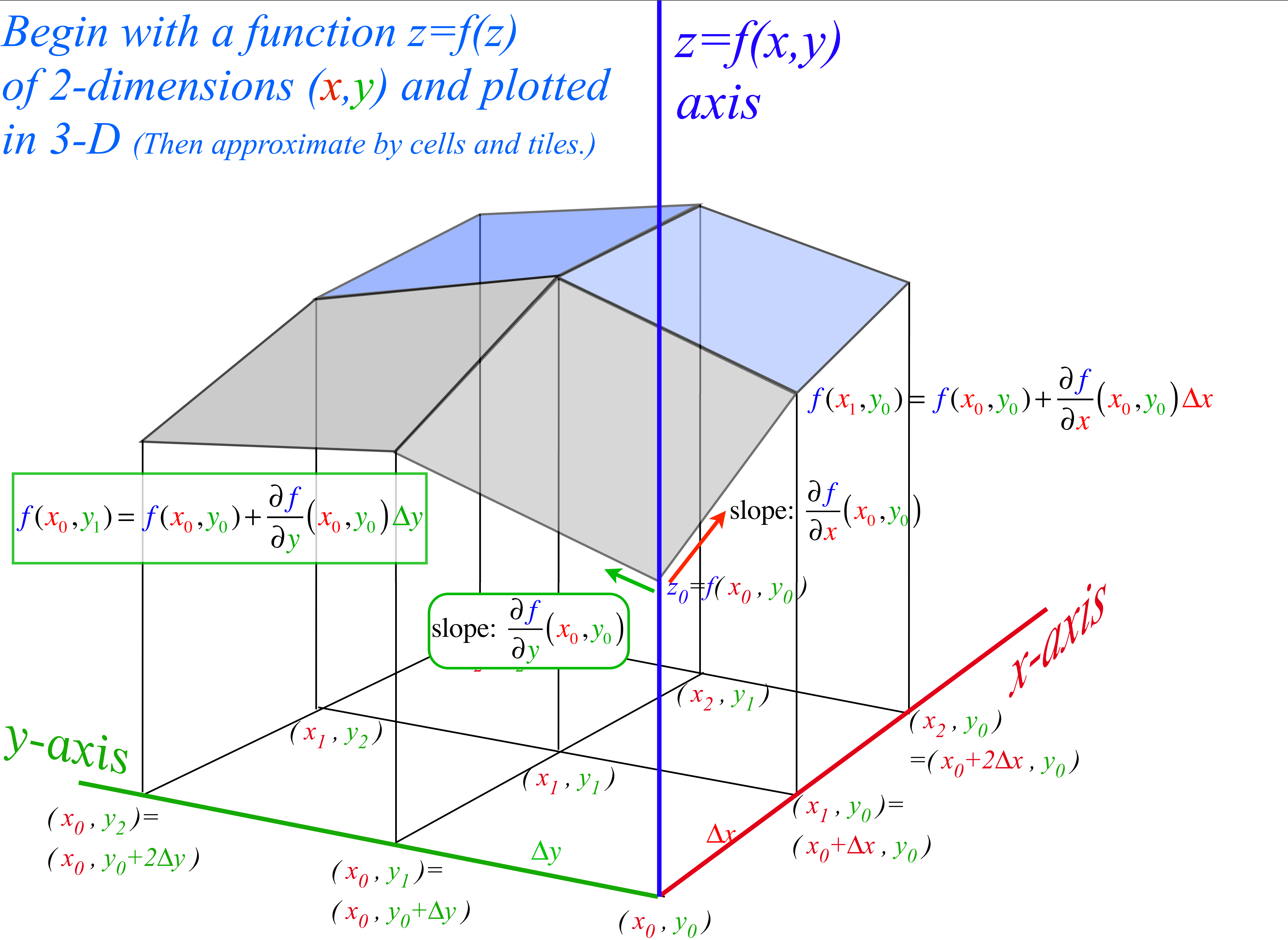

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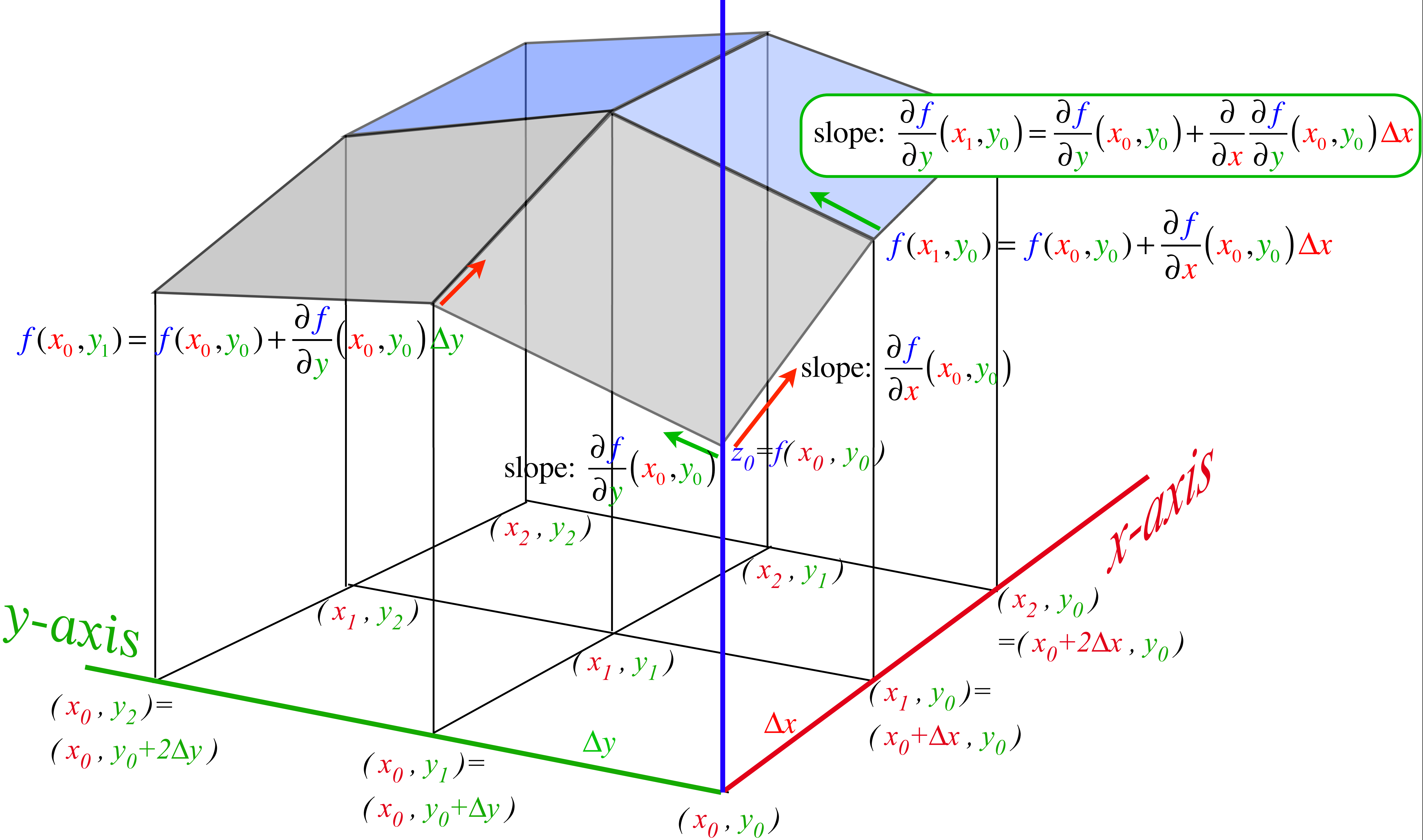

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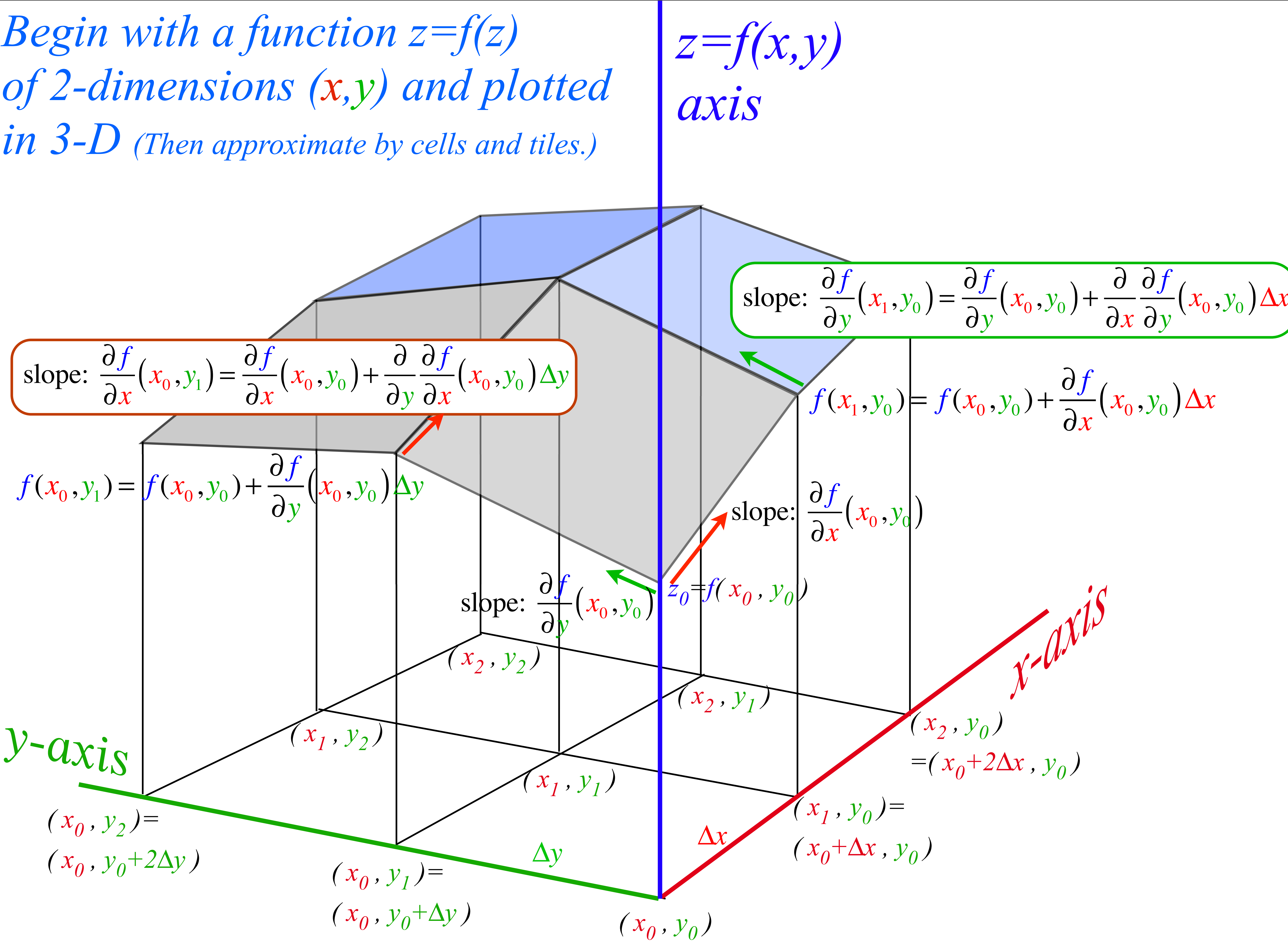
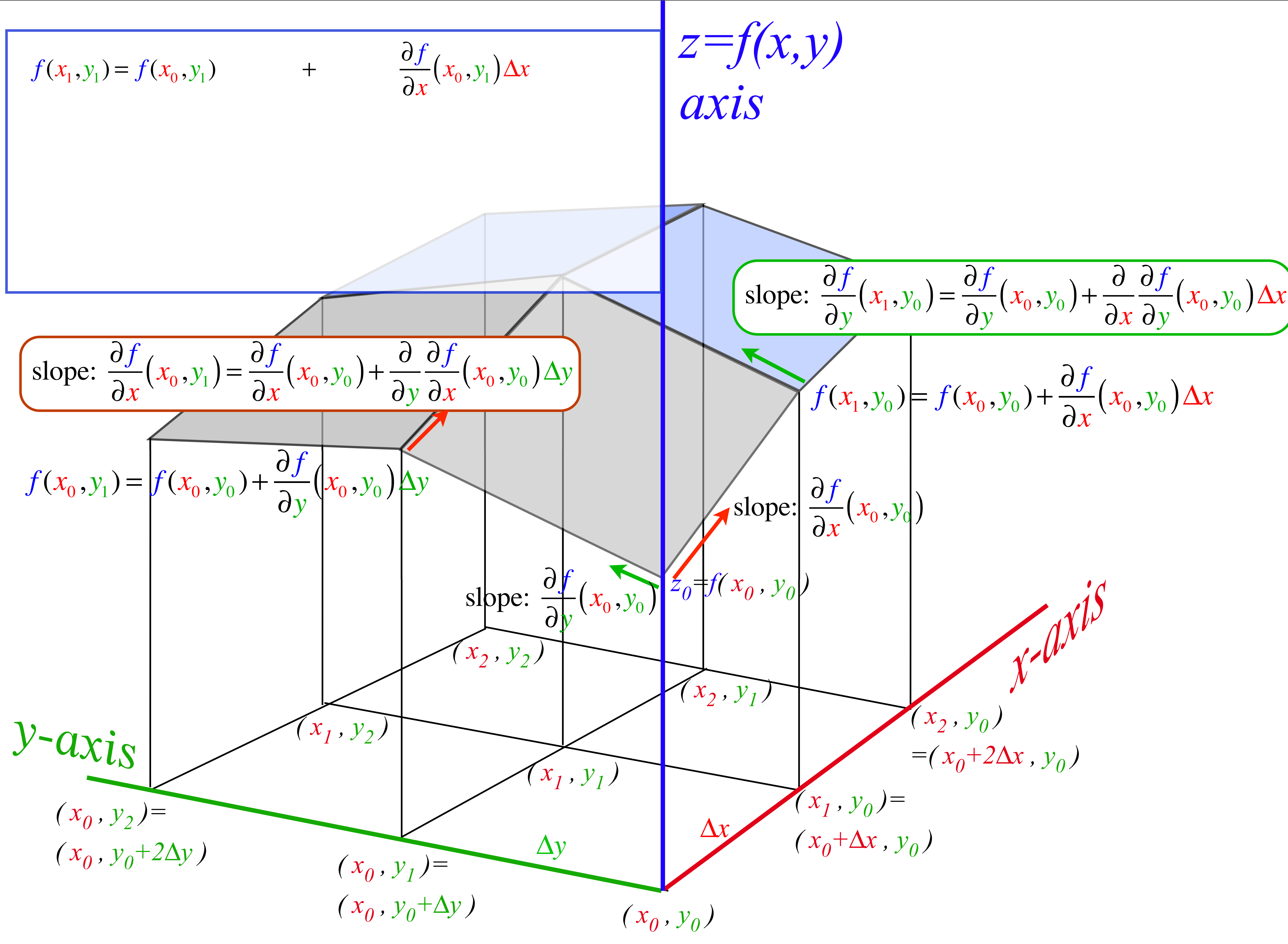
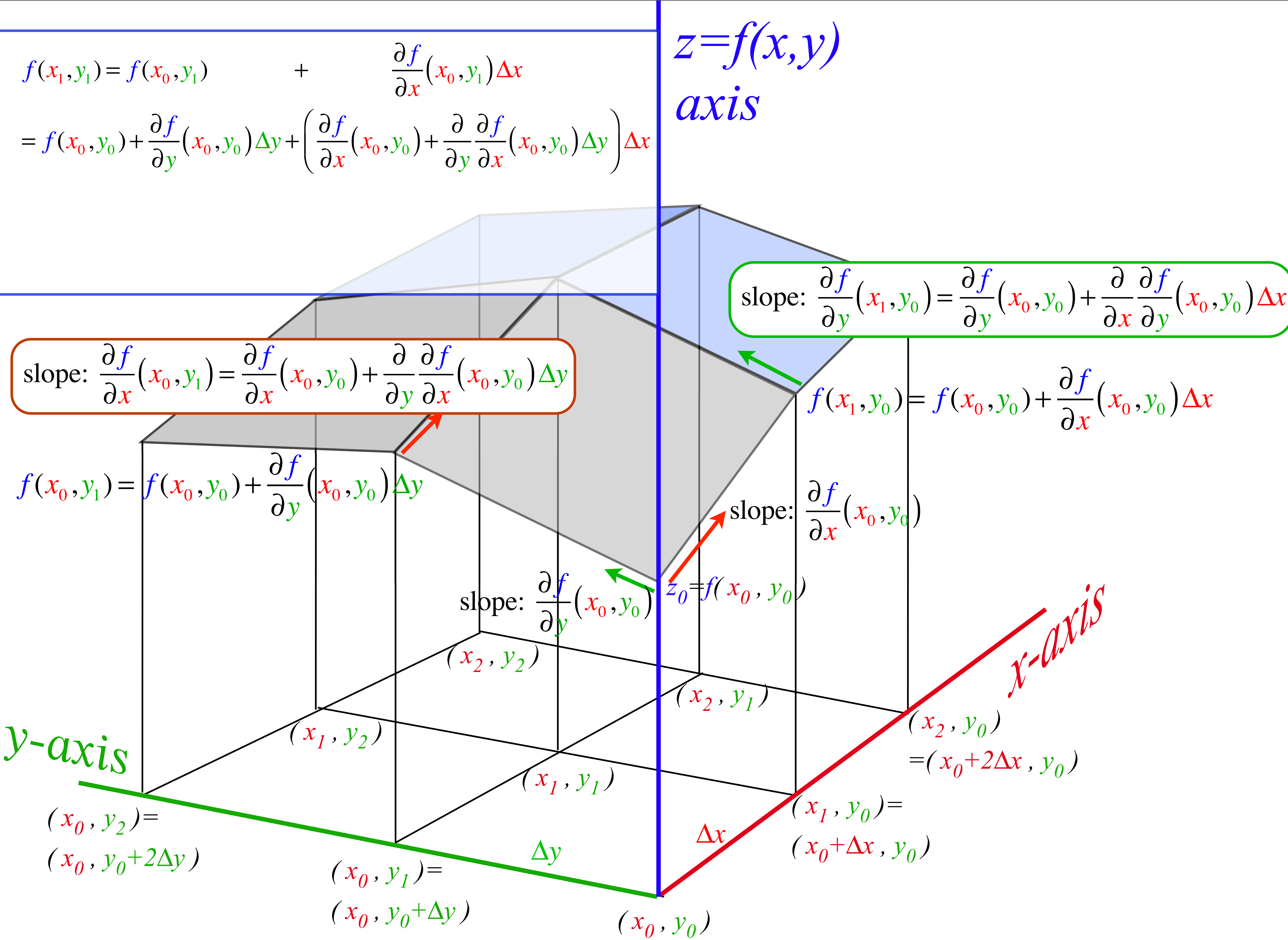




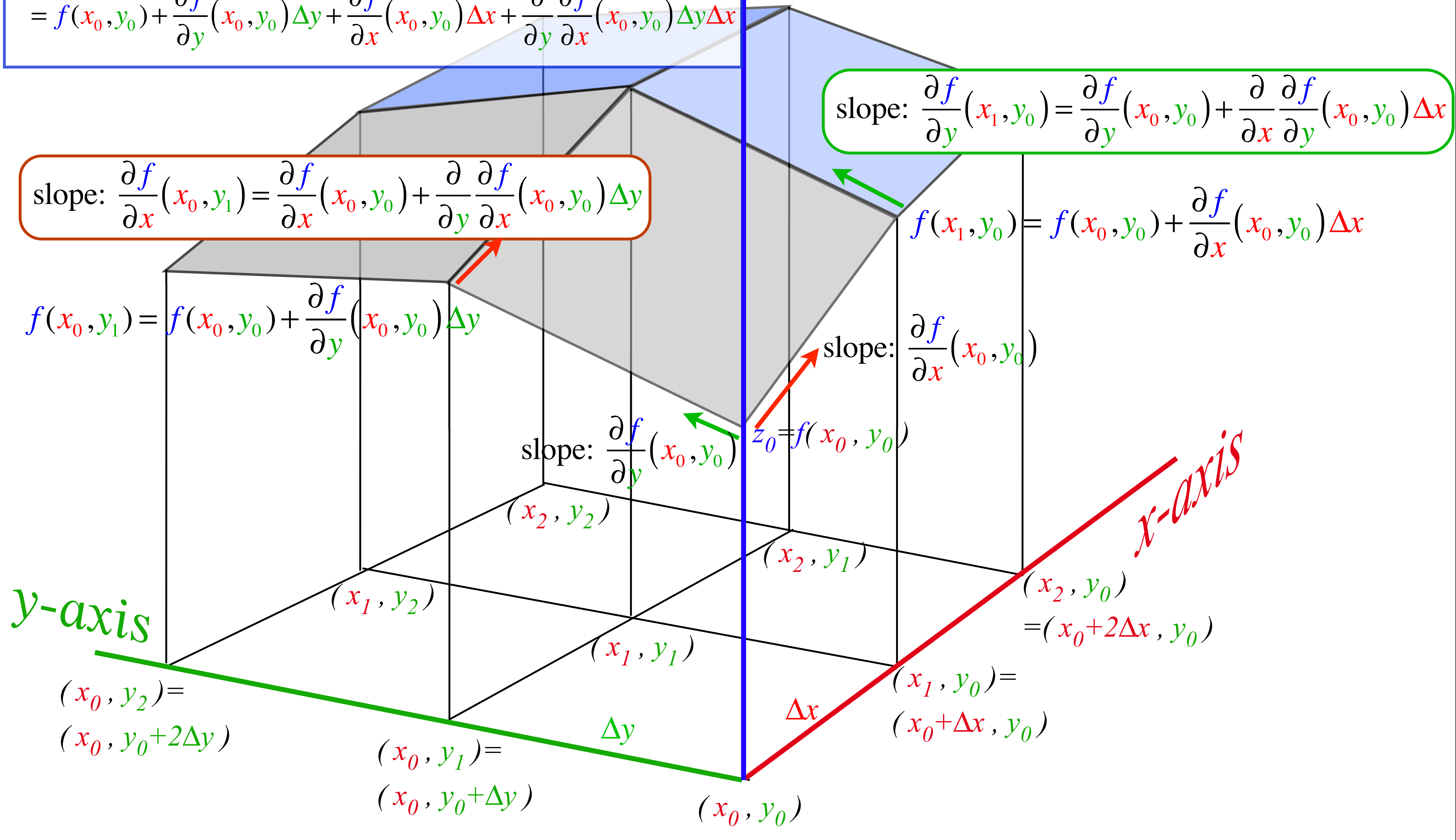
$f(x_1,y_1)=f(x_0,y_1) \quad + \quad \frac{\partial f}{\partial x}(x_0,y_1)\Delta x$
z=f(x,y) axis
slope: $\frac{\partial f}{\partial y}(x_1,y_0)=\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x$
slope: $\frac{\partial f}{\partial x}(x_0,y_1)=\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y$
$f(x_1,y_0)=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x$
$f(x_0,y_1)=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y$
slope: $\frac{\partial f}{\partial x}(x_0,y_0)$
slope: $\frac{\partial f}{\partial y}(x_0,y_0)$
$z_0=f(x_0,y_0)$
x-axis
y-axis
(x_2,y_2)
(x_2,y_1)
(x_1,y_2)
(x_1,y_1)
(x_2,y_0)
$=(x_0+2\Delta x,y_0)$
$(x_1,y_0)=$
$(x_0+\Delta x,y_0)$
$(x_0,y_2)=$
$(x_0,y_0+2\Delta y)$
$(x_0,y_1)=$
$(x_0,y_0+\Delta y)$
(x_0,y_0)
Δx
Δy

$$f(x_1,y_1) = f(x_0,y_1) + \frac{\partial f}{\partial x}(x_0,y_1)\Delta x$$

$$= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \left(\frac{\partial f}{\partial x}(x_0,y_0) + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x$$



$$
\begin{aligned}
f(x_1,y_1) &= f(x_0,y_1) \qquad + \qquad \frac{\partial f}{\partial x}(x_0,y_1)\Delta x \\
&= f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\left(\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x \\
&= f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\Delta x
\end{aligned}
$$

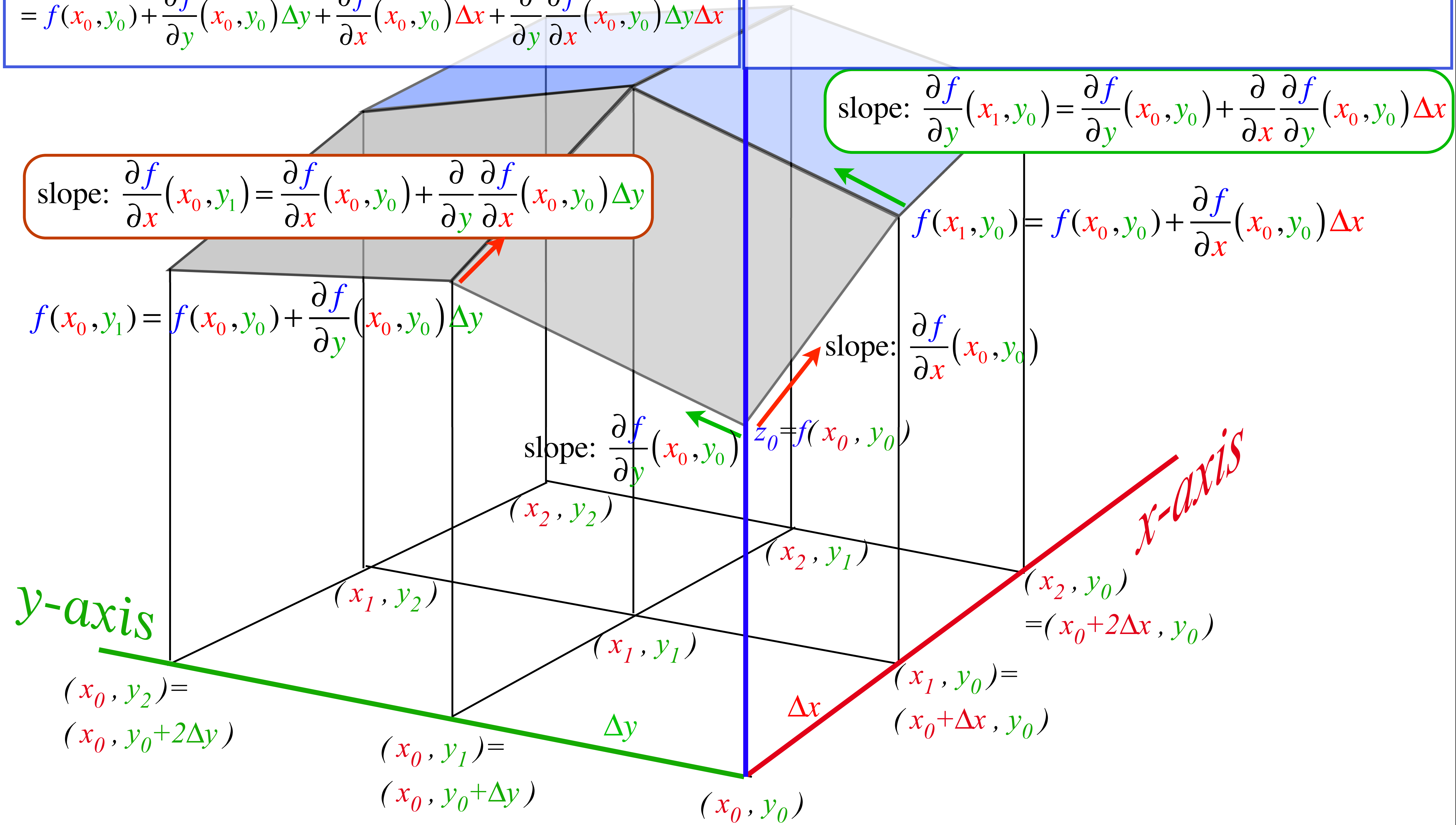


$$f(x_1,y_1) = f(x_0,y_1) + \frac{\partial f}{\partial x}(x_0,y_1)\Delta x$$

$$= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \left(\frac{\partial f}{\partial x}(x_0,y_0) + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x$$

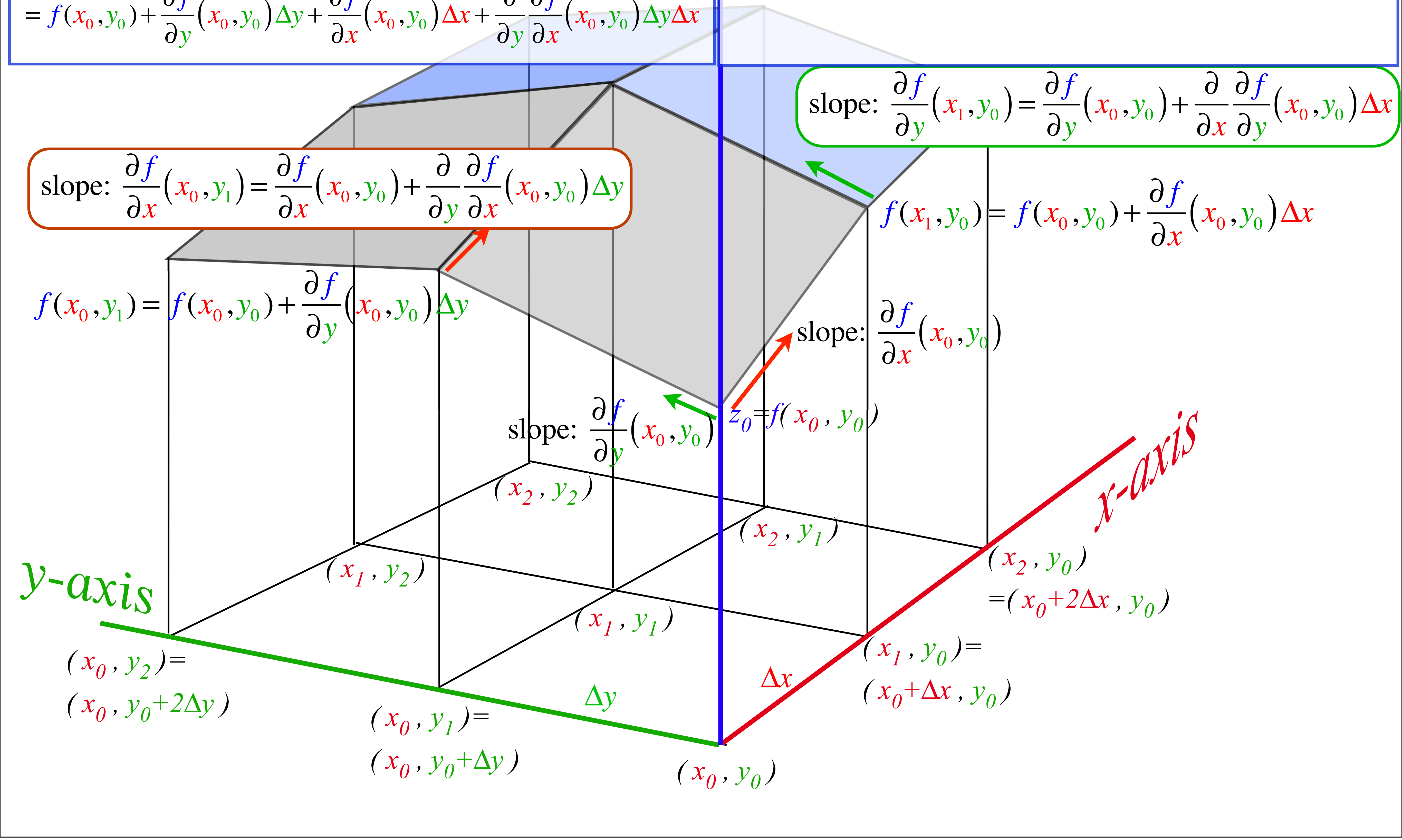
$$= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\Delta x$$

$$f(x_1,y_1) = f(x_1,y_0) + \frac{\partial f}{\partial y}(x_1,y_0)\Delta y$$



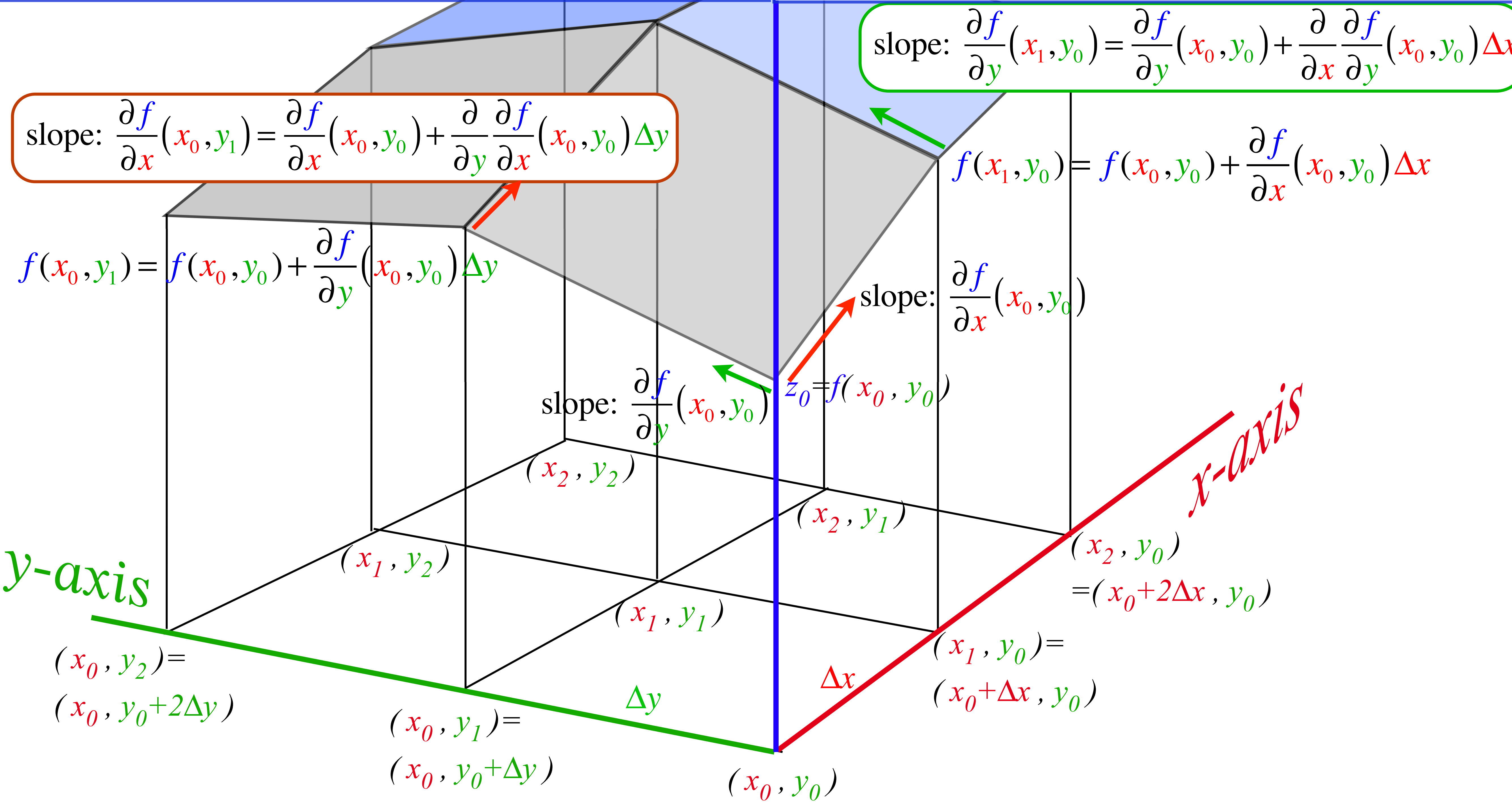
$$f(x_1,y_1)=f(x_0,y_1)+\frac{\partial f}{\partial x}(x_0,y_1)\Delta x$$
$$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\left(\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x$$
$$=f(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\Delta y+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\Delta x$$

$$f(x_1,y_1)=f(x_1,y_0)+\frac{\partial f}{\partial y}(x_1,y_0)\Delta y$$
$$=f(x_0,y_0)+\frac{\partial f}{\partial x}(x_0,y_0)\Delta x+\left(\frac{\partial f}{\partial y}(x_0,y_0)+\frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x\right)\Delta y$$



$$
\begin{aligned}
f(x_1,y_1) &= f(x_0,y_1) + \frac{\partial f}{\partial x}(x_0,y_1)\Delta x \\
&= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \left(\frac{\partial f}{\partial x}(x_0,y_0) + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\right)\Delta x \\
&= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta y\Delta x
\end{aligned}
$$

$$
\begin{aligned}
f(x_1,y_1) &= f(x_1,y_0) + \frac{\partial f}{\partial y}(x_1,y_0)\Delta y \\
&= f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \left(\frac{\partial f}{\partial y}(x_0,y_0) + \frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x\right)\Delta y \\
&= f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta x\Delta y
\end{aligned}
$$



Review of dual IHO elliptic orbits (Lecture 9)

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Introduction to dual matrix operator geometry

Quadratic form ellipse $\mathbf{r} \bullet Q \bullet \mathbf{r} = 1$ *vs.inverse form ellipse* $\mathbf{p} \bullet Q^{-1} \bullet \mathbf{p} = 1$

Duality norm relations ($\mathbf{r} \bullet \mathbf{p} = 1$)

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→ *Review of partial differential relations*

Chain rule and order $\partial^2\Psi/\partial x\partial y = \partial^2\Psi/\partial y\partial x$ *symmetry*

Duality relations of Lagrangian and Hamiltonian ellipse

Introducing the 1^{st} *(partial) differential equations of mechanics*

What the geometry indicates....(Two important results)

$$f(x_1,y_1) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta x \Delta y$$

$$= f(x_0,y_0) + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial}{\partial x}\frac{\partial f}{\partial y}(x_0,y_0)\Delta y \Delta x$$

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If $f(x,y)$ is contiuous around (x_0,y_0) and (x_1,y_1) then $\frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ equals $\frac{\partial}{\partial x}\frac{\partial f}{\partial y}$

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1. Chain rules

$$\left[f(x_1,y_1)-f(x_0,y_0)\right]=df=\frac{\partial f}{\partial x}(x_0,y_0)dx+\frac{\partial f}{\partial y}(x_0,y_0)dy\ldots_{(keep\ 1^{st}-order\ terms\ only!)}$$

$$\frac{df}{dt}=\frac{\partial f}{\partial x}(x_0,y_0)\frac{dx}{dt}+\frac{\partial f}{\partial y}(x_0,y_0)\frac{dy}{dt}$$

$$\dot{f}=\frac{\partial f}{\partial x}\dot{x}+\frac{\partial f}{\partial y}\dot{y}$$

(shorthand notation)

What the geometry indicates....(Two important results)

$$f(x_1,y_1) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta x \Delta y$$

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1. Chain rules

$$[f(x_1,y_1) - f(x_0,y_0)] = df = \frac{\partial f}{\partial x}(x_0,y_0)dx + \frac{\partial f}{\partial y}(x_0,y_0)dy \ldots_{(\text{keep } 1^{st}-\text{order terms only!})}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}(x_0,y_0)\frac{dx}{dt} + \frac{\partial f}{\partial y}(x_0,y_0)\frac{dy}{dt}$$

$$\dot{f} = \frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial y}\dot{y} \quad (\text{shorthand notation}) \quad = \partial_x f\,\dot{x} + \partial_y f\,\dot{y}$$

2. Symmetry of partial deriv. ordering

(pay attention to the 2^{nd} −order terms, too!)

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} \quad \text{or:} \quad \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} \quad \text{or:} \quad \partial_y \partial_x f = \partial_x \partial_y f$$

(shorthand notation)

What the geometry indicates....(Two important results)

$$f(x_1,y_1) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)\Delta x + \frac{\partial f}{\partial y}(x_0,y_0)\Delta y + \frac{\partial}{\partial y}\frac{\partial f}{\partial x}(x_0,y_0)\Delta x\Delta y$$

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If $f(x,y)$ is contiuous around (x_0,y_0) and (x_1,y_1) then $\frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ equals $\frac{\partial}{\partial x}\frac{\partial f}{\partial y}$

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$$[f(x_1,y_1) - f(x_0,y_0)] = df = \frac{\partial f}{\partial x}(x_0,y_0)dx + \frac{\partial f}{\partial y}(x_0,y_0)dy \ldots_{(\text{keep } 1^{st}-\text{order terms only!})}$$

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(shorthand notation)

$$Let: \vec{\nabla} = \begin{pmatrix} \partial_x & \partial_y \end{pmatrix} \quad so: \vec{\nabla} f \bullet \mathbf{dr} = \begin{pmatrix} \partial_x f & \partial_y f \end{pmatrix} \bullet \begin{pmatrix} dx \\ dy \end{pmatrix} = \partial_x f\, dx + \partial_y f\, dy = df$$

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→ *Duality relations of Lagrangian and Hamiltonian ellipse*

Introducing the 1^{st} *(partial) differential equations of mechanics*

Three ways to express energy: Consider kinetic energy (KE) first

*1. **Lagrangian** is explicit function of **velocity**:* $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$L(v_k \ldots) = \tfrac{1}{2}(m_1 v_1^2 + m_2 v_2^2 + \ldots) = L(\mathbf{v}\ldots) = \tfrac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v} + \ldots = \tfrac{1}{2}\begin{pmatrix} v_1 & v_2 \end{pmatrix}\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \ldots$$

*2. **"Estrangian"** is explicit function of **R**-rescaled velocity:*

(or l'Estrangian

*or: **"speedinum"** V* $\mathbf{V} = \mathbf{R}\bullet\mathbf{v}$ or: $\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} \sqrt{m_1} & 0 \\ 0 & \sqrt{m_2} \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$E(V_k \ldots) = \tfrac{1}{2}(V_1^2 + V_2^2 + \ldots) = E(\mathbf{V}\ldots) = \tfrac{1}{2}\mathbf{V}\bullet\mathbf{1}\bullet\mathbf{V} + \ldots = \tfrac{1}{2}\begin{pmatrix} V_1 & V_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + \ldots$$

*3. **Hamiltonian** is explicit function of **M=R**^{2}-rescaled velocity:*

*or: **momentum p*** $\mathbf{p} = \mathbf{M}\bullet\mathbf{v}$ or: $\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} m_1 v_1 \\ m_2 v_2 \end{pmatrix}$

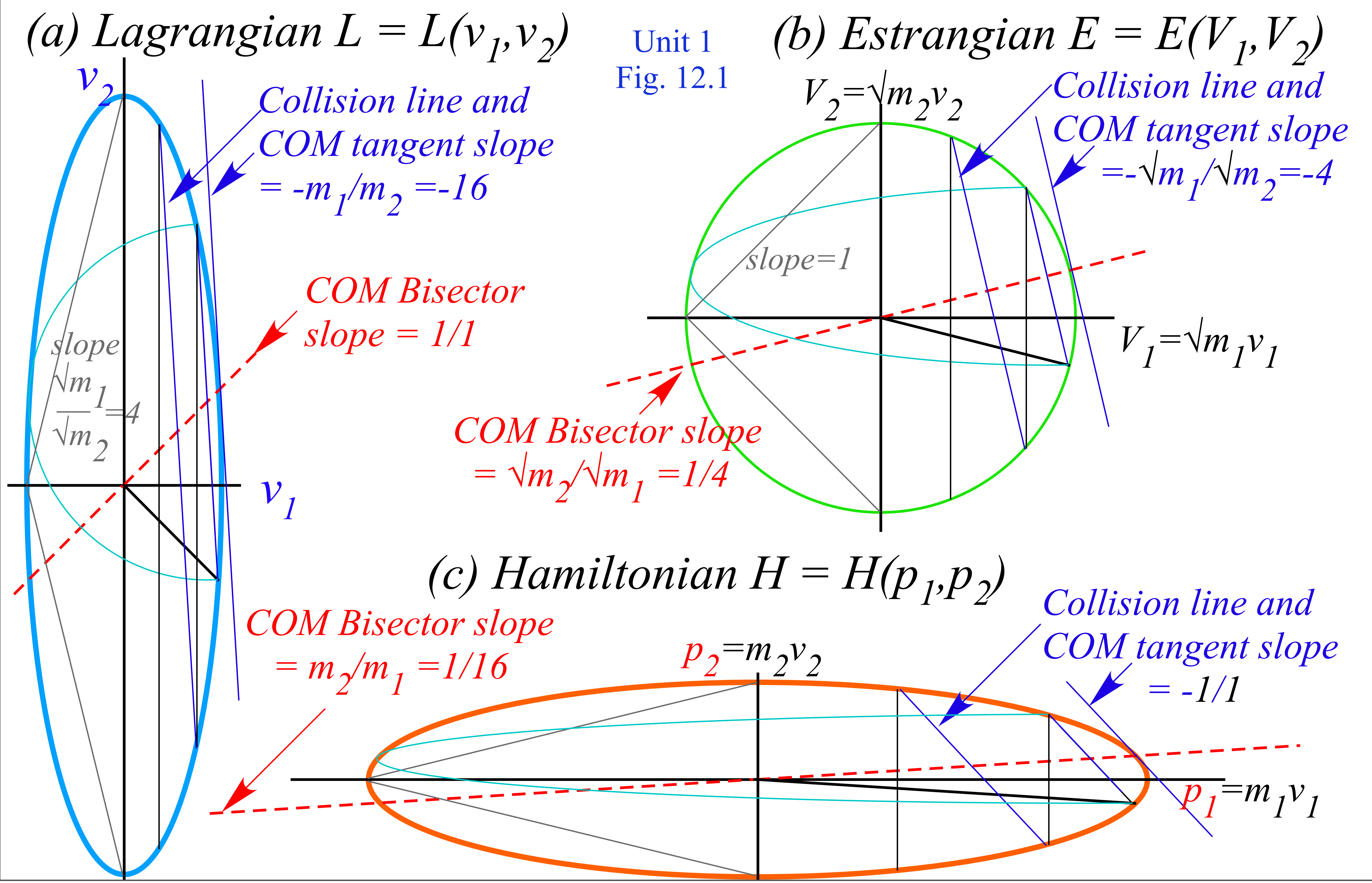
$$H(p_k \ldots) = \tfrac{1}{2}\left(\frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} + \ldots\right) = H(\mathbf{p}\ldots) = \tfrac{1}{2}\mathbf{p}\bullet\mathbf{M}^{-1}\bullet\mathbf{p} + \ldots = \tfrac{1}{2}\begin{pmatrix} p_1 & p_2 \end{pmatrix}\begin{pmatrix} 1/m_1 & 0 \\ 0 & 1/m_2 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} + \ldots$$

The R and Q matrix transformations are like the mechanics rescaling matrices $\sqrt{\mathbf{M}}$ *and* $\mathbf{M}$*:*

Like $Q=R^2$*:* $$\mathbf{M}=\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}=\mathbf{R}^2$$

Like $\sqrt{Q}=R$*:* $$\sqrt{\mathbf{M}}=\begin{pmatrix} \sqrt{m_1} & 0 \\ 0 & \sqrt{m_2} \end{pmatrix}=\mathbf{R}$$

Like $Q^{-1}=R^{-2}$*:* $$\mathbf{M}^{-1}=\begin{pmatrix} 1/m_1 & 0 \\ 0 & 1/m_2 \end{pmatrix}=\mathbf{R}^{-2}$$



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Duality relations of Lagrangian and Hamiltonian ellipse

→ *Introducing the 1st (partial) differential equations of mechanics*

Introducing the (partial$\frac{\partial ?}{\partial ?}$) differential equations of mechanics

Starts out with simple demands for explicit-dependence, "loyalty" or "fealty to the colors"

Lagrangian *and* ***Estrangian***
have no explicit dependence
on ***momentum*** **p**

$$\frac{\partial L}{\partial p_k} \equiv 0 \equiv \frac{\partial E}{\partial p_k}$$

Hamiltonian *and* ***Estrangian***
have no explicit dependence
on ***velocity*** **v**

$$\frac{\partial H}{\partial v_k} \equiv 0 \equiv \frac{\partial E}{\partial v_k}$$

Lagrangian *and* ***Hamiltonian***
have no explicit dependence
on ***speedinum*** **V**

$$\frac{\partial L}{\partial V_k} \equiv 0 \equiv \frac{\partial H}{\partial V_k}$$

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$$\frac{\partial L}{\partial V_k} \equiv 0 \equiv \frac{\partial H}{\partial V_k}$$

Such non-dependencies hold in spite of "under-the-table" matrix and partial-differential connections

$$\nabla_v L = \frac{\partial L}{\partial \mathbf{v}} = \frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{v} \bullet \mathbf{M} \bullet \mathbf{v}}{2} = \mathbf{M} \bullet \mathbf{v} = \mathbf{p}$$

$$\begin{pmatrix} \frac{\partial L}{\partial v_1} \\ \frac{\partial L}{\partial v_2} \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

$$\nabla_p H = \mathbf{v} = \frac{\partial H}{\partial \mathbf{p}} = \frac{\partial}{\partial \mathbf{p}} \frac{\mathbf{p} \bullet \mathbf{M}^{-1} \bullet \mathbf{p}}{2} = \mathbf{M}^{-1} \bullet \mathbf{p} = \mathbf{v}$$

$$\begin{pmatrix} \frac{\partial H}{\partial p_1} \\ \frac{\partial H}{\partial p_2} \end{pmatrix} = \begin{pmatrix} m_1^{-1} & 0 \\ 0 & m_2^{-1} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

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$$= \mathbf{M} \bullet \mathbf{v} = \mathbf{p}$$

$$\begin{pmatrix} \frac{\partial L}{\partial v_1} \\ \frac{\partial L}{\partial v_2} \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

Lagrange's 1st equation(s)

$$\frac{\partial L}{\partial v_k} = p_k \quad \text{or:} \quad \frac{\partial L}{\partial \mathbf{v}} = \mathbf{p}$$

$$\nabla_p H = \mathbf{v} = \frac{\partial H}{\partial \mathbf{p}} = \frac{\partial}{\partial \mathbf{p}} \frac{\mathbf{p} \bullet \mathbf{M}^{-1} \bullet \mathbf{p}}{2}$$
$$= \mathbf{M}^{-1} \bullet \mathbf{p} = \mathbf{v}$$

$$\begin{pmatrix} \frac{\partial H}{\partial p_1} \\ \frac{\partial H}{\partial p_2} \end{pmatrix} = \begin{pmatrix} m_1^{-1} & 0 \\ 0 & m_2^{-1} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Hamilton's 1st equation(s)

$$\frac{\partial H}{\partial p_k} = v_k \quad \text{or:} \quad \frac{\partial H}{\partial \mathbf{p}} = \mathbf{v}$$

(Forget Estrangian for now)

Unit 1
Fig. 12.2