Lecture 1
Tue. 8.26.2014

Axiomatic development of classical mechanics

(Ch. 1 and Ch. 2 of Unit 1)

Geometry of momentum conservation axiom
- *Totally Inelastic "ka-runch" collisions**
- *Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**
- *Comments on idealization in classical models*

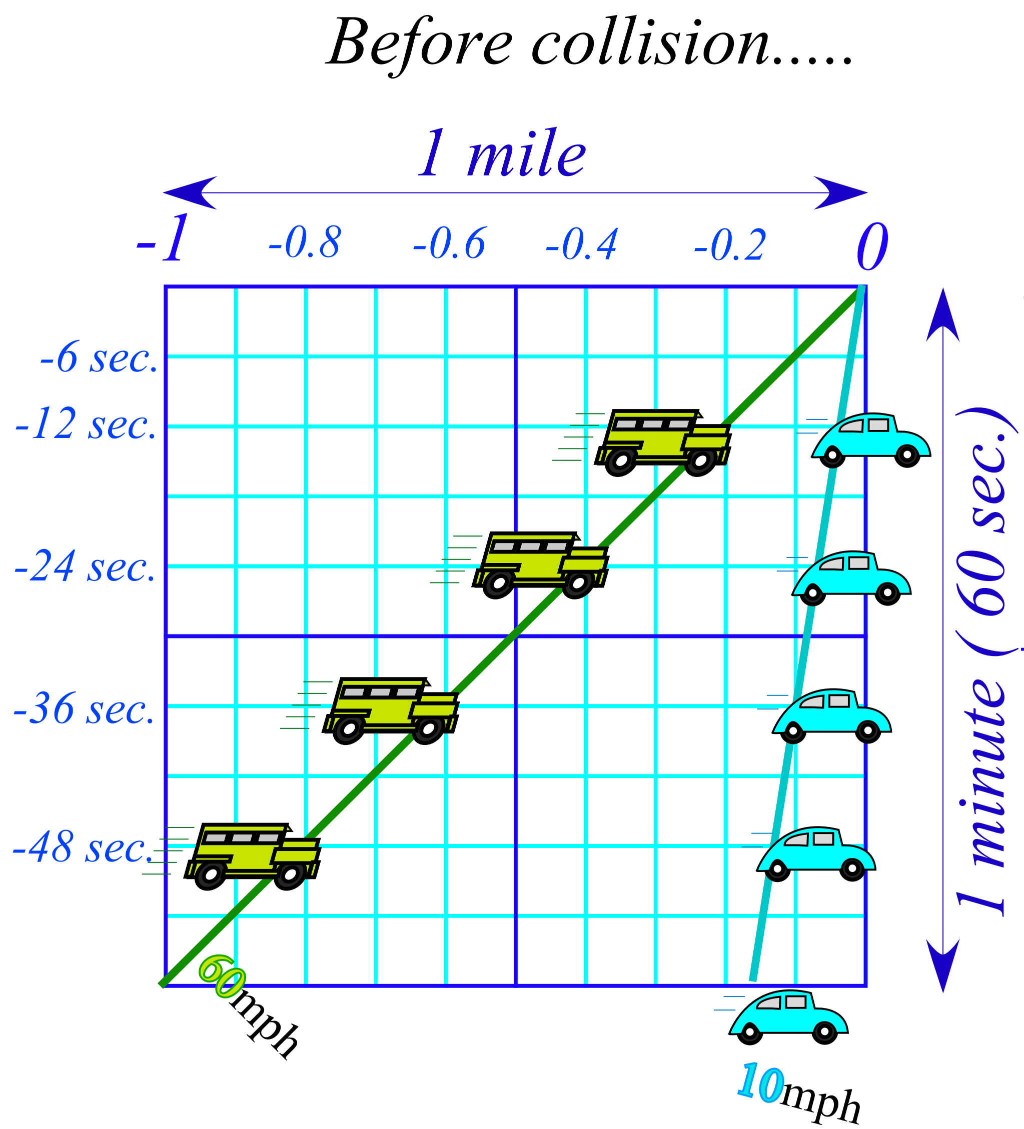
Geometry of Galilean translation symmetry
- *45° shift in (V_1,V_2)-space*
- *Time reversal symmetry*
- *...of COM collisions*

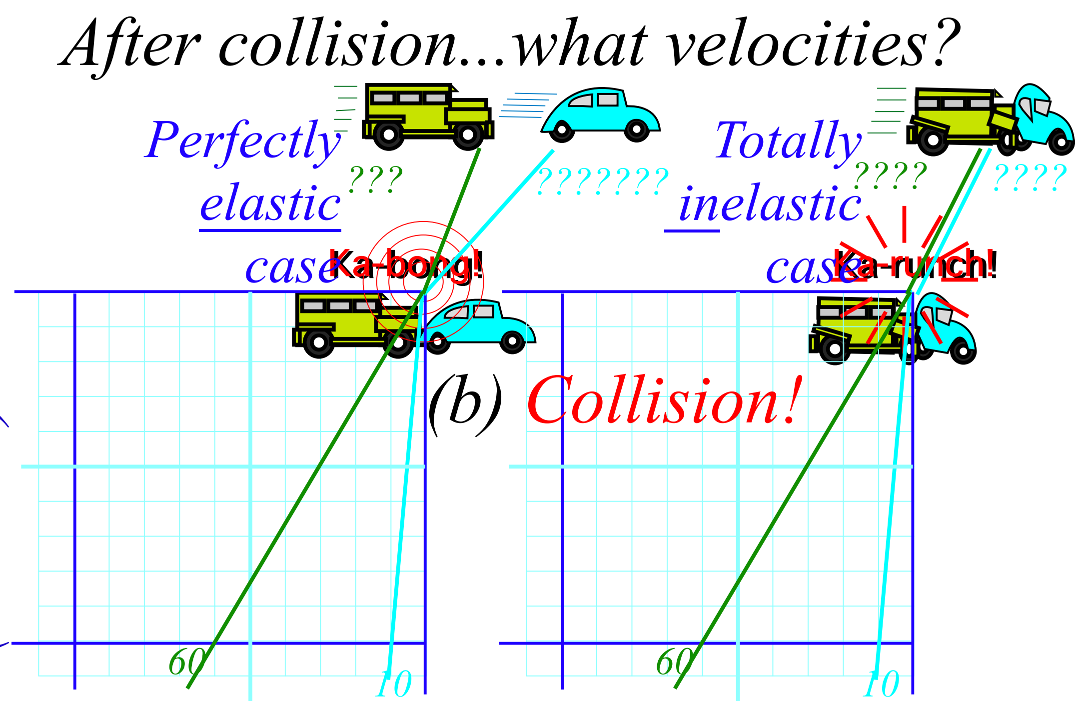
Algebra,Geometry, and Physics of momentum conservation axiom
- *Vector algebra of collisions*
- *Matrix or tensor algebra of collisions*
- *Deriving Energy Conservation Theorem*

Numerical details of collision tensor algebra

*Download Superball Collision Simulator http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)





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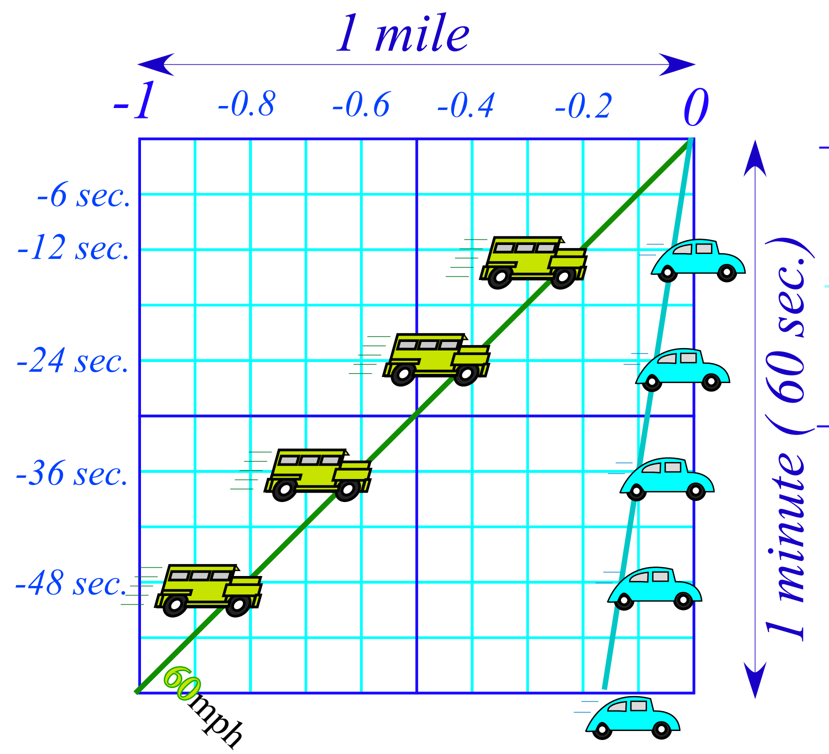
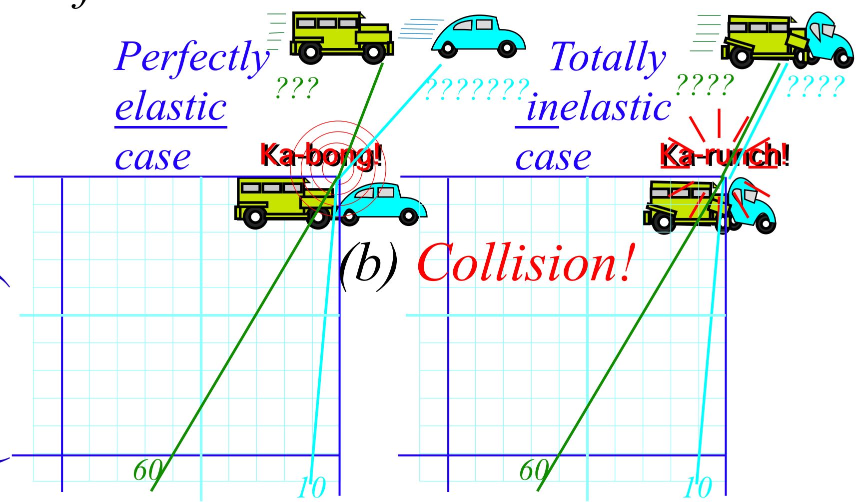
Before collision.....



After collision...what velocities?



Conventional solution:
Get out formulas:

$\Sigma mV(before) = \Sigma mV(before)$ *[momentum conservation]*

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etc.

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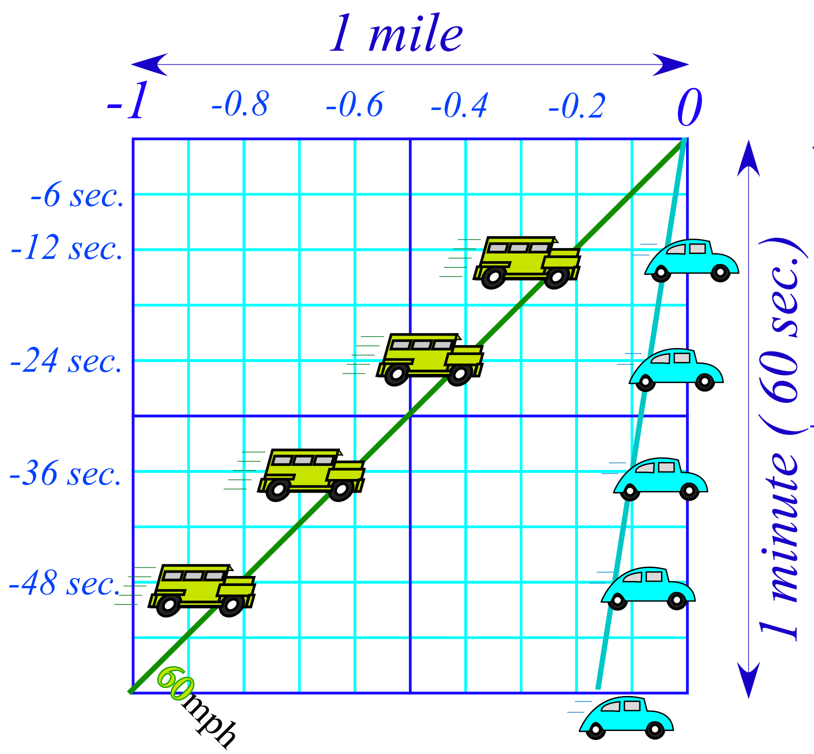

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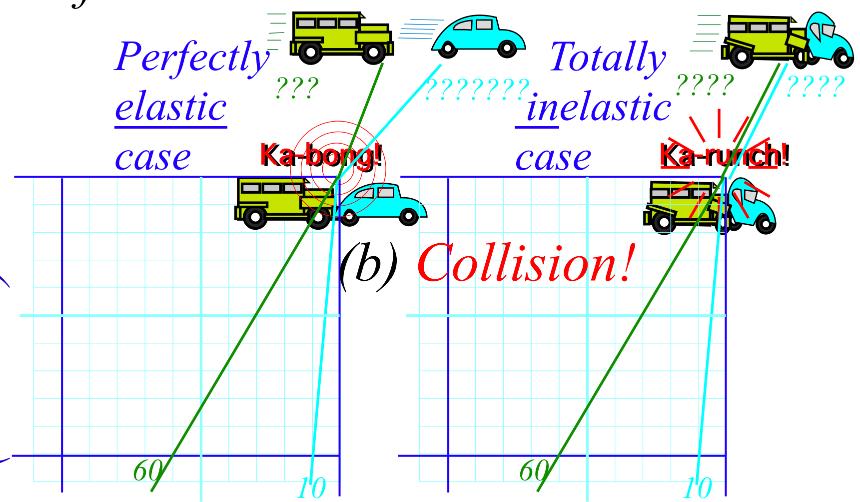

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etc.

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..... (Just have to draw 2 lines!)

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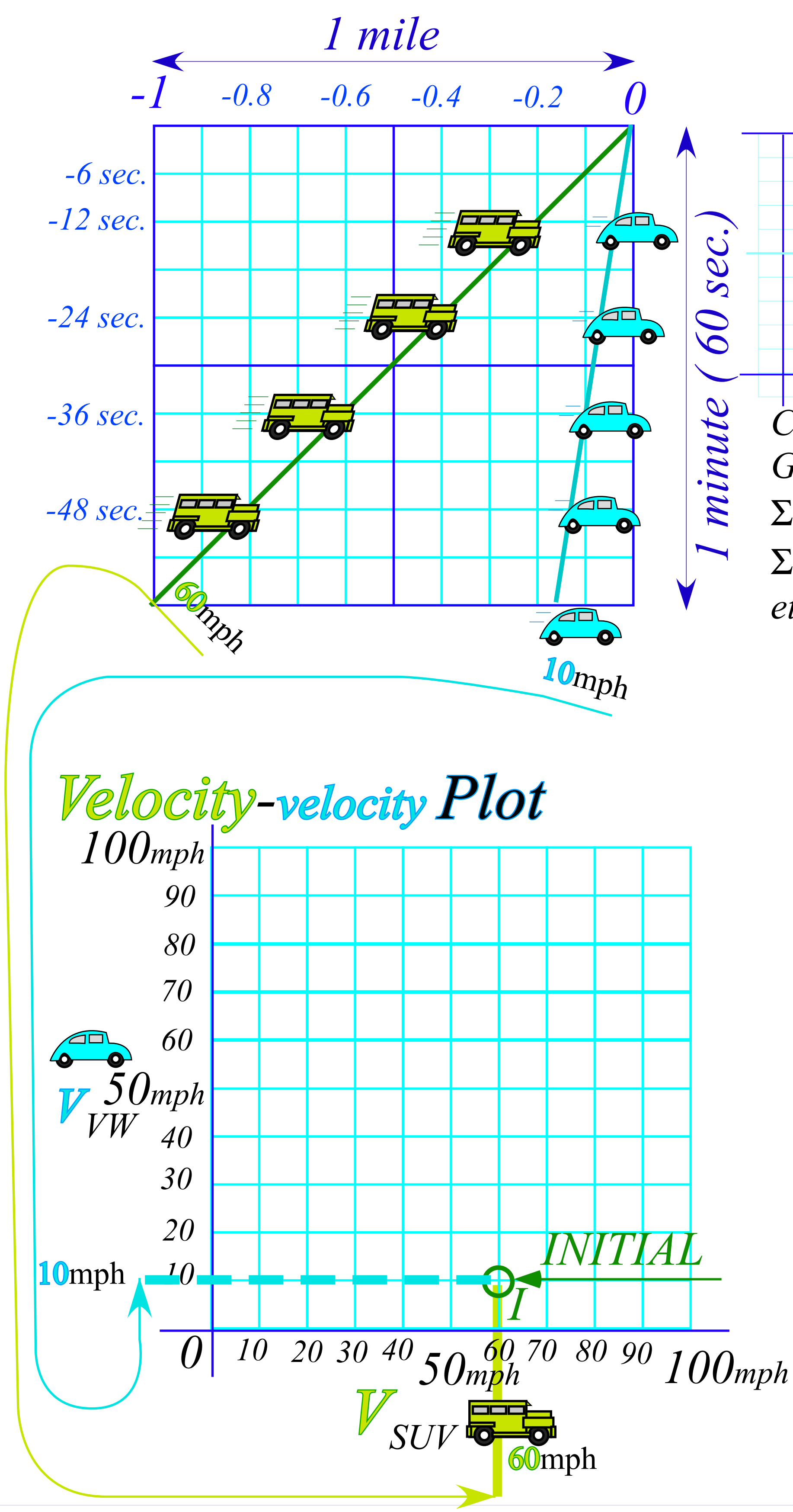

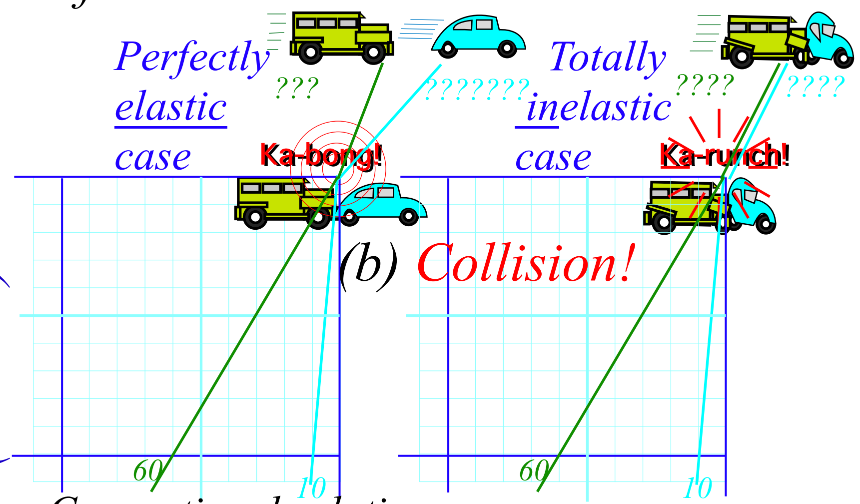

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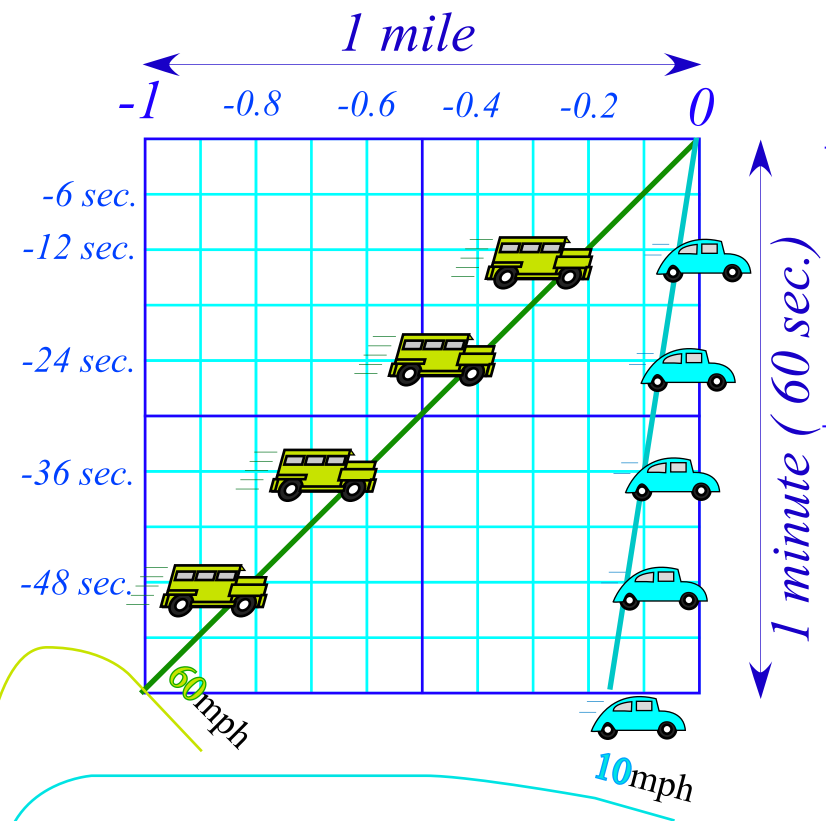

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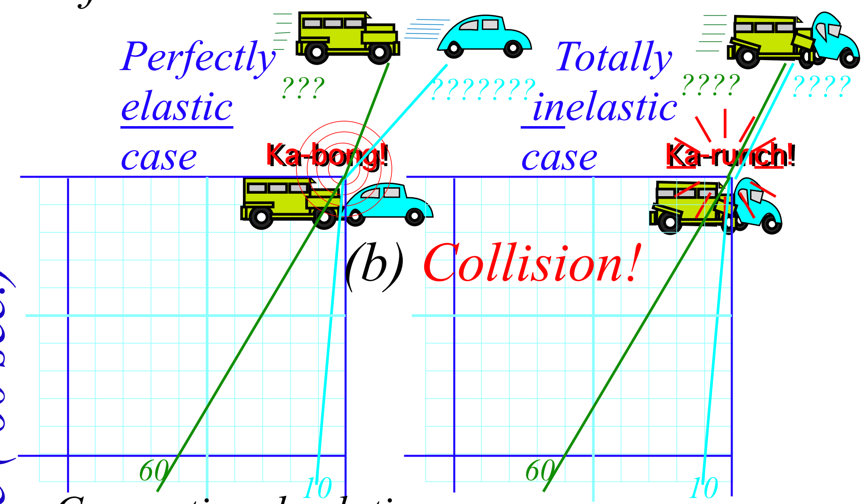

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Get out formulas:

$\Sigma mV(before)=\Sigma mV(before)$ *[momentum conservation]*

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etc.

$M_{SUV}V_{SUV}+M_{VW}V_{VW}$=*constant* is ***Axiom #1***

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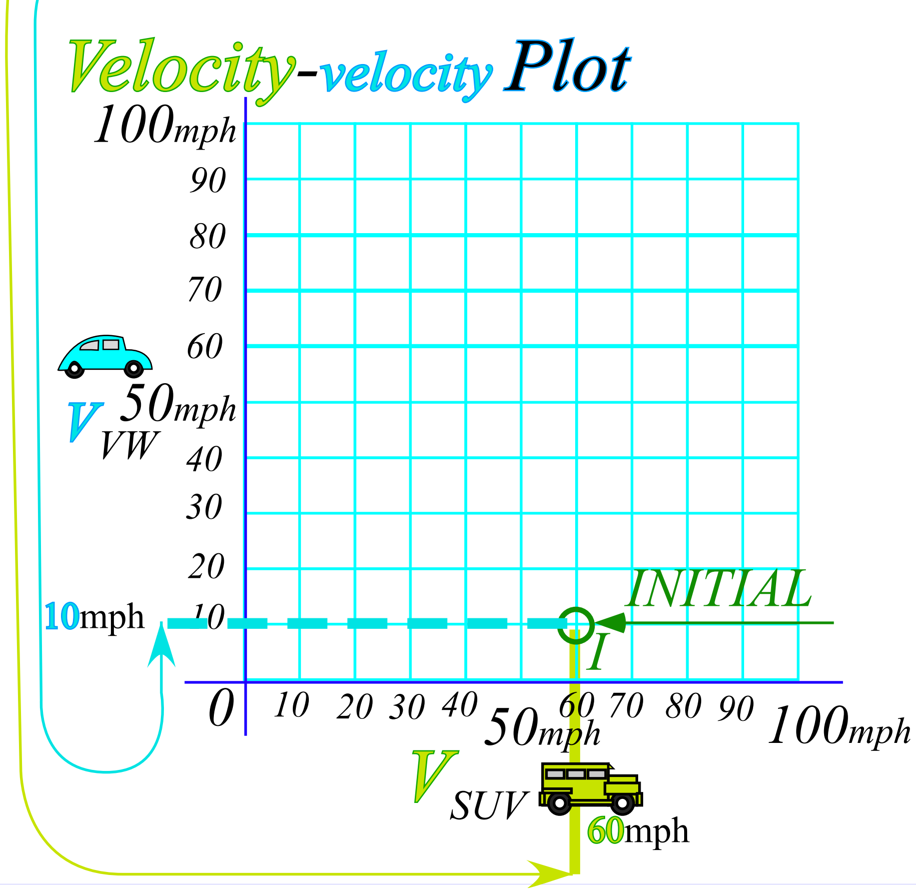

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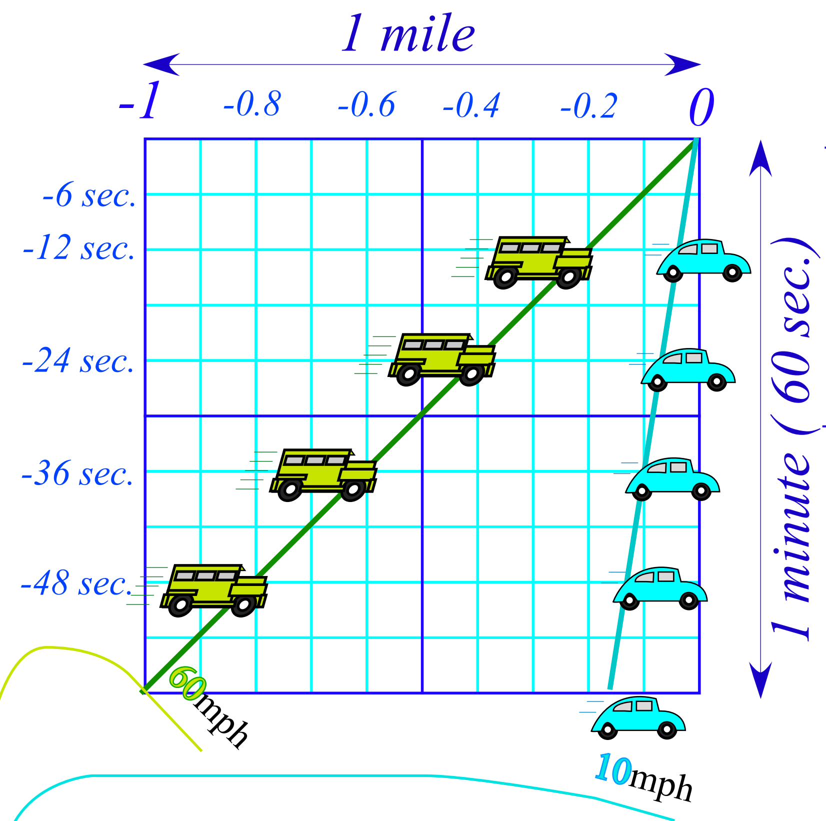

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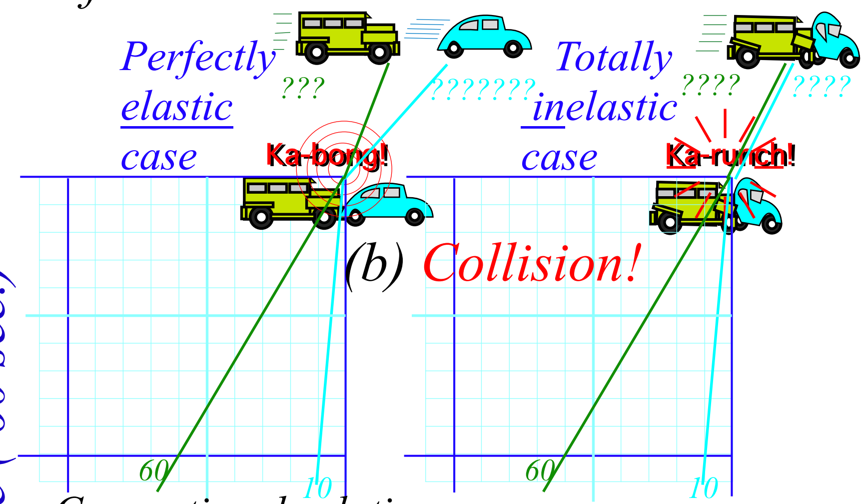

Conventional solution:
Get out formulas:

$\Sigma m V(\text{before}) = \Sigma m V(\text{before})$ [momentum conservation]

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etc.

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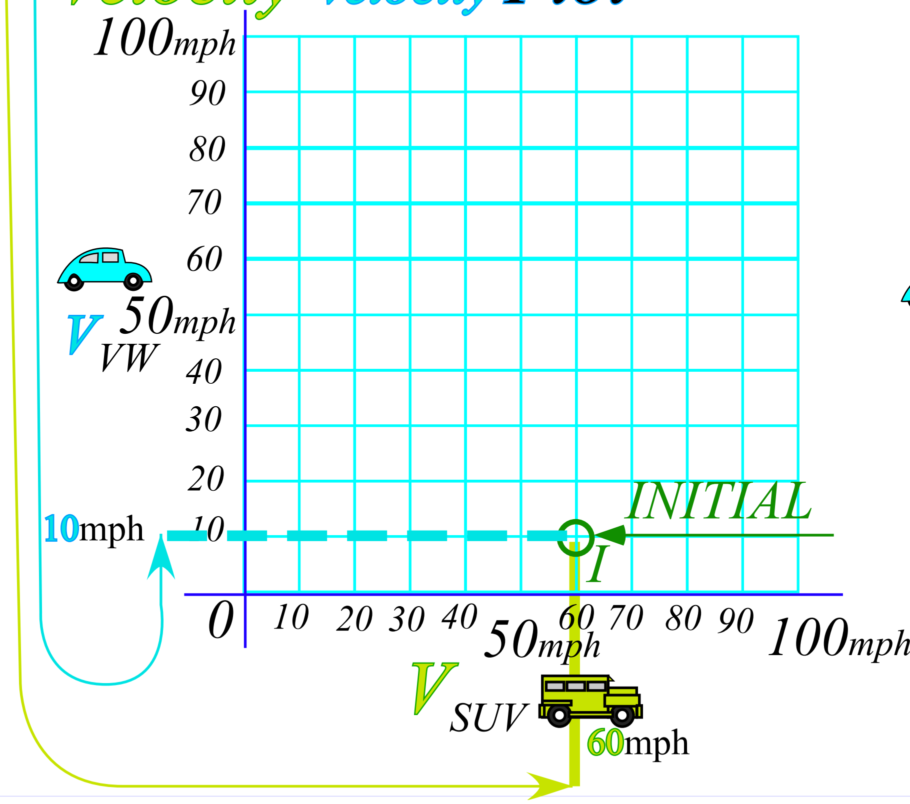
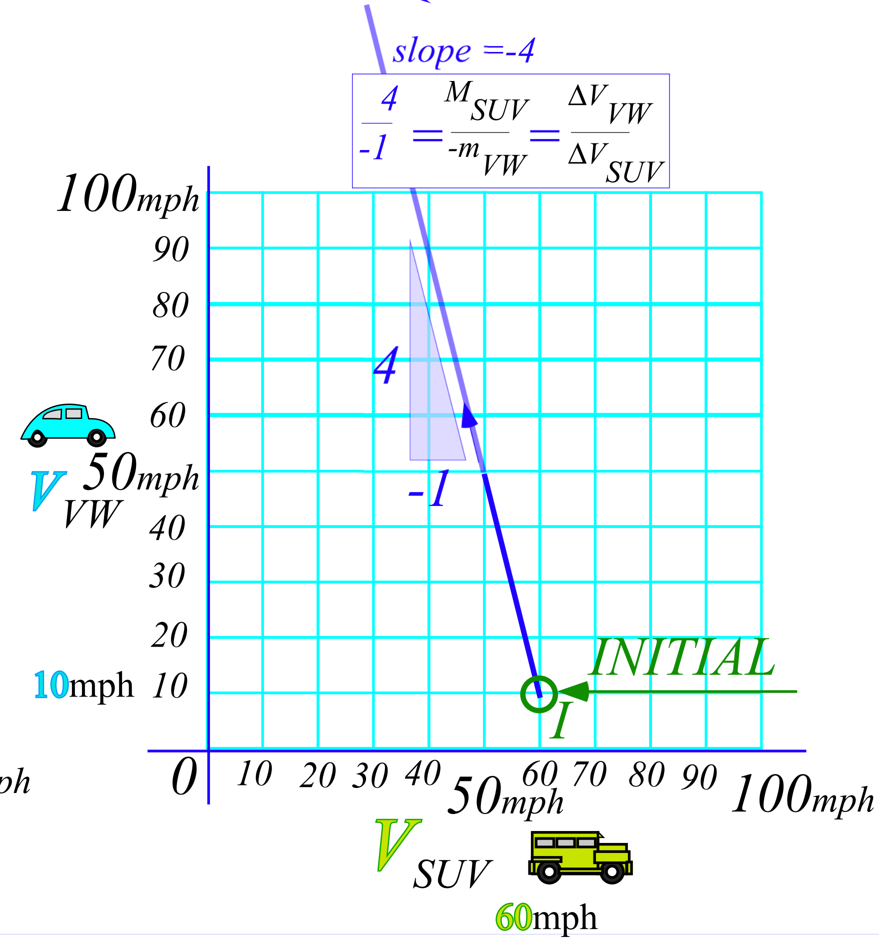
Velocity-velocity Plot





Geometry of momentum conservation axiom

→ *Totally Inelastic "ka-runch" collisions**

*Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**

Comments on idealization in classical models

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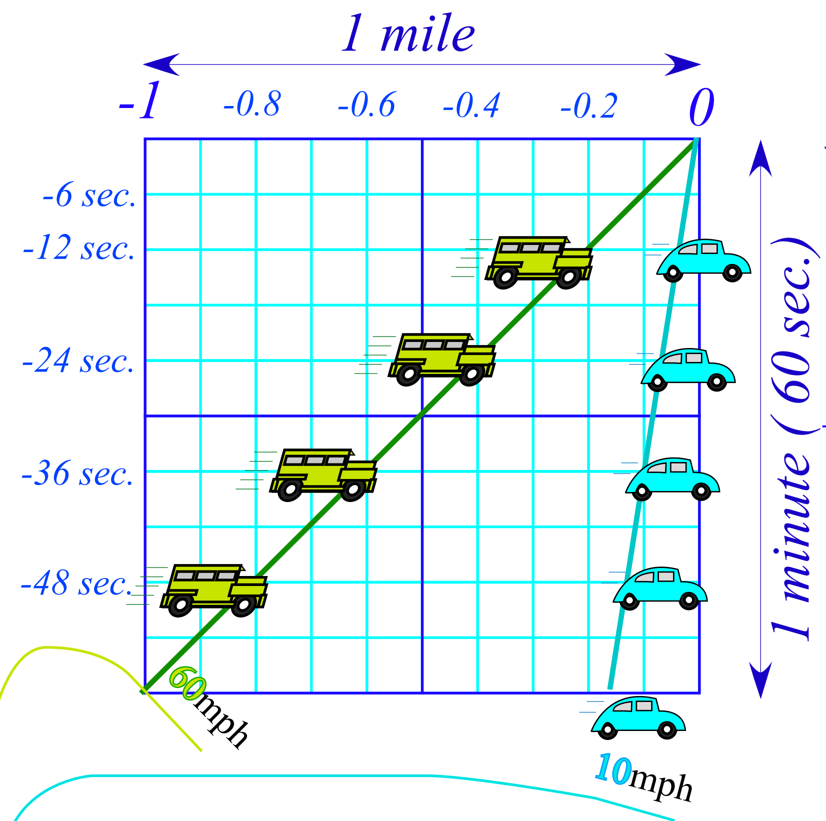

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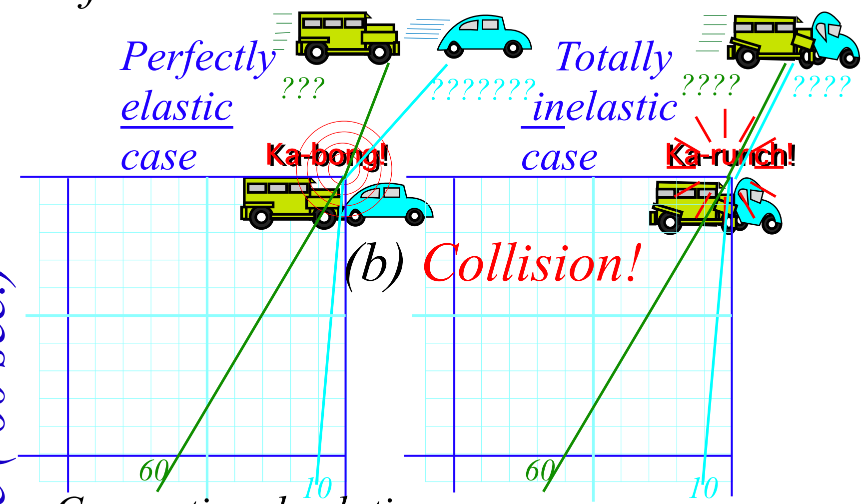

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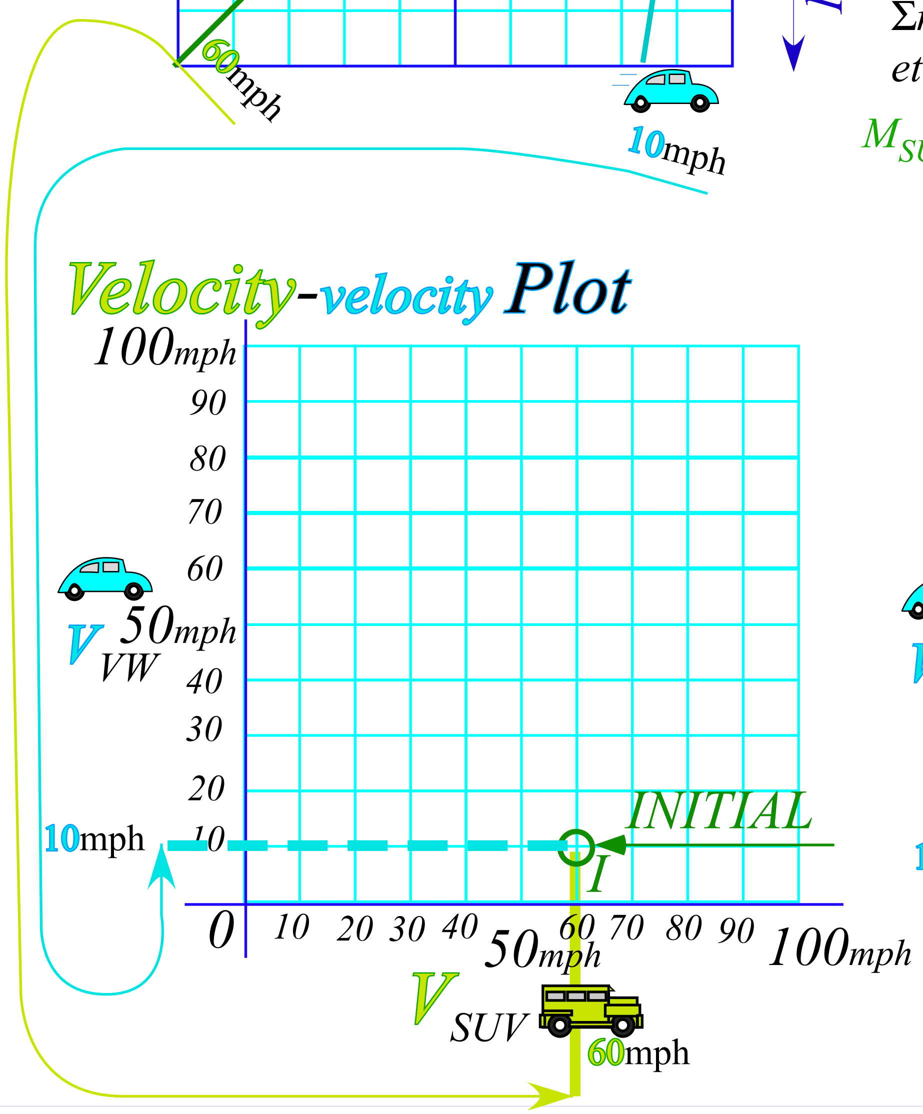

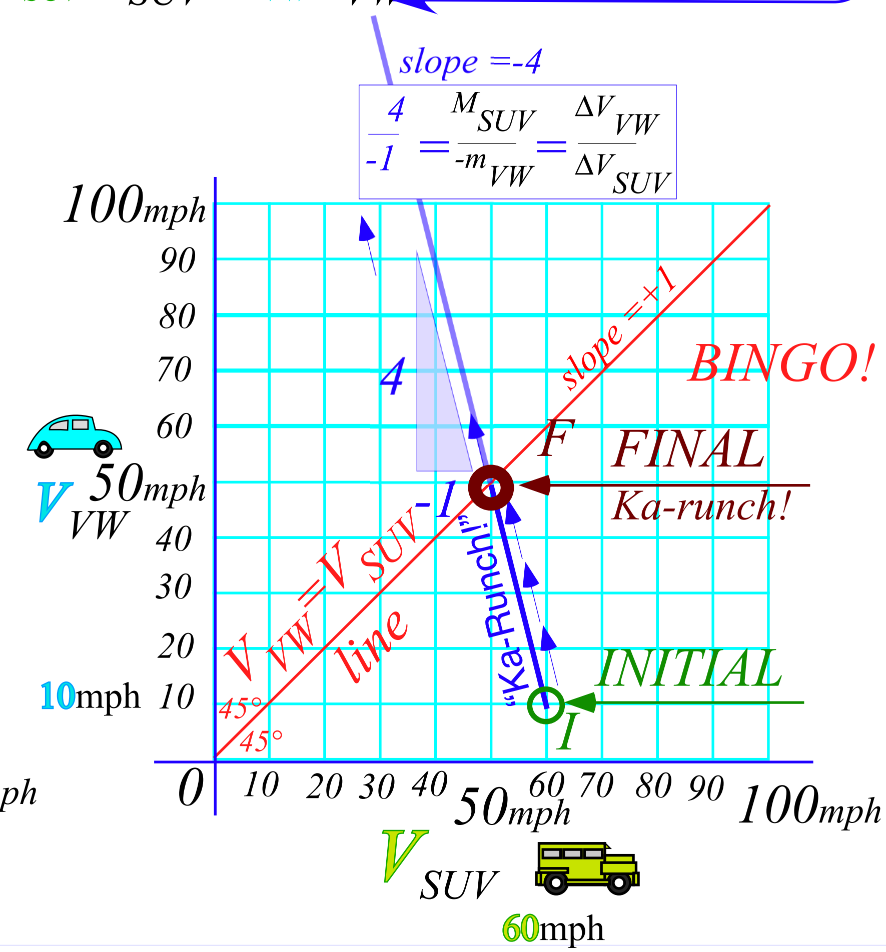

Totally Inelastic
(*Ka – Runch*!)

Geometry of momentum conservation axiom

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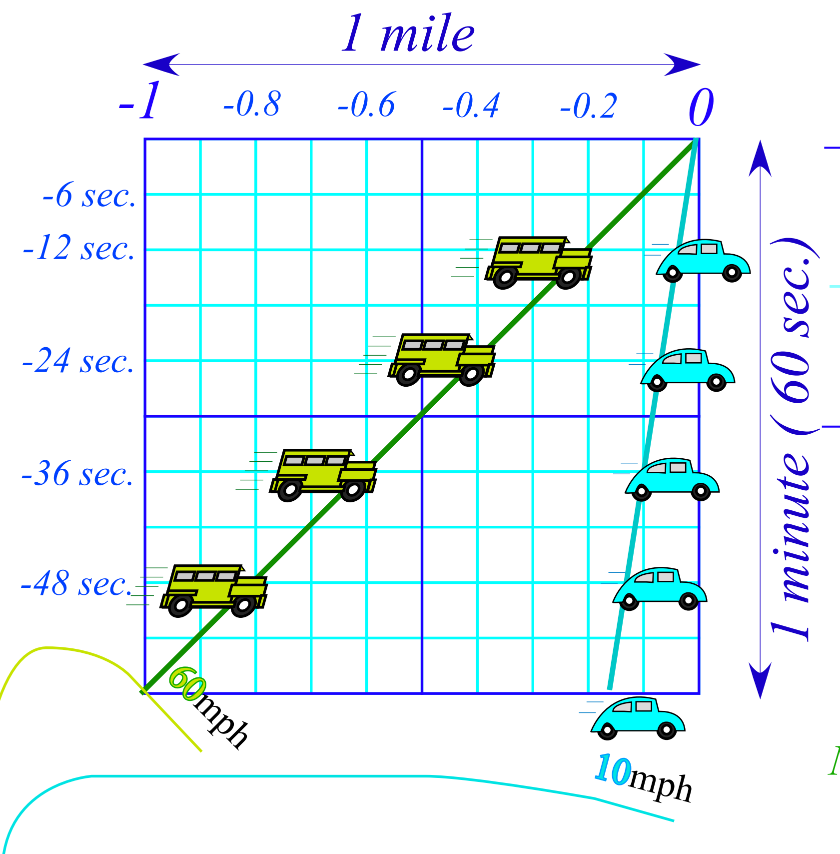

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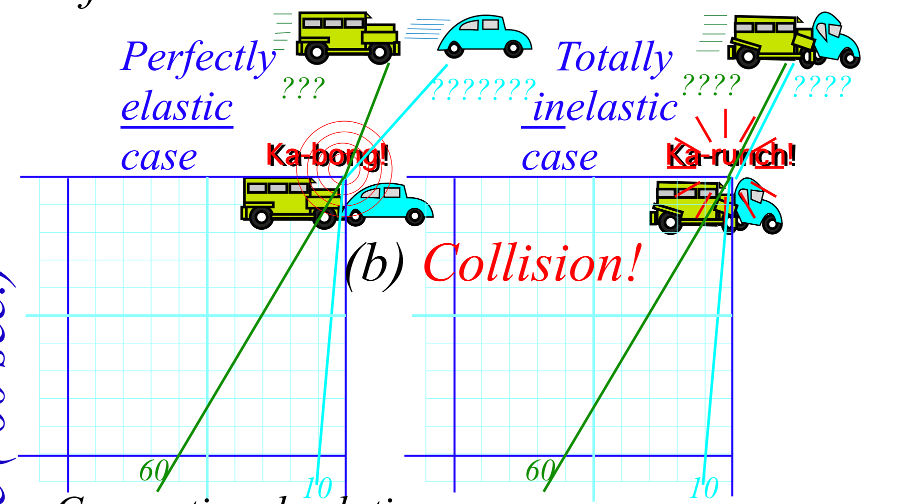

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etc.

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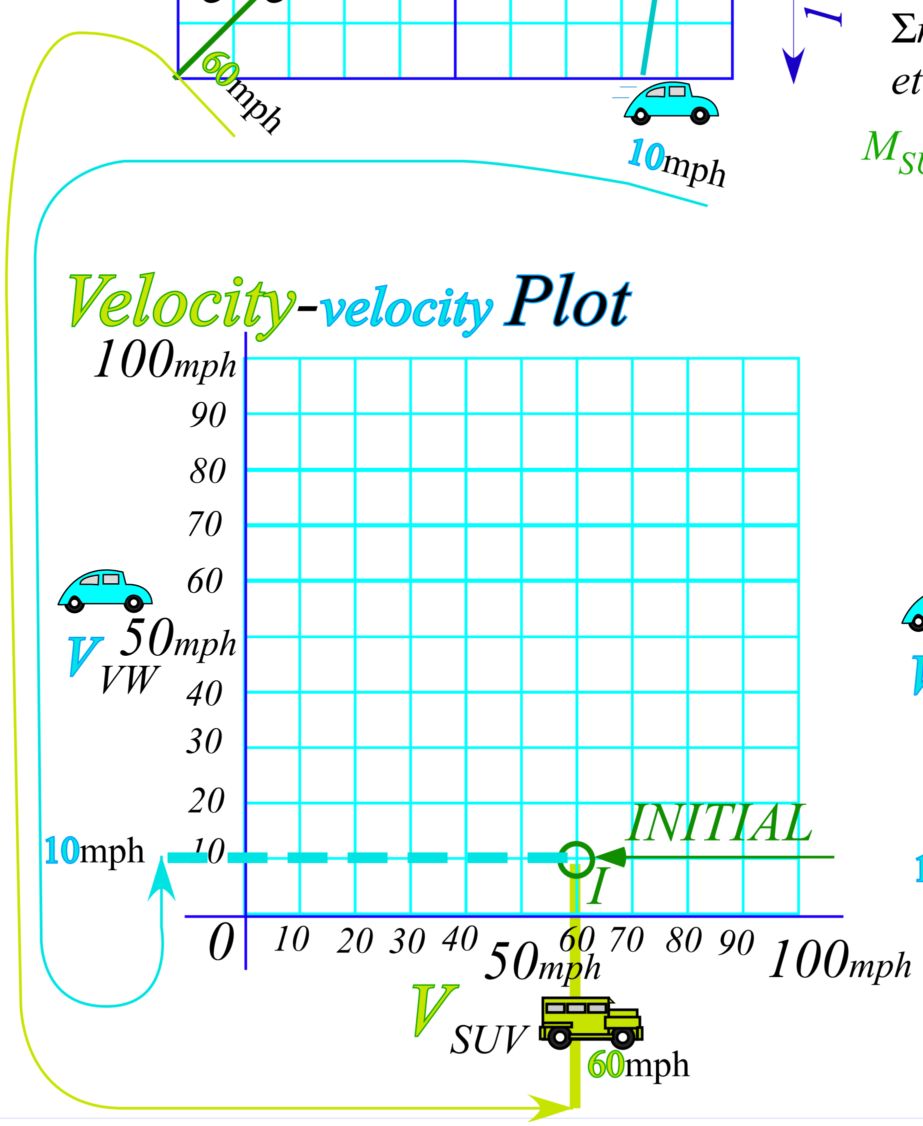

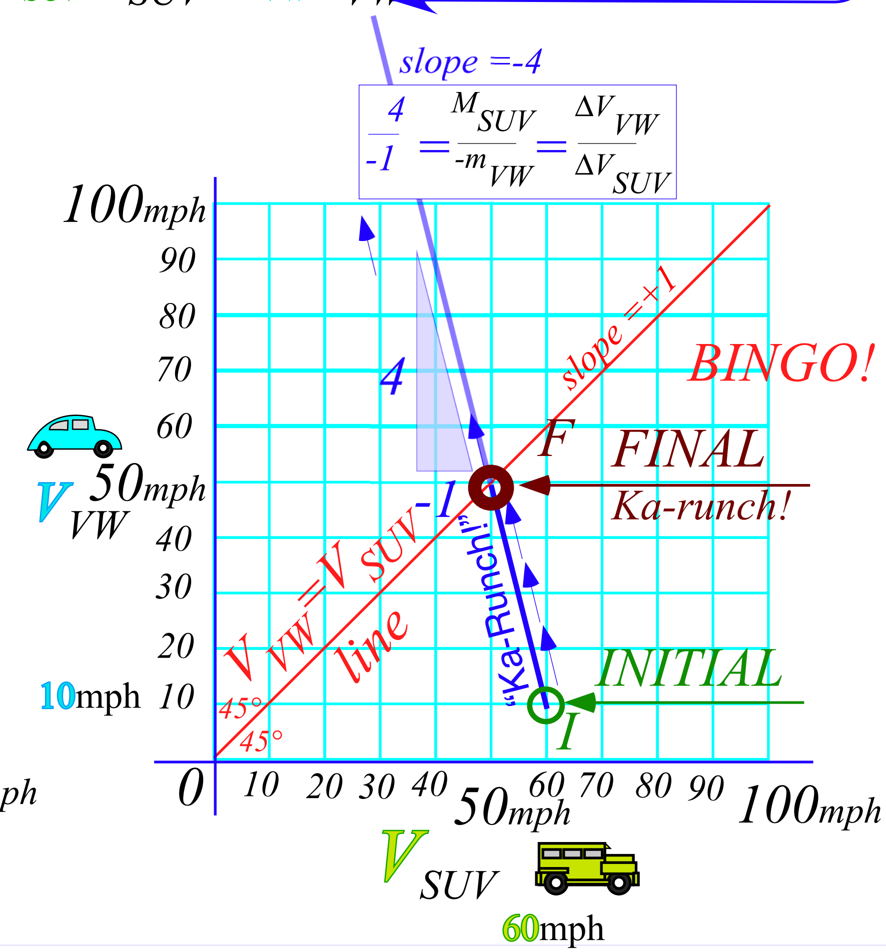

Totally Inelastic

$(Ka - Runch!)$

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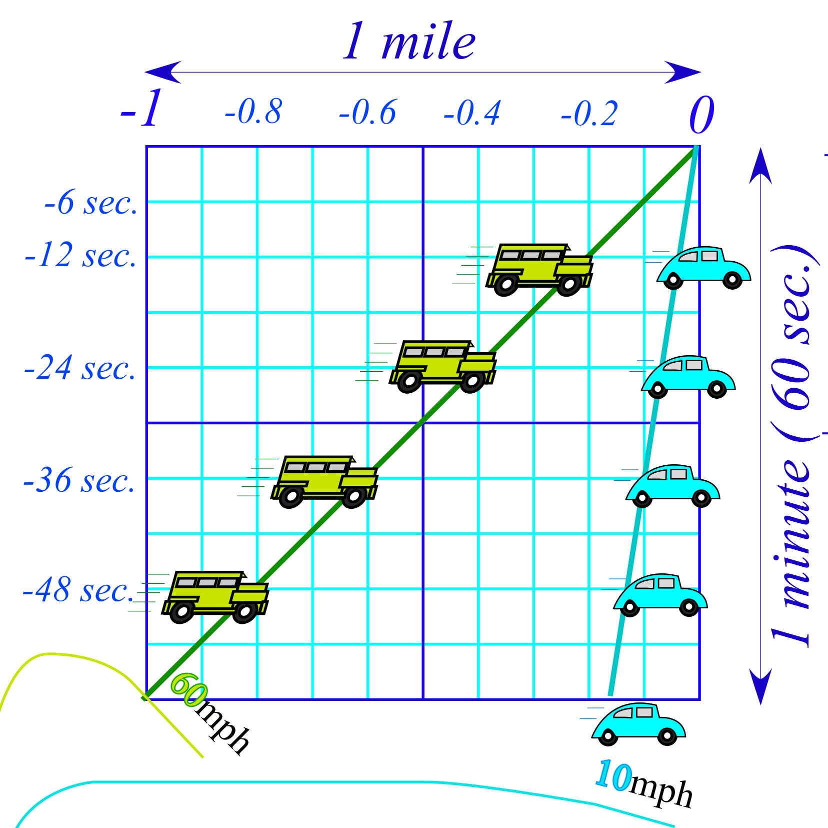

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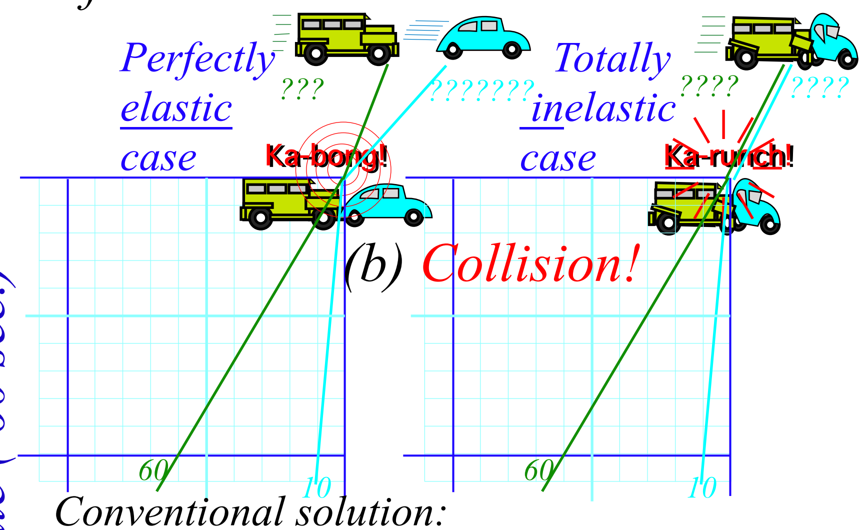

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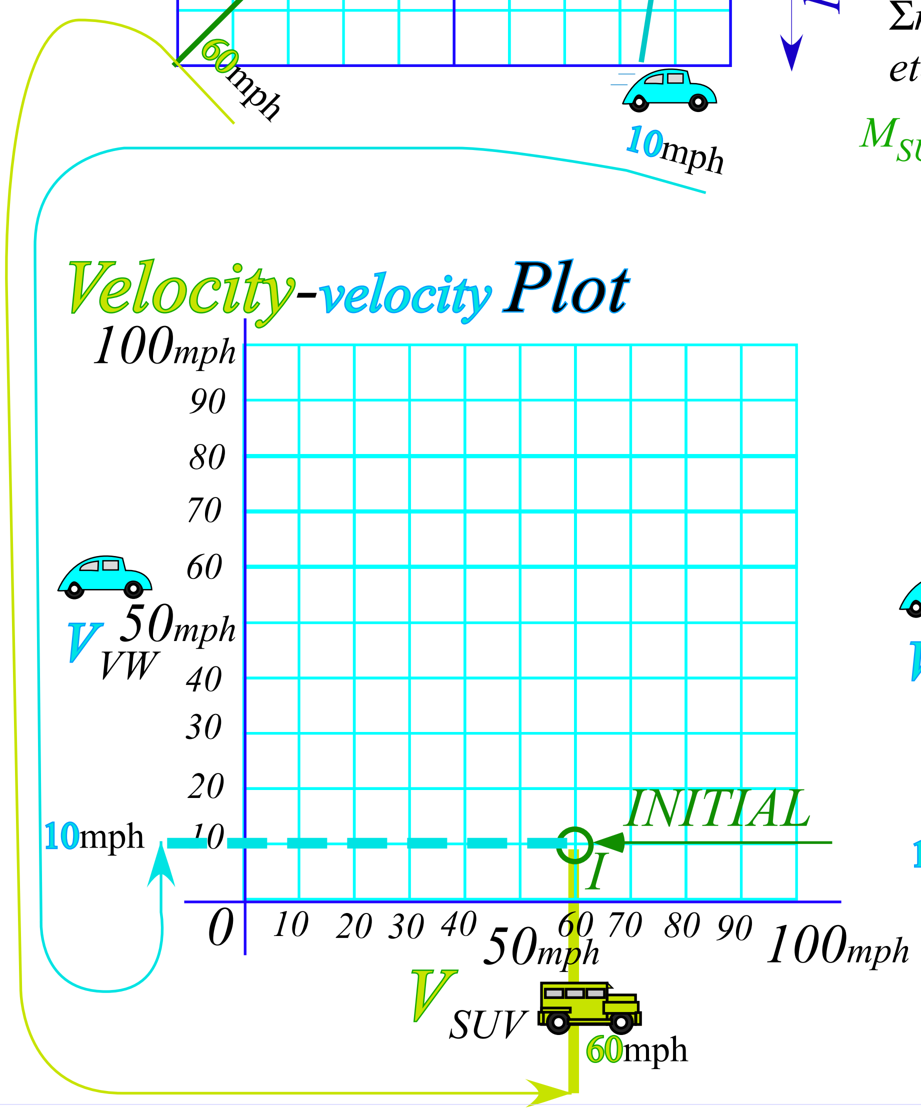
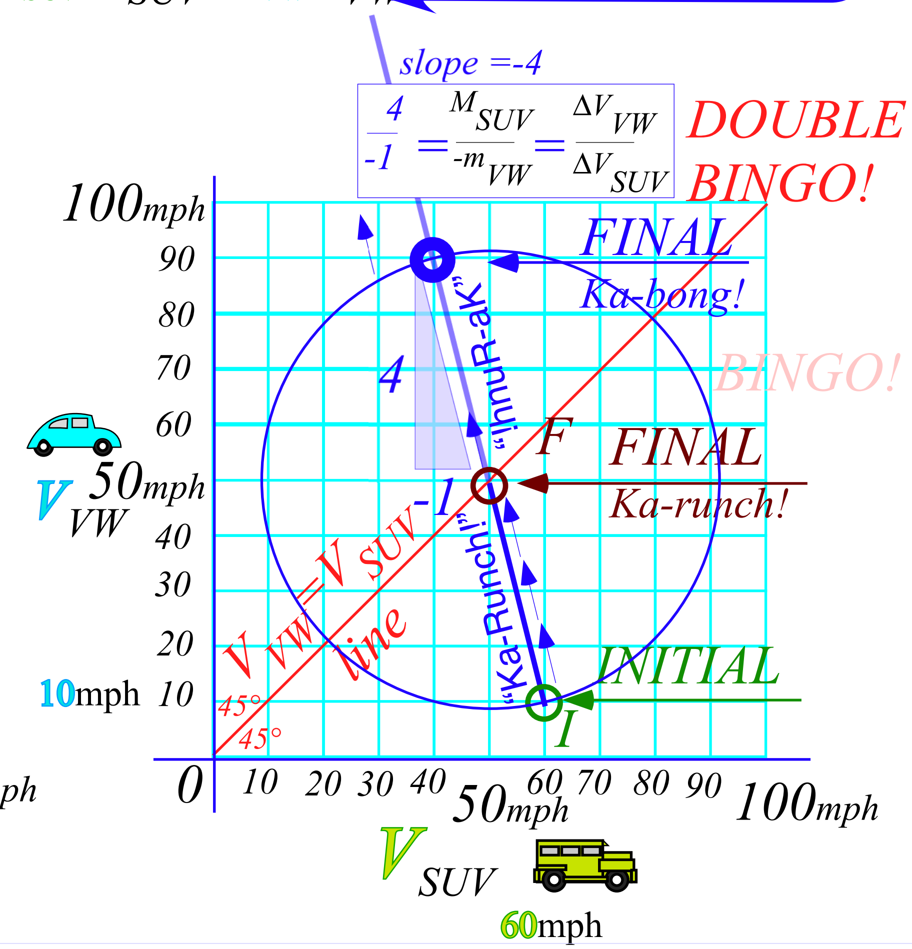
... (and a circle...)





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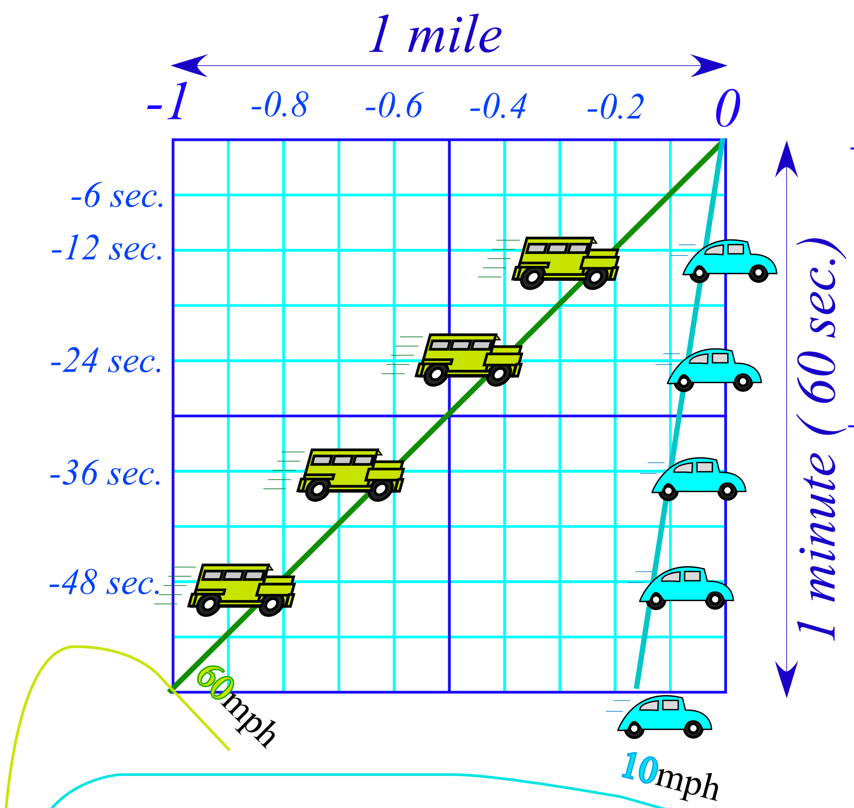

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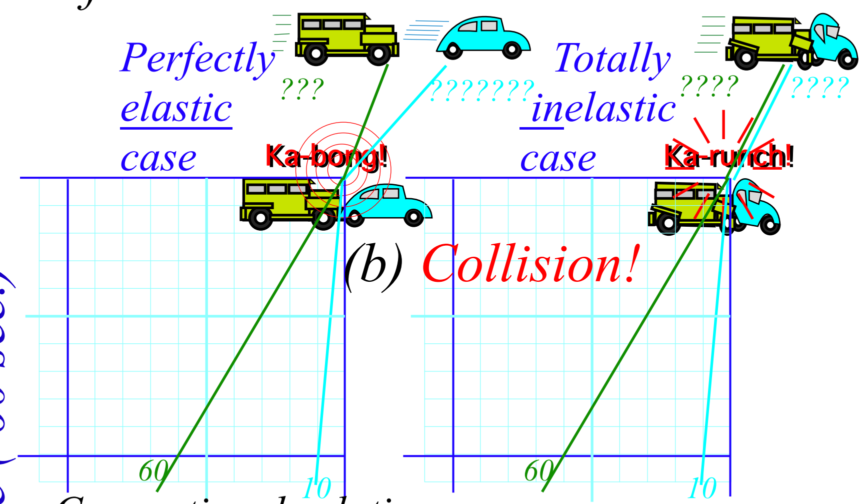

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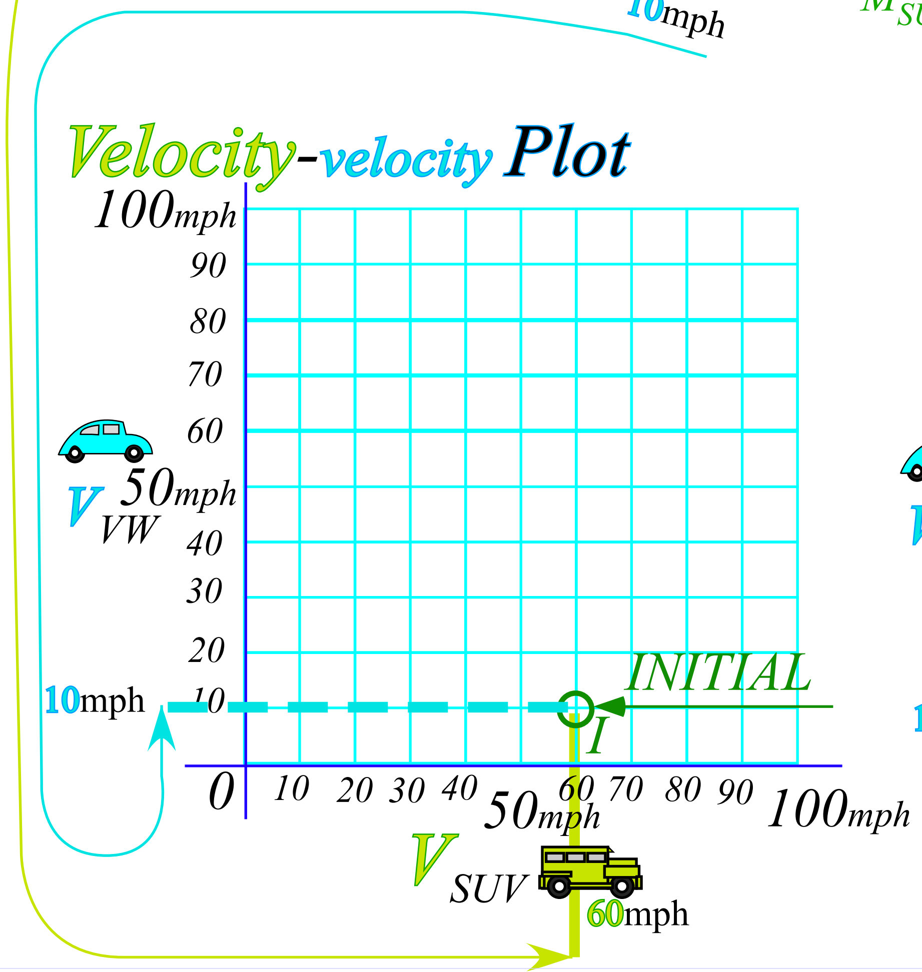
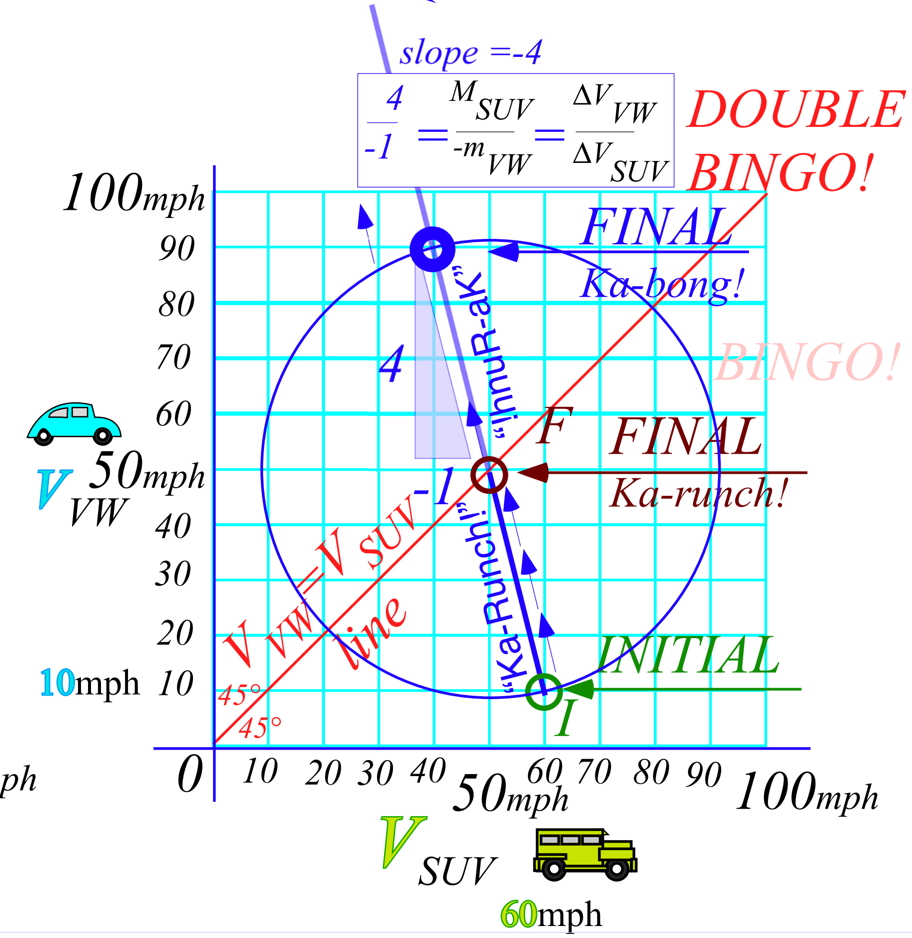
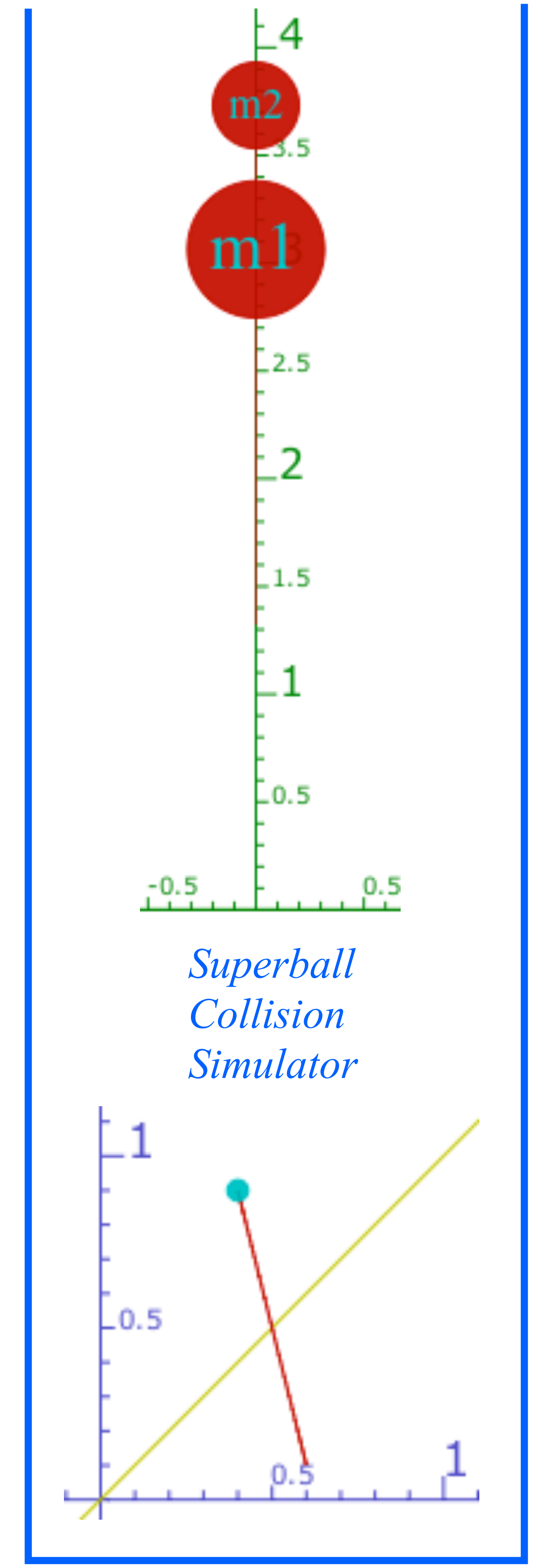


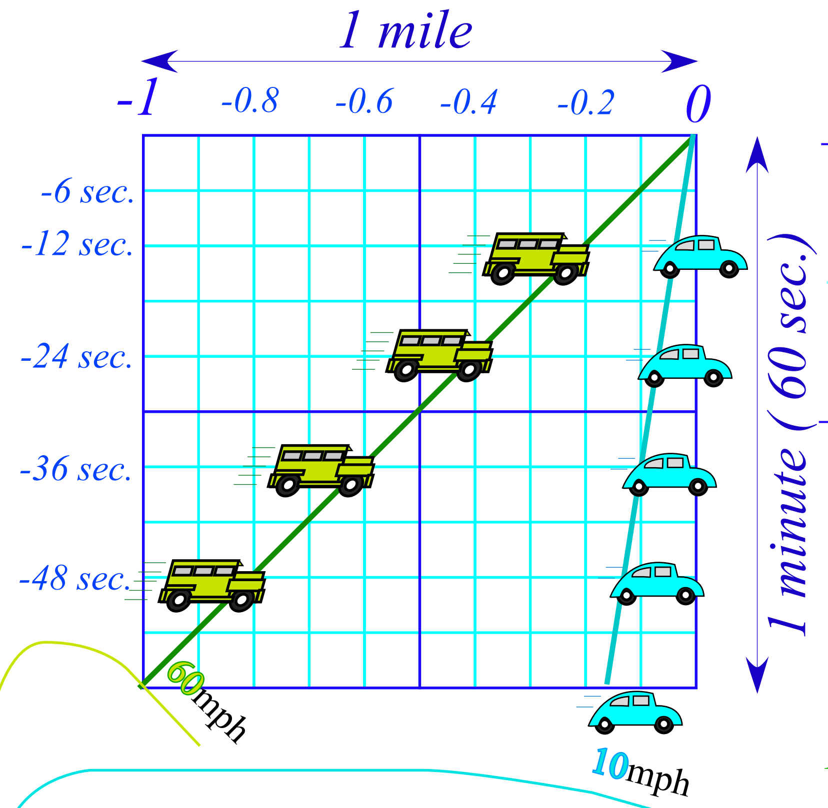




Superball Collision Simulator http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision.....



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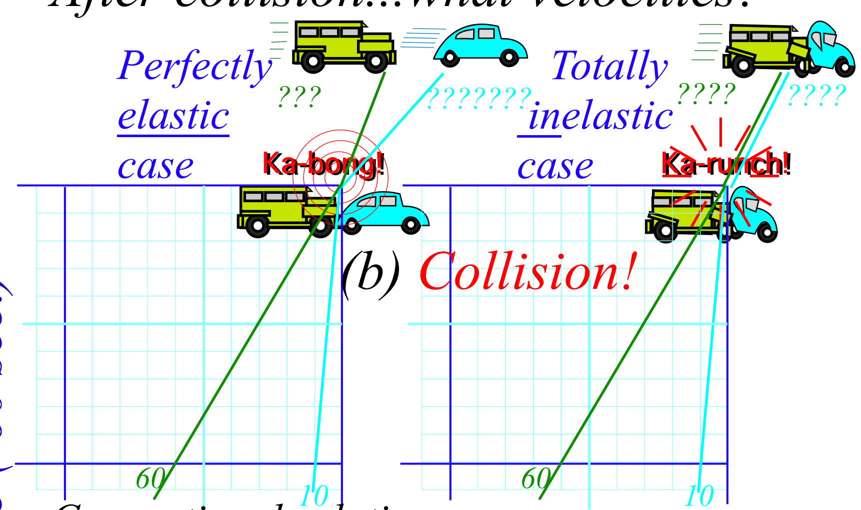

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slope =-4

...But an UNconventional way is quicker and slicker.....
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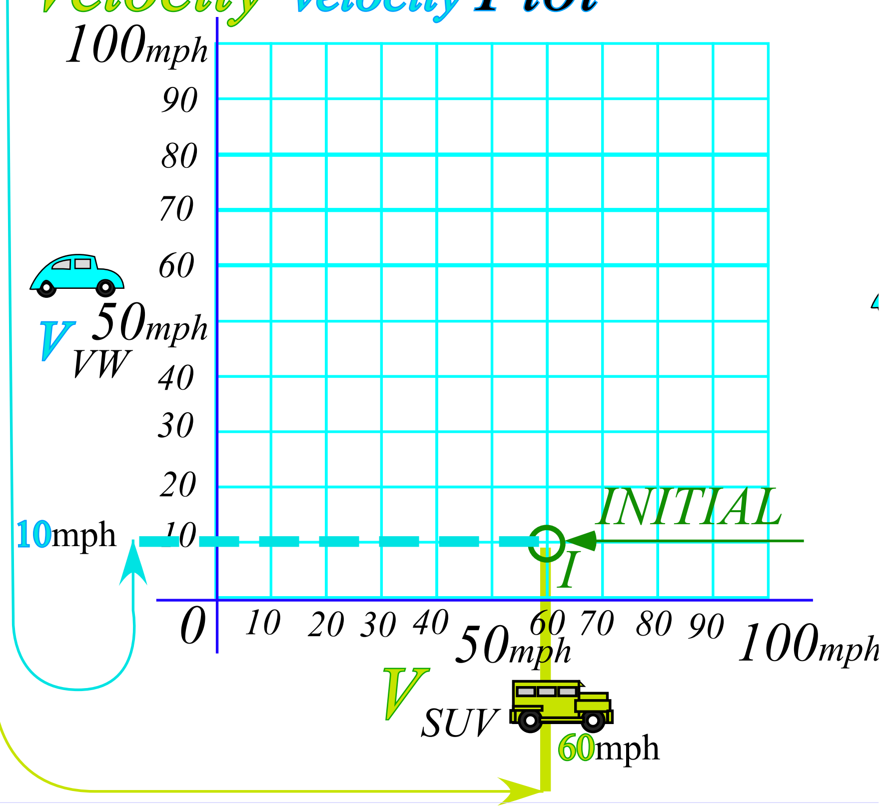
Velocity-velocity Plot



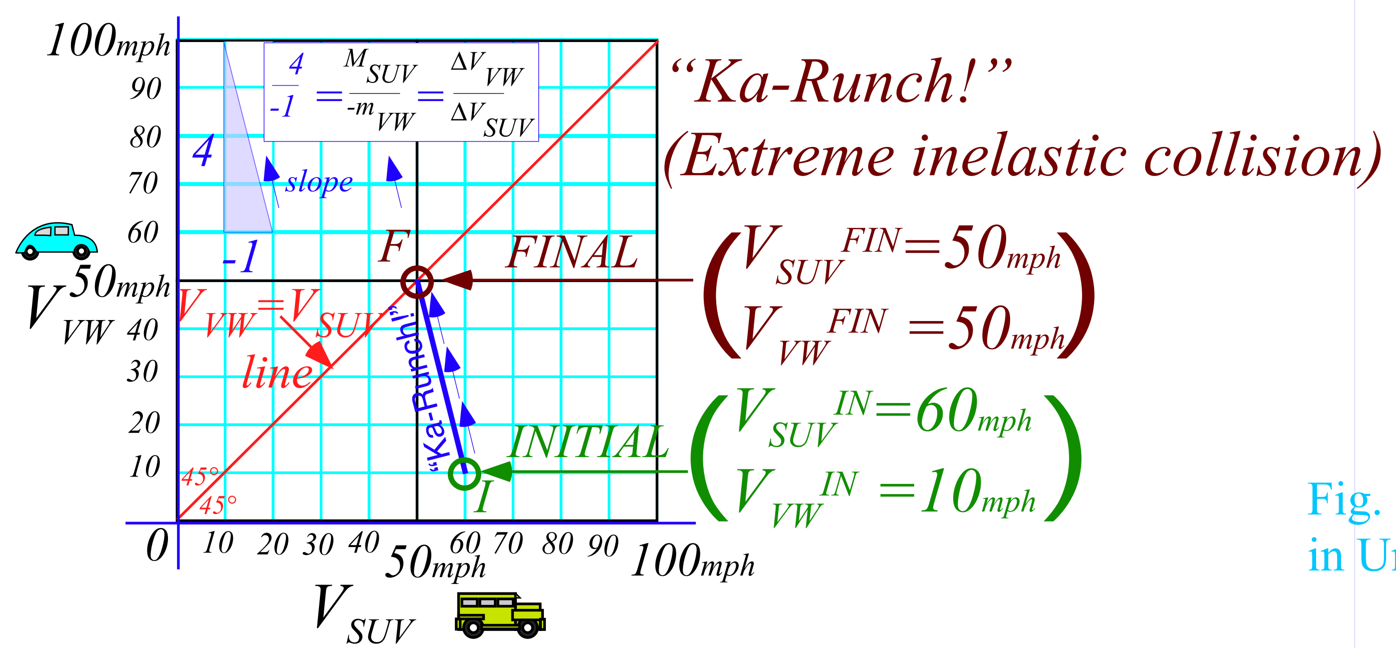
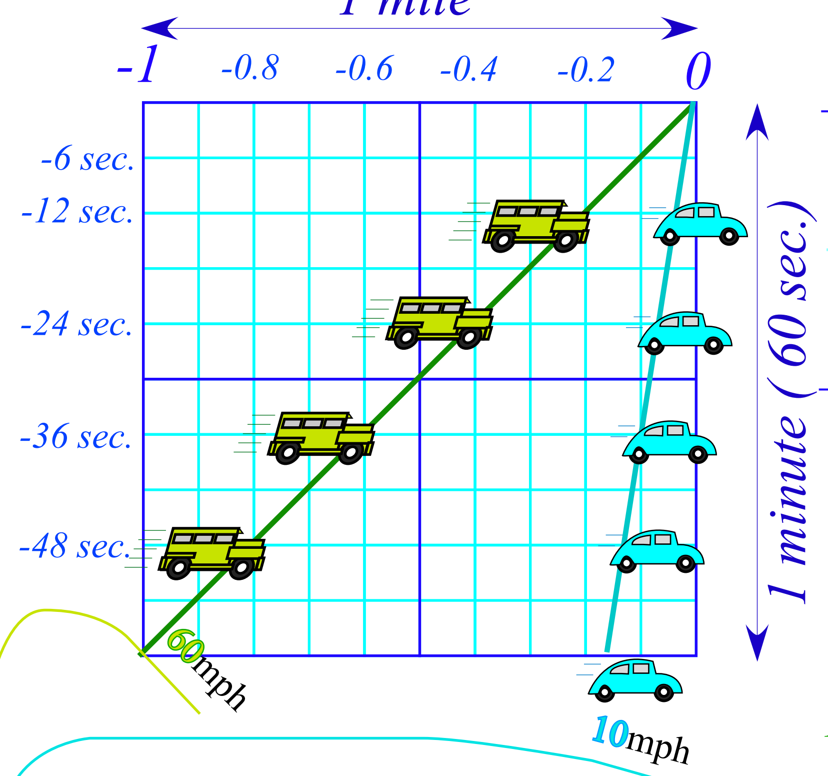


Fig. 2.1
in Unit 1

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Notice "Ka-Bong"
Figure 2.2 scaling
(ft./min. is more realistic)

Before collision.....



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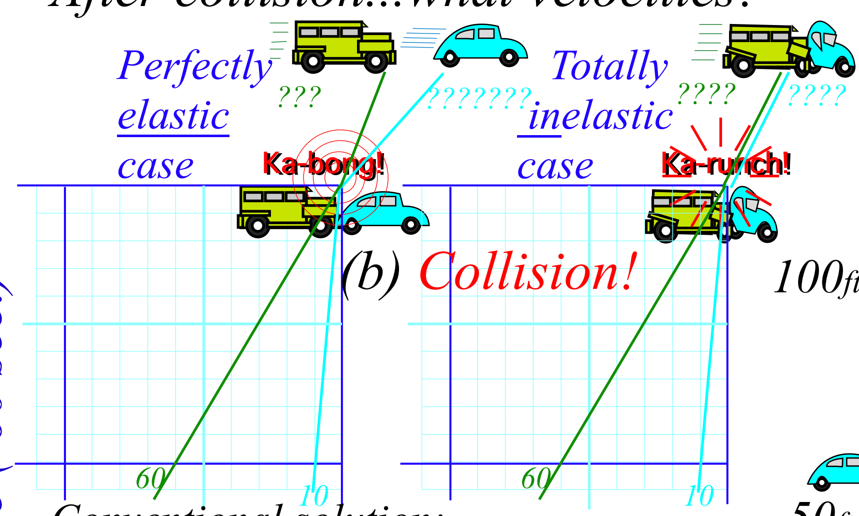

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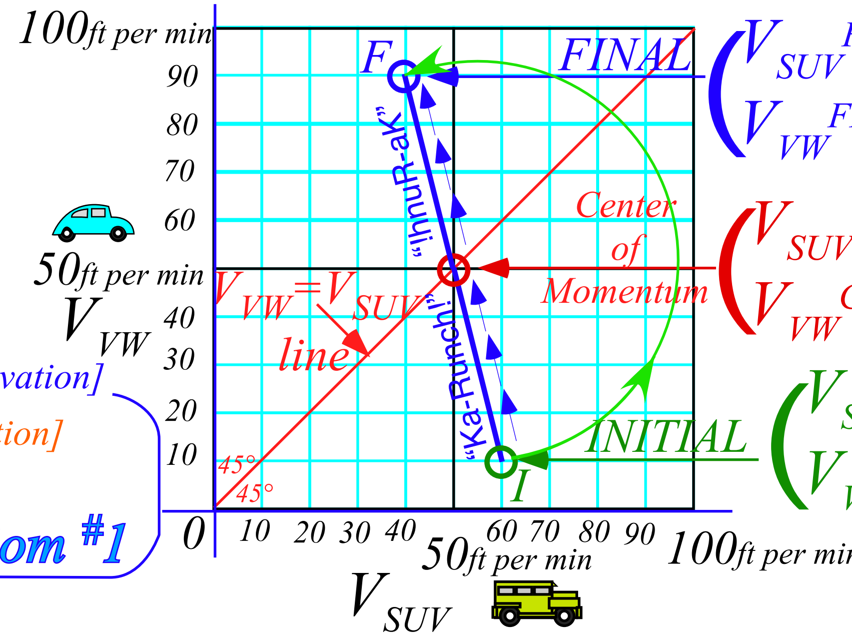
slope = -4

"Ka-Bong!" (Ideal elastic collision)



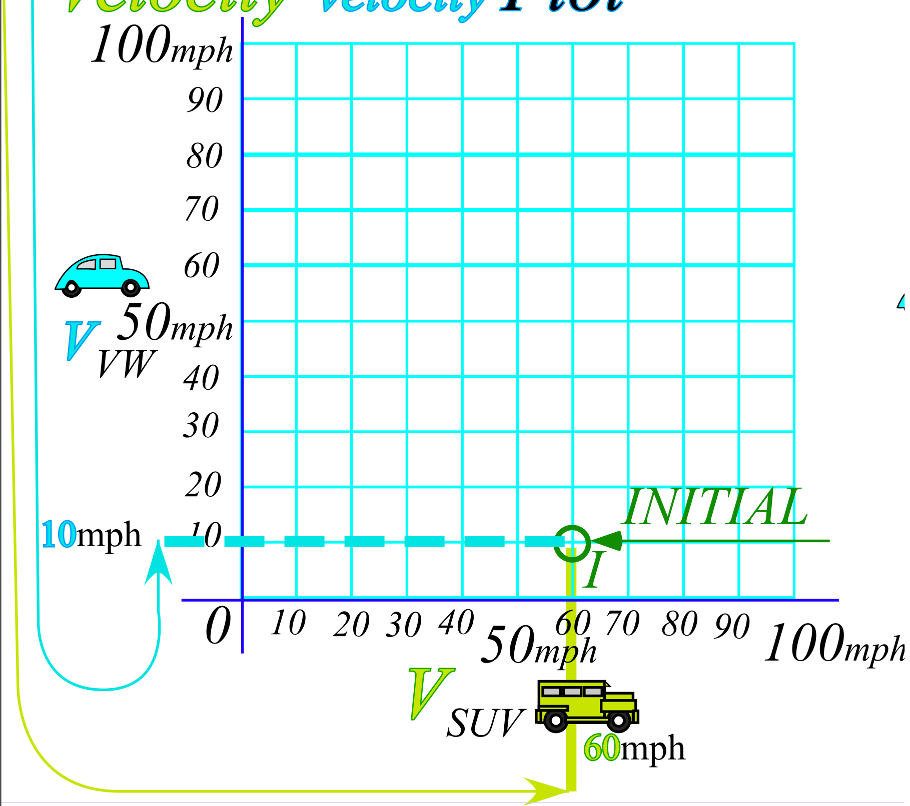
$\begin{pmatrix} V_{SUV}^{FIN} = 40_{\text{ft per min}} \\ V_{VW}^{FIN} = 90_{\text{ft per min}} \end{pmatrix}$

$\begin{pmatrix} V_{SUV}^{COM} = 50_{\text{ft per min}} \\ V_{VW}^{COM} = 50_{\text{ft per min}} \end{pmatrix}$

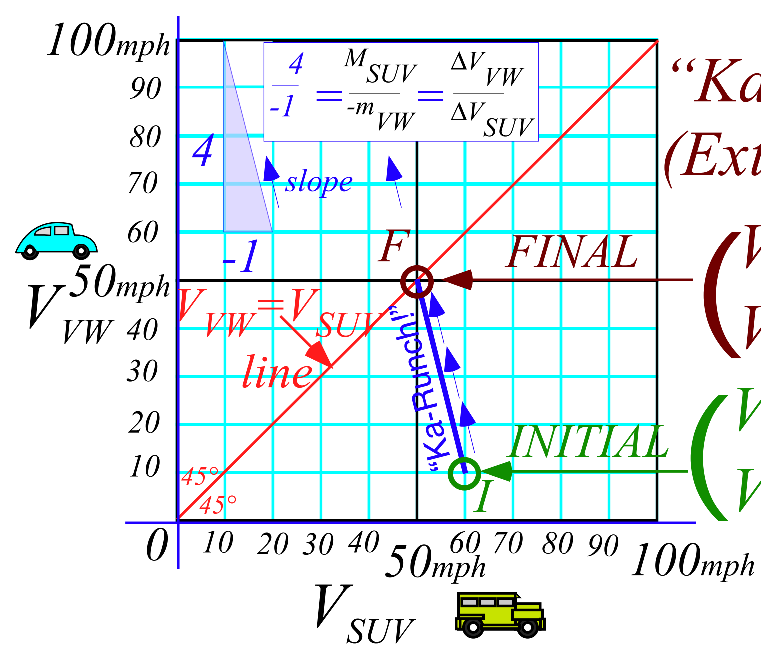
$\begin{pmatrix} V_{SUV}^{IN} = 60_{\text{ft per min}} \\ V_{VW}^{IN} = 10_{\text{ft per min}} \end{pmatrix}$

Fig. 2.2
in Unit 1

Velocity-velocity Plot



"Ka-Runch!"
(Extreme inelastic collision)



$\begin{pmatrix} V_{SUV}^{FIN} = 50_{mph} \\ V_{VW}^{FIN} = 50_{mph} \end{pmatrix}$

$\begin{pmatrix} V_{SUV}^{IN} = 60_{mph} \\ V_{VW}^{IN} = 10_{mph} \end{pmatrix}$

Fig. 2.1
in Unit 1

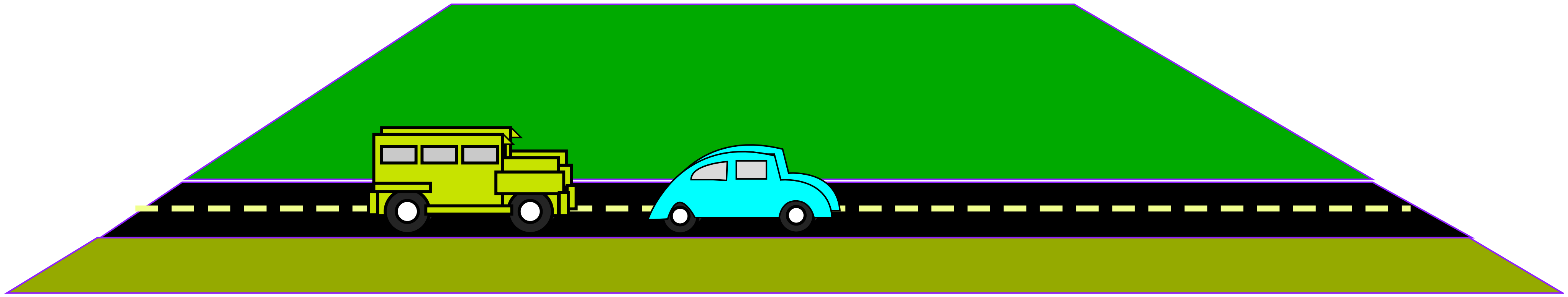
Geometry of momentum conservation axiom

*Totally Inelastic "ka-runch"collisions**

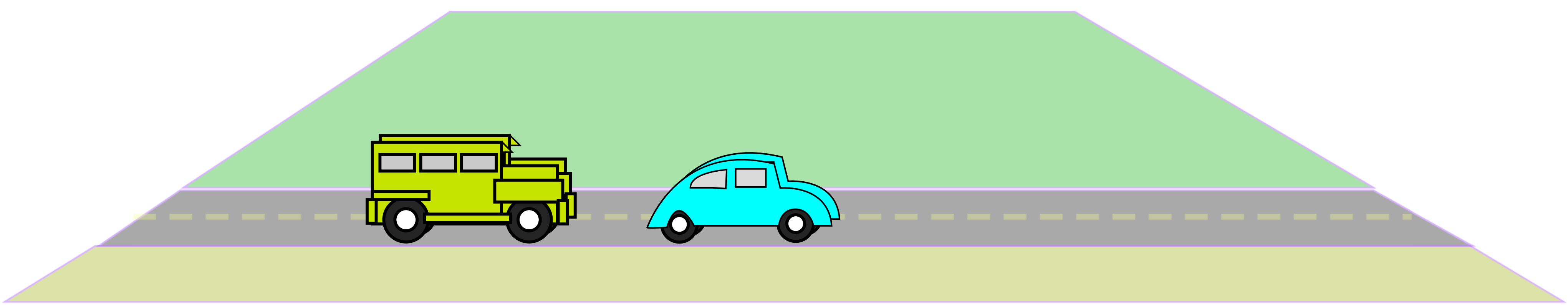
*Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**

→ *Comments on idealization in classical models*

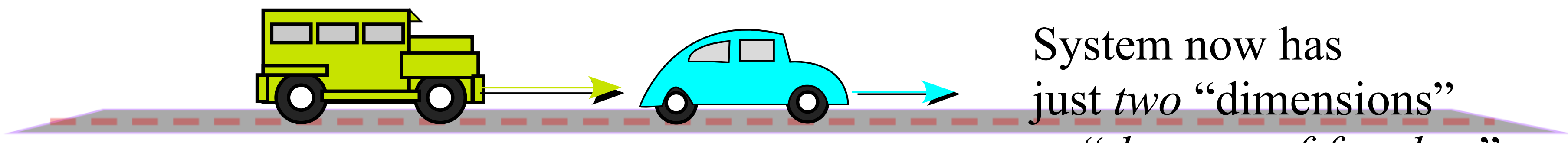
The SUV and VW *Idealized* thought experiments



Idealization 1. Ignore background.
(No rolling friction, air resistance, etc.)



Idealization 2. Make each 1-dimensional.
(Cars "constrained" to ride on frictionless rail)

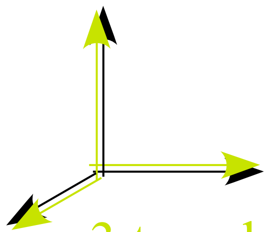


System now has just *two* "dimensions" or "*degrees-of-freedom*"

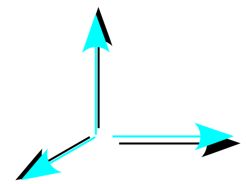
Landscape 1.1 Idealized model for collision model and thought experiments

Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)



3 translation dimensions

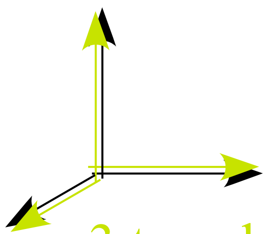


3 translation dimensions

6 translational degrees of freedom for SUV and VW.

Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)



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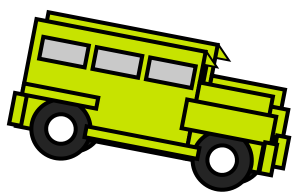
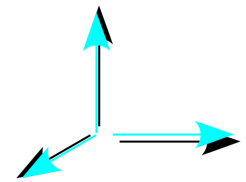
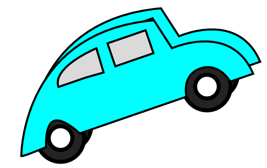


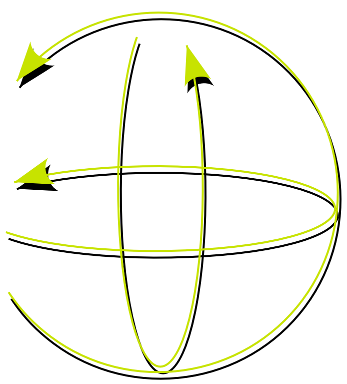


3 translation dimensions



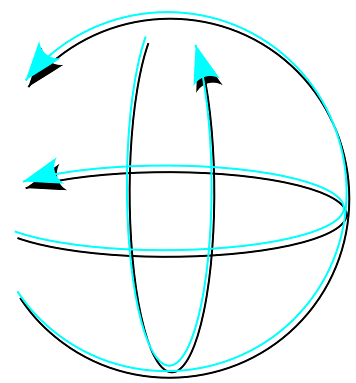
6 translational degrees of freedom for SUV and VW.

Rotation (Each body has 3 rotationaldegrees of freedom) (Intoduced in Units 3 and 7)



3 rotational dimensions

yaw-pitch-roll Euler angles



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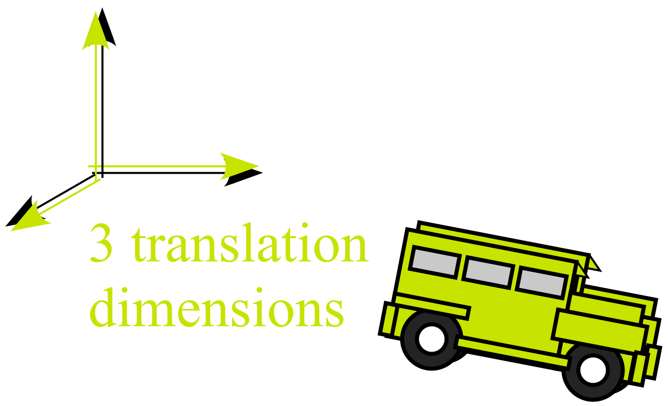
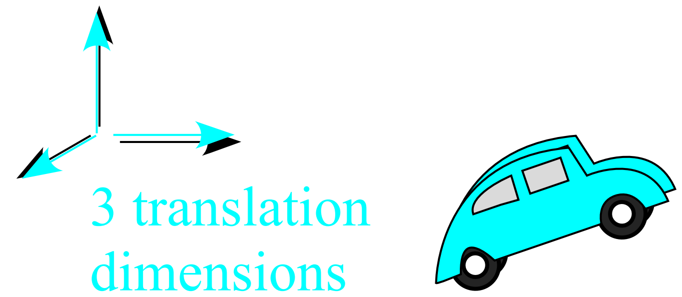
yaw-pitch-roll Euler angles

6 rotational degrees of freedom for SUV and VW.

Summary of Classical Mechanical Degrees of Freedom

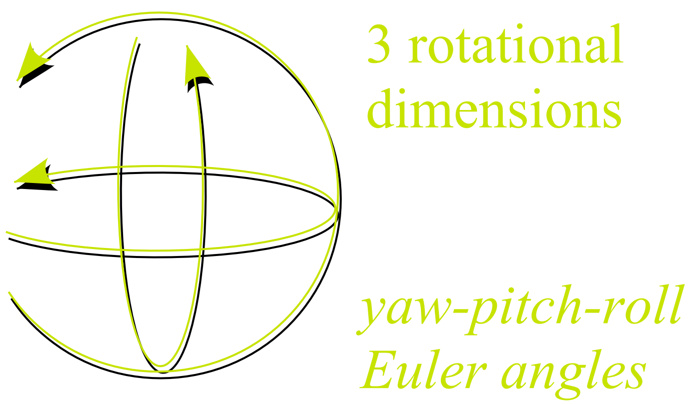
Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)

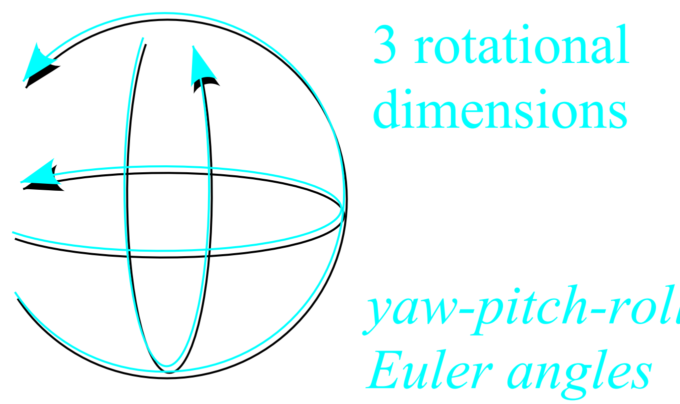




6 translational degrees of freedom for SUV and VW.

Rotation (Each body has 3 rotationaldegrees of freedom) (Intoduced in Units 3 and 7)



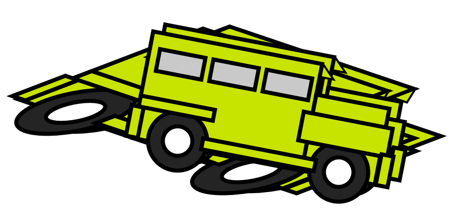


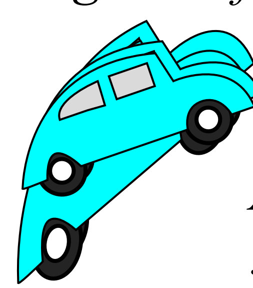
6 rotational degrees of freedom for SUV and VW.

SUV and VW system involves
12 rigid-body degrees of freedom

Vibration (Each body has many vibrational degrees of freedom) (Intoduced in Units 3-8)

Generalized Curvilinear Coordinates (GCC) introduced in Unit 1 Chapters 10 -12





An N-atom molecule has 3N-6 vibrational degrees of freedom

Landscape 1.2 Some idealized classical model degrees of freedom

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space
Time reversal symmetry
...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph slower

(a) Galileo transforms to COM frame

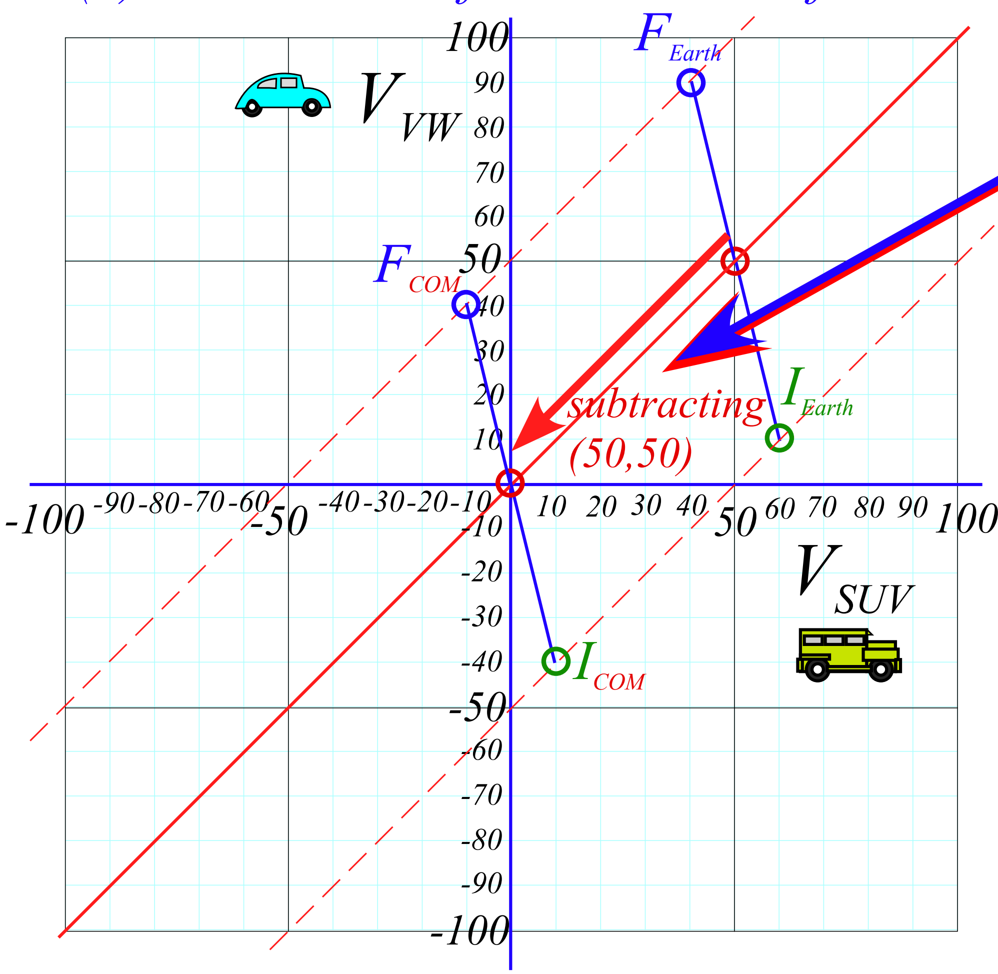


Fig. 2.5a
in Unit 1

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

*...the rest of the world appears to be 50 mph **slower***

(a) Galileo transforms to COM frame *(b) ... and to five or six other reference frames*

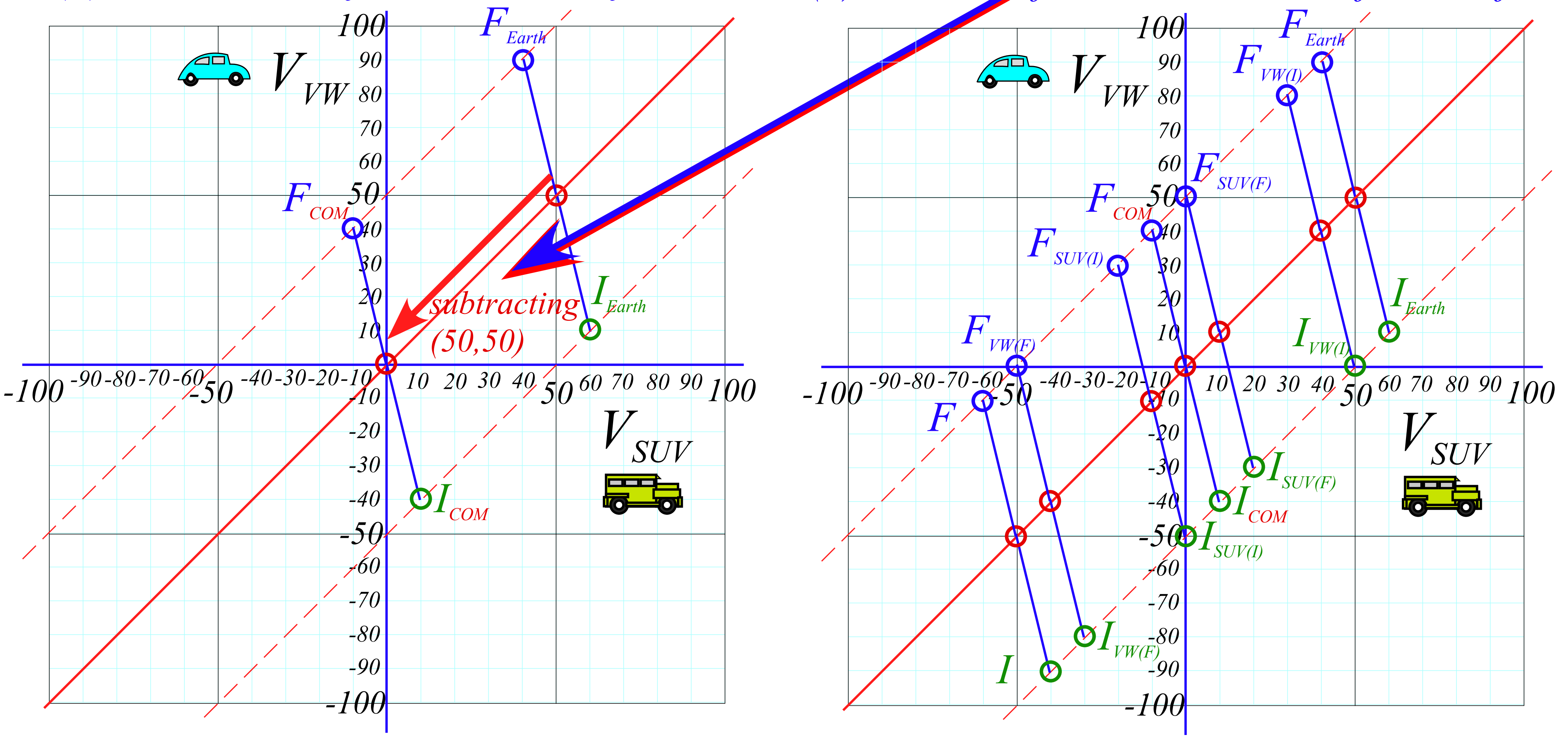


Fig. 2.5a in Unit 1

Fig. 2.5b in Unit 1

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

→ *Time reversal symmetry*

...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph slower

(a) Galileo transforms to COM frame

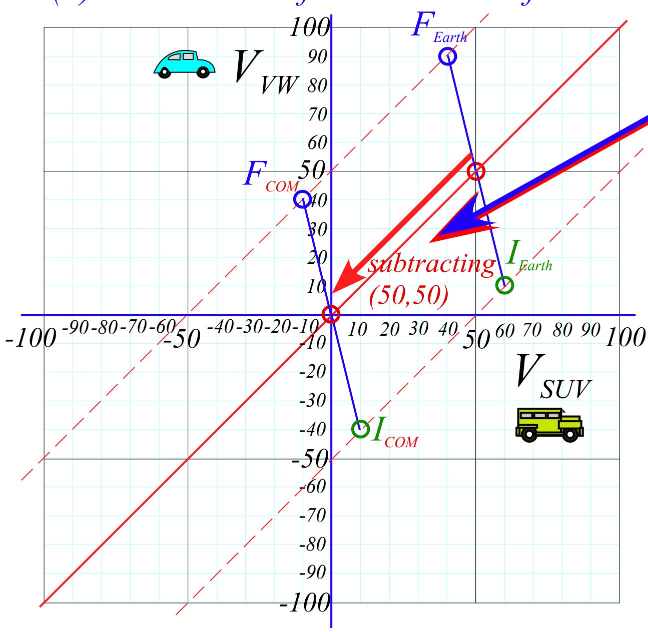


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

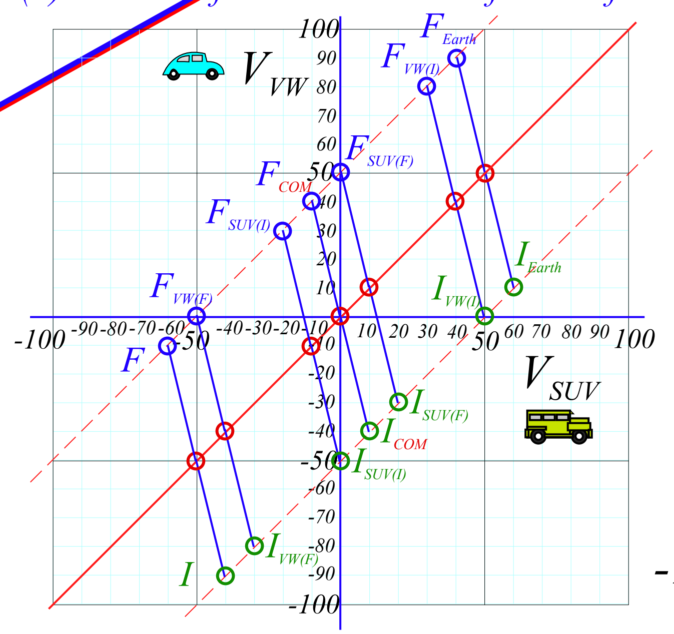
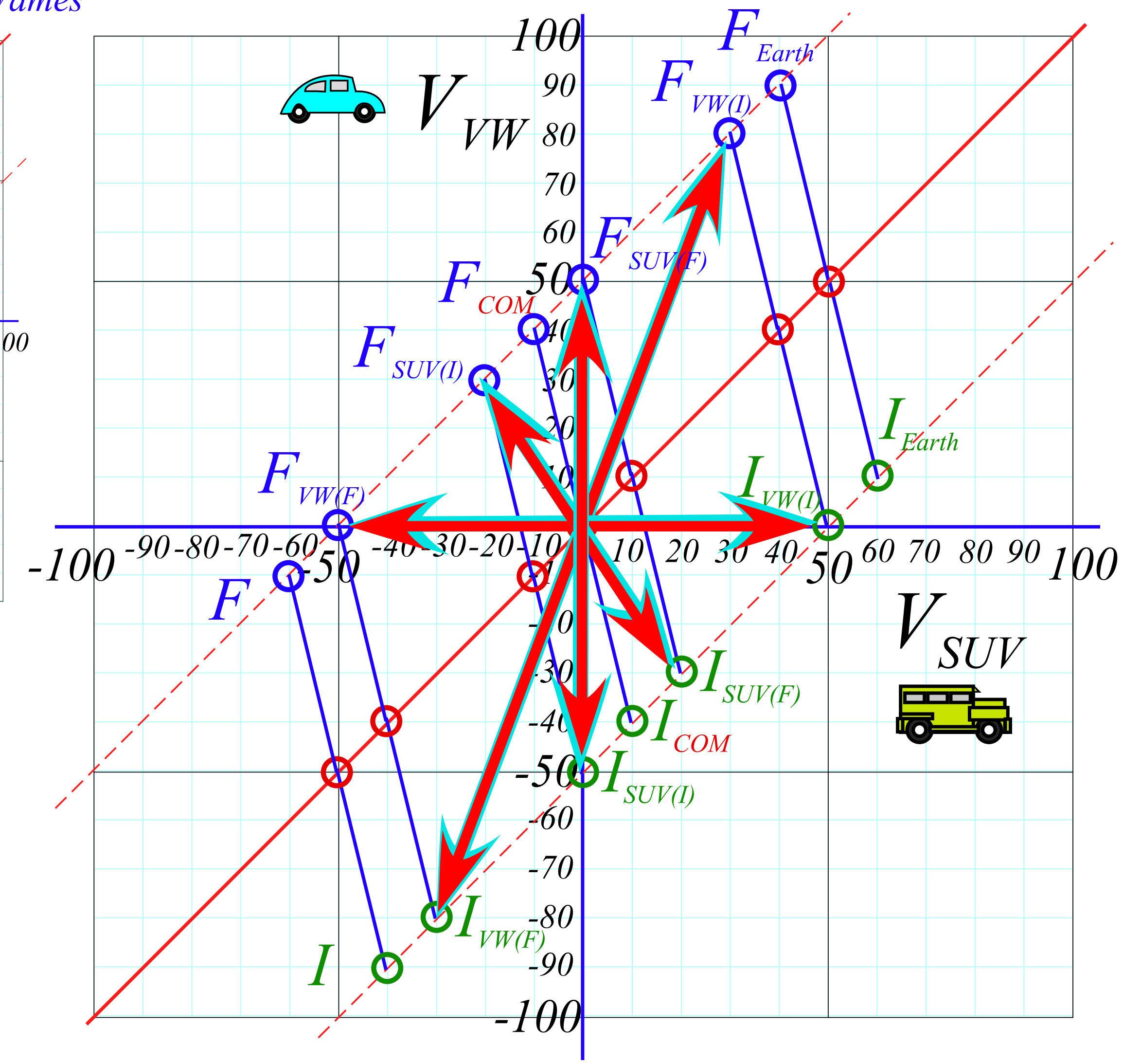


Fig. 2.5b
in Unit 1

Time-reversal (F-I) symmetry pairs (Four examples)



Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

Time reversal symmetry

→ *...of COM collisions*

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph slower

THE

COM Time-reversal symmetry pair (Just 1 case)

(a) Galileo transforms to COM frame

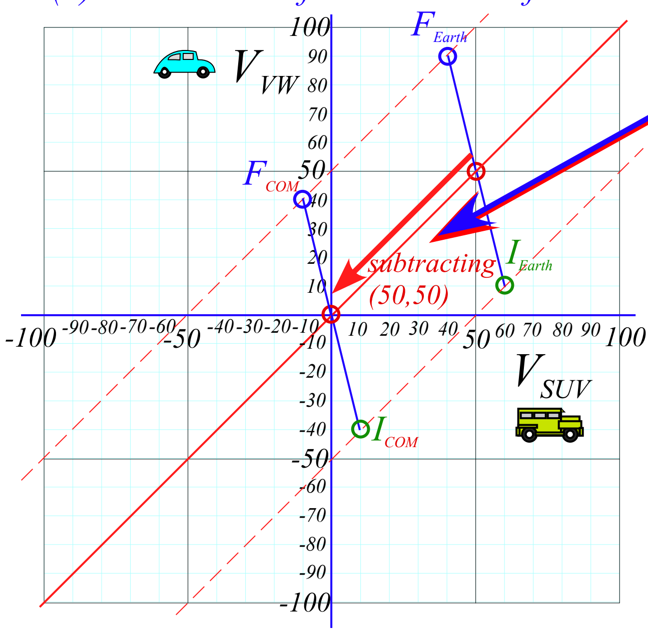


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

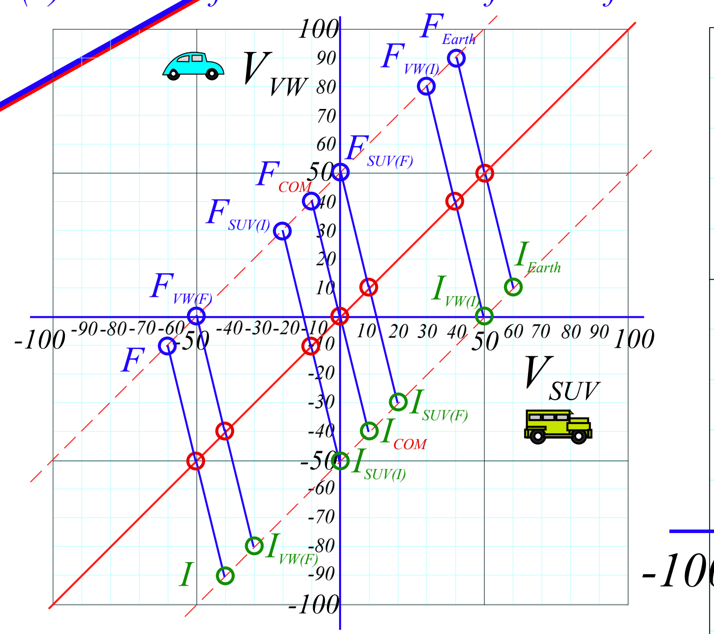
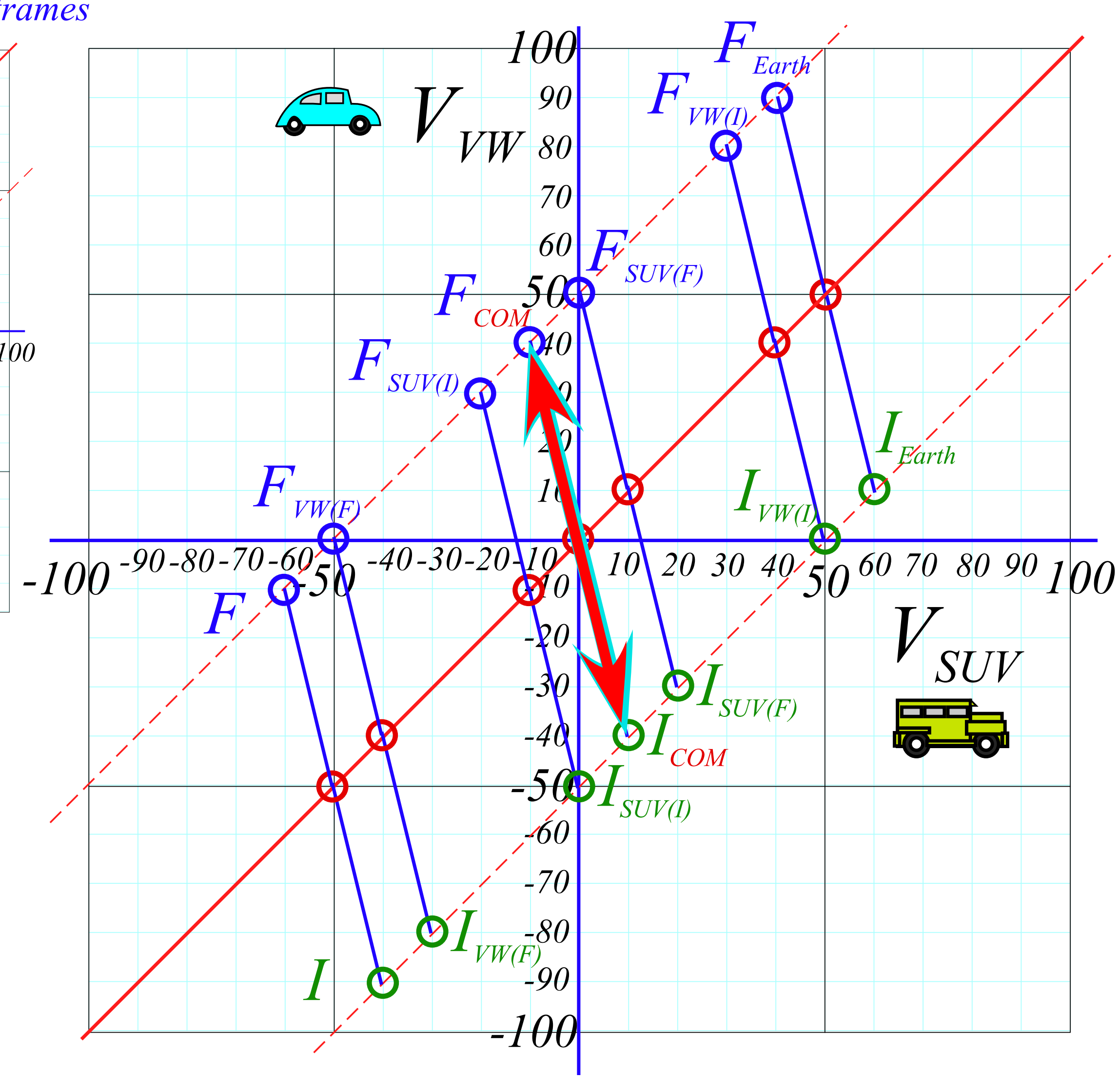


Fig. 2.5b
in Unit 1



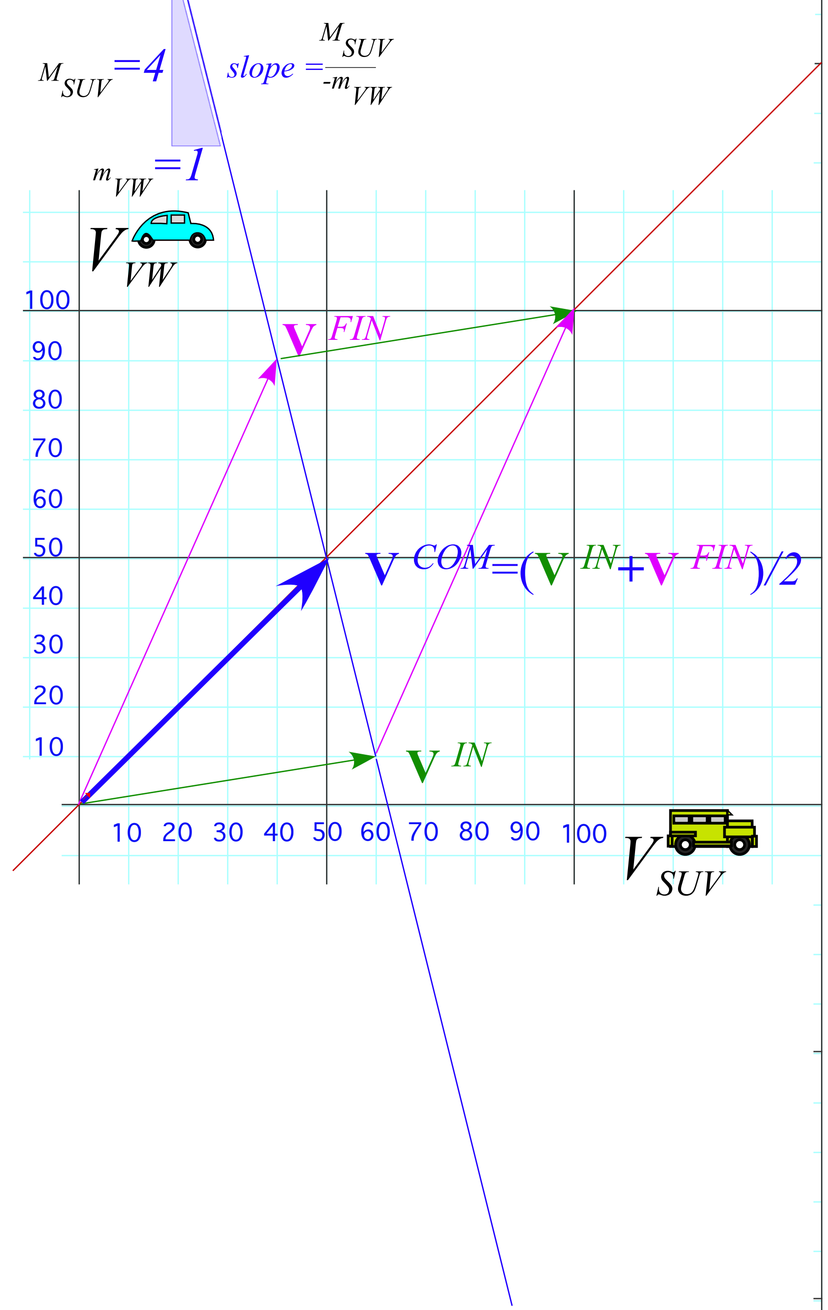
Algebra,Geometry, and Physics of momentum conservation axiom

- *Vector algebra of collisions*
- *Matrix or tensor algebra of collisions*
- *Deriving Energy Conservation* *Theorem*
- *Energy Ellipse geometry*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

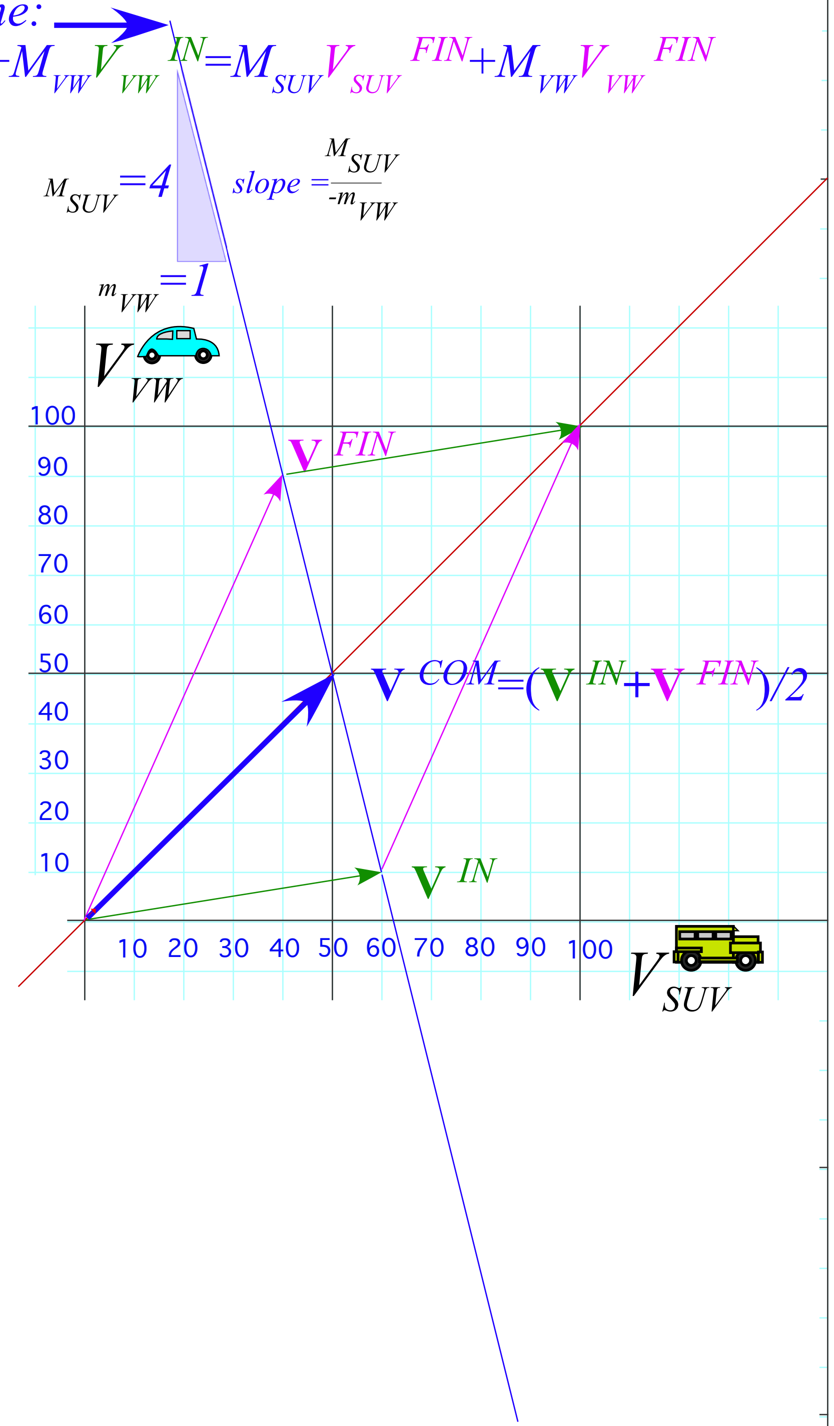
$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$



Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$



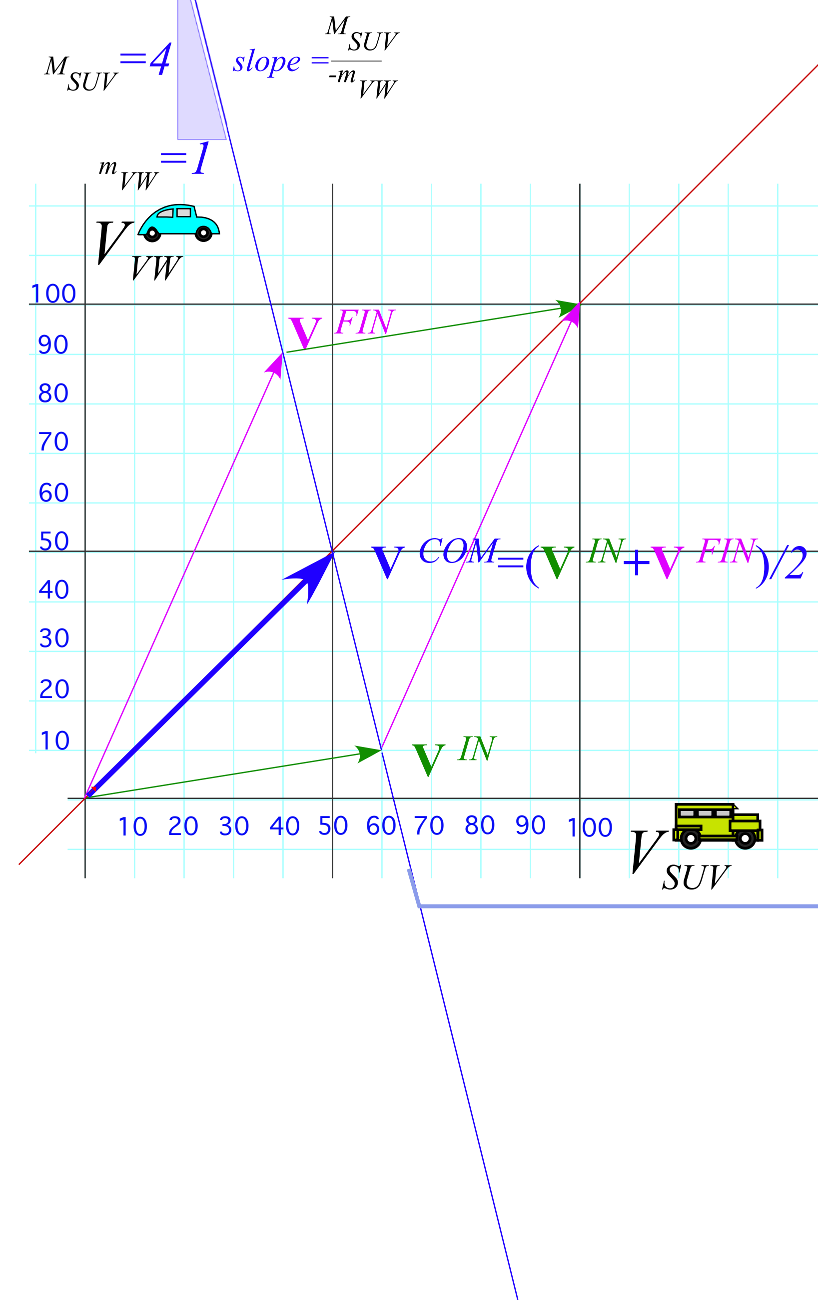
Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\ V_{VW}^{FIN}\end{pmatrix},\quad \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\ V_{VW}^{COM}\end{pmatrix},\quad \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\ V_{VW}^{IN}\end{pmatrix}$$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

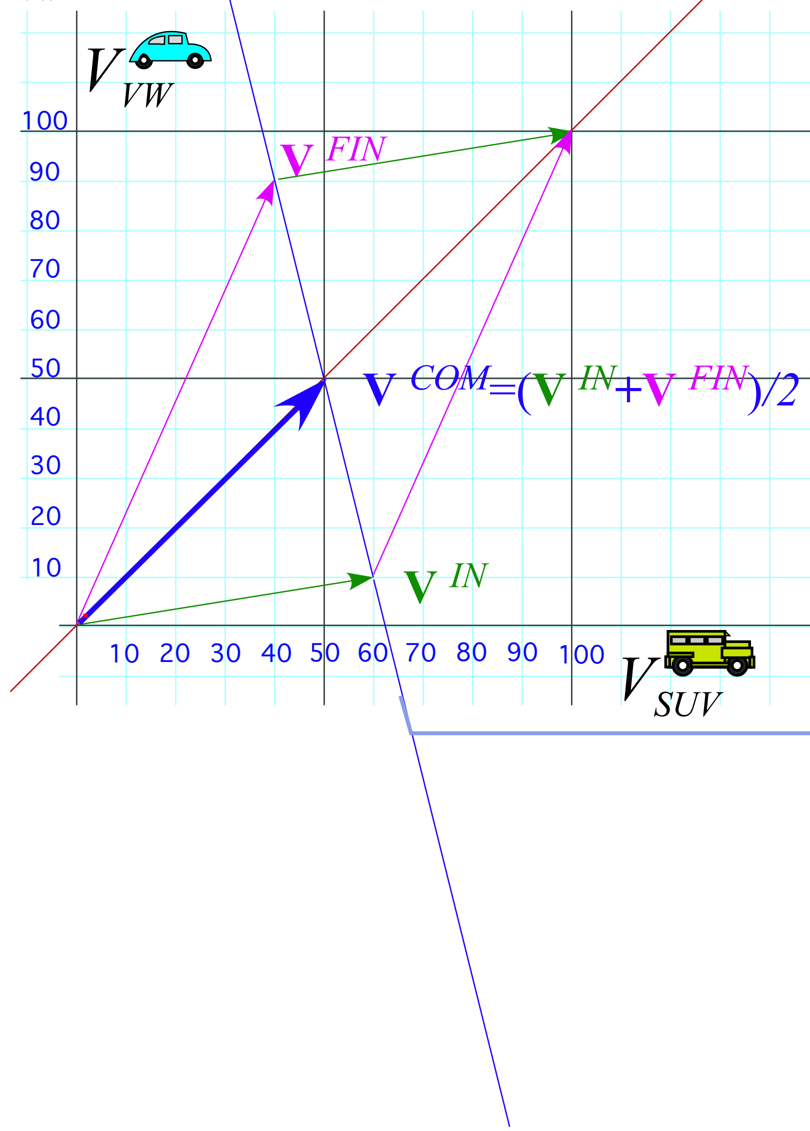
Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}, \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix}, \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

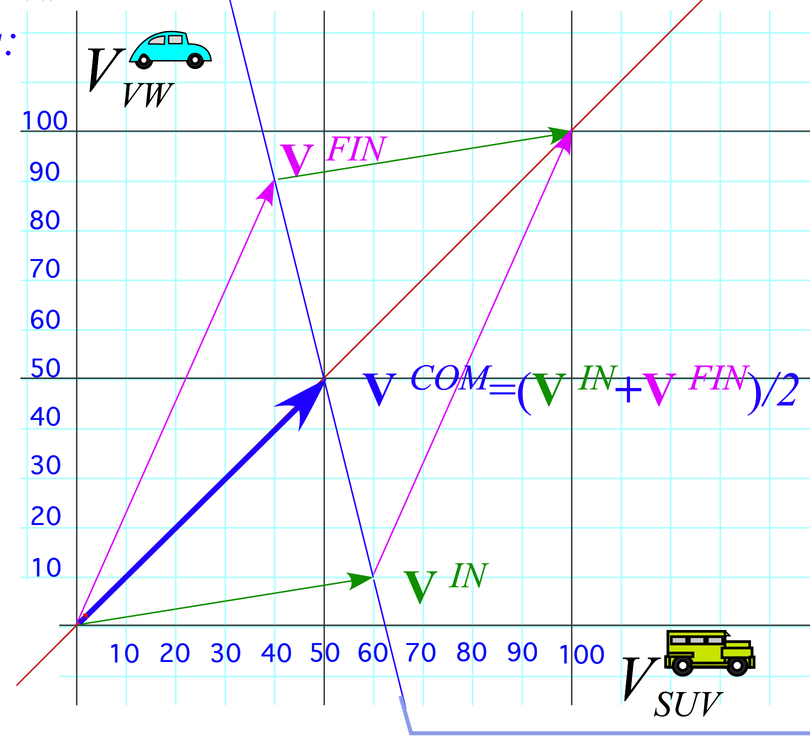
$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}=\begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$\mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}=\begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix}=V^{COM}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

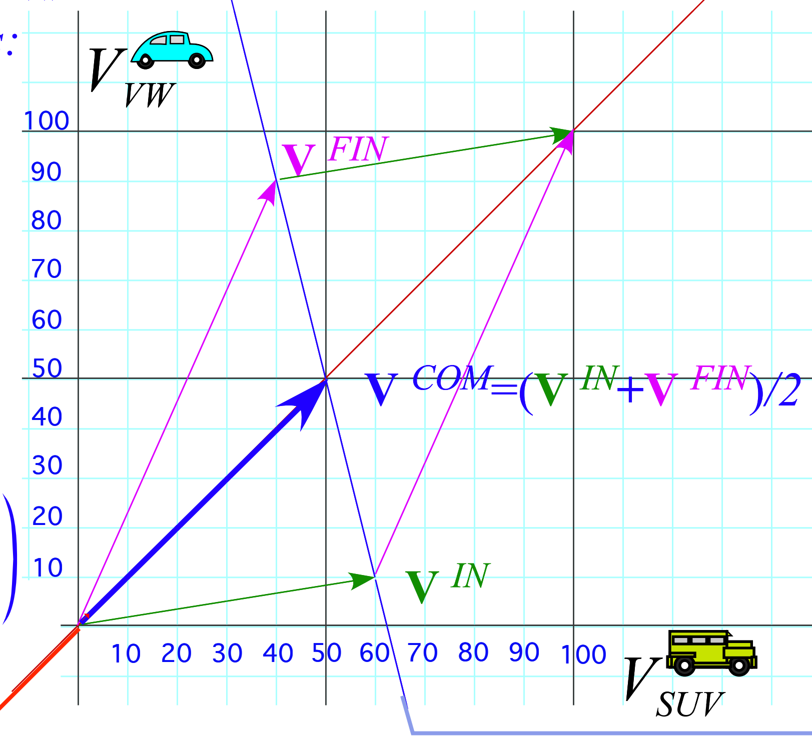
Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Define funny-unit vector : $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

→ *Matrix or tensor algebra of collisions*

Deriving Energy Conservation Theorem

Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

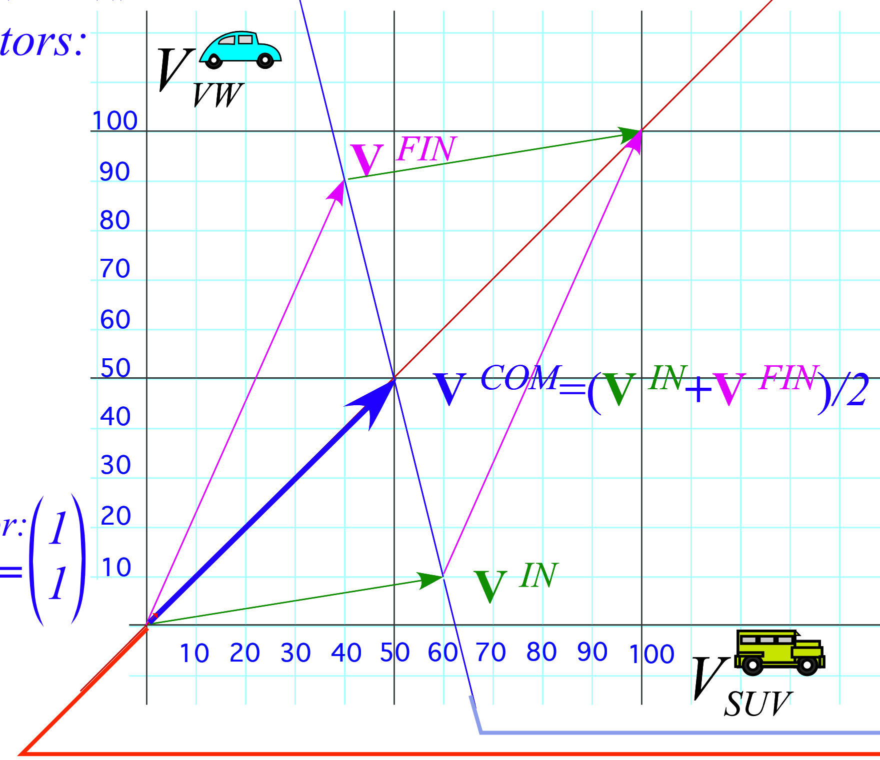
Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}=\begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$\mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}=\begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix}=V^{COM}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$=V^{COM}\mathbf{u}$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}, \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix}, \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Define funny-unit vector : $\mathbf{u}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

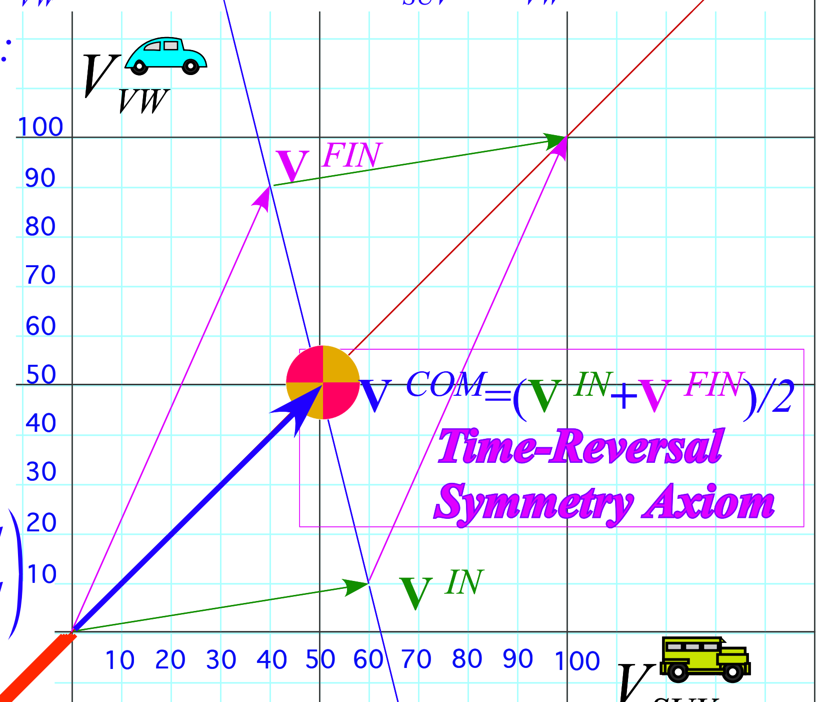
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}$$



...that give momentum vector:

$$\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$$

Define funny-unit vector : $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

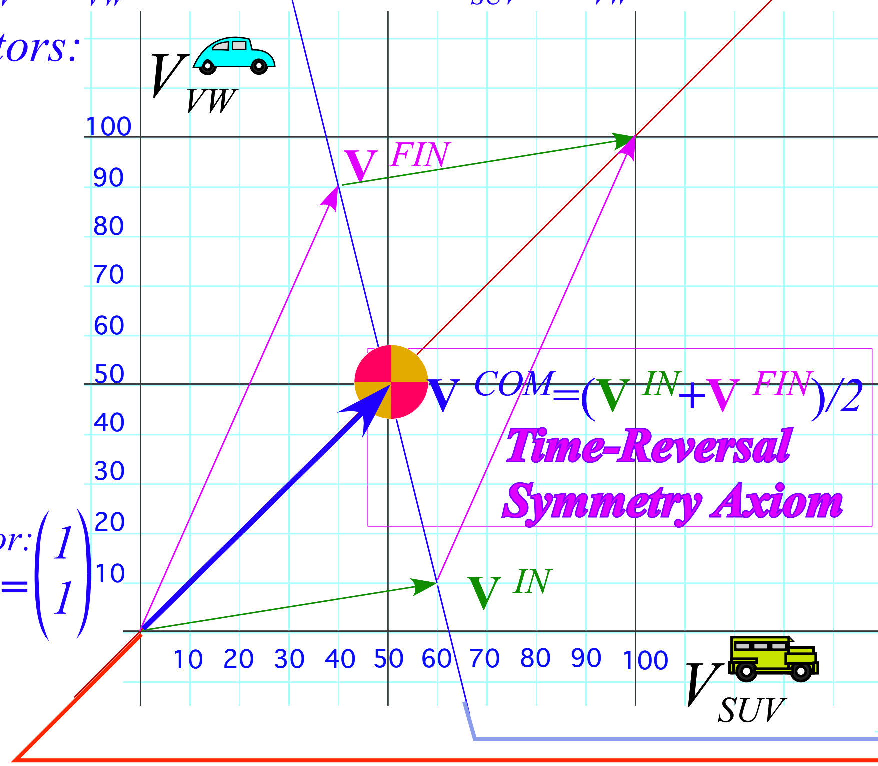
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$=V^{COM}\mathbf{u}$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}$$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by $\mathbf{u}\bullet$ *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Define funny-unit vector : $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

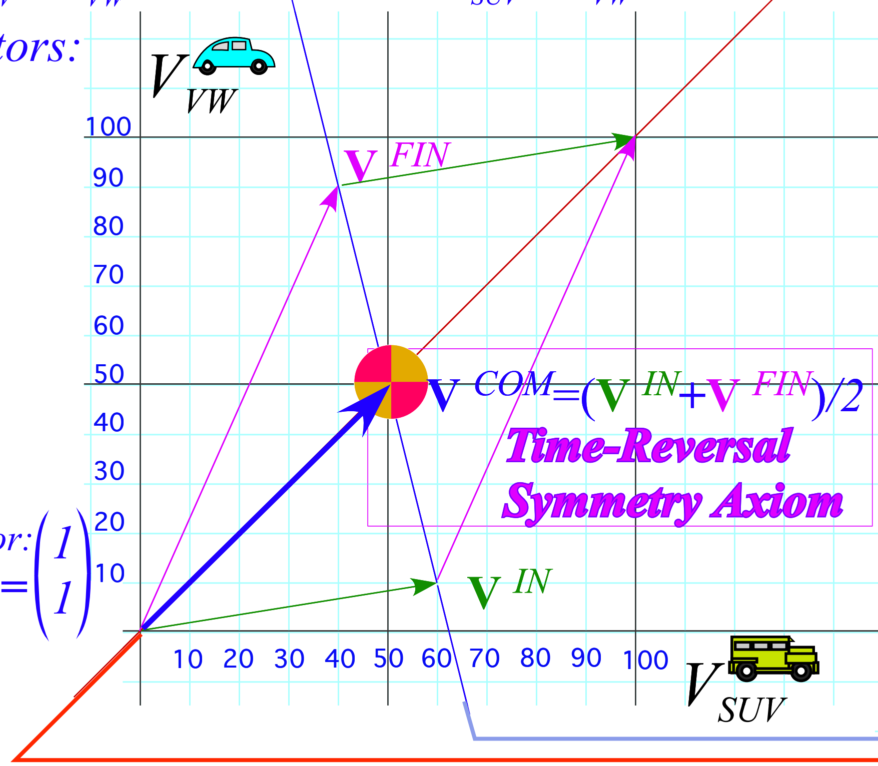
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$=V^{COM}\mathbf{u}$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}$$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

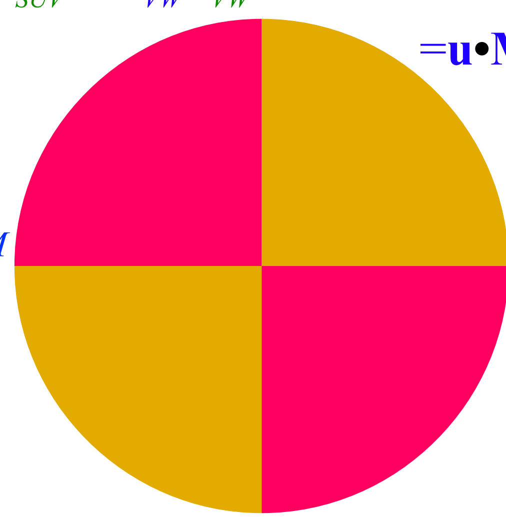
...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by $\mathbf{u}\bullet$ product)

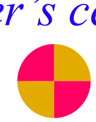
$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$



Denote Center of Momentum $\mathbf{V}^{COM}$

with engineer's centering symbol



Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

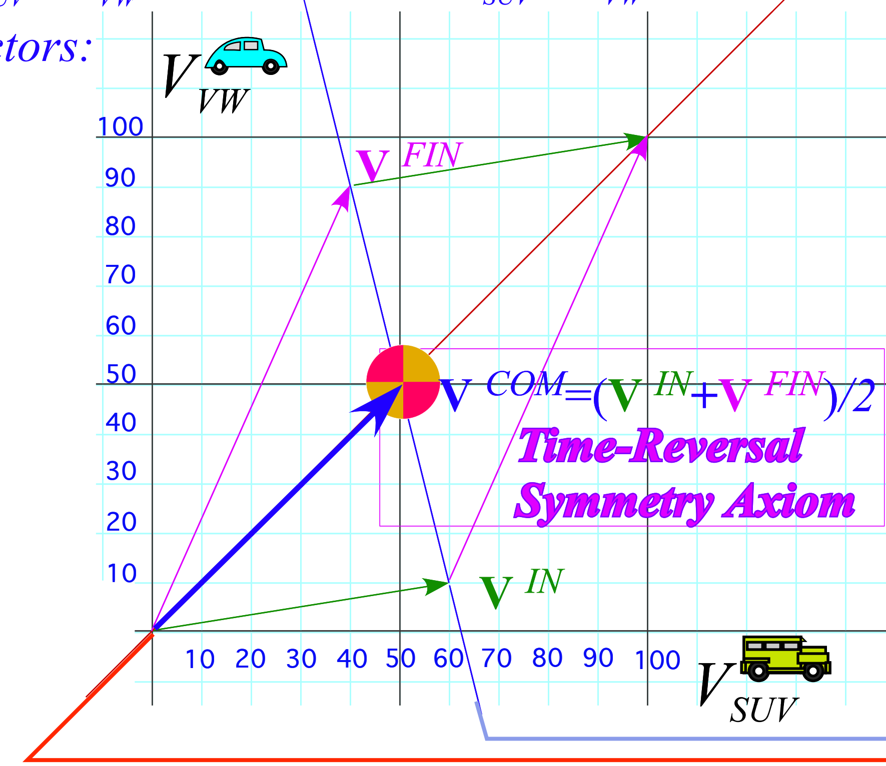
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}$$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by **u•** *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

Matrix or tensor algebra of collisions

→ *Deriving Energy Conservation <u>Theorem</u>*

Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\ V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\ 90\end{pmatrix}$$

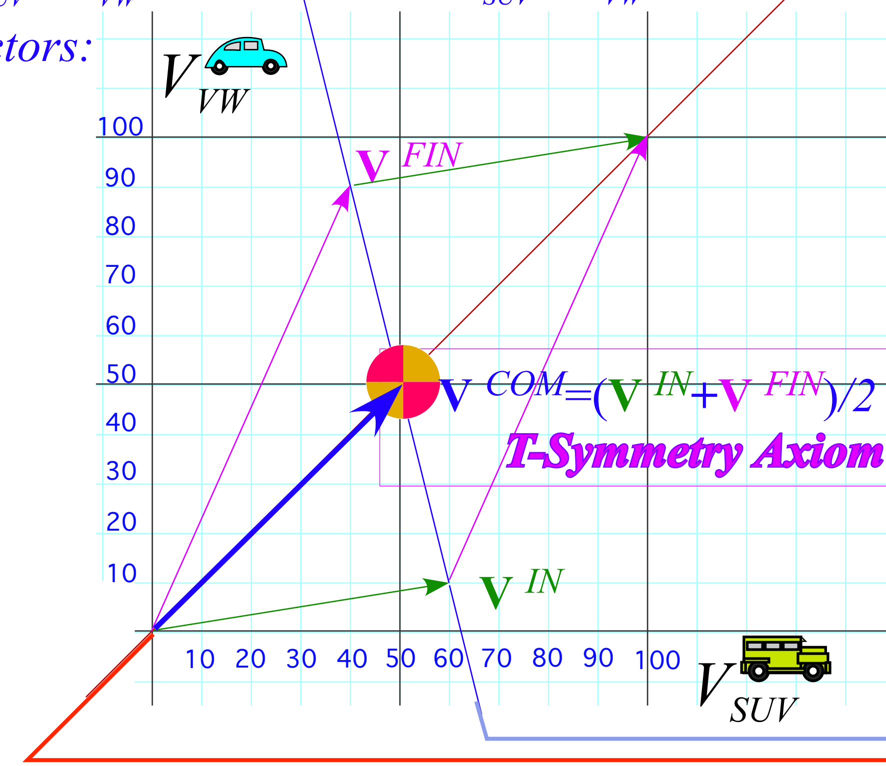
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\ V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\ 10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\ V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}$$



Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\ V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\ V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\ V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\ V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\ P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\ M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$.

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

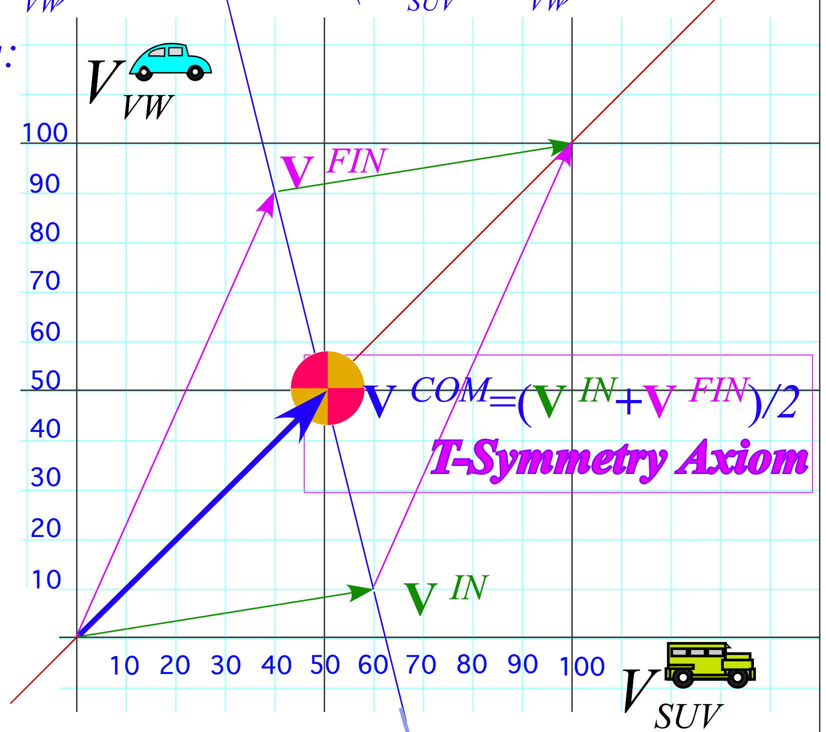
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$



M-symmetry $\mathbf{M}=\mathbf{M}^T$

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant. *(by* **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$.

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

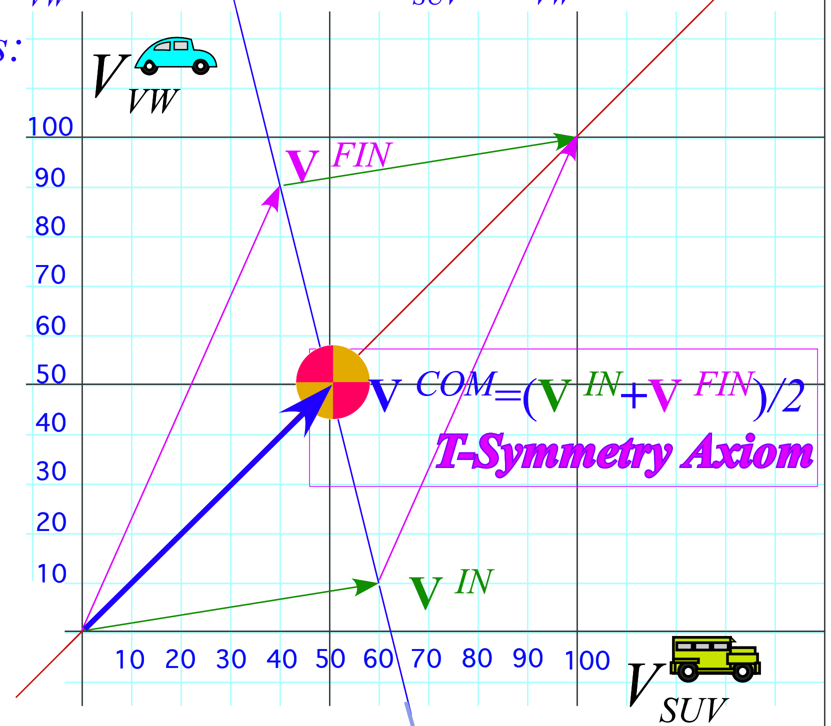
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$



M-symmetry $\mathbf{M}=\mathbf{M}^T$

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by **u•** *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. *Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}$$

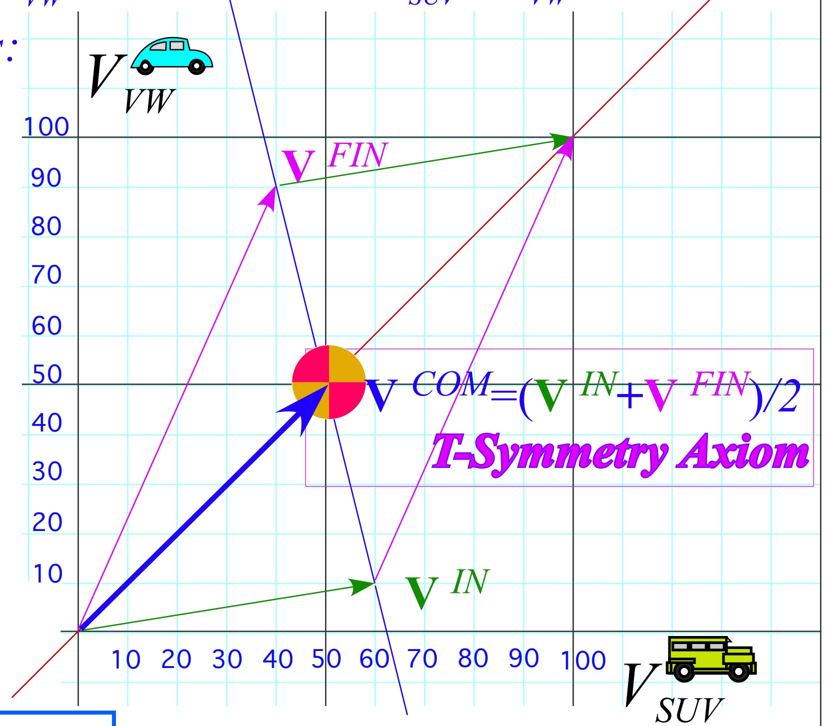
$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$

M-symmetry $\mathbf{M}=\mathbf{M}^T$



...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. *Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

By ***M-symmetry*** $\mathbf{M}=\mathbf{M}^T$: $\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$

this becomes:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}-1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=1/2\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
→ *Completing derivation of Energy Conservation <u>Theorem</u>*
Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

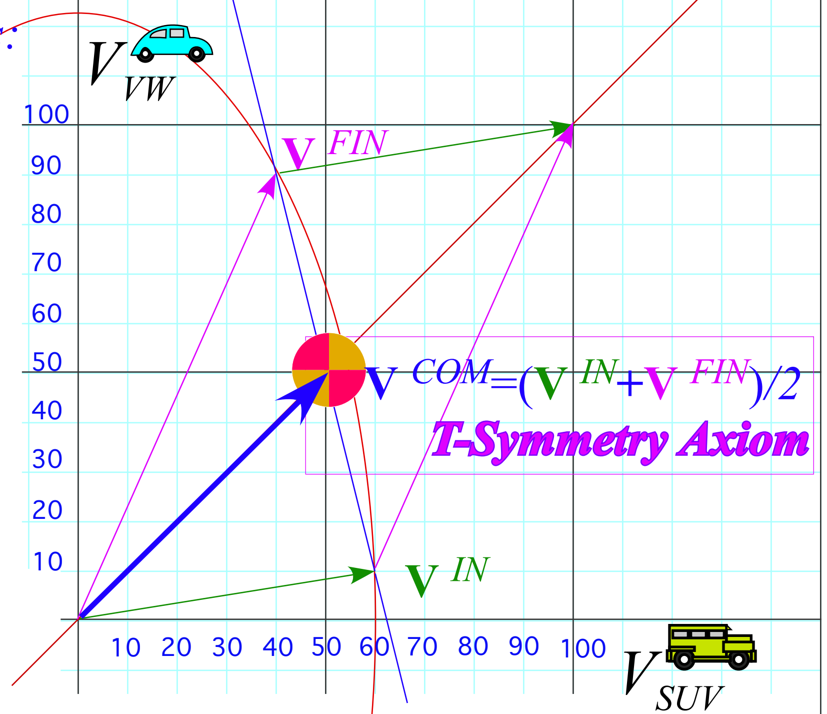
$$\mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix}=V^{COM}\begin{pmatrix} 1 \\ 1 \end{pmatrix}=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix} M_{SUV} & 0 \\ 0 & M_{VW} \end{pmatrix}=\mathbf{M}^{Transpose}$$



M-symmetry $\mathbf{M}=\mathbf{M}^T$

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix} M_{SUV} & 0 \\ 0 & M_{VW} \end{pmatrix}\begin{pmatrix} V_{SUV} \\ V_{VW} \end{pmatrix}=\begin{pmatrix} P_{SUV} \\ P_{VW} \end{pmatrix}=\begin{pmatrix} M_{SUV}V_{SUV} \\ M_{VW}V_{VW} \end{pmatrix}$

whose sum of components is constant.

(by $\mathbf{u}\bullet$ product)

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ gives:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. Substituting:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

By **M-symmetry** $\mathbf{M}=\mathbf{M}^T$: $\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$

this becomes:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}-1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=1/2\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

These are equations for energy conservation ellipse:

$$const.=\tfrac{1}{2}M_{SUV}V_{SUV}^{2}+\tfrac{1}{2}M_{VW}V_{VW}^{2}$$

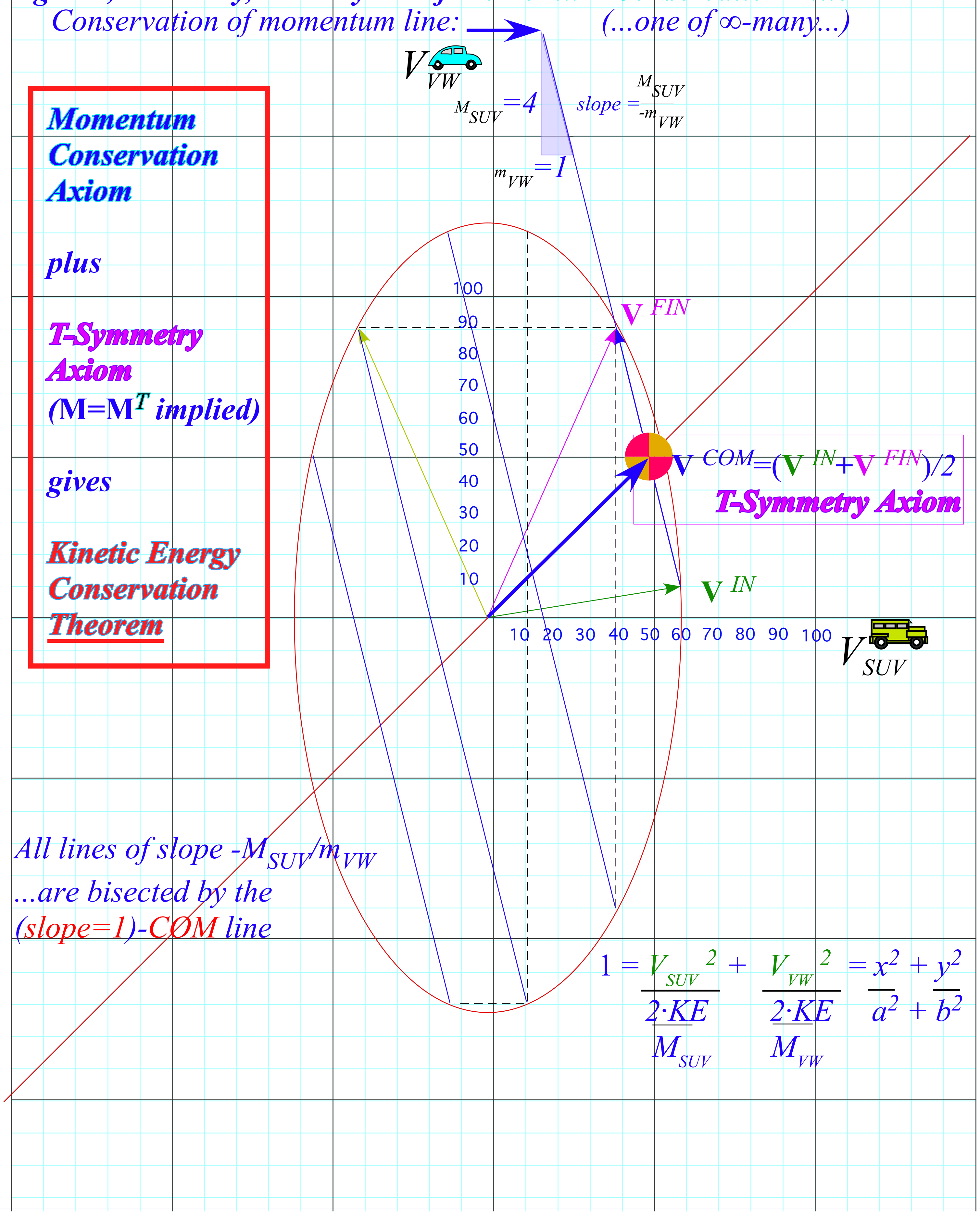
$$=\tfrac{1}{2}M_{SUV}V_{SUV}^{2}+\tfrac{1}{2}M_{VW}V_{VW}^{2}$$

=Kinetic Energy=KE is now defined
and proved a constant under T-Symmetry

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
→ *Energy Ellipse geometry*

Algebra, Geometry, and Physics of Momentum Conservation Axiom



$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM} - 1/2\,\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$= 1/2\,\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN} = 1/2\,\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

These are equations for energy conservation ellipse:

$$KE = {}^{1}/_{2} M_{SUV} V_{SUV}{}^{2} + {}^{1}/_{2} M_{VW} V_{VW}{}^{2}$$

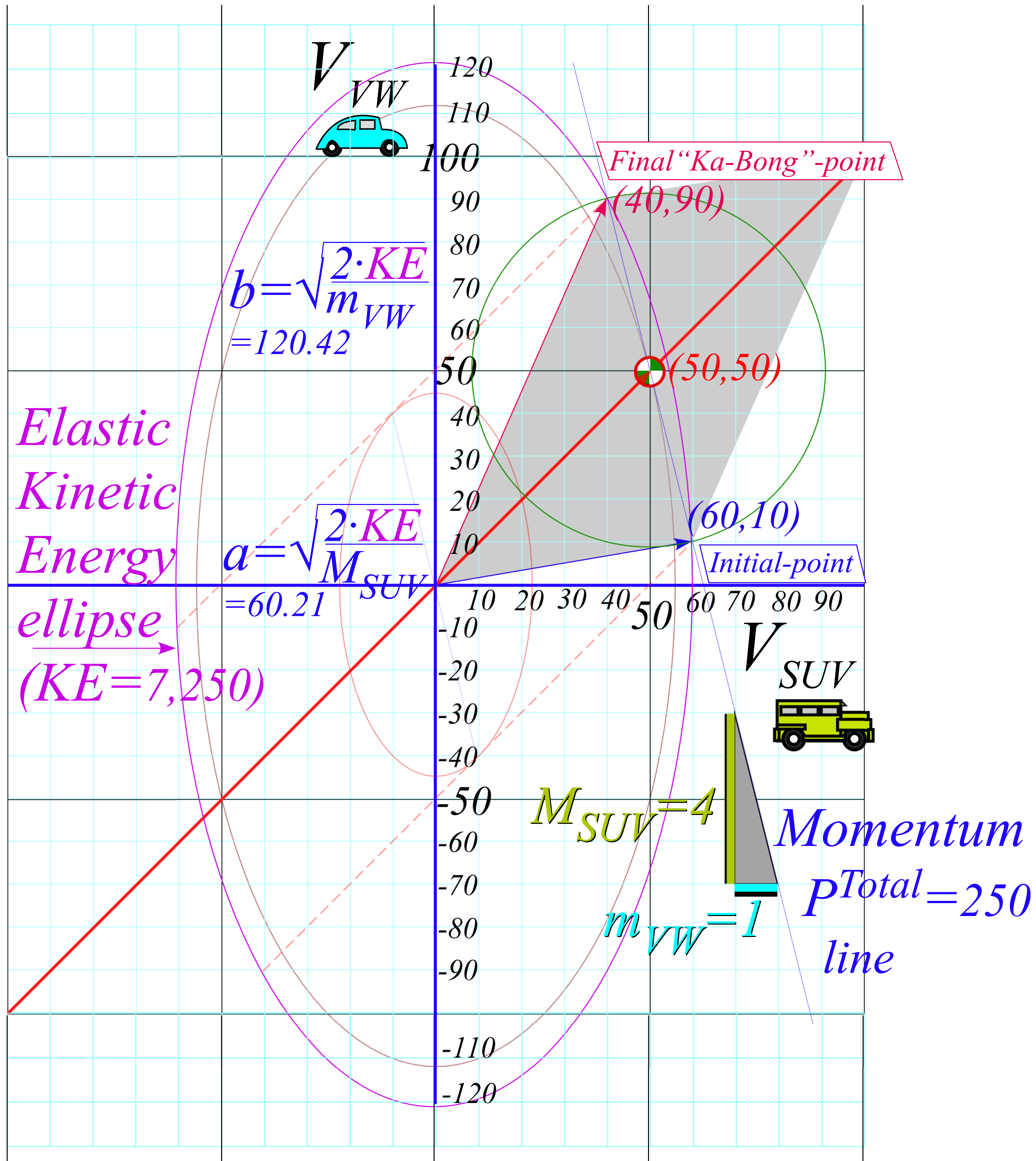


Fig. 3.1 a
in Unit 1

Fig. 3.1

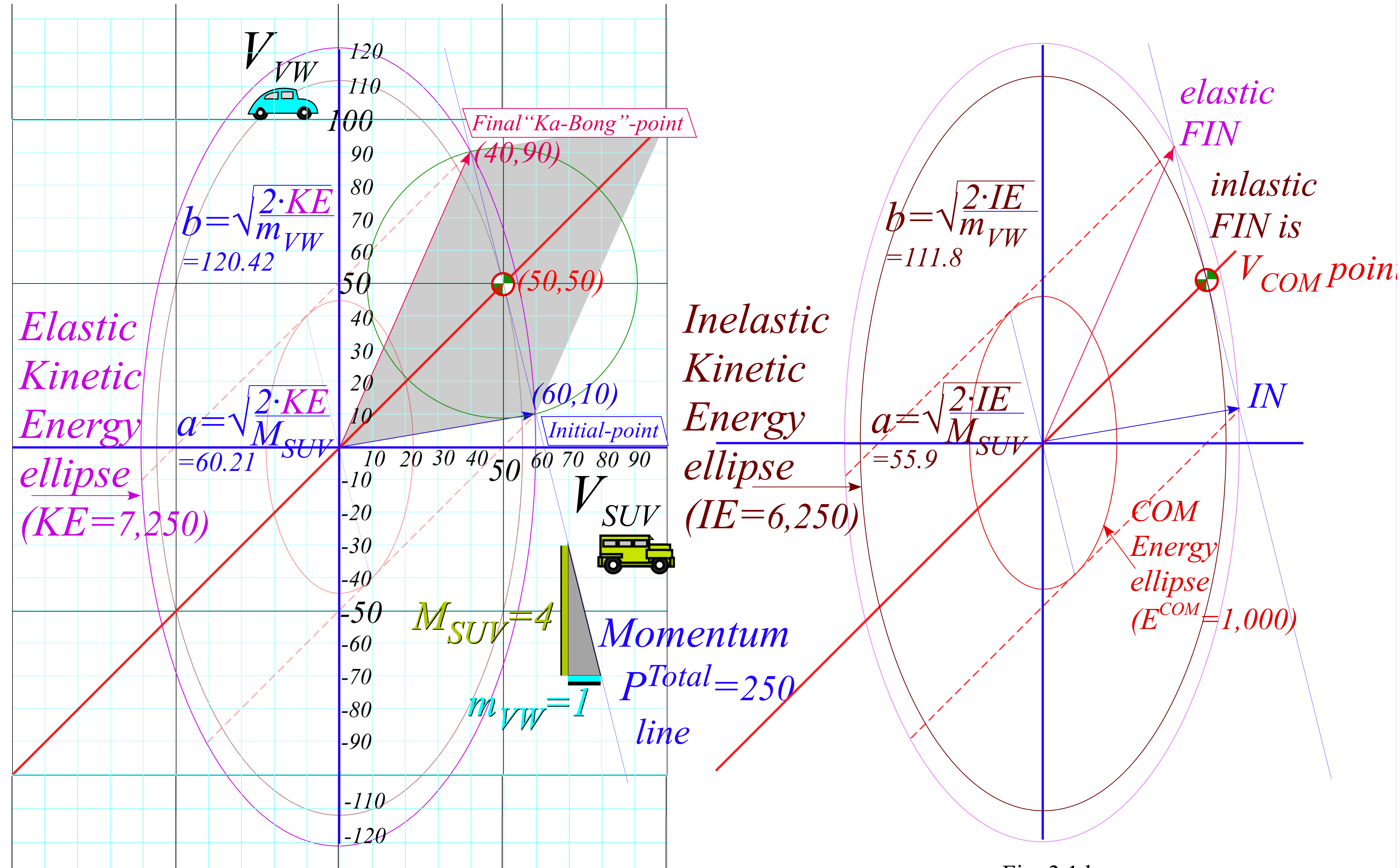


Fig. 3.1 a
in Unit 1

Fig. 3.1 b
in Unit 1

Fig. 3.1

As usual in physics, opposite extremes are easier to analyze
than the generic "real(er) world" in between!

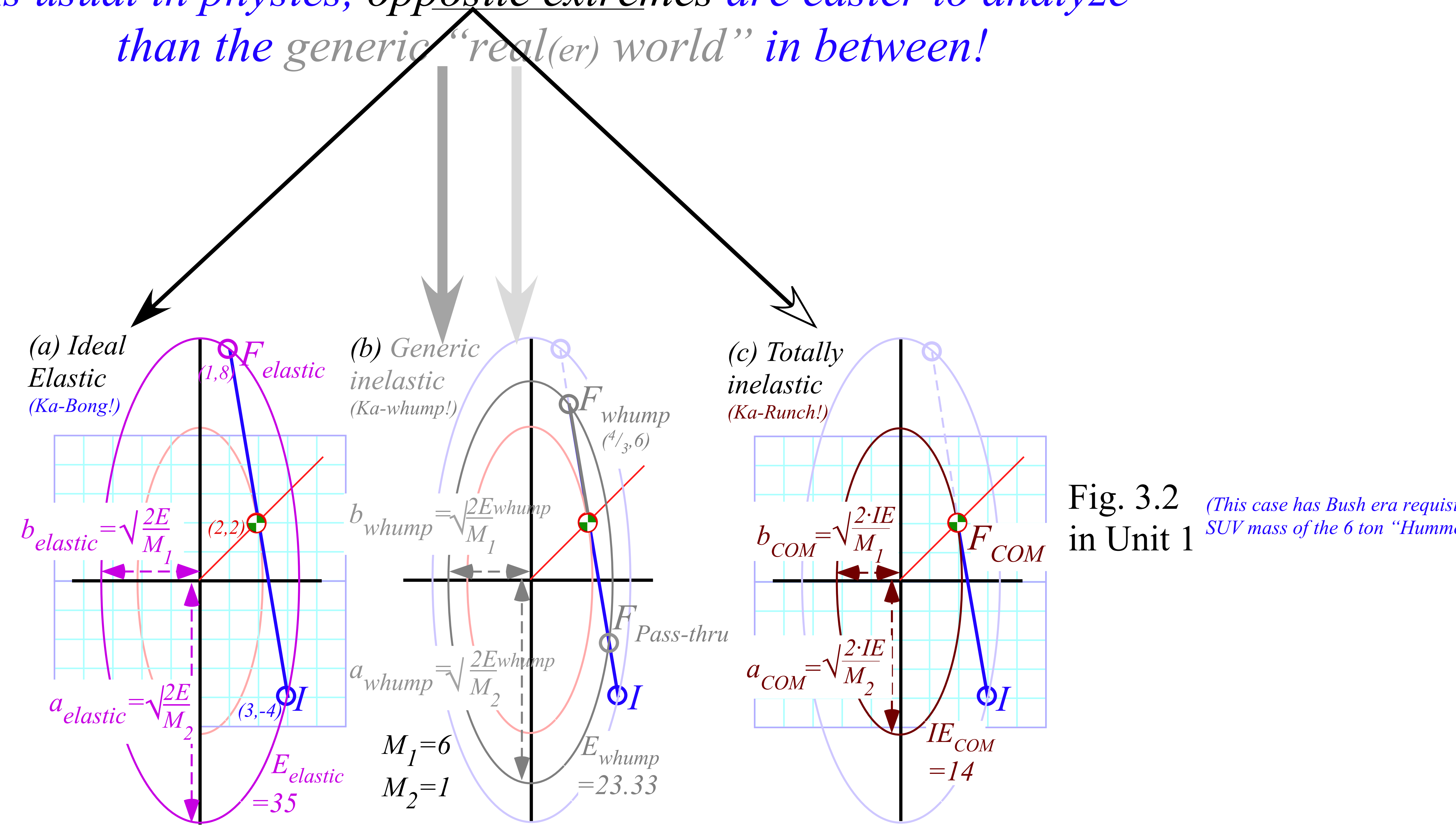


Fig. 3.2 in Unit 1

(This case has Bush era requisite SUV mass of the 6 ton "Hummer")

Next: **The X-2 pen-launcher**

Numerical details of collision tensor algebra

General Inertia Tensor M or inertia matrix of n^2 coefficients M_{jk} for dimension n=2, 3,....

$$\left.\begin{aligned} P_1 &= M_{11}V_1 + M_{12}V_2 \\ P_2 &= M_{21}V_1 + M_{22}V_2 \end{aligned}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

With 45° diagonal $\mathbf{V}^{COM}$ so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = M_{Total} V^{COM}$$

General Inertia Tensor M or inertia matrix of n^2 coefficients M_{jk} for dimension $n=2, 3,....$

$$\left.\begin{array}{l} P_1 = M_{11}V_1 + M_{12}V_2 \\ P_2 = M_{21}V_1 + M_{22}V_2 \end{array}\right\} \quad denoted : \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or : \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

With 45° diagonal $\mathbf{V}^{COM}$ so: . $\quad V_1^{COM} = V_2^{COM} \equiv V^{COM}$

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = M_{Total} V^{COM}$$

A product of total momentum P_{Total} and V^{COM} is expressed by *tensor quadratic forms* **v•M•u**

$$V^{COM} P_{Total} = \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = V^{COM} M_{Total} V^{COM}$$

General Inertia Tensor M or inertia matrix of n^2 coefficients M_{jk} for dimension n=2, 3,....

$$\left.\begin{aligned} P_1 &= M_{11}V_1 + M_{12}V_2 \\ P_2 &= M_{21}V_1 + M_{22}V_2 \end{aligned}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

With 45° diagonal $\mathbf{V}^{COM}$ so: . $\quad V_1^{COM} = V_2^{COM} \equiv V^{COM}$

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = M_{Total} V^{COM}$$

A product of total momentum P_{Total} and V^{COM} is expressed by *tensor quadratic forms* **v•M•u**

$$V^{COM} P_{Total} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = V^{COM} M_{Total} V^{COM}$$

Writing this out with the numbers appearing in Fig. 3.1 where $V^{COM} = 50$.

$$50 P_{Total} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix} = 50 M_{Total} 50 = 12{,}500$$

$$= 100 \cdot 125 = 100 \cdot 125 = 50 \cdot 250$$

General Inertia Tensor M or inertia matrix of n^2 coefficients M_{jk} for dimension n=2, 3,....

$$\left.\begin{array}{l} P_1 = M_{11}V_1 + M_{12}V_2 \\ P_2 = M_{21}V_1 + M_{22}V_2 \end{array}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

With 45° diagonal $\mathbf{V}^{COM}$ so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = M_{Total} V^{COM}$$

A product of total momentum P_{Total} and V^{COM} is expressed by *tensor quadratic forms* **v•M•u**

$$V^{COM} P_{Total} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = V^{COM} M_{Total} V^{COM}$$

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$$= 100 \cdot 125 = 100 \cdot 125 = 50 \cdot 250$$

P_{Total} =*250* is the same at IN, FIN, and COM. Now use *T*-symmetry: $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$

$$V^{COM} P_{Total} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2}$$

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$$\left.\begin{array}{l} P_1 = M_{11}V_1 + M_{12}V_2 \\ P_2 = M_{21}V_1 + M_{22}V_2 \end{array}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

With 45° diagonal $\mathbf{V}^{COM}$ so: . $\quad V_1^{COM} = V_2^{COM} \equiv V^{COM}$

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = M_{Total} V^{COM}$$

A product of total momentum P_{Total} and V^{COM} is expressed by *tensor quadratic forms* **v•M•u**

$$V^{COM} P_{Total} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = V^{COM} M_{Total} V^{COM}$$

Writing this out with the numbers appearing in Fig. 3.1 where $V^{COM} = 50$.

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$$= 100 \cdot 125 \qquad = 100 \cdot 125 \qquad = 50 \cdot 250$$

P_{Total} =*250* is the same at IN, FIN, and COM. Now use *T*-symmetry: $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$

$$V^{COM} P_{Total} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2}$$

$$V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{4}$$

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$$\left.\begin{array}{l} P_1 = M_{11}V_1 + M_{12}V_2 \\ P_2 = M_{21}V_1 + M_{22}V_2 \end{array}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

With 45° diagonal $\mathbf{V}^{COM}$ so: . $\quad V_1^{COM} = V_2^{COM} \equiv V^{COM}$

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = M_{Total} V^{COM}$$

A product of total momentum P_{Total} and V^{COM} is expressed by *tensor quadratic forms* **v•M•u**

$$V^{COM} P_{Total} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = V^{COM} M_{Total} V^{COM}$$

Writing this out with the numbers appearing in Fig. 3.1 where $V^{COM} = 50$.

$$50 P_{Total} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix} = 50 M_{Total} 50 = 12{,}500$$

$$= 100 \cdot 125 = 100 \cdot 125 = 50 \cdot 250$$

P_{Total} =250 is the same at IN, FIN, and COM. Now use T-symmetry: $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$

$$V^{COM} P_{Total} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2} \bullet \overleftrightarrow{\mathbf{M}} \bullet \frac{\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN}}{2}$$

$$V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{4}$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE_{Elastic} = \frac{1}{2} \overleftarrow{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at IN and FIN.

$$V^{COM} P_{Total} - \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE_{Elastic}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE_{Elastic}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE_{Elastic} = \frac{1}{2} \vec{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at IN and FIN.

$$V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE_{Elastic}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE_{Elastic}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

However, kinetic energy $IE = \frac{1}{2} \vec{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ in Fig. 3.1 at COM is reduced by *1,000* to zero.

$$KE_{Inelastic} = \frac{1}{2} V^{COM} P_{Total} = \frac{\vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE_{Elastic} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$\frac{12{,}500}{2} = 6{,}250 = 3{,}625 + 2{,}625 = IE$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE_{Elastic} = \frac{1}{2} \vec{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at IN and FIN.

$$V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE_{Elastic}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE_{Elastic}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

However, kinetic energy $IE = \frac{1}{2} \vec{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ in Fig. 3.1 at COM is reduced by *1,000* to <u>zero</u>.

$$KE_{Inelastic} = \frac{1}{2} V^{COM} P_{Total} = \frac{\vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE_{Elastic} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$\frac{12{,}500}{2} = 6{,}250 = 3{,}625 + 2{,}625 = IE$$

The difference is inelastic “crunch” energy $KE - IE$ or, for elastic cases, potential energy of compression.

$$KE_{Elastic} - KE_{Inelastic} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$1{,}000 = 3{,}625 - 2{,}625 = KE - IE$$

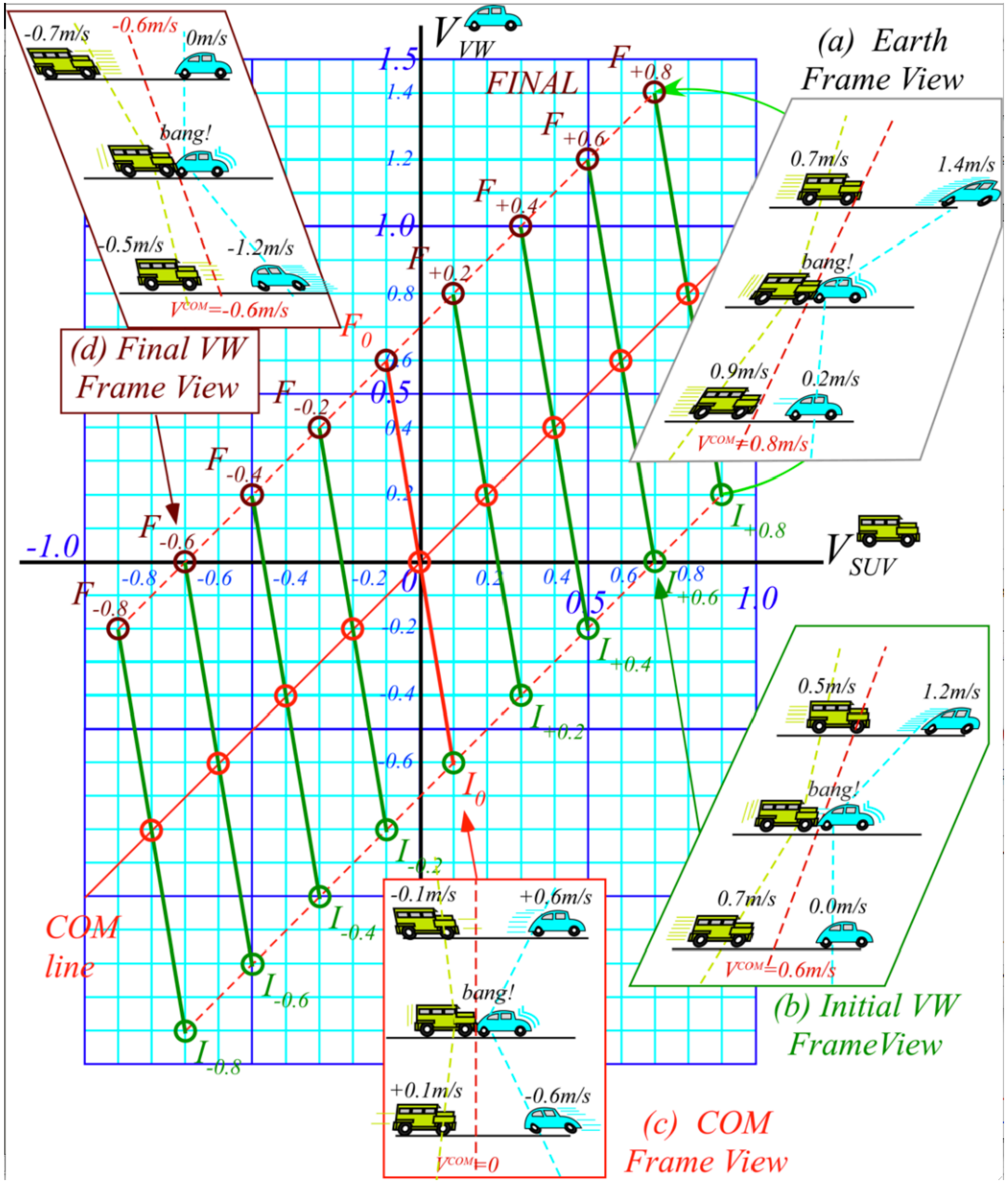

Fig. 3.4 Galilean Frame Views of collision like Fig. 2.5 or Fig. 3.1 with Bush (6:1) SUV.
(a) Earth frame view (b) Initial VW frame (VW initially fixed)
(c) COM frame view (d) Final VW frame (VW ends up fixed)

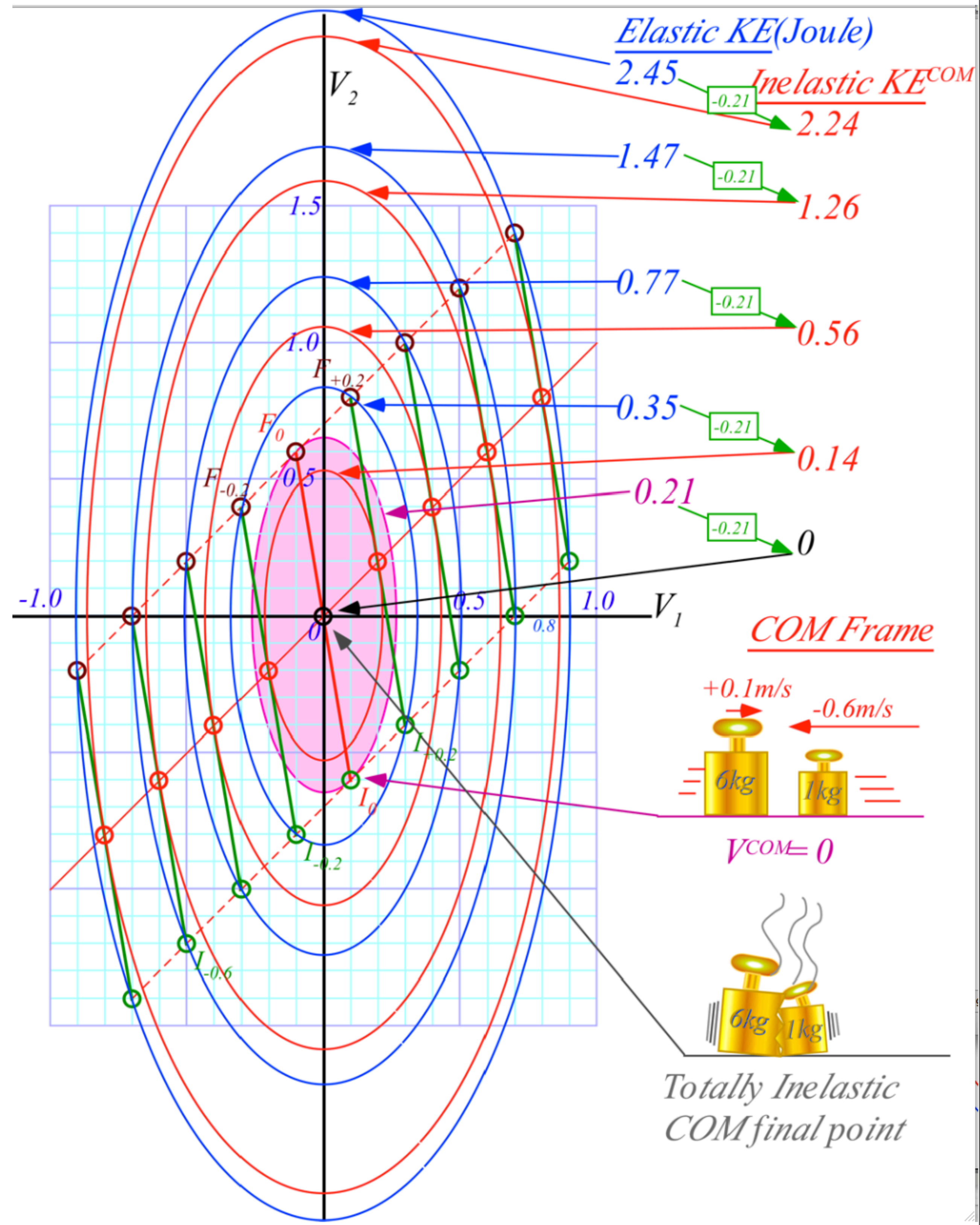

Fig. 3.5 Momentum (P=const.)-lines and energy (KE=const.)-ellipses appropriate for Fig. 3.4.