Lecture 8
Thur. 9.18.2014

Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

Compare mks units of Coulomb Electrostatic vs. Gravity

Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*

Contact-geometry of potential curve(s)

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Introducing 2D IHO orbits and phasor geometry

Phasor "clock" geometry

Geometry of common power-law potentials

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→ *"Zig-Zag" exponential geometry*

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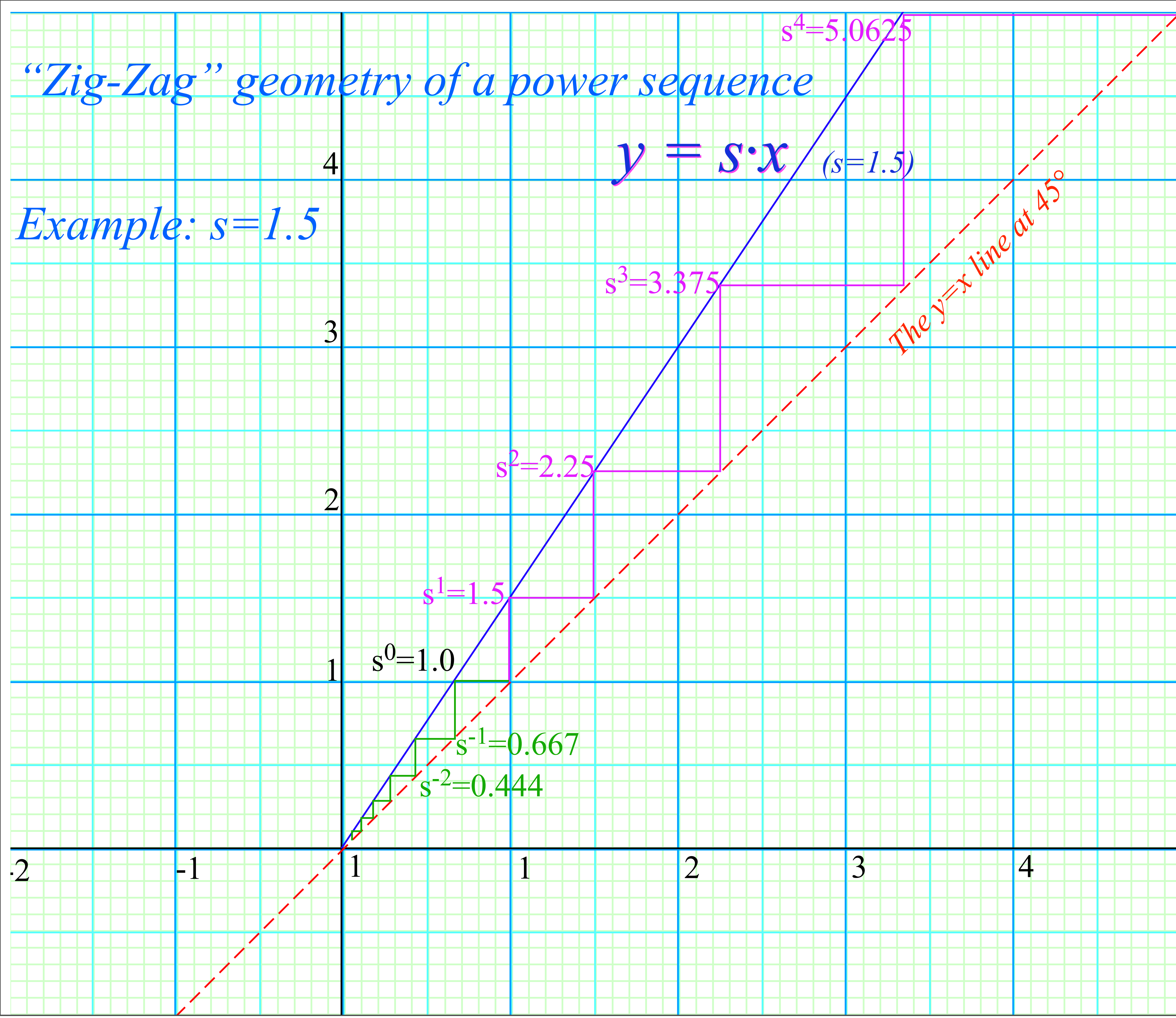
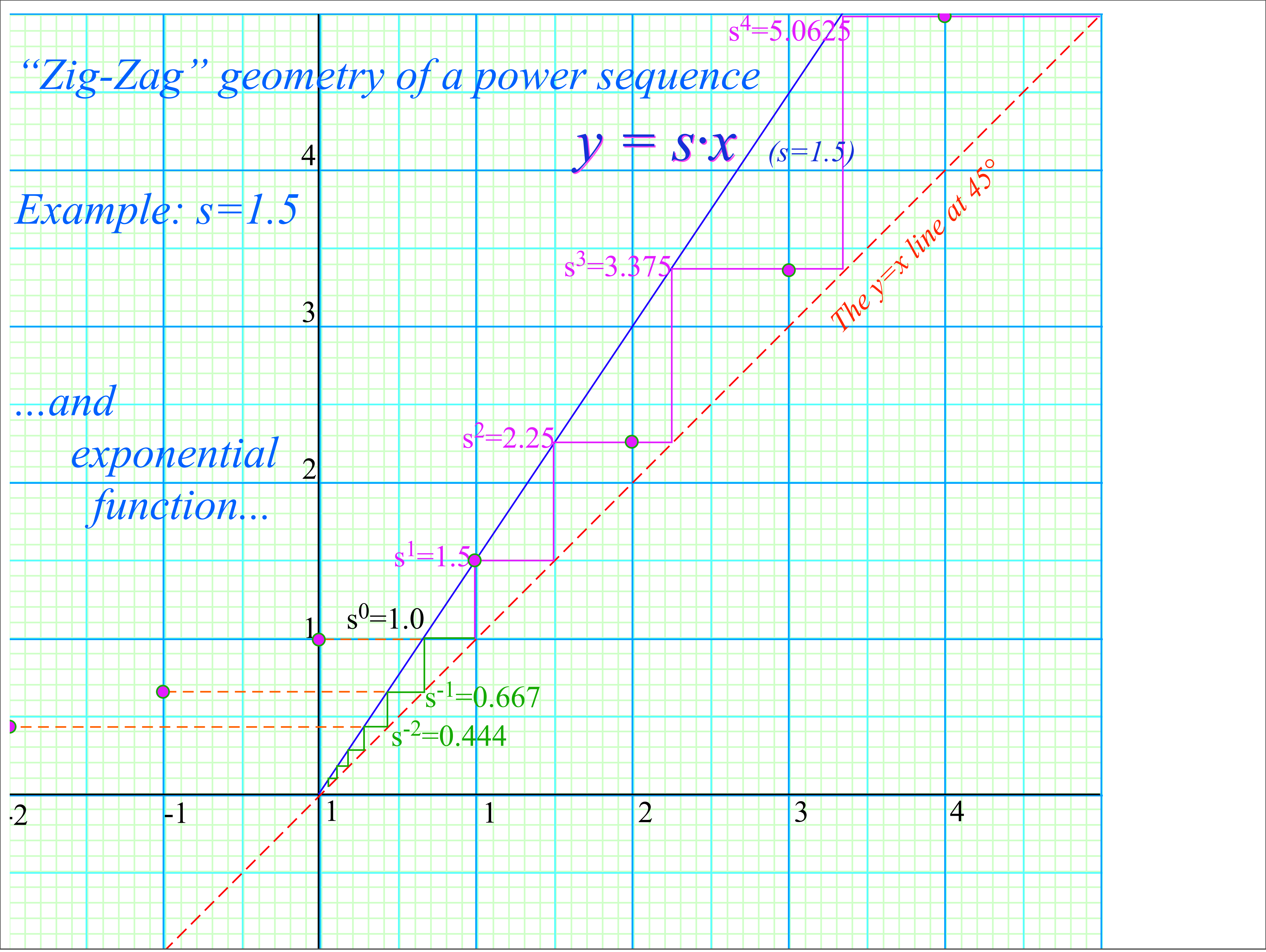


"Zig-Zag" geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
s4=5.0625
s3=3.375
s2=2.25
s1=1.5
s0=1.0
s-1=0.667
s-2=0.444
The y=x line at 45°
4
3
2
1
-2
-1
1
1
2
3
4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...and
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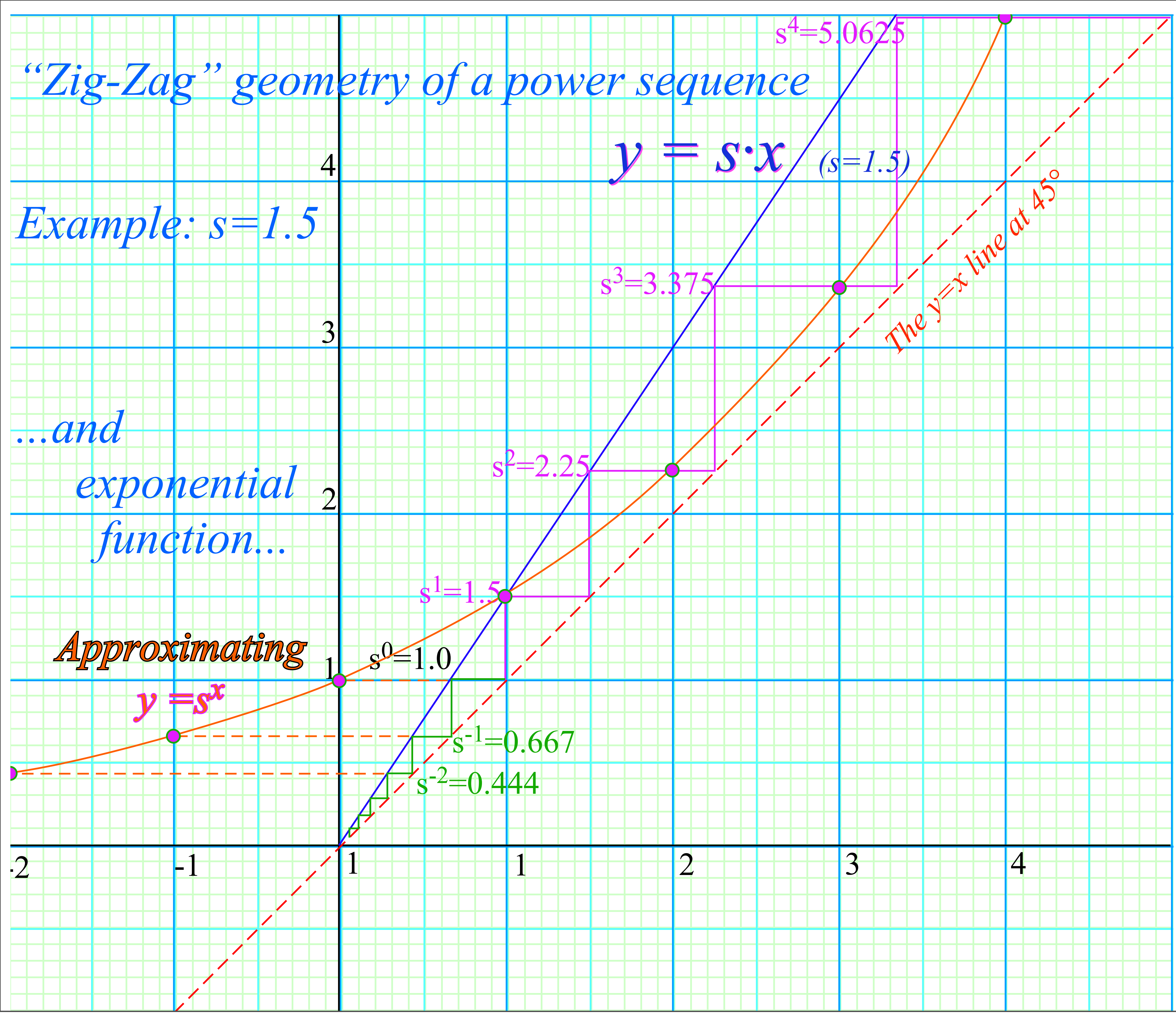
"Zig-Zag" geometry of a power sequence
Example: s=1.5
...and exponential function...
Approximating
y = s^x
y = s·x (s=1.5)
The y=x line at 45°
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s^3=3.375
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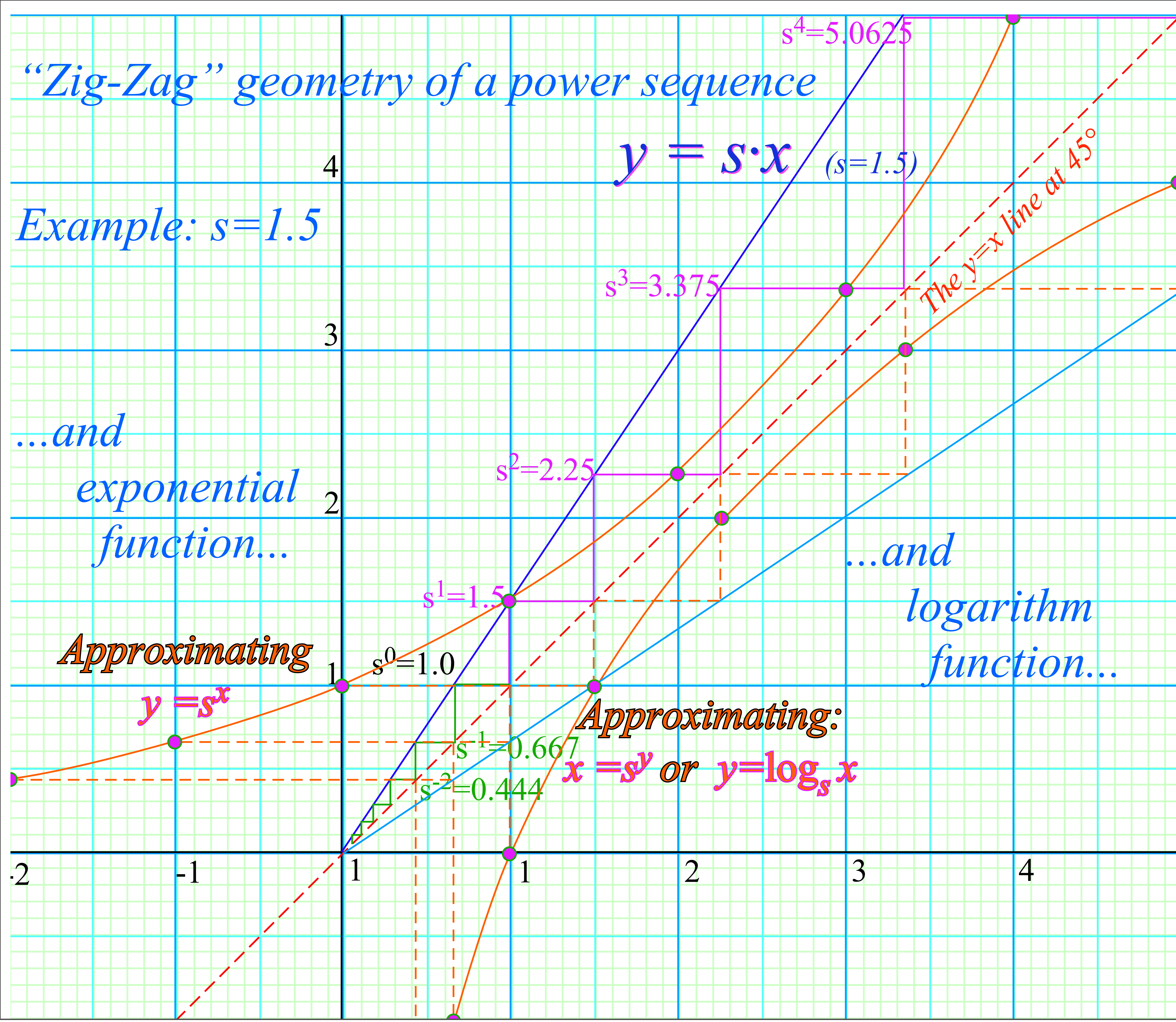

"Zig-Zag" geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
...and exponential function...
Approximating y =s^x
...and logarithm function...
Approximating: x =s^y or y=log_s x
The y=x line at 45°
s^4=5.0625
s^3=3.375
s^2=2.25
s^1=1.5
s^0=1.0
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Geometry of common power-law potentials

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→ *Projective or perspective geometry*

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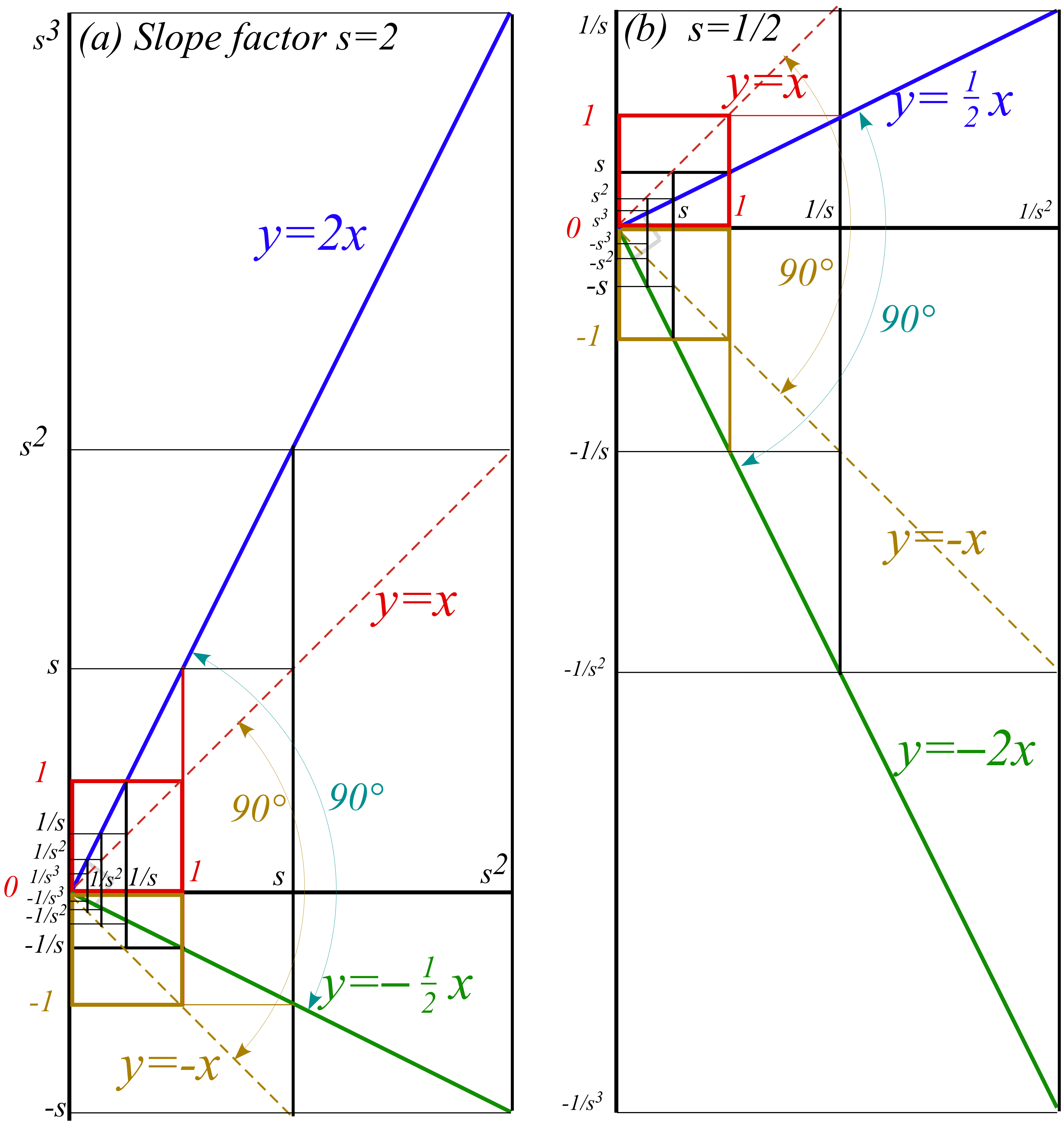


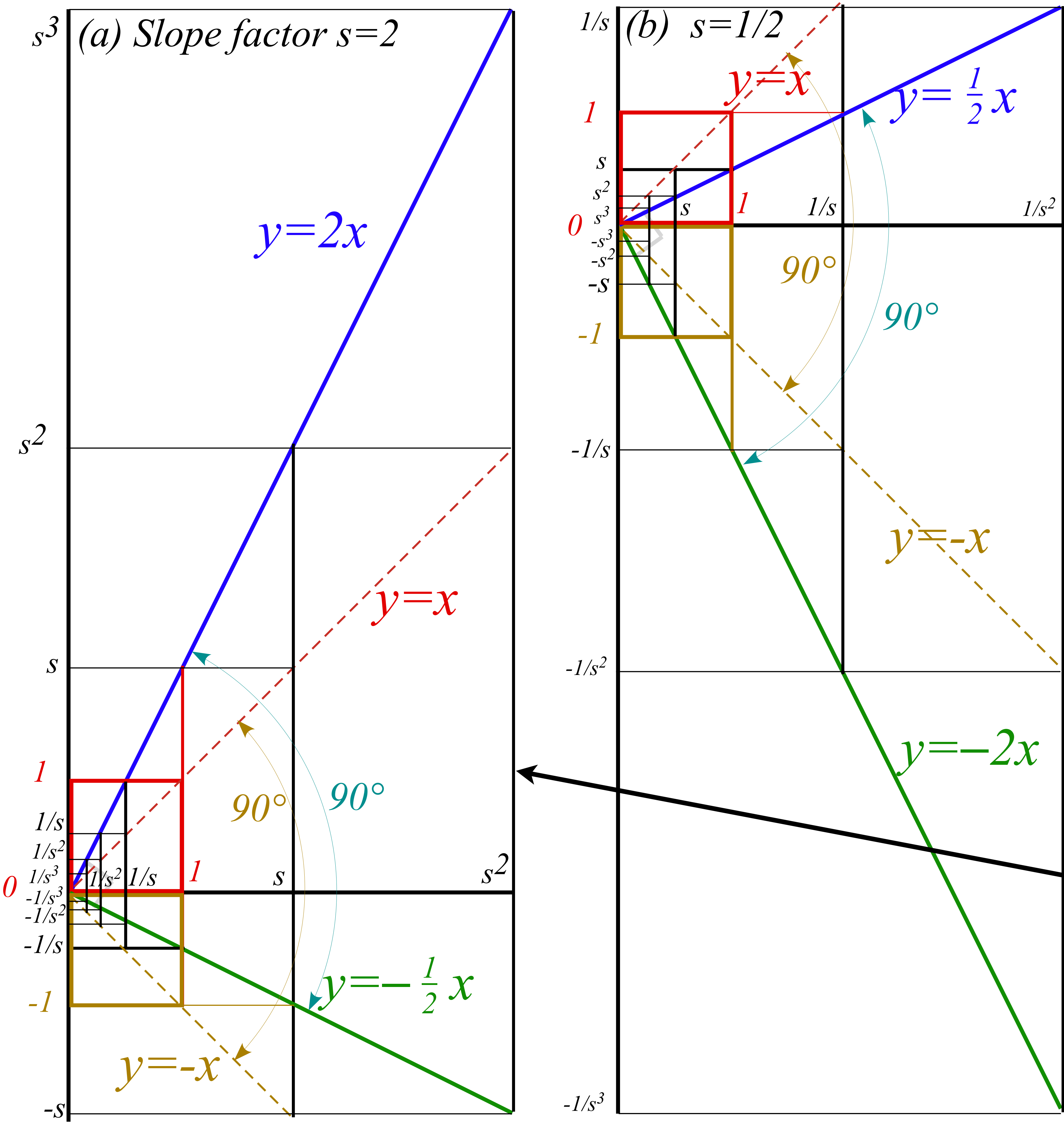
"Zig-Zags" give perspective geometry

(1D-vanishing point)

Unit 1
Fig. 9.2

"Zig-Zags" give perspective geometry

(1D-vanishing point)



Unit 1
Fig. 9.2

1st-day-of-school perspective of 12th-grader

1st-day-of-school perspective of 1st-grader

Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

Projective or perspective geometry

→ *Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields*

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

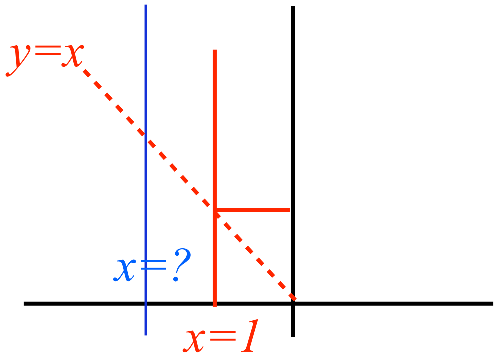
Compare mks units of Coulomb Electrostatic vs. Gravity

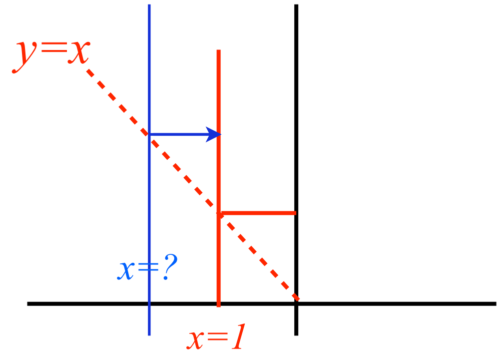
Each $y=x^2$ parabola point found by just one "Zig-Zag"

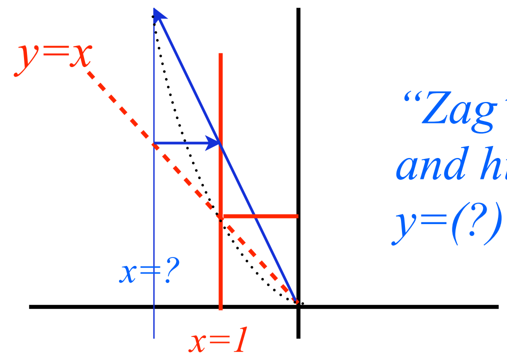
1. Pick an (x=?)-line

2. "Zig" from its y=x intersection to x=1 line

3. "Zag" from origin back to (x=?)-line







"Zag" line is $y=(?)\cdot x$ and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$

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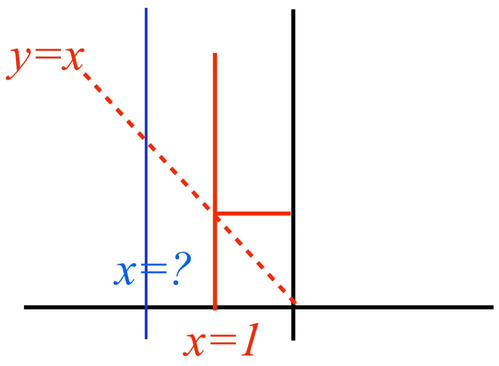
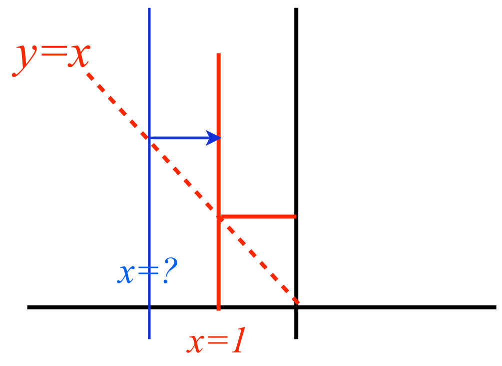
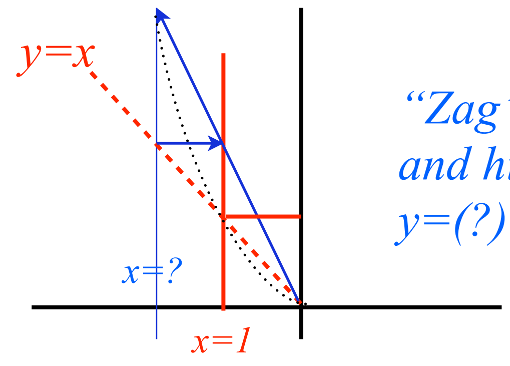
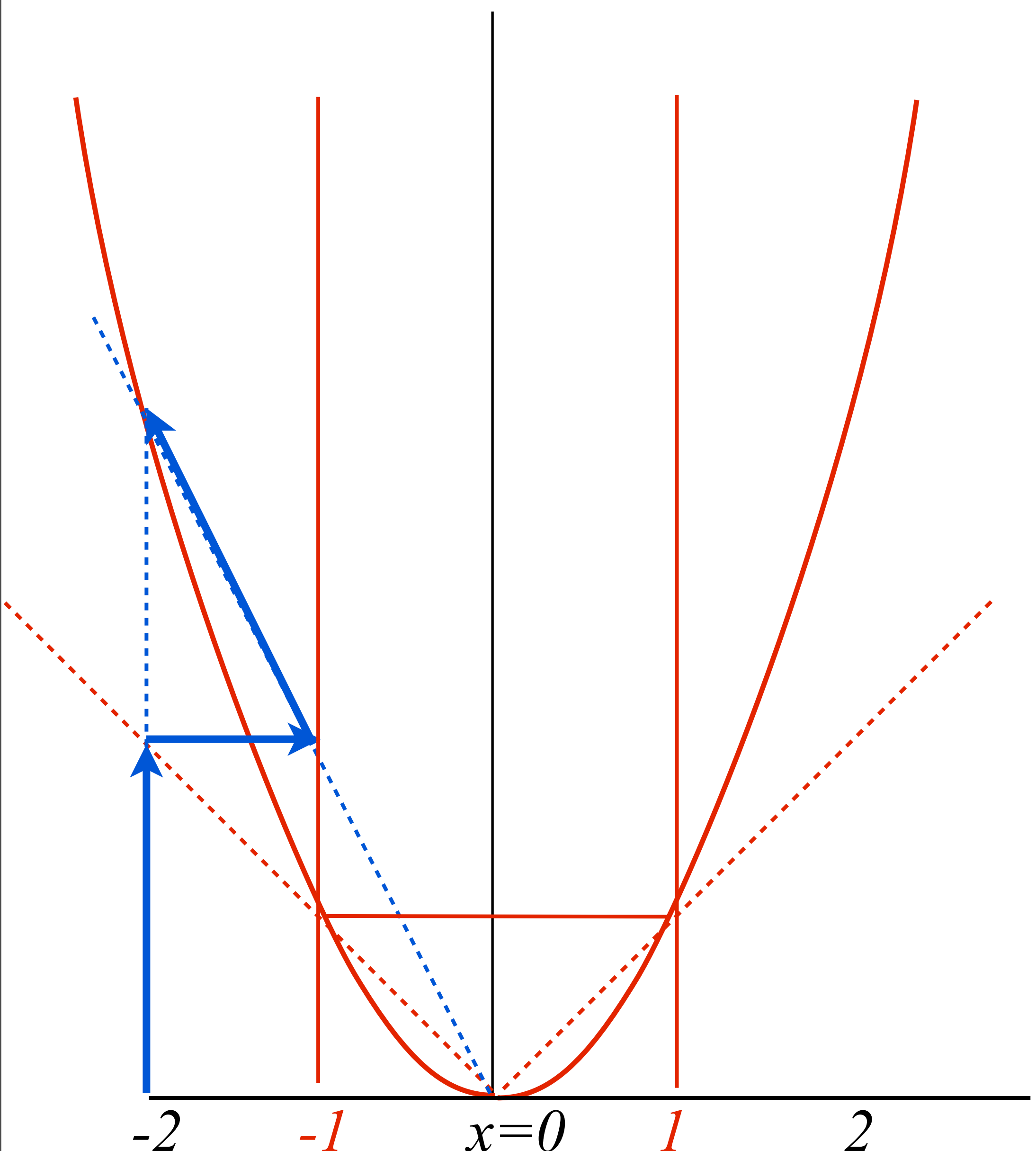






"Zag" line is y=(?)·x and hits (x=?)-line at y=(?)·(?)=(?)²



Unit 1
Fig. 9.1

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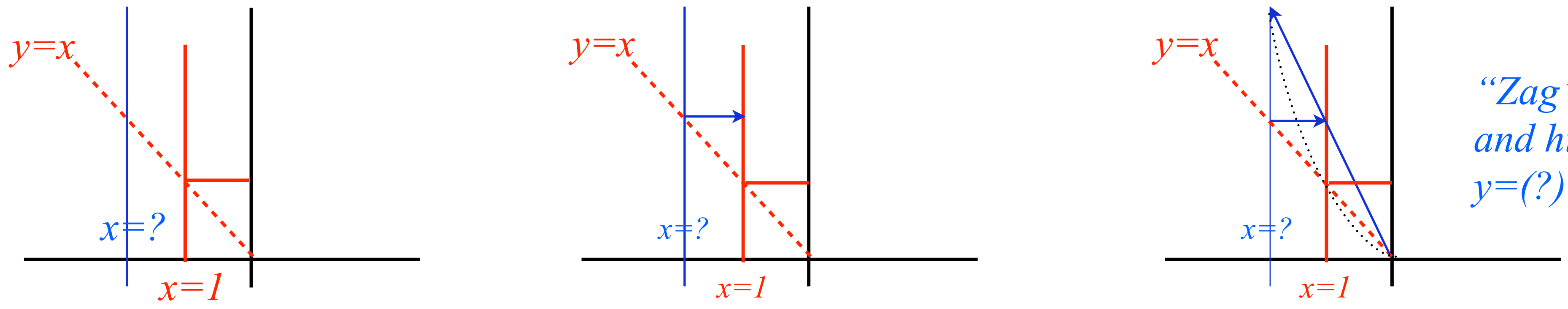

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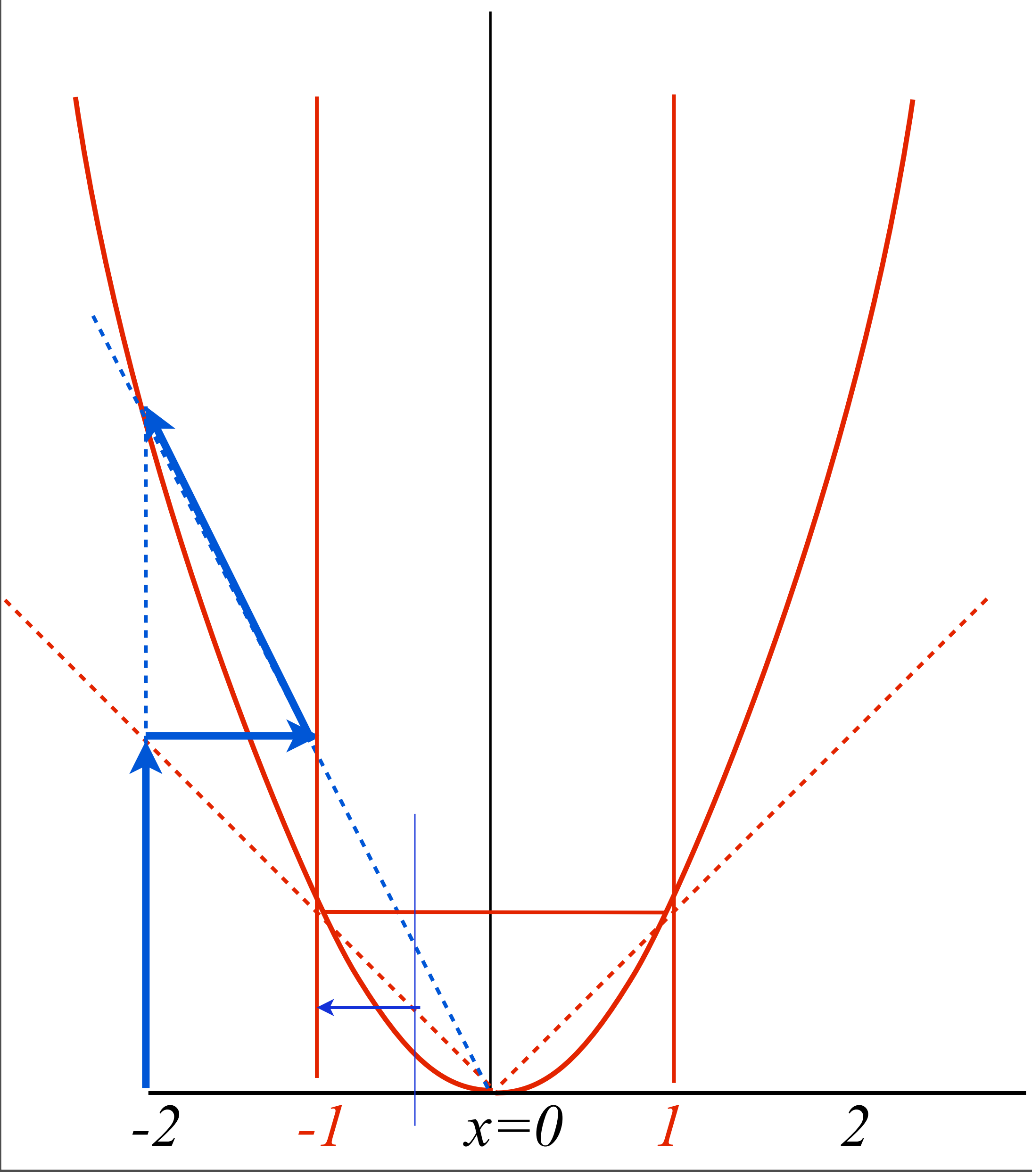

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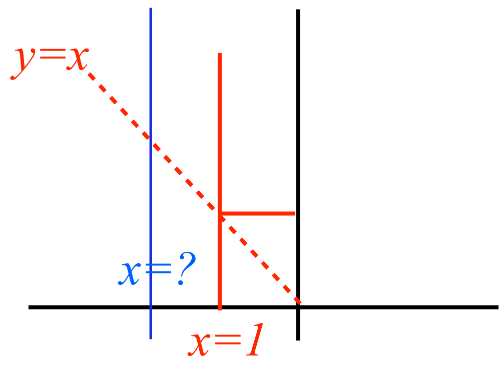

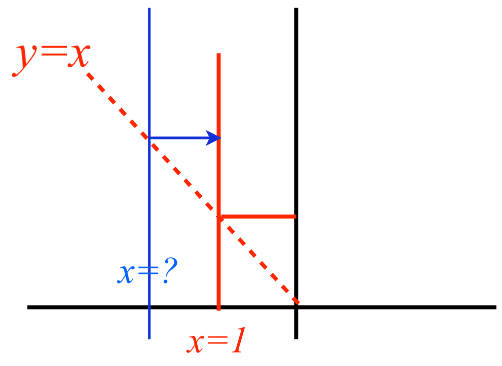

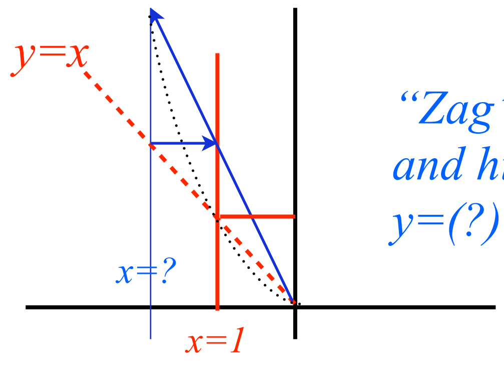

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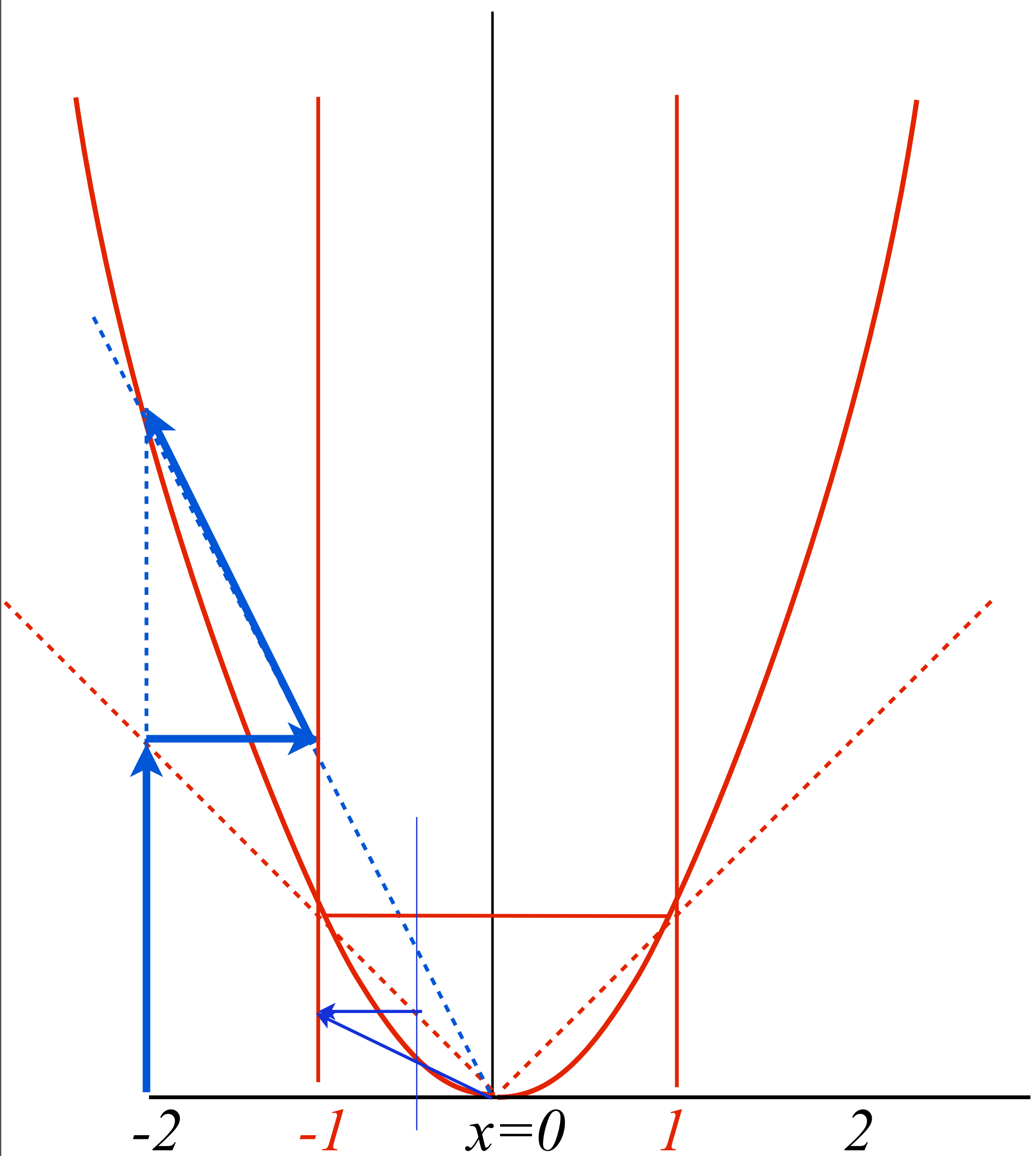

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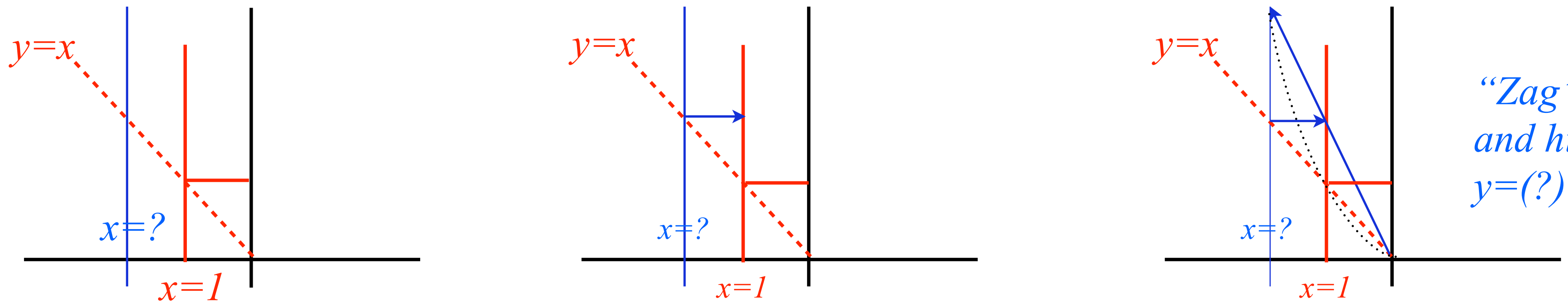
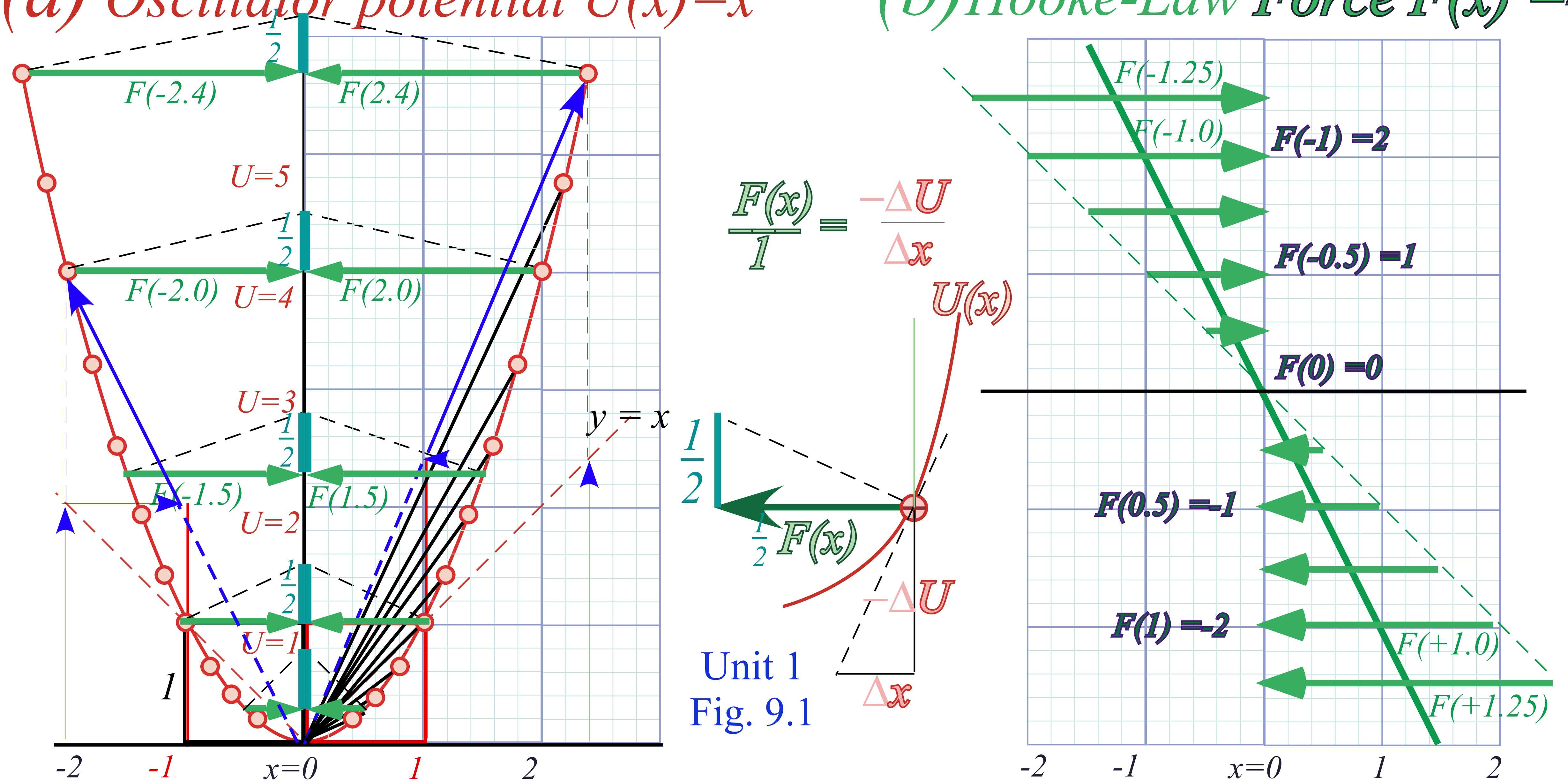


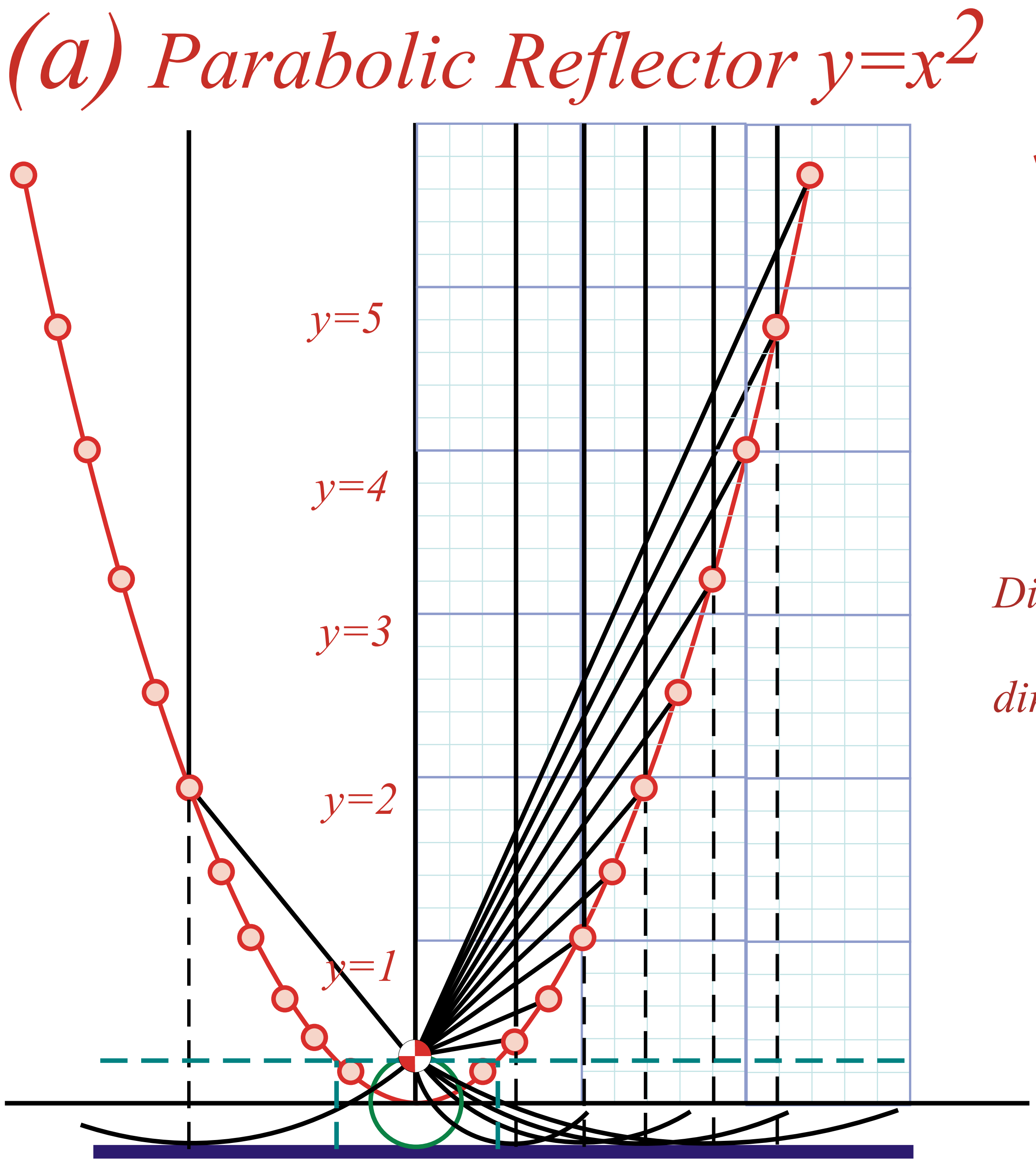
"Zag" line is $y=(?)\cdot x$ and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$

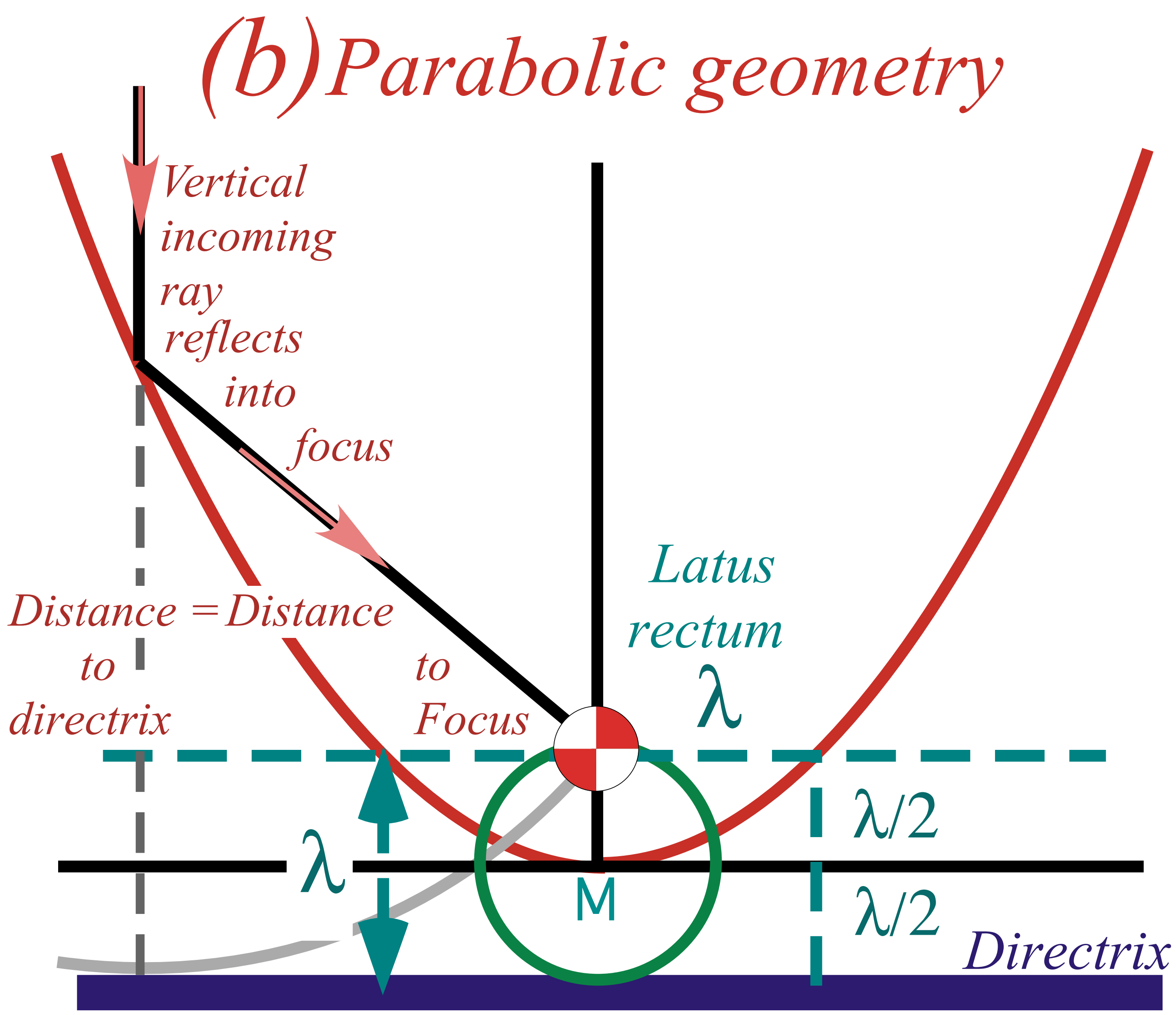
(a) Oscillator potential $U(x)=x^2$ *(b) Hooke-Law* **Force $F(x)$ =-2x**



Unit 1
Fig. 9.1

A more conventional parabolic geometry…(uses focal point)

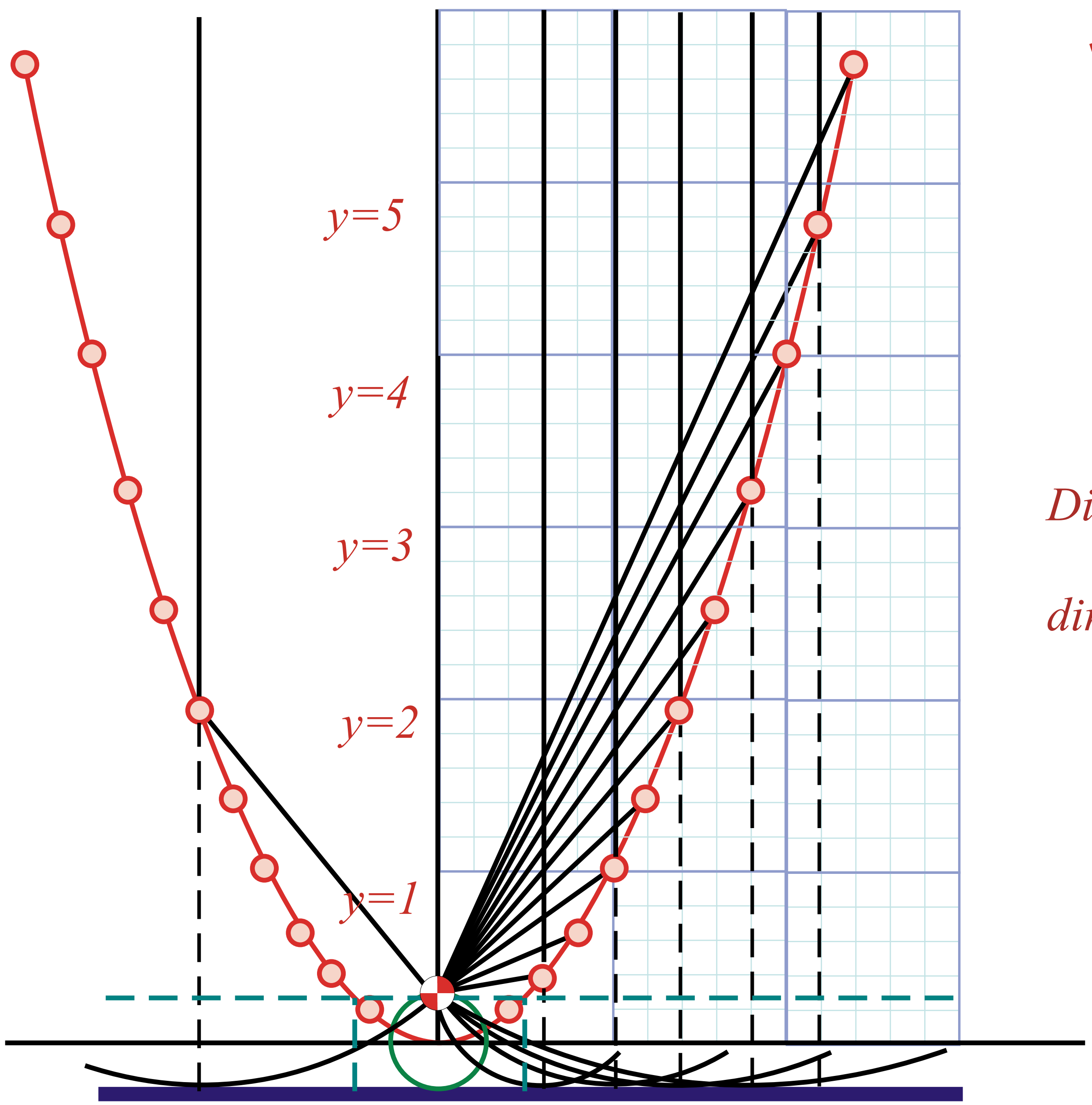




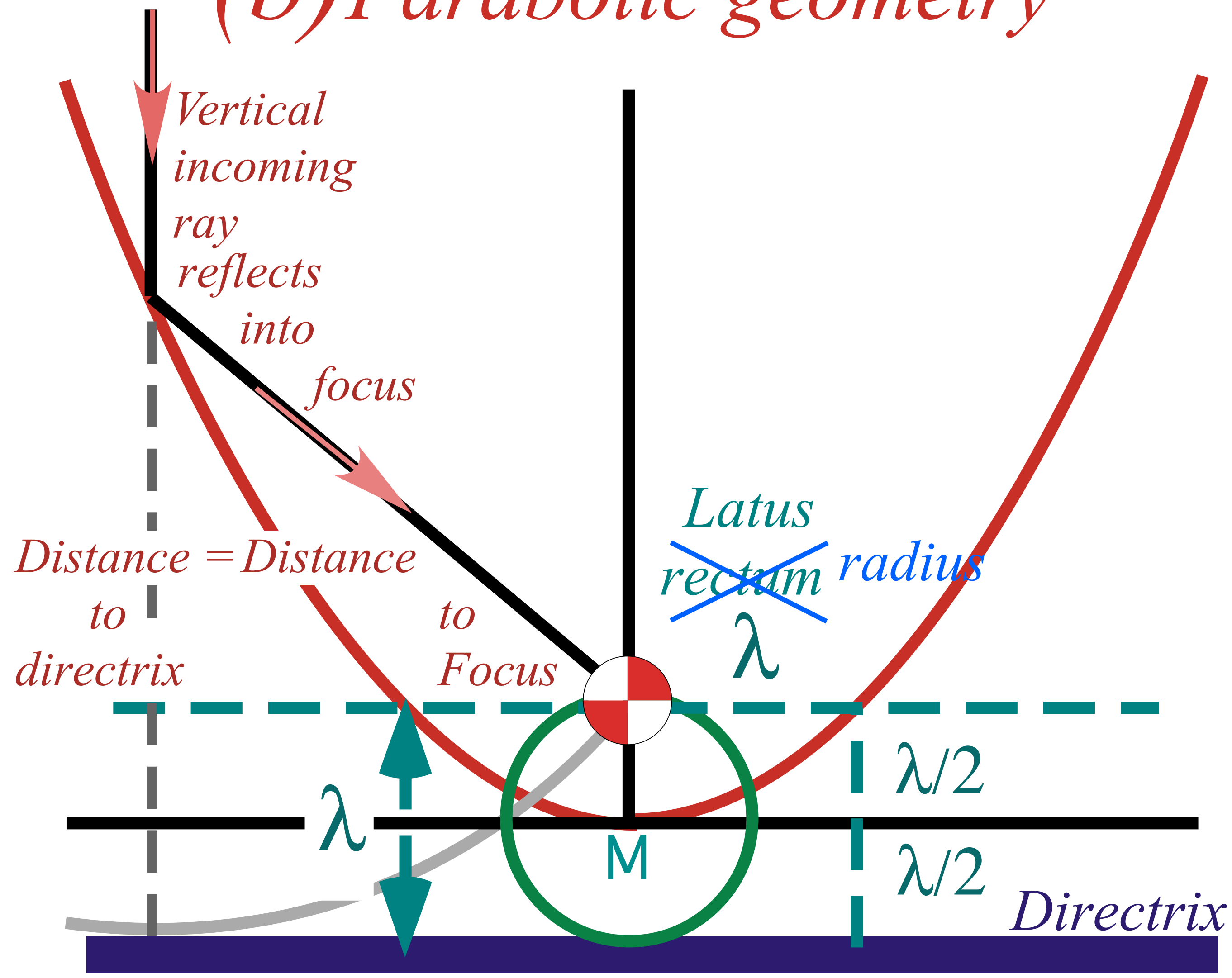
Unit 1
Fig. 9.3

A more conventional parabolic geometry...

(a) Parabolic Reflector $y=x^2$



(b) Parabolic geometry

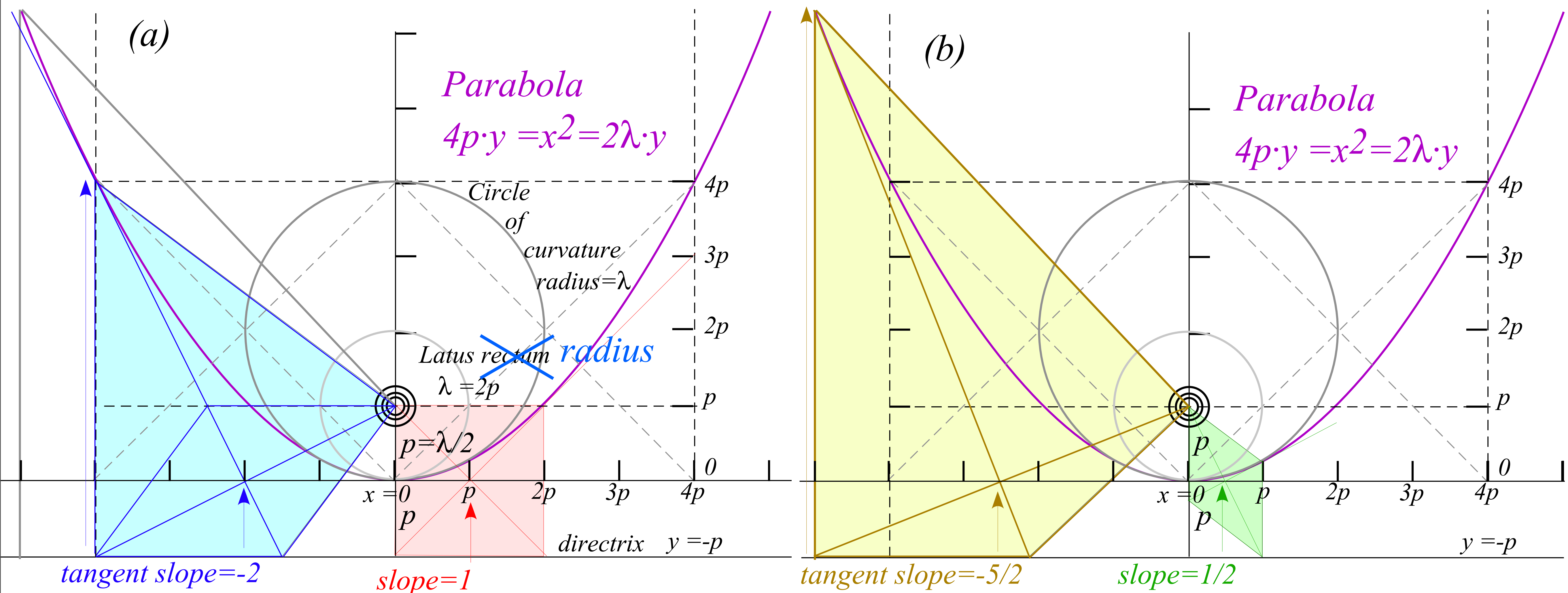


Better name† for λ : *latus radius*

†Old term *latus rectum* is exclusive copyright of
X-Treme Roidrage Gyms
Venice Beach, CA 90017

Unit 1
Fig. 9.3

...conventional parabolic geometry...carried to extremes...



Unit 1
Fig. 9.4

Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

Projective or perspective geometry

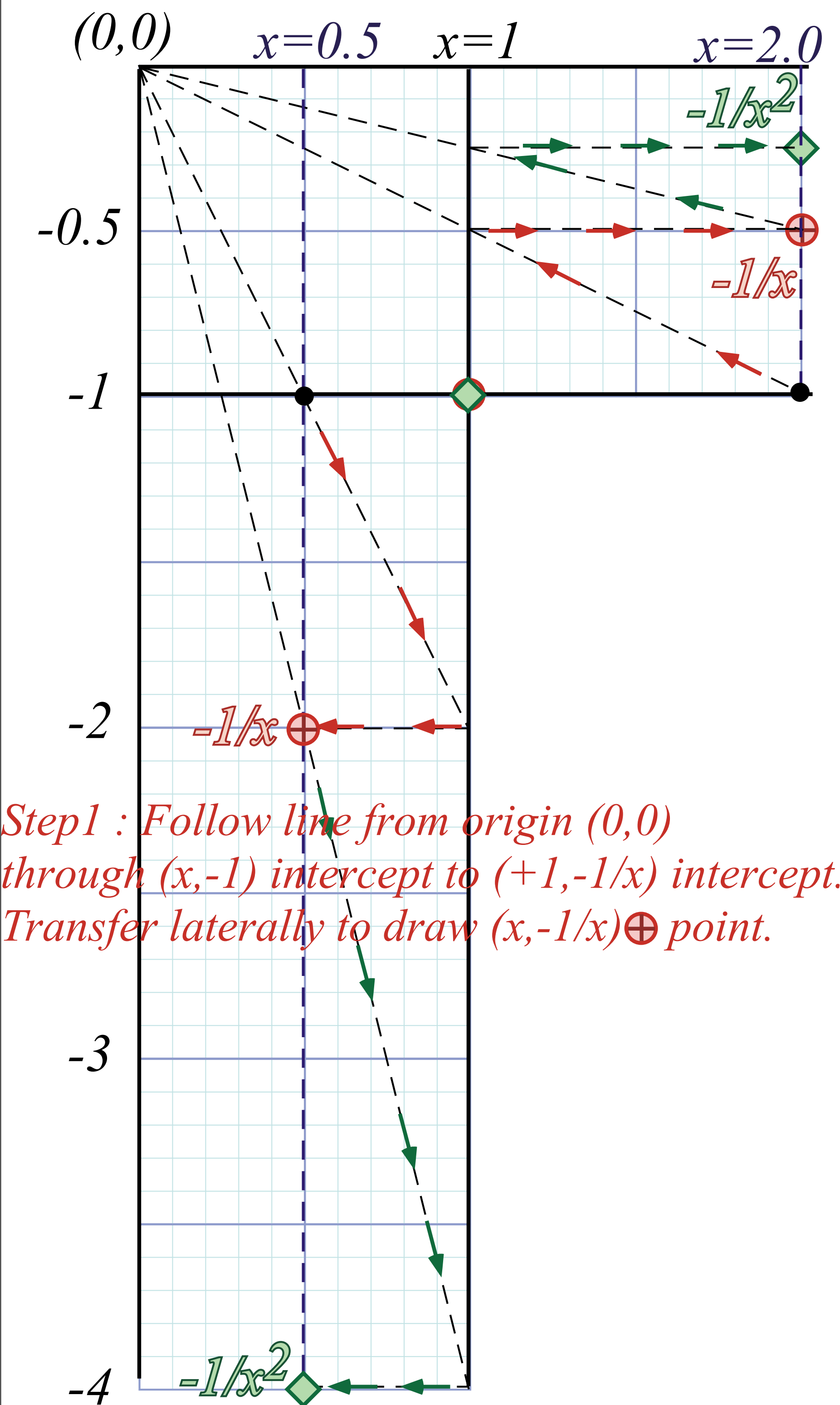
Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

→ *Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields*

Compare mks units of Coulomb Electrostatic vs. Gravity

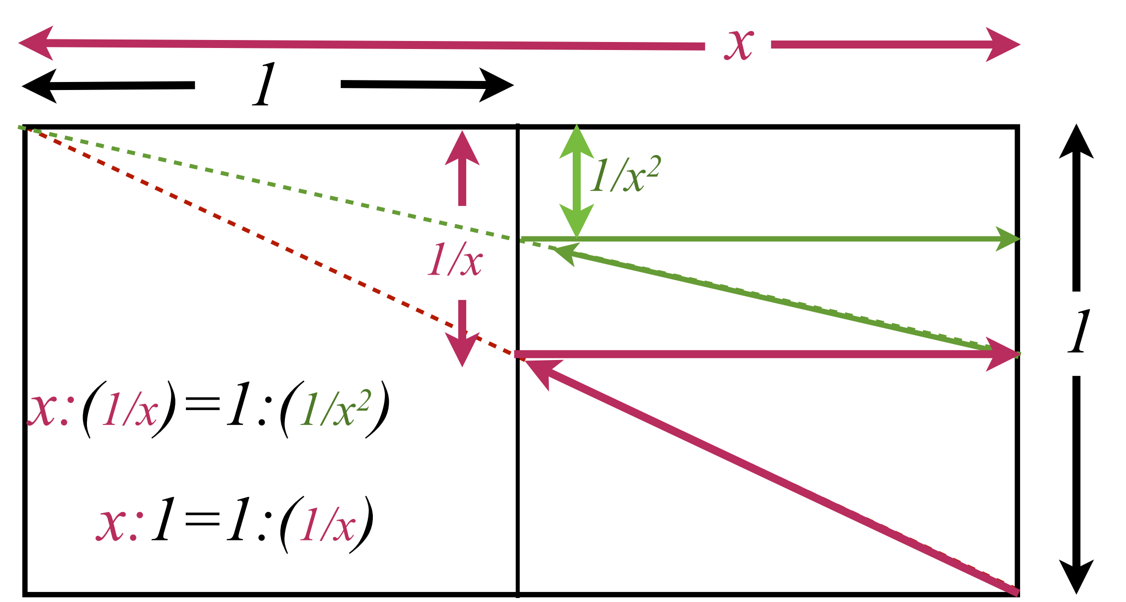
Unit 1
Fig. 9.4

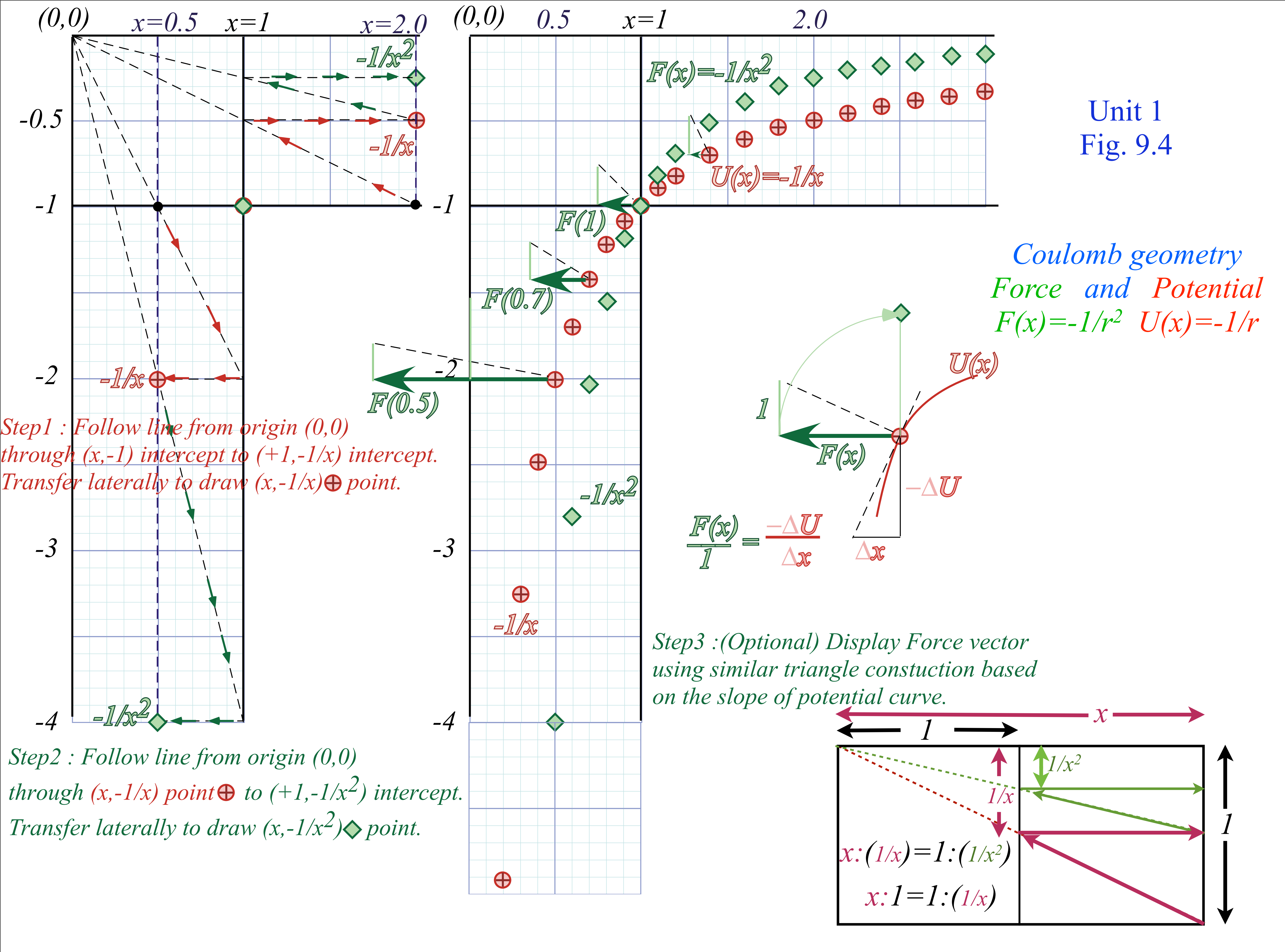
Coulomb geometry
Force and Potential
$F(x)=-1/r^2$ $U(x)=-1/r$



Step1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.

Step2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x^2) intercept. Transfer laterally to draw (x,-1/x^2)◇ point.





Unit 1
Fig. 9.4
Coulomb geometry
Force and Potential
F(x)=-1/r2 U(x)=-1/r
(0,0)
x=0.5
x=1
x=2.0
-0.5
-1
-2
-3
-4
-1/x2
-1/x
0.5
2.0
F(x)=-1/x2
U(x)=-1/x
F(1)
F(0.7)
F(0.5)
U(x)
1
F(x)
−ΔU
Δx
F(x)/1 = −ΔU/Δx
Step1 : Follow line from origin (0,0)
through (x,-1) intercept to (+1,-1/x) intercept.
Transfer laterally to draw (x,-1/x)⊕ point.
Step2 : Follow line from origin (0,0)
through (x,-1/x) point⊕ to (+1,-1/x2) intercept.
Transfer laterally to draw (x,-1/x2)◇ point.
Step3 :(Optional) Display Force vector
using similar triangle constuction based
on the slope of potential curve.
x
1
1/x2
1/x
x:(1/x)=1:(1/x2)
x:1=1:(1/x)

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Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

→ *Compare mks units of Coulomb Electrostatic vs. Gravity*

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

Compare mks units for Coulomb fields

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More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ *Coulomb*

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

...but 1 Ampere = 1 Coulomb/sec.

Compare mks units for Coulomb fields

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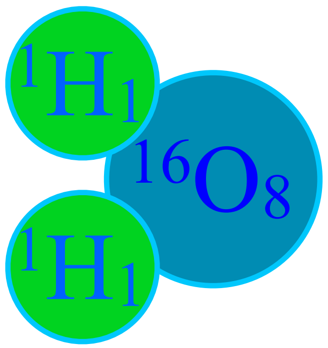
Attractive (+)(-) or (-)(+)

...but 1 Ampere = 1 Coulomb/sec.

"Fingertip Physics" of Ch. 9 notes that 1 $(cm)^3$ =1gm of water (1/18 Mole) has (1/18) $6\cdot 10^{23}$ molecules

$\sim 0.3\cdot 10^{23}$

H_2O Molecular weight~18



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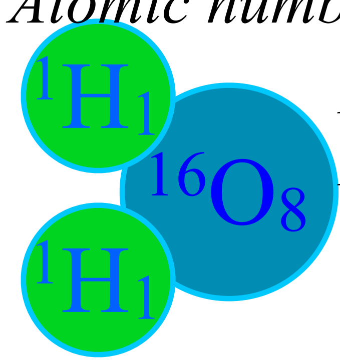
...but 1 Ampere = 1 Coulomb/sec.

"Fingertip Physics" of Ch. 9 notes that 1 $(cm)^3$ *=1gm of water (1/18 Mole) has (1/18)* $6\cdot 10^{23}$ *molecules or ~* $3\cdot 10^{23}$ *electrons* *and ~* $3\cdot 10^{23}$ *protrons.*

$\sim 0.3\cdot 10^{23}$

H_2O *Molecular weight~18*
Atomic number = 10



10 electrons
10 protons

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

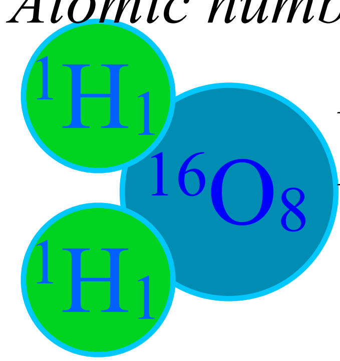
...but 1 Ampere = 1 Coulomb/sec.

"Fingertip Physics" of Ch. 9 notes that 1 $(cm)^3$ =1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules or ~ $3 \cdot 10^{23}$ electrons

$\sim 0.3 \cdot 10^{23}$ and ~ $3 \cdot 10^{23}$ protrons.

H_2O Molecular weight~18

Atomic number = 10



10 electrons *That is ~ − $3 \cdot 10^{23} \, 1.6022 \cdot 10^{-19}$ Coulomb or about $-0.5 \cdot 10^{+5}$ C or − 50,000 Coulomb*

10 protons *plus ~ $+3 \cdot 10^{23} \, 1.6022 \cdot 10^{-19}$ Coulomb or about $+0.5 \cdot 10^{+5}$ C or +50,000 Coulomb*

Equals zero total charge

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)
vs
Always Attractive (so far)

↓

↕ COMPARE! ↕

BIG
vs
small

2. Gravitational force between m(kilograms) and M(kg.)

$$F^{grav.}(r) = -G\frac{mM}{r^2} \quad where: G = 0.000{,}000{,}000{,}067 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

More precise value for gravitational constant : $G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

quantum of charge: $|e|=1.6022 \cdot 10^{-19}$ Coulomb

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q(Coulombs) and Q(C.)

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Joule}{per\ square\ Coulomb}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

quantum of charge: $|e|=1.6022 \cdot 10^{-19}$ Coulomb

Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q(Coulombs) and Q(C.)

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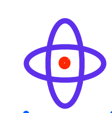
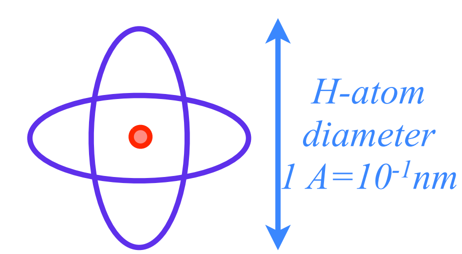


Nuclear size ~ 10^{-15} m = 1 femtometer =1fm

Atomic size ~ 1 Angstrom = 10^{-10} m



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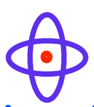

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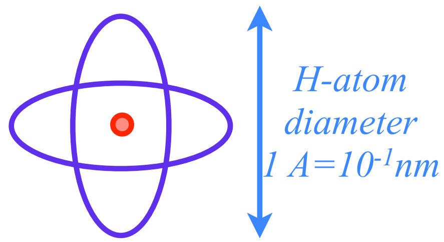

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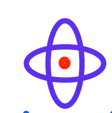

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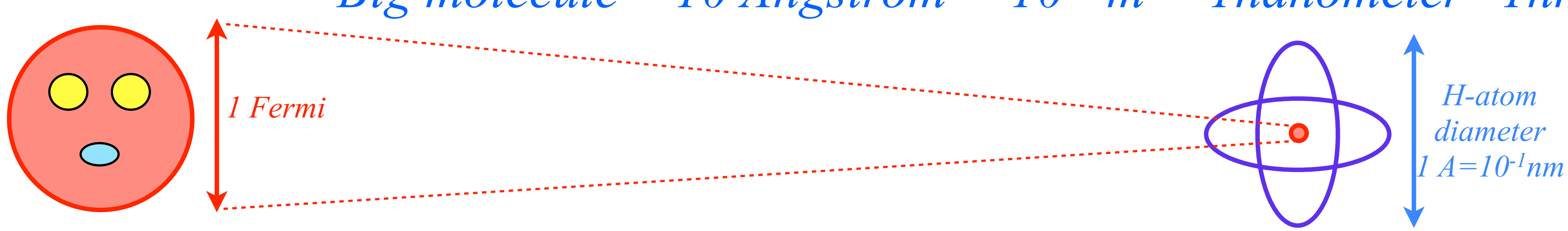
also: 1fm = 10^{-13} cm =1Fermi
=1Fm



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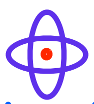

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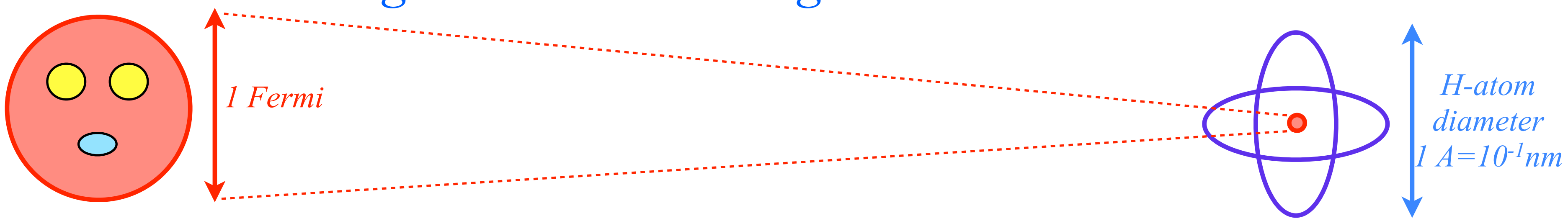


nuclear radii are 100,000 to 1,000,000 times smaller than atomic/chemical radii

...so nuclear qQ/r energy 100,000 to 1,000,000 times **bigger** that of atomic/chemical...

Geometry of idealized "Sophomore-physics Earth"

→ *Coulomb field outside* *Isotropic Harmonic Oscillator (IHO) field inside*

Contact-geometry of potential curve(s)

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Coulomb force vanishes inside-spherical shell (Gauss-law)

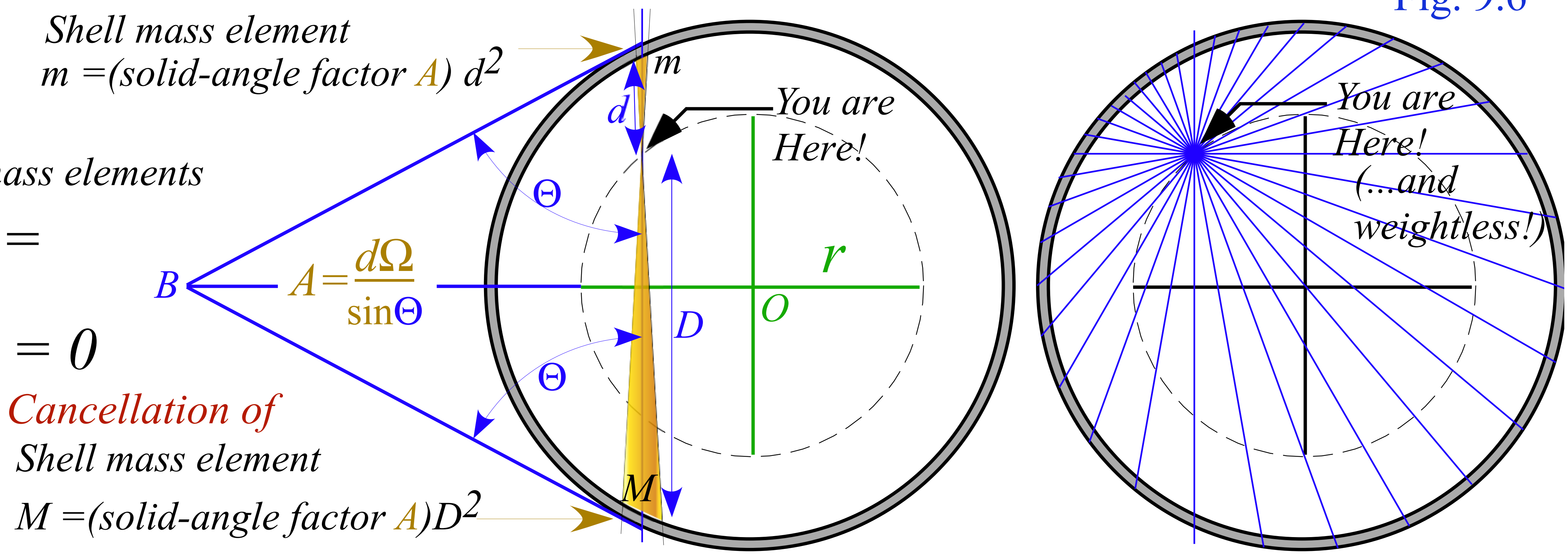
Unit 1
Fig. 9.6

Gravity at r
due to shell mass elements

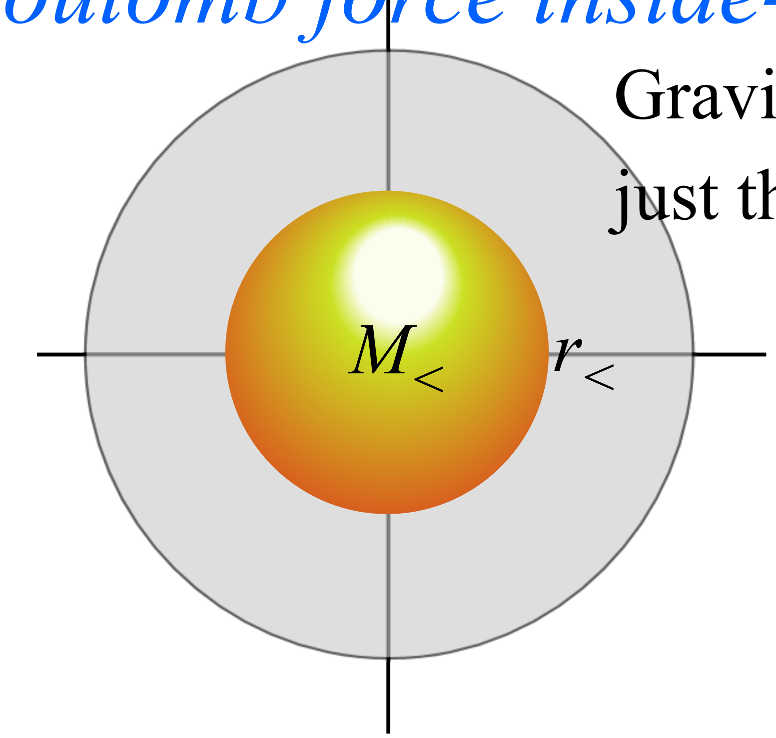
$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

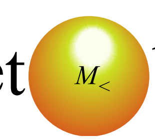
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

Cancellation of



Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet  below $r_<$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Shell mass element
m =(solid-angle factor A) d^2

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due to shell mass elements

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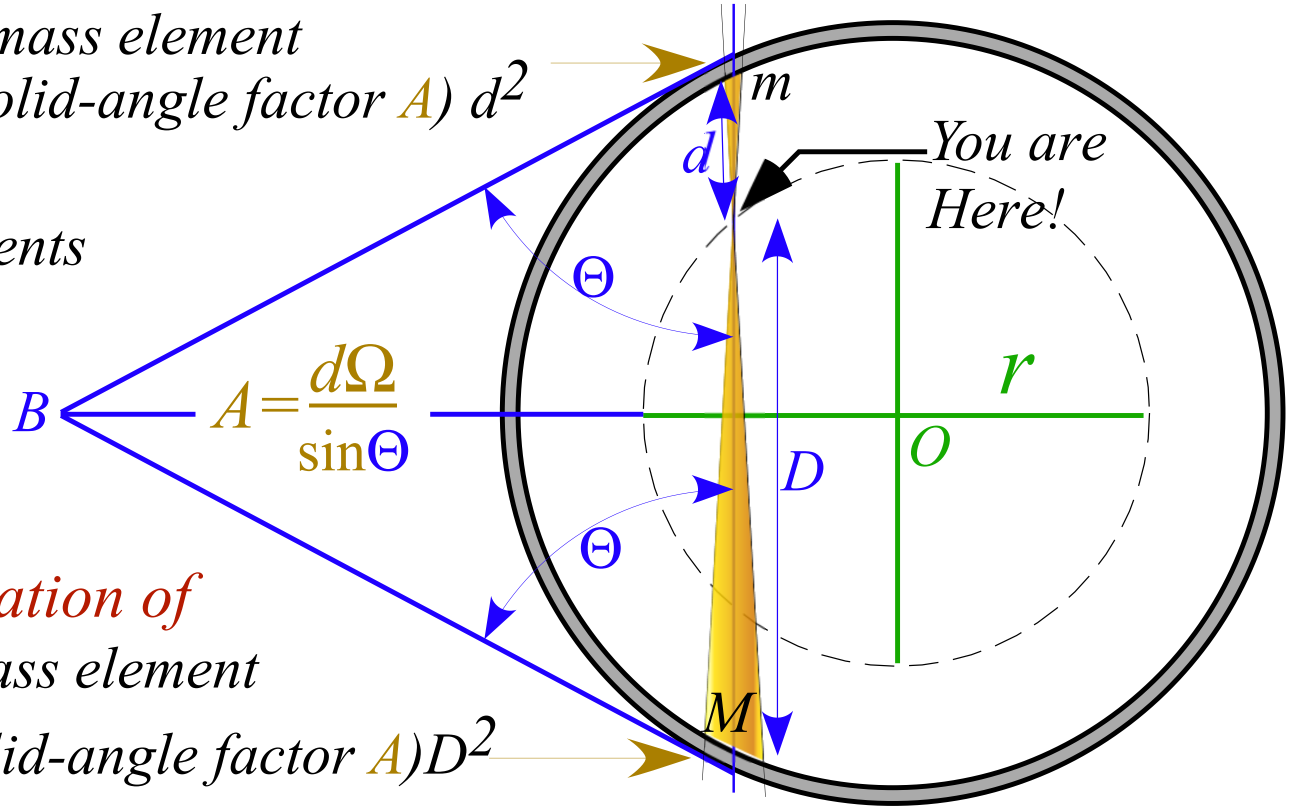

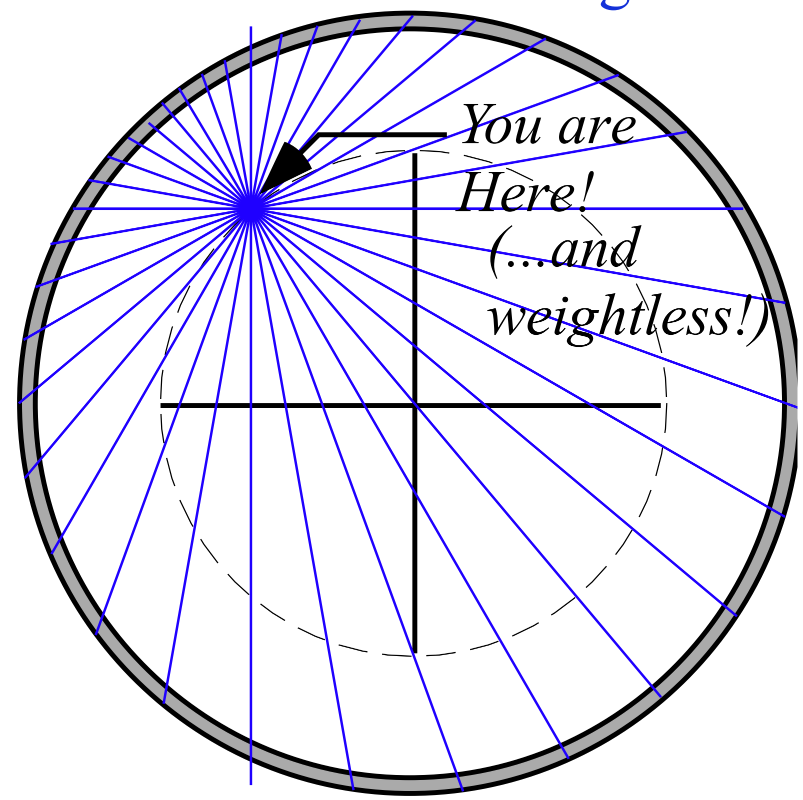

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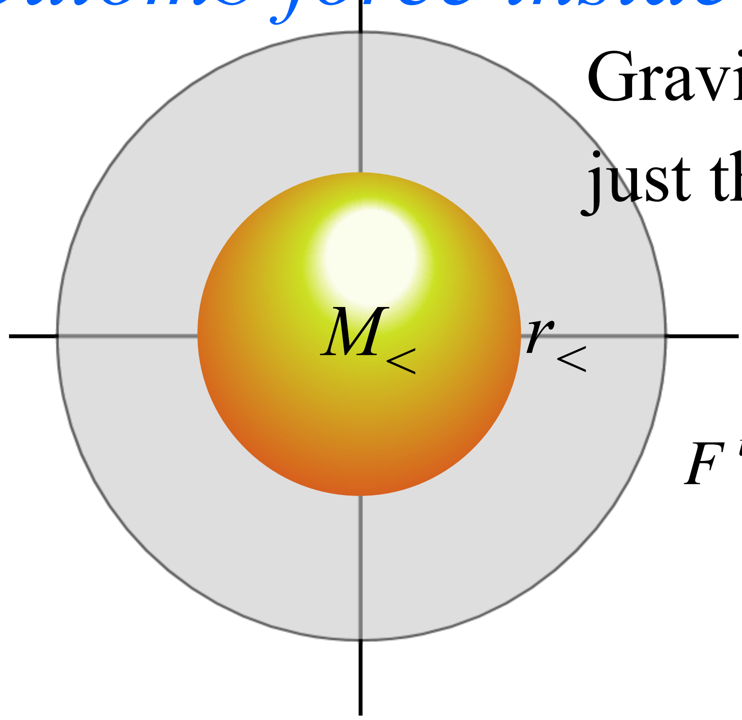

Gravitational force at $r_<$ is
just that of planet $M_<$ below $r_<$

Note:
Hooke's (linear) force law
for $r_<$ inside uniform body

$$F^{\,inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg\cdot x$$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

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due to shell mass elements

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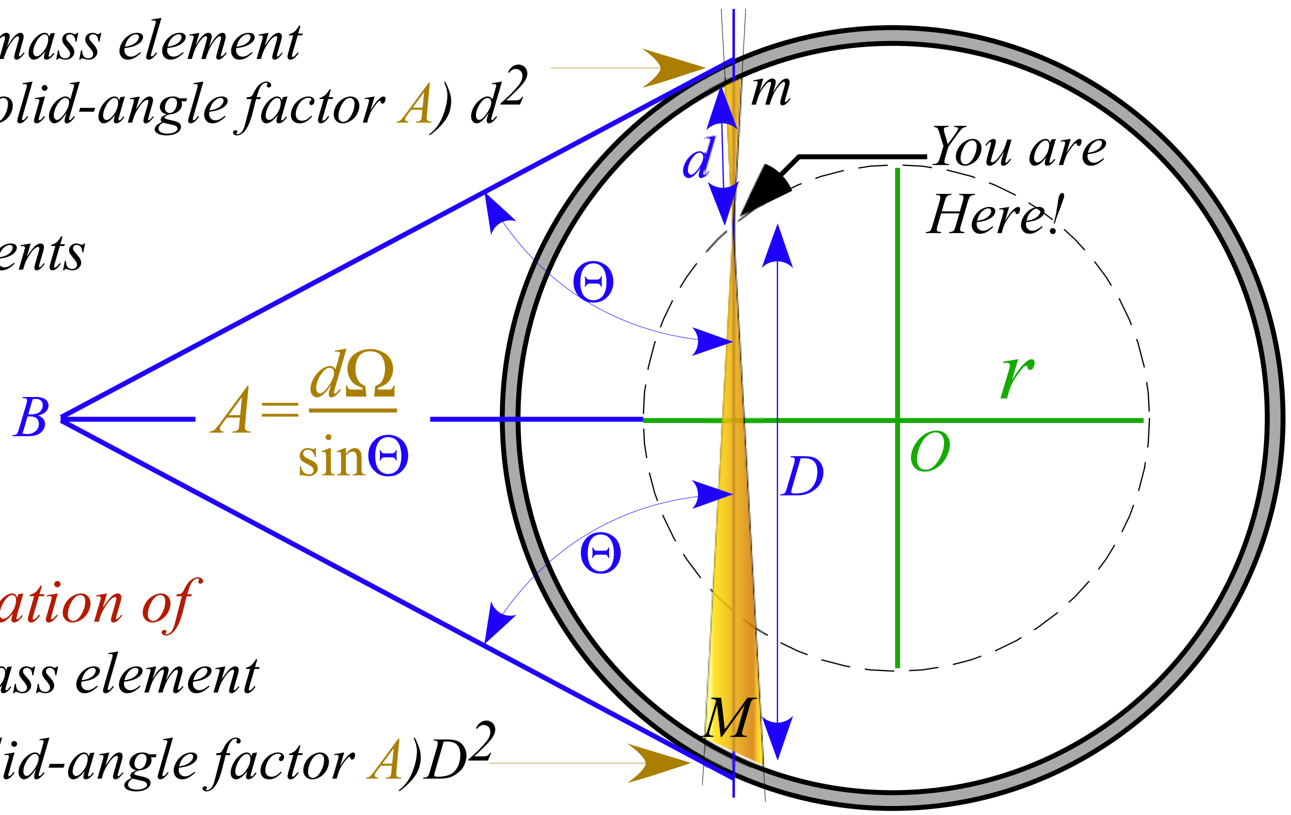
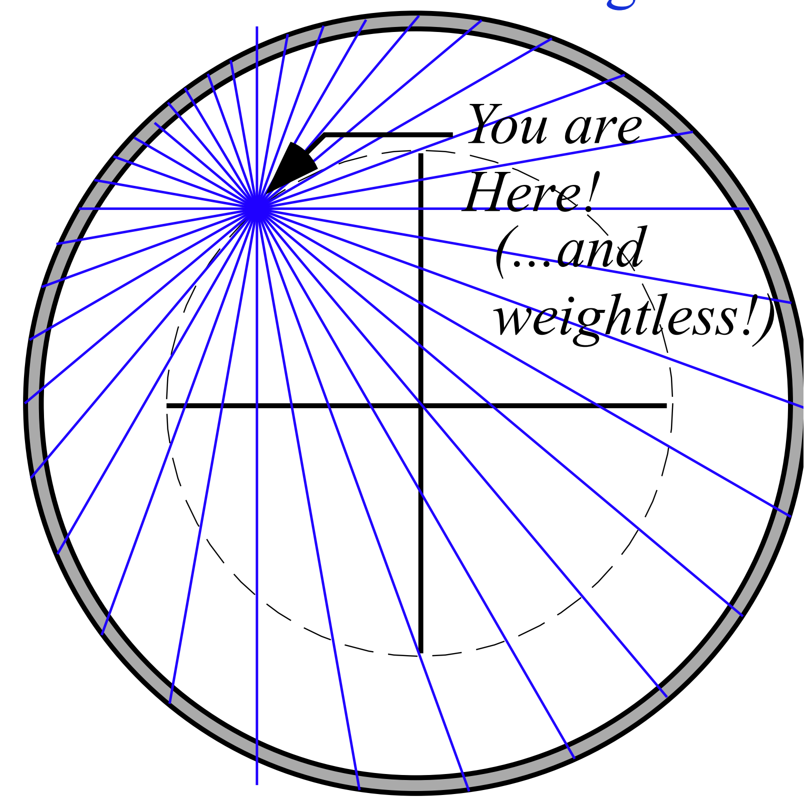
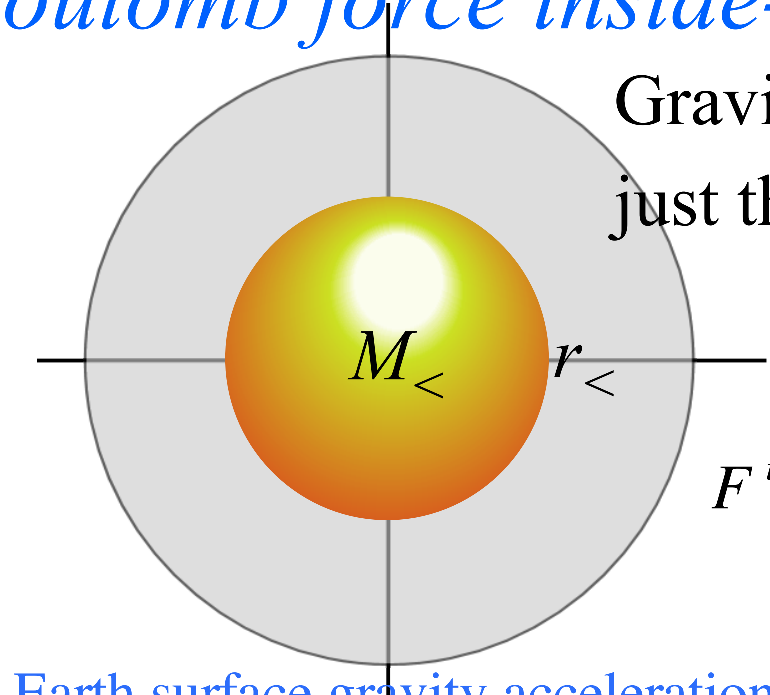
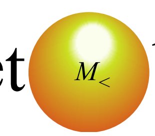




Cancellation of
Shell mass element
M =(solid-angle factor A)D^2

Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet  below $r_<$

Note:
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for $r_<$ inside uniform body

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg\cdot x$$

Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s$

$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Gravity at r
due to shell mass elements

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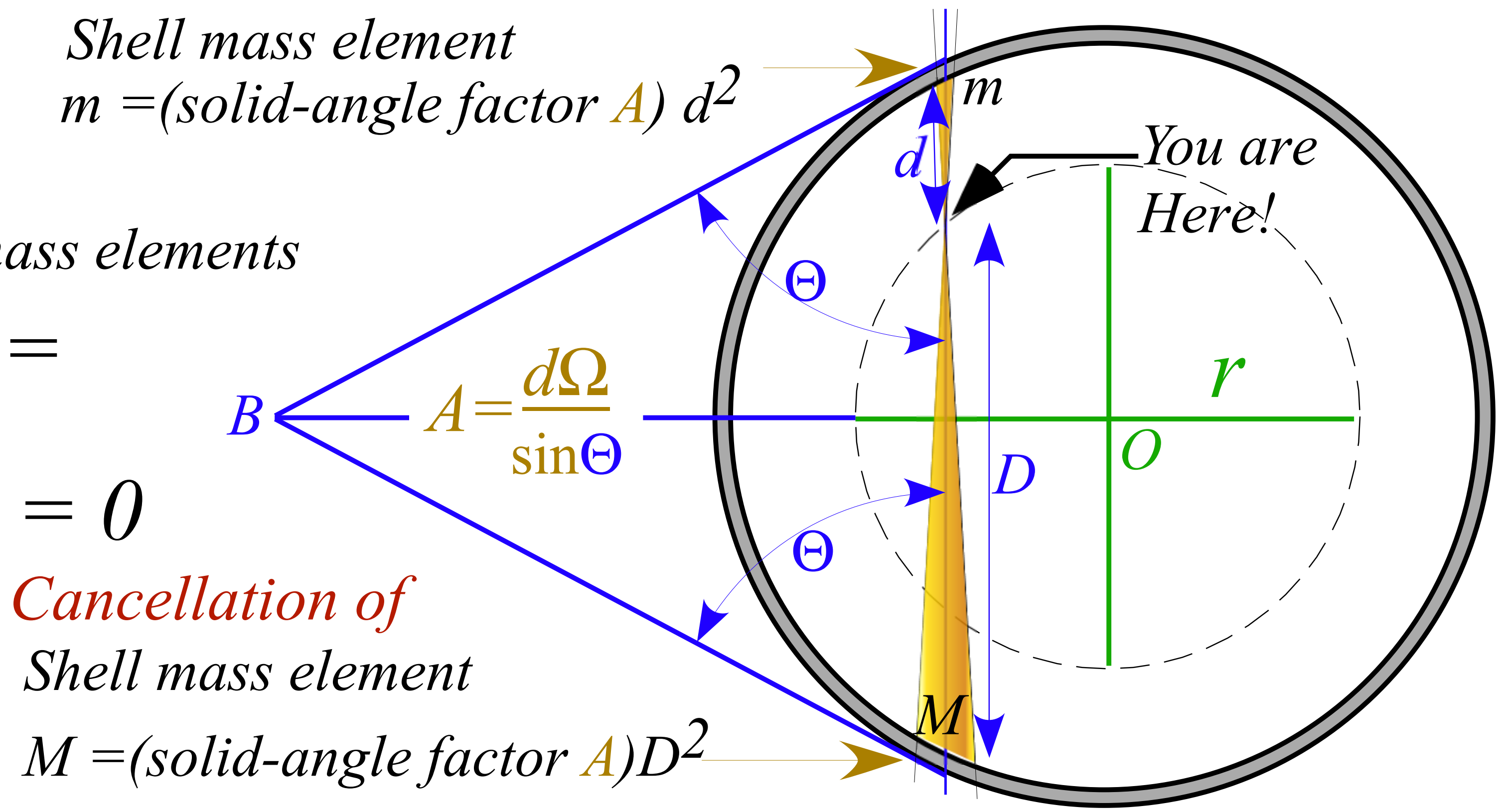
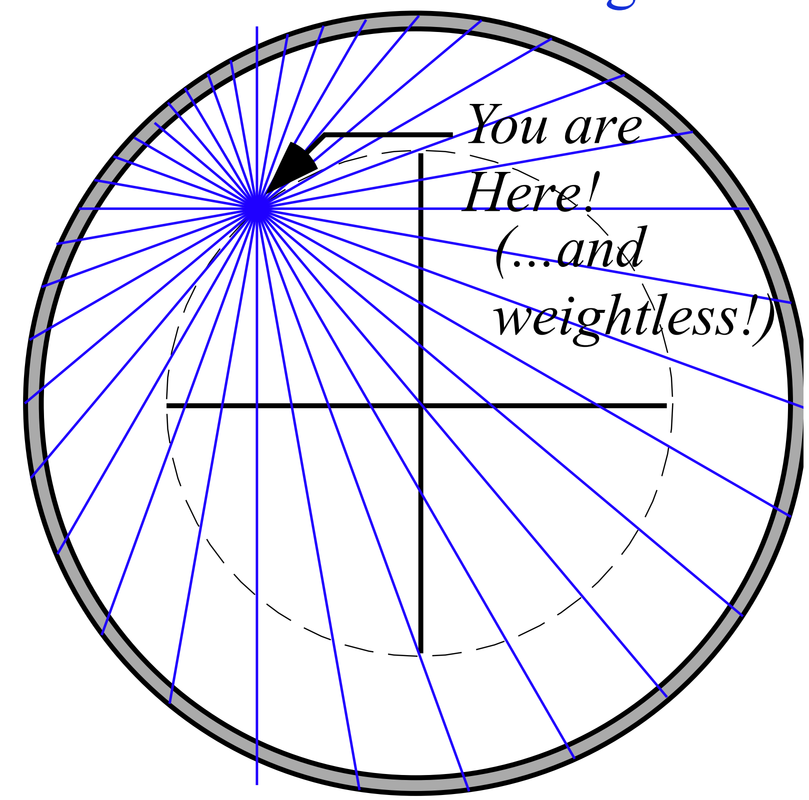
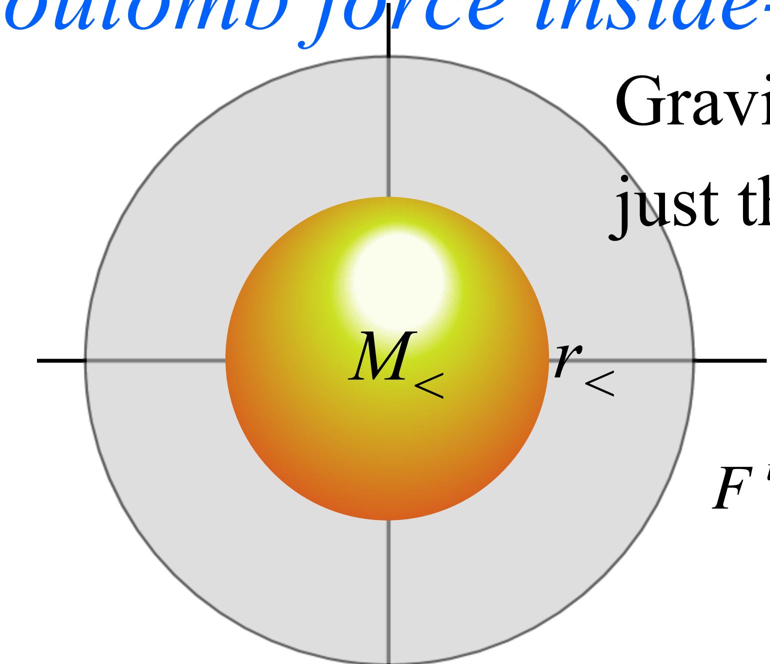




Coulomb force inside-spherical body due to stuff <u>below</u> you, <u>only</u>.



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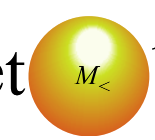
just that of planet  below $r_<$

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$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

Earth radius : $R_\oplus = 6.371\cdot 10^6\,m \simeq 6.4\cdot 10^6\,m$

Earth mass : $M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0\cdot 10^{24}\,kg.$

Solar radius : $R_\odot = 6.955 \times 10^8\,m. \simeq 7.0\cdot 10^8\,m.$

Solar mass : $M_\odot = 1.9889 \times 10^{30}\,kg. \simeq 2.0\cdot 10^{30}\,kg.$

Geometry of idealized "Sophomore-physics Earth"

Coulomb field <u>outside</u> *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

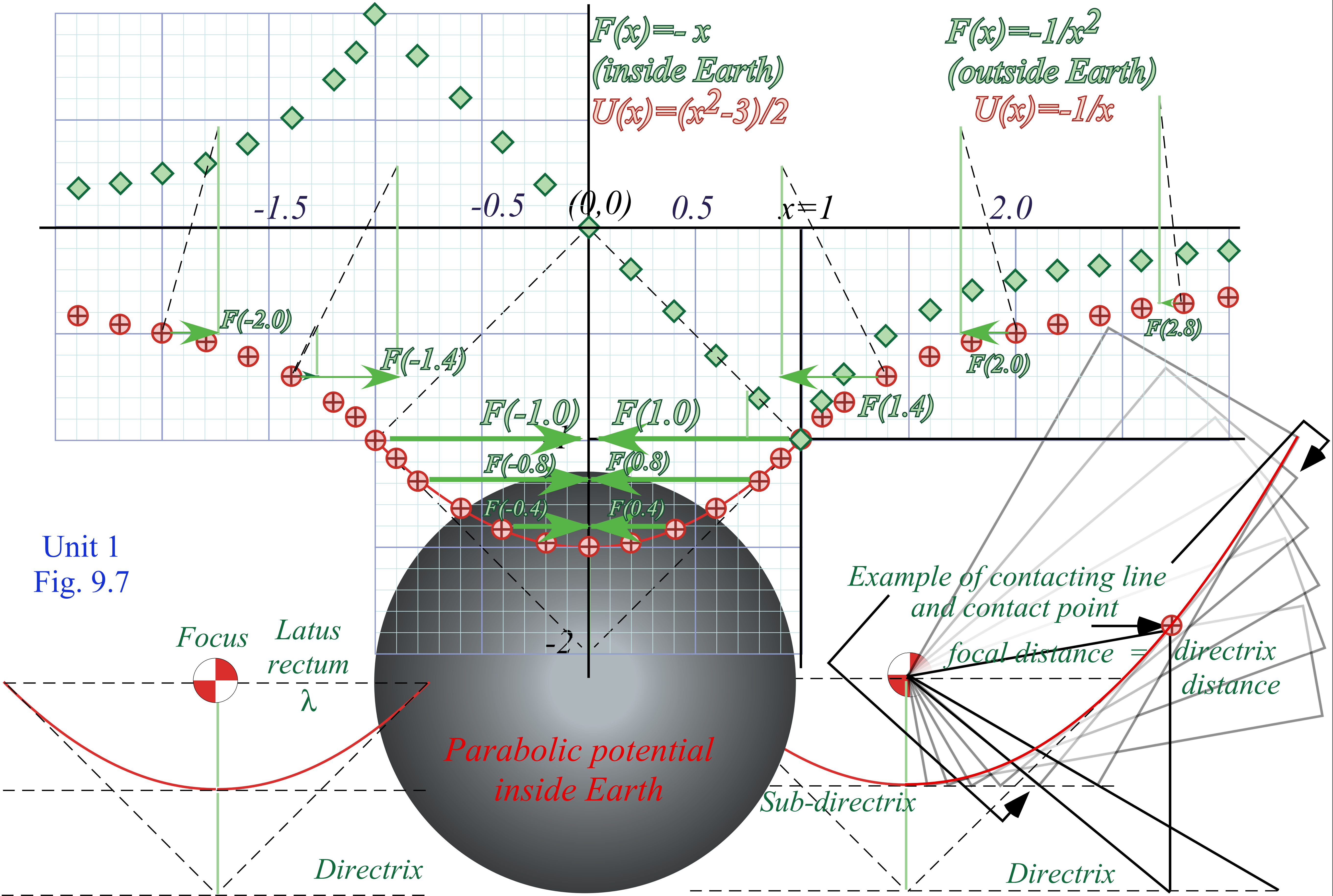
→ *Contact-geometry of potential curve(s)*

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

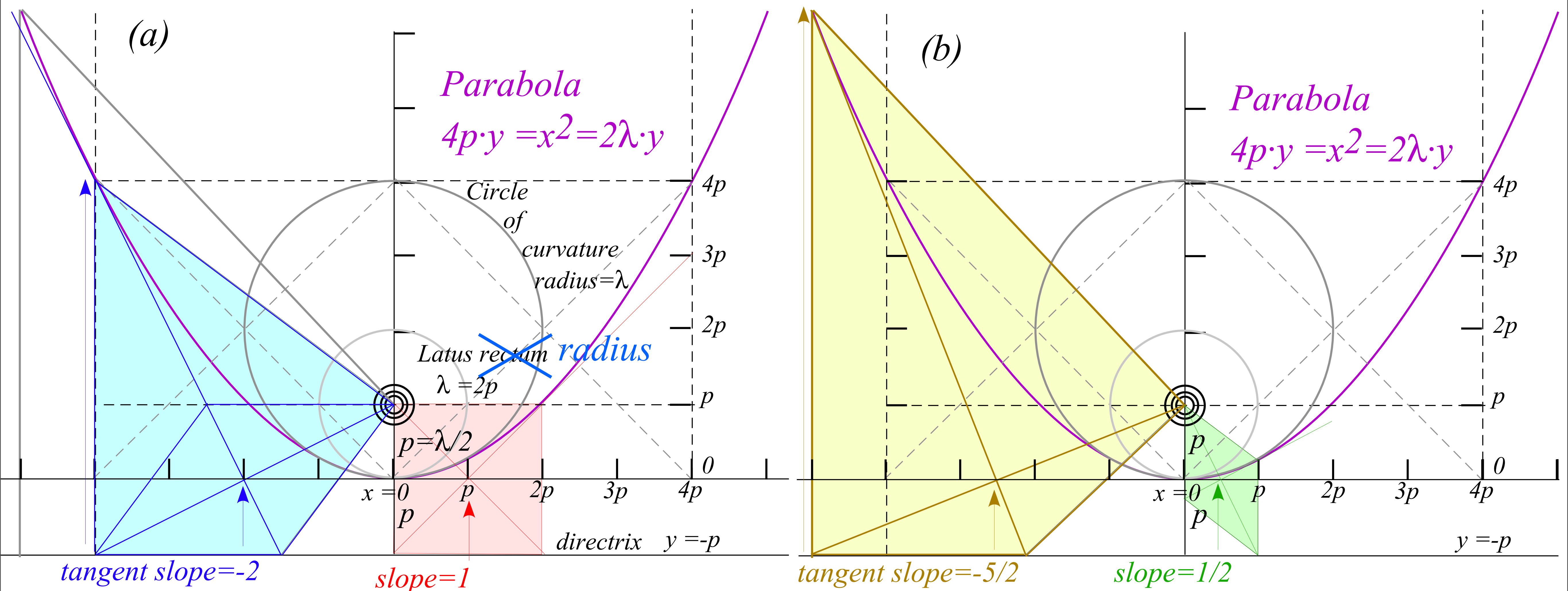
The ideal "Sophomore-Physics-Earth" model of geo-gravity



Unit 1
Fig. 9.7

...conventional parabolic geometry...carried to extremes...

(From p.18)



Unit 1
Fig. 9.4

Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside   *Isotropic Harmonic Oscillator (IHO) field inside*

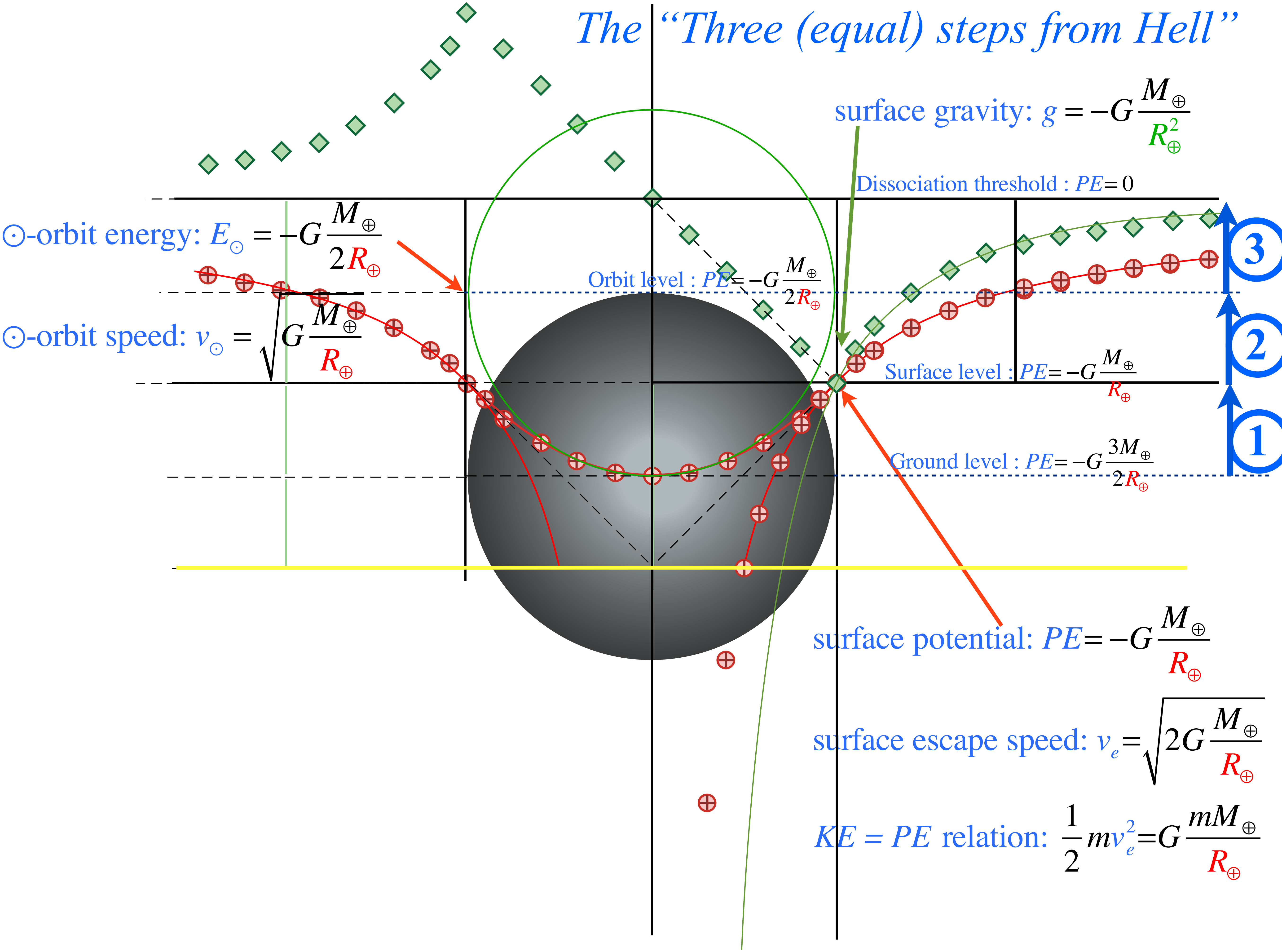
Contact-geometry of potential curve(s)

→ *"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

Earth matter vs nuclear matter:

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

The "Three (equal) steps from Hell"

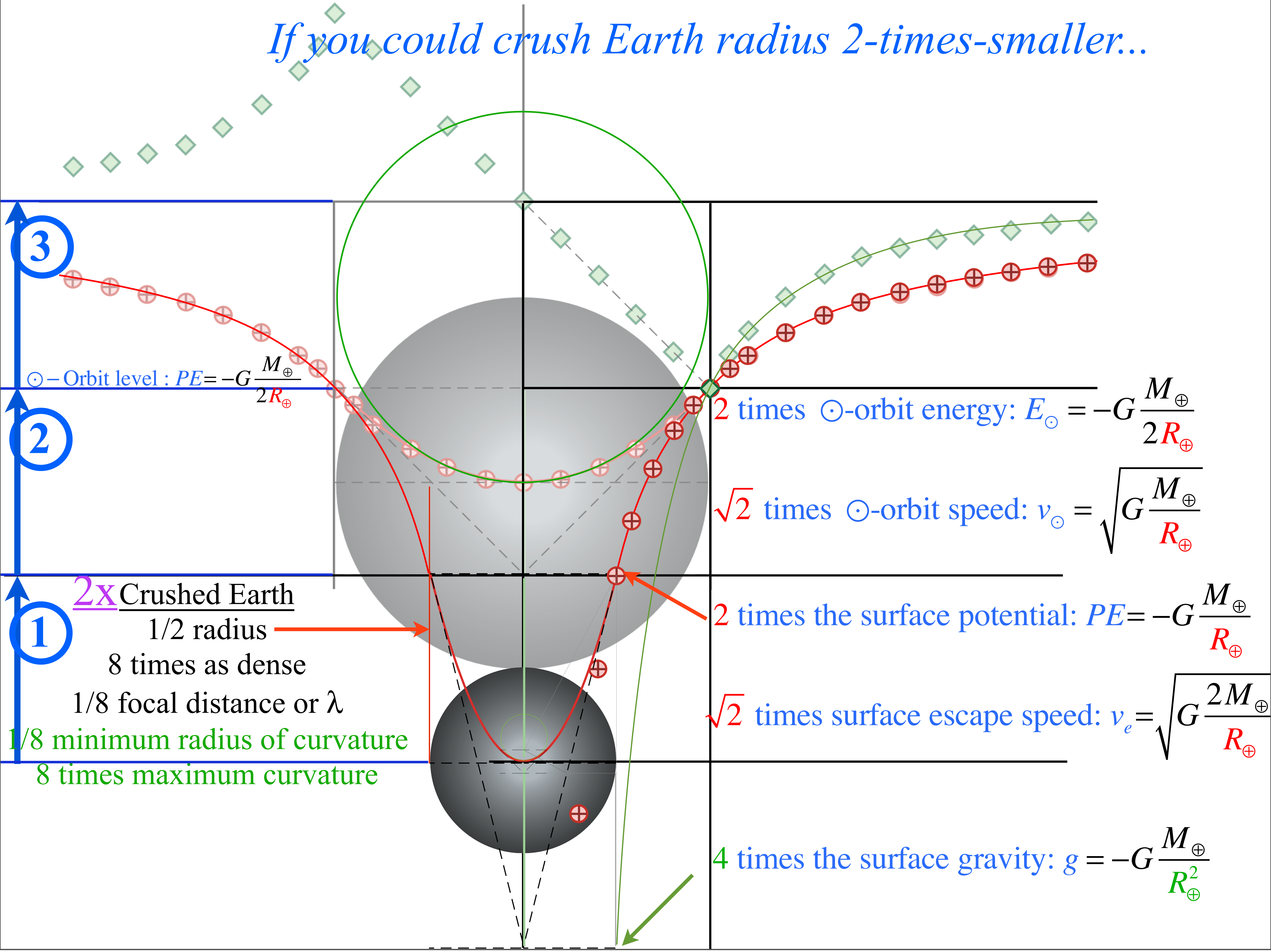


surface potential: $PE = -G\frac{M_\oplus}{R_\oplus}$

surface escape speed: $v_e = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$

$KE = PE$ relation: $\frac{1}{2}mv_e^2 = G\frac{mM_\oplus}{R_\oplus}$

If you could crush Earth radius 2-times-smaller...



Geometry of idealized "Sophomore-physics Earth"

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Earth matter vs nuclear matter:

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Examples of "crushed" matter

Earth matter

$Earth\, mass: M_{\oplus} = 5.9722\ \times\ 10^{24}\, kg. \simeq 6.0 \cdot 10^{24}\, kg.$ Density $\rho_{\oplus}$ $=??$

$Earth\, radius: R_{\oplus} = 6.371 \cdot 10^{6}\, m \simeq 6.4 \cdot 10^{6}\, m \quad Earth\, volume: (4\pi / 3) R_{\oplus}{}^{3} \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\, m^{3}$

$(6.4)^3 \sim 262$ *and* $(4\pi/3)260 = 1098 \sim 10^3$

Examples of "crushed" matter

Earth matter

Earth mass : $M_{\oplus} = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ Density $\rho_{\oplus} \sim 6.0 \cdot 10^{24\text{-}21} \sim 6 \cdot 10^{3} kg/m^{3}$

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Density of solid Fe$=7.9 \cdot 10^{3}$kg/m^{3}
Density of liquid Fe$=6.9 \cdot 10^{3}$kg/m^{3}

Examples of "crushed" matter

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Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

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That's $100 \cdot 10^{-27} = 10^{-25}\,kg$ packed into a volume of ${}^{4\pi}/_{3} r^{3} = {}^{4\pi}/_{3}\,(3 \cdot 10^{-15})^{3}\,m^{3}$ or about $10^{-43}\,m^{3}$.

$36\pi = 113 \sim 10^{2}$

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Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in the size of a fingertip (1cc).

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300 m$ would approach neutron-star density.

Examples of "crushed" matter

Earth matter

Earth mass: $M_{\oplus} = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg$. Density $\rho_{\oplus} \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^{3} kg/m^{3}$

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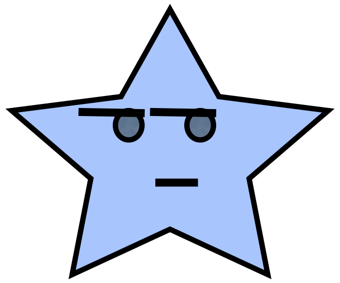
Introducing the "Neutron starlet"

$1\ cm^{3}$ *of nuclear matter: mass*$= 10^{12}\ kg.$



Examples of "crushed" matter

Earth matter

Earth mass : $M_{\oplus} = 5.9722 \times 10^{24} kg. \simeq 6.0 \cdot 10^{24} kg.$ Density $\rho_{\oplus} \sim 6.0 \cdot 10^{24\text{-}21} \sim 6 \cdot 10^{3} kg/m^{3}$

Earth radius : $R_{\oplus} = 6.371 \cdot 10^{6} m \simeq 6.4 \cdot 10^{6} m$ *Earth volume* : $(4\pi / 3) R_{\oplus}^{3} \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21} m^{3}$

Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

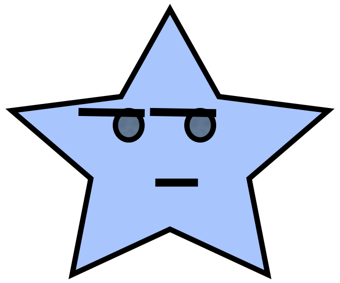
That's $100 \cdot 10^{-27} = 10^{-25}$ *kg* packed into a volume of $\frac{4\pi}{3} r^{3} = \frac{4\pi}{3} (3 \cdot 10^{-15})^{3}\ m^{3}$ or about $10^{-43}\ m^{3}$.

Nuclear density is $10^{-25+43} = 10^{18} kg/m^{3}$ or a *trillion* (10^{12}) kilograms in the size of a fingertip.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300m$ would approach neutron-star density.

Introducing the "Neutron starlet"

1 cm^{3} *of nuclear matter: mass*$= 10^{12}$ *kg.*



Introducing the "**Black Hole Earth**"

Suppose Earth is crushed so that its surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^{8} m/s$.

$c \equiv 299{,}792{,}458 m/s$ (EXACTLY)

$$V_{escape} = \sqrt{(2GM/R_{\otimes})}$$

(from p. 43)

$G = 6.67384(80) \cdot 10^{-11} Nm^{2}/C^{2} \sim (2/3) 10^{-10}$

Examples of "crushed" matter

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Introducing the "Neutron starlet"

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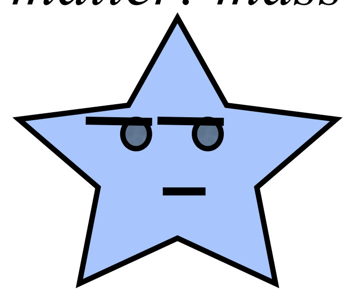

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(from p. 43)

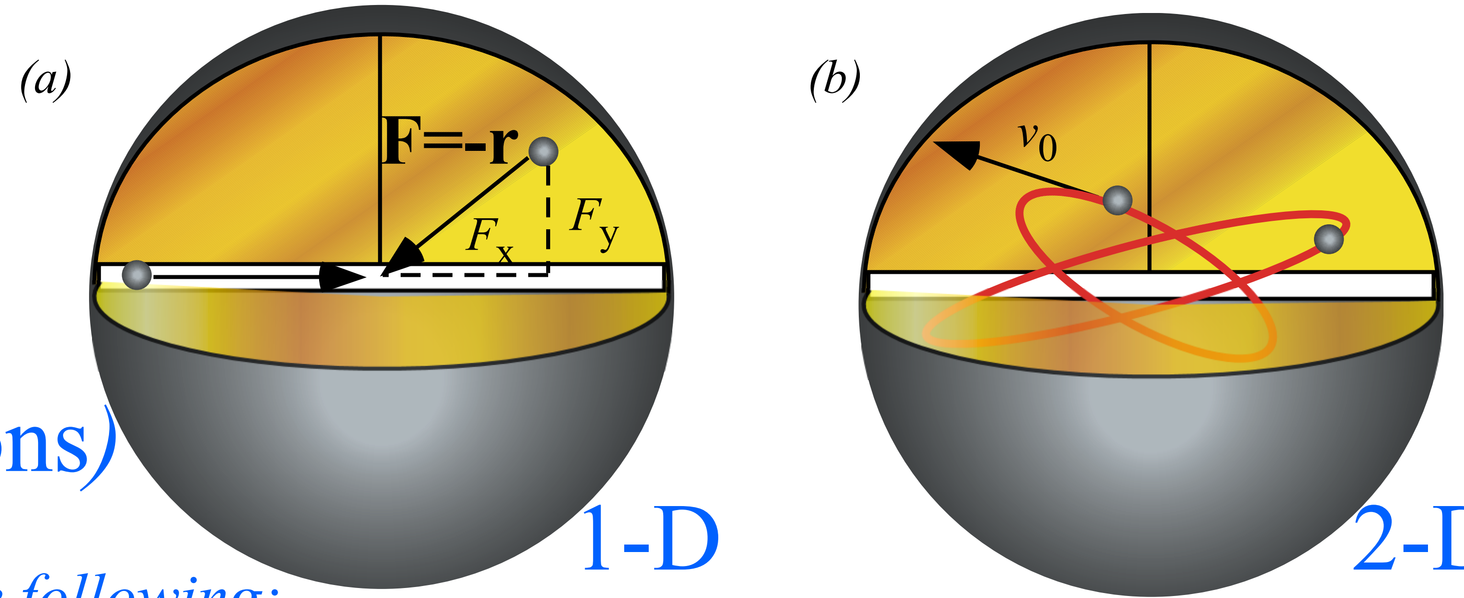
$R_{\otimes} = 2GM/c^{2} = 8.9 mm \sim 1 cm$ *(fingertip size!)*

$G = 6.67384(80) \cdot 10^{-11} Nm^{2}/C^{2} \sim (2/3) 10^{-10}$

→ *Introducing 2D IHO orbits and phasor geometry*

Phasor "clock" geometry

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

$F = -x$ (1-Dimension)

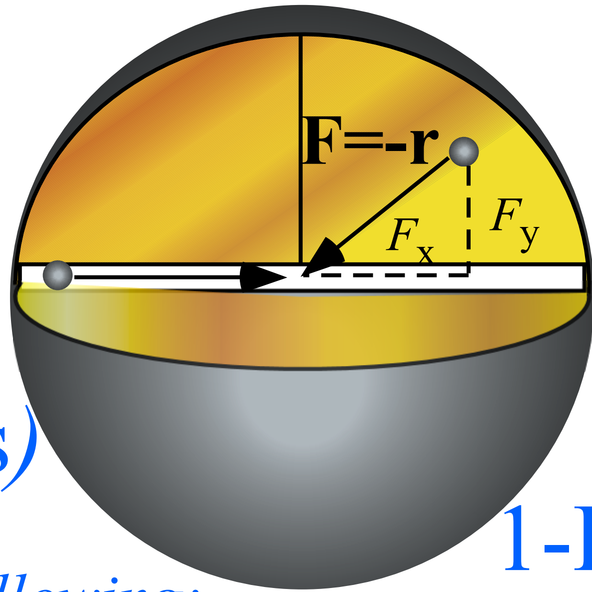
$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

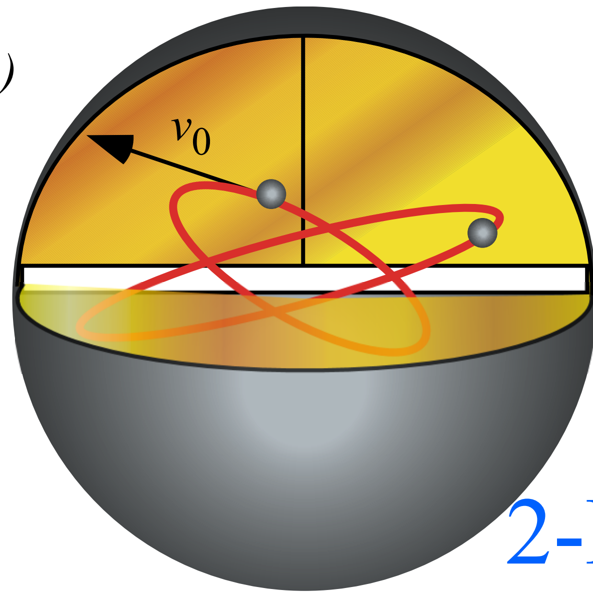
$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Isotropic Harmonic Oscillator *phase dynamics* *in uniform-body*

(a)



(b)



Unit 1
Fig. 9.10

I.H.O. Force law

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F =-**r** (2 or 3-Dimensions)

1-D

2-D or 3-D

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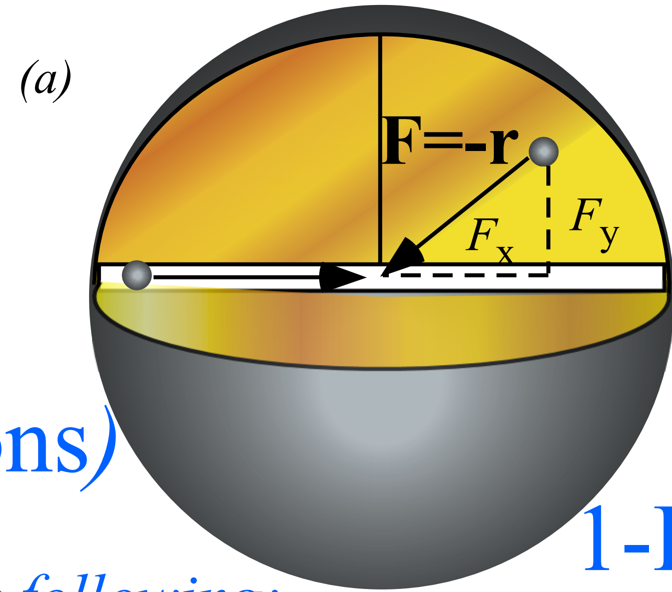
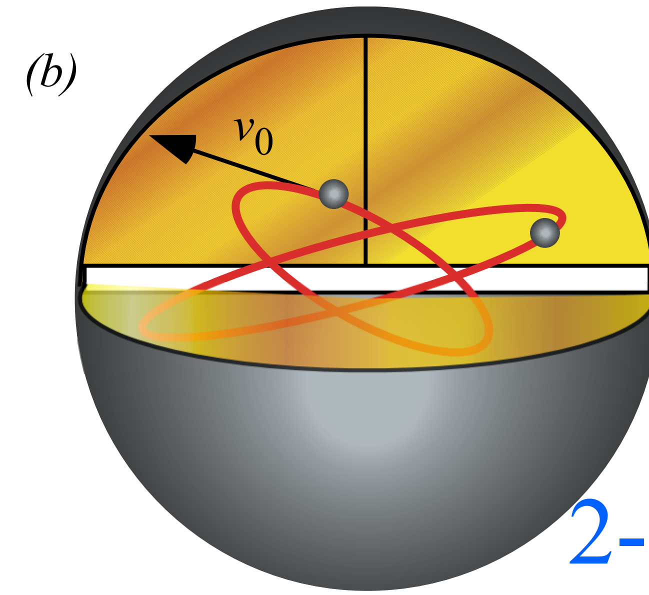
Isotropic Harmonic Oscillator phase dynamics in uniform-body



1-D



Unit 1
Fig. 9.10

2-D or 3-D

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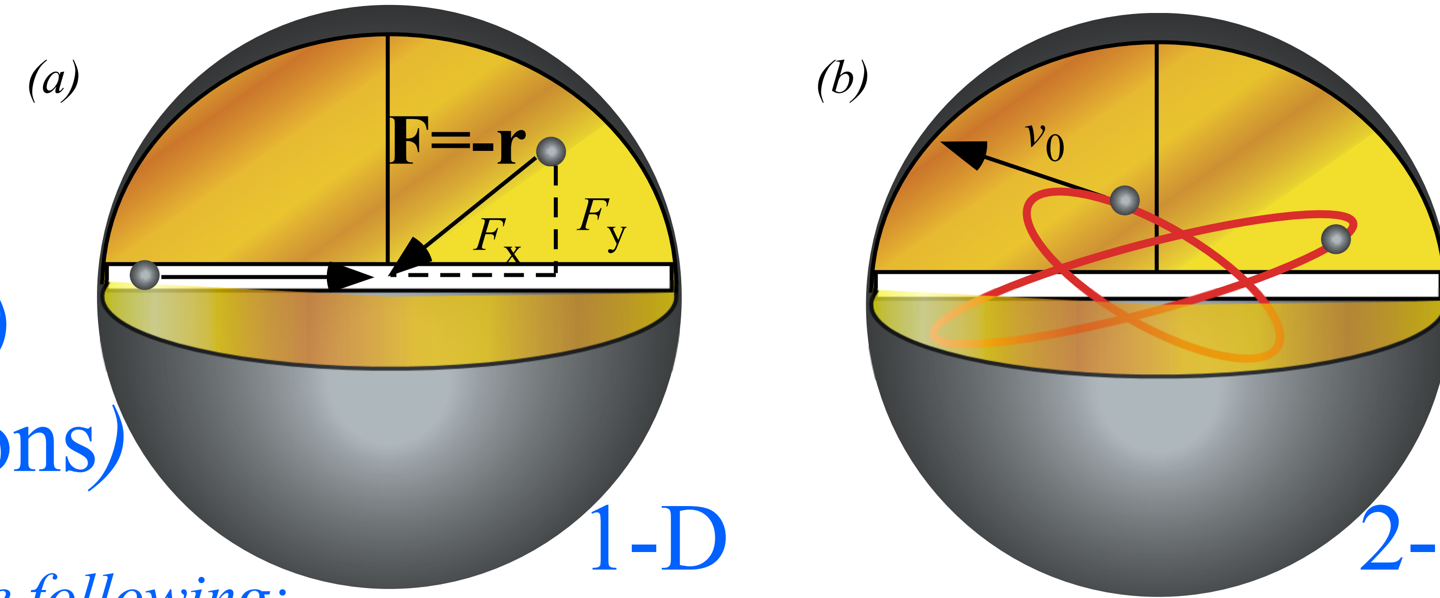
$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let:\ (1)\ v = \sqrt{2E/m}\cos\theta, \quad and:\ (2)\ x = \sqrt{2E/k}\sin\theta$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body



1-D

2-D or 3-D

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Unit 1
Fig. 9.10

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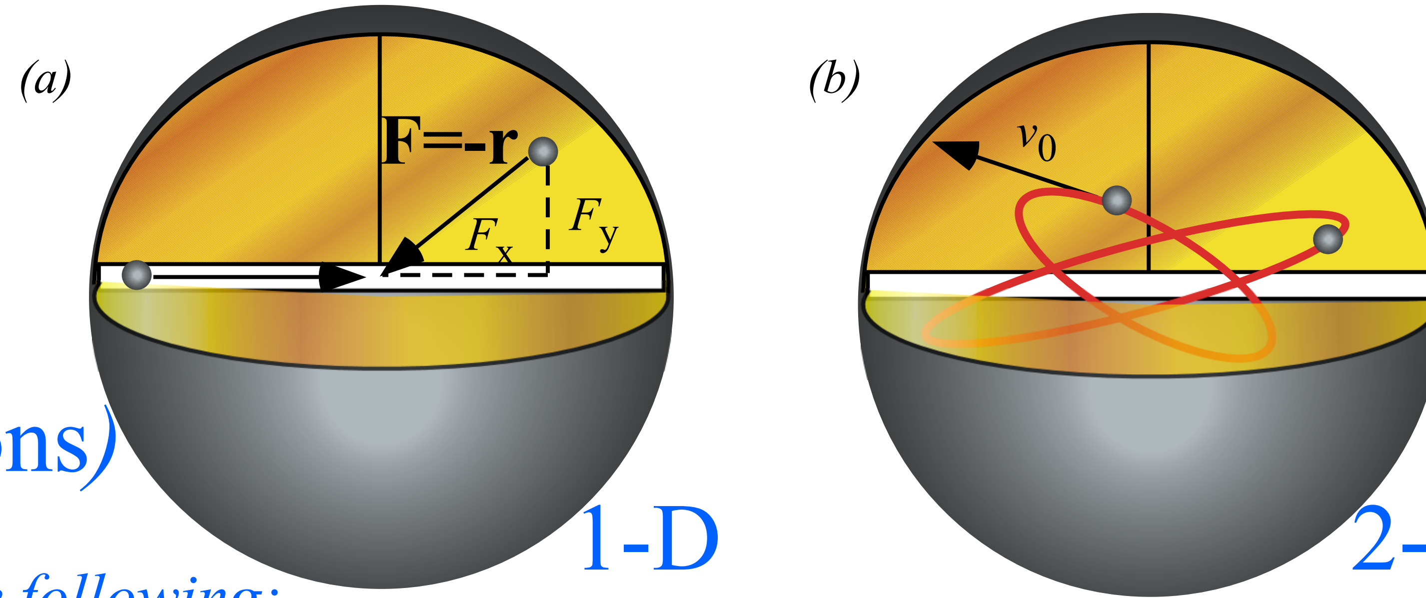
$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta}$$

(first equality: *by* (1); $\frac{d\theta}{dt}=\omega$: *by def.* (3))

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

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by (1) by def. (3) by (2)

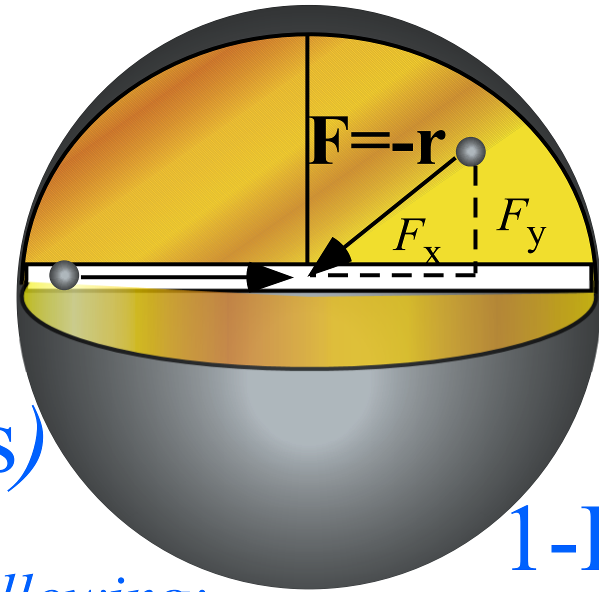
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

F =-x (1-Dimension)

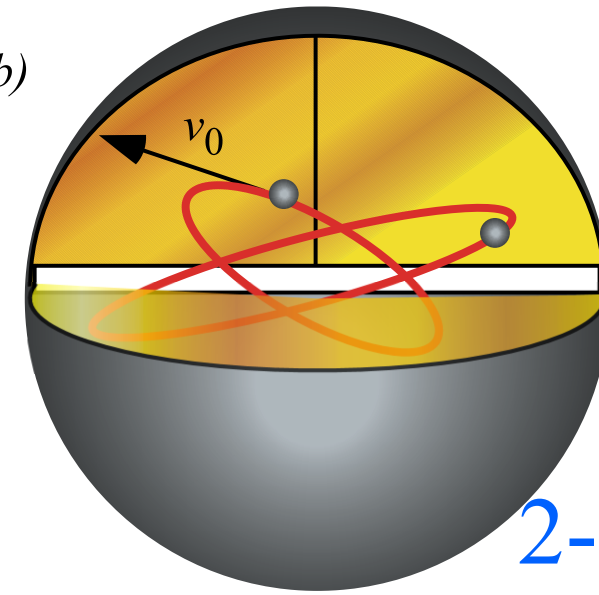
F =-**r** (2 or 3-Dimensions)

(a)



1-D

(b)



Unit 1
Fig. 9.10

2-D or 3-D

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by **(1)** — *by def.* **(3)** — *by* **(2)**

by def. **(3)**

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

divide this by **(1)**

Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

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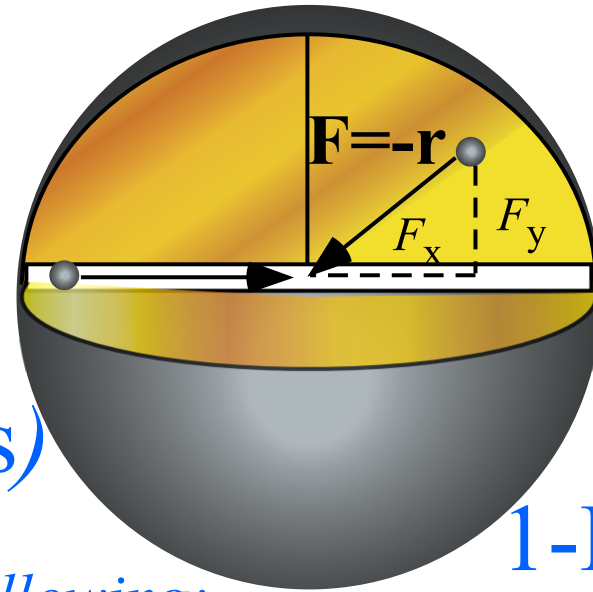
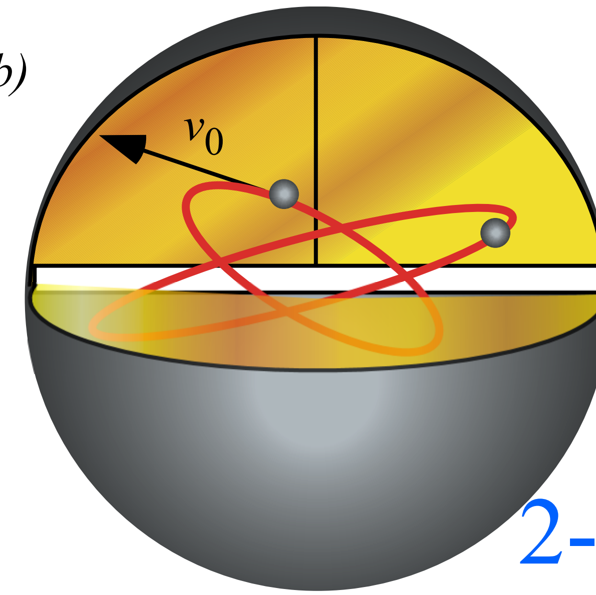


1-D

(b)



Unit 1
Fig. 9.10

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by **(1)** · *by def.* **(3)** · *by* **(2)**

by def. **(3)**

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

divide this by **(1)**

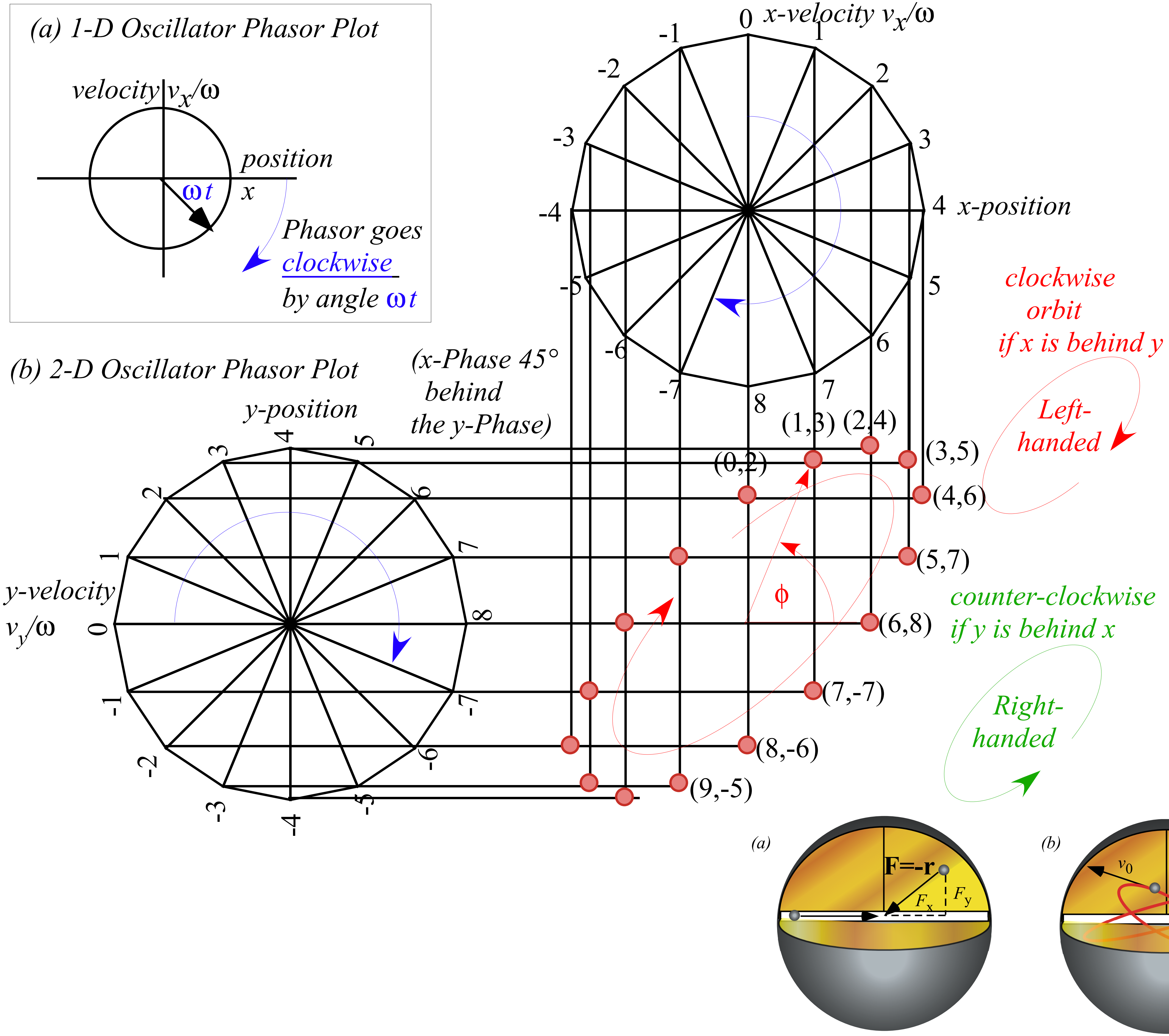
by integration given constant ω:

$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$

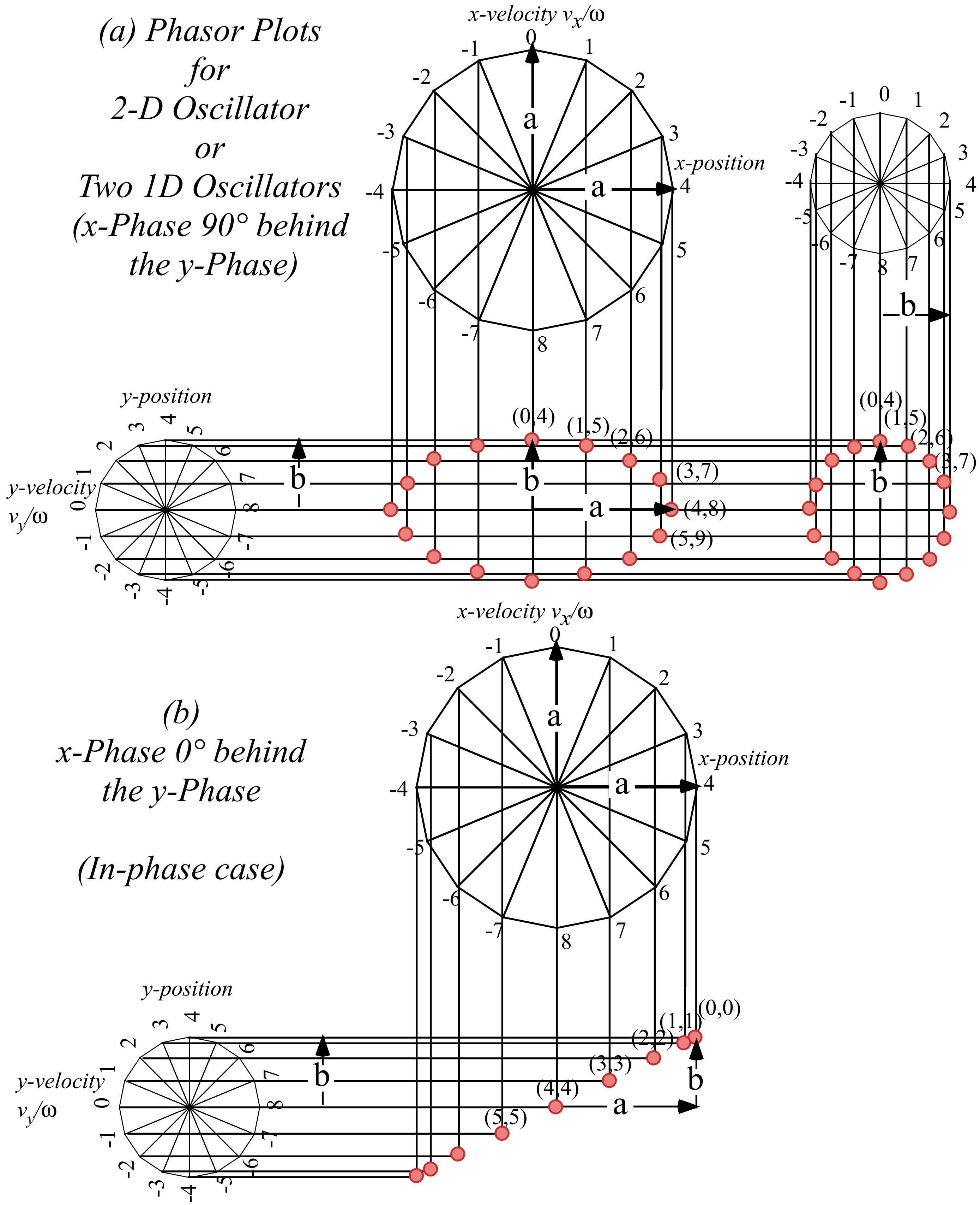
Introducing 2D IHO orbits and phasor geometry

→ *Phasor "clock" geometry*

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10



Unit 1
Fig. 9.12

These are more generic examples with radius of x-phasor differing from that of the y-phasor.