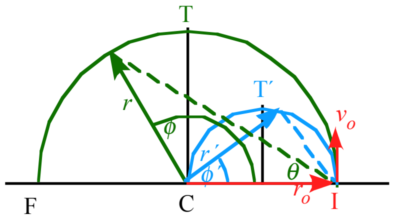
Self-Study Problems with solutions Set 16 - **Classical Mechanics 5103**

Involves Unit 5 Ch. 1-4 and Units 6 Ch. 1-6. Try to do exercises first before looking at solutions



Exercise 5.2.1 Circular firing squad: More Thales geometry problems

(a) What power-law PE(s) $V(r)=kr^n$ give circular paths ITF? For each relate k,n, and initial values (r,v) at I.

(b) What power-law PE(s) $V(r)=kr^n$ give circular paths IT′C? For each relate k,n, and initial values (r,v) at I.

(c) For each discuss time behavior of *C-pt* polar angle ϕ and *I-pt* polar angle θ. Which orbit radii sweep equal-area-in-equal-time (as Kepler would have it)?

Exercise 5.2.2 Repulsive oscillation

Derive formulas for the orbital path of a mass m in an isotropic repulsive quadratic potential

$V(\rho)=-\frac{1}{2}k\rho^2$ ($k>0$). Discuss analytic or geometric properties of the resulting orbits.

Exercise 5.2.3 Attractive oscillation

Verify oscillator equation of time in (5.2.11). Verify turning point formulas in terms of orbit radii a and b.

Exercise 5.2.4 Coulomb approach-avoidance

Derive an equation of time for (a) attractive Coulomb potential ($k<0$) (b) repulsive potential ($k>0$)

$$V(\rho)=-\frac{k}{\rho}$$

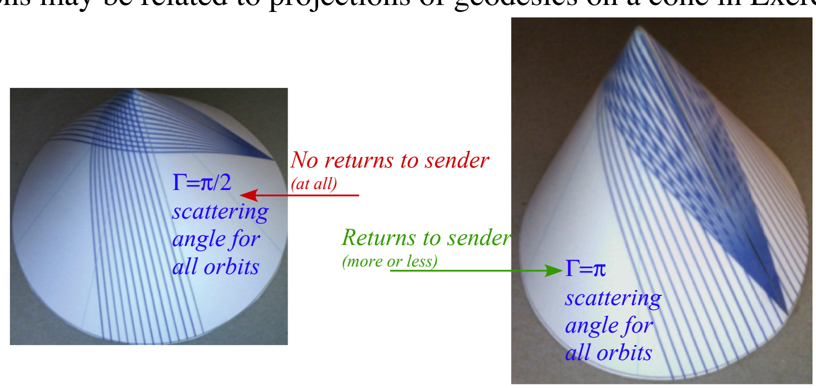
Exercise 5.2.5 Dyin' Ion

Suppose an atom of mass m is orbits a heavy atom of mass $M>>m$ (Assume M fixed) which is polarized so there is an attractive constant-dipole-like potential $V(r)=-A/r^2$ between the two particles.

(a) Derive the constant-momentum-p_ϕ effective radial potential form for the system.

(b) Solve and discuss the orbital path $r(\phi)$ for select values of A and p_ϕ. (Start with the $A=0$ solution.)

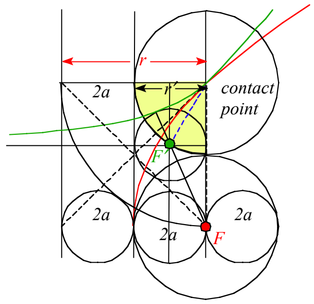
(c) The solutions may be related to projections of geodesics on a cone in Exercise5.2.6 *(Discuss)*.



Exercise 5.2.6 Space Bowling

Suppose a giant metal cone of polar half angle Θ is set up next to the space station for space bowling. (They have little else to do.) The polar angle is small enough that if the bowler misses the apex by a little bit, the ball orbits around and returns toward the bowler. *(See 2nd figure above.)* What value(s) of angle Θ allow this?

What if instead, NASA prefers ceiling-return, that is, along lines near azimuth of 180° from the bowler?

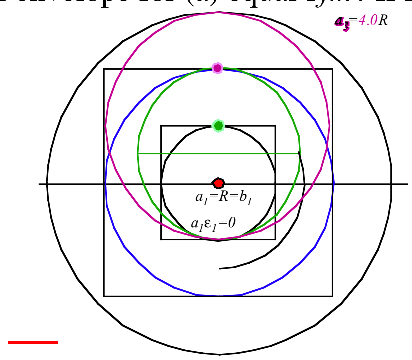


Exercise 5.3.1. Rutherford Coulomb

Use geometry to derive the equation of the Rutherford scattering caustic (contacting envelope) as a function of alpha particle mass m, Coulomb constant k and beam energy E. Include a description or construction of the contact point of a general trajectory with its envelope. Is the contact point ever a closest approach to F?

(a) How many eV of energy are needed to get an α^{+4} particle onto the Au^{+79} nuclear surface at $r= 1fm$?

(b) Does the minimum radius of curvature of envelope for (a) equal $1fm$? If not, what? And, so what?

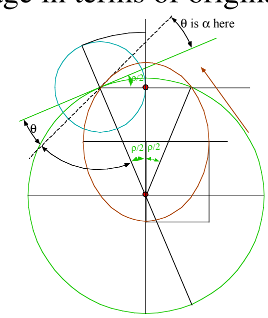


Exercise 5.3.4. Two burns

Space shuttle is in circular orbit of radius R_0 and in two burns moves to circular orbit of radius nR_0.

(a) Describe or sketch (for n=2 and 3) the quickest way to do this.

(b) Find energy and angular momentum of each stage in terms of original energy E_0 and momentum μ_o.



Exercise 5.3.6. Optimum range angle

For plane trajectories in uniform gravity a $\alpha=45°$ launch angle gives maximum range. Also, there is no maximum range (given by effective longitude angle ρ) for a given angle if you have enough v_0. For ballistic missiles traveling in space (or for war on the moon) all is different.

(a) Use geometry to derive the maximum range longitude angle ρ along the Earth's surface as a function of the launch elevation angle α above the horizon neglecting Earth spin. (First, why is ρ so limited?)

(b) Use geometry to derive launch angle α which throws a missile to a given range ρ with *minimum* energy. Compare result with that of part (a).