Lecture 31
Thur. 12.08.2016

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

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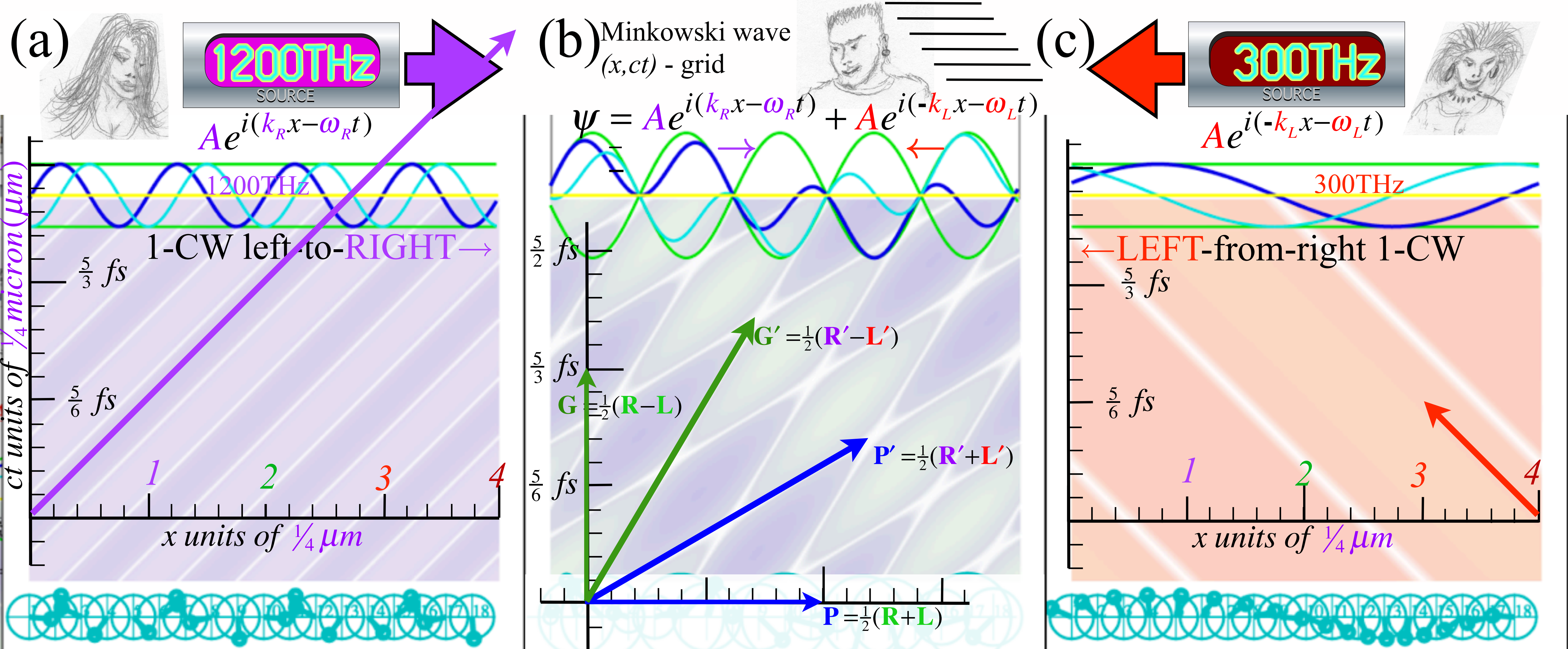


(d)

$$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$

$$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$$

$$=\psi'_{phase}\psi'_{group}$$

$R'=k_R x\text{-}\omega_R t$ and: $L'=\text{-}k_L x\text{-}\omega_L t$

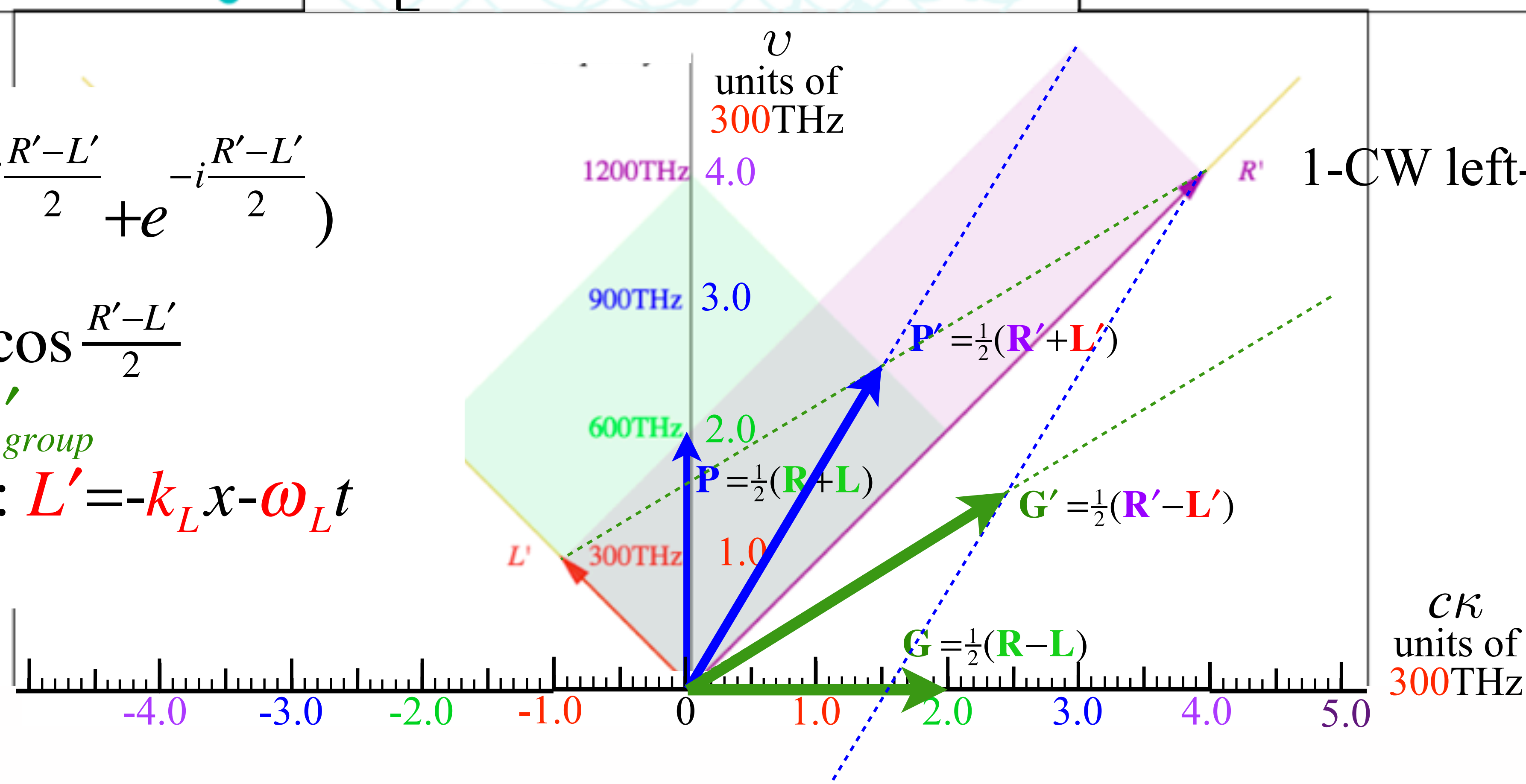
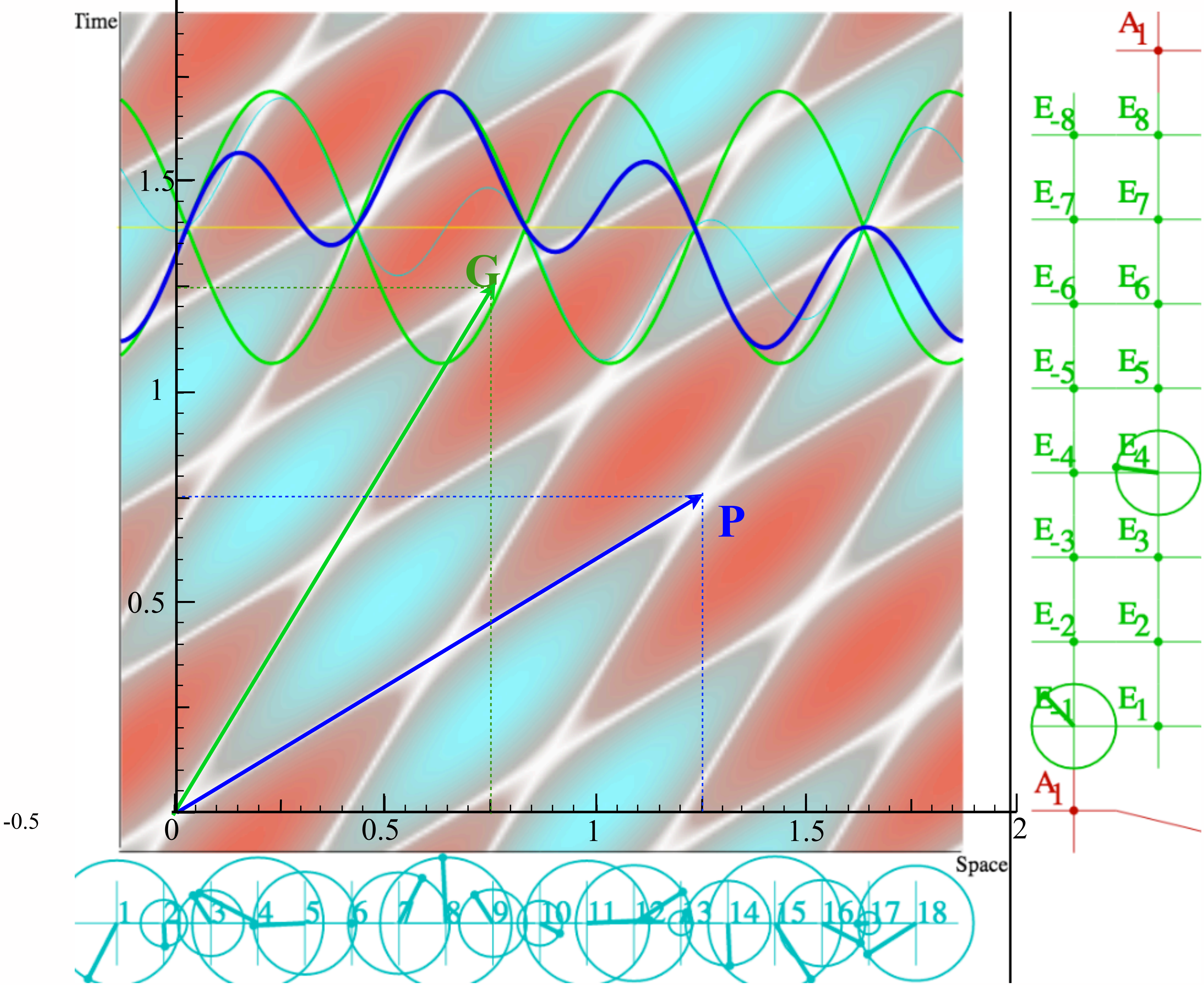
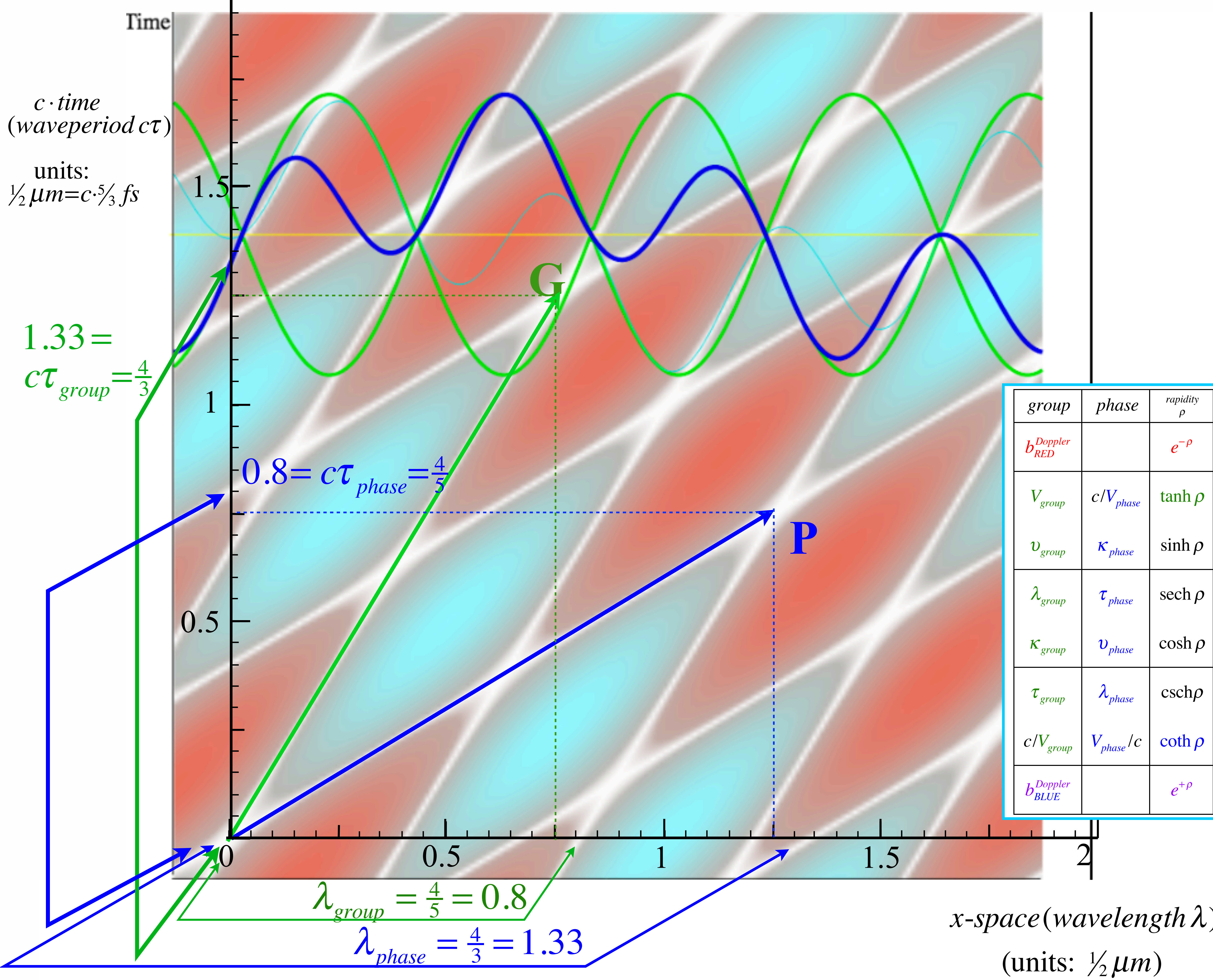
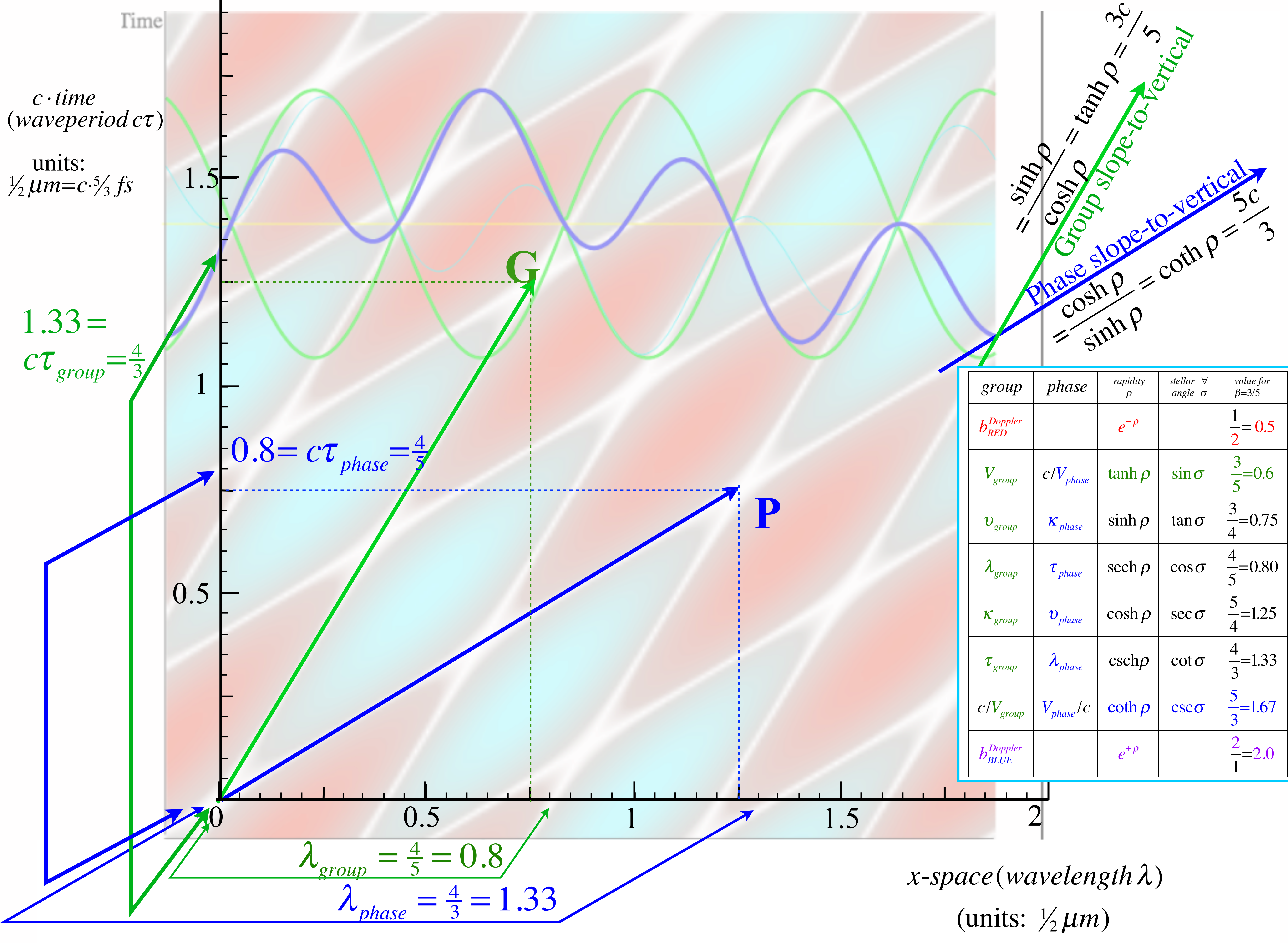


Fig. 10 in text
Relawavity...


Time
1.5
1
0.5
-0.5
0
0.5
1
1.5
2
Space
G
P
A1
E-8 E8
E-7 E7
E-6 E6
E-5 E5
E-4 E4
E-3 E3
E-2 E2
E-1 E1
A1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18



group	phase	rapidity ρ	stellar $\forall$ angle σ	value for β=3/5
$b^{Doppler}_{RED}$		$e^{-\rho}$		$\frac{1}{2}$= 0.5
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}$=0.6
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}$=0.75
λ_{group}	τ_{phase}	$\mathrm{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}$=0.80
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}$=1.25
τ_{group}	λ_{phase}	$\mathrm{csch}\rho$	$\cot\sigma$	$\frac{4}{3}$=1.33
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}$=1.67
$b^{Doppler}_{BLUE}$		$e^{+\rho}$		$\frac{2}{1}$=2.0



group	*phase*	*rapidity* ρ	*stellar* ∀ *angle* σ	*value for* β=3/5
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2}$= 0.5
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}$=0.6
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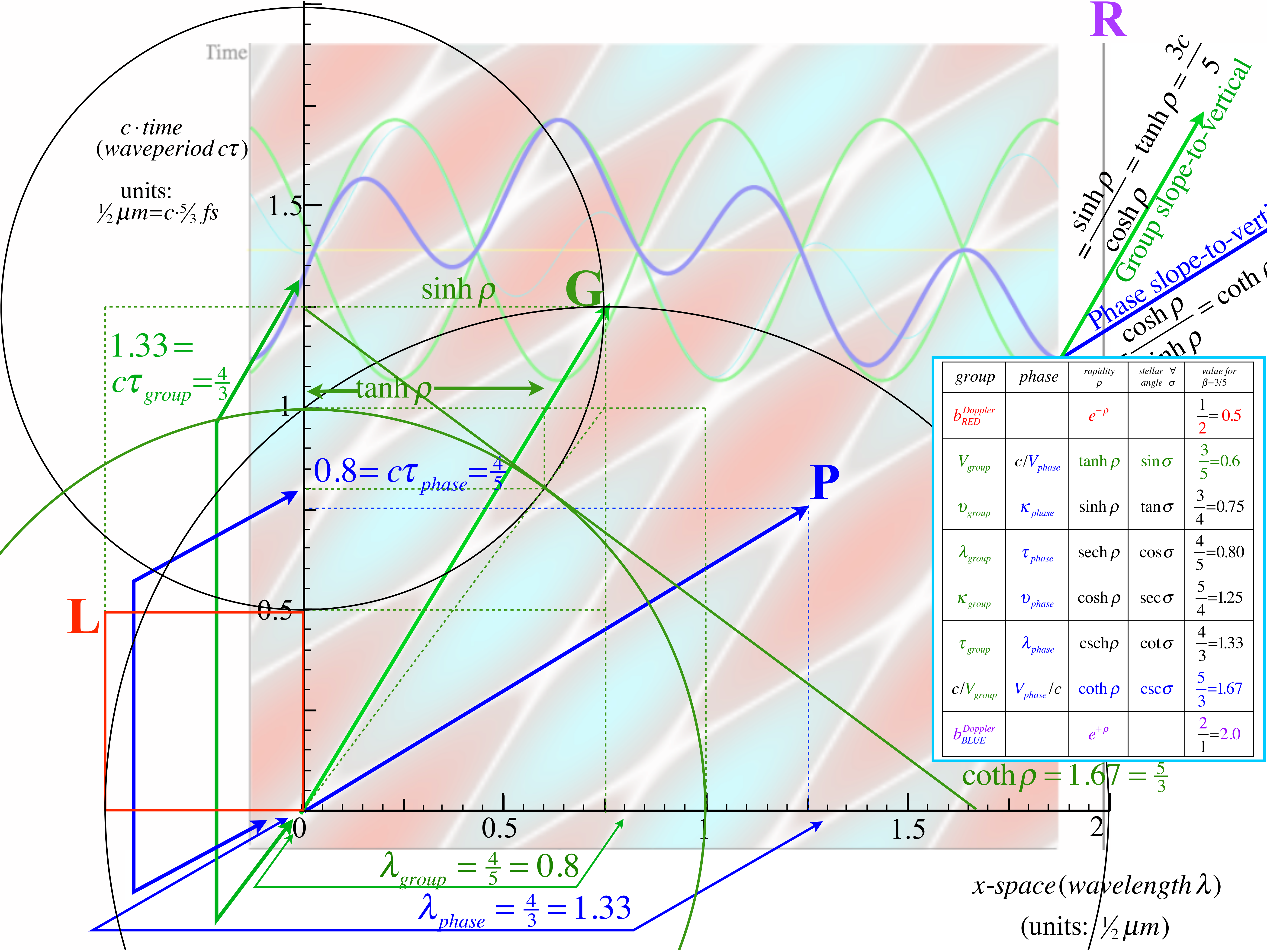
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}$=2.0



group	*phase*	*rapidity* ρ	*stellar angle* $\forall\ \sigma$	*value for* β=3/5
$b^{Doppler}_{RED}$		$e^{-\rho}$		$\frac{1}{2}$= 0.5
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}$=0.6
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c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}$=1.67
$b^{Doppler}_{BLUE}$		$e^{+\rho}$		$\frac{2}{1}$=2.0

This map has circle sector arc-area $\sigma = 0.6435$ set to angle $\angle\sigma = 36.87° = 0.6435\,radian$

Also it is set to hyperbola sector arc-area $\rho = 0.6931$ angle $\angle\rho = \nu = 30.96°$

sin(σ)=0.6000	=	tanh(ρ)	= 3/5
tan(σ)=0.7500	=	sinh(ρ)	= 3/4
sec(σ)=1.2500	=	cosh(ρ)	= 5/4
cos(σ)=0.8000	=	sech(ρ)	= 4/5
cot(σ)=1.3333	=	csch(ρ)	= 4/3
csc(σ)=1.6667	=	coth(ρ)	= 5/3

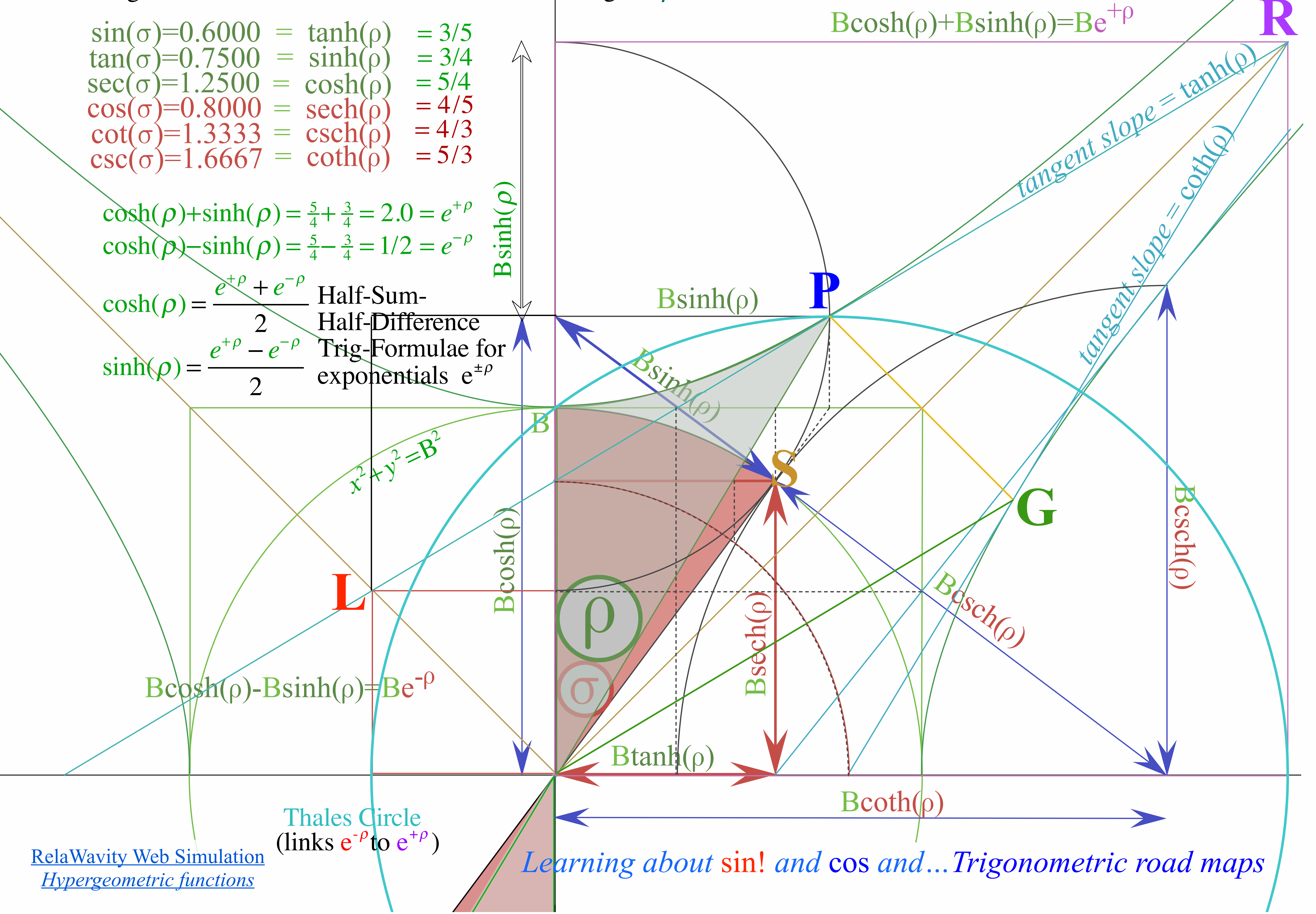
$\cosh(\rho)+\sinh(\rho) = \frac{5}{4} + \frac{3}{4} = 2.0 = e^{+\rho}$

$\cosh(\rho)-\sinh(\rho) = \frac{5}{4} - \frac{3}{4} = 1/2 = e^{-\rho}$

$$\cosh(\rho) = \frac{e^{+\rho} + e^{-\rho}}{2}$$

$$\sinh(\rho) = \frac{e^{+\rho} - e^{-\rho}}{2}$$

Half-Sum-Half-Difference Trig-Formulae for exponentials $e^{\pm\rho}$



RelaWavity Web Simulation
Hypergeometric functions

Learning about sin! and cos and…Trigonometric road maps

Fig. 11 in text <u>*Relawavity...*</u>

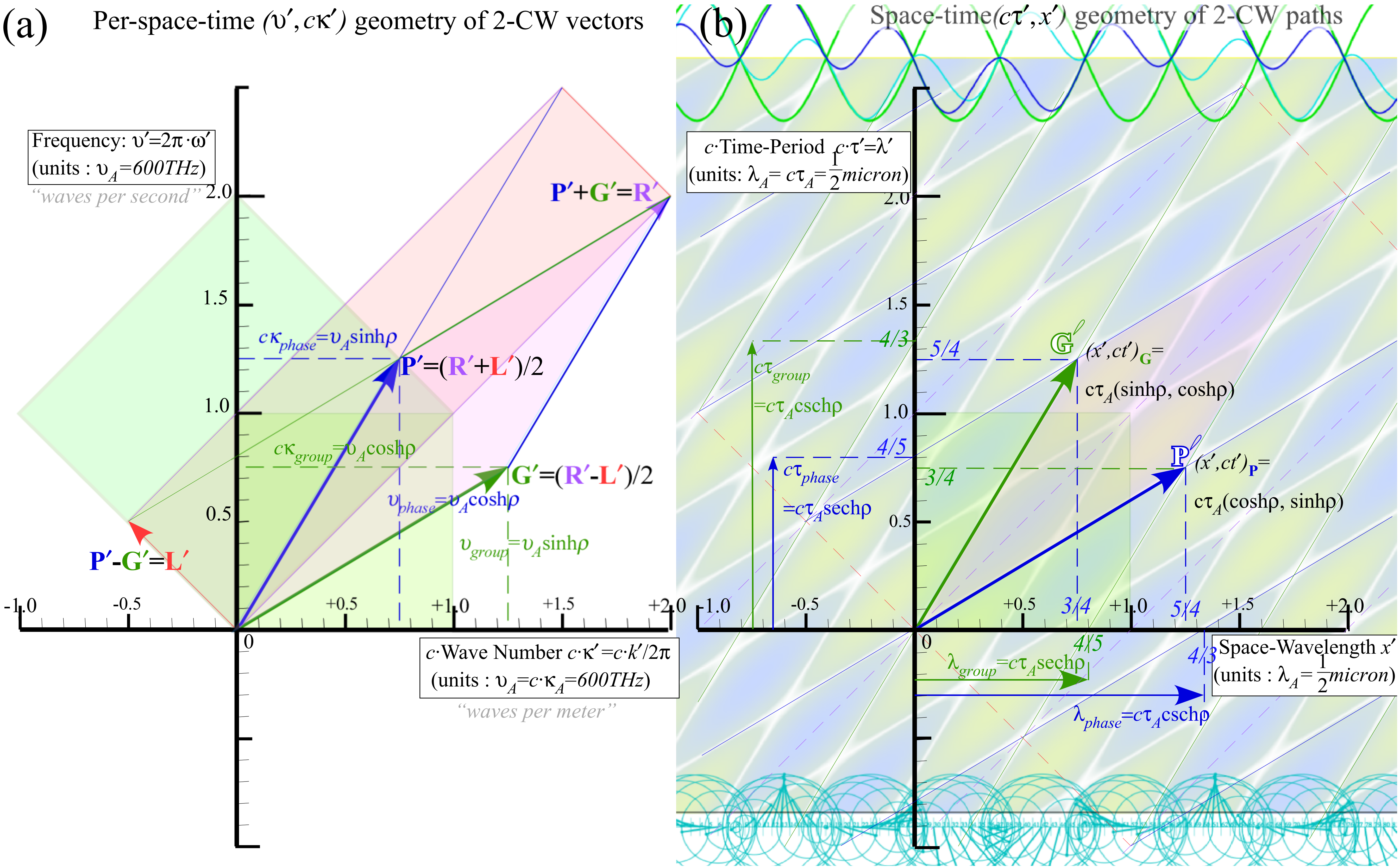
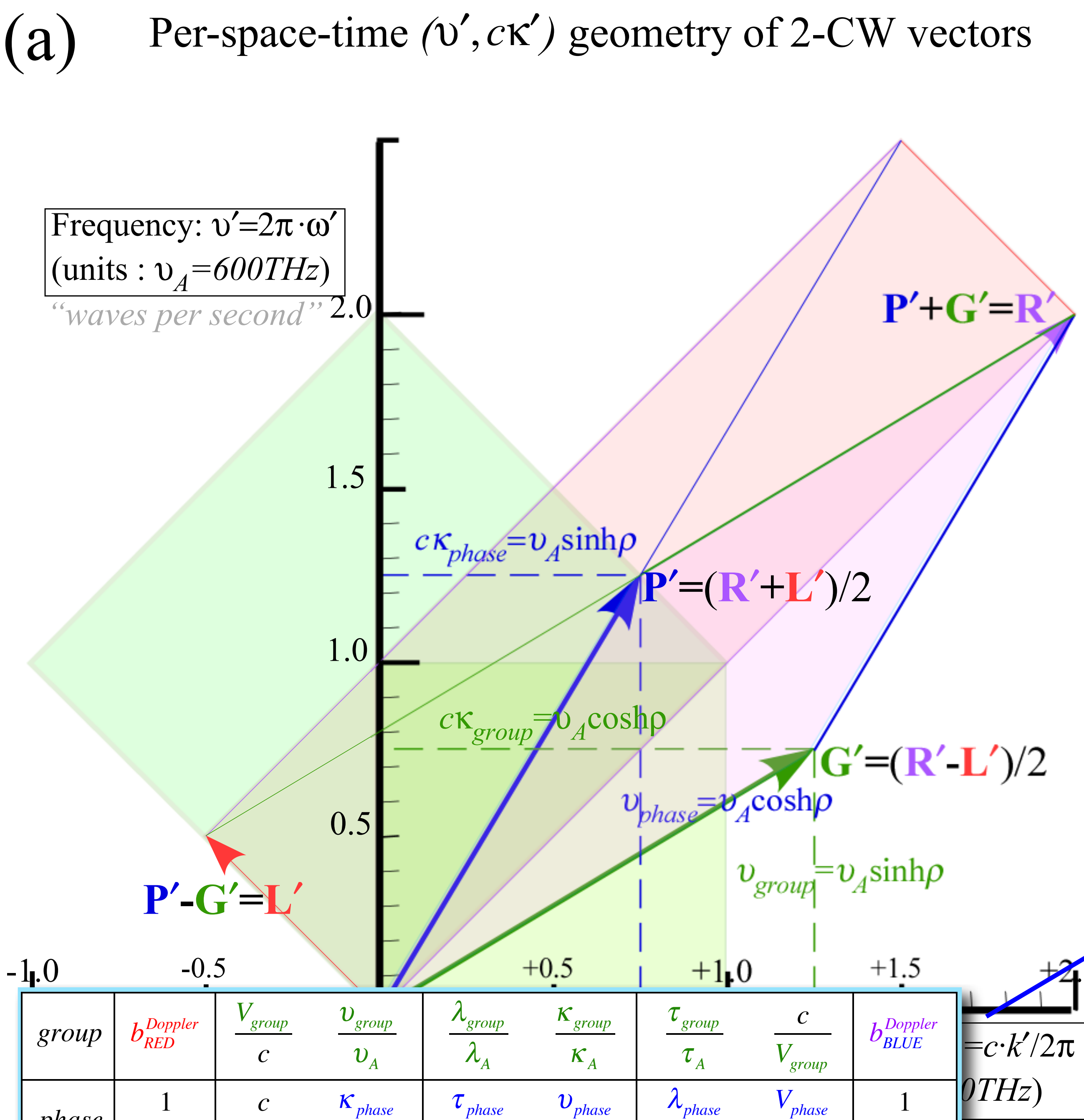
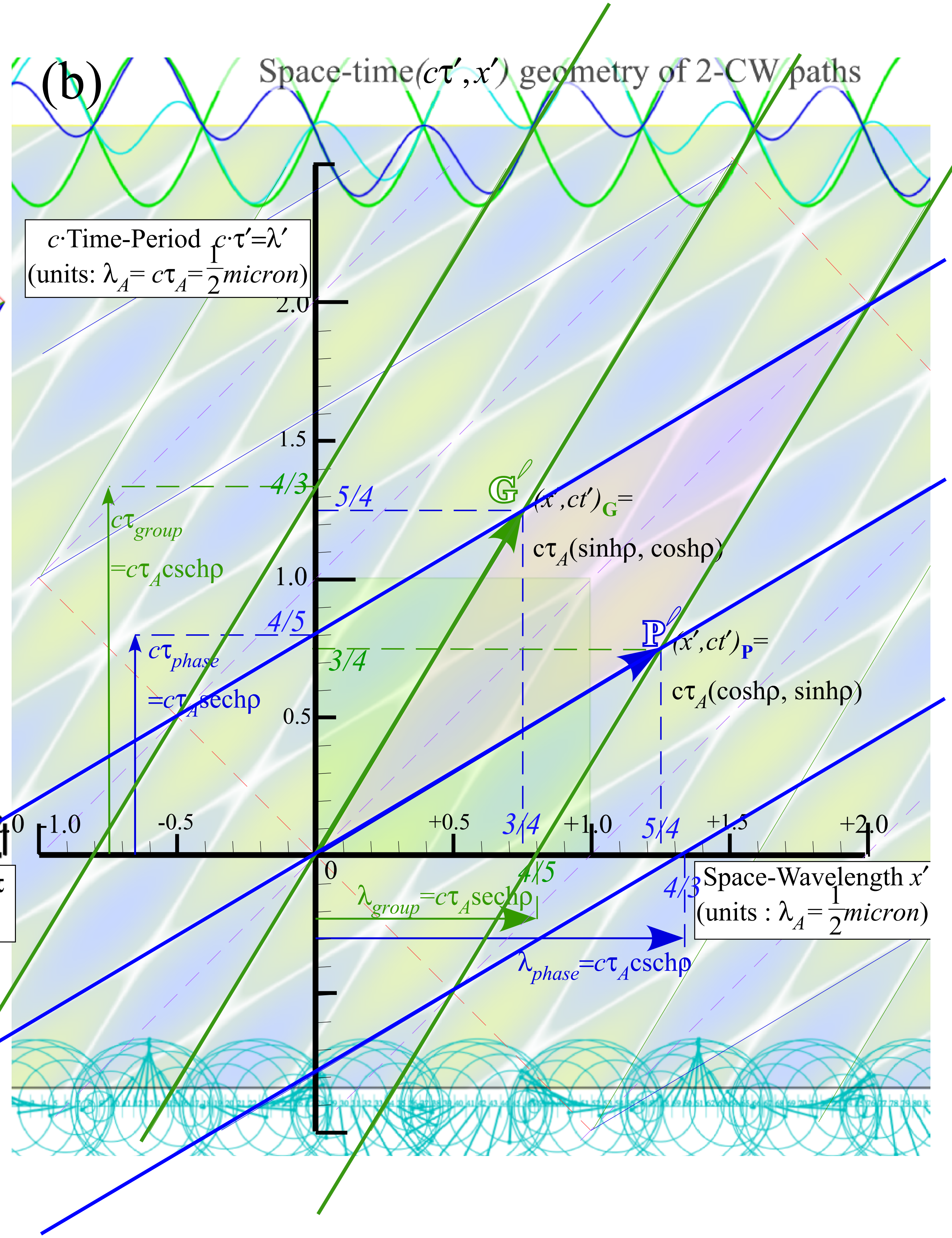


Fig. 11 in text *Relawavity...*





group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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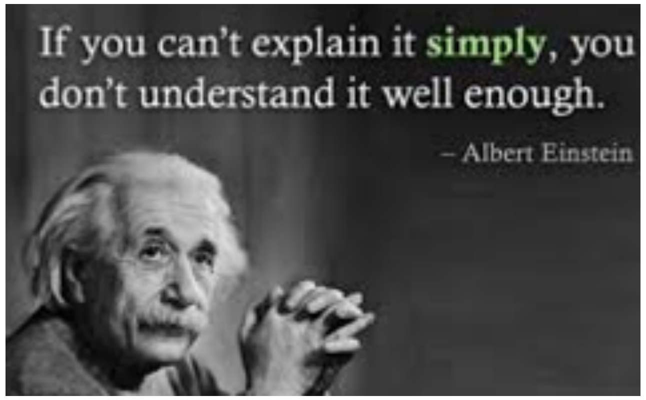
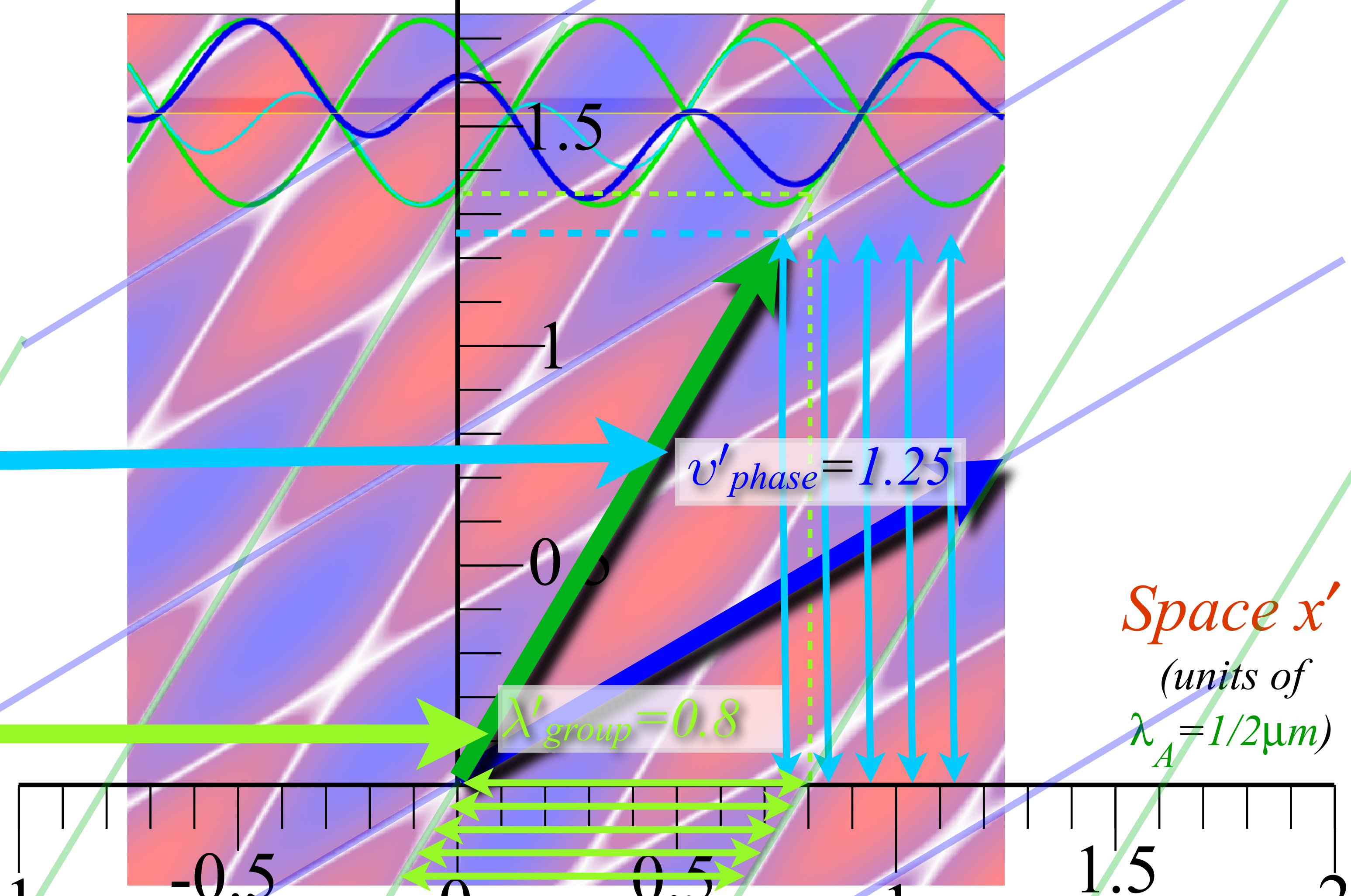
Two Famous-Name Coefficients

Review of Lect. 30 p.106

Albert Einstein 1859-1955



Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation**
(dilated by 25% here)

This number is called a: **Lorentz length-contraction**
(contracted by 20% here)

Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation - Relativistic *Terms*
(Expanded Table)

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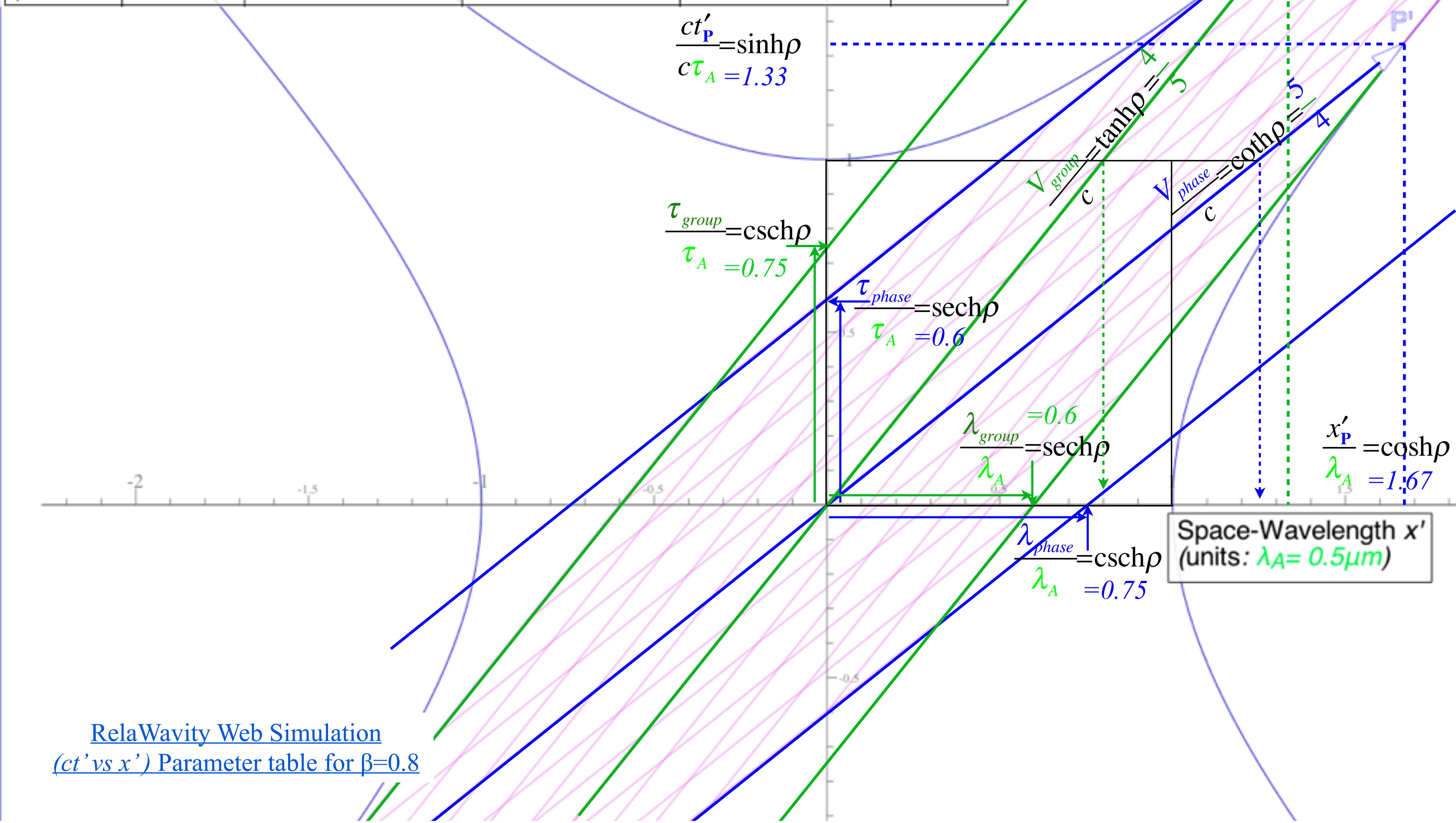
time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=0.80	0.33	0.80	1.34	0.60	1.67	0.75	1.25	3.01



[RelaWavity Web Simulation](#)
[(ct' vs x') Parameter table for β=0.8](#)

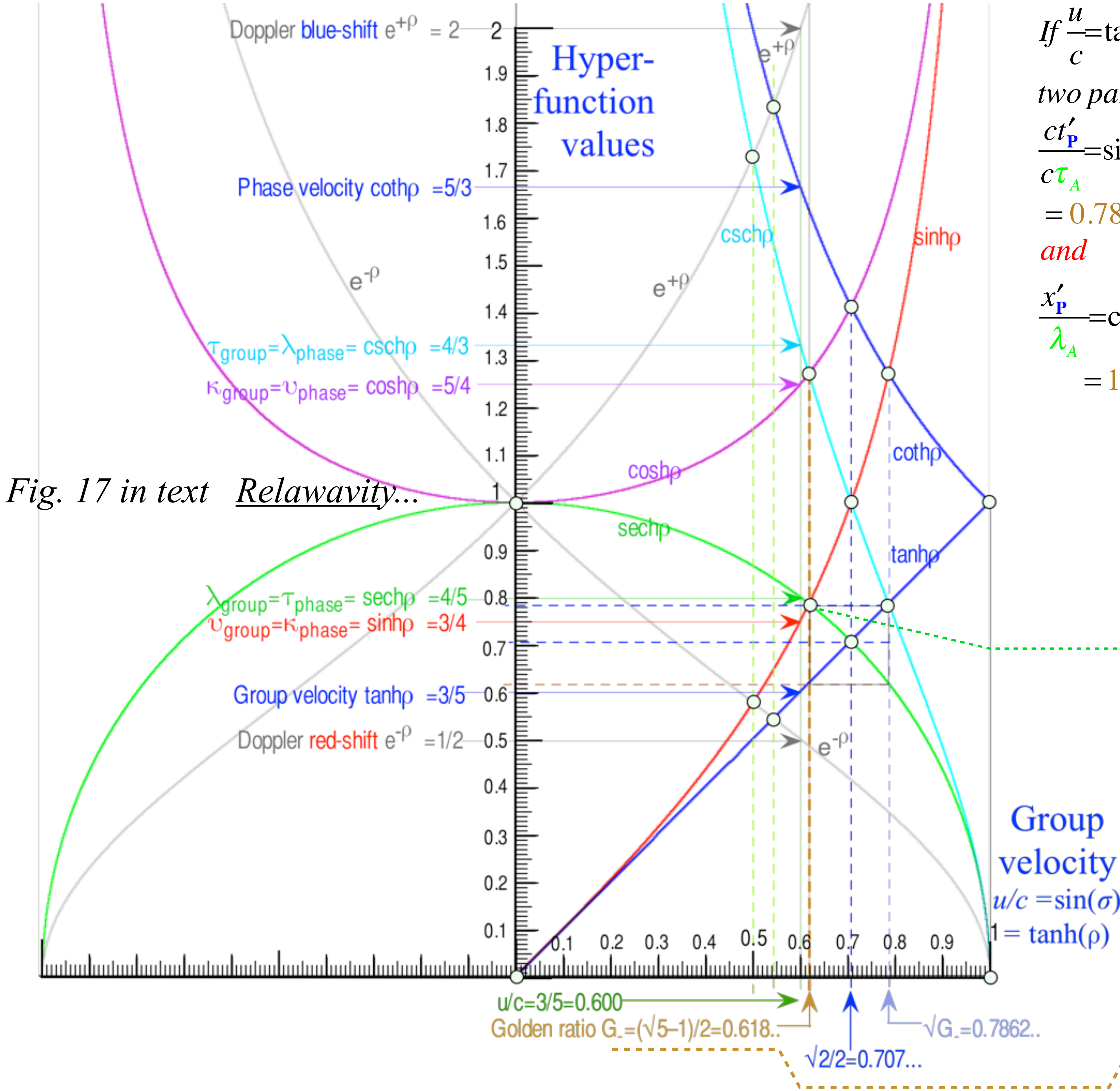


Fig. 17 in text *Relawavity...*

$$If\ \frac{u}{c}=\tanh\rho = 0.618\ldots (Golden\text{-}Mean\ G\_)$$

two parameters become exactly equal :

$$\frac{ct'_P}{c\tau_A}=\sinh\rho=\frac{\lambda_{group}}{\lambda_A}=\frac{\tau_{phase}}{\tau_A}=\mathrm{sech}\rho$$

$$=0.786..=\sqrt{G\_}\qquad =0.786..$$

and

$$\frac{x'_P}{\lambda_A}=\cosh\rho=\frac{\lambda_{phase}}{\lambda_A}=\frac{\tau_{group}}{\tau_A}=\mathrm{csch}\rho$$

$$=1.272..=1/\sqrt{G\_}\qquad =1.272..$$

Solve :

$\mathrm{sech}\rho=\sinh\rho$

or:

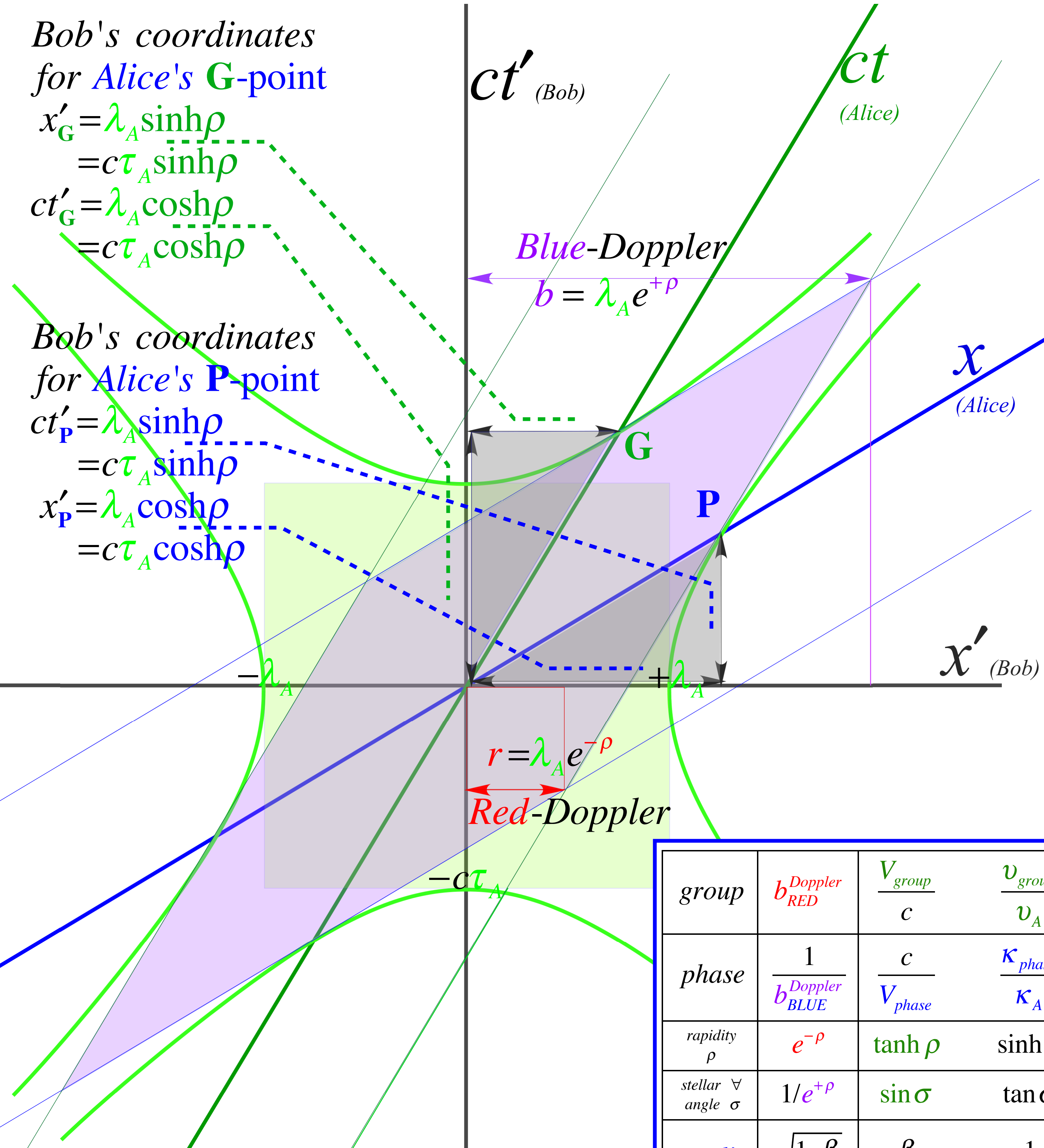
$\sinh\rho\cosh\rho=1$

or:

$\sinh 2\rho=2$

$$\rho=\frac{1}{2}\sinh^{-1}2=0.7218\ldots$$

$$\tanh\rho=0.618\ldots=\frac{\sqrt{5}-1}{2}$$



Space-time parameters

$\lambda_{phase}=\lambda_A \operatorname{csch}\rho$

$\lambda_{group}=\lambda_A \operatorname{sech}\rho$

$c\tau_{phase}=c\tau_A \operatorname{sech}\rho$

$c\tau_{group}=c\tau_A \operatorname{csch}\rho$

Per-space-time parameters

$c\kappa_{phase}=c\kappa_A \sinh\rho$

$c\kappa_{group}=c\kappa_A \cosh\rho$

$\upsilon_{phase}=\upsilon_A \cosh\rho$

$\upsilon_{group}=\upsilon_A \sinh\rho$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b^{Doppler}_{RED}$	V_{group}	past-future asymmetry (off-diagonal Lorentz-transform)	x-contraction(Lorentz) τ_{phase}-contraction	t-dilation(Einstein) υ_{phase}-dilation (on-diagonal Lorentz-transform)	inverse asymmetry	V_{phase}	$b^{Doppler}_{BLUE}$

RelaWavity Web Simulation
Comprehensive dual plots
with parameter table

RelaWavity Web Simulation
(ct' vs x') with parameter table

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Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

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(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

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Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R ?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R/\omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R ?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R/\omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

Geometric mean

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R ?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R/\omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

V_{group}/c is ratio of difference mean $\omega_{group} = \frac{\omega_R - \omega_L}{2}$ to arithmetic mean $\omega_{phase} = \frac{\omega_R + \omega_L}{2}$. Frequency $\omega_E = B$ is the geometric mean $\sqrt{\omega_R \cdot \omega_L}$ of left and right-moving frequencies defining the geometry

Geometric mean

Lecture 31
Thur. 12.08.2016

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Doppler jeopardy ➔ Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation
Learning about sin *and* cos *and...*

Derivation of relativistic quantum mechanics
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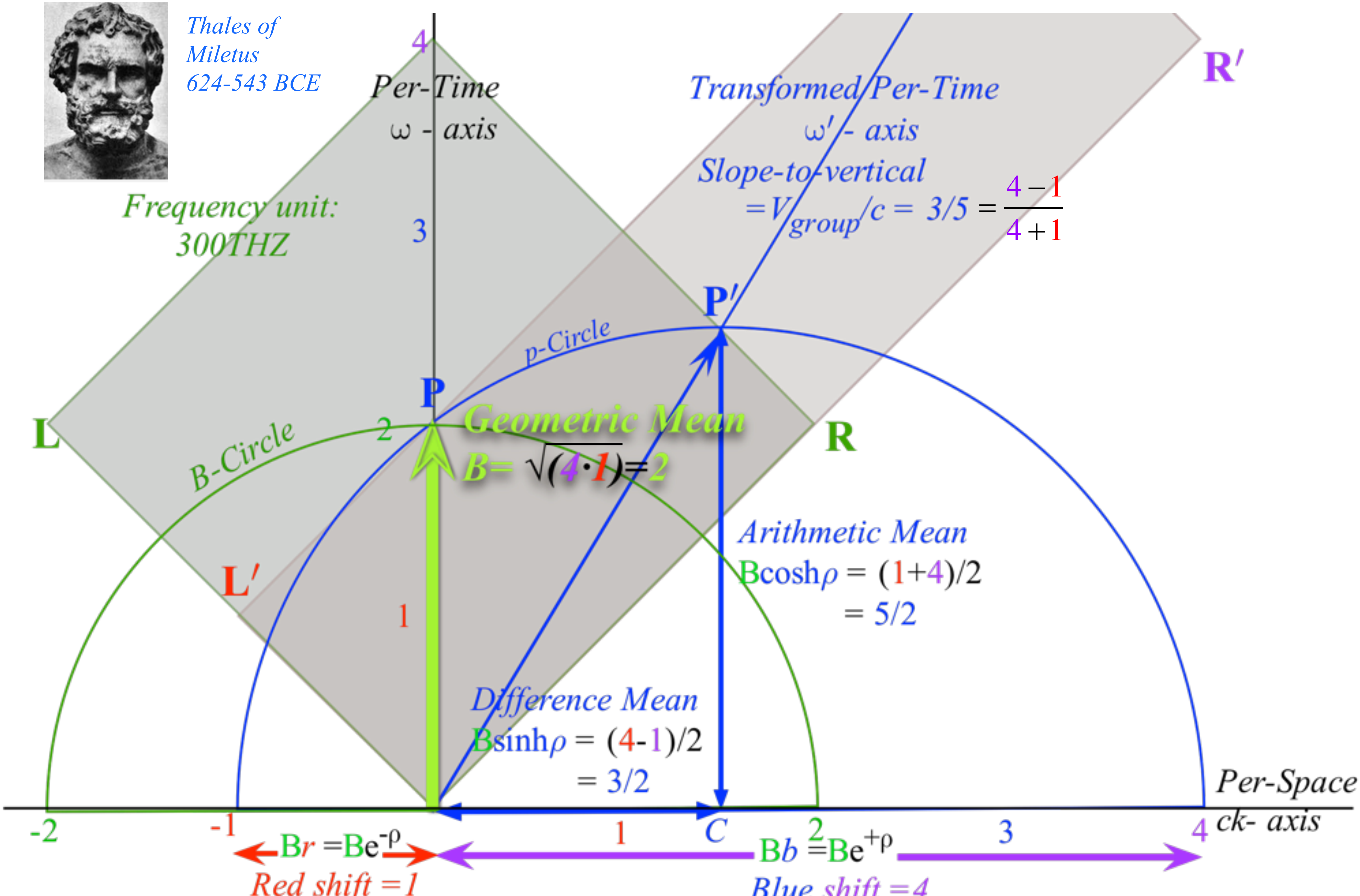
Thales Mean Geometry (600BCE)

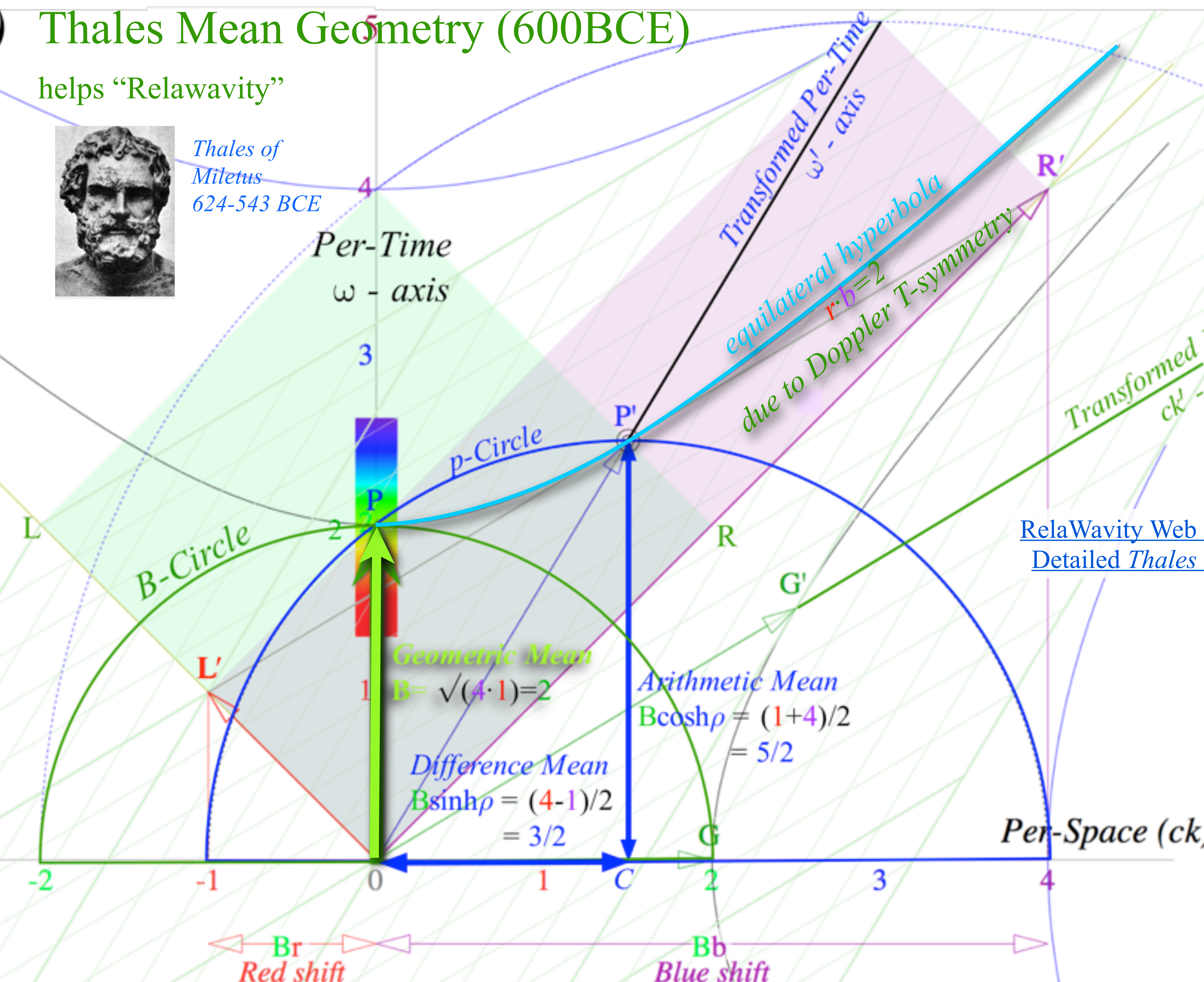
helps "Relawavity"



Thales Mean Geometry (600BCE)

helps "Relawavity"

Thales of Miletus 624-543 BCE



RelaWavity Web Simulation
Detailed *Thales Geometry*

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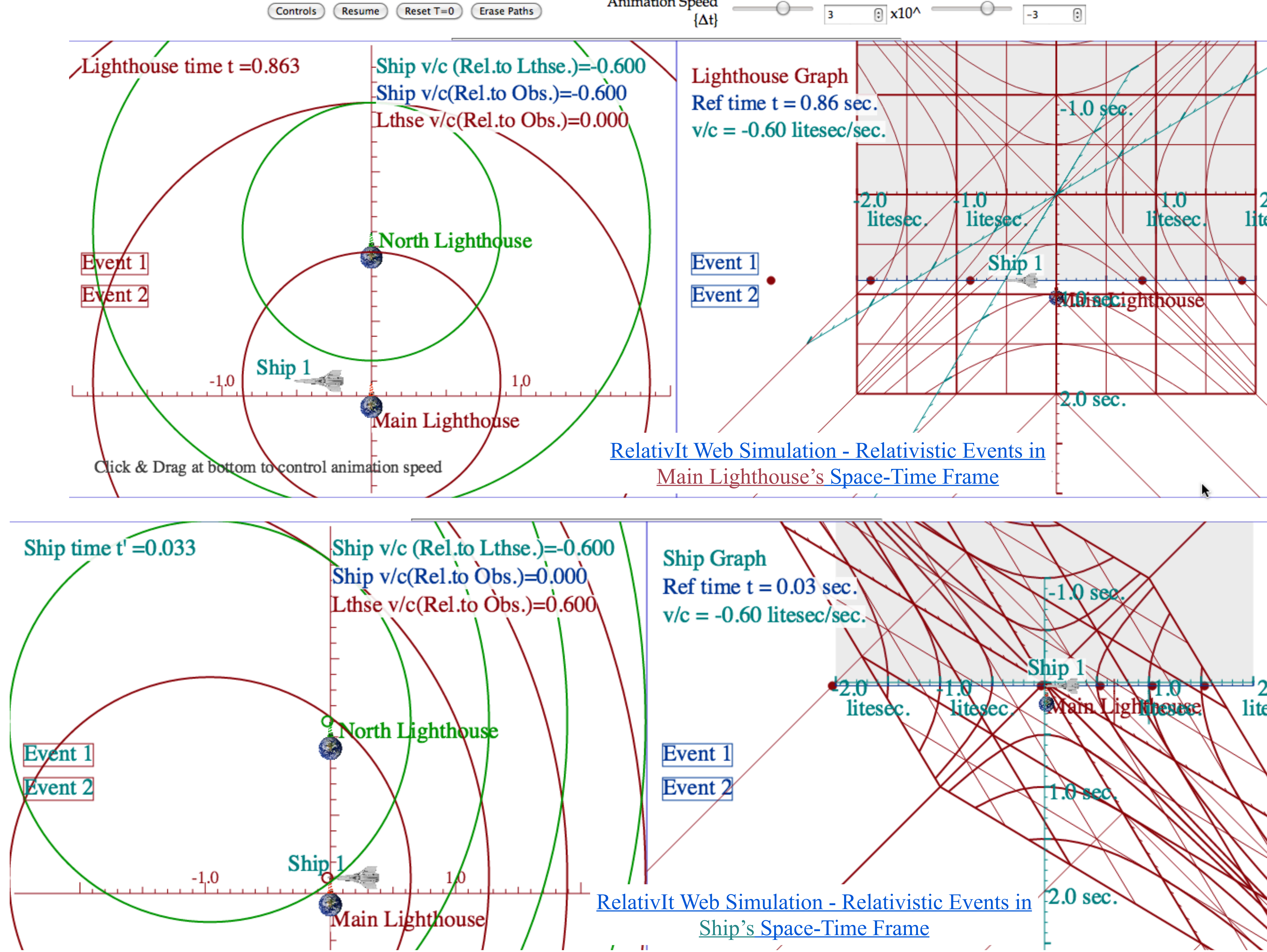

Controls
Resume
Reset T=0
Erase Paths
Animation Speed
{Δt}
x10^
Lighthouse time t =0.863
Ship v/c (Rel.to Lthse.)=-0.600
Ship v/c(Rel.to Obs.)=-0.600
Lthse v/c(Rel.to Obs.)=0.000
North Lighthouse
Event 1
Event 2
Ship 1
-1.0
1.0
Main Lighthouse
Click & Drag at bottom to control animation speed
Lighthouse Graph
Ref time t = 0.86 sec.
v/c = -0.60 litesec/sec.
-1.0 sec.
-2.0 litesec.
-1.0 litesec.
1.0 litesec.
2.0 sec.
RelativIt Web Simulation - Relativistic Events in
Main Lighthouse's Space-Time Frame
Ship time t' =0.033
Ship v/c (Rel.to Lthse.)=-0.600
Ship v/c(Rel.to Obs.)=0.000
Lthse v/c(Rel.to Obs.)=0.600
Ship Graph
Ref time t = 0.03 sec.
v/c = -0.60 litesec/sec.
-1.0 sec.
1.0 sec.
2.0 sec.
RelativIt Web Simulation - Relativistic Events in
Ship's Space-Time Frame

Lecture 31
Thur. 12.10.2015

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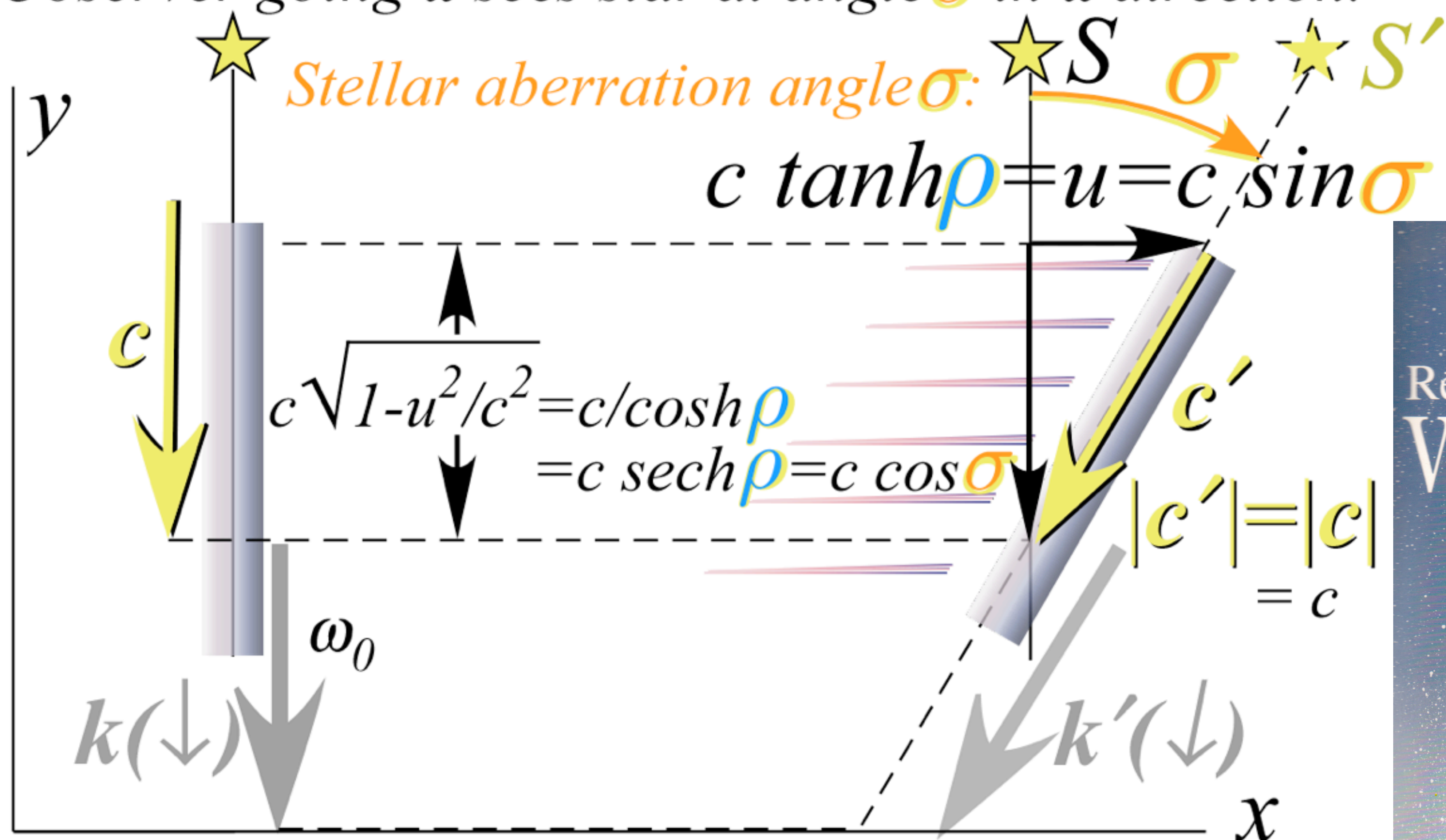
Comparing <u>Longitudinal</u> relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a <u>Transverse</u>*relativity parameter: Stellar aberration angle σ

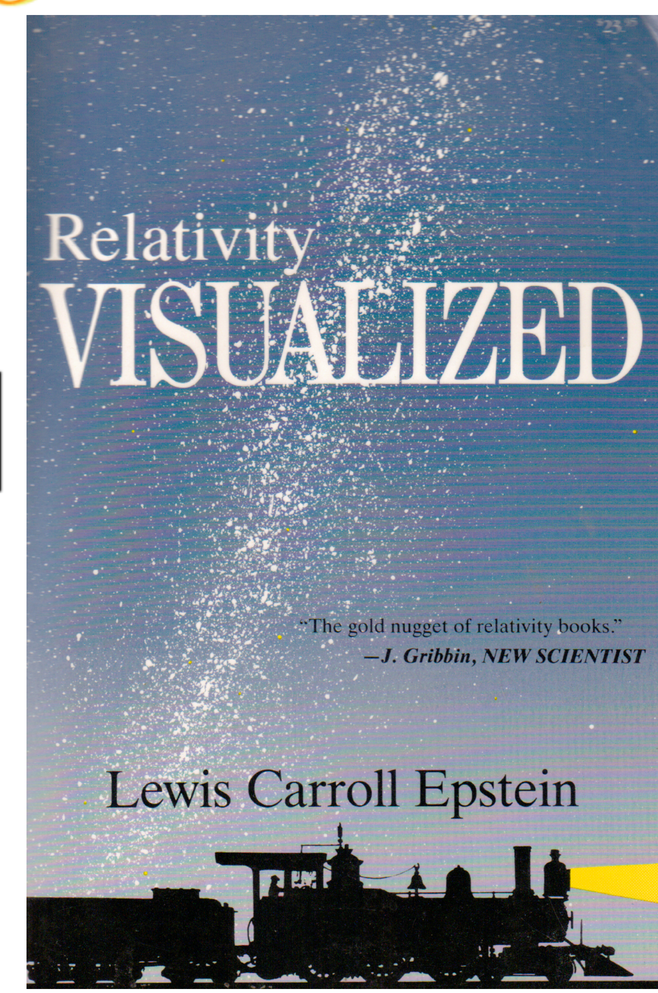
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



We used notion σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.



Purchase at: AllBookstores.com amazon

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ

Review of 16 relawavity functions of ρ and related geometric approach to relativity

Doppler jeopardy Geometric mean and Relativistic hyperbolas

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This map has circle sector arc-area $\sigma = 0.6435$ set to angle $\angle\sigma = 36.87° = 0.6435\,radian$

Also it is set to hyperbola sector arc-area $\rho = 0.6931$ angle $\angle\rho = \nu = 30.96°$

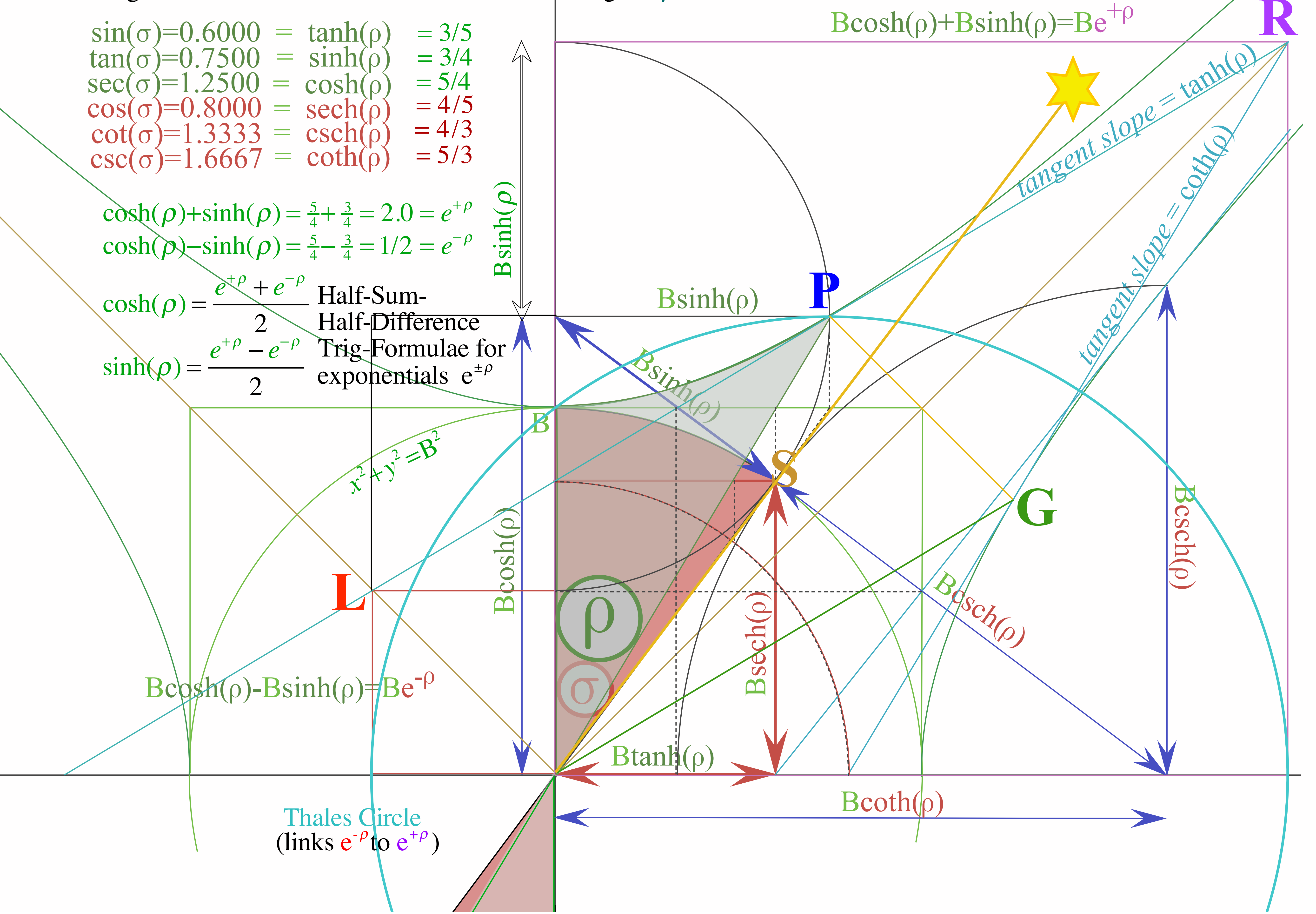
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3

$\cosh(\rho)+\sinh(\rho) = \frac{5}{4}+\frac{3}{4} = 2.0 = e^{+\rho}$

$\cosh(\rho)-\sinh(\rho) = \frac{5}{4}-\frac{3}{4} = 1/2 = e^{-\rho}$

$\cosh(\rho) = \frac{e^{+\rho}+e^{-\rho}}{2}$ Half-Sum-Half-Difference Trig-Formulae for exponentials $e^{\pm\rho}$

$\sinh(\rho) = \frac{e^{+\rho}-e^{-\rho}}{2}$



Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

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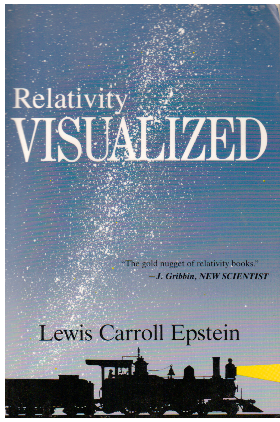
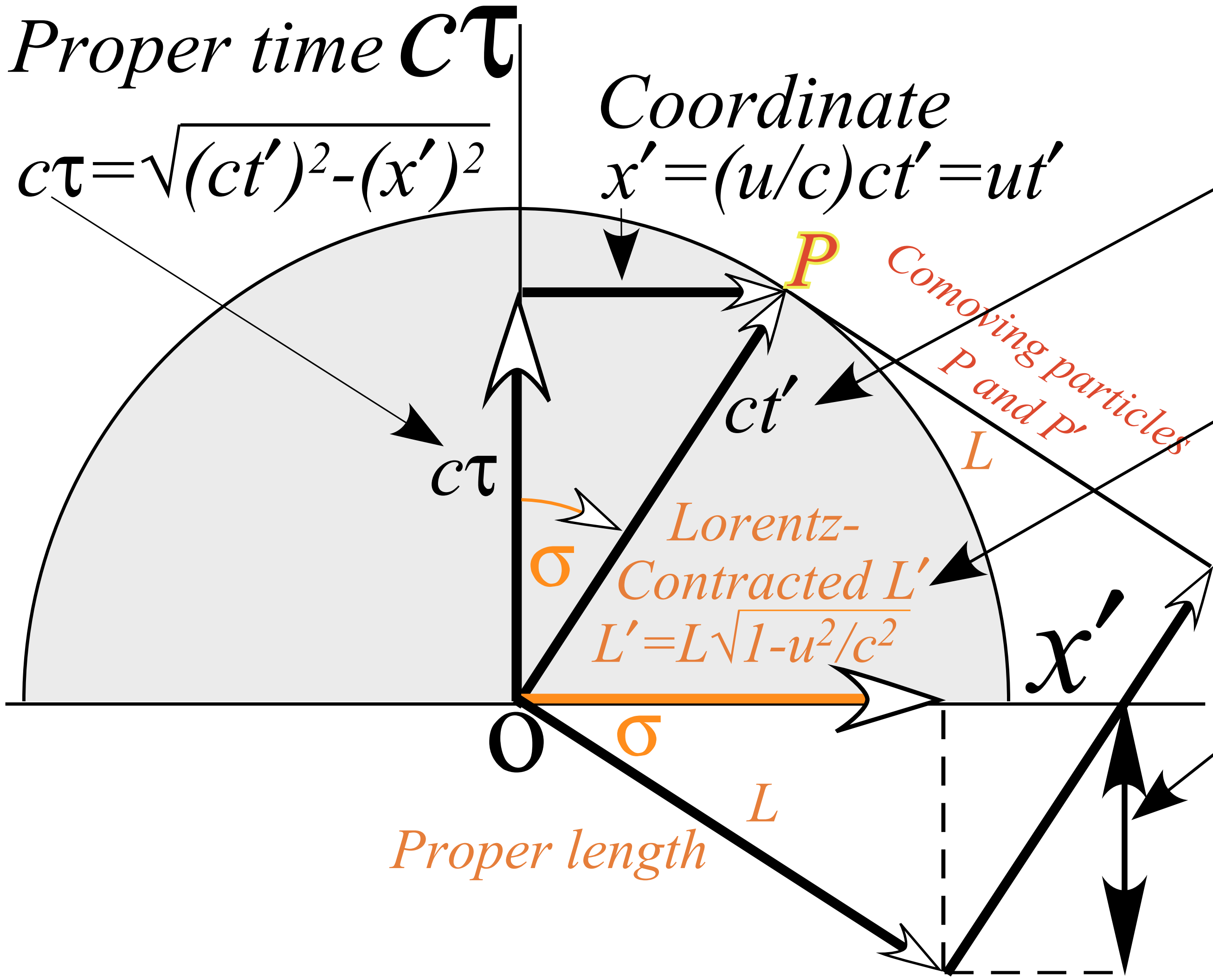


Proper time cτ vs. coordinate space x - *(L. C. Epstein's "Cosmic Speedometer")*

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L \operatorname{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L' \sinh\rho = L \cos\sigma \sinh\rho$$
$$= L \cos\sigma \tan\sigma$$
$$= L \sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

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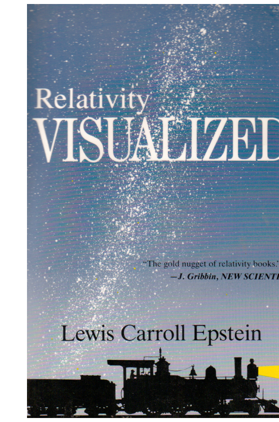

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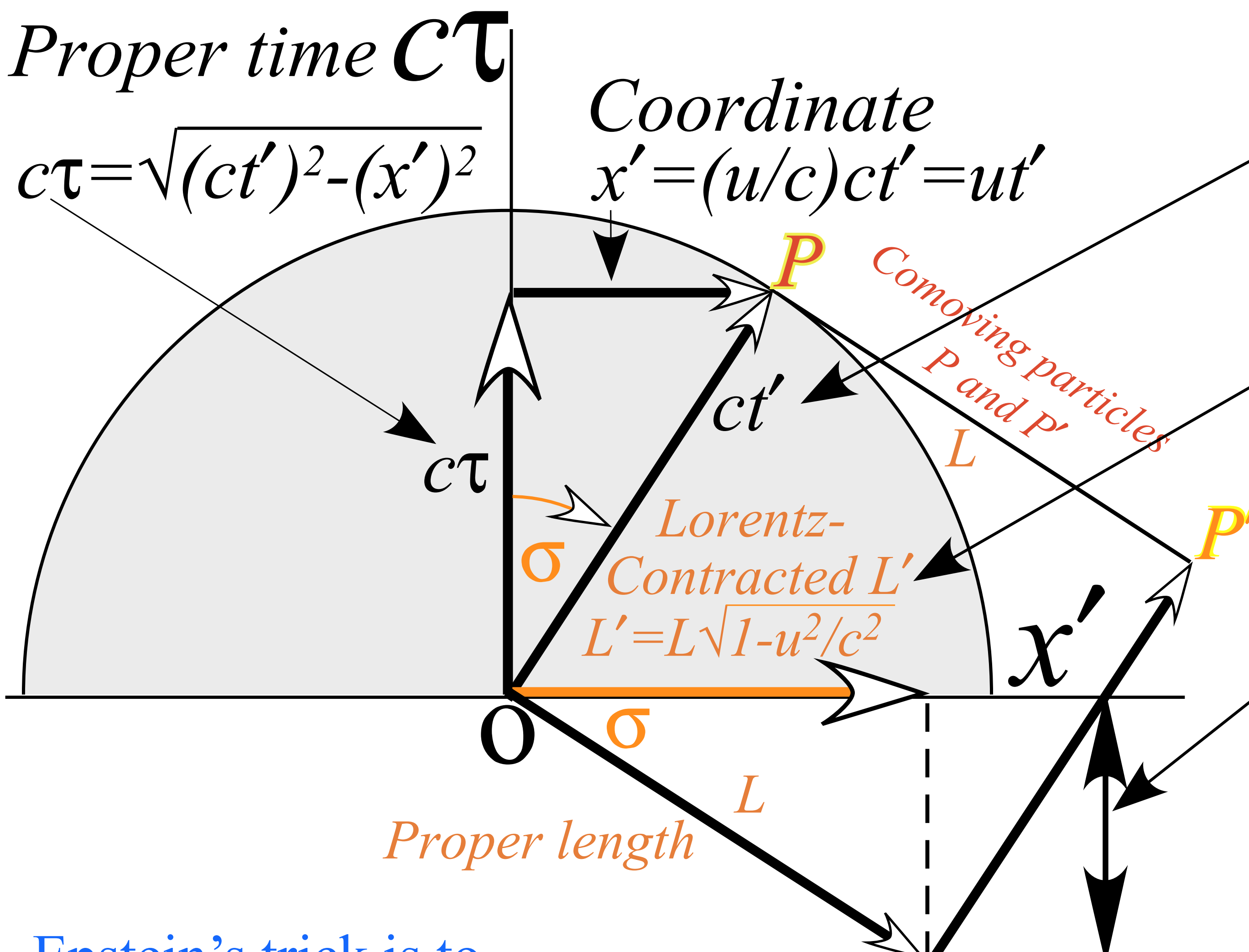

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$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2 + (x')^2} = (ct')$$

Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

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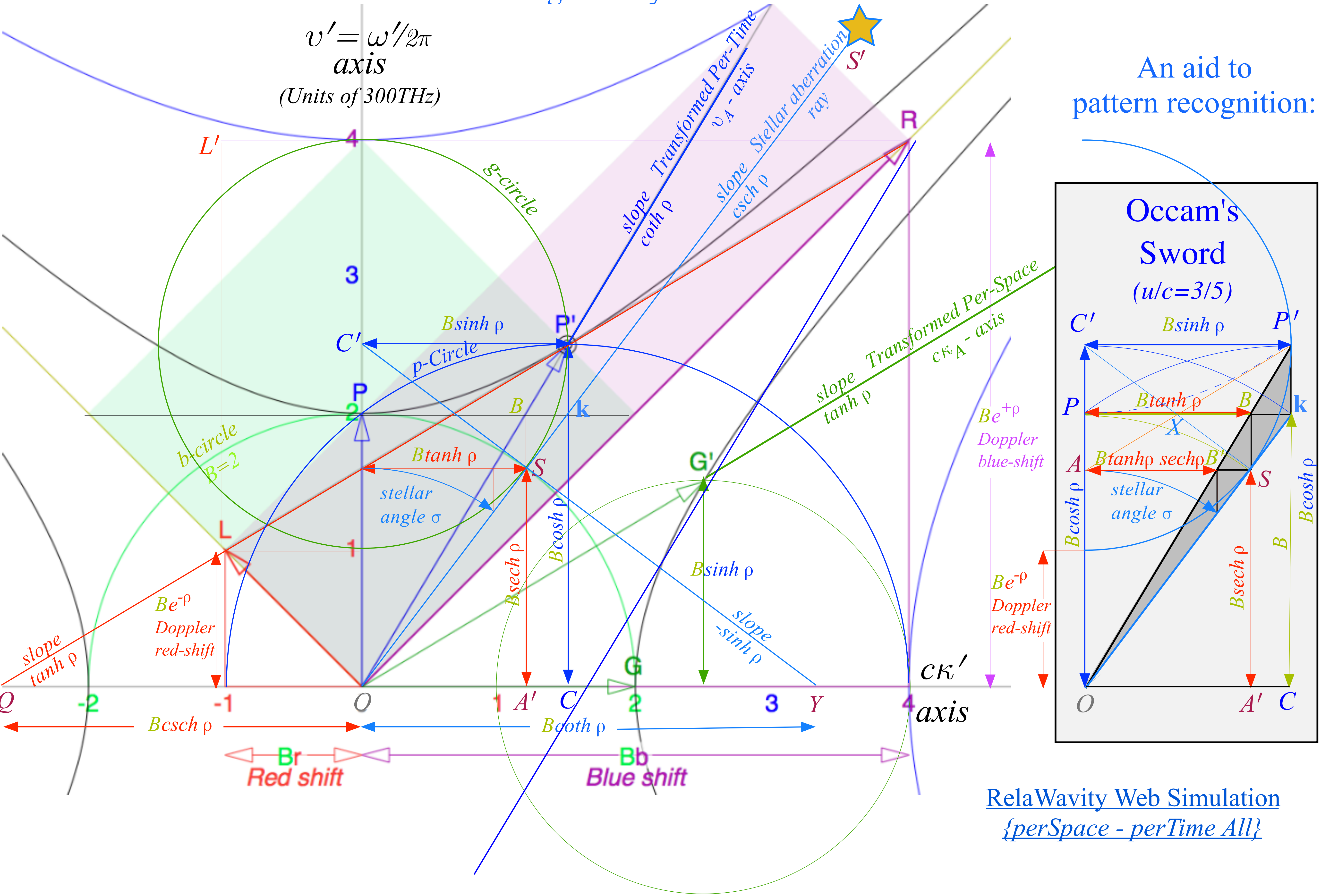
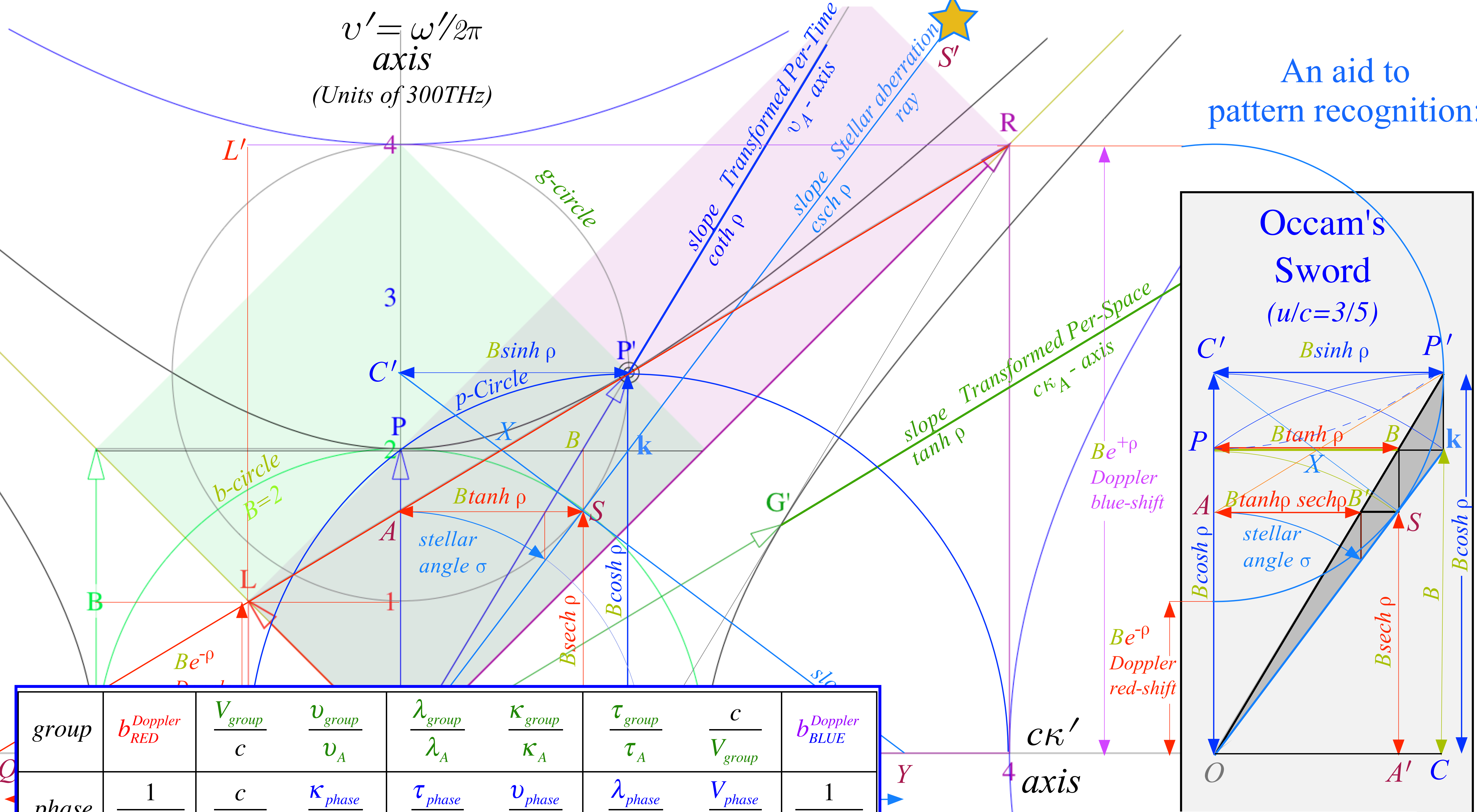
Summary of optical wave parameters for relativity and QM

...and their geometry



RelaWavity Web Simulation
{perSpace - perTime All}

An aid to pattern recognition:



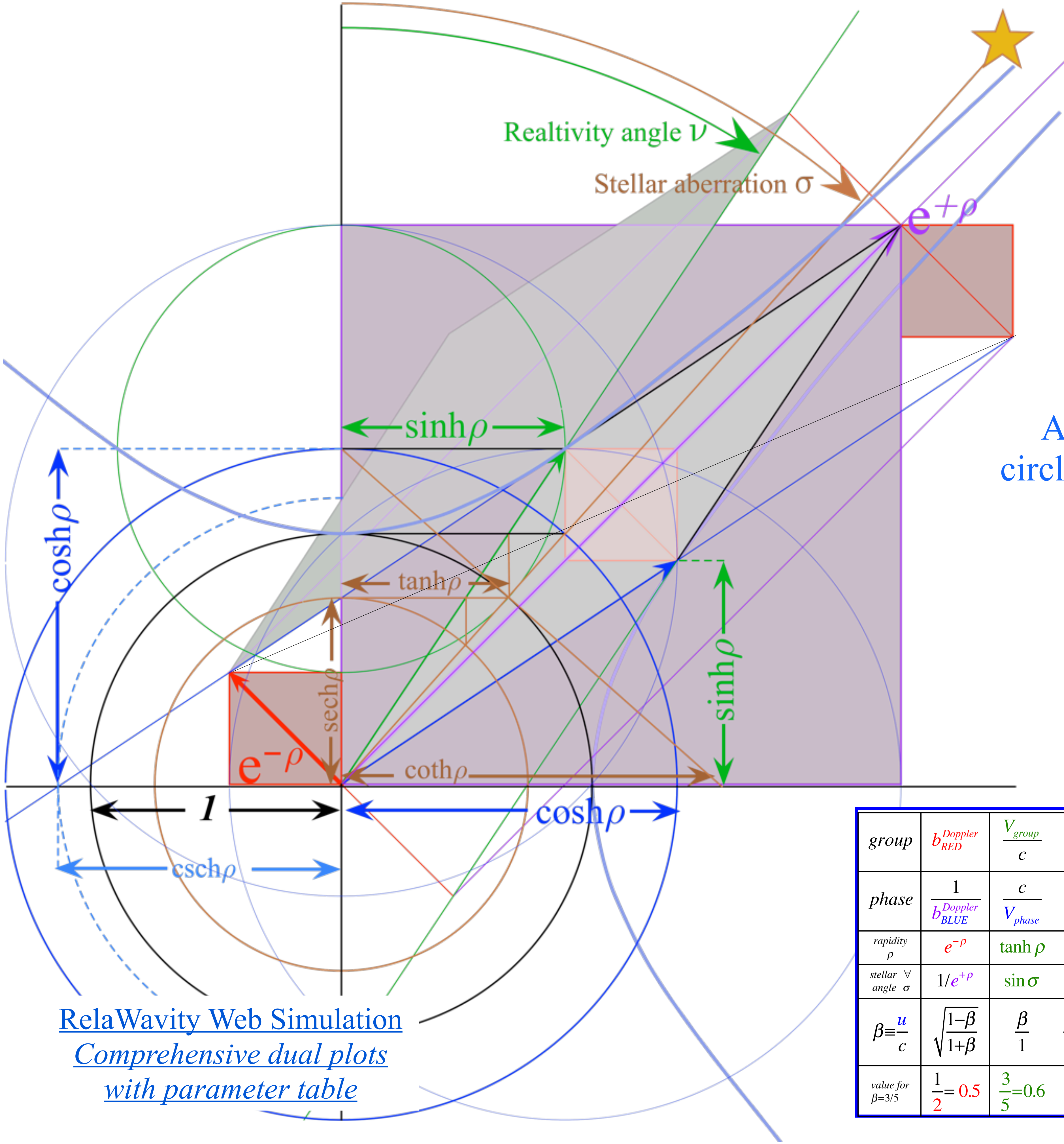
group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation
{perSpace - perTime All}

Table of 12 wave parameters (includes inverses) for relativity

...and values for $u/c=3/5$

RelaWavity Web Simulation
Expanded *Table of Relativistic* Relations



A more compact circle-based geometry

RelaWavity Web Simulation
Comprehensive dual plots with parameter table

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
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stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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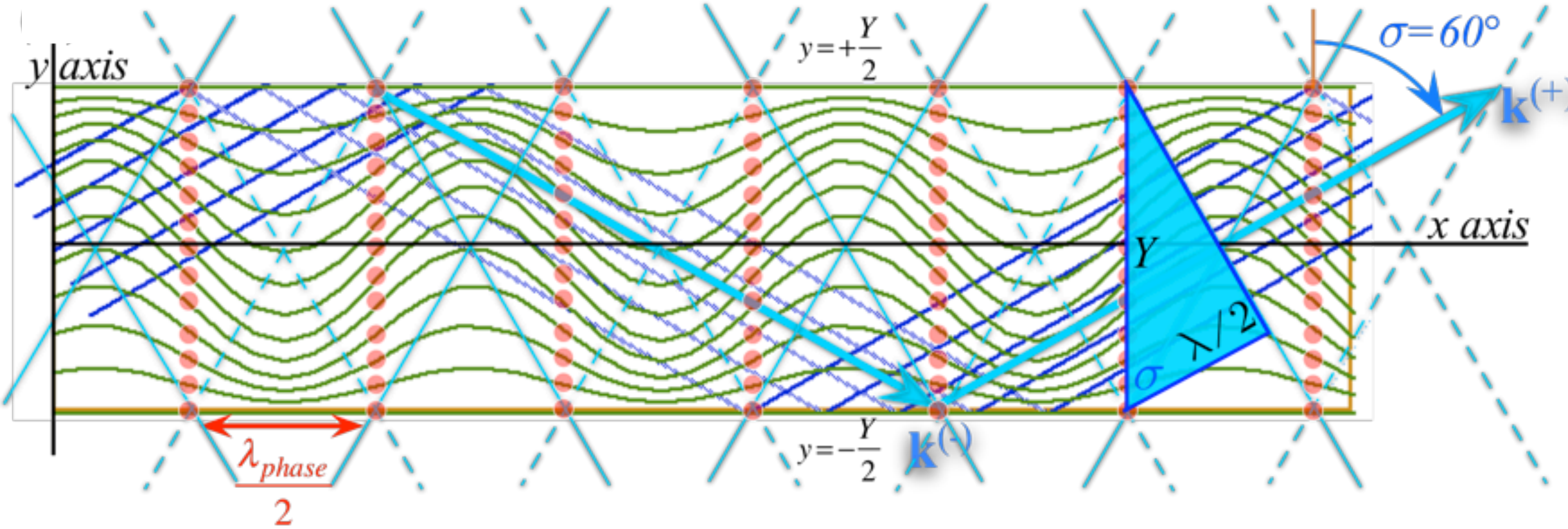
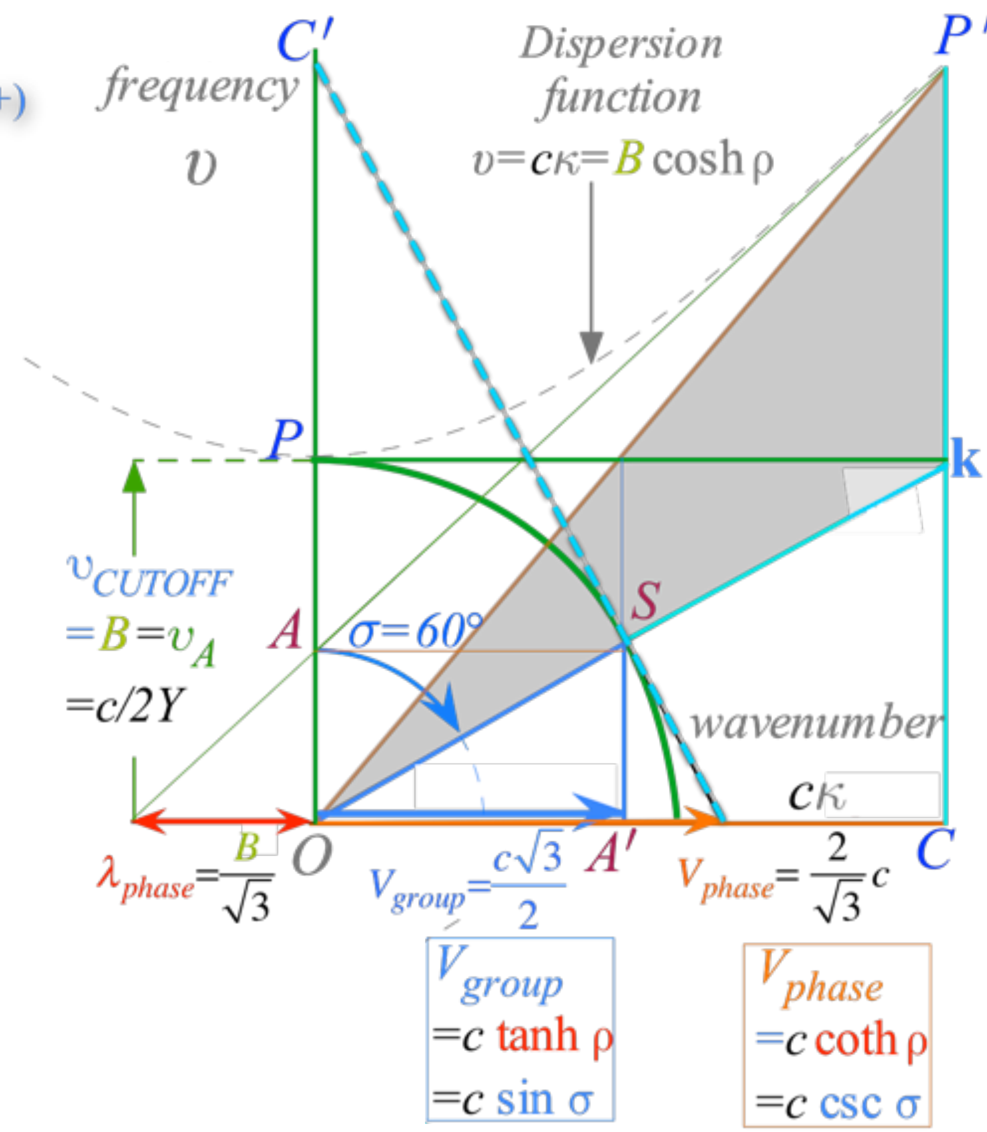
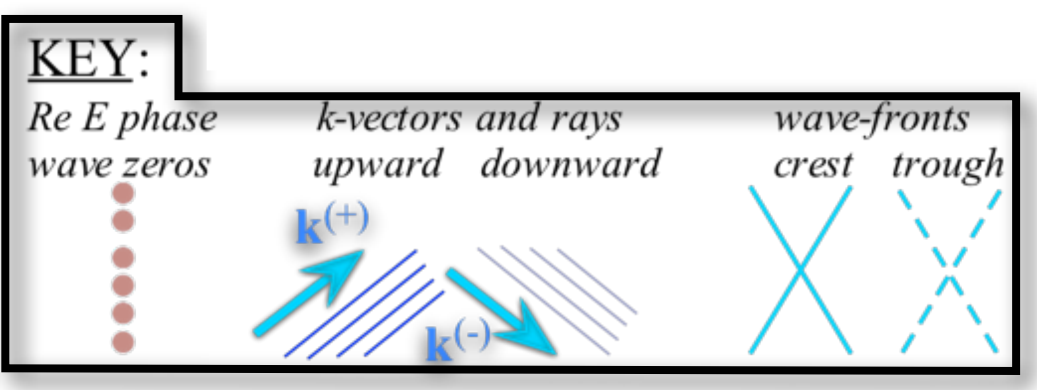
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)







Optical wave guide relativistic geometry aided by Occam's Sword

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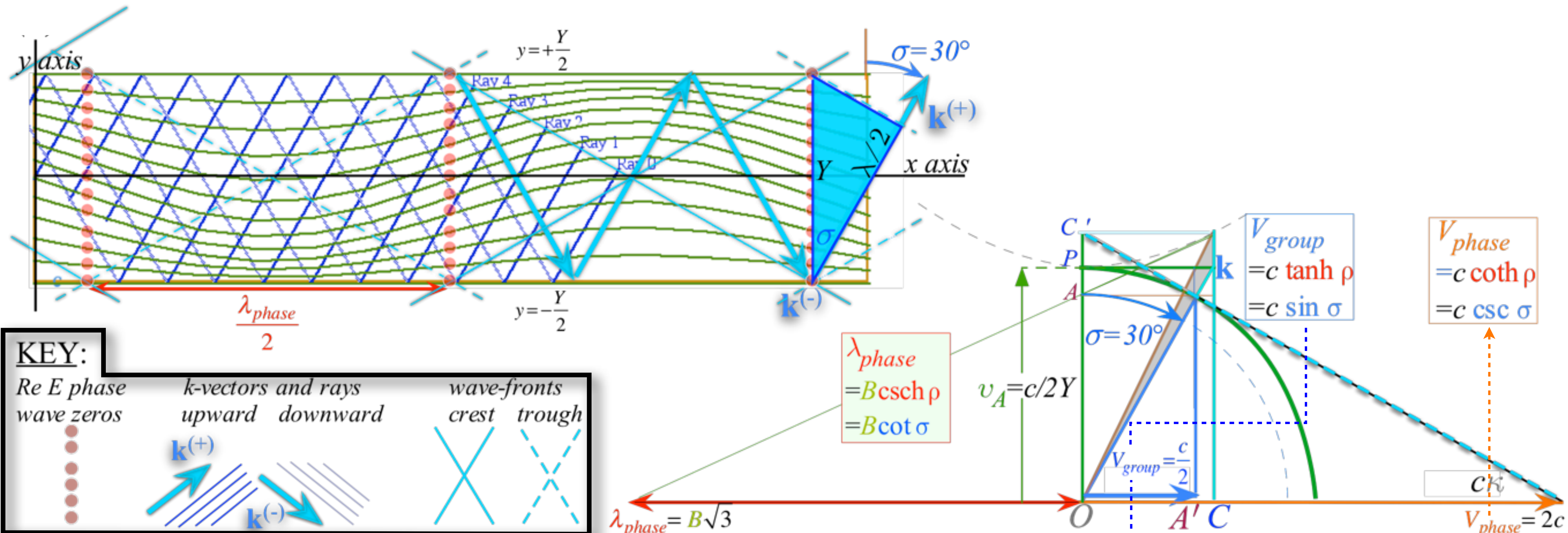

Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

Optical wave guide relativistic geometry aided by Occam's Sword

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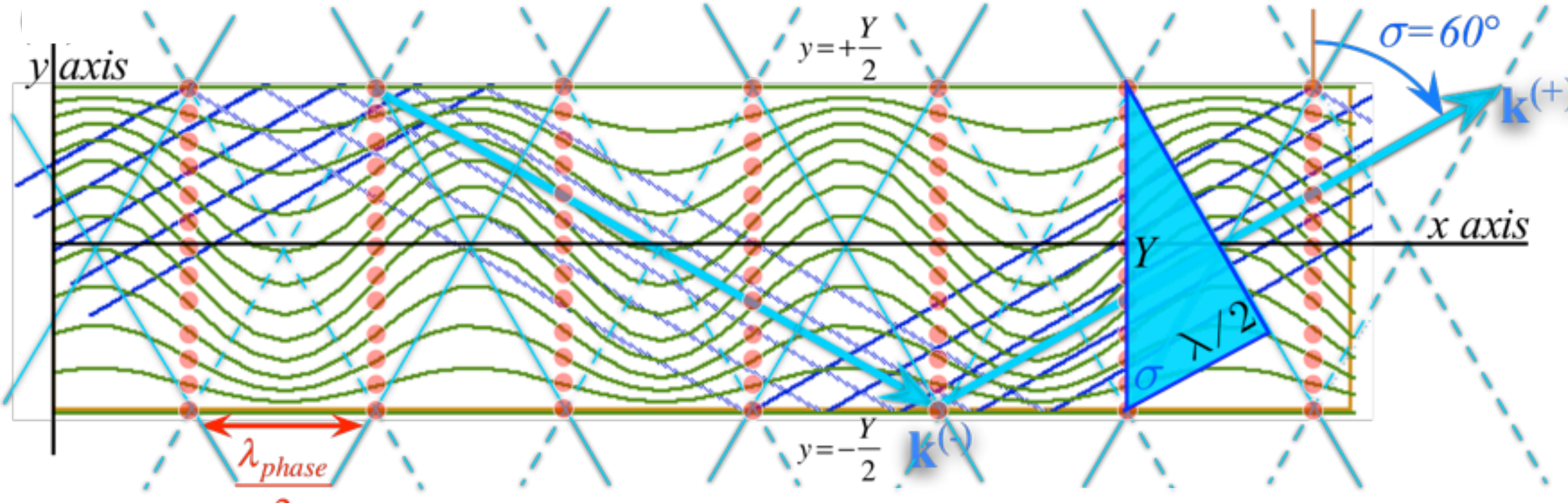
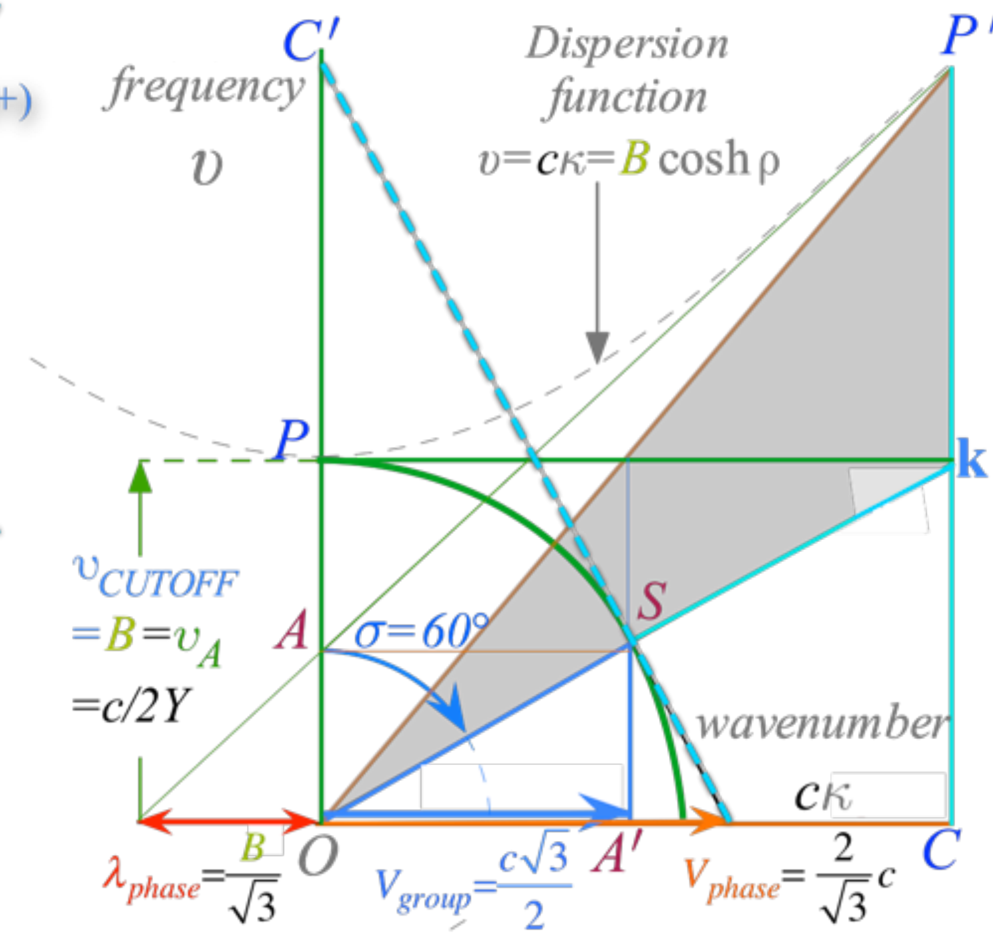
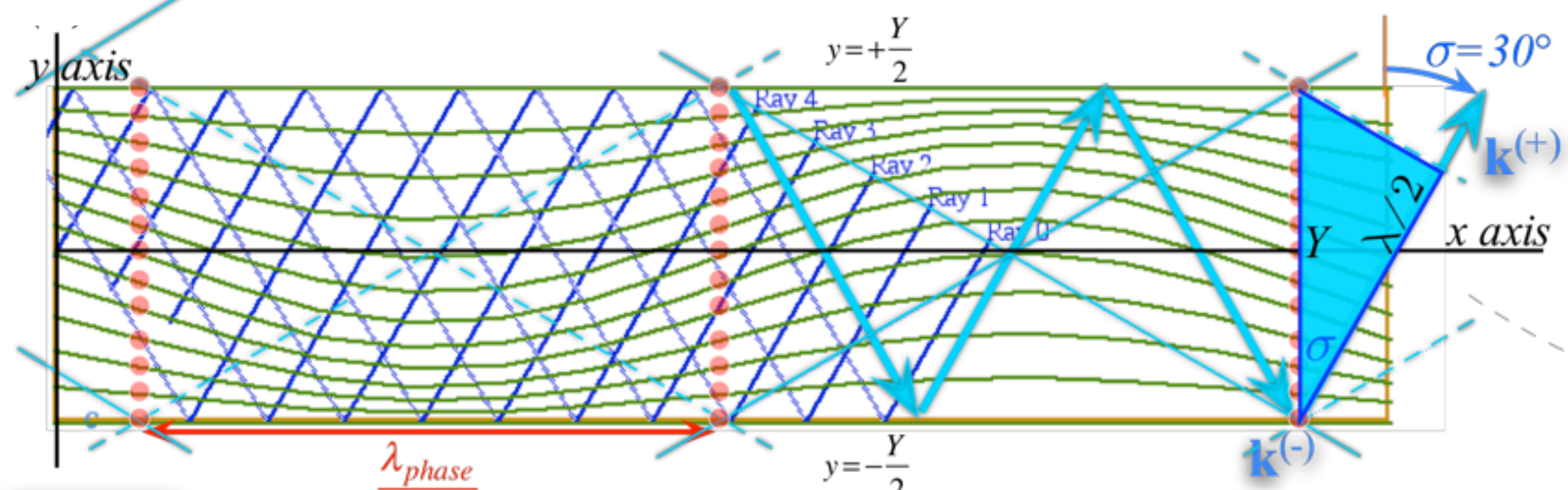
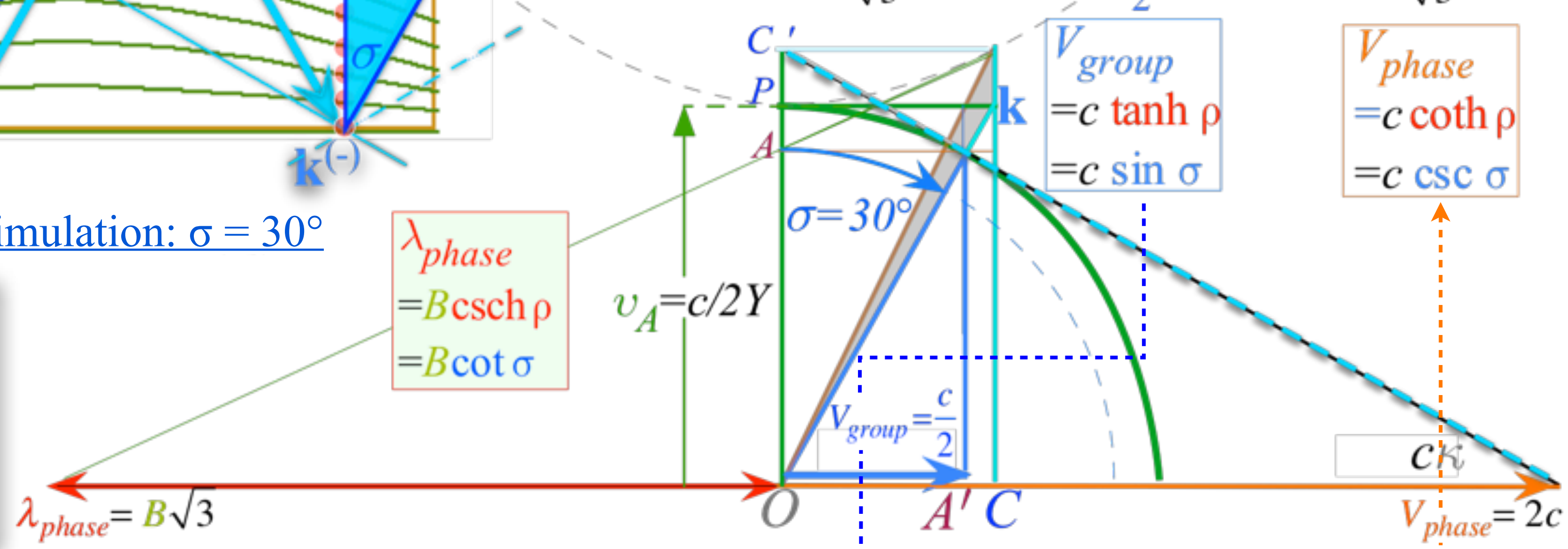


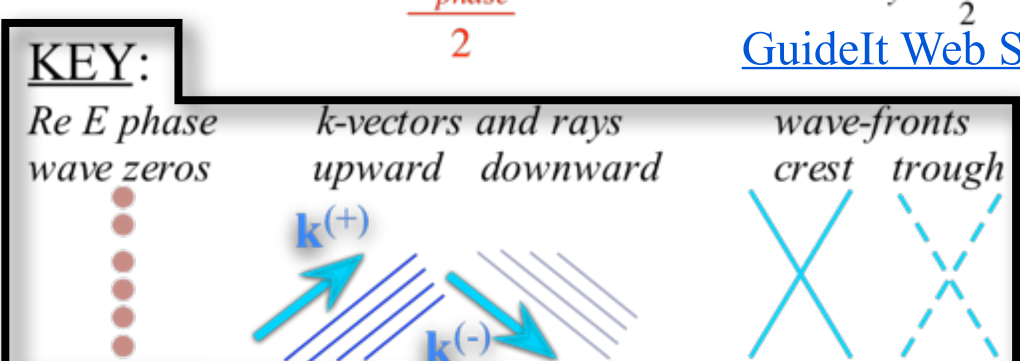
GuideIt Web Simulation: σ = 60°





GuideIt Web Simulation: σ = 30°





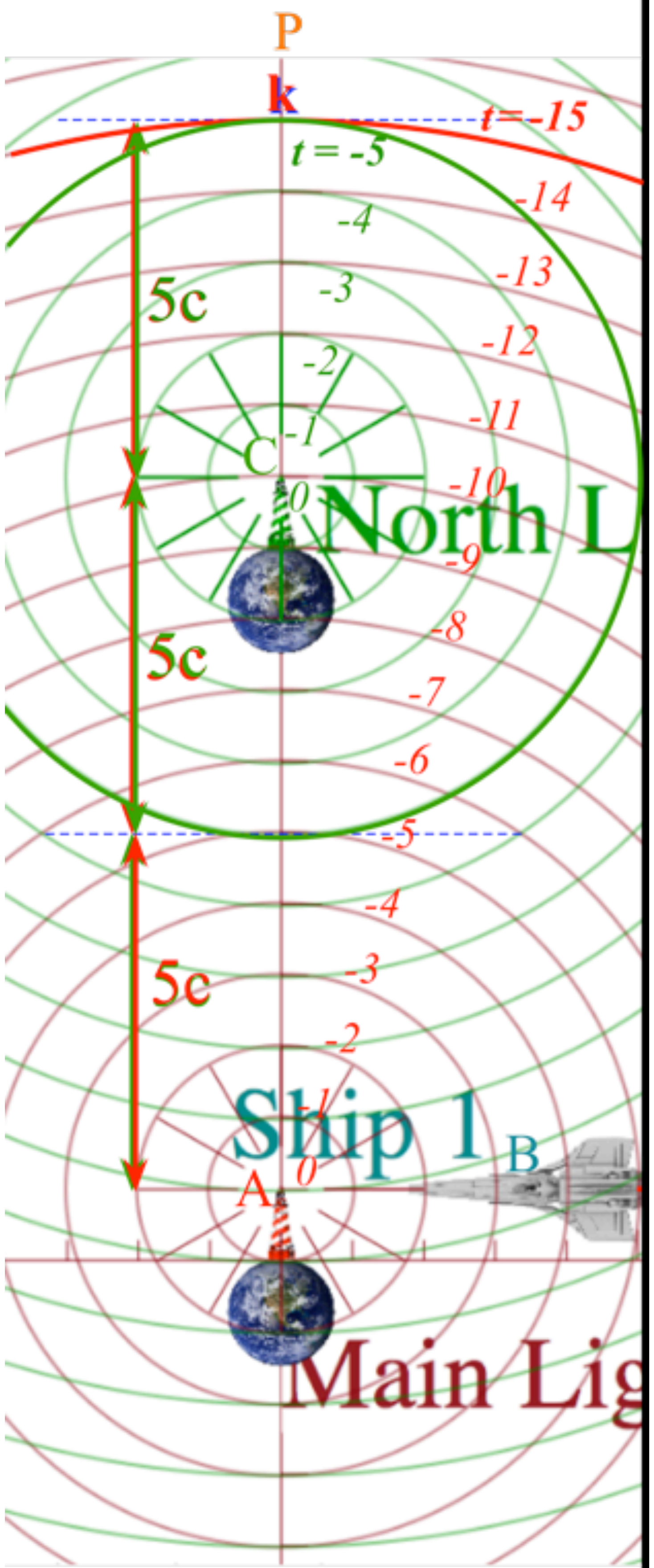
Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

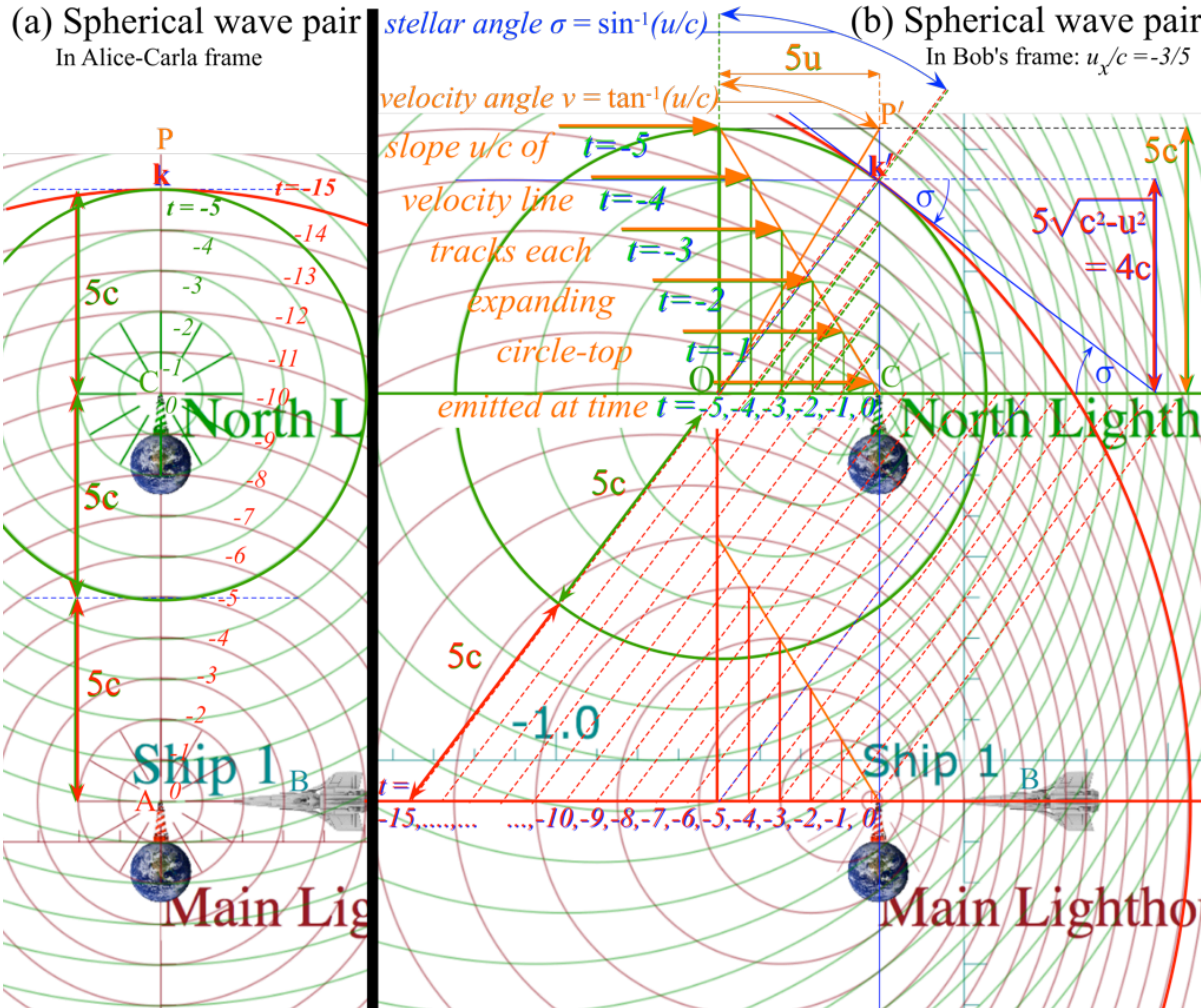
Spherical wave relativistic geometry

Also, aided by Occam's Sword

(a) Spherical wave pair

In Alice-Carla frame



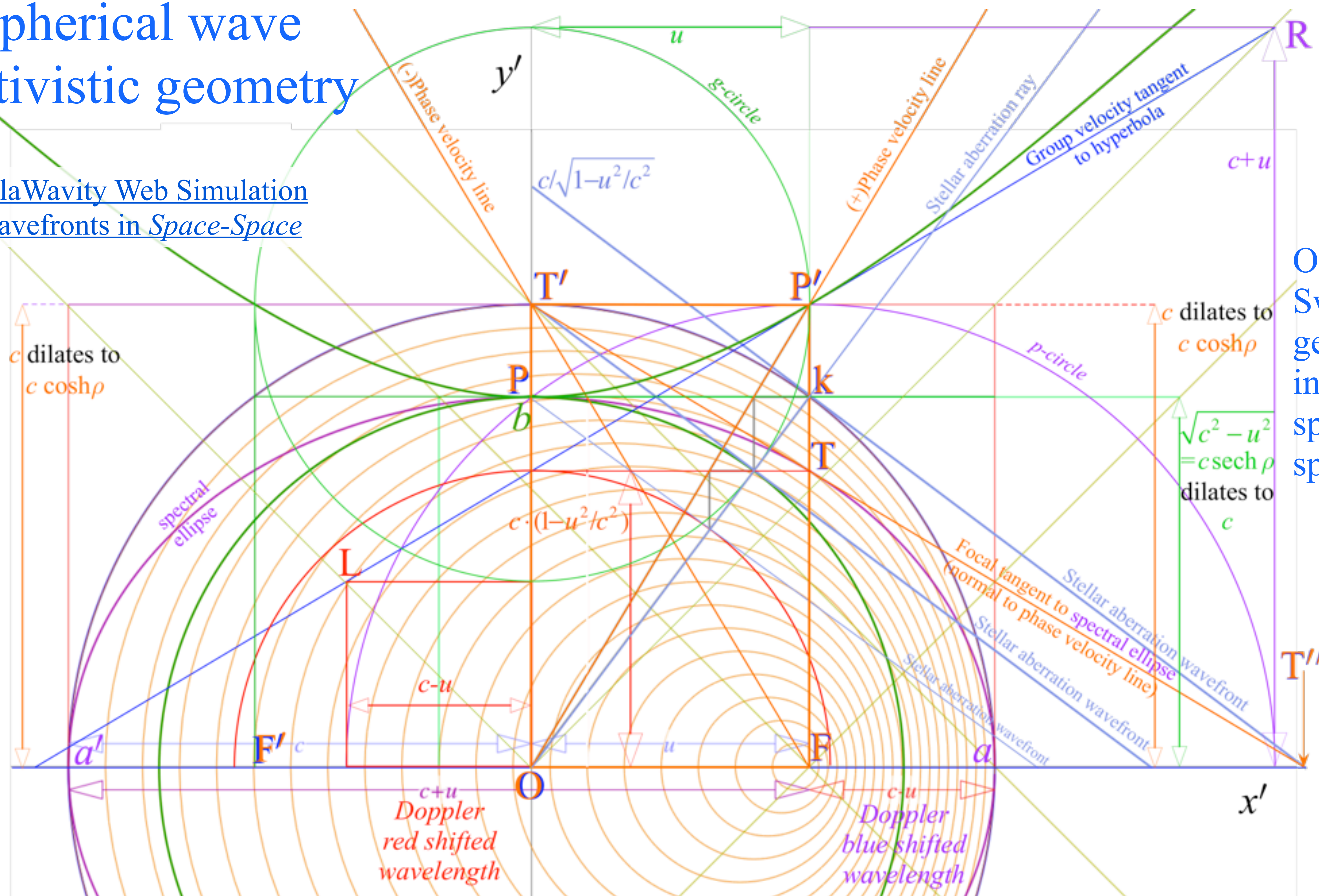


Occam Sword geometry in (x,y) space-space

Spherical wave relativistic geometry

RelaWavity Web Simulation
Wavefronts in *Space-Space*

Occam Sword geometry in (x,y) space-space



Doppler Red $\lambda=c+u$
dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse focal length **FO**$= u = c\tanh\rho$
dilates to: $u\cosh\rho = c\sinh\rho$

Doppler Blue $\lambda=c-u$
dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

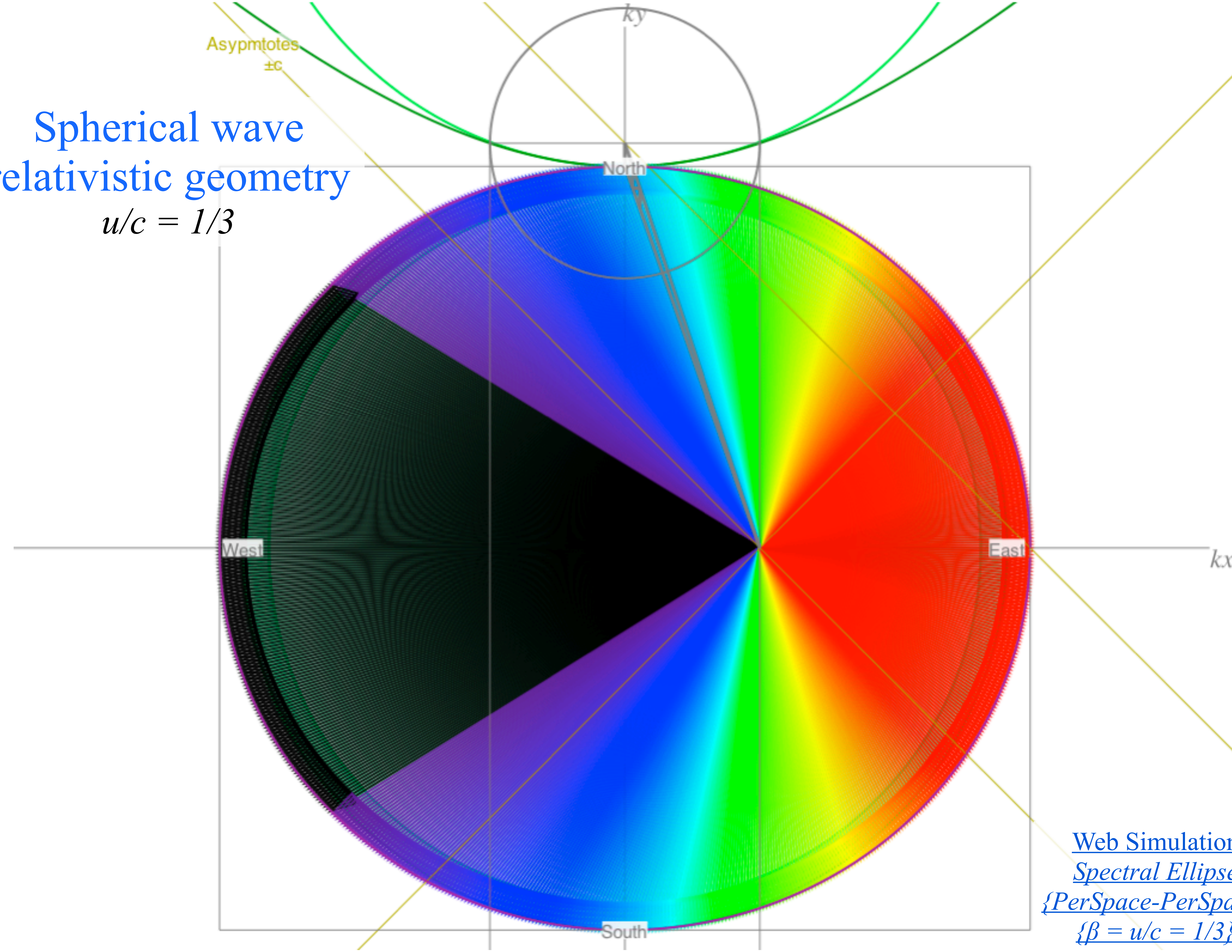
ellipse major radius a=**OF**a=c
dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor:
$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse latus radius **FT**$=c(1-u^2/c^2)$
dilates to: $c(1-u^2/c^2)\cosh\rho$
$= c\sqrt{1-u^2/c^2} = c\ \text{sech}\rho$

Base height **FTk**$=\sqrt{c^2-u^2}$
dilates to: $\sqrt{c^2-u^2}\ \cosh\rho = c$
(equal to ellipse minor radius b)

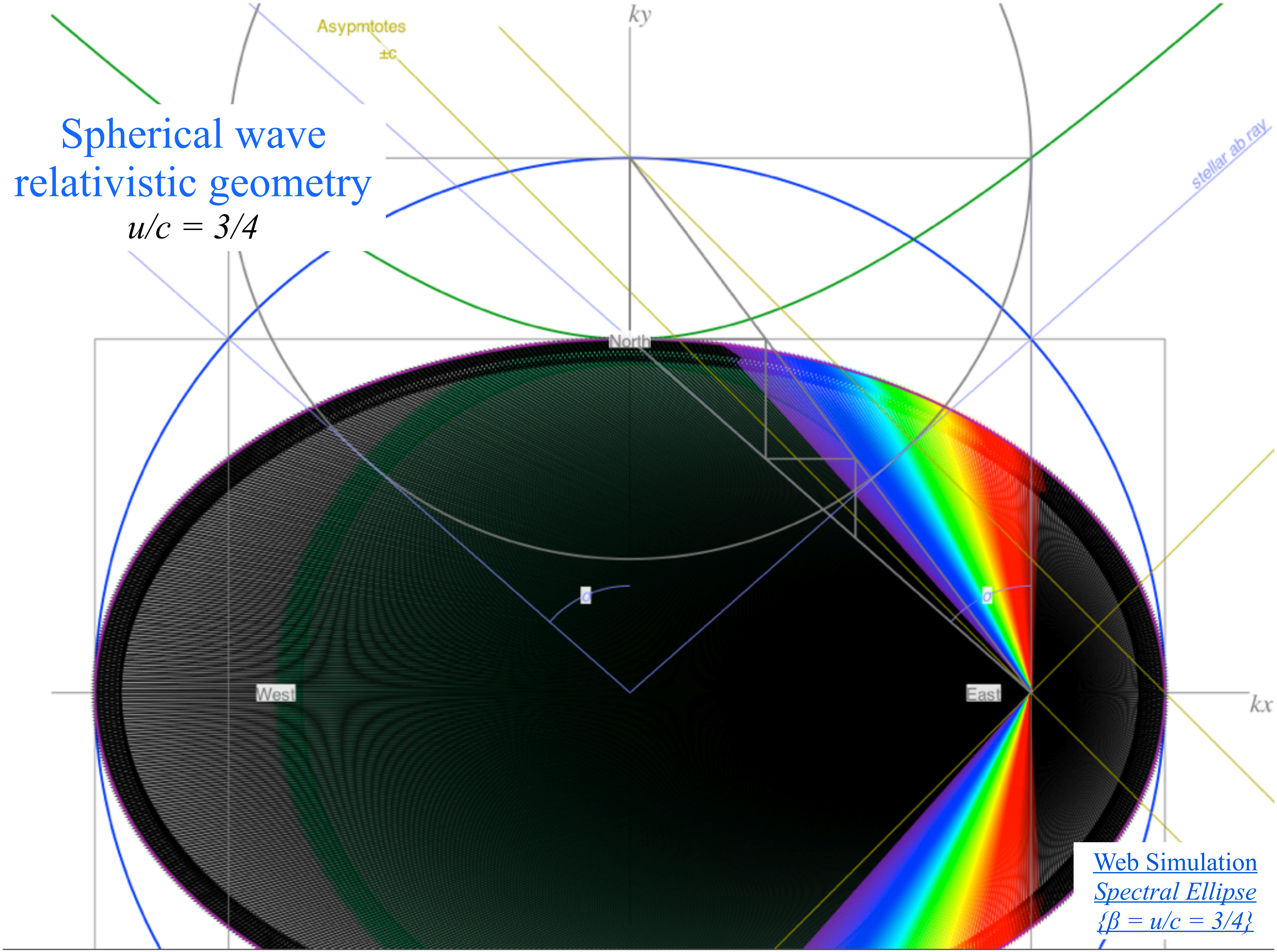
Spherical wave relativistic geometry
u/c = 1/3



[Web Simulation](#)
[*Spectral Ellipse*](#)
[*{PerSpace-PerSpace}*](#)
[*{β = u/c = 1/3}*](#)

Spherical wave relativistic geometry

u/c = 3/4



[*Web Simulation*](#)
[*Spectral Ellipse*](#)
[*{β = u/c = 3/4}*](#)

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and summary of 16 parameter functions of ρ and σ

Applications to optical waveguide, spherical waves, and accelerator radiation

Learning about sin! *and* cos *and...*

➡ Derivation of relativistic quantum mechanics
What's the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \qquad \text{(for } u \ll c)$$

At low speeds:

$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2$

$\sinh\rho \approx \rho$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation - Relativistic *Terms (Expanded Table)*

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c)$$

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

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space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

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So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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RelaWavity Web Simulation - Relativistic *Terms*
(Expanded Table)

Using (some) wave parameters to develop relativistic quantum theory

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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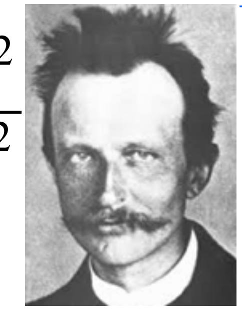

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This motivates the "particle" normalization $\int \Psi^*\Psi \, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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For more visit the Pirelli Challenge Site
Quantized amplitude

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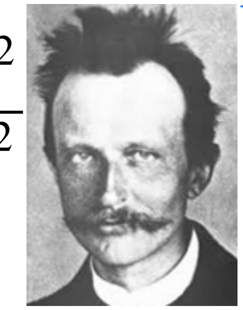

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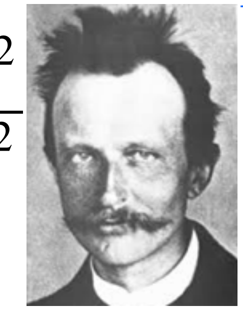

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Resolution and dirty secret: $\mathbf{E}$, N, and υ_{phase} are all frequencies!

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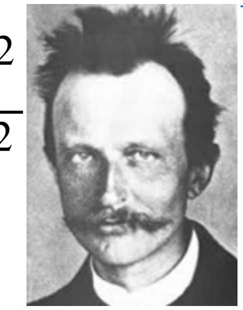

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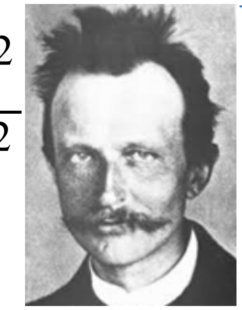

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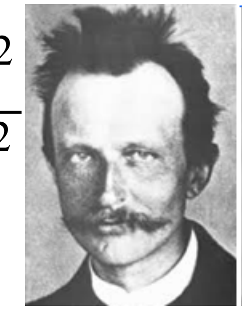

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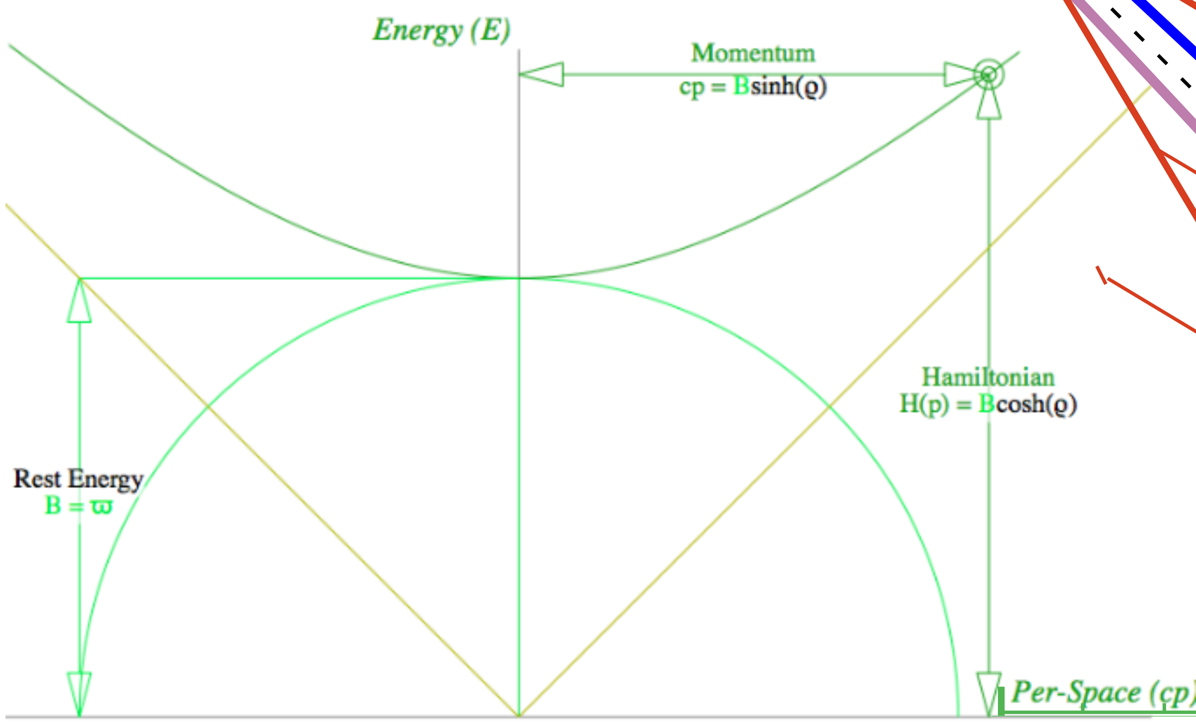
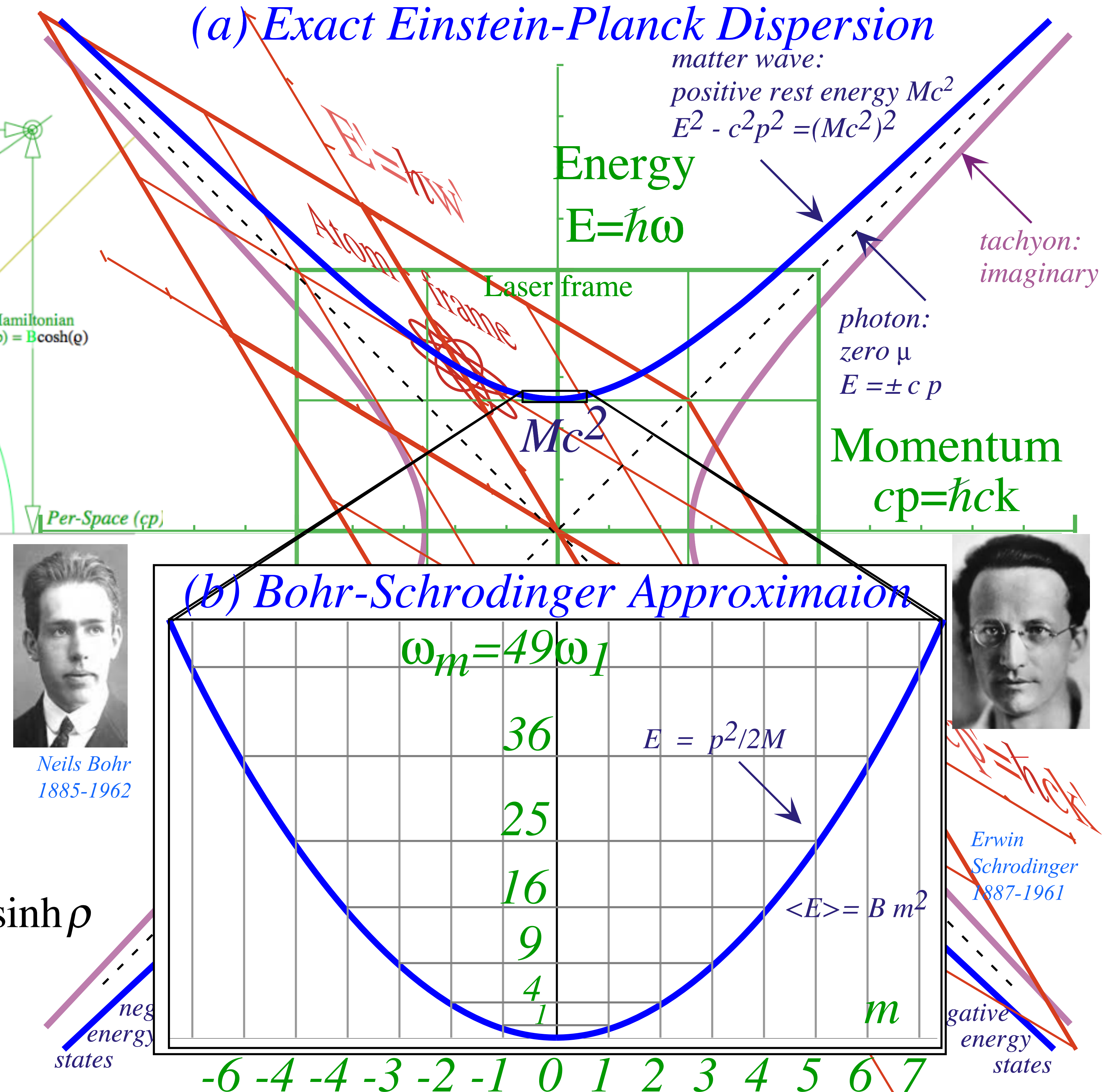
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave coordinates for relativistic quantum theory





Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

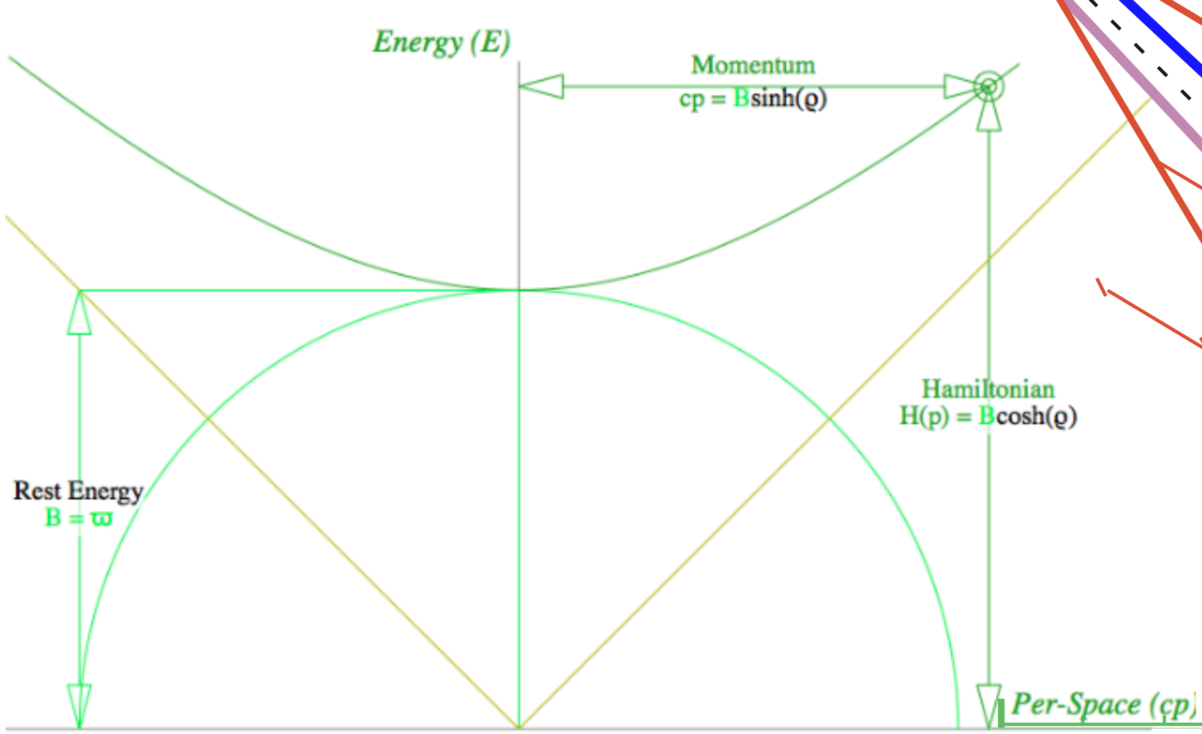
$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

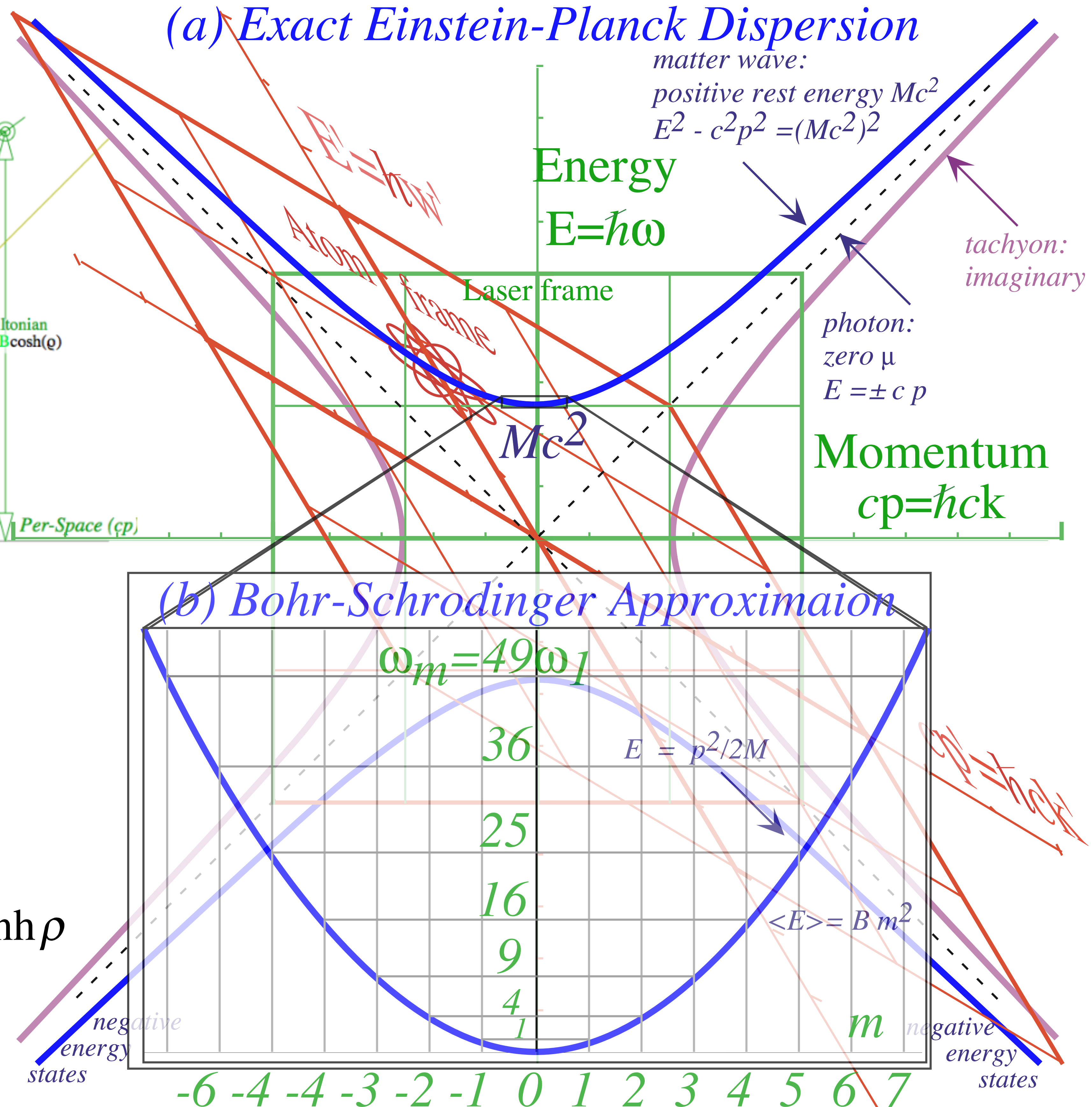
Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2+(cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

Using (some) wave coordinates for relativistic quantum theory





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low speed approximation

Relawavity variable tables

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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effects	$b^{Doppler}_{RED}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*(Lorentz) τ_{phase}-*contraction*	*t-dilation*(Einstein) υ_{phase}-*dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b^{Doppler}_{BLUE}$

Relativistic quantum mechanics variable tables

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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functions		$V_{group}= c\tanh\rho$	*momentum* $cp=Mc^2\sinh\rho$	*-Lagrangian* $L=-Mc^2\operatorname{sech}\rho$	*Hamiltonian* $H=Mc^2\cosh\rho$	*DeBroglie* $\lambda=\alpha\operatorname{csch}\rho$	$V_{phase}= c\coth\rho$	

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
➔ What's the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

Definition(s) of mass for relativity/quantum

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$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

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velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

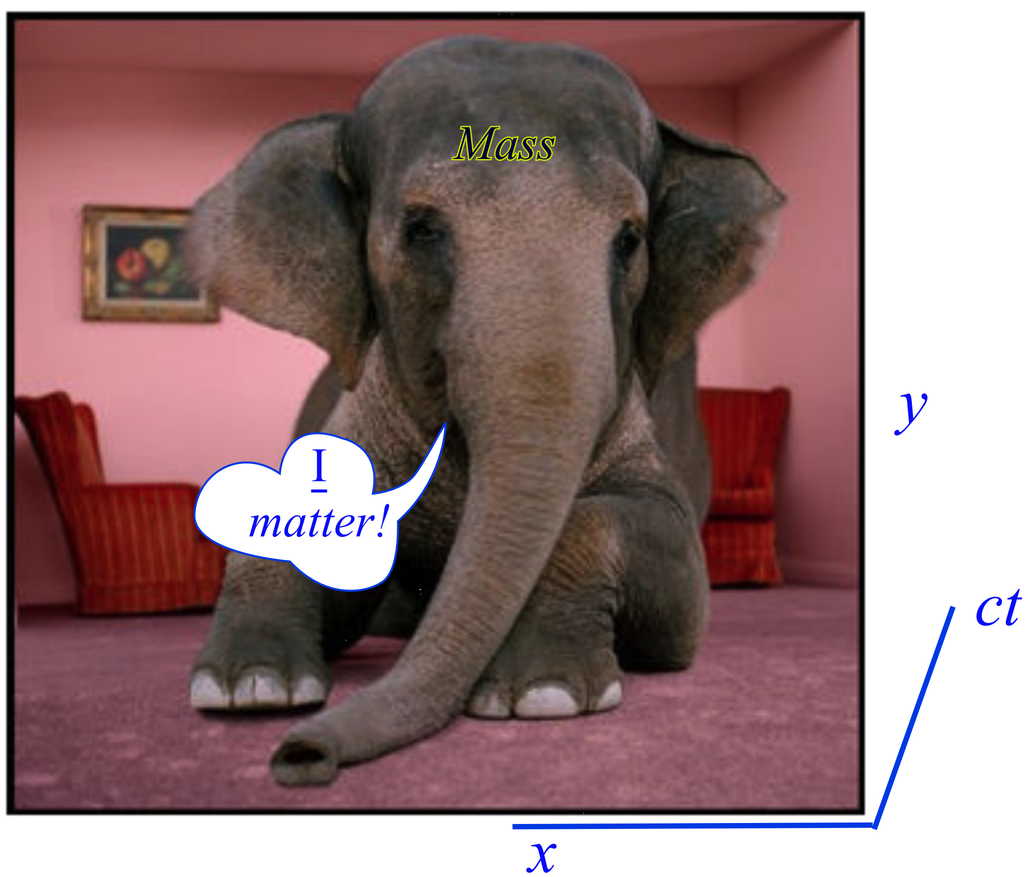
Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

- *What's the matter with Mass?*



Shining some light on the elephant in the spacetime room

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$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c\sinh\rho}{c\tanh\rho}$$

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$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

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Effective Mass

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More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}}$$

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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Definition(s) of mass for relativity/quantum

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$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2}$$ *Rest Mass*

Momentum Mass M_{mom} *(Galileo's mass)* Defined by p/u

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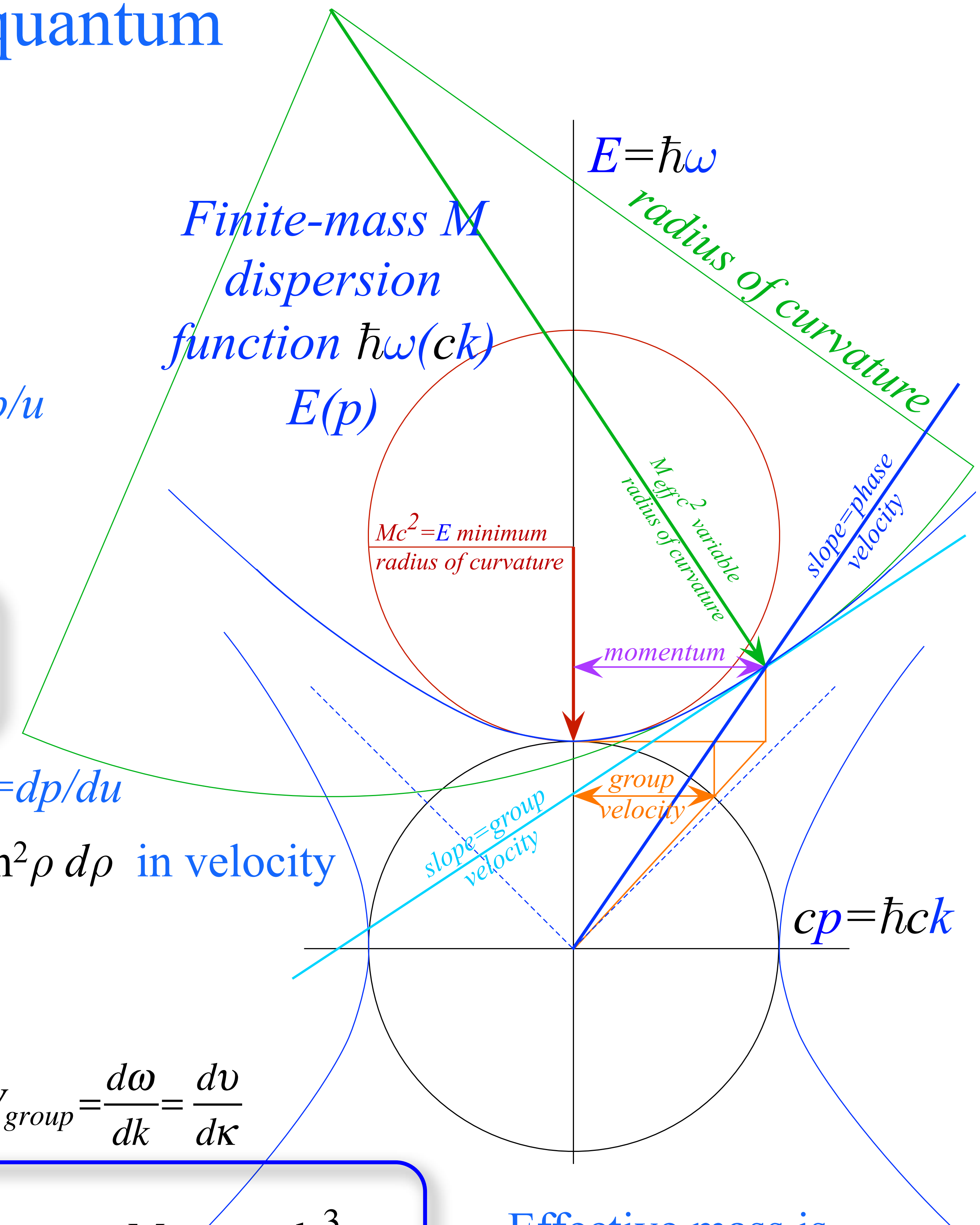

Effective mass is proportional to the *radius of curvature* of $\omega(k)$ *dispersion.*

Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

Rest Mass $(a)\gamma$-rest mass: $M^{\gamma}_{rest} = 0,$

Momentum Mass $(b)\gamma$-momentum mass: $M^{\gamma}_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\upsilon}{c^2},$

Effective Mass $(c)\gamma$-effective mass: $M^{\gamma}_{eff} = \infty.$

Newton complained about *his* "corpuscles" of light having *"fits"* (going *crazy*).

(All *this* would be evidence of *triple Schizophrenia*.)

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2\cdot 10^{-51})kg\cdot s = 4.5\cdot 10^{-36}\,kg \quad \text{(for: } \upsilon\text{=600THz)}$$

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➔ Relativistic action and Lagrangian-Hamiltonian relations
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Compton recoil related to rocket velocity formula
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Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor format→

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
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$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

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Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho$

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Legendre transformation

Use *Group velocity* : $u=\frac{dx}{dt}=c\tanh\rho$

$$p=\hbar k = Mc\sinh\rho \qquad E=\hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$$= Mc^2\frac{\sinh^2\rho-\cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

$$L \text{ is} : Mc^2\frac{-1}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

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$h\upsilon_A=Mc^2=hc\kappa_A$	$\hbar\omega_A=Mc^2=\hbar ck_A$
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Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

Note: $Mcu = Mc^2\tanh\rho$

$$= Mc^2\frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho$$

Prior wave relations

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Note: $Mcu = Mc^2 \tanh\rho$

$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

with *Hamiltonian* $H = E$

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Also: $cp = Mc^2\sinh\rho$

Compare *Lagrangian* L

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho$$

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$$H = \hbar\omega = Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho$$

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$\hbar = h/2\pi$

Prior wave relations

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Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c\tanh\rho$

$= c\sin\sigma$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

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Note: $Mcu = Mc^2\tanh\rho$

$= Mc^2\sin\sigma$

Also: $cp = Mc^2\sinh\rho$

$= \hbar ck = Mc^2\tan\sigma$

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

with *Hamiltonian* $H = E$

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Define *Action* $S = \hbar\Phi$

Including stellar angle σ

Prior wave relations

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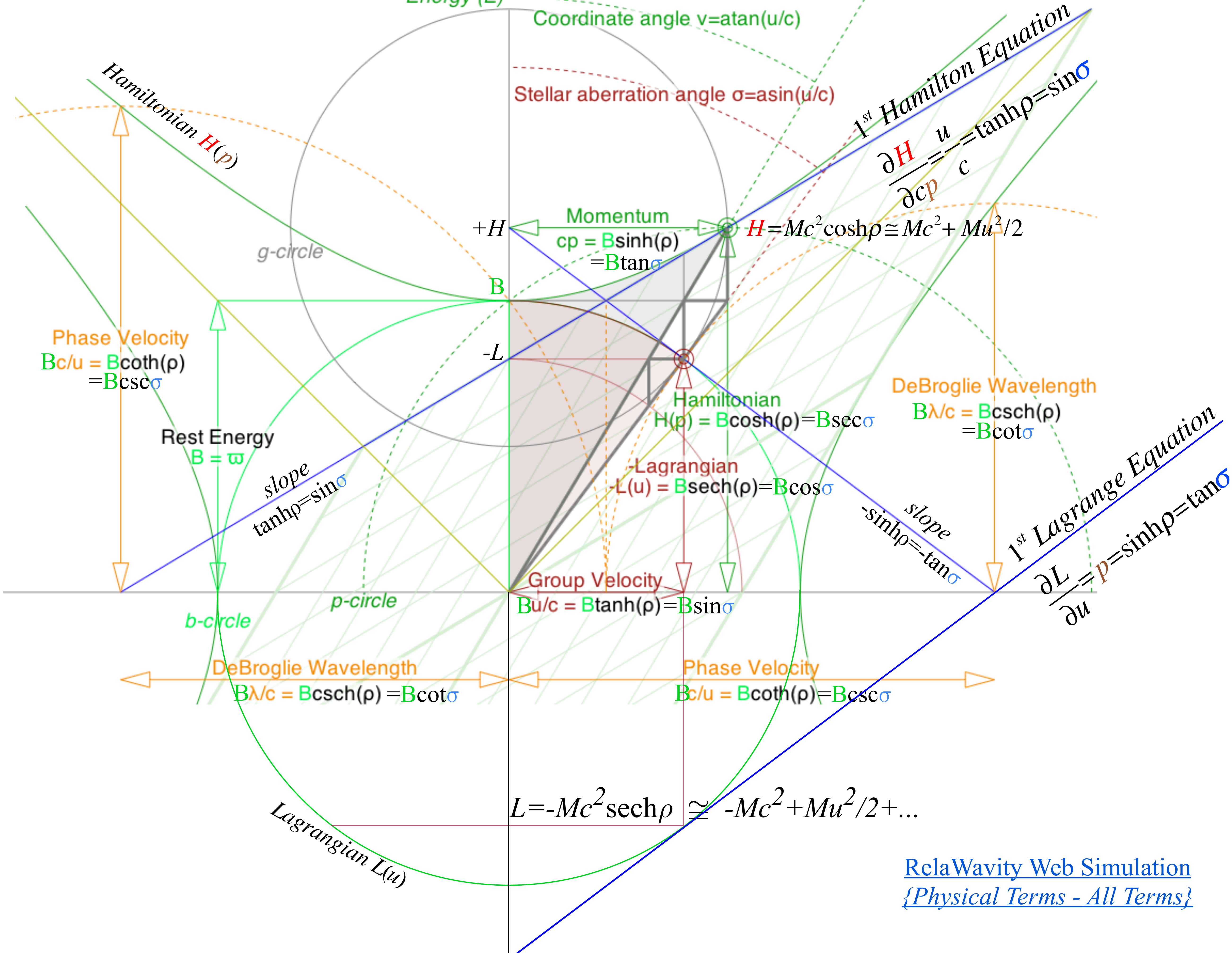

[RelaWavity Web Simulation](#)
[{Physical Terms - All Terms}](#)

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
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Legendre transformation

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\,\mathrm{sech}\rho = -Mc^2\cos\sigma$$

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$$H = \hbar\omega = Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho = Mc^2\sec\sigma$$

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Define *Action* $S = \hbar\Phi$

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Legendre transformation

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\, dx - H\, dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho = -Mc^2 \cos\sigma$$

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Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c\tanh\rho$

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\,dx - \hbar\omega\,dt = p\,dx - H\,dt$$

Poincare Invariant action differential

$$\frac{\partial S}{\partial x} = p \qquad \frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\,\mathrm{sech}\rho = -Mc^2\cos\sigma$$

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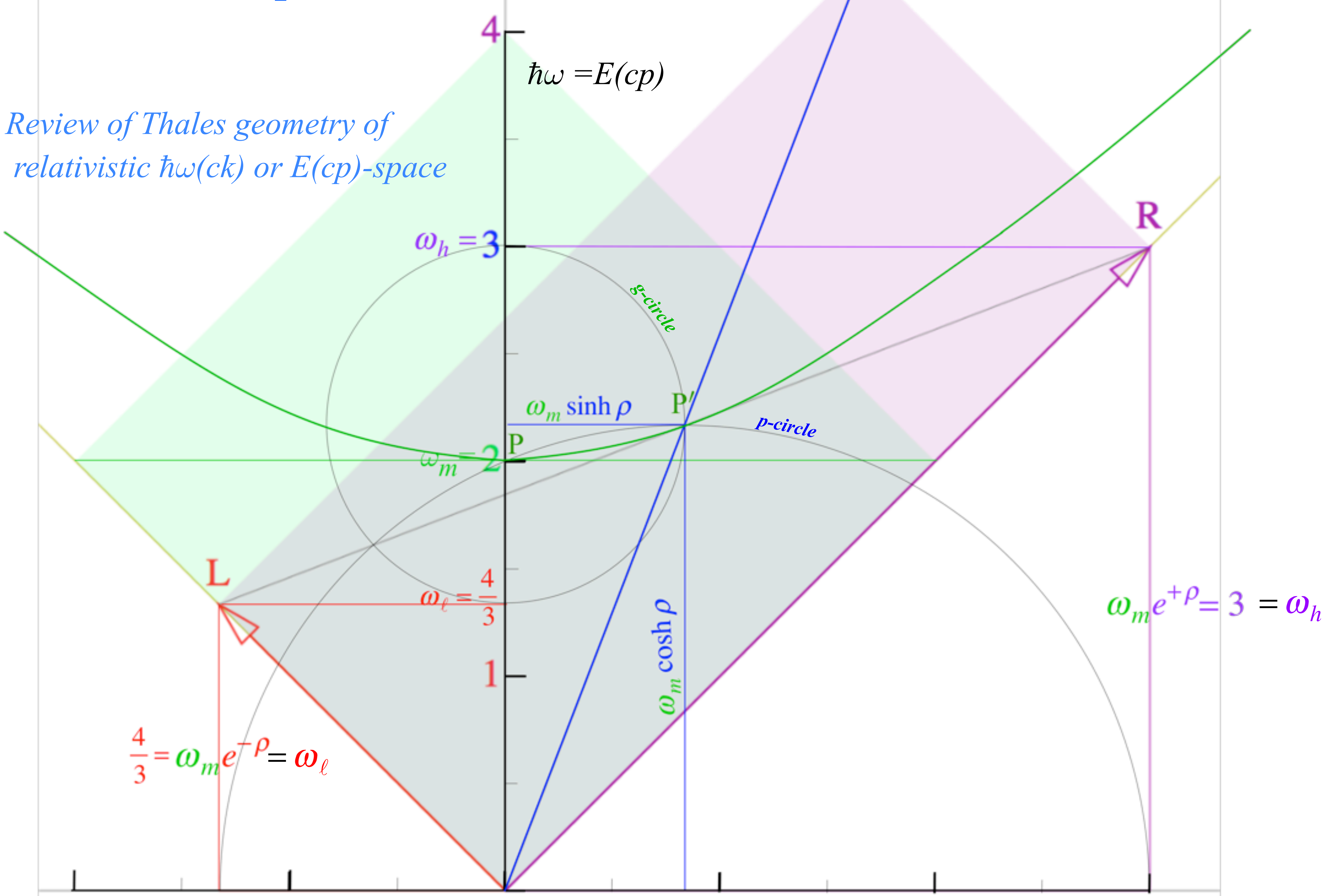
Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

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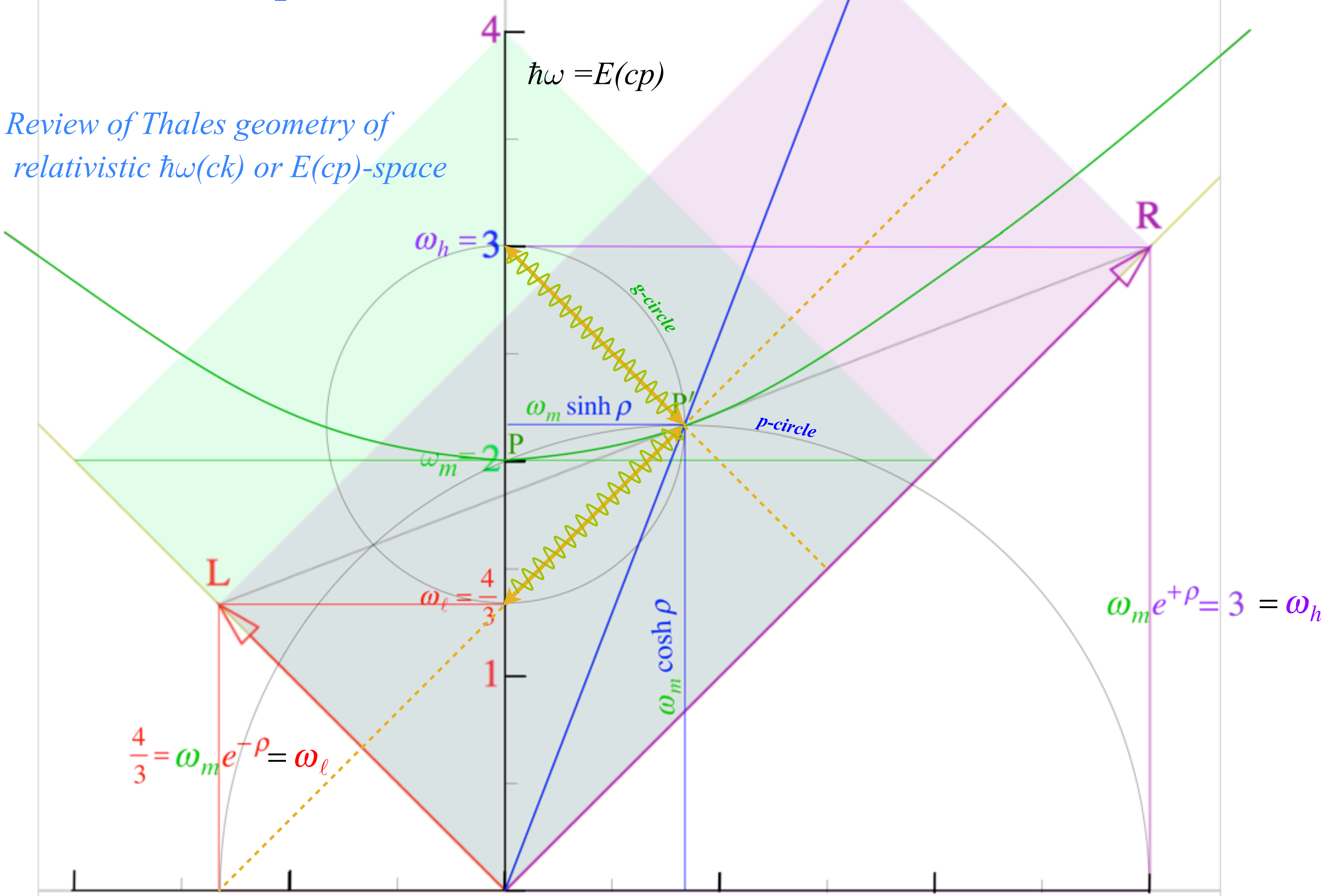
Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



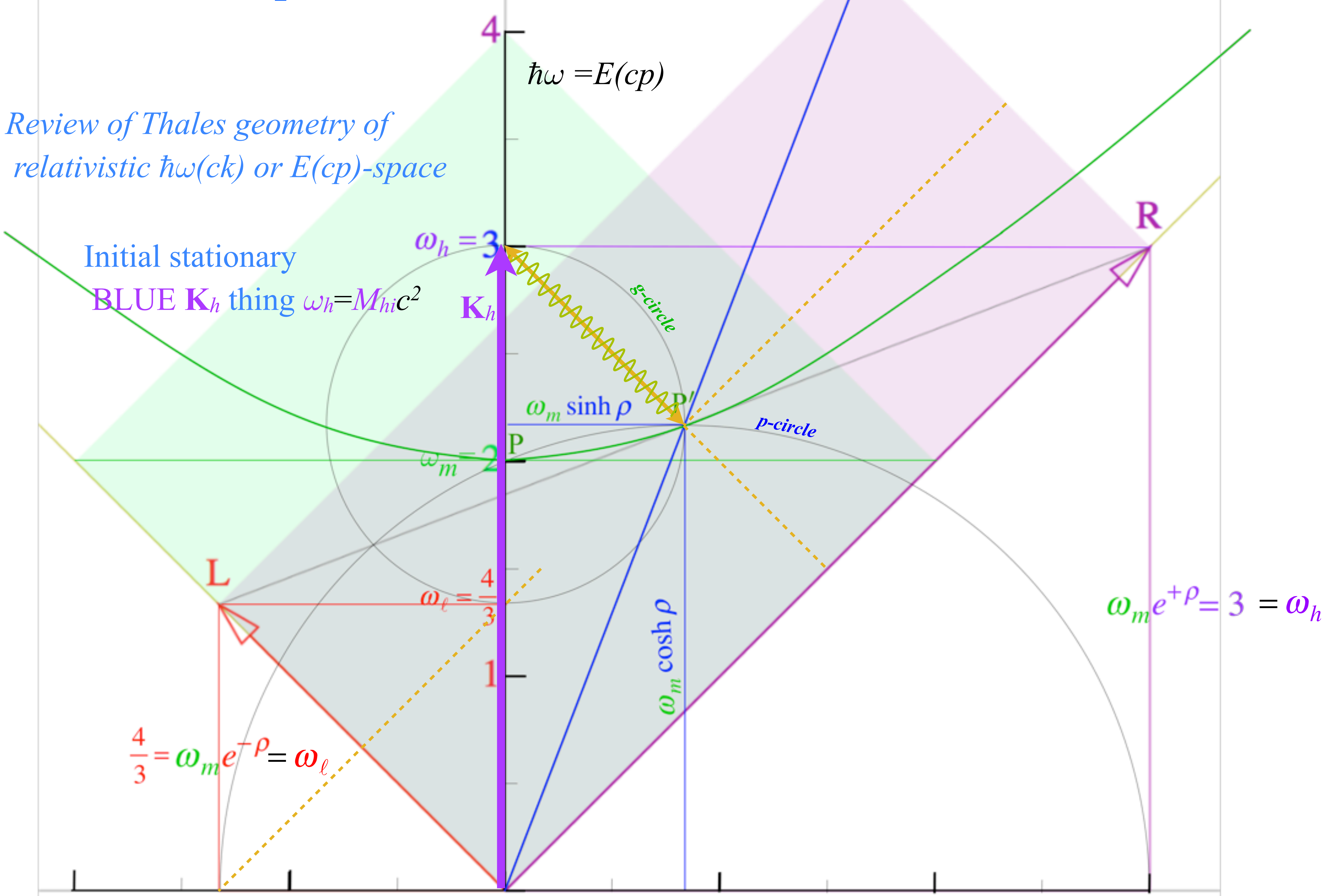
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

Initial stationary BLUE $\mathbf{K}_h$ thing $\omega_h = M_{hi}c^2$



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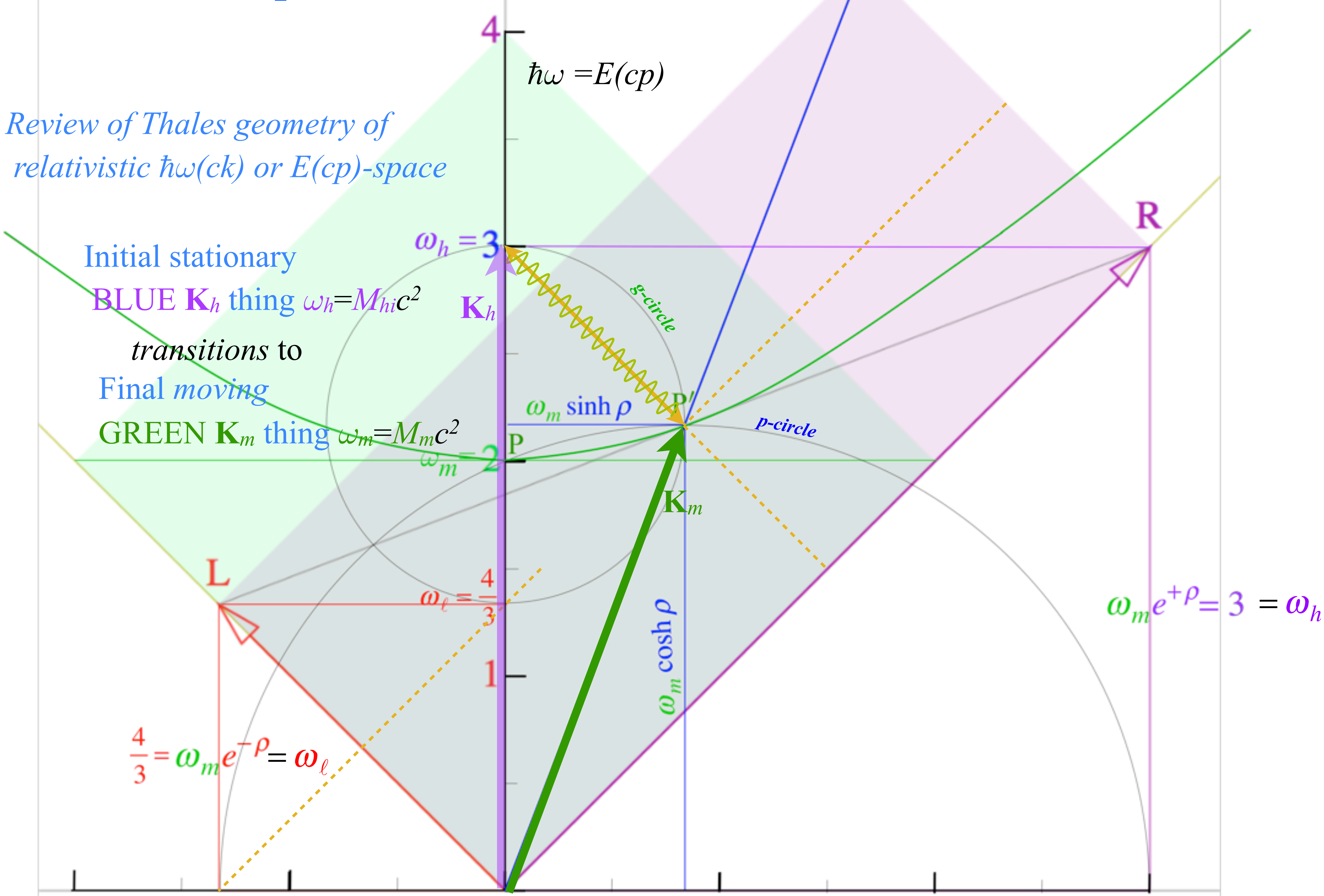
Initial stationary BLUE $\mathbf{K}_h$ thing $\omega_h = M_{hi}c^2$ *transitions* to Final *moving* GREEN $\mathbf{K}_m$ thing $\omega_m = M_m c^2$



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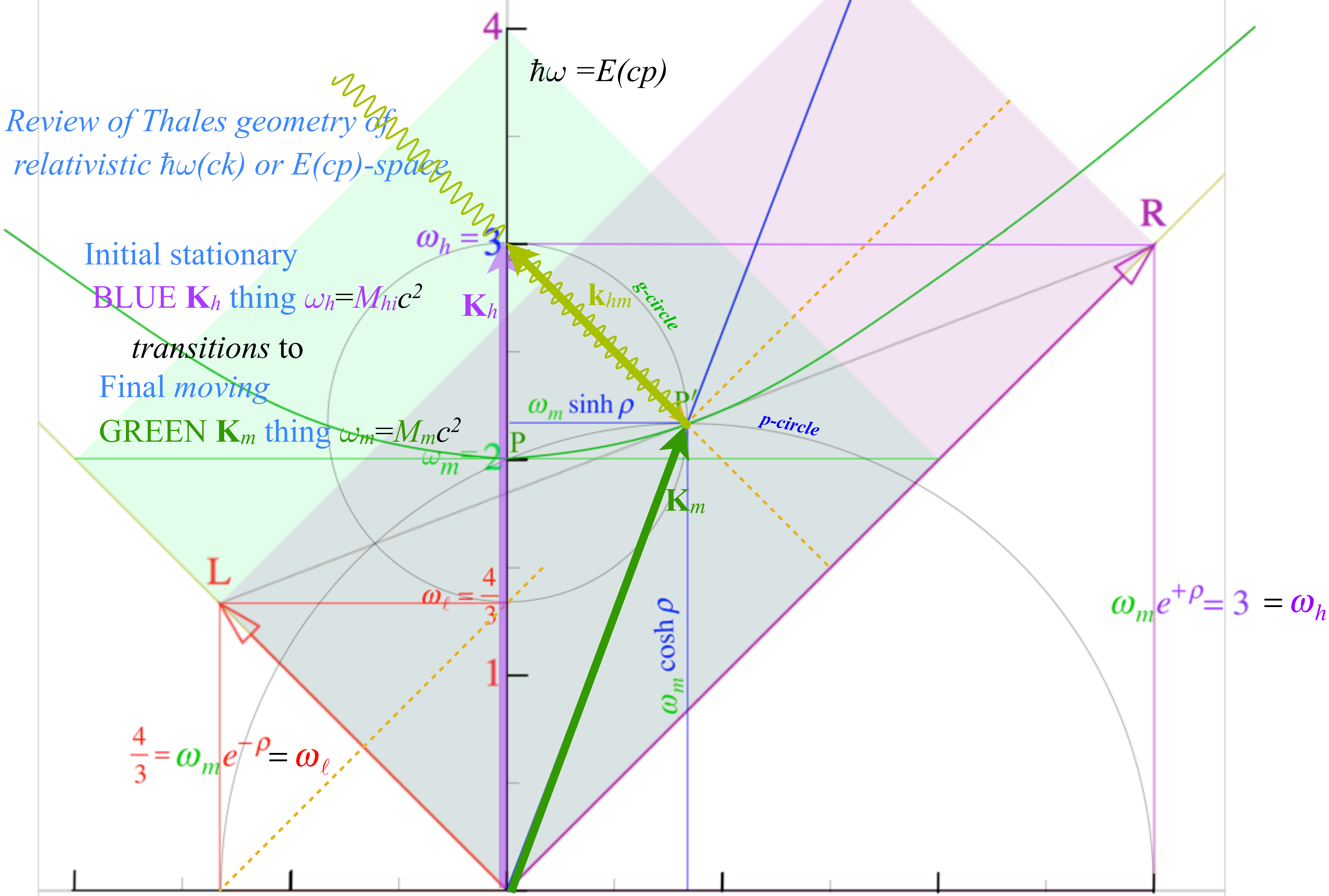

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Lecture 31
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➔ Feynman diagram geometry

Compton recoil related to rocket velocity formula

Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

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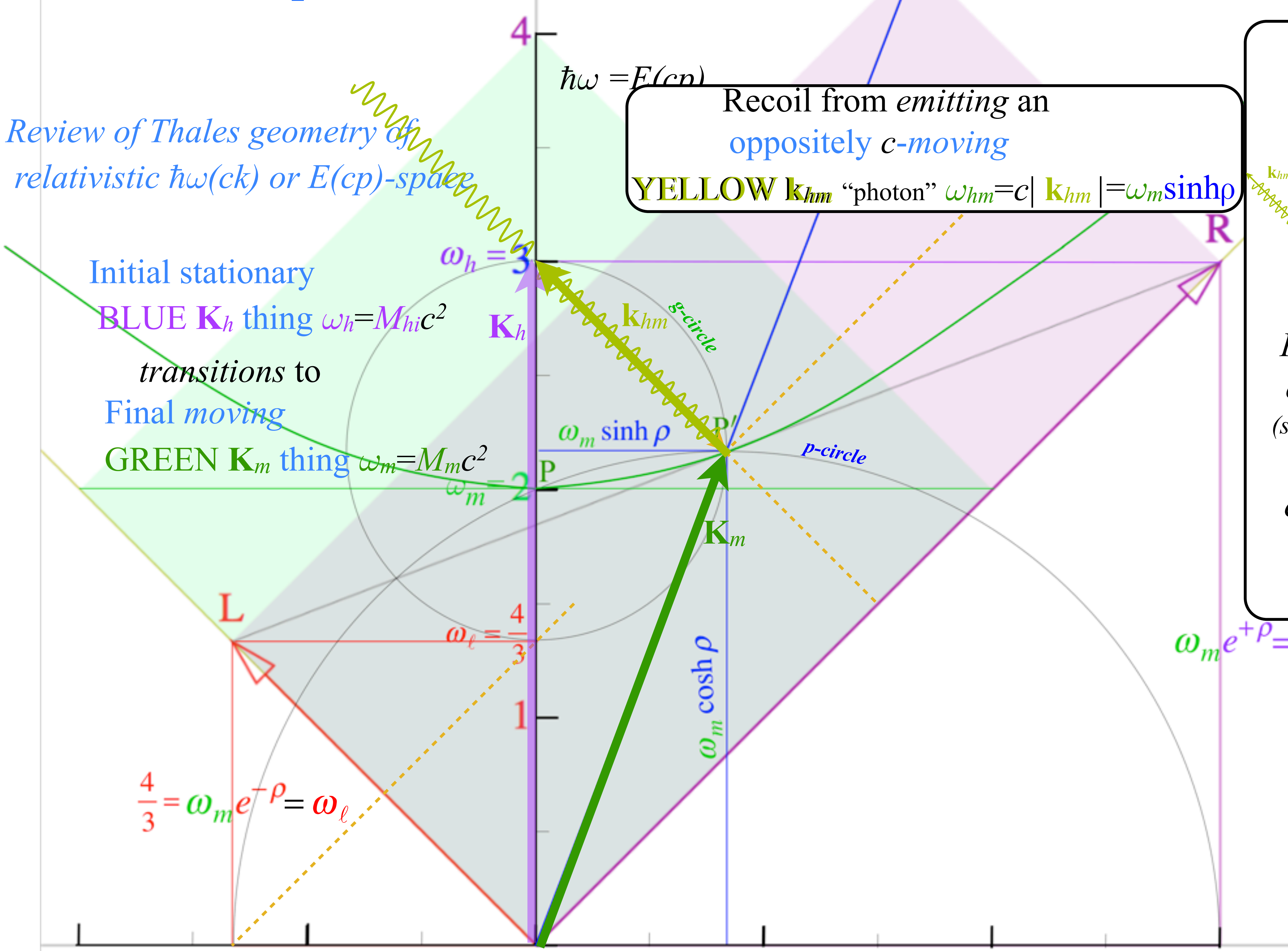
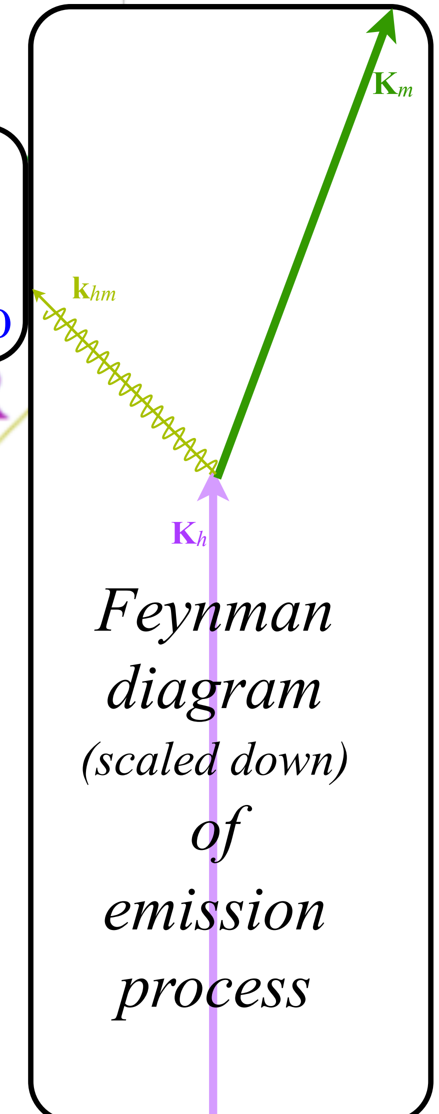
Initial stationary BLUE $\mathbf{K}_h$ thing $\omega_h=M_{hi}c^2$ *transitions* to Final *moving* GREEN $\mathbf{K}_m$ thing $\omega_m=M_m c^2$

Recoil from *emitting* an oppositely *c-moving* YELLOW $\mathbf{k}_{hm}$ "photon" $\omega_{hm}=c|\mathbf{k}_{hm}|=\omega_m \sinh\rho$





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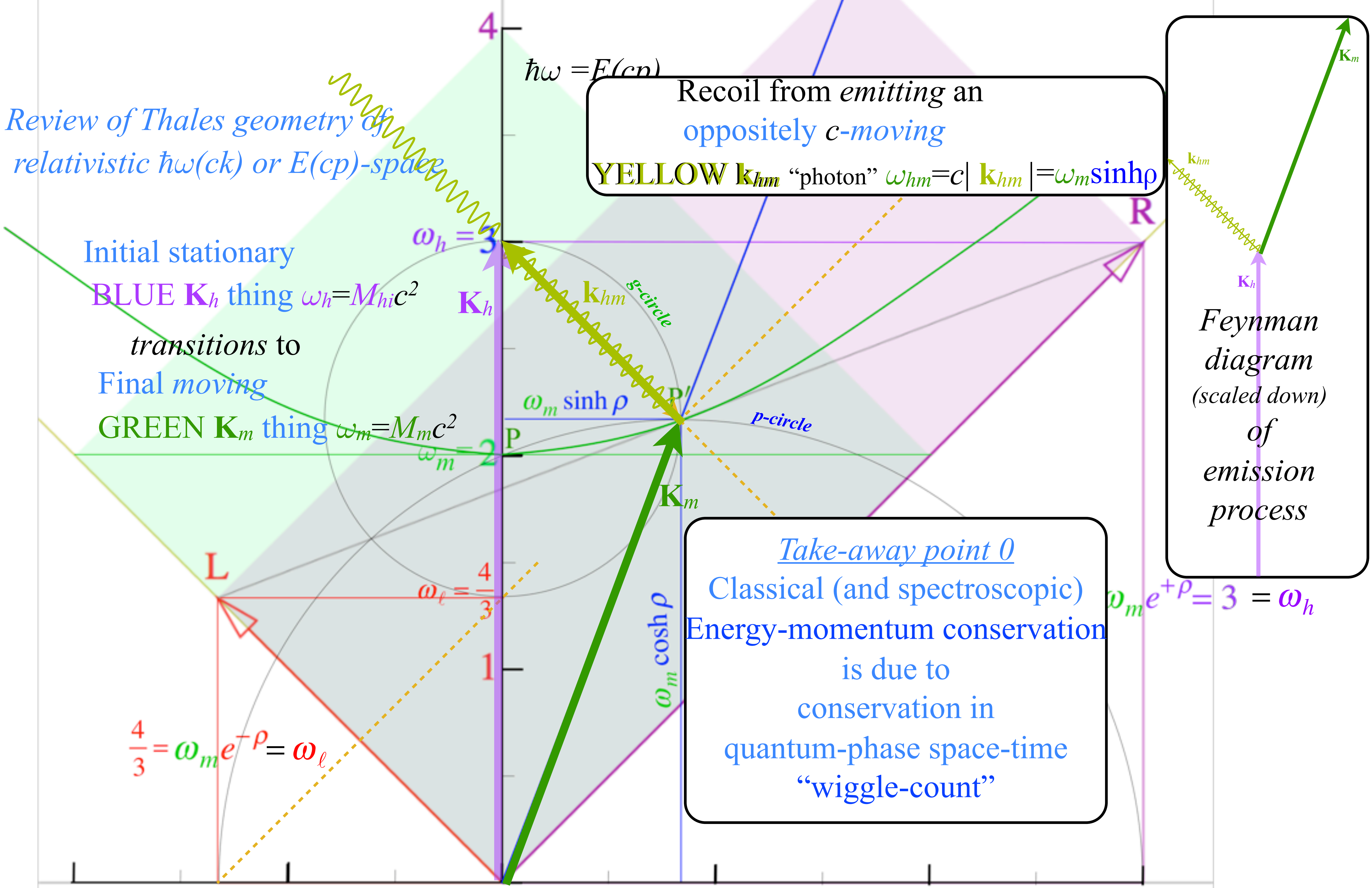

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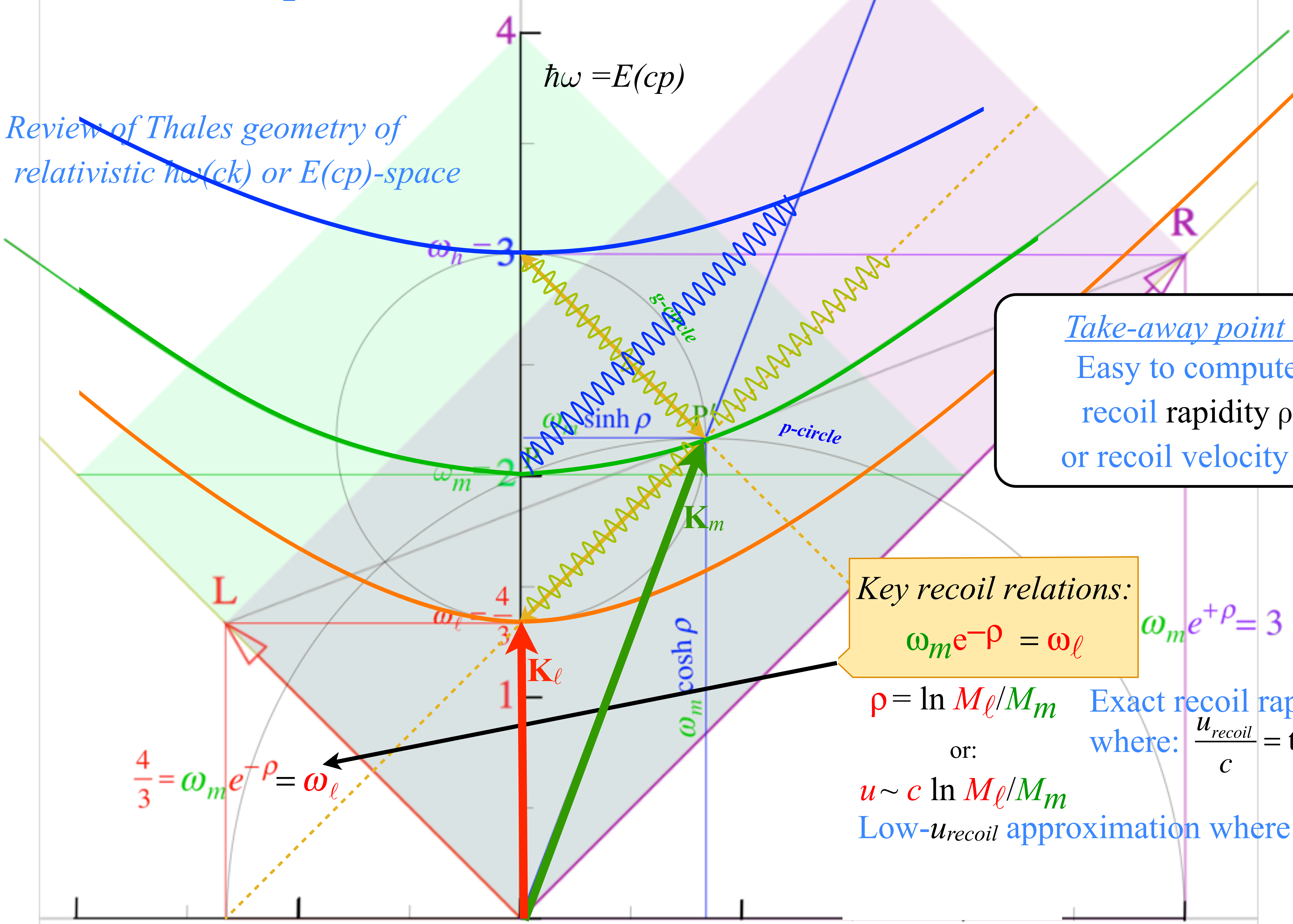

Take-away point 2

Easy to compute recoil rapidity ρ or recoil velocity u

Key recoil relations:

$\omega_m e^{-\rho} = \omega_\ell$

$\omega_m e^{+\rho} = 3 = \omega_h$

$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$

$\rho = \ln M_\ell / M_m$

or:

$u \sim c \ln M_\ell / M_m$

Exact recoil rapidity where: $\frac{u_{recoil}}{c} = \tanh \rho$

Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

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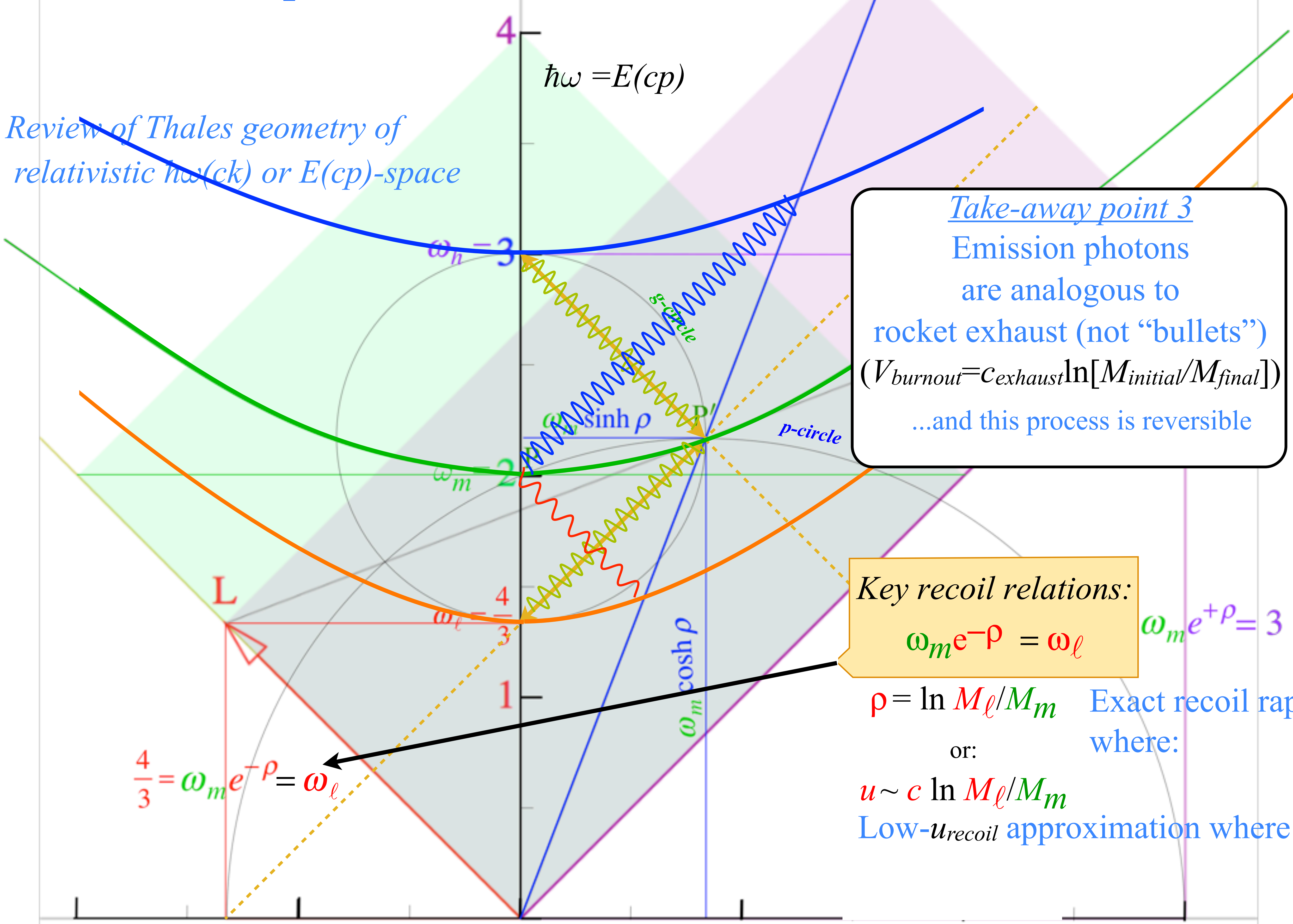

<u>*Take-away point 3*</u>

Emission photons are analogous to rocket exhaust (not “bullets”)

$(V_{burnout} = c_{exhaust} \ln[M_{initial}/M_{final}])$

...and this process is reversible

Key recoil relations:

$\omega_m e^{-\rho} = \omega_\ell$

$\omega_m e^{+\rho} = 3 = \omega_h$

$\rho = \ln M_\ell / M_m$ Exact recoil rapidity where:

or:

$u \sim c \ln M_\ell / M_m$

Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

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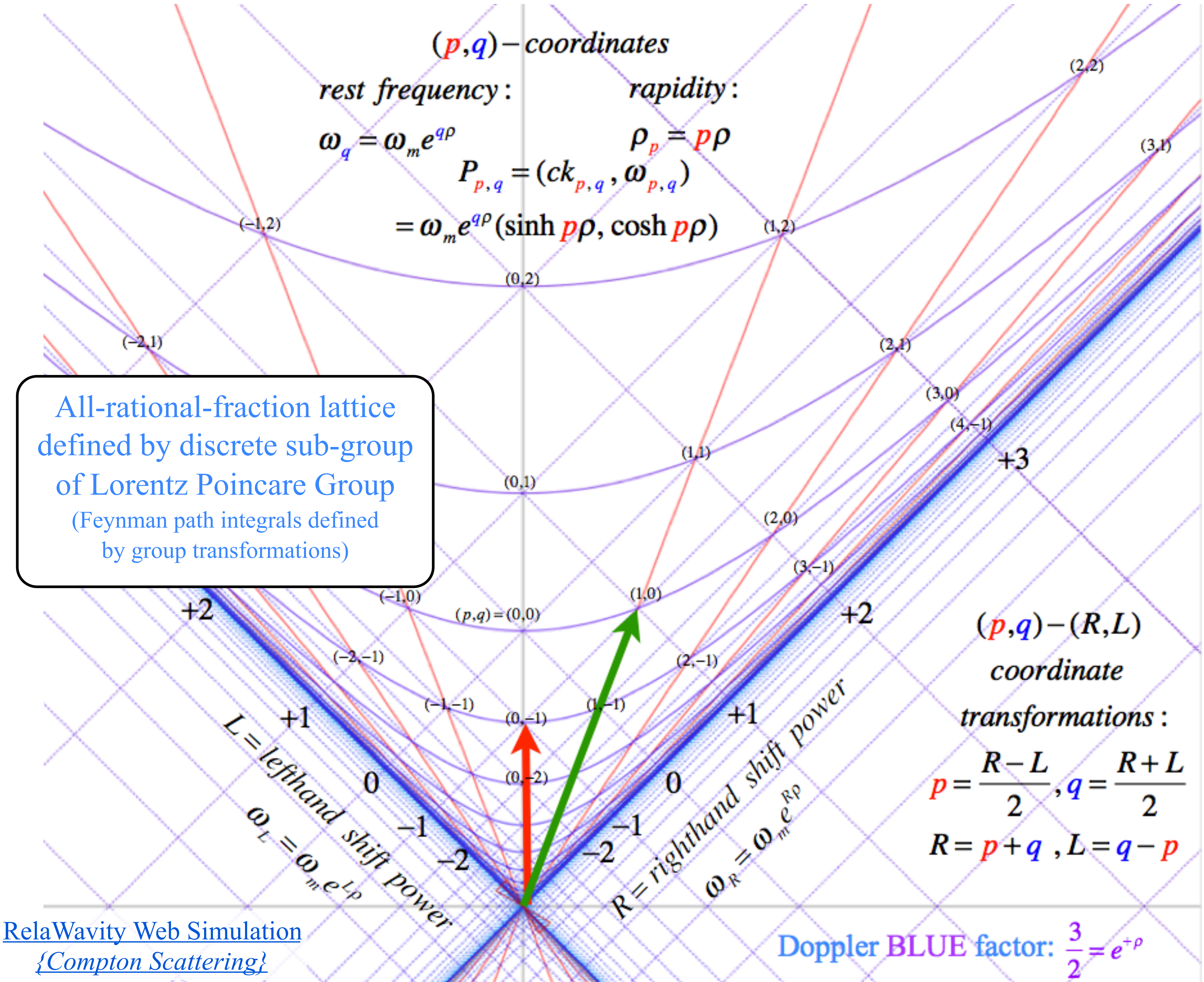

(p,q) − coordinates
rest frequency:
$\omega_q = \omega_m e^{q\rho}$
rapidity:
$\rho_p = p\rho$
$P_{p,q} = (ck_{p,q}, \omega_{p,q})$
$= \omega_m e^{q\rho}(\sinh p\rho, \cosh p\rho)$
(2,2)
(3,1)
(−1,2)
(1,2)
(0,2)
(−2,1)
(2,1)
(3,0)
(4,−1)
+3
(1,1)
(0,1)
(2,0)
(3,−1)
(1,0)
(−1,0)
(p,q)=(0,0)
+2
+2
(−2,−1)
(2,−1)
(−1,−1)
(1,−1)
(0,−1)
+1
+1
(0,−2)
0
0
−1
−1
−2
−2
L = lefthand shift power
$\omega_L = \omega_m e^{L\rho}$
R = righthand shift power
$\omega_R = \omega_m e^{R\rho}$
All-rational-fraction lattice
defined by discrete sub-group
of Lorentz Poincare Group
(Feynman path integrals defined
by group transformations)
(p,q) − (R,L)
coordinate
transformations:
$p = \frac{R-L}{2}, q = \frac{R+L}{2}$
$R = p + q, L = q - p$
RelaWavity Web Simulation
{Compton Scattering}
Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

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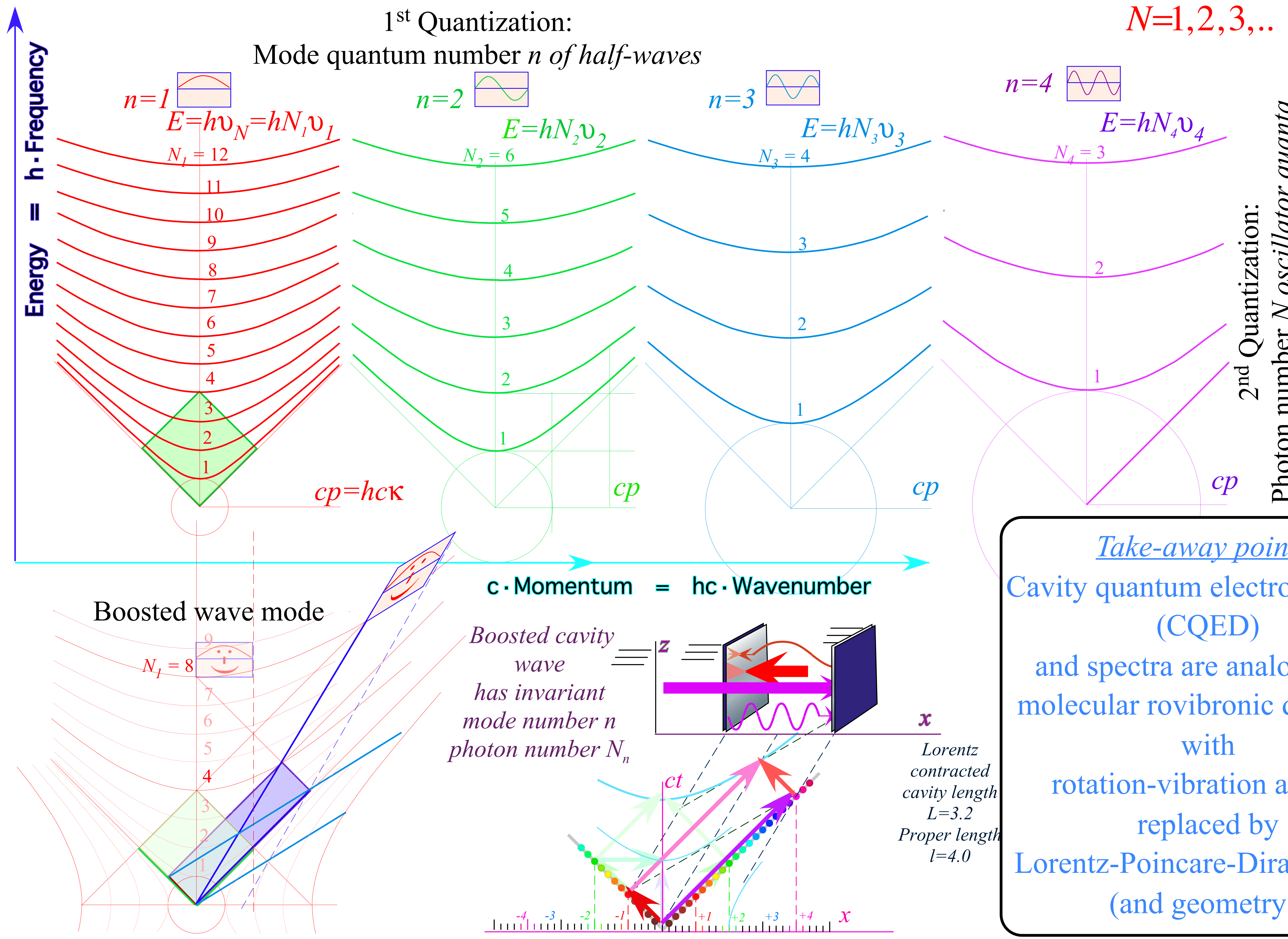
2nd Quantization:

$h\upsilon$ *is actually* $hN\upsilon$

$\left(h\upsilon_{phase} = E = h\upsilon_A \cosh\rho \right)$ *is actually* $\left(hN\upsilon_{phase} = E_N = hN\upsilon_A \cosh\rho \right.$ with *quantum* numbers$\left.\right)$

$N=1,2,3,..$



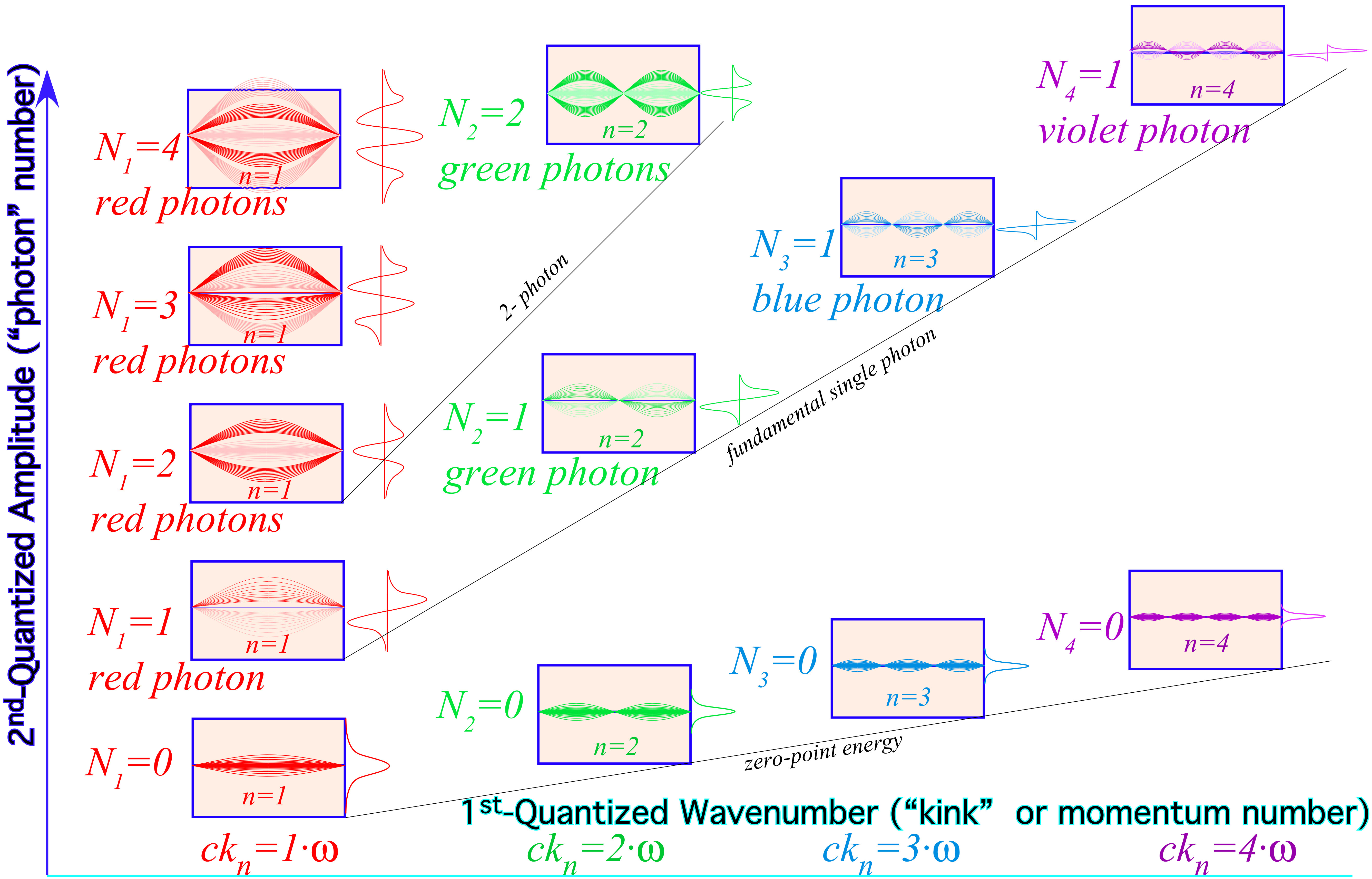
Take-away point 4

Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2^{nd} Quantization:

$h\upsilon$ *is actually* $hN\upsilon$

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ *is actually* $(hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho \quad (N=1,2,..))$



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Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid

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Acceleration by chirping laser pairs

Varying acceleration (General case)

From Lect. 35 ModPhys (2012)

Varying local acceleration $\rho = \rho(\tau)$

$$u = \frac{dx}{dt} = c\tanh(\tau)$$

$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

$$ct = c\int\cosh\rho(\tau)\,d\tau \qquad x = c\int\sinh\rho(\tau)\,d\tau$$

Constant local acceleration $\rho = \frac{g\tau}{c}$ "*Einstein Elevator*"

$$ct = c\int\cosh\frac{g\tau}{c}\,d\tau \qquad x = c\int\sinh\frac{g\tau}{c}\,d\tau$$

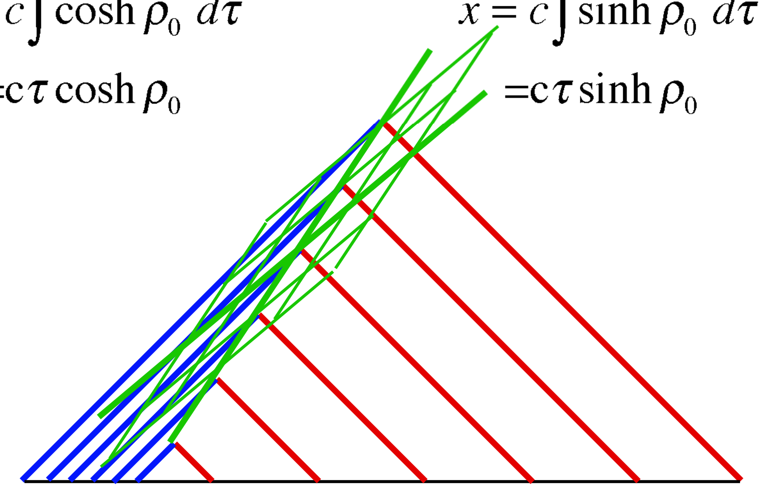
$$= \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$

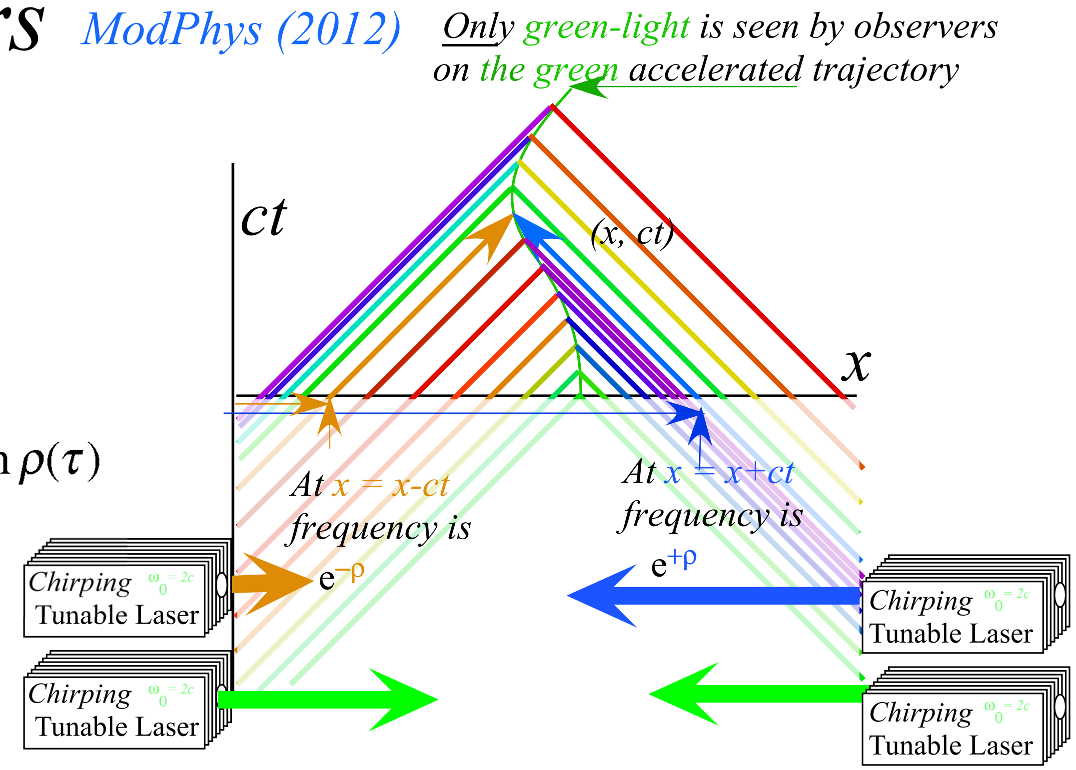
Previous examples involved constant velocity

Constant velocity $\rho = \rho_0 = const.$ "*Lorentztransformation*"

$$ct = c\int\cosh\rho_0\,d\tau \qquad x = c\int\sinh\rho_0\,d\tau$$

$$= c\tau\cosh\rho_0 \qquad = c\tau\sinh\rho_0$$





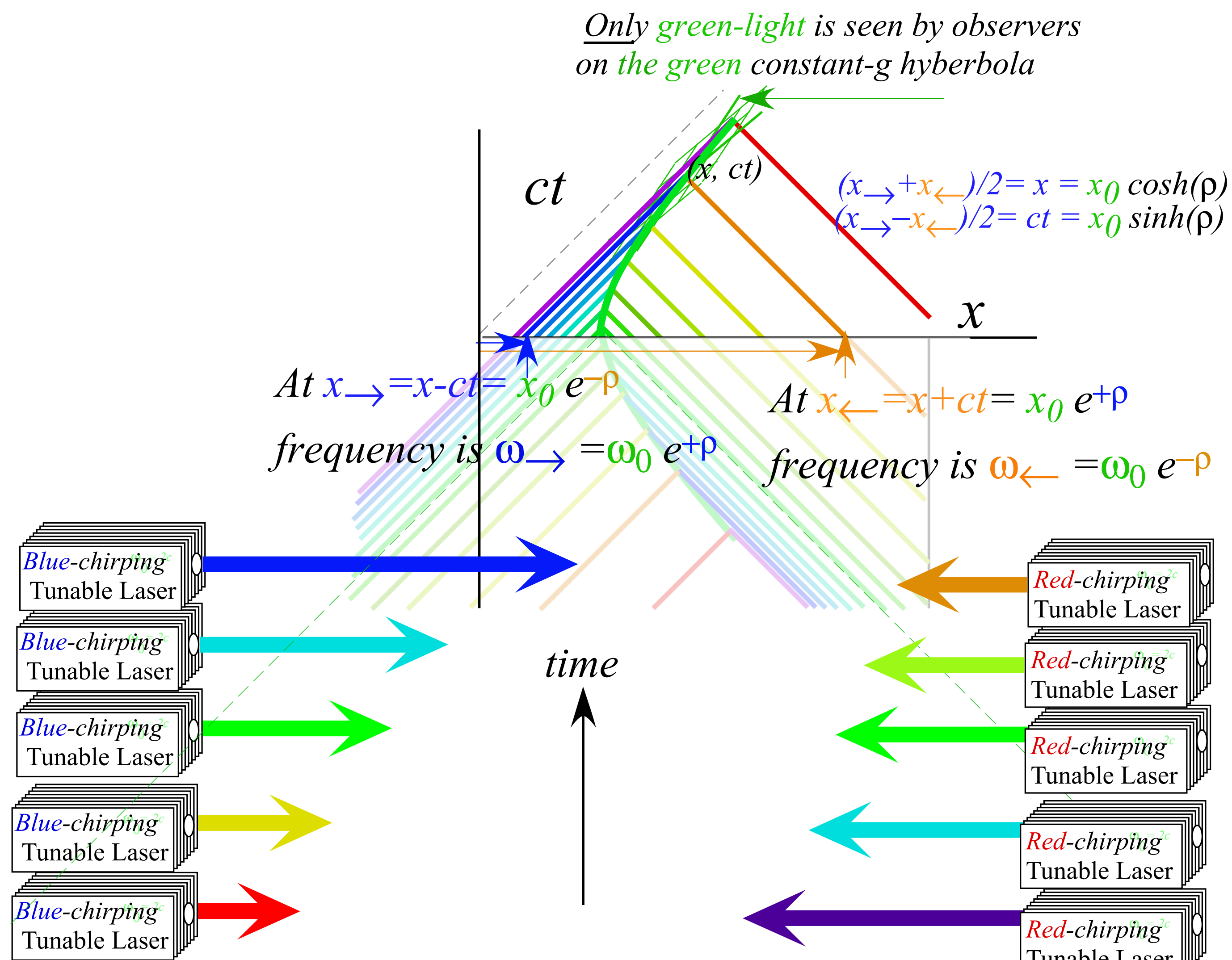
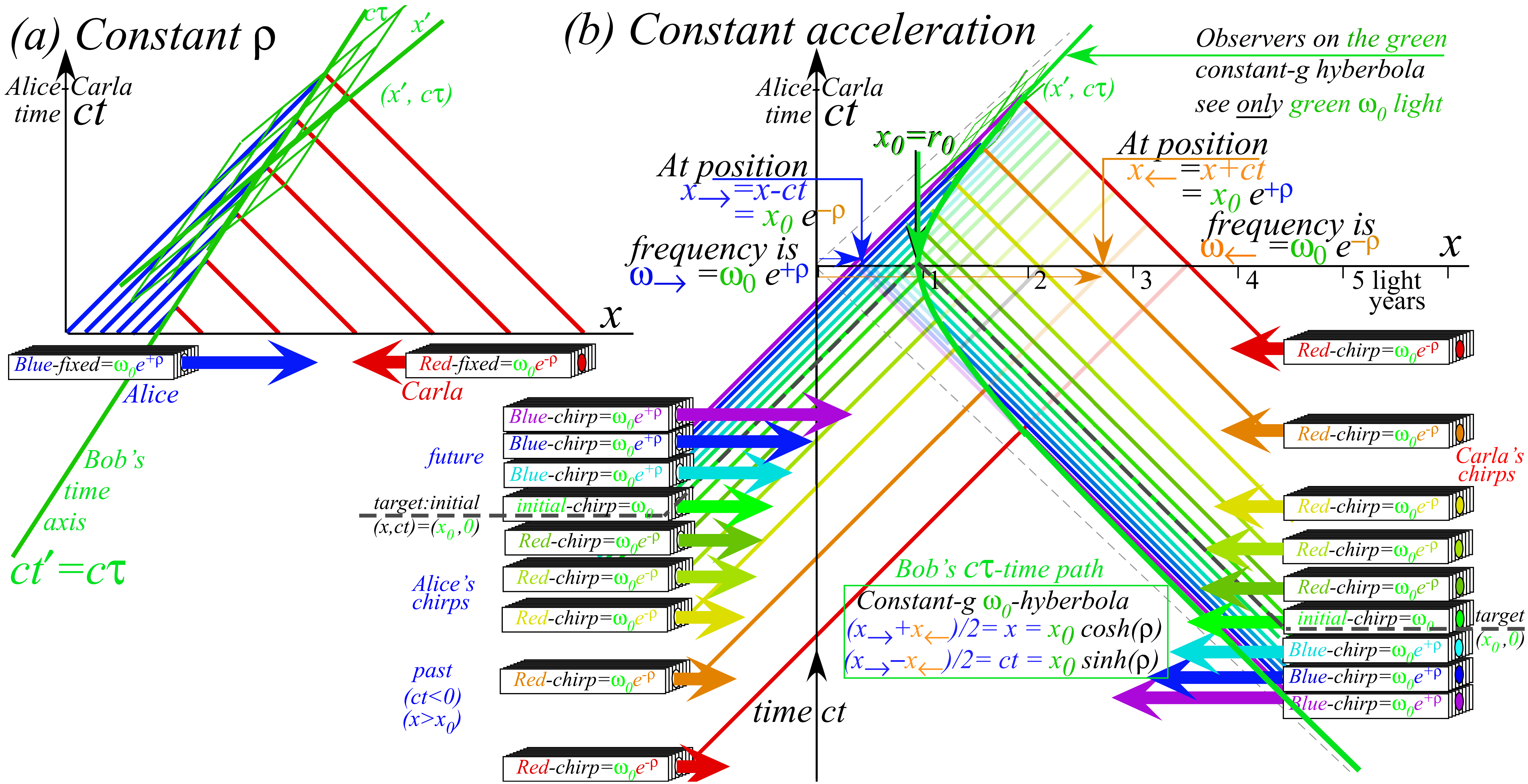
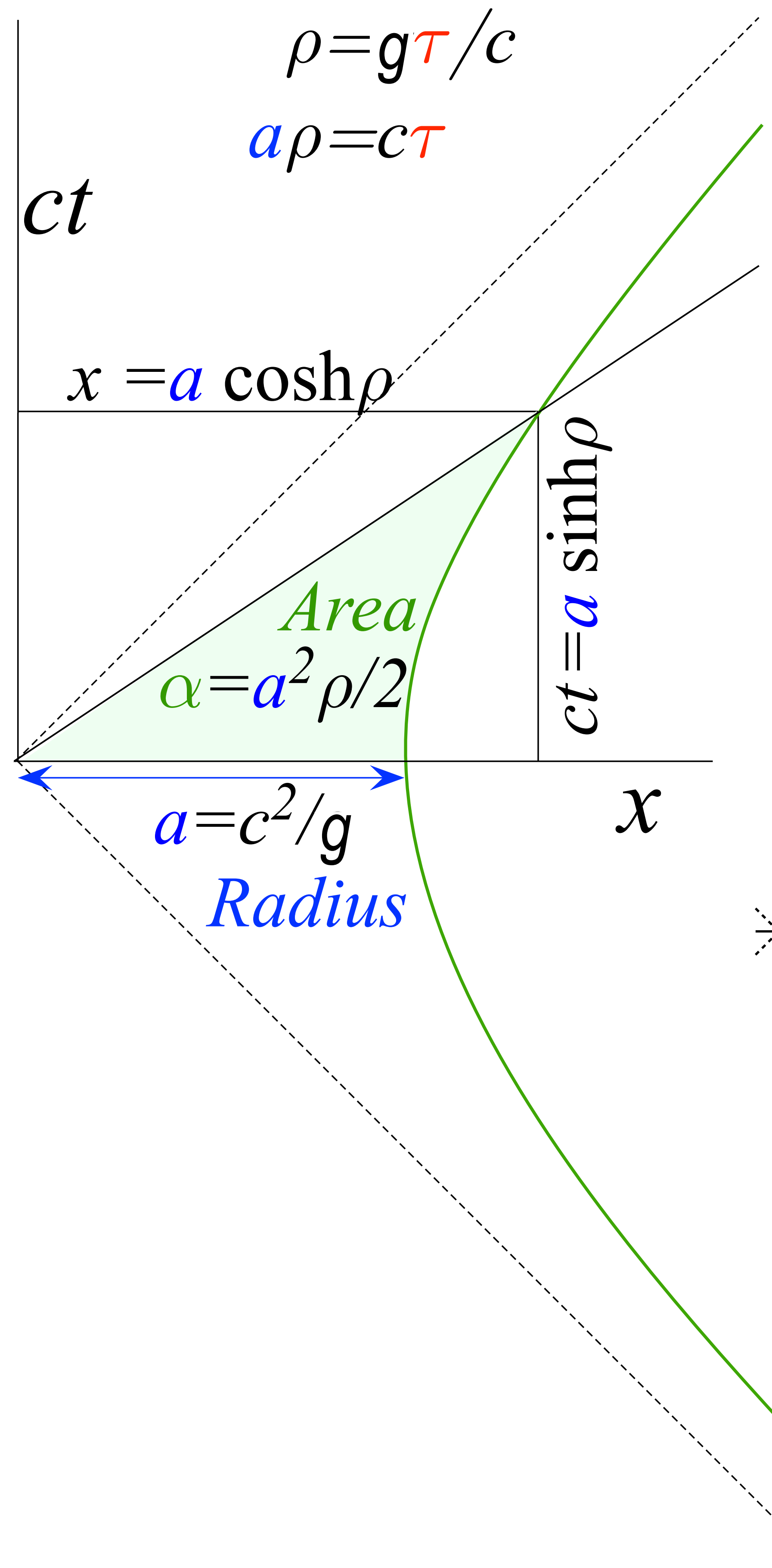


Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

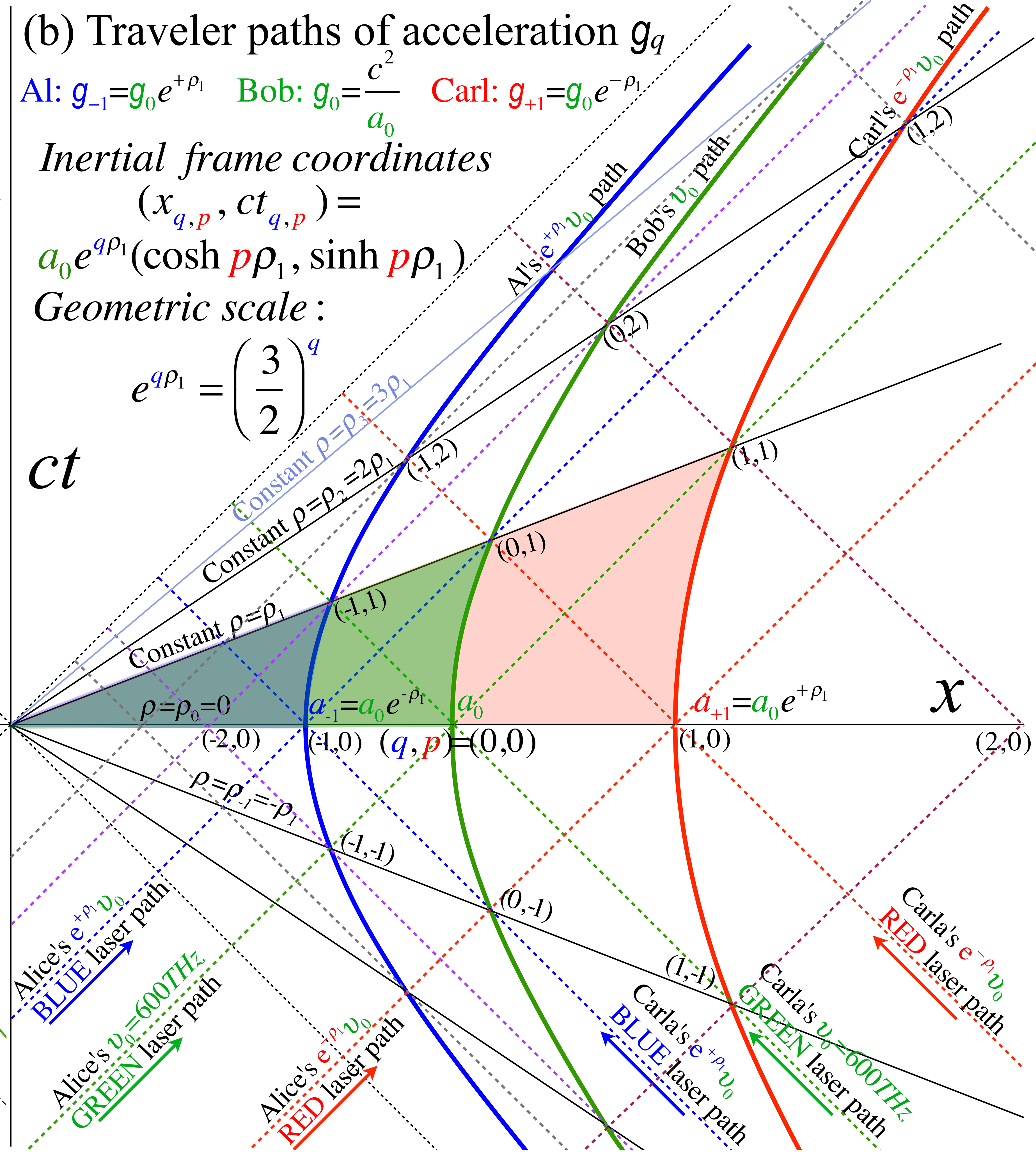

(a) Constant ρ
Alice-Carla time ct
x
ct′=cτ
(x′, cτ)
Blue-fixed=ω0e+ρ
Red-fixed=ω0e-ρ
Alice
Carla
Bob's time axis
ct′=cτ
(b) Constant acceleration
Alice-Carla time ct
x0=r0
(x′, cτ)
Observers on the green constant-g hyberbola see only green ω0 light
At position
x→=x-ct
= x0 e−ρ
frequency is
ω→ =ω0 e+ρ
At position
x←=x+ct
= x0 e+ρ
frequency is
ω← =ω0 e−ρ
x
1
2
3
4
5 light years
Blue-chirp=ω0e+ρ
Blue-chirp=ω0e+ρ
Blue-chirp=ω0e+ρ
initial-chirp=ω0
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
future
target:initial
(x,ct)=(x0,0)
Alice's chirps
past
(ct<0)
(x>x0)
time ct
Bob's cτ-time path
Constant-g ω0-hyberbola
(x→+x←)/2= x = x0 cosh(ρ)
(x→−x←)/2= ct = x0 sinh(ρ)
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Carla's chirps
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
Red-chirp=ω0e-ρ
initial-chirp=ω0
target
(x0,0)
Blue-chirp=ω0e+ρ
Blue-chirp=ω0e+ρ
Blue-chirp=ω0e+ρ

(a) Constant acceleration g

Rapidity ρ *vs proper time* τ



(b) Traveler paths of acceleration g_q



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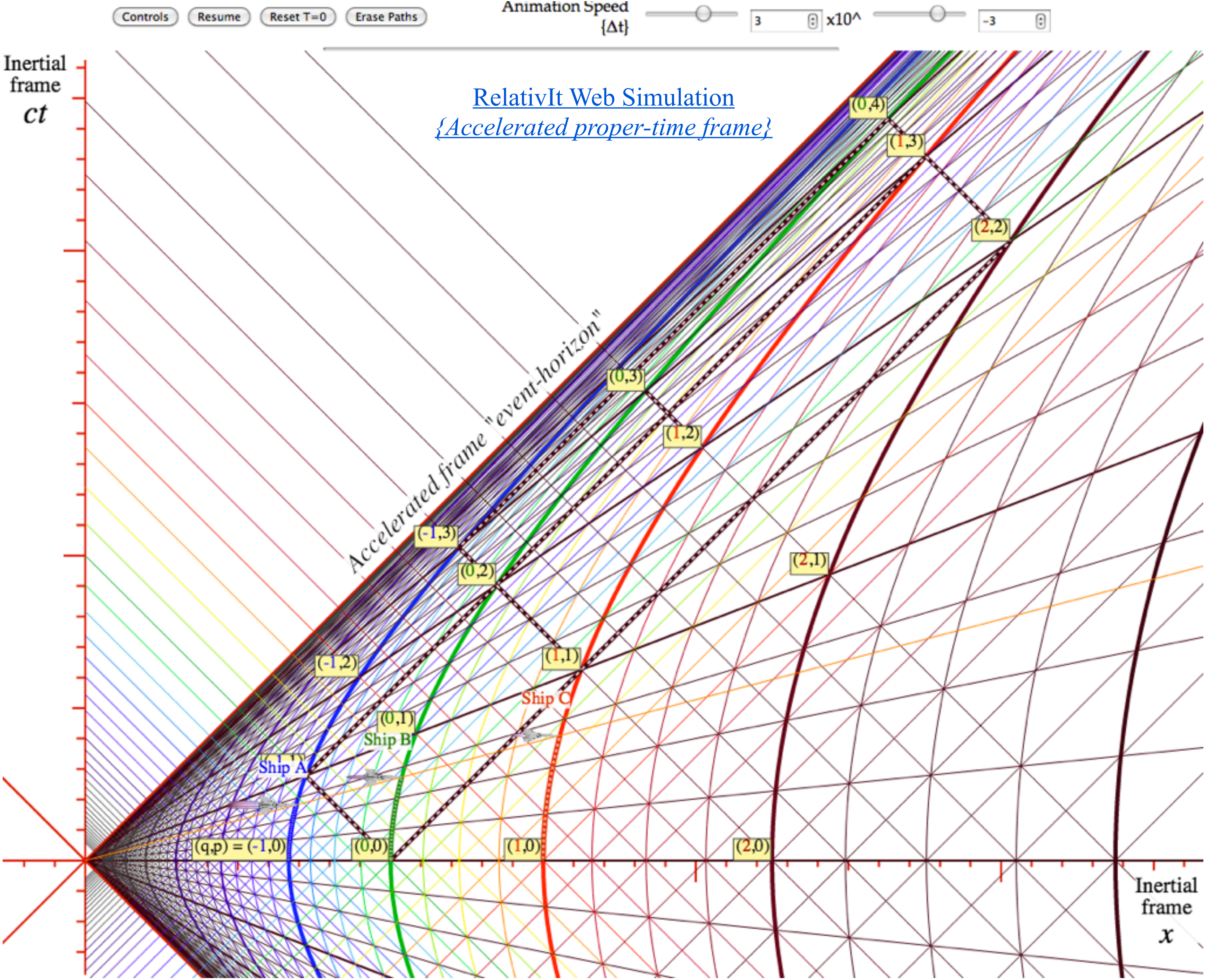
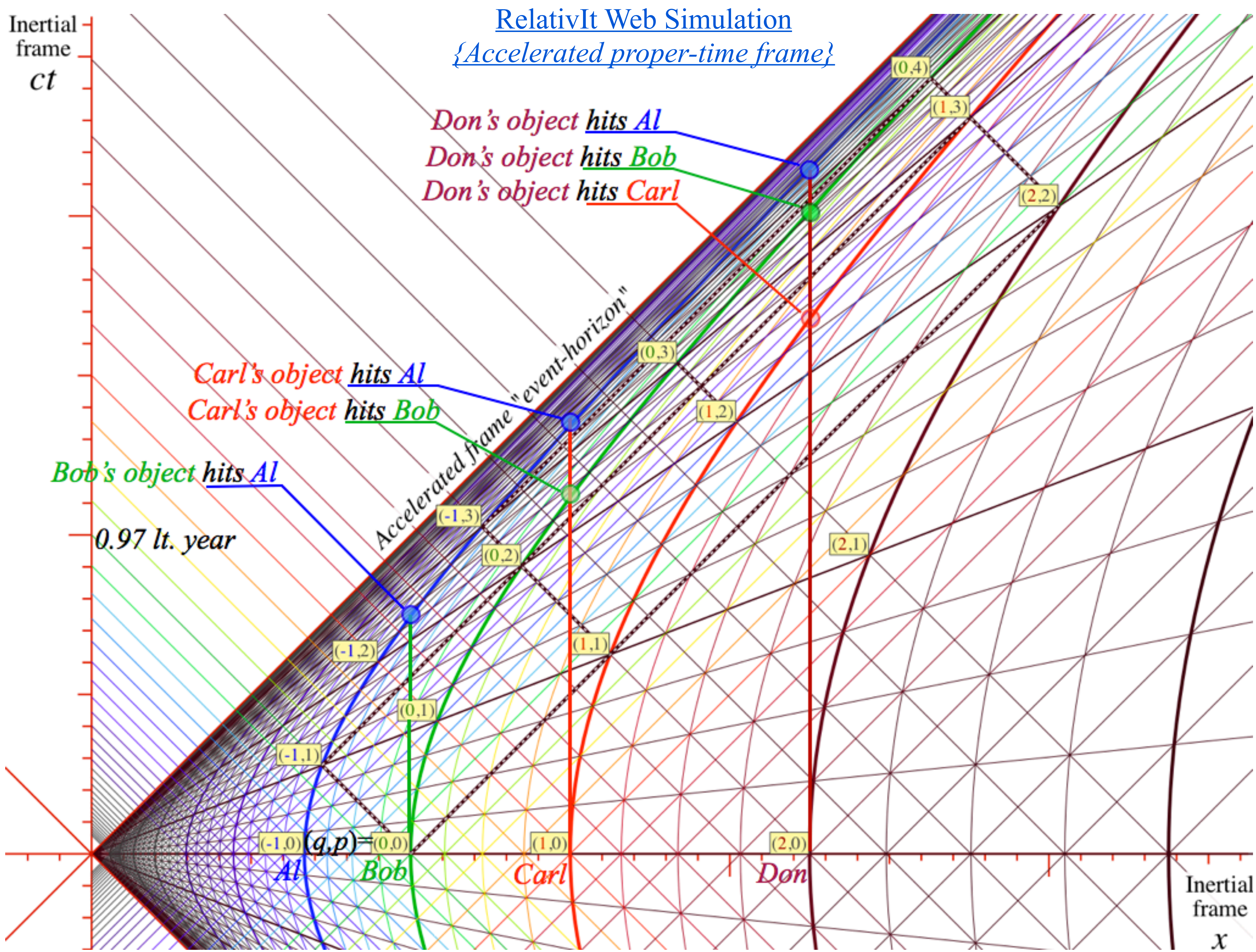

Controls
Resume
Reset T=0
Erase Paths
Animation Speed {Δt}
3
x10^
-3
Inertial frame ct
RelativIt Web Simulation
{Accelerated proper-time frame}
Accelerated frame "event-horizon"
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(1,1)
(-1,2)
Ship C
(0,1)
Ship B
Ship A
(q,p) = (-1,0)
(0,0)
(1,0)
(2,0)
Inertial frame x



RelativIt Web Simulation
{Accelerated proper-time frame}
Inertial frame
ct
Don's object hits Al
Don's object hits Bob
Don's object hits Carl
Carl's object hits Al
Carl's object hits Bob
Bob's object hits Al
0.97 lt. year
Accelerated frame "event-horizon"
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(-1,2)
(1,1)
(0,1)
(-1,1)
(-1,0)
(q,p)=(0,0)
(1,0)
(2,0)
Al
Bob
Carl
Don
Inertial frame
x

From Lect. 35
ModPhys (2012)

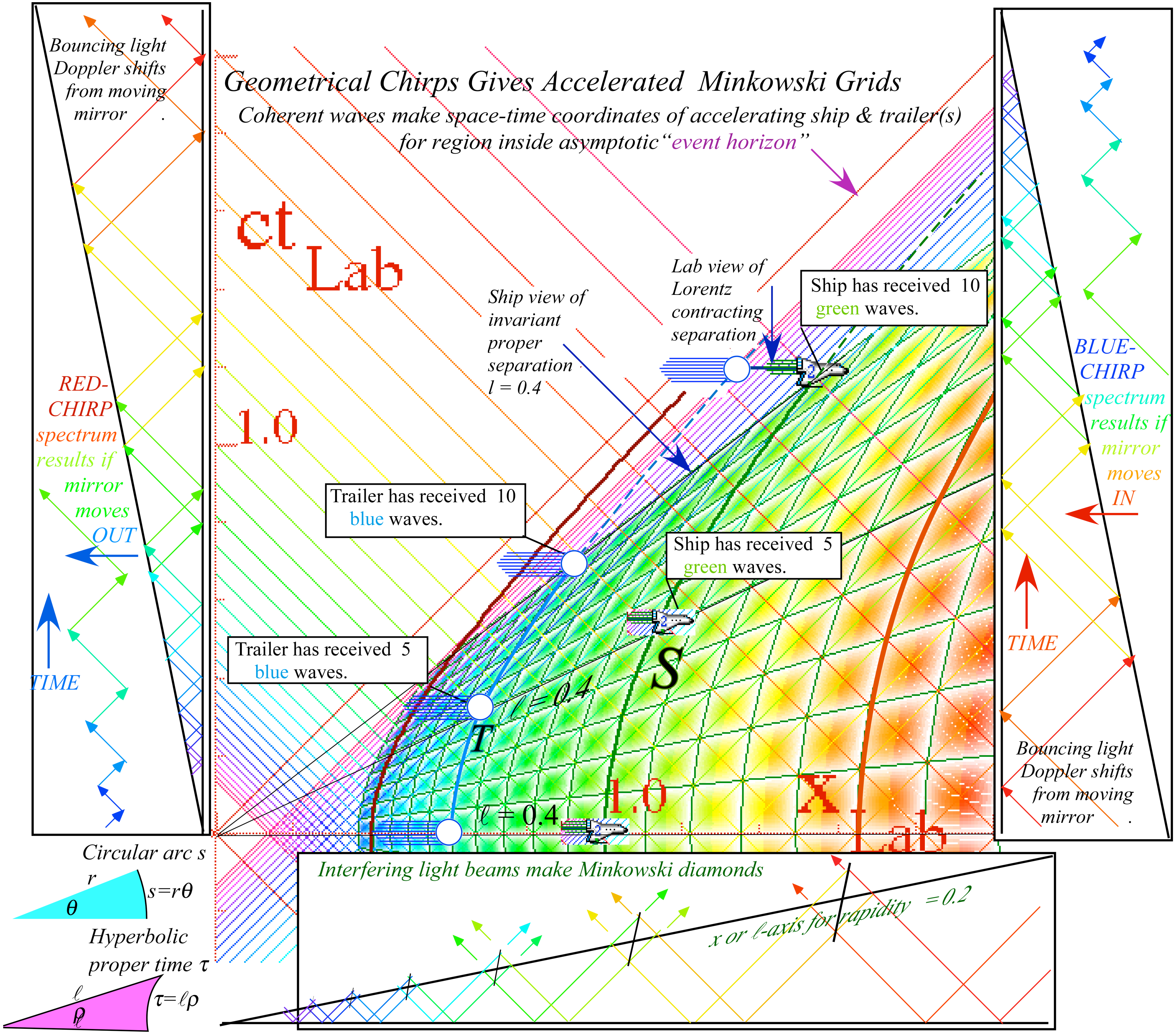


Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light