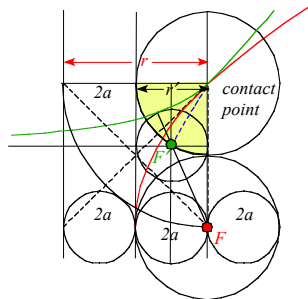


**Ex.1** Two burns

Space shuttle in circular orbit of radius  $R_1$  uses two burns to transit to circular orbit of radius  $R_n = nR_1$ .

- (a) Derive orbital parameters ( $a, b, \epsilon, \lambda$ ) involved in transit 1to2, 2to3, and direct transit 1to3. Derive and plot focal points, latus radius and minor radius of each transit orbit. (Are ellipses in sketch too fat? If so, plot correct paths.)
- (b) Find energy and angular momentum of each of five stages in terms of original energy  $E_1$  and momentum  $L_1$ .
- (c) How much must shuttle pilot increase? or decrease? velocity to enter orbit 1to3. How much?  $v_{13\text{perigee}}/v_1 = \underline{\hspace{2cm}}$
- (d) What velocity change must be done to finally enter orbit 3?  $v_3/v_{13\text{apogee}} = \underline{\hspace{2cm}}$



**Ex.2** Enveloping Rutherford's Coulomb orbits

This exercise mostly concerns  $\alpha^{++}$  orbits repelled by positively charged nucleus but also considers anti- $\alpha^-$  orbits attracted to the nucleus.

- (a) By ruler&compass geometry construct Rutherford orbits of radius  $a=1$  for impact parameter  $b=2a$  and  $b=3a$ . Show left-incoming and upper-right-outgoing asymptotes and scattering angles. Check angle geometry algebraically. Be sure to include axis lines containing atomic nuclear focal point **F**, orbit center **C**, and 2<sup>nd</sup> focus **F''** for each orbit. First find what  $b$ -value makes  $\alpha^{++}$  fly away at  $\Phi=90^\circ = \Theta$  to beam.

Hint: Geometry of forward scattering half-angle  $\Phi/2$  is convenient. Its center is nucleus point **F** at center of protractor graph paper.

- (b) Each repelling  $\alpha^{++}$  orbit has a symmetric attracting  $\alpha^-$  orbit on the other side. Draw their paths and asymptotes.
- (c) On a separate graph superimpose the repelling orbits of (a) and use their geometry to help derive the locus of the Rutherford scattering caustic that is the contact envelope of all repelling alpha paths for a given  $a$ . Describe how to locate a contact point for a given  $b$ -trajectory with the envelope for  $a=1$ . Could **F** ever be a contact point?

Hints: Envelope geometry uses "kite" structure of parabolic tangents. See Lect. 26 and 27.

Recall that contact point always shares a line with focal point of envelope and focal point of contacting curve.

- (c) Could Rutherford's alpha beams conceivably osculate (kiss) the nucleus? Could the contact envelope ever touch a nucleus and match its curvature at  $b=0$ ? Calculate minimum envelope Radius of Curvature  $R_{ofC} = \underline{\hspace{2cm}} a$ .
- (d) How many eV of energy does an  $\alpha^{+4}$  particle need to touch a  $Au^{+79}$  nucleus with an assumed radius  $r = 1fm$ ? Discuss whether Rutherford may have been able to accomplish this. ( $1eV = 1.6 \cdot 10^{-19} \text{Joule}$ . Gold has 79 protons while each  $\alpha^{++}$  has just two, each with charge  $1.6 \cdot 10^{-19} \text{Coulomb}$  equal in magnitude to that of the electron. Highest energy alpha emitter is Astatine-217 with  $7.067 \text{MeV}$ . Neglect electronic screening effects.)

