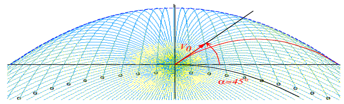
Assignment 5 - due Wed Sept. 26 - Mainly Chapters 9 - 12. "Families of orbits and their contacting envelopes."



The atoms of NIST or volcanoes of Io

1. Suppose one of the volcanoes on Jupter's moon *Io* detonates in a constant gravity-$g(m{\cdot}s^{-2})$ vacuum sending equi-velocity $\pm v_0(m{\cdot}s^{-1})$ fragments off at initial elevation angles $\alpha=0°, 15°, 30°, \ldots, 75°$, and $90°$ with the latter one going straight up to an altitude of $y=h_0=1$-*unit* in the attached graph and falling straight down.

   (a.) That one distance unit has what *mks*-value in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1})$? $h_0=$____________( ).

   (b.) Derive the parabolic time-coordinates $x(t)=$ ____________, $y(t)=$ ____________ in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1})$ and elevation angle α.

   (c.) Derive the parabolic focus-locus coordinates $x_{foc}=$ ____________, $y_{foc}=$ ____________ in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1}$ and elevation angle α) for $h_0=1$ and construct its curve on graph. (This curve has aspects of Thales geometry (subtended angle of circle diameter) that relate to trajectories. If you can show these below.)

   (d.) Derive the parabolic directrix coordinate $y_{dir}=$ ____________ in terms of $h_0=1$ and elevation angle α and construct this directrix line on graph for the cases $\alpha=0°$-$90°$ listed above.

   (e.) Give general parabolic trajectory curve function $y(x)=$ ________________ in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1})$ and α for $h_0=1$. For the cases $\alpha=0°, 30°, 45°$, and $90°$ construct enough of their curve points and tangents to accurately represent them on the graph.

   (f.) Locate the envelope contact points for the cases $\alpha=0°, 30°, 45°$, and $90°$ and construct enough of the envelope points and tangents to accurately represent the envelope on the graph. If a contact point lies off the graph indicate where. Deduce $y_{envelope}(x)=$ ________________ in terms of $h_0=1$.

   (g.) Each parabola trajectory has kite-like structure (Recall Fig. 9.4.) as does envelope. Draw and relate them.

   (h.) Do any of the α-trajectories have the same shape as the envelope? If so, tell which one.

2. Now consider time behavior implicit in problem 1. In a "snapshot" of each moment, volcano fragments lie on "blast-front" curve. A geometric time unit T_1 is the time for the $\alpha=90°$ fragment to reach its peak.

   (a.) That one time unit has what *mks*-value in terms of $g(m{\cdot}s^{-2})$ and $v_0(m{\cdot}s^{-1})$? $T_1=$______________( ).

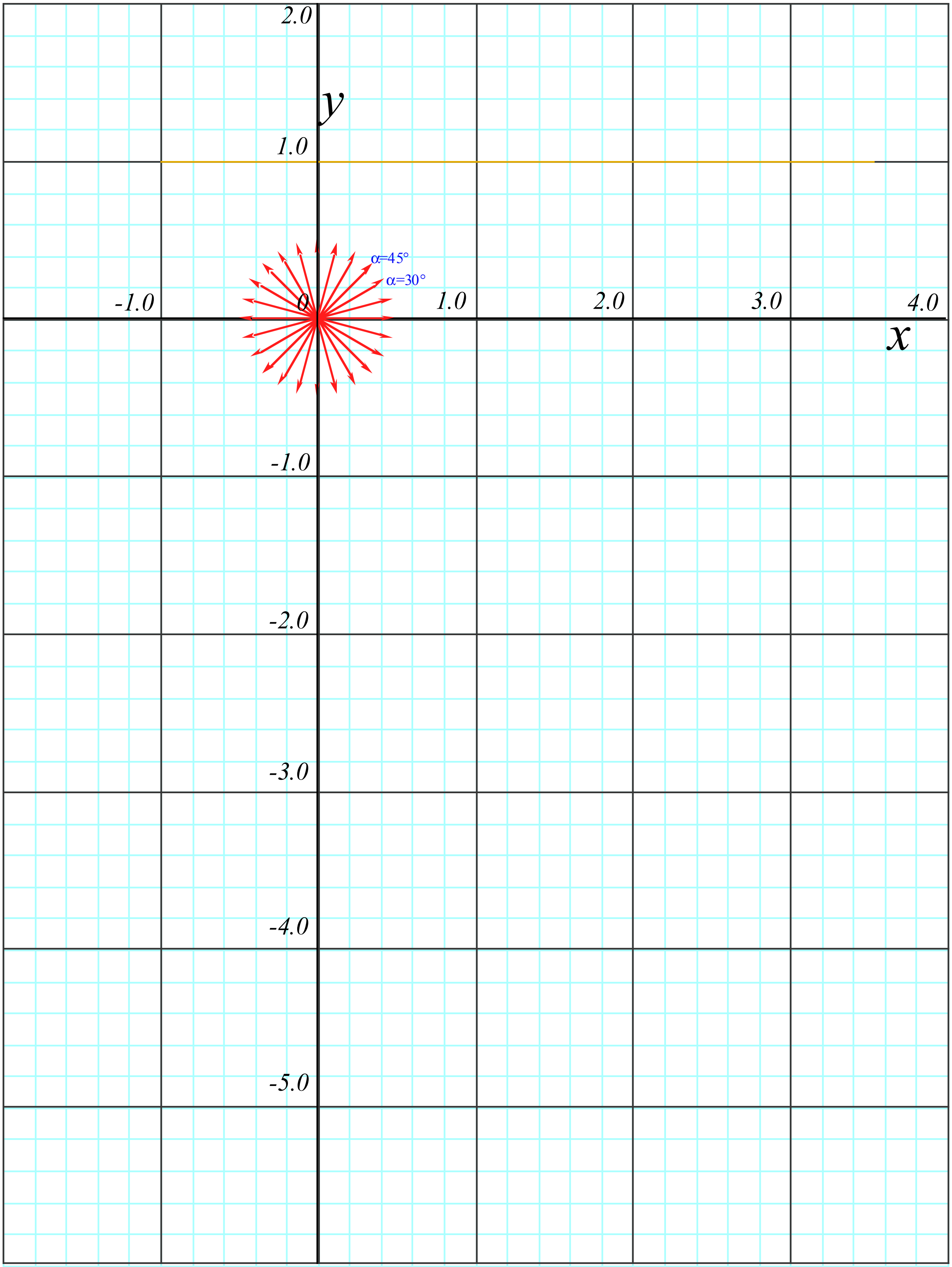
   (b.) Give a brief explanation addressing why this "snapshot" curve or locus has to be (whichever): a parabola? ___ straight line?___ circle?____ ellipse?___(Check one and explain choice on graph.)

   (c.) Derive and/or construct the "blast-front" curve for the case $\alpha=90°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{90°}=$____ Find polar angle of contact normal.

   (d.) Derive and/or construct the "blast-front" curve for the case $\alpha=45°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{45°}=$____ Find polar angle of contact normal.

   (e.) Derive and/or construct the "blast-front" curve for the case $\alpha=30°$ at the moment when that fragment first contacts volcano envelope. Give time in T_1 units. $T_{30°}=$____ Find polar angle of contact normal.

3. Suppose fragments continue falling into a tunnel through moon-*Io* that has radius $R_{Io}=0.5{\cdot}10^6\ h_0$. Estimate radius of tunnel at widest point if it just big enough to let all fragments orbit without hitting its walls. $R_{tunnel}=$_____(h_0)
   Note: For this problem the gravity is not uniform constant $g=9.8ms^{-2}$ except near surface. (Ellipse geometry.)


2.0
y
1.0
α=45°
α=30°
-1.0
0
1.0
2.0
3.0
4.0
x
-1.0
-2.0
-3.0
-4.0
-5.0