Assignment 8 Oct 10 , 2018 Due Wednesday Oct 17: Based on Unit 2 Chapter 1-3 and Unit 3 Chapter 1-3.

Well-known Coordinates (OCC) *NOTE: Save copy of solution to this Ex.***1**(b) *for next Assignment 9.*

1. Find Jacobian, Kajobian, $\mathbf{E}_m$, $\mathbf{E}^m$, metric tensors g_{mn} and g^{mn} for OCC (a) and (b). (*You may do (b) then reduce to (a).*)

(a) Cylindrical coordinates $\{q^1=\rho, q^2=\phi, q^3=z\}$: $x=x^1=\rho\cos\phi$, $y=x^2=\rho\sin\phi$, $z=x^3$.

(b) Spherical coordinates: $\{q^1=r, q^2=\theta, q^3=\phi\}$: $x=x^1=r\sin\theta\cos\phi$, $y=x^2=r\sin\theta\sin\phi$, $z=x^3=r\cos\theta$.

"Plopped" Parabolic Coordinates (GCC) *(In attached figure)*

2. Consider the GCC(Cartesian) definition: $q^1 = (x)^2 + y \quad q^2 = (y)^2 - x$

(a) Does an analytic Cartesian coordinate definition $x^j = x^j(q^m)$ exist? If so, show.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

(c) On the appropriate graph on attached pages sketch the unitary vectors at the point $(x=1, y=1)$ (Arrow) and at the point $(x=1, y=0)$. Where, if anywhere, is the grid an OCC however briefly? Indicate loci on graph.

(d) Find and indicate where, if anywhere, are there Jacobian or Kajobian singularities of this GCC. Show on graph.

"Sliding" Parabolic Coordinates (GCC) *(In attached figure)*

3. Consider the Cartesian(GCC) definition: $x = 0.4\,(q^1)^2 - q^2\,, \quad y = q^1 - 0.4\,(q^2)^2$

(a) Does an analytic GCC coordinate definition $q^m = q^m(x^j)$ exist? If so, show.

(b) Derive the Jacobian, Kajobian, unitary vectors $\mathbf{E}_m$, $\mathbf{E}^m$, and metric tensors for this GCC.

(c) On the appropriate graph on attached pages sketch the unitary vectors near point $(x=1, y=1)$ (Arrow) and near point $(x=1, y=0)$. Where, if anywhere, is the grid an OCC however briefly? Indicate loci on graph.

(d) Find and indicate where, if anywhere, are there Jacobian or Kajobian singularities of this GCC. Show on graph.

4. *Covariant vs Contravariant Geometry* *(In attached figure)*

GCC components of a vector **V** in attached figure are realized by line segments OA, BV, etc. Give each segment length by single terms of the form V_m or V^m times $(\sqrt{g_{mm}})^{+1}$, $(\sqrt{g_{mm}})^{-1}$, $(\sqrt{g^{mm}})^{+1}$, or $(\sqrt{g^{mm}})^{-1}$ with the correct $m=1$ or 2. Also label each unitary vector as $\mathbf{E}_1$, $\mathbf{E}^1$, $\mathbf{E}_2$, or $\mathbf{E}^2$, whichever it is.

You should be able to do this quickly without looking at the text figures.

"Unprofessional" Paraboloidal Coordinates (GCC) *(In attached figure)*

5. The surface $z = f(x,y) = \frac{1}{2}x^2 + y^2$ (See *xyz*-plot) introduces 3D partial derivative chain rules. It is the ($q^3=0$)-surface in a 3D GCC coordinate grid $q^1=x,\ q^2=y,\ q^3=\frac{1}{2}x^2+y^2-z$. It contains a projection of an orthogonal (x,y) Cartesian coordinate grid on the surface that is obviously *not* orthogonal most places.

a. Derive the *3-by-3* Jacobian $J(x,y,z)$ and Kajobian $K(x,y,z)$ for ($q^3=0$).

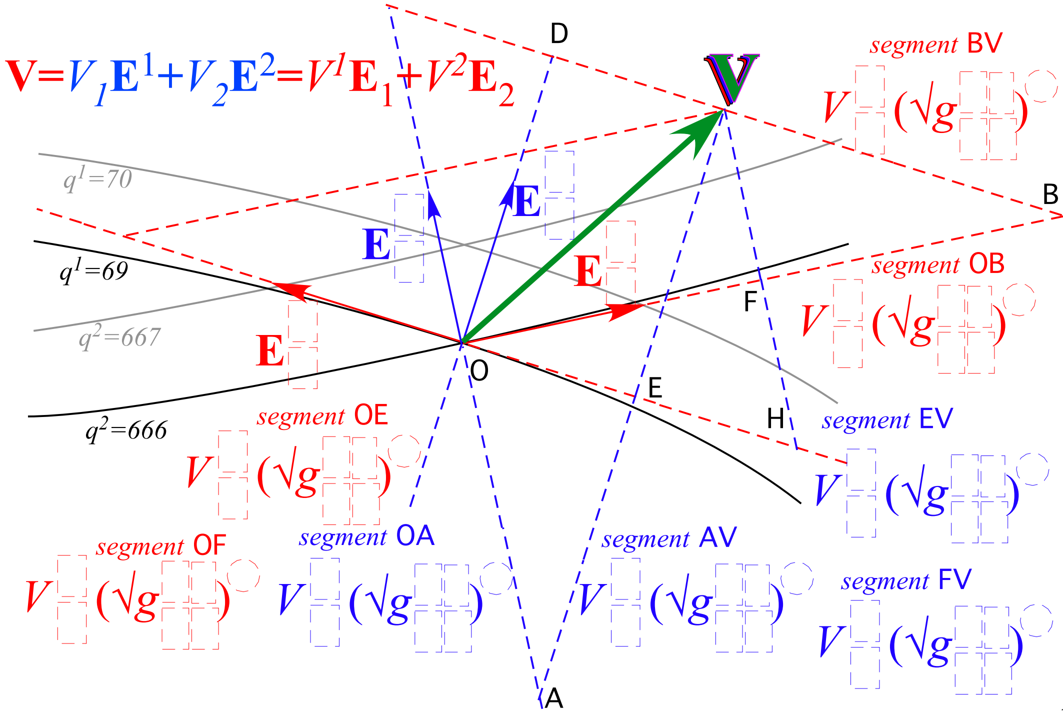
b. Extract covariant $\{\mathbf{E}_1,\mathbf{E}_2,\mathbf{E}_3\}$ and contravariant $\{\mathbf{E}^1,\mathbf{E}^2,\mathbf{E}^3\}$ vectors represented in Cartesian (x,y,z) basis.

c. Derive the *3-by-3* covariant metric $g_{\upsilon\nu}(x,y)$ and contravariant metric $g^{\upsilon\nu}(x,y)$ for ($q^3=0$) and tell which if any points on the surface have grids that are locally *orthogonal* and which if any are locally *orthonormal*.

*(Larger graph provided separately for Ex.***5***d and Ex.***5***e.*

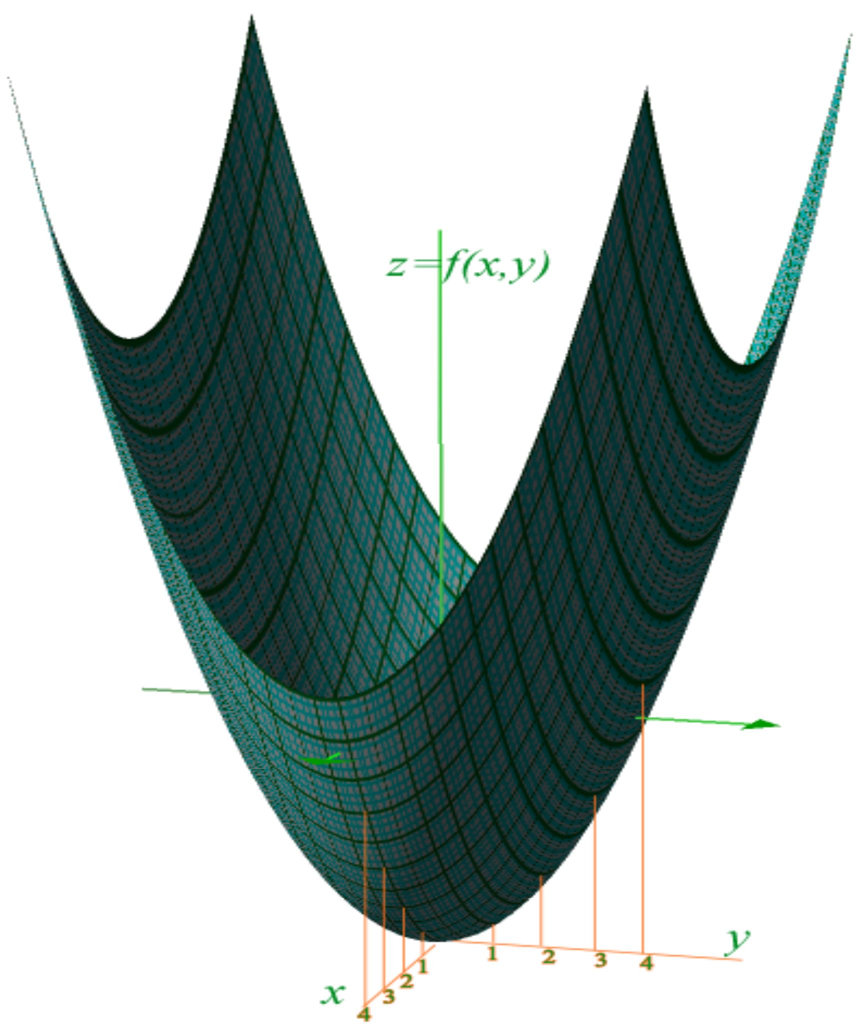
d. Calculate and sketch covariant $\{\mathbf{E}_1,\mathbf{E}_2,\mathbf{E}_3\}$ on ($q^3=0$) surface where $(x=4,y=-2)$ and where $(x=3,y=+2)$.

e. Calculate and sketch contravariant $\{\mathbf{E}^1,\mathbf{E}^2,\mathbf{E}^3\}$ on ($q^3=0$) surface where $(x=4,y=+2)$ and where $(x=0,y=+4)$.

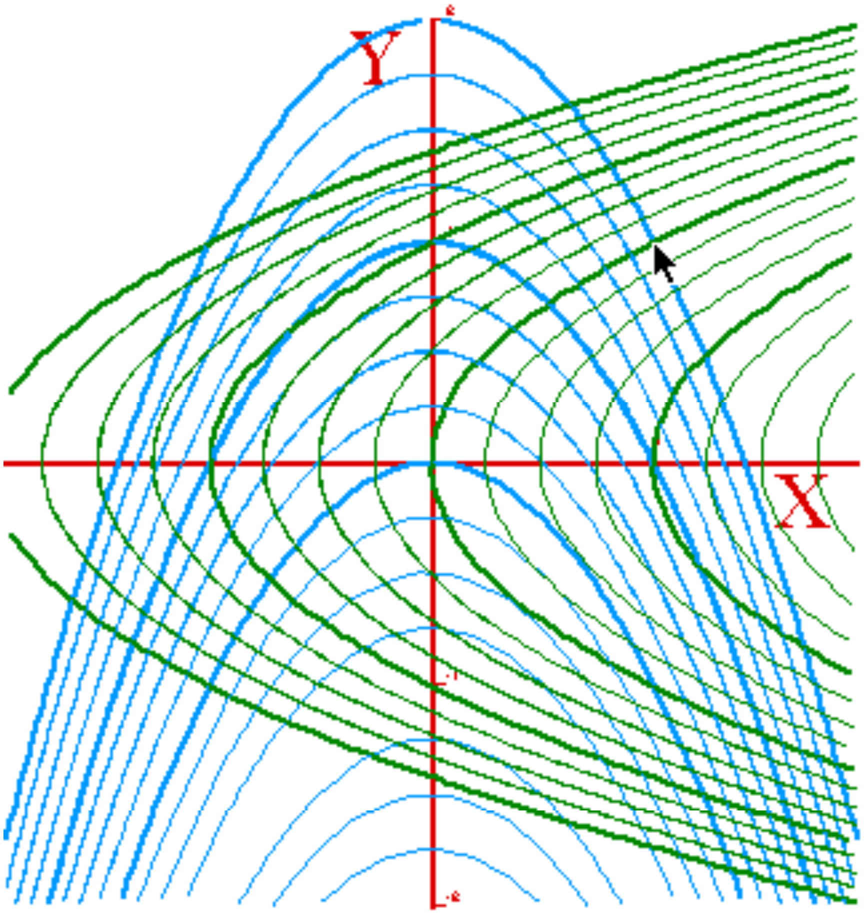


Above is 2D plot for Ex. **4**

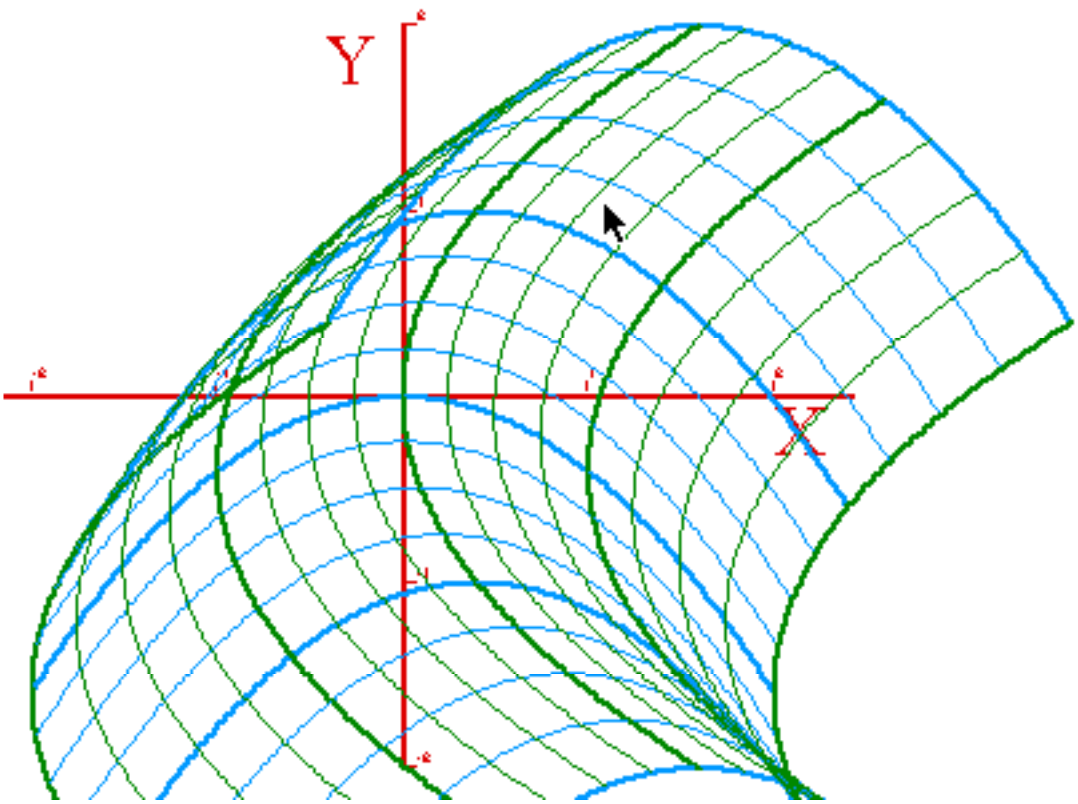
*Below is 3D (xyz) plot for Ex.***5**.



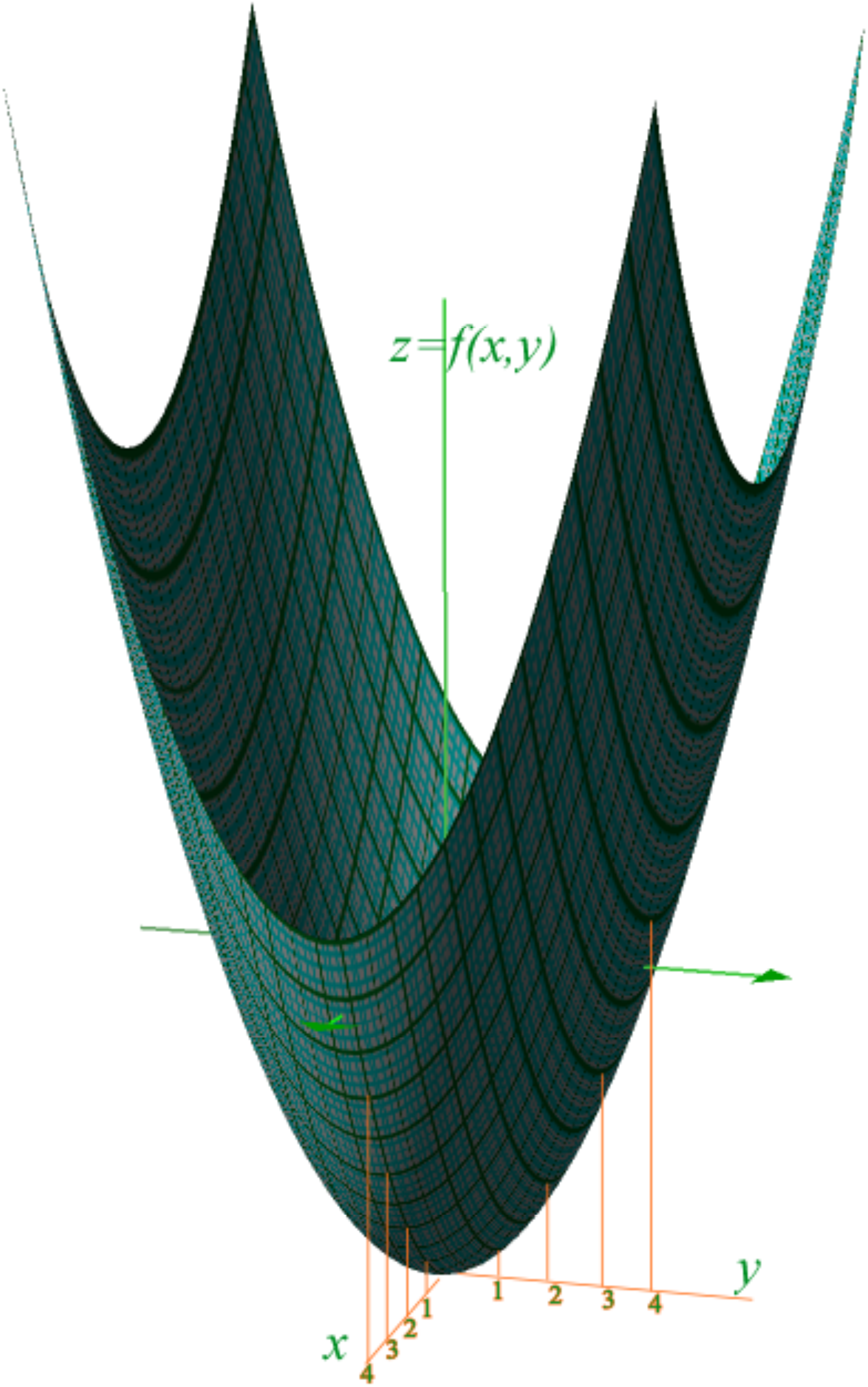
*"Plopped" and"Sliding" Parabolic Coordinates are 2D (xy) plots for Ex.***2** *and Ex.***3** *(despite 3D appearance of latter.)*



*"Plopped" Parabolic Coordinates for Ex.***2**



*"Sliding" Parabolic Coordinates for Ex.***3**



Assignment 8(contd.) - Ex. **5**
"Unprofessional" Paraboloidal Coordinates