

Lecture 1

Mon. 8.20.2018

1st axioms and theorems of classical mechanics

(Ch. 1 thru Ch. 3 of Unit 1)

A preface comment: Text by Eric Heller (for Semester II of Advanced Mechanics)

Geometry of momentum conservation axiom (aka Occam's Razor)

Totally Inelastic "ka-runch" collisions (begin 4:1 graph project)*

*Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**

+Intro to weighty-averages and vector notation

Comments on idealization in classical models

Geometry of Galilean translation symmetry

45° shift in (V_1, V_2) -space

Time reversal symmetry

...of COM collisions

Algebra, Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

Matrix or tensor algebra of collisions

Deriving Energy Conservation Theorem

Numerical details of collision tensor algebra

After-class
Bull-session

$\frac{2}{3}$ of a PhD is Ph
that is Philosophy

Should physicists
worry about *philosophy*?
...about *economics*?
...about *current events*?
...about *politics*?...

Note - Many of the underlined phrases throughout these lecture files link to Simulations of those specific, selected cases viewed within your default Web browser

2018 UA PHYS 5103 Advanced Mechanics Class web site

*Launch Generic Car Collision Web Simulator

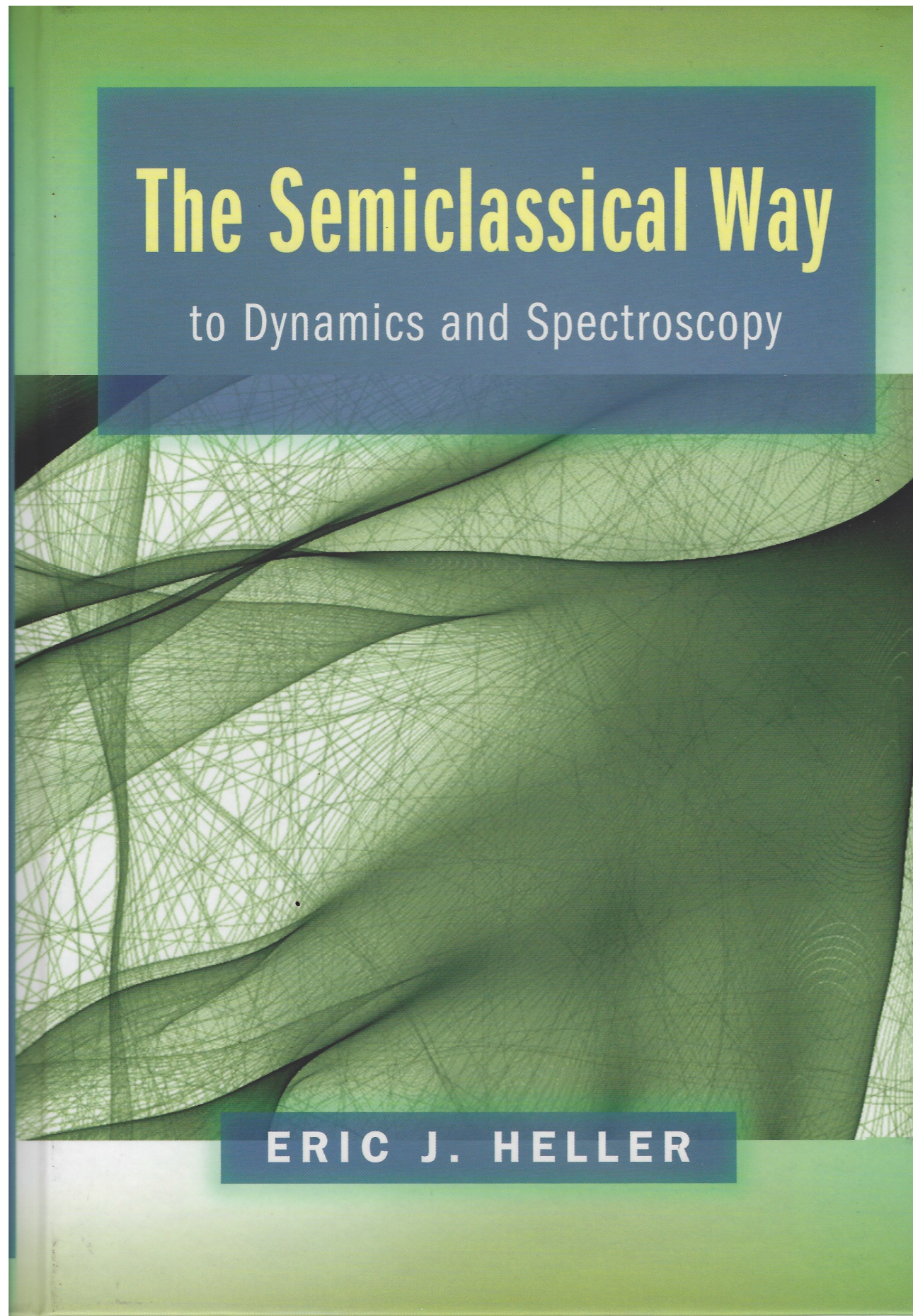
*Launch Generic Superball Collision Web Simulator

https://modphys.hosted.uark.edu/markup/CMwBang_TitlePage.html

<https://modphys.hosted.uark.edu/markup/CMMotionWeb.html>

<https://modphys.hosted.uark.edu/markup/BounceItWeb.html>

A preface comment: [Text by Rick Heller](#) that would be used for Semester II of Advanced Mechanics



Expect Semester I to also reference gems of theory from this excellent text.

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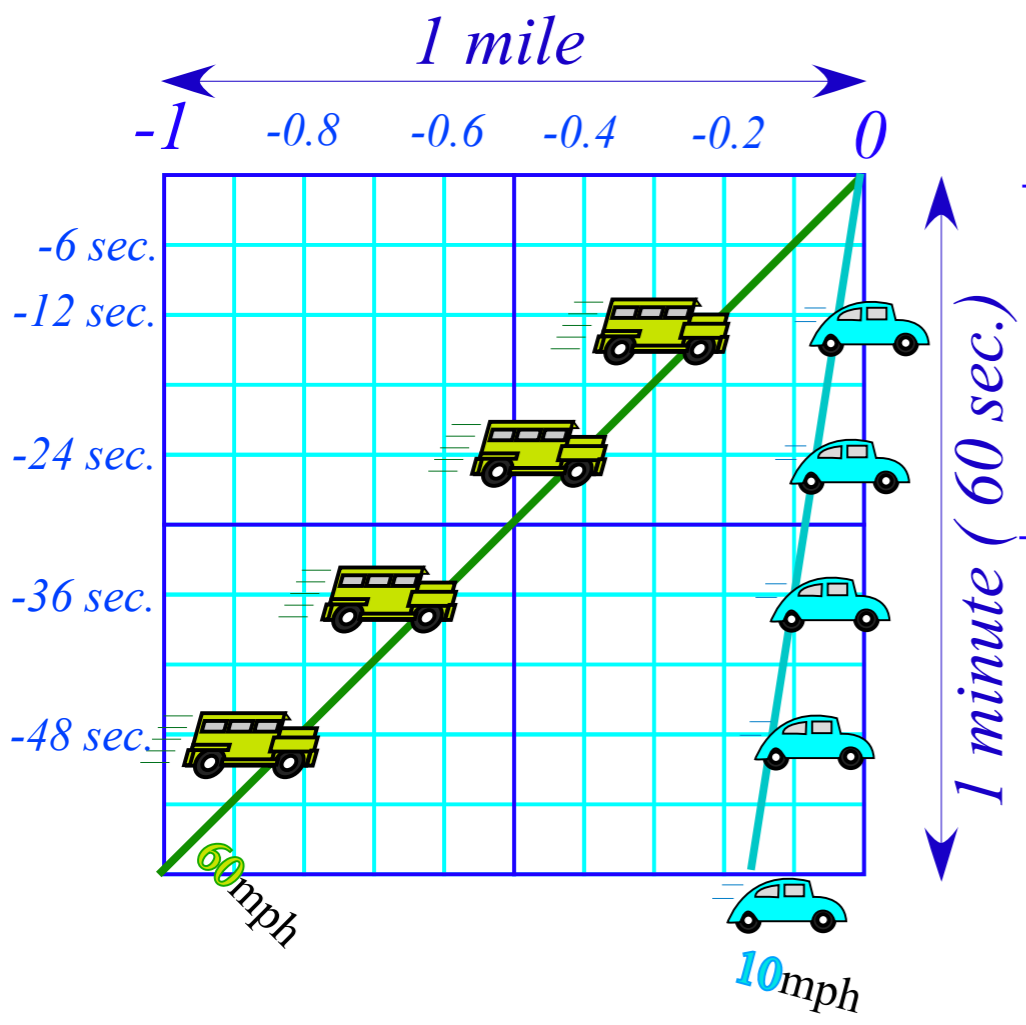
[Destination, rates & speeds](#)

 **Add to basket**

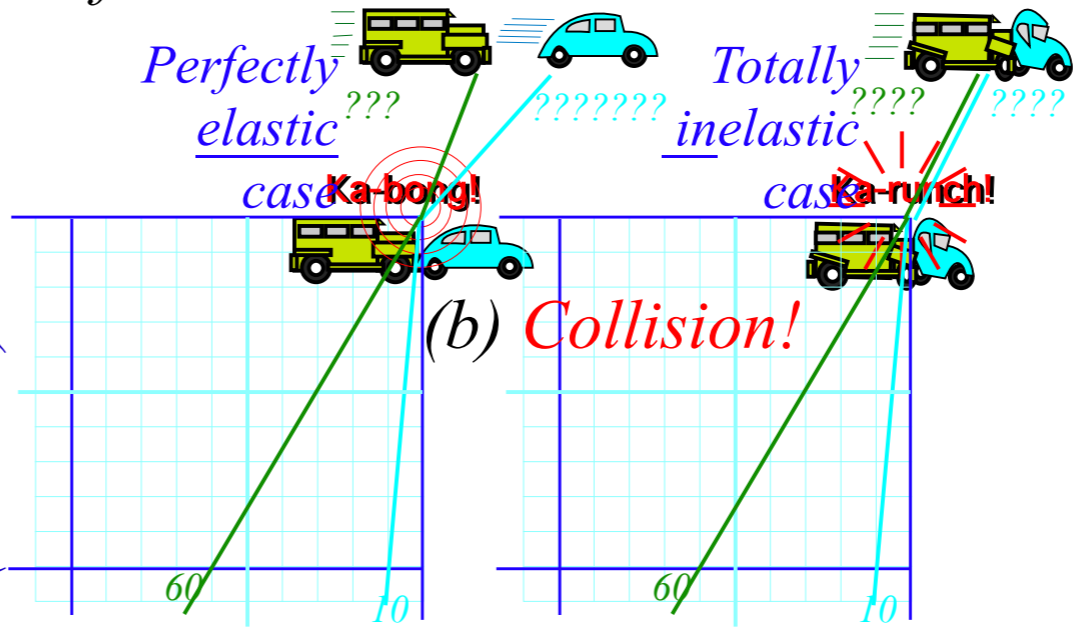
 **30 Day Return Policy**

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision.....



After collision...what velocities?



Car Simulator links*

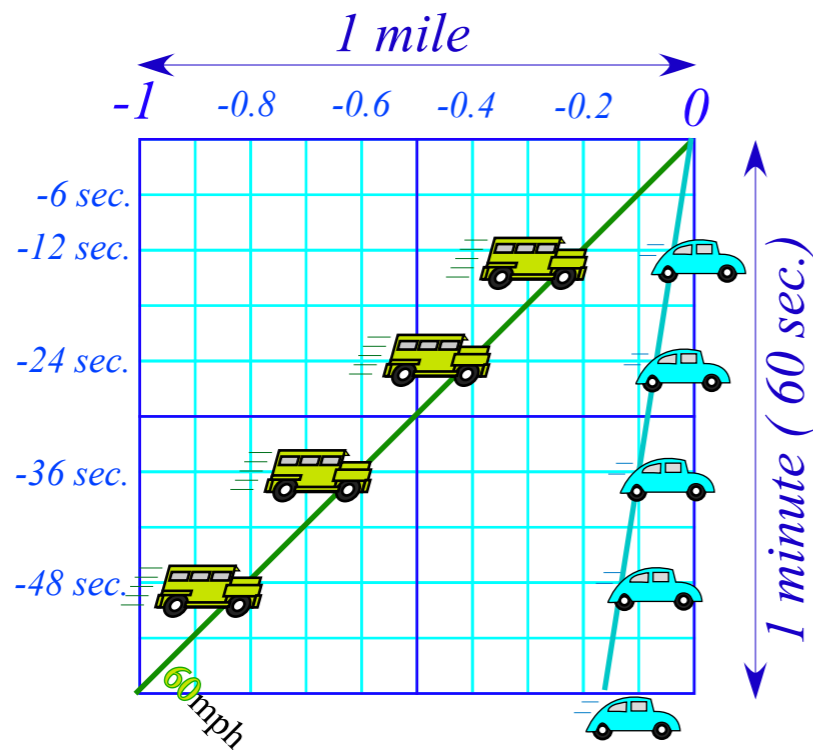
- | | |
|--|--|
| <u>Space vs Space Elastic</u> | <u>Space vs Space Inelastic</u> |
| <u>Elastic Collision Dual Panel Space vs Space and Space vs Time (Newton)</u> | <u>Inelastic Collision Dual Panel Space vs Space and Space vs Time (Newton)</u> |
| <u>Elastic Collision Dual Panel Space vs Space and Time vs. Space(Minkowski)</u> | <u>Inelastic Collision Dual Panel Space vs Space and Time vs. Space(Minkowski)</u> |

Our web apps (and course site) are *FREE*, and available *world wide* through most connected modern web browsers!!
 The links can be copy and pasted and emailed, so with a simple click one has a whole application suite at your fingertips - One *full* of *Physics*
Underlines usually denote active web links

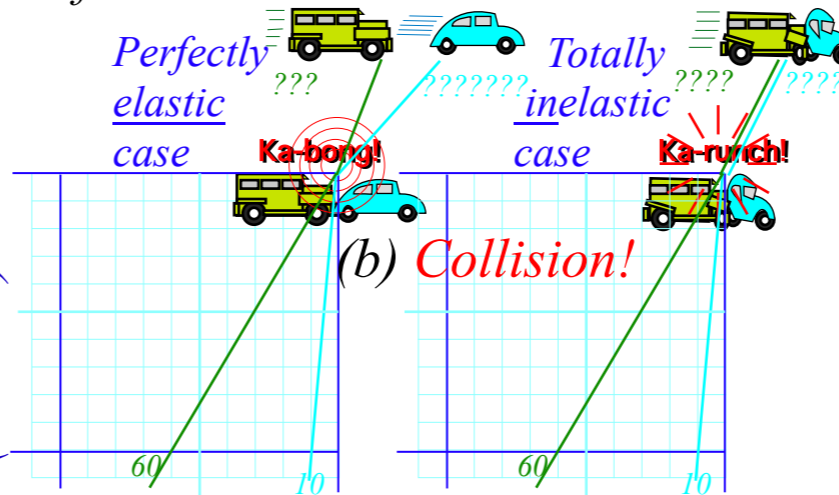
Modern syntax: <https://modphys.hosted.uark.edu/markup/CMMotionWeb.html>
 Older Cavern links are to redirects: <http://www.uark.edu/ua/modphys/markup/CMMotionWeb.html>
 Links to "Testing Area": <http://www.uark.edu/ua/modphys/testing/markup/CMMotionWeb.html>

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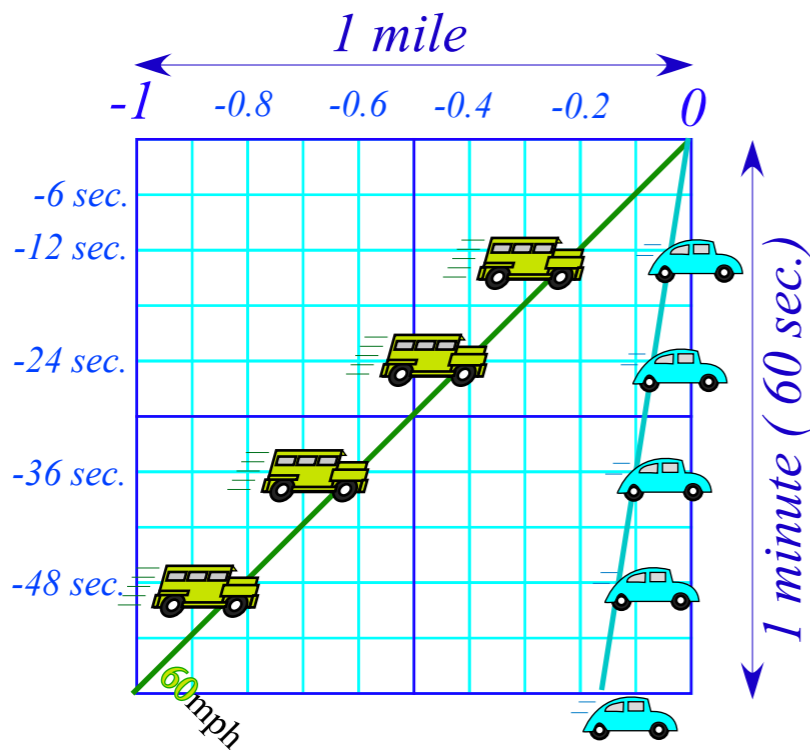
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 $\sum_i m V_i(\text{initial}) = \sum_i m V_i(\text{final})$
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 and solve...

Let's see if we can solve this *easily* with just *one* (or one-and-a-half) axiom(s)

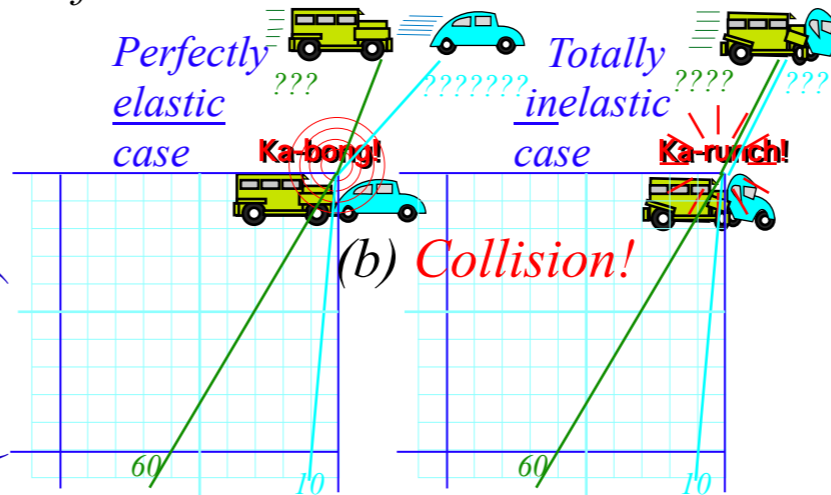
Axiom-1: All mass or masses keep their total momentum until it is changed by some outsider.

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After collision...what velocities?



V_{SUV} and V_{VW} change violently
but NOT **total momentum**
 $P_{Total} = M_{SUV} V_{SUV} + M_{VW} V_{VW}$

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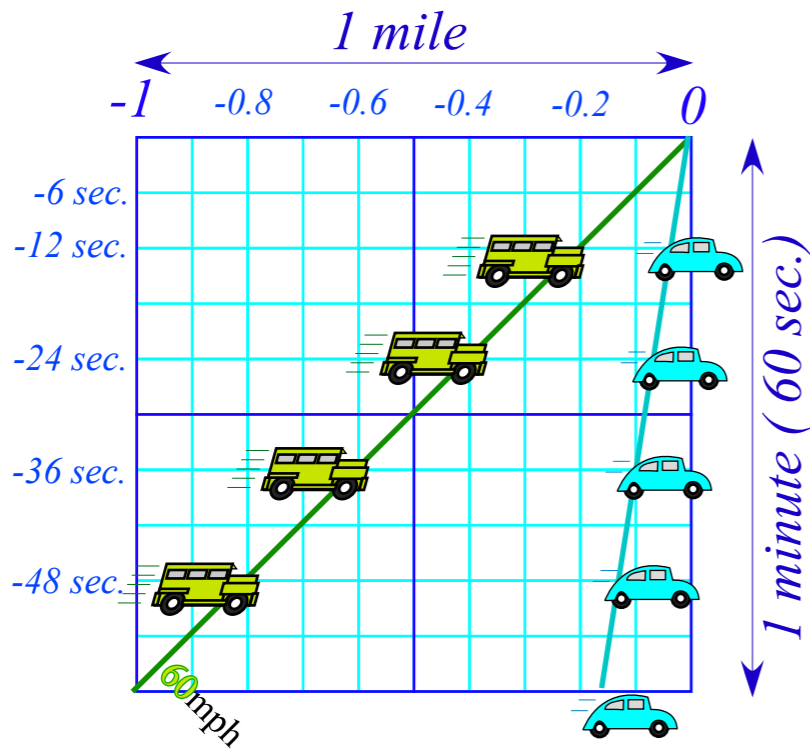
...But an UNconventional way is quicker and slicker....
 (Just have to draw 2 lines!)

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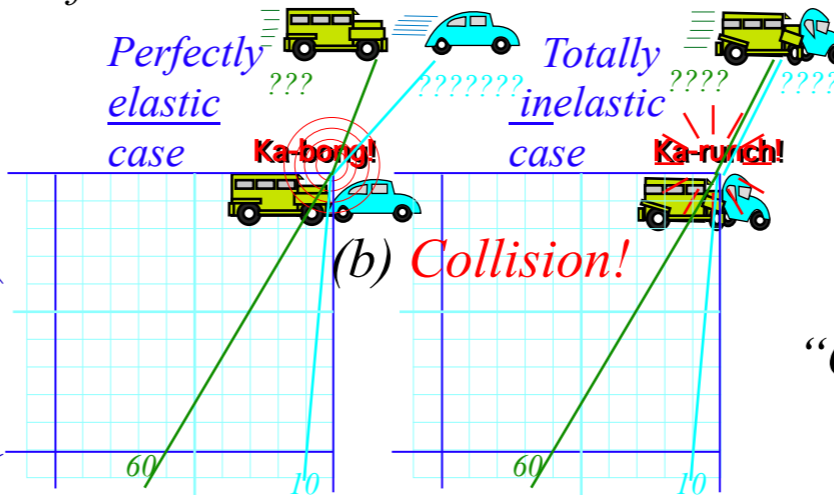
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Inventor of
 "Occam's Razor"



William of Ockham
 1285-1349

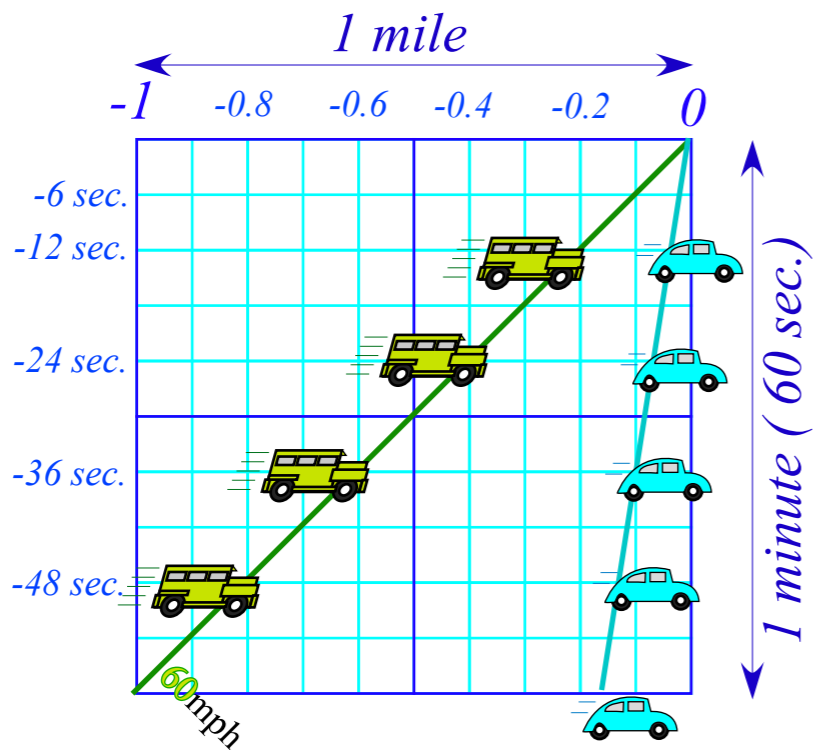
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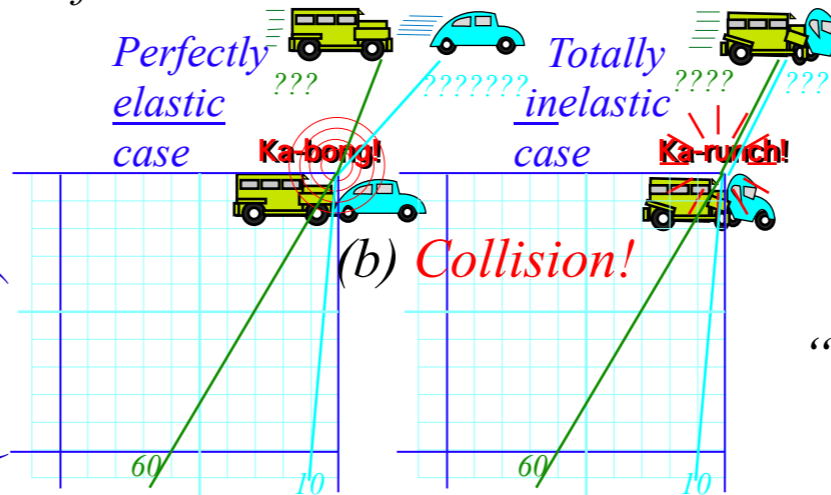
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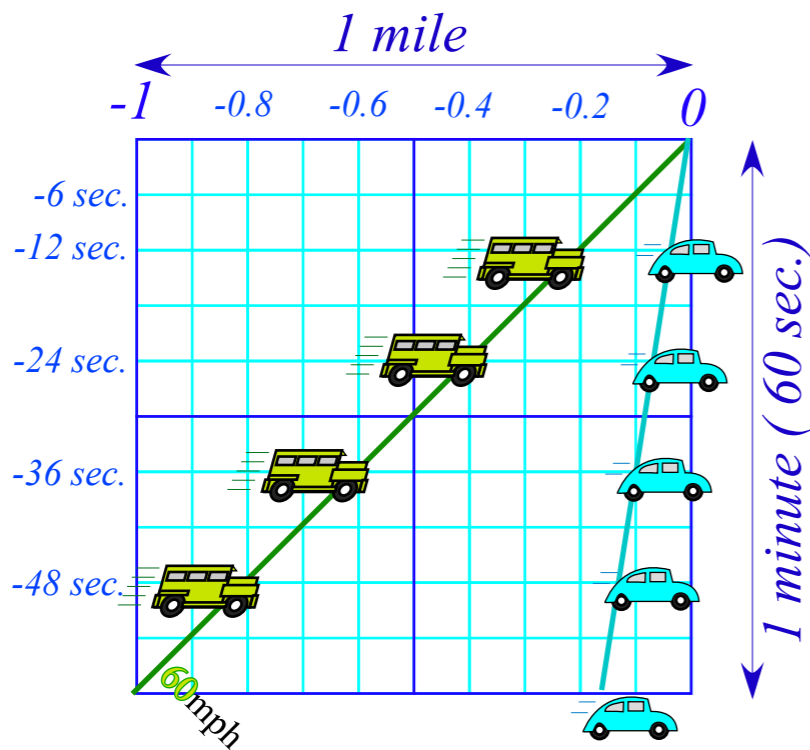
"*Pluralitas non set ponedda sine necessitate.*"

and has a number of interpretations:

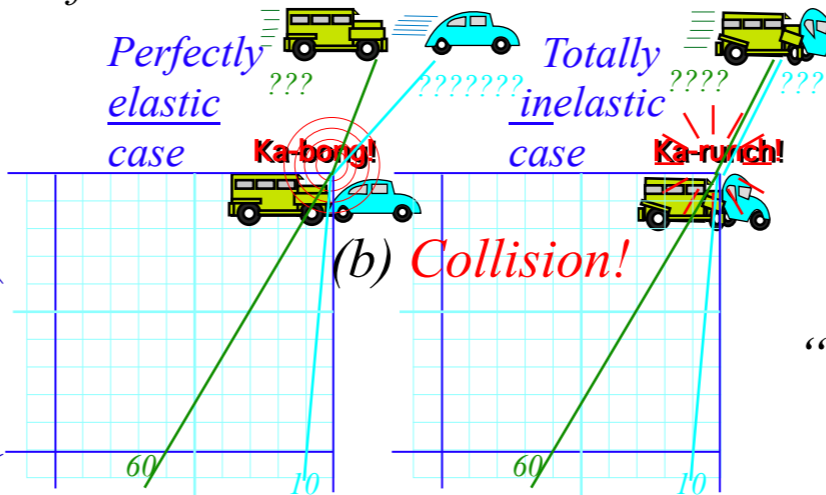
1. Literally: "Don't make pluralities of conjectures without necessity."
2. Logically: "Assume less to prove more."
3. Practical coding advice: "Keep it simple, make it powerful."

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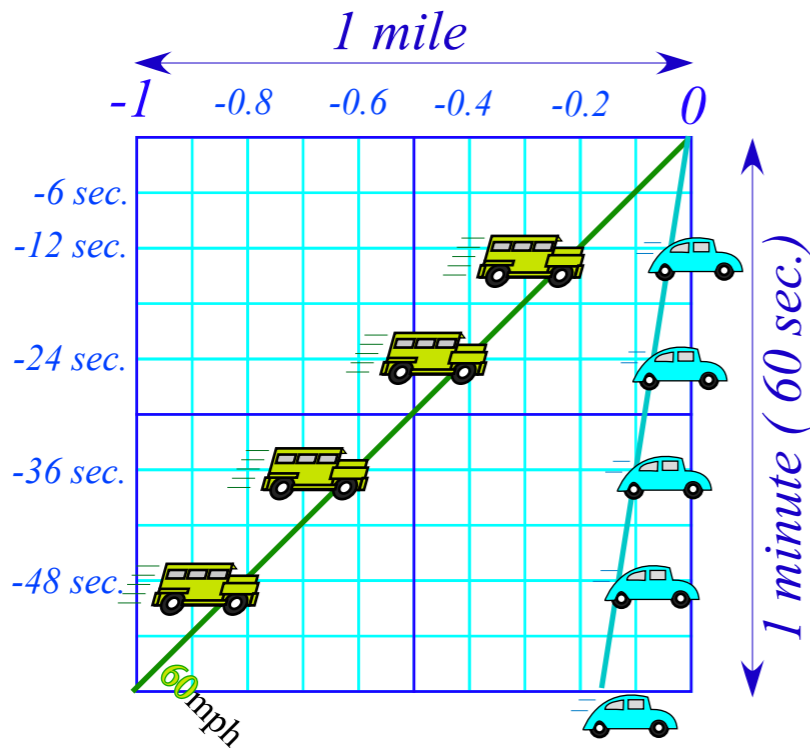
BEFORE: (INITIAL or IN)

AFTER: (FINAL or FIN)

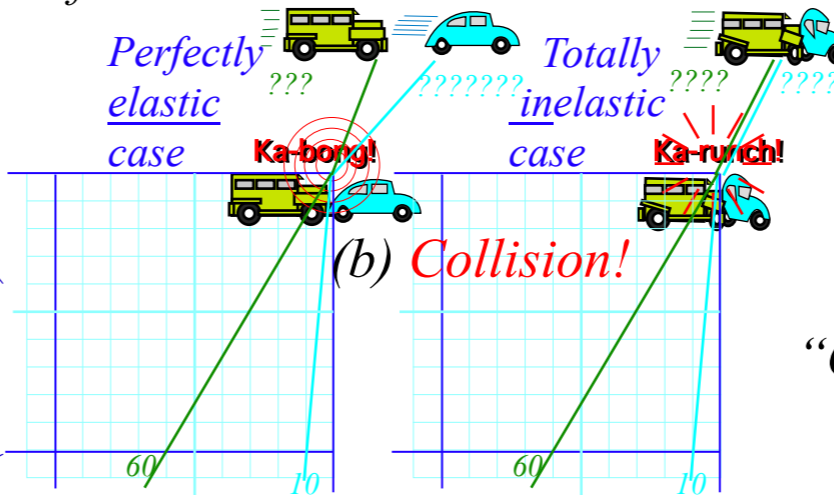
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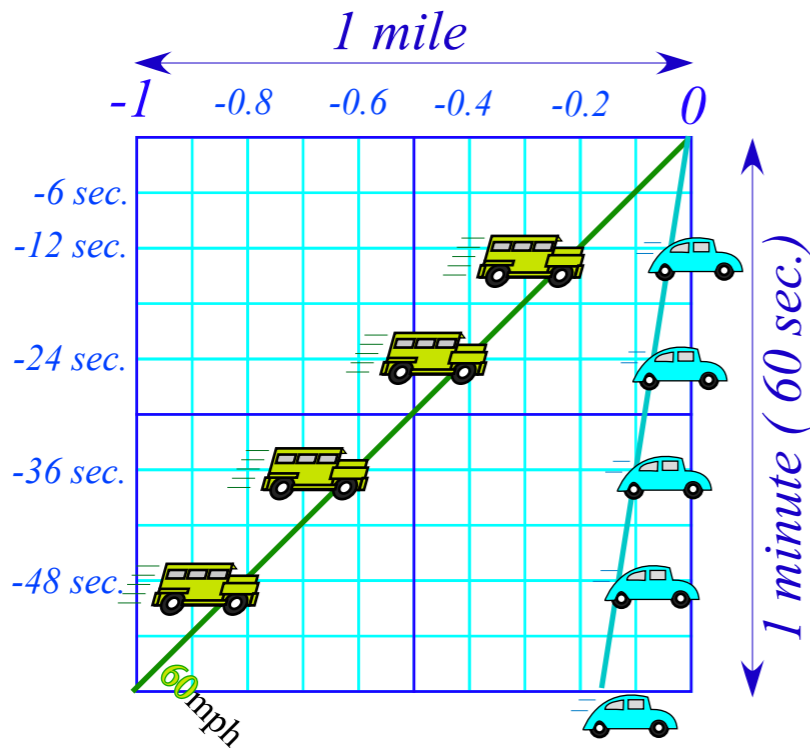
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$$4 \cdot 60 + 1 \cdot 10 = 4 \cdot ? + 1 \cdot ?? = 250$$

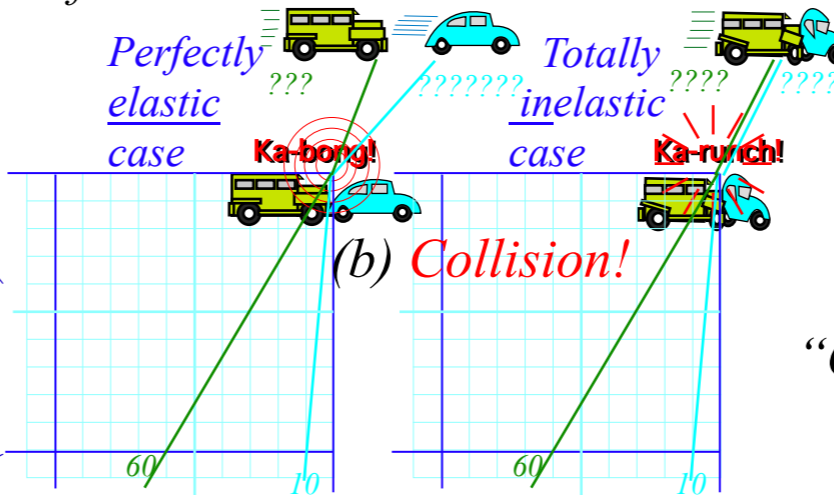
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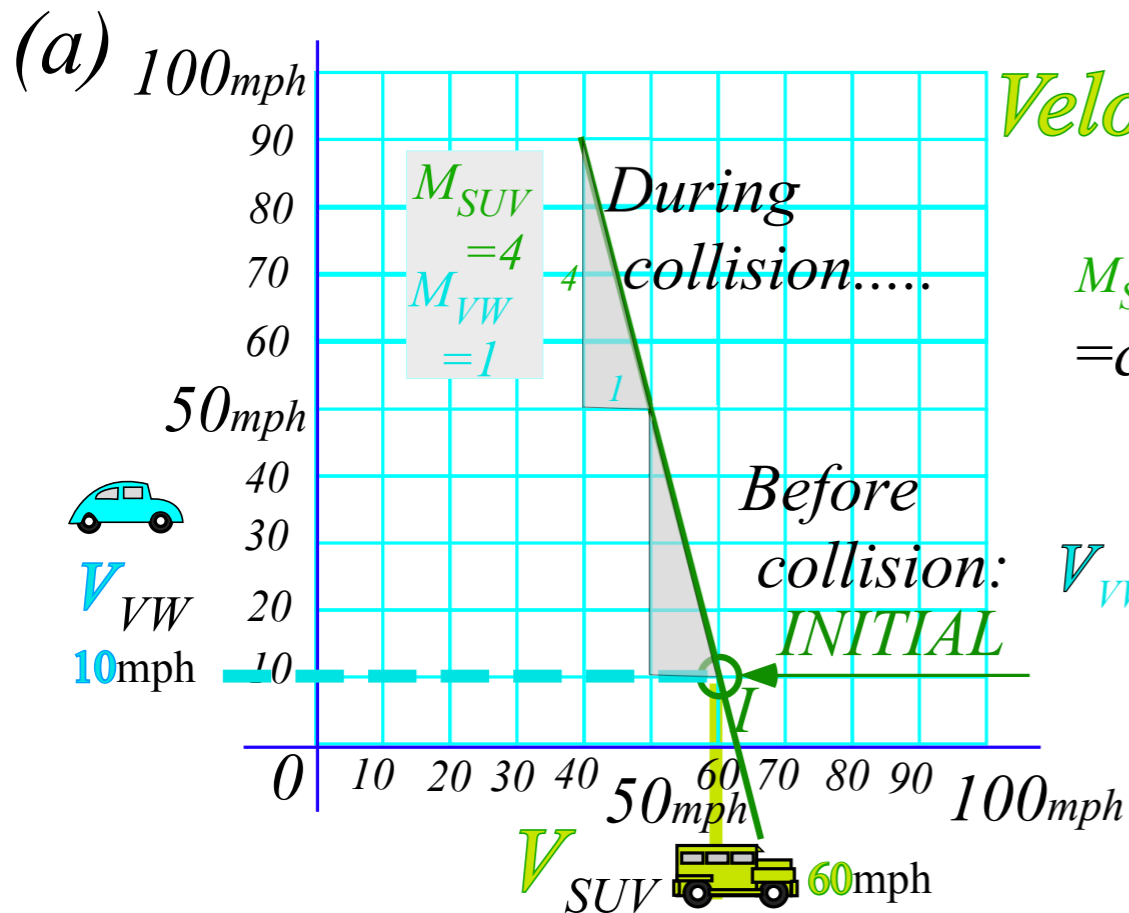
$$4 \cdot 60 + 1 \cdot 10 = 4 \cdot ? + 1 \cdot ?? = 250$$

It's a simple Cartesian equation

$$4 \cdot x + 1 \cdot y = 250$$



Rene Descartes
1596-1650

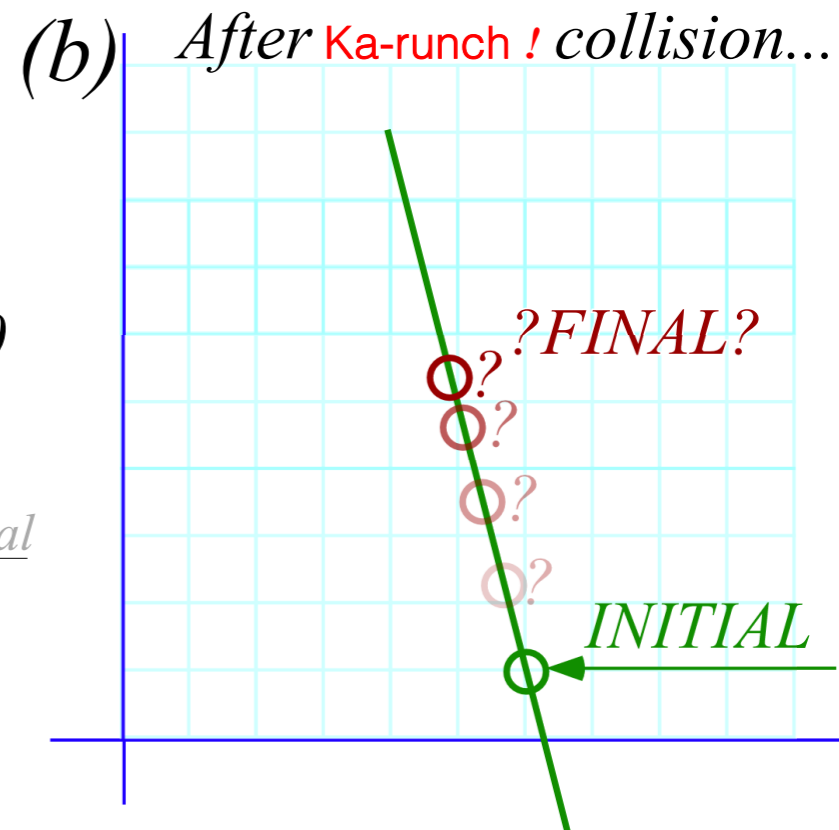


Velocity-velocity plot of Axiom-1:

$$M_{SUV}V_{SUV} + M_{VW}V_{VW} = \text{constant} = P_{Total} = 250$$

$$V_{VW} = -\frac{M_{SUV}}{M_{VW}}V_{SUV} + \frac{P_{Total}}{M_{VW}}$$

$$= -4V_{SUV} + 250$$



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Geometry of momentum conservation axiom

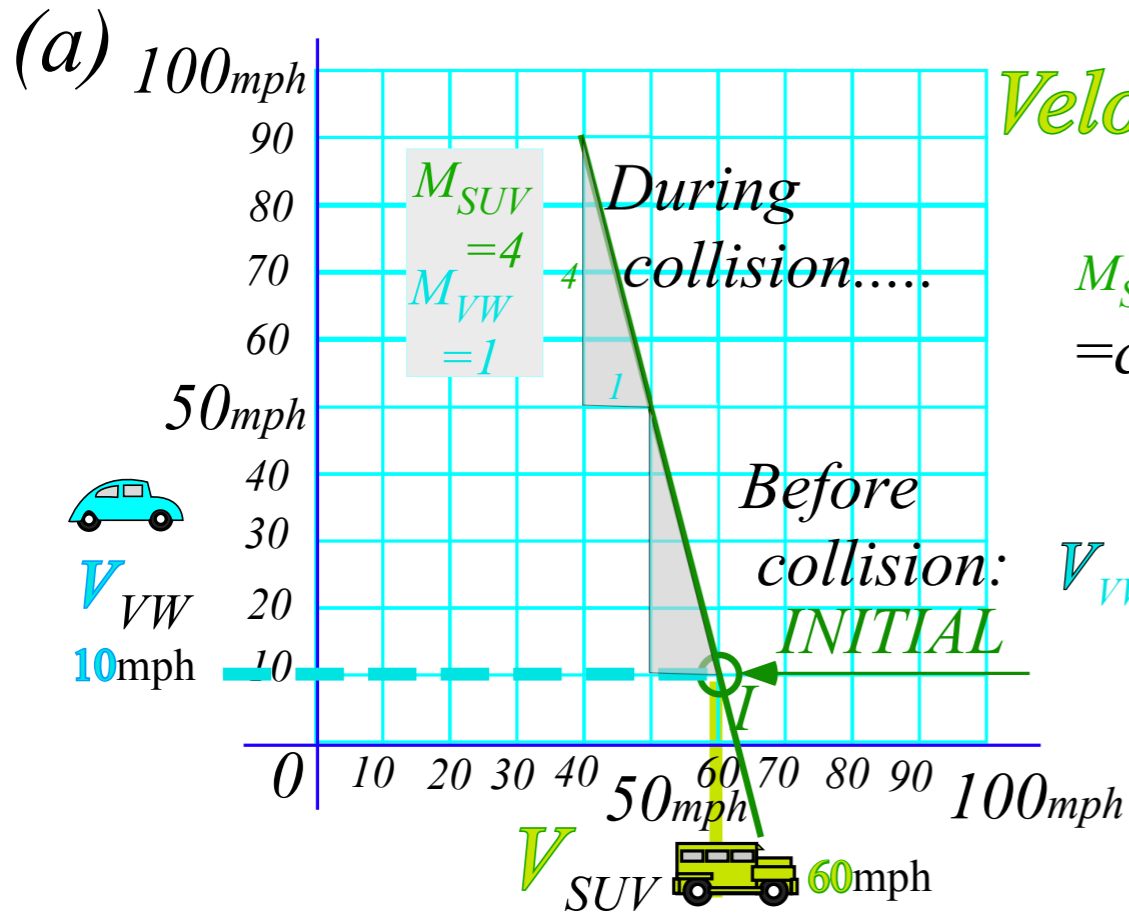


Totally Inelastic “ka-runch” collisions

Perfectly Elastic “ka-bong” and Center Of Momentum (COM) symmetry

+Intro to weighted-averages and vector notation

Comments on idealization in classical models

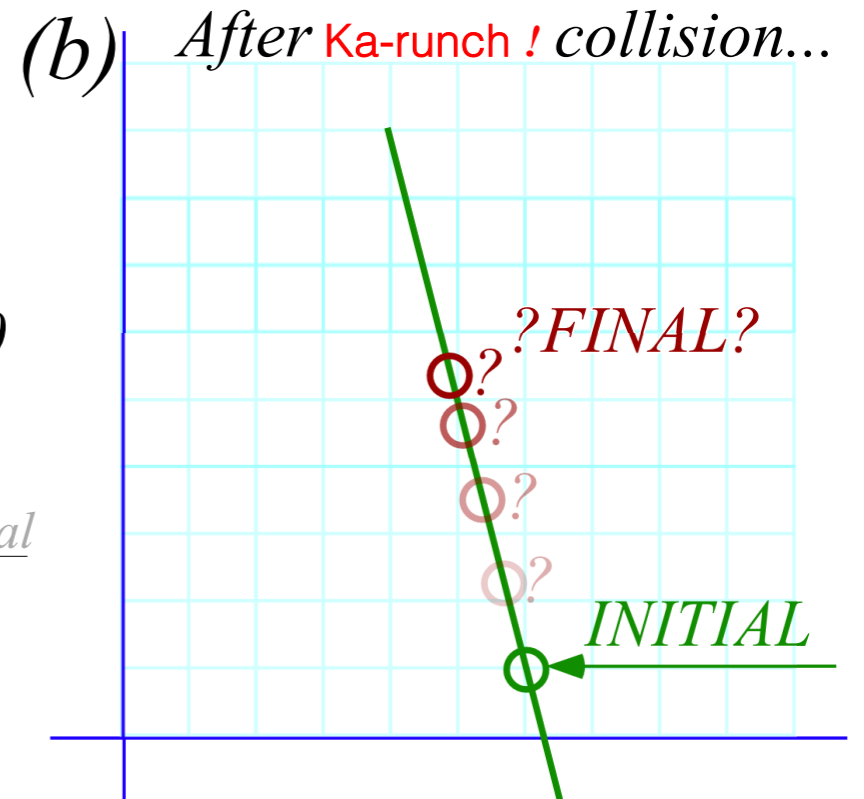


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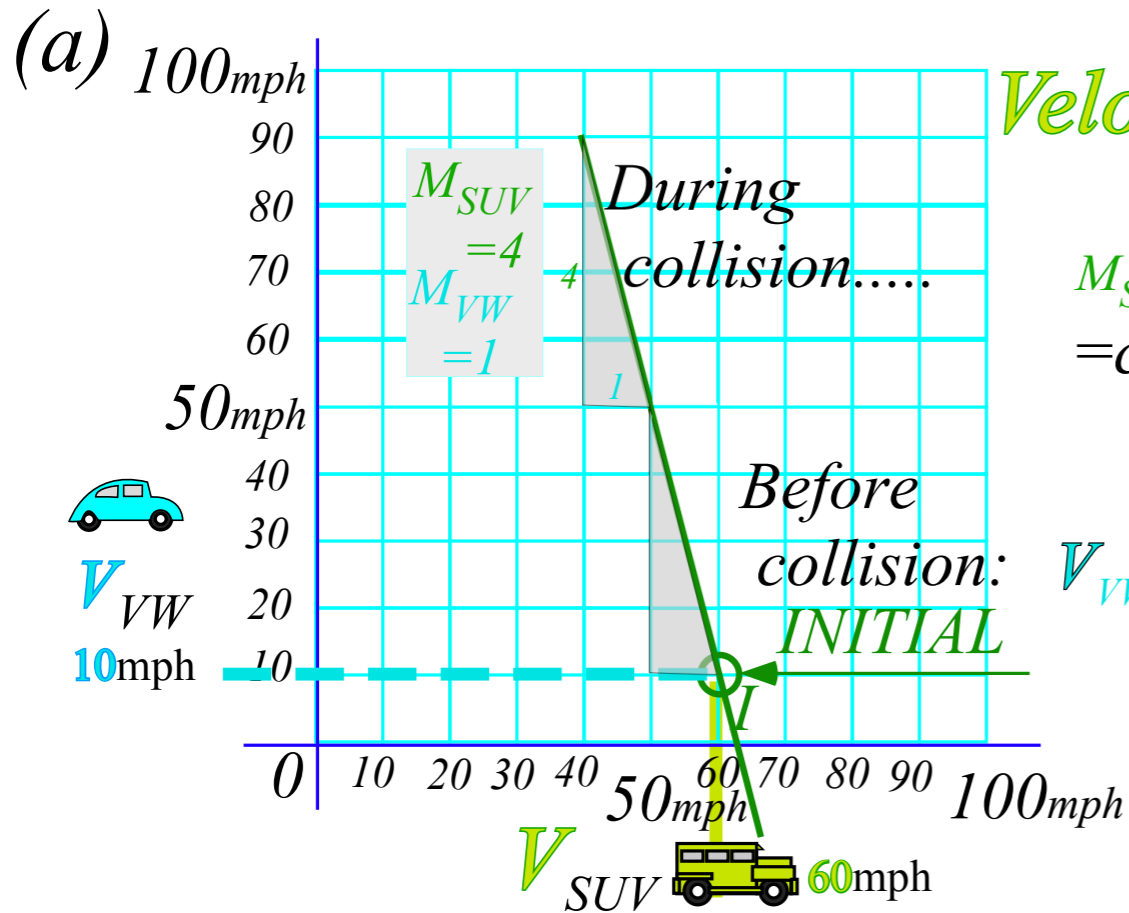
$$4 \cdot x + 1 \cdot y = 250$$

$$y = 250 - 4 \cdot x$$

...with a simple Cartesian line-plot.



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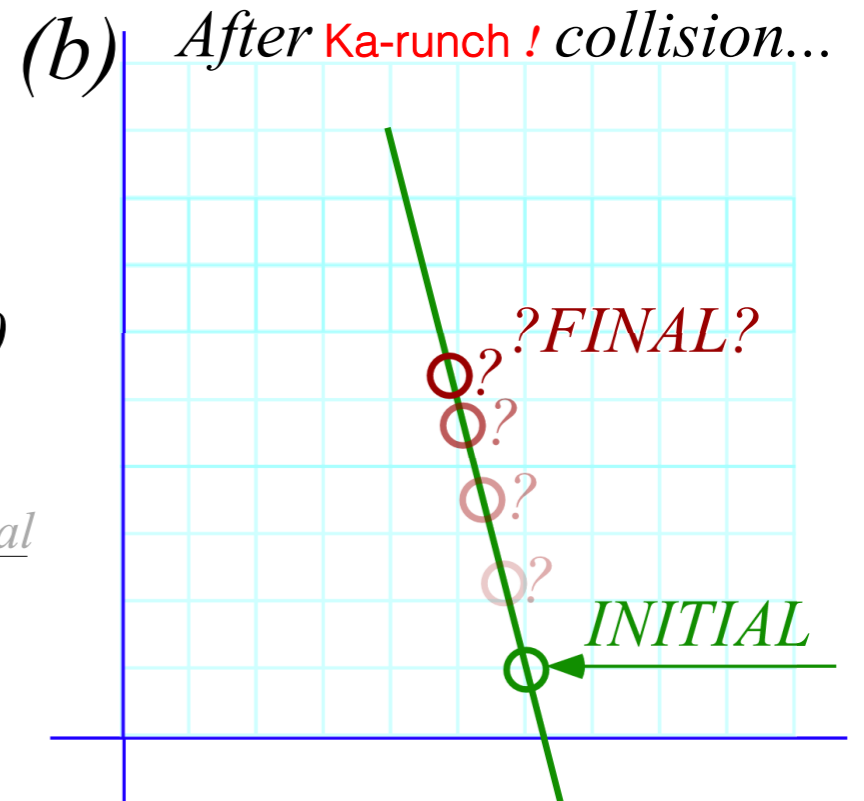


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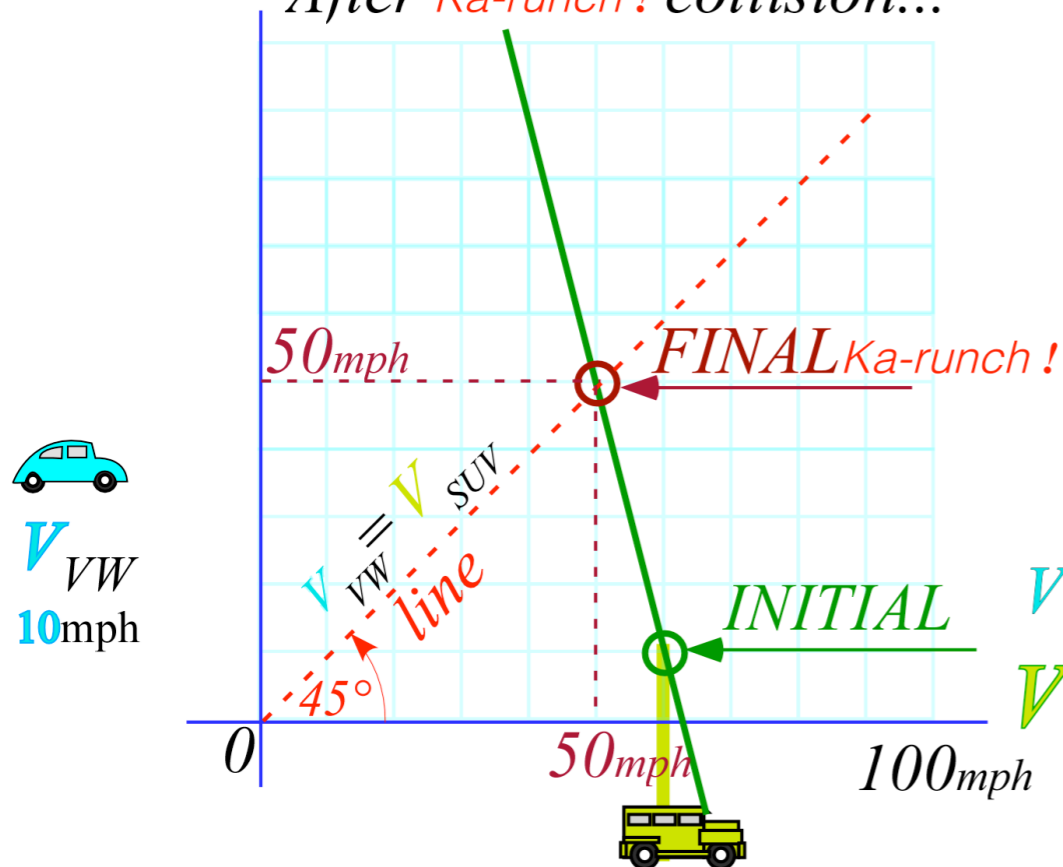
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$$= -4V_{SUV} + 250$$



After Ka-runch! collision...



It's a simple *Cartesian* equation...

$$4 \cdot x + 1 \cdot y = 250$$

$$y = 250 - 4 \cdot x$$

$$5 \cdot x = 250 \text{ ...with } y = x = 50$$

...with a simple *Cartesian* line-plot.



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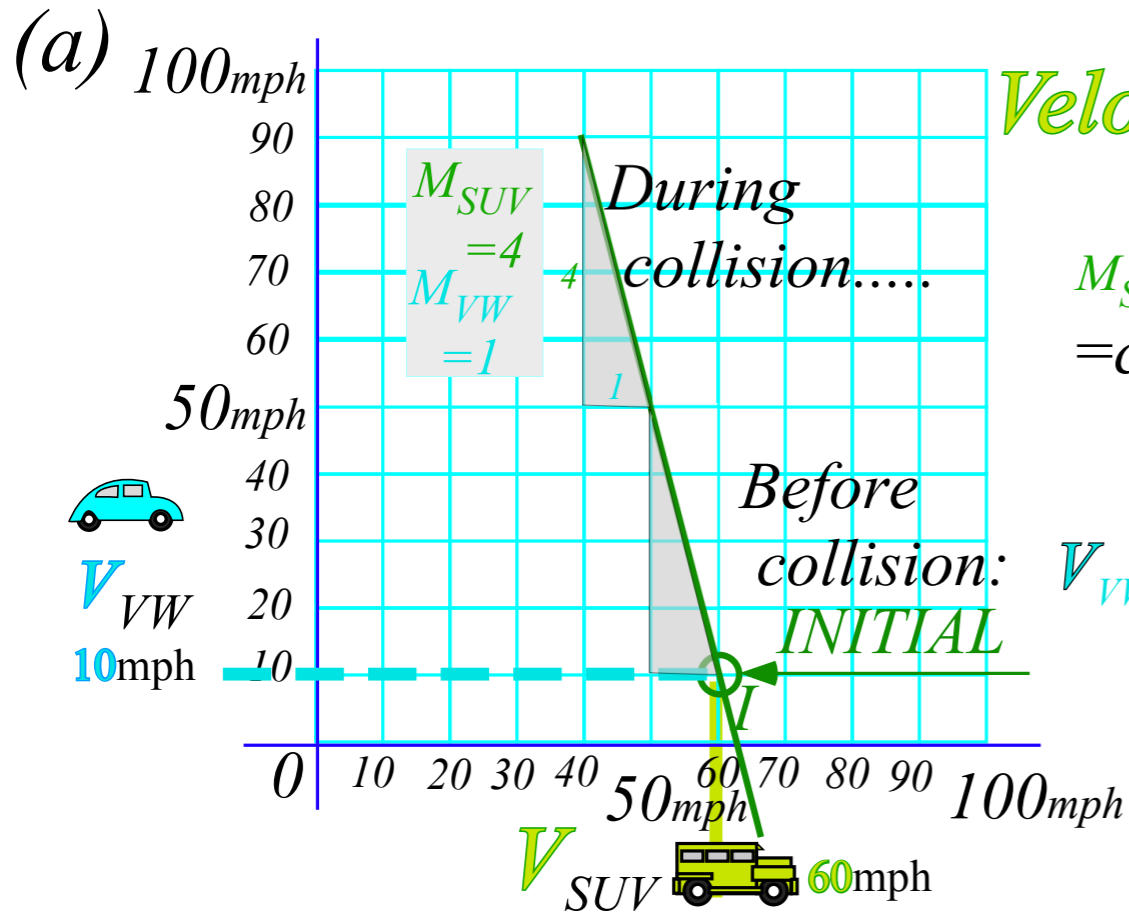
Geometry of momentum conservation axiom

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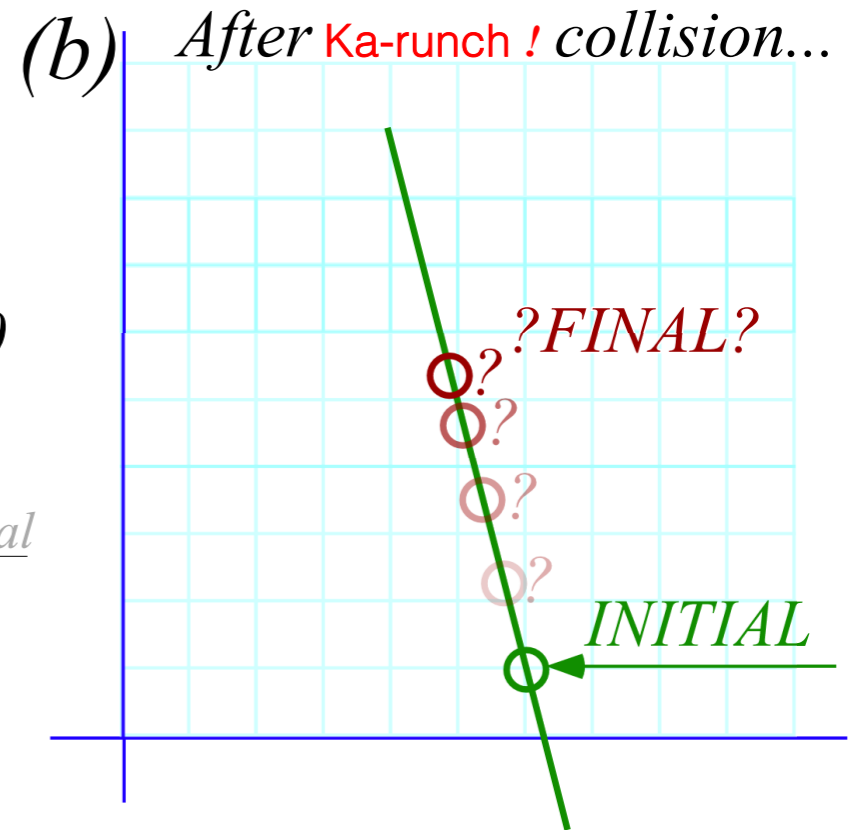


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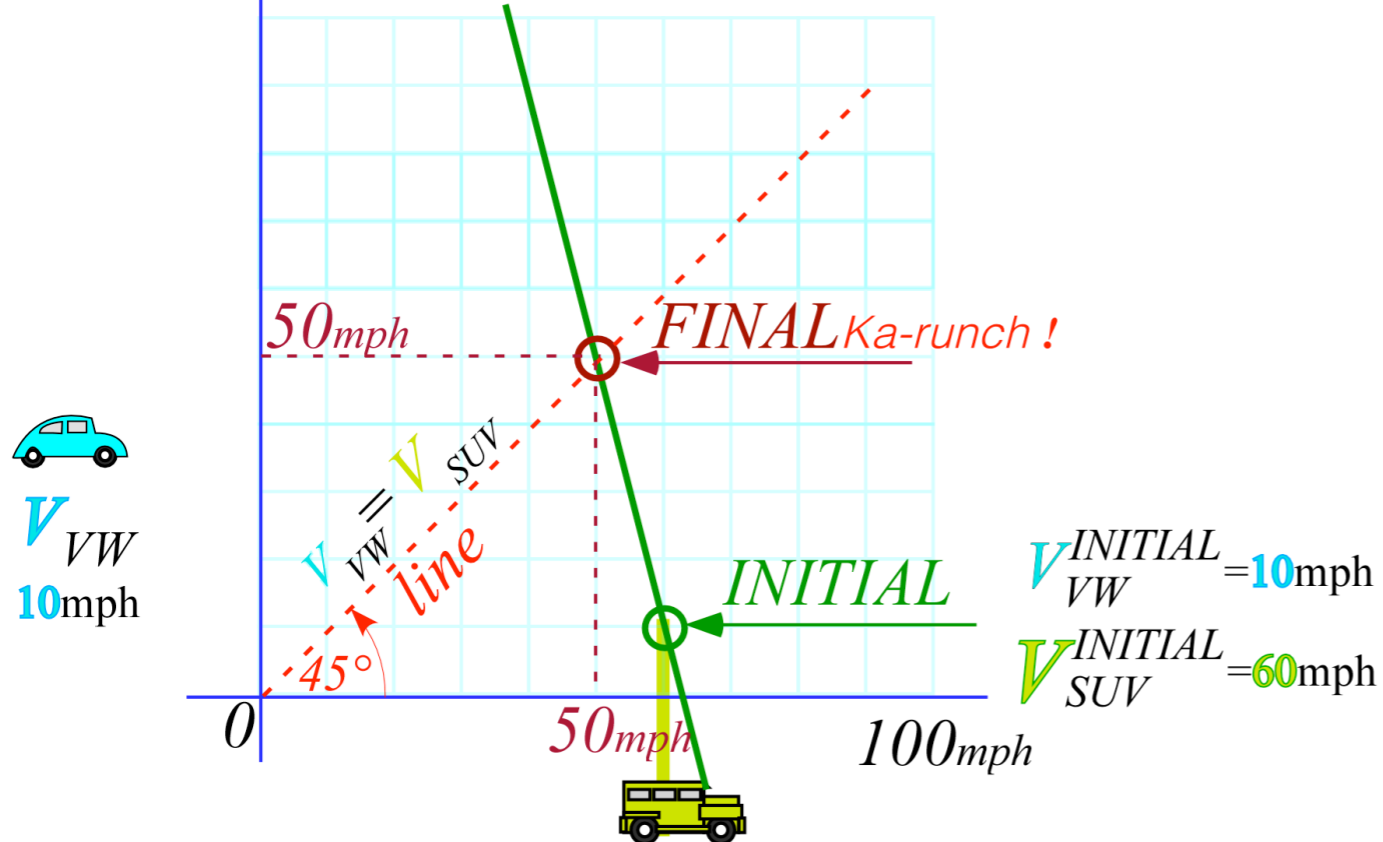
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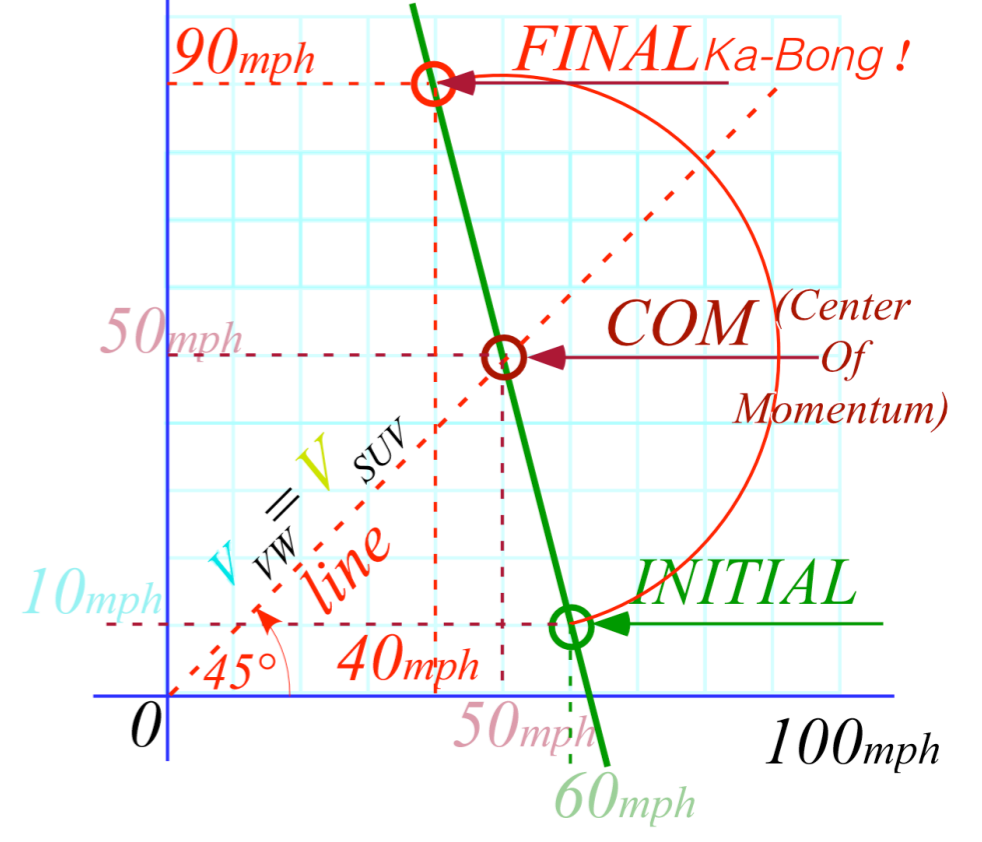
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After Ka-runch! collision...



After Ka-Bong! collision...



Geometry of momentum conservation axiom

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Geometry of Momentum Conservation Axiom - 1

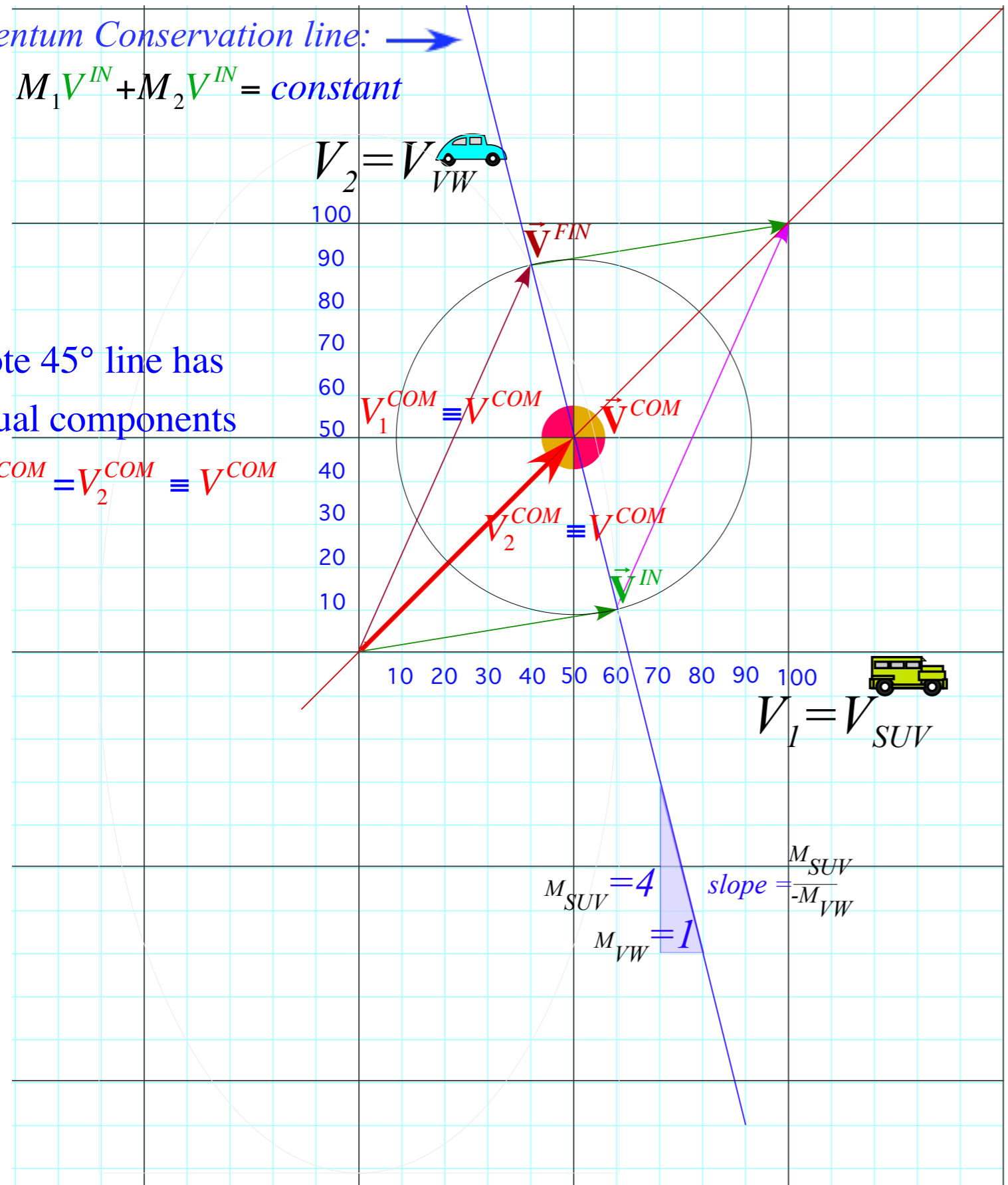
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Momentum Conservation line: \rightarrow

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$

Note 45° line has equal components

$$V_1^{COM} = V_2^{COM} \equiv V^{COM}$$



Geometry of Momentum Conservation Axiom-1

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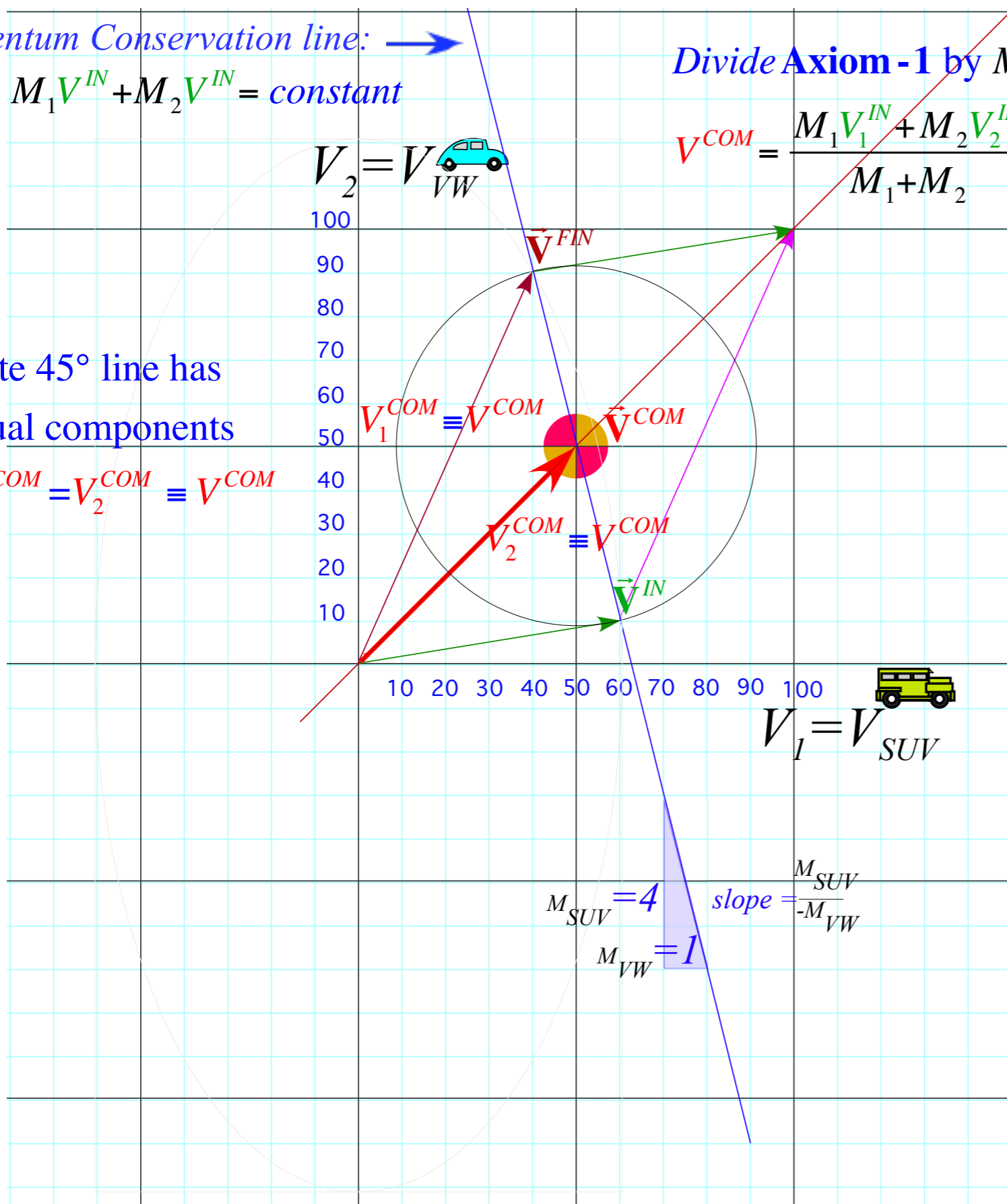
$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$

Divide Axiom-1 by $M_{Total} = (M_1+M_2)$

$$V^{COM} = \frac{M_1V_1^{IN} + M_2V_2^{IN}}{M_1+M_2} = \frac{M_1V_1^{COM} + M_2V_2^{COM}}{M_1+M_2} = \frac{M_1V_1^{FIN} + M_2V_2^{FIN}}{M_1+M_2} = 50$$

Note 45° line has equal components

$$V_1^{COM} = V_2^{COM} \equiv V^{COM}$$



$$M_{SUV} = 4 \quad \text{slope} = \frac{M_{SUV}}{-M_{VW}}$$

$$M_{VW} = 1$$

Geometry of Momentum Conservation Axiom-1

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{COM} + M_2V_2^{COM} = M_1V_1^{FIN} + M_2V_2^{FIN} = M_{Total}V^{COM}$$

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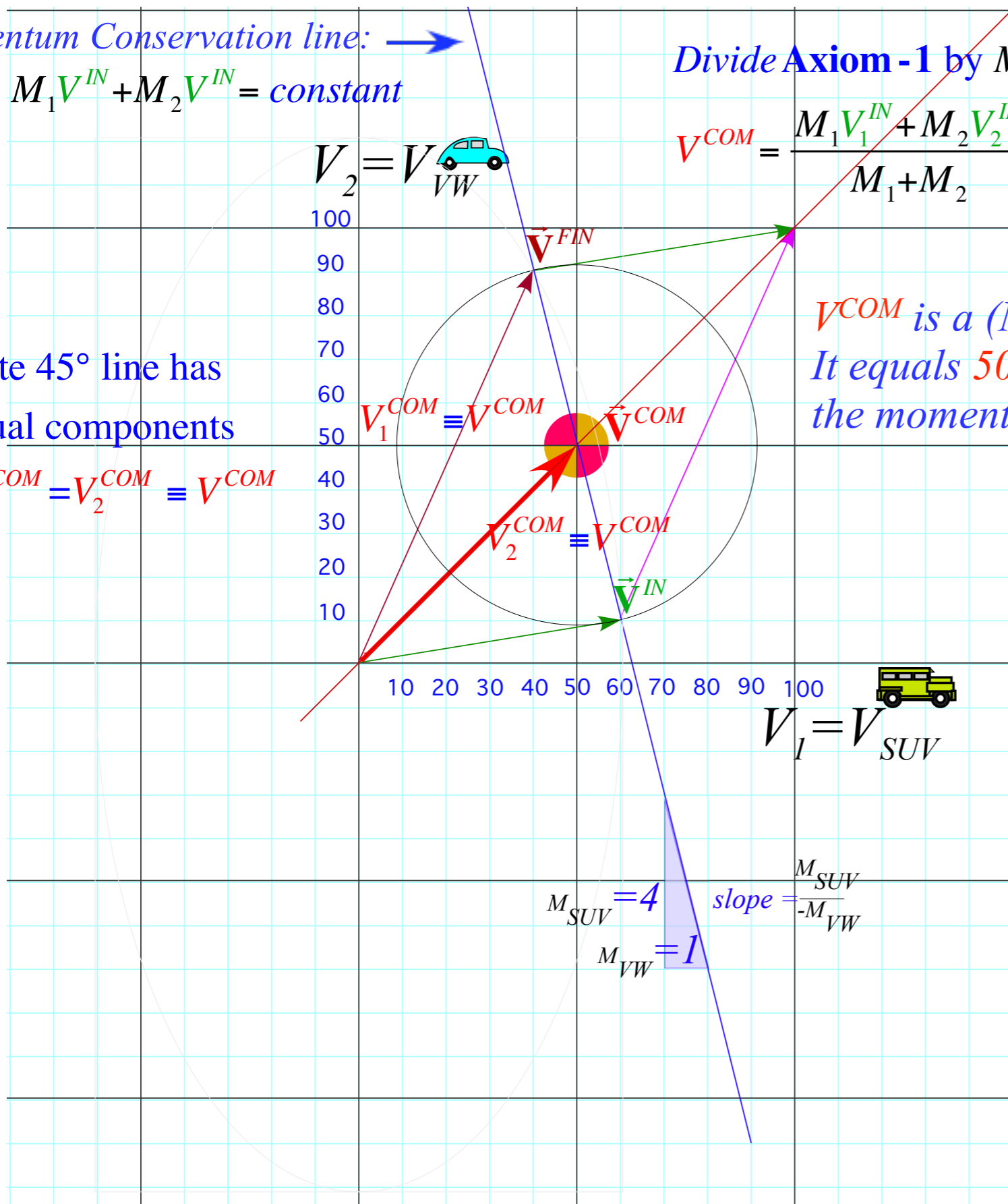
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Note 45° line has equal components

$$V_1^{COM} = V_2^{COM} \equiv V^{COM}$$

V^{COM} is a (M_1, M_2) Weighted Average of V_1 and V_2
It equals 50 for every point (V_1, V_2) on the momentum line



$V_2 = V_{VW}$

$V_1 = V_{SUV}$

$M_{SUV} = 4$
 $M_{VW} = 1$
slope = $-\frac{M_{SUV}}{M_{VW}}$

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Vector notation

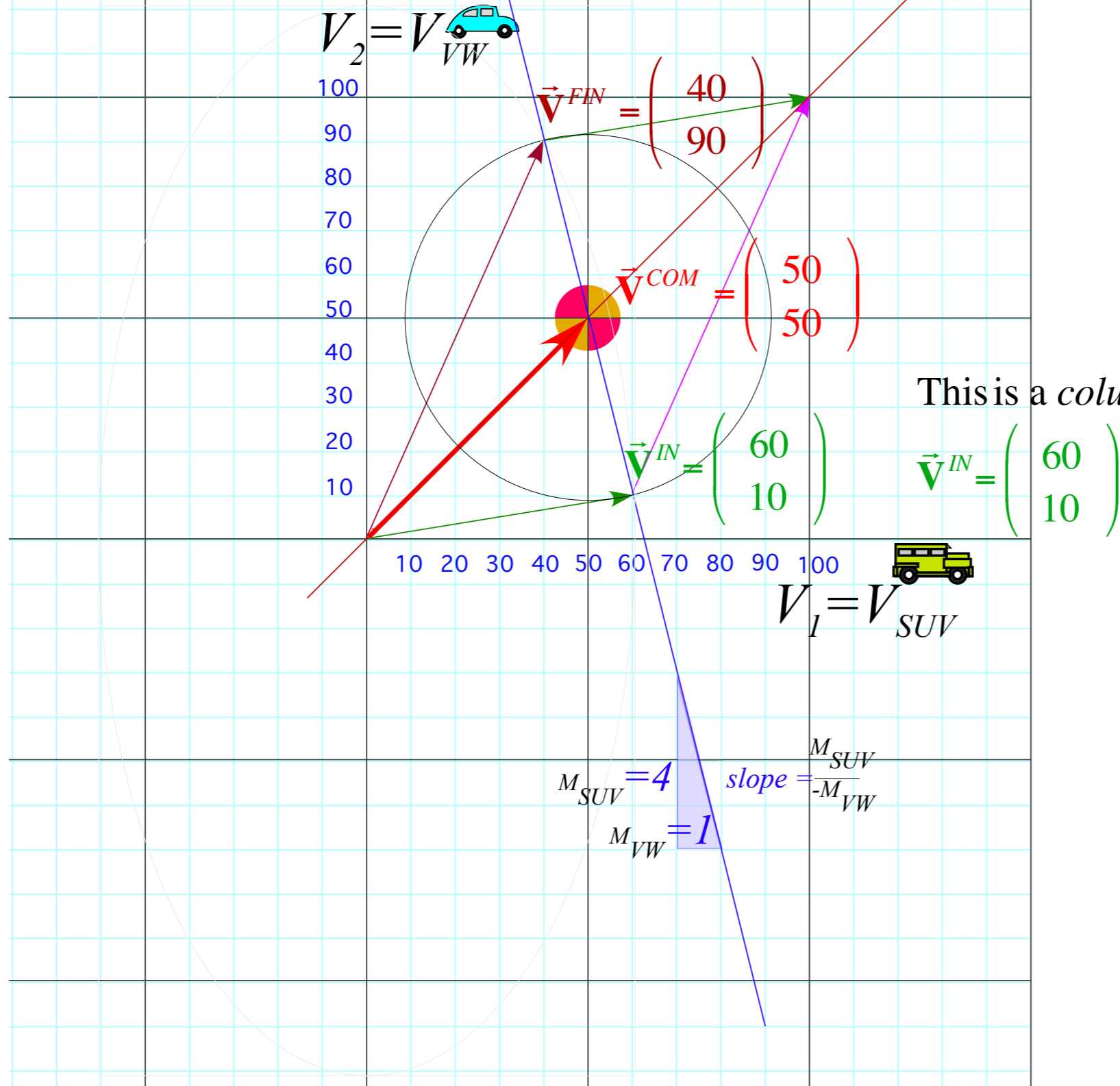
...But not the x,y,.. components

Introducing SUV, VW .. components

...pretty WEIRD?!

Momentum Conservation line: \rightarrow

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$



This is a column-vector (or ket $|IN\rangle$ in QM)

Geometry of Momentum Conservation Axiom-1

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

Vector notation

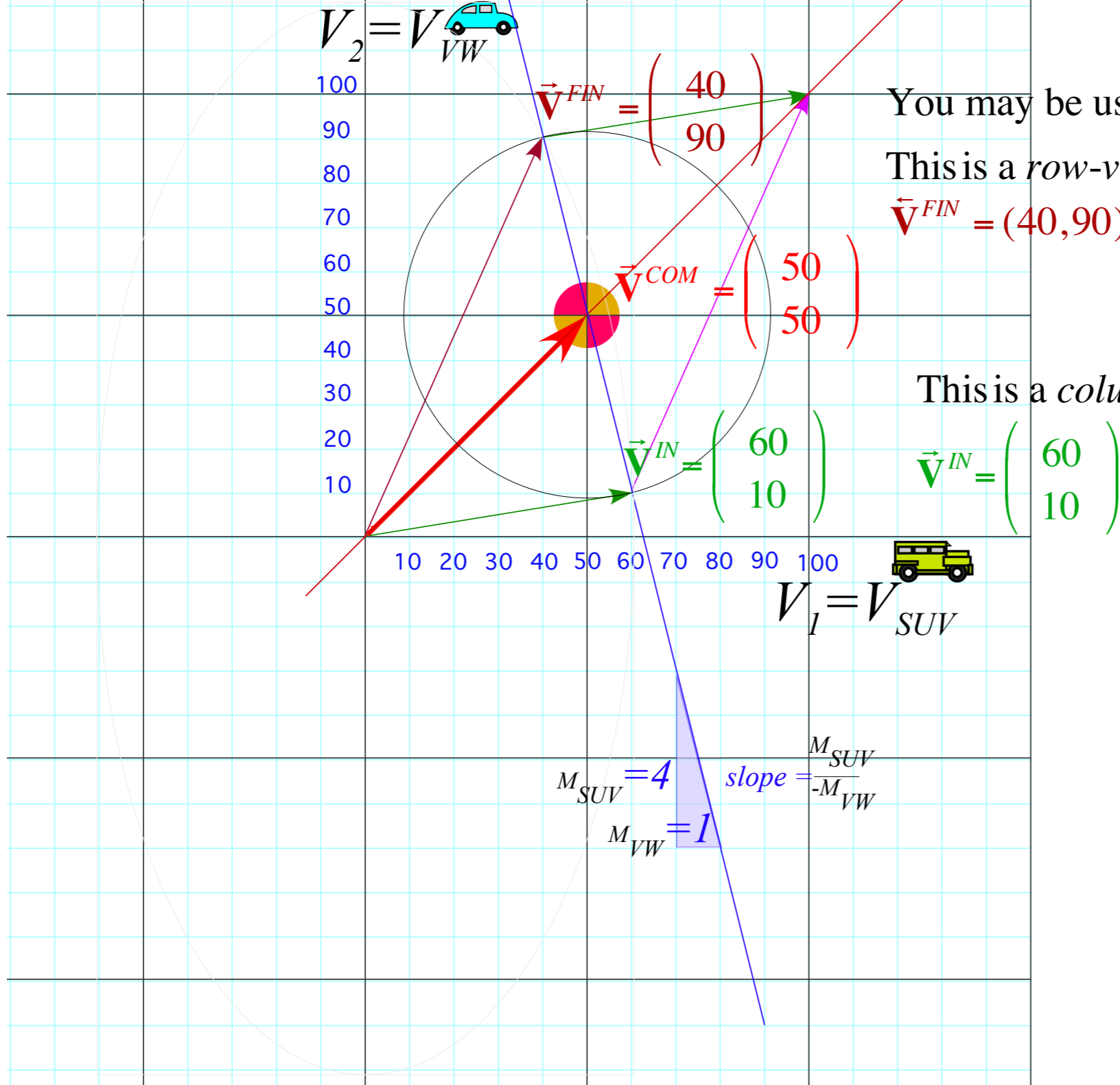
...But not the x,y,.. components

Introducing SUV, VW .. components

...pretty WEIRD?!

Momentum Conservation line: →

$$M_1V^{IN} + M_2V^{IN} = \text{constant}$$



You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

$$\vec{V}^{FIN} = (40, 90)$$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$$\vec{V}^{IN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

Geometry of Momentum Conservation Axiom-1

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

Vector notation

...But not the x,y,.. components

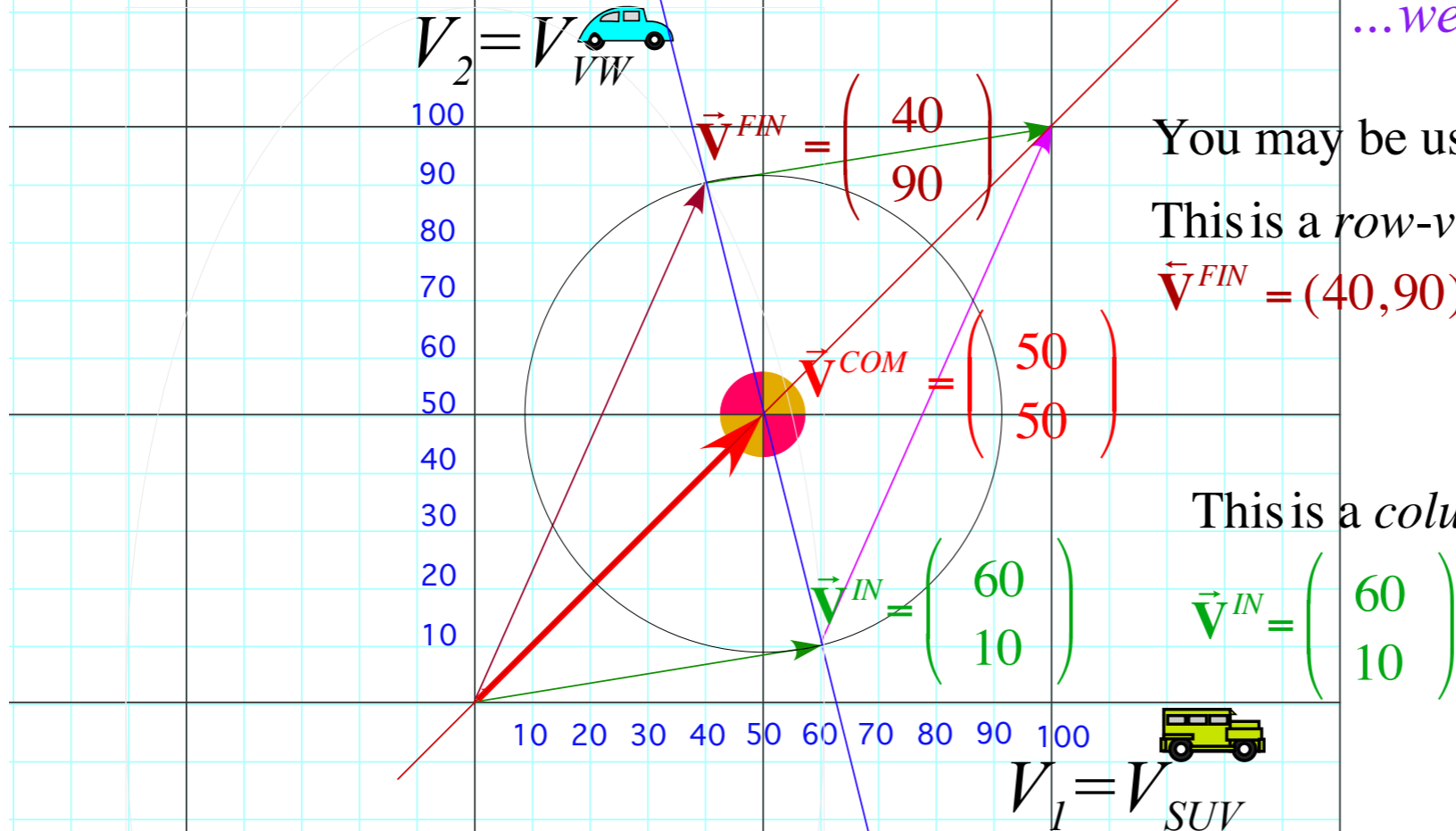
Introducing SUV, VW .. components

...pretty WEIRD?!

...well, not too weird, actually...

Momentum Conservation line: →

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$



You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

$$\vec{V}^{FIN} = (40, 90)$$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$$\vec{V}^{IN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

This is a *dot* product (or *scalar* product)

$$\vec{V}^{FIN} \cdot \vec{V}^{IN} = (40, 90) \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \langle FIN|IN\rangle = 40 \cdot 60 + 90 \cdot 10 = 2400 + 900 = 3300$$

of a *row-vector* $\vec{V}^{FIN} = (40, 90)$ (or *bra* $\langle FIN|$)

with *column-vector* $= \begin{pmatrix} 60 \\ 10 \end{pmatrix}$ (or *ket* $|IN\rangle$)

Geometry of Momentum Conservation Axiom-1

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

Vector notation

...But not the x,y,.. components

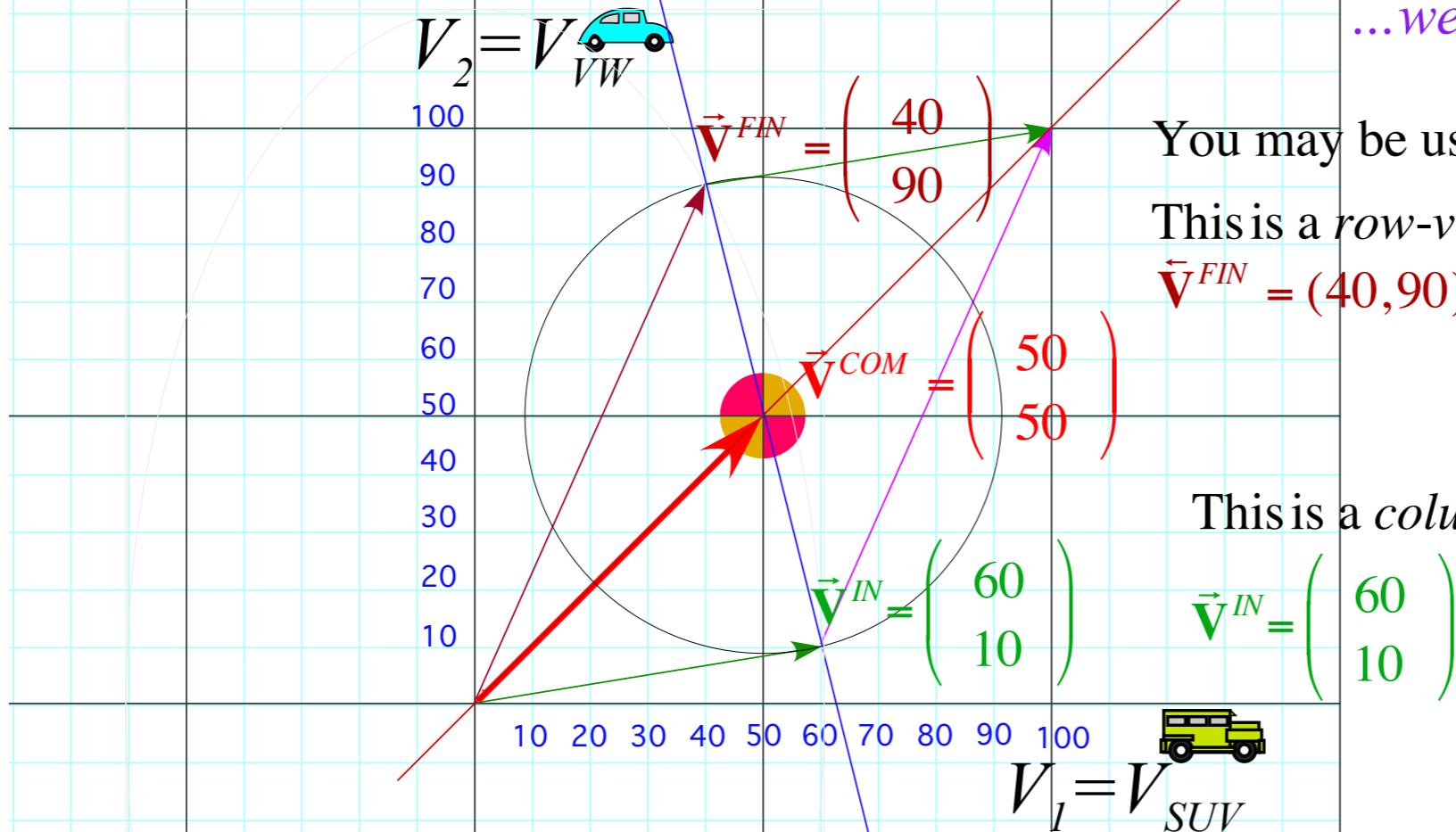
Introducing SUV, VW .. components

...pretty WEIRD?!

...well, not too weird, actually...

Momentum Conservation line: →

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$



You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

$$\vec{V}^{FIN} = (40, 90)$$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$$\vec{V}^{IN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

This is a *outer product* (or *tensor product*)

$$\vec{V}^{IN} \otimes \vec{V}^{FIN} = \begin{pmatrix} 60 \\ 10 \end{pmatrix} \otimes (40, 90) = |IN\rangle\langle FIN| = \begin{pmatrix} 60 & 40 & 60 & 90 \\ 10 & 40 & 10 & 90 \end{pmatrix} = \begin{pmatrix} 2400 & 5400 \\ 400 & 900 \end{pmatrix}$$

of a *column-vector* = $\begin{pmatrix} 60 \\ 10 \end{pmatrix}$ (or *ket* $|IN\rangle$)

with a *row-vector* $\vec{V}^{FIN} = (40, 90)$ (or *bra* $\langle FIN|$)

Geometry of momentum conservation axiom

Totally Inelastic “ka-runch” collisions

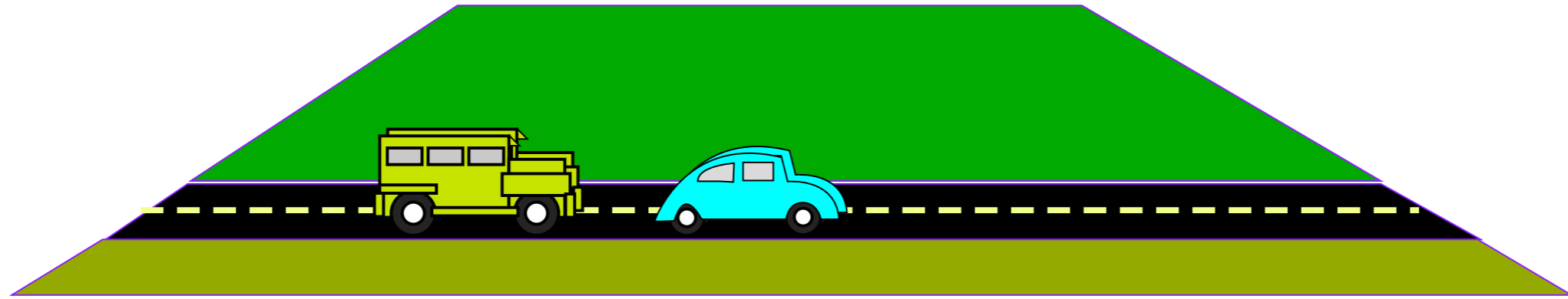
Perfectly Elastic “ka-bong” and Center Of Momentum (COM) symmetry

+Intro to weighty-averages and vector notation

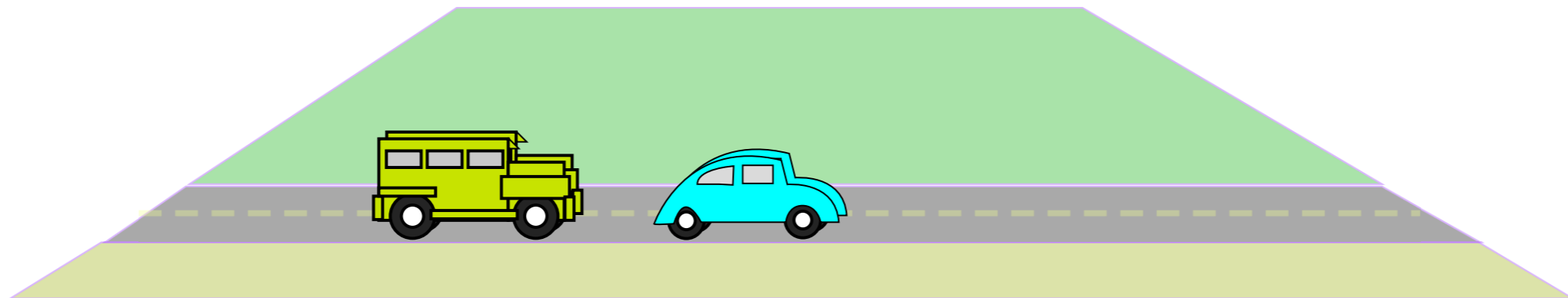
Comments on idealization in classical models



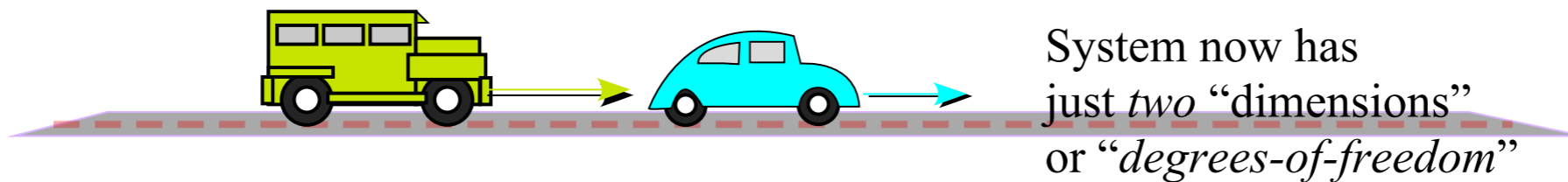
The SUV and VW *Idealized* thought experiments



Idealization 1. Ignore background.
(No rolling friction, air resistance, etc.)



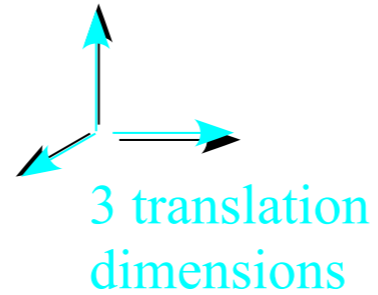
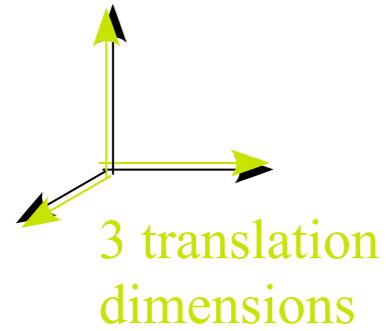
Idealization 2. Make each 1-dimensional.
(Cars “constrained” to ride on frictionless rail)



System now has
just *two* “dimensions”
or “*degrees-of-freedom*”

Summary of Classical Mechanical Degrees of Freedom

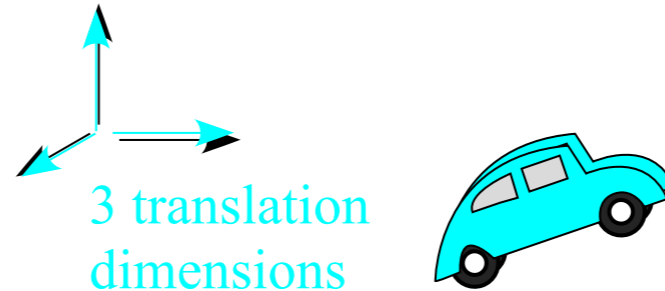
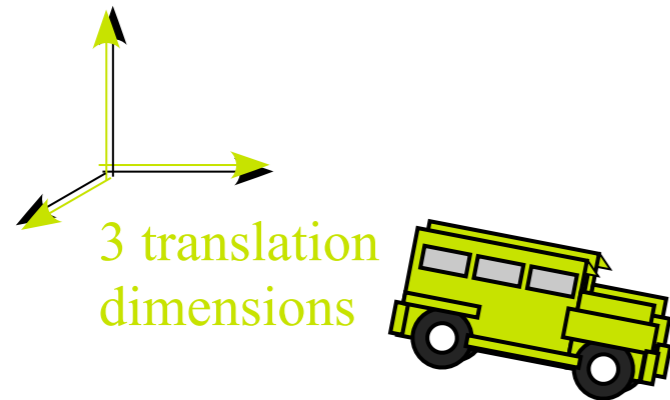
Translation (Each body has 3 translational degrees of freedom) (Introduced in Units 1 and 2)



6 translational degrees of freedom for SUV and VW.

Summary of Classical Mechanical Degrees of Freedom

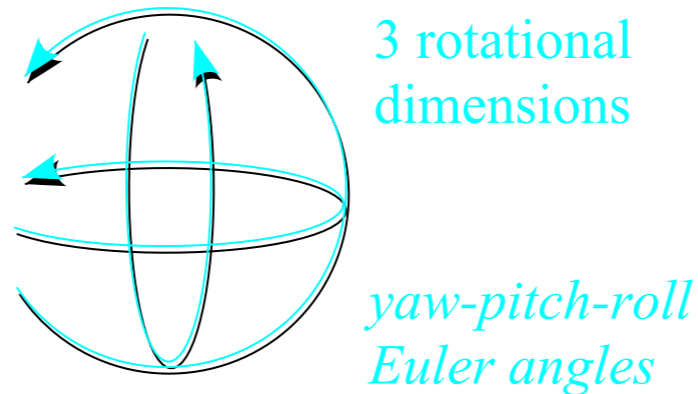
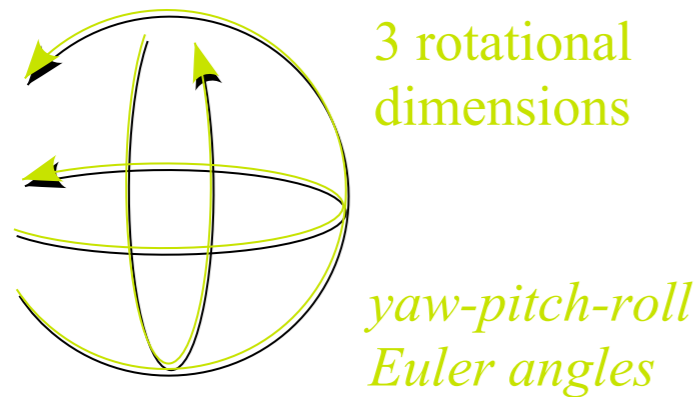
Translation (Each body has 3 translational degrees of freedom) (Introduced in Units 1 and 2)



6 translational degrees of freedom for SUV and VW.

Rotation (Each body has 3 rotational degrees of freedom)

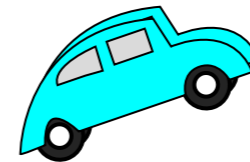
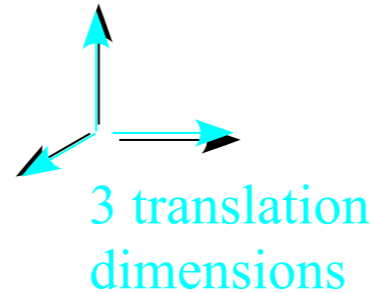
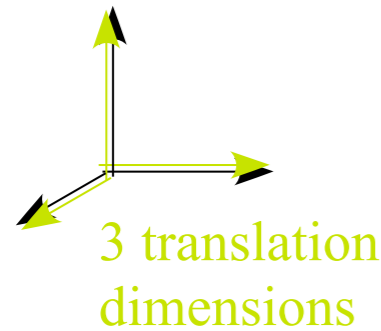
(Introduced in Units 3 and 7)



6 rotational degrees of freedom for SUV and VW.

Summary of Classical Mechanical Degrees of Freedom

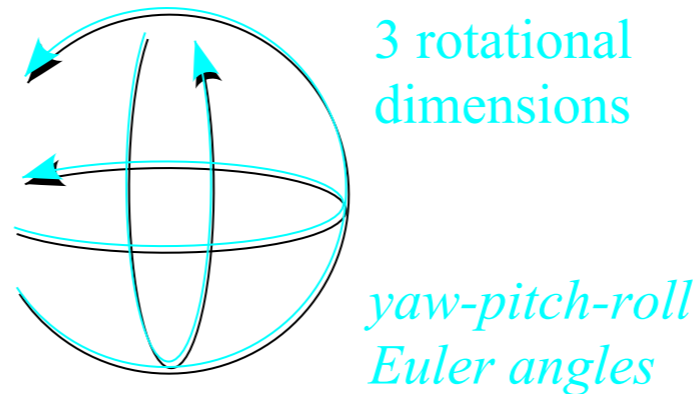
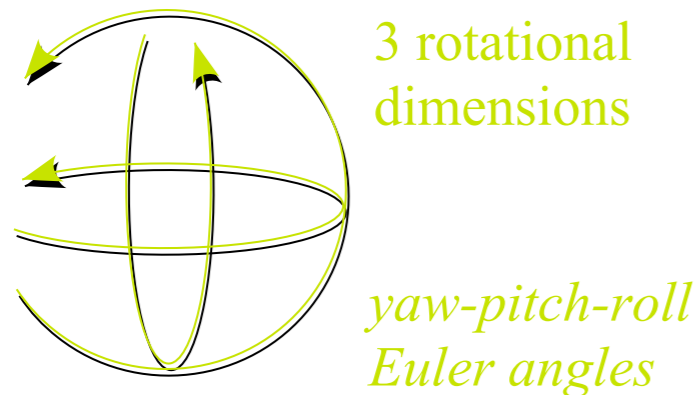
Translation (Each body has 3 translational degrees of freedom) (Introduced in Units 1 and 2)



6 translational degrees of freedom for *SUV* and *VW*.

Rotation (Each body has 3 rotational degrees of freedom)

(Introduced in Units 3 and 7)

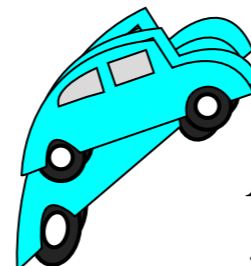
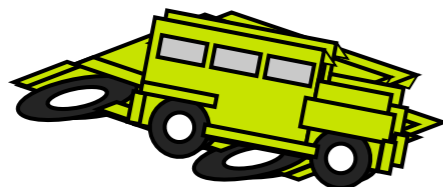


6 rotational degrees of freedom for *SUV* and *VW*.

SUV and VW system involves 12 rigid-body degrees of freedom


Vibration (Each body has many vibrational degrees of freedom) (Introduced in Units 3-8)

Generalized Curvilinear Coordinates (GCC) introduced in Unit 1 Chapters 10-12



An N-atom molecule has $3N-6$ vibrational degrees of freedom

Geometry of Galilean translation symmetry

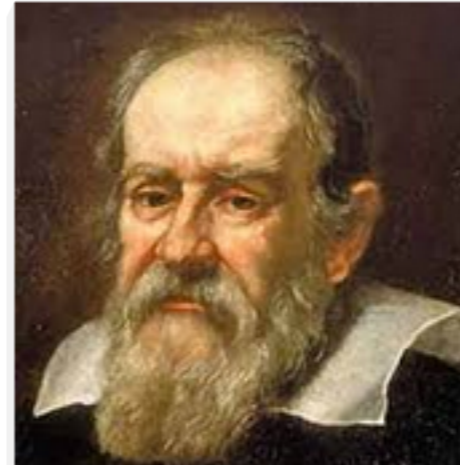
 *45° shift in (V_1, V_2) -space*
Time reversal symmetry
...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

*Geometry of Galilean translation (A **symmetry transformation**)*

If you increase your velocity by 50 mph,... (In some direction x,y, or z...)

*...the rest of the world appears to be 50 mph **slower** (In that direction...)*



*Galileo Galilei
1564-1642*

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

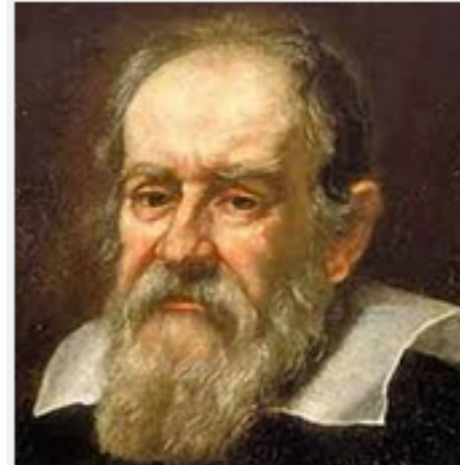
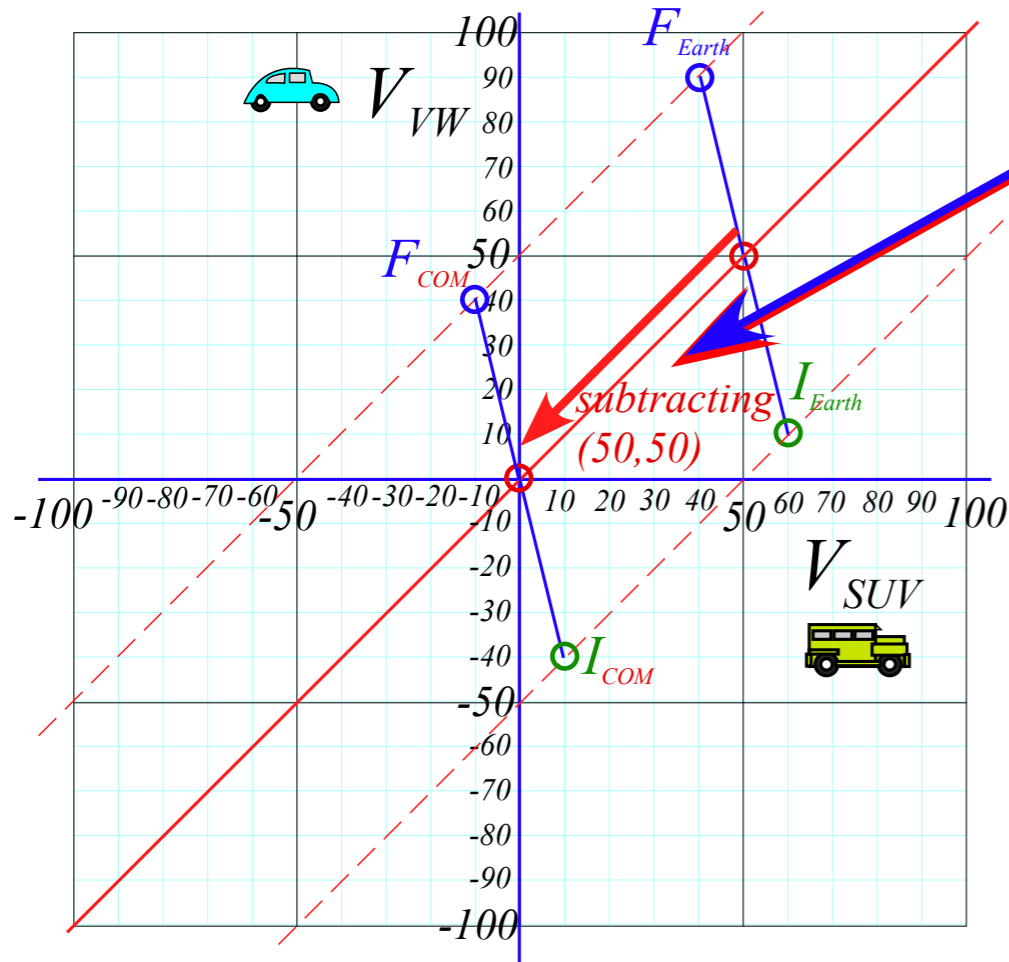
Geometry of Galilean translation (A *symmetry transformation*)

If you increase your velocity by 50 mph,...

(In some direction x, y , or $z...$)

...the rest of the world appears to be 50 mph *slower* (In that direction...)

(a) Galileo transforms to *COM* frame



Galileo Galilei
1564-1642

Fig. 2.5a
in Unit 1

Q: Sir, does that include a light pulse?

A: Well, yeah...kinda...

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A *symmetry transformation*)

If you increase your velocity by 50 mph,...

(In some direction x, y , or z ...)

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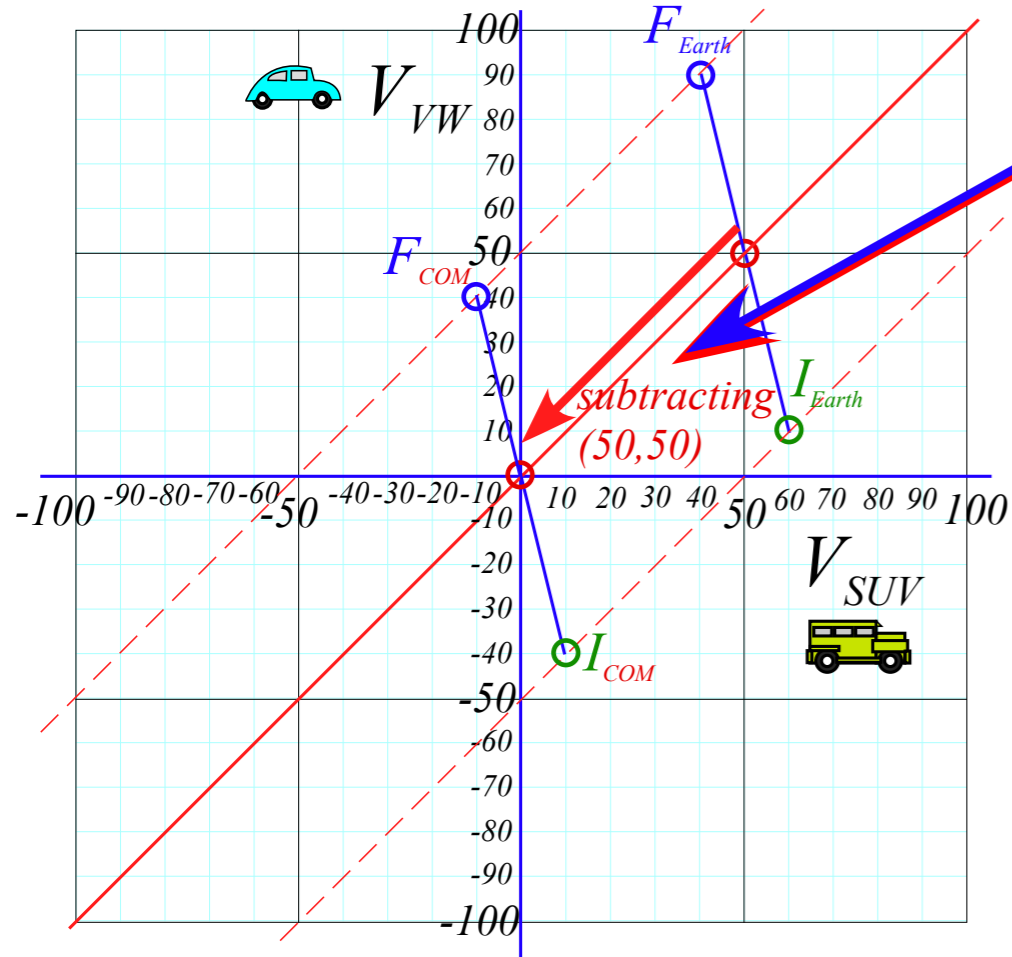


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

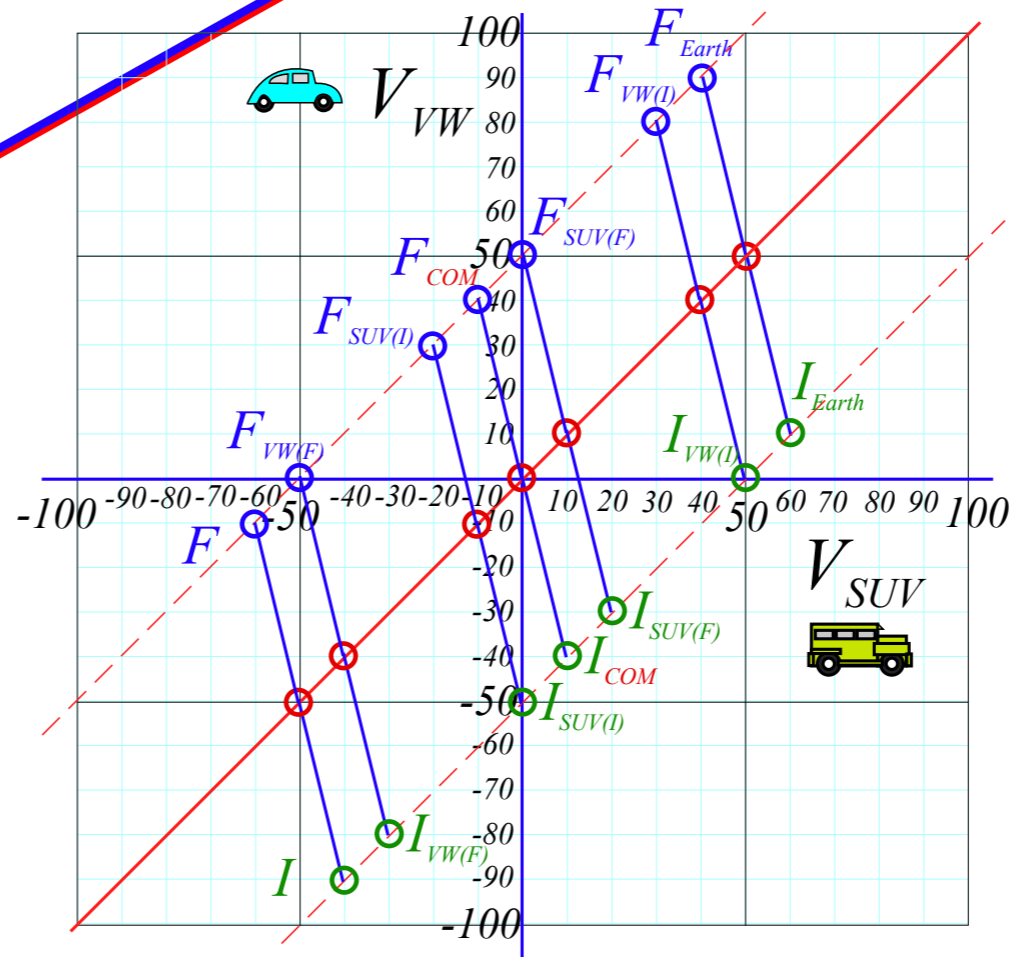
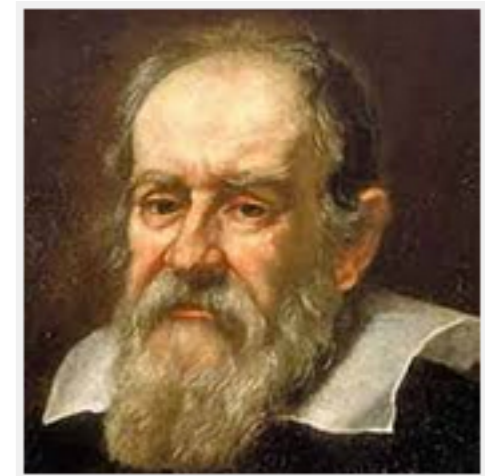


Fig. 2.5b
in Unit 1



Galileo Galilei
1564-1642

(Five of these have 0 for a velocity coord.)

Geometry of Galilean translation symmetry

45° shift in (V_1, V_2) -space

Time reversal symmetry

...of COM collisions



A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph VW)

Final F and Initial I trade places ...

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph slower

Time-reversal (F-I) symmetry pairs (Four examples)

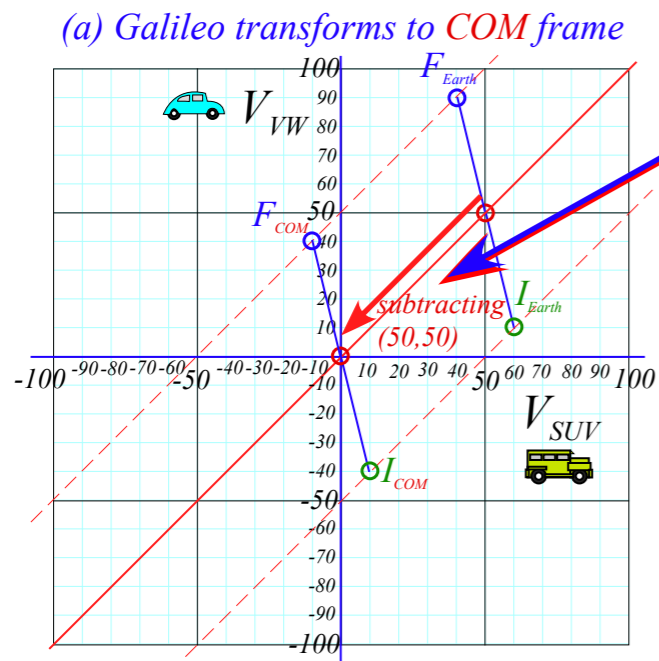


Fig. 2.5a
in Unit 1

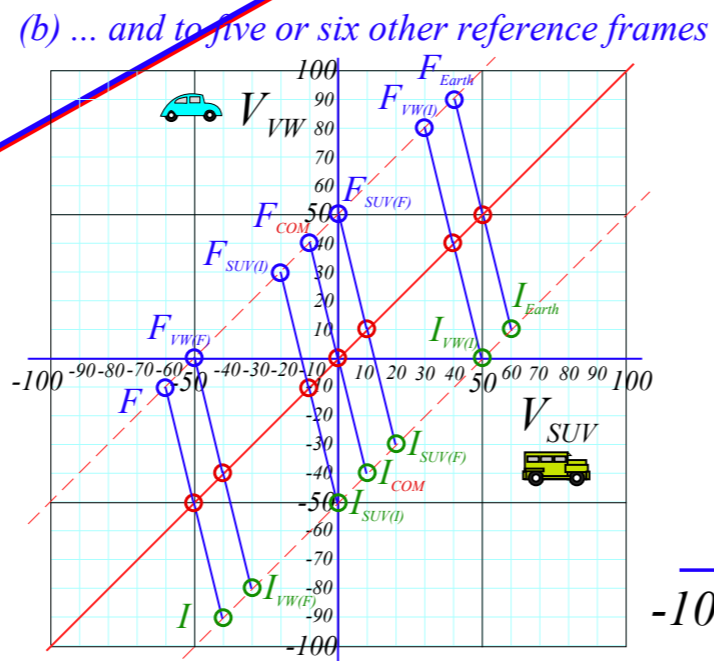
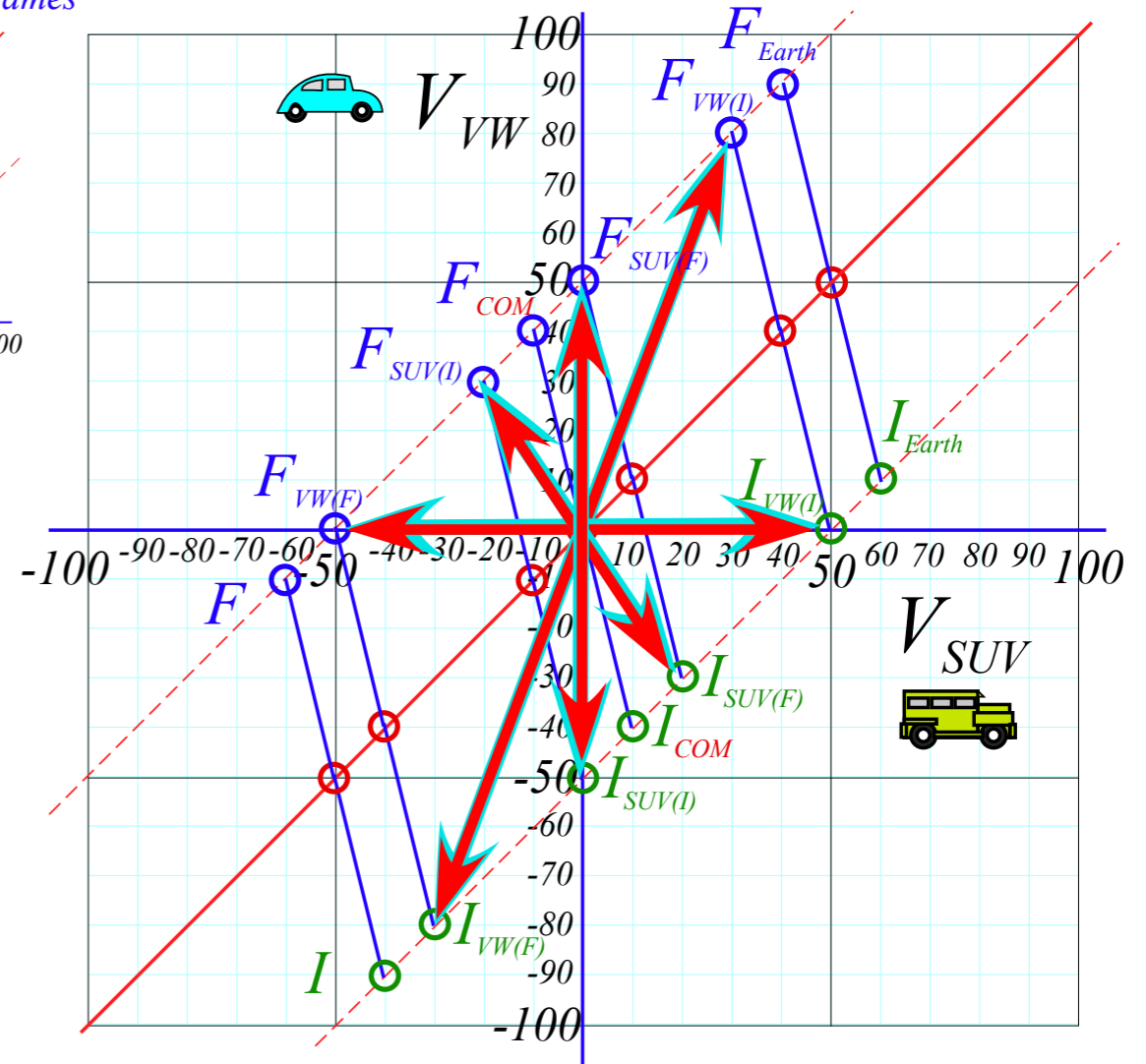


Fig. 2.5b
in Unit 1



*Time-reversal means flip t to $-t$...
(Run a movie backwards)*

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Final F and Initial I trade places ...

Geometry of Galilean translation (A symmetry transformation)

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Time-reversal (F-I) symmetry pairs (Four examples)

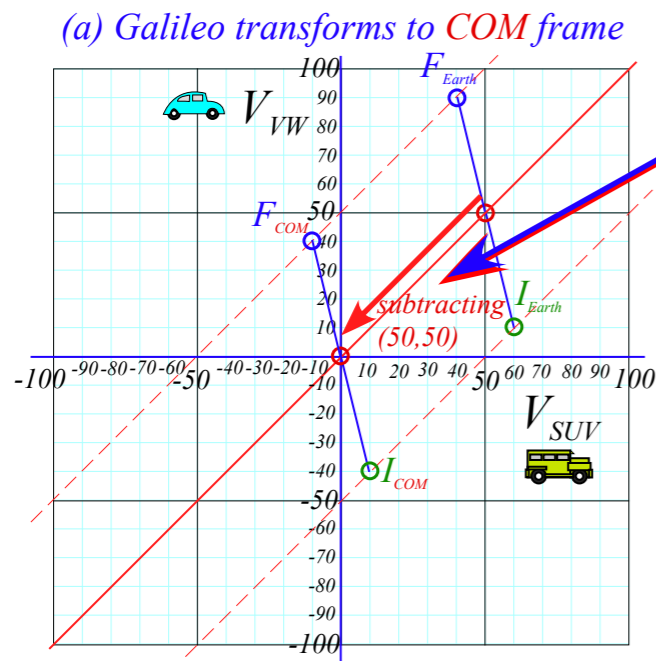


Fig. 2.5a
in Unit 1

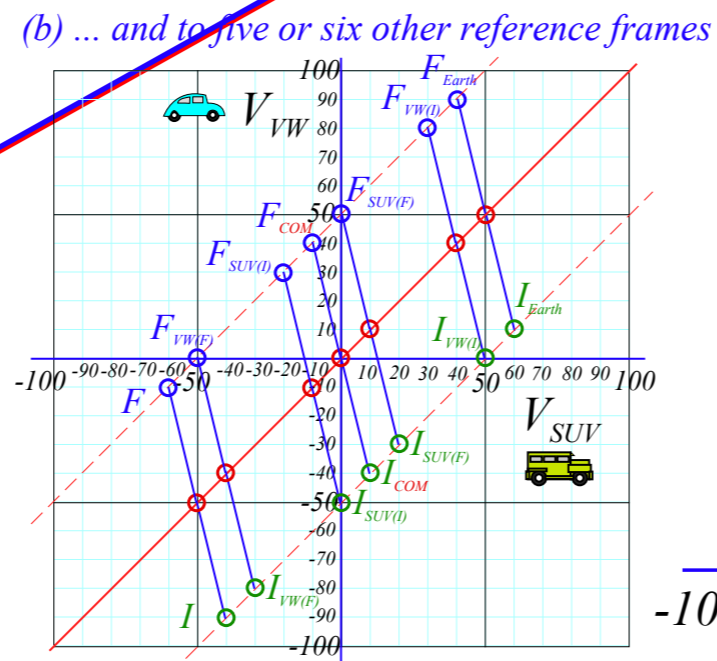
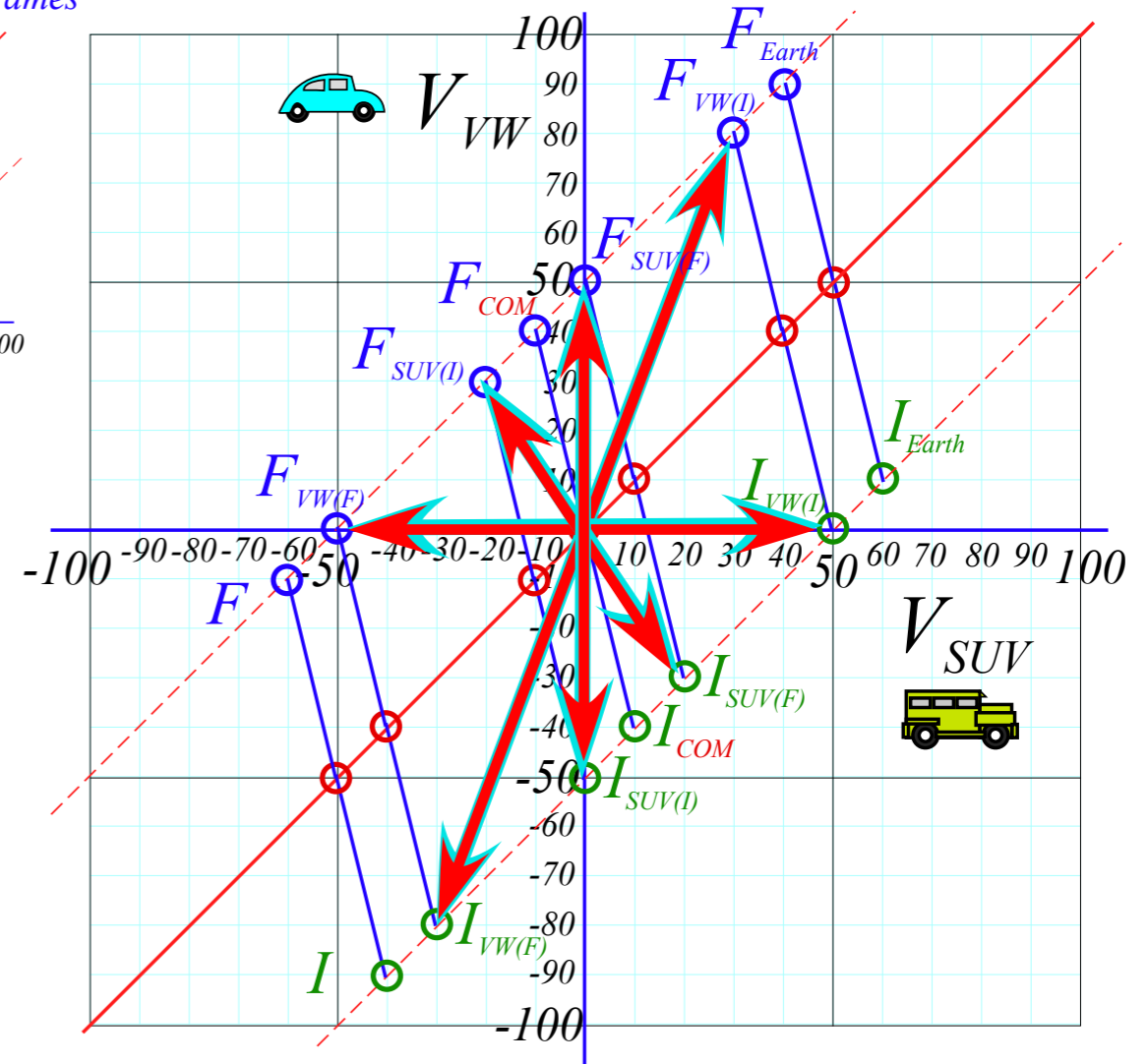


Fig. 2.5b
in Unit 1



*Time-reversal means flip t to $-t$...
(Run a movie backwards)*

*That means you flip Velocity V to $-V$...
(Everything goes backwards)*

Geometry of Galilean translation symmetry

45° shift in (V_1, V_2) -space

Time reversal symmetry

...of COM collisions



A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph slower

THE
COM Time-reversal
symmetry pair
(Just 1 case)

(a) Galileo transforms to COM frame

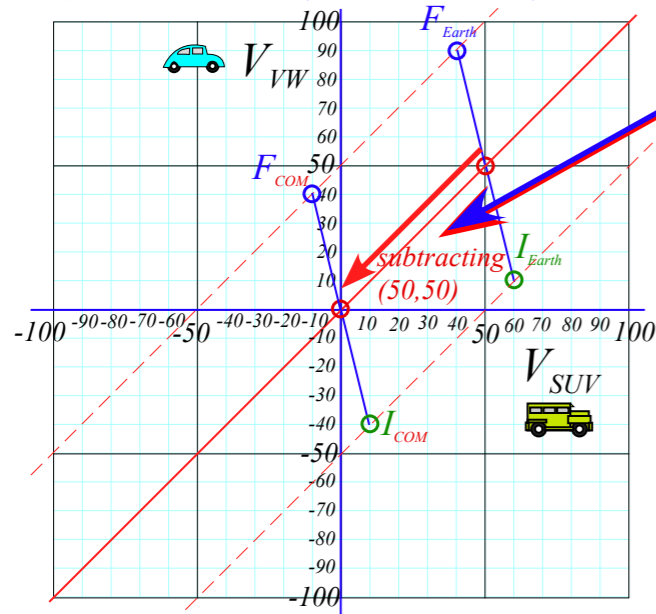


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

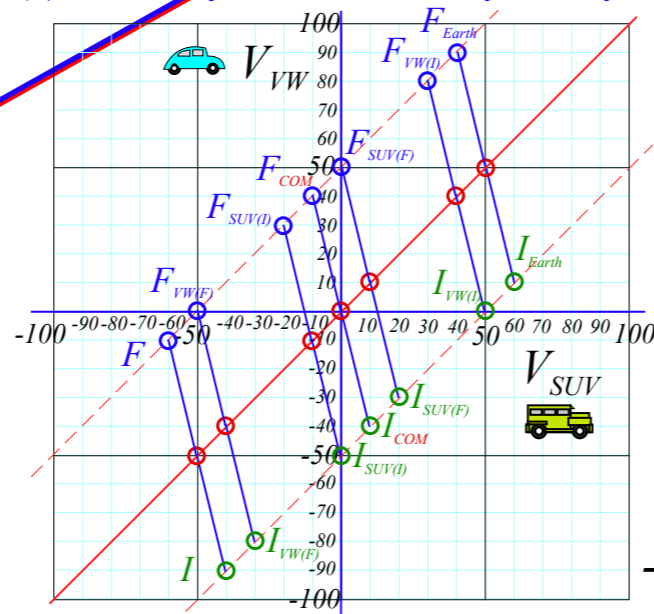
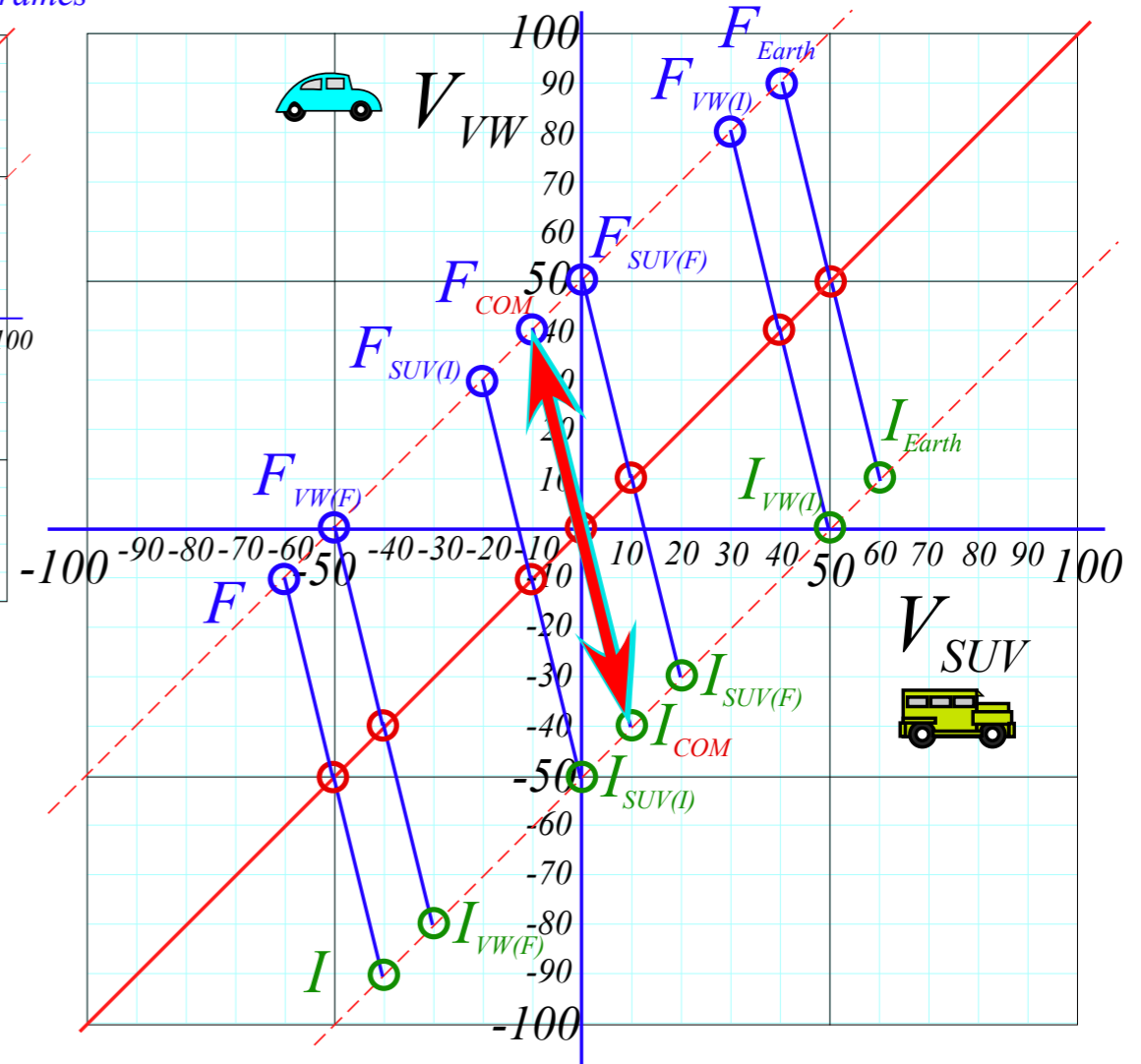


Fig. 2.5b
in Unit 1



*There is just one velocity frame
in which the time-reversed collision
looks just like the original collision*

*That is the
Center-of-Momentum
(COM)-frame*

*Time-reversal means flip t to $-t$...
(Run a movie backwards)*

*That means you flip Velocity V to $-V$...
(Everything goes backwards)*

Algebra, Geometry, and Physics of momentum conservation axiom

→ *Vector algebra of collisions*

Matrix or tensor algebra of collisions

Deriving Energy Conservation Theorem

Energy Ellipse geometry

Quick lesson on

Gibb's notation for

dot (\bullet) product of matrix operator \mathbf{M} and column vector \mathbf{V}^{IN} :

$$\begin{aligned} & \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} \\ & \begin{pmatrix} A & B \\ C & D \end{pmatrix} \bullet \begin{pmatrix} x^{IN} \\ y^{IN} \end{pmatrix} \\ & = \begin{pmatrix} Ax^{IN} + By^{IN} \\ Cx^{IN} + Dy^{IN} \end{pmatrix} \end{aligned}$$

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Quick lesson on

Dirac notation is

much simpler:

$$\begin{aligned} & M |IN\rangle \\ & \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \langle x | IN \rangle \\ \langle y | IN \rangle \end{pmatrix} \end{aligned}$$

Algebra, Geometry, and Physics of momentum conservation axiom

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Gibb's notation for

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$$\vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}$$
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \bullet \begin{pmatrix} x^{IN} \\ y^{IN} \end{pmatrix}$$
$$= \begin{pmatrix} Ax^{IN} + By^{IN} \\ Cx^{IN} + Dy^{IN} \end{pmatrix}$$

Quick lesson on

Dirac notation is

much simpler:

$M|IN\rangle$ (...at first!)

$$\begin{pmatrix} \langle x|M|x\rangle & \langle x|M|y\rangle \\ \langle y|M|x\rangle & \langle y|M|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|IN\rangle \\ \langle y|IN\rangle \end{pmatrix}$$
$$= \begin{pmatrix} \langle x|M|x\rangle\langle x|IN\rangle + \langle x|M|y\rangle\langle y|IN\rangle \\ \langle y|M|x\rangle\langle x|IN\rangle + \langle y|M|y\rangle\langle y|IN\rangle \end{pmatrix}$$

Algebra, Geometry, and Physics of momentum conservation axiom

→ *Vector algebra of collisions*

Matrix or tensor algebra of collisions

Deriving Energy Conservation Theorem

Energy Ellipse geometry

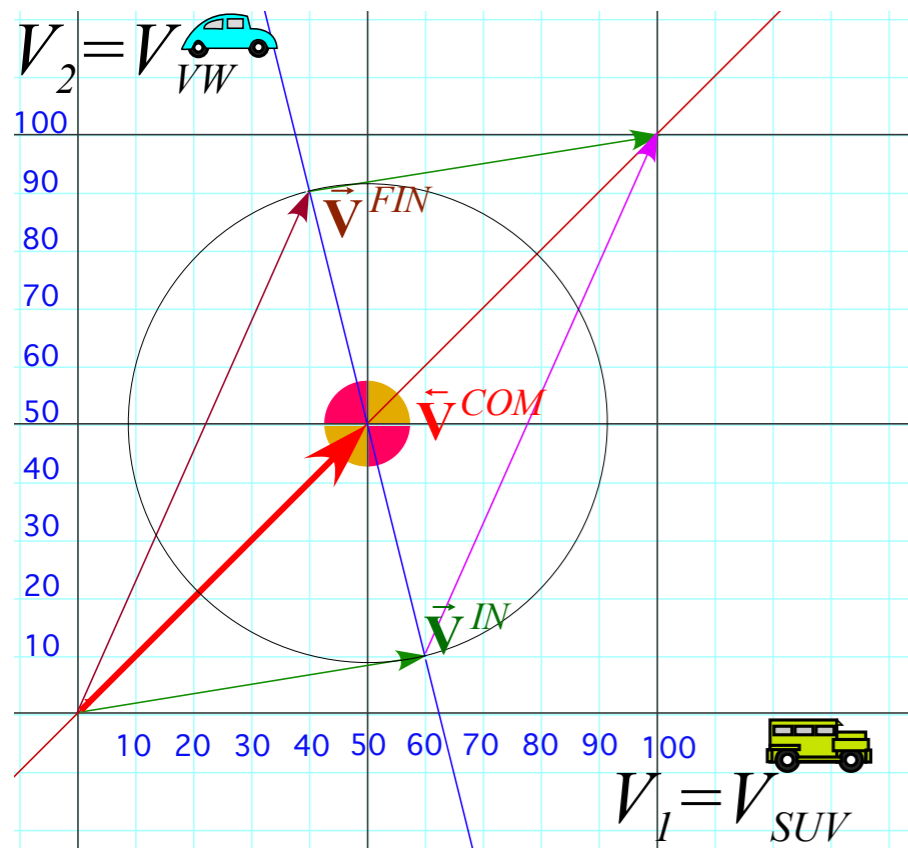
General Inertia Tensor \mathbf{M} or inertia matrix of 2 coefficients $M_{11}=M_1$ and $M_{22}=M_2$ for 2 dimensions

$$\left. \begin{aligned} P_1 &= M_1 V_1 + M_{12} V_2 \\ P_2 &= M_{21} V_1 + M_2 V_2 \end{aligned} \right\} \text{denoted : } \vec{\mathbf{P}} = \vec{\mathbf{M}} \cdot \vec{\mathbf{V}} \quad \text{or : } \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Generalizing the definition of momentum...

With 45° diagonal \mathbf{V}^{COM} so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = (M_1 + M_2) V^{COM} = M_{Total} V^{COM}$$



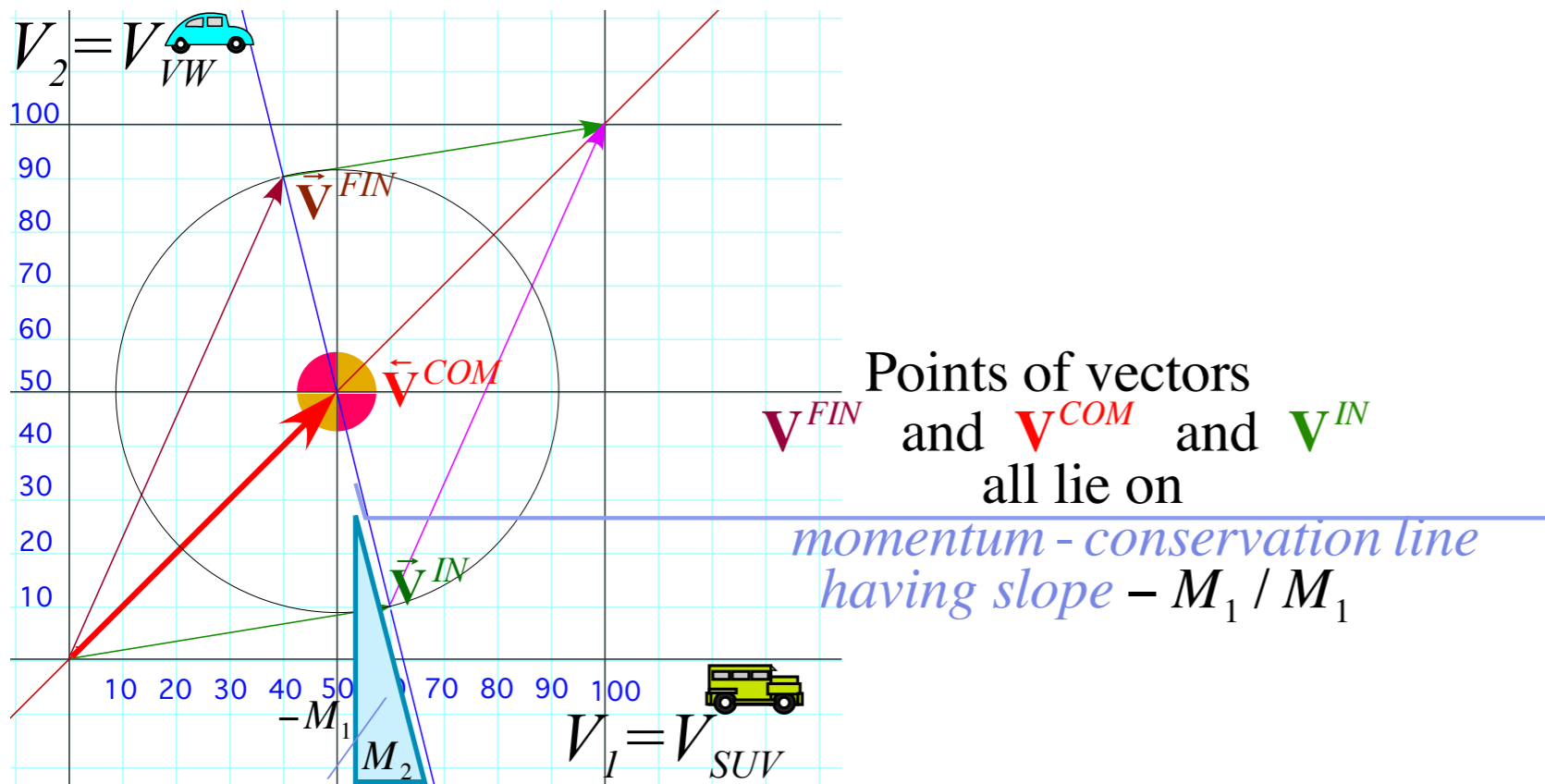
General Inertia Tensor \mathbf{M} or inertia matrix of 2 coefficients $M_{11}=M_1$ and $M_{22}=M_2$ for 2 dimensions

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Generalizing the definition of momentum...

With 45° diagonal \mathbf{V}^{COM} so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = (M_1 + M_2) V^{COM} = M_{Total} V^{COM}$$



General Inertia Tensor \mathbf{M} or inertia matrix of 3 coefficients M_{11} , M_{22} and $M_{12}=M_{21}$ for 2 dimensions

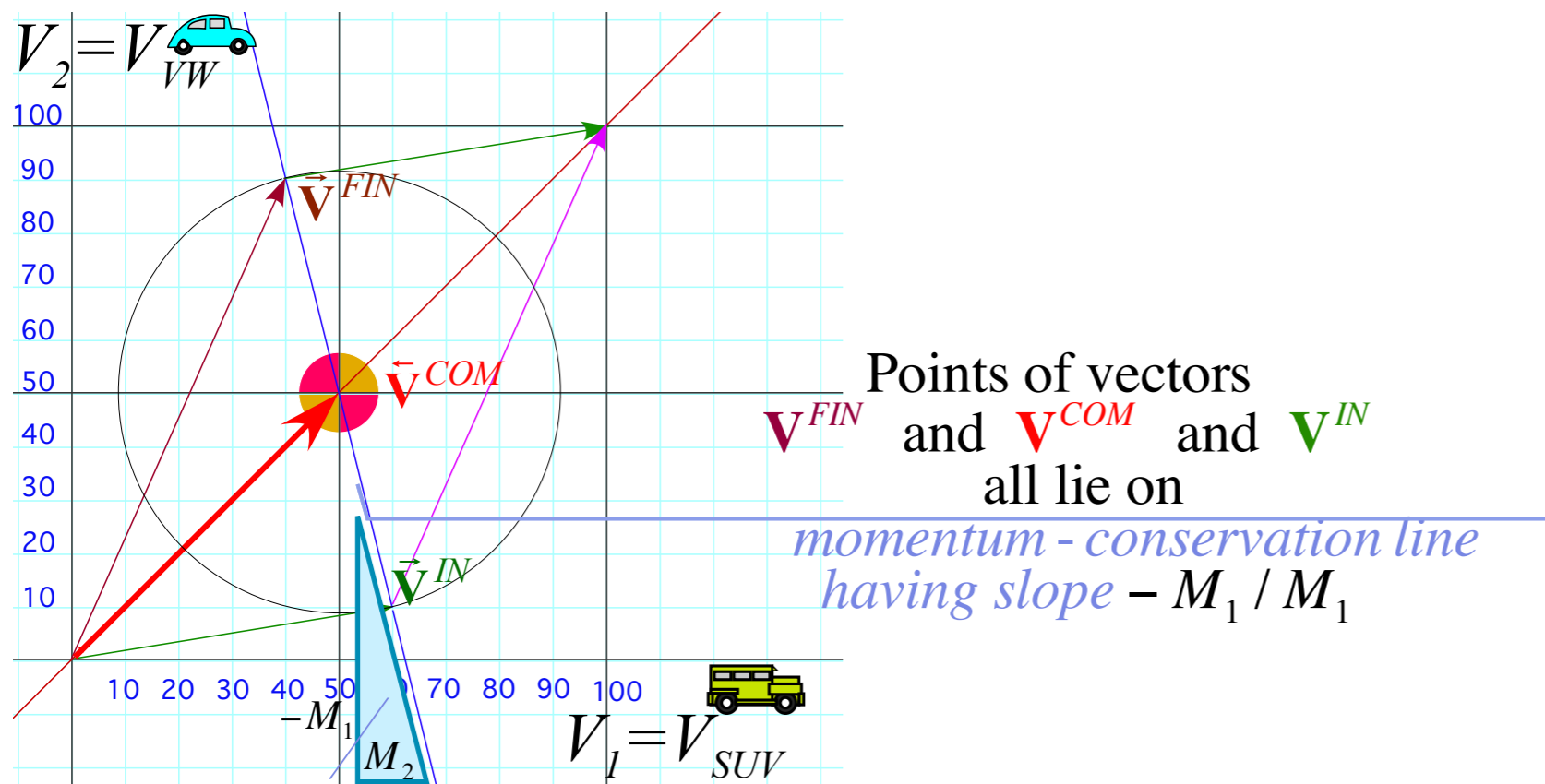
$$\left. \begin{aligned} P_1 &= M_{11}V_1 + M_{12}V_2 \\ P_2 &= M_{21}V_1 + M_{22}V_2 \end{aligned} \right\} \text{denoted : } \vec{\mathbf{P}} = \vec{\mathbf{M}} \cdot \vec{\mathbf{V}} \text{ or : } \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Generalizing the definition of momentum...some more...

With 45° diagonal \mathbf{V}^{COM} so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = (M_1 + M_2) V^{COM} = M_{Total} V^{COM}$$

Here
 $M_{ij} = 0$
 for $i \neq j$



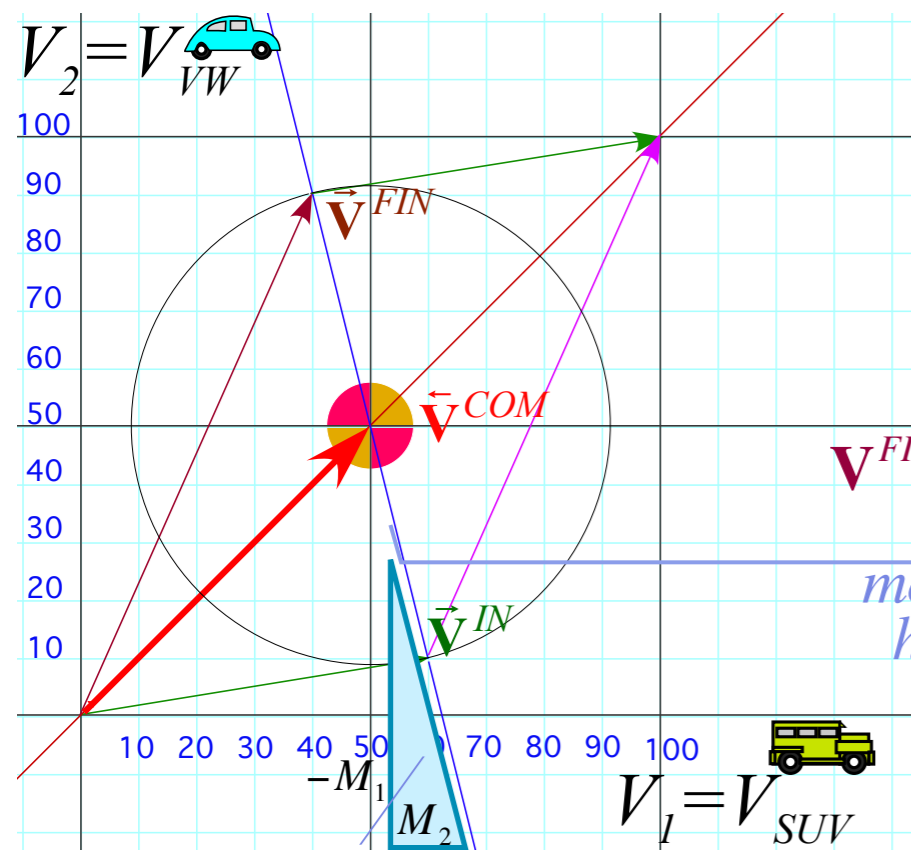
General Inertia Tensor \mathbf{M} or inertia matrix of $(n^2+n)/2$ coefficients $M_{jk} = M_{kj}$ for dimension $n=2, 3, \dots$

$$\left. \begin{aligned} P_1 &= M_{11}V_1 + M_{12}V_2 \dots \\ P_2 &= M_{21}V_1 + M_{22}V_2 \dots \\ \vdots &= \vdots \quad \vdots \quad \ddots \end{aligned} \right\} \begin{array}{l} \text{denoted: } \vec{\mathbf{P}} = \vec{\mathbf{M}} \cdot \vec{\mathbf{V}} \text{ or:} \\ \text{Generalizing the definition} \\ \text{of momentum...some more...and more} \end{array} \left(\begin{array}{c} P_1 \\ P_2 \\ \vdots \end{array} \right) = \left(\begin{array}{ccc} M_{11} & M_{12} & \dots \\ M_{21} & M_{22} & \dots \\ \vdots & \vdots & \ddots \end{array} \right) \left(\begin{array}{c} V_1 \\ V_2 \\ \vdots \end{array} \right)$$

With 45° diagonal $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$ so: $V_1^{COM} = V_2^{COM} \equiv V^{COM}$...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = (M_1 + M_2) V^{COM} = M_{Total} V^{COM}$$

Here
 $M_{ij} = 0$
for $i \neq j$



Points of vectors
 \mathbf{V}^{FIN} and \mathbf{V}^{COM} and \mathbf{V}^{IN}
all lie on
momentum - conservation line
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Algebra, Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

→ *Matrix or tensor algebra of collisions*

Deriving Energy Conservation Theorem

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General Inertia Tensor \mathbf{M} or inertia matrix of 2 coefficients $M_{11}=M_1$ and $M_{22}=M_2$ for 2 dimensions

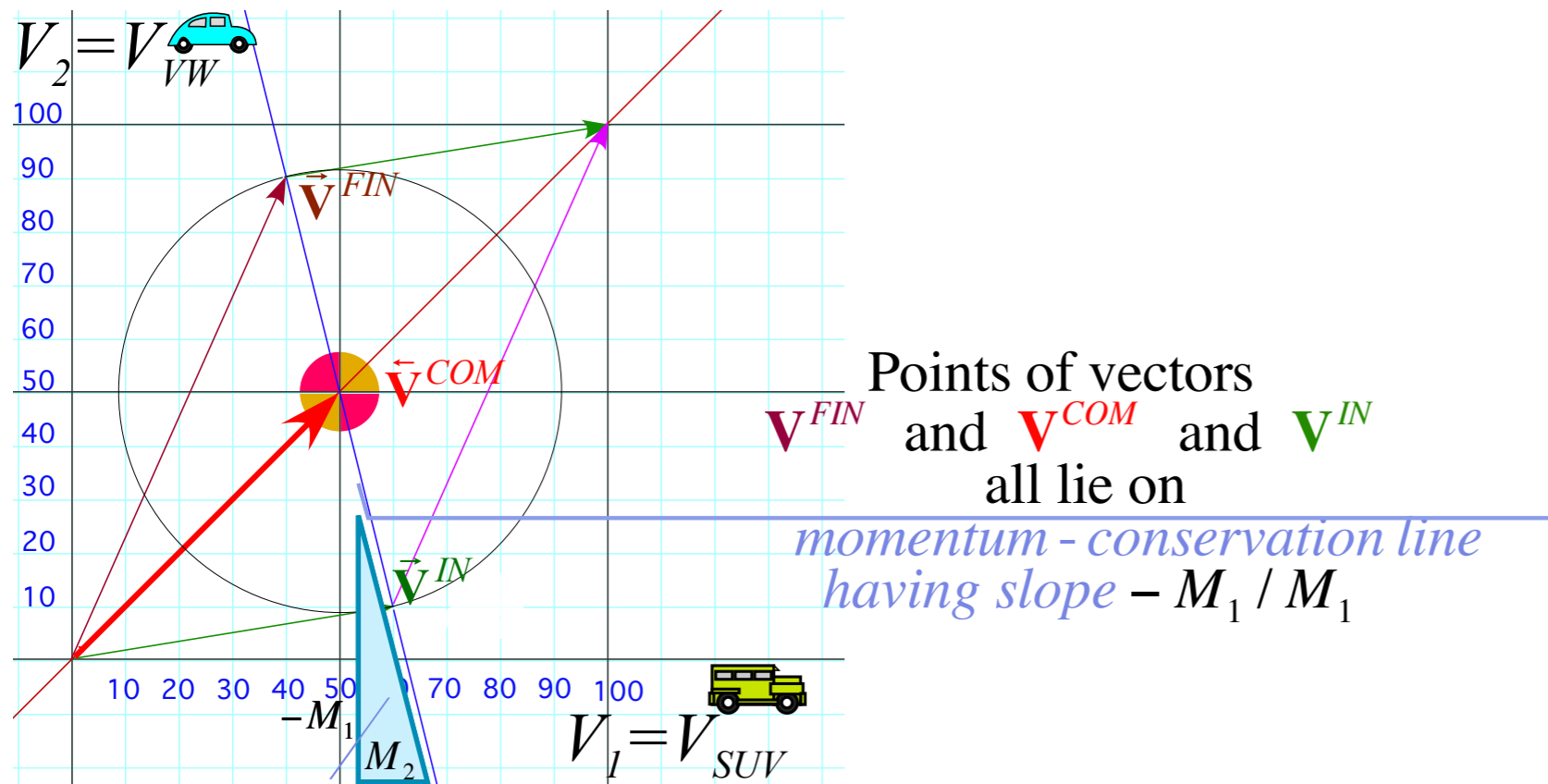
$$\left. \begin{aligned} P_1 &= M_1 V_1 + M_{12} V_2 \\ P_2 &= M_{21} V_1 + M_2 V_2 \end{aligned} \right\} \text{denoted: } \vec{\mathbf{P}} = \vec{\mathbf{M}} \cdot \vec{\mathbf{V}} \text{ or: } \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \quad \begin{array}{l} \text{Here} \\ M_{ij}=0 \\ \text{for } i \neq j \end{array}$$

Generalizing the definition of momentum...

With 45° diagonal $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$ so: $V_1^{COM} = V_2^{COM} \equiv V^{COM}$...and 4 or 5 ways to write Axiom-1

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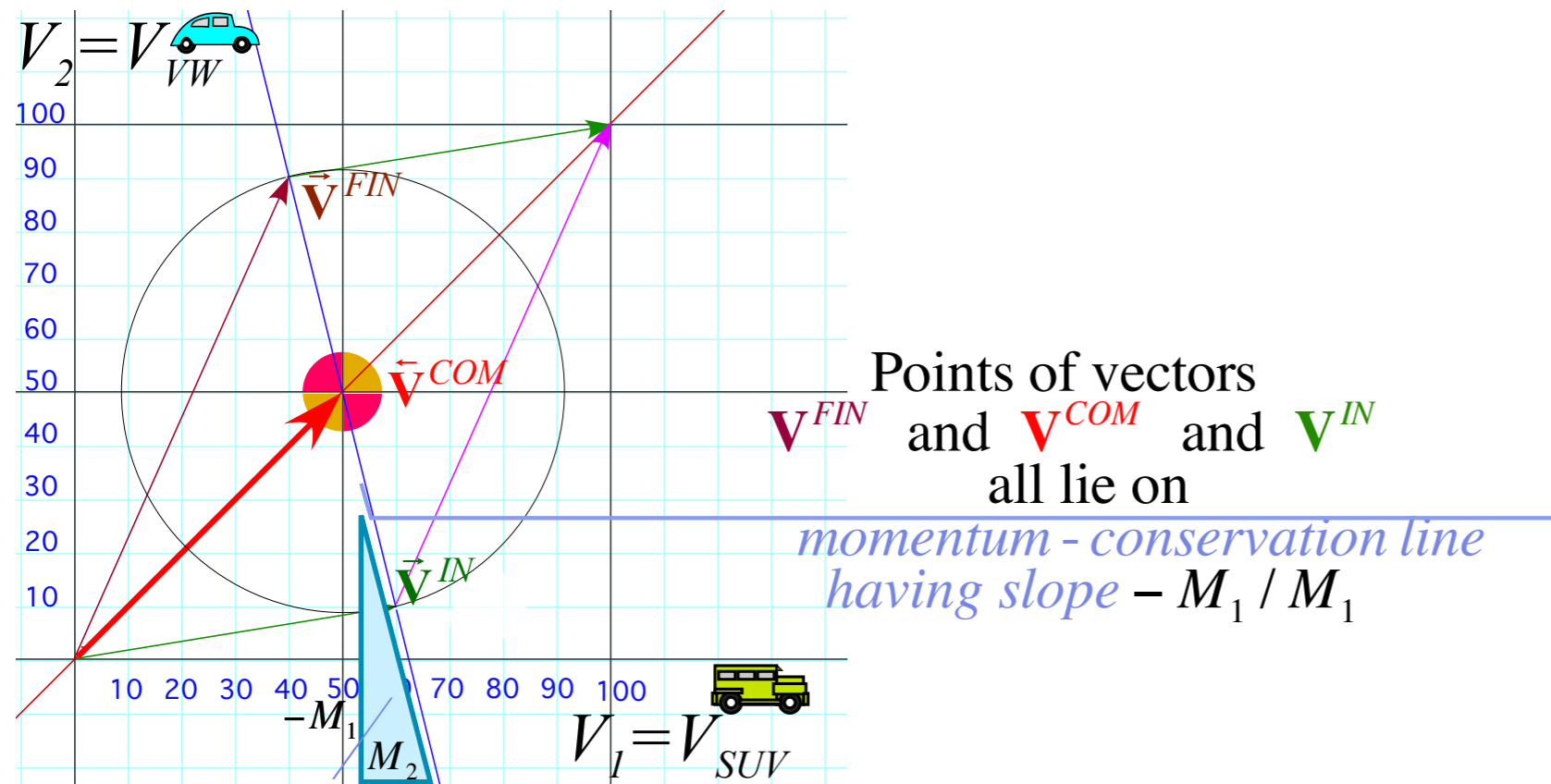
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$$V^{COM} P_{Total} = \vec{\mathbf{V}}^{COM} \cdot \vec{\mathbf{M}} \cdot \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{COM} \cdot \vec{\mathbf{M}} \cdot \vec{\mathbf{V}}^{FIN} = \vec{\mathbf{V}}^{COM} \cdot \vec{\mathbf{M}} \cdot \vec{\mathbf{V}}^{COM} = V^{COM} M_{Total} V^{COM} \quad \begin{array}{l} \text{Here} \\ M_{ij}=0 \\ \text{for } i \neq j \end{array}$$



Numerical details of collision tensor algebra

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Generalizing the definition of momentum...

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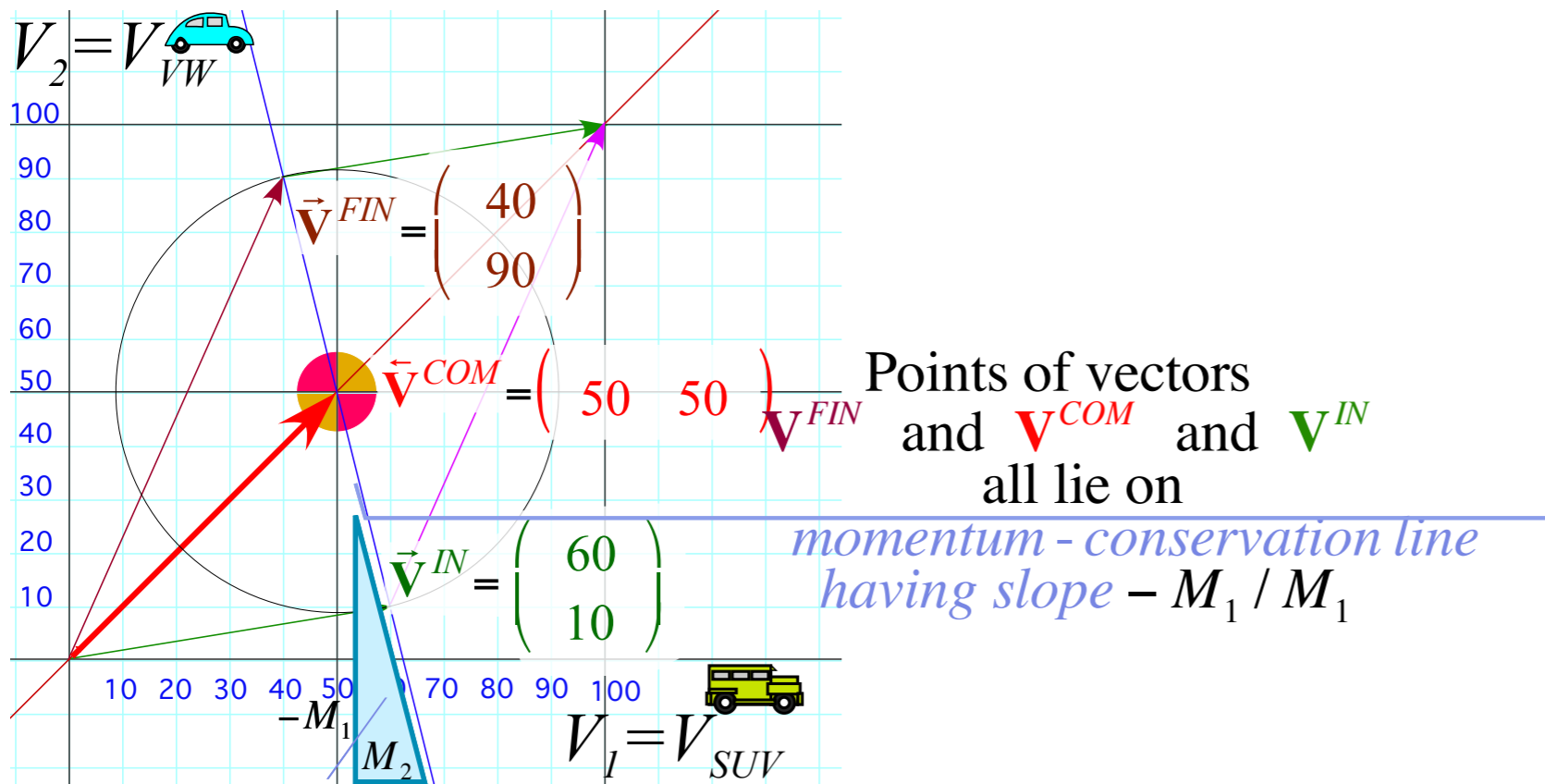
$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = (M_1 + M_2) V^{COM} = M_{Total} V^{COM}$$

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Write this out with the numbers used in Fig. 1.3 where $V^{COM} = 50$.

$$50 P_{Total} = \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} = 50 M_{Total} 50 = 12,500$$



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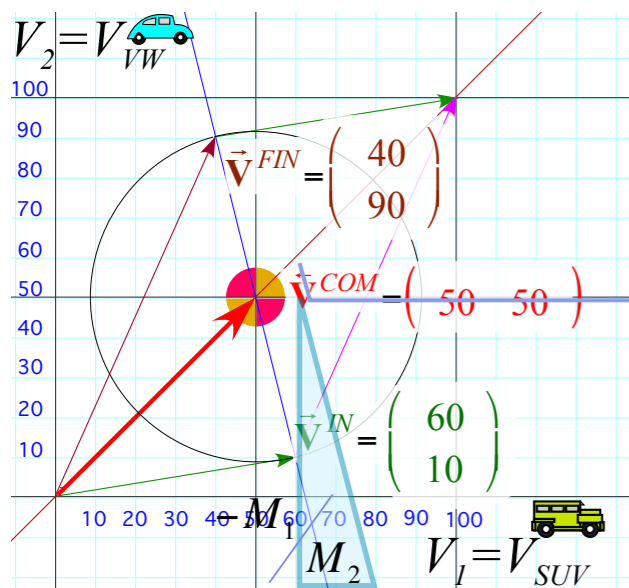
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Points of vectors \mathbf{V}^{FIN} and \mathbf{V}^{COM} and \mathbf{V}^{IN} all lie on momentum - conservation line having slope $-M_1 / M_2$

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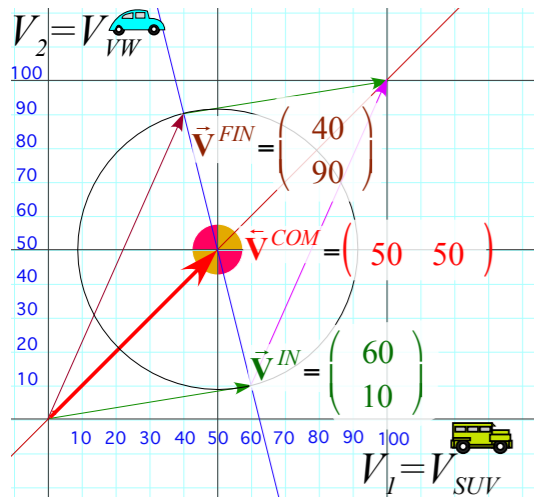
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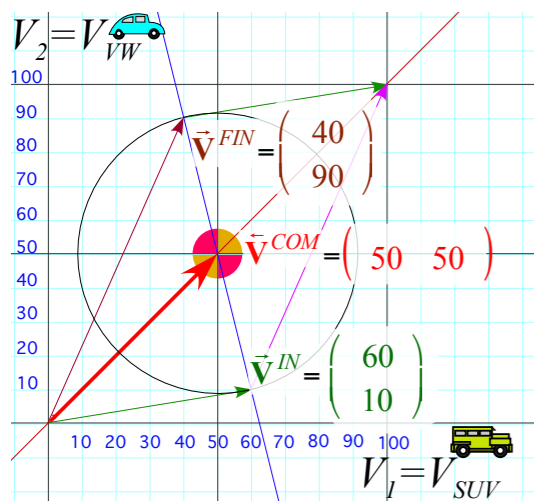
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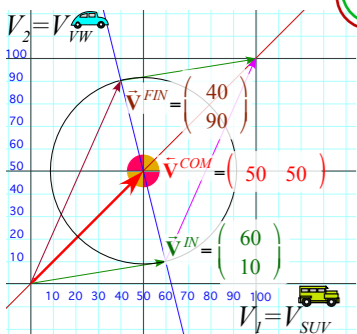
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Transpose symmetry ($M_{jk} = M_{kj}$) of **M**-matrix makes 'lopsided' **FIN-IN**-terms equal:

$$\begin{aligned} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} &= \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{FIN} \\ \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} &= \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ &= 100 \cdot 105 = 100 \cdot 105 = 10,500 \end{aligned}$$



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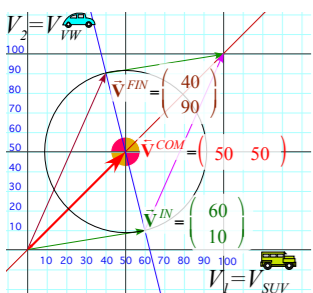
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$$V^{COM} P_{Total} - \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} = \frac{1}{2} \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN} = \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN}$$

FIN-IN-term is subtracted to give

Conservation of Kinetic Energy

$$KE = \frac{1}{2} \vec{V} \cdot \vec{M} \cdot \vec{V} = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2$$



$$\begin{aligned} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} &= \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{FIN} \\ \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} &= \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ &= 100 \cdot 105 = 100 \cdot 105 = 10,500 \end{aligned}$$

General Inertia Tensor M or inertia matrix of 2 coefficients $M_{11}=M_1$ and $M_{22}=M_2$ for 2 dimensions

$$\left. \begin{aligned} P_1 &= M_1 V_1 + M_{12} V_2 \\ P_2 &= M_{21} V_1 + M_2 V_2 \end{aligned} \right\} \text{denoted: } \vec{P} = \vec{M} \cdot \vec{V} \text{ or: } \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \quad \begin{array}{l} \text{Here} \\ M_{ij}=0 \\ \text{for } i \neq j \end{array}$$

Generalizing the definition of momentum...

With 45° diagonal $\vec{V}^{COM} = (\vec{V}^{FIN} + \vec{V}^{IN})/2$ so: $V_1^{COM} = V_2^{COM} \equiv V^{COM}$...and 4 or 5 ways to write **Axiom-1**

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = (M_1 + M_2) V^{COM} = M_{Total} V^{COM}$$

A product of total momentum P_{Total} and V^{COM} is expressed by *tensor quadratic forms* $\mathbf{v} \cdot \mathbf{M} \cdot \mathbf{u}$

$$V^{COM} P_{Total} = \vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{IN} = \vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{FIN} = \vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{COM} = V^{COM} M_{Total} V^{COM}$$

Write this out with the numbers used in Fig. 1.3 where $V^{COM} = 50$.

$$\begin{aligned} 50 P_{Total} &= \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} = 50 M_{Total} 50 = 12,500 \\ &= 100 \cdot 125 = 100 \cdot 125 = 50 \cdot 250 \end{aligned}$$

$P_{Total} = 250$ is the same at **IN**, **FIN**, and **COM**. Now use *T*-symmetry: $\vec{V}^{COM} = (\vec{V}^{FIN} + \vec{V}^{IN})/2$ (**Axiom-2**)

$$\begin{aligned} V^{COM} P_{Total} &= \frac{\vec{V}^{FIN} + \vec{V}^{IN}}{2} \cdot \vec{M} \cdot \vec{V}^{IN} = \frac{\vec{V}^{FIN} + \vec{V}^{IN}}{2} \cdot \vec{M} \cdot \vec{V}^{FIN} = \frac{\vec{V}^{FIN} + \vec{V}^{IN}}{2} \cdot \vec{M} \cdot \frac{\vec{V}^{FIN} + \vec{V}^{IN}}{2} \\ &= \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} + \frac{1}{2} \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN} = \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN} + \frac{1}{2} \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{FIN} \end{aligned}$$

$$V^{COM} P_{Total} - \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} = \frac{1}{2} \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN} = \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN}$$

$$50 \cdot 250 - \frac{1}{2} \cdot 10,500$$

$$\frac{1}{2} \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

FIN-IN-term is subtracted to give

Conservation of Kinetic Energy

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$$\begin{aligned} 50 \cdot 250 &- \frac{1}{2} \cdot 10,500 \\ 12,500 &- 5,250 \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN} &= \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN} \\ \frac{1}{2} \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ = \frac{1}{2} (60 \cdot 4 \cdot 60 + 10 \cdot 1 \cdot 10) &= \frac{1}{2} (40 \cdot 4 \cdot 40 + 90 \cdot 1 \cdot 90) \end{aligned}$$

FIN-IN-term is subtracted to give

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$$V^{COM} P_{Total} - \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} = \frac{1}{2} \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN} = \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN}$$

$$\begin{aligned} 50 \cdot 250 - \frac{1}{2} \cdot 10,500 \\ 12,500 - 5,250 = 7,250 \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN} &= \frac{1}{2} \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} \\ &= \frac{1}{2} (60 \cdot 4 \cdot 60 + 10 \cdot 1 \cdot 10) \\ &= 2 \cdot 3600 + 50 = 7250 \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN} &= \frac{1}{2} \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ &= \frac{1}{2} (40 \cdot 4 \cdot 40 + 90 \cdot 1 \cdot 90) \\ &= 2 \cdot 1600 + \frac{1}{2} 8100 = 7250 \end{aligned}$$

FIN-IN-term is subtracted to give

Conservation of Kinetic Energy

$$\begin{aligned} KE &= \frac{1}{2} \vec{V} \cdot \vec{M} \cdot \vec{V} = \\ &= \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 \end{aligned}$$

Algebra, Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

Matrix or tensor algebra of collisions

Deriving Energy Conservation Theorem

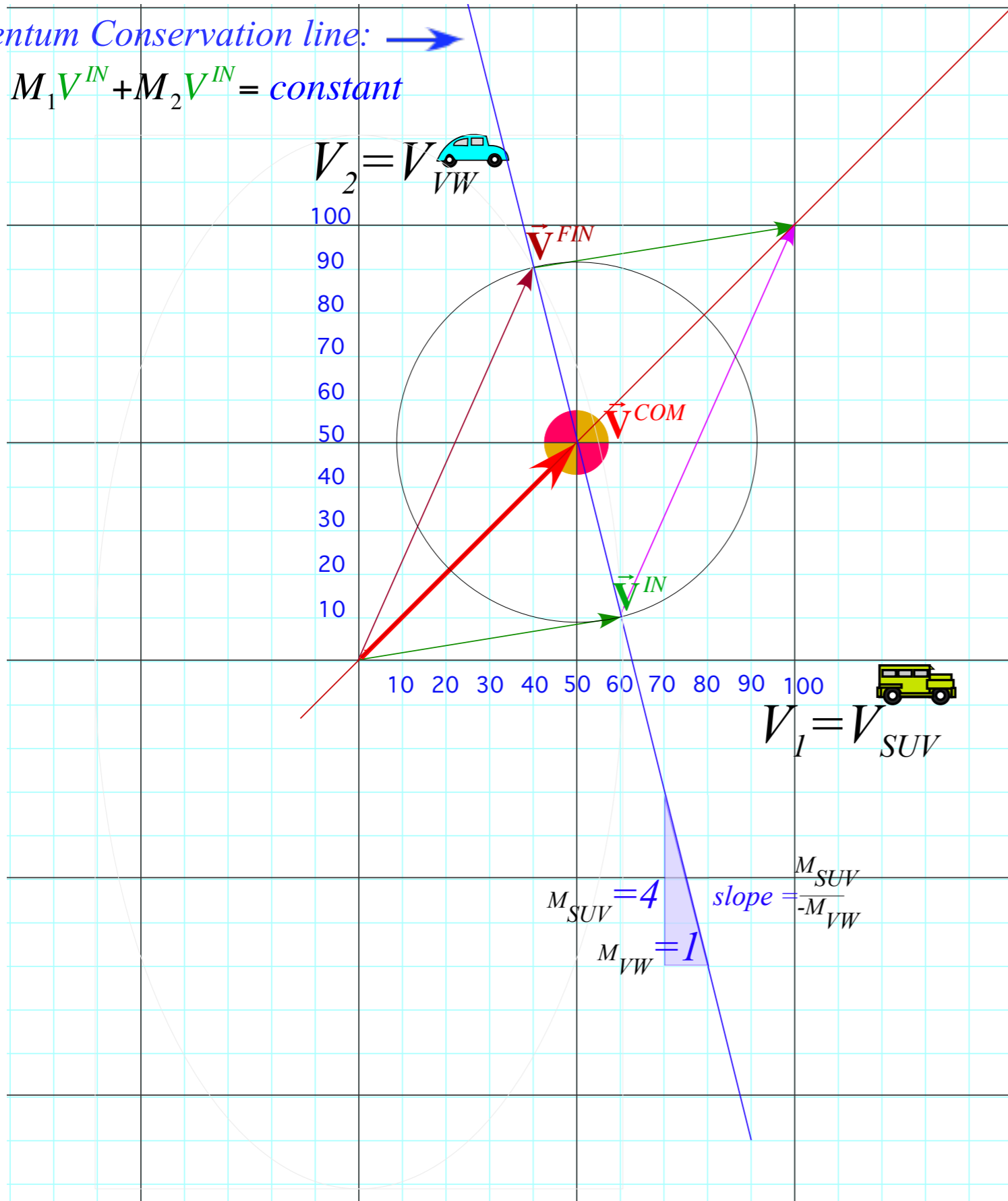
 *Energy Ellipse geometry*

Geometry of Momentum Conservation Axiom -1

$$(M_1 + M_2) \vec{V}^{COM} = M_1 \vec{V}_1^{IN} + M_2 \vec{V}_2^{IN} = M_1 \vec{V}_1^{FIN} + M_2 \vec{V}_2^{FIN}$$

Momentum Conservation line: \rightarrow

$$M_1 \vec{V}_1^{IN} + M_2 \vec{V}_2^{IN} = \text{constant}$$



Collision Web Simulator
Basic elastic Collision
Dual Panel
Space vs Space
 and
 $V(VW)$ vs. $V(SUV)$

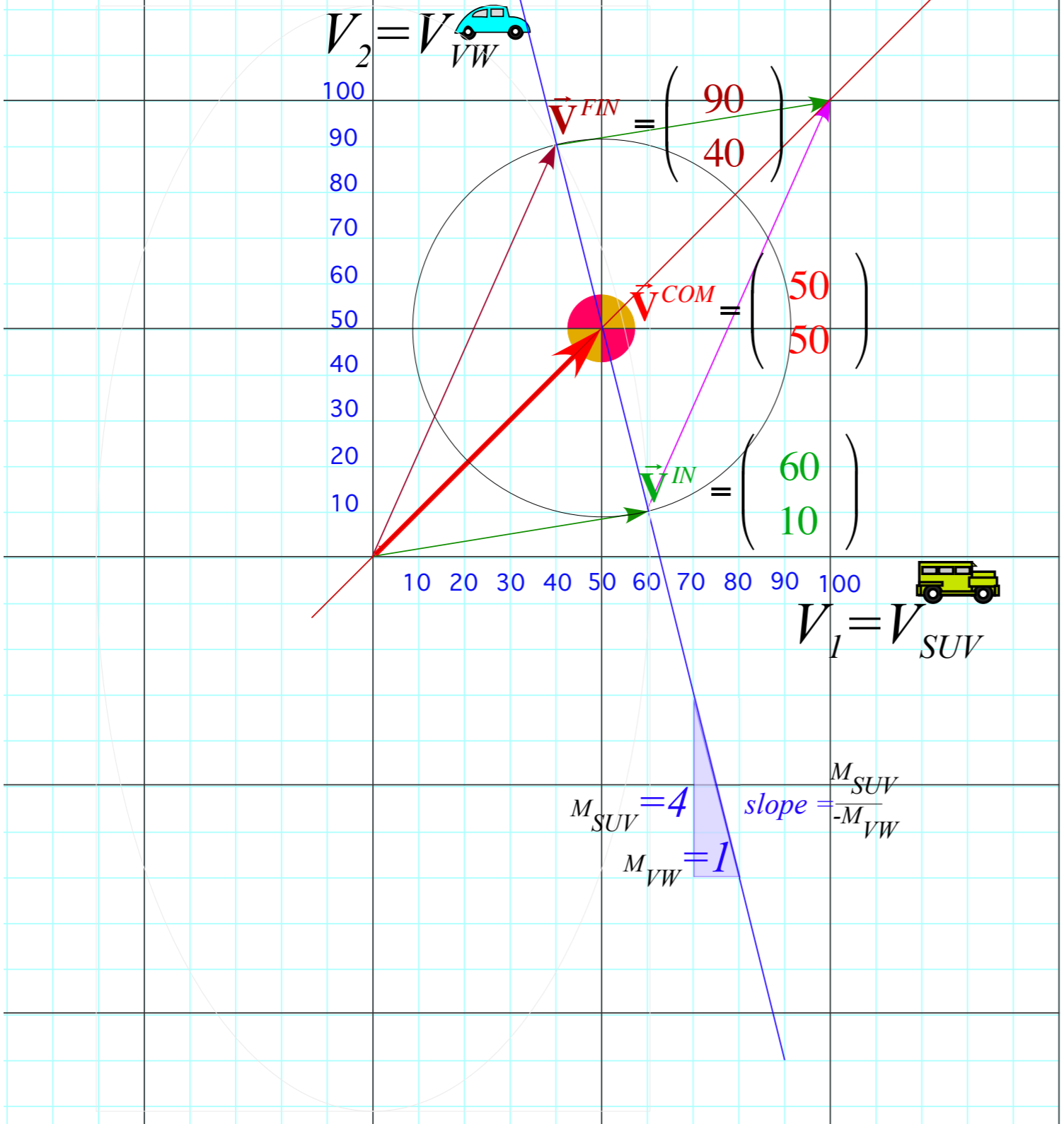
BounceIt
Superball Web Simulator
Basic elastic Collision
Dual Panel
Space vs Space
 and
 $V(VW)$ vs. $V(SUV)$

Geometry of Momentum Conservation Axiom - 1

$$(M_1 + M_2) \vec{V}^{COM} = M_1 \vec{V}_1^{IN} + M_2 \vec{V}_2^{IN} = M_1 \vec{V}_1^{FIN} + M_2 \vec{V}_2^{FIN}$$

Momentum Conservation line: \rightarrow

$$M_1 \vec{V}_1^{IN} + M_2 \vec{V}_2^{IN} = \text{constant}$$



Collision Web Simulator
Basic elastic Collision
Dual Panel
Space vs Space
 and
V(VW) vs. V(SUV)

BounceIt
Superball Web Simulator
Basic elastic Collision
Dual Panel
Space vs Space
 and
V(VW) vs. V(SUV)

Geometry of Momentum Conservation Axiom-1

Geometry of Kinetic Energy Conservation Axiom-2/Theorem-1

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

$$KE^{IN} = \frac{1}{2}M_1(V_1^{IN})^2 + \frac{1}{2}M_2(V_2^{IN})^2 = \frac{1}{2}M_1(V_1^{FIN})^2 + \frac{1}{2}M_2(V_2^{FIN})^2 = KE^{FIN}$$

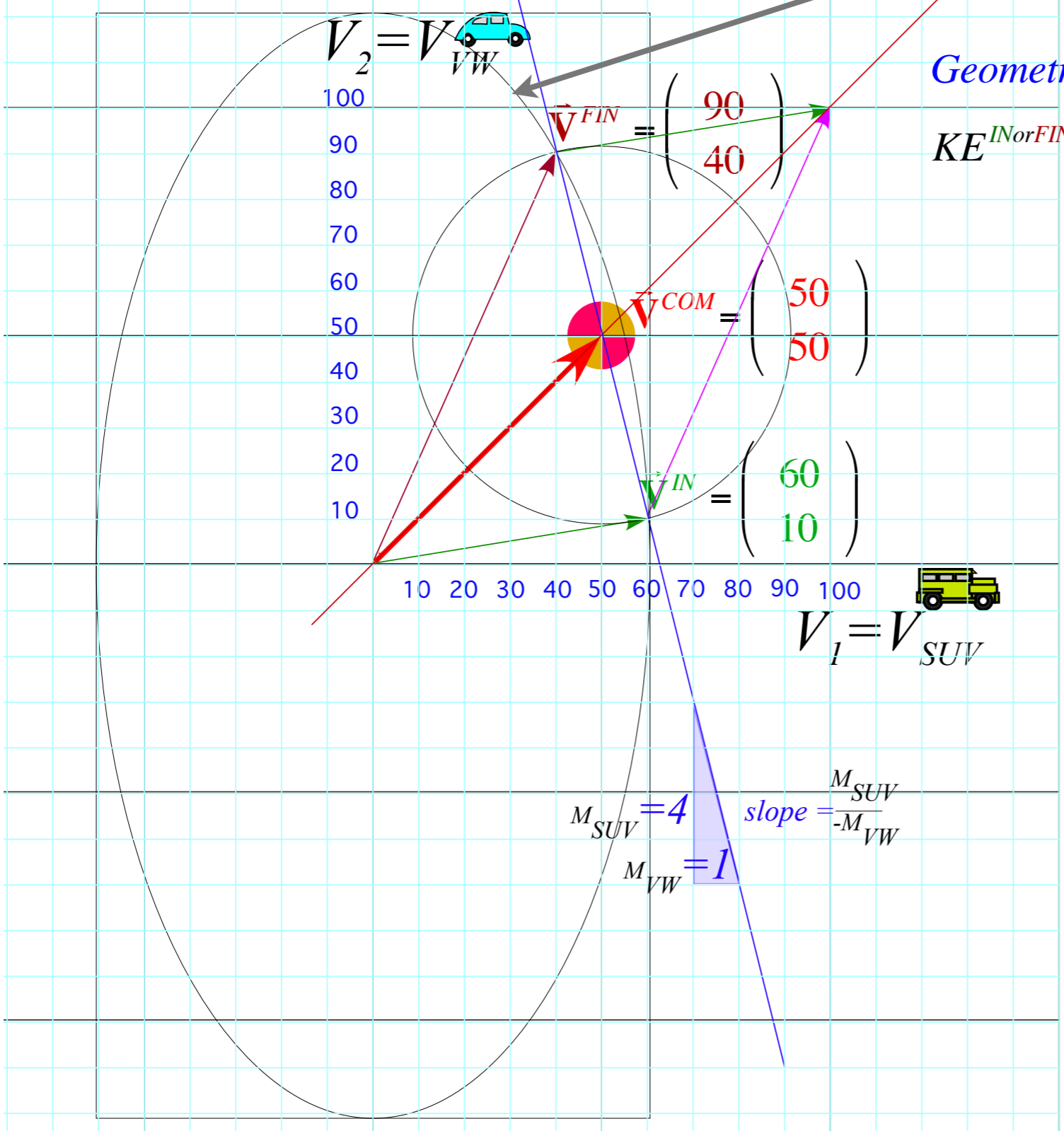
Momentum Conservation line: \rightarrow

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$

KE Conservation ellipse:

Geometry of KE Conservation Theorem -1

$$KE^{INorFIN} = \frac{1}{2}M_1(V_1)^2 + \frac{1}{2}M_2(V_2)^2 = \frac{1}{2}4(60)^2 + \frac{1}{2}1(10)^2 = 7,250$$



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 V(VW) vs. V(SUV)

Geometry of Momentum Conservation Axiom-1

Geometry of Kinetic Energy Conservation Axiom-2/Theorem-1

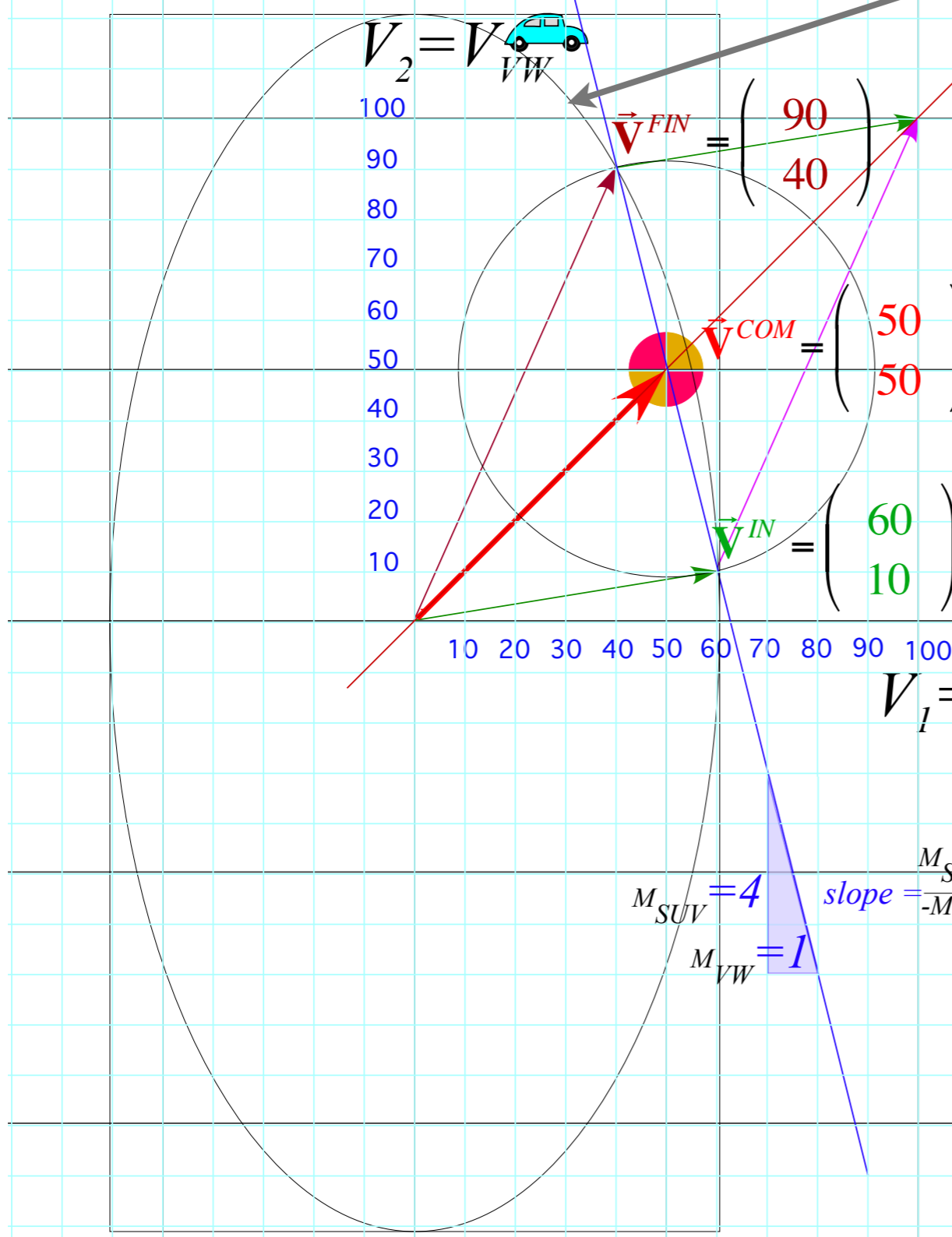
$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

$$KE^{IN} = \frac{1}{2}M_1(V_1^{IN})^2 + \frac{1}{2}M_2(V_2^{IN})^2 = \frac{1}{2}M_1(V_1^{FIN})^2 + \frac{1}{2}M_2(V_2^{FIN})^2 = KE^{FIN}$$

Momentum Conservation line: \rightarrow

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$

KE Conservation ellipse:



Geometry of KE Conservation Theorem -1

$$KE^{INorFIN} = \frac{1}{2}M_1(V_1)^2 + \frac{1}{2}M_2(V_2)^2 = \frac{1}{2}4(60)^2 + \frac{1}{2}1(10)^2 = 7,250$$

$$\text{or : } 1 = \frac{V_1^2}{\left(\frac{2 \cdot KE^{IFN}}{M_1}\right)} + \frac{V_2^2}{\left(\frac{2 \cdot KE^{IFN}}{M_2}\right)} = \frac{V_1^2}{\left(\sqrt{\frac{2 \cdot KE^{IFN}}{M_1}}\right)^2} + \frac{V_2^2}{\left(\sqrt{\frac{2 \cdot KE^{IFN}}{M_2}}\right)^2}$$

Collision Web Simulator
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Geometry of Momentum Conservation Axiom-1

Geometry of Kinetic Energy Conservation Axiom-2/Theorem-1

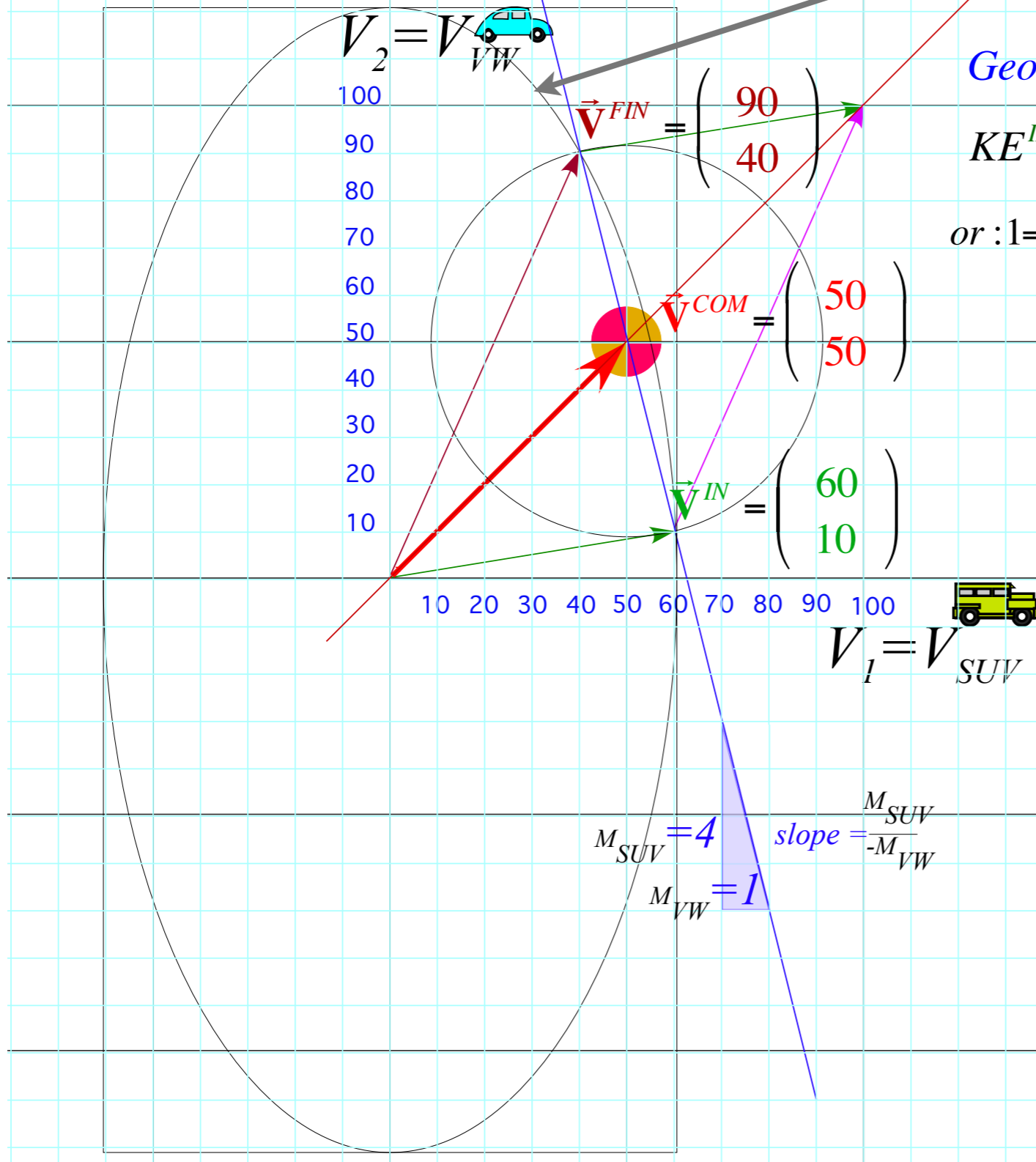
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Momentum Conservation line: \rightarrow

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KE Conservation ellipse:



Geometry of KE Conservation Theorem -1

$$KE^{INorFIN} = \frac{1}{2}M_1(V_1)^2 + \frac{1}{2}M_2(V_2)^2 = \frac{1}{2}4(60)^2 + \frac{1}{2}1(10)^2 = 7,250$$

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Has the algebraic form: $1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

[Collision Web Simulator](#)

[Basic elastic Collision](#)

[Dual Panel](#)

[Space vs Space](#)

and

[V\(VW\) vs. V\(SUV\)](#)

[BounceIt](#)

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[Basic elastic Collision](#)

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Geometry of Momentum Conservation Axiom-1

Geometry of Kinetic Energy Conservation Axiom-2/Theorem-1

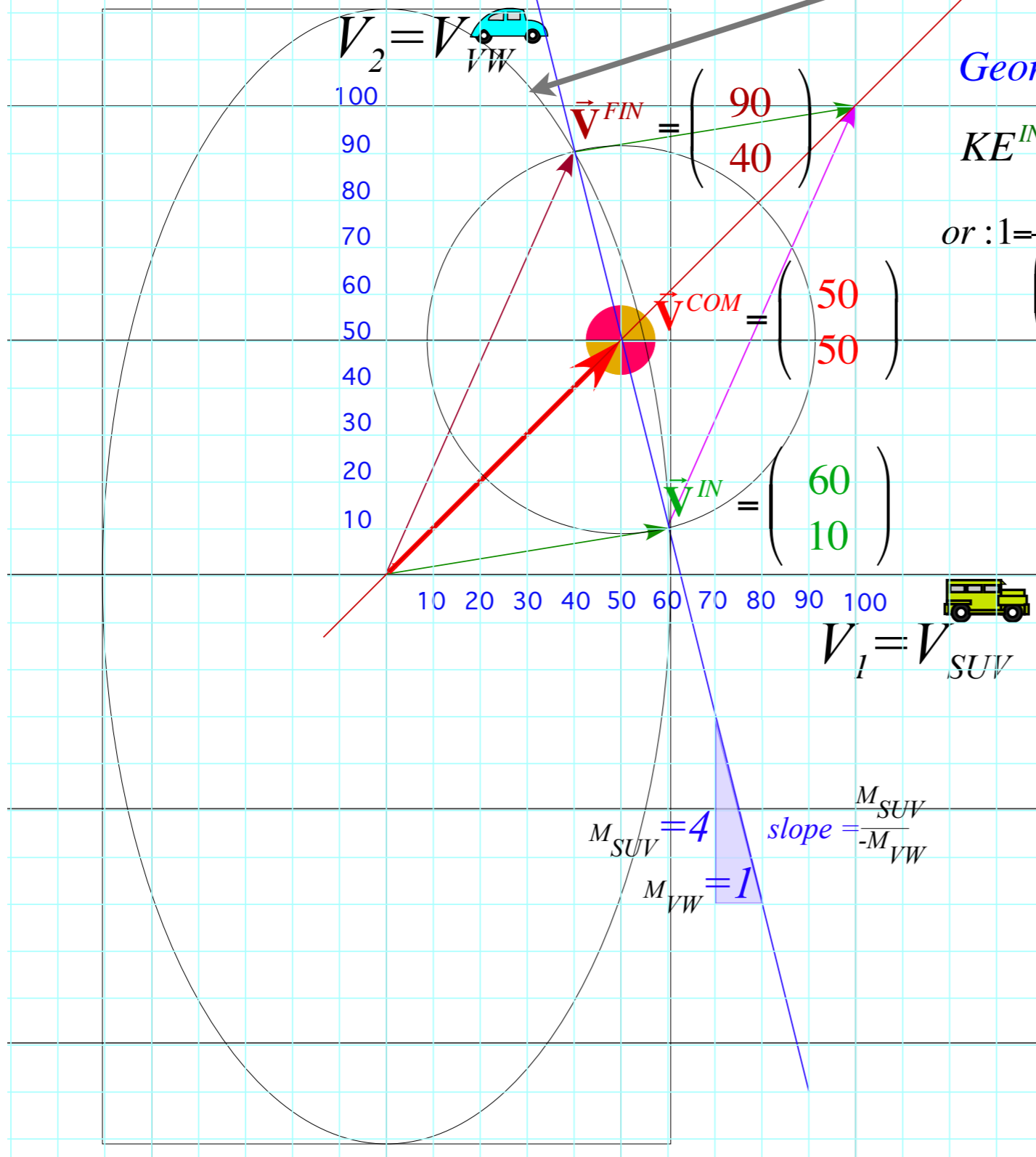
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$$KE^{IN} = \frac{1}{2}M_1(V_1^{IN})^2 + \frac{1}{2}M_2(V_2^{IN})^2 = \frac{1}{2}M_1(V_1^{FIN})^2 + \frac{1}{2}M_2(V_2^{FIN})^2 = KE^{FIN}$$

Momentum Conservation line: \rightarrow

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$

KE Conservation ellipse:



Geometry of KE Conservation Theorem -1

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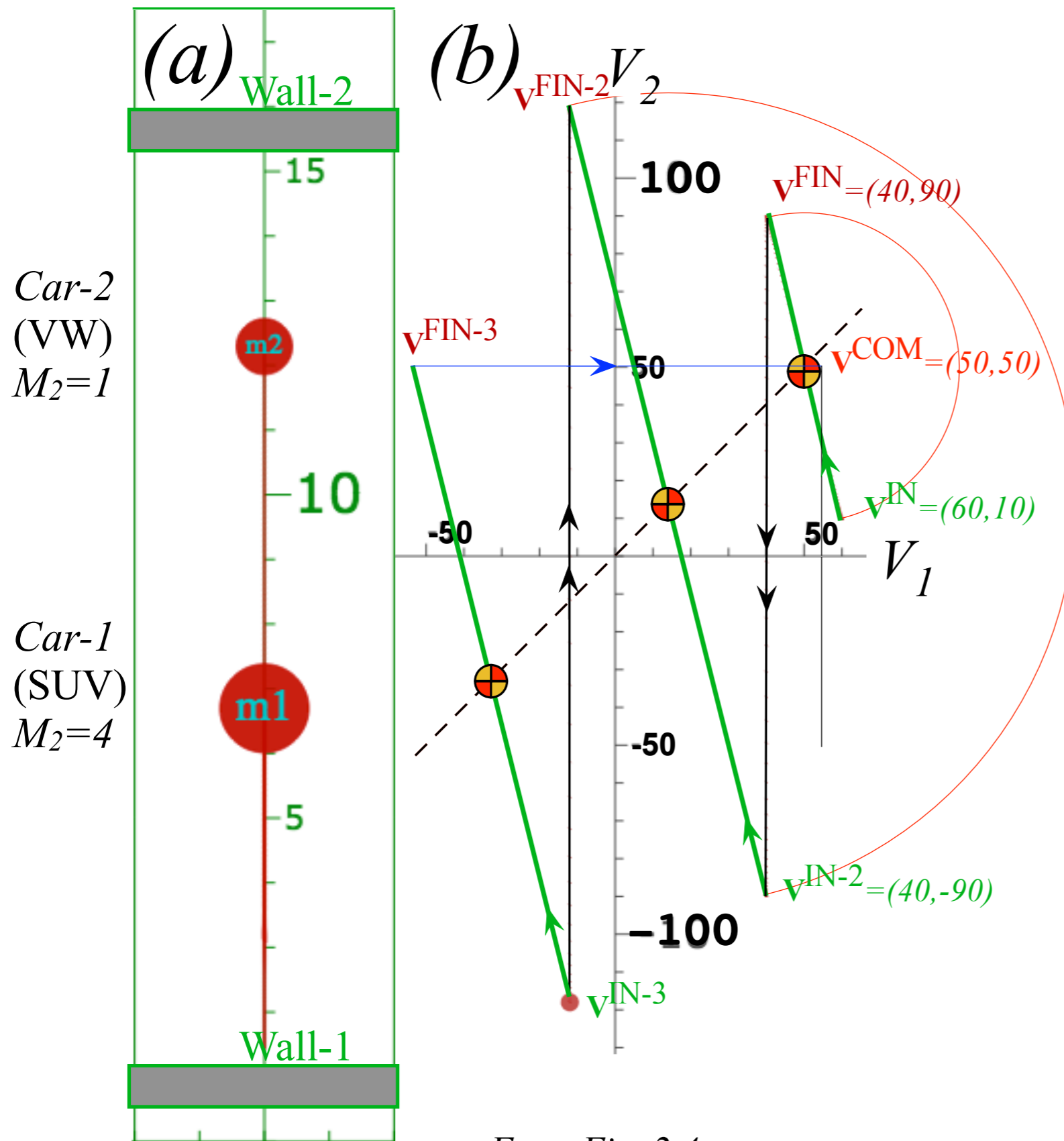
Has the algebraic form: $1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

$$\text{elliptic radii : } a = \sqrt{\frac{2KE^{INorFIN}}{M_1}} \quad b = \sqrt{\frac{2KE^{INorFIN}}{M_2}}$$

$$= \sqrt{\frac{2 \cdot 7,250}{4}} \quad = \sqrt{\frac{2 \cdot 7,250}{1}}$$

$$= 60.21 \quad = 120.42$$

BounceIt Simulation: frictionless 1D-track with elastic bumper cars bouncing between walls



From Fig. 2.4

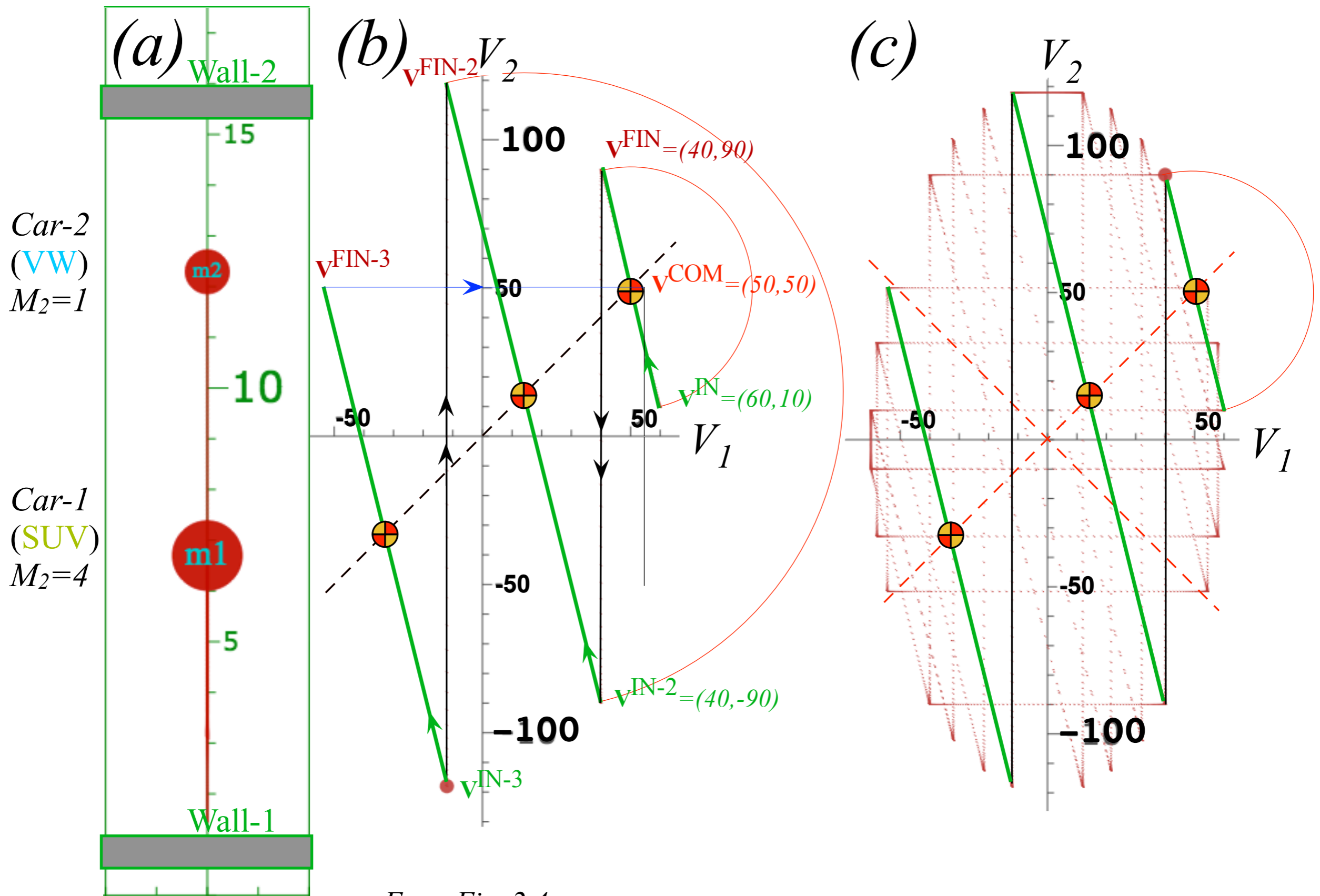
BounceIt
Superball Web Simulator

Basic elastic Collision
Dual Panel
Space vs Space
and
 $V(VW)$ vs. $V(SUV)$

Repeated elastic Collisions
Dual Panel
Space vs Space
and
 $V(VW)$ vs. $V(SUV)$

Collision Web Simulator
Basic elastic Collision
Dual Panel
Space vs Space
and
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BounceIt Simulation: frictionless 1D-track with elastic bumper cars bouncing between walls



From Fig. 2.4

Transpose symmetry ($M_{jk} = M_{kj}$) of the \mathbf{M} -matrix implies: $\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} = \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{FIN}$

$$\begin{aligned} \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} &= \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ &= 100 \cdot 105 = 100 \cdot 105 = 10,500 \end{aligned}$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \vec{V} \cdot \vec{M} \cdot \vec{V}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$\begin{aligned} V^{COM} P_{Total} - \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{2} &= \frac{\vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN}}{2} = \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN}}{2} = KE^{INorFIN} \\ 12,500 - \frac{10,500}{2} &= \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN} \\ 12,500 - 5,250 &= 7,250 = 7,250 \end{aligned}$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the \mathbf{M} -matrix implies: $\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} = \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{FIN}$

$$\begin{aligned} \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} &= \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ &= 100 \cdot 105 = 100 \cdot 105 = 10,500 \end{aligned}$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \vec{V} \cdot \vec{M} \cdot \vec{V}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$\begin{aligned} V^{COM} P_{Total} - \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{2} &= \frac{\vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN}}{2} = \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN}}{2} = KE^{INorFIN} \\ 12,500 - \frac{10,500}{2} &= \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN} \\ 12,500 - 5,250 &= 7,250 = 7,250 \end{aligned}$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from *7,250* to *6,250*.

$$\begin{aligned} KE^{COM} &= \frac{1}{2} \vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{COM} = \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 50 \\ 50 \end{pmatrix} \\ &= \frac{1}{2} (50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \frac{1}{2} \cdot 2500 \\ &= 5000 + \frac{1}{2} \cdot 1250 = 6,250 \end{aligned}$$

Geometry of Momentum Conservation Axiom-1

Geometry of Kinetic Energy Conservation Axiom-2/Theorem-1

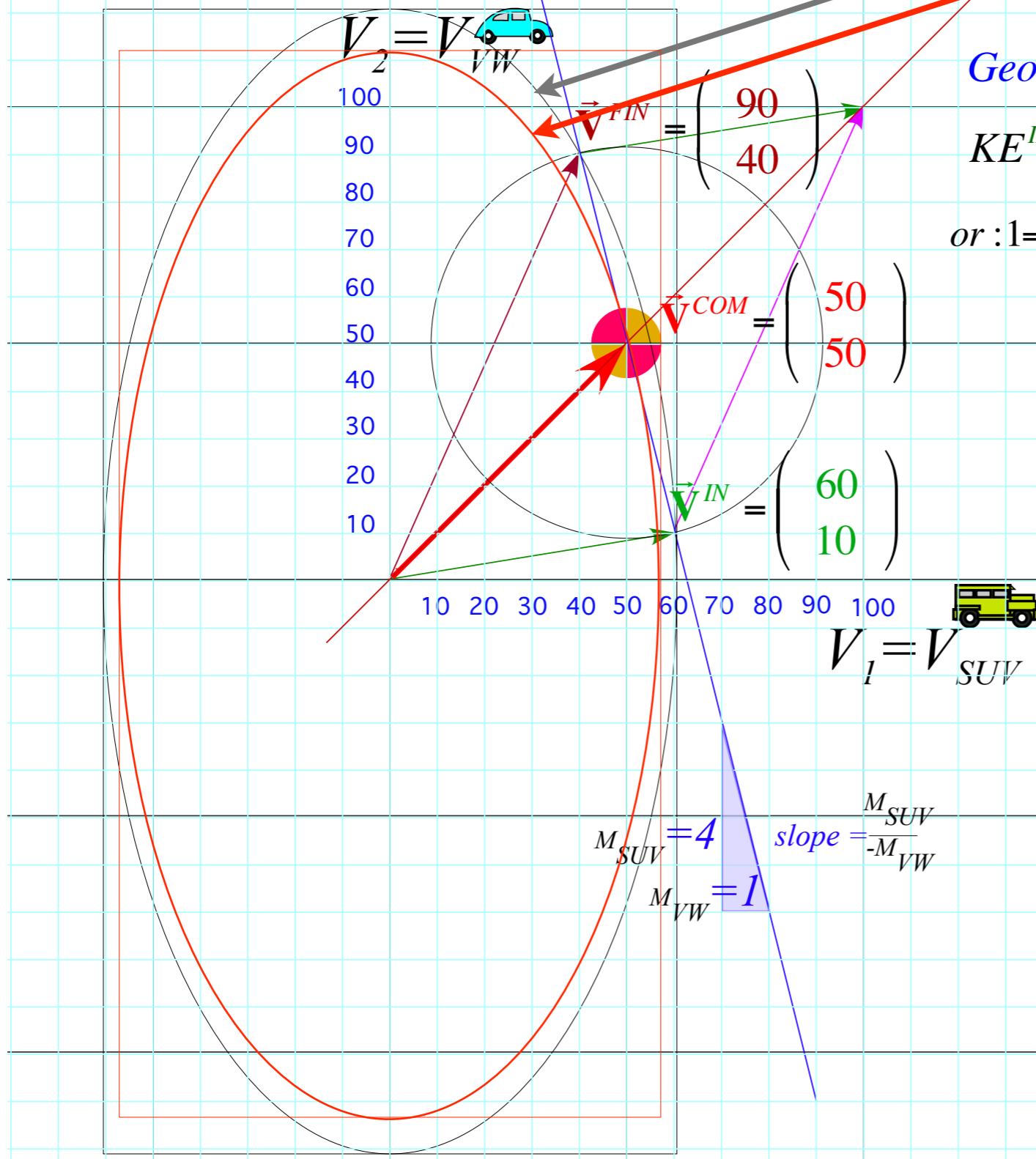
$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

$$KE^{IN} = \frac{1}{2}M_1(V_1^{IN})^2 + \frac{1}{2}M_2(V_2^{IN})^2 = \frac{1}{2}M_1(V_1^{FIN})^2 + \frac{1}{2}M_2(V_2^{FIN})^2 = KE^{FIN}$$

Momentum Conservation line: \rightarrow

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$

KE^{IN} or KE^{FIN} Conservation ellipse:
 KE^{COM} Ka-runch ellipse:



Geometry of KE Conservation Theorem-1

$$KE^{INorFIN} = \frac{1}{2}M_1(V_1)^2 + \frac{1}{2}M_2(V_2)^2$$

$$\text{or : } 1 = \frac{V_1^2}{\left(\frac{2 \cdot KE^{IFN}}{M_1}\right)} + \frac{V_2^2}{\left(\frac{2 \cdot KE^{IFN}}{M_2}\right)} = \frac{V_1^2}{\left(\sqrt{\frac{2 \cdot KE^{IFN}}{M_1}}\right)^2} + \frac{V_2^2}{\left(\sqrt{\frac{2 \cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Geometry of KE^{COM} at Center Of Momentum

$$KE^{COM} = \frac{1}{2}M_1(V_1^{COM})^2 + \frac{1}{2}M_2(V_2^{COM})^2$$

$$= \frac{1}{2}(M_1+M_2)(V^{COM})^2$$

elliptic radii: $a^{COM} = \sqrt{\frac{KE^{COM}}{M_1}}$ $b^{COM} = \sqrt{\frac{KE^{COM}}{M_2}}$

$$= \sqrt{\frac{2 \cdot 6,250}{4}}$$

$$= 55.90$$

$$= \sqrt{\frac{2 \cdot 6,250}{1}}$$

$$= 111.80$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the \mathbf{M} -matrix implies: $\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} = \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{FIN}$

$$\begin{aligned} \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} &= \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ &= 100 \cdot 105 = 100 \cdot 105 = 10,500 \end{aligned}$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \vec{V} \cdot \vec{M} \cdot \vec{V}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$\begin{aligned} V^{COM} P_{Total} - \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{2} &= \frac{\vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN}}{2} = \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN}}{2} = KE^{INorFIN} \\ 12,500 - \frac{10,500}{2} &= \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN} \\ 12,500 - 5,250 &= 7,250 = 7,250 \end{aligned}$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by $1,000$ from $7,250$ to $6,250$.

$$\begin{aligned} KE^{COM} = V^{COM} P_{Total} - \frac{\vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{COM}}{2} &= \frac{1}{2} V^{COM} P_{Total} = \frac{\vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{COM}}{2} = \frac{\vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN}}{4} + \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{4} = \frac{1}{2} KE^{INorFIN} + \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{4} \\ 12,500 - \frac{12,500}{2} &= 6,250 = 6,250 = 3,625 + 2,625 = 3,625 + 2,625 \\ KE^{COM} = \frac{1}{2} \vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{COM} &= \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 50 \\ 50 \end{pmatrix} \\ &= \frac{1}{2} (50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \frac{1}{2} \cdot 2500 \\ &= 5000 + \frac{1}{2} \cdot 1250 = 6,250 \end{aligned}$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the \mathbf{M} -matrix implies: $\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN} = \vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{FIN}$

$$\begin{aligned} \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} &= \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ &= 100 \cdot 105 = 100 \cdot 105 = 10,500 \end{aligned}$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \vec{V} \cdot \vec{M} \cdot \vec{V}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$\begin{aligned} V^{COM} P_{Total} - \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{2} &= \frac{\vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN}}{2} = \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{FIN}}{2} = KE^{INorFIN} \\ 12,500 - \frac{10,500}{2} &= \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN} \\ 12,500 - 5,250 &= 7,250 = 7,250 \end{aligned}$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by $1,000$ from $7,250$ to $6,250$.

$$\begin{aligned} KE^{COM} = V^{COM} P_{Total} - \frac{\vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{COM}}{2} &= \frac{1}{2} V^{COM} P_{Total} = \frac{\vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{COM}}{2} = \frac{\vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN}}{4} + \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{4} = \frac{1}{2} KE^{INorFIN} + \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{4} \\ 12,500 - \frac{12,500}{2} &= 6,250 = 6,250 = 3,625 + 2,625 = 3,625 + 2,625 \end{aligned}$$

$$\begin{aligned} KE^{COM} = \frac{1}{2} \vec{V}^{COM} \cdot \vec{M} \cdot \vec{V}^{COM} &= \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 50 \\ 50 \end{pmatrix} \\ &= \frac{1}{2} (50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \frac{1}{2} \cdot 2500 \\ &= 5000 + \frac{1}{2} \cdot 1250 = 6,250 \end{aligned}$$

Introducing
Potential Energy = PE

Difference is inelastic "ka-Runch" $KE^{INorFIN} - KE^{COM}$. For elastic "ka-Bong" the $1,000$ is PE^{COM} of compression.

$$\begin{aligned} KE^{INorFIN} - KE^{COM} &= \frac{\vec{V}^{IN} \cdot \vec{M} \cdot \vec{V}^{IN}}{4} - \frac{\vec{V}^{FIN} \cdot \vec{M} \cdot \vec{V}^{IN}}{4} \\ 1,000 &= 3,625 - 2,625 \end{aligned}$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the \mathbf{M} -matrix implies: $\tilde{\mathbf{V}}^{FIN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN} = \tilde{\mathbf{V}}^{IN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{FIN}$

$$\begin{aligned} \begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix} &= \begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix} \\ &= 100 \cdot 105 = 100 \cdot 105 = 10,500 \end{aligned}$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$\begin{aligned} V^{COM} P_{Total} - \frac{\tilde{\mathbf{V}}^{FIN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{2} &= \frac{\tilde{\mathbf{V}}^{IN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{2} = \frac{\tilde{\mathbf{V}}^{FIN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN} \\ 12,500 - \frac{10,500}{2} &= \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN} \\ 12,500 - 5,250 &= 7,250 = 7,250 \end{aligned}$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by $1,000$ from $7,250$ to $6,250$.

$$\begin{aligned} KE^{COM} = V^{COM} P_{Total} - \frac{\tilde{\mathbf{V}}^{COM} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{COM}}{2} &= \frac{1}{2} V^{COM} P_{Total} = \frac{\tilde{\mathbf{V}}^{COM} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{COM}}{2} = \frac{\tilde{\mathbf{V}}^{IN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{4} + \frac{\tilde{\mathbf{V}}^{FIN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE^{INorFIN} + \frac{\tilde{\mathbf{V}}^{FIN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{4} \\ 12,500 - \frac{12,500}{2} &= 6,250 = 6,250 = 3,625 + 2,625 = 3,625 + 2,625 \end{aligned}$$

$$\begin{aligned} KE^{COM} = \frac{1}{2} \tilde{\mathbf{V}}^{COM} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{COM} &= \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 50 \\ 50 \end{pmatrix} \\ &= \frac{1}{2} (50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \frac{1}{2} \cdot 2500 \\ &= 5000 + \frac{1}{2} \cdot 1250 = 6,250 \end{aligned}$$

Introducing
Potential Energy = PE

Difference is inelastic "ka-Runch" $KE^{INorFIN} - KE^{COM}$. For elastic "ka-Bong" the $1,000$ is PE^{COM} of compression.

$$\begin{aligned} KE^{INorFIN} - KE^{COM} &= \frac{\tilde{\mathbf{V}}^{IN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{4} - \frac{\tilde{\mathbf{V}}^{FIN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{4} & KE^{COM} &= \frac{\tilde{\mathbf{V}}^{IN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{4} + \frac{\tilde{\mathbf{V}}^{FIN} \cdot \tilde{\mathbf{M}} \cdot \tilde{\mathbf{V}}^{IN}}{4} \\ 1,000 &= 3,625 - 2,625 & 6,250 &= 3,625 + 2,625 \end{aligned}$$

Difference $KE^{INorFIN} - KE^{COM} = 1,000$ is the same in *all* frames including *COM*-frame where $\mathbf{V}^{COM} = \mathbf{0}$.

Geometry of Momentum Conservation Axiom-1

Geometry of Kinetic Energy Conservation Axiom-2/Theorem-1

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

$$KE^{IN} = \frac{1}{2}M_1(V_1^{IN})^2 + \frac{1}{2}M_2(V_2^{IN})^2 = \frac{1}{2}M_1(V_1^{FIN})^2 + \frac{1}{2}M_2(V_2^{FIN})^2 = KE^{FIN}$$

Momentum Conservation line: \rightarrow

$$M_1V_1^{IN} + M_2V_2^{IN} = \text{constant}$$

KE^{IN} or KE^{FIN} Conservation ellipse:
 KE^{COM} Ka-runch ellipse:

Geometry of KE Conservation Theorem-1

$$KE^{INorFIN} = \frac{1}{2}M_1(V_1)^2 + \frac{1}{2}M_2(V_2)^2$$

$$\text{or : } 1 = \frac{V_1^2}{\left(\frac{2 \cdot KE^{IFN}}{M_1}\right)} + \frac{V_2^2}{\left(\frac{2 \cdot KE^{IFN}}{M_2}\right)} = \frac{V_1^2}{\left(\sqrt{\frac{2 \cdot KE^{IFN}}{M_1}}\right)^2} + \frac{V_2^2}{\left(\sqrt{\frac{2 \cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Geometry of KE^{COM} at Center Of Momentum

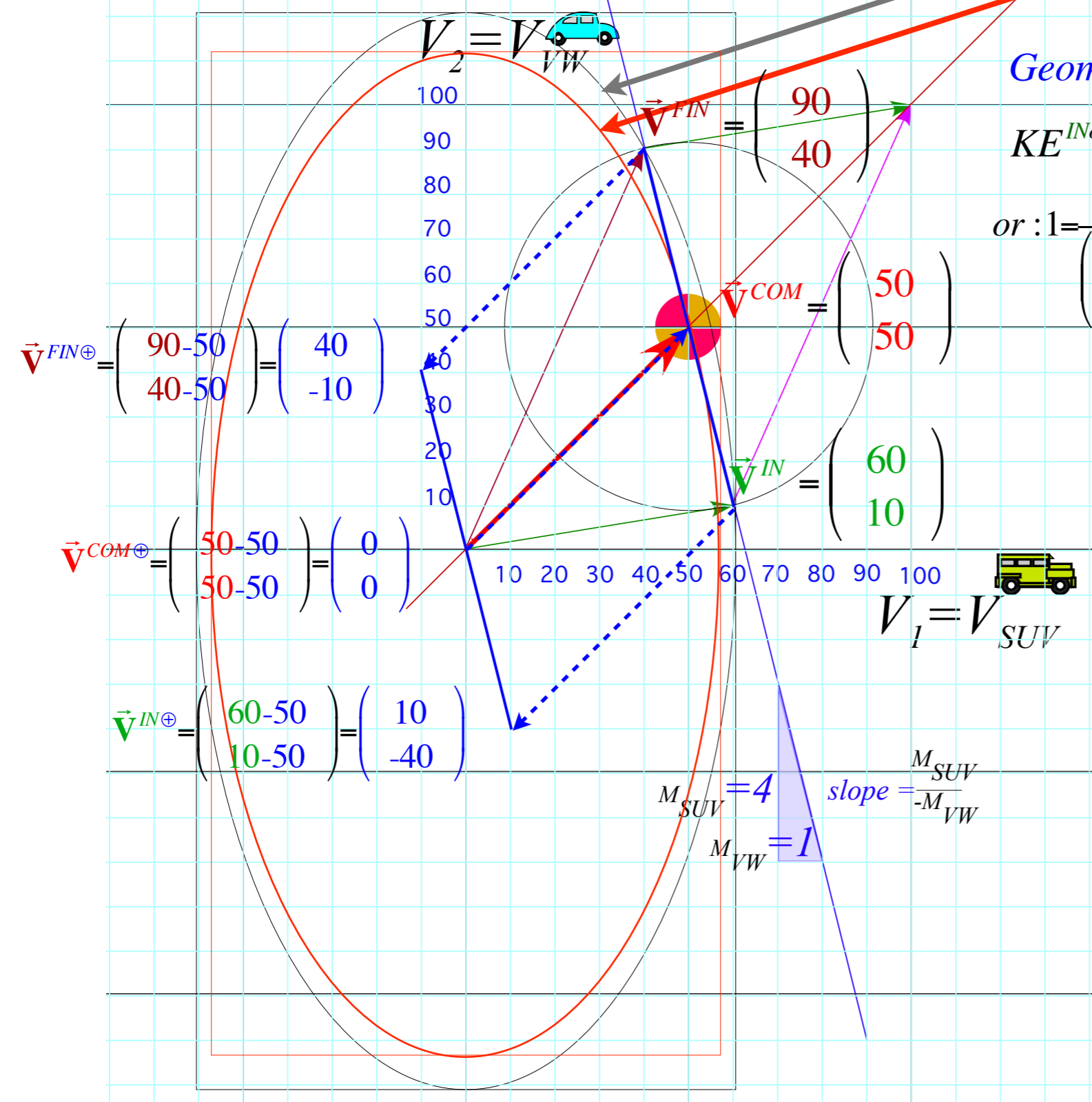
$$KE^{COM} = \frac{1}{2}M_1(V_1^{COM})^2 + \frac{1}{2}M_2(V_2^{COM})^2$$

$$= \frac{1}{2}(M_1+M_2)(V^{COM})^2$$

elliptic radii: $a^{COM} = \sqrt{\frac{KE^{COM}}{M_1}}$ $b^{COM} = \sqrt{\frac{KE^{COM}}{M_2}}$

$$= \sqrt{\frac{2 \cdot 6,250}{4}} \qquad = \sqrt{\frac{2 \cdot 6,250}{1}}$$

$$= 55.90 \qquad = 111.80$$



Geometry of Momentum Conservation Axiom-1

Geometry of Kinetic Energy Conservation Axiom-2/Theorem-1

$$(M_1+M_2)V^{COM} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN}$$

$$KE^{IN} = \frac{1}{2}M_1(V_1^{IN})^2 + \frac{1}{2}M_2(V_2^{IN})^2 = \frac{1}{2}M_1(V_1^{FIN})^2 + \frac{1}{2}M_2(V_2^{FIN})^2 = KE^{FIN}$$

Momentum Conservation line: \rightarrow

$$M_1V^{IN} + M_2V^{IN} = \text{constant}$$

$KE^{INorFIN}$ Conservation ellipse:
 KE^{COM} Ka-runch ellipse:

Geometry of KE Conservation Theorem-1

$$KE^{INorFIN} = \frac{1}{2}M_1(V_1)^2 + \frac{1}{2}M_2(V_2)^2$$

$$\text{or : } 1 = \frac{V_1^2}{\left(\frac{2 \cdot KE^{IFN}}{M_1}\right)} + \frac{V_2^2}{\left(\frac{2 \cdot KE^{IFN}}{M_2}\right)} = \frac{V_1^2}{\left(\sqrt{\frac{2 \cdot KE^{IFN}}{M_1}}\right)^2} + \frac{V_2^2}{\left(\sqrt{\frac{2 \cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Geometry of KE^{COM} at Center Of Momentum

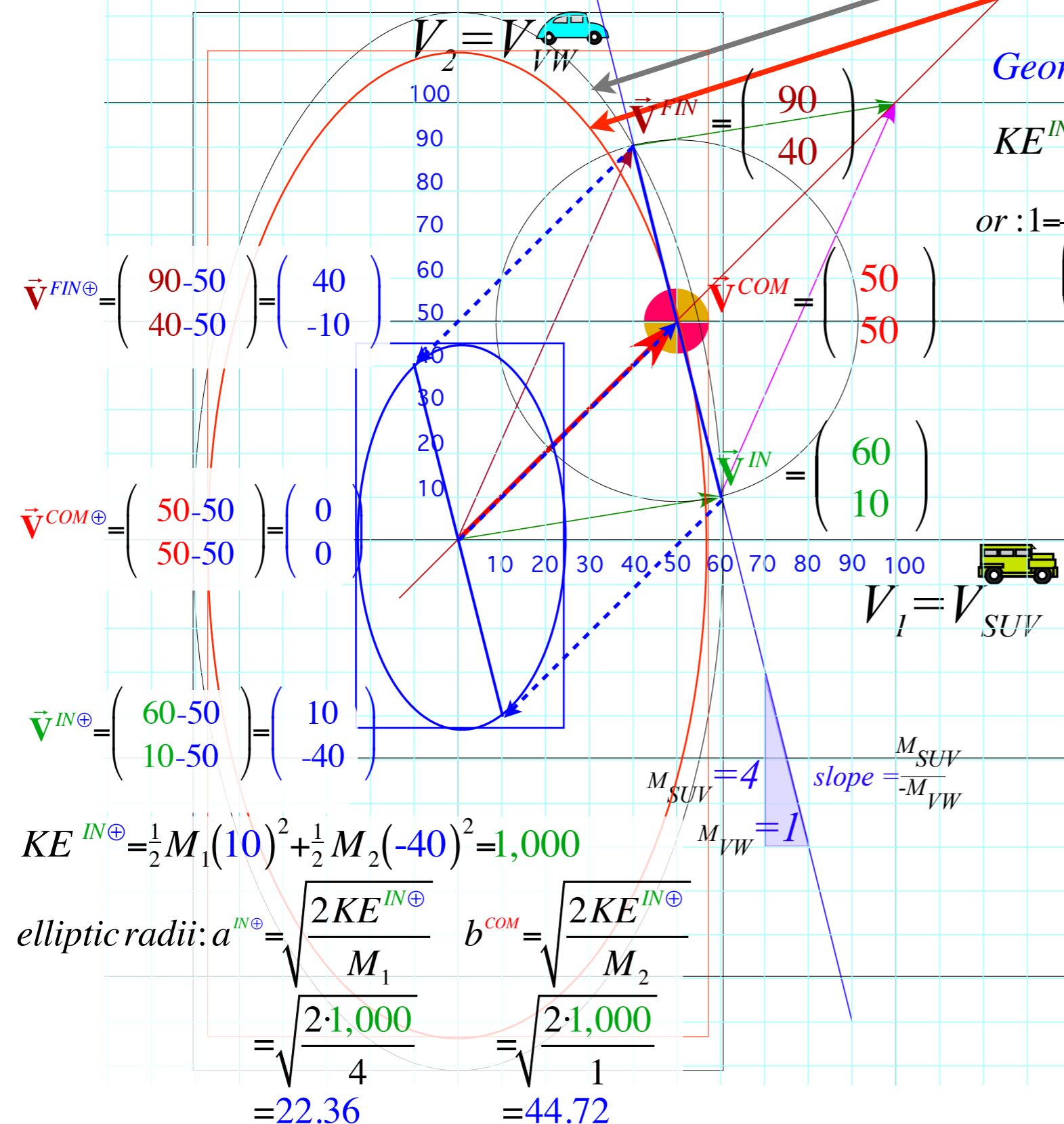
$$KE^{COM} = \frac{1}{2}M_1(V_1^{COM})^2 + \frac{1}{2}M_2(V_2^{COM})^2$$

$$= \frac{1}{2}(M_1+M_2)(V^{COM})^2$$

elliptic radii: $a^{COM} = \sqrt{\frac{KE^{COM}}{M_1}}$ $b^{COM} = \sqrt{\frac{KE^{COM}}{M_2}}$

$$= \sqrt{\frac{2 \cdot 6,250}{4}} \qquad = \sqrt{\frac{2 \cdot 6,250}{1}}$$

$$= 55.90 \qquad = 111.80$$



$$\vec{V}_{FIN\oplus} = \begin{pmatrix} 90-50 \\ 40-50 \end{pmatrix} = \begin{pmatrix} 40 \\ -10 \end{pmatrix}$$

$$\vec{V}_{COM\oplus} = \begin{pmatrix} 50-50 \\ 50-50 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\vec{V}_{IN\oplus} = \begin{pmatrix} 60-50 \\ 10-50 \end{pmatrix} = \begin{pmatrix} 10 \\ -40 \end{pmatrix}$$

$$KE^{IN\oplus} = \frac{1}{2}M_1(10)^2 + \frac{1}{2}M_2(-40)^2 = 1,000$$

$$\text{elliptic radii: } a^{IN\oplus} = \sqrt{\frac{2KE^{IN\oplus}}{M_1}} = \sqrt{\frac{2 \cdot 1,000}{4}} = 22.36$$

$$b^{COM} = \sqrt{\frac{2KE^{IN\oplus}}{M_2}} = \sqrt{\frac{2 \cdot 1,000}{1}} = 44.72$$

Developing
Conservation-of-Momentum
 The key axiom of mechanics
 leading to
Conservation-of-Energy Theorem

If and only if...
 there is **T-Symmetry**

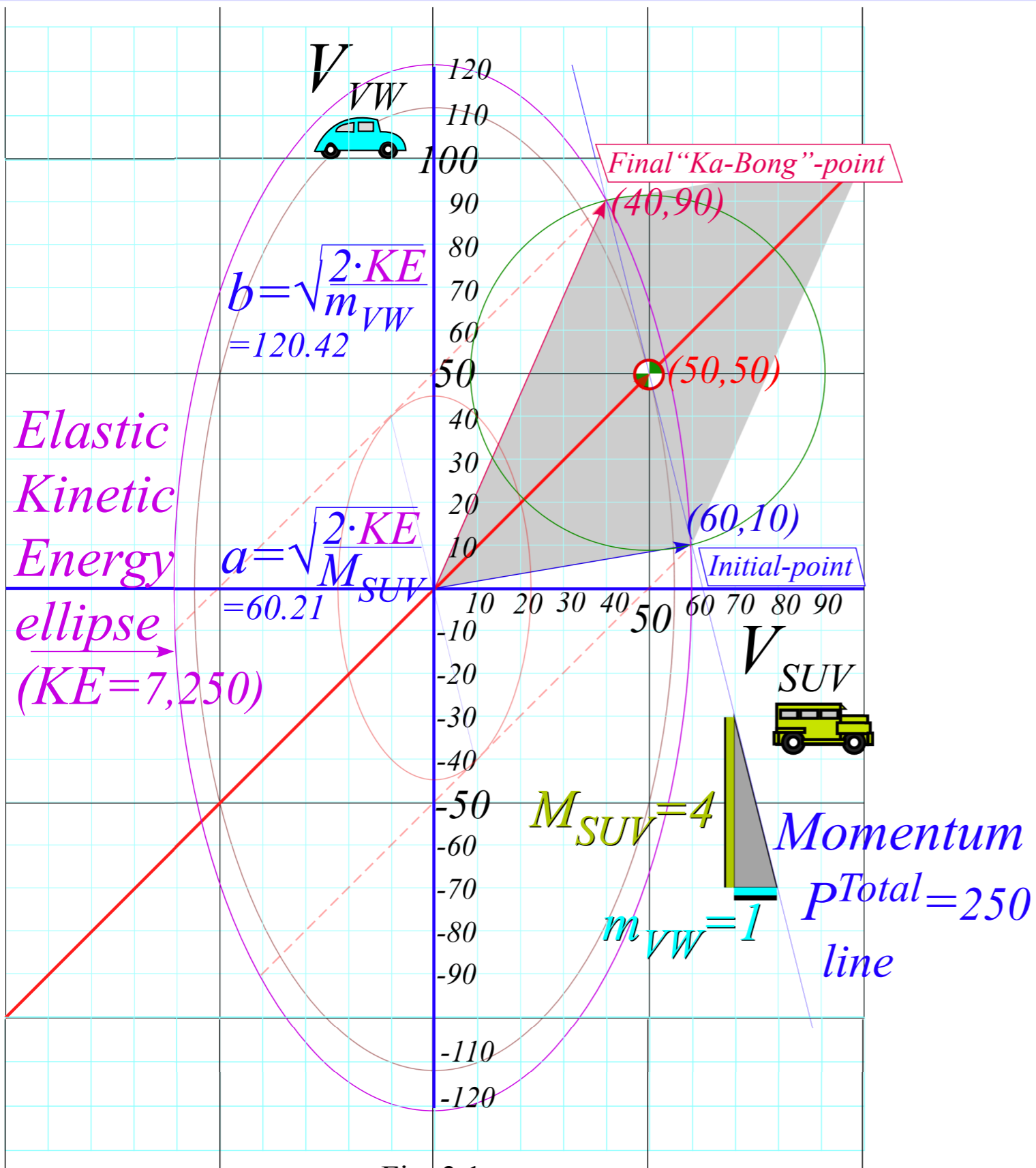


Fig. 3.1 a
 in Unit 1

Fig. 3.1

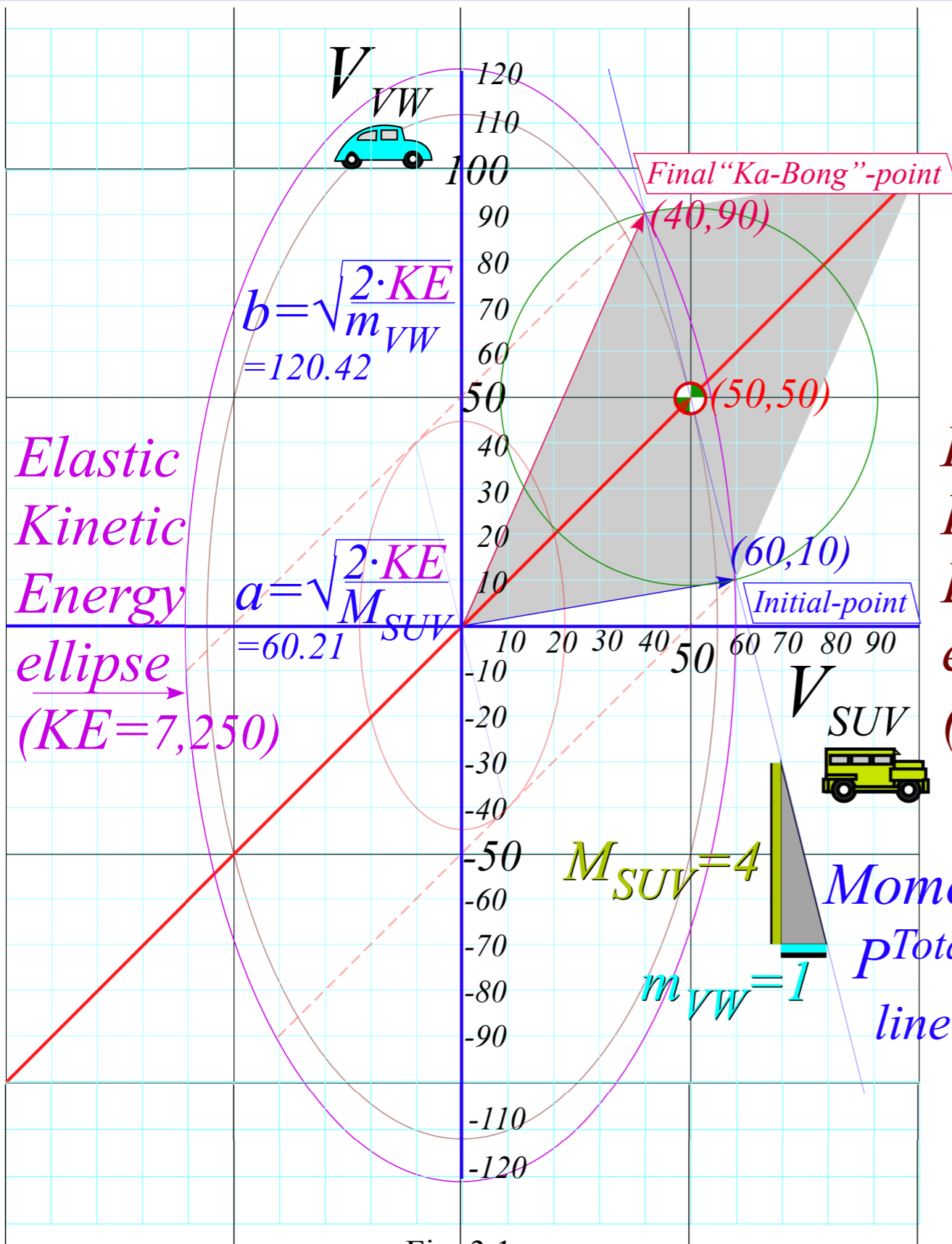


Fig. 3.1 a
in Unit 1

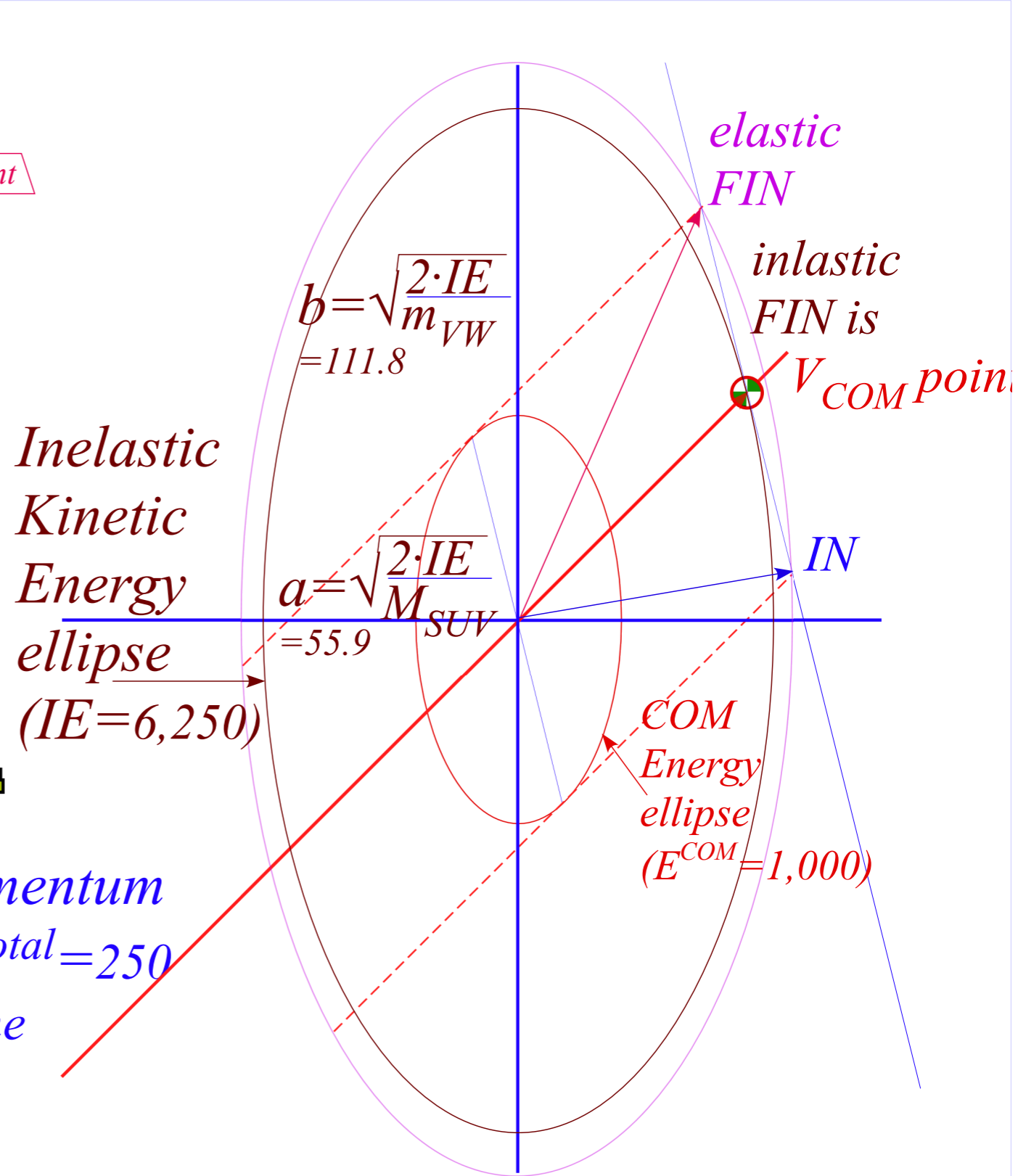


Fig. 3.1 b
in Unit 1

Fig. 3.1

As usual in physics, opposite extremes are easier to analyze than the generic “real(er) world” in between!

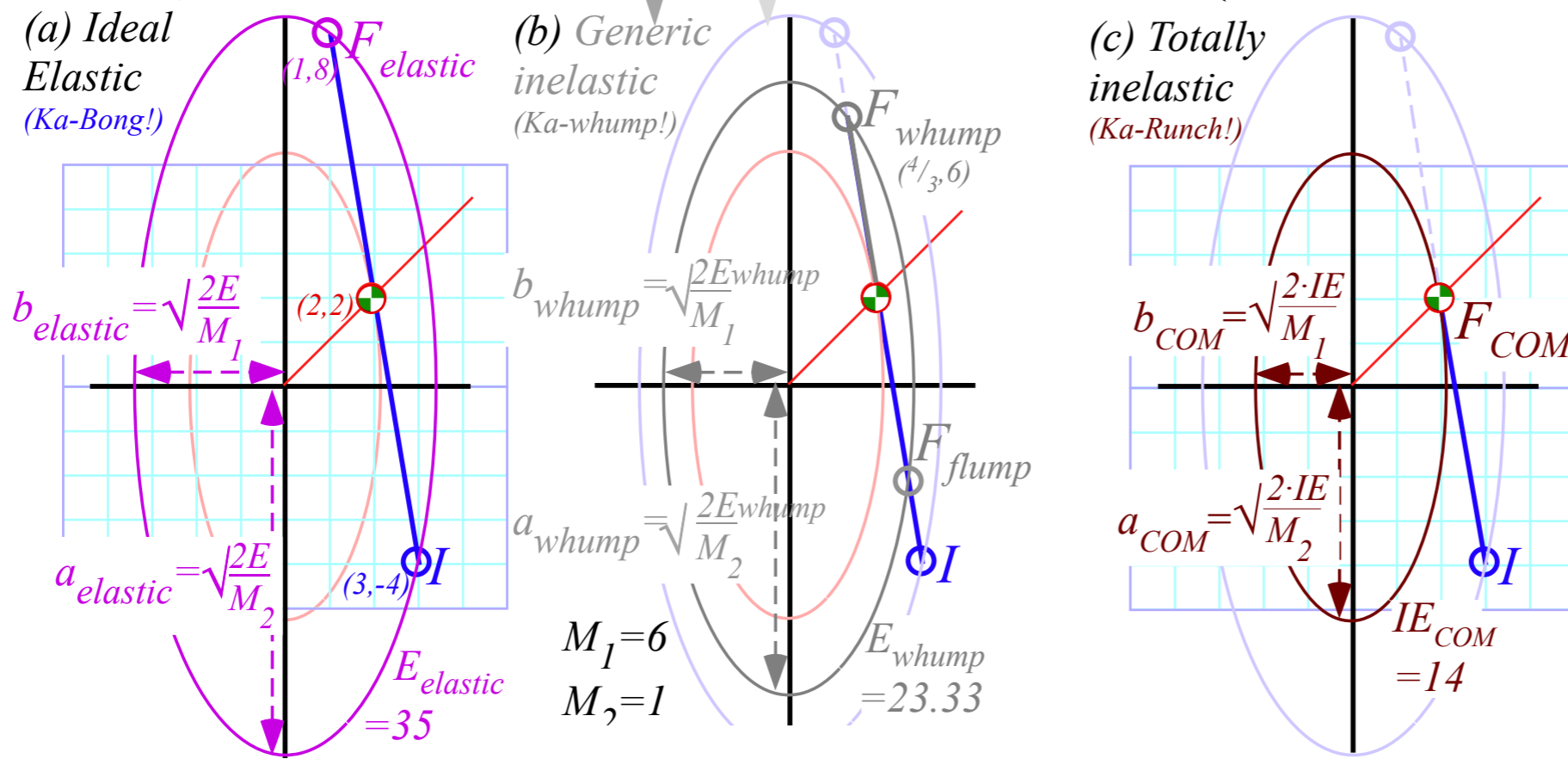


Fig. 2.3 (6-Ton SUV)

(During Bush II era an SUV with a mass of at least 6 tons allowed its owner to take a 100% write-off (up to \$100,000) on Federal Income Tax.)

Here *T-Symmetry* is best

Here *T-Symmetry* is less

Here *T-Symmetry* is least

Graph paper facilitates construction of energy ellipses given the two radii a and b in KE equation.

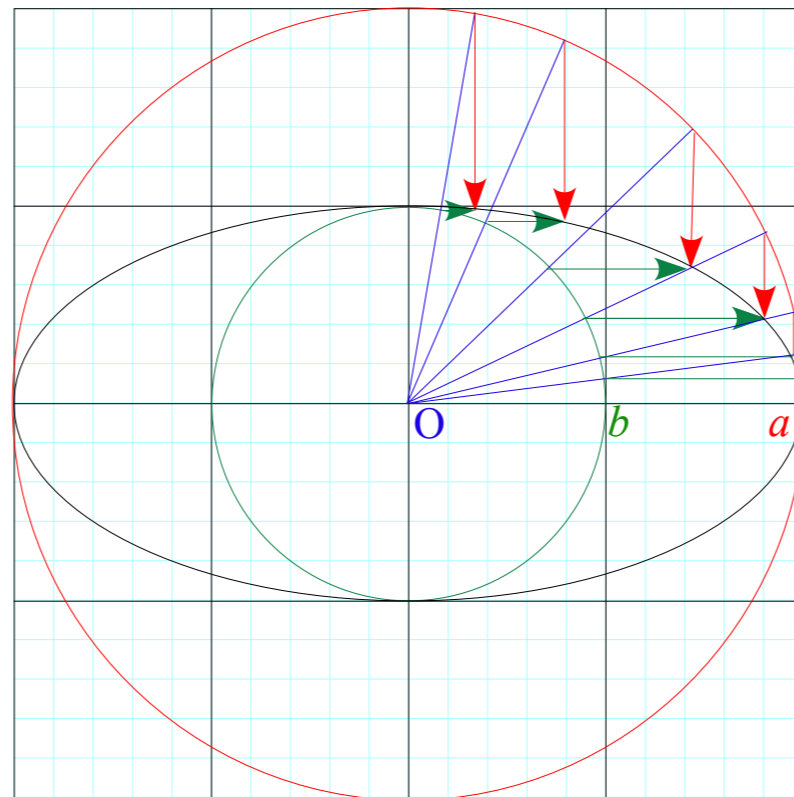
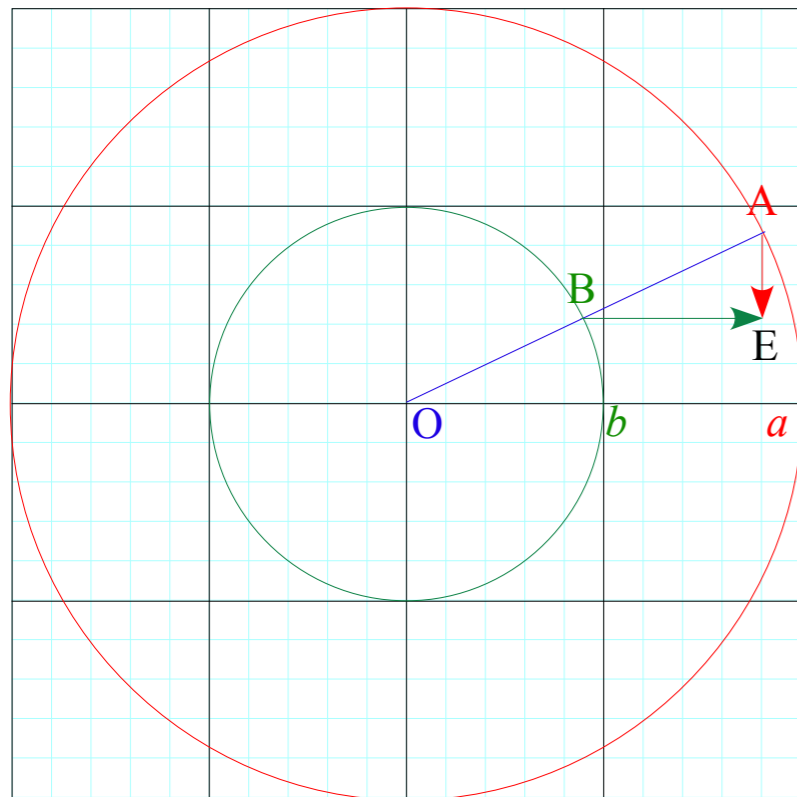
First step: draw concentric circles of radius a and b .

Then any radial line OBA “points” to point E on the ellipse.

Ellipse point E lies at the intersection of a vertical line AE thru radial intersection A with circle a and a horizontal line BE thru radial intersection B with circle b .

Graph grid helps locate E for a radius OBA , and usually there is no need to draw AE or BE .

You can pick x and find y or else *vice-versa*.

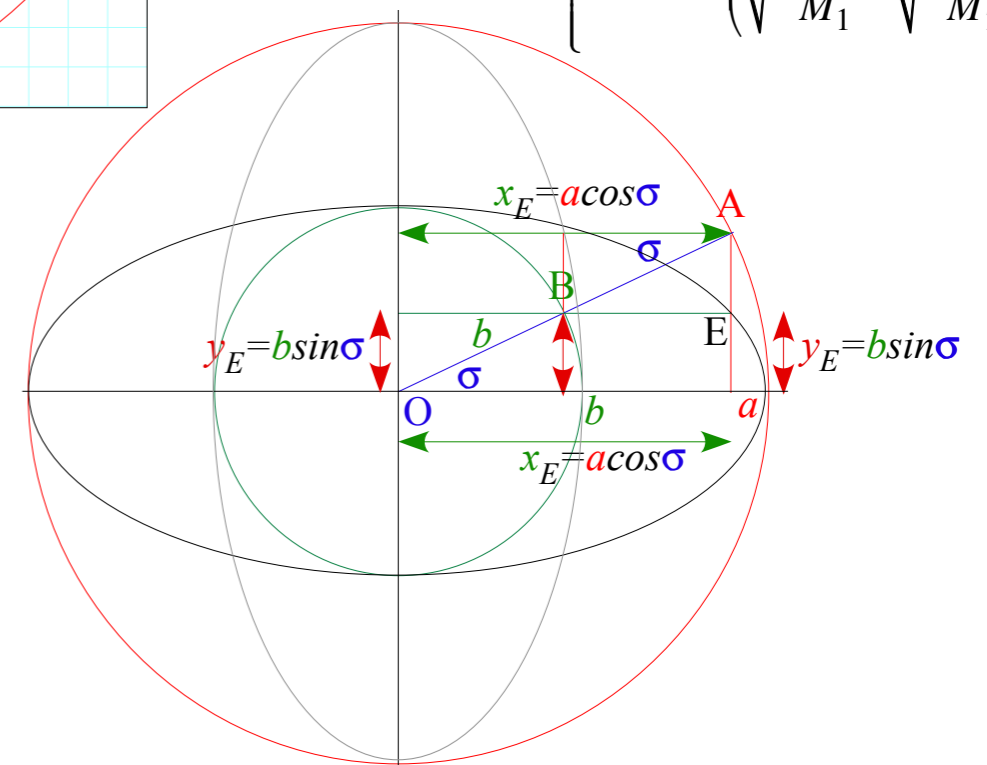


$$\frac{1}{2}M_1 \cdot V_1^2 + \frac{1}{2}M_2 \cdot V_2^2 = KE$$

$$\frac{V_1^2}{\left(\frac{2 \cdot KE}{M_1}\right)} + \frac{V_2^2}{\left(\frac{2 \cdot KE}{M_2}\right)} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\begin{cases} (x,y) = (V_1, V_2) \\ (a,b) = \left(\sqrt{\frac{2 \cdot KE}{M_1}}, \sqrt{\frac{2 \cdot KE}{M_2}} \right) \end{cases}$$

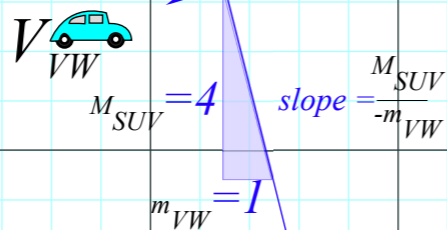


Ellipse coordinates ($x_E = a \cdot \cos \sigma$, $y_E = b \cdot \sin \sigma$) are rescaled base and altitude

($x_r = r \cdot \cos \sigma$, $y_r = r \cdot \sin \sigma$) of Fig. 2.6.

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line: \rightarrow (...one of ∞ -many...)



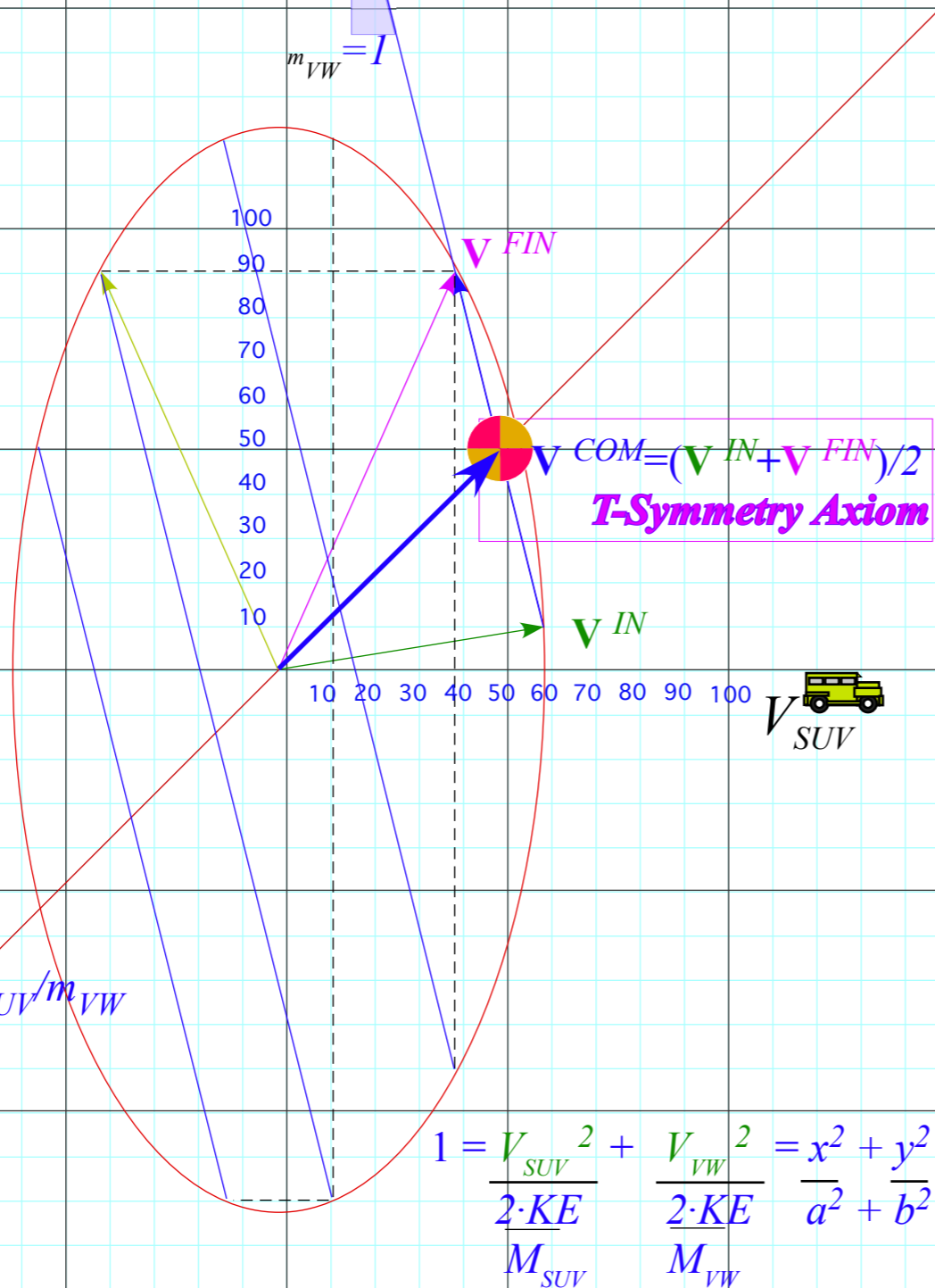
Momentum Conservation Axiom

plus

T-Symmetry Axiom
($M=M^T$ implied)

gives

Kinetic Energy Conservation Theorem



All lines of slope $-M_{SUV}/m_{VW}$
...are bisected by the
(slope=1)-COM line

$$1 = \frac{V_{SUV}^2}{\frac{2 \cdot KE}{M_{SUV}}} + \frac{V_{VW}^2}{\frac{2 \cdot KE}{M_{VW}}} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

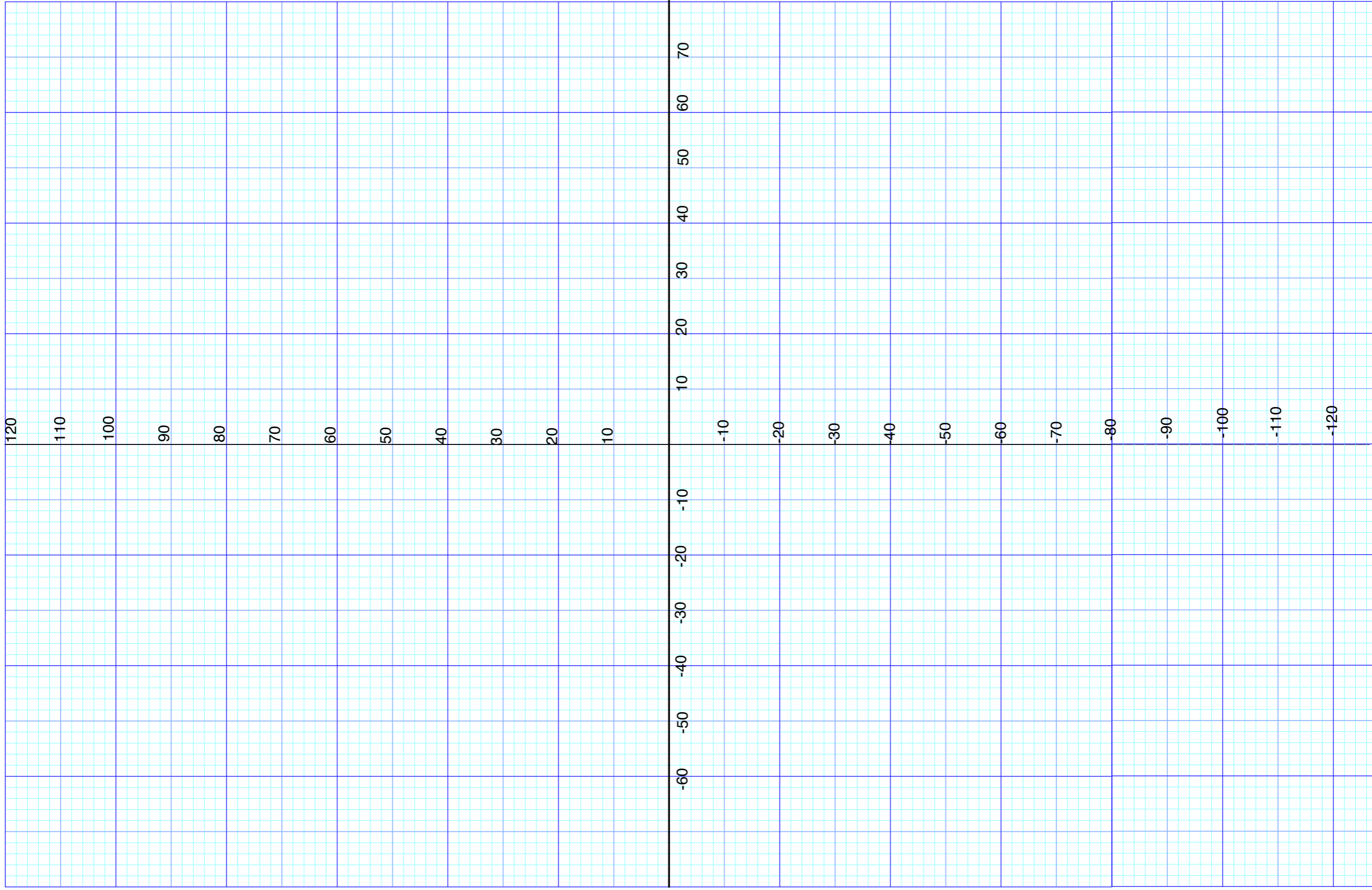
Developing
Conservation-of-Momentum
The key axiom of mechanics
leading to
Conservation-of-Energy Theorem

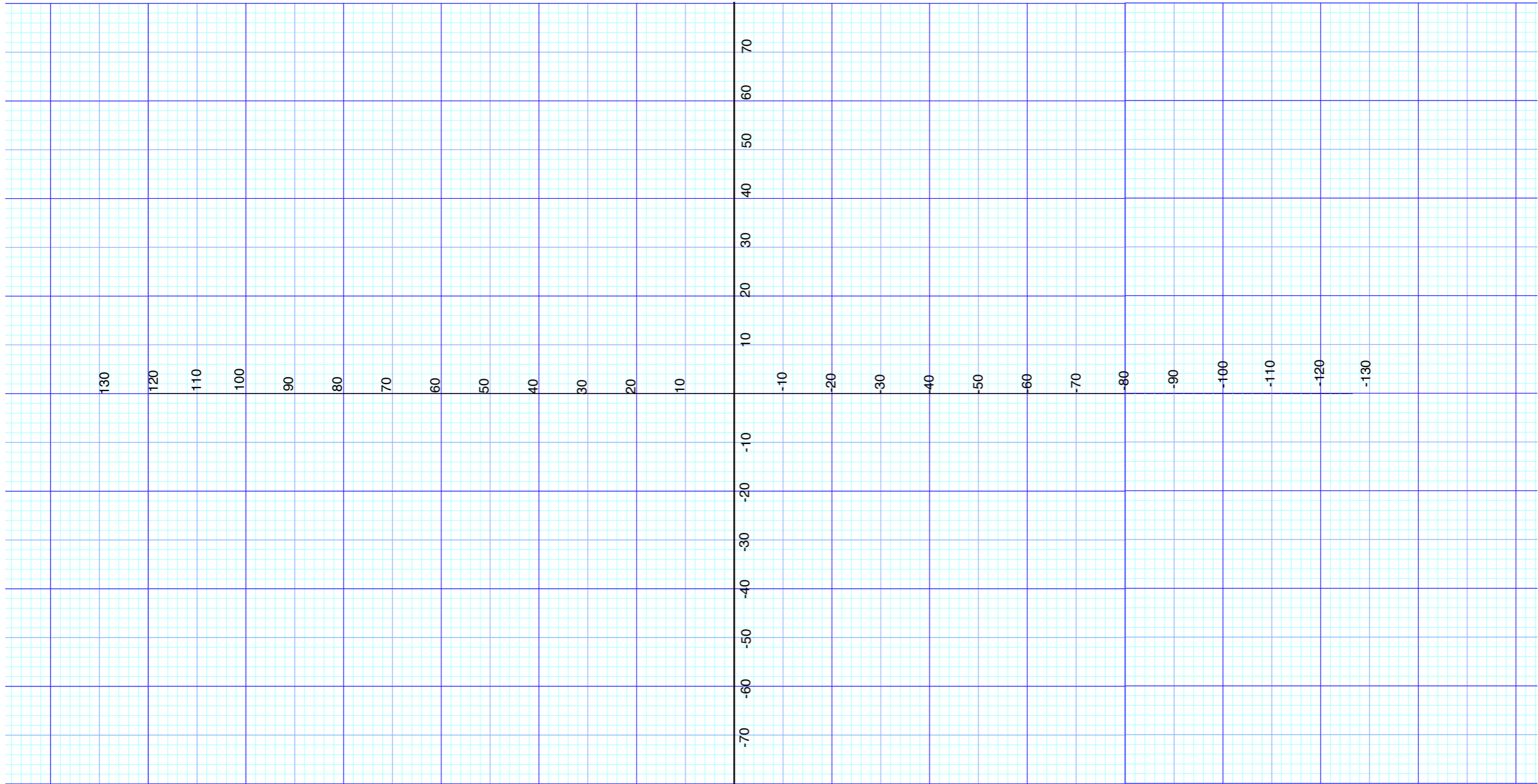
If and only if...
there is **T-Symmetry**

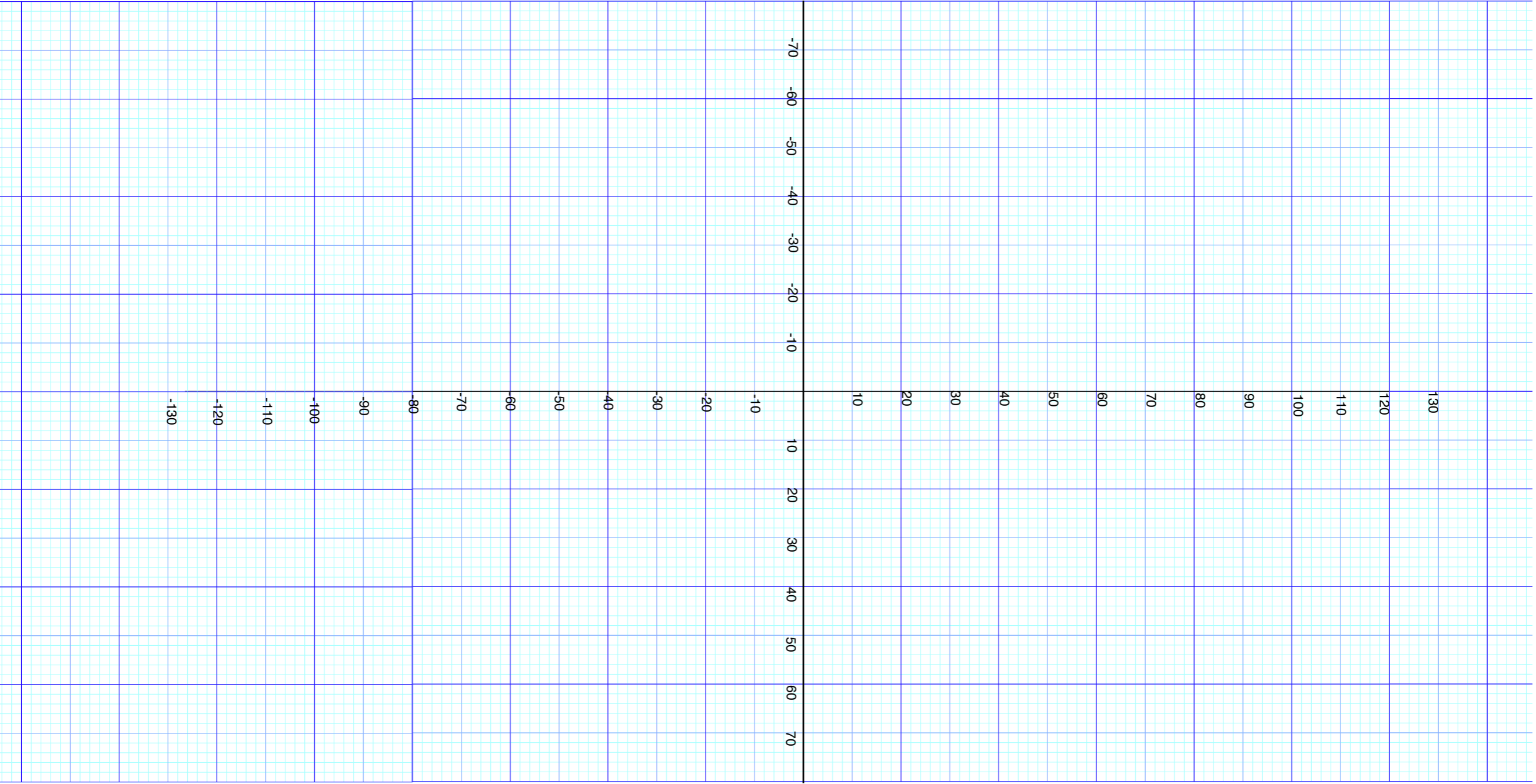
$$V_{COM} \cdot M \cdot V_{COM} - 1/2 V_{FIN} \cdot M \cdot V_{IN} = 1/2 V_{IN} \cdot M \cdot V_{IN} = 1/2 V_{FIN} \cdot M \cdot V_{FIN}$$

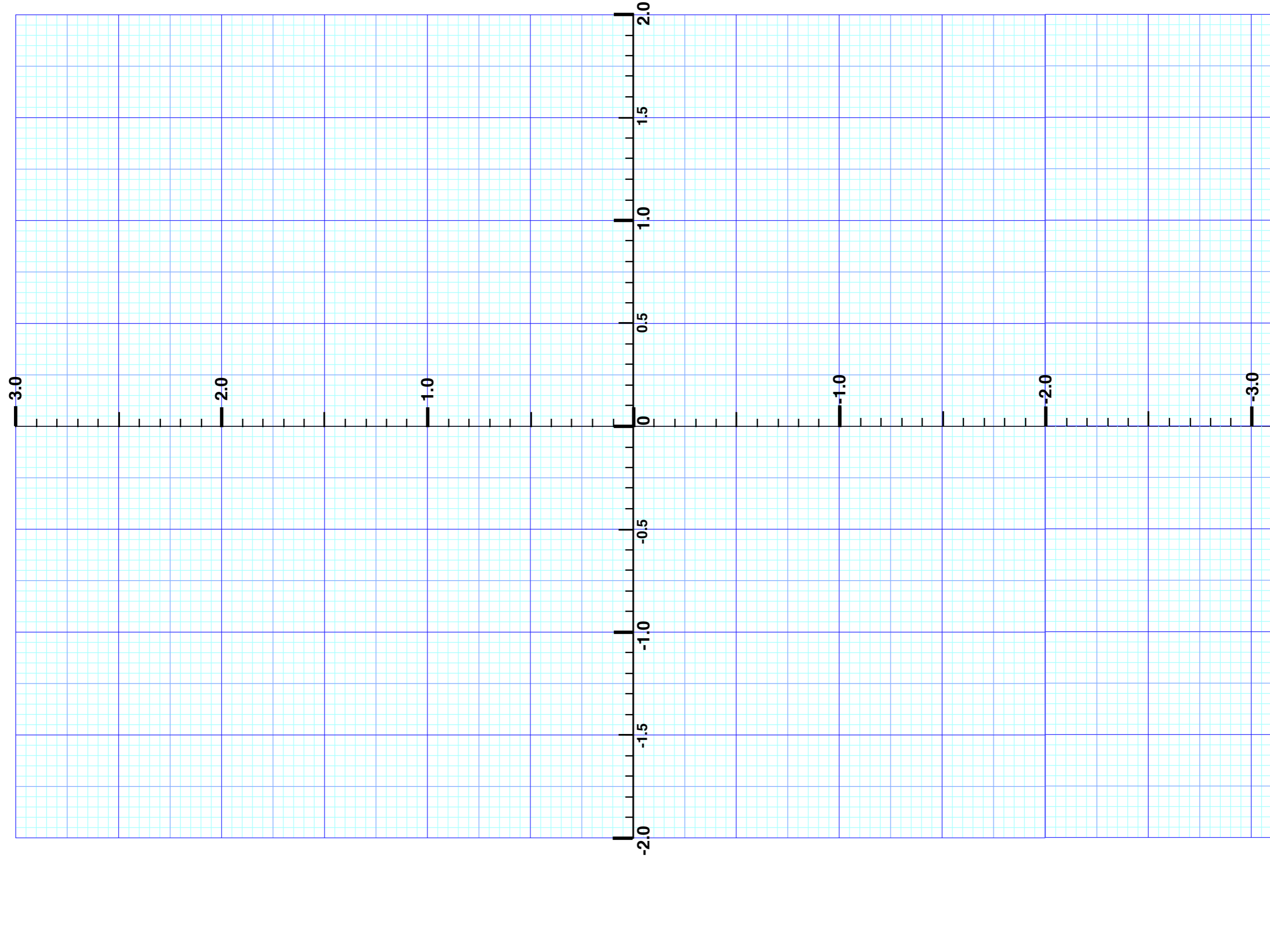
These are equations for energy conservation ellipse:

$$KE = 1/2 M_{SUV} V_{SUV}^2 + 1/2 M_{VW} V_{VW}^2$$









Note “crunch” energy $Elastic KE - inelastic IE = 0.21$ is the same in all frames including COM-frame.

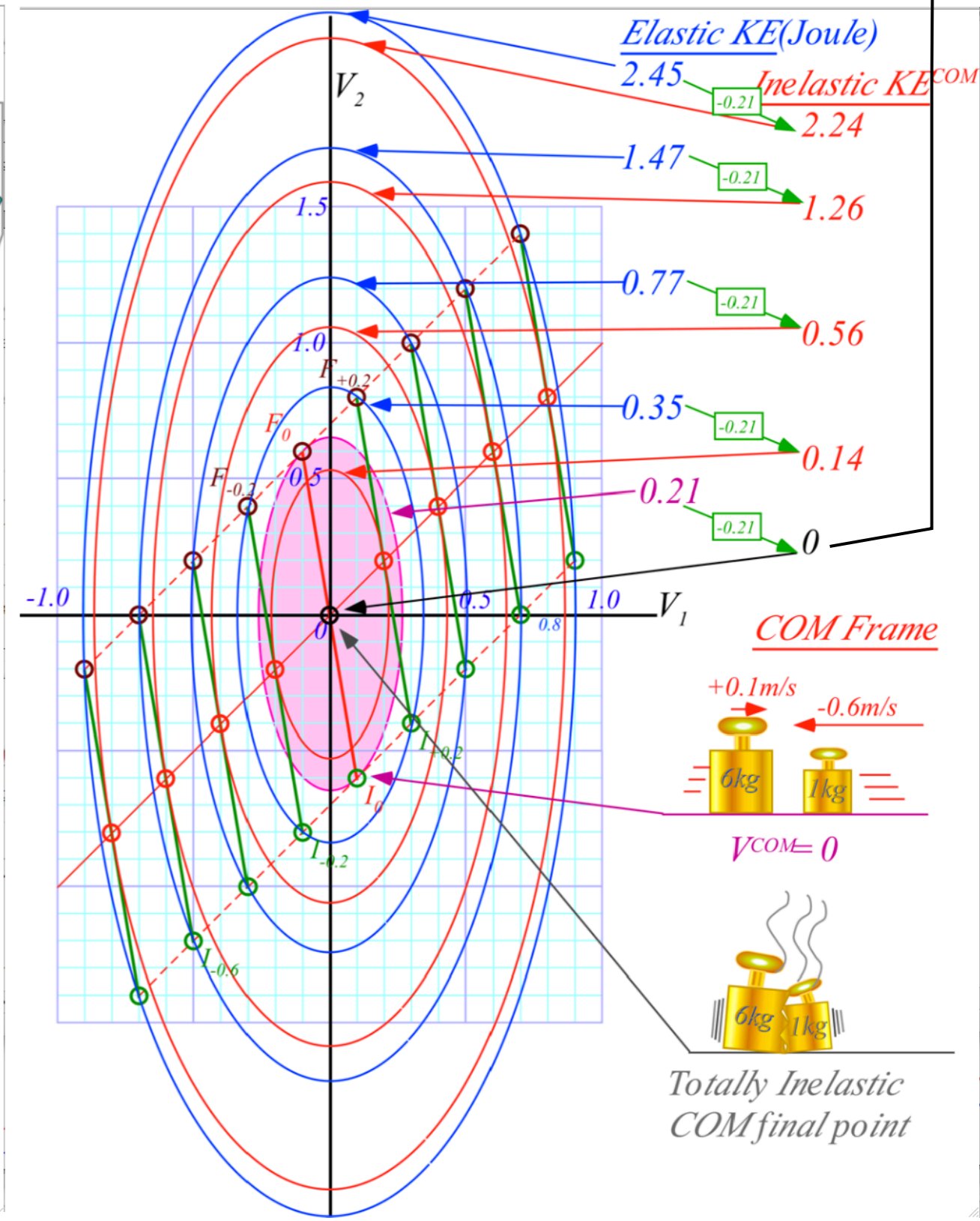
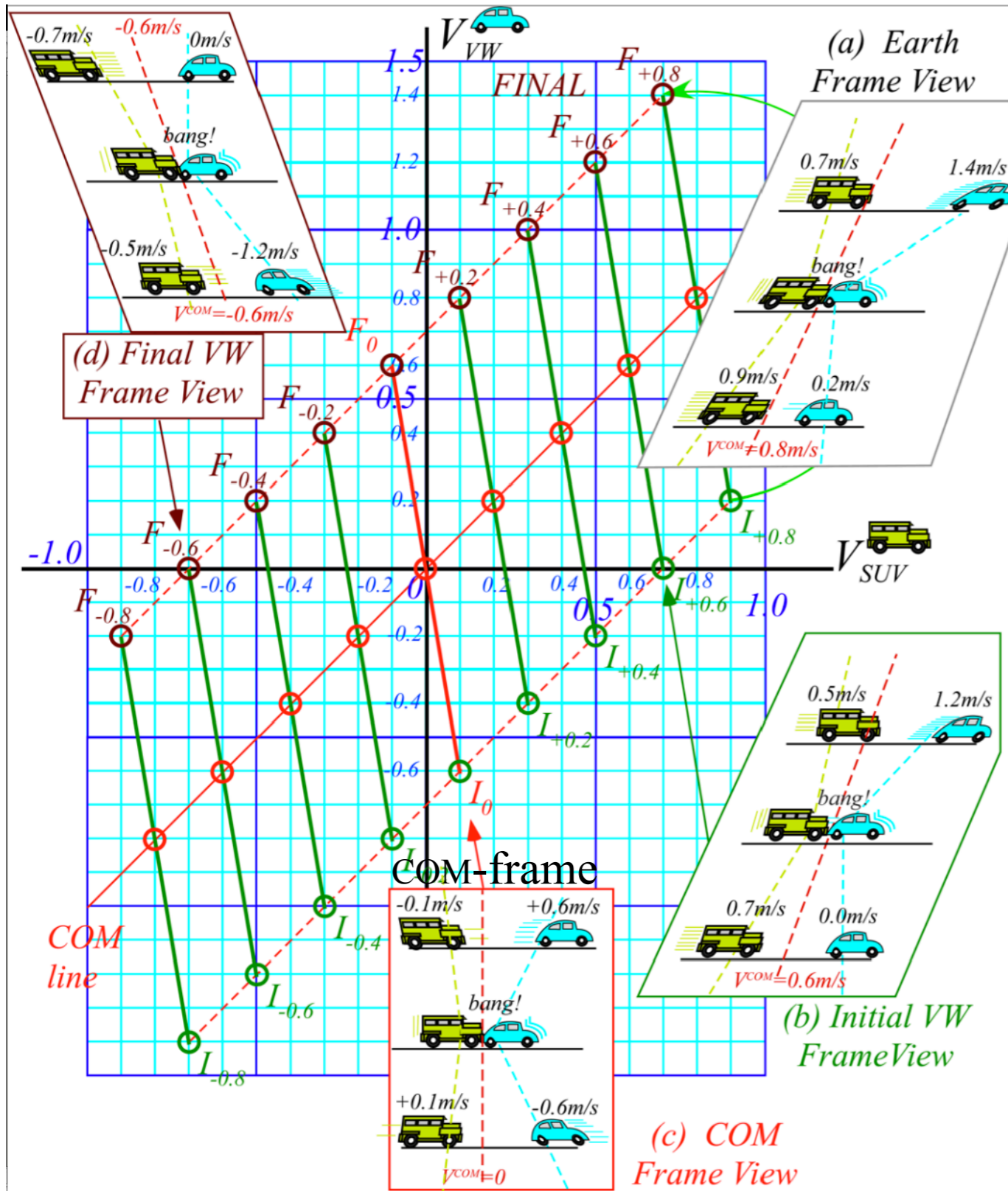


Fig. 3.4 Galilean Frame Views of collision like Fig. 2.5 or Fig. 3.1 with Bush (6:1) SUV.
 (a) Earth frame view (b) Initial VW frame (VW initially fixed)
 (c) COM frame view (d) Final VW frame (VW ends up fixed)

Fig. 3.5 Momentum ($P=const.$)-lines and energy ($KE=const.$)-ellipses appropriate for Fig. 3.4.

After-class
Bull-session

$\frac{2}{3}$ of a *PhD* is *Ph*
that is *Philosophy*

Should physicists
worry about *philosophy*?
...about *economics*?
...about *current events*?
...about *politics*?...

After-class
Bull-session

$\frac{2}{3}$ of a *PhD* is *Ph*
that is *Philosophy*

Should physicists
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Excerpt of Preface to *CMwithBANG!* (What does *Liberal* mean?)

Humans attempts to develop the *CC* are so sporadic at first it is impossible to label its emergence. Traditionally one points to the *Seven Liberal Arts* as our break with pre-medieval superstition. The seven consisted of the *Trivium*: (*Grammar, Logic, and Rhetoric*), and the *Quadrivium*: (*Arithmetic, Geometry, Astronomy, and Music*). The term *Liberal* is interchangeable with *Liberating* and probably was used to designate a pathway to avoid slavery. It appears that the *Trivium* contains *drivers* of the creative results in the *Quadrivium*. Indeed the latter has grown to more like *Seven Thousand Liberal Arts and Sciences* in just a few centuries. It's an explosion! You'll have to excuse physics and chemistry for not making the first cut.

(What are alternatives to *Liberal*?)

After-class
Bull-session

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(What are famous historical alternatives to *Liberal*?)

Luther, on the other hand, was more anti-scholarly, at least with regard to Copernicus. His *LLL* attitude was less *Seven Liberal Arts* and more *Seven Deadly Sins*. These may also be divided into a *Trivium* and a *Quadrivium*, however now the latter (*Greed, Envy, Lust, and Gluttony*) are drivers of the former (*Pride, Wrath, and Sloth*), that is, *Pride* or “*Gloating*” if your *Greed, Envy,..etc.* yields success, or else *Wrath* or “*Rage*” if you are unsuccessful, followed by *Sloth* or “*Depression.*” These are just drives and responses of *LLL* acquisition processes involving short-term ebb and flow of our small 3-to-5-ring molecules called *neurotransmitters*.

Should physicists worry about *philosophy*? ...about *economics*? ...about *current events*? ...about *politics*?...

“All the News
That’s Fit to Print”

The New York Times

National Edition

Periodic clouds and sunshine. A few afternoon showers and thunderstorms west. Highs in the upper 70s to the 80s. Rain and thunder tonight. Details in SportsSunday, Page 10.

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Trump Tax Cut Pays Dividends For the G.O.P.

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AP/ANDREW HARNIK

White House counsel Don McGahn listens during a Cabinet meeting Thursday. Fearing President Donald Trump was setting him up for blame, people close to him say, McGahn has been cooperating with the special counsel investigation to show he did nothing wrong.

TOP TRUMP AIDE GIVES MUELLER COVETED DETAILS

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By **MICHAEL S. SCHMIDT**
and **MAGGIE HABERMAN**

WASHINGTON — The White House counsel, Donald F. McGahn II, has cooperated extensively in the special counsel investigation, sharing detailed accounts about

Pictured in yesterday’s NYTimes are two Don’s: Don McGahn, and Don Trump. Which is on the better path?