Lecture 29
Wed. 11.28.2018

Relawavity : a novel introduction to relativistic mechanics I.

(CMwBang! Unit 8 , AMOP Ch.0,)

Why ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra

1CW wavefunctions and phasors

Per-space-per-time vs Space-time (How to understand wave parameters)

Wave velocity formulas

Introducing Doppler shifting

Why is c *so* constant?!

Introducing Doppler Arithmetic and *Rapidity* ρ

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

Thales geometry of Lorentz transformation

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of 16 parameter functions of ρ and σ

Application to TE-Waveguide modes and synchrotron beam relativity

A running collection of links to course-relevant sites and articles

Physics Web Resources

Comprehensive Harter-Soft Resource Listing

UAF Physics YouTube channel

LearnIt Physics Web Applications

"Texts"

Classical Mechanics with a Bang!

Quantum Theory for the Computer Age

Principles of Symmetry, Dynamics, and Spectroscopy

Modern Physics and its Classical Foundations

Classes

2014 AMOP

2017 Group Theory for QM

2018 AMOP

2018 Adv Mechanics

Neat external material to start the class:

- AIP publications
- AJP article on superball dynamics
- AAPT summer reading

These *are* hot off the presses:

- Sorting ultracold atoms in a 3D optical lattice in a realization of Maxwell's demon - Kumar-Nature-Letters-2018
- Synthetic three-dimensional atomic structures assembled atom by atom - Berredo-Nature-Letters-2018

Slightly Older ones:

- Wave–particle duality of C60 molecules
- Optical vortex knots – One Photon at a Time

AMOP Detailed Development of Relawavity

AMOP Ch 0 Space-Time Symmetry - 2019

Seminar at Rochester Institute of Optics, Auxiliary slides, June 19, 2018

Springer AMO Handbook - Ch 32 - Harter-Reimer-2019

"*Relawavity*" and quantum basis of *Lagrangian* & *Hamiltonian* mechanics:

- 2-CW laser wave - BohrIt Web App
- Lagrangian vs Hamiltonian - RelaWavity Web App

Older Links from Lectures 14-20

http://thearmchaircritic.blogspot.com/2011/11/punkin-chunkin.html
http://www.sussexcountyonline.com/news/photos/punkinchunkin.html
Shooting-range-for-medieval-siege-weapons-Anybody-knows
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html?scenario=MontezumasRevenge
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html?scenario=SeigeOfKenilworth
The trebuchet, Chevedden, Sci Am 1995
'*Simple*' *Pendulum* Sim: https://modphys.hosted.uark.edu/markup/PendulumWeb.html
'*Cycloid*' *Pendulum*: https://modphys.hosted.uark.edu/markup/CycloidulumWeb.html
Google search on: "Satelite view of Patricia" (Images)
Physics Girl Channel - Fun with Vortex Rings in the Pool
iBall demo - Quasi-periodicity: https://youtu.be/_jntDtULxDc
https://modphys.hosted.uark.edu/markup/CoulItWeb.html?scenario=SynchrotronMotion
https://modphys.hosted.uark.edu/markup/CoulItWeb.html?scenario=SynchrotronMotion2
Mechanical Analog to EM Motion (YouTube video) - https://youtu.be/hTd5FTJ-vRk
CoulIt Web Simulation: Bound-state motion in parabolic coordinates
CoulIt Web Simulation: Bound-state motion in hyperbolic coordinates
OscillIt Web App: Simulations of various types of resonance: 18, 27, 31, 35, 38, 39
Smith Chart
http://nobelprize.org/

AnalyIt Web Application, posted 10/22/2018 in our *testing* area:
https://modphys.hosted.uark.edu/testing/markup/AnalyItBJS.html

Older Links from Lectures 21-23

Advanced Atomic and Molecular Optical Physics 2018 Class #9, pages: 5, 61

BoxIt Web Simulations

- Pure A-Type w/Cosine
- Pure B-Type w/Cosine
- Pure B-Type w/Freq ratios
- Mixed AB-Type 2:1 Freq ratio
- Pure A-Type A=4.9, B=0 ,C=0, & D=4.0
- Pure B-Type: A=4.0, B=-0.2, C=0, & D=4.0
- Pure C-Type A,D=4.055, B=0, C=0.1
- Mixed AB-Type w/Cosine
- Mixed AB Type A=4.0, BU2=0.866…, CU2=0, & D=1.0 w/Stokes & Freq rats
- Mixed AB Type A=5.086 B=-0.27 C=0 D=2.024 w/Stokes plot
- Mixed ABC Type A=4.833 B=0.2403 C=0.4162 D=4.277 w/Stokes plot
- Recent mixed ABC Type A=0.325 B=0.375 C=0.825 D=0.05 w/Stokes plot

Classical Mechanics with a Bang! 2018

- Lectures 8, 9, 23 page 93
- Text Unit 6, page=27

ColorU2 for the Web - in development

Group Theory for Quantum Mechanics - 2017 Lectures: 6, 7, 8,
and the combined 9-10

Quantum Theory for the Computer Age Unit 3 Ch.7-10, page=90

Web based 3D & XR (x∈{A,M,V}, R=Reality) https://www.babylonjs.com/

Web based 3D graphics WebGL API (Graphics Layer modeled after OpenGL)

Wiki on Pafnuty Chebyshev

continued ↘

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Repeated from previous page

Older Links from Lectures 24-27

JerkIt Web App: 2-, 2+, Amp50Omega147-, Amp50Omega296, Amp50Omega602, Gap(1)
MolVibes Web App: C3vN3
WaveIt Web App:
- Dim = 3 w/Wave Components;
- Static Char Table: 6, 12, 12(b), 16, 36, 256
- Quantum Carpet with N=20: Gaussian, Boxcar

Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-CPL-2015
QTCA Unit_5 Ch14 2013
Lester. R. Ford, Am. Math. Monthly 45,586(1938)
John Farey, Phil. Mag.(1816) Wolfram
Harter, J. Mol. Spec. 210, 166-182 (2001)
Harter, Li IMSS (2013)
Li, Harter, Chem.Phys.Letters (2015)
OscillatorPE Web App: IHO Scenario 2, Coulomb Scenario 3
RelaWavity Web App/Simulator/Calculator: Elliptical - IHO orbits
CoulIt Web App Simulations: p19, p32, p72, p73, p92, R=-0.375, R=+0.5, Rutherford

Older Links from Lectures 28

CMwBang Text 2012 Unit 6 page=5
BounceIt Web App/Scenarios:
- 5002, 5003

CoulIt Web App/Scenarios:
- TwoParticleCollision_LToR, TwoParticleCollision_LToR_CM, TwoParticleOrbit_Coulomb, TwoParticleOrbit_Coulomb_CM, TwoParticleOrbit_Hooke, TwoParticleOrbit_Hooke_CM

Singular Motion of Asymetric Rotators AJP 44, 11 p1080 Harter-Kim-1976
Molecular Eigensolution Symmetry Analysis and Fine Structure - Int.J.MolSci1.4.13 Harter-Mitchell-IJMS-2013
Lenz Vector and Orbital Analog Computers - AJP 44 p348 1976
Some Geometric Aspects of Classical Coulomb Scattering AJP 40 4 p1852 1972
How Molecules do Self-NMR - Harter-Mitchell-Columbus-2009
Classical Mechanics with a Bang! - Asymmetric Top Demo
Allbookstores.com - Compare for Heller's SemiClassical Way - 0691163731
"My Bomerang Won't Come Back" (YouTube: Playlist)
Rotating Solid Bodies in Microgravity (YouTube)
Dancing T-handle in zero-g (YouTube)

Links to supplement Lecture 29

Ruler & Compass - Relawavity Exercise
2018 RelaWavity Portal Page
- ***AMOP Chapter 0: Space-Time Symmetry***
- AMOP Detailed Development of RelaWavity
- 2018 Rochester Talk (Auxilary Slides)
- Special Relativity and Quantum Theory by Ruler and Compass - Earlier, expanded draft

Pirelli Relativity Challenge Web Site:
- Title Page, Clocks_12_hr, Clocks_24_hr_QT, Phasors Addition

BohrIt Web App/Simulations: -130022, -30001, -30104, 30004, 30022
GuideIt Web App/Scenarios: 230, 260
RelativIt Web App/Scenarios: 22, 24
RelaWavity Web App/Scenarios: 0,9, 3,6, 3,6 NoMink, 4,8, 6,1, 6,3a, 6,3b, 7,1, 7,2,1, 7,2,2, 7,2,3, 7,2,7, 8,3, 8,5, 8,7, 8,8

➡ *Why* ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

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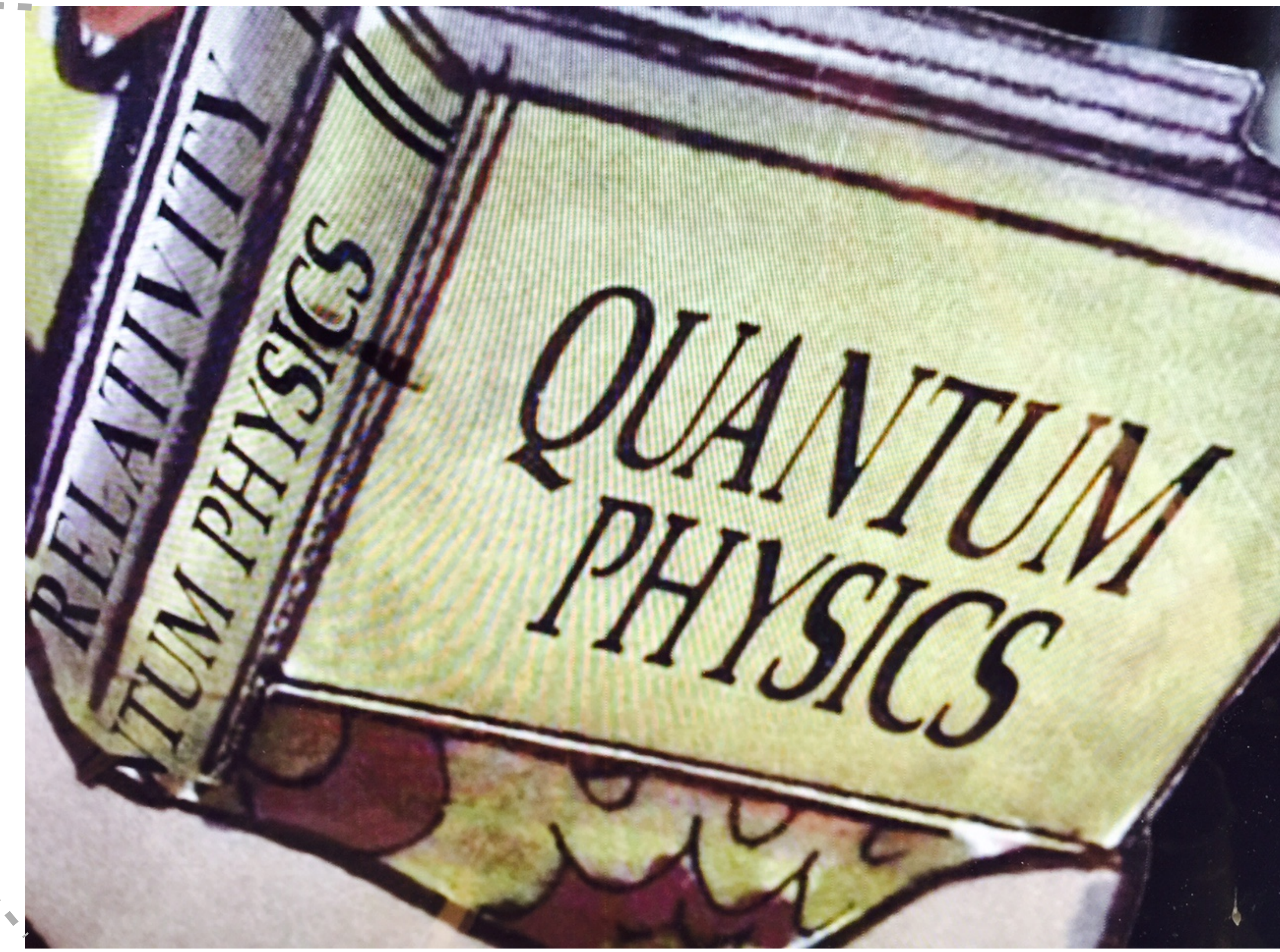
For an introductory, web based development of this and other concepts in special relativity see our entrant in the 2005 Pirelli Challenge:

A Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers

From AMOP Ch.0 article.

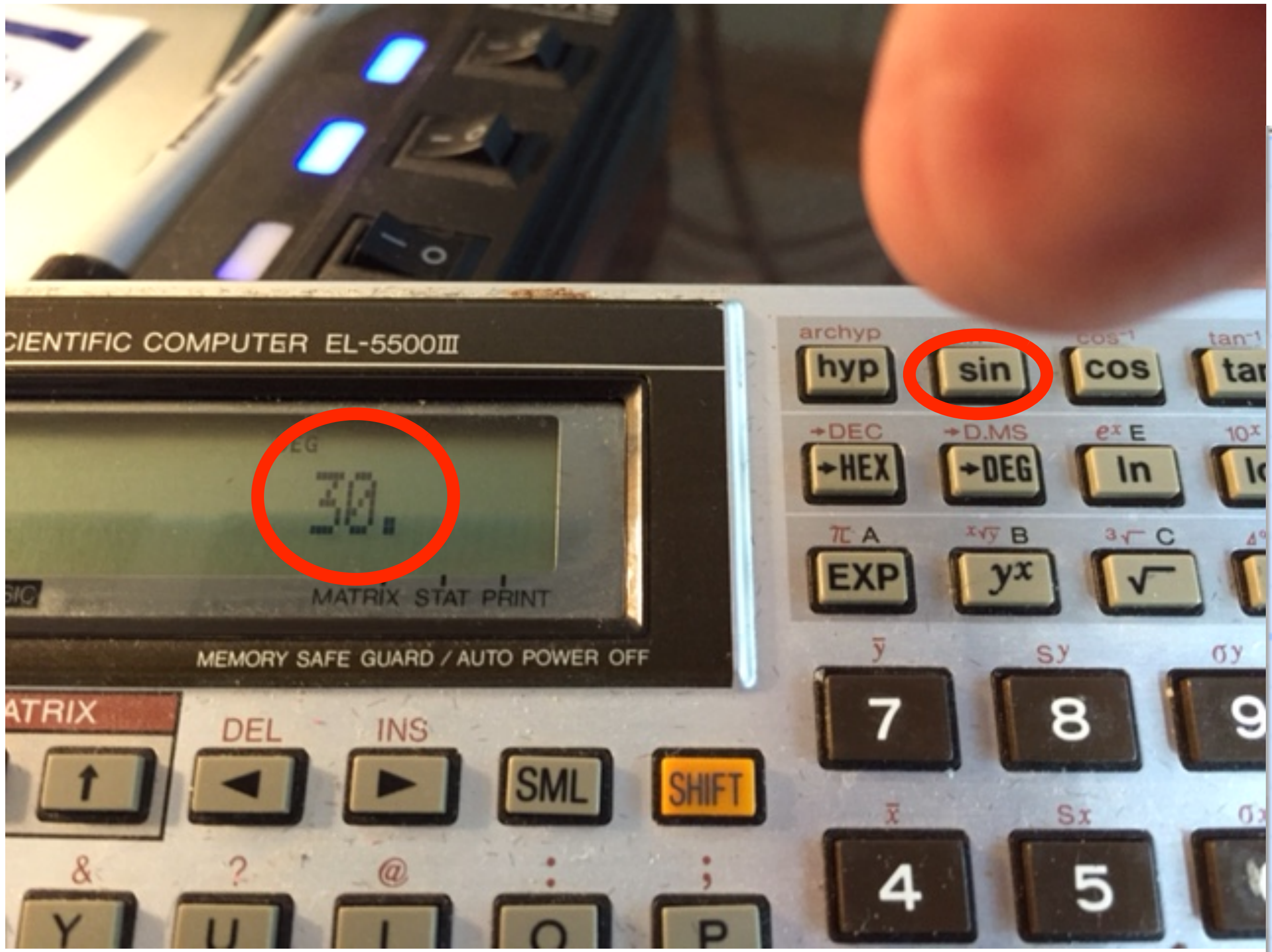
Why did a ***Men In Black*** candidate shoot little Suzy?

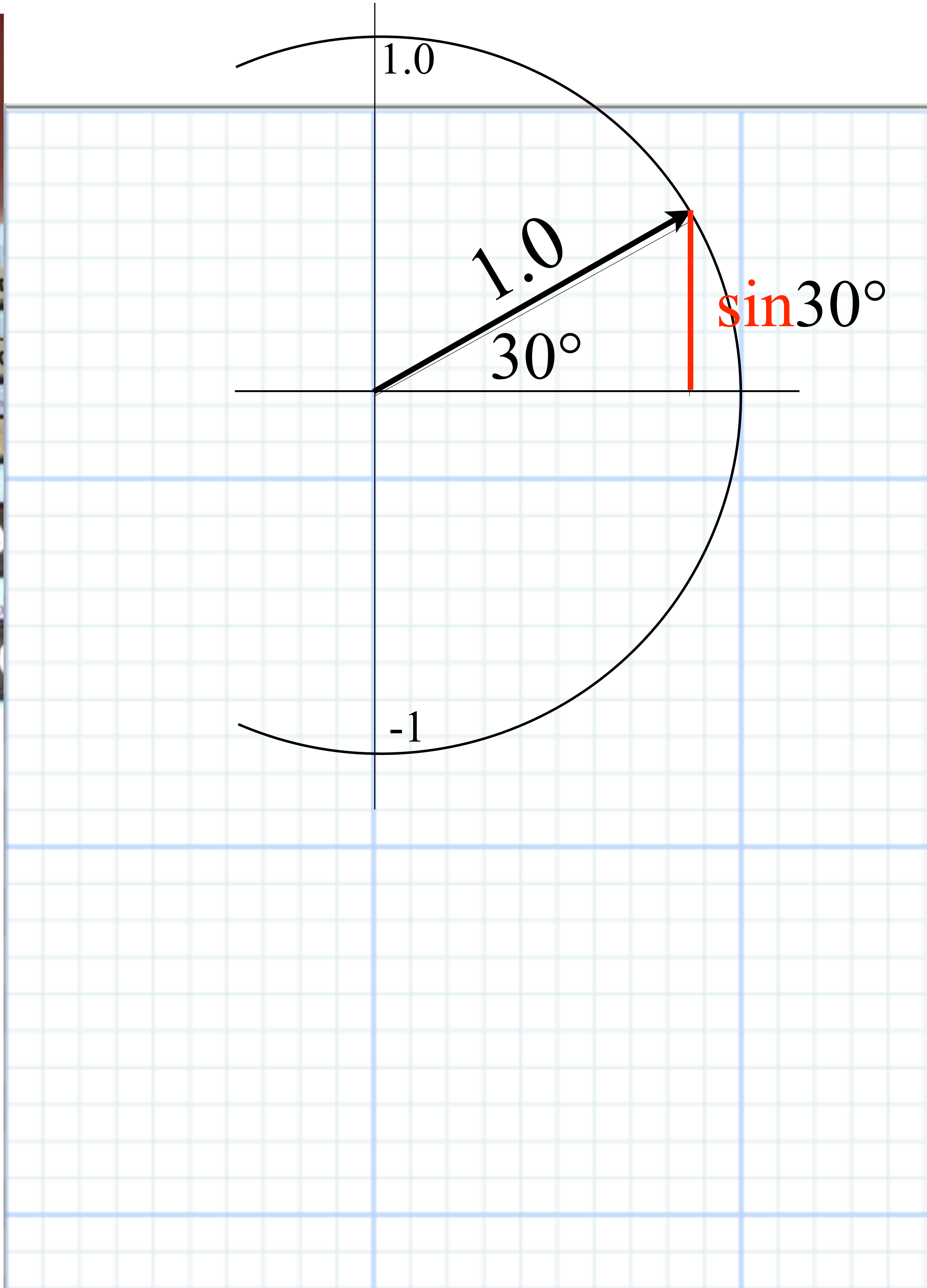


Bad Suzy!
Relativity and Quantum Theory need to be unified in *one* book *half* the size of those old tomes!

We call that a ***Relawavity*** *book.*
(It's a *lot* lighter!)

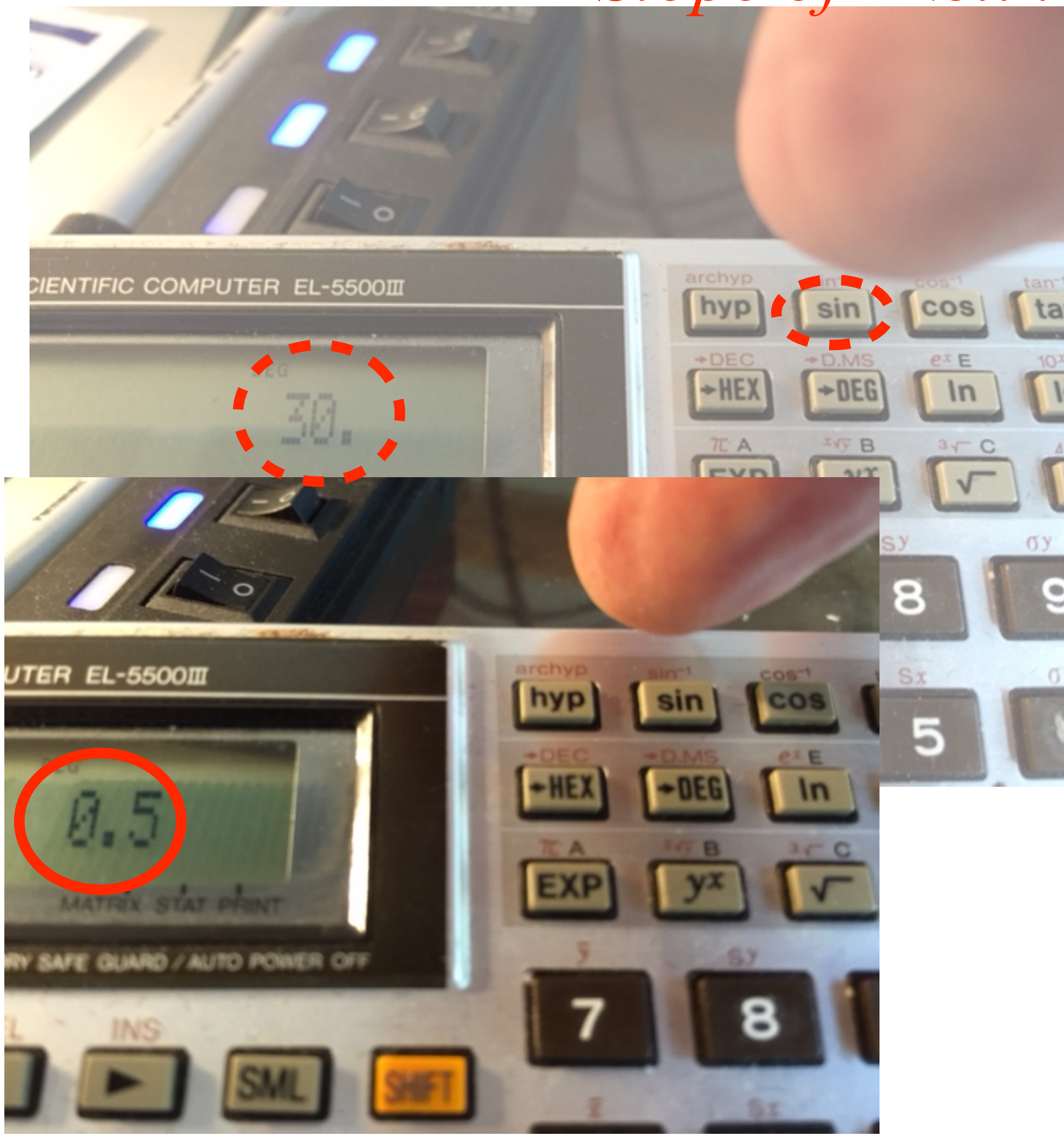
Learning about SIN

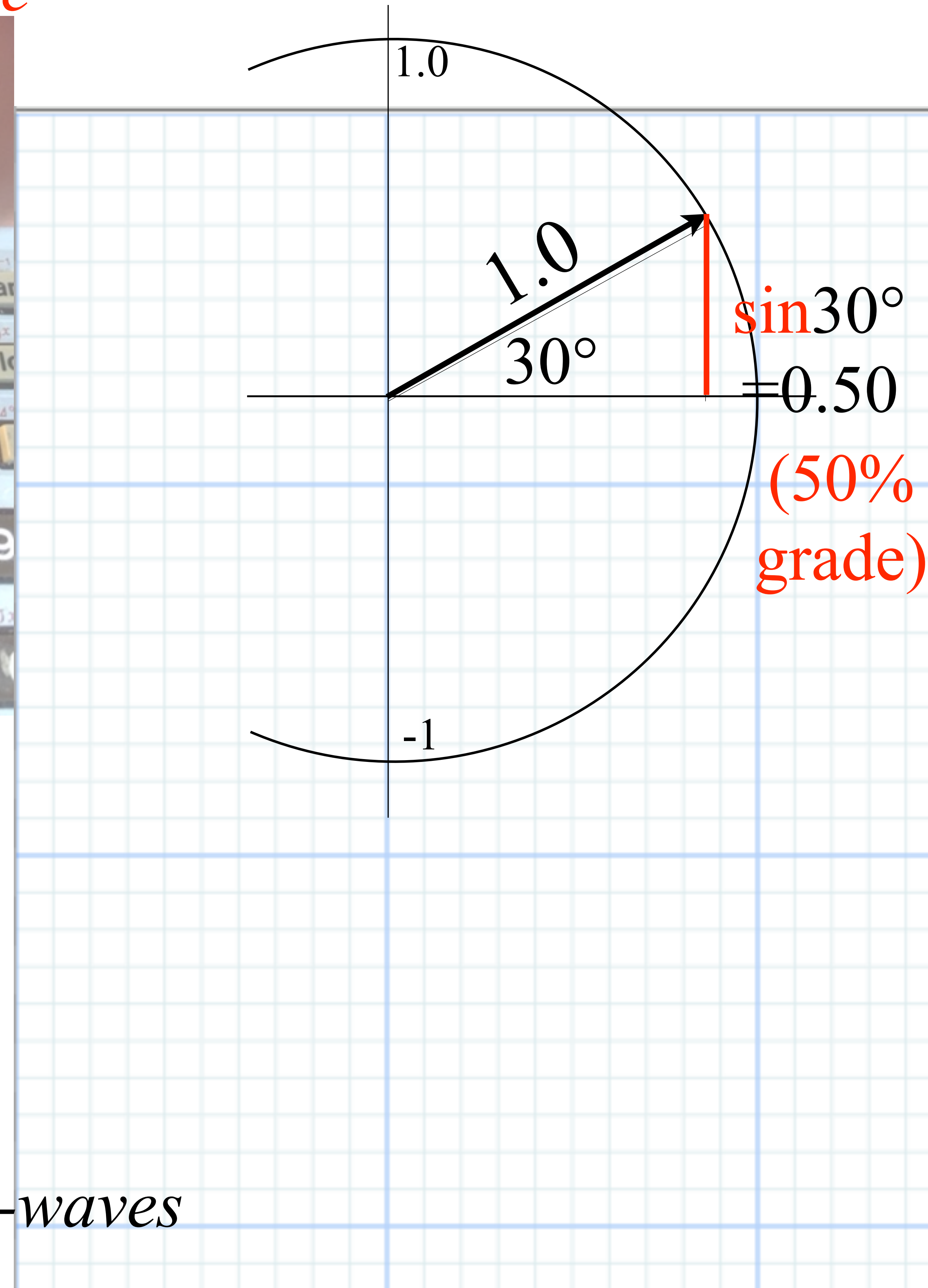




Learning about SIN

"Slope of INcline"

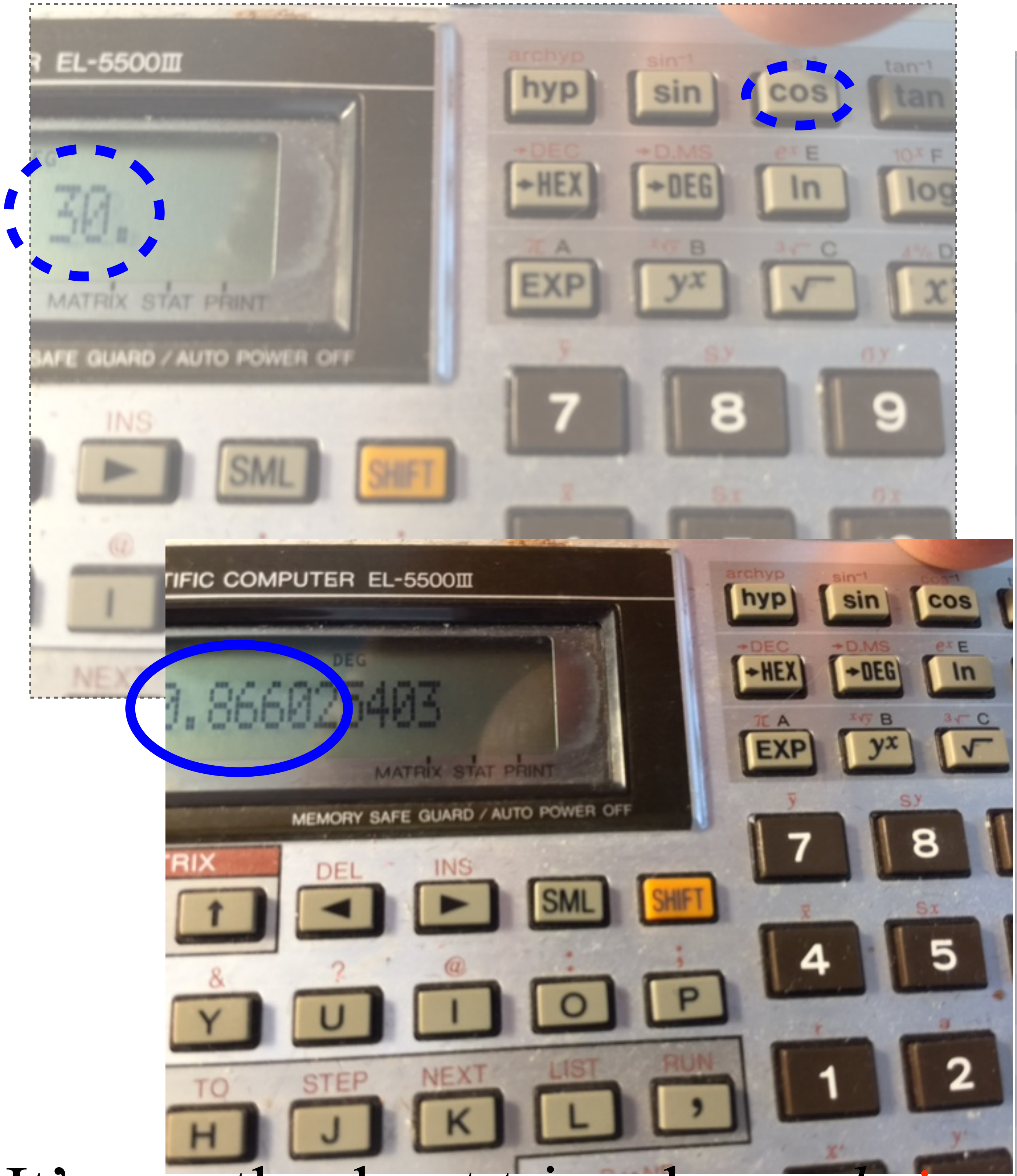


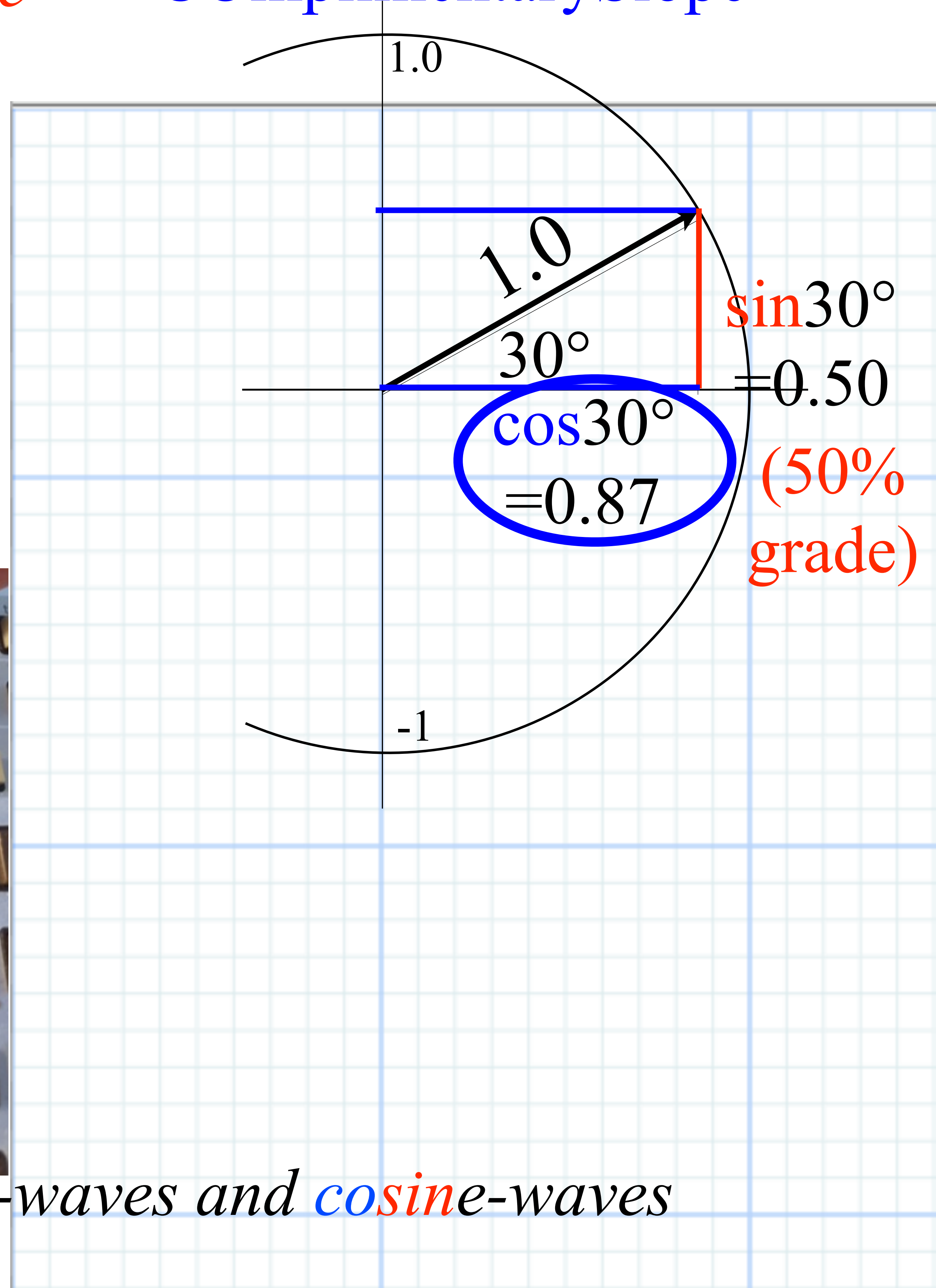


It's mostly about triangles *and sine-waves*

Learning about SIN and the COSin

“*Slope of INcline*” “COmplimentarySlope”

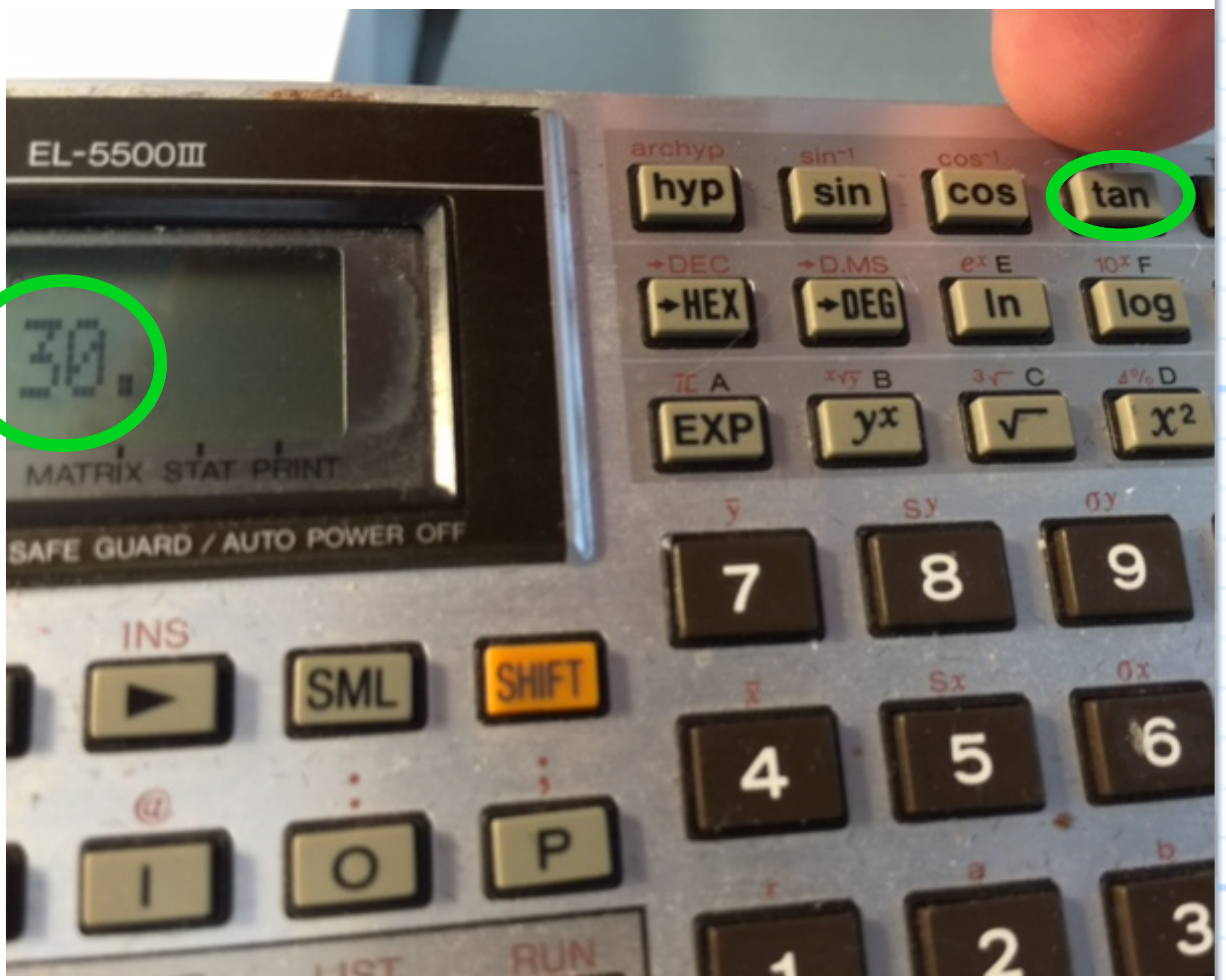


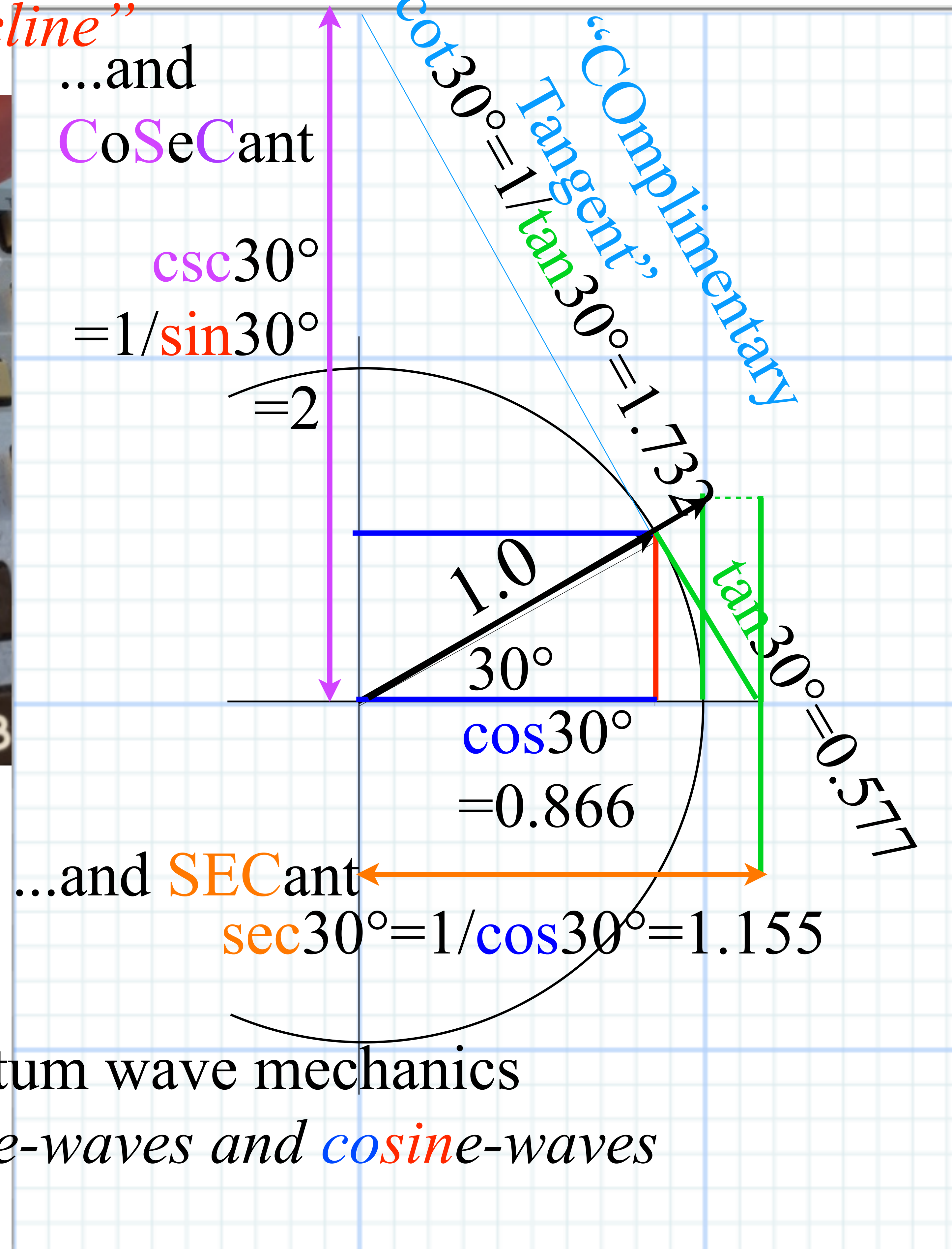


It’s mostly about triangles *and sine-waves and cosine-waves*

Learning about SIN and the COSin and TANgent and COTangent

"Slope of INcline"





Fundamental relativity and quantum wave mechanics is mostly about triangles *and sine-waves and cosine-waves*

Why ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

➔ Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra

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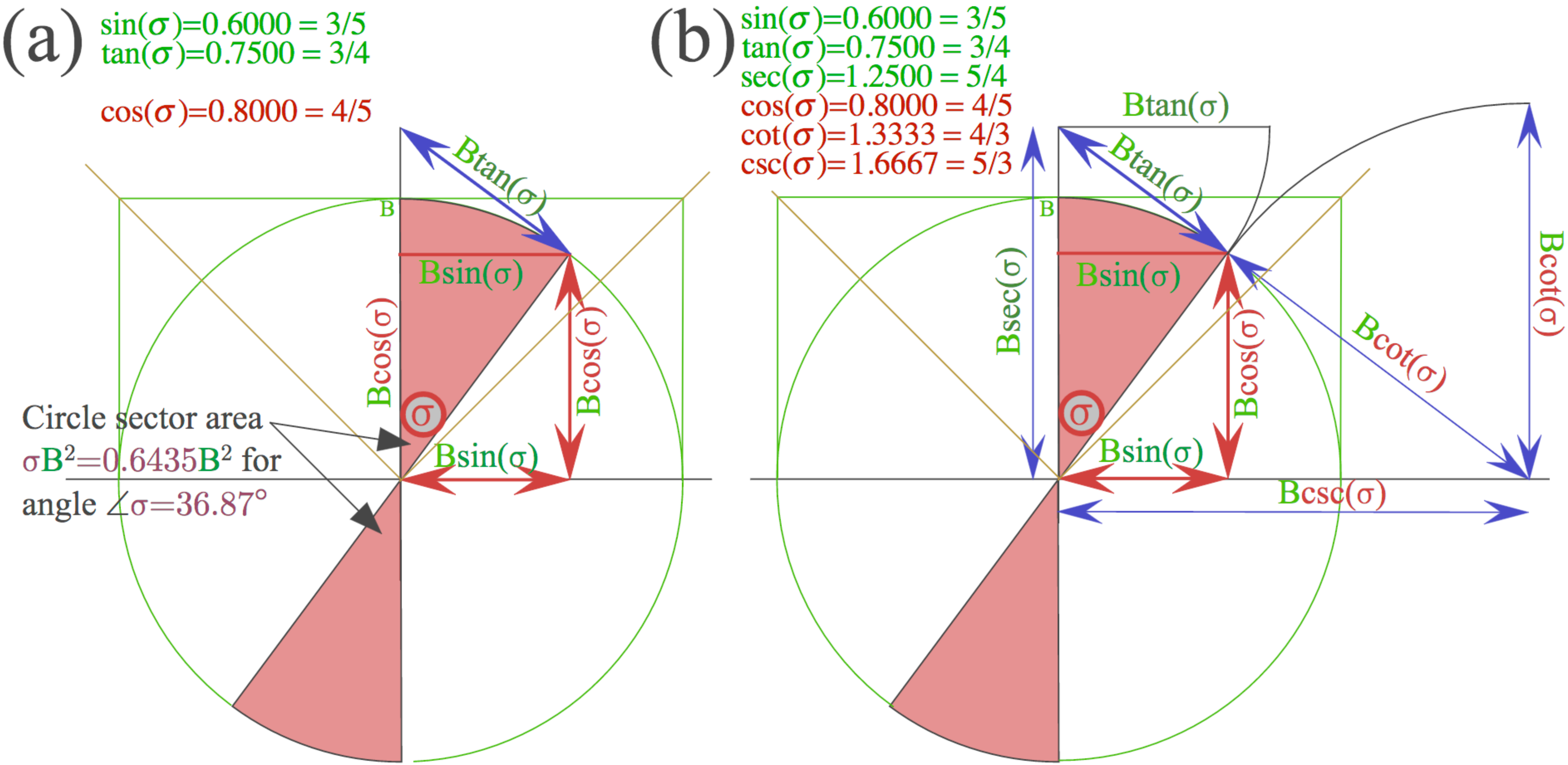
Trigonometric road maps



All this physics of relativity is mostly simple trigonometry of optical wave interference!

And, it derives fundamentals of quantum theory, too!

RelaWavity Web Simulation
Relations between
Hypergeometric and
Hypergeometric functions

Trigonometric road maps become hyperbolic trig maps…

[RelaWavity Web Simulation
Relations between
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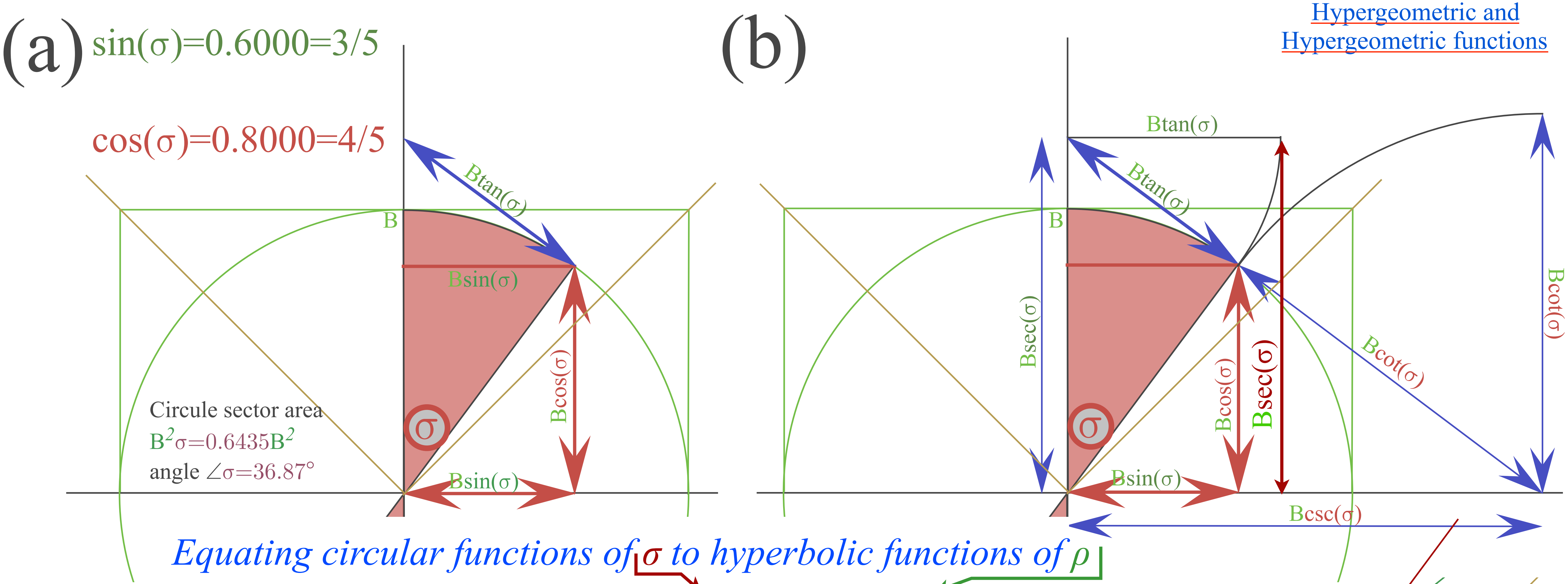

AMOP Ch.0 article p.9.

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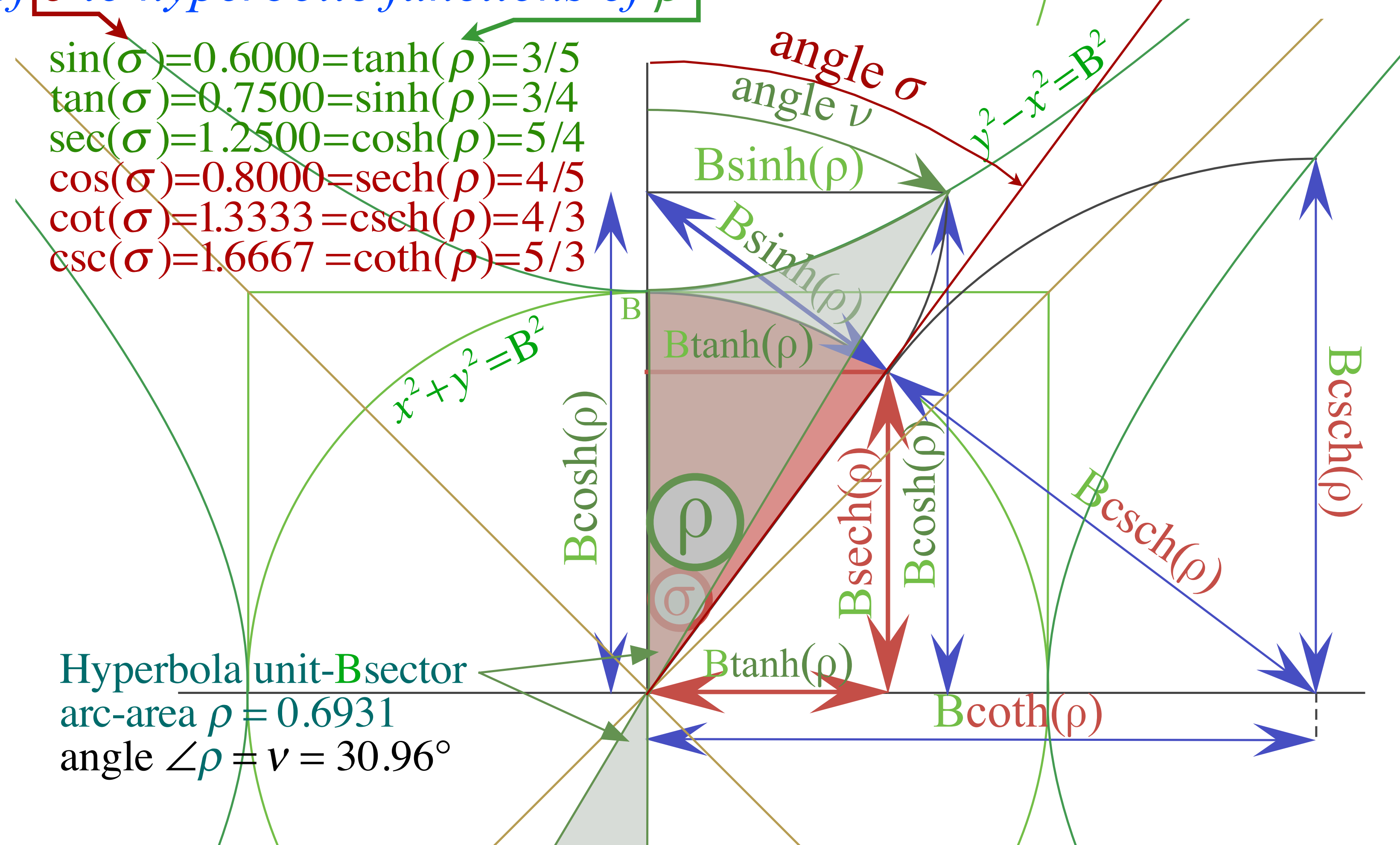

Trigonometric road maps become hyperbolic trig maps…

Need to see how trig road map matches the physical map on following page.

RelaWavity Web Simulation
Relations between
Hypergeometric and
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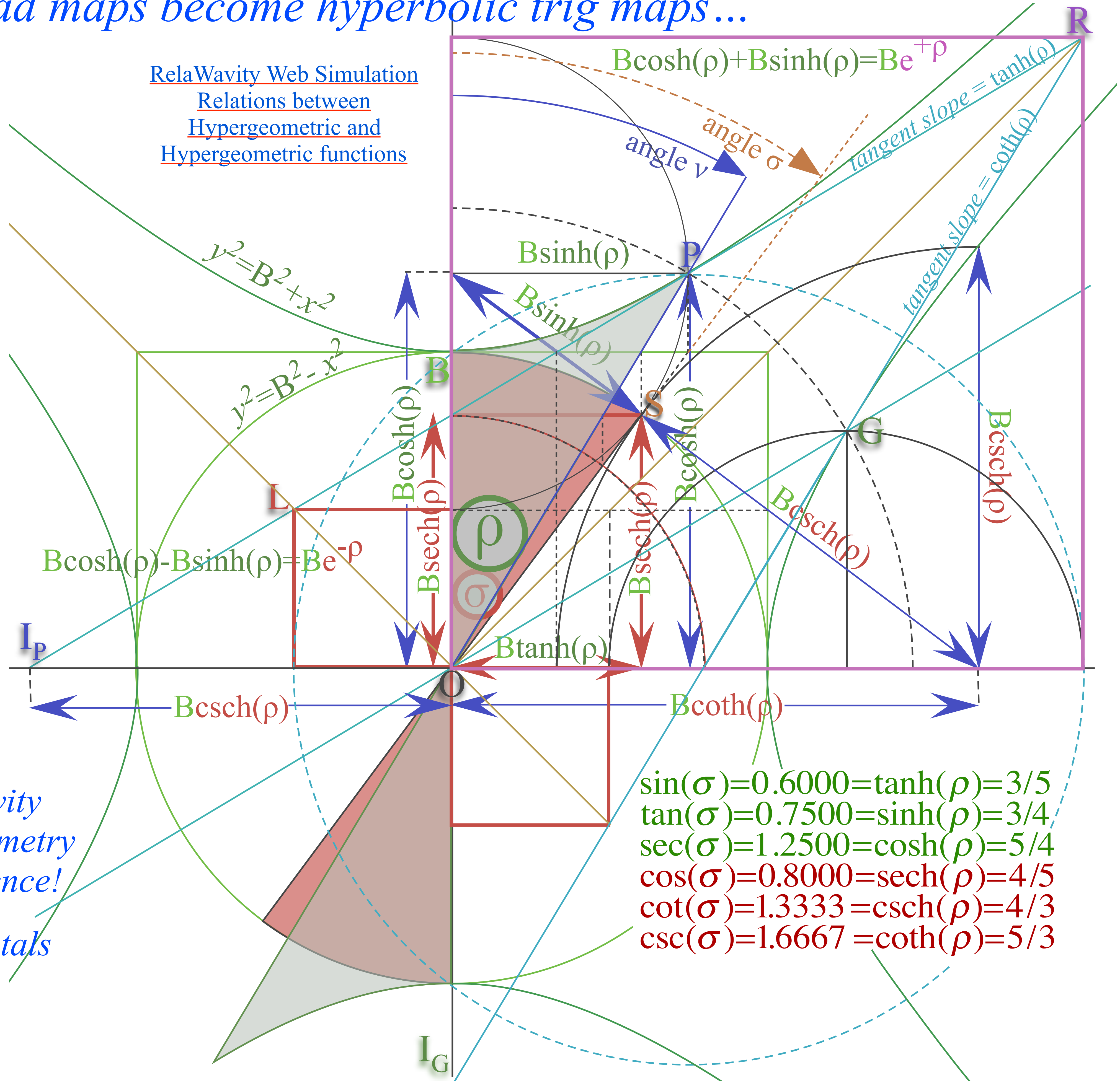

AMOP Ch.0 article p.10.

$\sin(\sigma)=0.6000=\tanh(\rho)=3/5$
$\tan(\sigma)=0.7500=\sinh(\rho)=3/4$
$\sec(\sigma)=1.2500=\cosh(\rho)=5/4$
$\cos(\sigma)=0.8000=\text{sech}(\rho)=4/5$
$\cot(\sigma)=1.3333=\text{csch}(\rho)=4/3$
$\csc(\sigma)=1.6667=\coth(\rho)=5/3$

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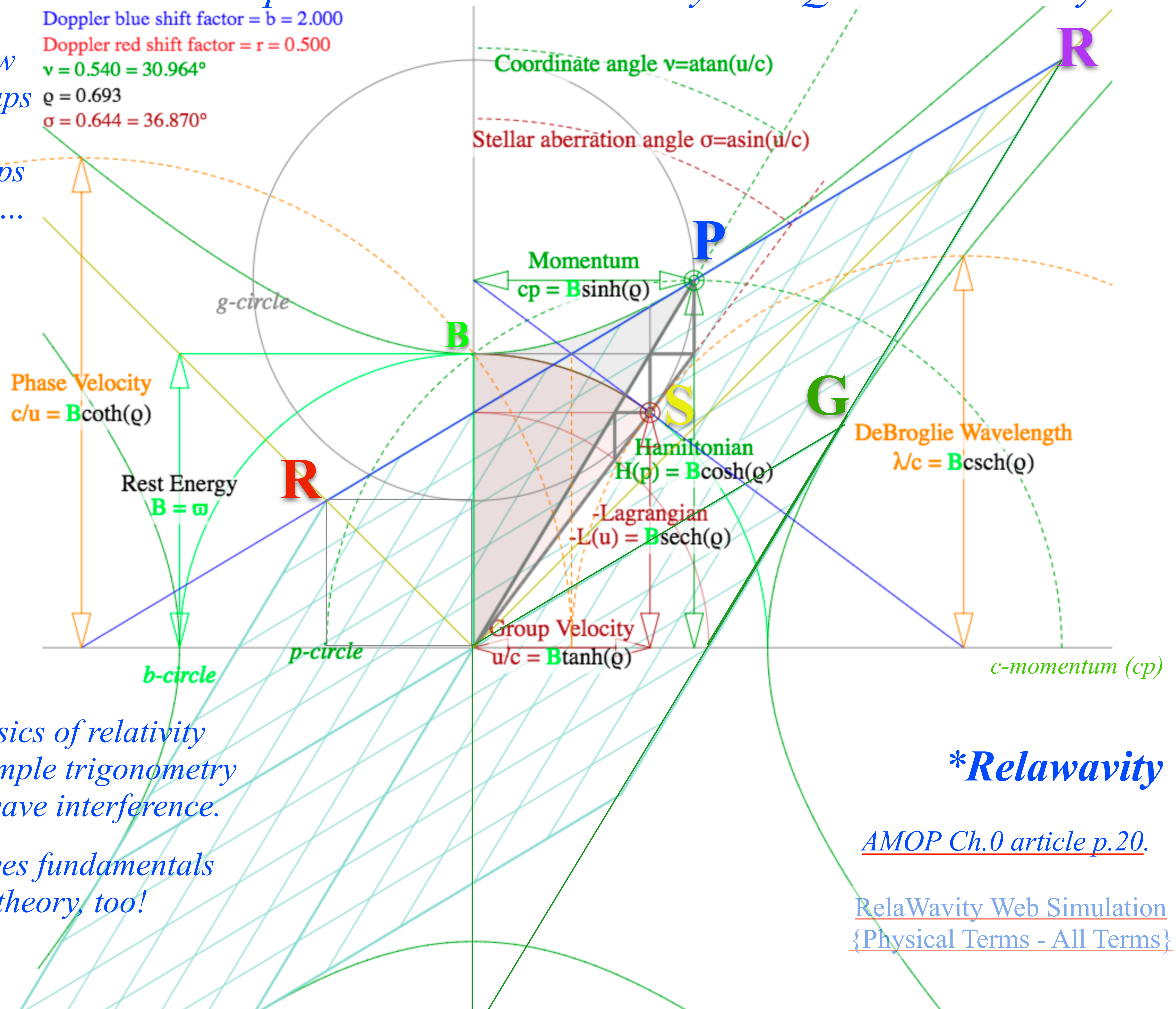
*Trigonometric road maps…. to Relativity and Quantum Theory**

Need to show trig road maps can match physical maps like this one…



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Relawavity

AMOP Ch.0 article p.20.

RelaWavity Web Simulation
{Physical Terms - All Terms}

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Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra ←

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Hyper-Trigonometric algebra easily derives *Circular*-Trigonometric-algebra

Exponential derived by infinite-n-compounding limit of the interest rate-r formula.

$$e^{rt} = \lim_{n\to\infty}\left(1+\frac{rt}{n}\right)^n$$

Infinite-n limit of binomial series is an exponential power-p series of $(rt)^p$ with $1/p!$ coefficients.

$$e^{rt} = 1 + rt + \frac{(rt)^2}{2} + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \ldots$$

$$e^{-rt} = 1 - rt + \frac{(rt)^2}{2} - \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots$$

Half-sum and half difference of $e^{\pm rt}$ series define the hyperbolic cosine ($\cosh(rt)$) and sine ($\sinh(rt)$).

$$\frac{e^{+rt}+e^{-rt}}{2} = 1 + \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cosh(rt)$$

$$\frac{e^{+rt}-e^{-rt}}{2} = rt + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \ldots = \sinh(rt)$$

Hyper-Trig

$\cosh\rho$ and $\sinh\rho$

Hyper-Trigonometric algebra easily derives *Circular*-Trigonometric-algebra

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$$e^{-rt} = 1 - rt + \frac{(rt)^2}{2} - \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cosh(rt) - \sinh(rt)$$

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$$\frac{e^{+rt}-e^{-rt}}{2} = rt + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \ldots = \sinh(rt)$$

Hyper-Trig

coshρ and sinhρ

Replace rate r with imaginary rate ir and $i=\sqrt{-1}$ powers $i^0=1$, $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$, $i^5=i$, $i^6=-1$, $i^7=-i$,…

Then *hyper*-sine-cosine becomes the *circular*-sine-cosine.

$$\frac{e^{+i\,rt}+e^{-i\,rt}}{2} = 1 - \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cos rt$$

$$\frac{e^{+i\,rt}-e^{-i\,rt}}{2} = i\,rt - i\frac{(rt)^3}{2\cdot 3} + i\frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} - \ldots = i \sin rt$$

Circular-Trig

cosσ and sinσ

Sum and difference of this pair gives the Euler-DeMoivre relations of exponentials vs trig-functions.

$$e^{+i\sigma} = \cos\sigma + i\sin\sigma\ , \qquad e^{-i\sigma} = \cos\sigma - i\sin\sigma .$$

$$e^{+\rho} = \cosh\rho + \sinh\rho\ , \qquad e^{-\sigma} = \cosh\rho - \sinh\rho .$$

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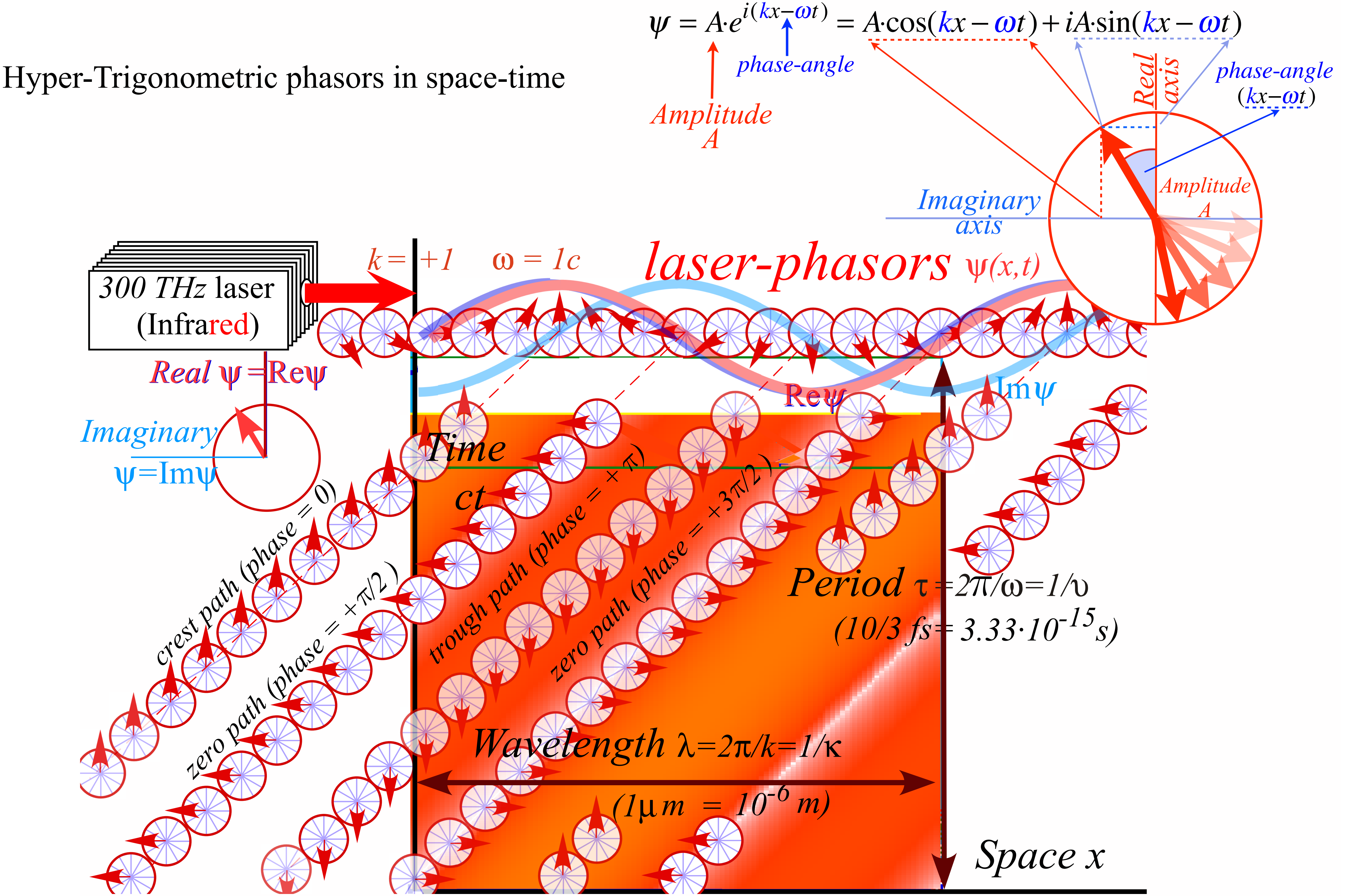
1CW Laser-phasor wave function

Hyper-Trigonometric phasors in space-time



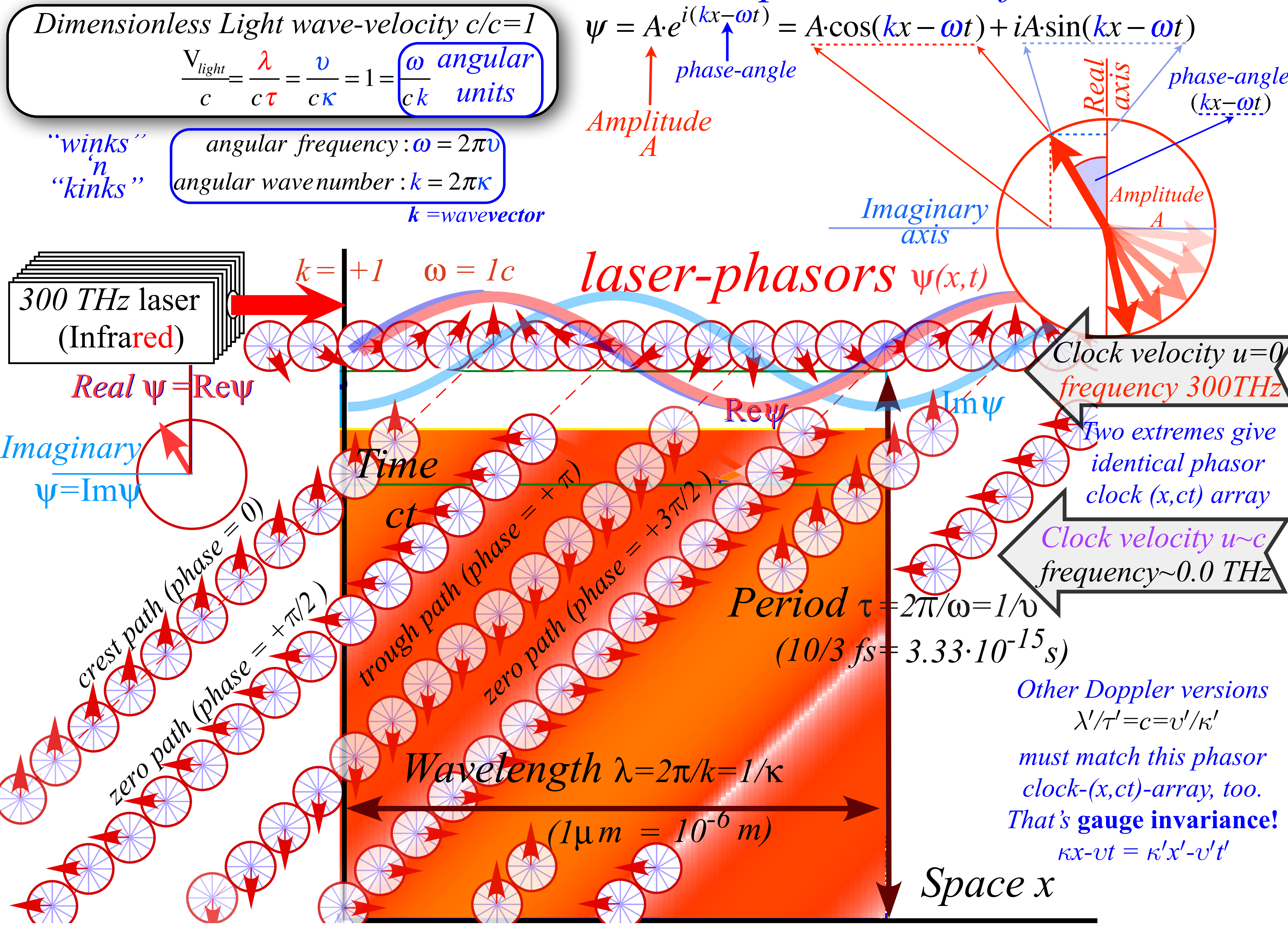
(a) Single-phasor plot of wave-function at (x,ct). (b) Array of phasors at many (x,ct)-points.

Pirelli Site
World of Clocks
Animations

24 hr. clock

12 hr. clock

1CW Laser-phasor wave function



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Application to TE-Waveguide modes and synchrotron beam relativity

The "Keyboard of the gods" : per-space-per-time plot *versus* space-time Minkowski plot

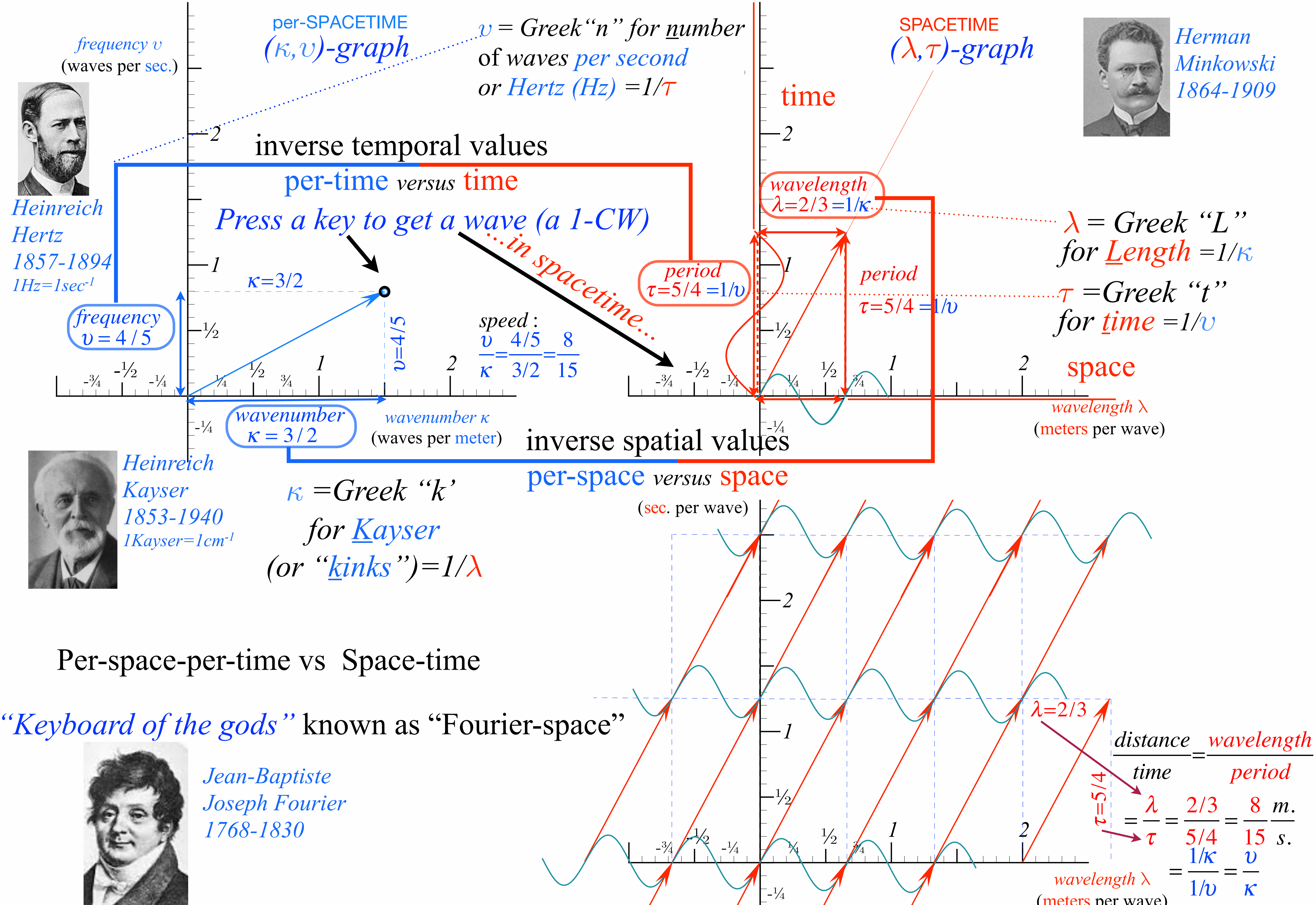
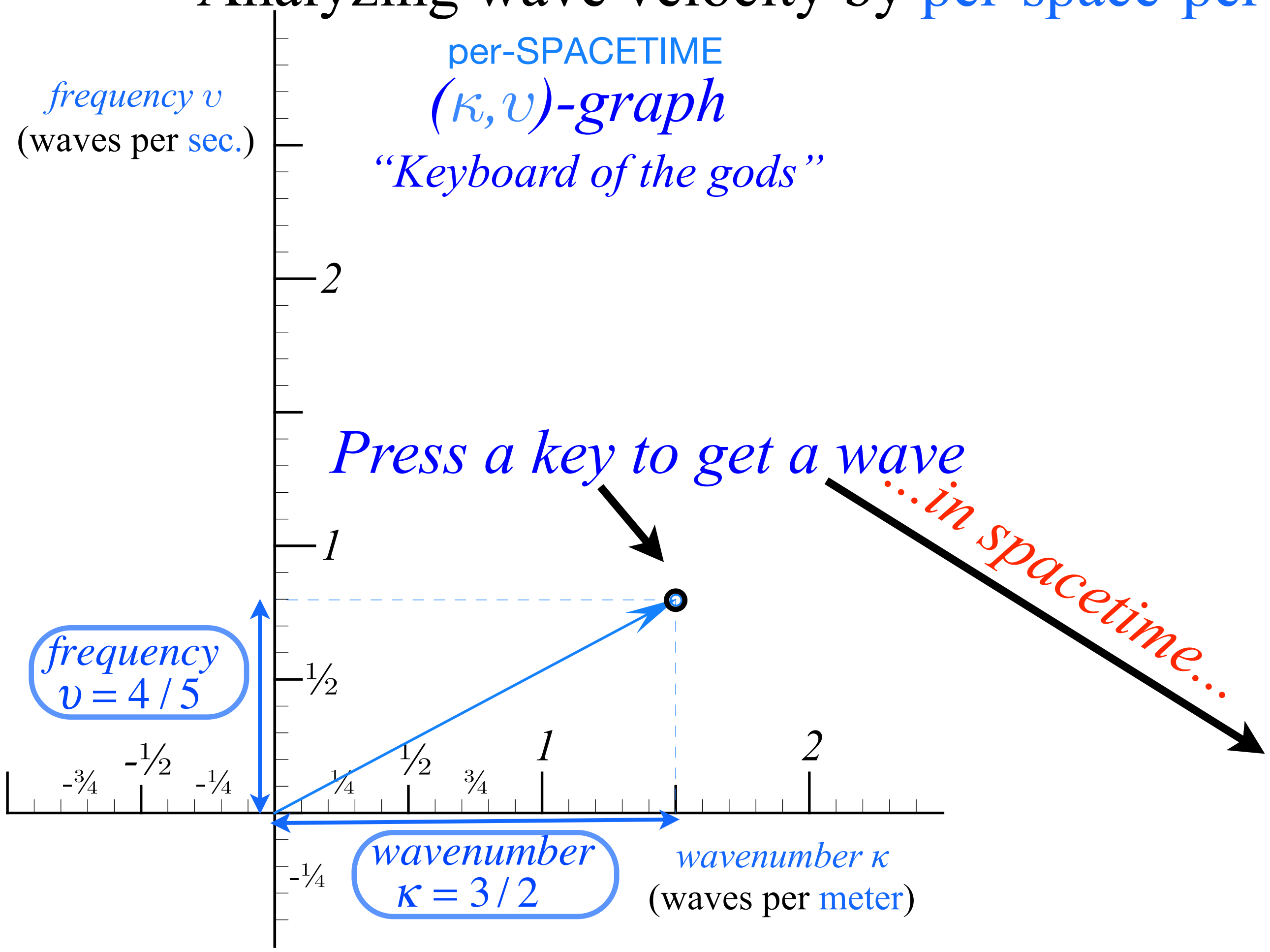


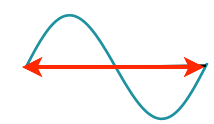
Fig. 5 Comparing a wave point in Kaiser-Hertz per-space-time to its Minkowski space-time view.

Analyzing wave velocity by per-space-per-time *and* space-time graphs



SPACETIME
(λ,τ)-graph

Herman Minkowski 1864-1909



"Keyboard of the gods" is known as "Fourier-space"

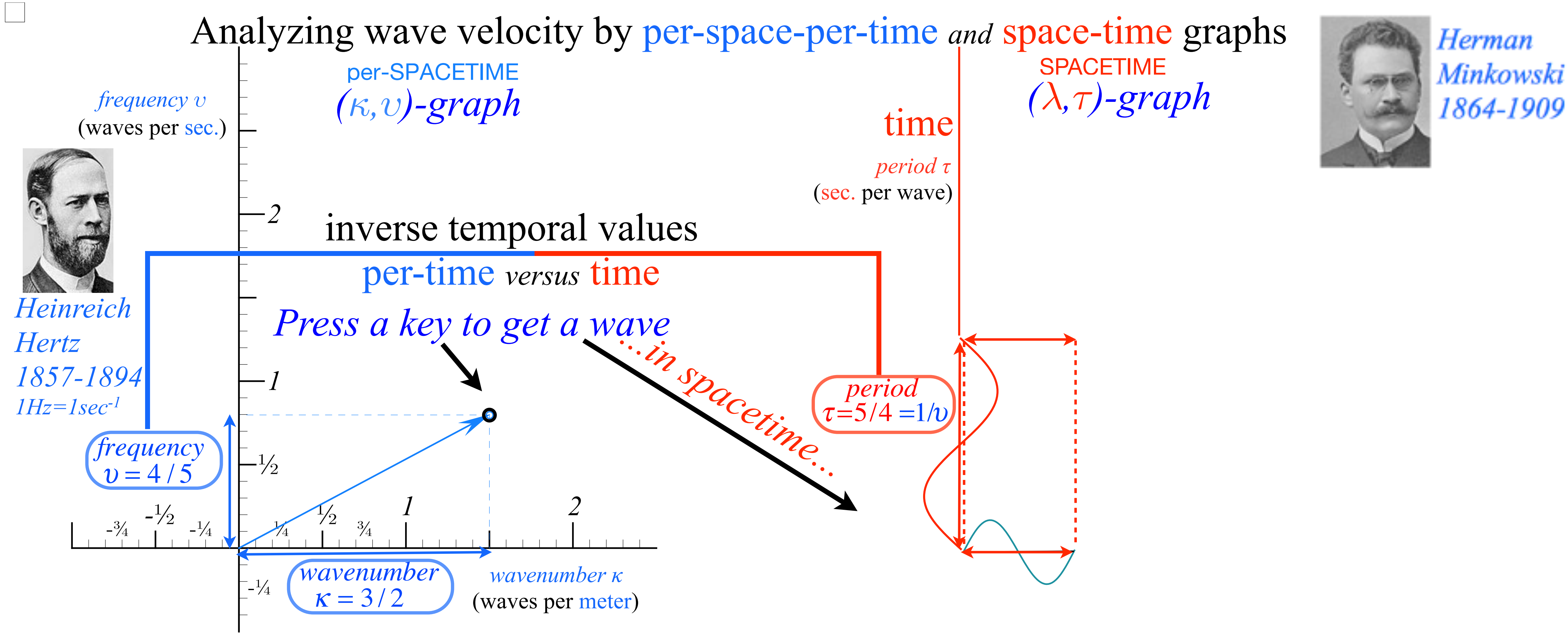
Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave parameters

wave frequency υ	*wave period τ*
wavenumber κ	*wavelength λ*

RelaWavity Web Simulation
Keyboard of the Gods (per-Time vs per-Space)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



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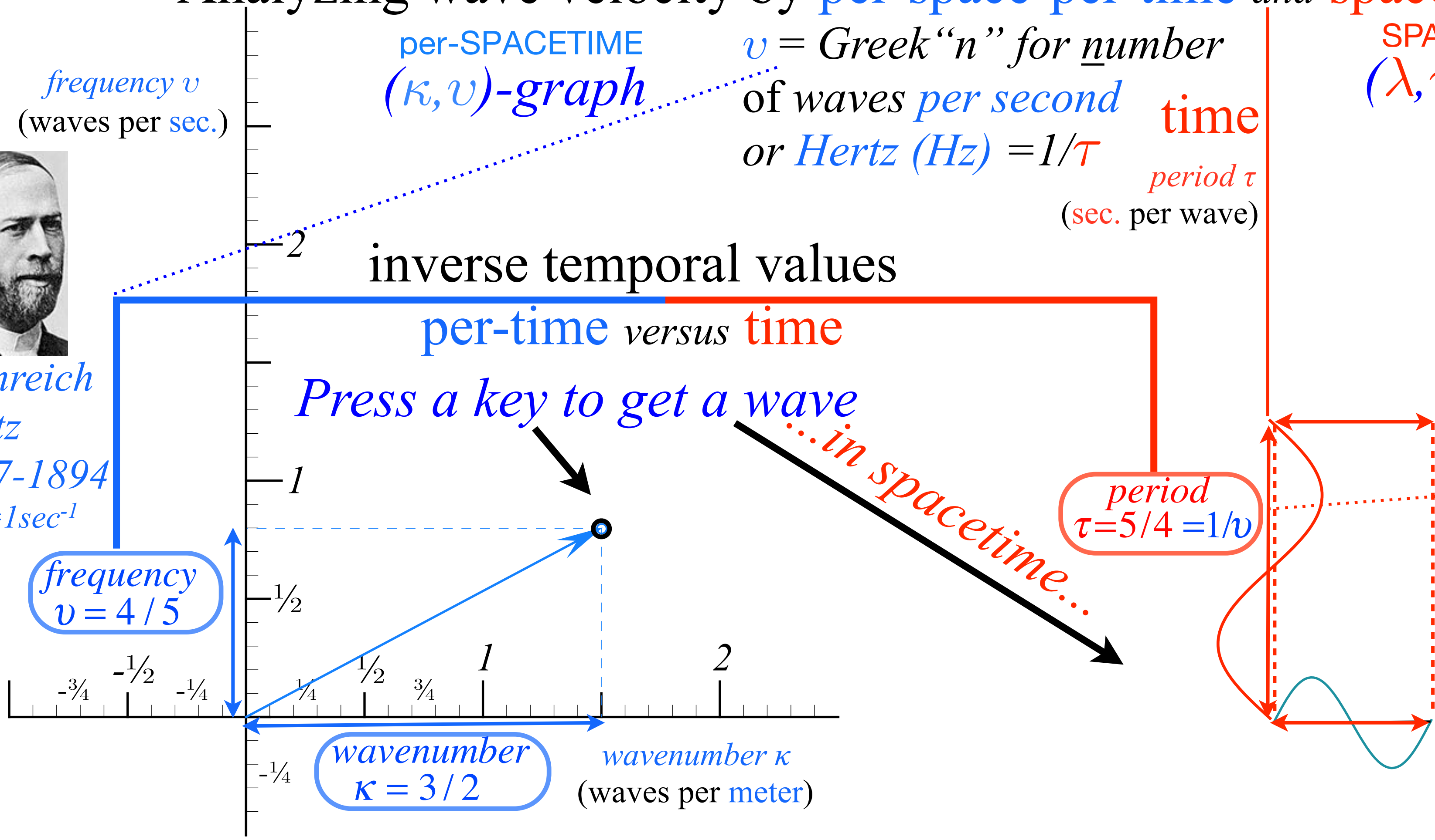

υ = Greek "n" for number of waves per second or Hertz (Hz) =1/τ

Heinreich Hertz 1857-1894

1Hz=1sec^{-1}

Herman Minkowski 1864-1909

τ =Greek "t" for time =1/υ

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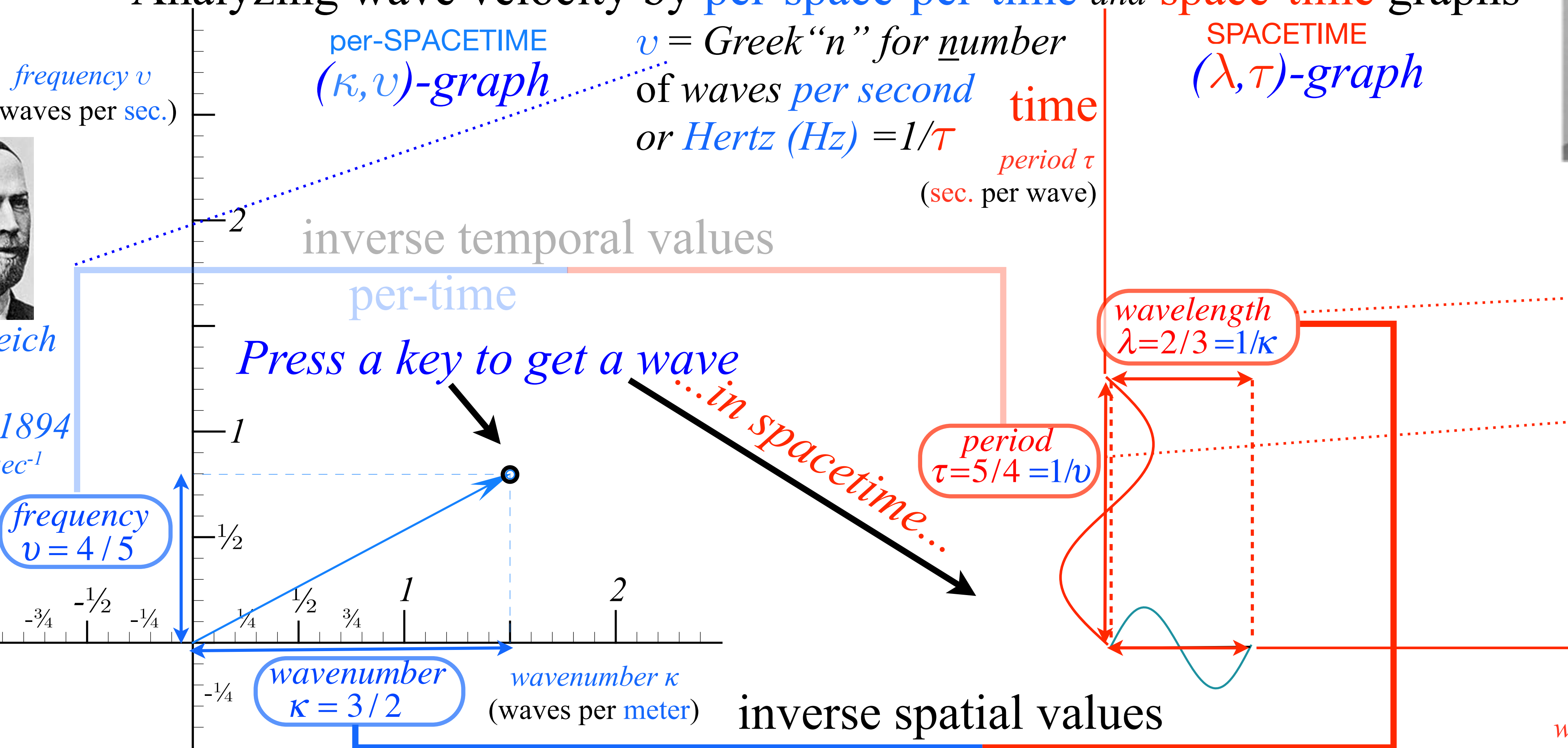
RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot #1)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



Heinreich Hertz 1857-1894
1Hz=1sec⁻¹

Herman Minkowski 1864-1909

υ = Greek "n" for number of *waves per second* or *Hertz (Hz) =1/τ*

λ = Greek "L" for Length =1/κ

τ =Greek "t" for time =1/υ

κ =Greek "k" for Kayser (or "kinks")=1/λ

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wave frequency υ	*wave period τ*
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Herman Minkowski 1864-1909

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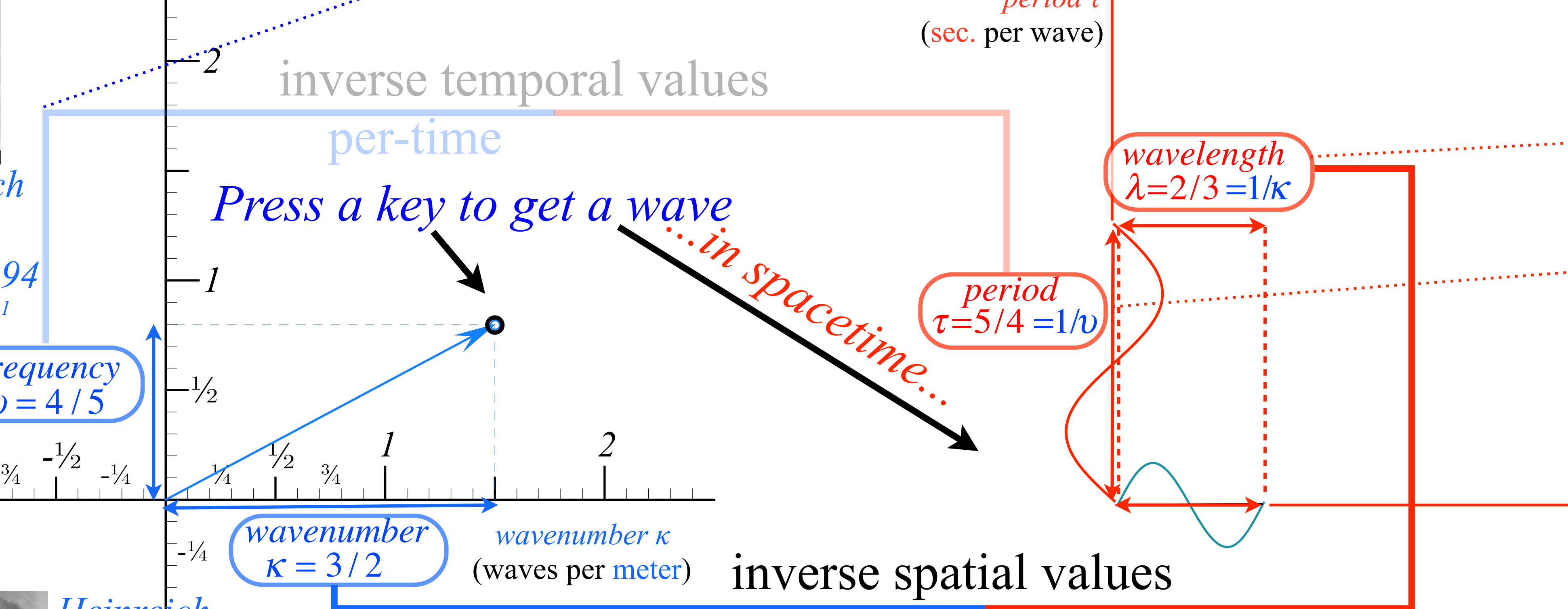
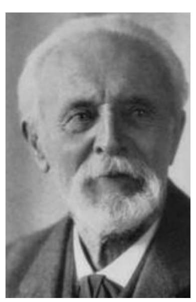


λ = Greek "L"
for Length =1/κ

τ =Greek "t"
for time =1/υ



Heinreich Kayser 1853-1940
1Kayser=1cm^{-1}

κ =Greek "k"
for Kayser
(or "kinks")=1/λ

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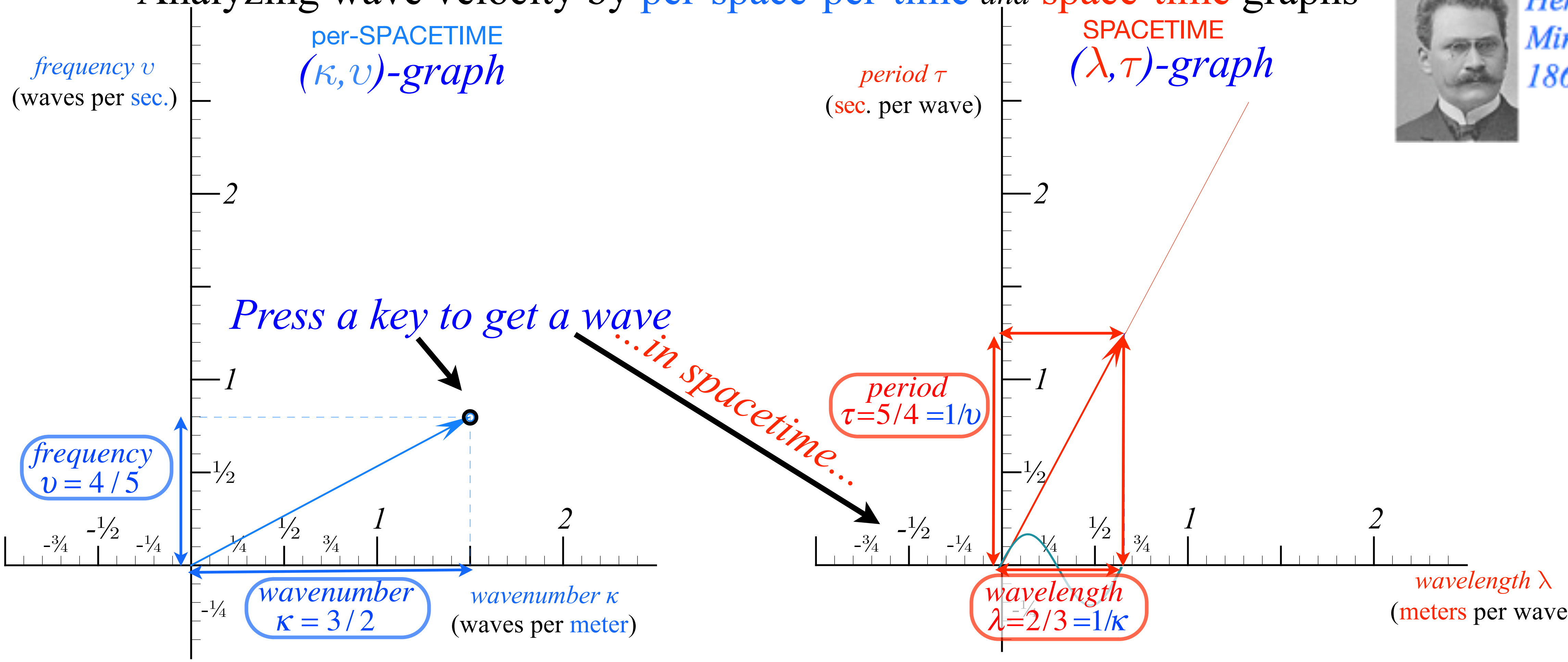
RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot #1)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909



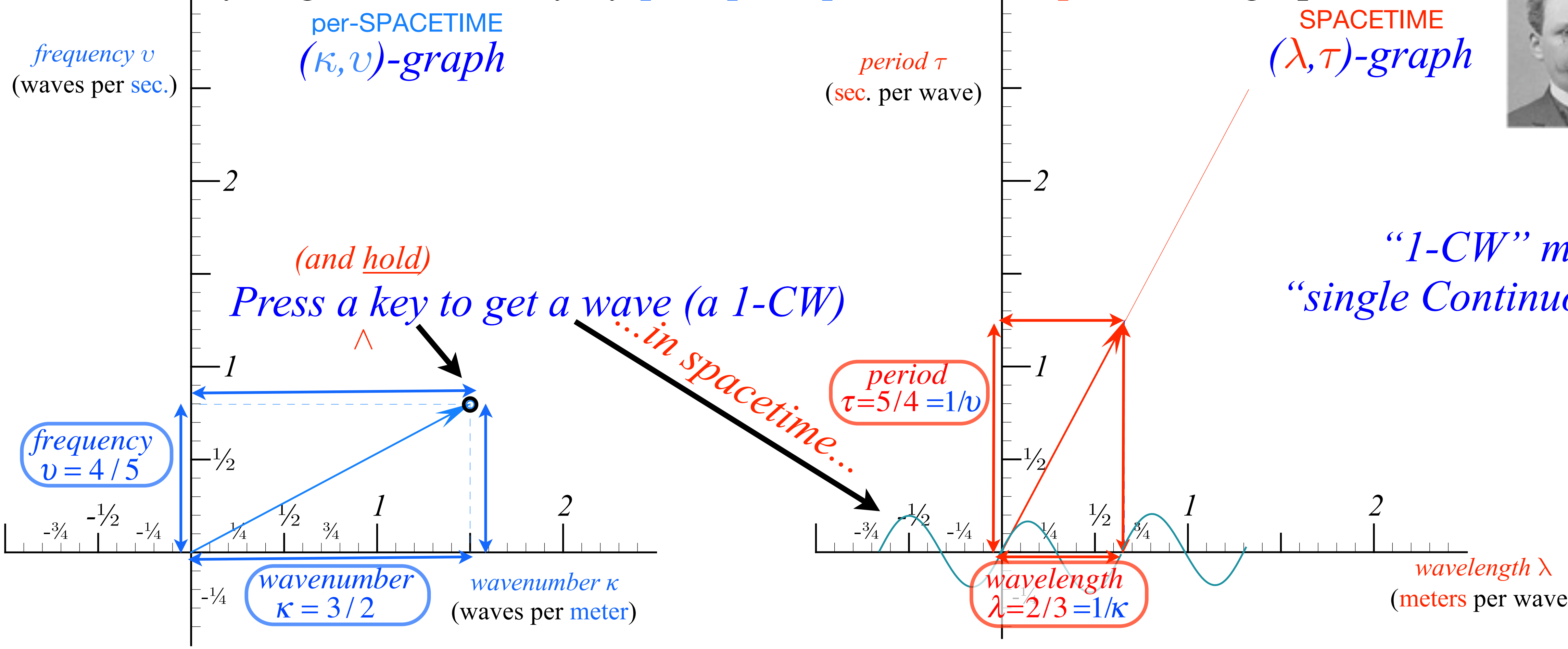
"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave parameters

wave frequency υ	*wave period τ*
wavenumber κ	*wavelength λ*

Analyzing wave velocity by per-space-per-time *and* space-time graphs



"1-CW" means "single Continuous Wave"

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- *How to understand waves and wave parameters*

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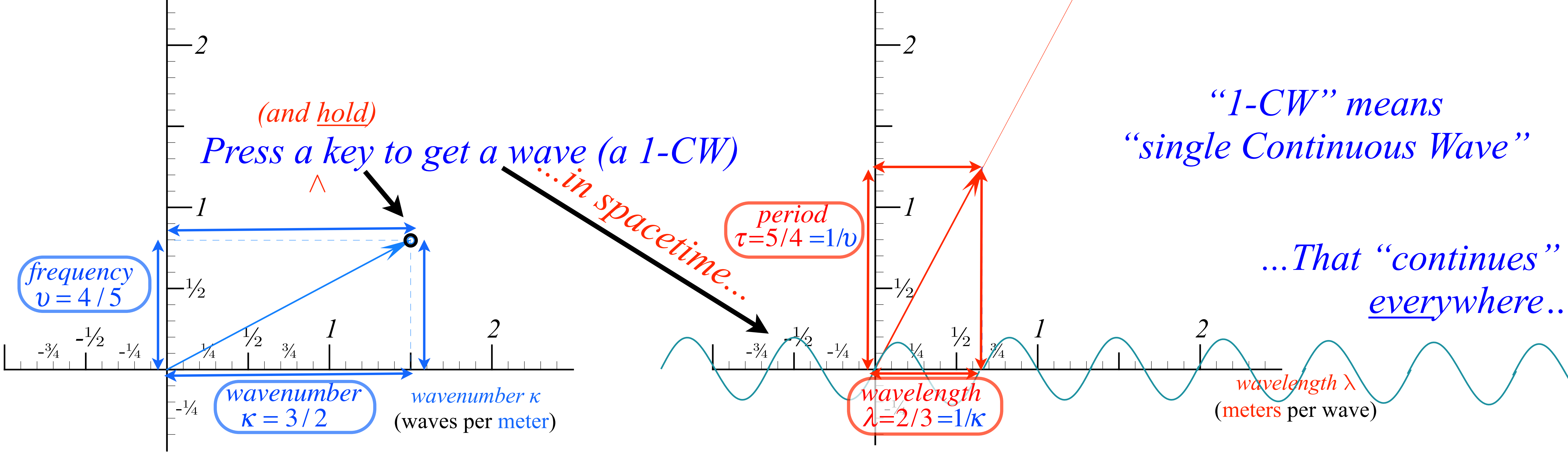
RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot #2)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909



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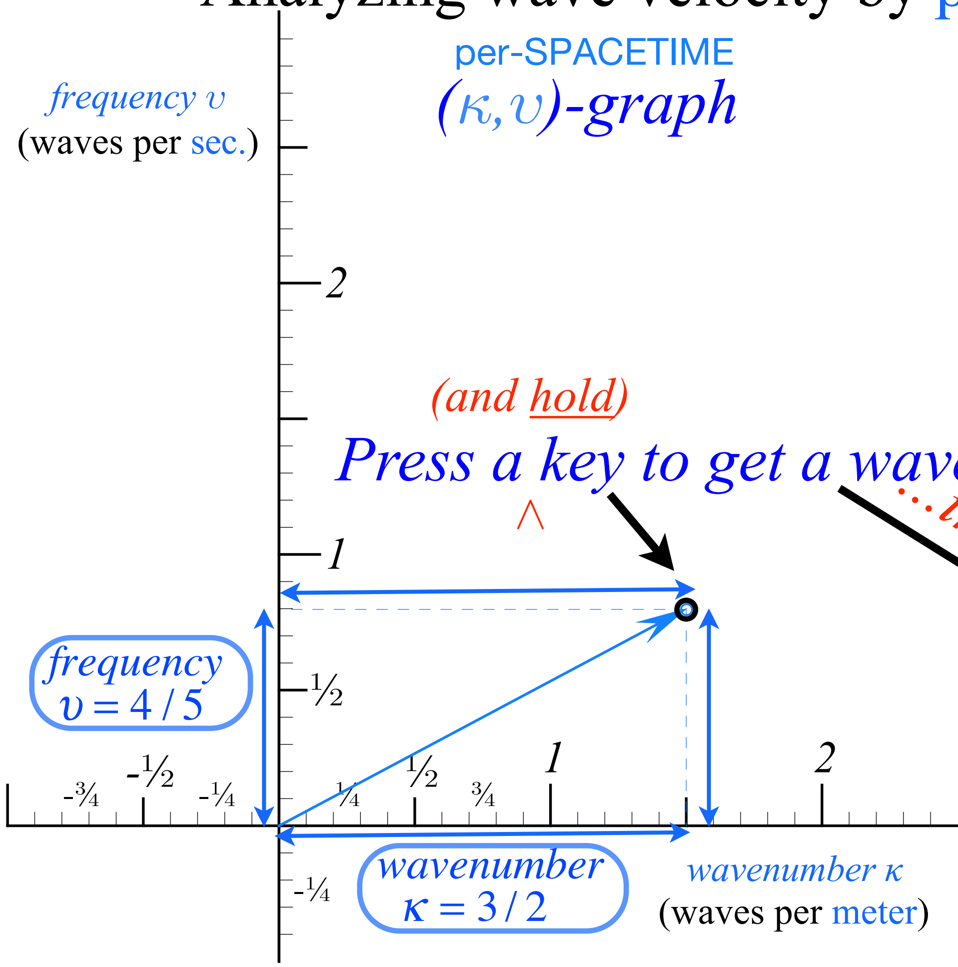
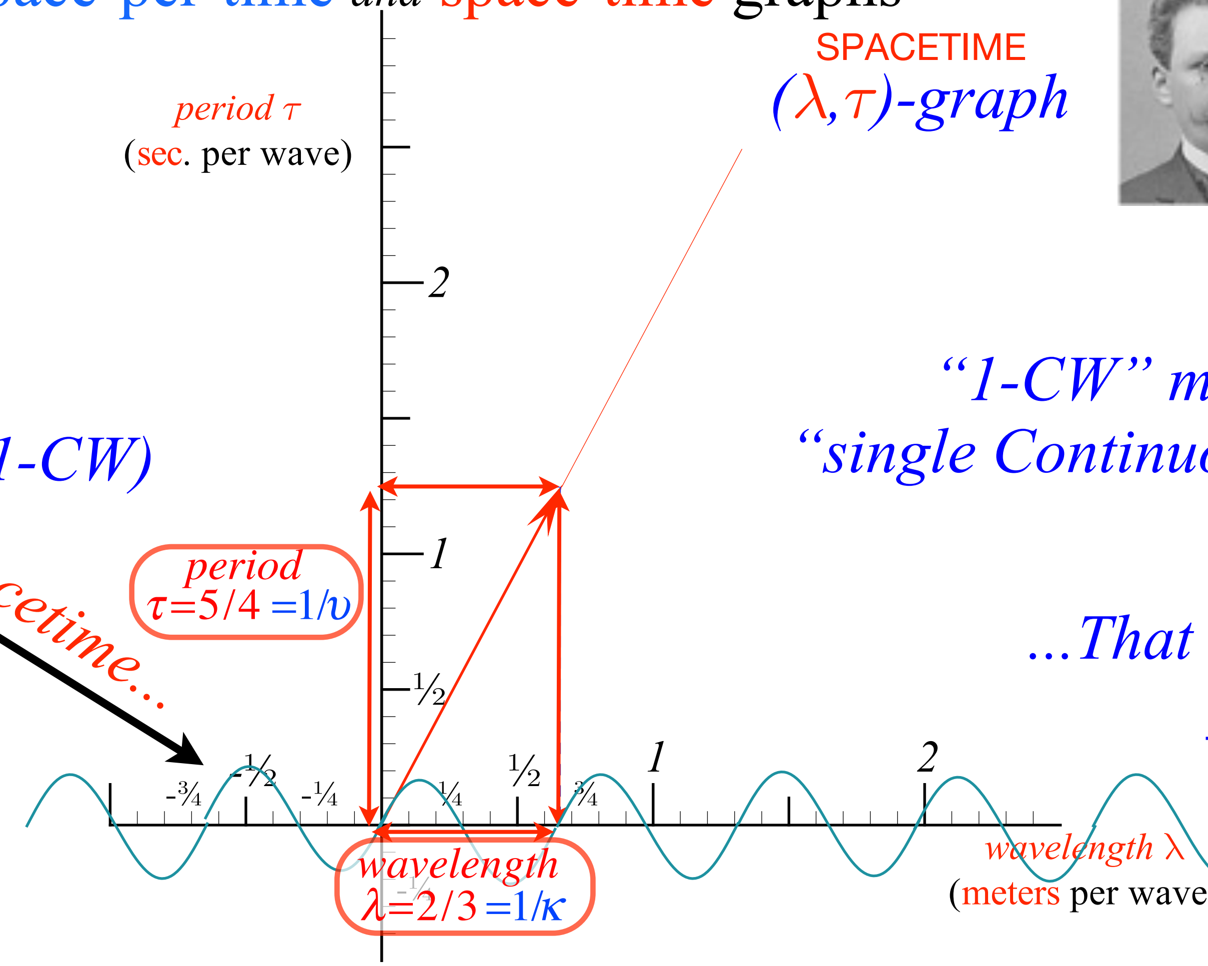
RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot #3)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



(and hold)
Press a key to get a wave (a 1-CW)
...in spacetime...

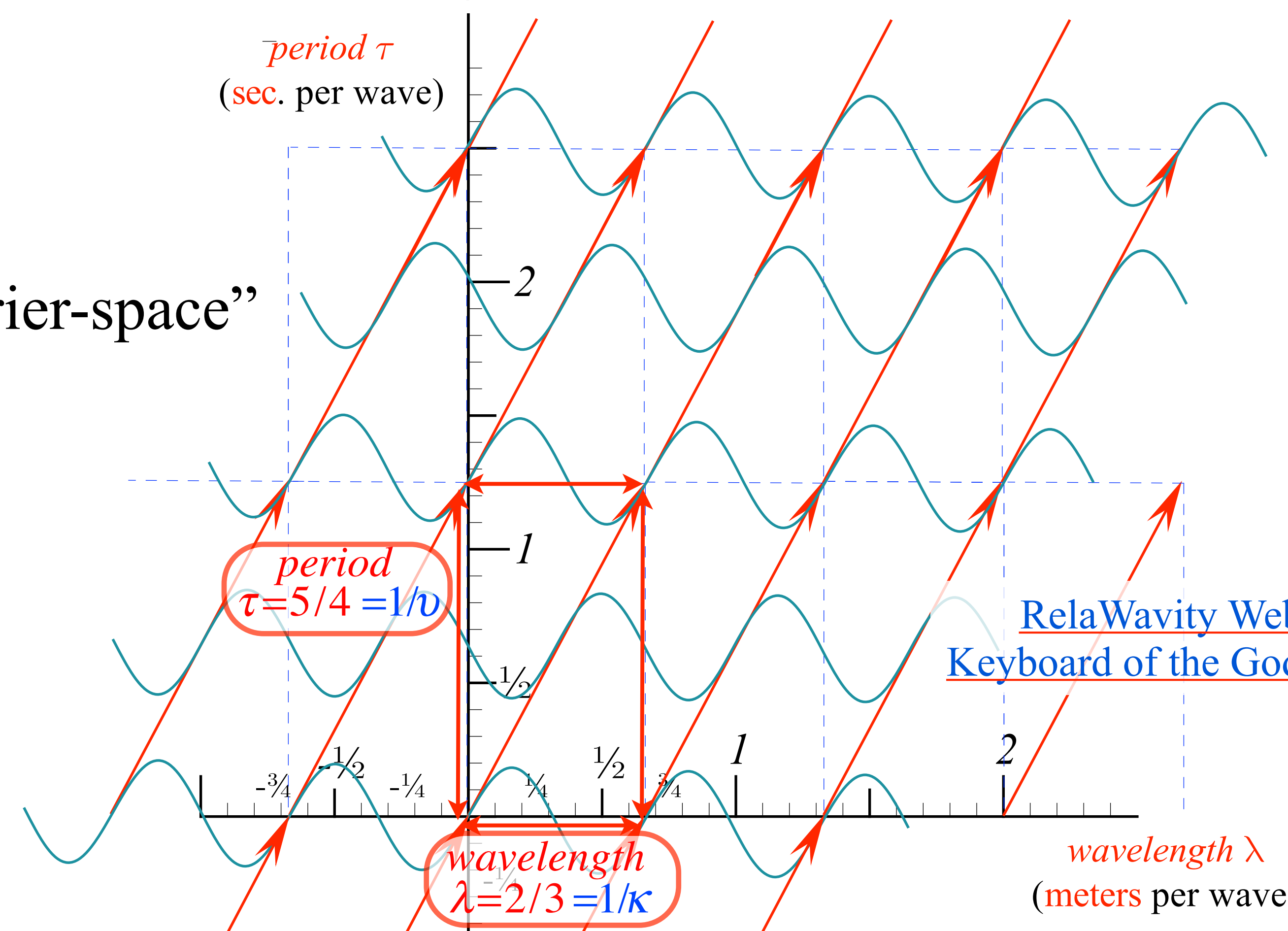


Herman Minkowski 1864-1909

"1-CW" means "single Continuous Wave"

...That "continues" everywhere...

...for all time...



RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot #7)

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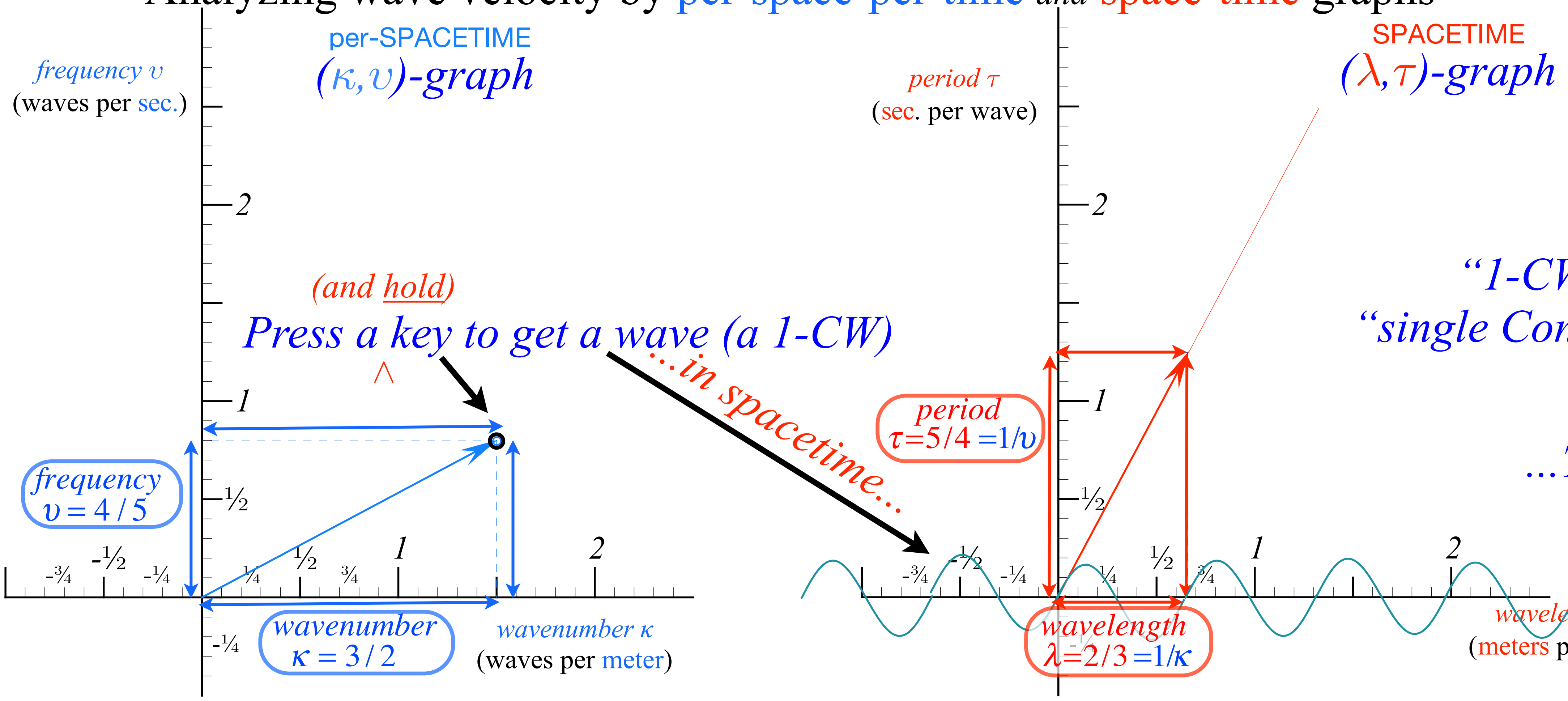

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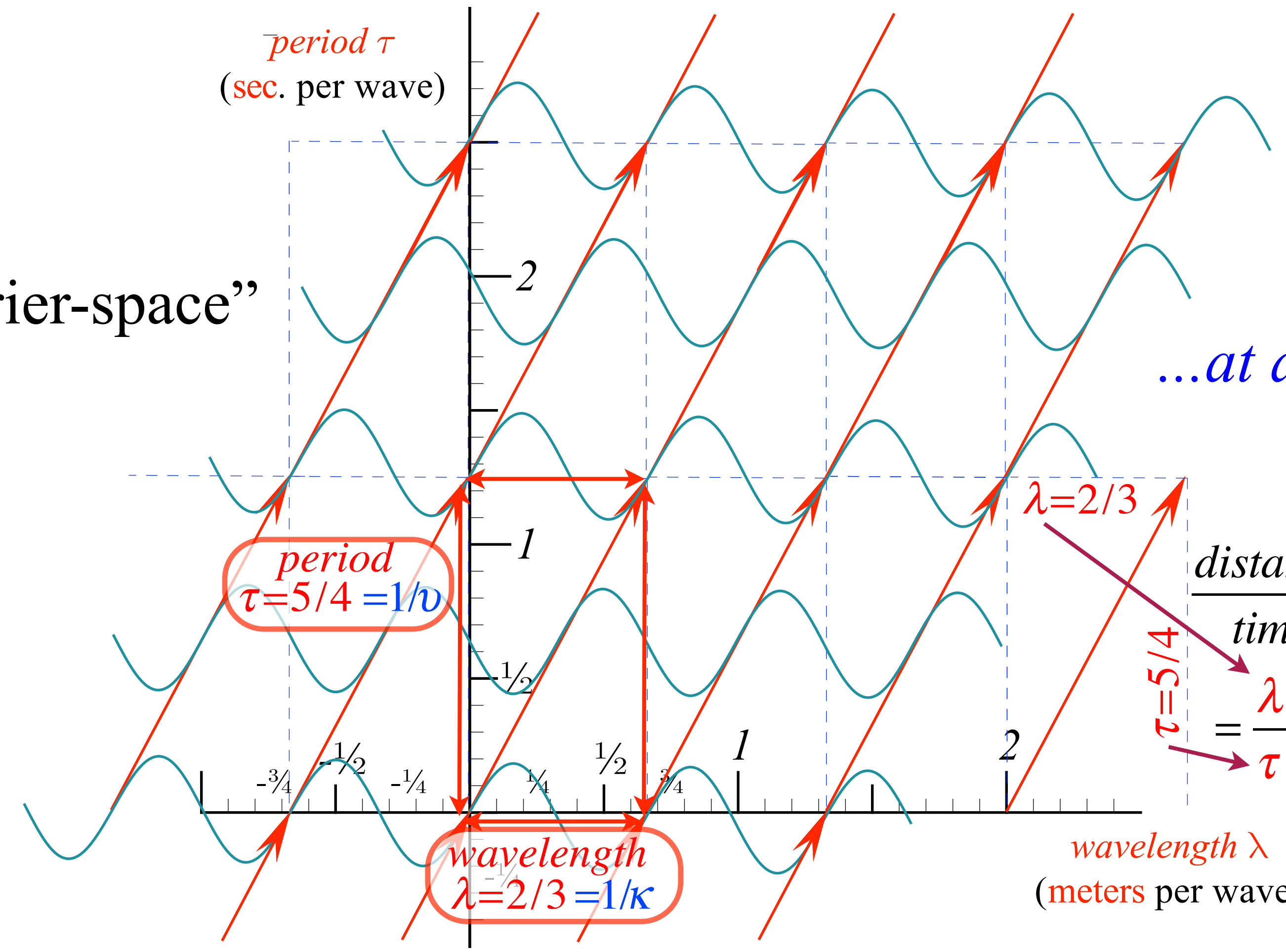
...That "continues" everywhere...

...for all time...

...at a speed of:

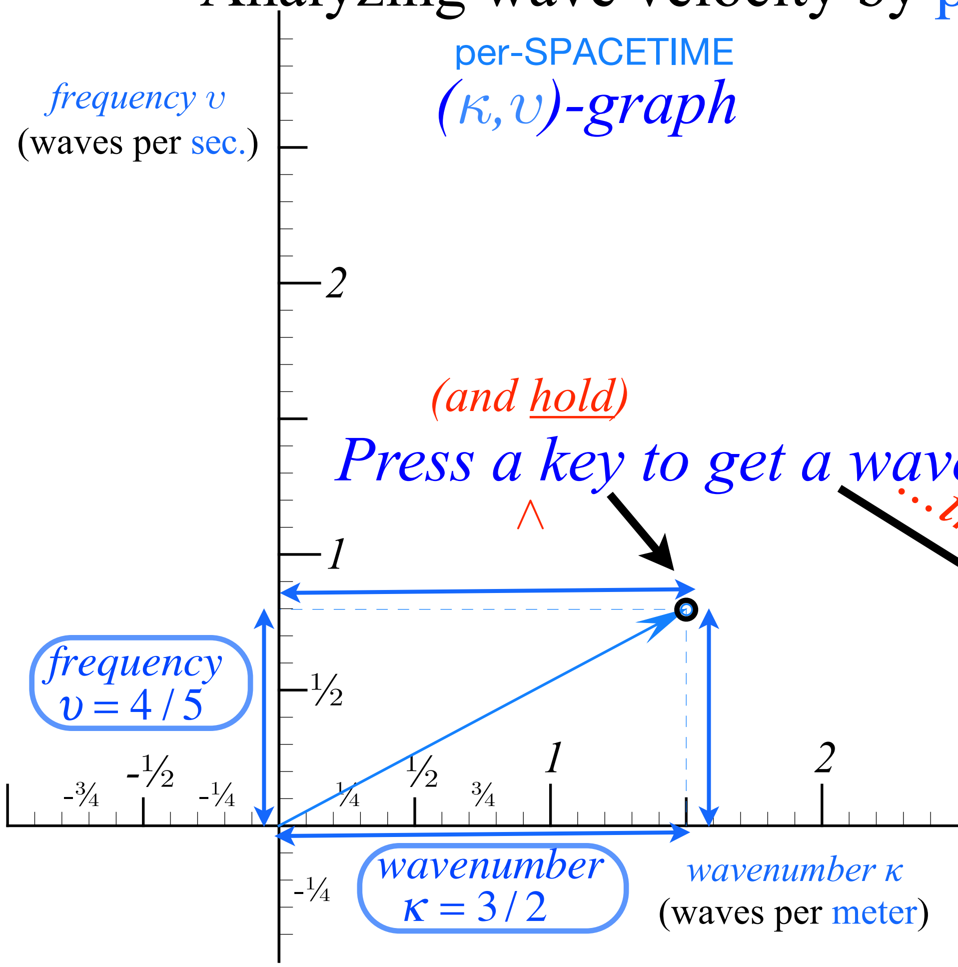


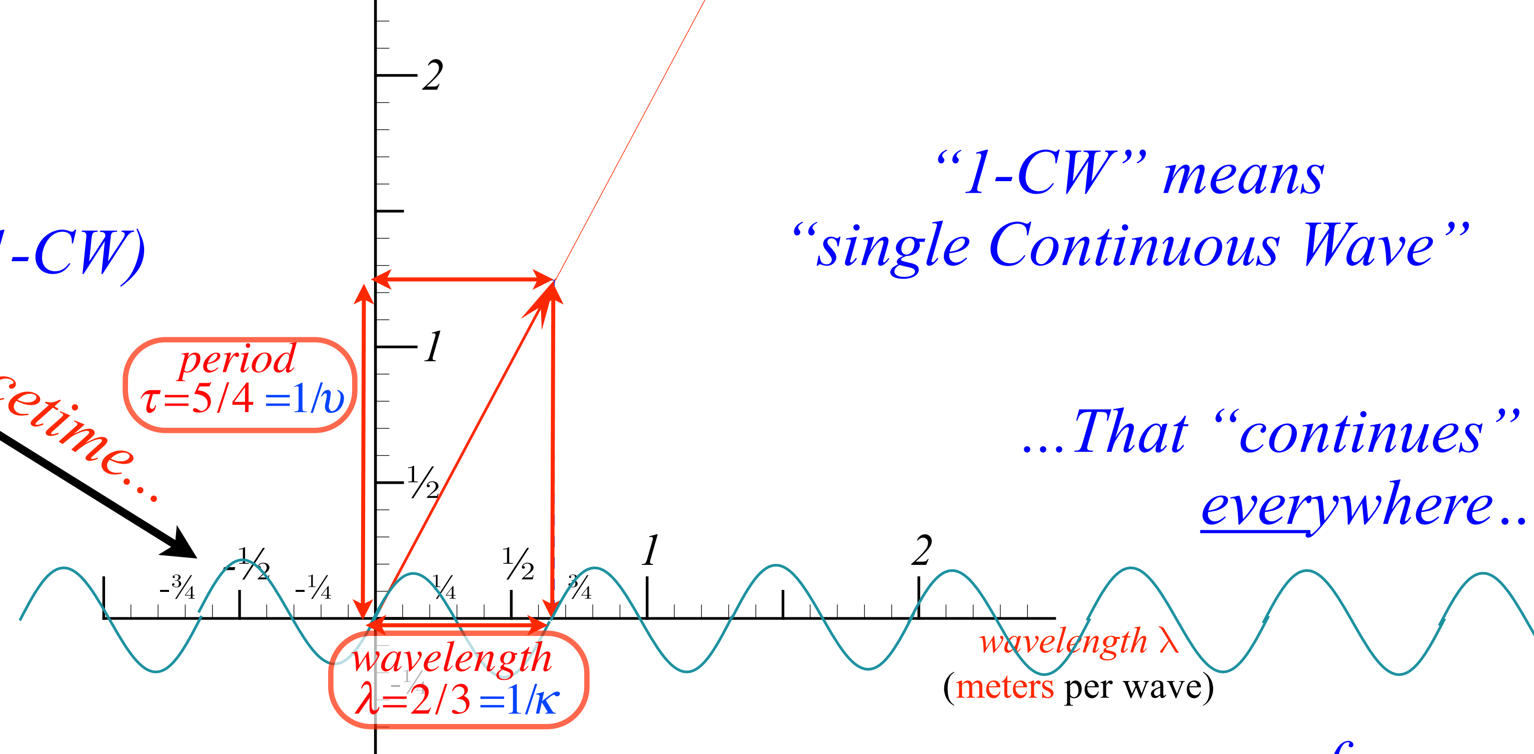
$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau}$$

"Keyboard of the gods" is known as "Fourier-space"

•How to understand waves and wave velocity V_{wave}

Analyzing wave velocity by per-space-per-time *and* space-time graphs





Herman Minkowski 1864-1909

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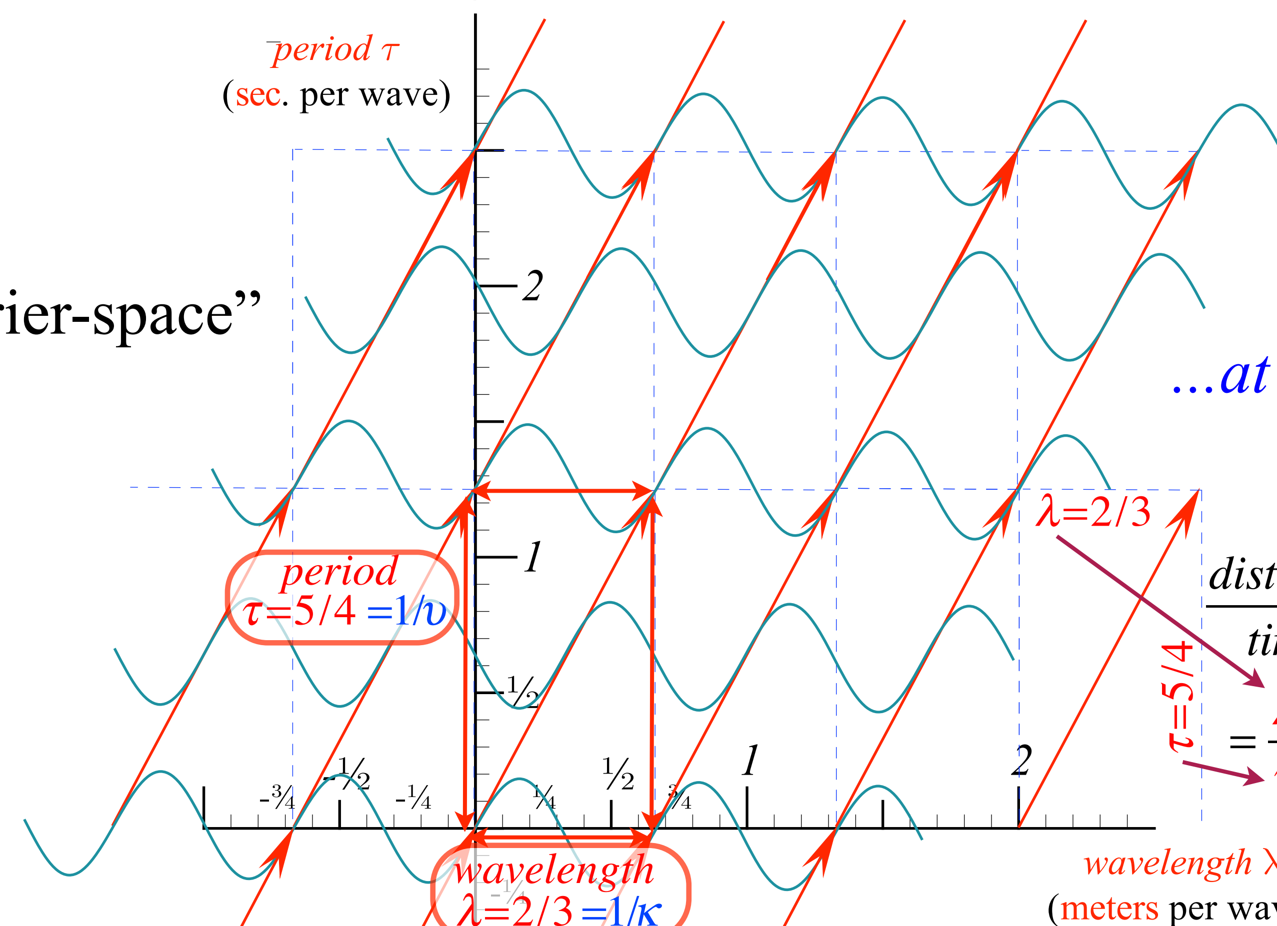

…for all time…

…at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

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Jean-Baptiste Joseph Fourier 1768-1830

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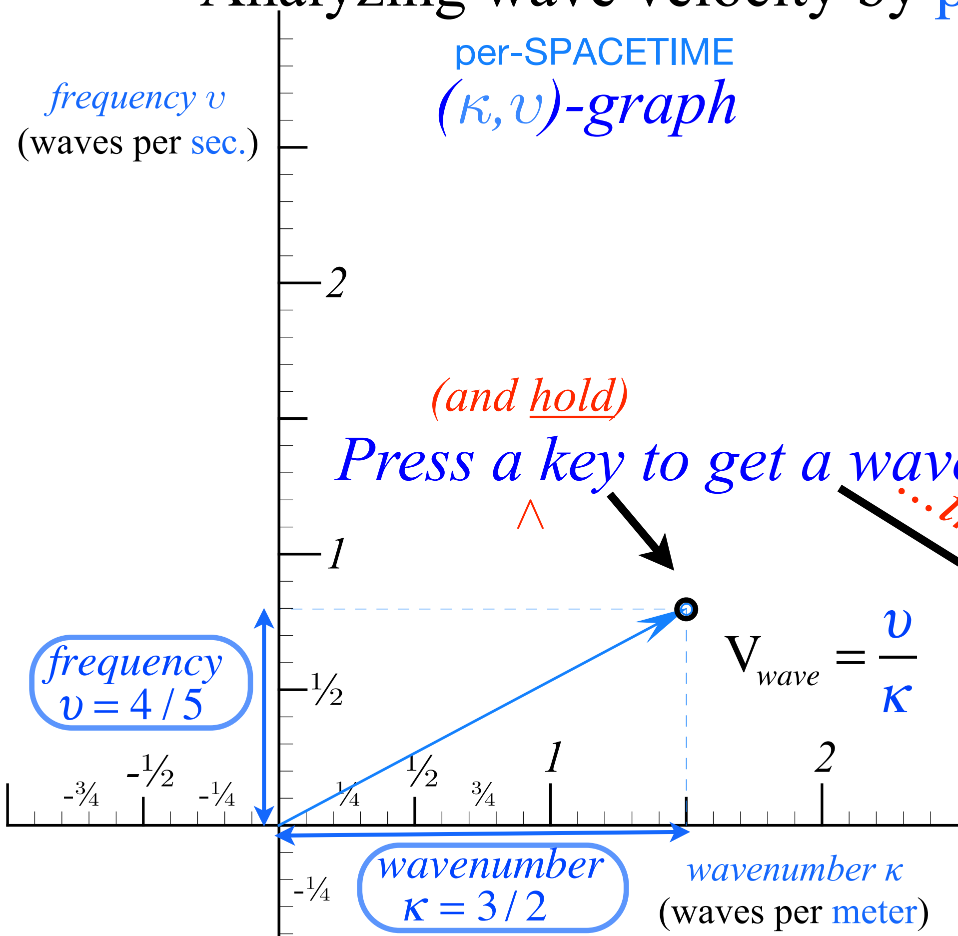
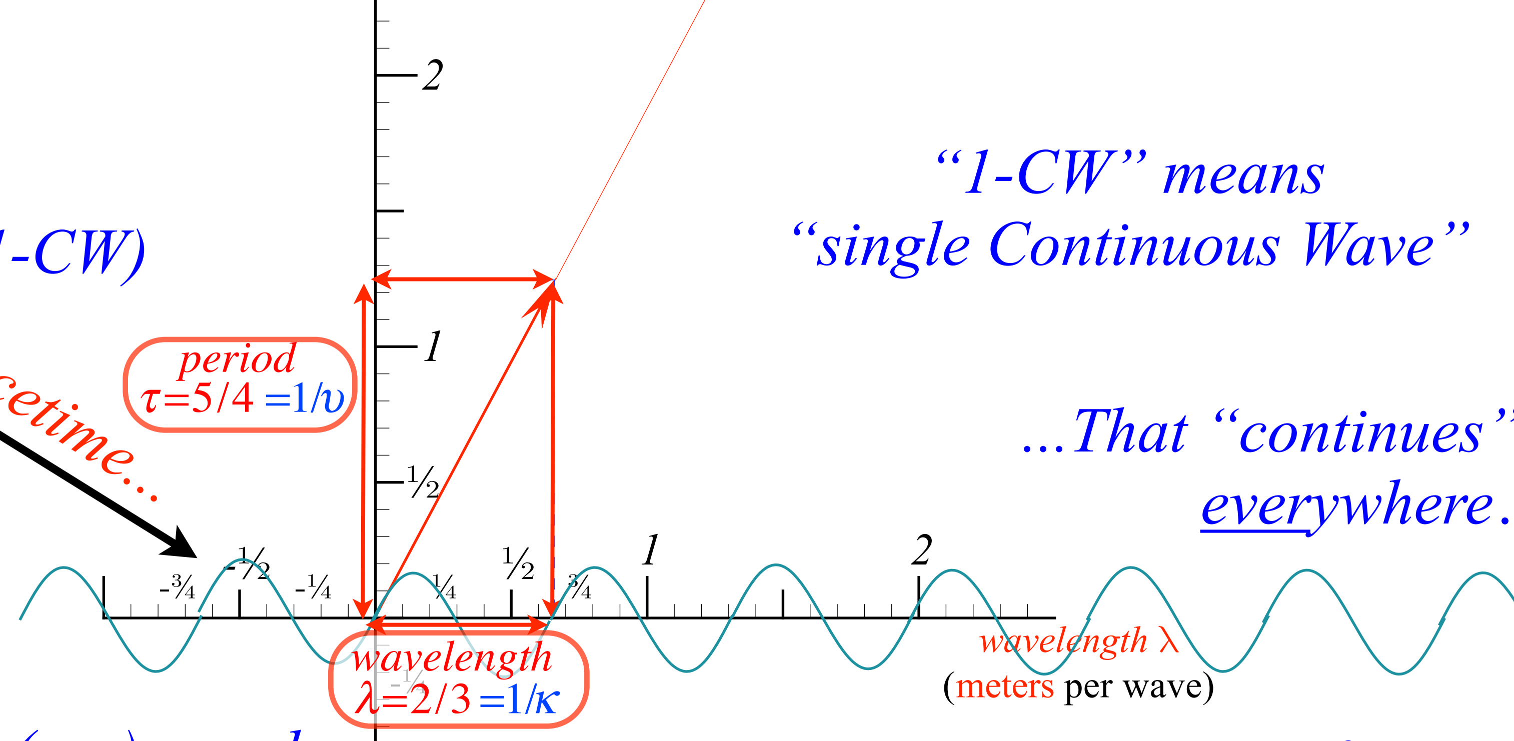
wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909





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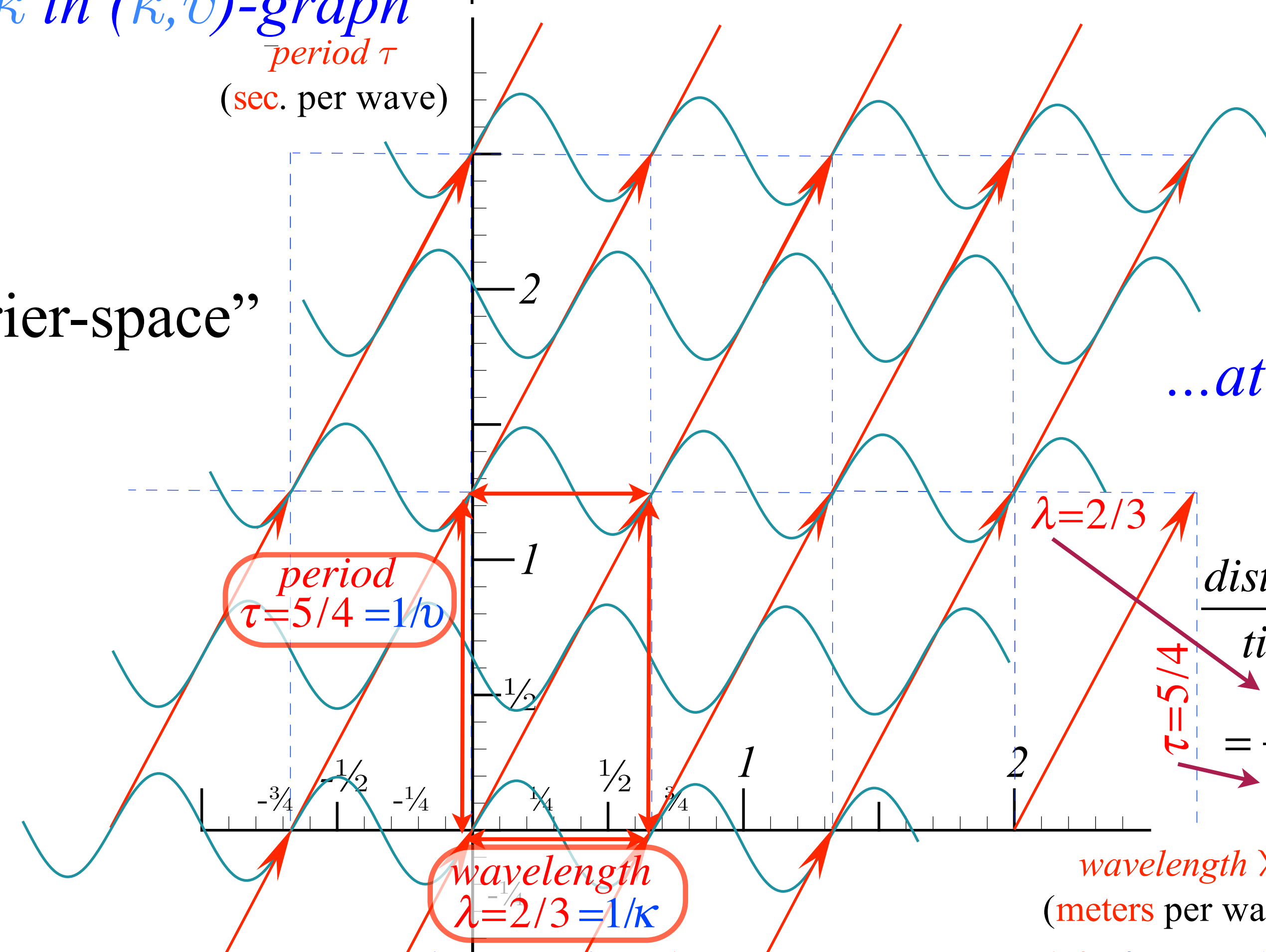
...That "continues" everywhere...

wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph

...for all time...



...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\ \frac{m.}{s.} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa}$$

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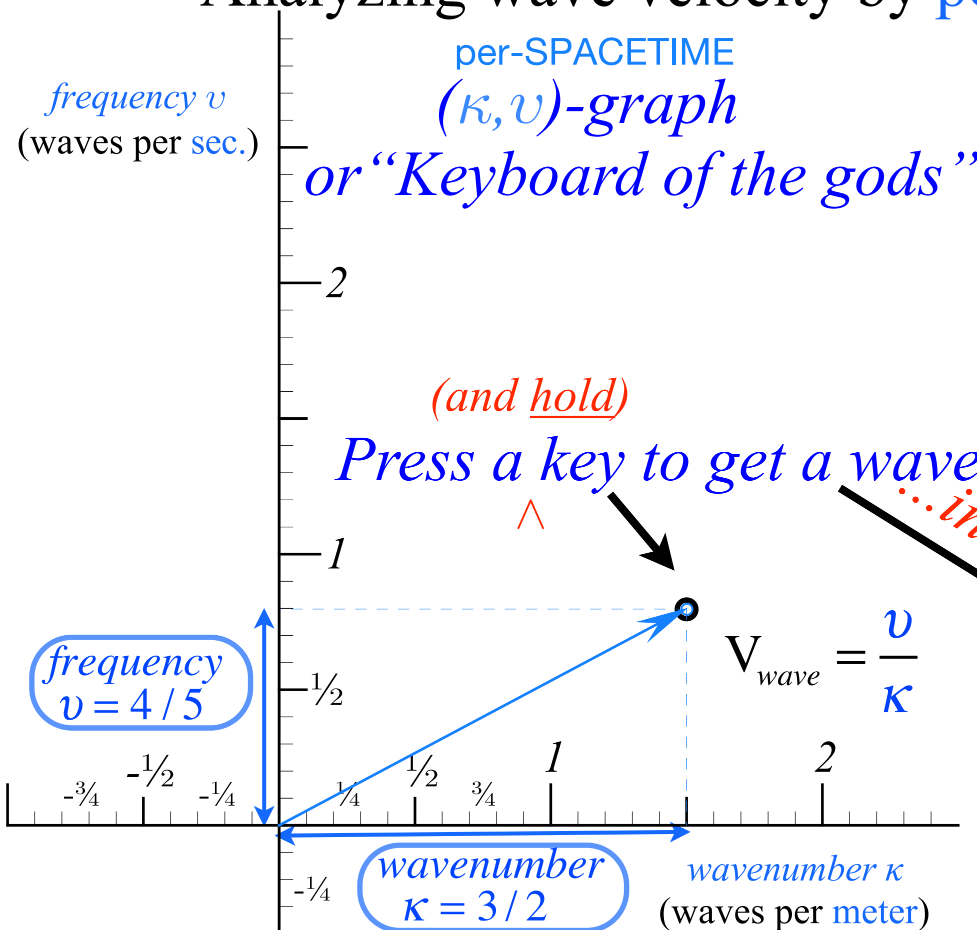

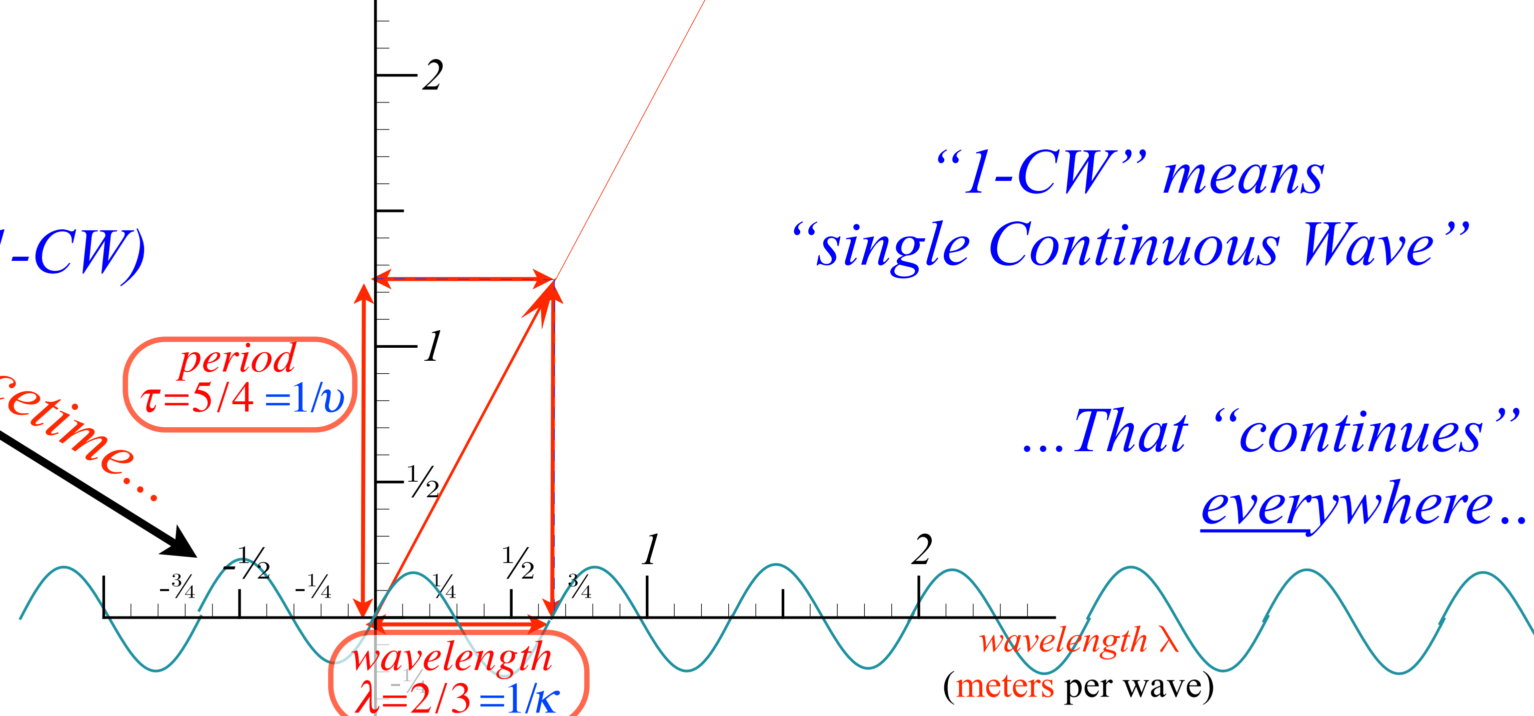

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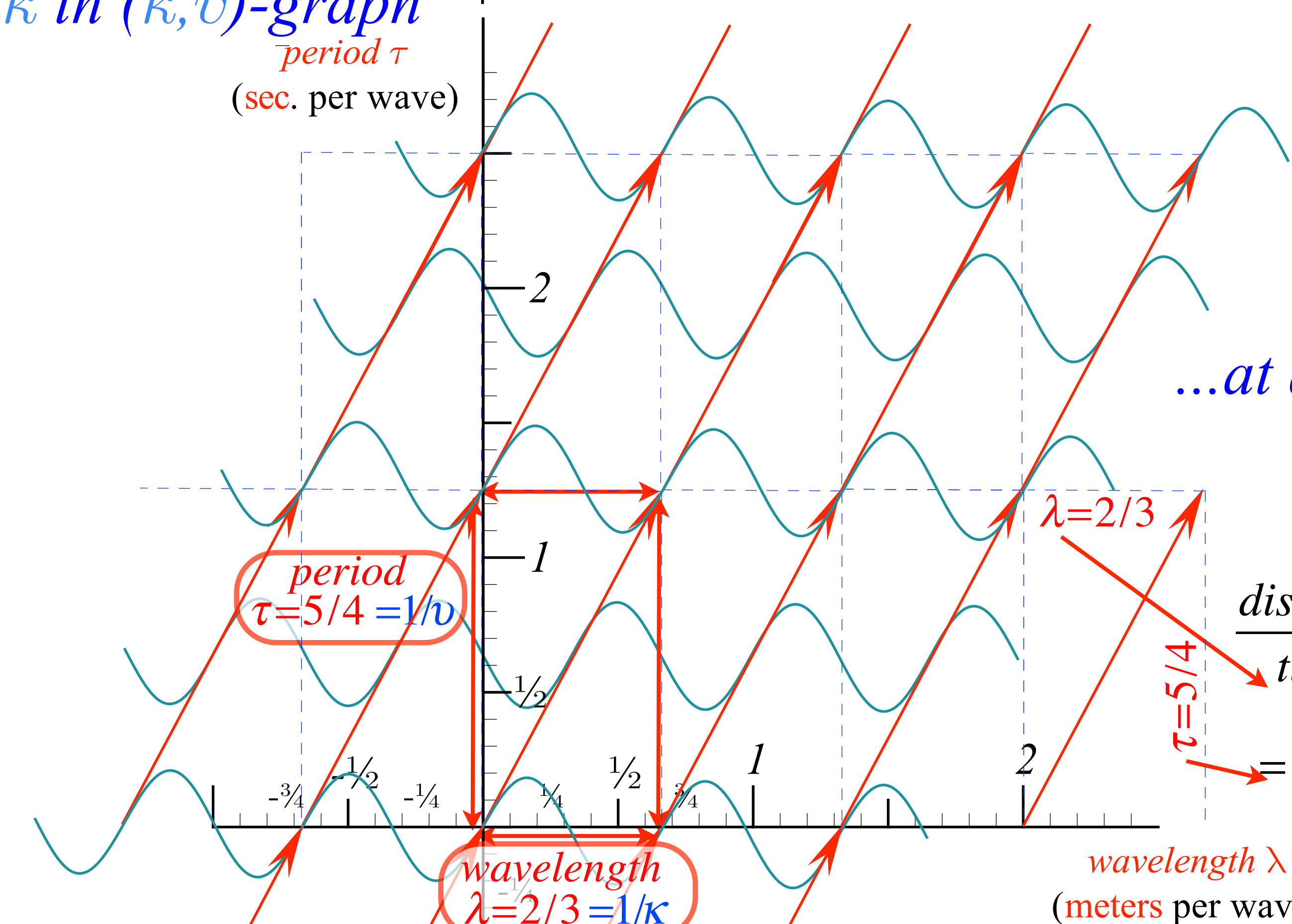

...at a speed of:

$$\frac{distance}{time}=\frac{wavelength}{period}=\frac{\lambda}{\tau}=\frac{2/3}{5/4}=\frac{8}{15}\frac{m.}{s.}$$

wave-velocity formulas

$$\frac{distance}{time}=\frac{wavelength}{period}=\frac{frequency}{wavenumber}$$

$$V_{wave}=\frac{\lambda}{\tau}=\frac{1/\kappa}{1/\upsilon}=\frac{\upsilon}{\kappa}=\frac{1/\tau}{1/\lambda}$$

$$=\frac{2/3}{5/4}=\frac{4/5}{3/2}=\frac{8}{15}\frac{m.}{s.}$$

wave arithmetic is simpler to explain using fractions

•How to understand waves and wave velocity V_{wave}

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

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➔ Introducing Doppler shifting

Why is c *so* constant?!

Introducing Doppler Arithmetic and *Rapidity* ρ

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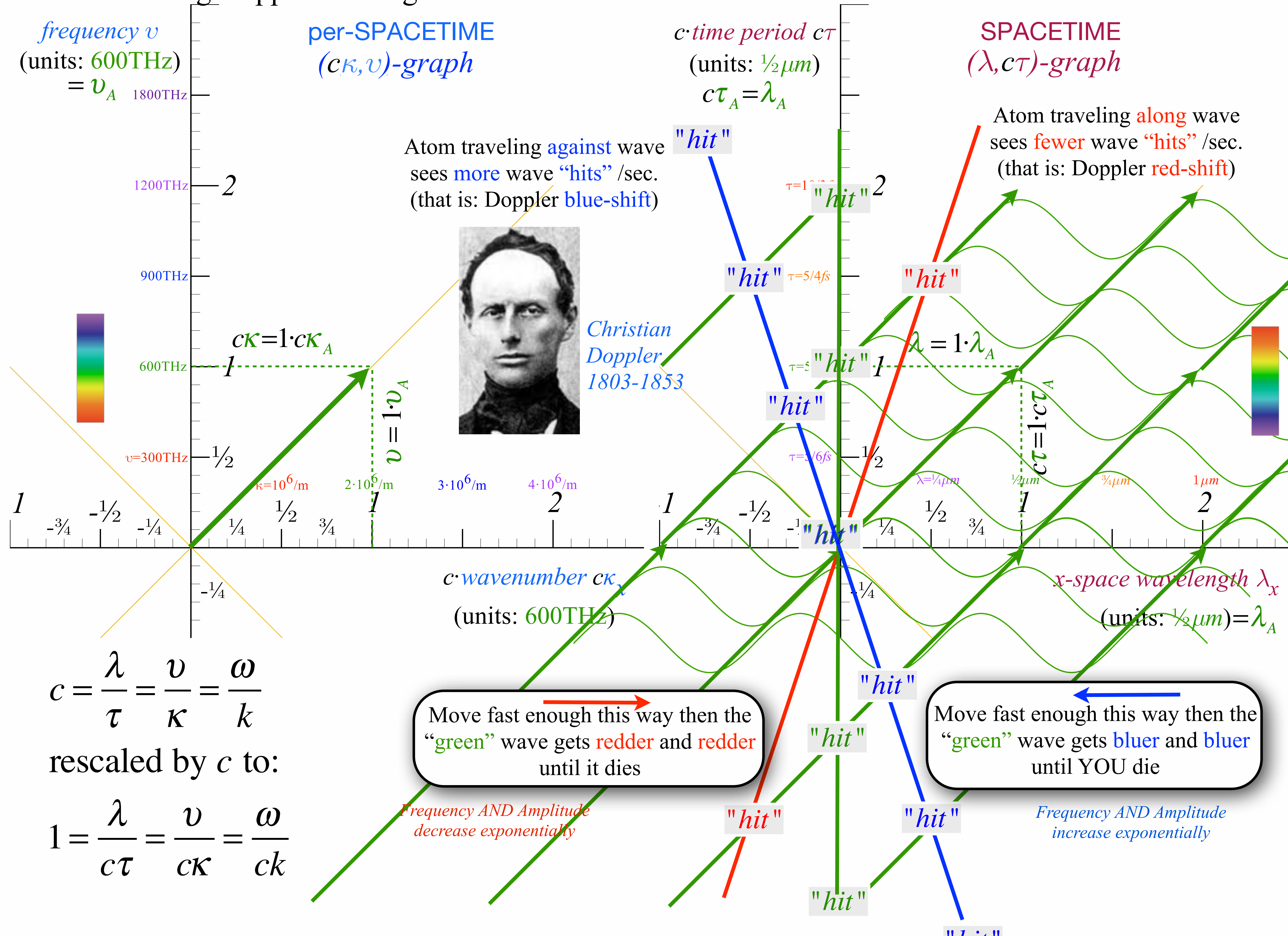

$$c=\frac{\lambda}{\tau}=\frac{\upsilon}{\kappa}=\frac{\omega}{k}$$

rescaled by c to:

$$1=\frac{\lambda}{c\tau}=\frac{\upsilon}{c\kappa}=\frac{\omega}{ck}$$

Move fast enough this way then the "green" wave gets redder and redder until it dies

Frequency AND Amplitude decrease exponentially

Move fast enough this way then the "green" wave gets bluer and bluer until YOU die

Frequency AND Amplitude increase exponentially

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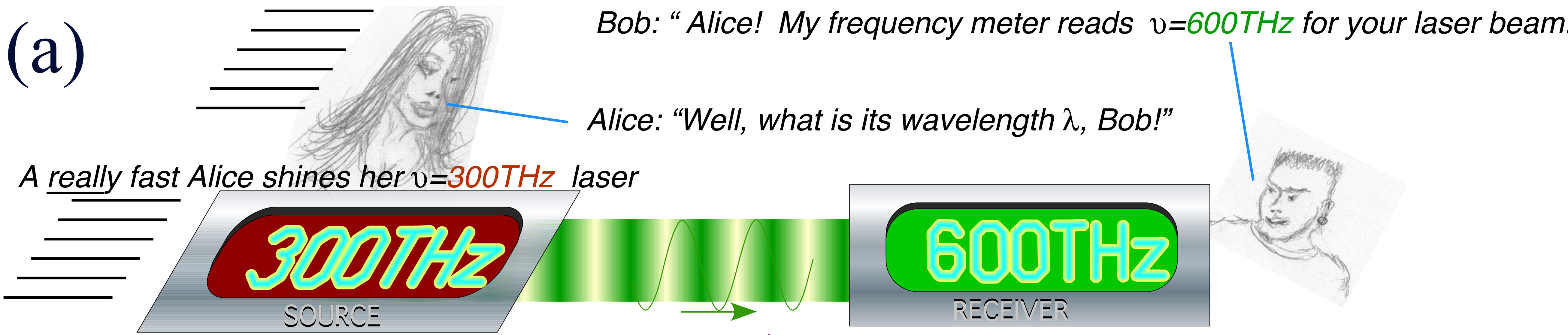
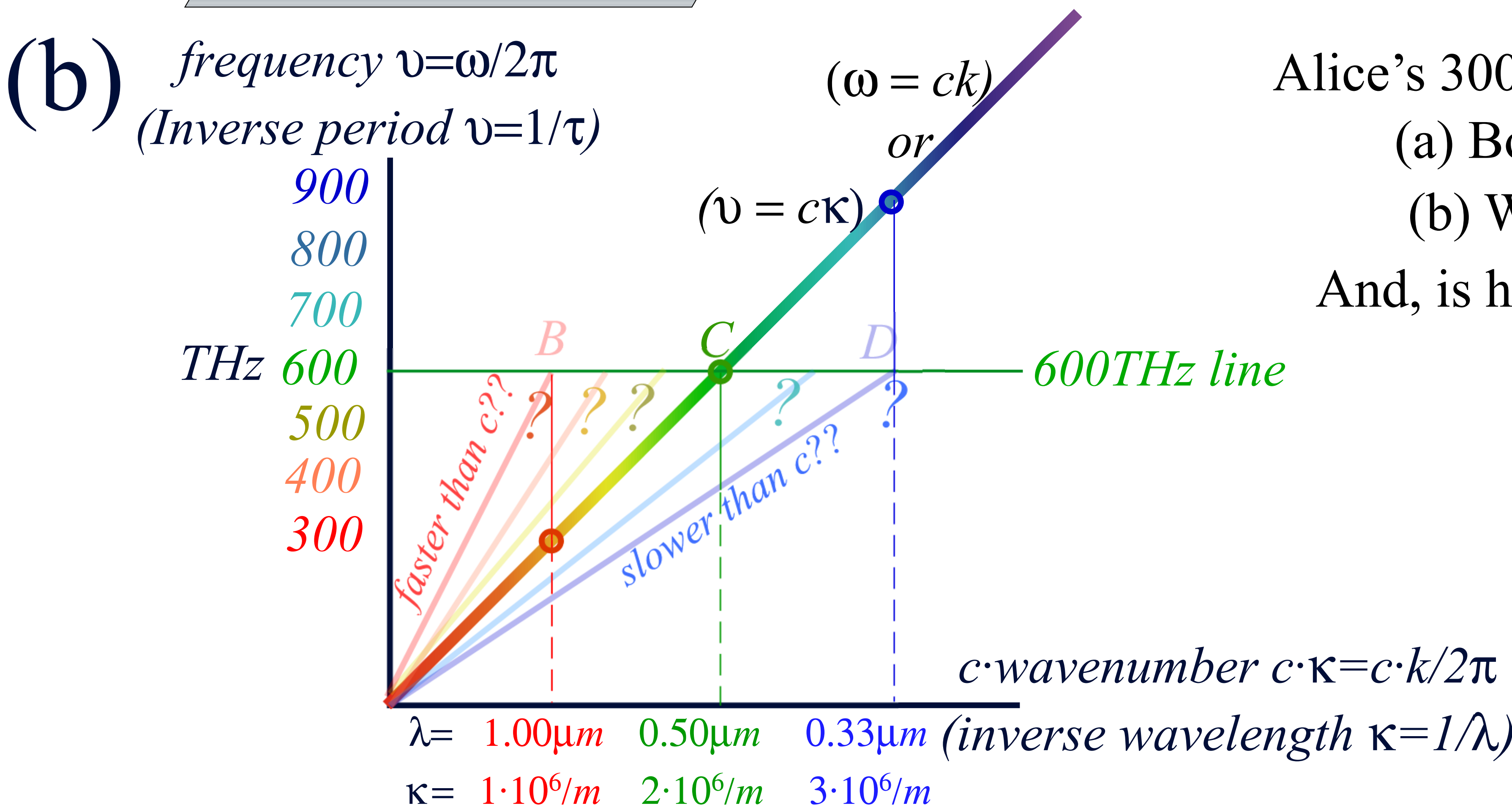
Introducing Doppler shifting and why *c* is *so* constant (and *so slow*)





Alice's 300THz laser approaches Bob.

(a) Bob sees υ=600THz.

(b) What $\lambda=1/\kappa$ does Bob measure?

And, is he seeing a '*phony*' *green*?

Introducing Doppler shifting and why c is constant

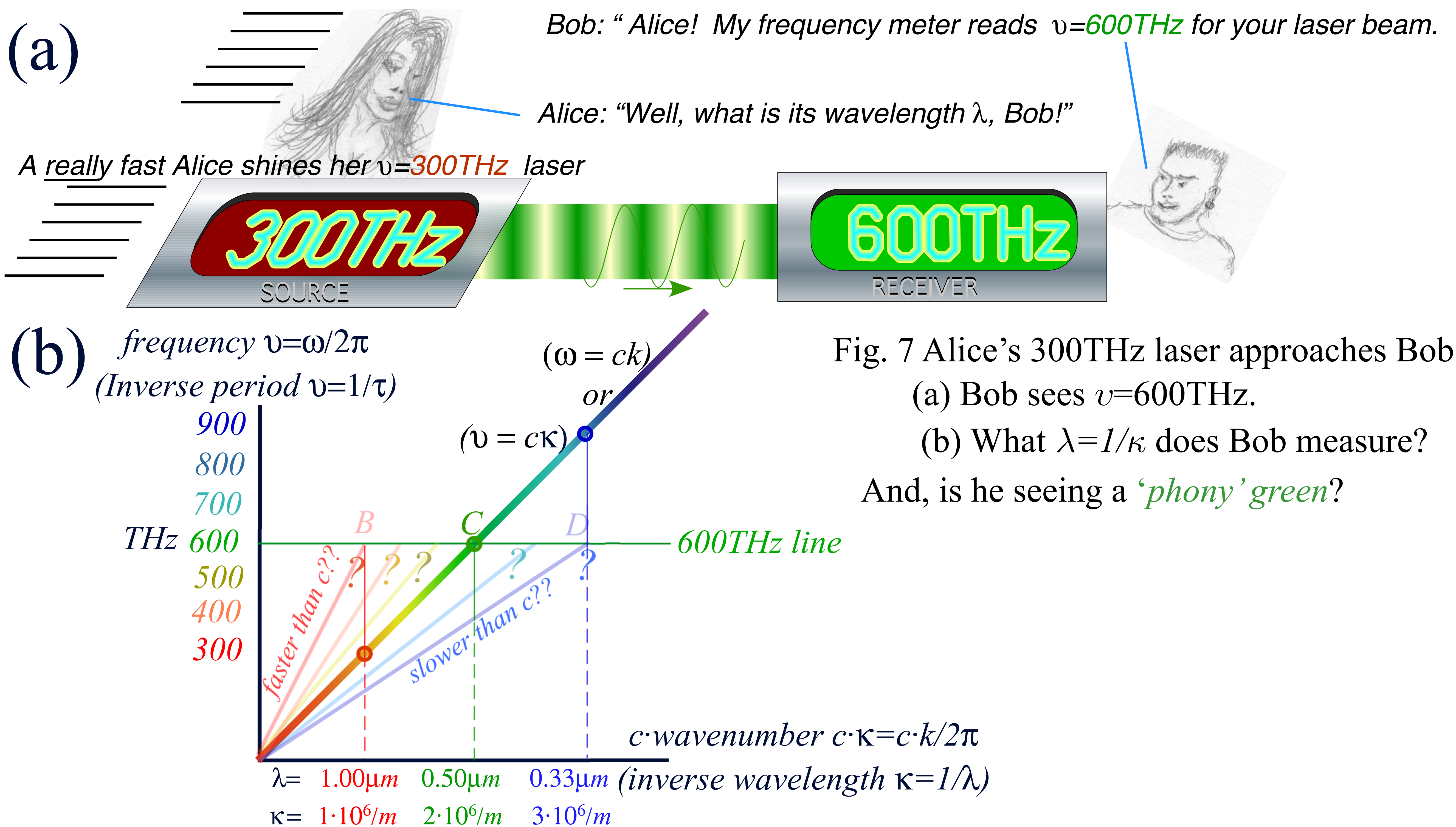
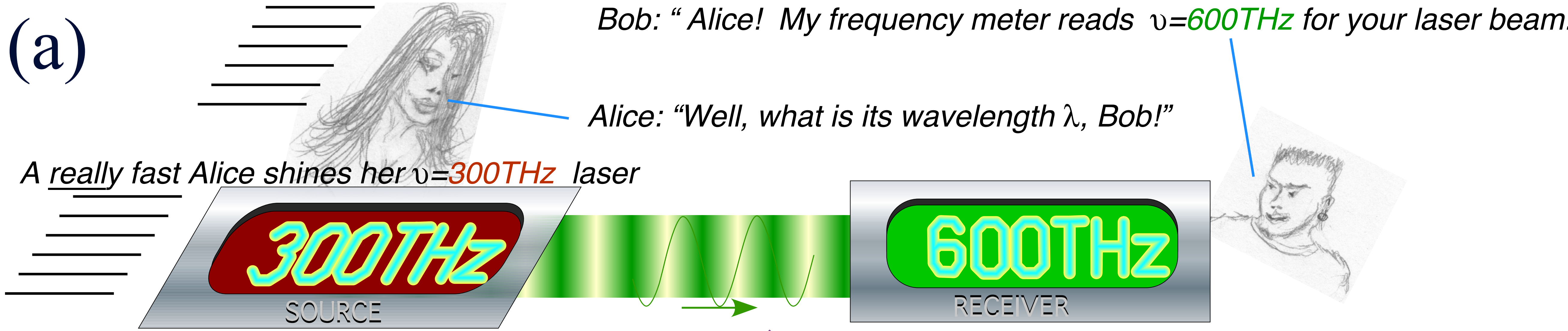


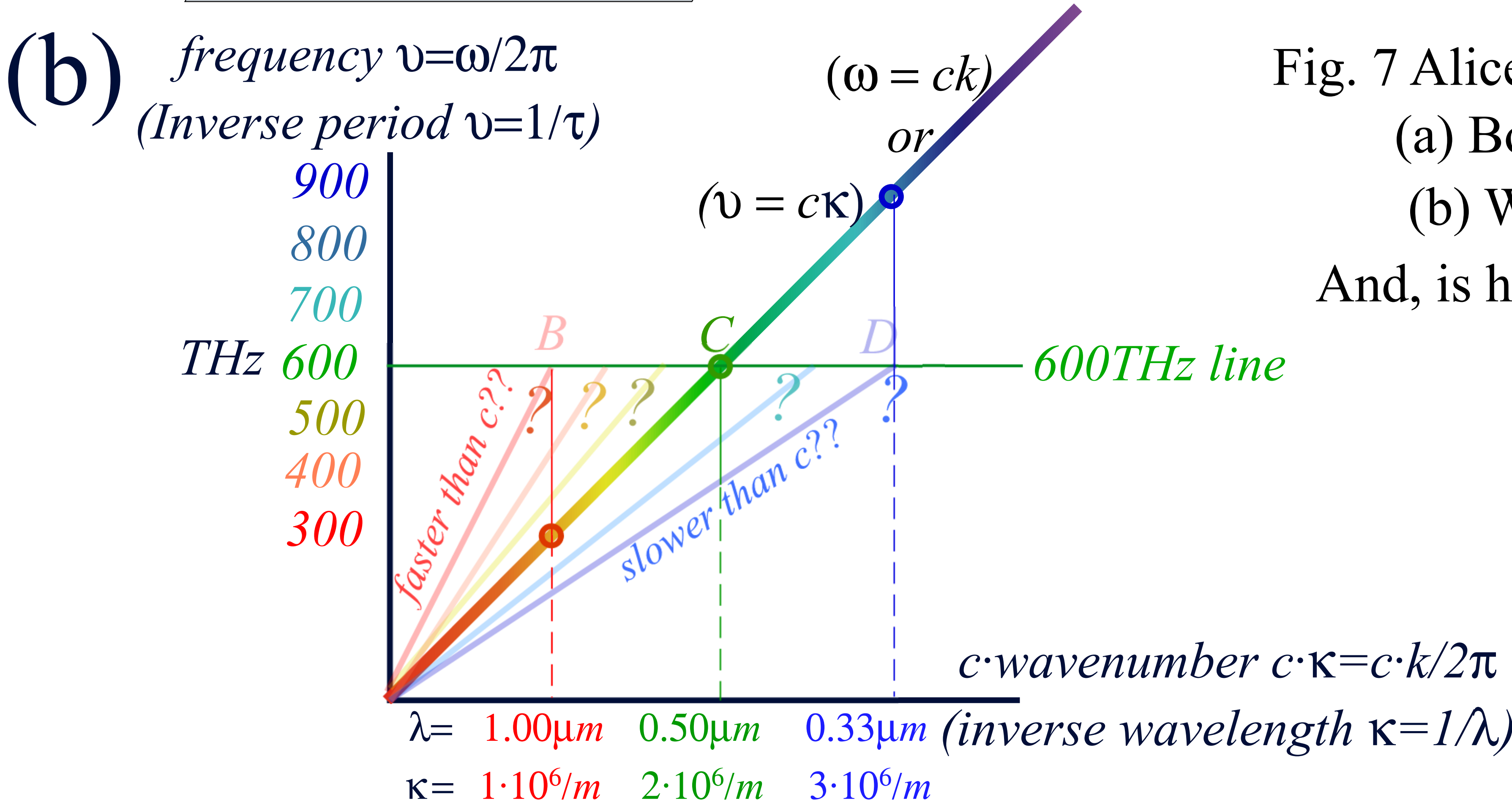
Fig. 7 Alice's 300THz laser approaches Bob.
(a) Bob sees υ=600THz.
(b) What $\lambda=1/\kappa$ does Bob measure?
And, is he seeing a '*phony' green*?

Years of spectroscopy rule out 'phony' 600THz blue-green that do not have wavelength λ=0.5*micron.*

The only choice is C.

Introducing Doppler shifting and why c is constant





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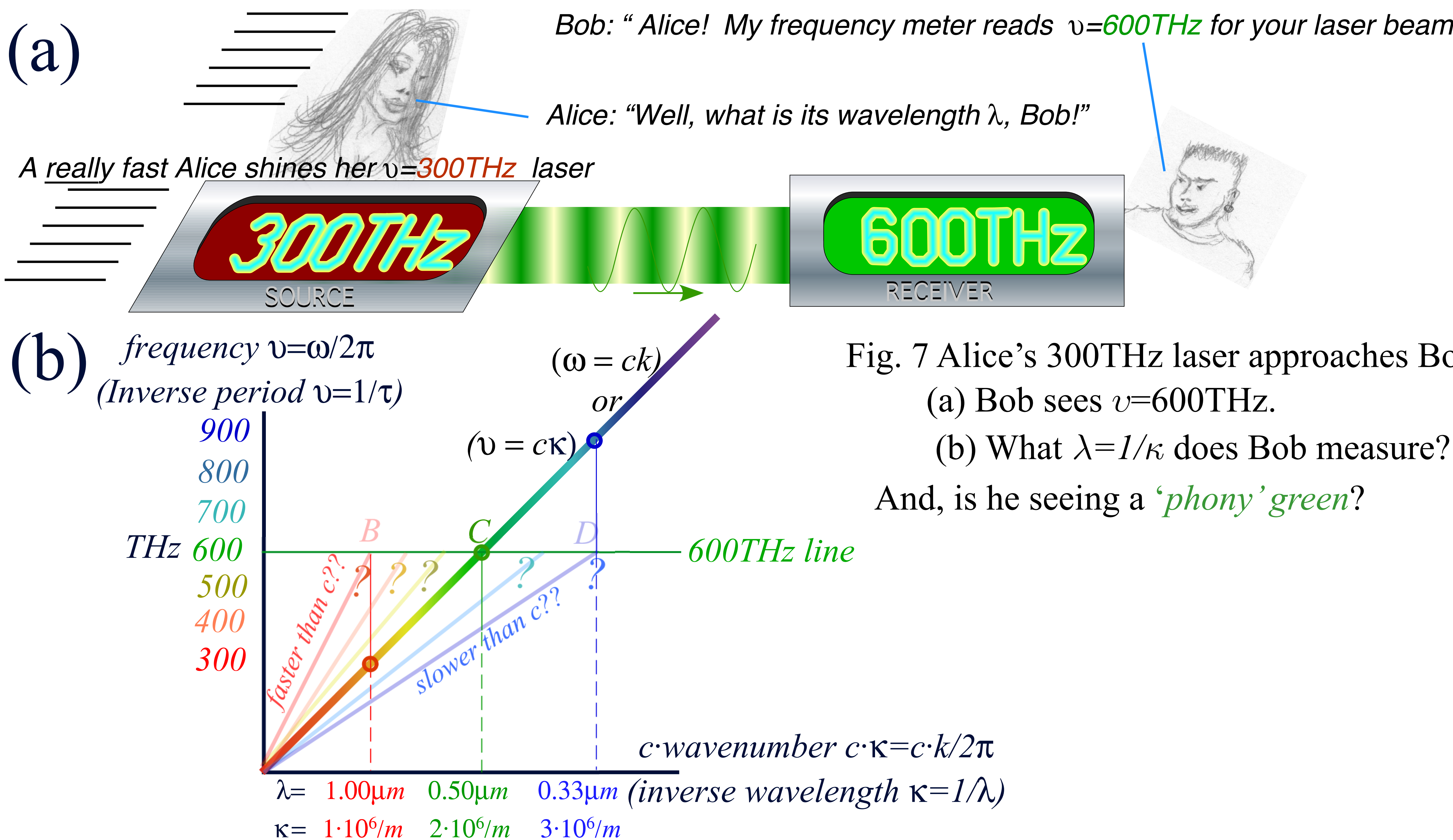
Introducing Doppler shifting and why c is constant



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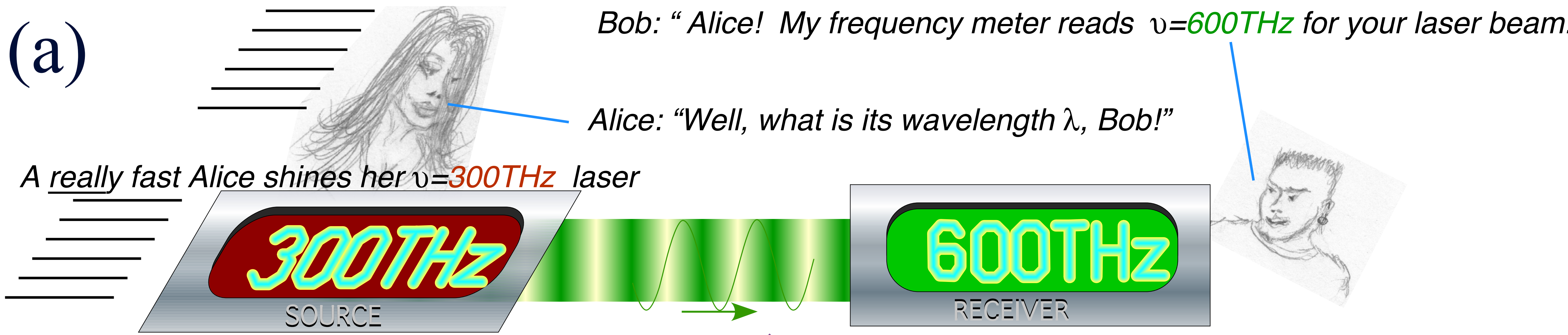
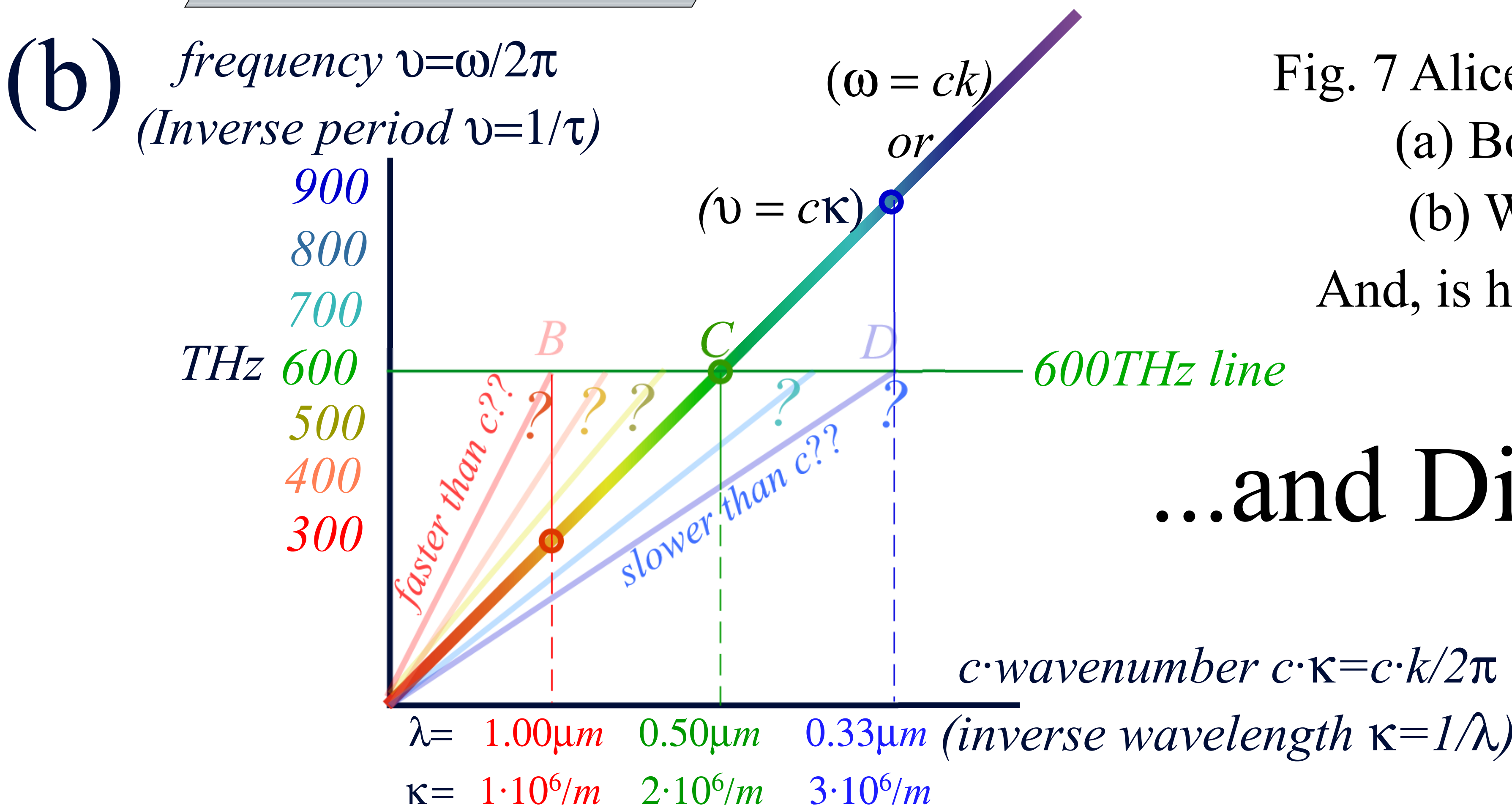
Actually: $2.99792458\cdot 10^{8}\mathrm{m\cdot s^{-1}}$

Introducing Doppler shifting and why c is constant





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And, is he seeing a *'phony' green*?

...and Dispersion-Free!

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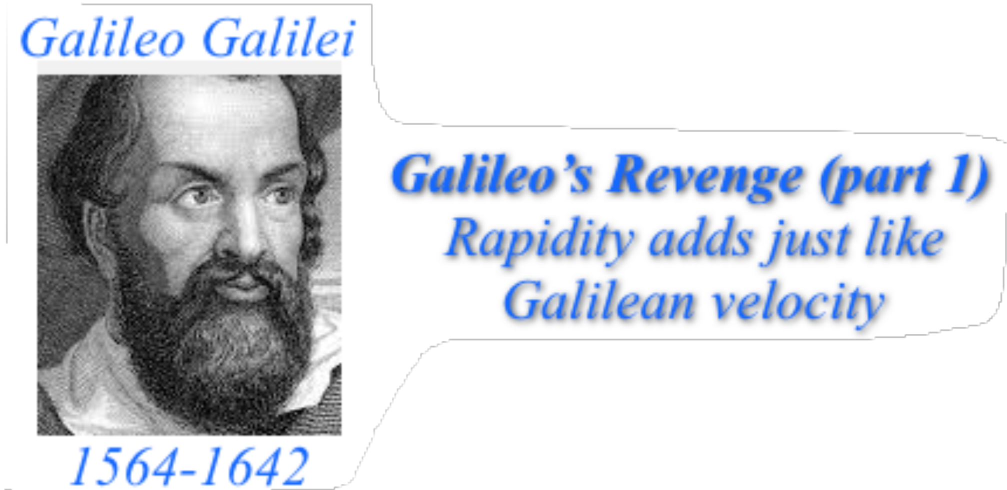

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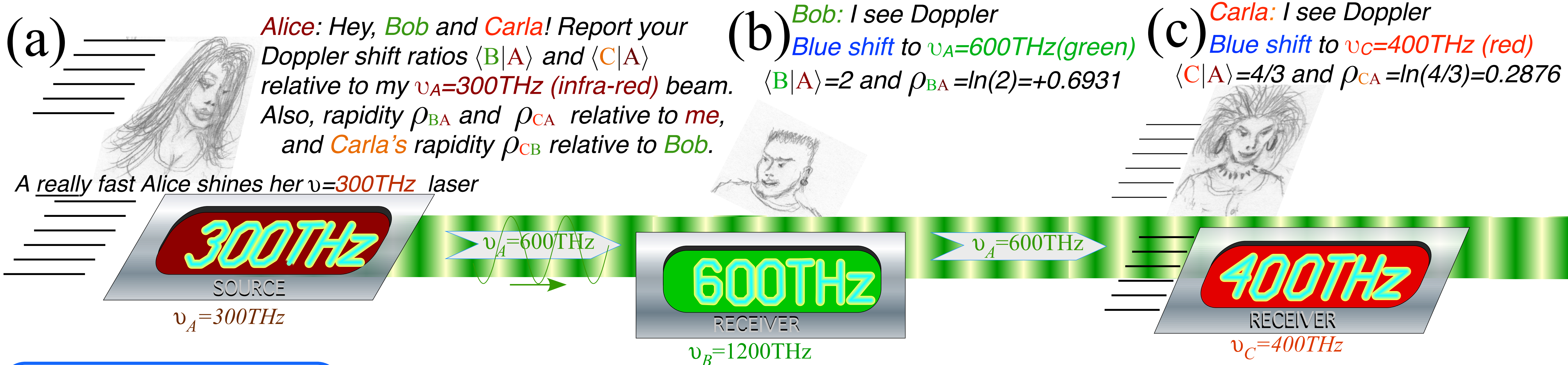


Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity ρ_{RS}

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \ln\langle B|A\rangle = \ln\frac{2}{1} = 0.6931$$

$$\rho_{AB} = \ln\langle A|B\rangle = \ln\frac{1}{2} = -0.6931 = -\rho_{BA}$$

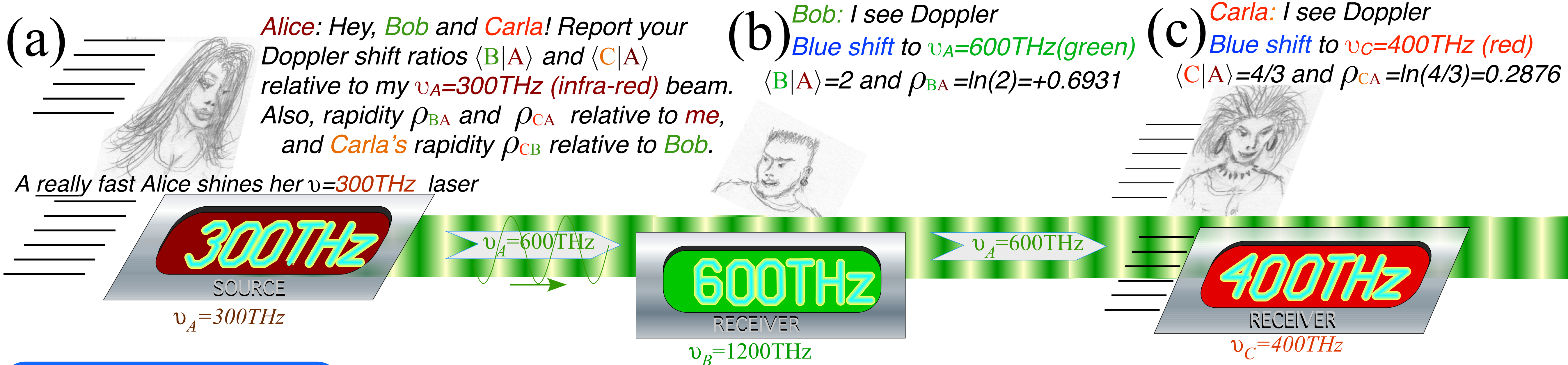
Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$

Introducing Doppler Arithmetic and rapidity ρ



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity ρ_{RS}

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \ln\langle B|A\rangle = \ln\frac{2}{1} = 0.6931$$

$$\rho_{AB} = \ln\langle A|B\rangle = \ln\frac{1}{2} = -0.6931 = -\rho_{BA}$$

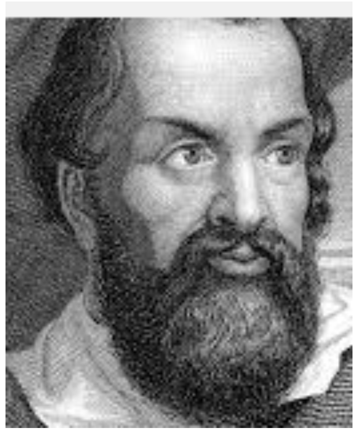
Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$

Galileo Galilei



1564-1642

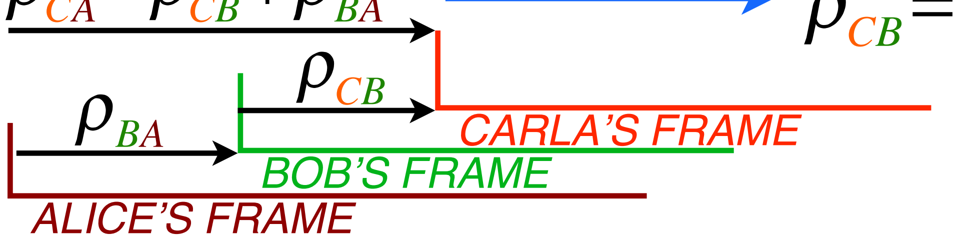
Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle = \frac{4}{3}\frac{1}{2} = \frac{2}{3}$$

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CA} = \rho_{CB} + \rho_{BA}$$

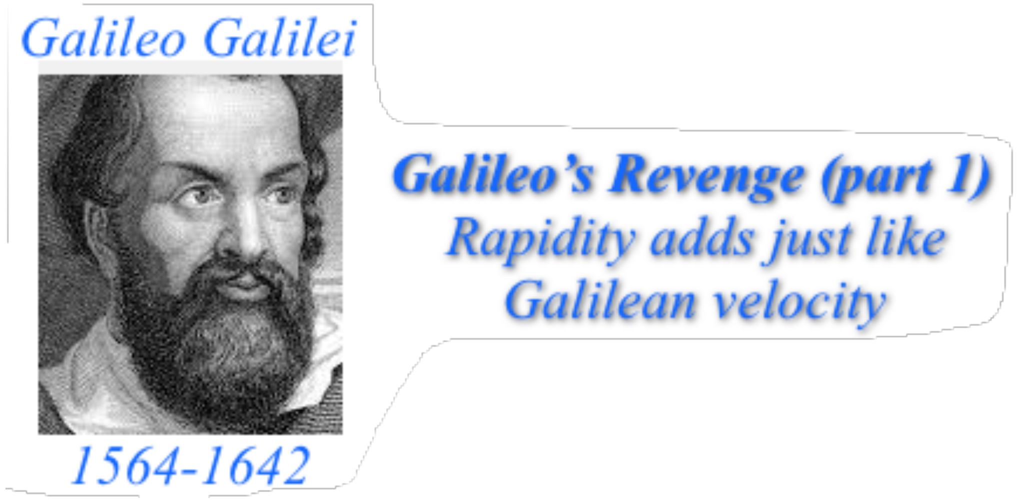


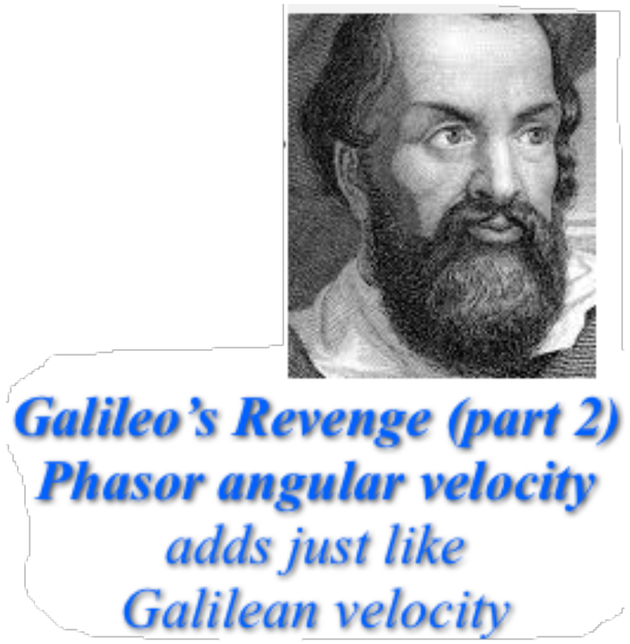
Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

$$\rho_{CB} = \rho_{CA} + \rho_{AB} = 0.2876 - 0.6931 = -0.4055$$

$$= \ln\frac{4}{3} + \ln\frac{1}{2} = \ln\frac{2}{3}$$





➔ Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

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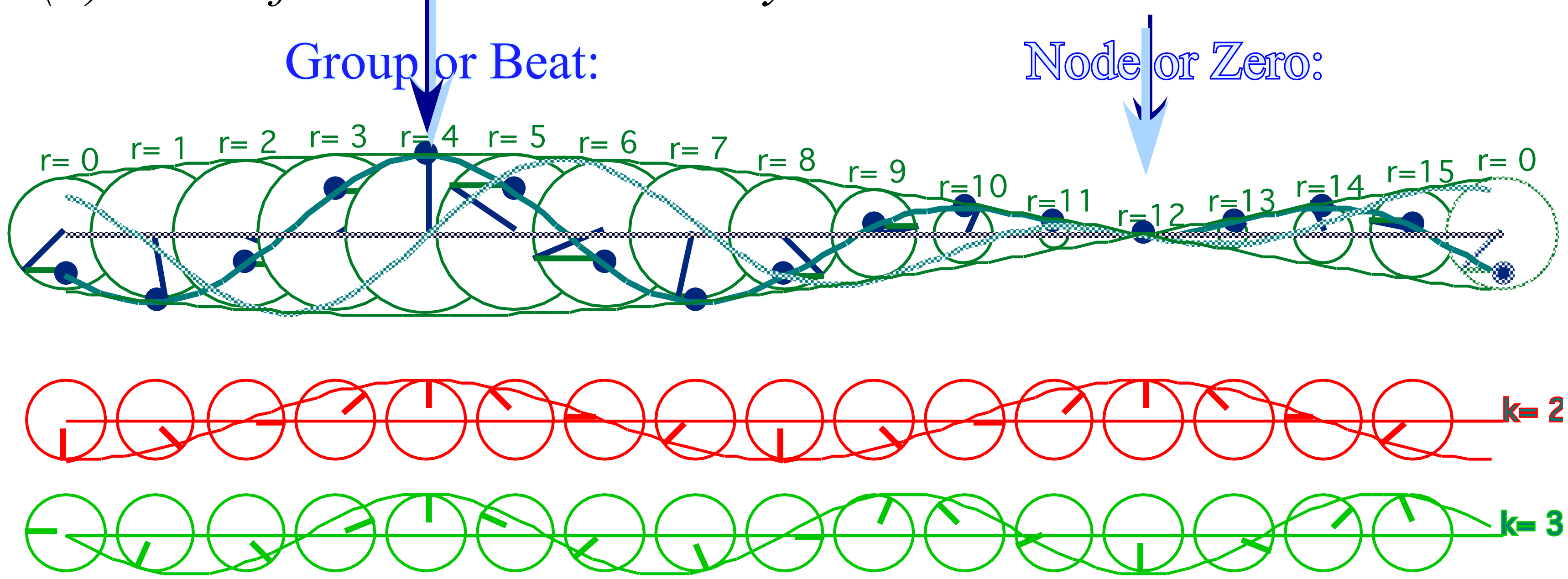
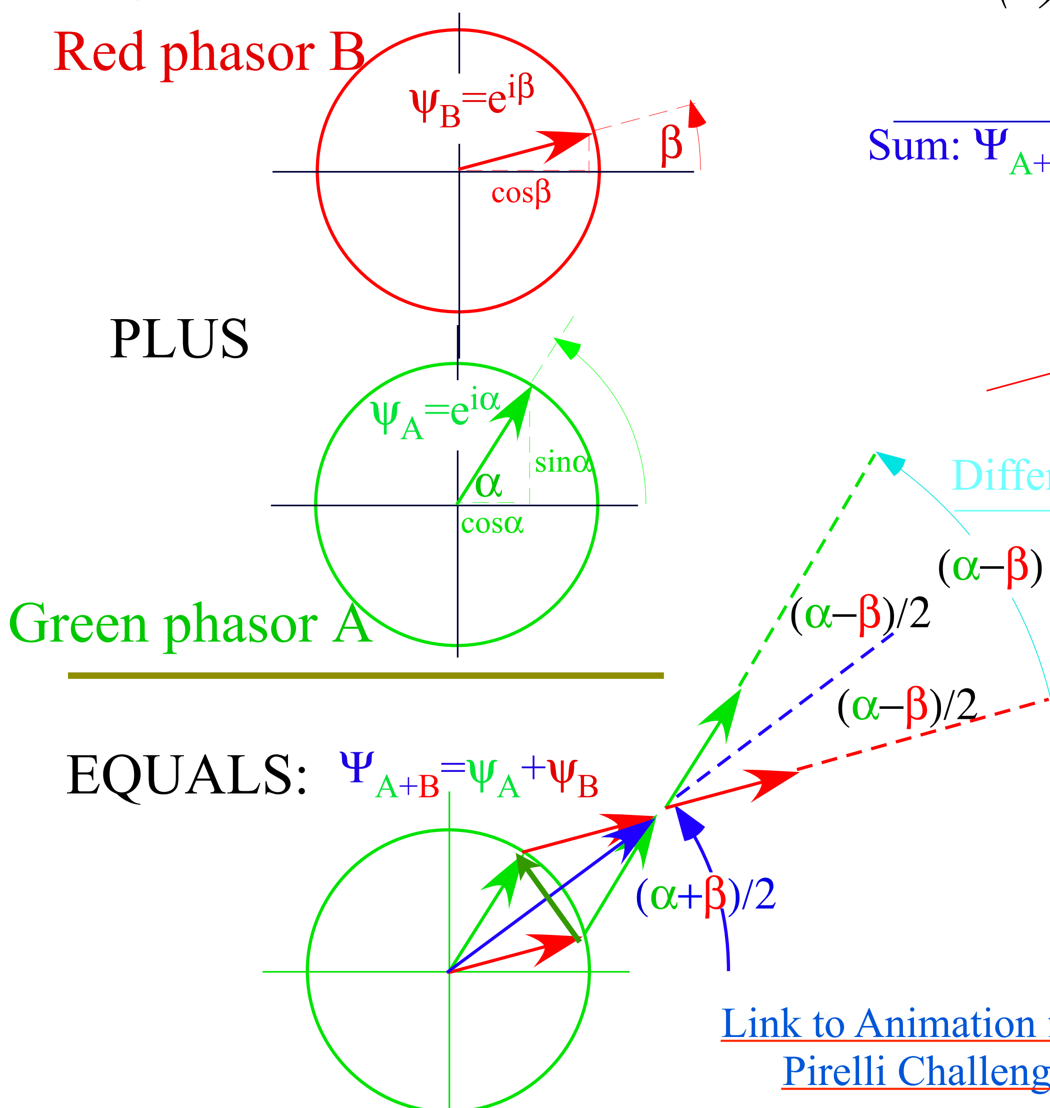
Application to TE-Waveguide modes and synchrotron beam relativity

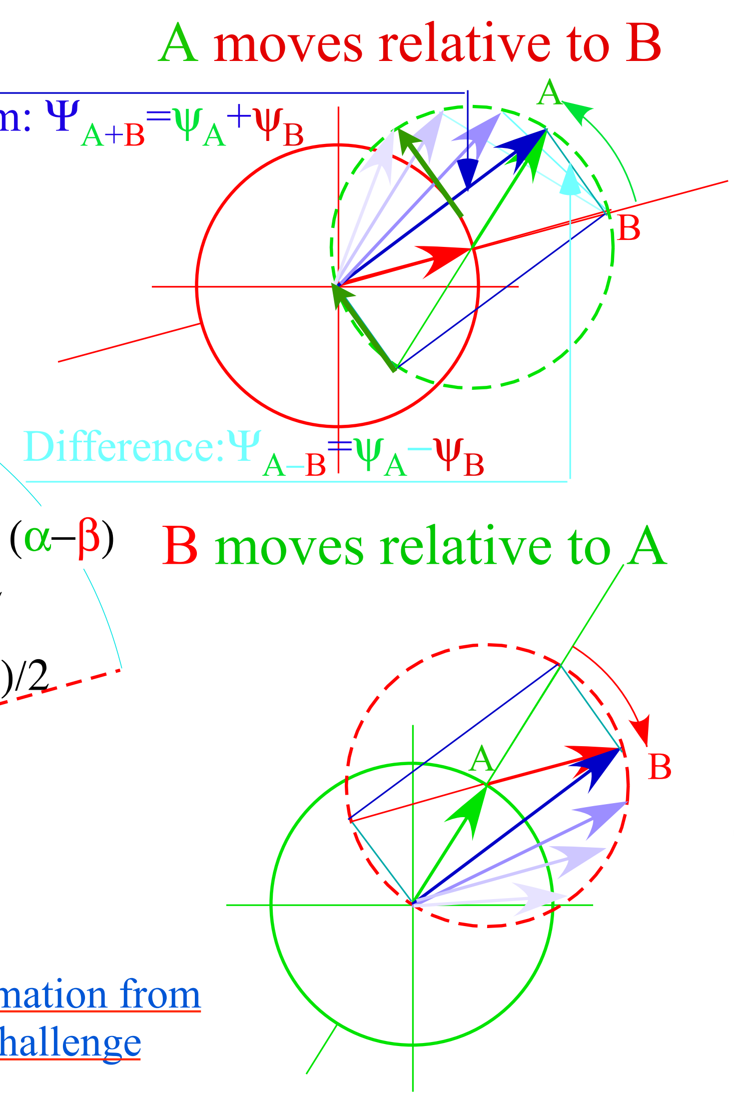
(a) Sum of Wave Phasor Array



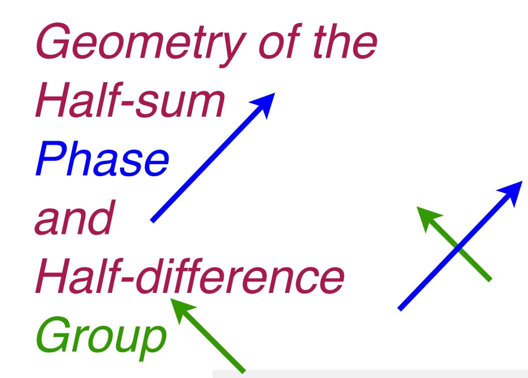
(b) Typical Phasor Sum:

(c) Phasor-relative views

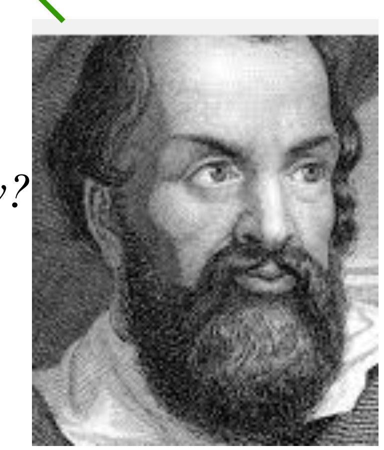




[Link to Animation from Pirelli Challenge]

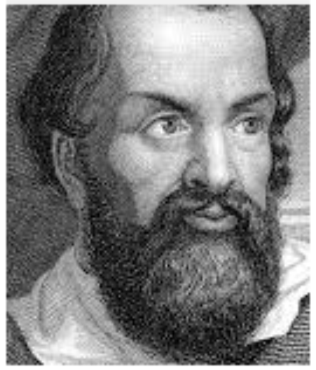


Happy now?



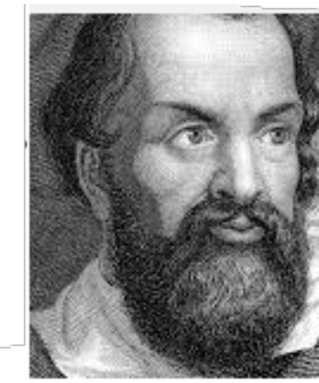
Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

➔ Details of 2CW wavefunctions in rest frame

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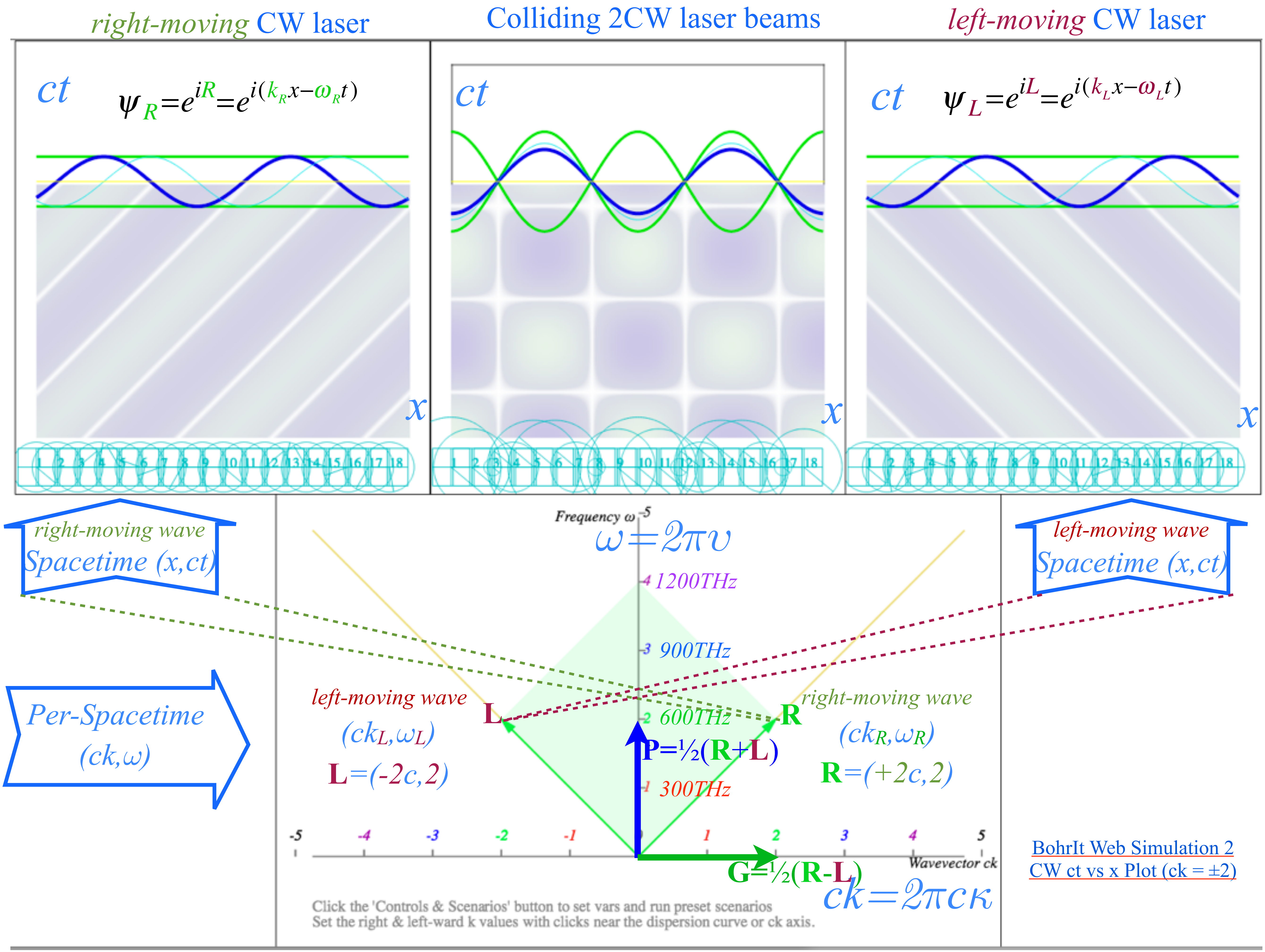
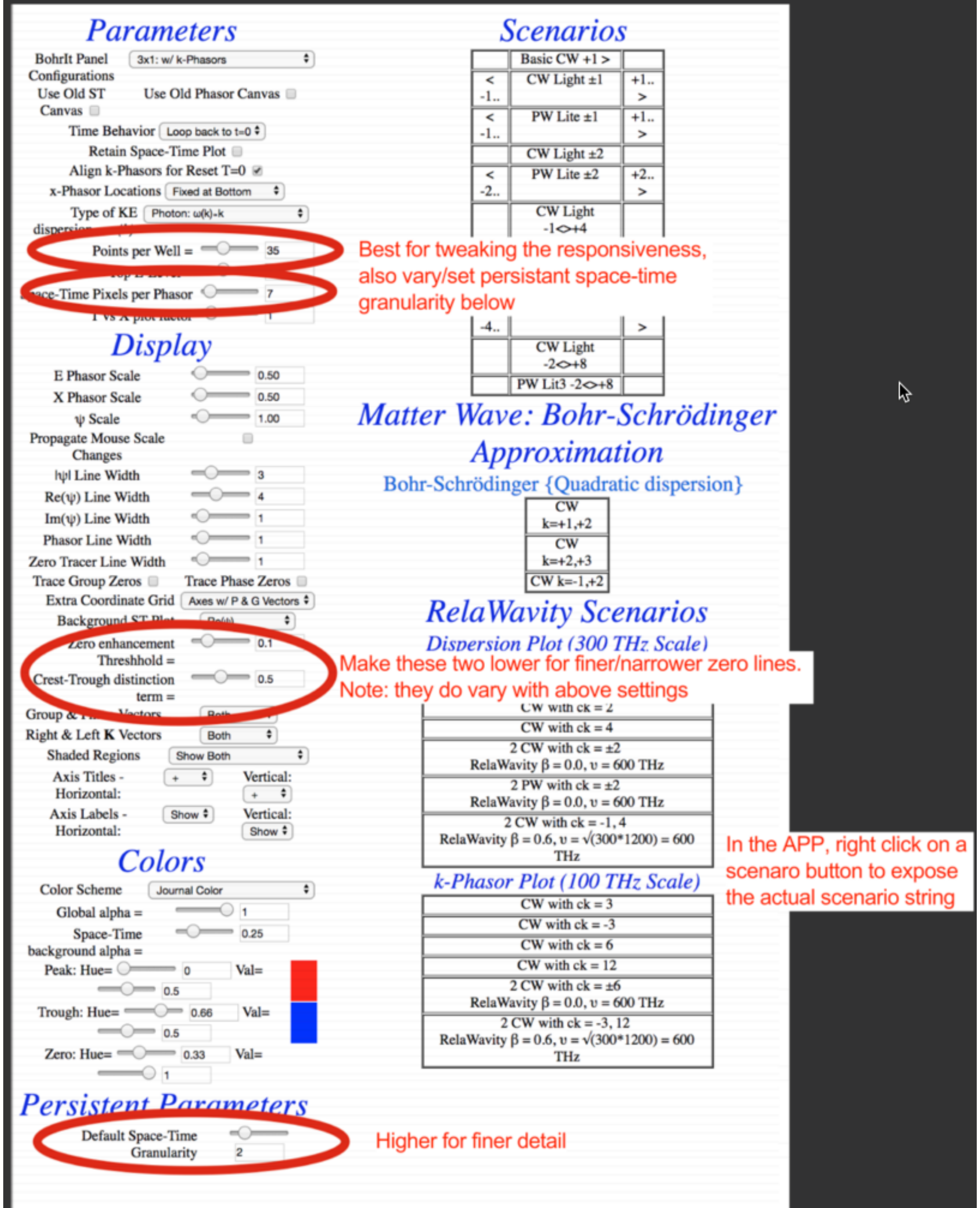
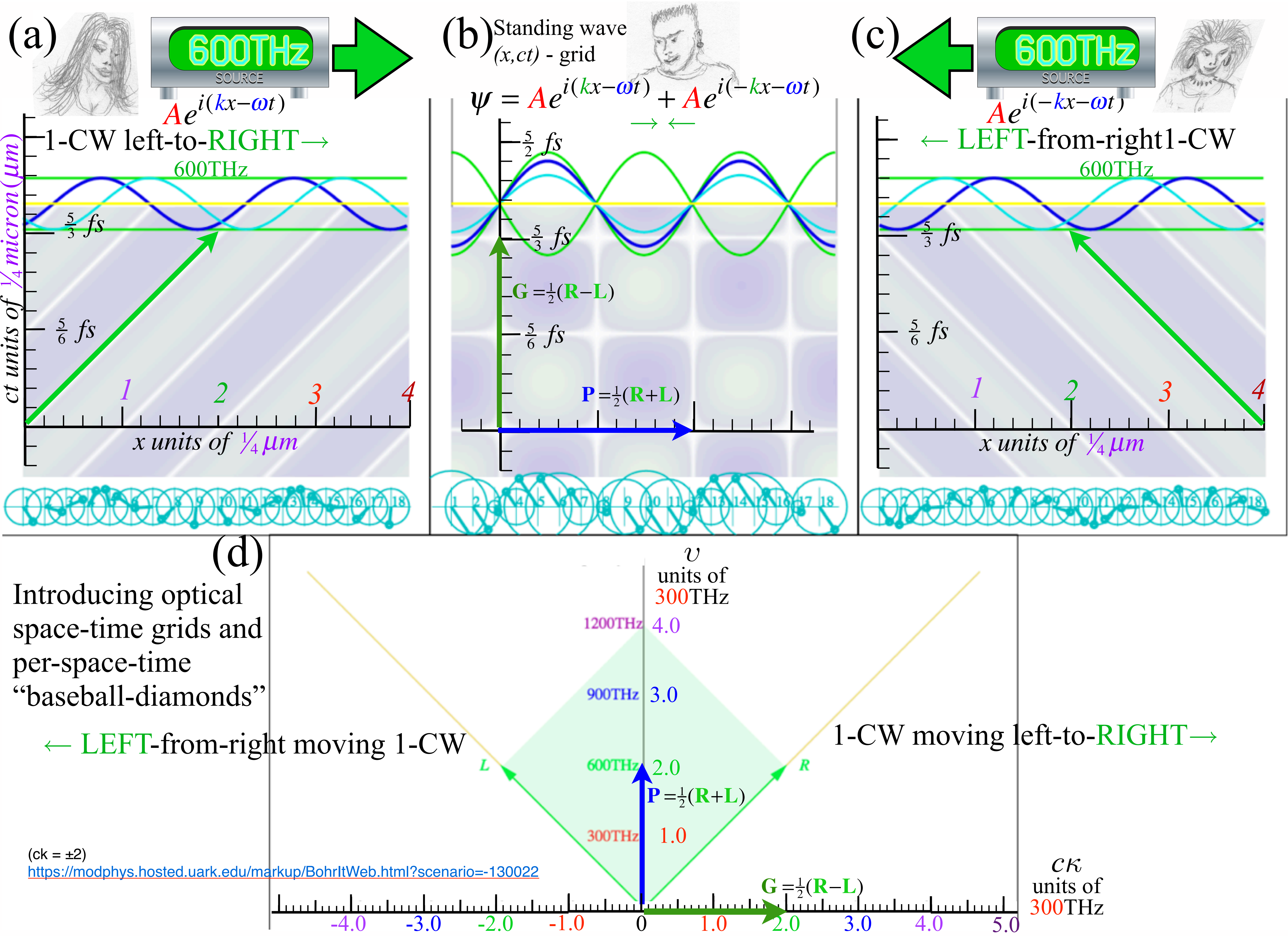
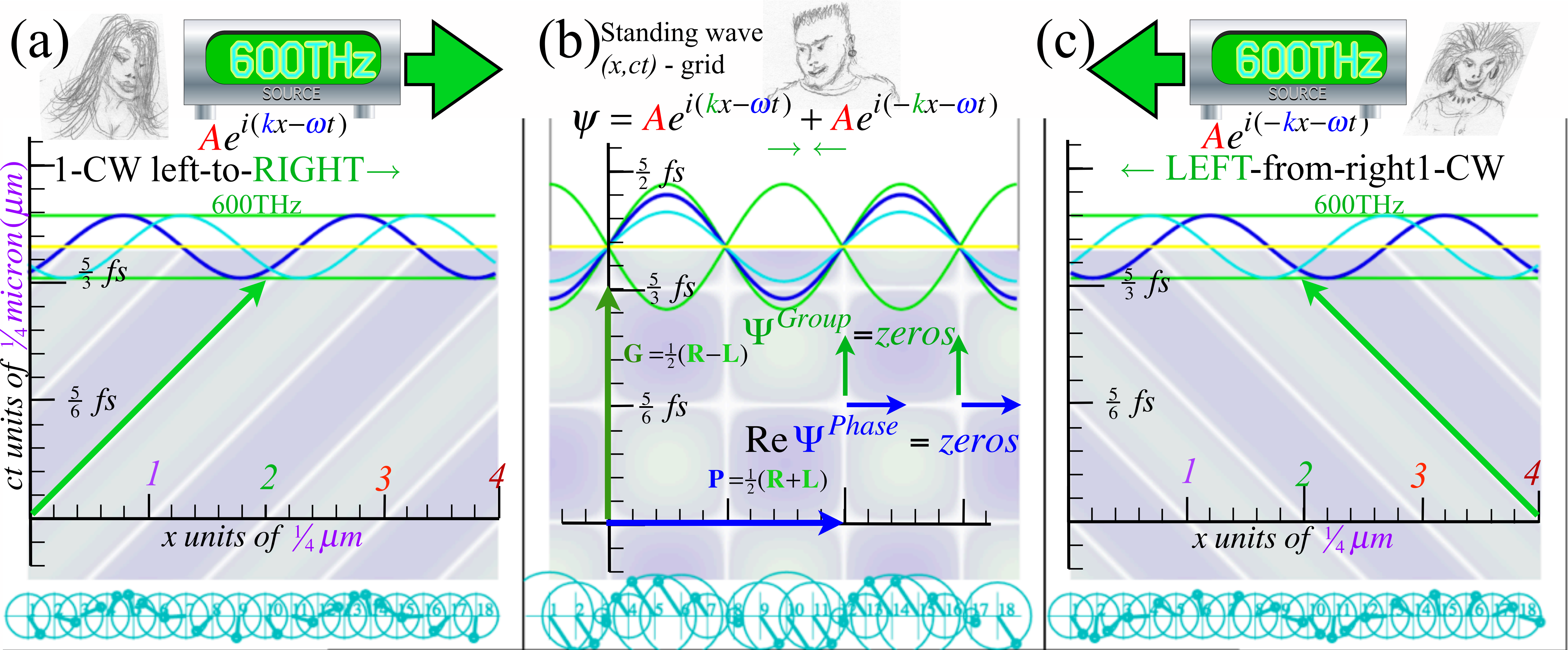

right-moving CW laser
Colliding 2CW laser beams
left-moving CW laser
ct
ψR=e^iR=e^i(kRx−ωRt)
ct
ct
ψL=e^iL=e^i(kLx−ωLt)
x
x
x
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
ω=2πυ
1200THz
900THz
600THz
300THz
left-moving wave
(ckL,ωL)
L=(-2c,2)
L
R
right-moving wave
(ckR,ωR)
R=(+2c,2)
P=½(R+L)
G=½(R-L)
-5 -4 -3 -2 -1 0 1 2 3 4 5
Wavevector ck
ck=2πcκ
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
BohrIt Web Simulation 2
CW ct vs x Plot (ck = ±2)


Parameters
BohrIt Panel Configurations
3x1: w/ k-Phasors
Use Old ST Canvas
Use Old Phasor Canvas
Time Behavior
Loop back to t=0
Retain Space-Time Plot
Align k-Phasors for Reset T=0
x-Phasor Locations
Fixed at Bottom
Type of KE
Photon: ω(k)-k
Points per Well =
35
Space-Time Pixels per Phasor
7
Best for tweaking the responsiveness, also vary/set persistant space-time granularity below
Display
E Phasor Scale
0.50
X Phasor Scale
0.50
ψ Scale
1.00
Propagate Mouse Scale Changes
|ψ| Line Width
3
Re(ψ) Line Width
4
Im(ψ) Line Width
1
Phasor Line Width
1
Zero Tracer Line Width
1
Trace Group Zeros
Trace Phase Zeros
Extra Coordinate Grid
Axes w/ P & G Vectors
Zero enhancement Threshhold =
0.1
Crest-Trough distinction term =
0.5
Make these two lower for finer/narrower zero lines. Note: they do vary with above settings
Right & Left K Vectors
Both
Shaded Regions
Show Both
Axis Titles - Horizontal:
Vertical:
Axis Labels - Horizontal:
Show
Vertical:
Show
Colors
Color Scheme
Journal Color
Global alpha =
1
Space-Time background alpha =
0.25
Peak: Hue=
0
Val=
0.5
Trough: Hue=
0.66
Val=
0.5
Zero: Hue=
0.33
Val=
1
Persistent Parameters
Default Space-Time Granularity
2
Higher for finer detail
Scenarios
Basic CW +1 >
< -1..
CW Light ±1
+1.. >
< -1..
PW Lite ±1
+1.. >
CW Light ±2
< -2..
PW Lite ±2
+2.. >
CW Light -1<>+4
-4..
>
CW Light -2<>+8
PW Lit3 -2<>+8
Matter Wave: Bohr-Schrödinger Approximation
Bohr-Schrödinger {Quadratic dispersion}
CW k=+1,+2
CW k=+2,+3
CW k=-1,+2
RelaWavity Scenarios
Dispersion Plot (300 THz Scale)
CW with ck = 4
2 CW with ck = ±2 RelaWavity β = 0.0, υ = 600 THz
2 PW with ck = ±2 RelaWavity β = 0.0, υ = 600 THz
2 CW with ck = -1, 4 RelaWavity β = 0.6, υ = √(300*1200) = 600 THz
k-Phasor Plot (100 THz Scale)
CW with ck = 3
CW with ck = -3
CW with ck = 6
CW with ck = 12
2 CW with ck = ±6 RelaWavity β = 0.0, υ = 600 THz
2 CW with ck = -3, 12 RelaWavity β = 0.6, υ = √(300*1200) = 600 THz
In the APP, right click on a scenaro button to expose the actual scenario string

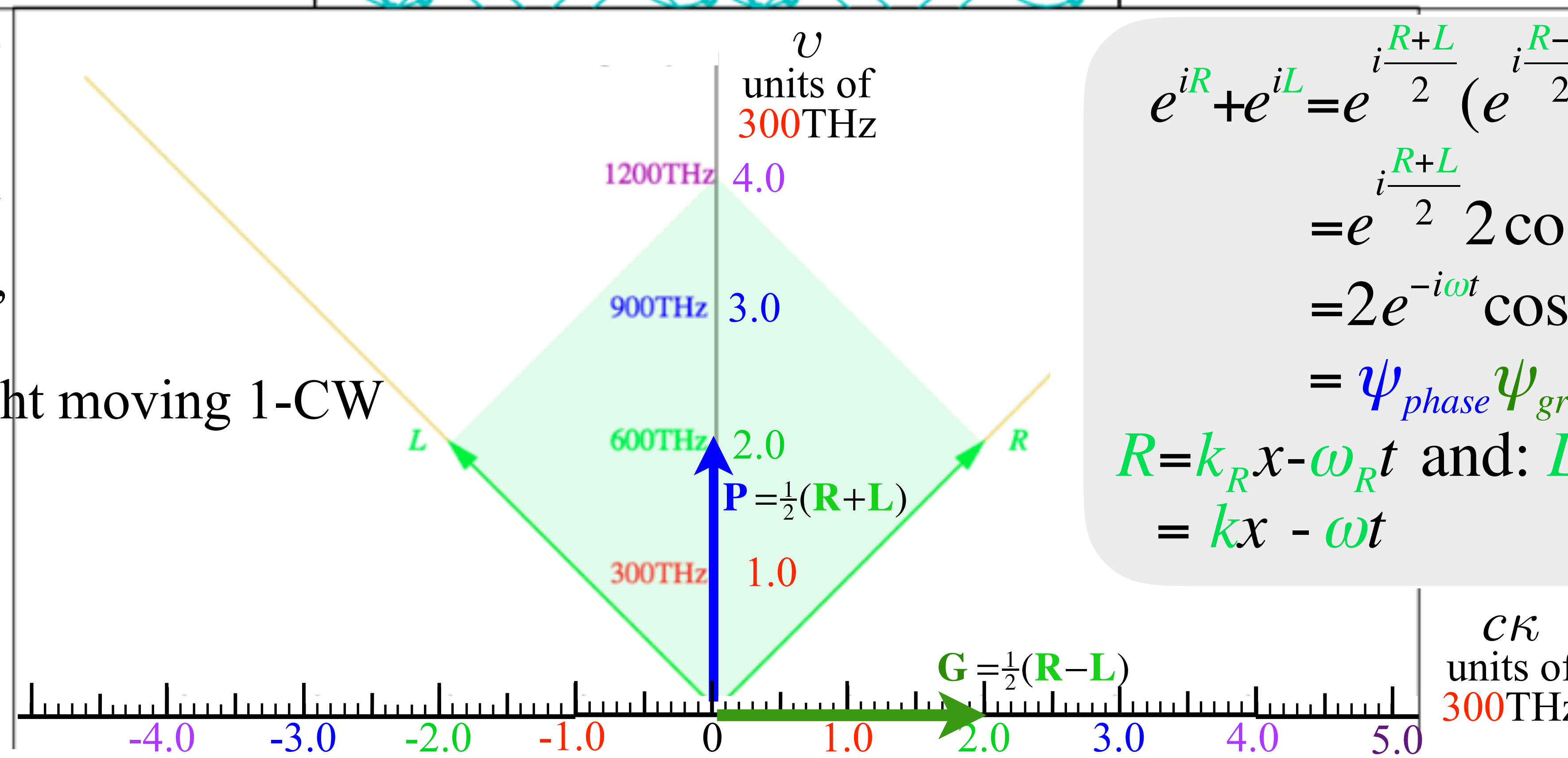

(a)
600THz
SOURCE
$Ae^{i(kx-\omega t)}$
1-CW left-to-RIGHT→
600THz
ct units of ¼ micron (μm)
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1 2 3 4
x units of ¼ μm
(b) Standing wave
(x,ct) - grid
$\psi = Ae^{i(kx-\omega t)} + Ae^{i(-kx-\omega t)}$
→ ←
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
$\frac{5}{6}$ fs
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
(c)
600THz
SOURCE
$Ae^{i(-kx-\omega t)}$
← LEFT-from-right1-CW
600THz
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1 2 3 4
x units of ¼ μm
(d)
Introducing optical space-time grids and per-space-time "baseball-diamonds"
← LEFT-from-right moving 1-CW
1-CW moving left-to-RIGHT→
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
L
R
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
cκ units of 300THz
-4.0 -3.0 -2.0 -1.0 0 1.0 2.0 3.0 4.0 5.0
(ck = ±2)
https://modphys.hosted.uark.edu/markup/BohrItWeb.html?scenario=-130022



(d)

Introducing optical space-time grids and per-space-time "baseball-diamonds"

← LEFT-from-right moving 1-CW

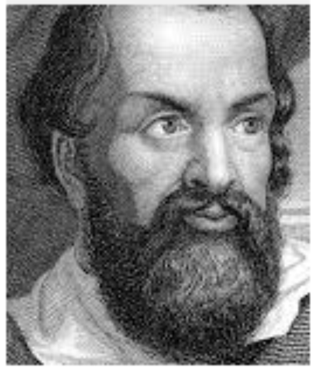


$$e^{iR}+e^{iL}=e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}}\right)$$
$$=e^{i\frac{R+L}{2}}\,2\cos\frac{R-L}{2}$$
$$=2e^{-i\omega t}\cos kx$$
$$=\psi_{phase}\psi_{group}$$

$R=k_R x-\omega_R t$ and: $L=-k_L x-\omega_L t$

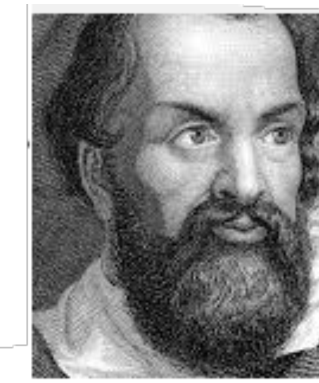
$= kx - \omega t$ $\quad = -kx - \omega t$

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

➔ Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

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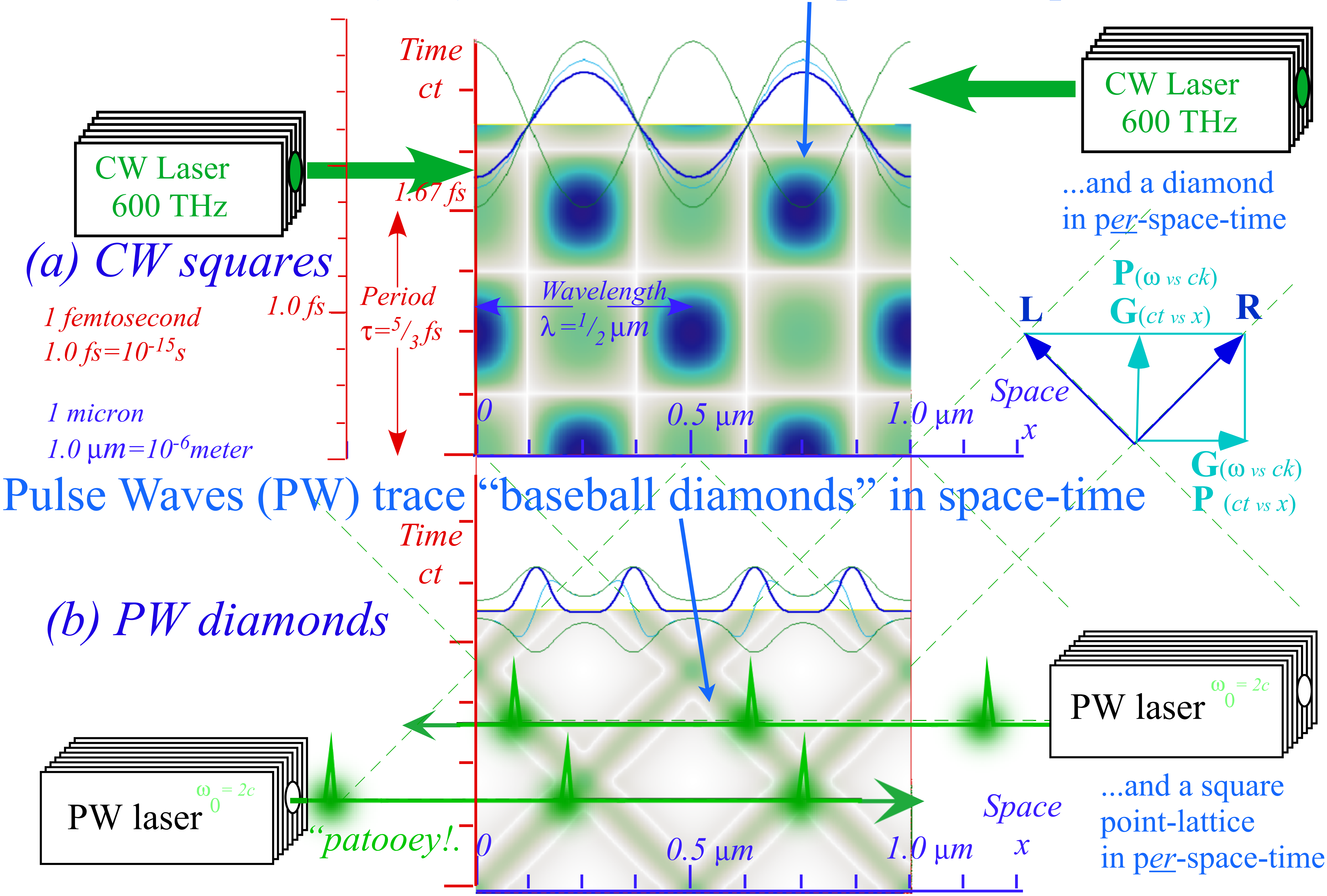
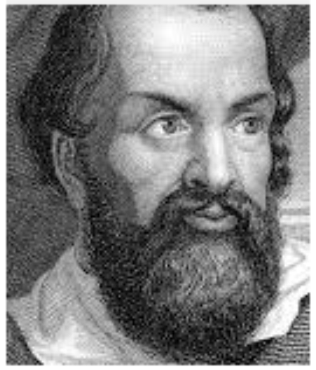
Application to TE-Waveguide modes and synchrotron beam relativity

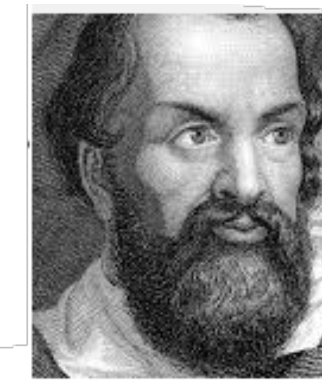
Continuous Waves (CW) trace "Cartesian squares" in space-time



BohrIt Web Simulation: *Multi-Panel 2 PW ct vs x Plot*



Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



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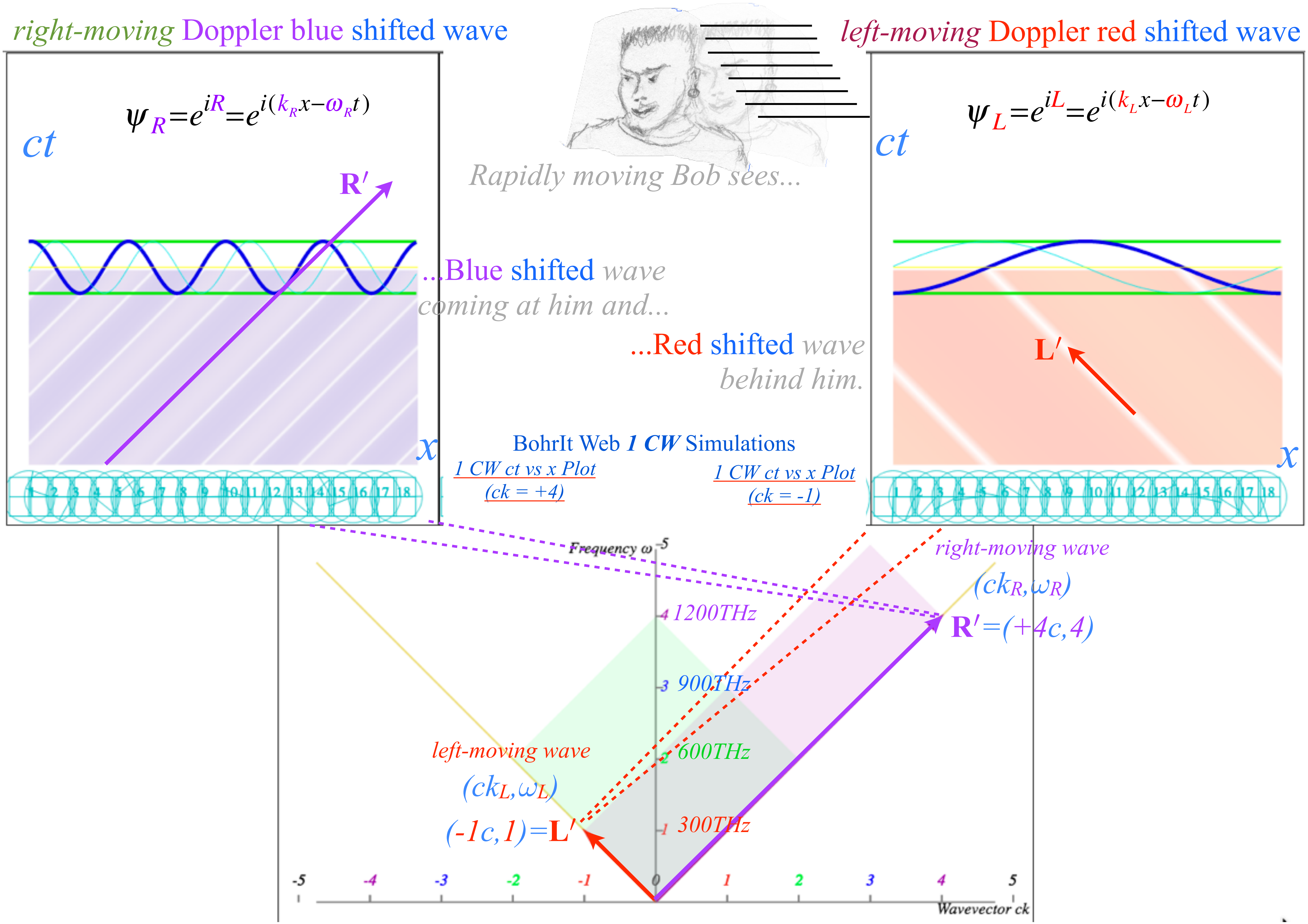
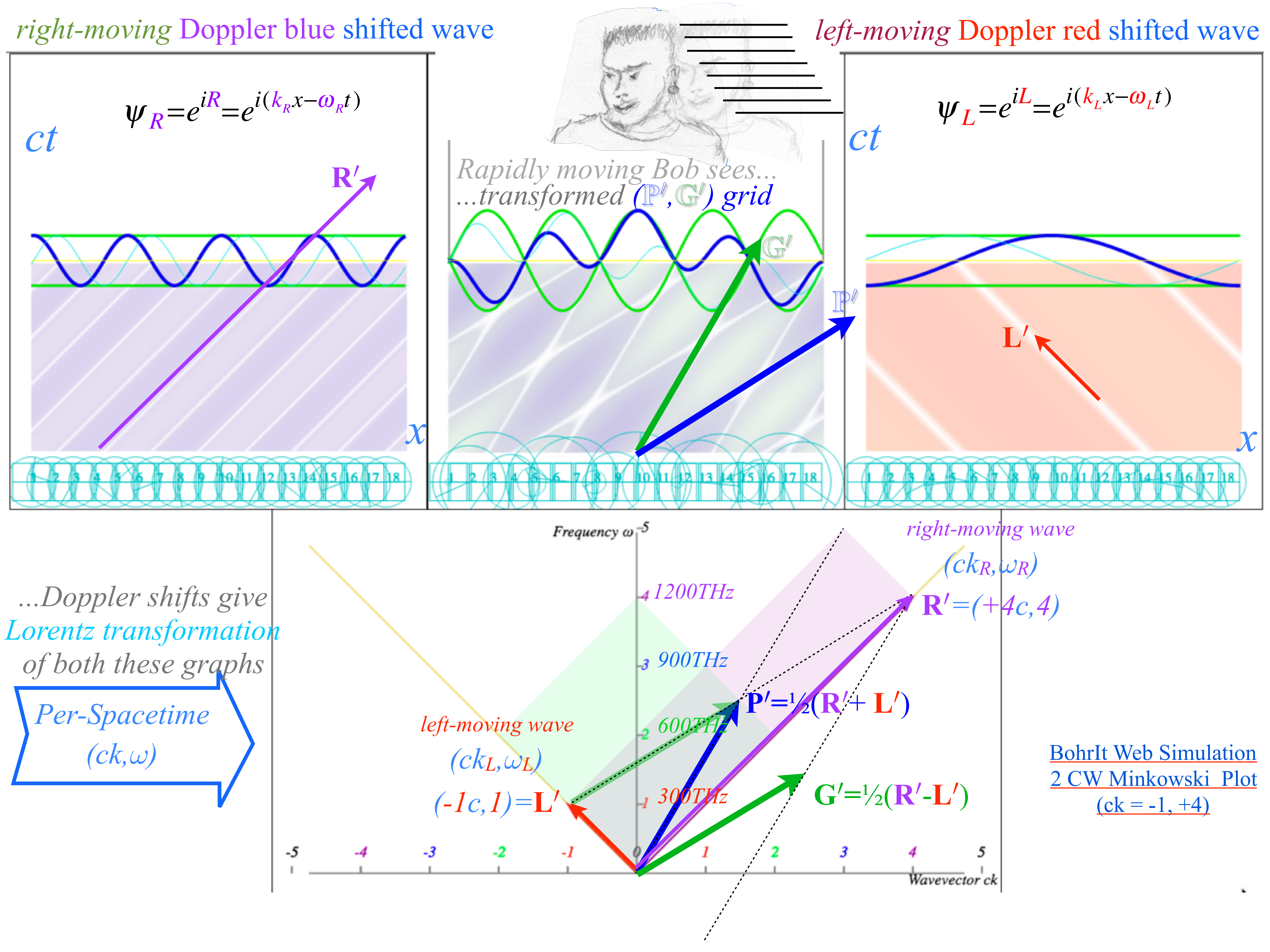
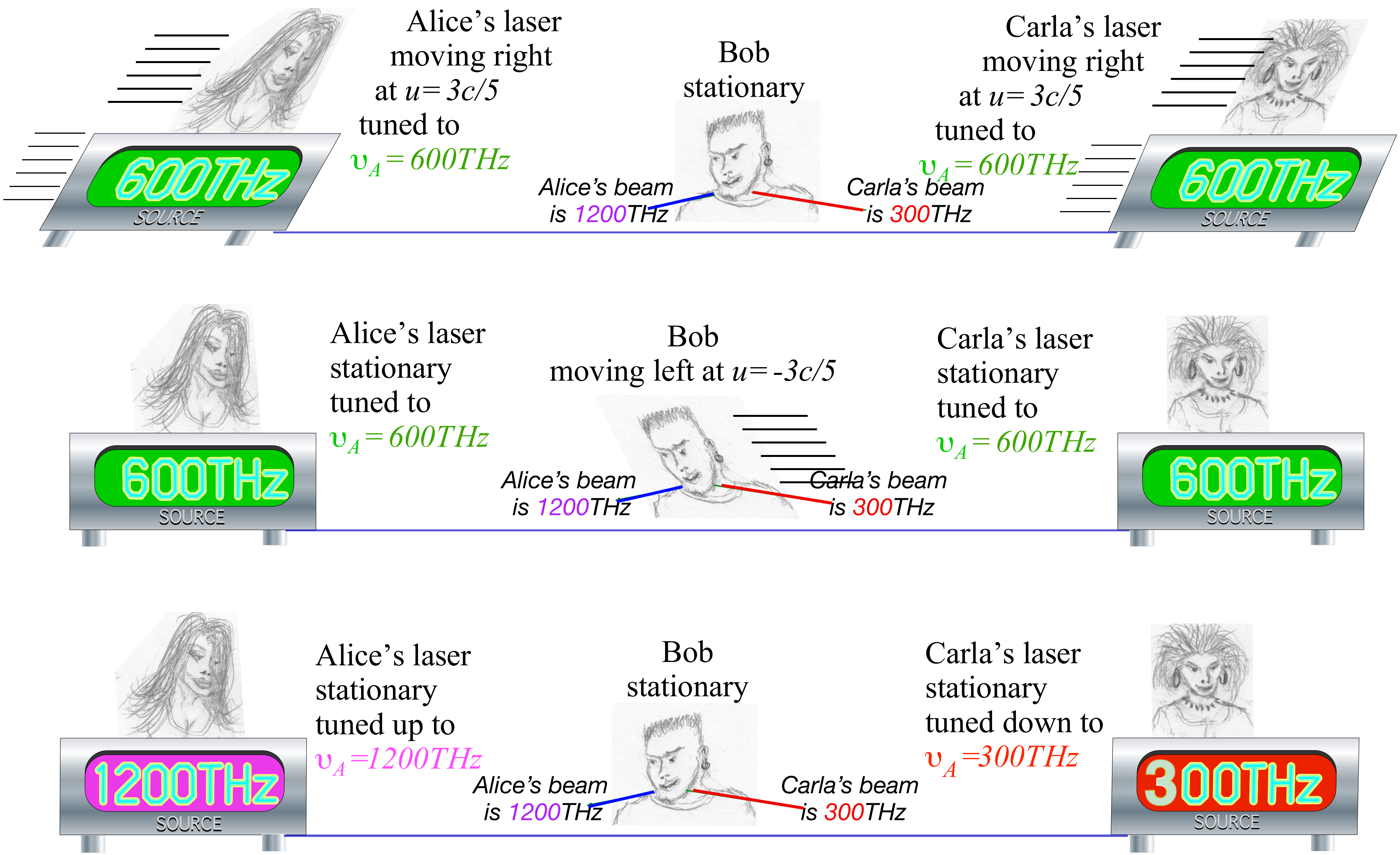


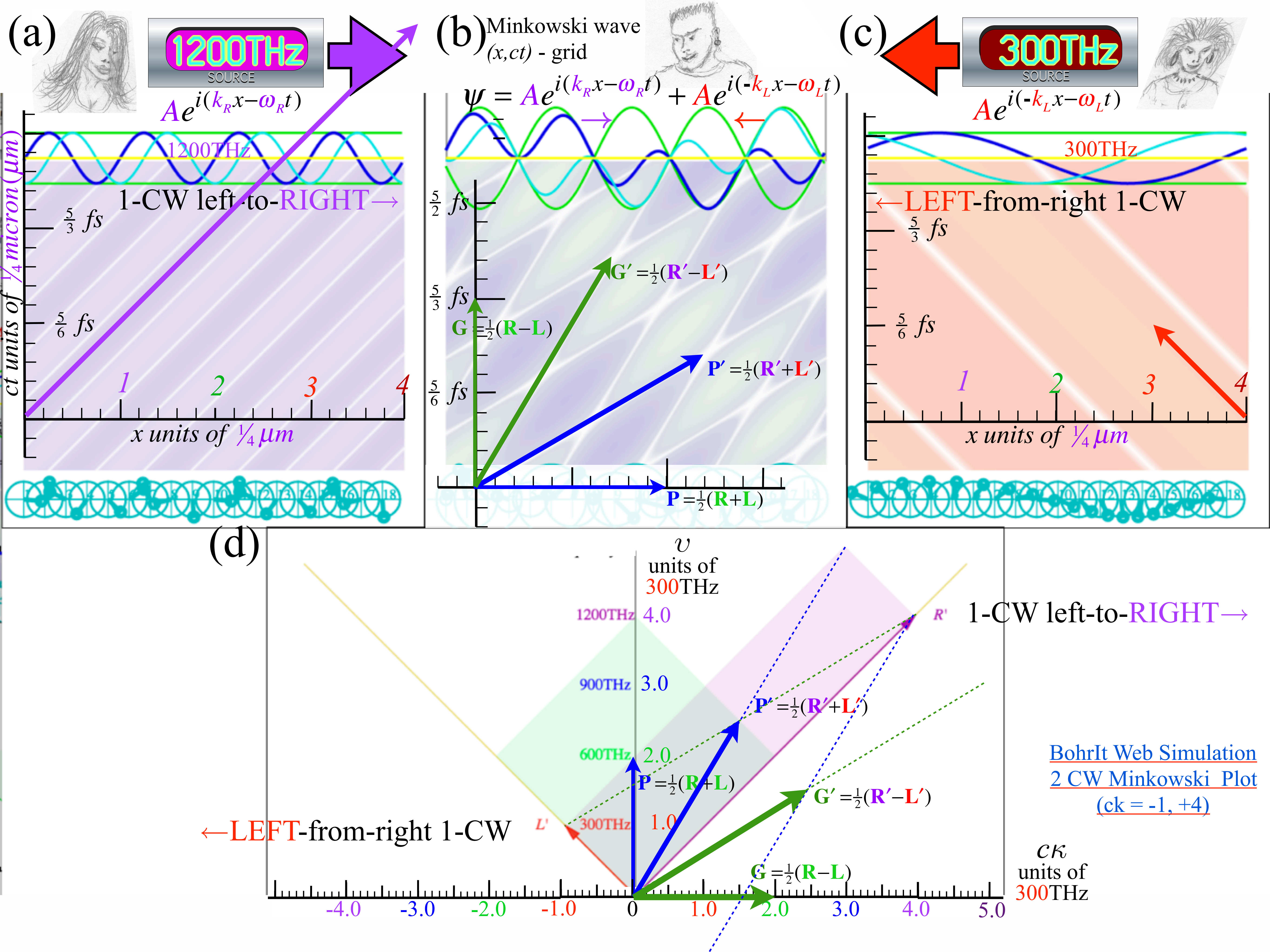
right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
x
R′
L′
Rapidly moving Bob sees...
...Blue shifted wave coming at him and...
...Red shifted wave behind him.
BohrIt Web 1 CW Simulations
1 CW ct vs x Plot (ck = +4)
1 CW ct vs x Plot (ck = -1)
Frequency ω
right-moving wave
(ck_R, ω_R)
R′=(+4c,4)
1200THz
900THz
600THz
300THz
left-moving wave
(ck_L, ω_L)
(-1c,1)=L′
Wavevector ck


right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
x
R′
L′
Rapidly moving Bob sees...
...transformed (P′,G′) grid
G′
P′
Frequency ω
1200THz
900THz
600THz
300THz
Wavevector ck
right-moving wave
(ckR,ωR)
R′=(+4c,4)
P′=½(R′+ L′)
G′=½(R′-L′)
left-moving wave
(ckL,ωL)
(-1c,1)=L′
...Doppler shifts give
Lorentz transformation
of both these graphs
Per-Spacetime
(ck,ω)
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

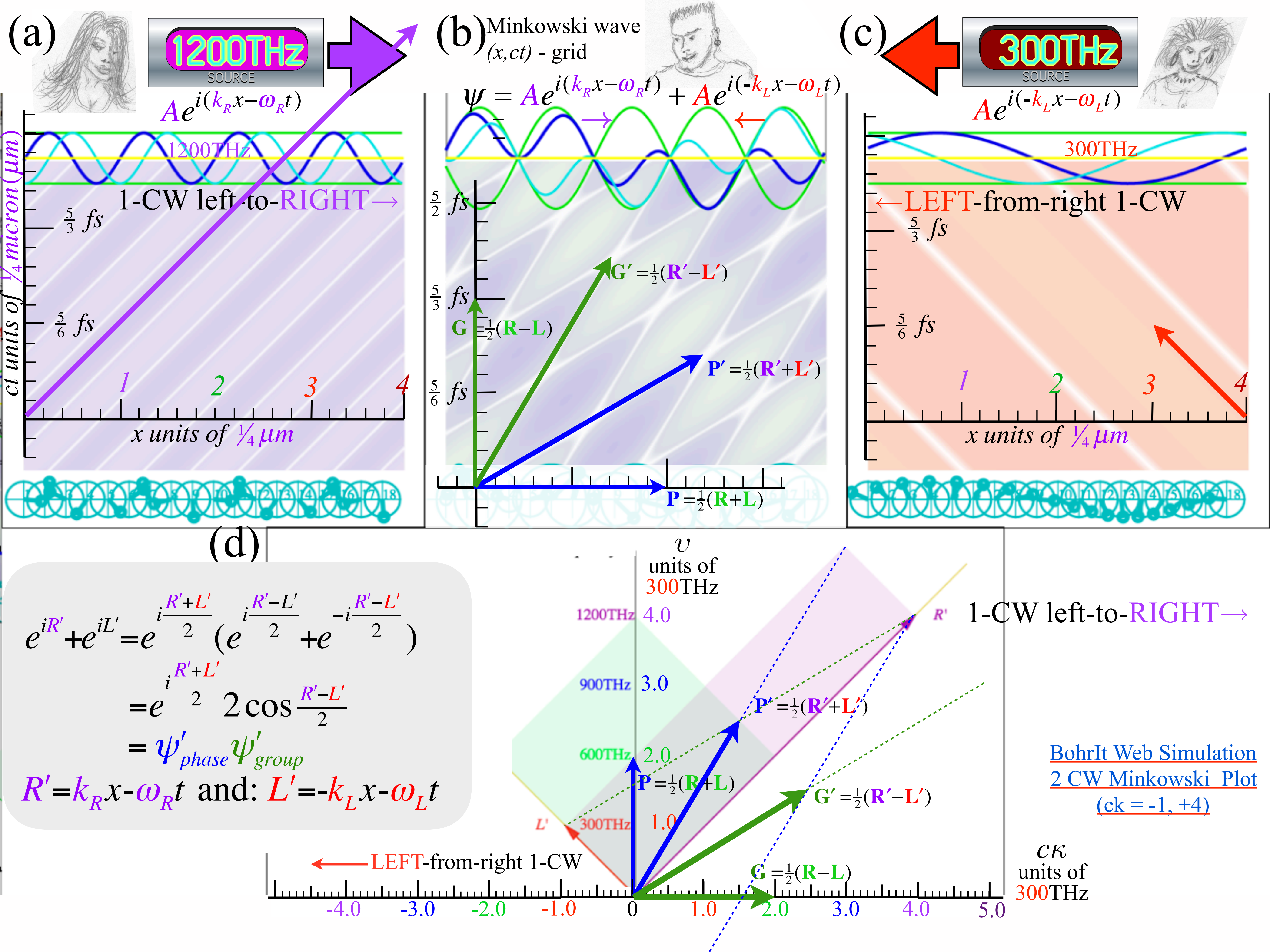
Three scenarios that look the same to Bob



Much cheaper (and safer) to do the 3rd scenario!$!

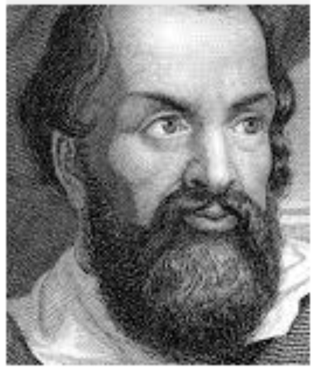


(a)
1200THz
SOURCE
$Ae^{i(k_R x-\omega_R t)}$
1200THz
1-CW left-to-RIGHT→
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
ct units of ¼ micron (μm)
x units of ¼ μm
1
2
3
4
(b)
Minkowski wave
(x,ct) - grid
$\psi = Ae^{i(k_R x-\omega_R t)} + Ae^{i(-k_L x-\omega_L t)}$
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
$\mathbf{G}' = \frac{1}{2}(\mathbf{R}'-\mathbf{L}')$
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
$\mathbf{P}' = \frac{1}{2}(\mathbf{R}'+\mathbf{L}')$
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
(c)
300THz
SOURCE
$Ae^{i(-k_L x-\omega_L t)}$
300THz
←LEFT-from-right 1-CW
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(d)
υ
units of
300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
R'
L'
1-CW left-to-RIGHT→
←LEFT-from-right 1-CW
$\mathbf{P}' = \frac{1}{2}(\mathbf{R}'+\mathbf{L}')$
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
$\mathbf{G}' = \frac{1}{2}(\mathbf{R}'-\mathbf{L}')$
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
cκ
units of
300THz
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
5.0
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)



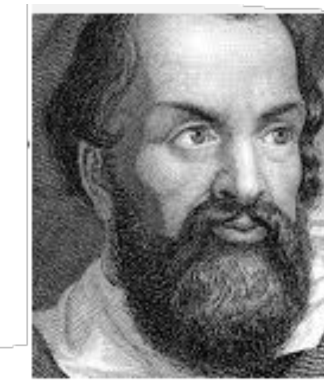
(a)
1200THz
SOURCE
$Ae^{i(k_R x-\omega_R t)}$
1200THz
1-CW left-to-RIGHT→
ct units of ¼ micron (μm)
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(b)
Minkowski wave
(x,ct) - grid
$\psi = Ae^{i(k_R x-\omega_R t)} + Ae^{i(-k_L x-\omega_L t)}$
→
←
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
G′ =½(R′−L′)
G =½(R−L)
P′ =½(R′+L′)
P =½(R+L)
(c)
300THz
SOURCE
$Ae^{i(-k_L x-\omega_L t)}$
300THz
←LEFT-from-right 1-CW
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(d)
$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$
$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$
$=\psi'_{phase}\psi'_{group}$
$R'=k_R x-\omega_R t$ and: $L'=-k_L x-\omega_L t$
υ
units of
300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
R'
L'
1-CW left-to-RIGHT→
P′ =½(R′+L′)
P =½(R+L)
G′ =½(R′−L′)
G =½(R−L)
←LEFT-from-right 1-CW
cκ
units of
300THz
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
5.0
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

Galileo Galilei



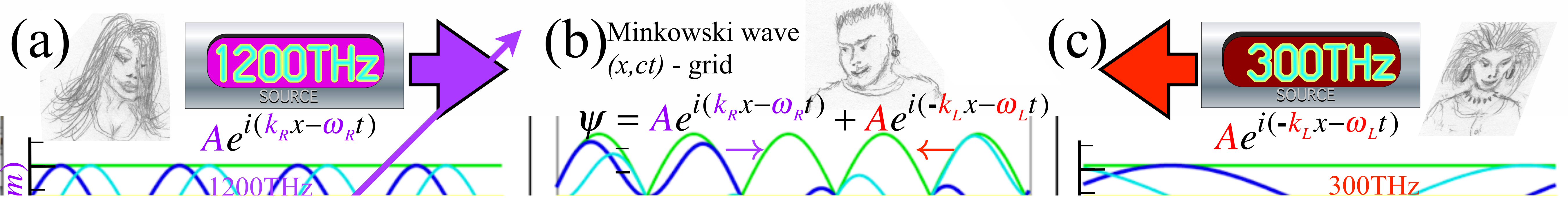
1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

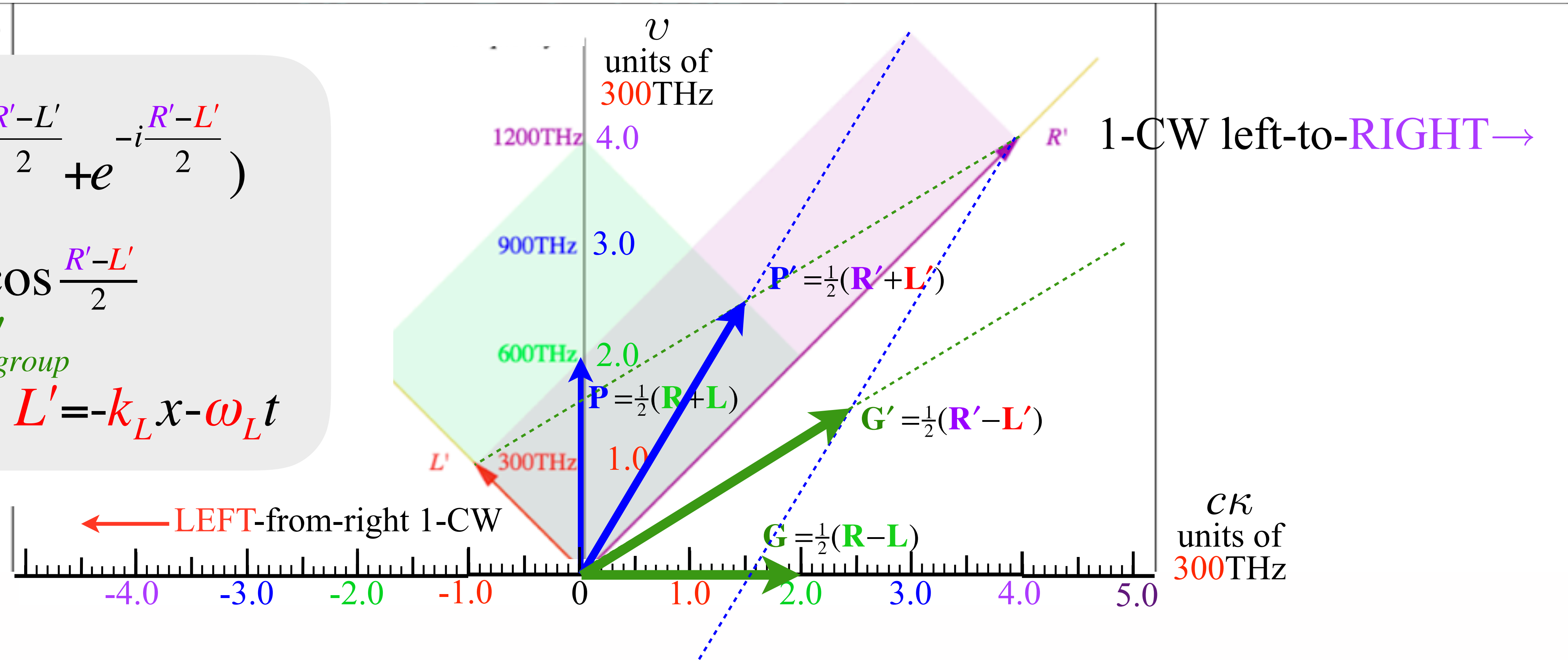
Optical interference "baseball-diamond" displays *phase* and *group* velocity
Details of 2CW wavefunctions in rest frame
Pulse waves (PW) versus Continuous Waves (CW)
Doppler shifted "baseball-diamond" displays Lorentz frame transformation
➔ Analyzing wave velocity by per-space-per-time *and* space-time graphs
16 coefficients of relativistic 2CW interference
Two "famous-name" coefficients and the Lorentz transformation
Thales geometry of Lorentz transformation
Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and geometry of 16 parameter functions of ρ and σ
Application to TE-Waveguide modes and synchrotron beam relativity



$$\mathbf{P}' = \begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix} = \tfrac{1}{2}(\mathbf{R}'+\mathbf{L}') = \upsilon_A \begin{pmatrix} \frac{1}{2}(e^{\rho}+e^{-\rho}) \\ \frac{1}{2}(e^{\rho}-e^{-\rho}) \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}_{Bob's\ View} \text{ or: } \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{Alice's\ View}$$

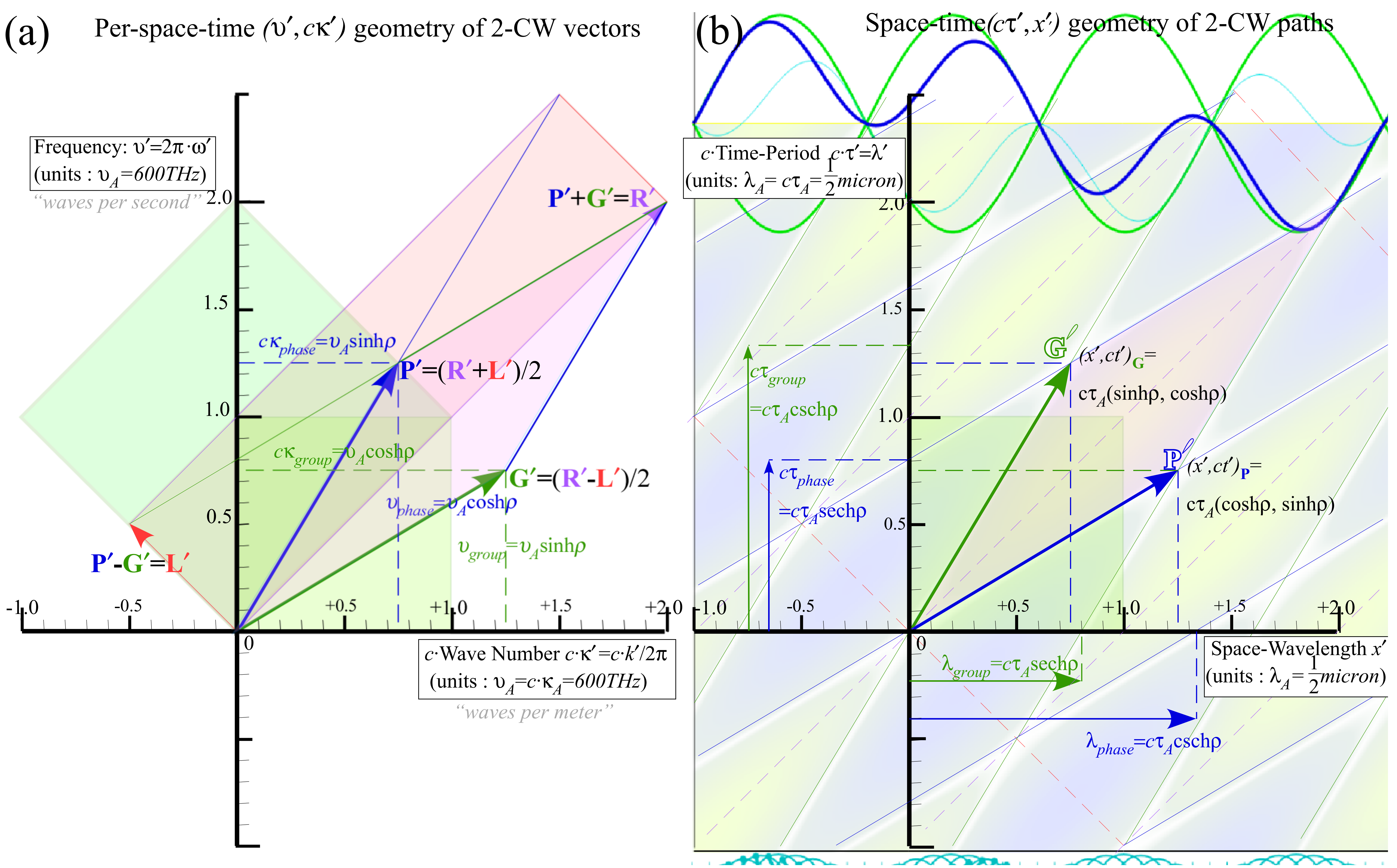
$$\mathbf{G}' = \begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix} = \tfrac{1}{2}(\mathbf{R}'-\mathbf{L}') = \upsilon_A \begin{pmatrix} \frac{1}{2}(e^{\rho}-e^{-\rho}) \\ \frac{1}{2}(e^{\rho}+e^{-\rho}) \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{3}{2} \\ \frac{5}{2} \end{pmatrix}_{Bob's\ View} \text{ or: } \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{Alice's\ View}$$

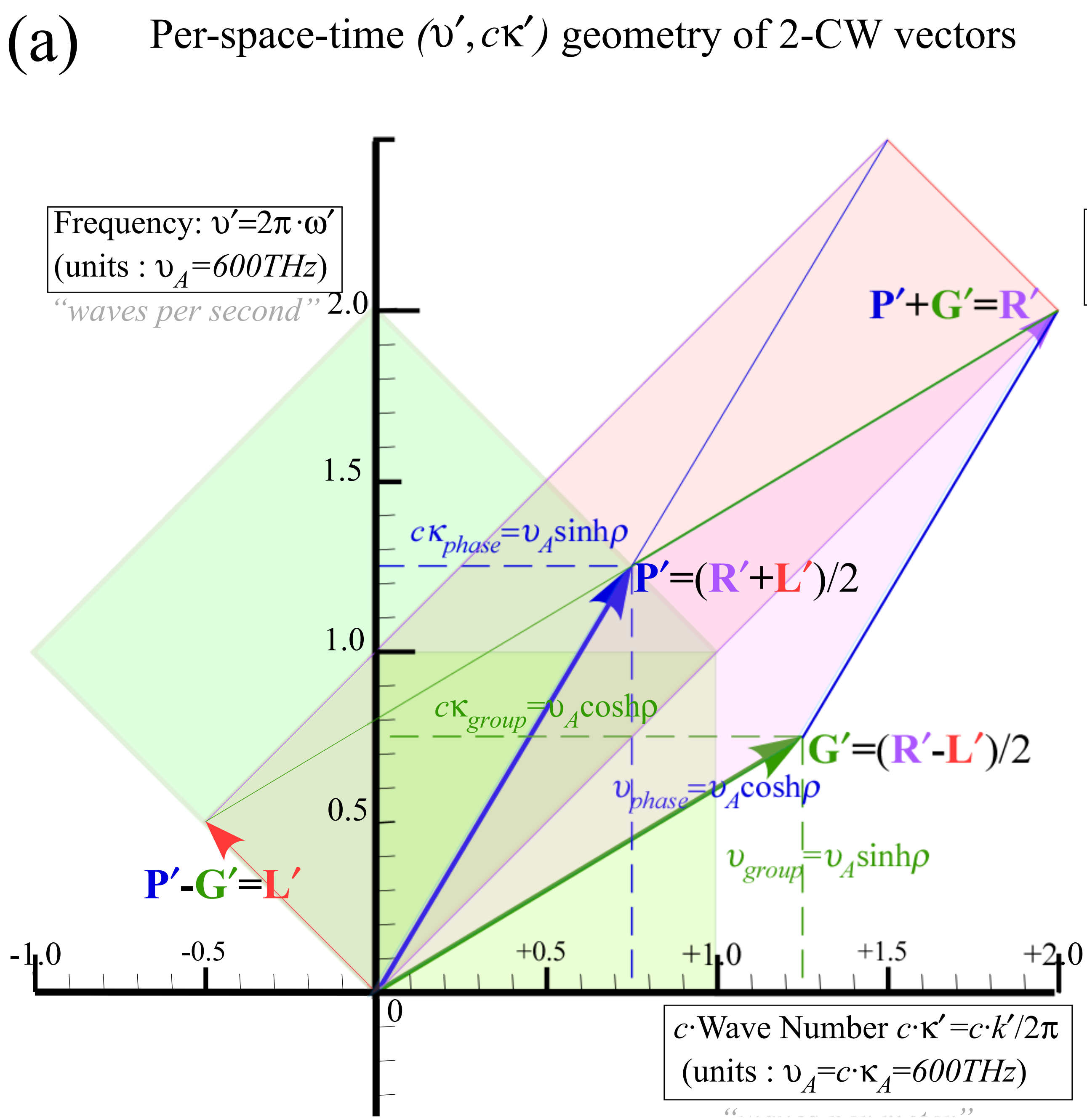
(d)

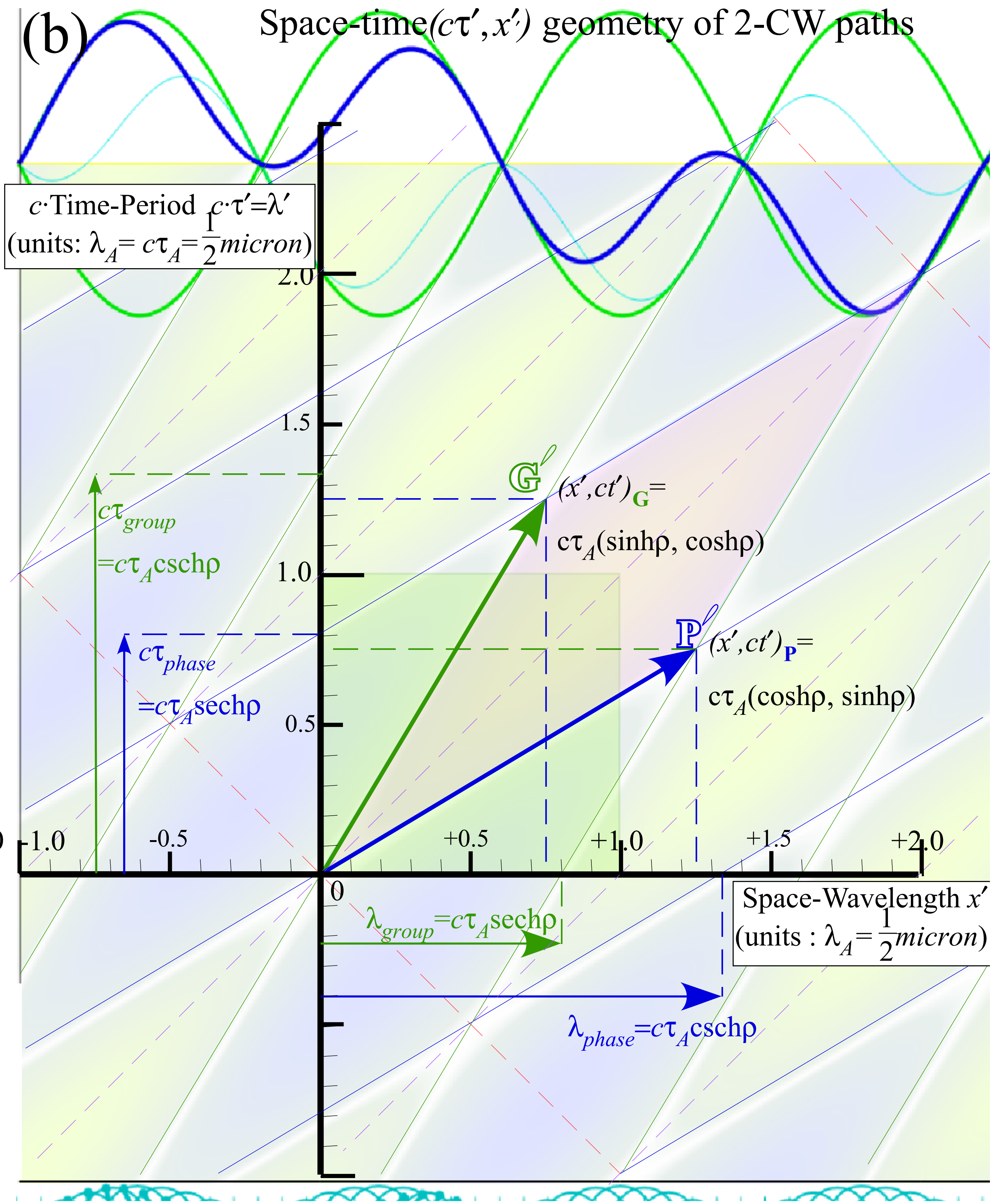


$$e^{iR'}+e^{iL'} = e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$
$$= e^{i\frac{R'+L'}{2}}\, 2\cos\frac{R'-L'}{2}$$
$$= \psi'_{phase}\psi'_{group}$$

$R' = k_R x - \omega_R t$ and: $L' = -k_L x - \omega_L t$


(a) Per-space-time (υ′, cκ′) geometry of 2-CW vectors
Frequency: υ′=2π·ω′
(units : υ_A=600THz)
"waves per second"
P′+G′=R′
cκ_phase=υ_A sinhρ
P′=(R′+L′)/2
cκ_group=υ_A coshρ
G′=(R′-L′)/2
υ_phase=υ_A coshρ
υ_group=υ_A sinhρ
P′-G′=L′
c·Wave Number c·κ′=c·k′/2π
(units : υ_A=c·κ_A=600THz)
"waves per meter"
(b) Space-time (cτ′, x′) geometry of 2-CW paths
c·Time-Period c·τ′=λ′
(units: λ_A= cτ_A=1/2 micron)
cτ_group
=cτ_A cschρ
cτ_phase
=cτ_A sechρ
G′ (x′,ct′)_G=
cτ_A(sinhρ, coshρ)
P′ (x′,ct′)_P=
cτ_A(coshρ, sinhρ)
λ_group=cτ_A sechρ
λ_phase=cτ_A cschρ
Space-Wavelength x′
(units : λ_A=1/2 micron)



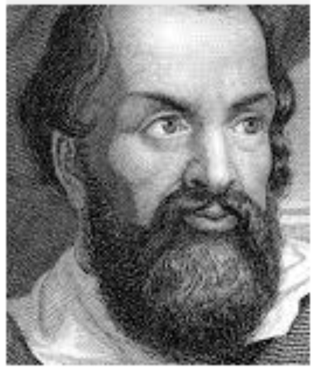


The slope of Bob's group vector $\mathbf{G}'$ in $(c\kappa, \upsilon)$-plot is actual group wave velocity in c-units.

$$\frac{V^{group}}{c} = \frac{\upsilon'_{group}}{c\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \tanh\rho = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5} \equiv \frac{u}{c} \equiv \beta$$

Group vector $\mathbb{G}'$ in (x,ct)-plot has 3/5 slope relative to time axis

Galileo Galilei



1564-1642

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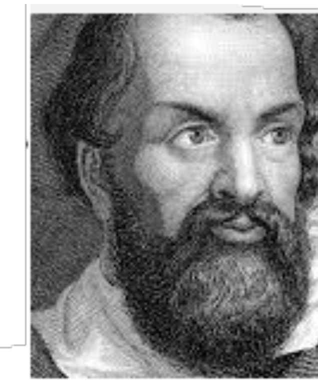

Galileo's Revenge (part 2)
Phasor angular velocity
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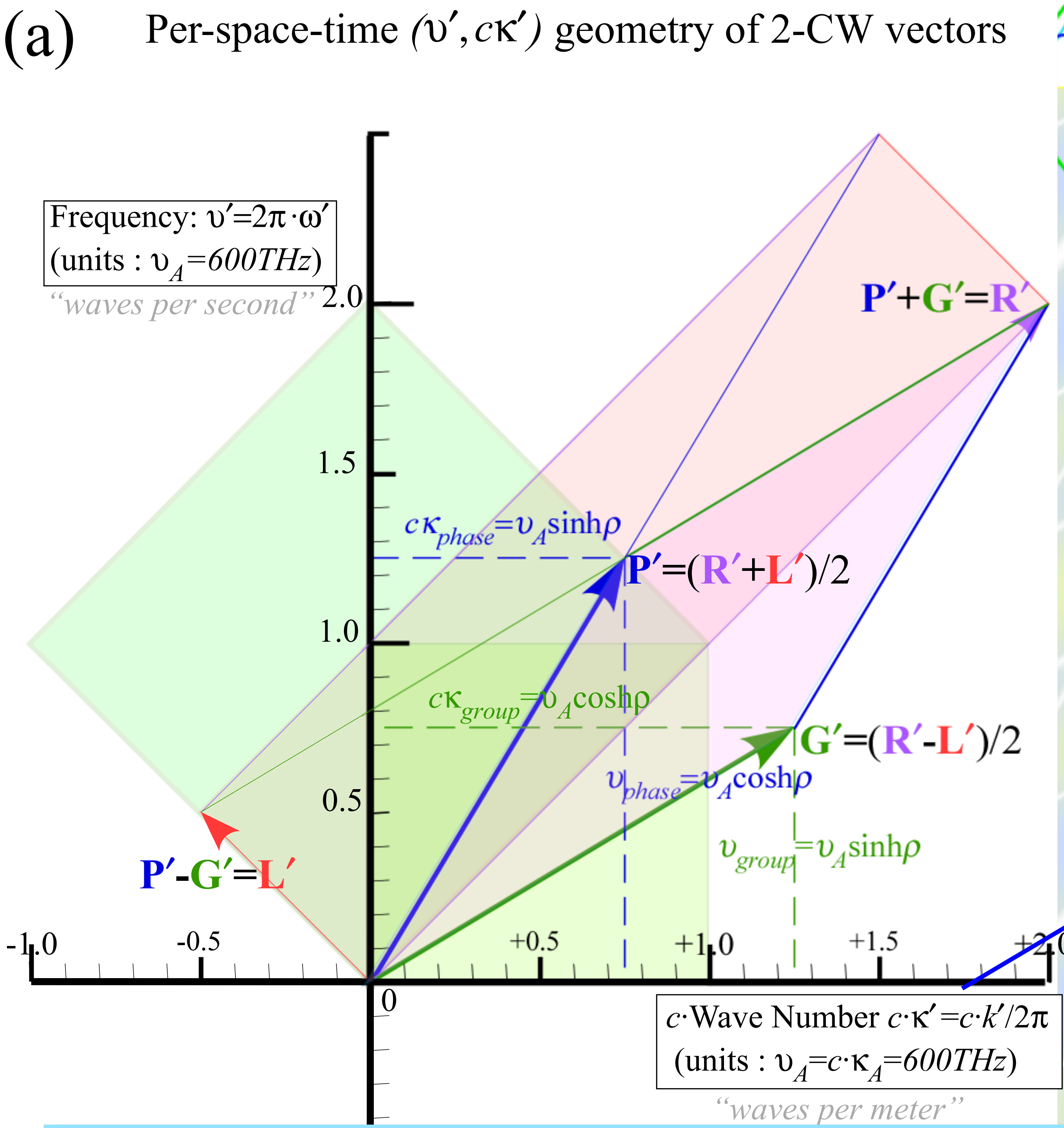
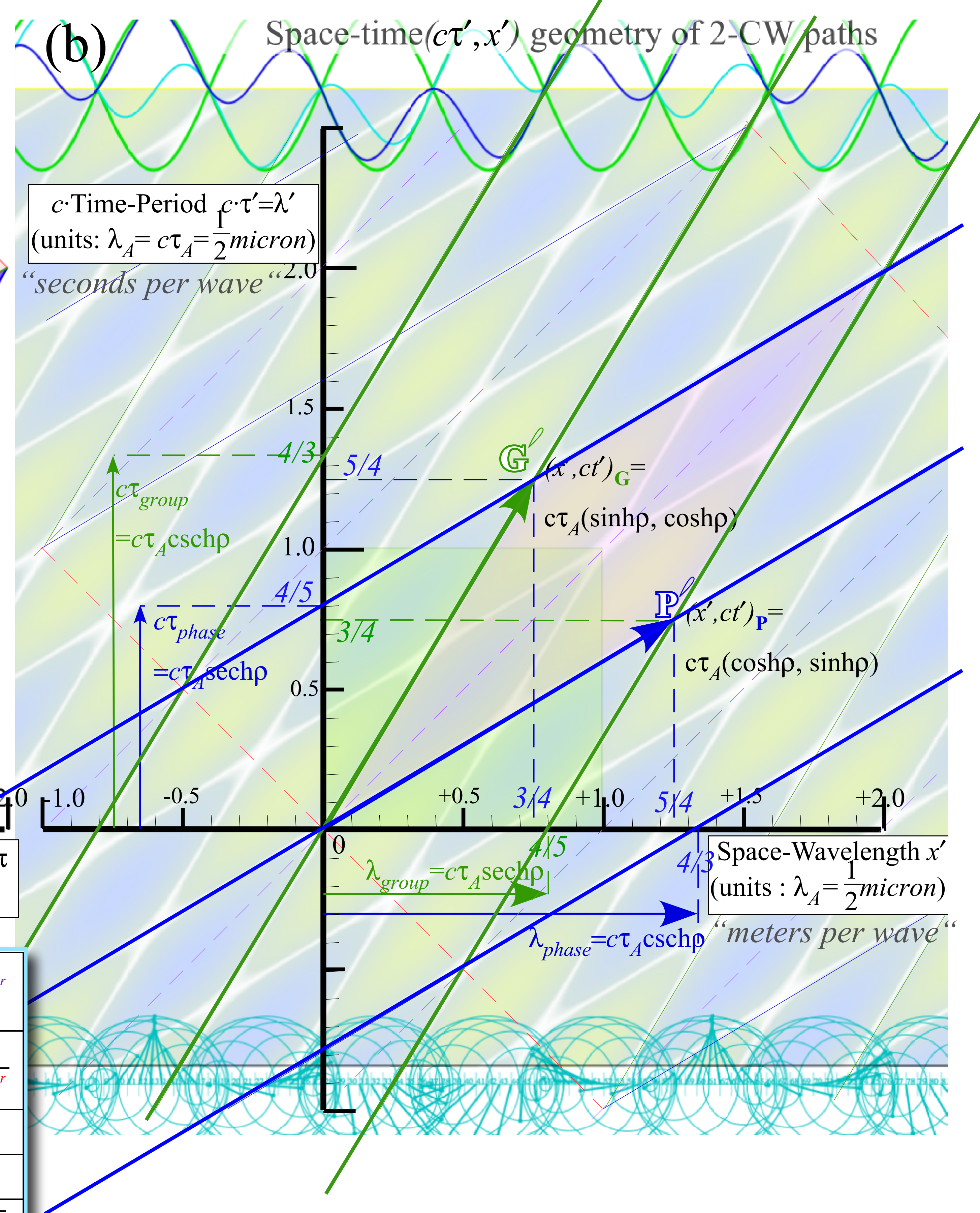




group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Fig. 11 in text Relawavity...

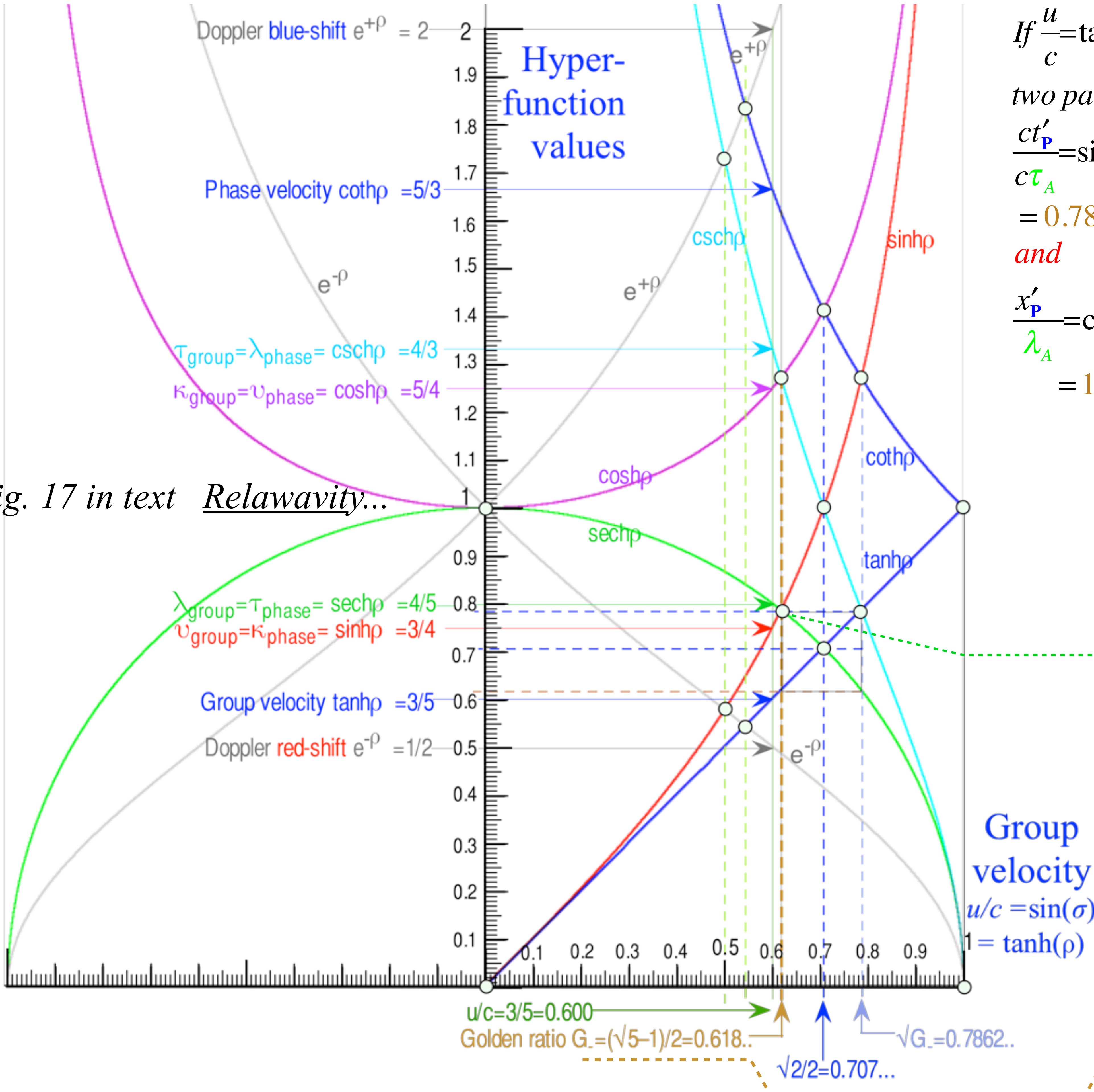


Fig. 17 in text *Relawavity...*

If $\frac{u}{c} = \tanh\rho = 0.618...(\textit{Golden-Mean } G\_)$

two parameters become exactly equal :

$$\frac{ct'_P}{c\tau_A} = \sinh\rho = \frac{\lambda_{group}}{\lambda_A} = \frac{\tau_{phase}}{\tau_A} = \text{sech}\rho$$

$$= 0.786.. = \sqrt{G\_} \qquad = 0.786..$$

and

$$\frac{x'_P}{\lambda_A} = \cosh\rho = \frac{\lambda_{phase}}{\lambda_A} = \frac{\tau_{group}}{\tau_A} = \text{csch}\rho$$

$$= 1.272.. = 1/\sqrt{G\_} \qquad = 1.272..$$

Solve :

$\text{sech}\rho = \sinh\rho$

or:

$\sinh\rho\cosh\rho = 1$

or:

$\sinh 2\rho = 2$

$\rho = \frac{1}{2}\sinh^{-1}2 = 0.7218...$

$\tanh\rho = 0.618... = \frac{\sqrt{5}-1}{2}$

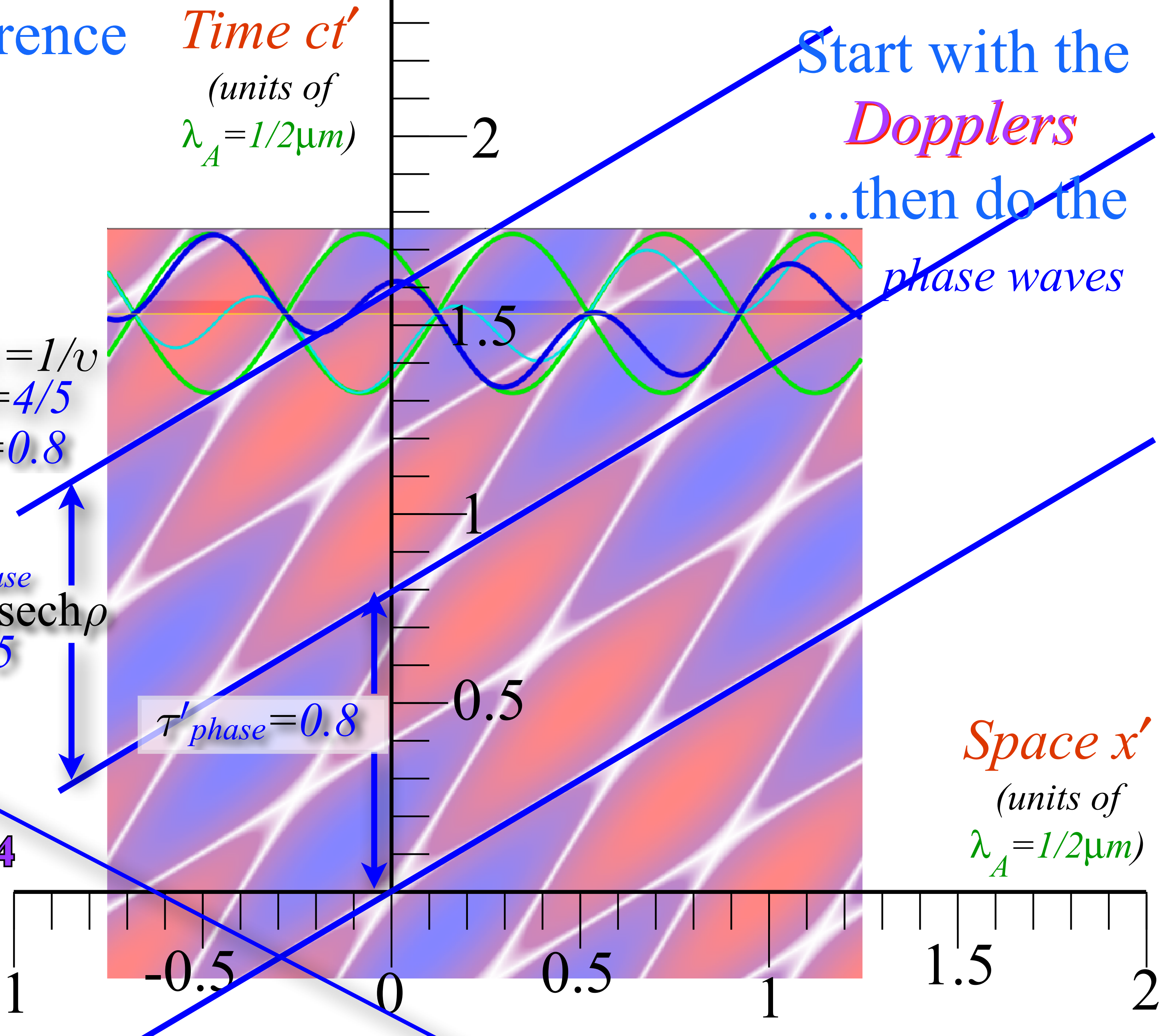
The 16 dimensions of 2CW interference

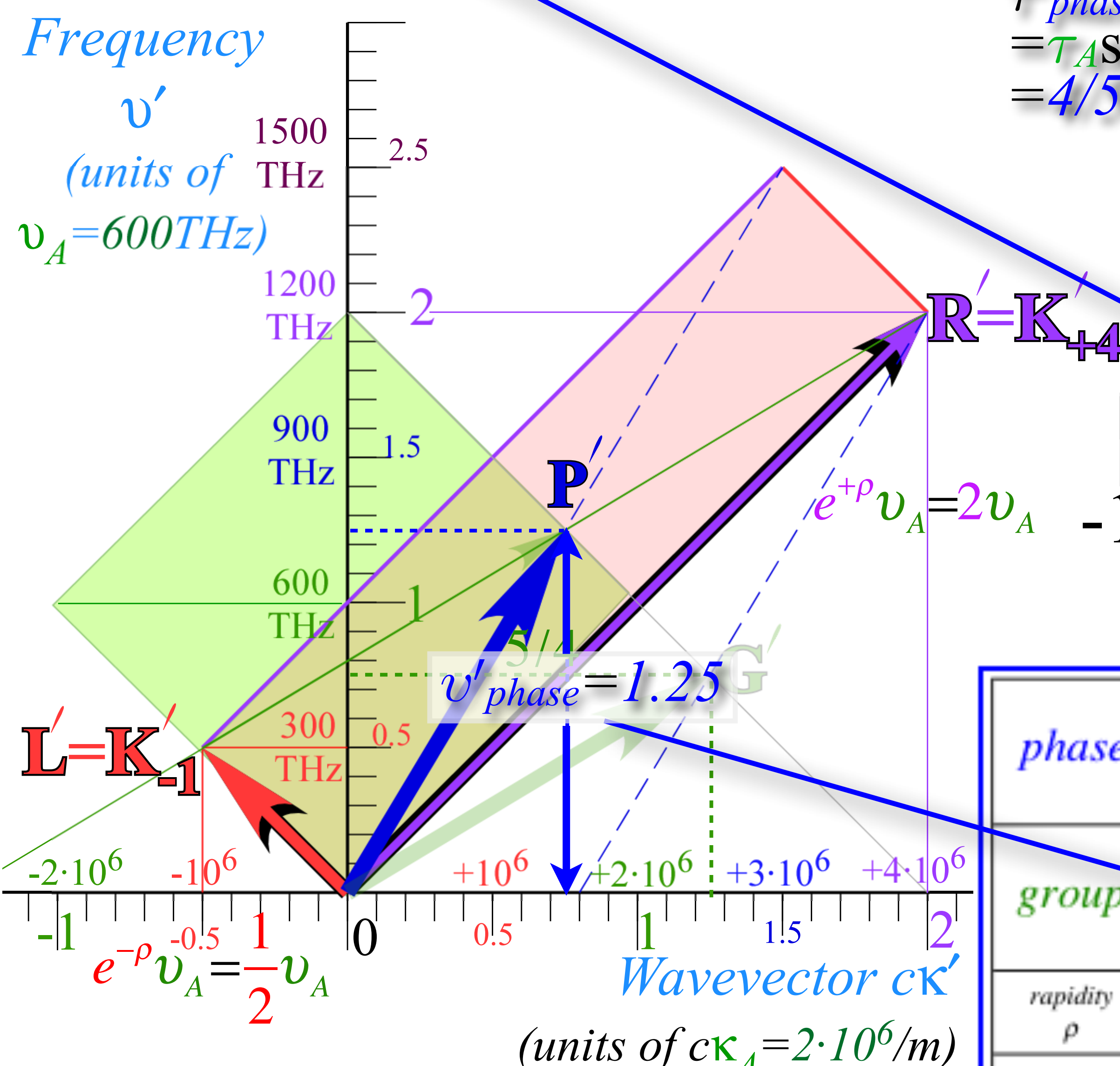
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \mathrm{sech}\rho = 4/5 = 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

The 16 dimensions of 2CW interference

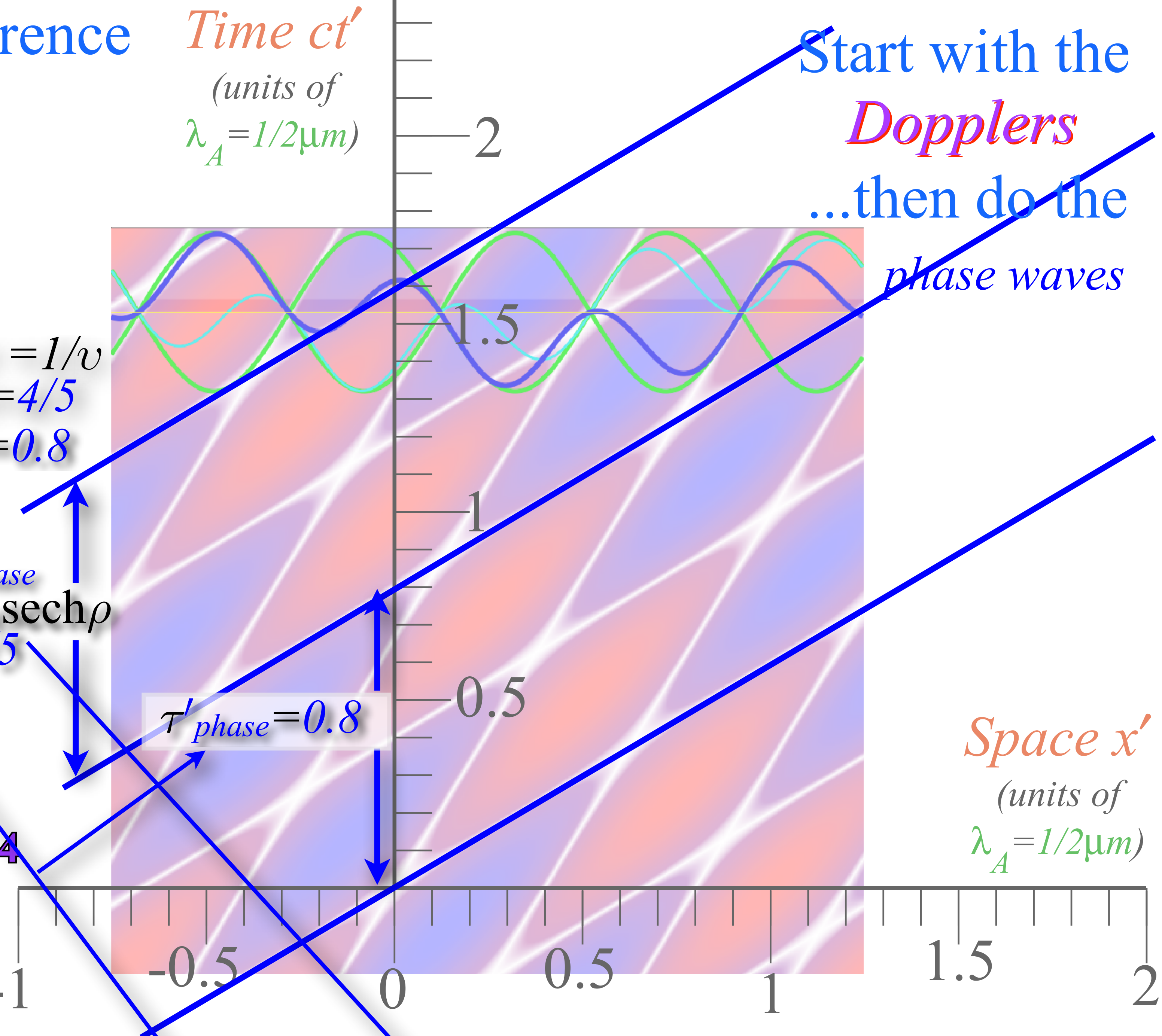
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

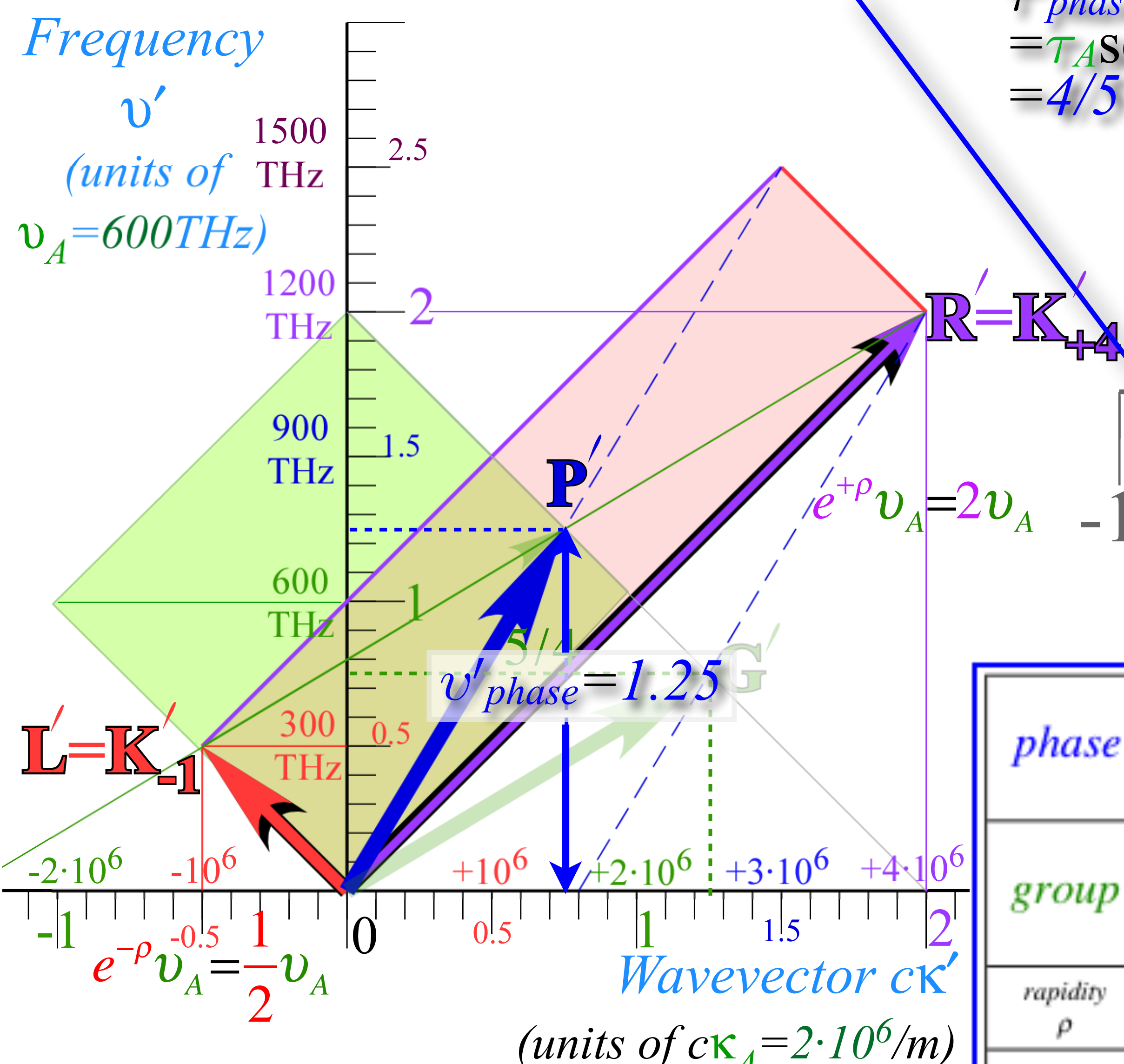
Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$
$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

Start with the *Dopplers* …then do the *phase waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

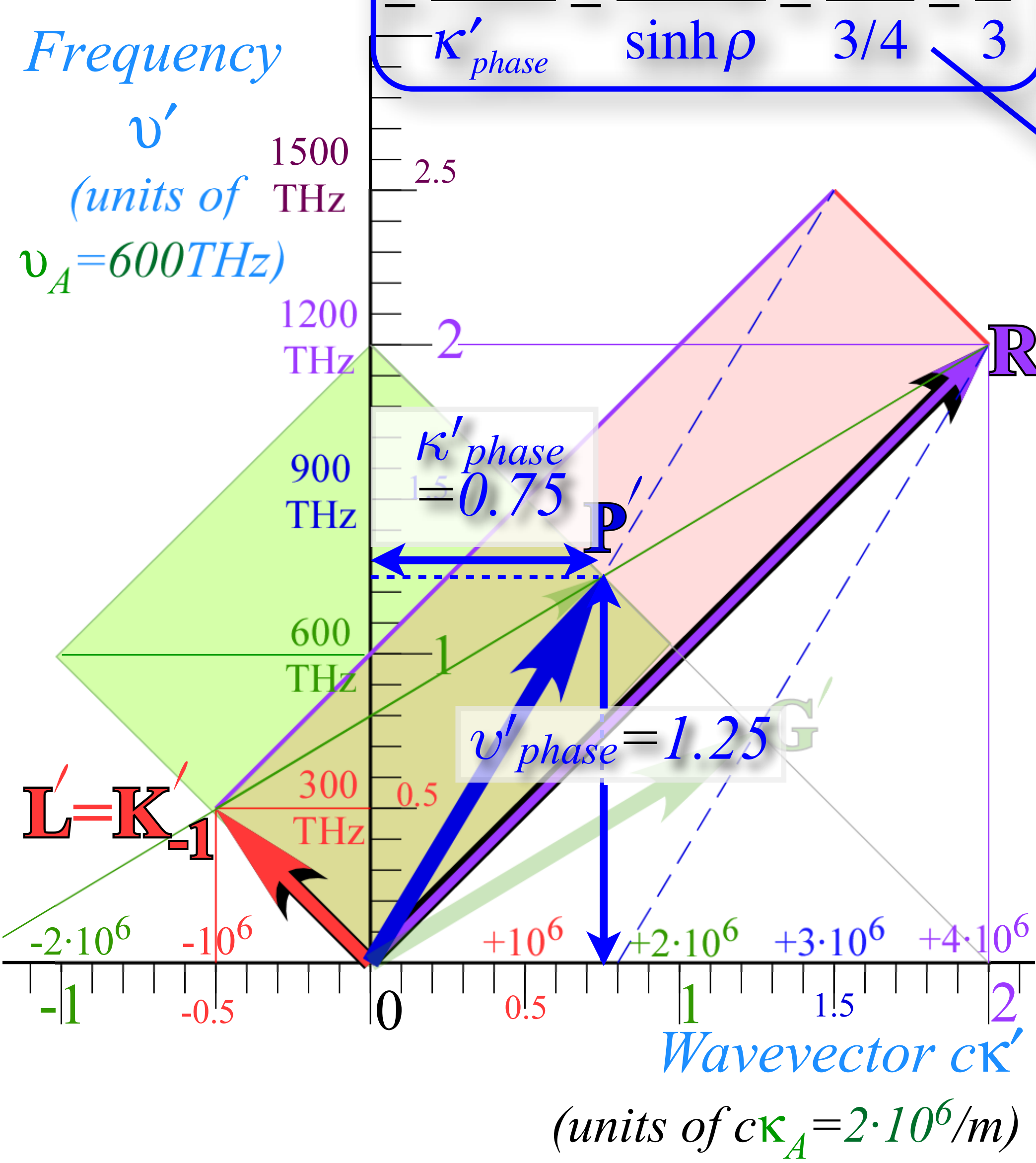
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

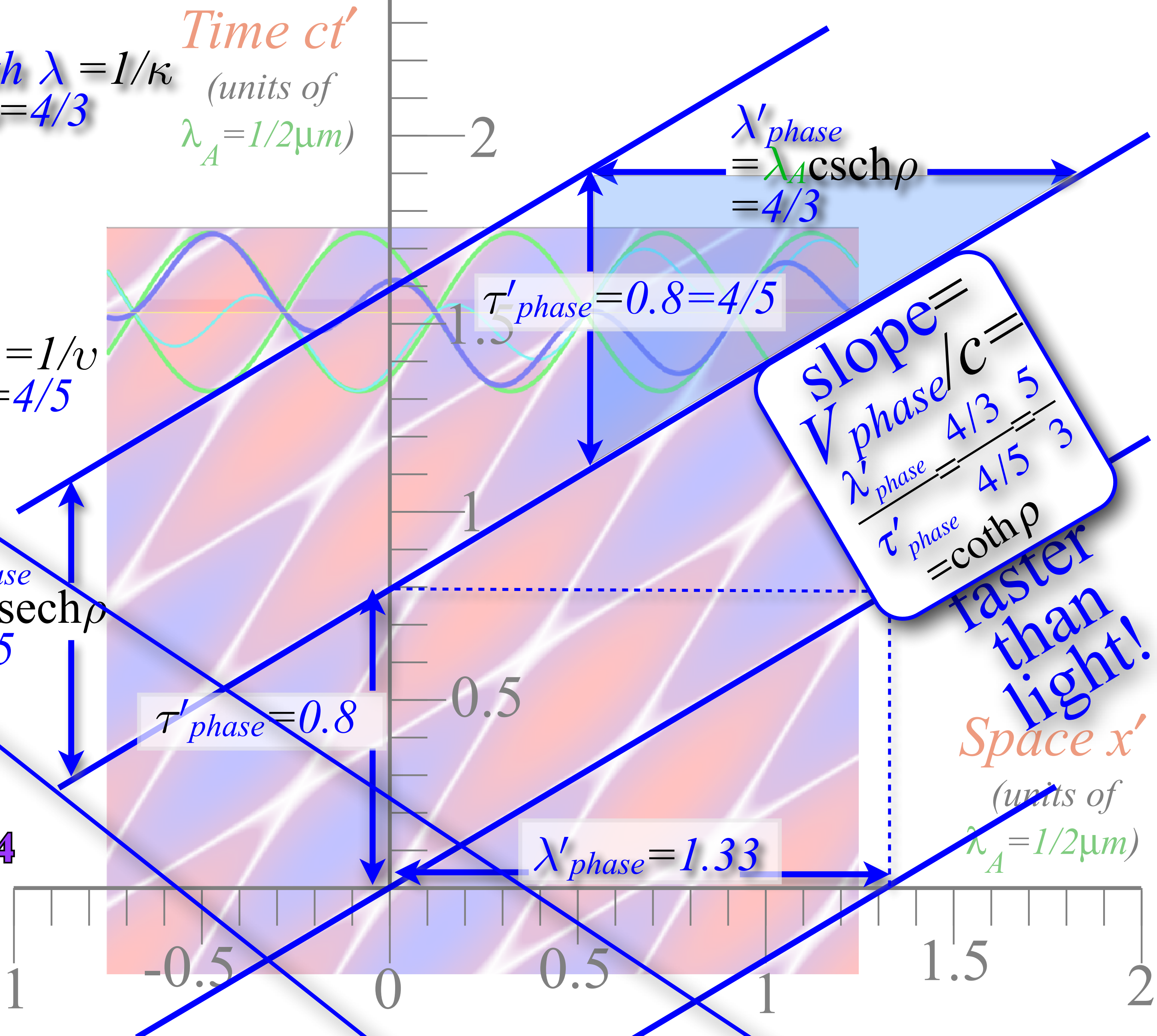
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

P-slope $= V_{phase}/c = \dfrac{\upsilon'_{phase}}{\kappa'_{phase}} = \dfrac{\cosh\rho}{\sinh\rho} = \dfrac{5/4}{3/4} = \dfrac{5}{3}$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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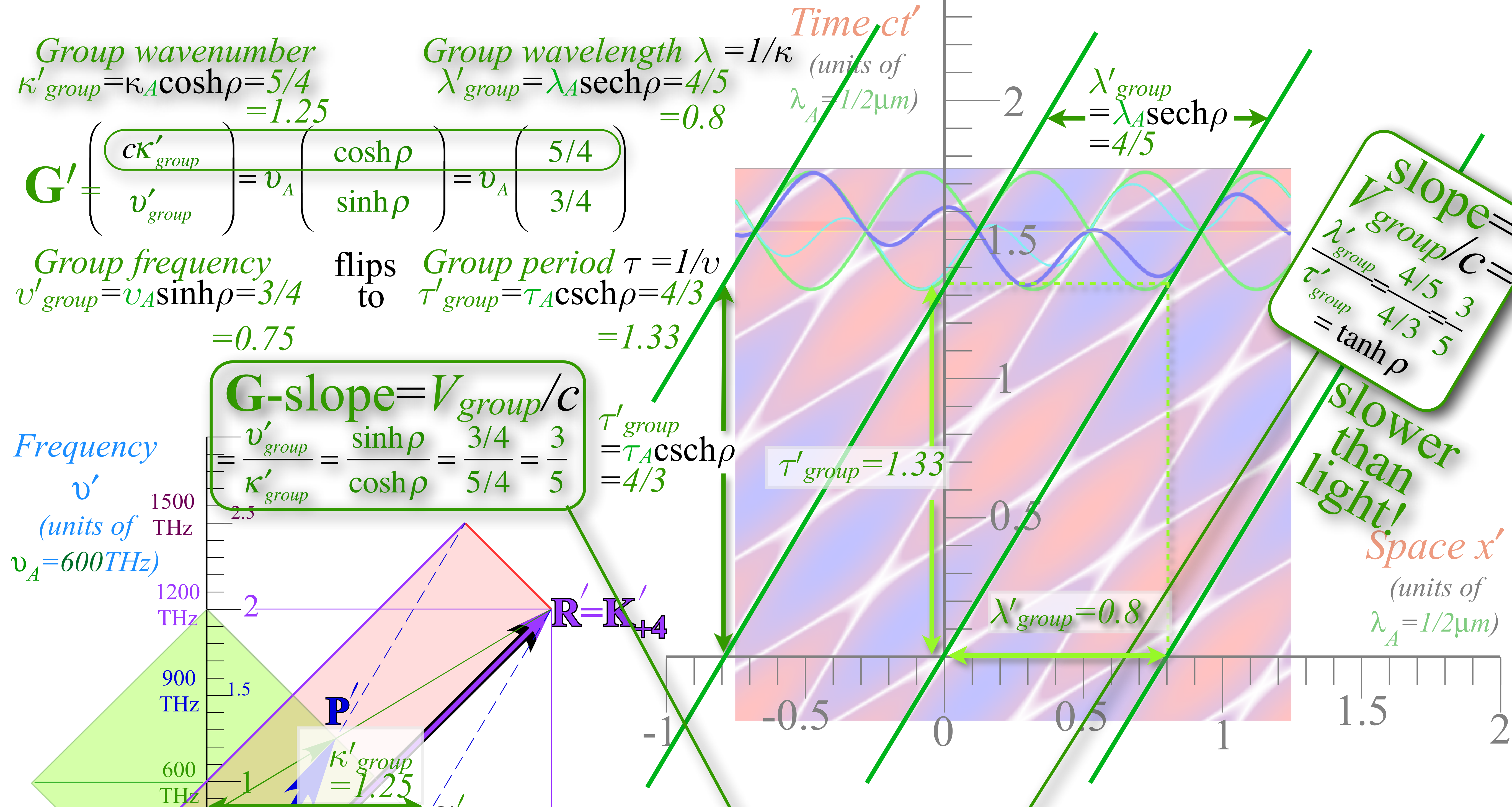

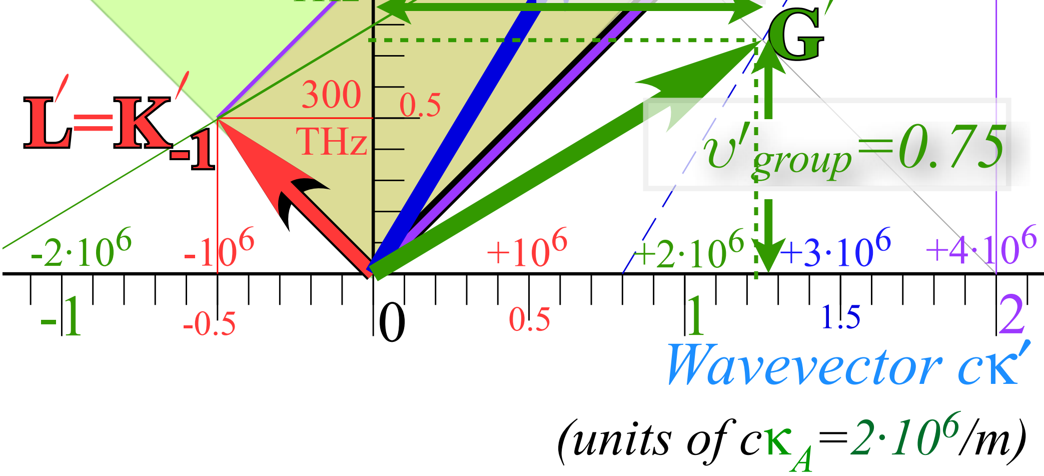

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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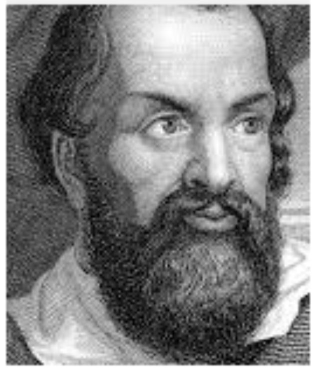
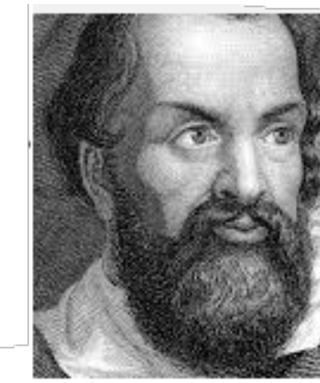
value for $\beta=3/5$	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
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Lorentz transformations...

write $\mathbf{G}'$ *and* $\mathbf{P}'$ *in terms of* $\mathbf{G}$ *and* $\mathbf{P}$ *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

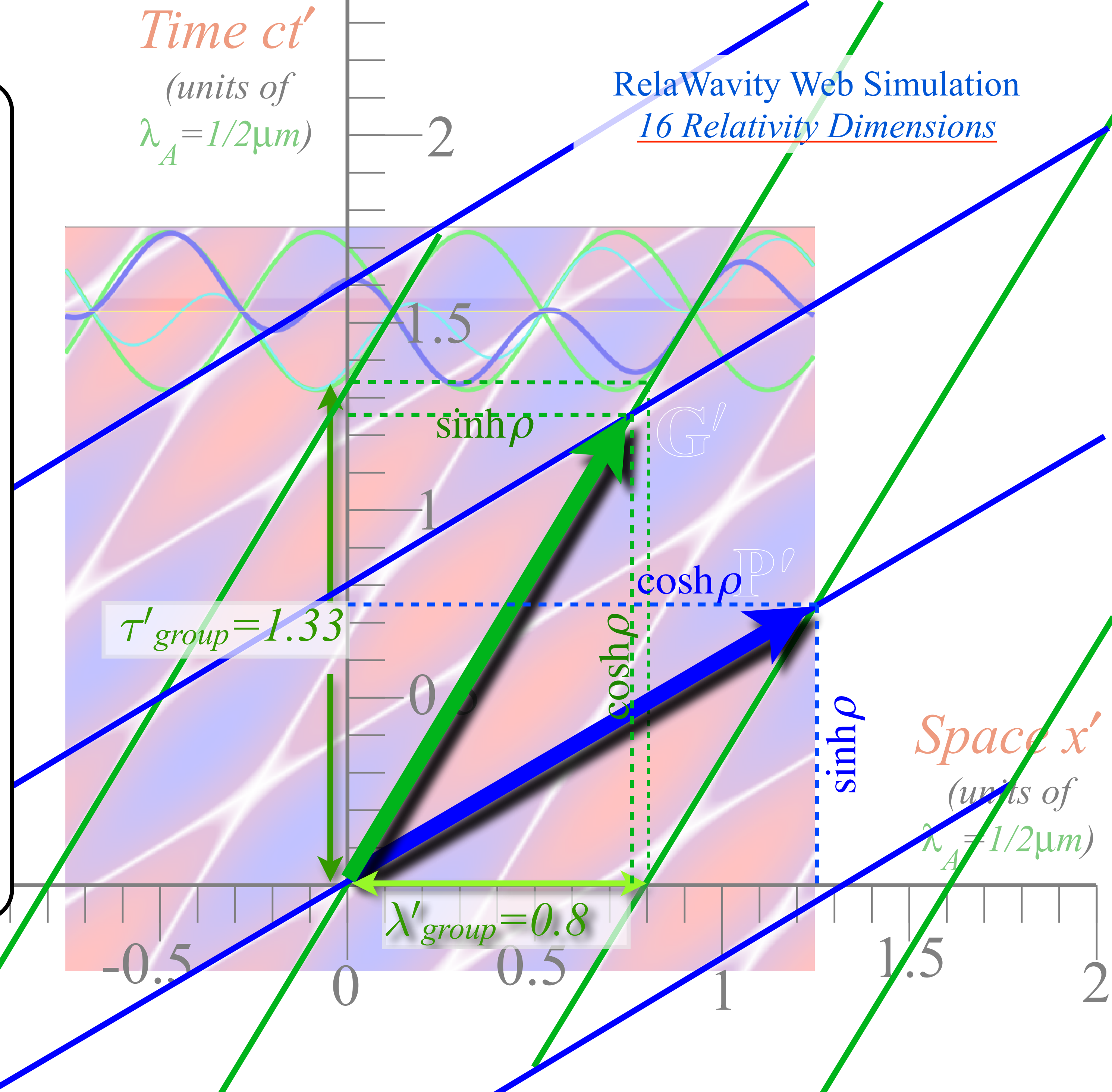
$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

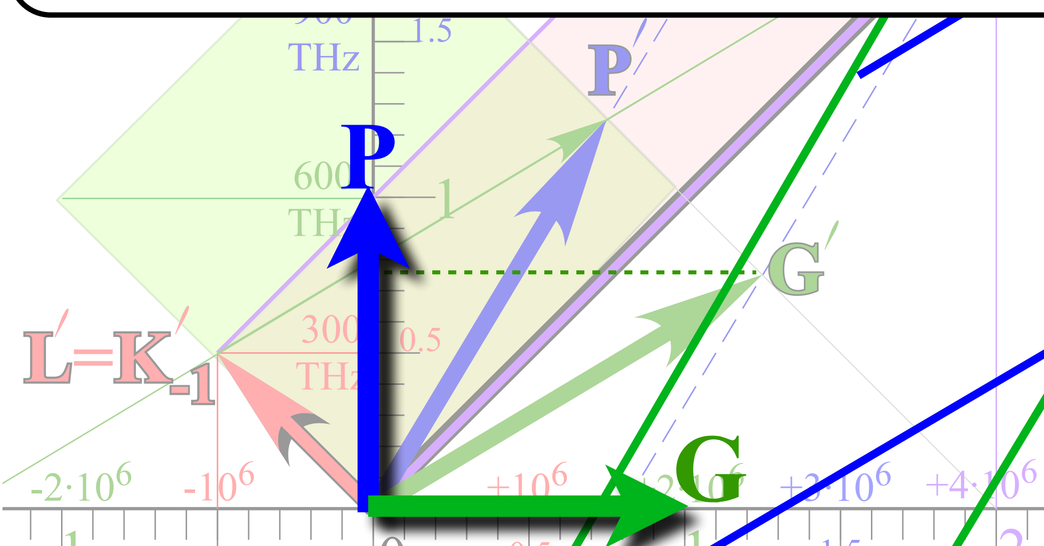
$$\mathbf{G}' = \mathbf{G} \cosh\rho + \mathbf{P} \sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sinh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cosh\rho$$

$$\mathbf{P}' = \mathbf{G} \sinh\rho + \mathbf{P} \cosh\rho$$



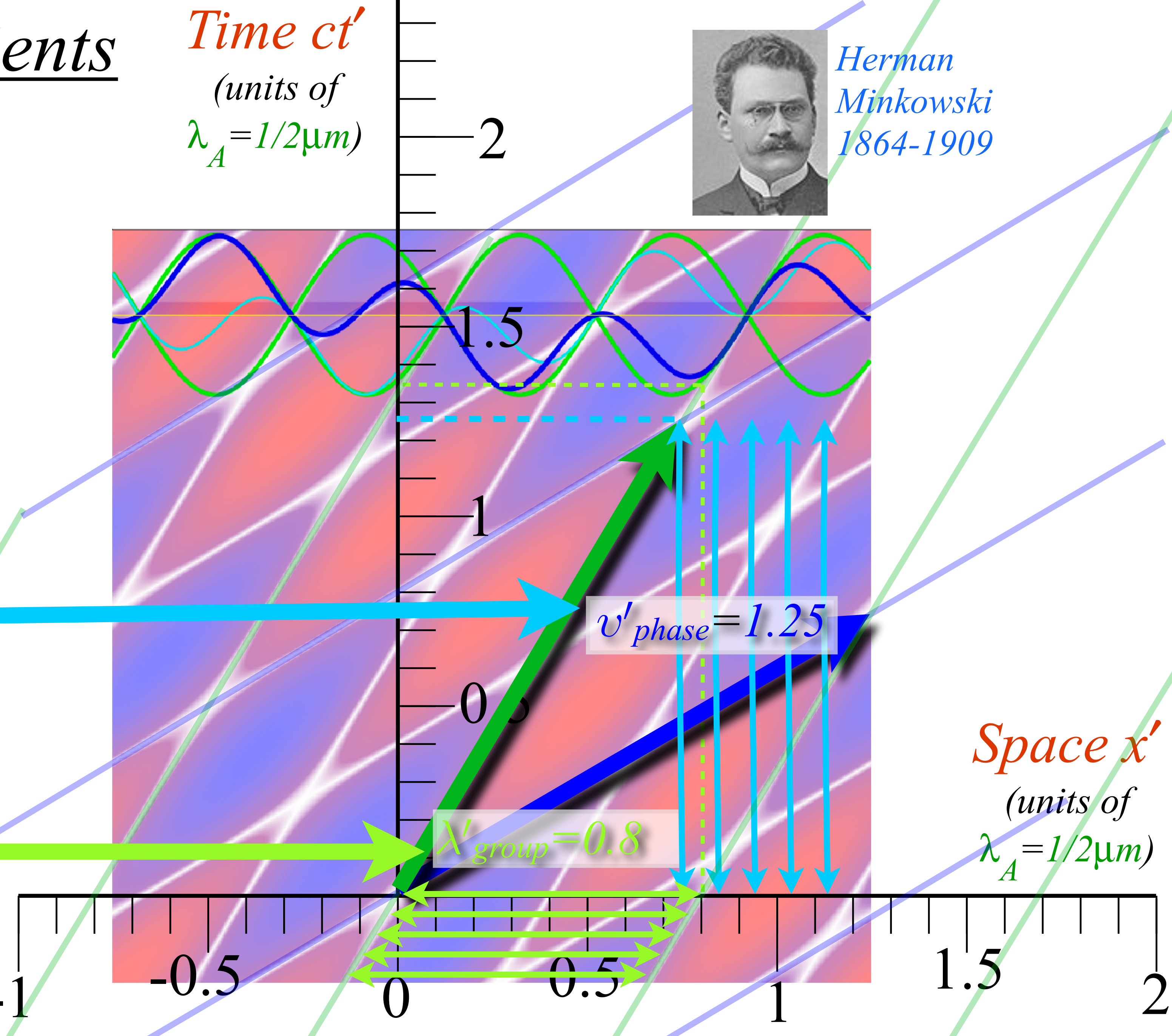


$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$

Lorentz transform matrix

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

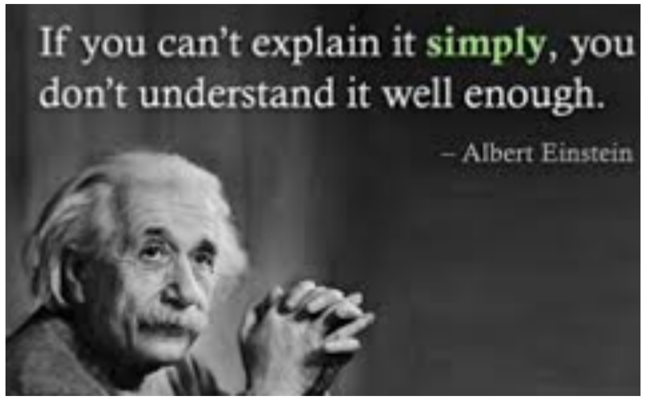
Two Famous-Name Coefficients



This number
is called an: Einstein
time-dilation
(dilated by 25% here)

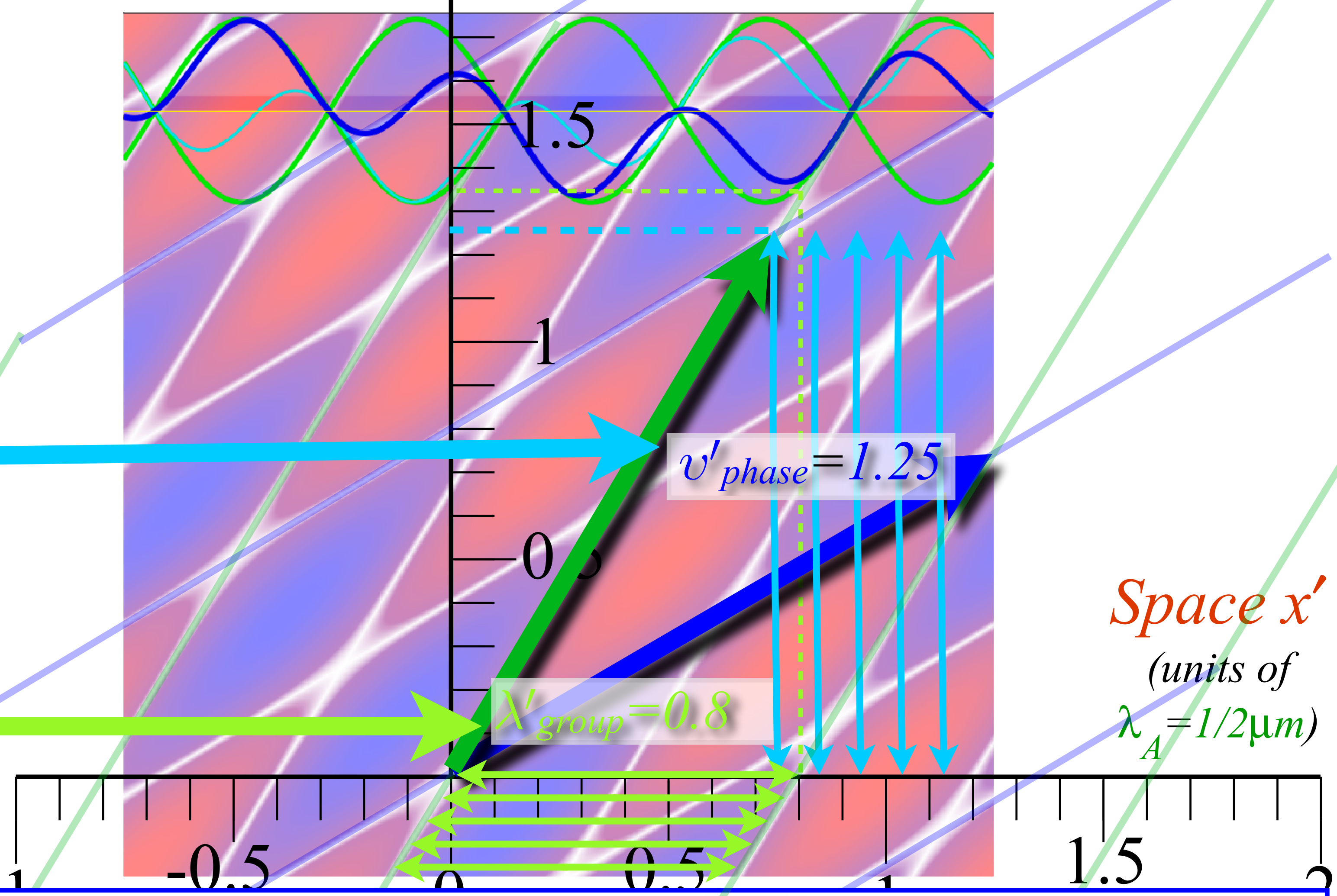
This number
is called a: Lorentz
length-contraction
(contracted by 20% here)

Two Famous-Name Coefficients



Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation** (dilated by 25% here)

This number is called a: **Lorentz length-contraction** (contracted by 20% here)

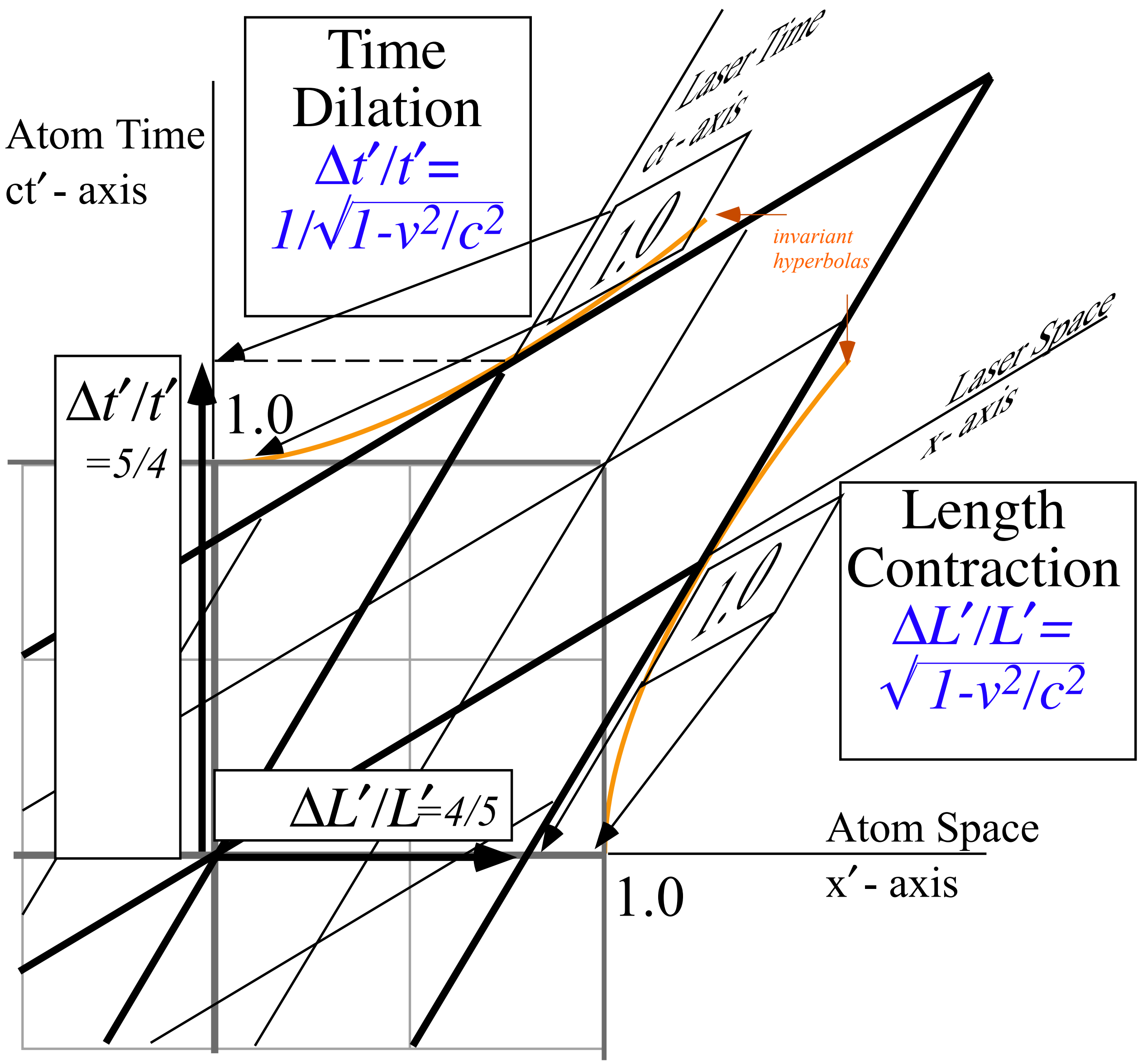
Hendrik A. Lorentz 1853-1928

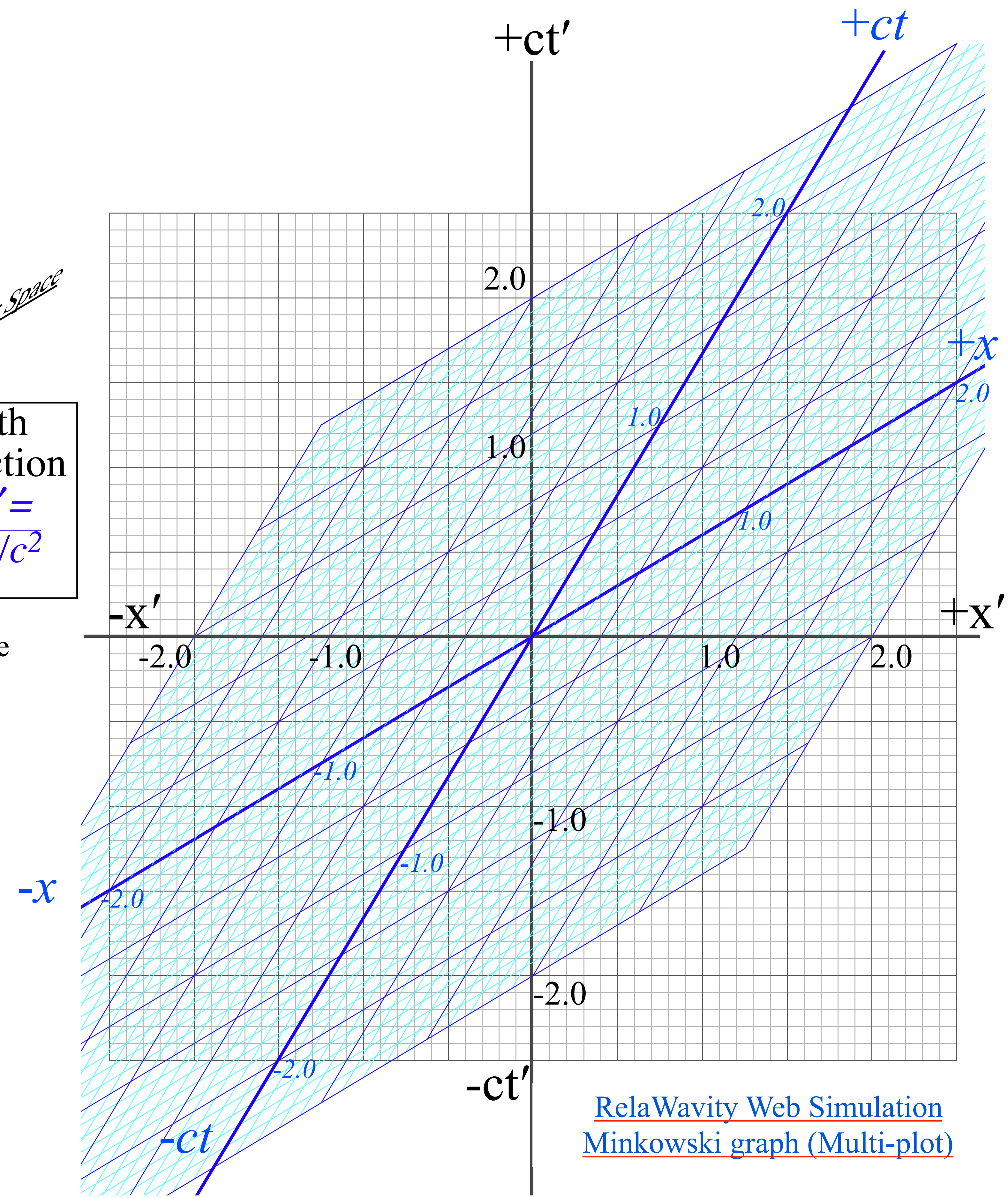
Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

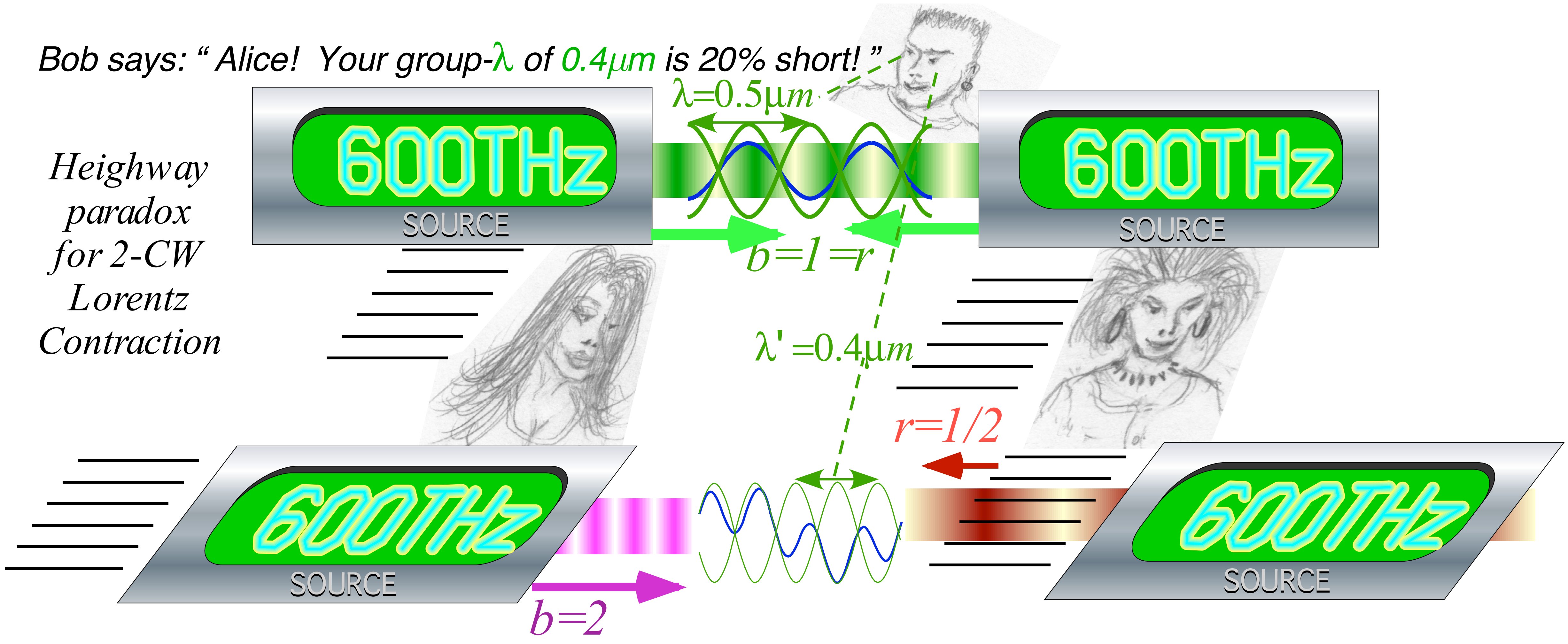
RelaWavity Web Simulation
Relativistic Terms (Expanded Table)

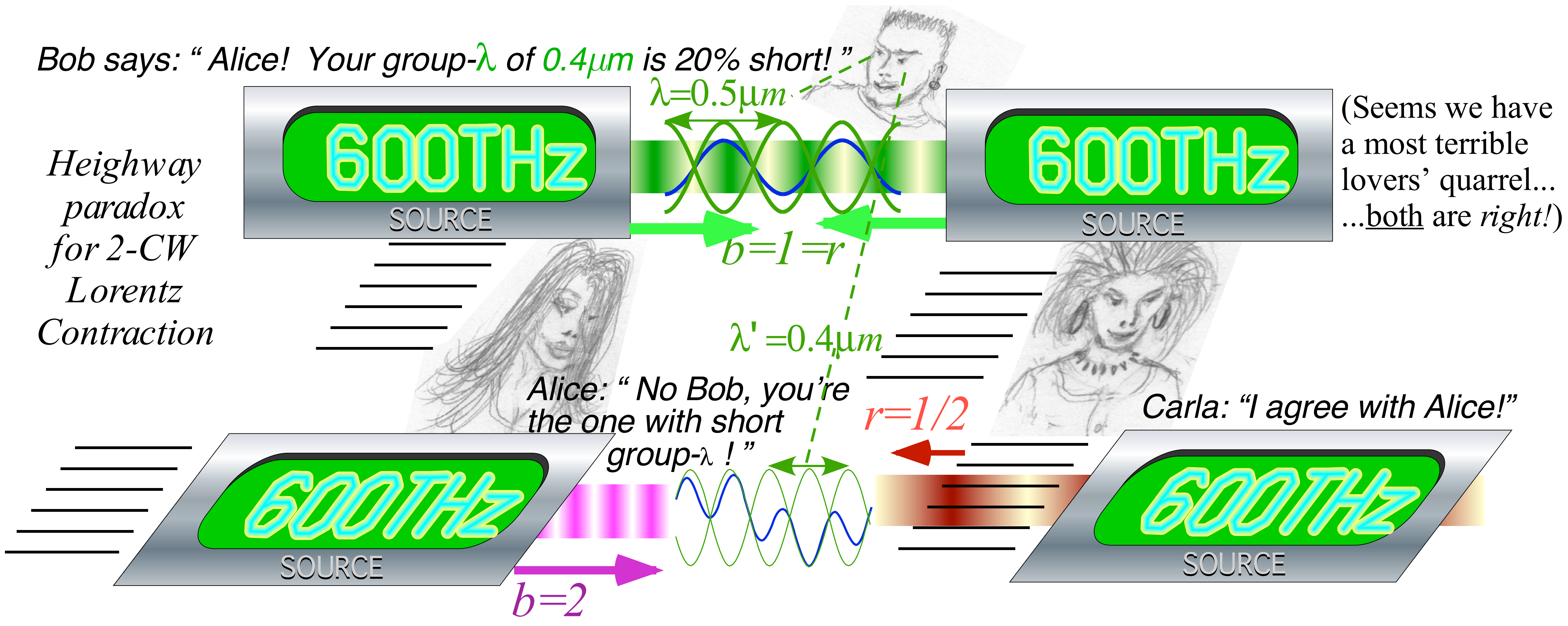
Reading Minkowski graph plots



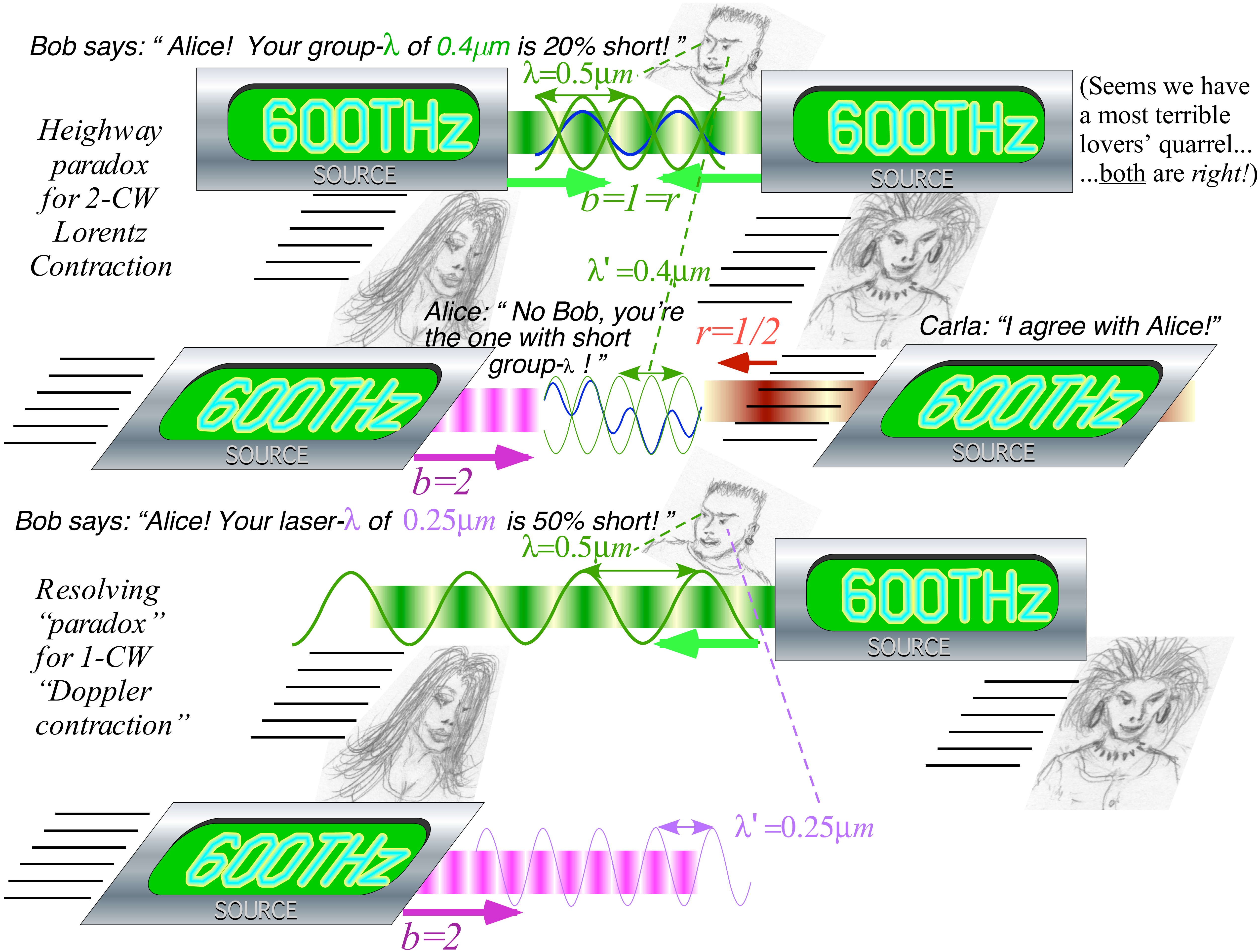


[RelaWavity Web Simulation
Minkowski graph (Multi-plot)]


Bob says: " Alice! Your group-λ of 0.4μm is 20% short! "
Heighway paradox for 2-CW Lorentz Contraction
λ=0.5μm
600THz
SOURCE
600THz
SOURCE
b=1=r
λ' =0.4μm
r=1/2
600THz
SOURCE
600THz
SOURCE
b=2



Bob says: “ Alice! Your group-λ of 0.4μm is 20% short! ”
λ=0.5μm
Heighway paradox for 2-CW Lorentz Contraction
600THz
SOURCE
600THz
SOURCE
(Seems we have a most terrible lovers’ quarrel... ...both are right!)
b=1=r
λ' =0.4μm
Alice: “ No Bob, you’re the one with short group-λ ! ”
r=1/2
Carla: “I agree with Alice!”
600THz
SOURCE
600THz
SOURCE
b=2



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600THz
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b=1=r
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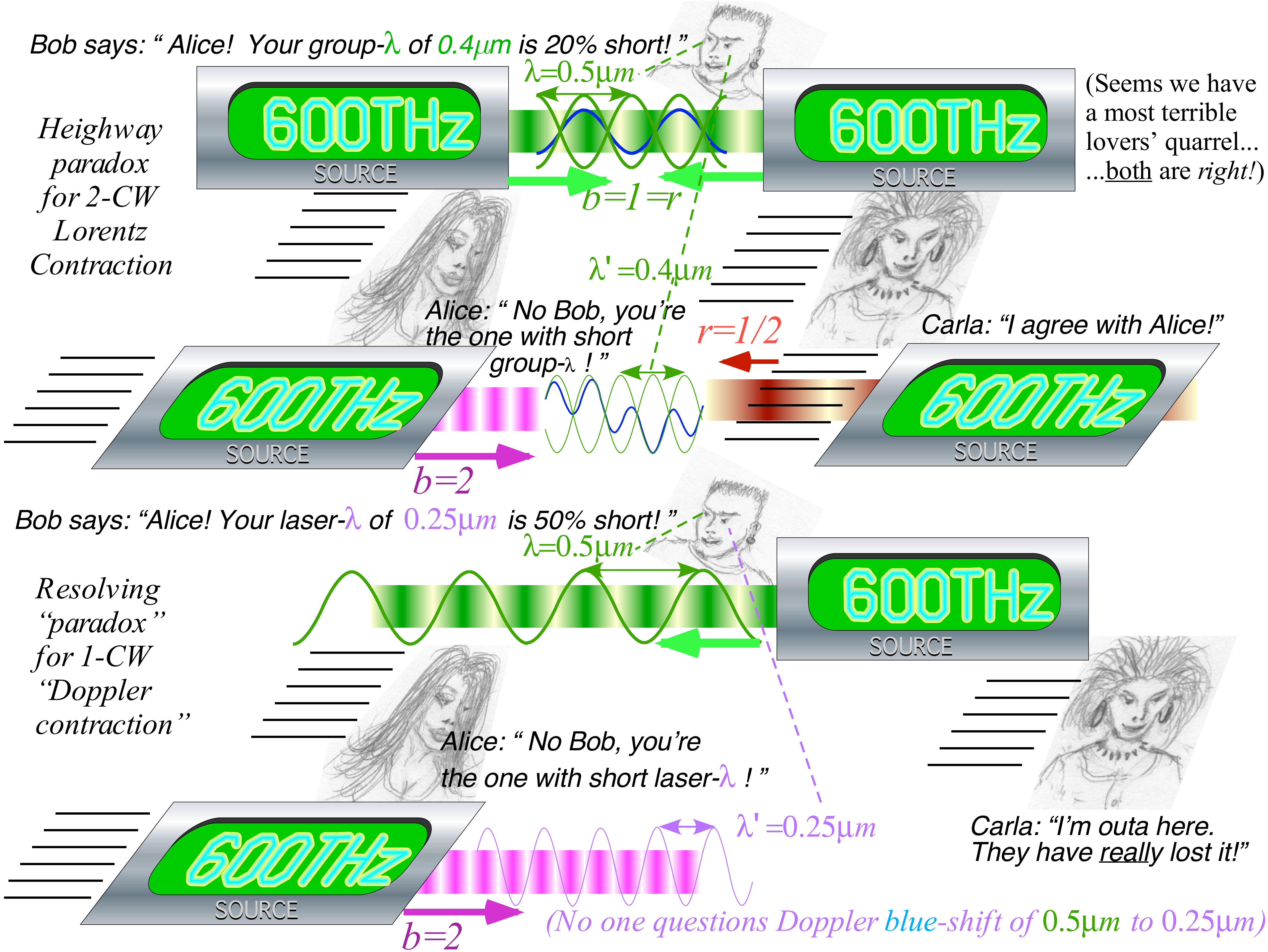
Alice: " No Bob, you're the one with short group-λ ! "
r=1/2
Carla: "I agree with Alice!"
600THz
SOURCE
600THz
SOURCE
b=2
Bob says: "Alice! Your laser-λ of 0.25μm is 50% short! "
λ=0.5μm
600THz
SOURCE
Resolving "paradox" for 1-CW "Doppler contraction"
λ'=0.25μm
600THz
SOURCE
b=2



Bob says: " Alice! Your group-λ of 0.4μm is 20% short! "
λ=0.5μm
600THz
SOURCE
600THz
SOURCE
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Heighway paradox for 2-CW Lorentz Contraction
b=1=r
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600THz
SOURCE
600THz
SOURCE
b=2
Bob says: "Alice! Your laser-λ of 0.25μm is 50% short! "
λ=0.5μm
600THz
SOURCE
Resolving "paradox" for 1-CW "Doppler contraction"
Alice: " No Bob, you're the one with short laser-λ ! "
λ'=0.25μm
Carla: "I'm outa here. They have really lost it!"
600THz
SOURCE
b=2
(No one questions Doppler blue-shift of 0.5μm to 0.25μm)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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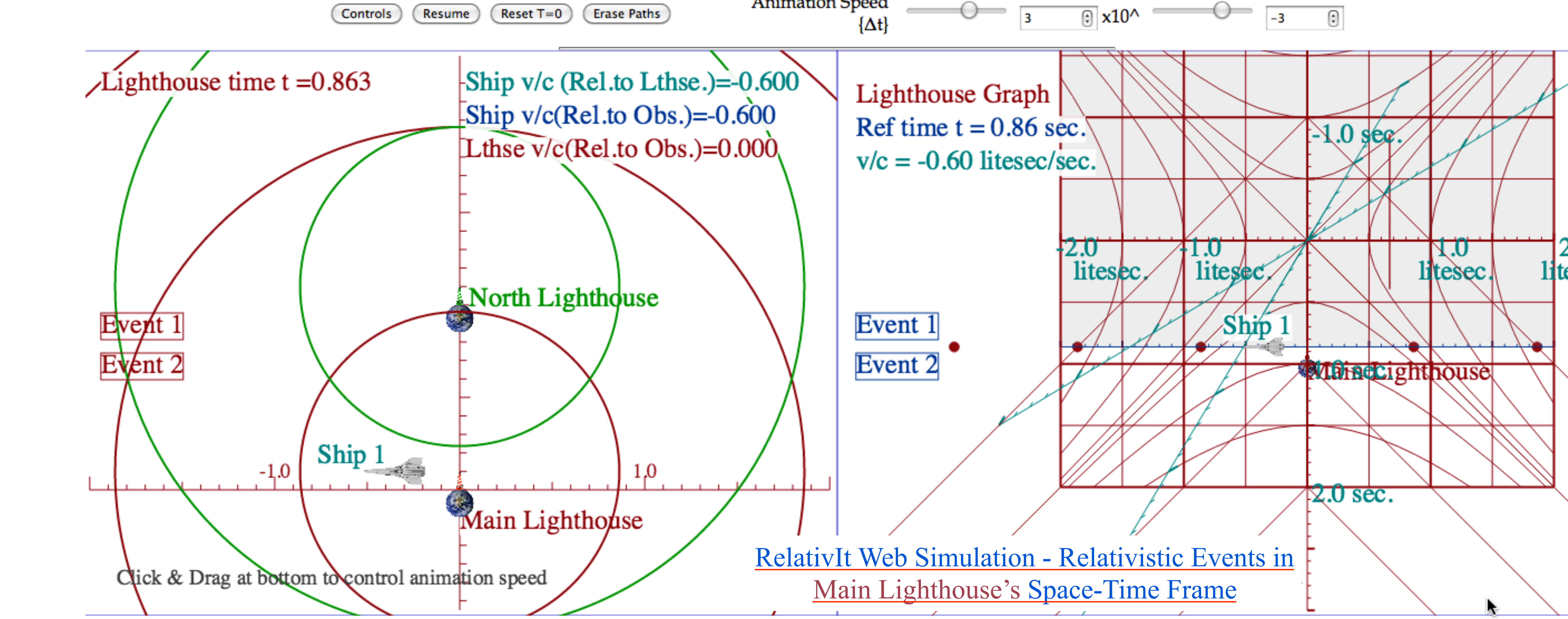
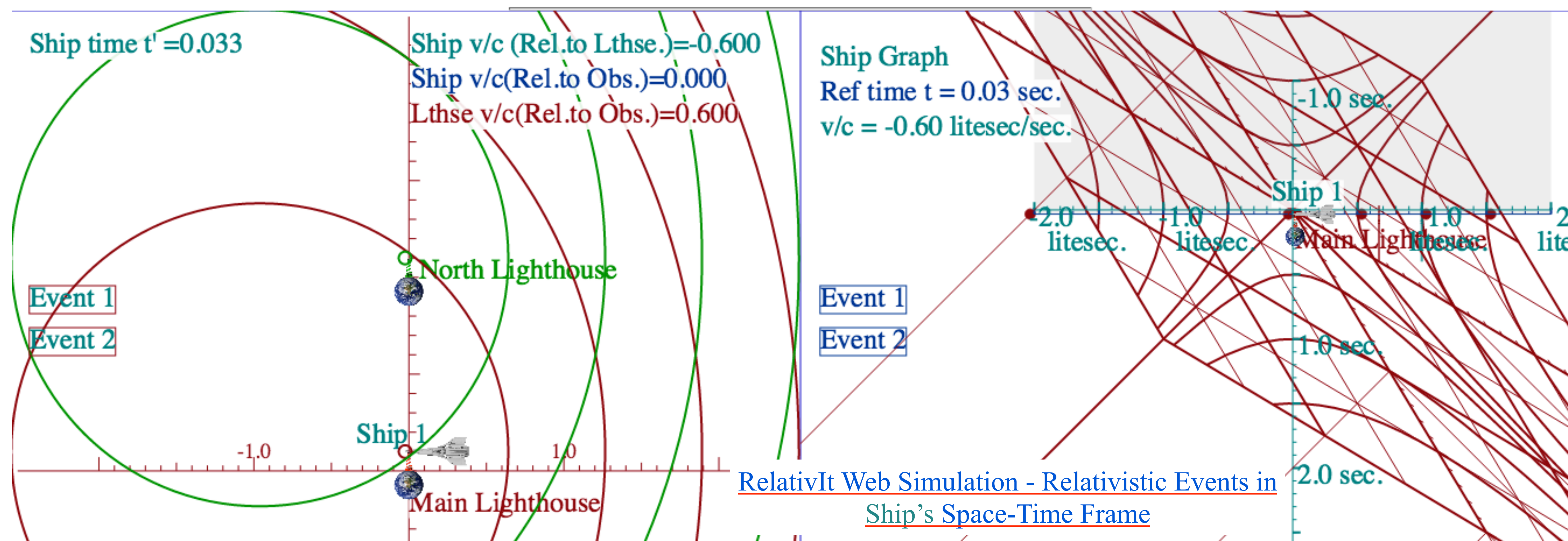
effects	$b_{RED}^{Doppler}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*$^{(Lorentz)}$ τ_{phase}*-contraction*	*t-dilation*$^{(Einstein)}$ υ_{phase}*-dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b_{BLUE}^{Doppler}$

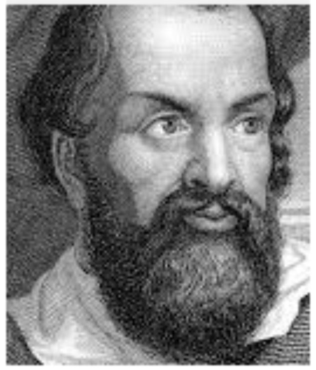


RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame



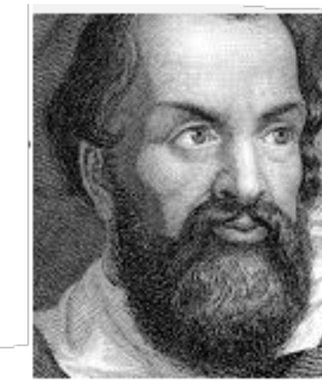
RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
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Optical interference "baseball-diamond" displays *phase* and *group* velocity
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Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Doppler Jeopardy

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(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

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Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$\frac{300}{900}$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

$$V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

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Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

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$$\sqrt{6\cdot 3} = 3\sqrt{2} = 4.24$$

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

Geometric mean

$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

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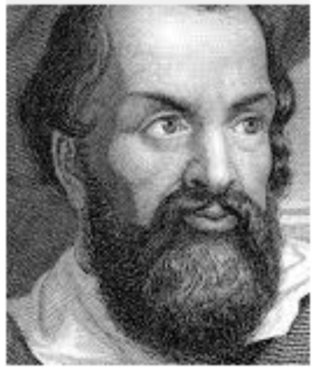
$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

V_{group}/c is ratio of difference mean $\omega_{group} = \frac{\omega_R - \omega_L}{2}$ to arithmetic mean $\omega_{phase} = \frac{\omega_R + \omega_L}{2}$. Frequency $\omega_E = B$ is the geometric mean $\sqrt{\omega_R \cdot \omega_L}$ of left and right-moving frequencies defining the geometry

Geometric mean

Galileo Galilei



1564-1642

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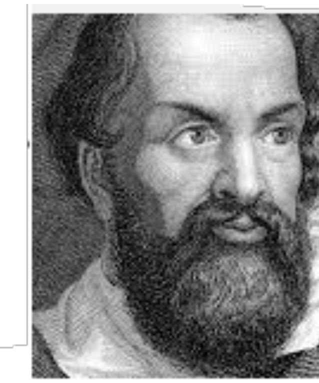

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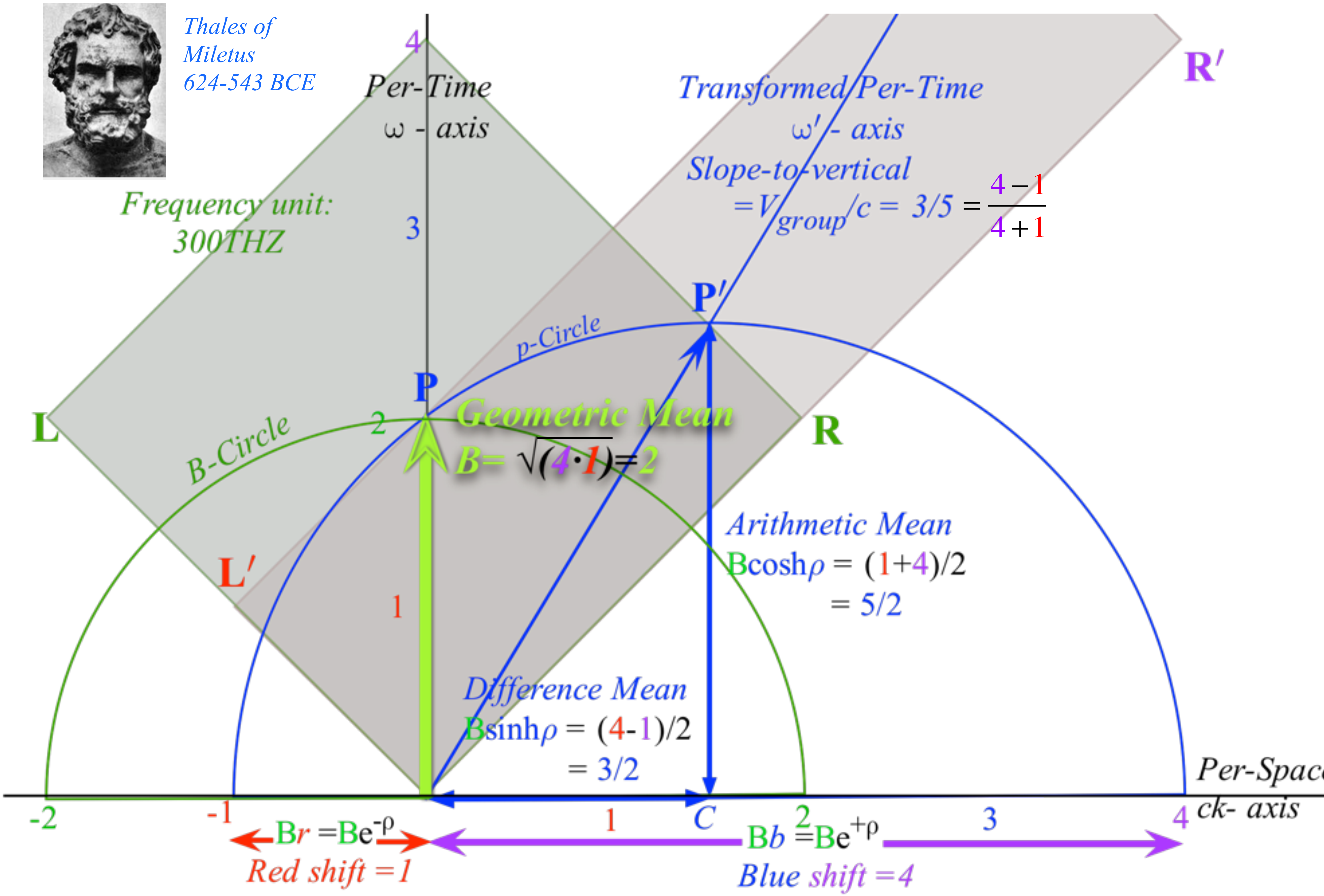
Thales Mean Geometry (600BCE)

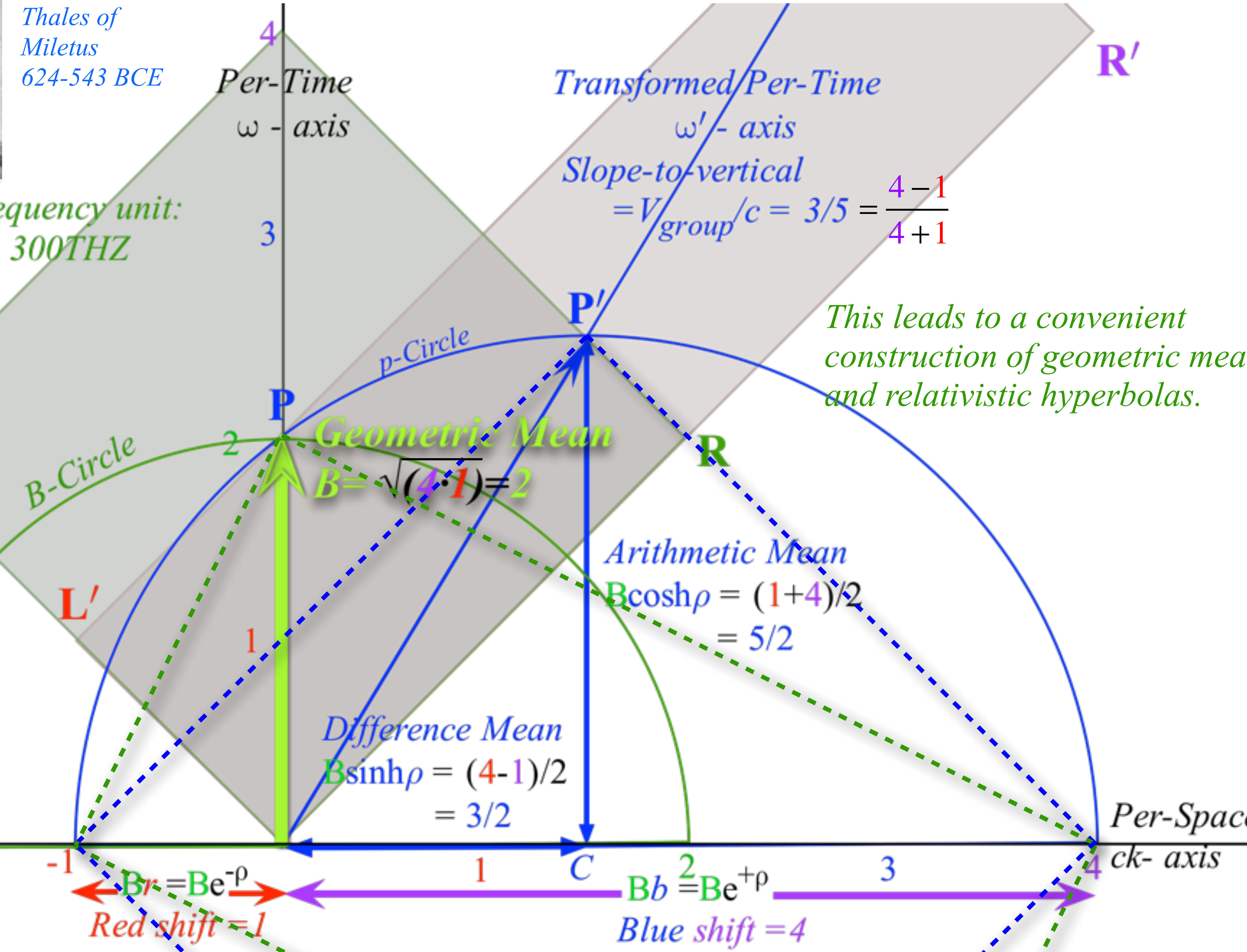
helps "Relawavity"



Thales Mean Geometry (600BCE)

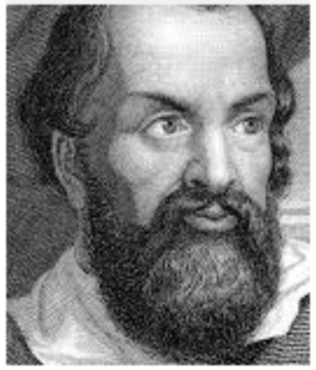
helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*

Thales of Miletus 624-543 BCE



This leads to a convenient construction of geometric means and relativistic hyperbolas.

Galileo Galilei



1564-1642

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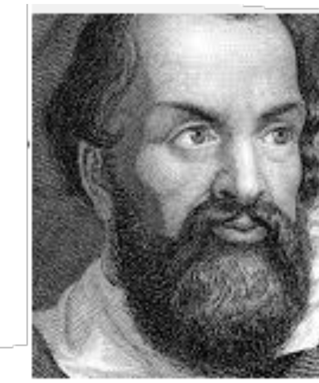

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Thales geometry of Lorentz transformation → ...and invariant hyperbolas ←

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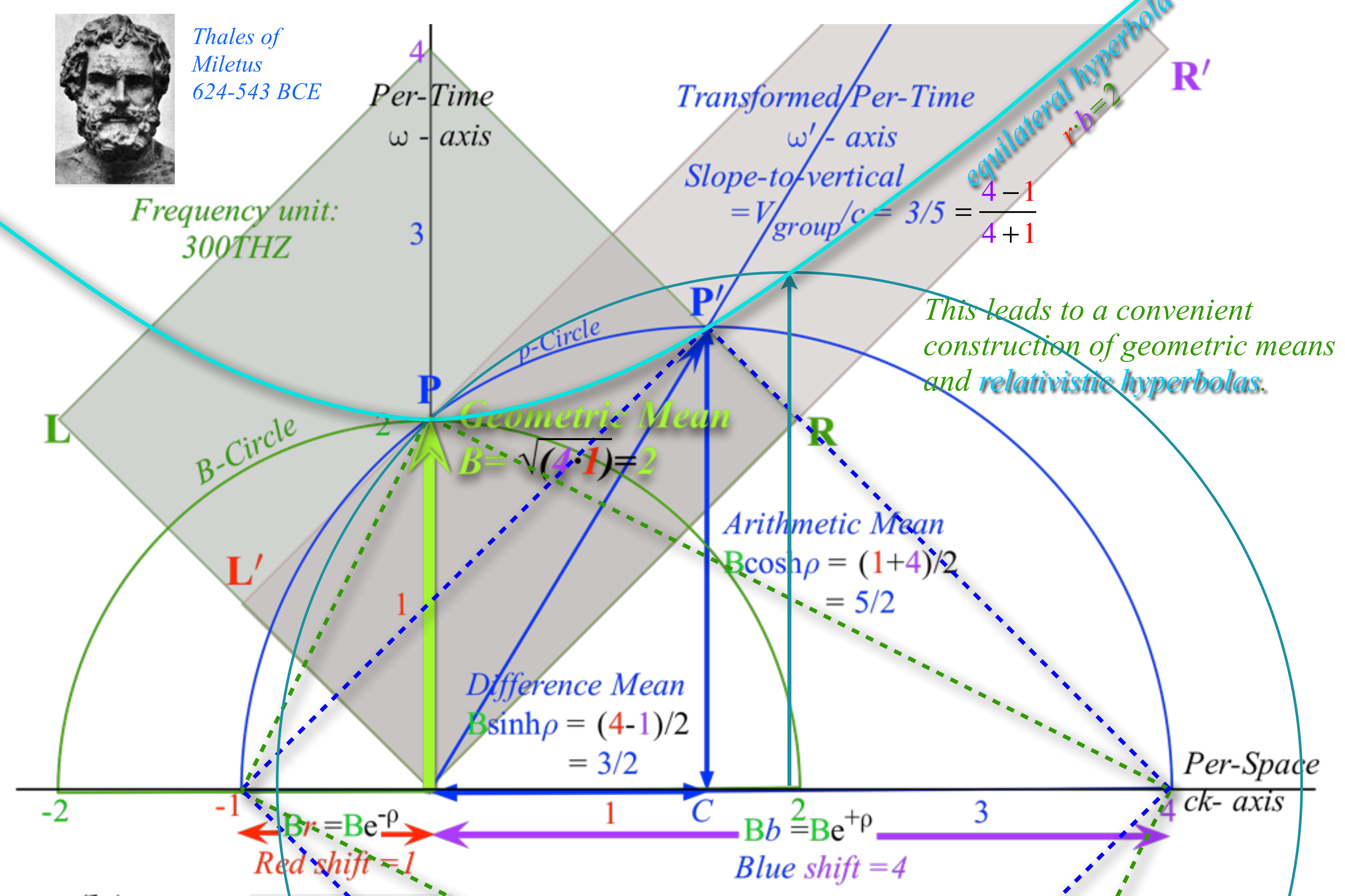

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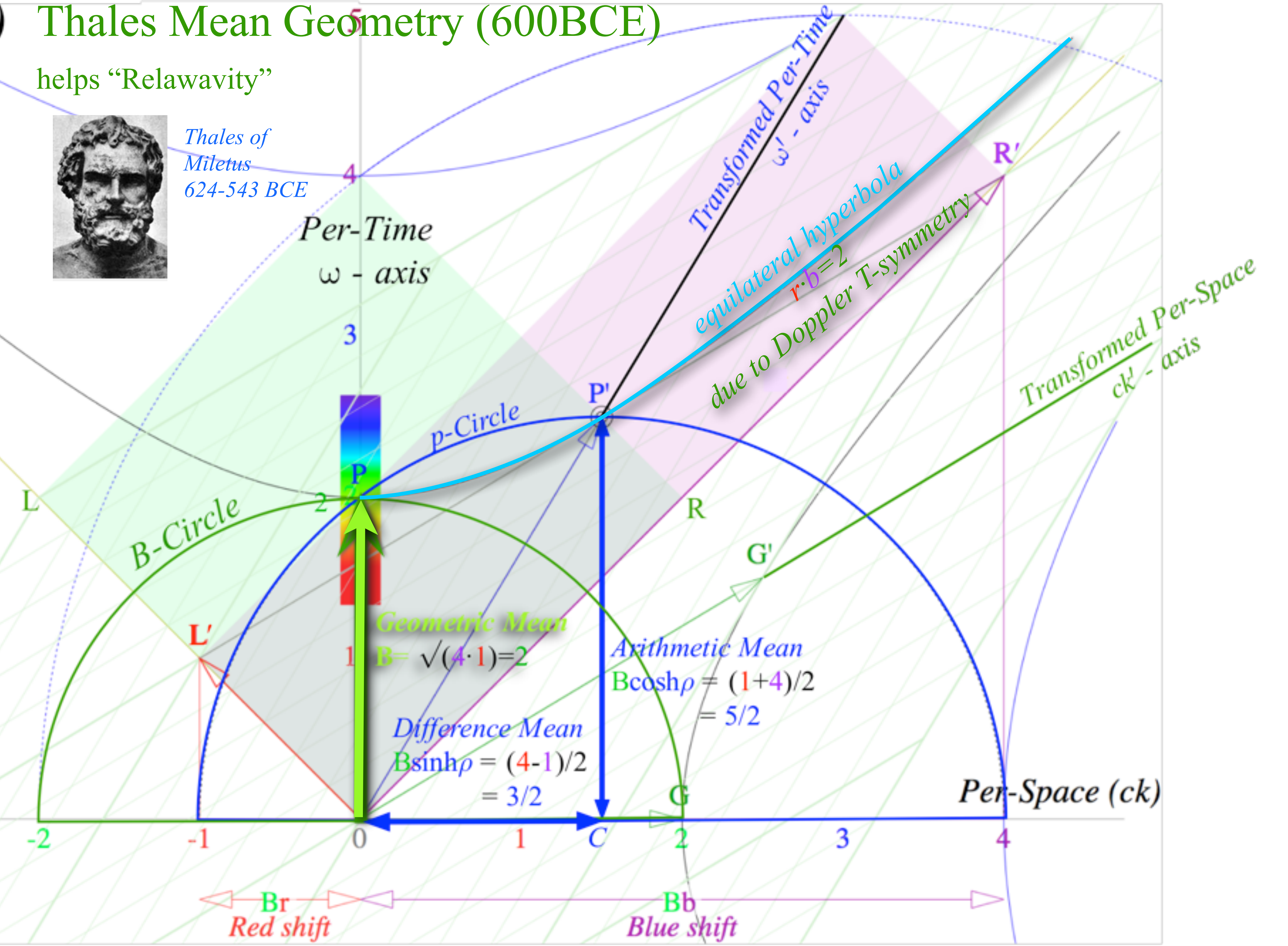
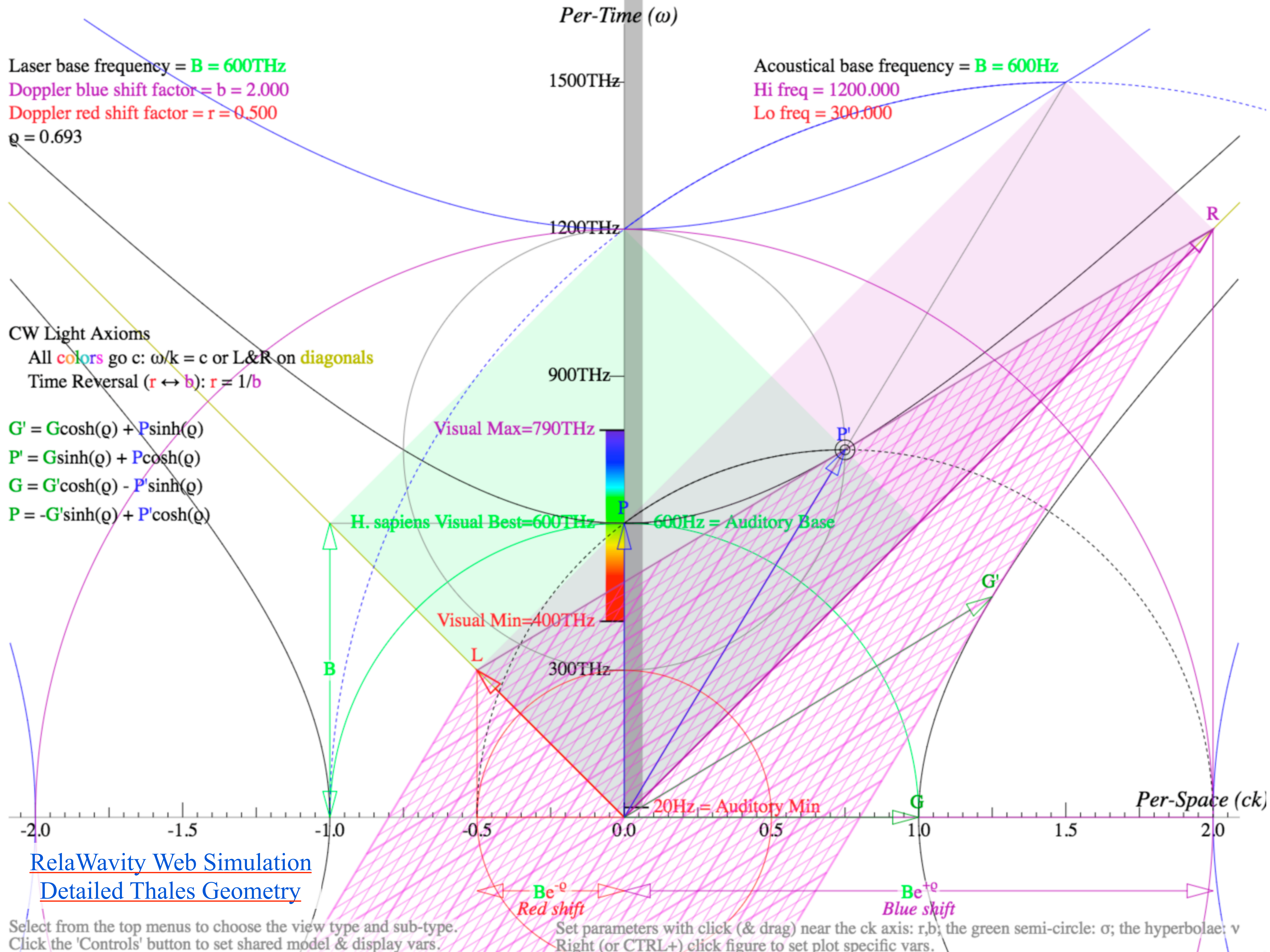
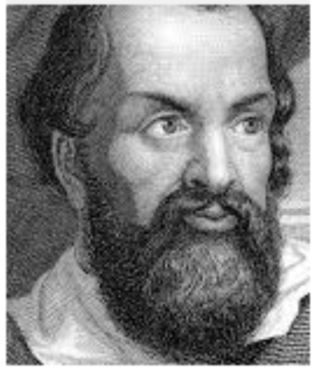




Per-Time (ω)
Laser base frequency = B = 600THz
Doppler blue shift factor = b = 2.000
Doppler red shift factor = r = 0.500
ϱ = 0.693
Acoustical base frequency = B = 600Hz
Hi freq = 1200.000
Lo freq = 300.000
1500THz
1200THz
900THz
300THz
CW Light Axioms
All colors go c: ω/k = c or L&R on diagonals
Time Reversal (r ↔ b): r = 1/b
G' = Gcosh(ϱ) + Psinh(ϱ)
P' = Gsinh(ϱ) + Pcosh(ϱ)
G = G'cosh(ϱ) - P'sinh(ϱ)
P = -G'sinh(ϱ) + P'cosh(ϱ)
Visual Max=790THz
H. sapiens Visual Best=600THz
600Hz = Auditory Base
Visual Min=400THz
20Hz = Auditory Min
R
P'
P
G'
G
L
B
Per-Space (ck)
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
Be-ϱ
Red shift
Be+ϱ
Blue shift
RelaWavity Web Simulation
Detailed Thales Geometry
Select from the top menus to choose the view type and sub-type.
Click the 'Controls' button to set shared model & display vars.
Set parameters with click (& drag) near the ck axis: r,b; the green semi-circle: σ; the hyperbolae: v
Right (or CTRL+) click figure to set plot specific vars.

Galileo Galilei



1564-1642

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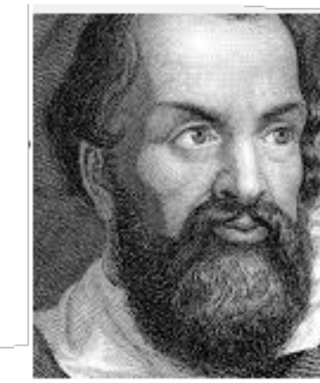

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Application to TE-Waveguide modes and synchrotron beam relativity

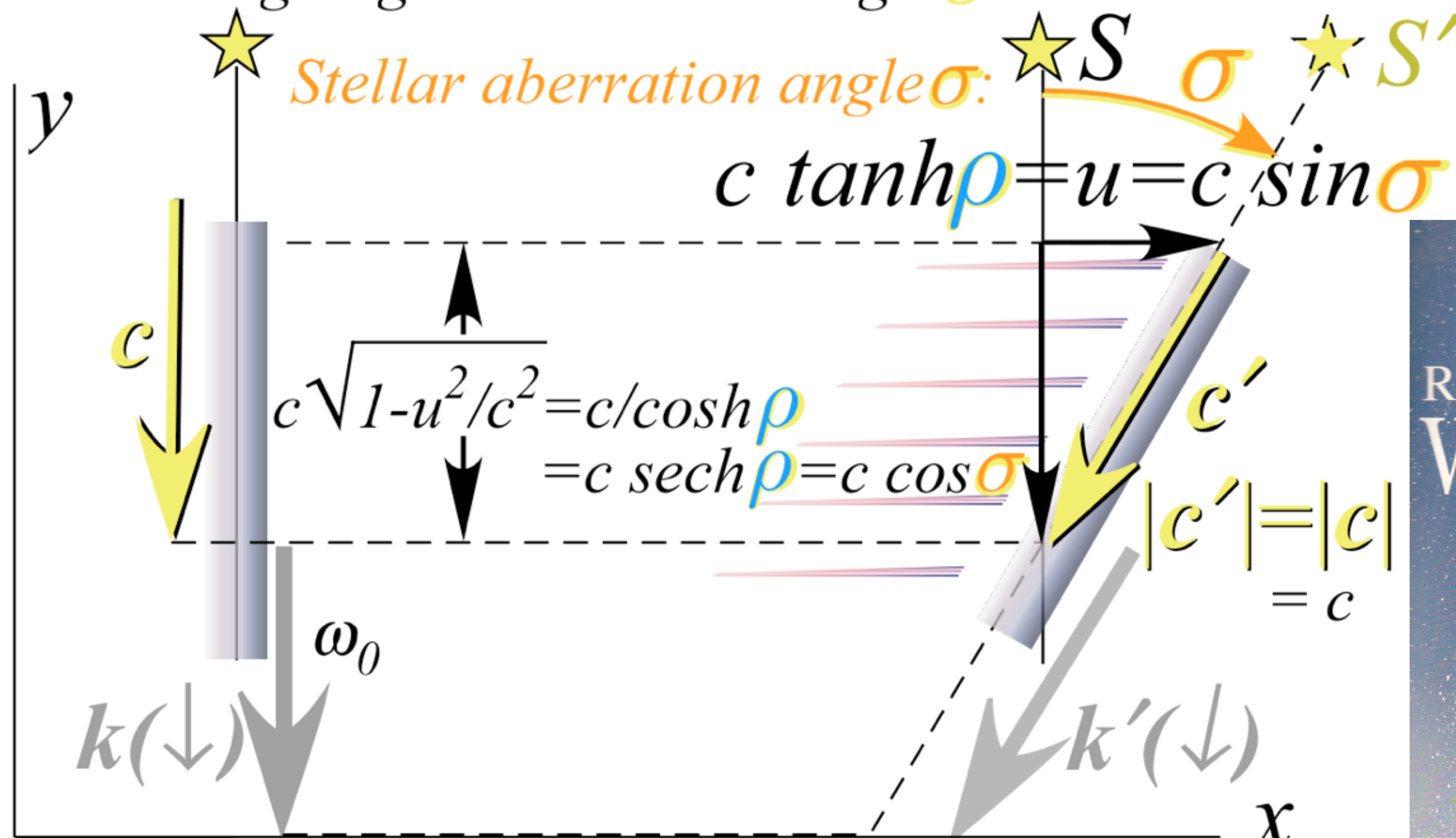
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

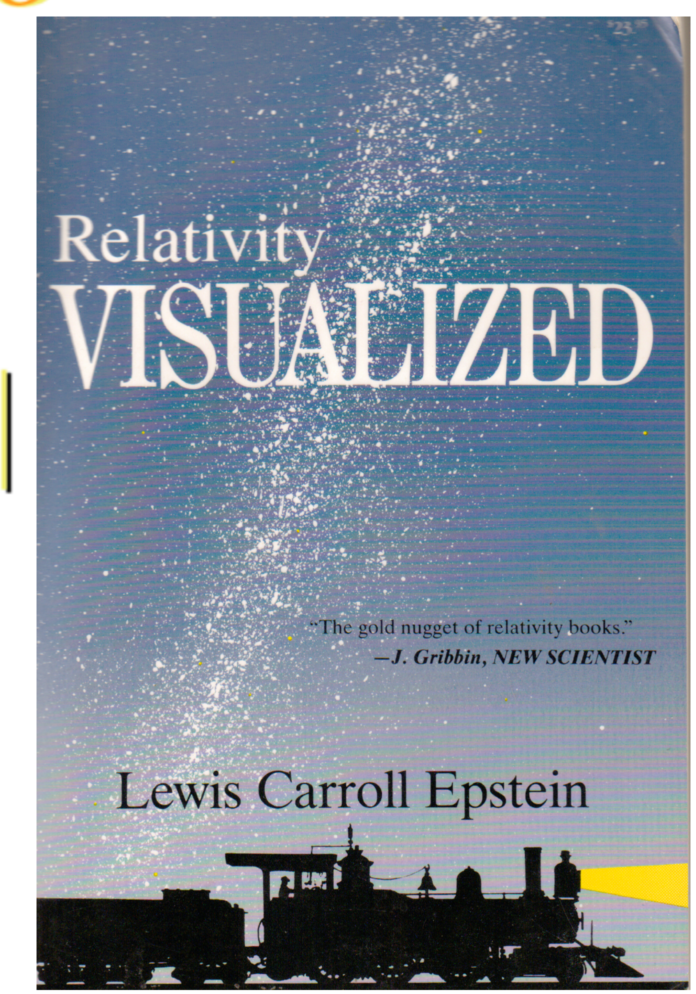
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

We used notation σ for stellar-ab-angle, (a "flipped-out" ρ).
Epstein not interested in ρ analysis or in relation of σ and ρ.

Observer fixed below star sees it directly overhead.
*Observer going **u** sees star at angle σ in **u** direction.*





Purchase at: AllBookstores.com amazon

Details of stellar aberration angle σ of K-vexctor rotation

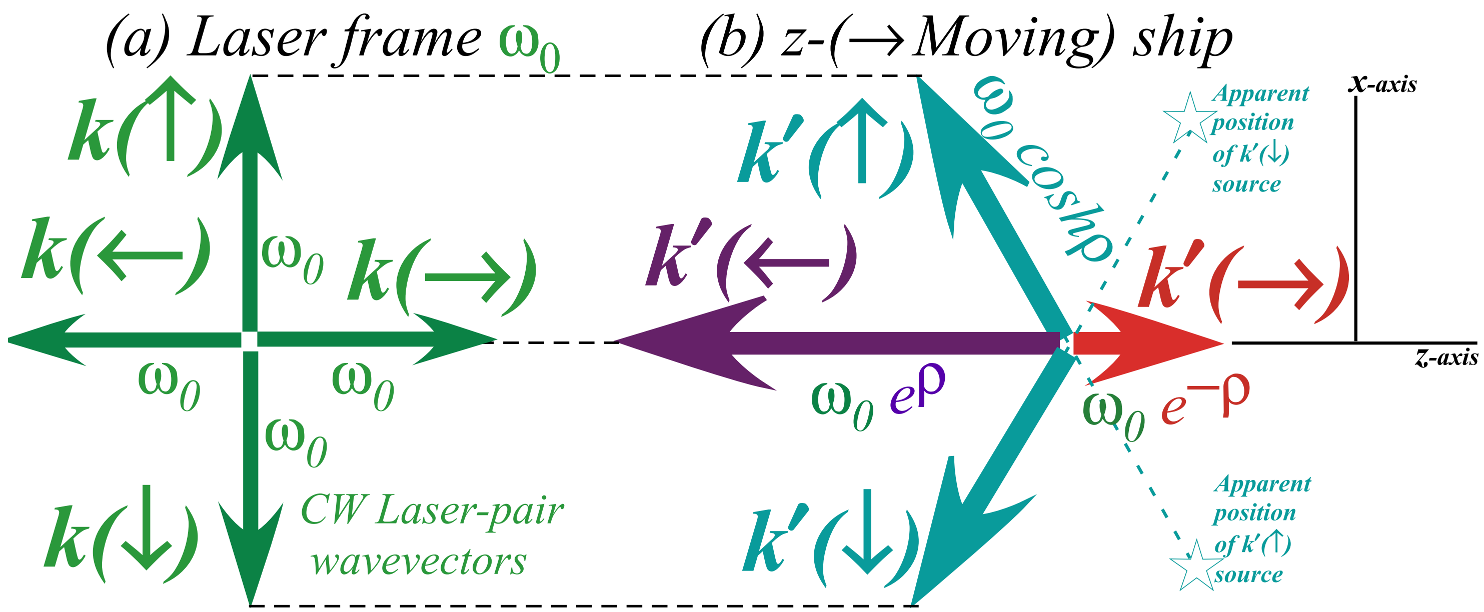


Fig. 8.5.7
Unit 8

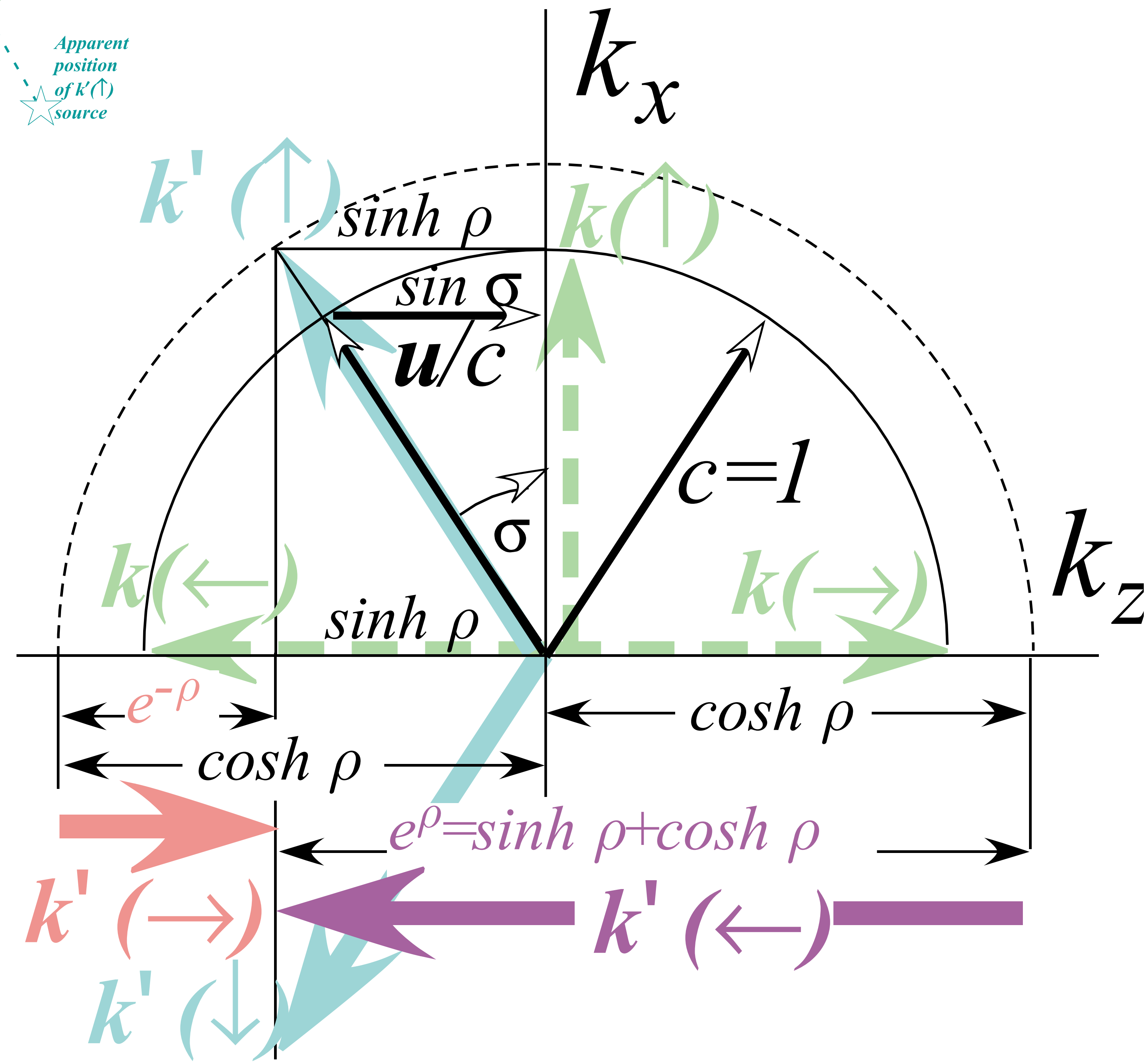


Fig. 8.5.10
Unit 8

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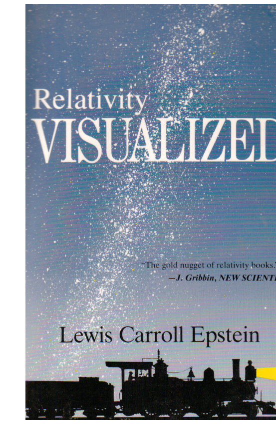
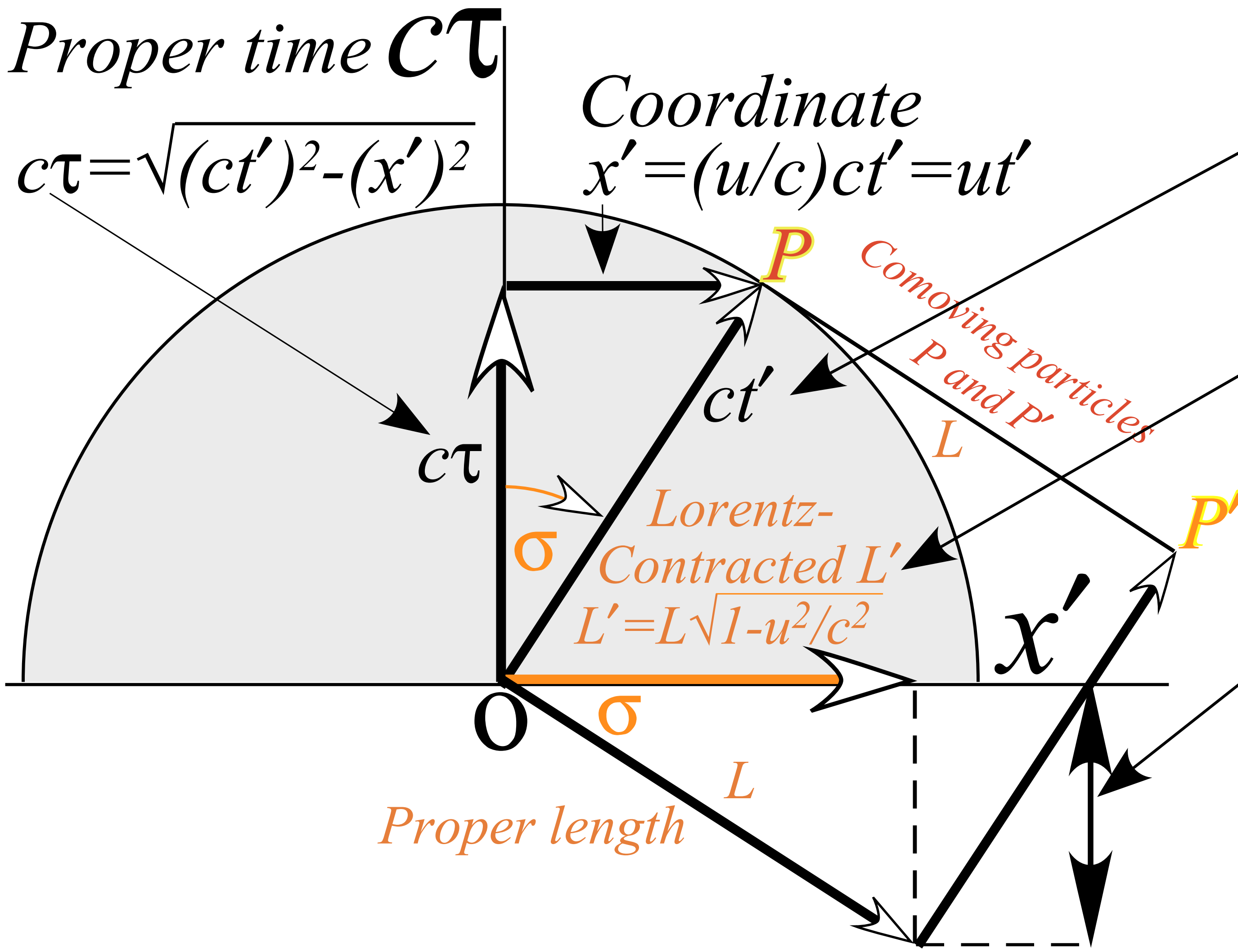


Proper time $c\tau$ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x',ct') and speed c in $(x, c\tau)$



Einstein time dilation:

$$ct'=c\tau\sec\sigma=c\tau\cosh\rho=c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L'=L\,\mathrm{sech}\rho=L\cos\sigma=L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau=L'\sinh\rho=L\cos\sigma\sinh\rho$$

$$=L\cos\sigma\tan\sigma$$

$$=L\sin\sigma\quad=L/\sqrt{c^2/u^2-1}\sim L\,u/c$$

RelativIt Web Simulation
Epstein plot

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

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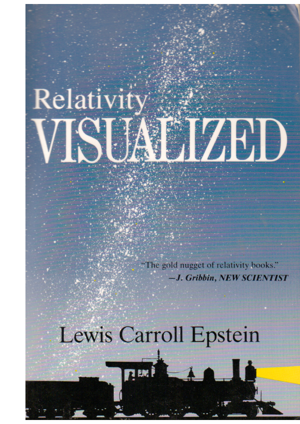

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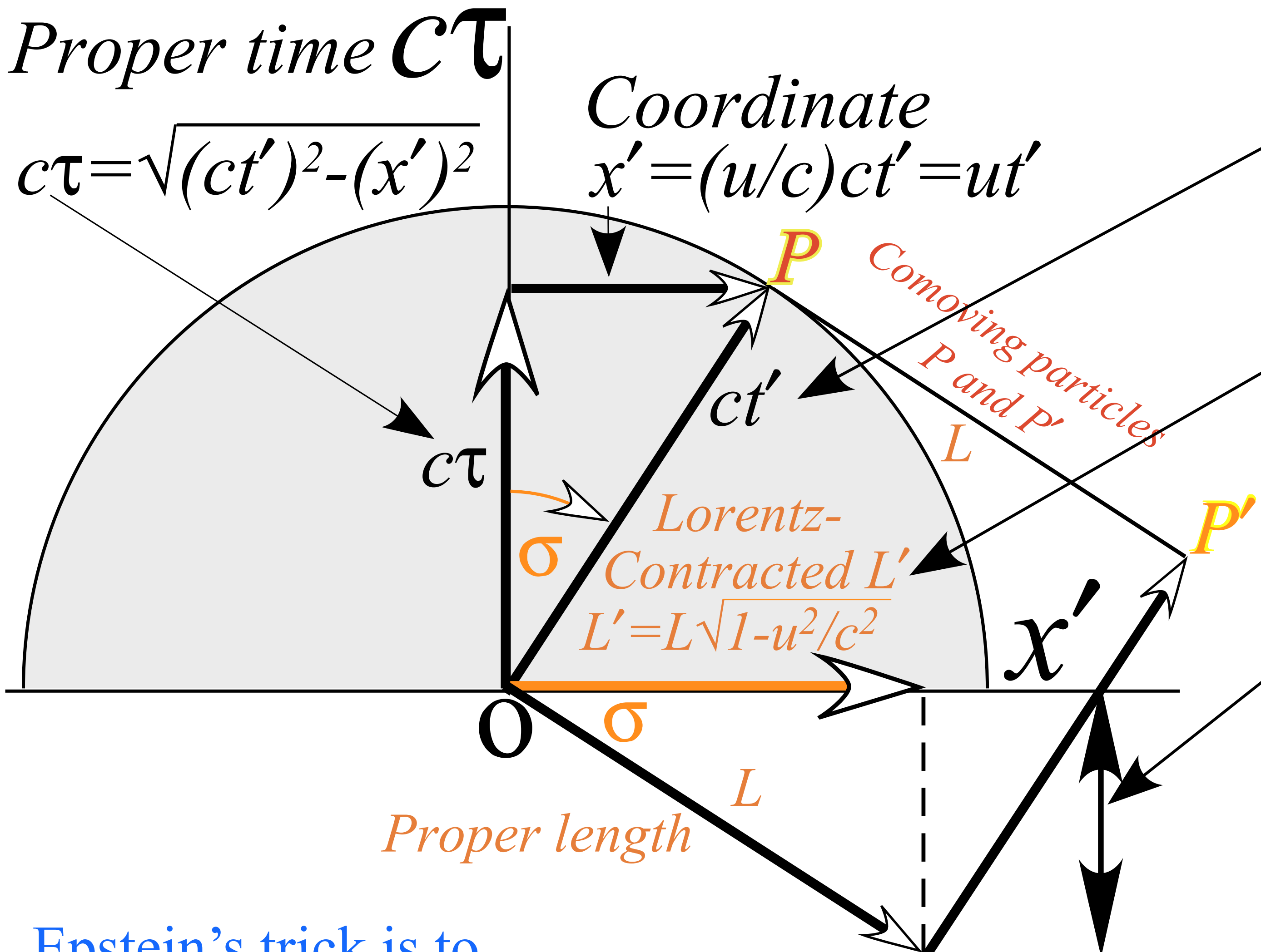

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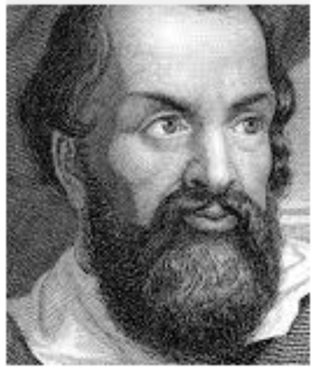
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau=\sqrt{(ct')^2-(x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2+(x')^2}=(ct')$$

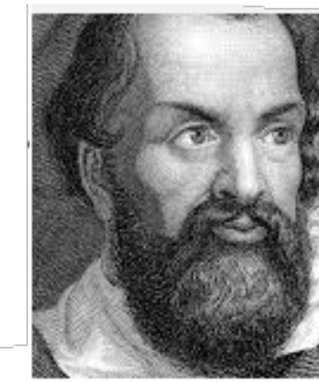
Then everything (and everybody) always goes speed c through $(x',c\tau)$ space!

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

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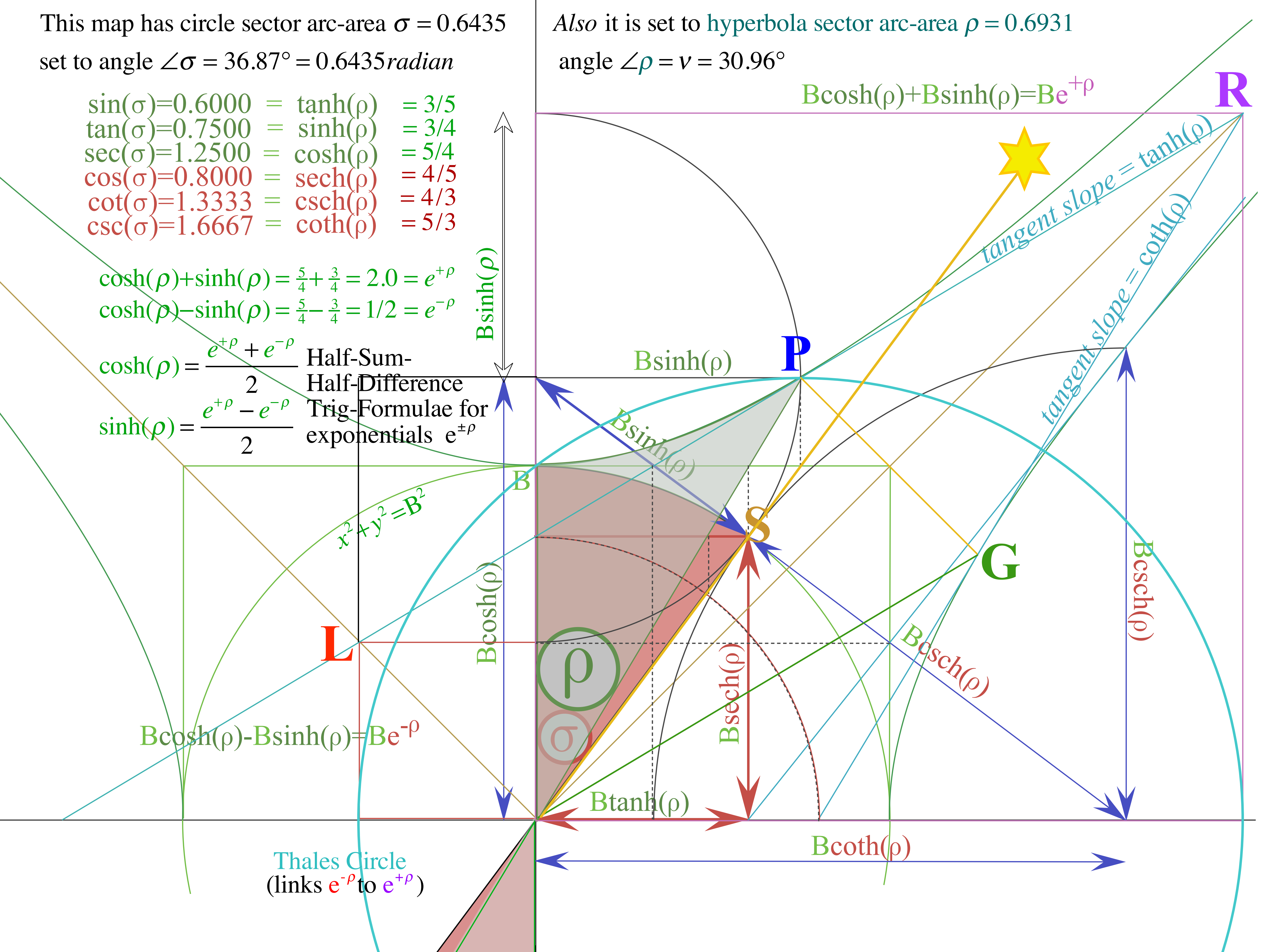
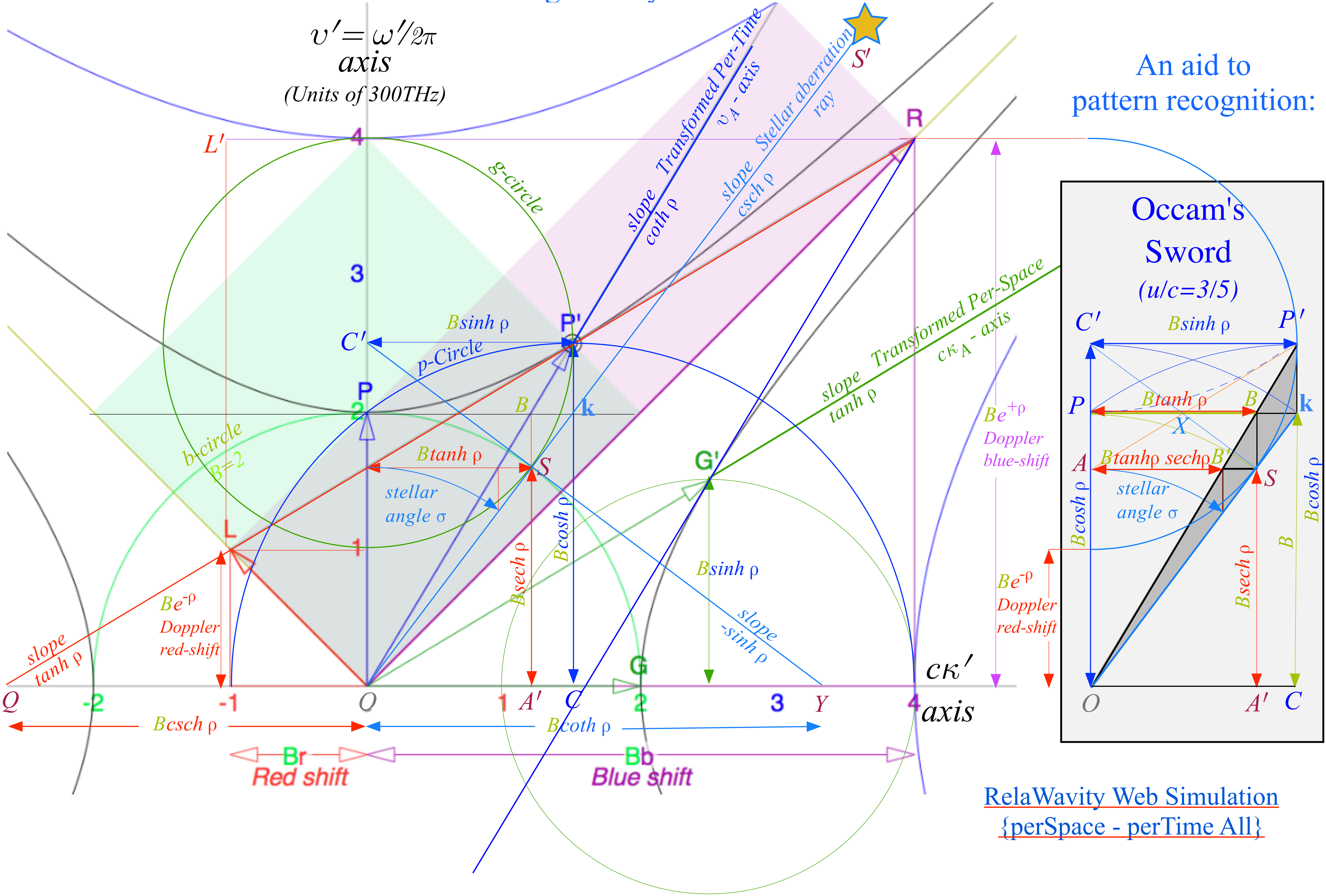
➤ "Occams Sword" and geometry of 16 parameter functions of ρ and σ

Application to TE-Waveguide modes and synchrotron beam relativity

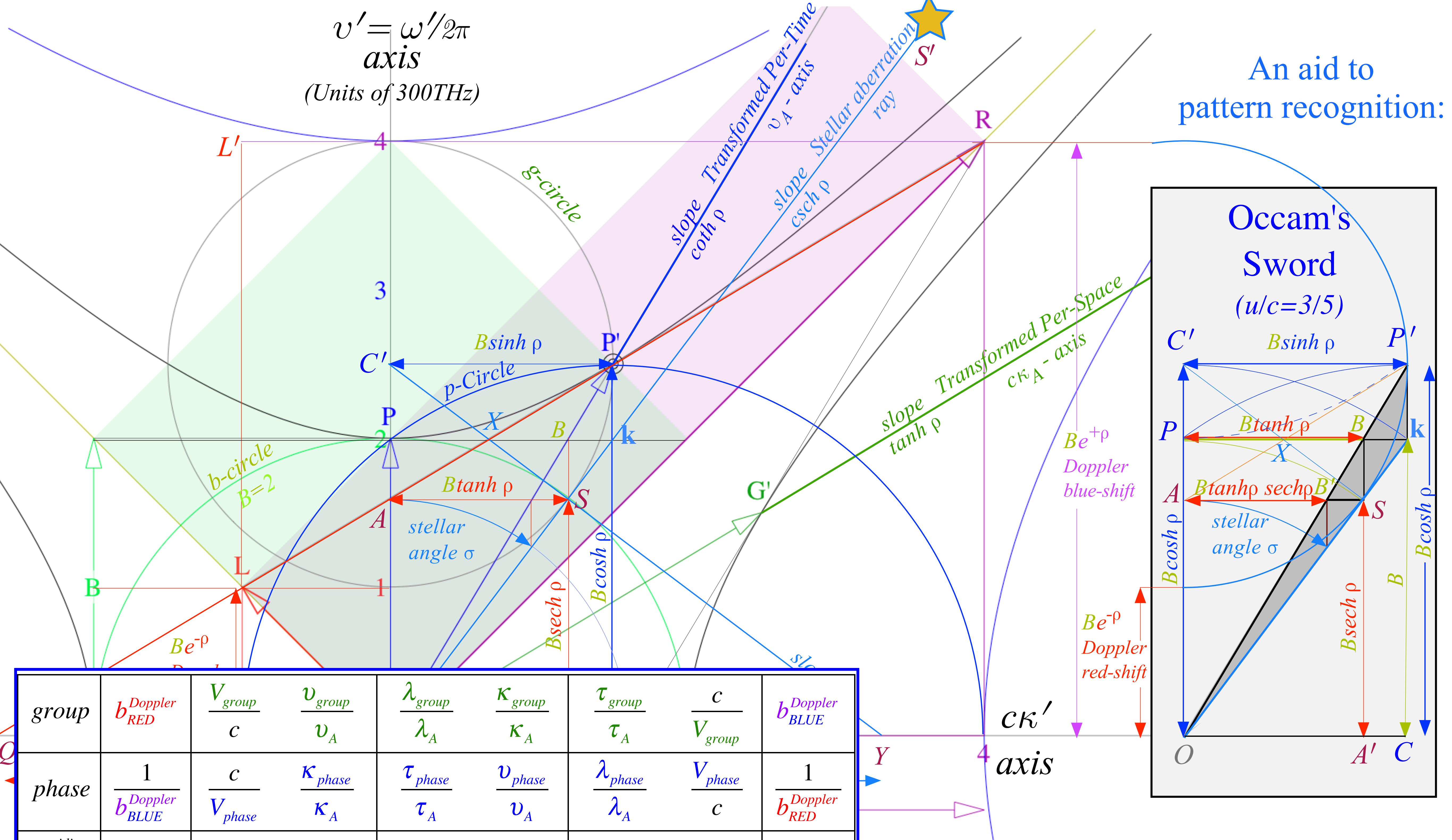

This map has circle sector arc-area $\sigma = 0.6435$
set to angle $\angle\sigma = 36.87° = 0.6435\,radian$
Also it is set to hyperbola sector arc-area $\rho = 0.6931$
angle $\angle\rho = \nu = 30.96°$
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3
$\cosh(\rho)+\sinh(\rho) = \frac{5}{4}+\frac{3}{4} = 2.0 = e^{+\rho}$
$\cosh(\rho)-\sinh(\rho) = \frac{5}{4}-\frac{3}{4} = 1/2 = e^{-\rho}$
$\cosh(\rho) = \frac{e^{+\rho}+e^{-\rho}}{2}$
$\sinh(\rho) = \frac{e^{+\rho}-e^{-\rho}}{2}$
Half-Sum-Half-Difference Trig-Formulae for exponentials $e^{\pm\rho}$
Bcosh(ρ)+Bsinh(ρ)=Be$^{+\rho}$
R
tangent slope = tanh(ρ)
tangent slope = coth(ρ)
Bsinh(ρ)
Bsinh(ρ)
P
Bsinh(ρ)
B
$x^2+y^2=B^2$
S
G
Bcsch(ρ)
L
Bcosh(ρ)
ρ
σ
Bsech(ρ)
Bcsch(ρ)
Bcosh(ρ)-Bsinh(ρ)=Be$^{-\rho}$
Btanh(ρ)
Bcoth(ρ)
Thales Circle
(links e$^{-\rho}$ to e$^{+\rho}$)

Summary of optical wave parameters for relativity and QM

...and their geometry



RelaWavity Web Simulation
{perSpace - perTime All}



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

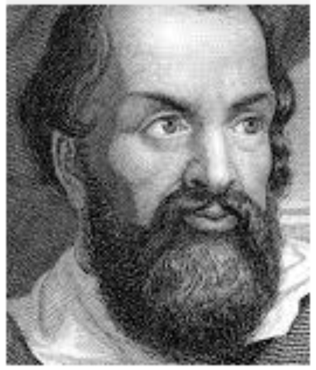
Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

RelaWavity Web Simulation

Expanded Relativistic Relations

Galileo Galilei



1564-1642

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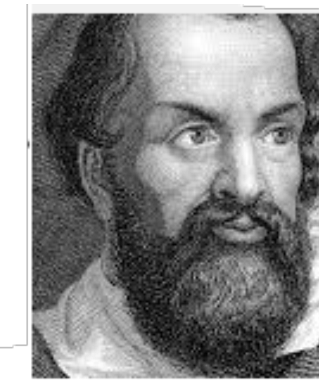

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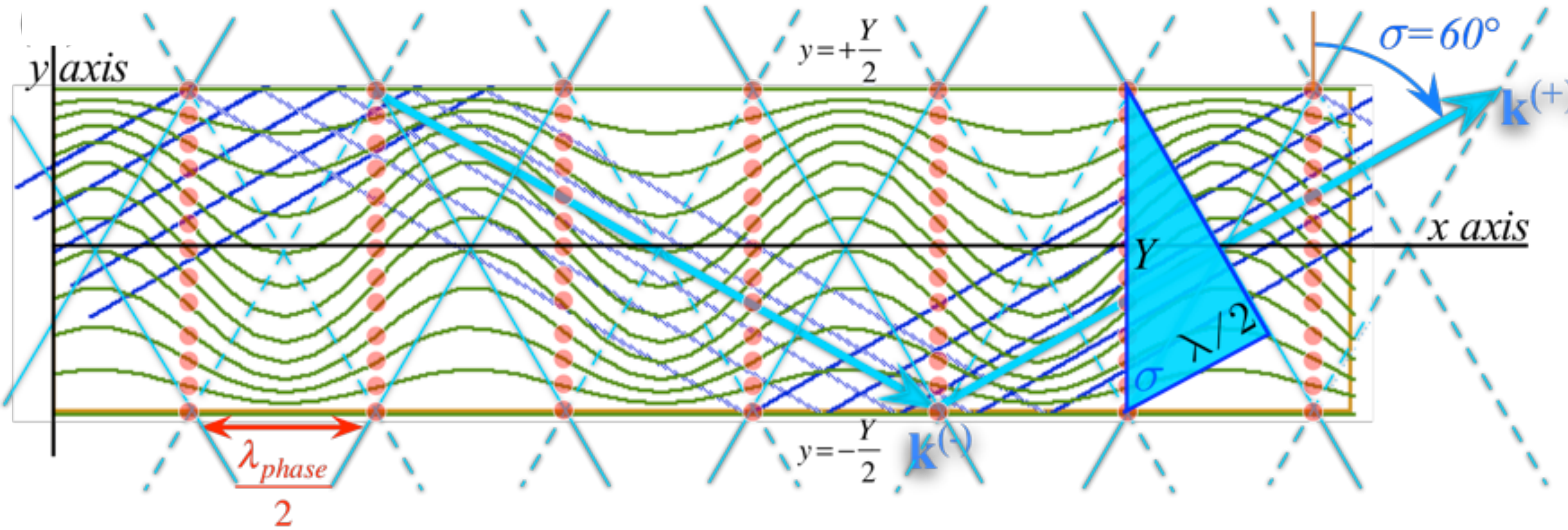
Optical wave guide relativistic geometry aided by Occam's Sword

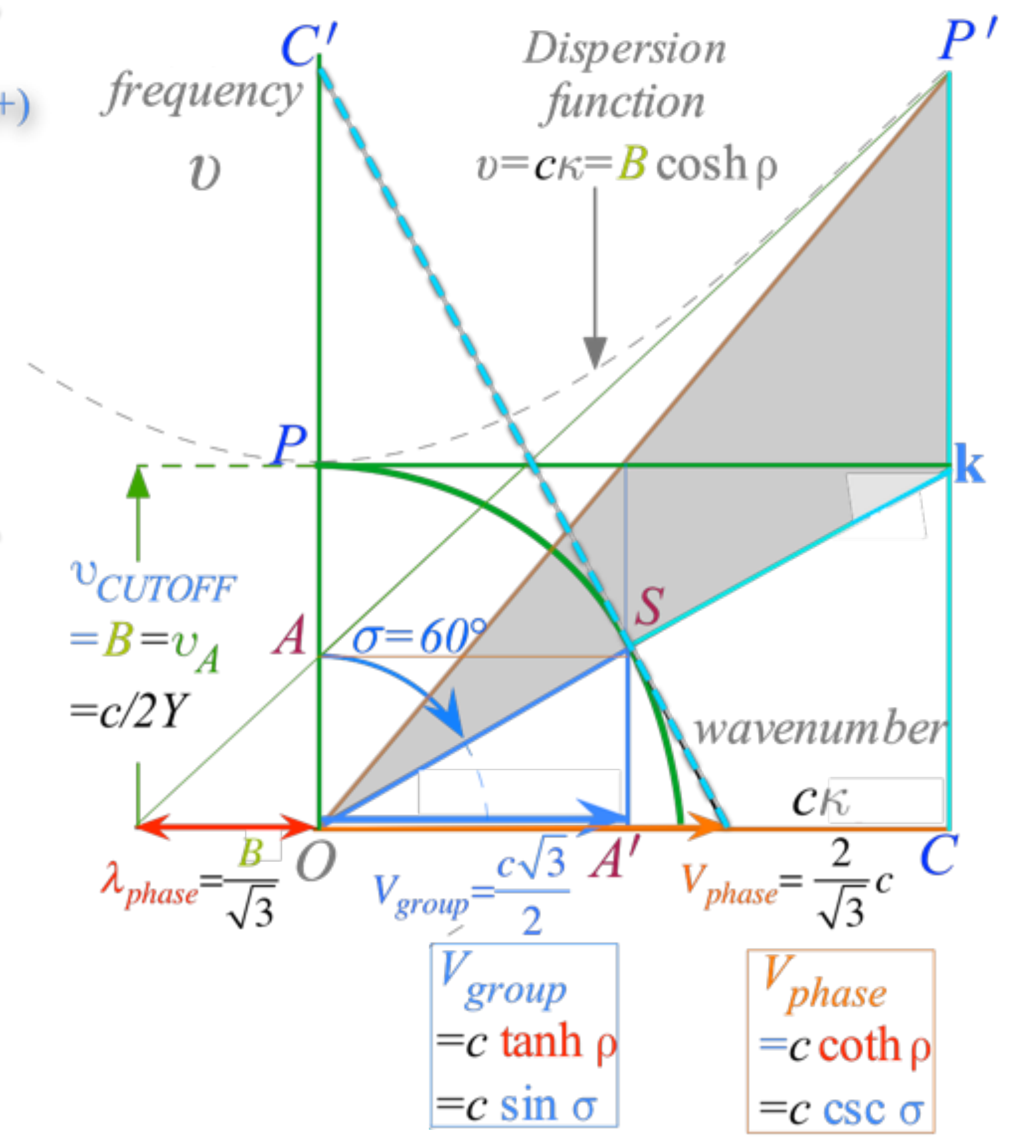
geometry applies to (x,y) space-space

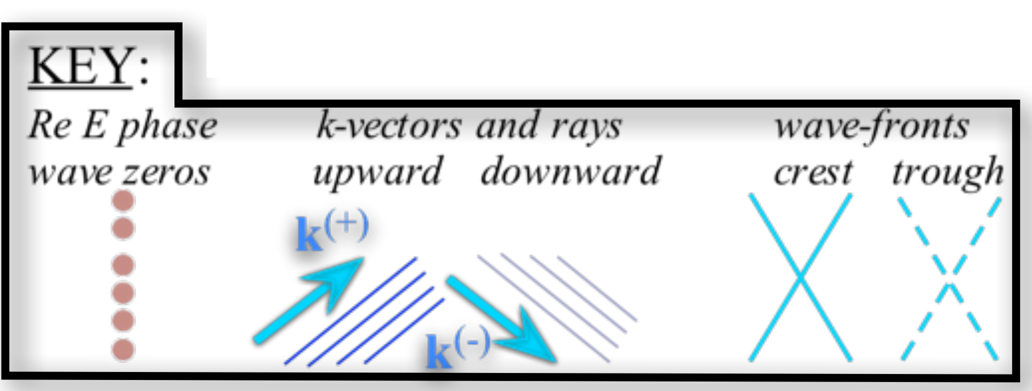
to (k_x,k_y)per-space-per-space

to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)







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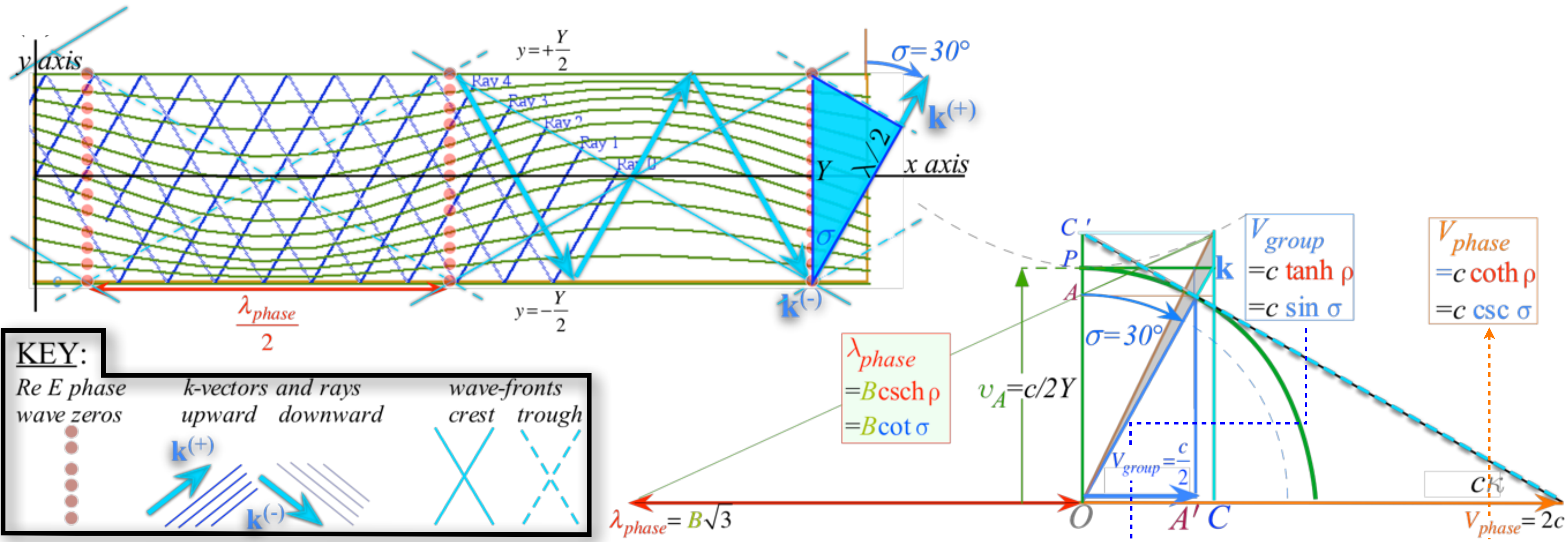

Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

Optical wave guide relativistic geometry aided by Occam's Sword

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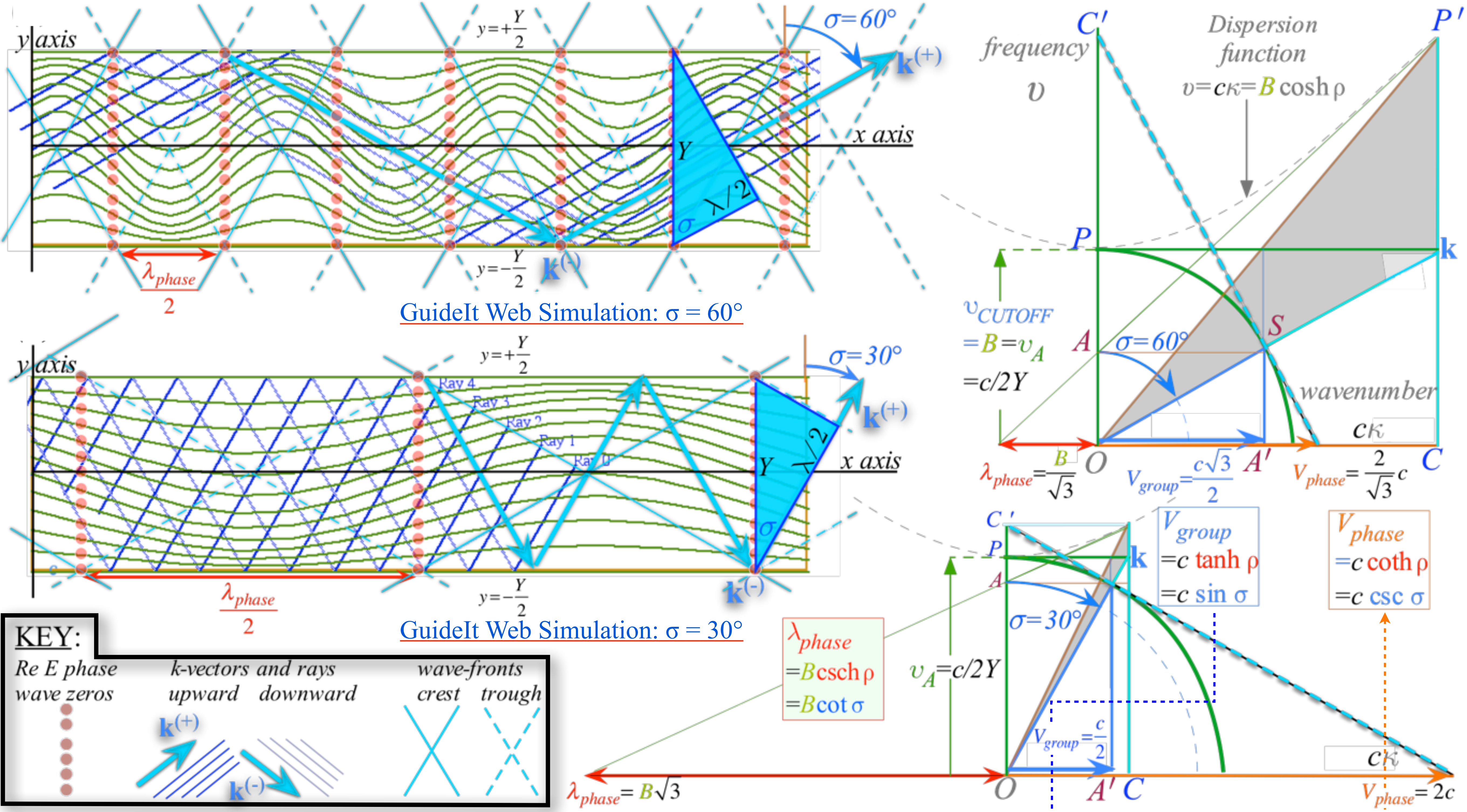

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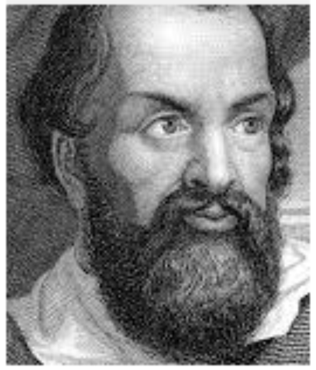

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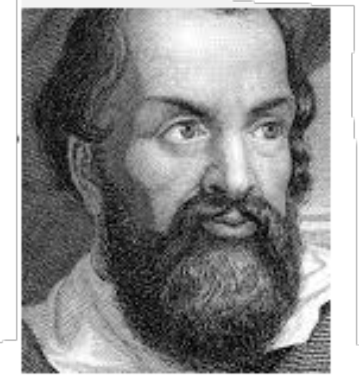

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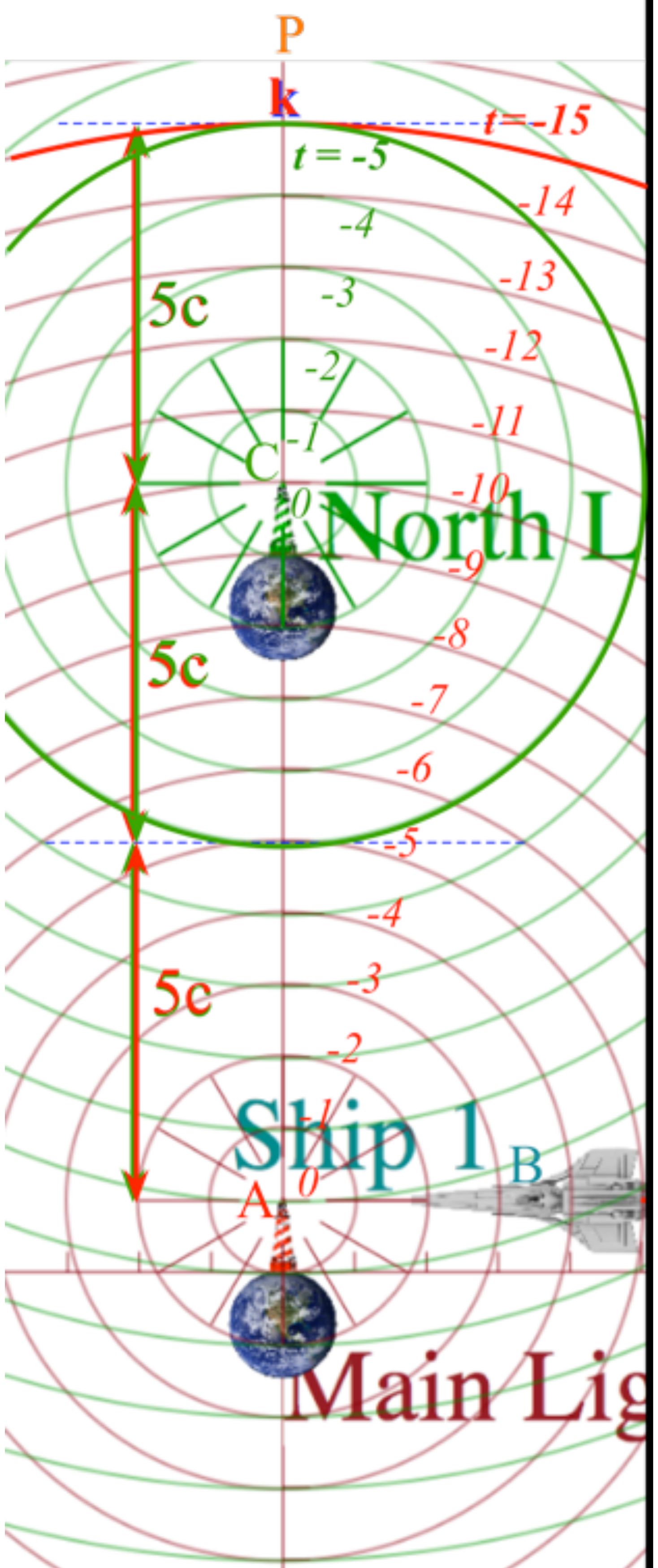
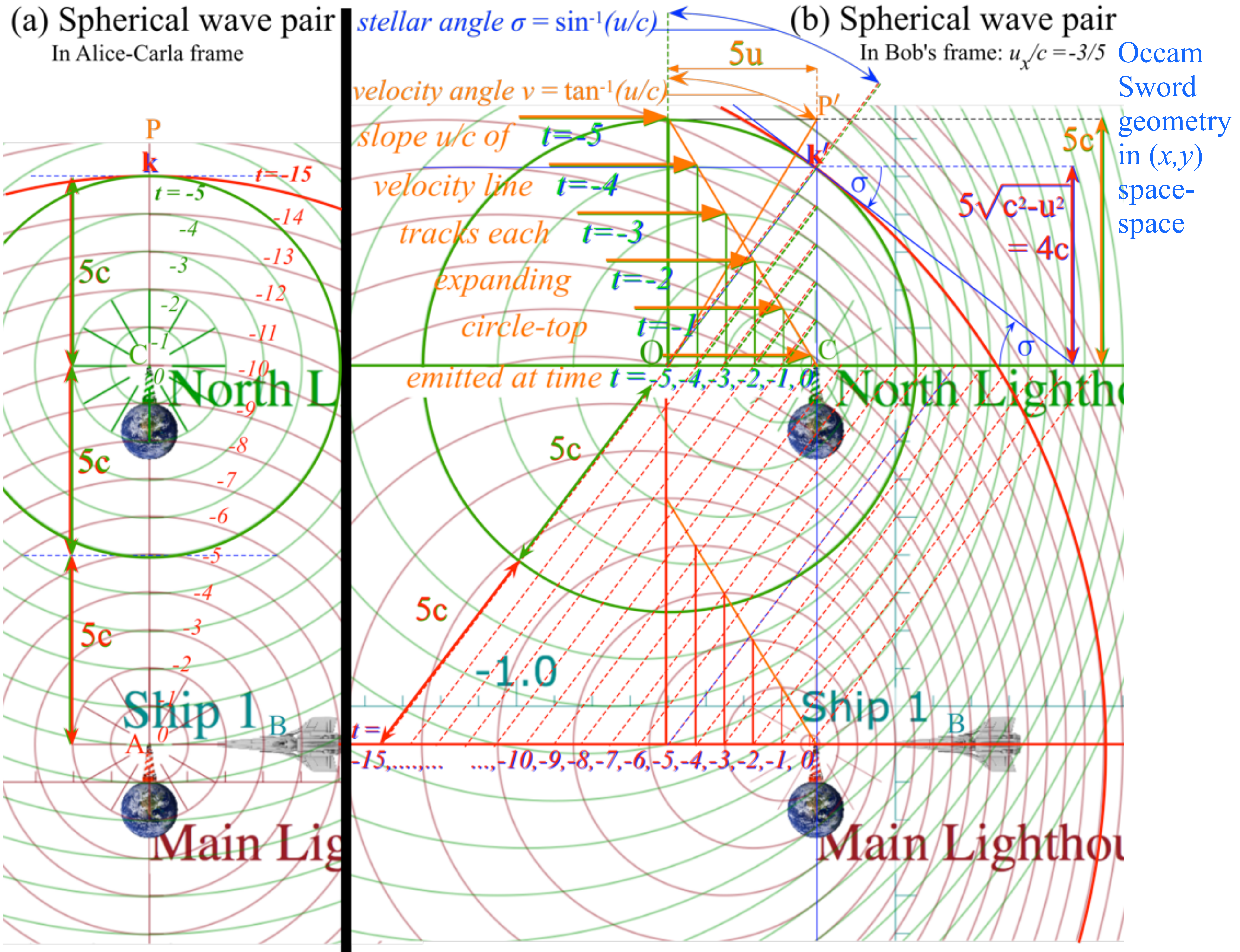
Spherical wave relativistic geometry

Also, aided by Occam's Sword

(a) Spherical wave pair

In Alice-Carla frame

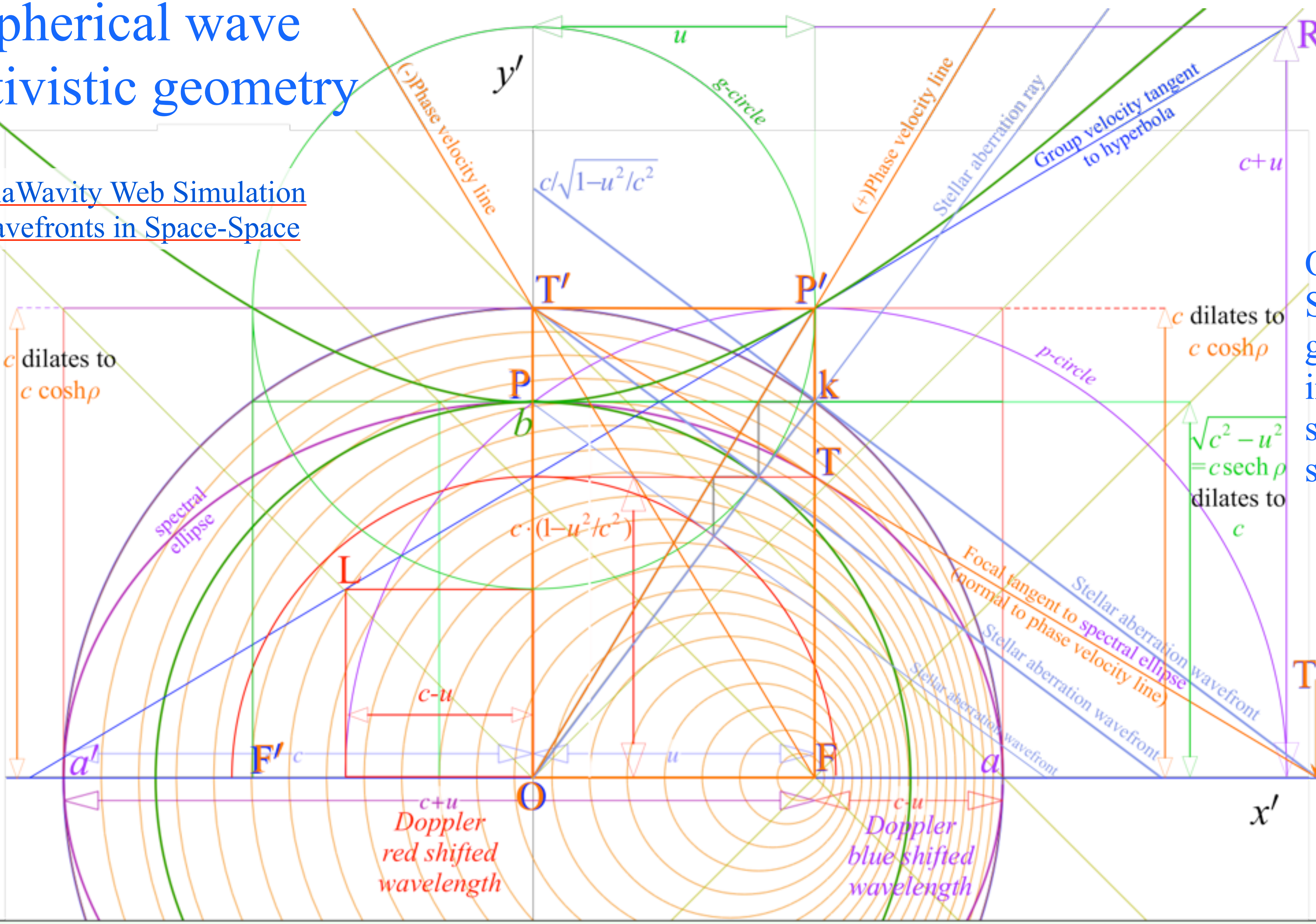




(a) Spherical wave pair
In Alice-Carla frame
stellar angle σ = sin⁻¹(u/c)
(b) Spherical wave pair
In Bob's frame: u_x/c = -3/5
Occam
Sword
geometry
in (x,y)
space-
space
5u
velocity angle ν = tan⁻¹(u/c)
slope u/c of
velocity line
tracks each
expanding
circle-top
emitted at time t = -5,-4,-3,-2,-1,0
5√c²-u² = 4c
North Lighthouse
Main Lighthouse
Ship 1
-15,........
...,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1, 0
-1.0

Spherical wave relativistic geometry

[RelaWavity Web Simulation Wavefronts in Space-Space]



Occam Sword geometry in (x,y) space-space

Doppler Red λ=c+u dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse focal length **FO**= $u = c\tanh\rho$ dilates to: $u\cosh\rho = c\sinh\rho$

Doppler Blue λ=c−u dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

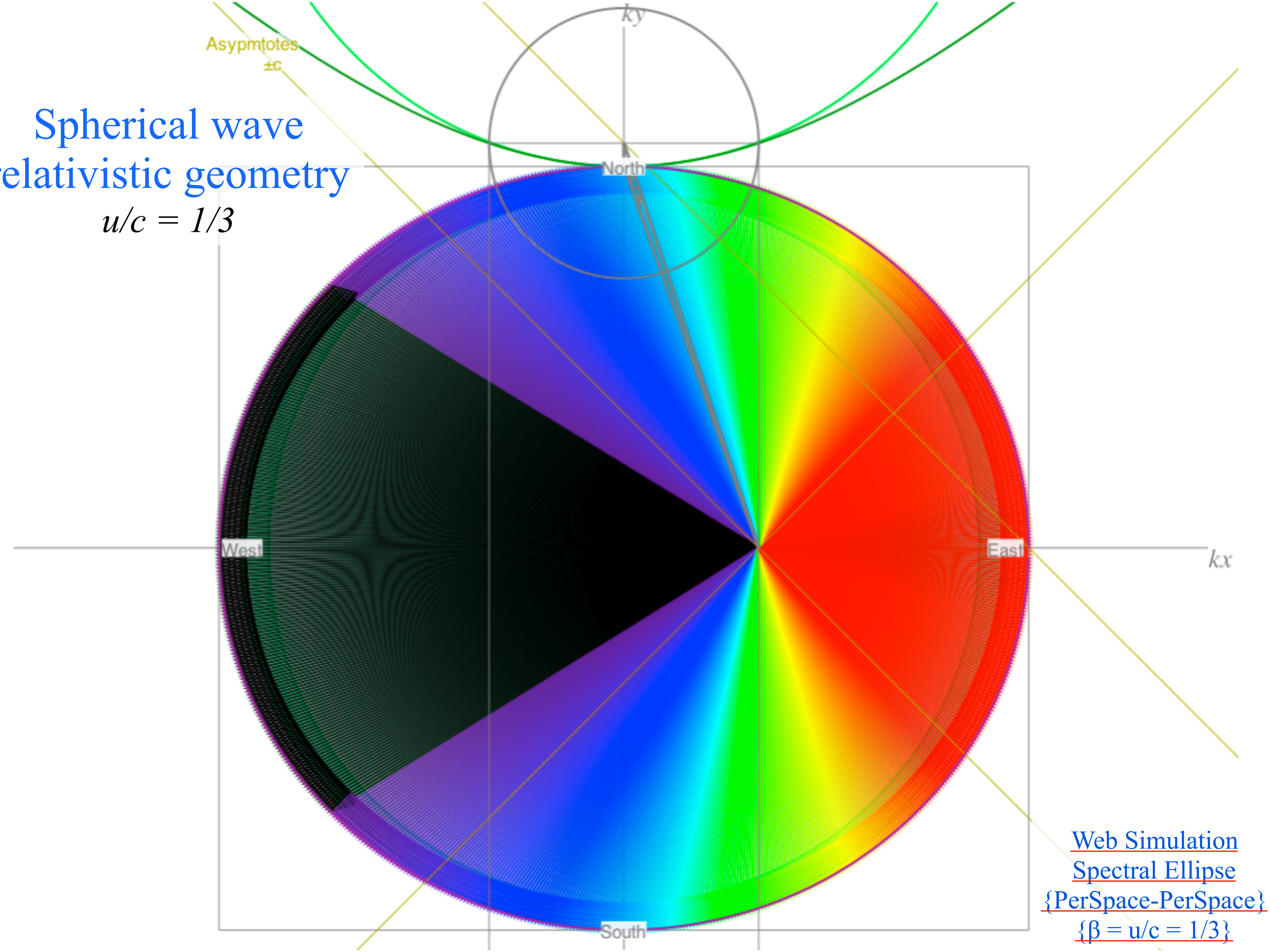
ellipse major radius a=**OF**a=c dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor: $\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse latus radius **FT**=$c(1-u^2/c^2)$ dilates to: $c(1-u^2/c^2)\cosh\rho$ $= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

Base height **FTk**=$\sqrt{c^2-u^2}$ dilates to: $\sqrt{c^2-u^2}\cosh\rho = c$ (equal to ellipse minor radius b)

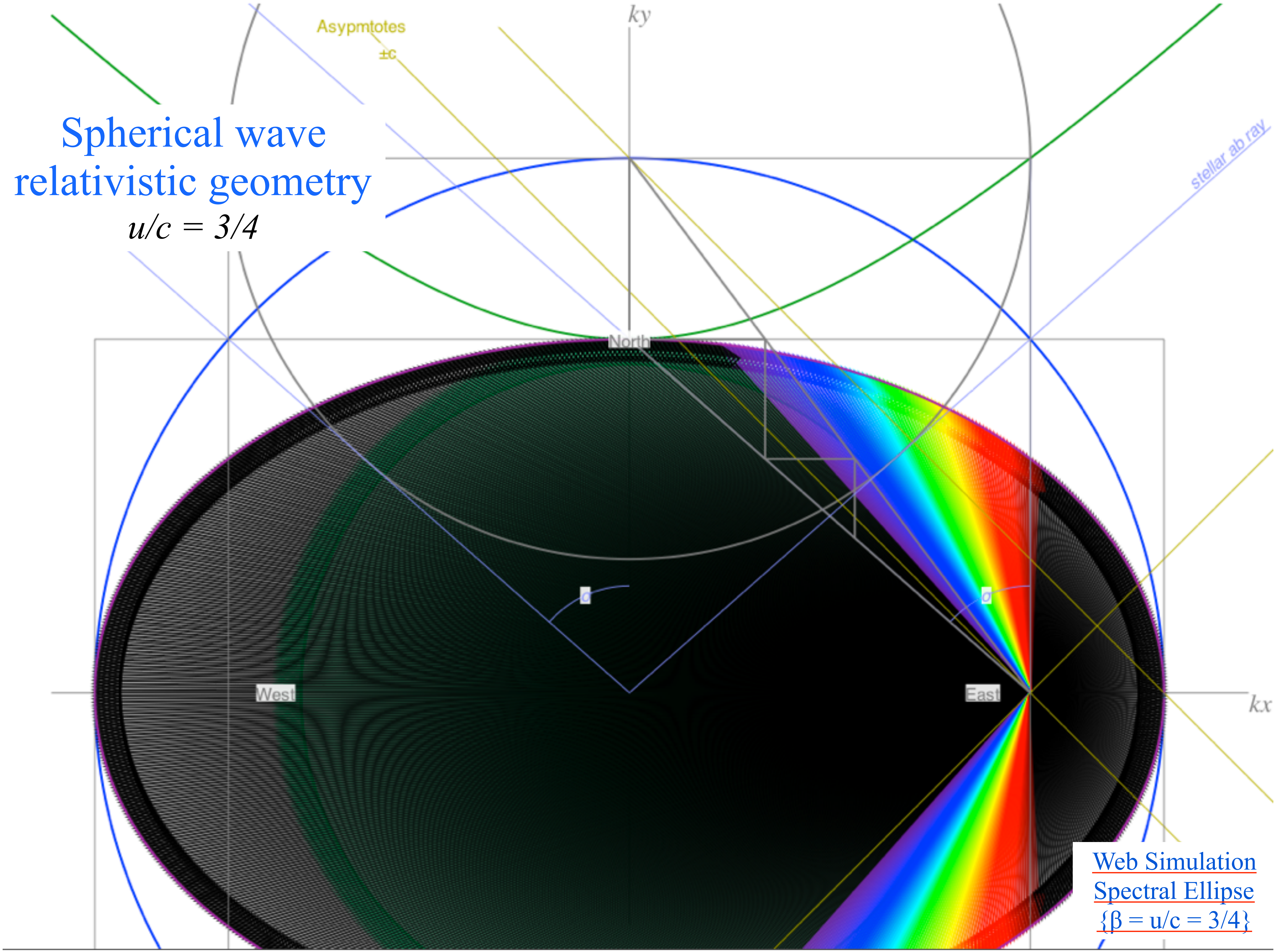
Spherical wave relativistic geometry
u/c = 1/3



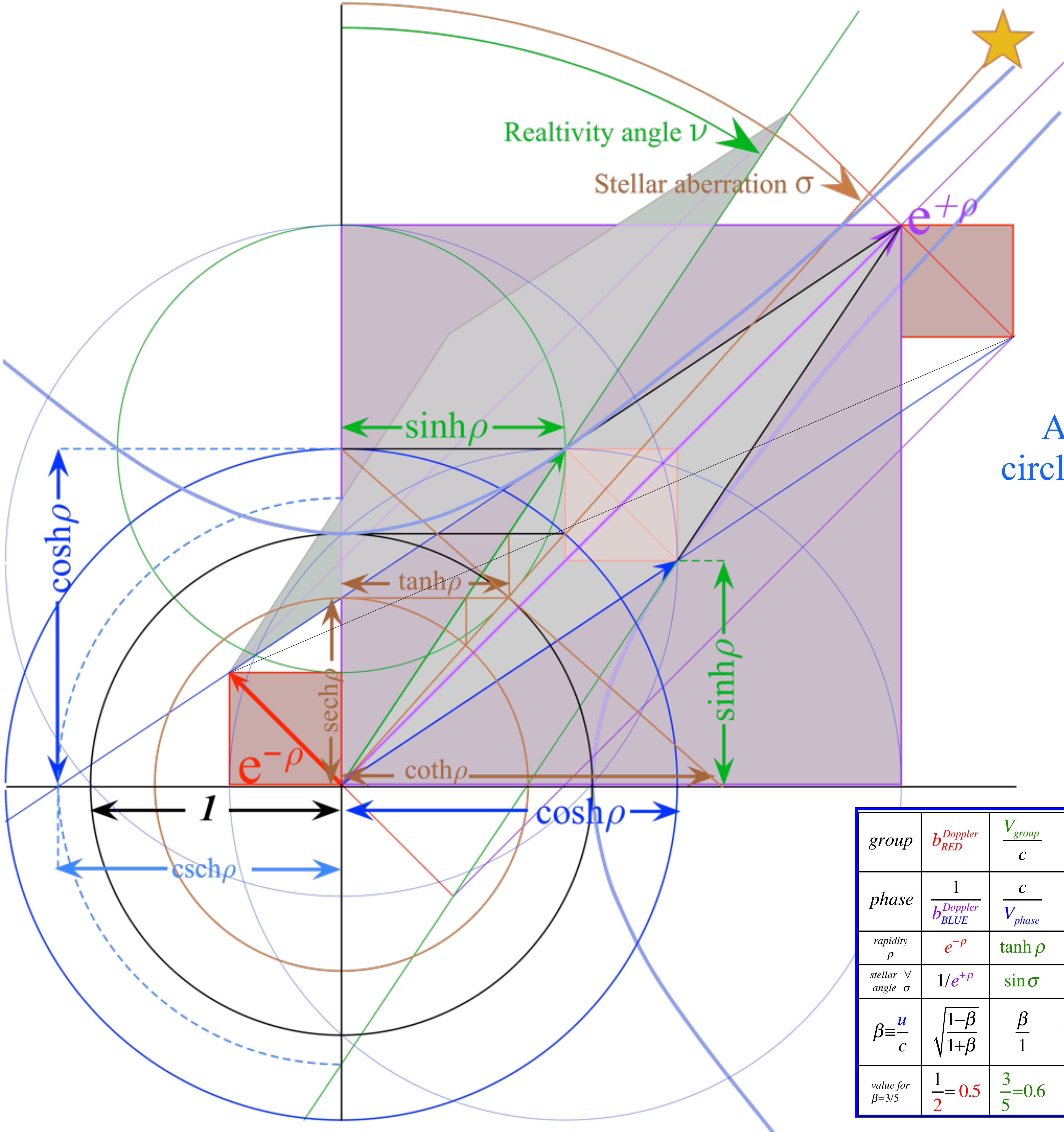
Web Simulation
Spectral Ellipse
{PerSpace-PerSpace}
{β = u/c = 1/3}

Spherical wave relativistic geometry

u/c = 3/4



Web Simulation
Spectral Ellipse
{β = u/c = 3/4}



A more compact circle-based geometry

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$