Lecture 2
Wed 8.22.2018

Analysis of 1D 2-Body Collisions
(Ch. 2 to Ch. 4 of Unit 1)

NOTICE THIS: *AIP-AAPT* Cool demos

Review: COM Momentum line, elastic vs inelastic kinetic energy ellipse geometry

The X2 Superball pen launcher

Perfectly elastic "ka-bong" velocity amplification effects (Faux-Flubber)

Geometry of X2 launcher bouncing in box (gravity-free)

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(s)

Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots

Integration of (V_1,V_2) data to space-space plots (y_1, y_2) *Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$*

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C•M**

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics in Ch. 12

AAPT PHYSICS EDUCATION
The Physics Teacher

Even though school is out, physics learning doesn't have to stop! Many summertime activities provide fun opportunities to explore science in the real world. Check out the papers below to see how you and your students can continue to study physics all year long.

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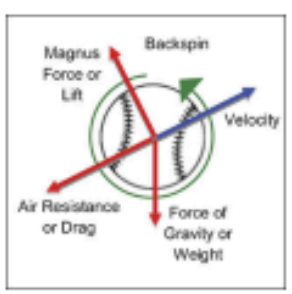

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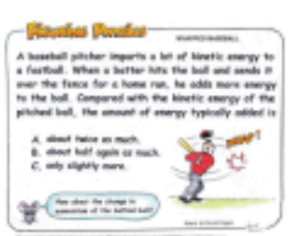

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https://aip-info.org/37VS-QW7L-1462CY2628/cr.aspx?v=1

Baseball Physics: Physics and the boys of summer, phys.csuchico.edu:16080/baseball/
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Review of Momentum line and COM point geometry

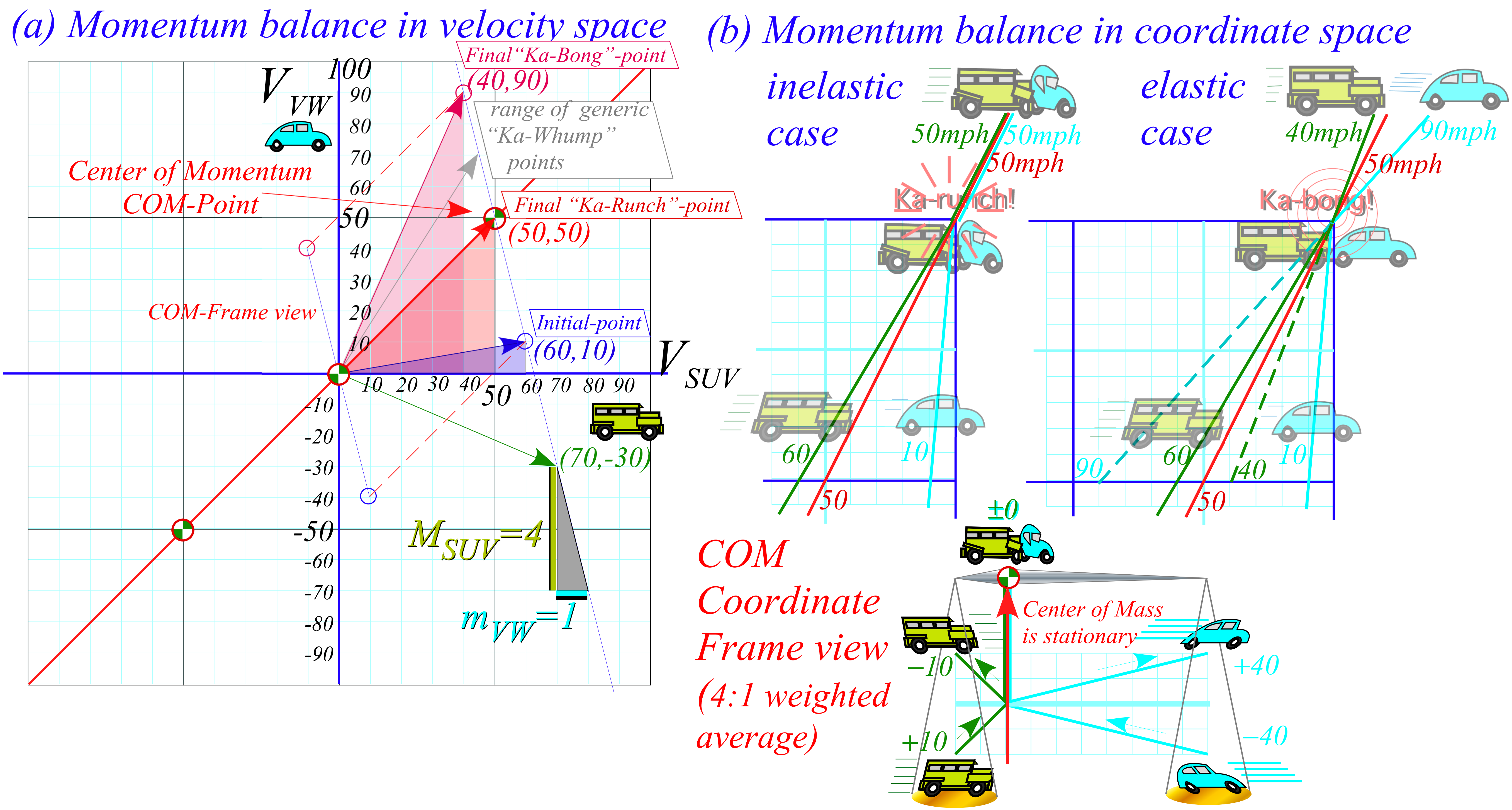


Fig. 1.2.6ab
(Unit 1)

Review of Kinetic Energy ellipse geometry

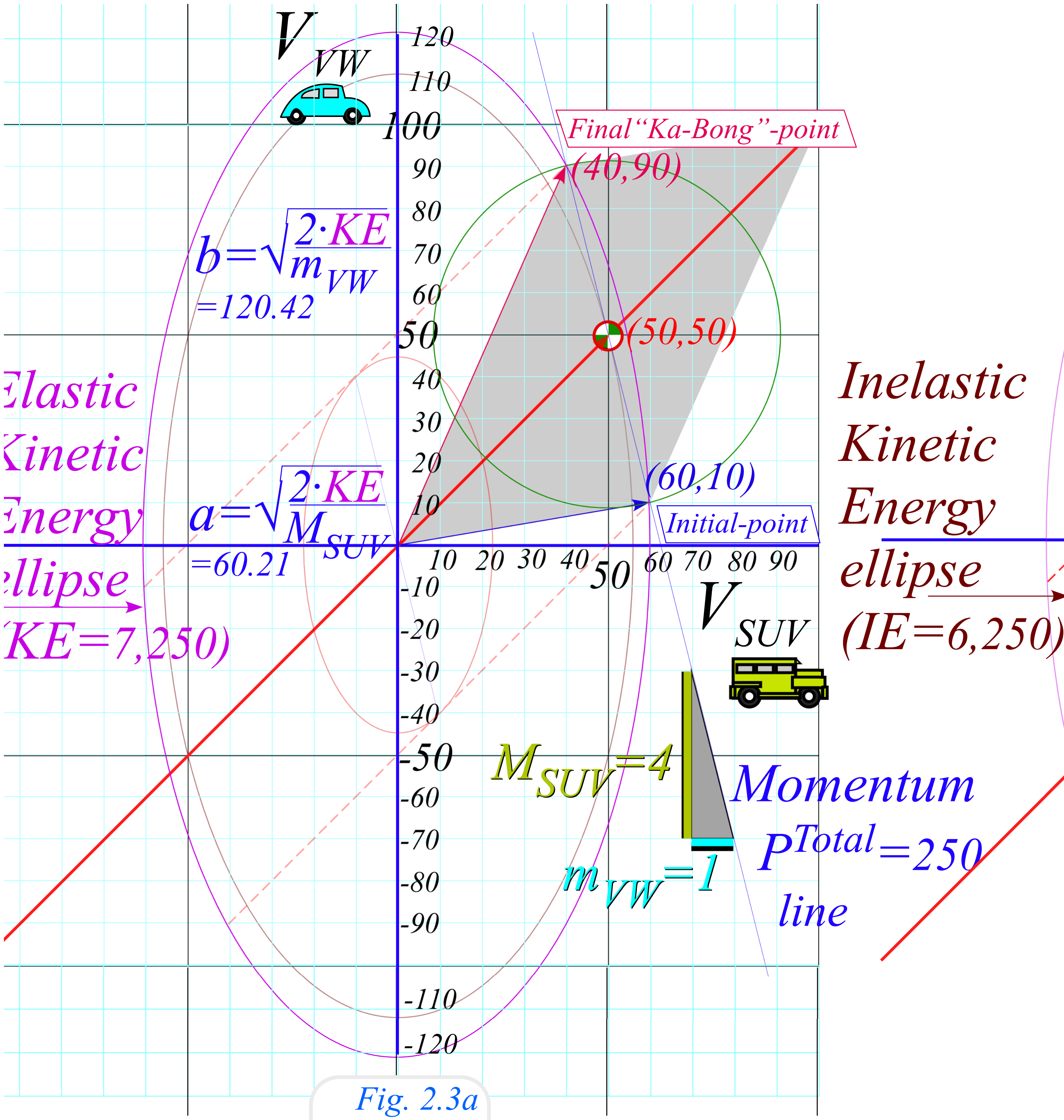


Fig. 2.3a (Unit 1)

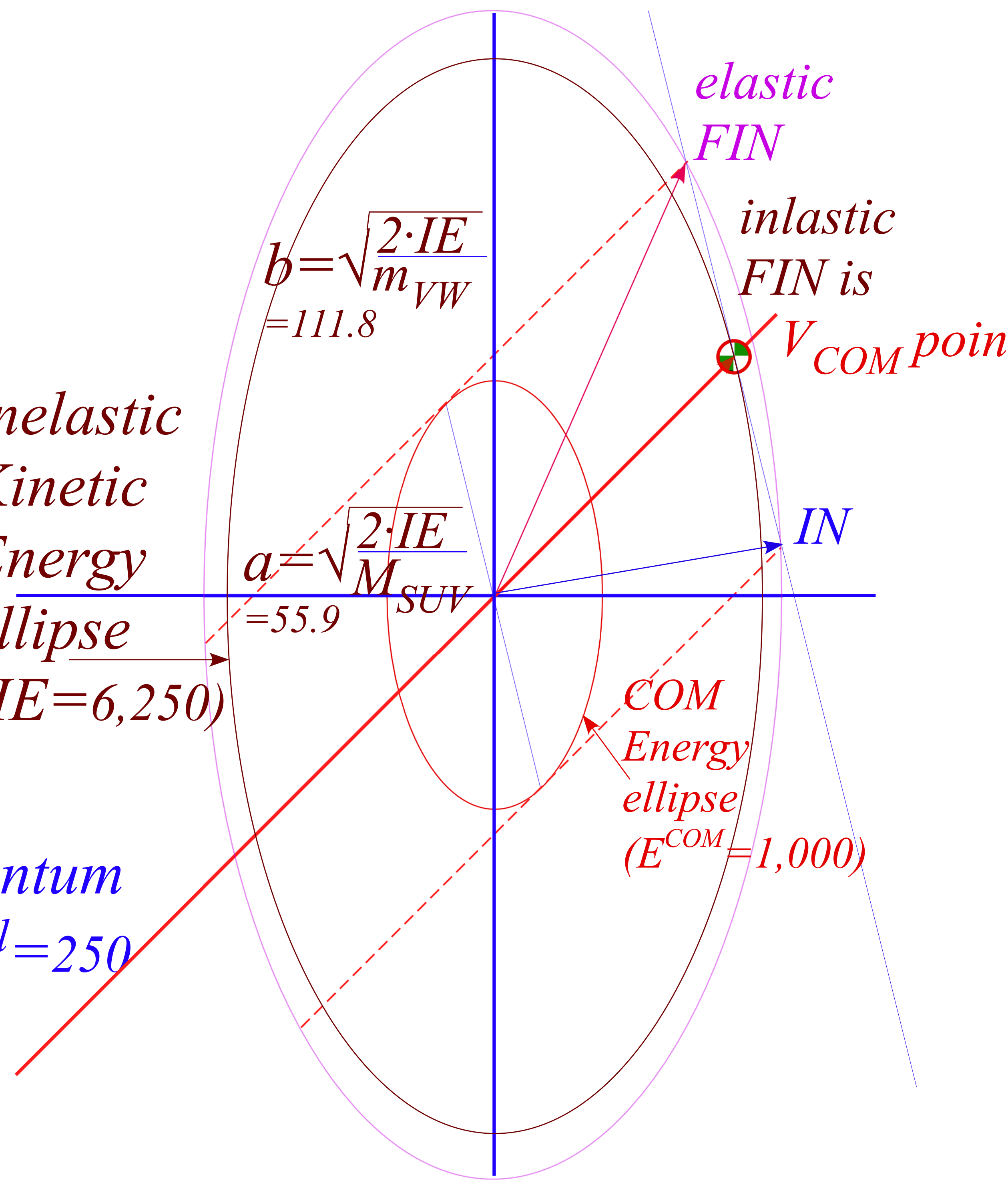


Fig. 2.3b (Unit 1)

Review of Elastic vs. Ineleastic Kinetic Energy ellipse geometry

Same collision viewed from nine different COM reference frames

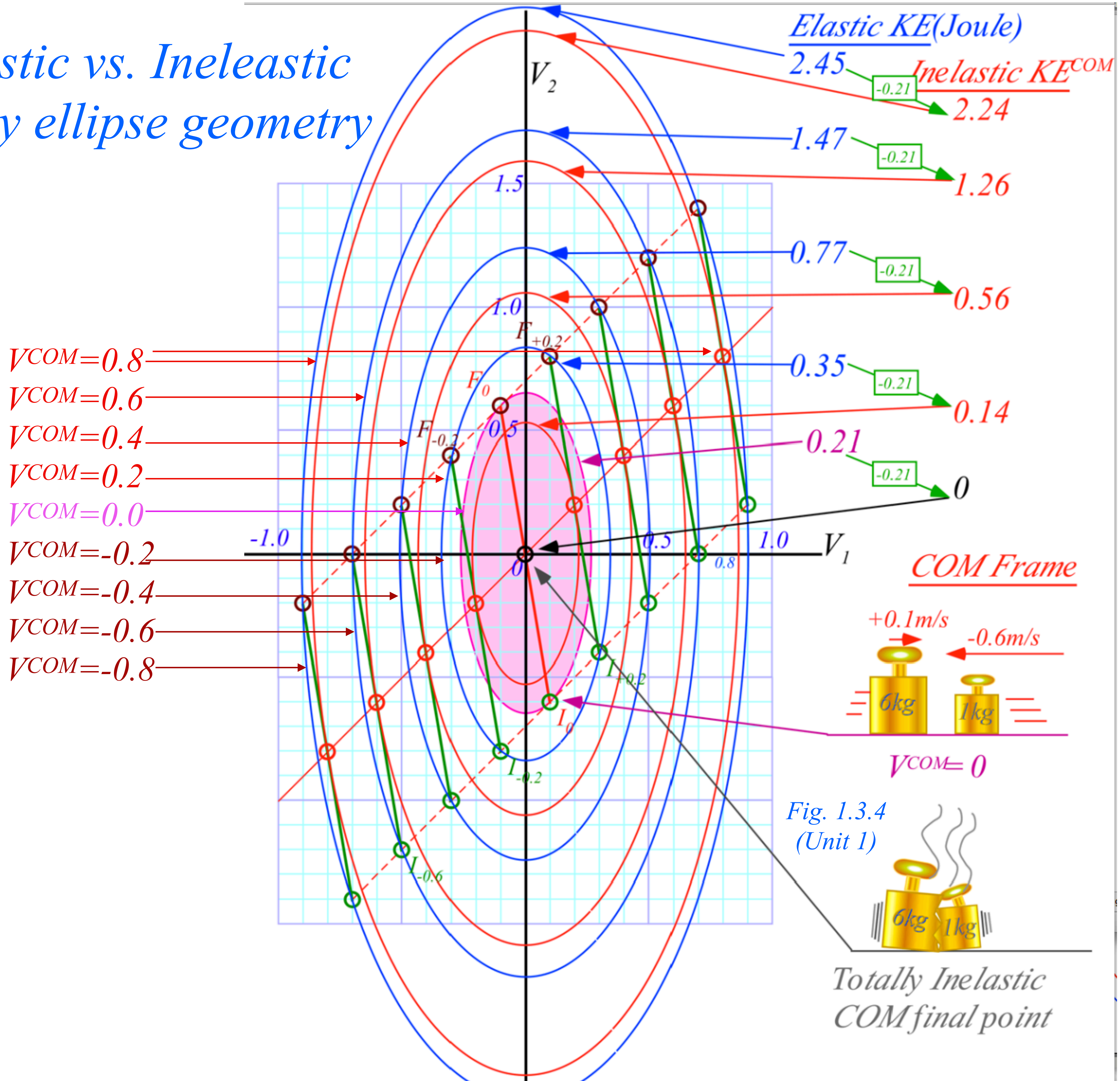


Fig. 1.3.4 (Unit 1)

Geometry of X2 launcher bouncing in box *(gravity-free)*

- *Independent Bounce Model (IBM)*
- *Geometric optimization and range-of-motion calculation(t)*
- *Integration of (V_1,V_2) data to space-time plots ($y_1(t)$,t) and ($y_2(t)$,t) plots*
- *Integration of (V_1,V_2) data to space-space plots (y_1, y_2)*

The X-2 Pen launcher and Superball Collision Simulator*

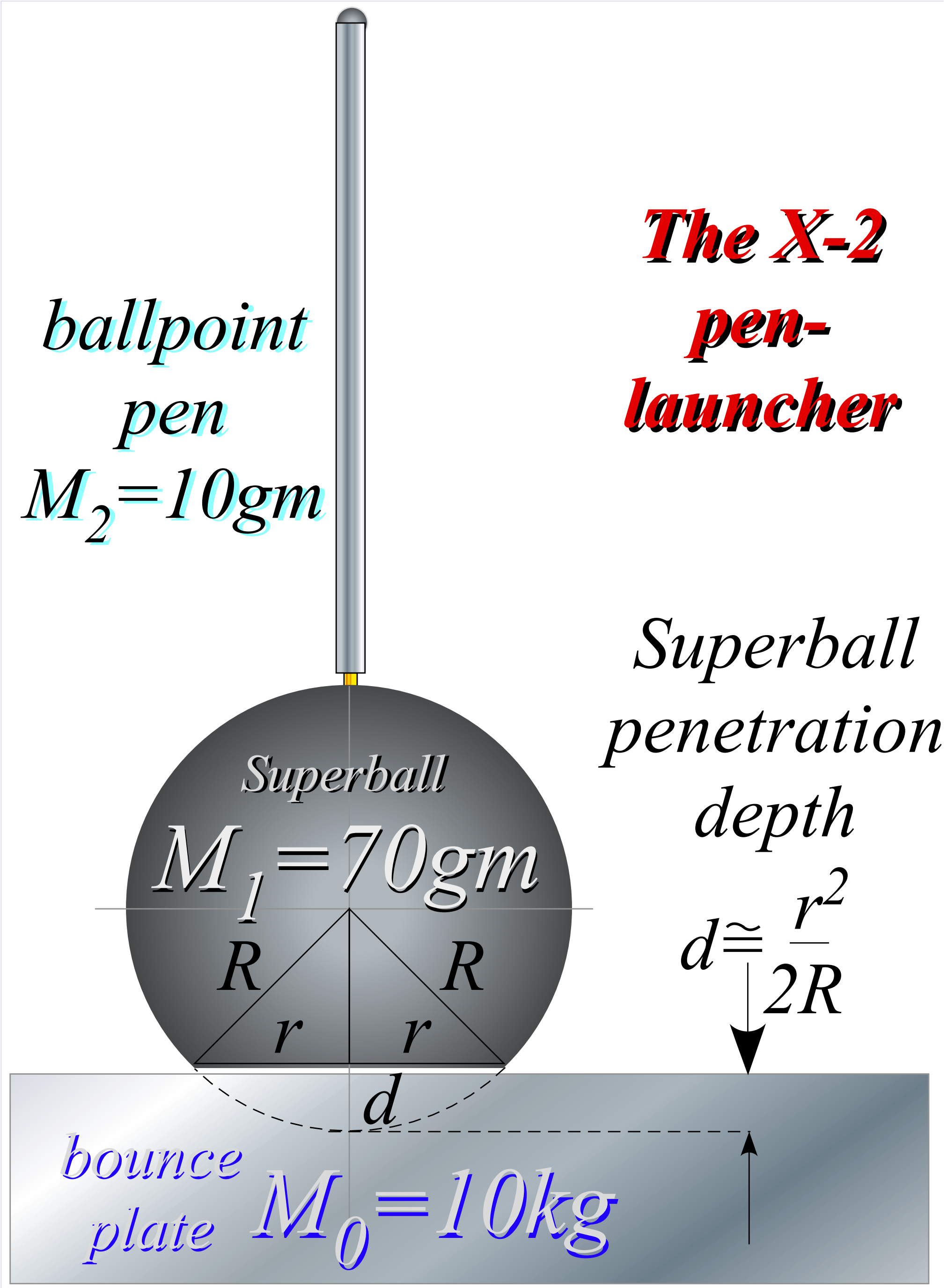
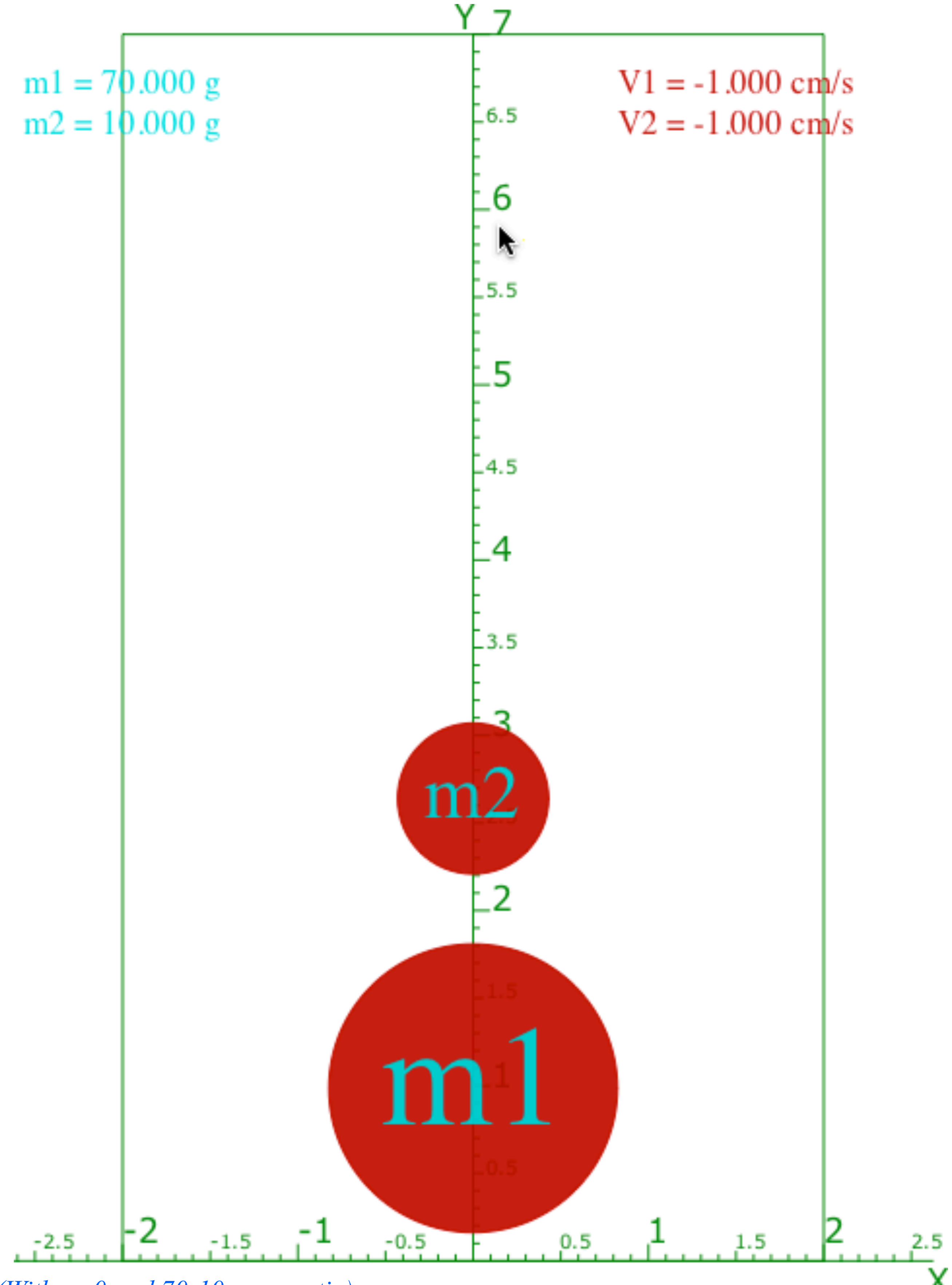


Fig. 3.1 (Unit 1)



(With g=0 and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007

The X-2 Pen launcher and Superball Collision Simulator*

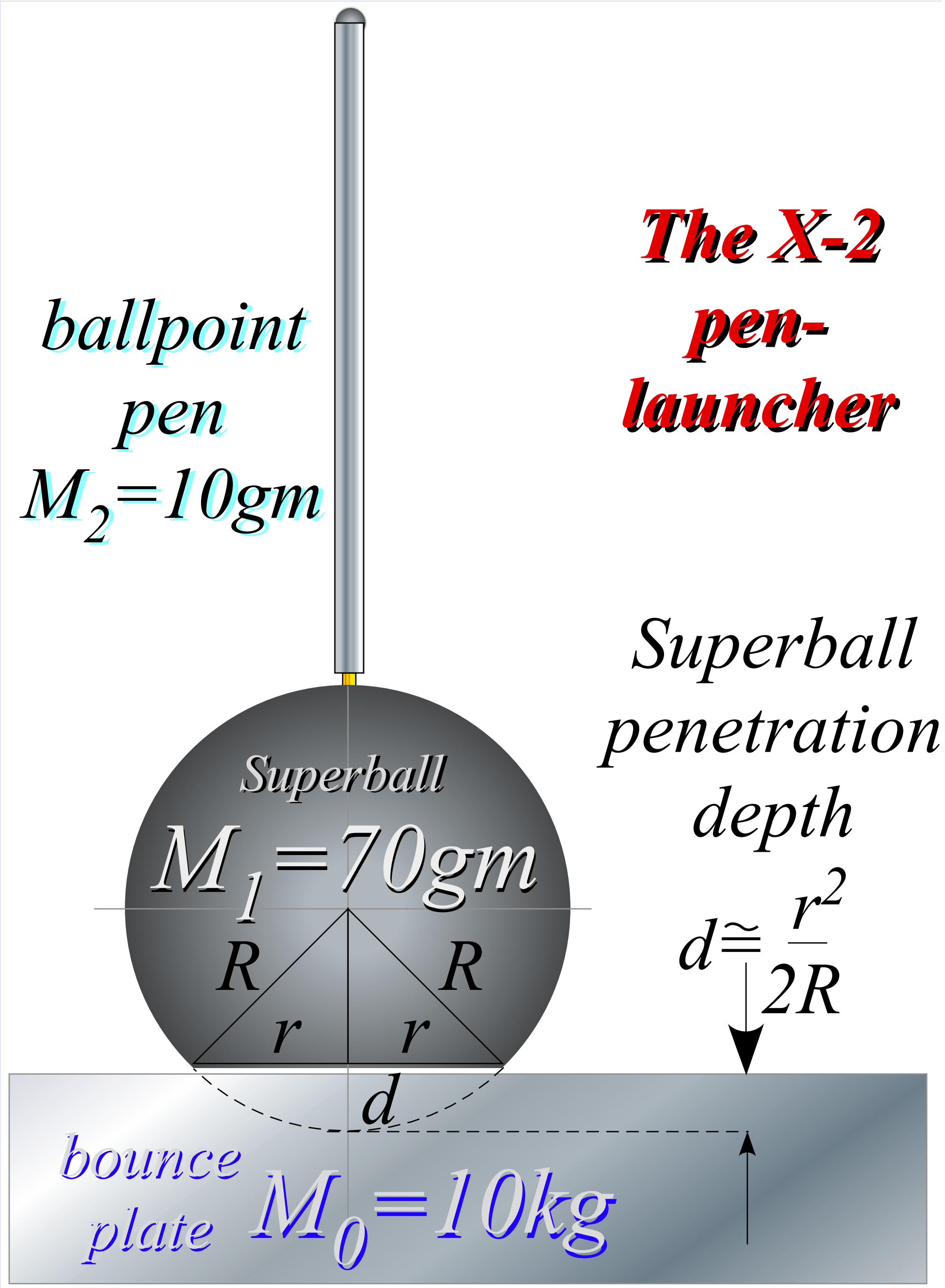
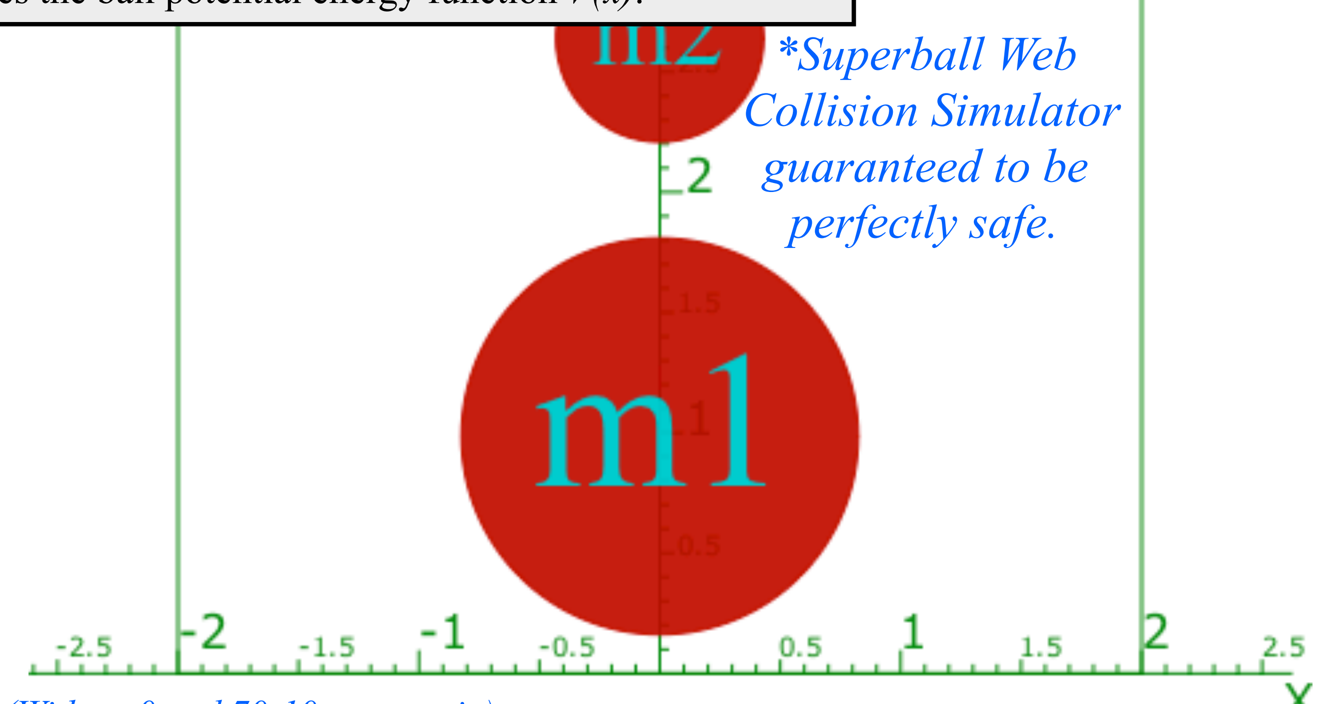


Fig. 3.1 (Unit 1)

Caution: Product Liablility Disclaimer
This ballpoint pen could be hazardous to your health!
The experiments which are the subject of this discussion are both spectacular and potentially dangerous, and care to protect one's eyes should be taken. The simplest experiment involves sticking a ball point pen into a superball or other hard rubber ball and dropping the two onto a hard floor.
If done correctly the pen will eject the ball with such force it may stick in the ceiling of the room. Obviously you want to be careful with this weapon. And, this goes doubly and triply for the more advanced models that may be developed in the course of studying this stuff. It is recommended that experimenters wear safety glasses when doing these experiments with pens. (We could just say don't use pens, but that's an easy way to do this experiment and probably the way most people will go about it.) Some of the tangential experiments associated with this development are less hazardous. To measure the potential force function of a ball one may simply paint the ball and measure the spot size as a function of drop height *h*.

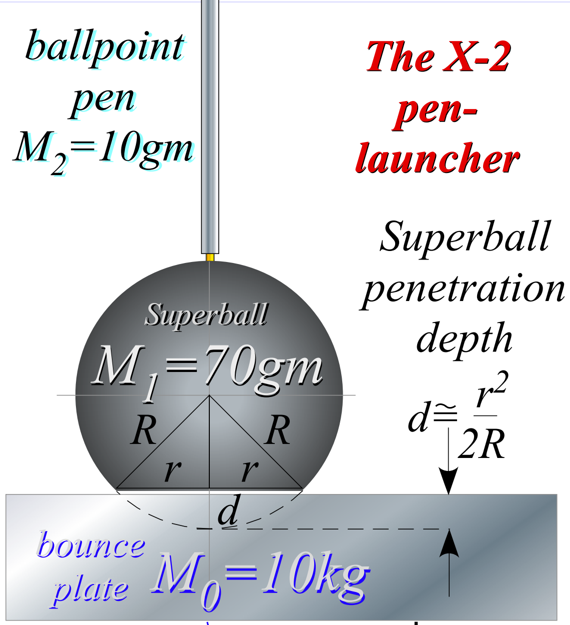
The saggital approximation $d=r^2/2R$ allows one to quickly convert spot radius *r* to penetration depth *x* for a superball of radius *R* as shown in the figure. Equating this to *Mgh* gives the ball potential energy function *V(x)*.



*Superball Web Collision Simulator guaranteed to be perfectly safe.

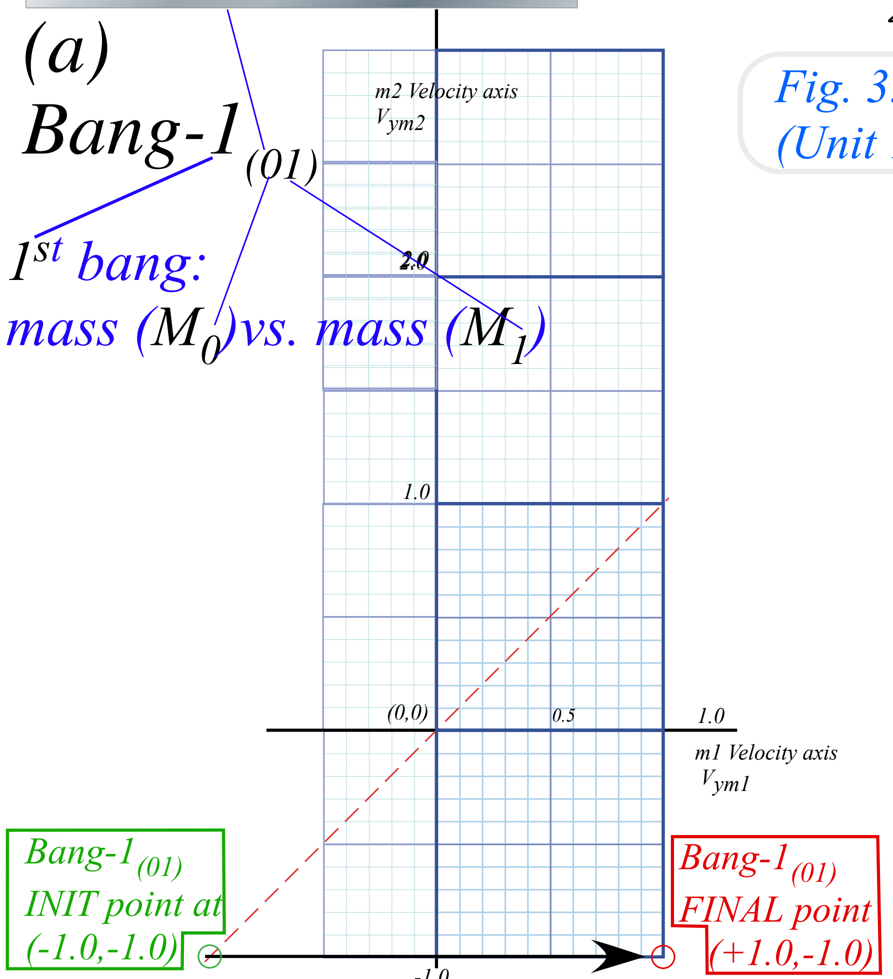
(With g=0 and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator https://modphys.hosted.uark.edu/markup/BounceItWeb.html?scenario=1007



(a) Bang-$1_{(01)}$

1^{st} bang: mass (M_0) vs. mass (M_1)



This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Fig. 3.3 (Unit 1)

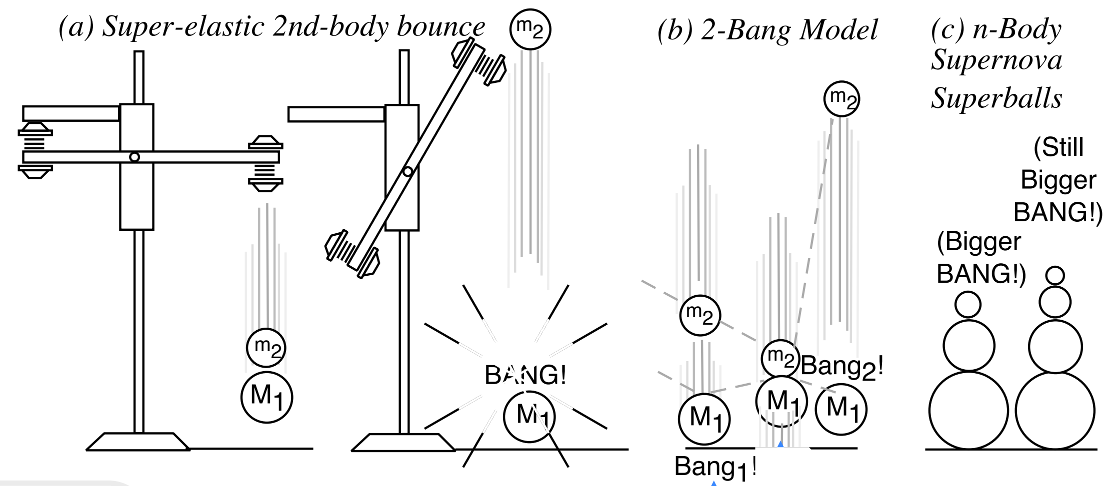
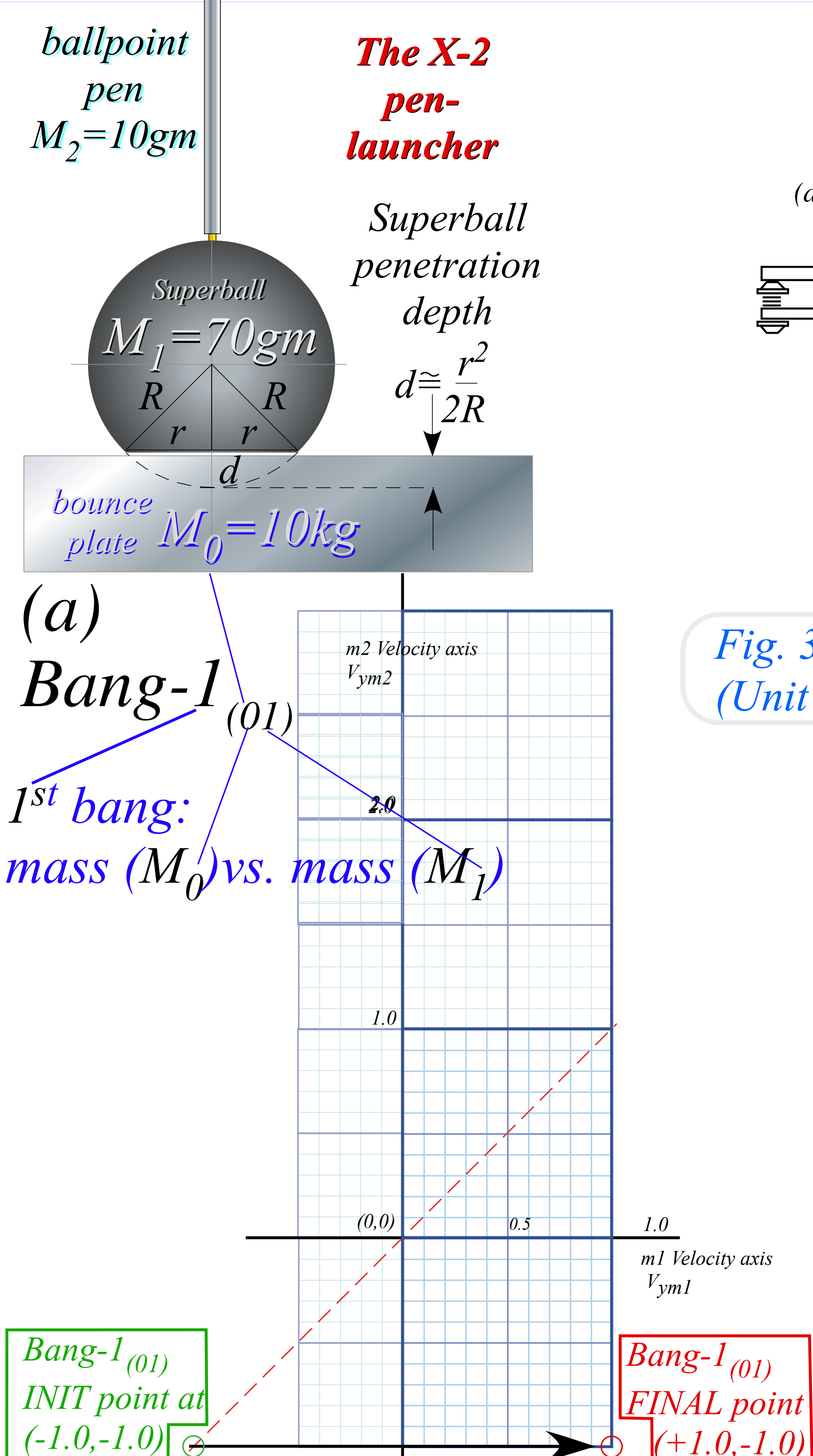


Fig. 3.4 (Unit 1)

1st bang: M_1 off floor

(With g=0 and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator https://modphys.hosted.uark.edu/markup/BounceItWeb.html?scenario=1007



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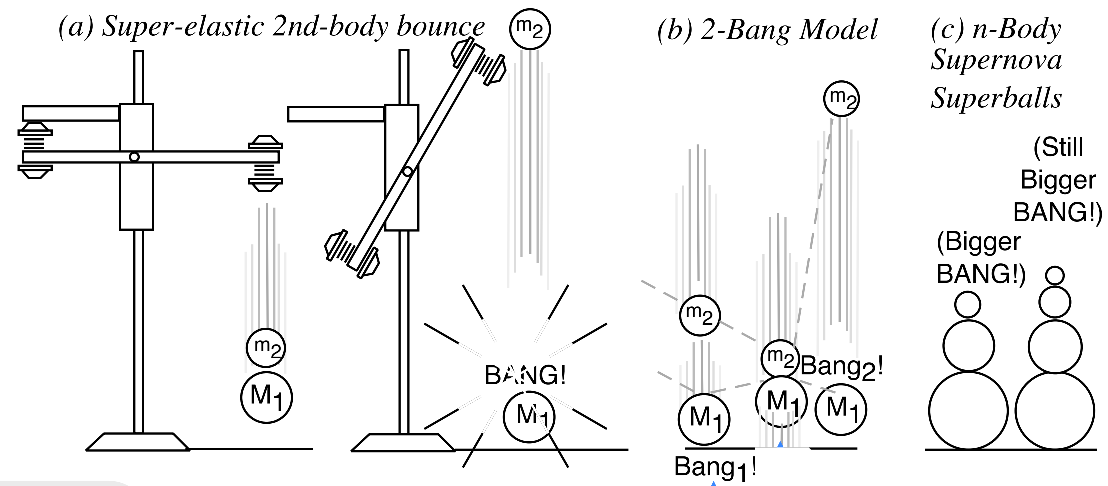

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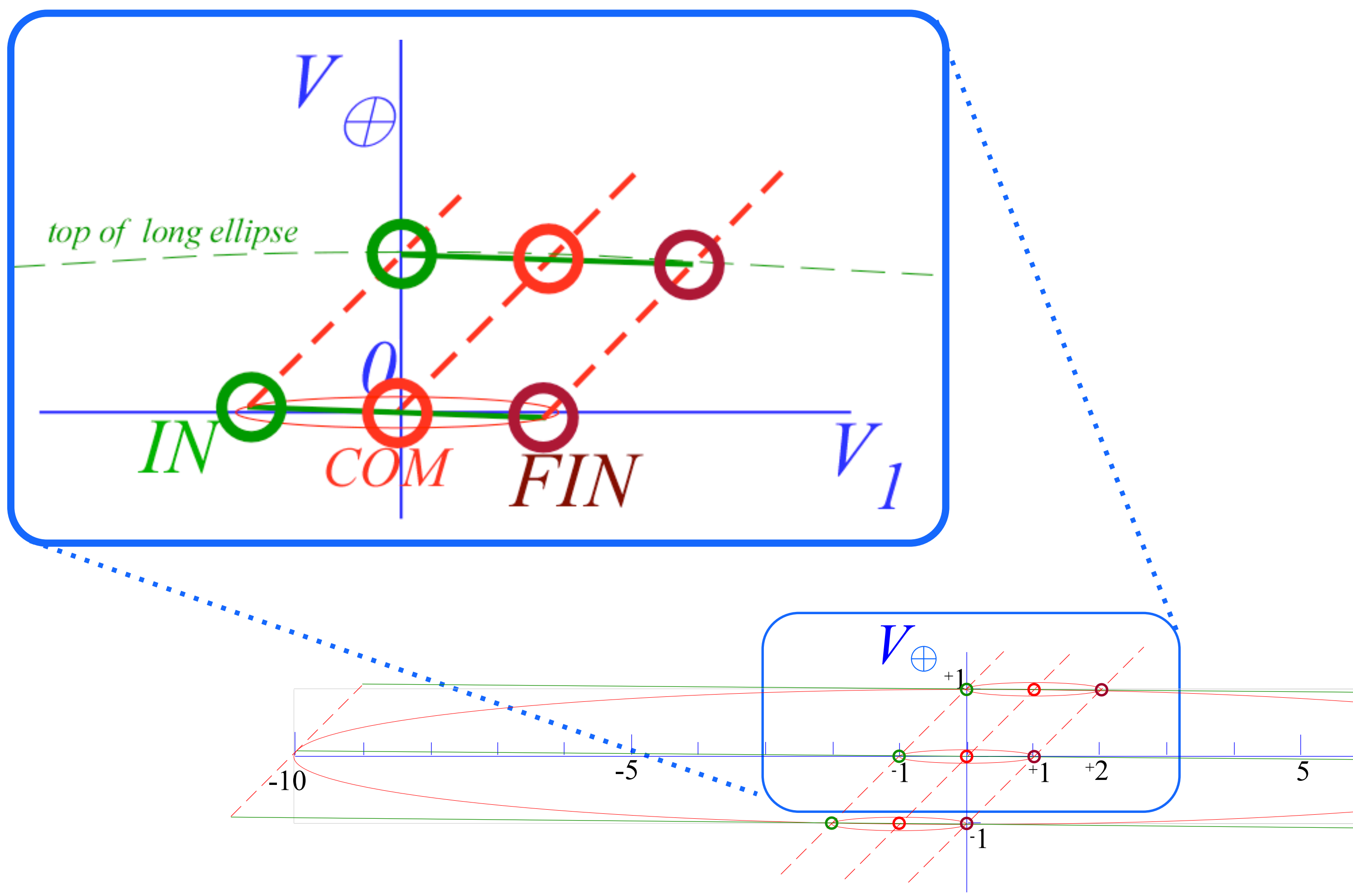


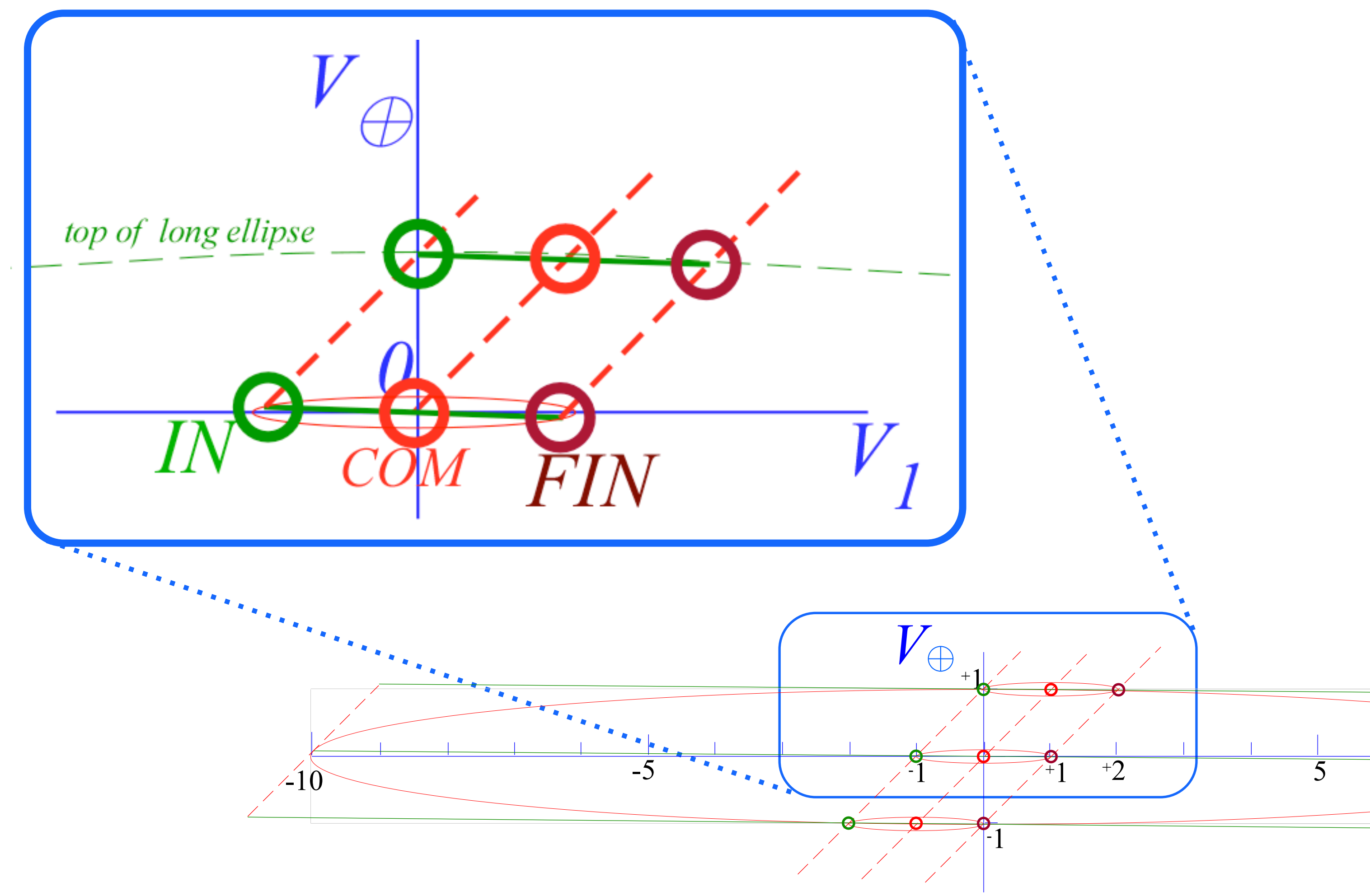
[BounceIt web simulation with g=0 and 70:10 mass ratio]

[With non zero g, velocity dependent damping and mass ratio of 70:35]

(a) 1st bang of M_1 off floor plate $M_\oplus$ =100 M_1 along (V_1,$V_\oplus$)-momentum line of slope

$-M_1/M_\oplus$ =-1/100

from IN-end to COM to FIN-end of (a/b $=\sqrt{M_\oplus}/\sqrt{M_1}$=10) ellipse



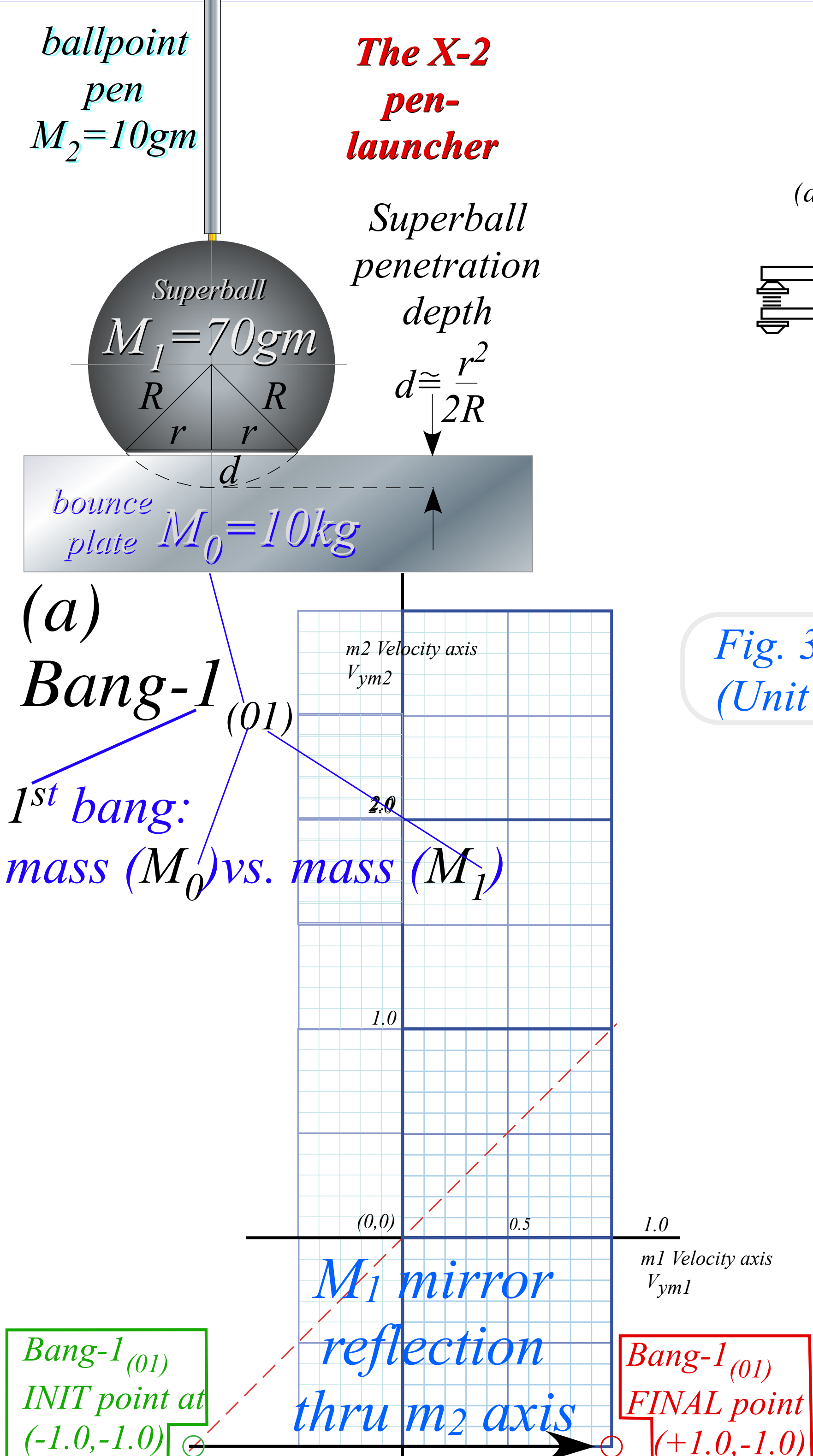


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Fig. 3.3 (Unit 1)

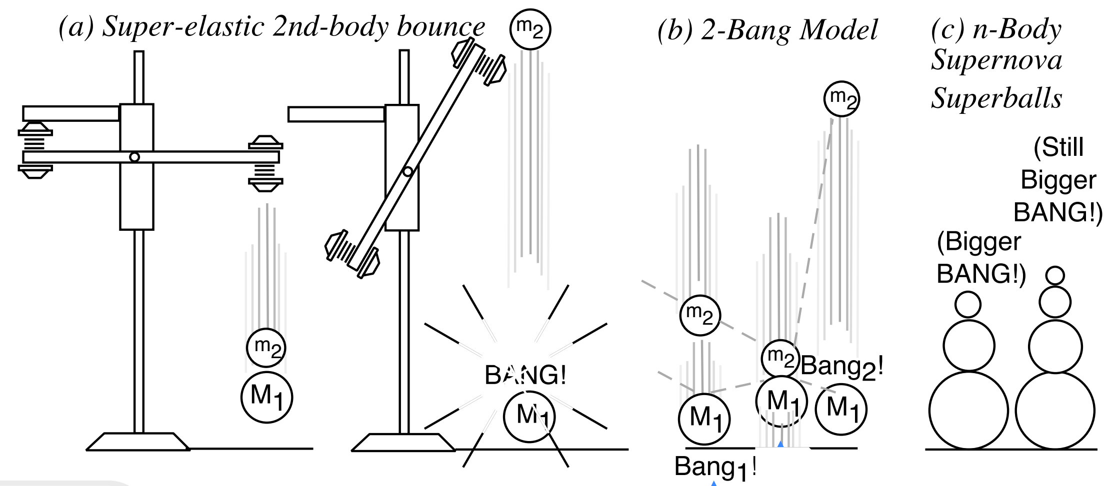
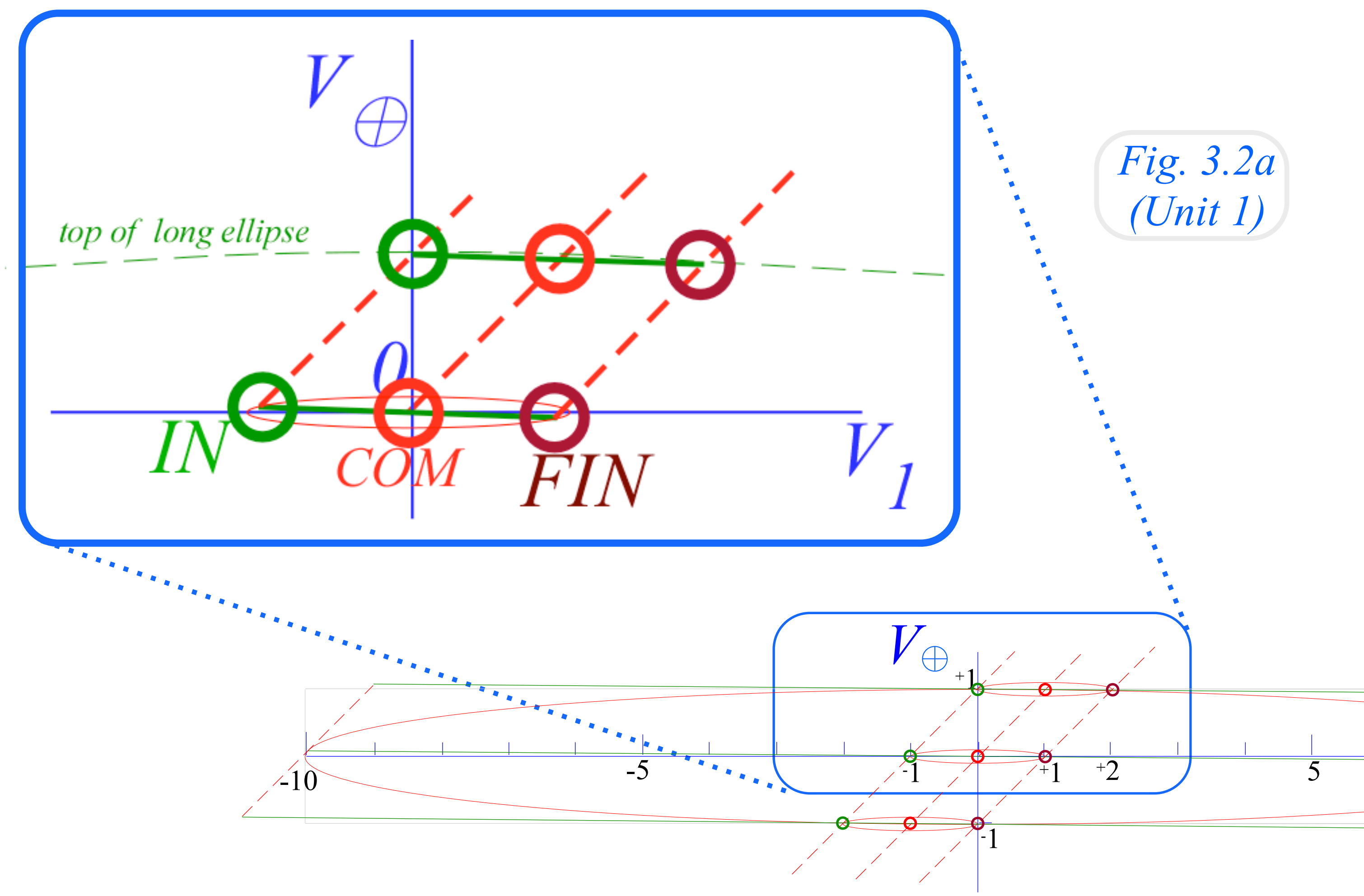


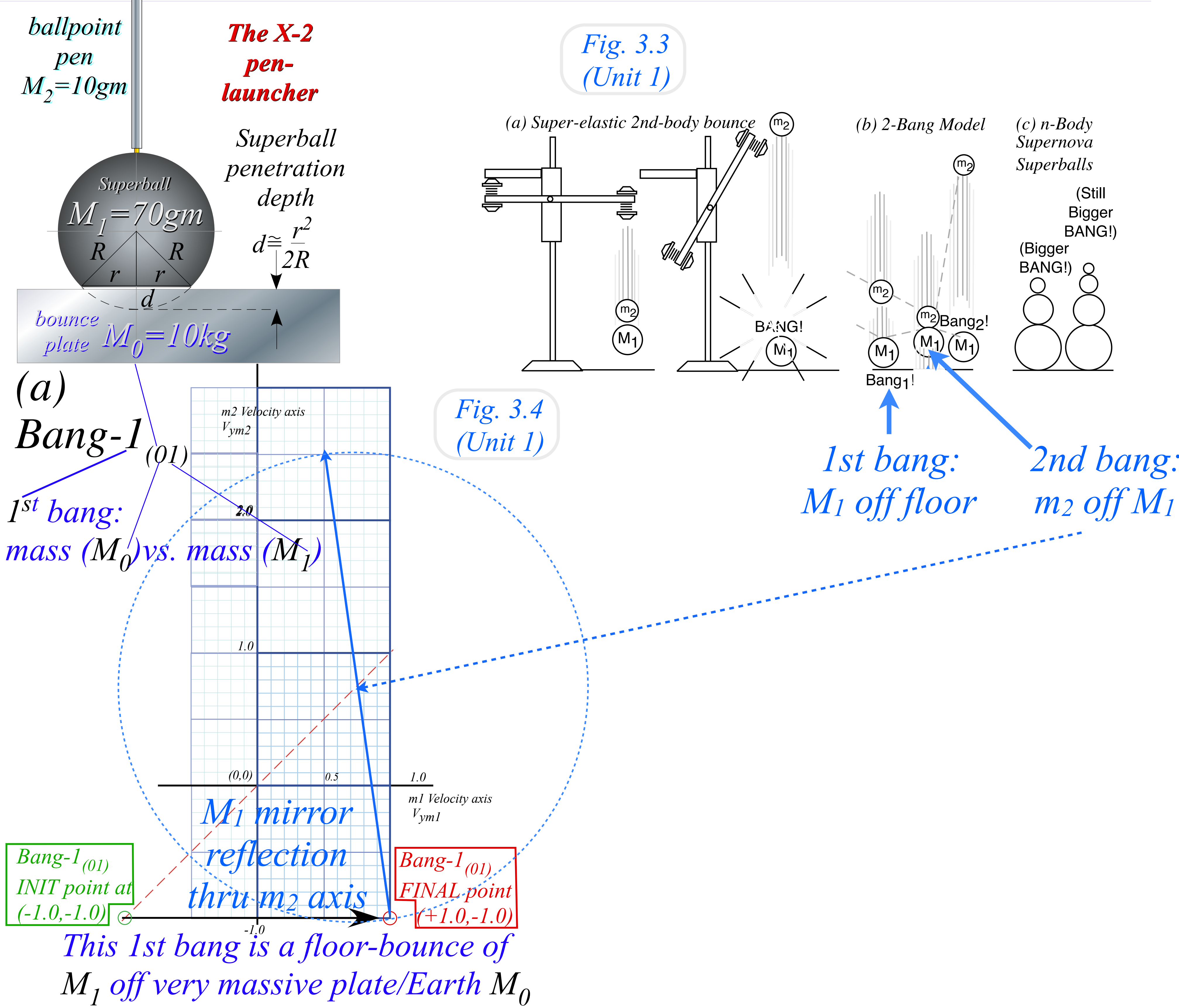
Fig. 3.4 (Unit 1)

1st bang: M_1 off floor

Fig. 3.2a (Unit 1)

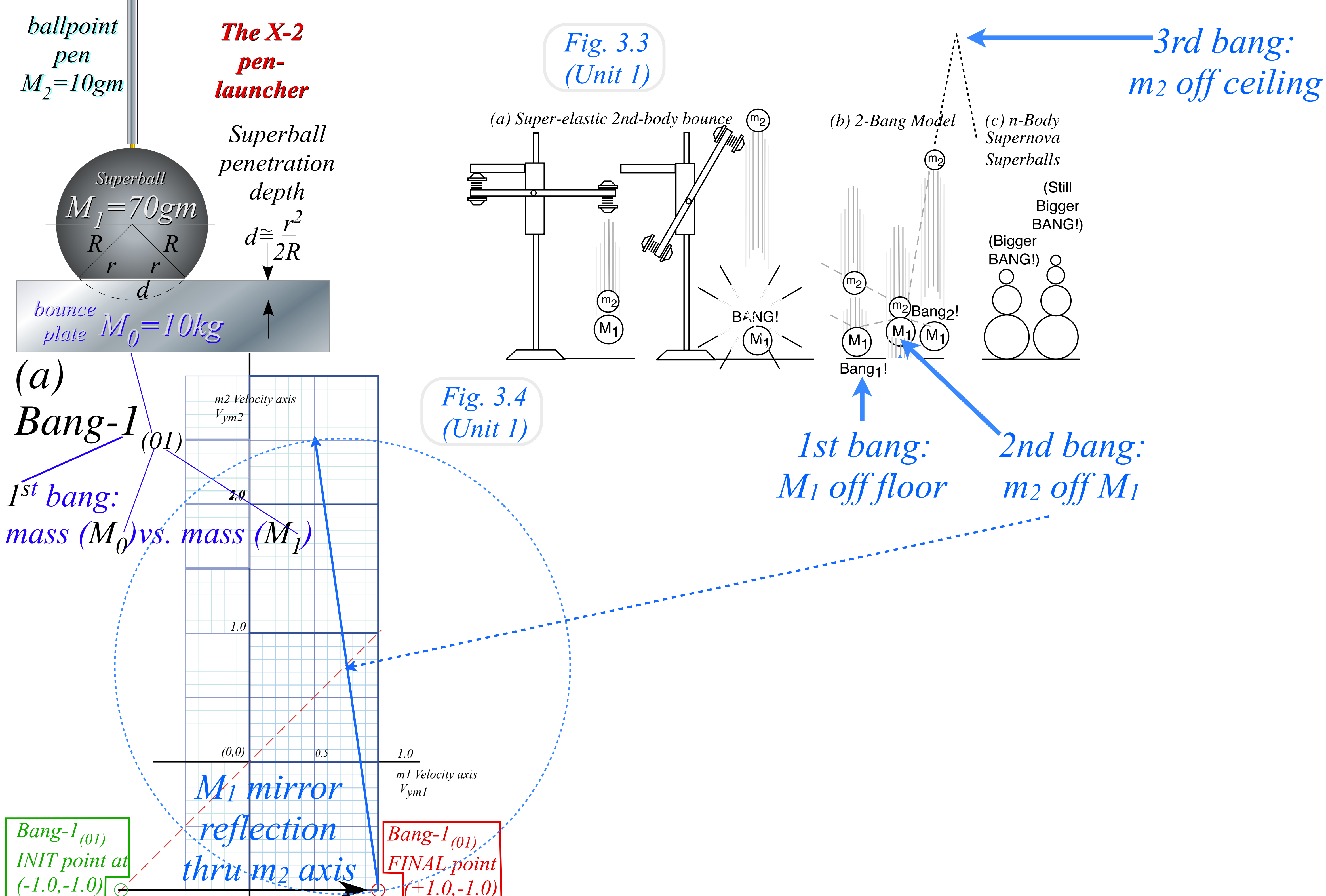


BounceIt web simulation with g=0 and 70:10 mass ratio



BounceIt web simulation with g=0 and 70:10 mass ratio

With non zero g, velocity dependent damping and mass ratio of 70:35



This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

[BounceIt web simulation with g=0 and 70:10 mass ratio]

[With non zero g, velocity dependent damping and mass ratio of 70:35]

Geometry of X2 launcher bouncing in box *(gravity-free)*

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots

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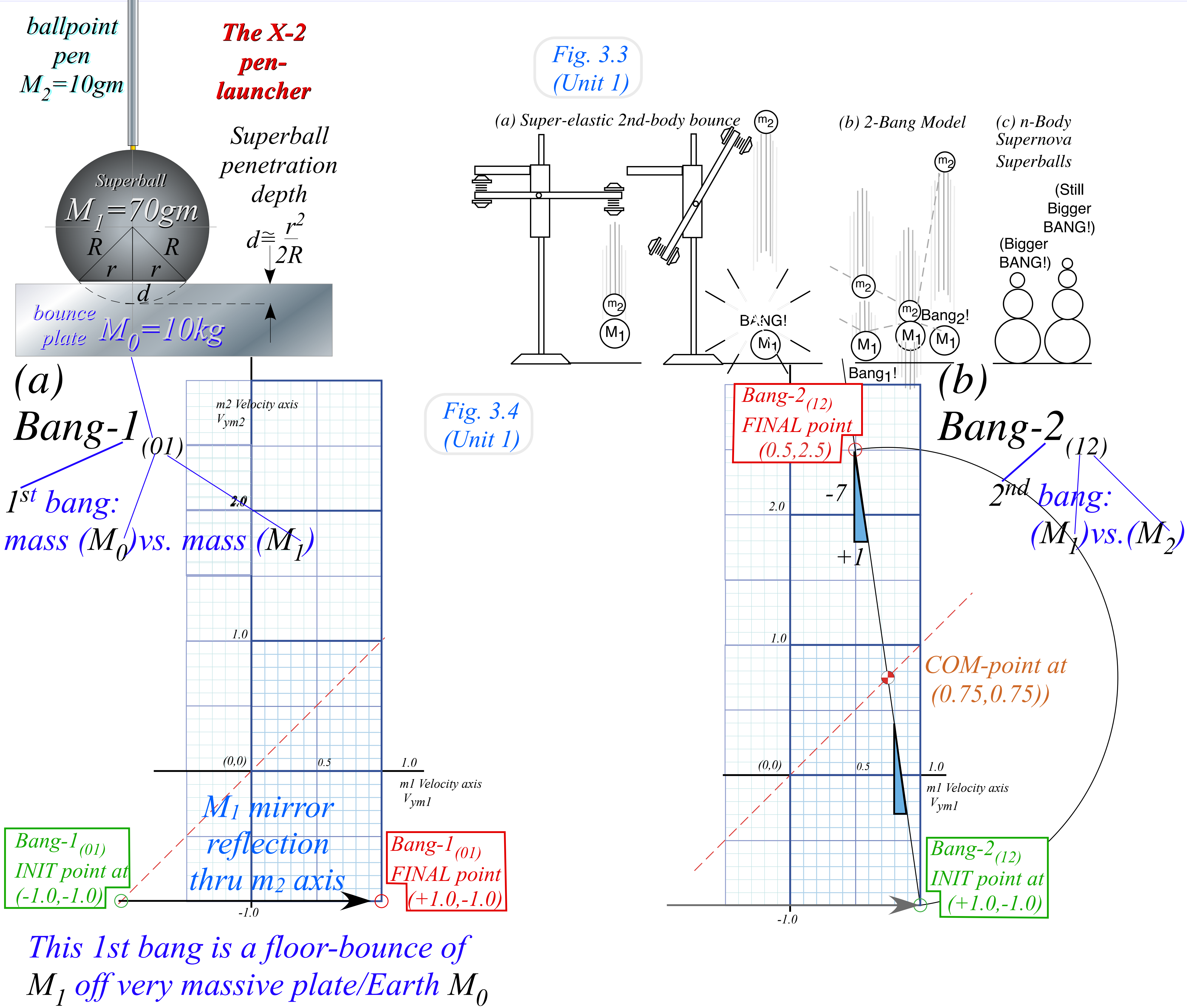

ballpoint pen
M2=10gm
The X-2 pen-launcher
Superball penetration depth
d≅ r2/2R
Superball
M1=70gm
R
R
r
r
d
bounce plate
M0=10kg
(a)
Bang-1(01)
1st bang: mass (M0)vs. mass (M1)
m2 Velocity axis
Vym2
2.0
1.0
(0,0)
0.5
1.0
m1 Velocity axis
Vym1
-1.0
M1 mirror reflection thru m2 axis
Bang-1(01) INIT point at (-1.0,-1.0)
Bang-1(01) FINAL point (+1.0,-1.0)
This 1st bang is a floor-bounce of M1 off very massive plate/Earth M0
Fig. 3.3 (Unit 1)
(a) Super-elastic 2nd-body bounce
(b) 2-Bang Model
(c) n-Body Supernova Superballs
m2
M1
BANG!
Bang2!
Bang1!
(Still Bigger BANG!)
(Bigger BANG!)
Fig. 3.4 (Unit 1)
(b)
Bang-2(12)
2nd bang: (M1)vs.(M2)
Bang-2(12) FINAL point (0.5,2.5)
-7
+1
COM-point at (0.75,0.75))
Bang-2(12) INIT point at (+1.0,-1.0)

Geometry of X2 launcher bouncing in box *(gravity-free)*

Independent Bounce Model (IBM)

→ *Geometric optimization and range-of-motion calculation(s)*

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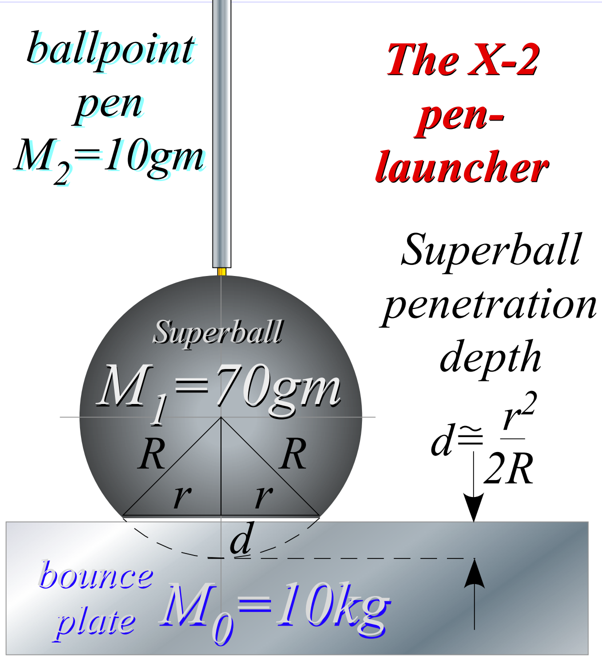
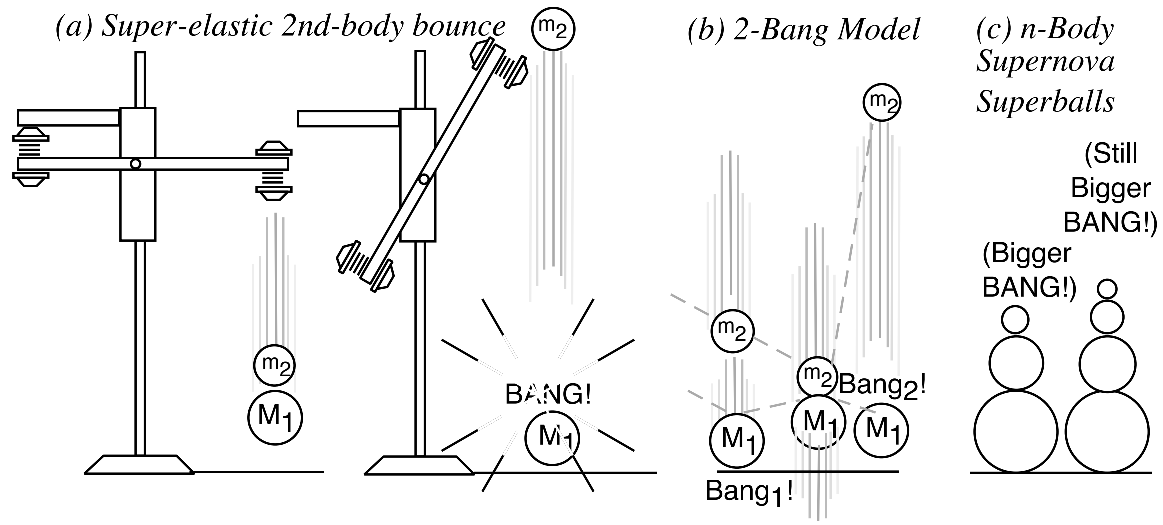


Fig. 3.3 (Unit 1)



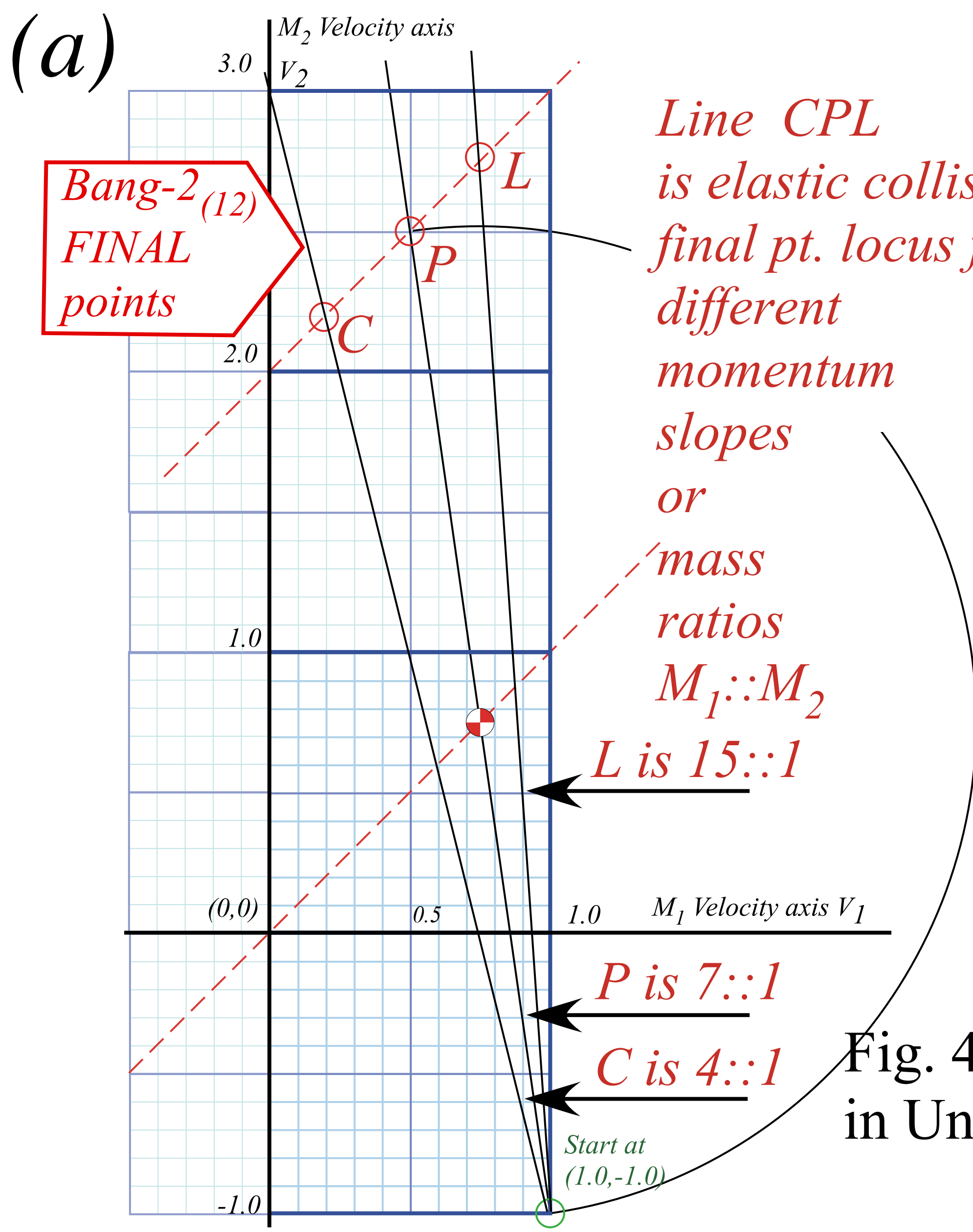


Fig. 4.5a in Unit 1

[BounceIt web simulation with g=0 and 70:10 mass ratio]

[With non zero g, velocity dependent damping and mass ratio of 70:35]

http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html?smallMass=1&massRatio=100&scenario=7000

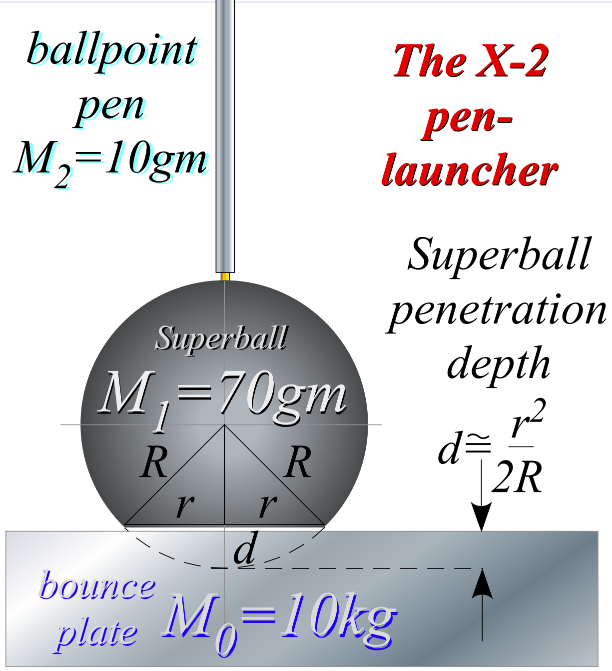
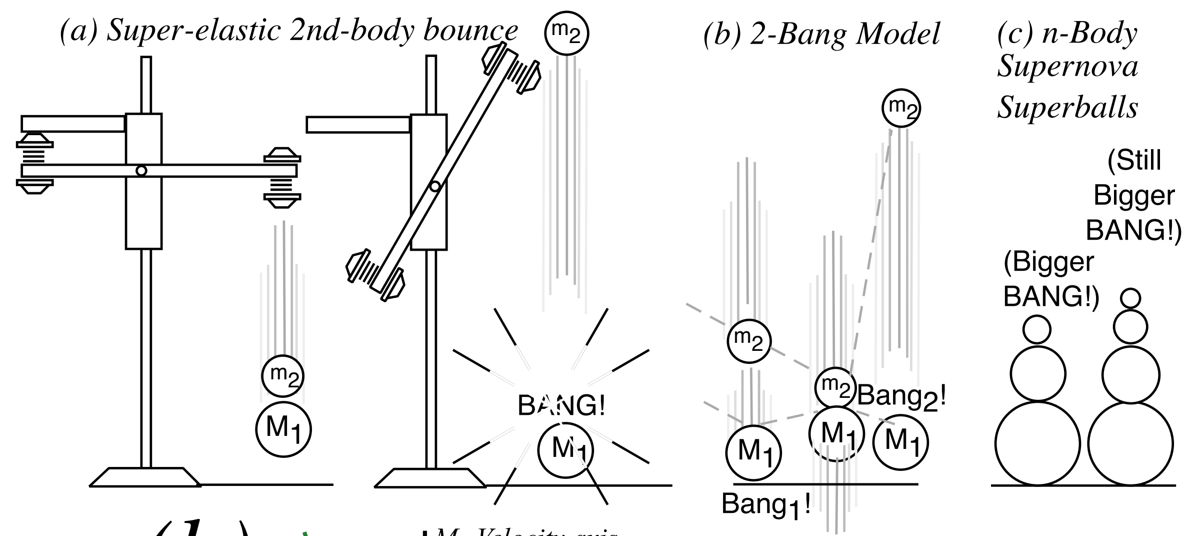


Fig. 3.3 (Unit 1)



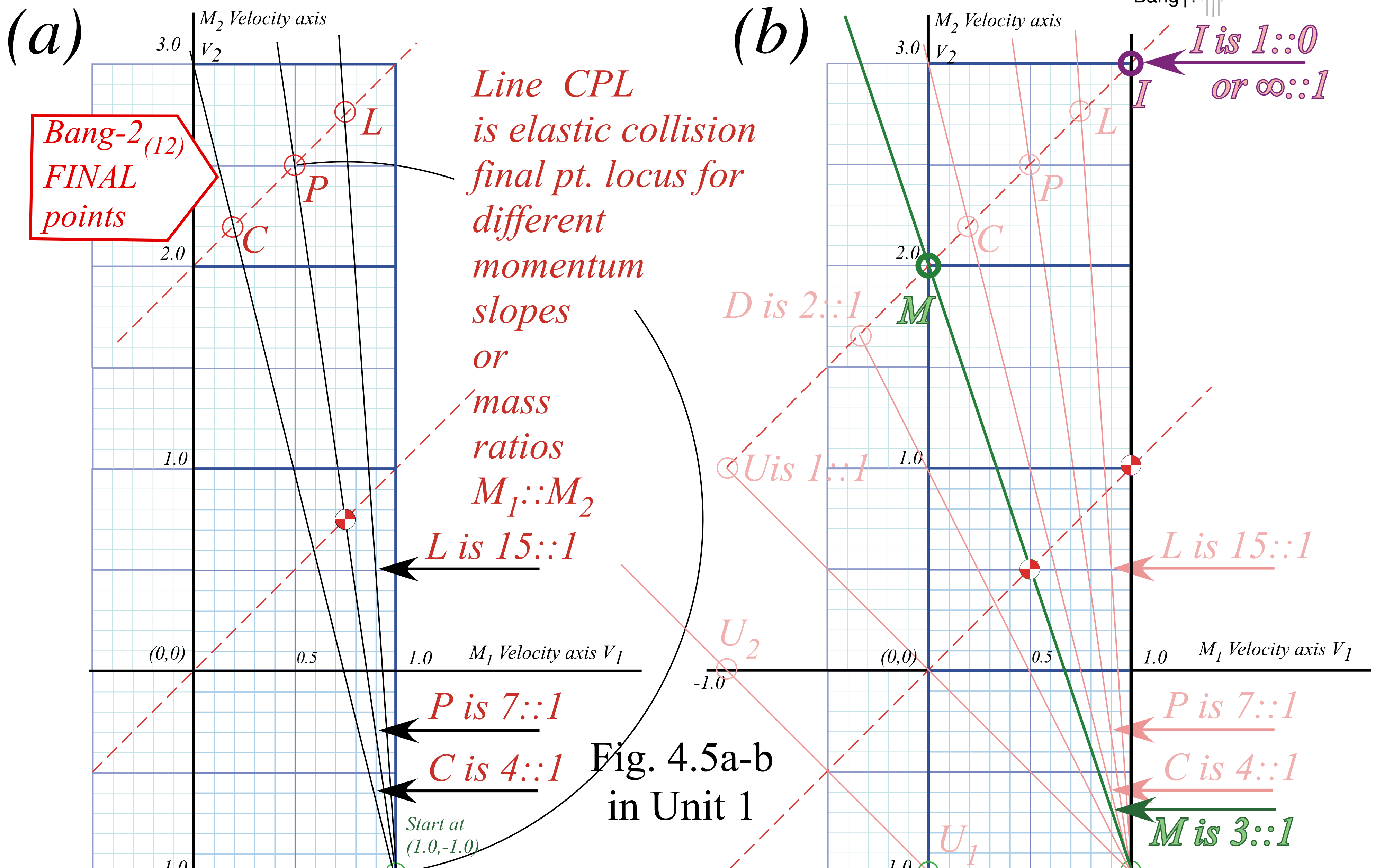


Fig. 4.5a-b in Unit 1

BounceIt web simulation with g=0 and 70:10 mass ratio

With non zero g, velocity dependent damping and mass ratio of 70:35

Geometry of X2 launcher bouncing in box *(gravity-free)*

Independent Bounce Model (IBM)

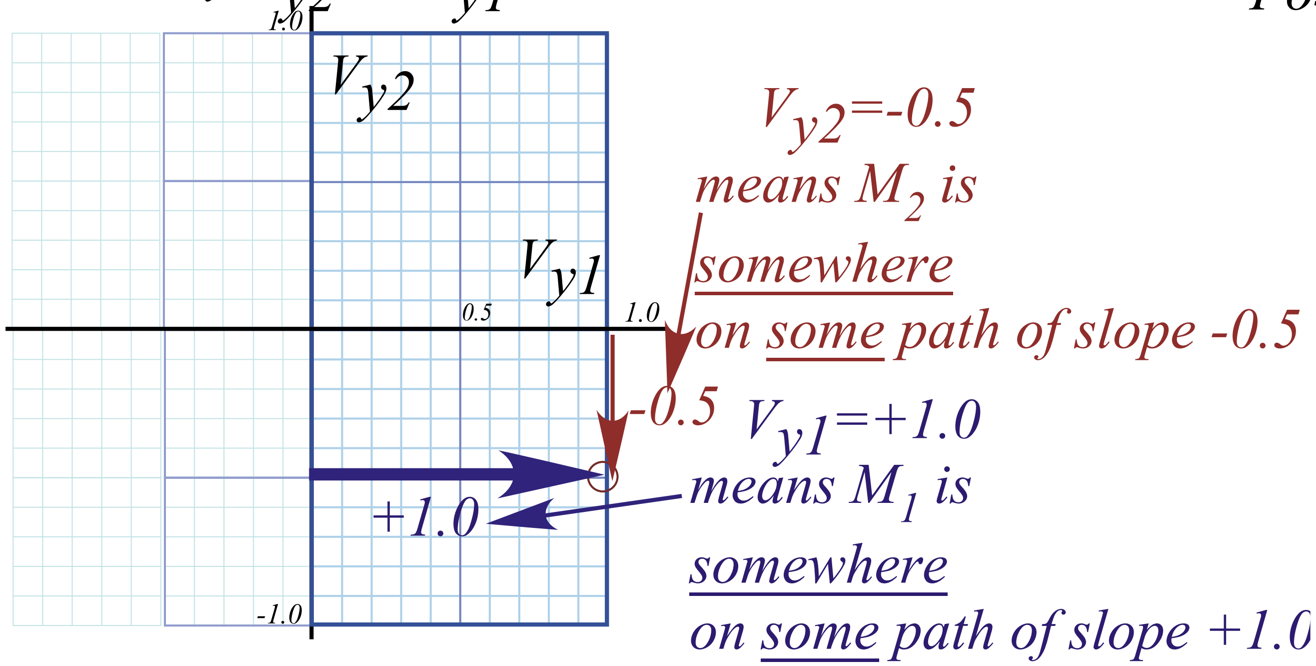
Geometric optimization and range-of-motion calculation(s)

→ *Integration of* (V_1,V_2) *data to space-time plots* $(y_1(t),t)$ *and* $(y_2(t),t)$ *plots*

Integration of (V_1,V_2) *data to space-space plots* (y_1, y_2) *Examples* $(M_1=7, M_2=1)$ *and* $(M_1=49, M_2=1)$

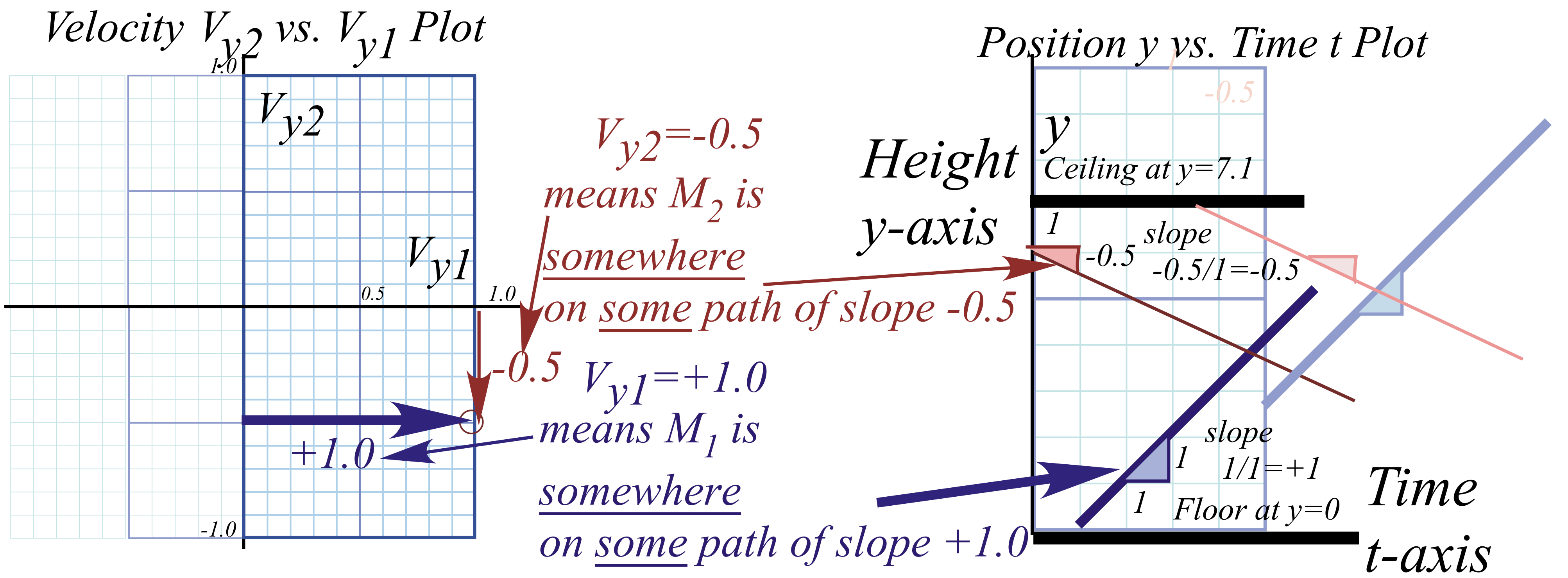
Geometric "Integration" (Converting Velocity data to Spacetime)

Velocity V_{y2} vs. V_{y1} Plot



Position y vs. Time t Plot

Geometric "Integration" (Converting Velocity data to Spacetime)



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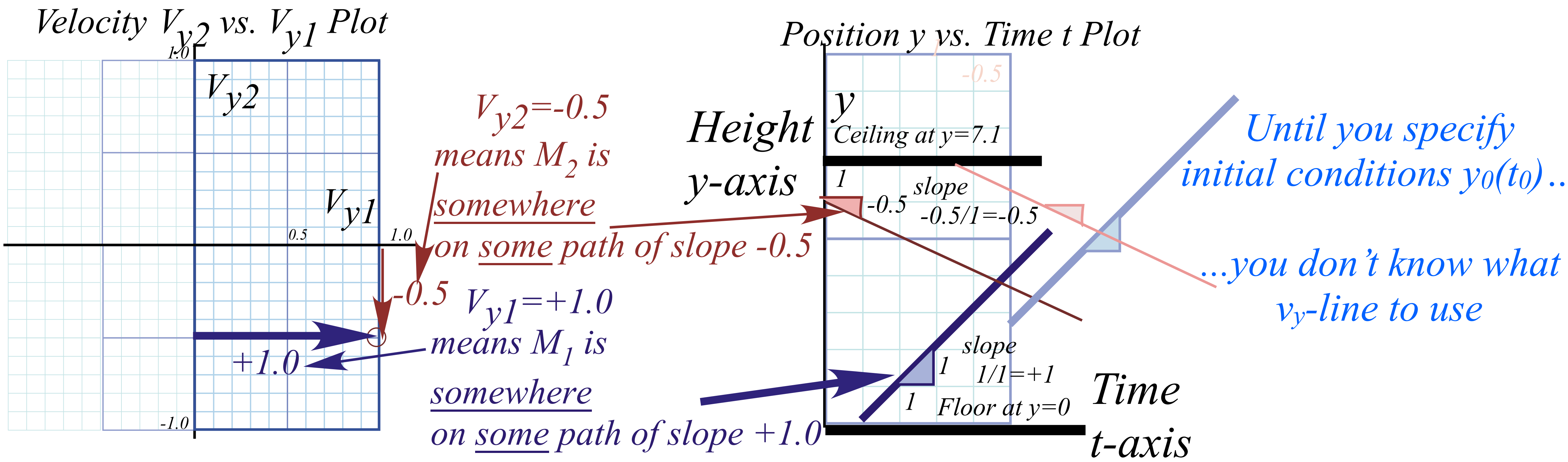

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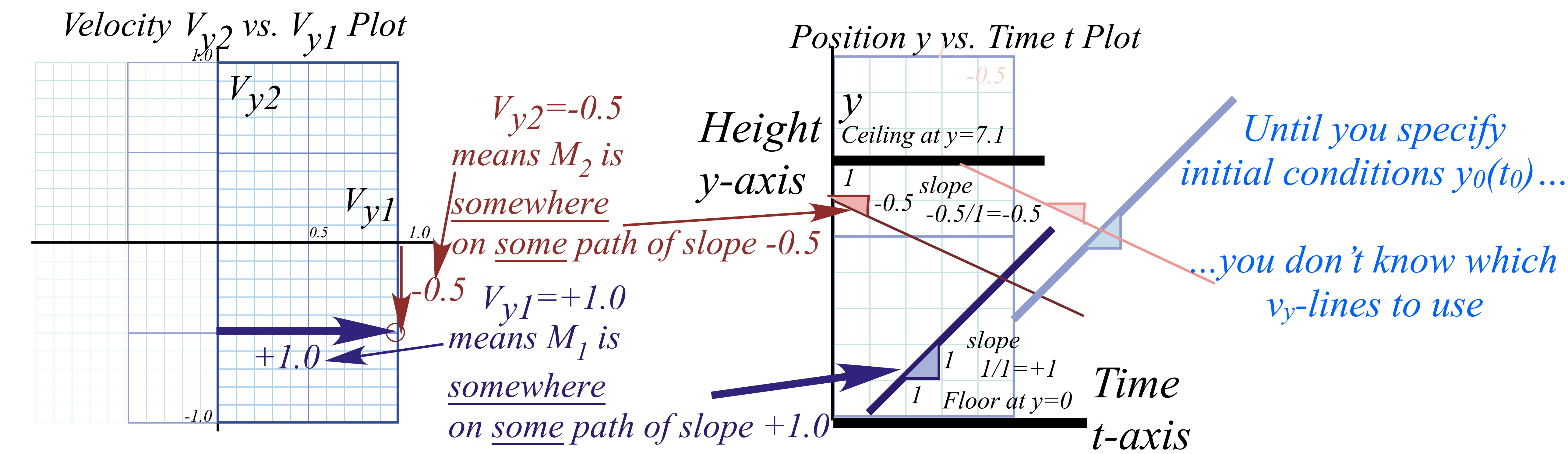
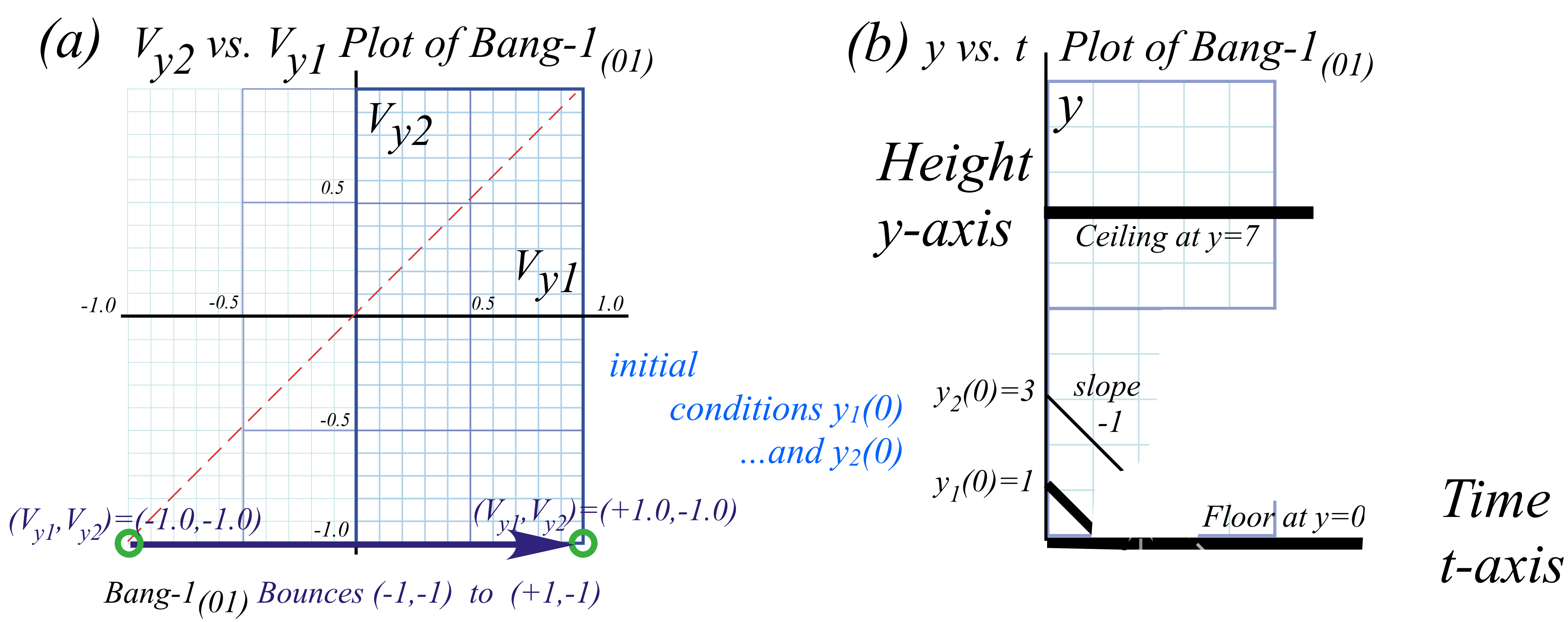


Fig. 4.6a-b
in Unit 1



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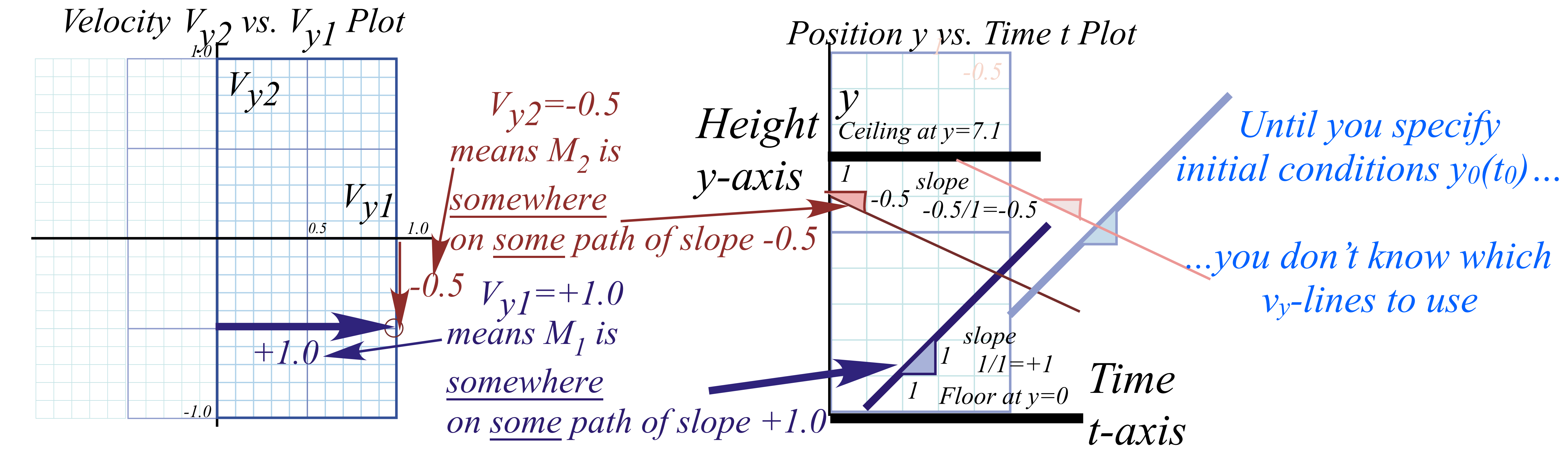

Fig. 4.6a-b
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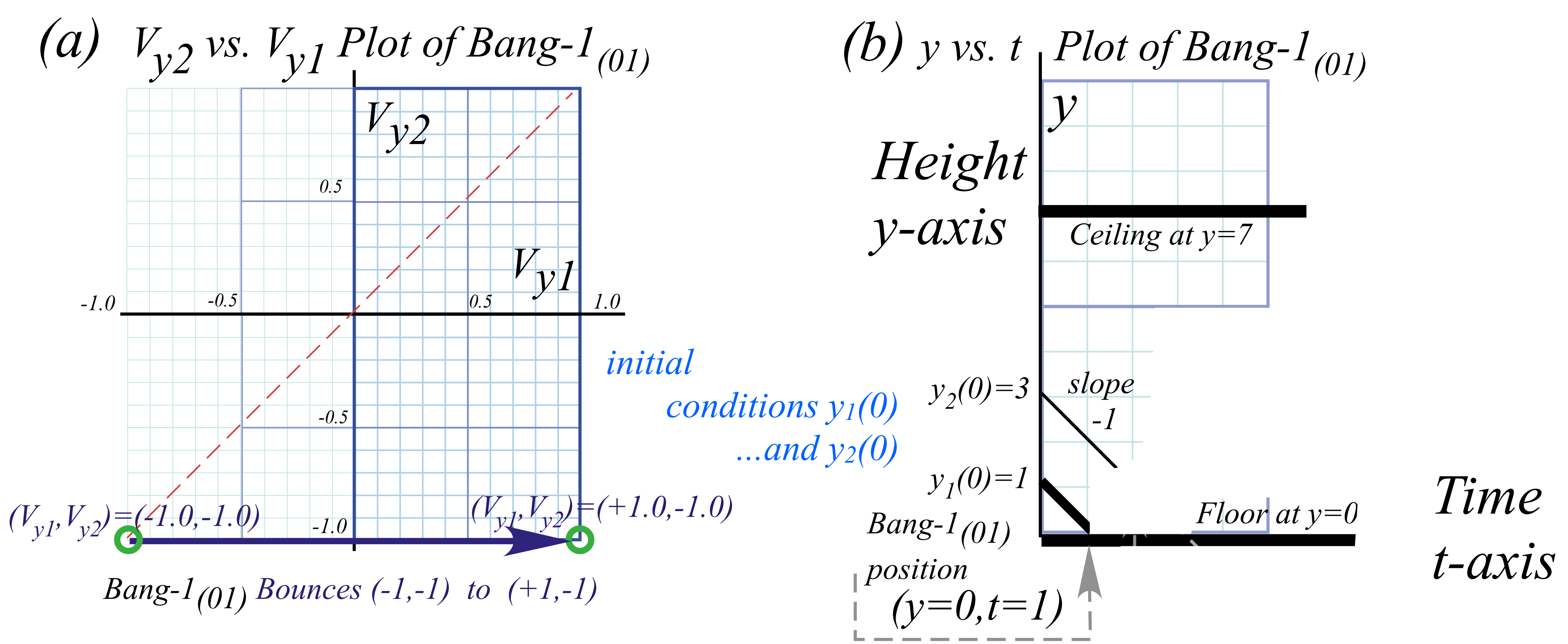

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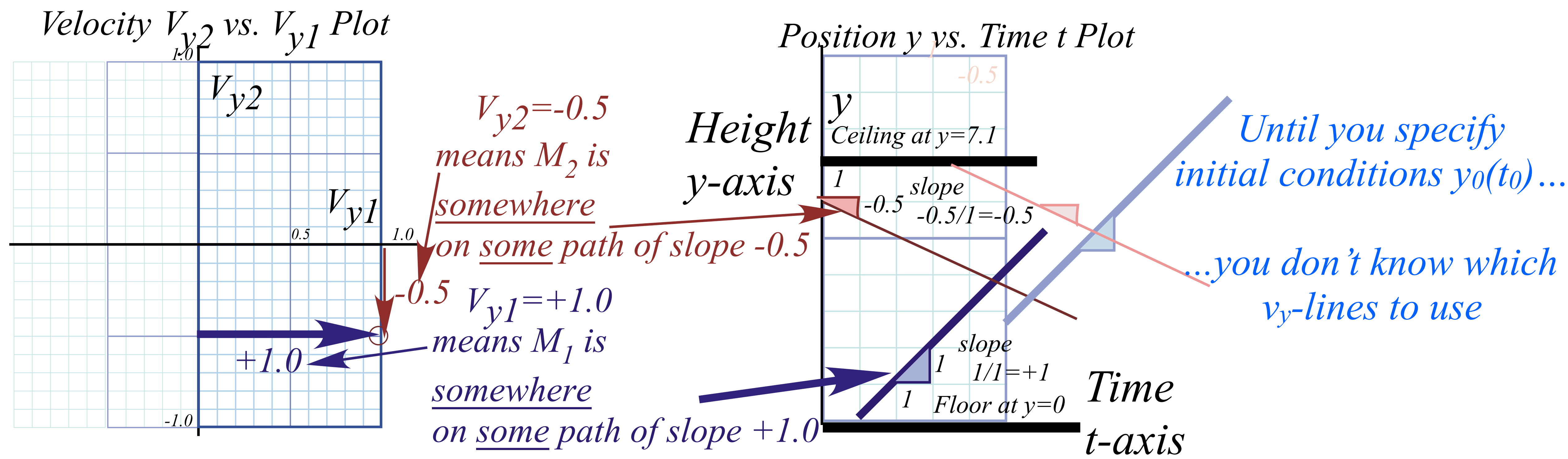
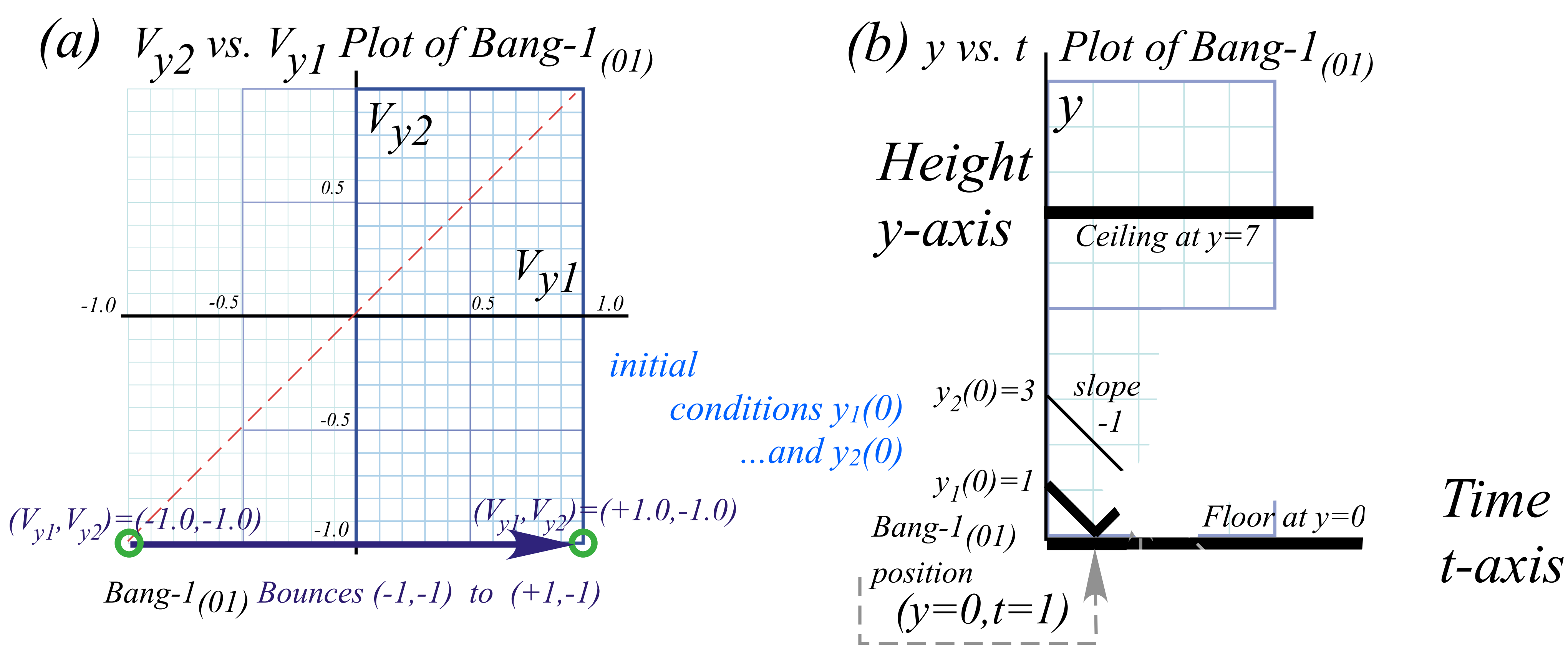


Fig. 4.6a-b
in Unit 1



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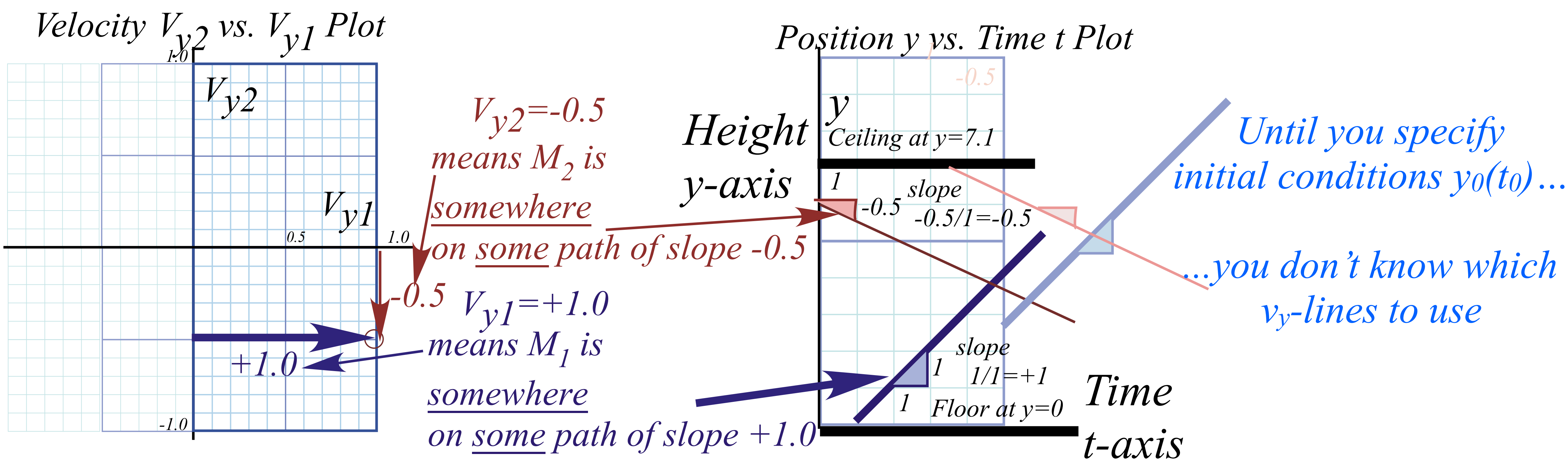

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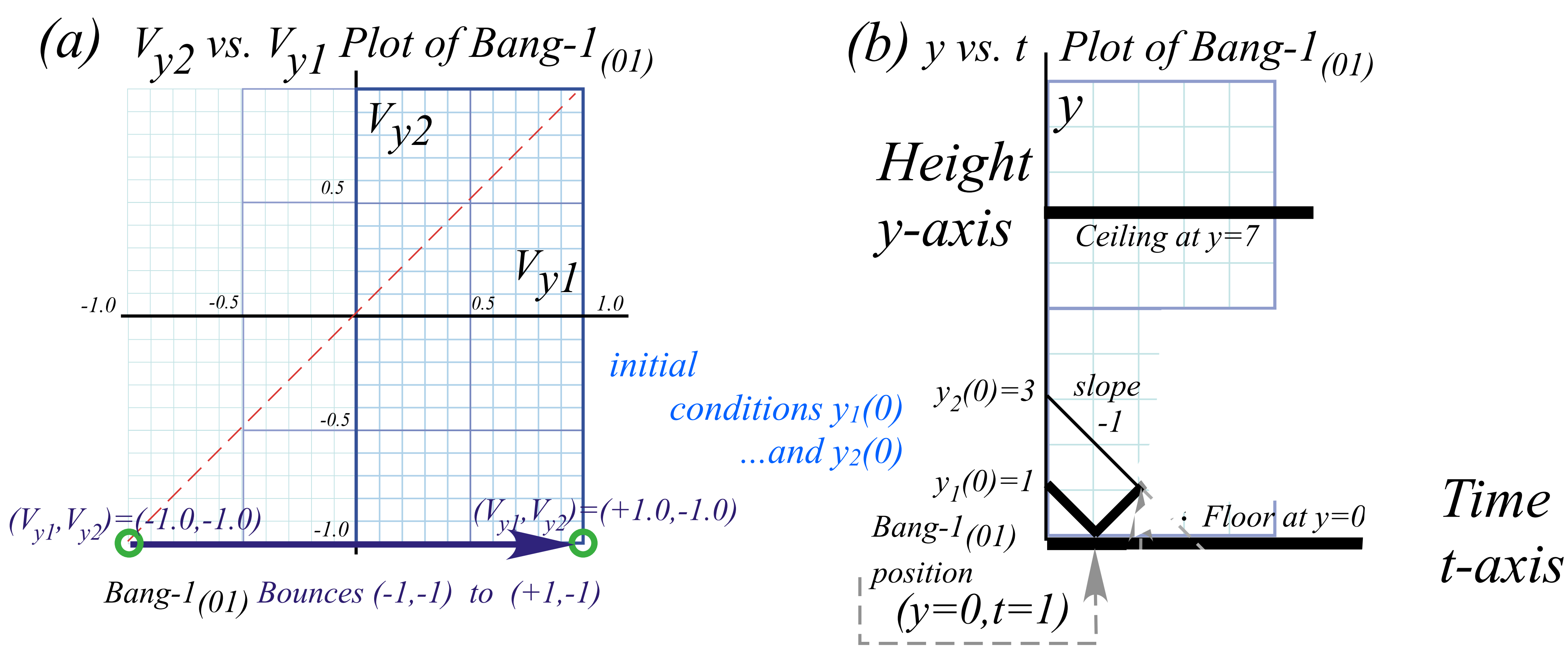

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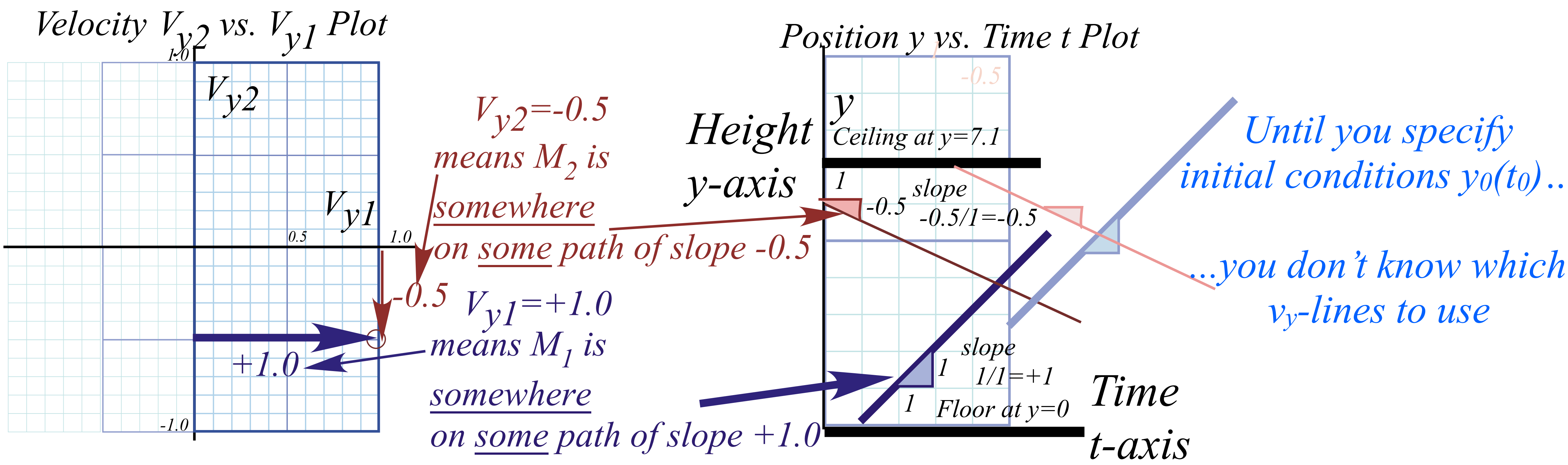
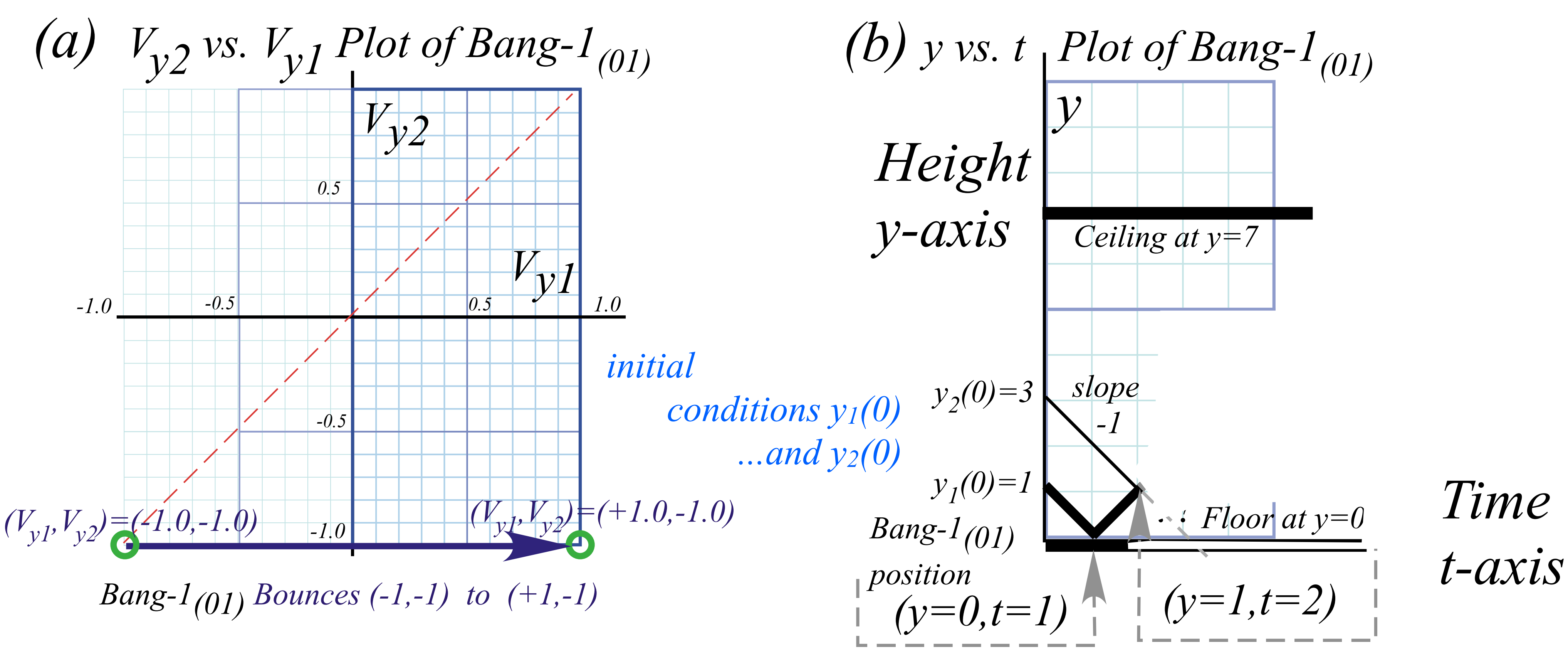


Fig. 4.6a-b
in Unit 1



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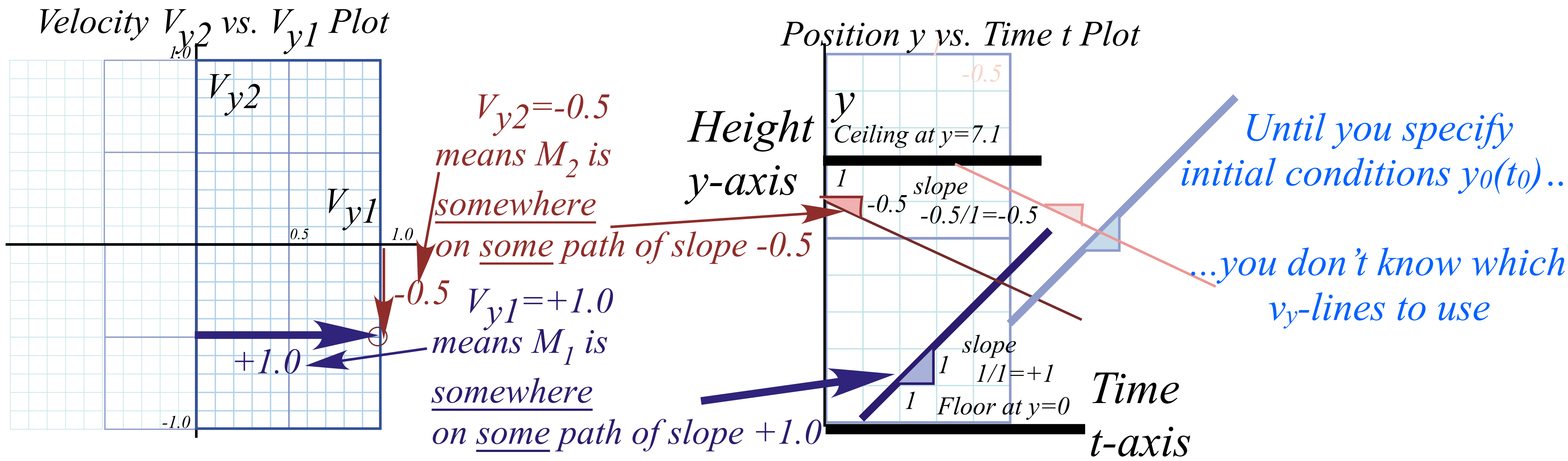

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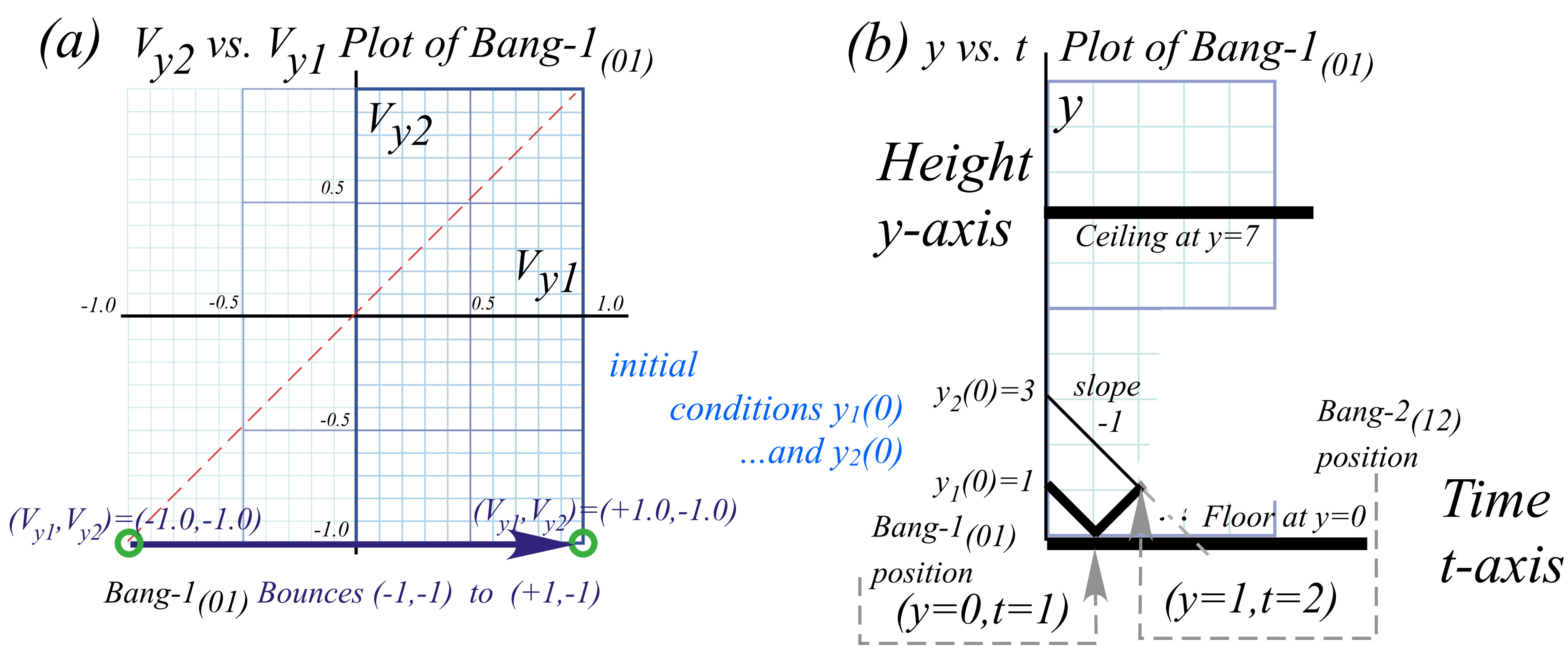

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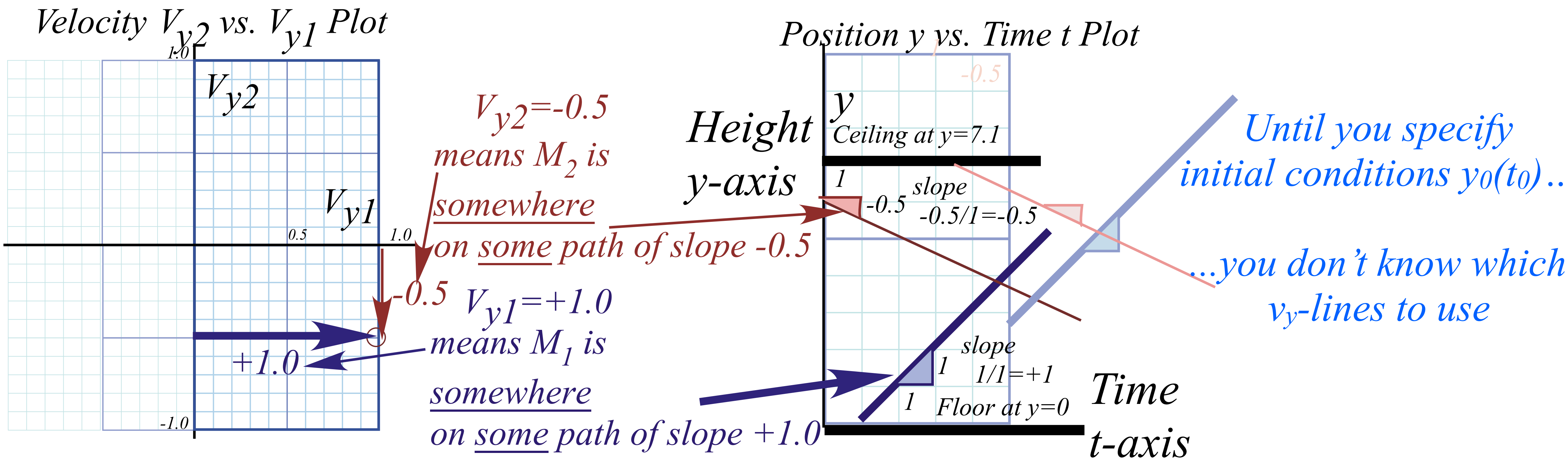

Fig. 4.6a-b
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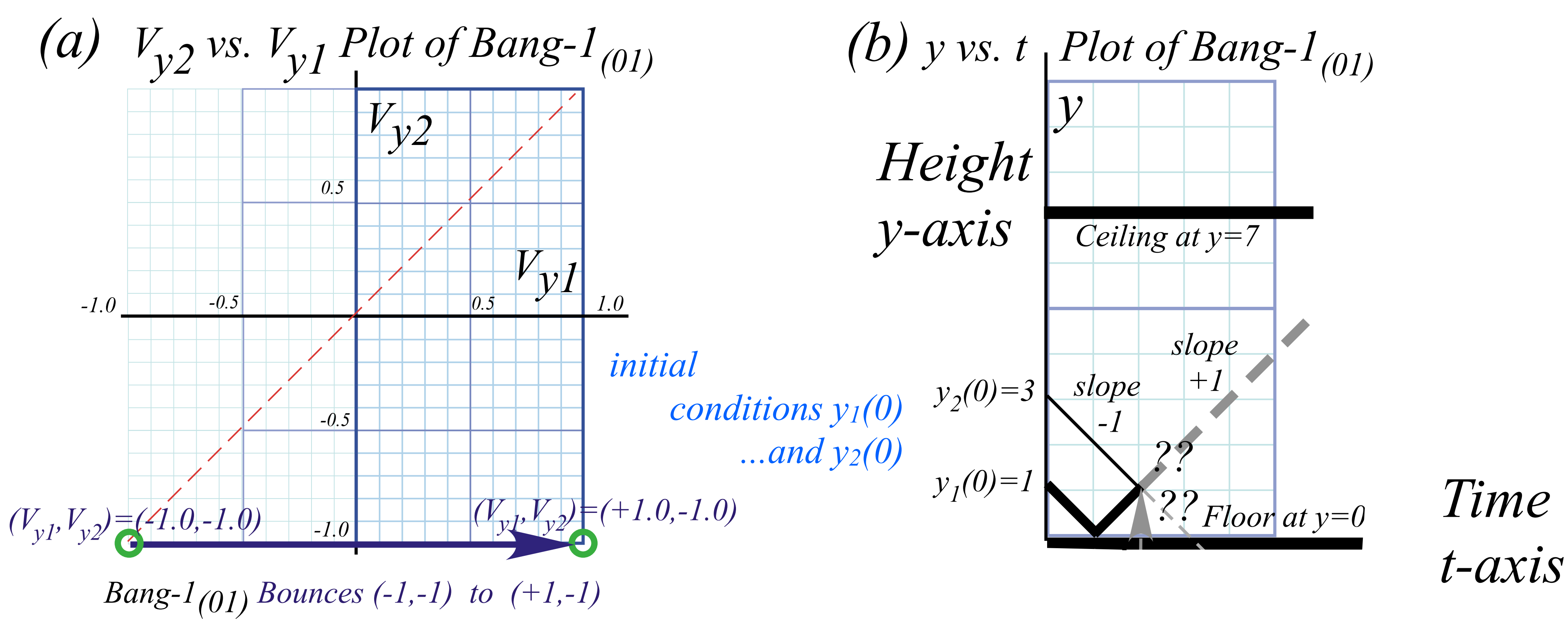

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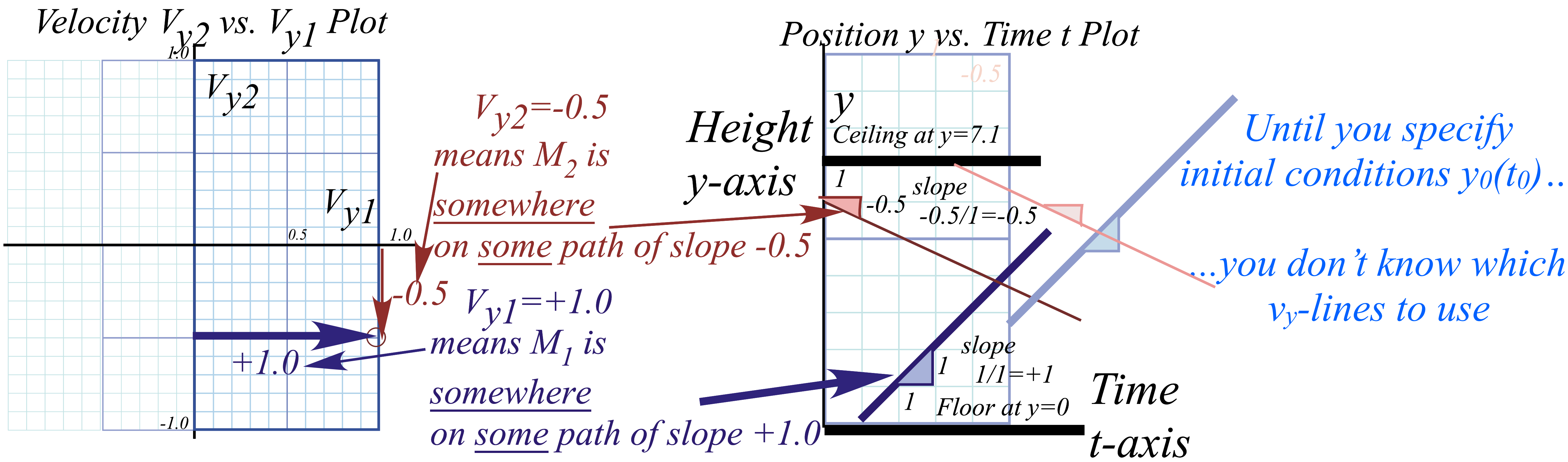

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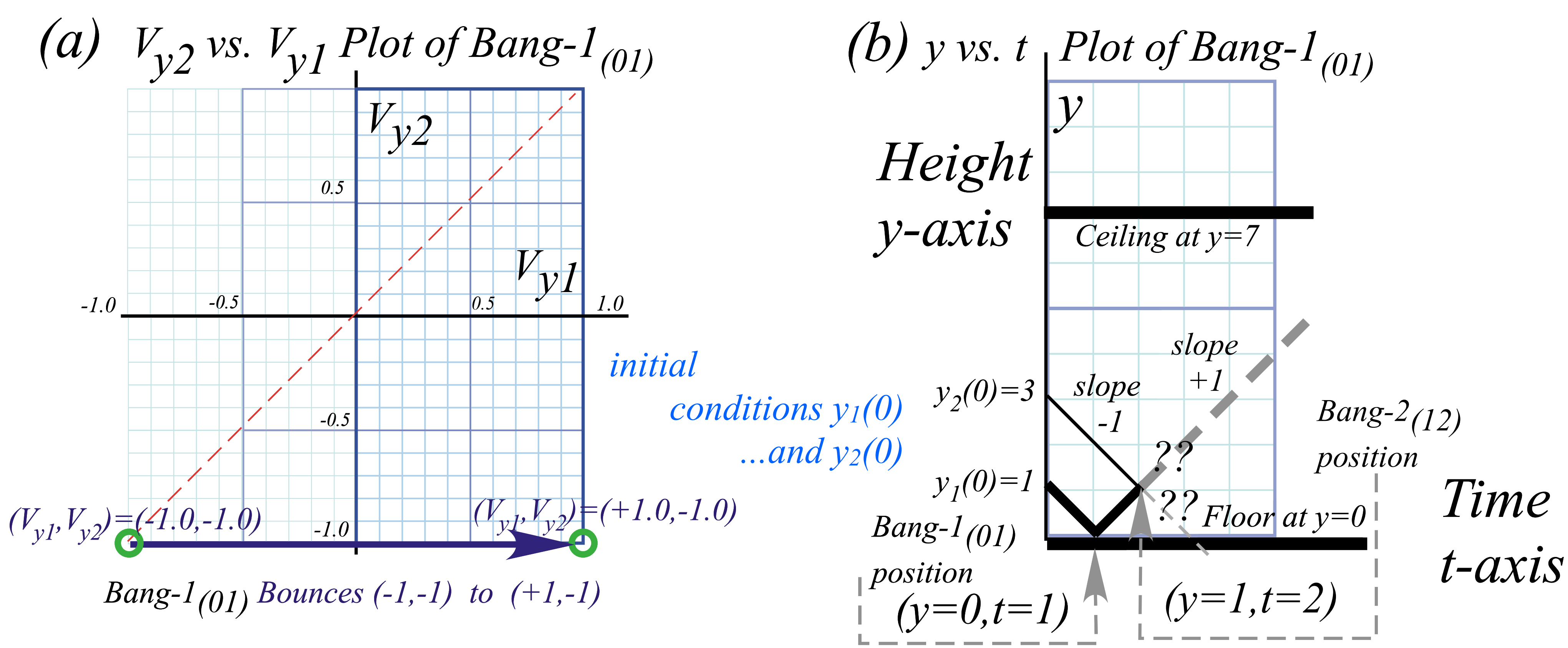
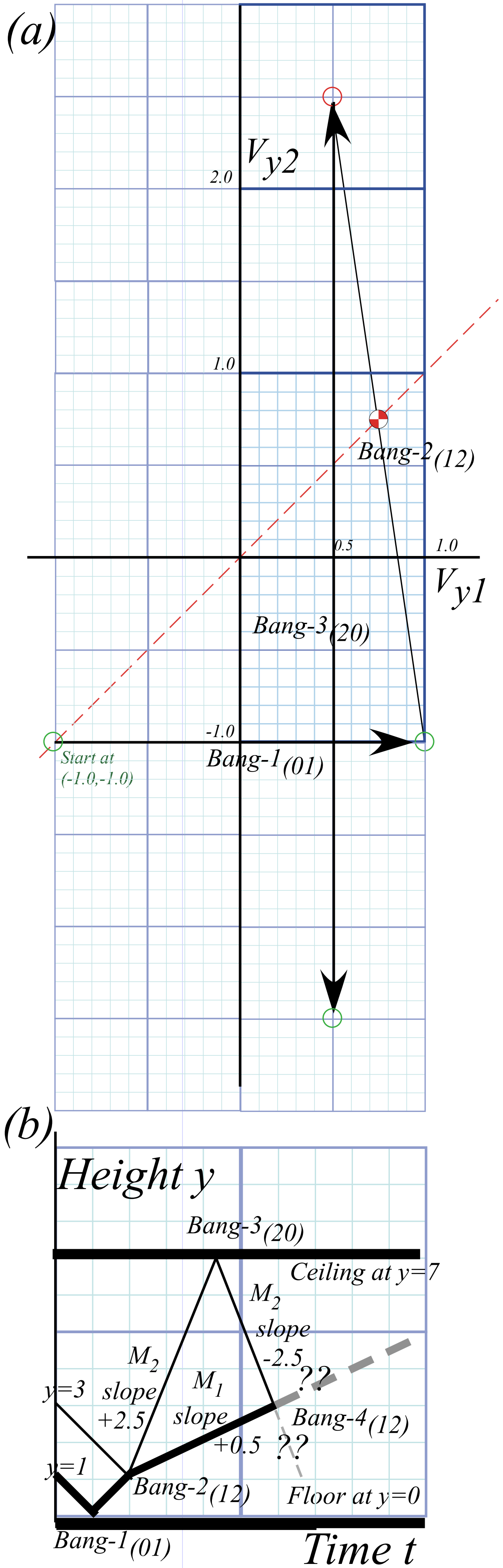
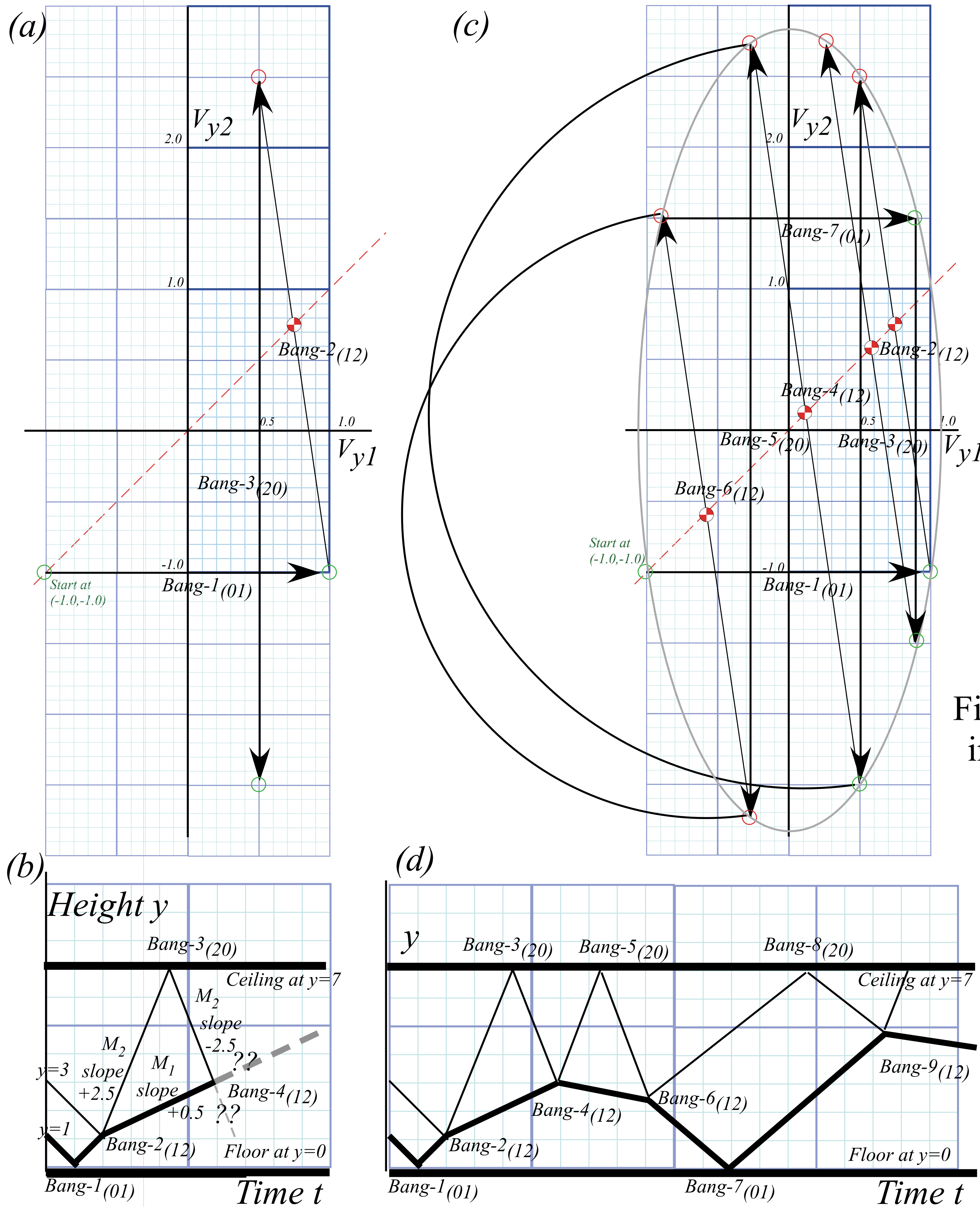
in Unit 1



Geometric "Integration" (Converting Velocity data to Spacetime)



Geometric "Integration" (Converting Velocity data to Spacetime)



Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{1}{2} + \frac{7}{2} = 4$$

$$1 = \frac{V_1^2}{2KE / M_1} + \frac{V_2^2}{2KE / M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Fig. 4.7a-d in Unit 1

Geometric "Integration" (Converting Velocity data to Spacetime)

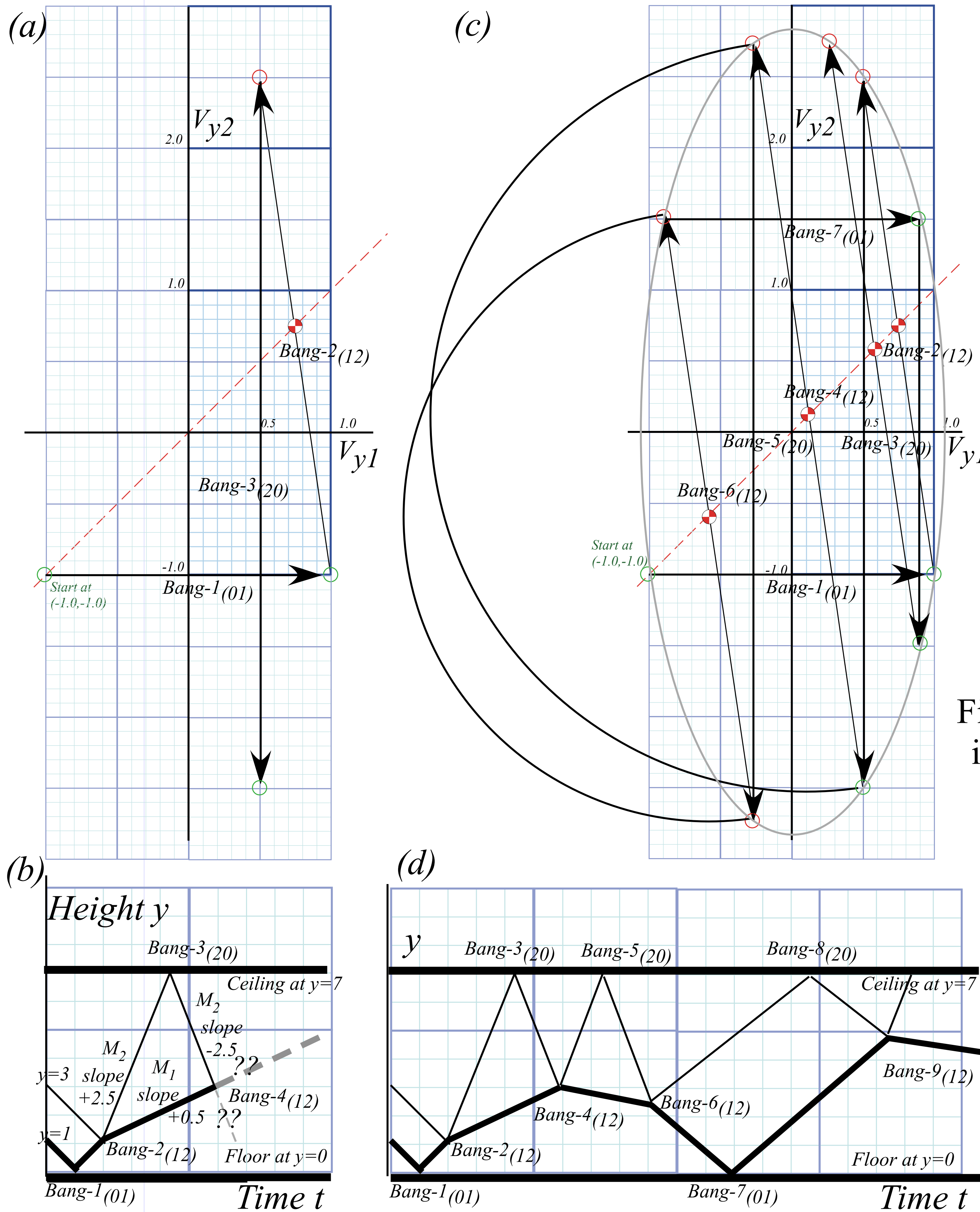


Fig. 4.7a-d in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

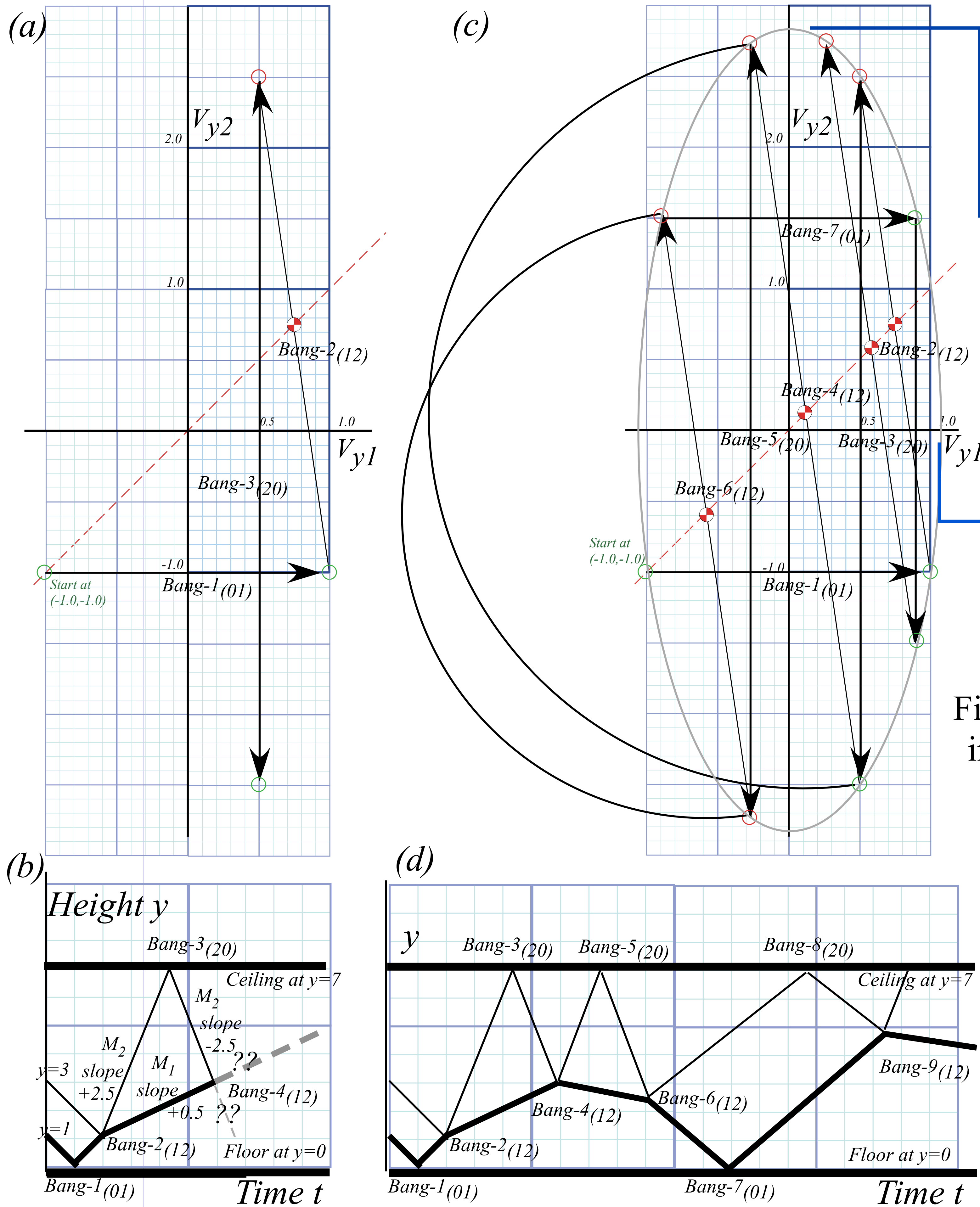
Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1}$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_2}$$

Geometric "Integration" (Converting Velocity data to Spacetime)



Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE/M_1} + \frac{V_2^2}{2KE/M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_2} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Fig. 4.7a-d
in Unit 1

Geometric "Integration" (Converting Velocity data to Spacetime)

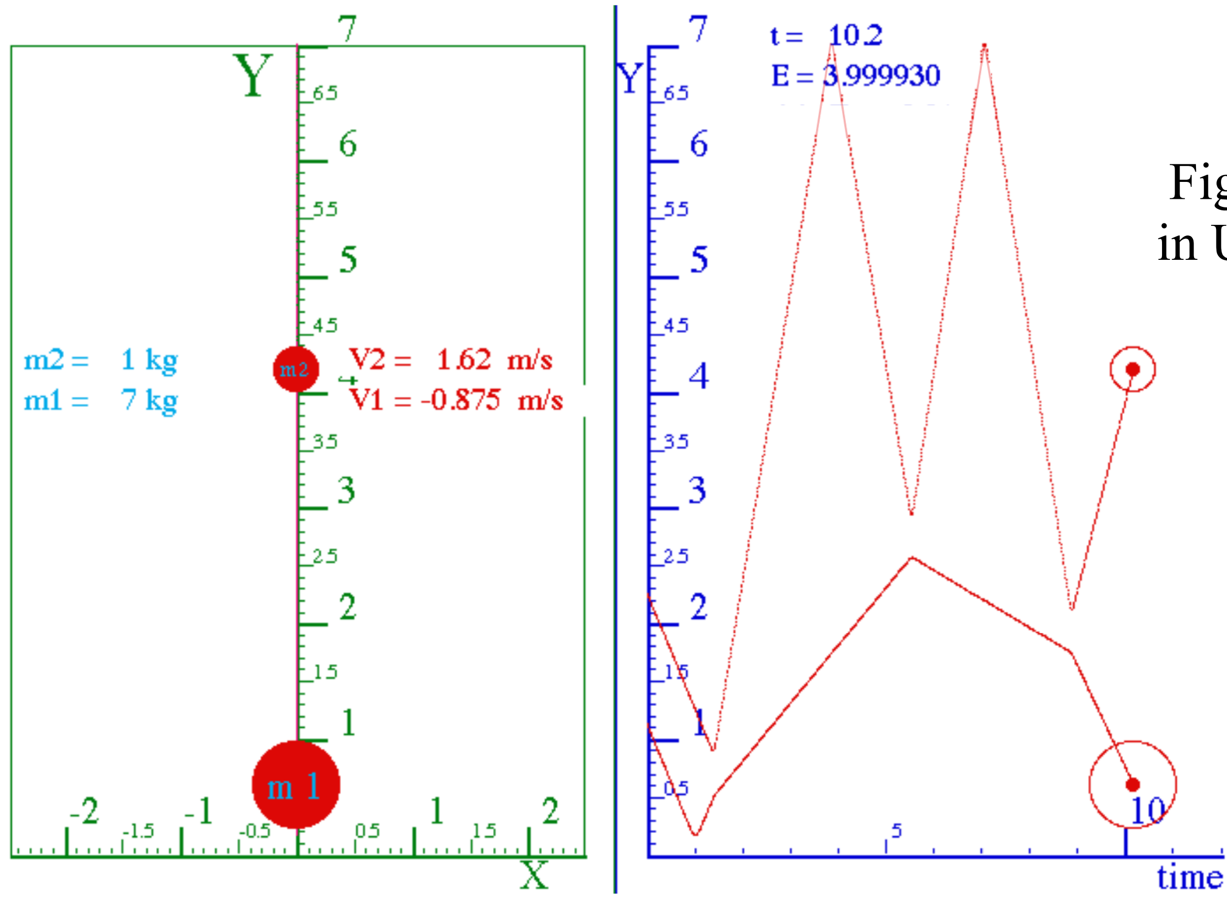


Fig. 4.8
in Unit 1

BounceIt Superball Collision Web Simulator:
[M₁=70, M₂=10 with Newtonian time plot]

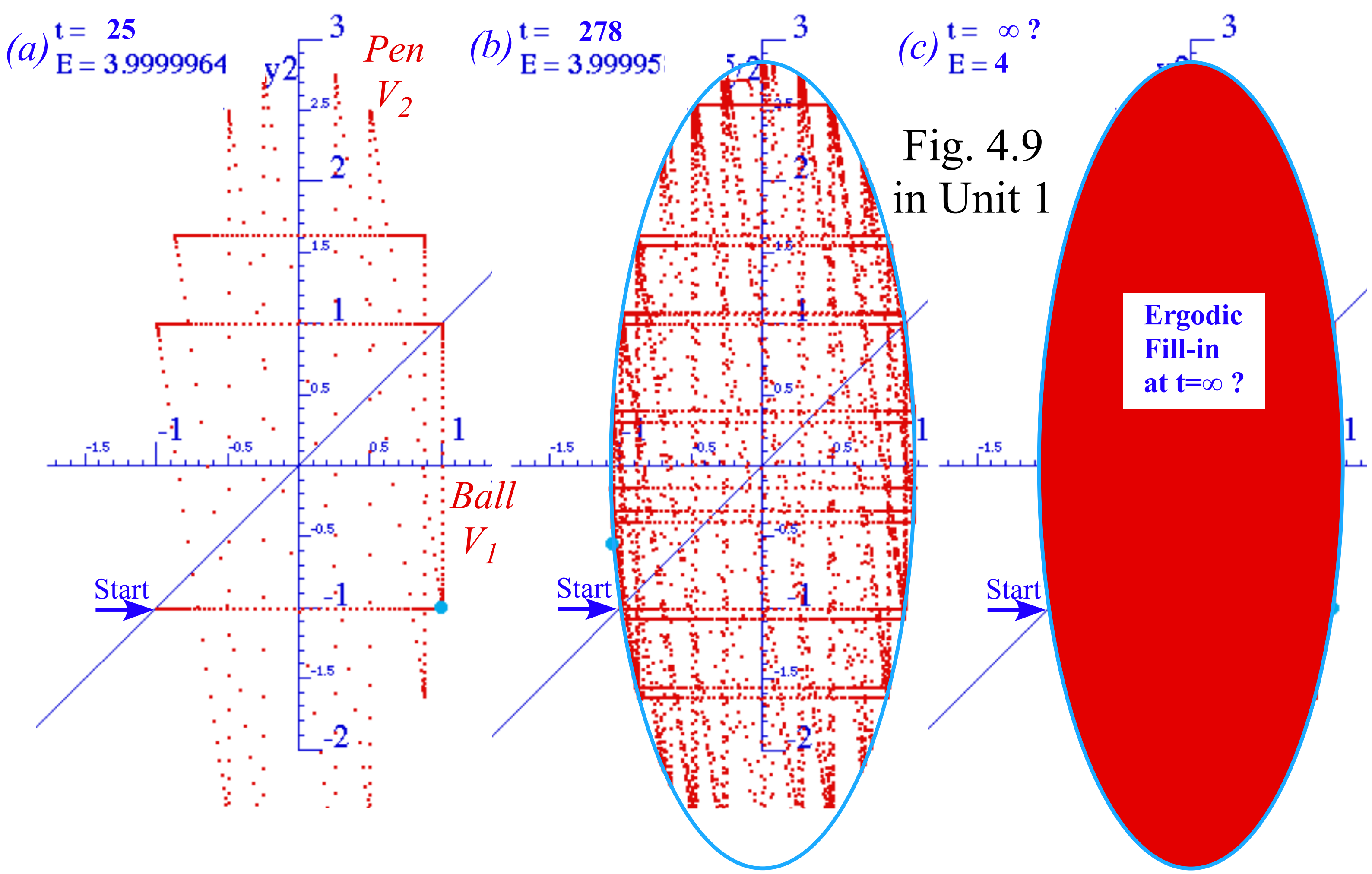


Fig. 4.9
in Unit 1

BounceIt Superball Collision Web Simulator:
[M_1=70, M_2=10 with V_2 vs V_1 plot]

Geometry of X2 launcher bouncing in box *(gravity-free)*

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1,V_2) data to space-time plots $(y_1(t),t)$ and $(y_2(t),t)$ plots

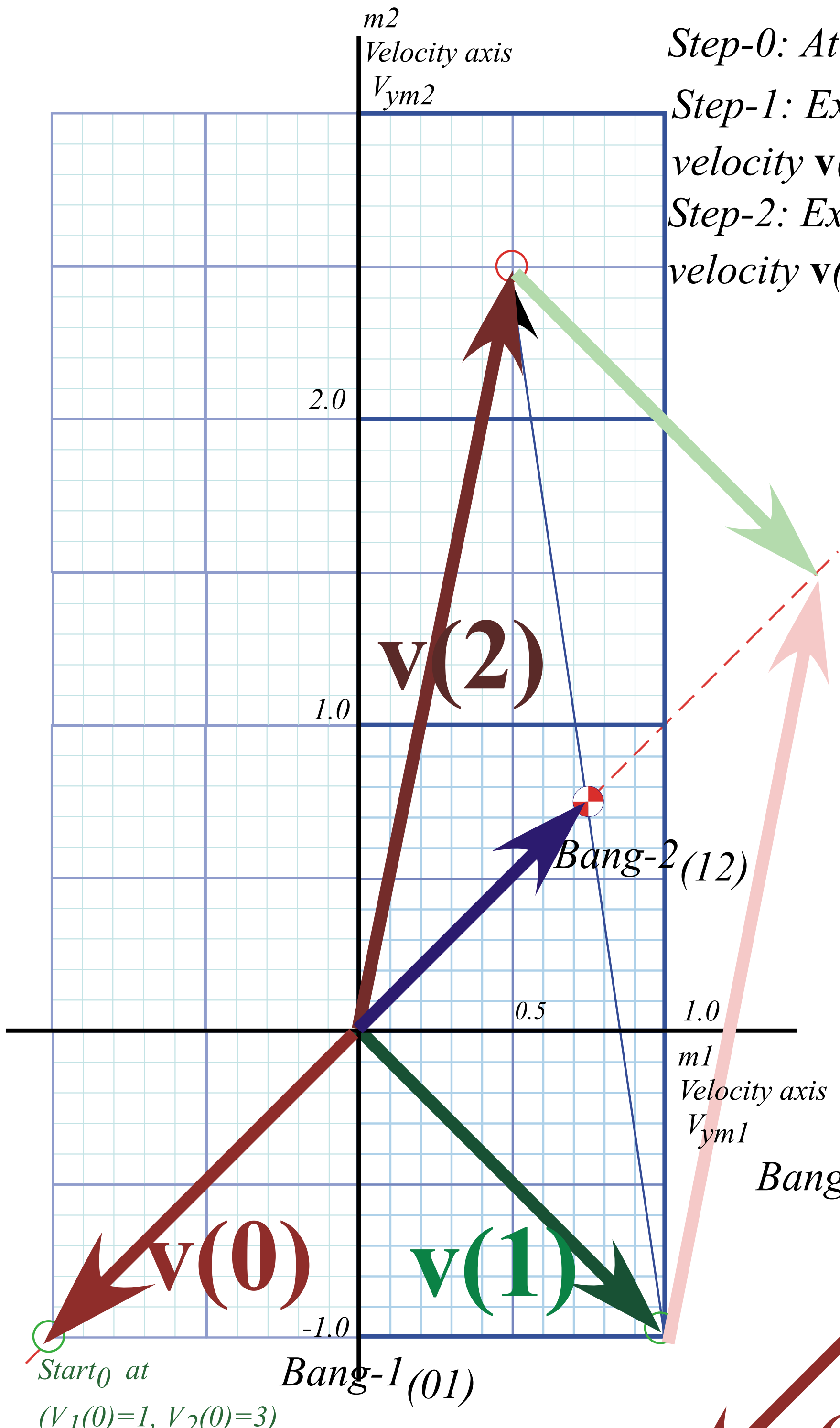
→ *Integration of (V_1,V_2) data to space-space plots (y_1, y_2)* *Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$*

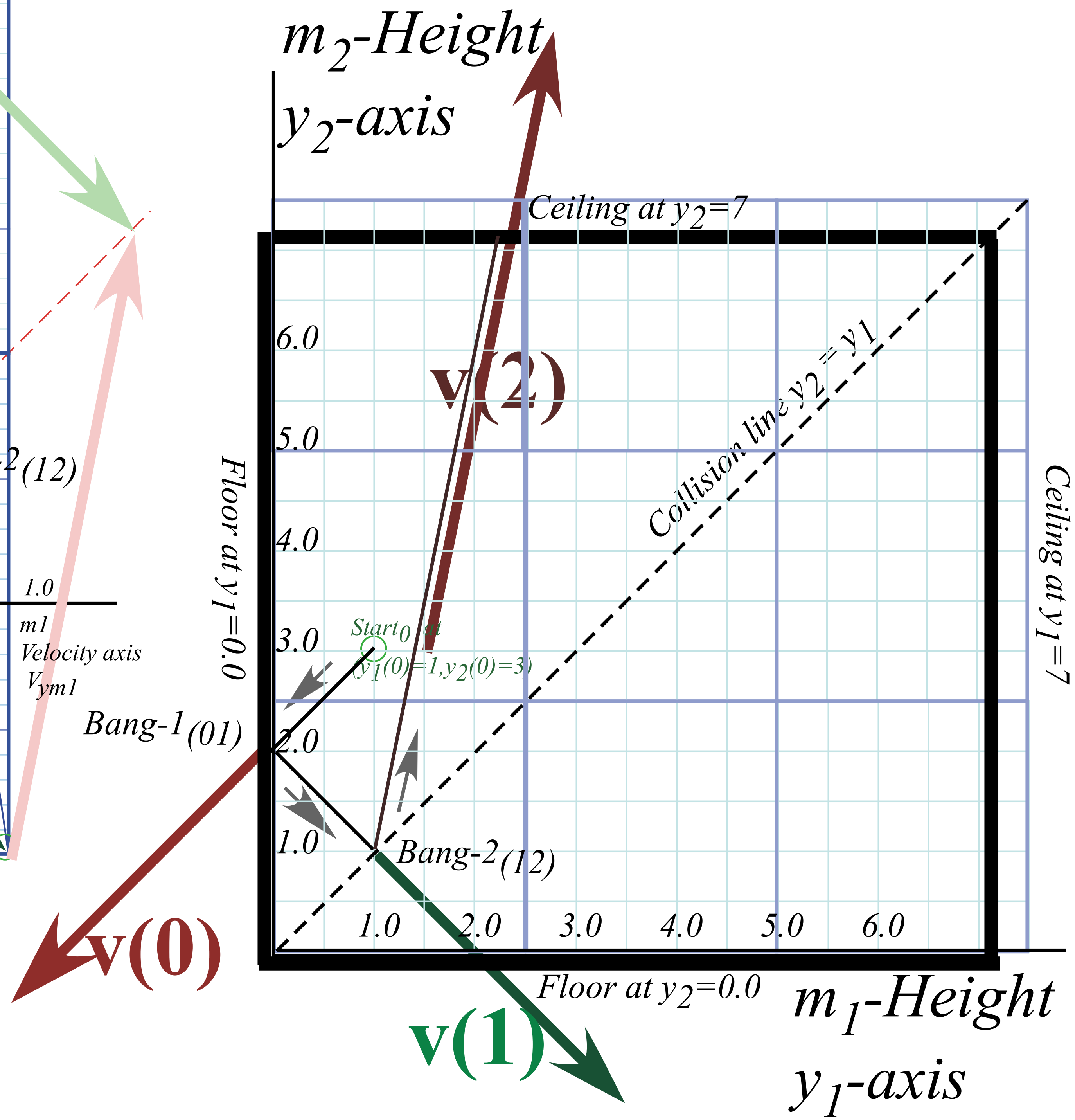
Geometric "Integration" (Converting Velocity data to Space-space trajectory)

Fig. 4.11
in Unit 1

Step-0: At starting position **y**$(0)=(1,3)$ *draw initial velocity* **v**$(0)=(-1,-1)$ *line.*
Step-1: Extend **v**(0) *line to floor point* **y**$(0)=(0,?)$ *and draw Bang-1*$_{(01)}$ *velocity* **v**$(1)=(1,-1)$ *line. (Find* **v**(1) *using V-V plot.)*
Step-2: Extend **v**(1) *line to collision point* **y**$(0)=(?,?)$ *and draw Bang-2*$_{(12)}$ *velocity* **v**$(2)=(0.5,2.5)$*. (Find* **v**(2) *using V-V plot.)*

Ellipse radius 1	*Ellipse radius 2*
$a_1=\sqrt{2KE/M_1}$	$a_2=\sqrt{2KE/M_2}$
$=\sqrt{2KE/7}$	$=\sqrt{2KE/1}$
$=\sqrt{8/7}$	$=\sqrt{8/1}$
$=1.07$	$=2.83$





Geometric "Integration" (Converting Velocity data to Space-space trajectory)

Fig. 4.11
in Unit 1

Ellipse radius 1

$a_1 = \sqrt{2KE/M_1}$
$= \sqrt{2KE/7}$
$= \sqrt{8/7}$
$= 1.07$

Ellipse radius 2

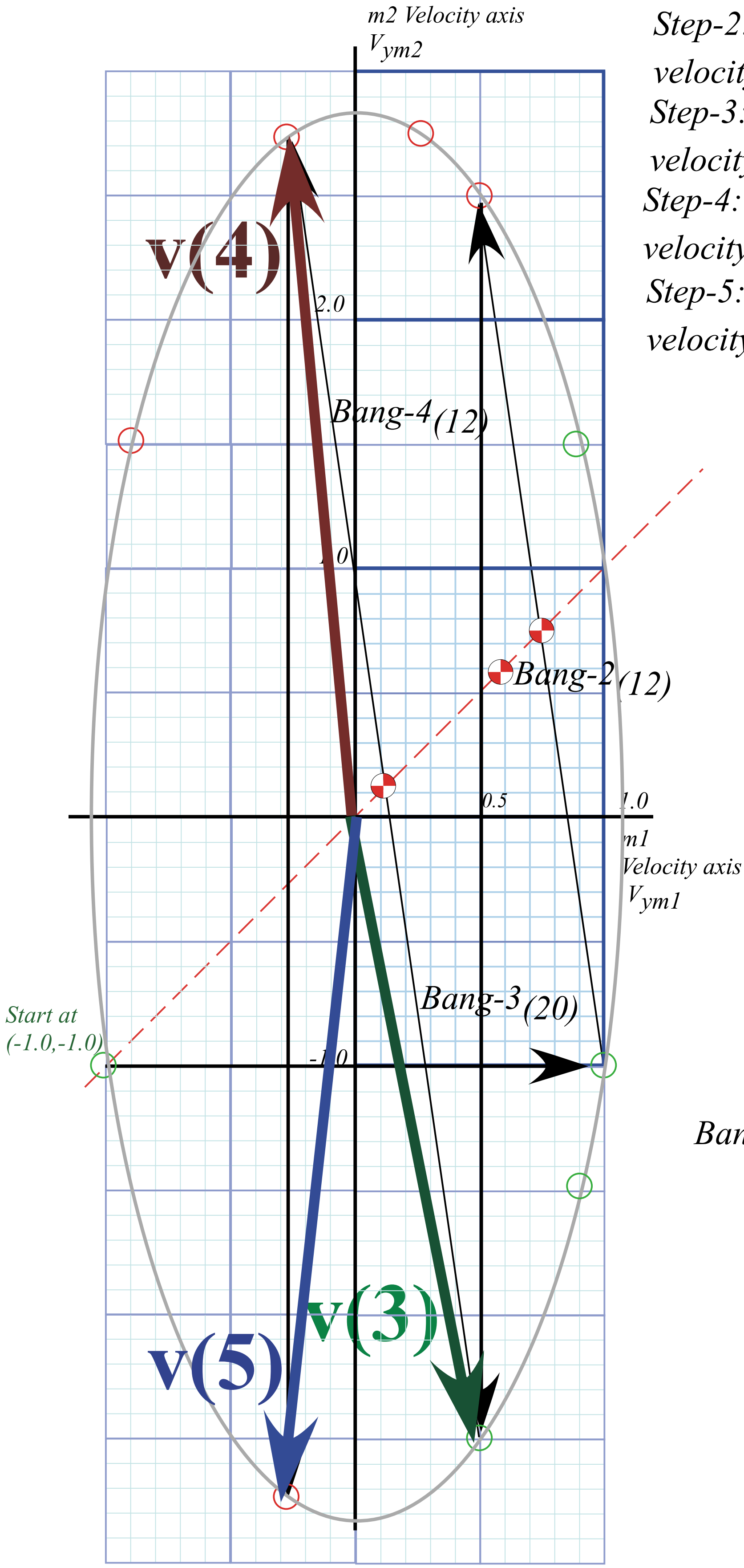
$a_2 = \sqrt{2KE/M_2}$
$= \sqrt{2KE/1}$
$= \sqrt{8/1}$
$= 2.83$

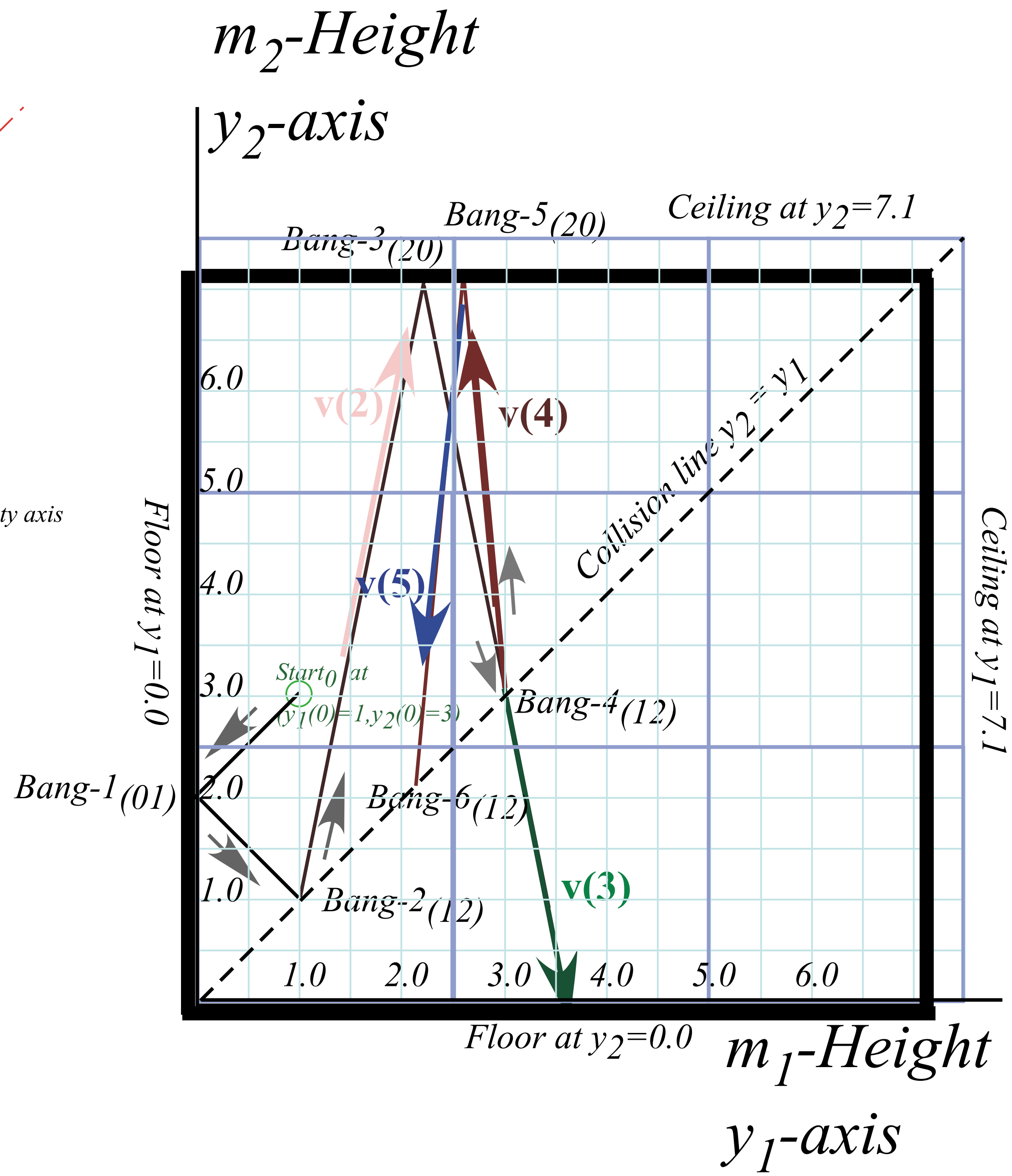
Step-2: Extend **v**(2) *line to ceiling point* **y**(3)=(?,7.1) *and draw Bang-3*$_{(20)}$ *velocity* **v**(3)=(1,-1) *line. (Find* **v**(3) *using V-V plot.)*

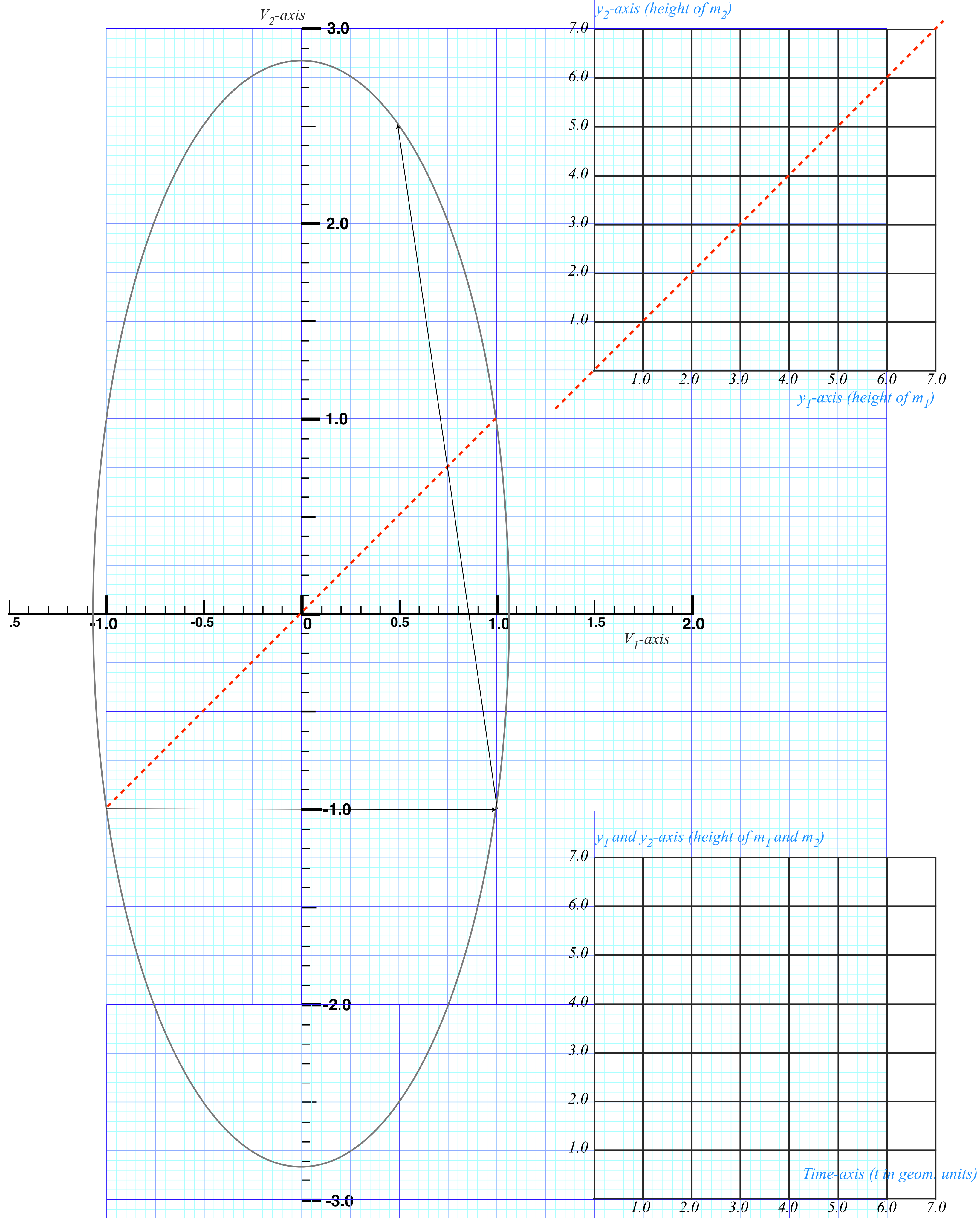
Step-3: Extend **v**(3) *line to collision point* **y**(4)=(?,?) *and draw Bang-4*$_{(12)}$ *velocity* **v**(4)=(0.5,2.5). *(Find* **v**(4) *using V-V plot.)*

Step-4: Extend **v**(4) *line to ceiling point* **y**(4)=(?,7.1) *and draw Bang-5*$_{(20)}$ *velocity* **v**(5)=(1,-1) *line. (Find* **v**(5) *using V-V plot.)*

Step-5: Extend **v**(5) *line to collision point* **y**(6)=(?,?) *and draw Bang-6*$_{(12)}$ *velocity* **v**(6)=(0.5,2.5). *(Find* **v**(6) *using V-V plot.)*







Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_2} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)

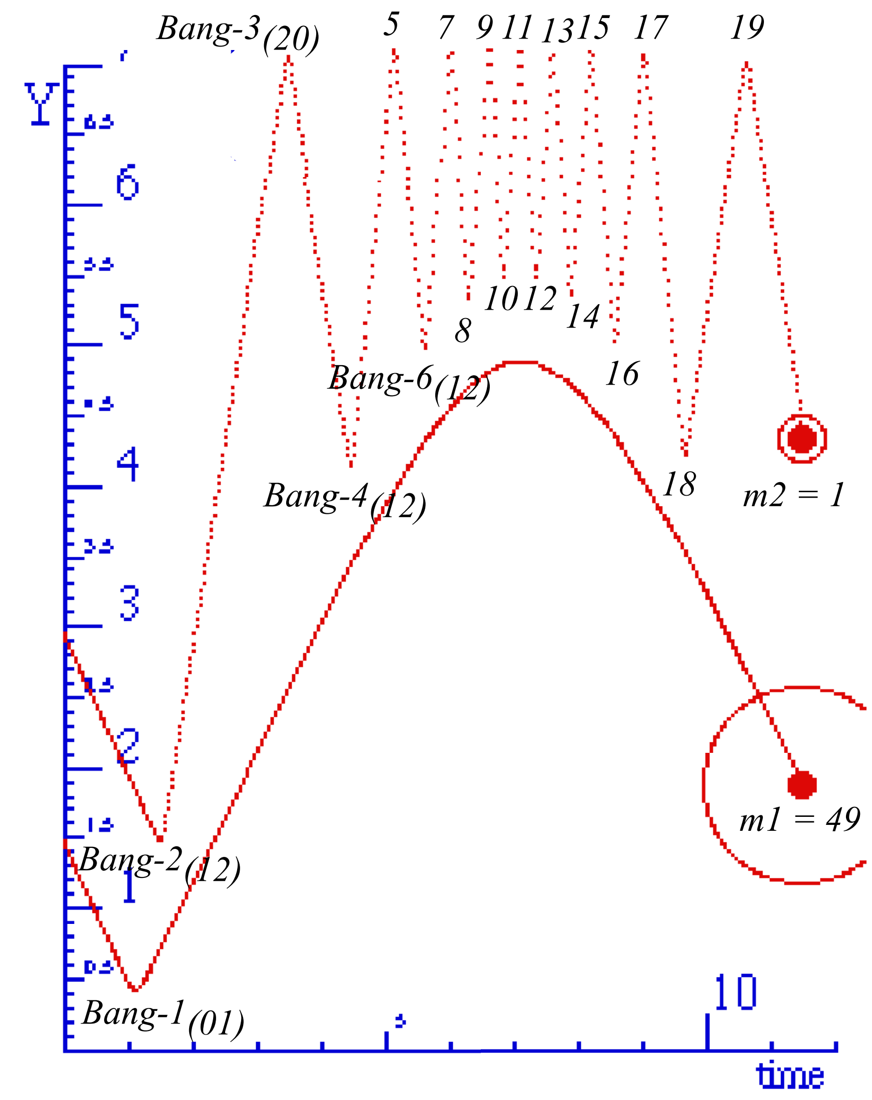
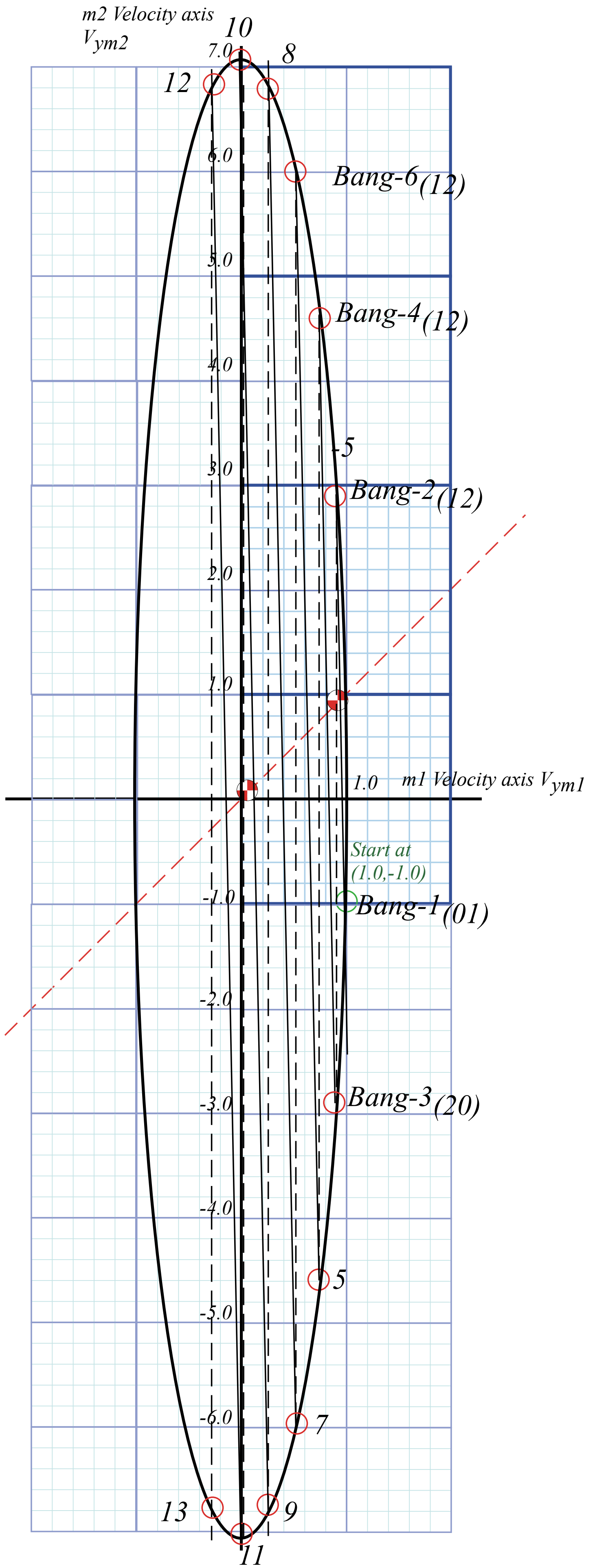


Fig. 5.1
in Unit 1

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with Newtonian time plot]
[M_1=49, M_2=1 with V_2 vs V_1 plot]

Geometry of 1-D 2-body collisions (Example with masses: $m_1=49$ and $m_2=1$)



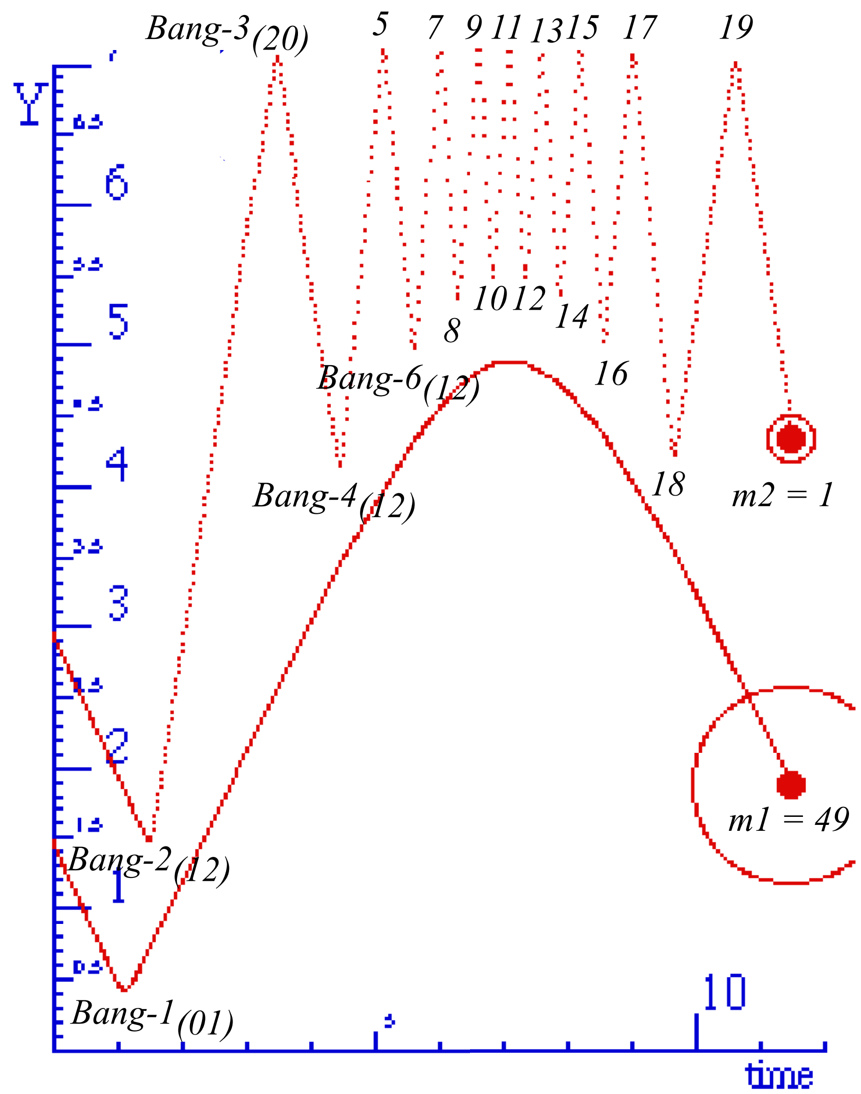
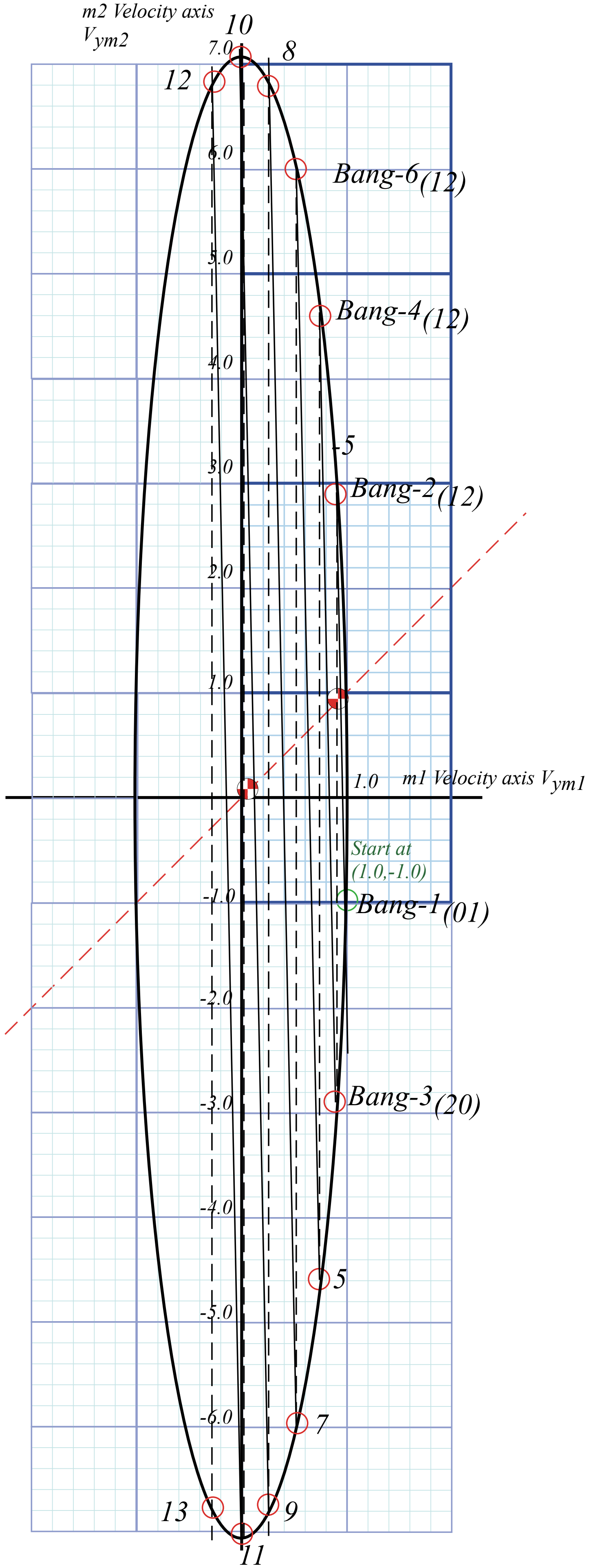


Fig. 5.1
in Unit 1

BounceIt Superball Collision Web Simulator:
$M_1=49$, $M_2=1$ with Newtonian time plot
$M_1=49$, $M_2=1$ with V_2 vs V_1 plot

Geometry of 1-D 2-body collisions (Example with masses: m_1=49 and m_2=1)



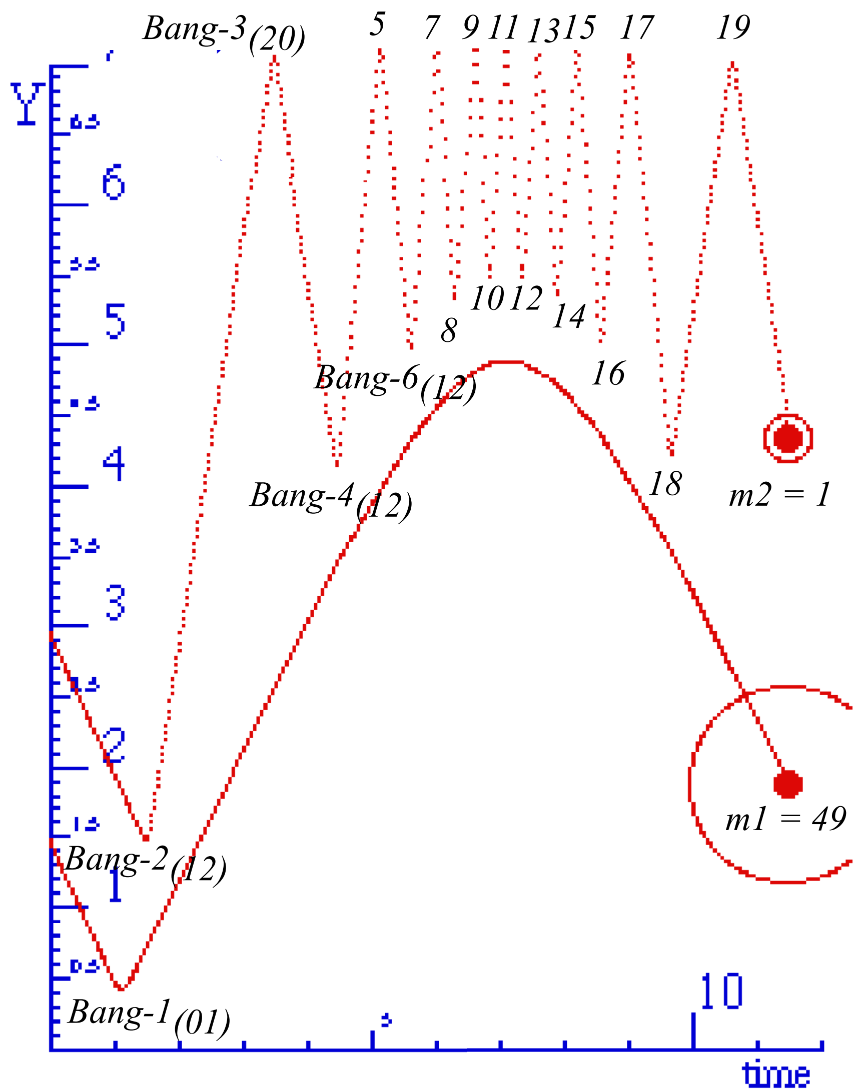


Fig. 5.1
in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{V_1^2}{2KE/m_1} + \frac{V_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R** = **C•M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:
$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2} \begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} =$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM}=\frac{\mathbf{V}^{FIN}+\mathbf{V}^{IN}}{2}=\frac{m_1v_1+m_2v_2}{m_1+m_2}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=\tfrac{1}{2}\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}+\tfrac{1}{2}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN}=\mathbf{M}\bullet\mathbf{v}^{IN}$...

$$\begin{pmatrix}v_1^{FIN}\\v_2^{FIN}\end{pmatrix}=\begin{pmatrix}2V^{COM}-v_1^{IN}\\2V^{COM}-v_2^{IN}\end{pmatrix}=\begin{pmatrix}2\dfrac{m_1v_1^{IN}+m_2v_2^{IN}}{m_1+m_2}-v_1^{IN}\\2\dfrac{m_1v_1^{IN}+m_2v_2^{IN}}{m_1+m_2}-v_2^{IN}\end{pmatrix}=\frac{\begin{pmatrix}m_1v_1^{IN}-m_2v_1^{IN}+2m_2v_2^{IN}\\2m_1v_1^{IN}+m_2v_2^{IN}-m_1v_2^{IN}\end{pmatrix}}{m_1+m_2}=\frac{\begin{pmatrix}m_1-m_2 & 2m_2\\2m_1 & m_2-m_1\end{pmatrix}\begin{pmatrix}v_1^{IN}\\v_2^{IN}\end{pmatrix}}{m_1+m_2}$$

$$=\begin{pmatrix}\dfrac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_1^{IN}-m_2v_1^{IN}}{m_1+m_2}\\\dfrac{2m_1v_1^{IN}+2m_2v_2^{IN}-m_1v_2^{IN}-m_2v_2^{IN}}{m_1+m_2}\end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:

$$\mathbf{V}^{COM} = \frac{\mathbf{V}^{FIN} + \mathbf{V}^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix} = \tfrac{1}{2}\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\dfrac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\dfrac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

$$= \begin{pmatrix} \dfrac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_1^{IN} - m_2 v_1^{IN}}{m_1 + m_2} \\ \dfrac{2m_1 v_1^{IN} + 2m_2 v_2^{IN} - m_1 v_2^{IN} - m_2 v_2^{IN}}{m_1 + m_2} \end{pmatrix}$$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \dfrac{m_1 - m_2}{m_1 + m_2} & \dfrac{2m_2}{m_1 + m_2} \\ \dfrac{2m_1}{m_1 + m_2} & \dfrac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

→ *What about that 2nd quadratic solution?*

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C**•**M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

What about that 2nd quadratic solution?

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

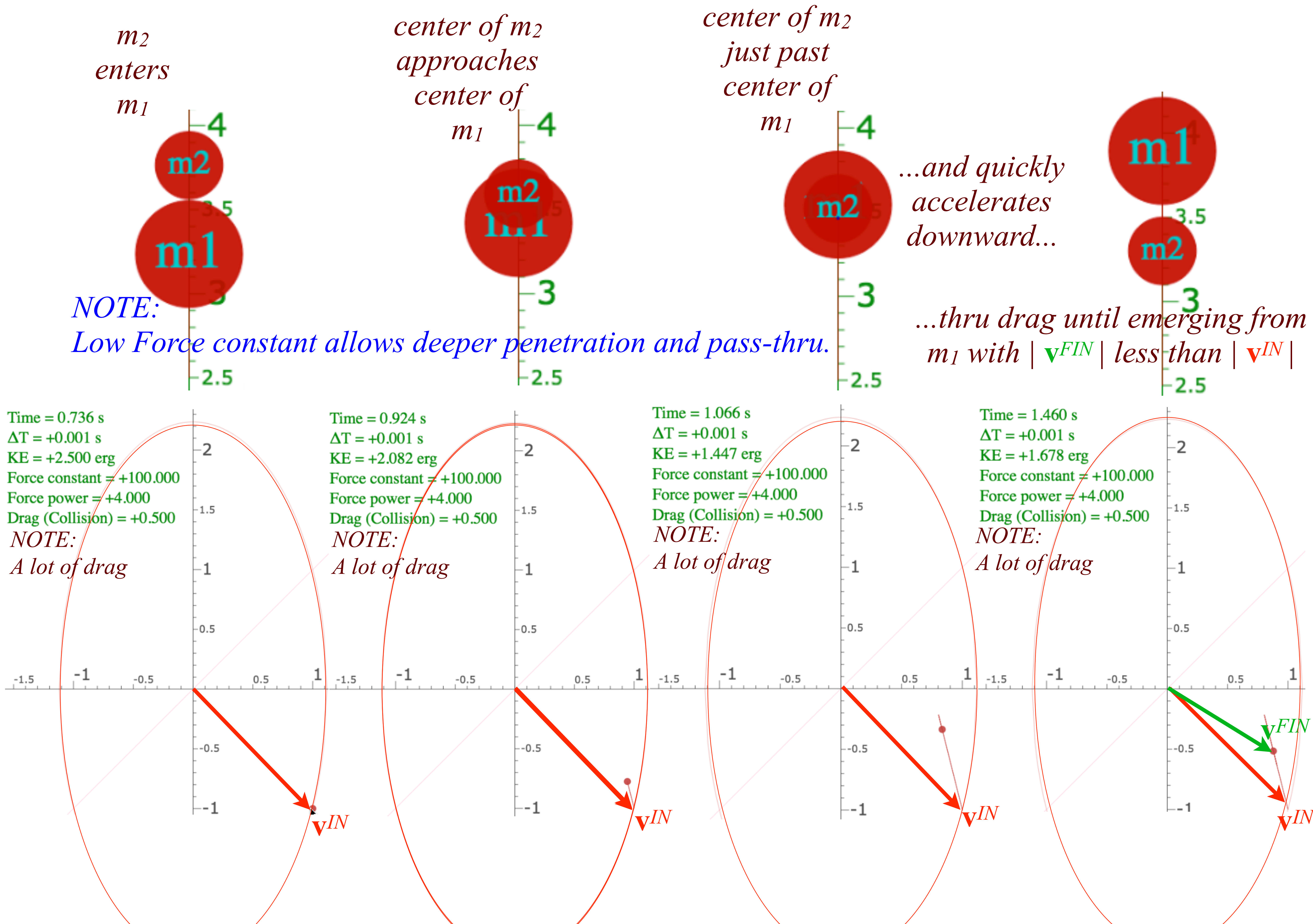
$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

What about that 2nd quadratic solution?

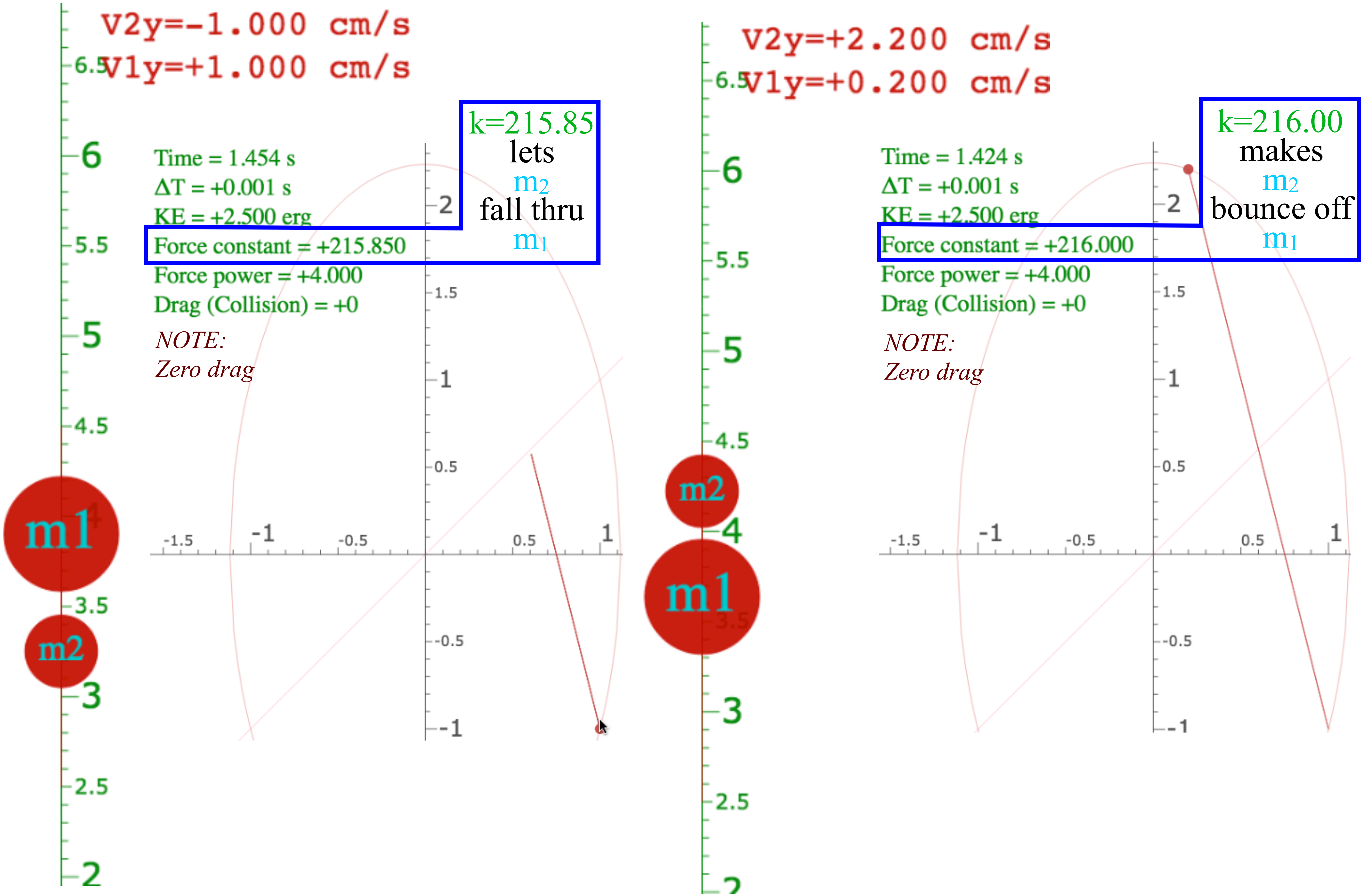
Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

Q: What is the *second* solution and to what simple process would it correspond?

Example with friction



[BounceIt Web Simulation - *Low force constant with drag displaying a Pass-thru*]



Fall-Thru | BounceIt Superball Collision Web Simulations with Low force constant & Zero drag | *Bounce-Off*

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C•M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix}$$

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$= \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ and $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

What about that 2nd quadratic solution?

"Mass-bang" matrix **M**, *"Floor-bang" matrix* **F**, *"Ceiling-bang" matrix* **C**.

Geometry and algebra of "ellipse-Rotation" group product: **R**= **C**•**M**

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives $\mathbf{v}^{FIN}$ in terms of $\mathbf{v}^{IN}$...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \bullet \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2\frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang **F** of m_1:

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang **M** of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang **C** of m_2:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1 = 49$ *and* $m_2 = 1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Define *"ellipse-Rotation"* **R** as *group product*: $\mathbf{R} = \mathbf{C} \bullet \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}$

$$|FIN^9\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \; |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL } (0))}$$

$$|FIN^9\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL }(0))}$$

$$|FIN^9\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{(\text{after Bang-1})}$$

"ellipse-Rotation" group product: **R**= **C**•**M**

$$|FIN^9\rangle = \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \; |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL } (0))}$$

$$|FIN^9\rangle = \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} \, |IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{(\text{after Bang-1})}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{(\text{after Bang-9})}$$

"ellipse-Rotation" group product: **R**= **C•M**

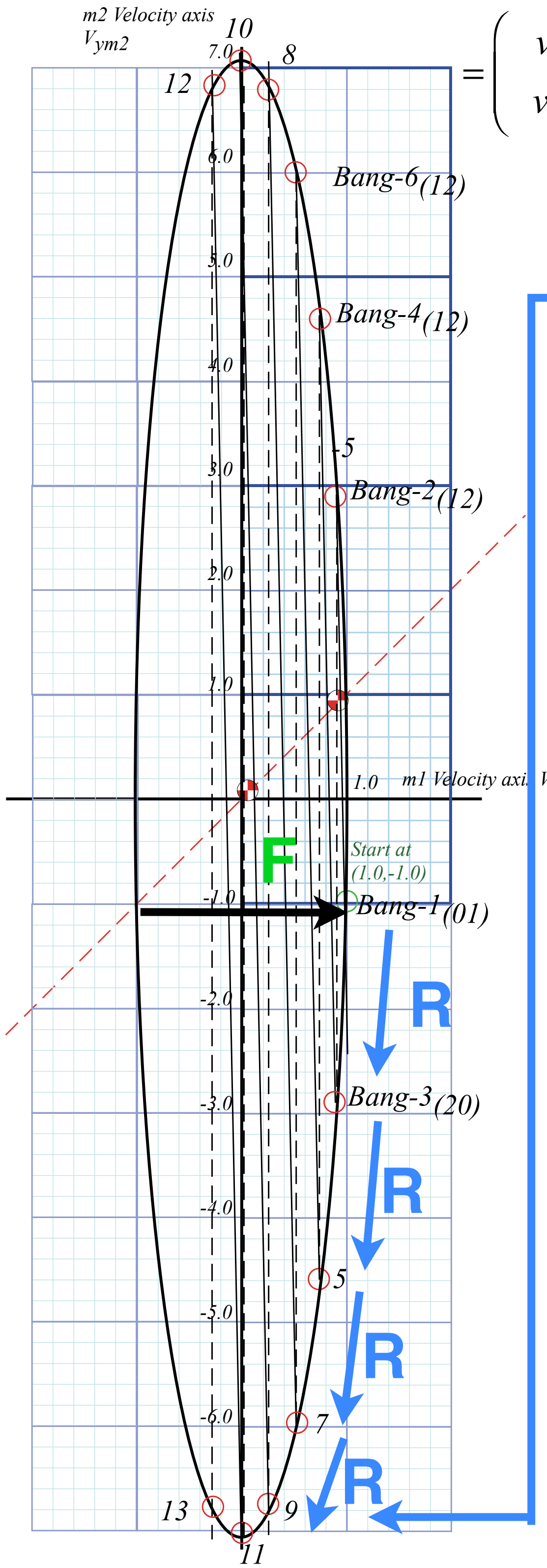
$$\left|FIN^9\right\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN}=-1 \\ v_2^{IN}=-1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

$$\left|FIN^9\right\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\left|IN^0\right\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1=1 \\ v_2=-1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1=0.2925 \\ v_2=-6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$



"ellipse-Rotation" group product: **R= C•M**

Collisions for mass ratio m_1:m_2= 49:1

Fig. 5.1a (revised)

$$|FIN^9\rangle = \mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{C}\cdot\mathbf{M}\cdot\mathbf{F}\,|IN^0\rangle$$

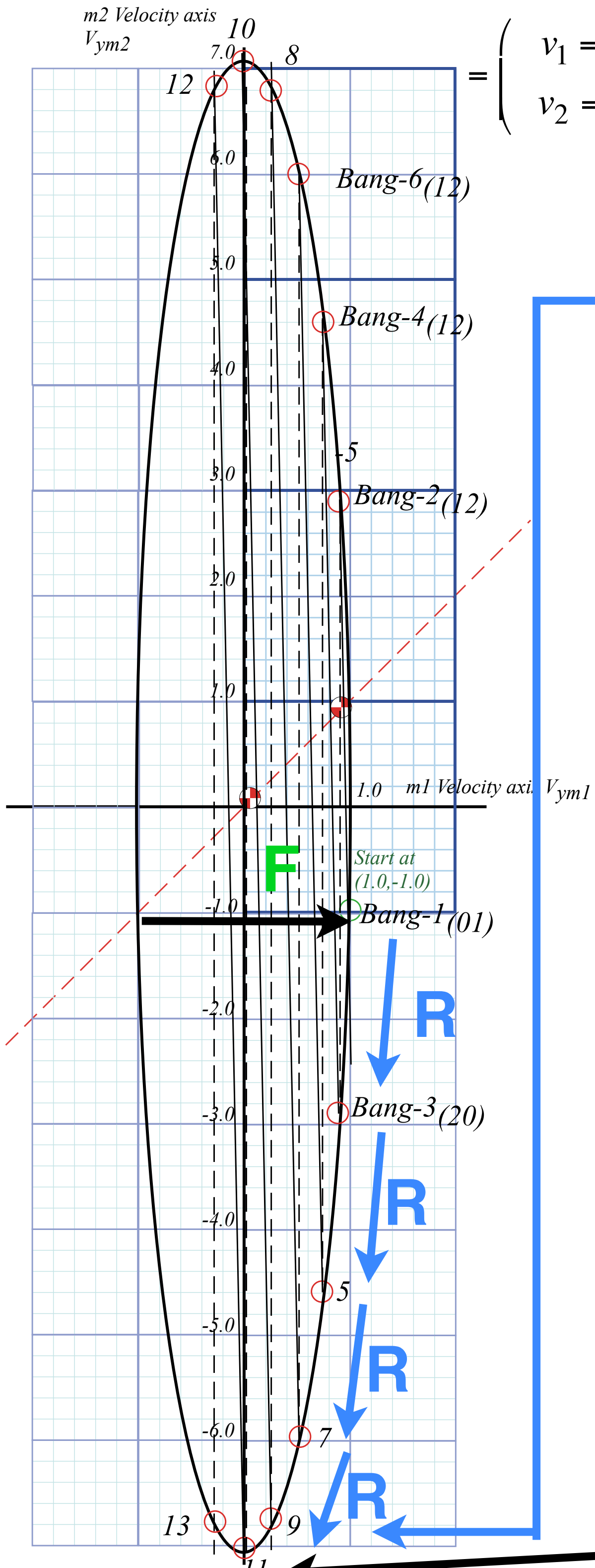
$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}}_{\mathbf{R}}\cdot\begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix}\begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{\text{(INITIAL (0))}}$$

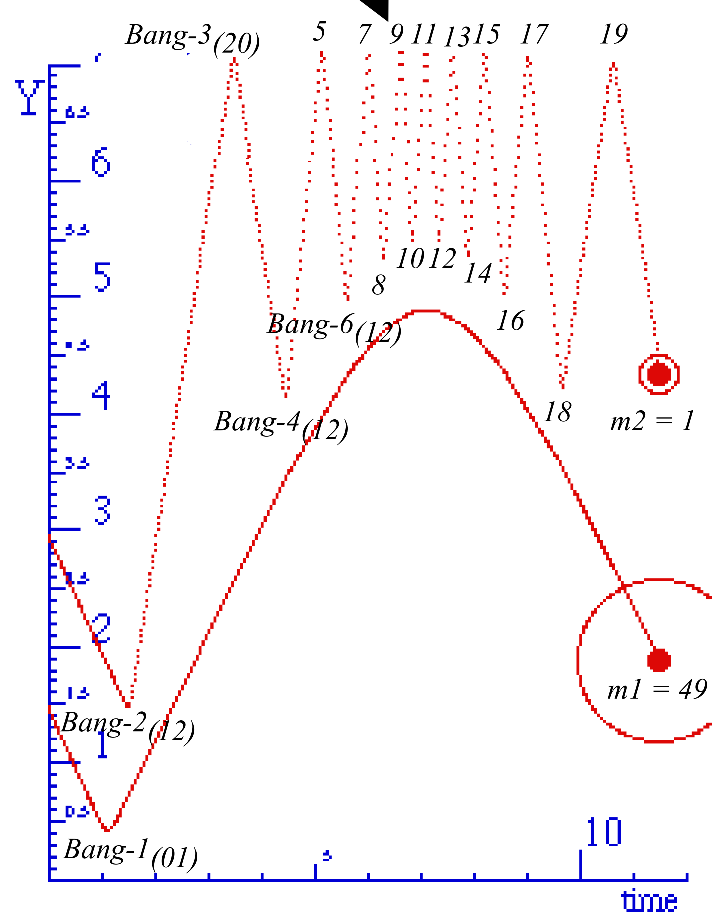
$$|FIN^9\rangle = \mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{R}\cdot\mathbf{F}\,|IN^0\rangle$$

$$\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{\text{(after Bang-1)}}$$

$$= \begin{pmatrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{pmatrix}_{\text{(after Bang-9)}}$$

"ellipse-Rotation" group product: **R= C•M**





$$\begin{pmatrix} v_1^{FIN-11} \\ v_2^{FIN-11} \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}\cdot\begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix}$$

$$= \begin{pmatrix} v_1 = 0.0100 \\ v_2 = -7.071 \end{pmatrix}_{\text{(after Bang-11)}}$$

Collisions for mass ratio $m_1{:}m_2 = 49{:}1$

BounceIt Superball Collision Web Simulator:
M_1=49, M_2=1 with Newtonian time plot

M_1=49, M_2=1 with V_2 vs V_1 plot

Fig. 5.1a-b (revised)

<<Under Construction>>
Matrix Collision Web Simulator:
M_1=49, M_2=1 V_2 vs V_1 plot

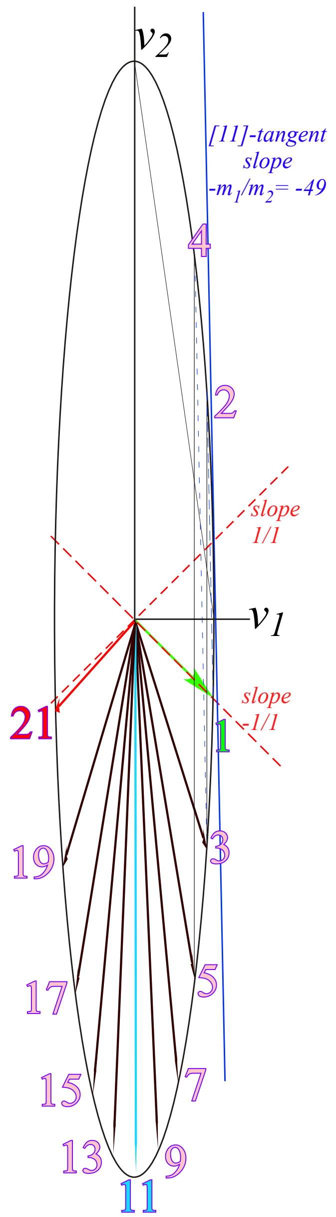
Ellipse rescaling-geometry and reflection-symmetry analysis

→ *Rescaling KE ellipse to circle*

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$



Collisions for mass ratio m_1:m_2= 49:1

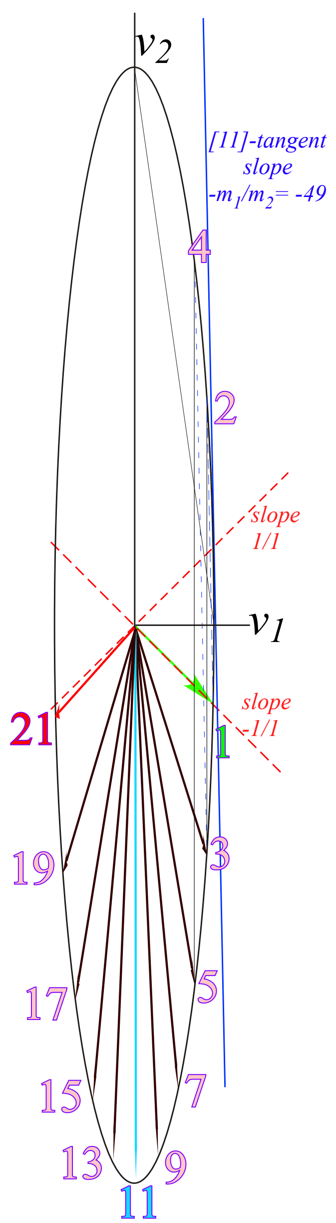
Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

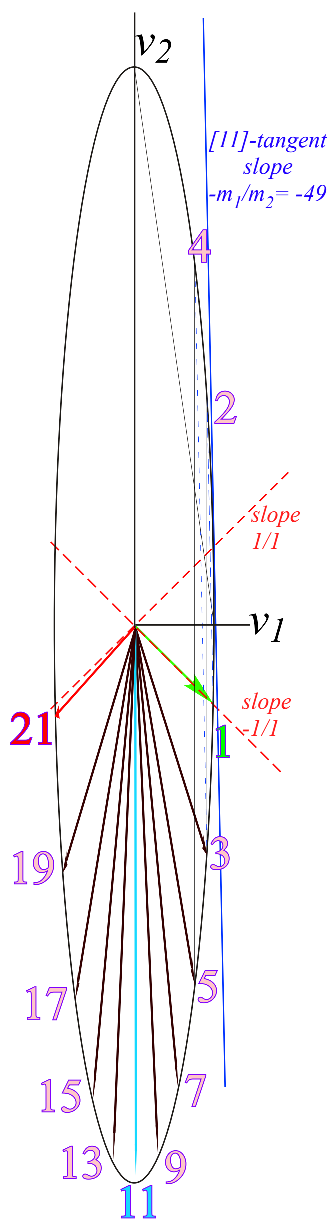
$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} V_1^{FIN_1} / \sqrt{m_1} \\ V_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

or: $$\begin{pmatrix} V_1^{FIN_1} \\ V_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \bullet \vec{V}$$,

or: $$\begin{pmatrix} V_1^{FIN_2} \\ V_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \bullet \mathbf{M} \bullet \vec{V}$$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

or:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{M} \bullet \vec{\mathsf{V}}$$

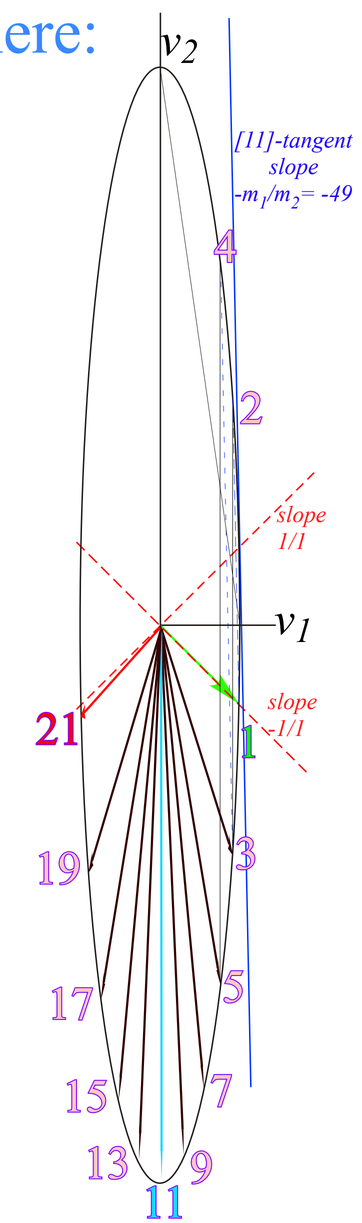
or:

$$\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathsf{C} \bullet \mathsf{M} \bullet \vec{\mathsf{V}}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where:

$$\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \quad \text{and:} \quad \sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right) \quad \text{with:} \quad \left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$$



Collisions for mass ratio m_1:m_2= 49:1

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\mathsf{V}_1^2 + \frac{1}{2}\mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

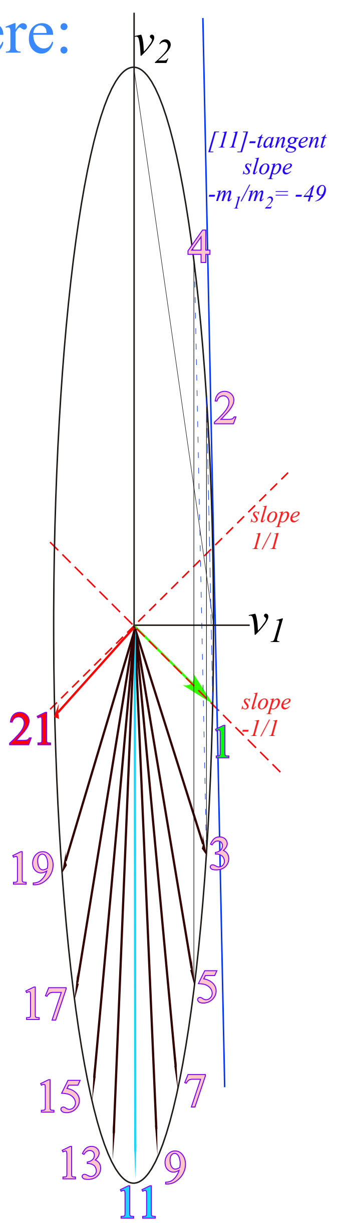
$$\begin{pmatrix} \mathsf{V}_1^{FIN_1}/\sqrt{m_1} \\ \mathsf{V}_2^{FIN_1}/\sqrt{m_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1/\sqrt{m_1} \\ \mathsf{V}_2/\sqrt{m_2} \end{pmatrix}$$

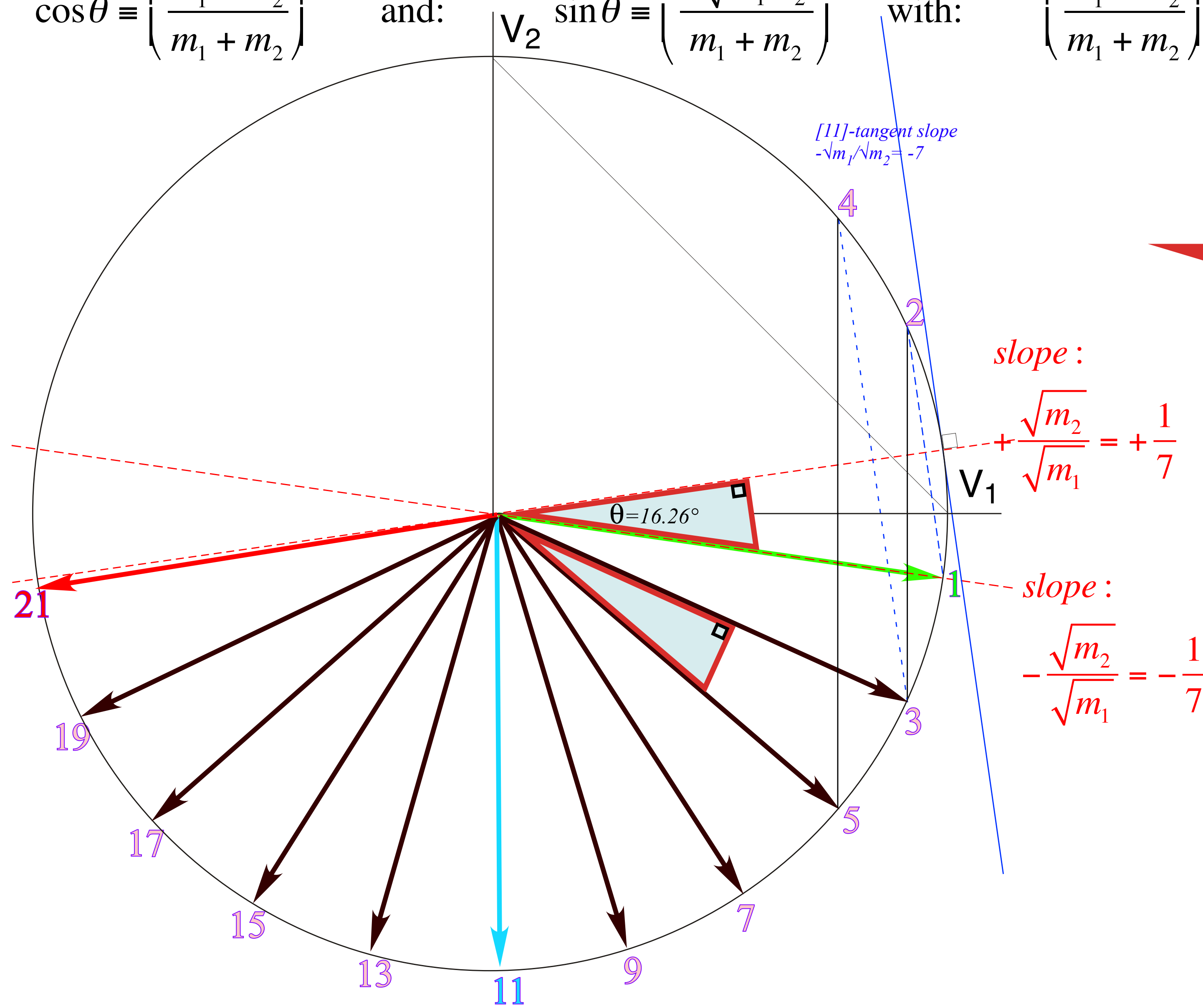
or: $$\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{M}\bullet\vec{\mathsf{V}}$$

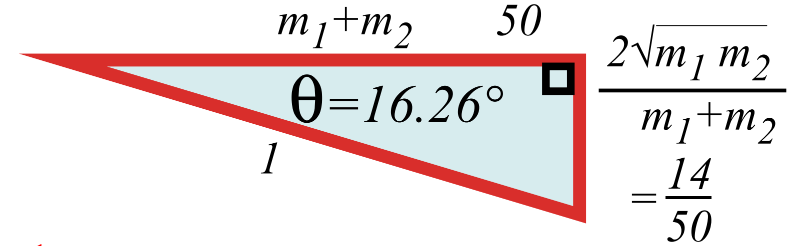
or: $$\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M}\begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix}\begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{C}\bullet\mathbf{M}\bullet\vec{\mathsf{V}}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$







Collisions for mass ratio $m_1 : m_2 = 49:1$

Fig. 5.2a-c

(revised)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \mathsf{V}_1^2 + \frac{1}{2} \mathsf{V}_2^2$

$$\begin{pmatrix} v_1^{FIN_1} \\ v_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

becomes:

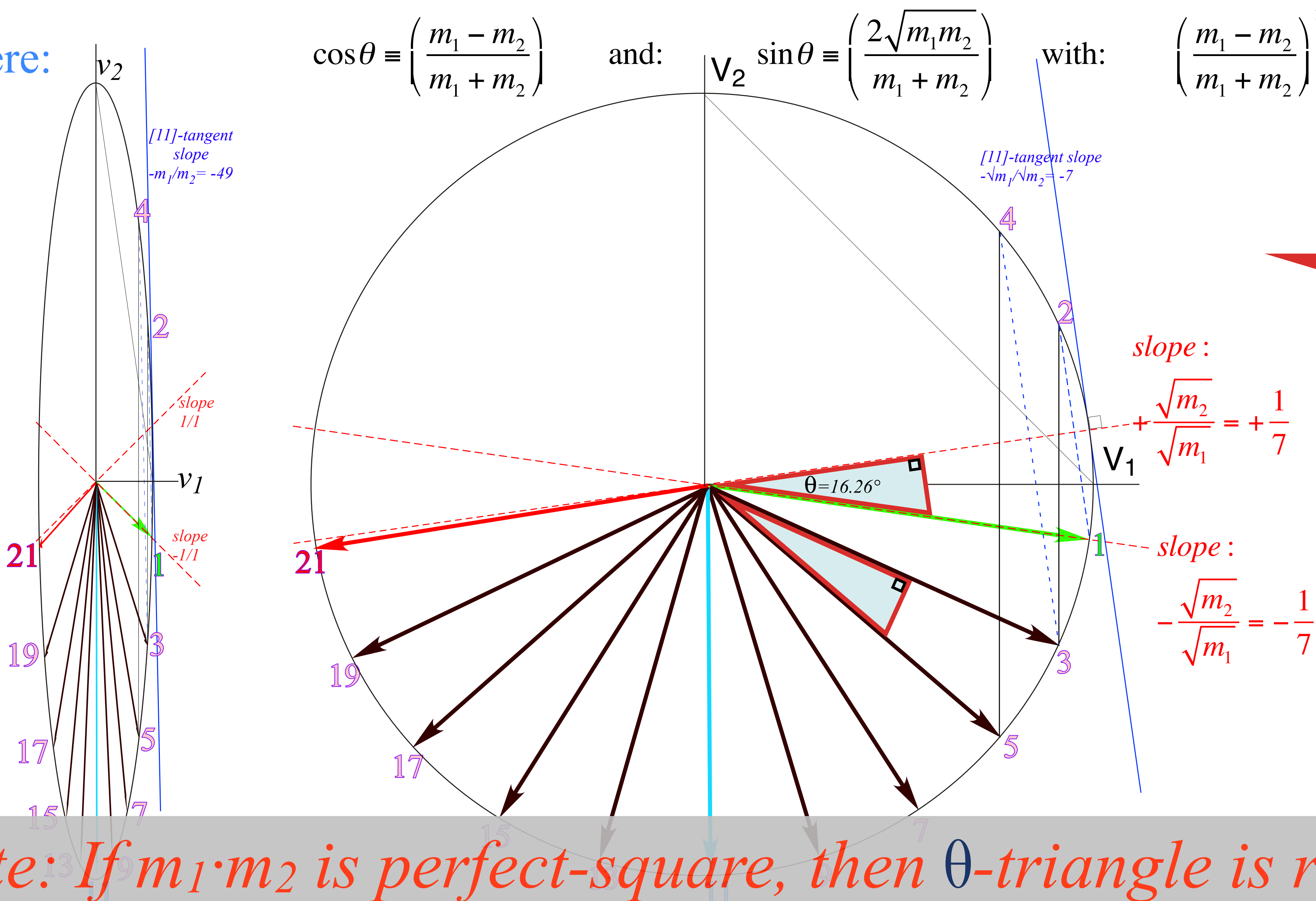
$$\begin{pmatrix} \mathsf{V}_1^{FIN_1} / \sqrt{m_1} \\ \mathsf{V}_2^{FIN_1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 / \sqrt{m_1} \\ \mathsf{V}_2 / \sqrt{m_2} \end{pmatrix}$$

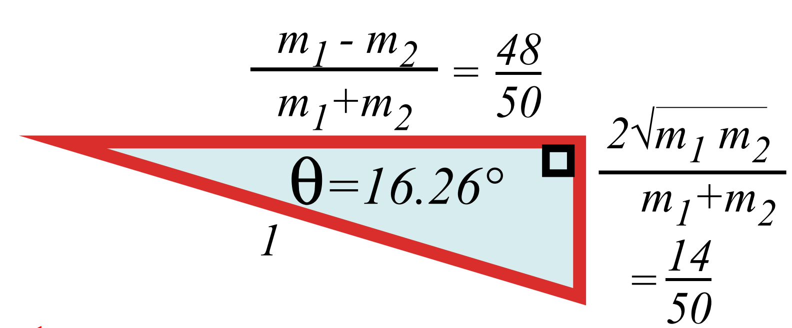
or: $$\begin{pmatrix} \mathsf{V}_1^{FIN_1} \\ \mathsf{V}_2^{FIN_1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{M} \bullet \vec{\mathsf{V}}$$,

or: $$\begin{pmatrix} \mathsf{V}_1^{FIN_2} \\ \mathsf{V}_2^{FIN_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} \mathsf{V}_1 \\ \mathsf{V}_2 \end{pmatrix} = \mathbf{C} \bullet \mathbf{M} \bullet \vec{\mathsf{V}}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$





Collisions for mass ratio $m_1 : m_2 = 49:1$

Fig. 5.2a-c

(revised)

Note: If $m_1 \cdot m_2$ is perfect-square, then θ-triangle is rational *($3^2+4^2=5^2$, etc.)*

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $\mathsf{V}_1 = v_1 \cdot \sqrt{m_1}$, $\mathsf{V}_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\mathsf{V}_1^2 + \frac{1}{2}\mathsf{V}_2^2$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations*

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$

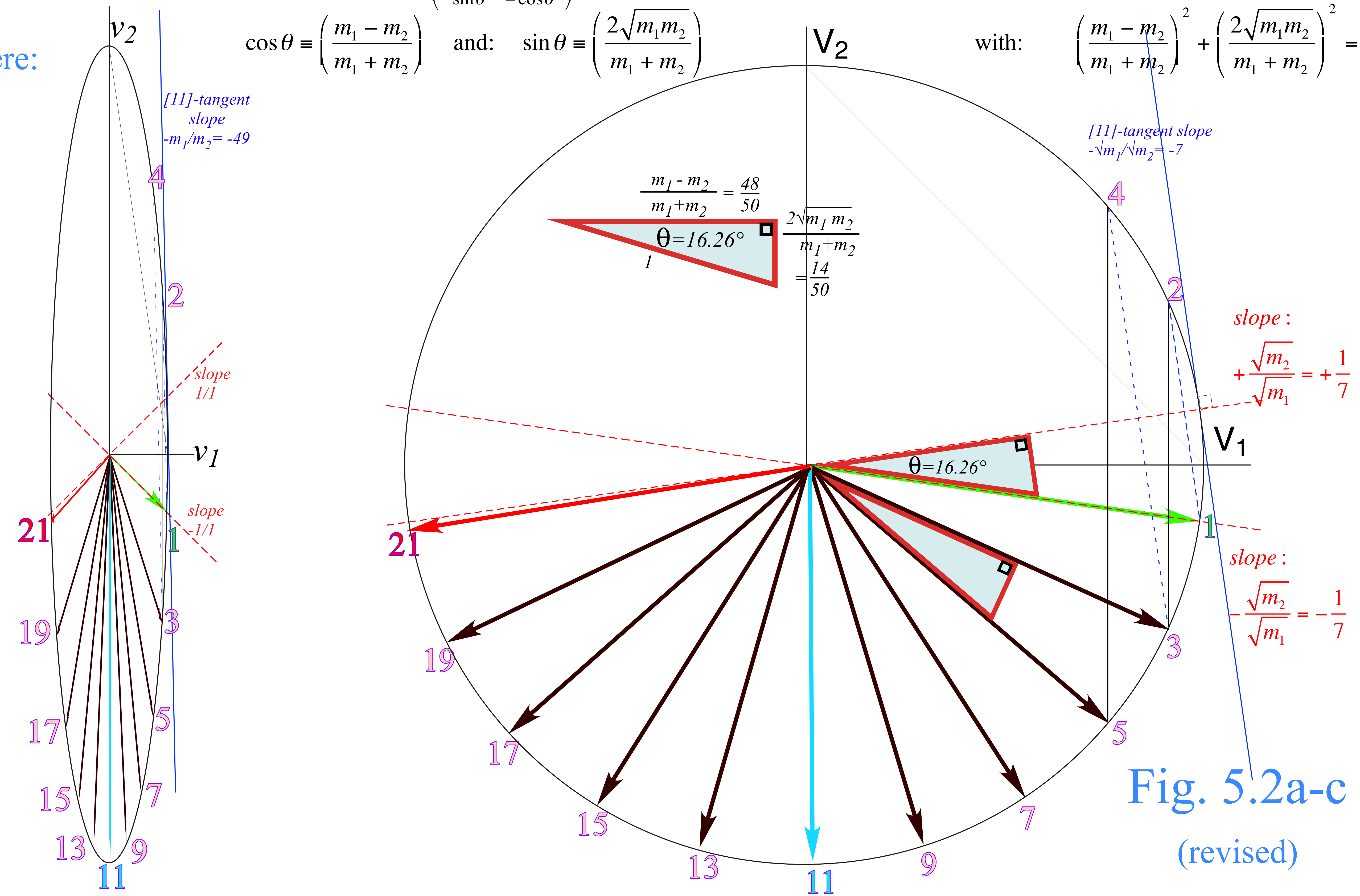


Fig. 5.2a-c

(revised)

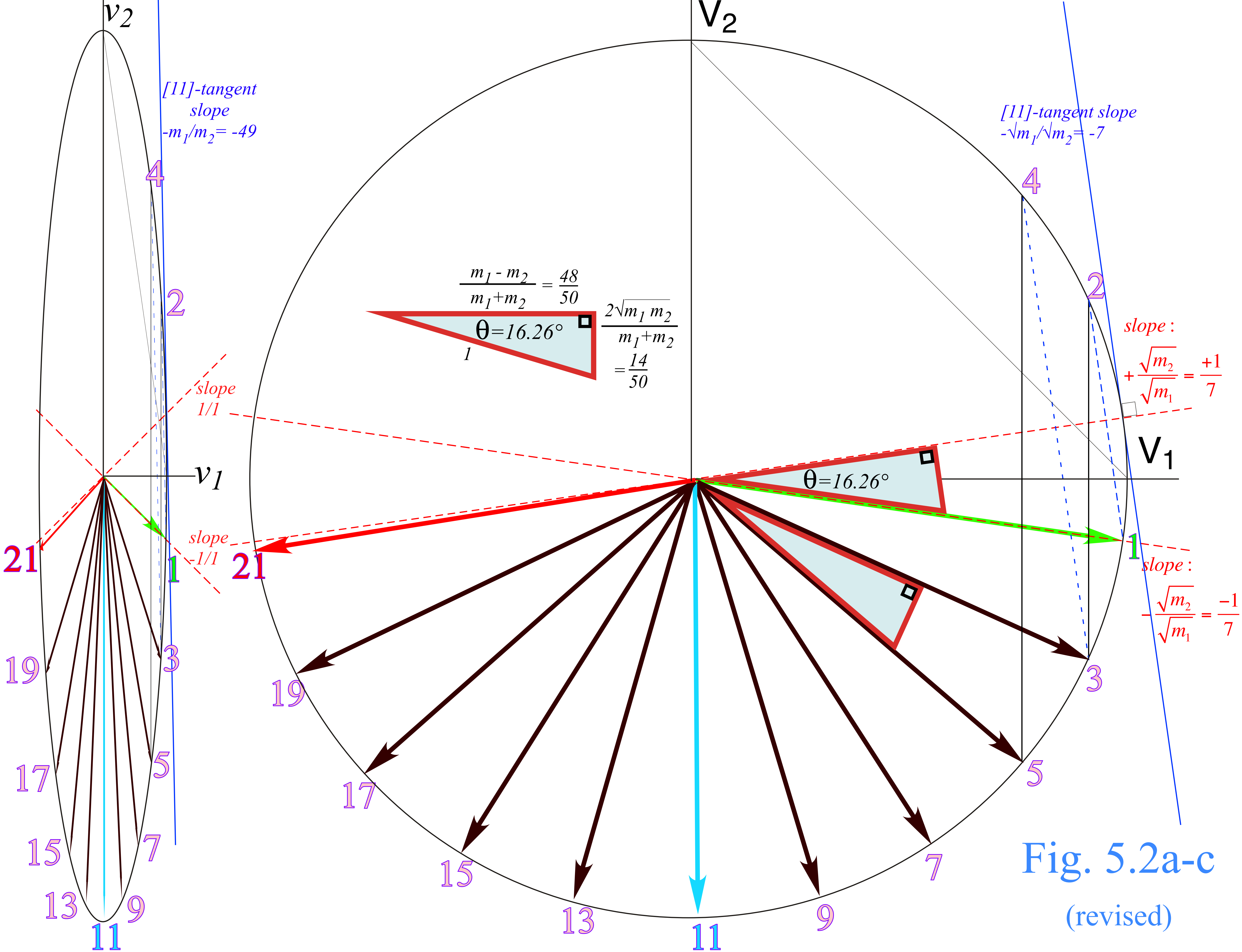


Fig. 5.2a-c

(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Reflections in the clothing store: "It's all done with mirrors!"

Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting for $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Other not-so-symmetric examples: $m_1/m_2=4$ and $m_1/m_2=7$

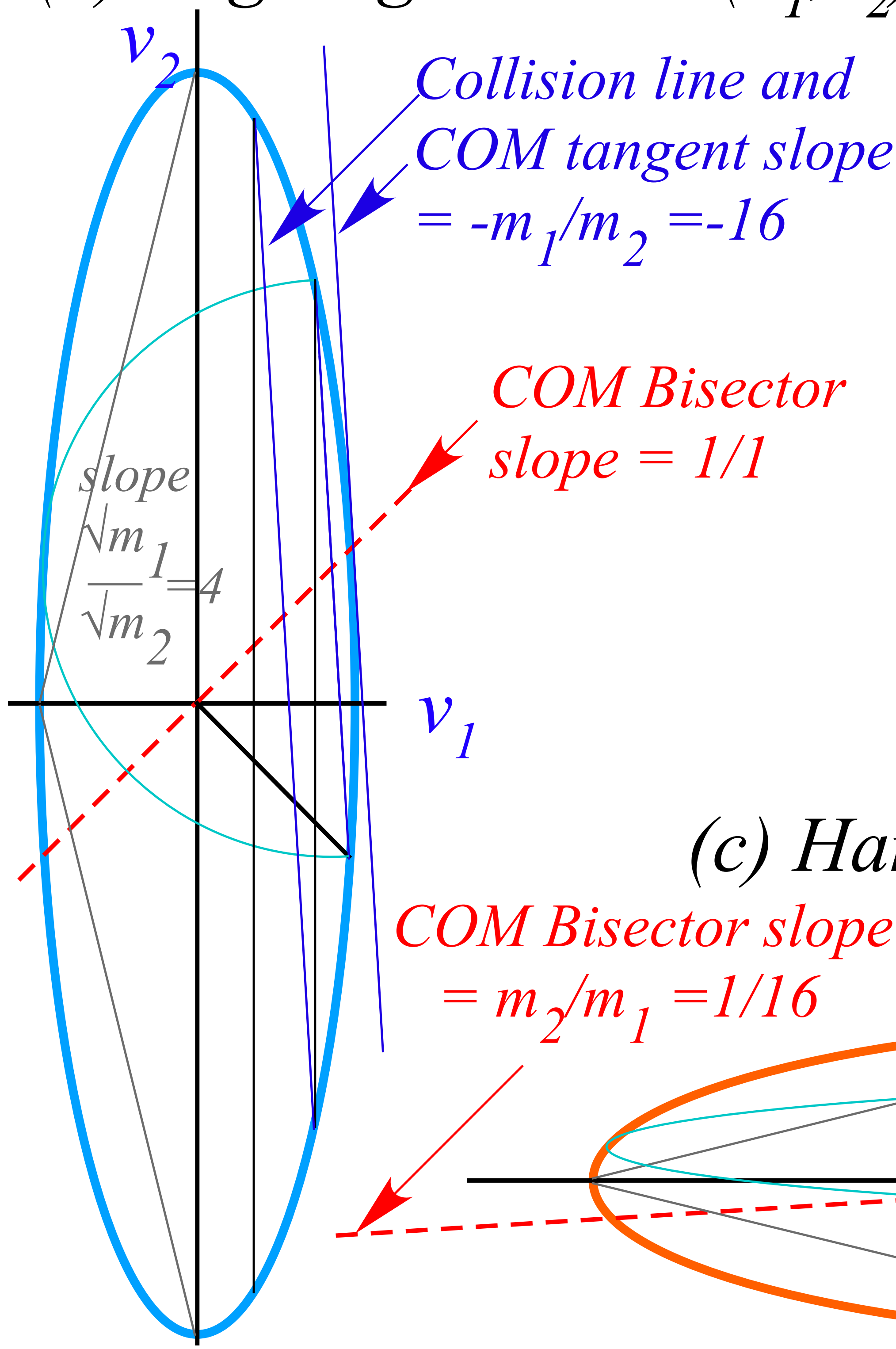
What ellipse rescaling leads to...

How this relates to Lagrangian, and Hamiltonian mechanics later on

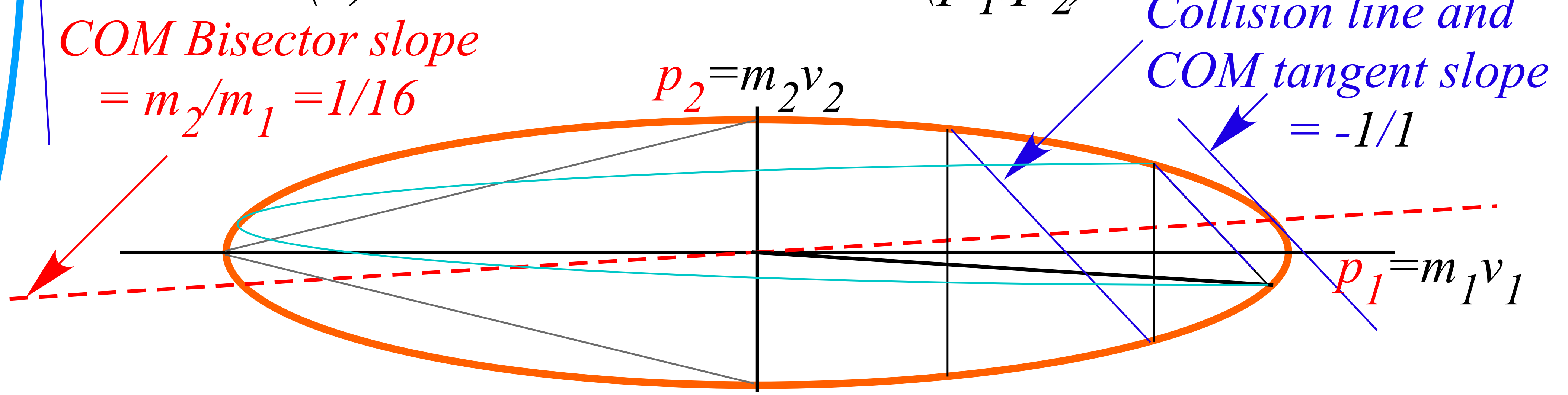
velocity v_1 rescaled to momentum: $p_1 = m_1 v_1$

velocity v_2 rescaled to momentum: $p_2 = m_2 v_2$

(a) Lagrangian $L = L(v_1, v_2)$



(c) Hamiltonian $H = H(p_1, p_2)$

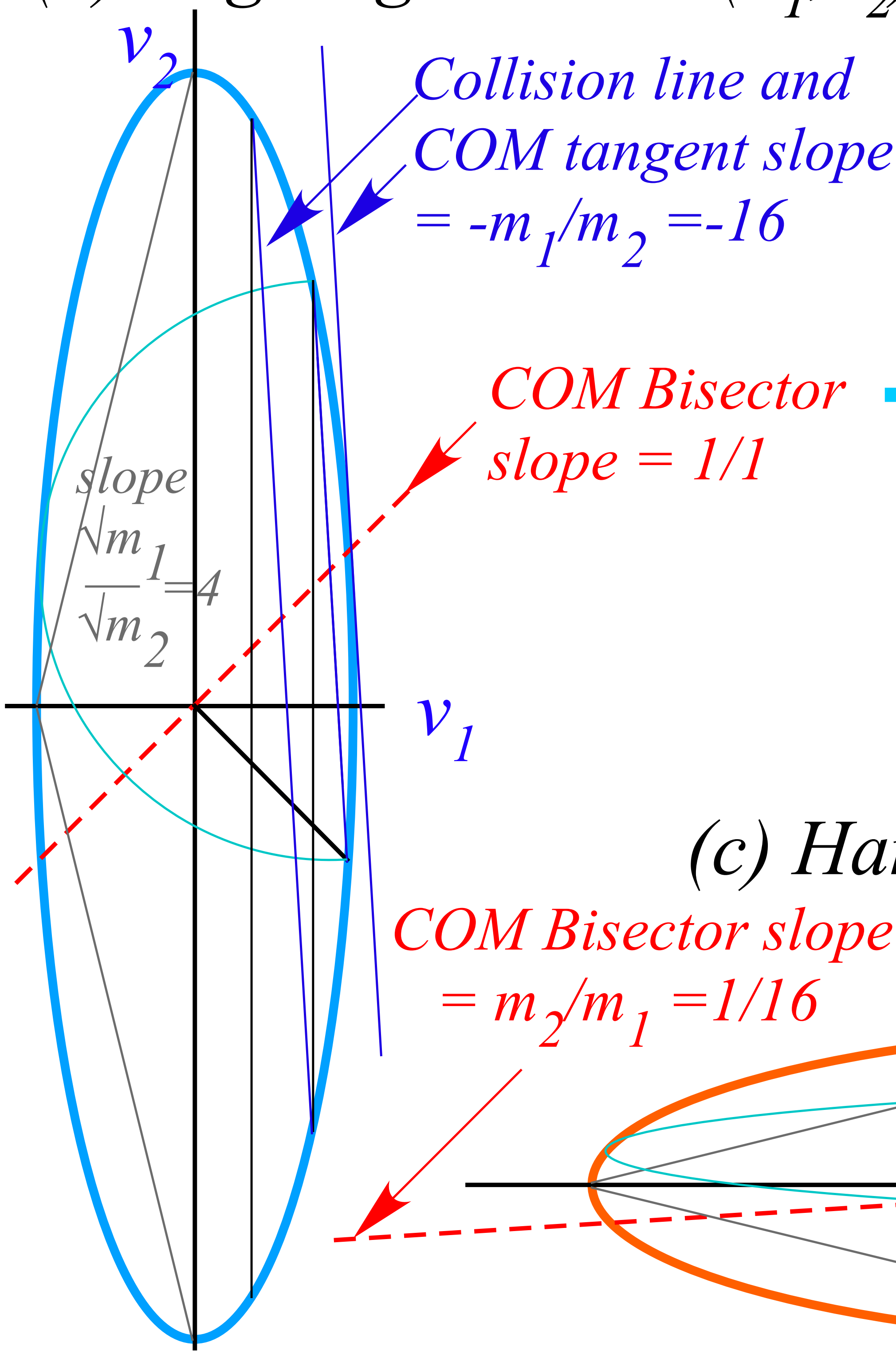


What ellipse rescaling leads to...

Fig. 12.1 (Unit 1)

How this relates to Lagrangian, and Hamiltonian mechanics later on

(a) Lagrangian $L = L(v_1,v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$

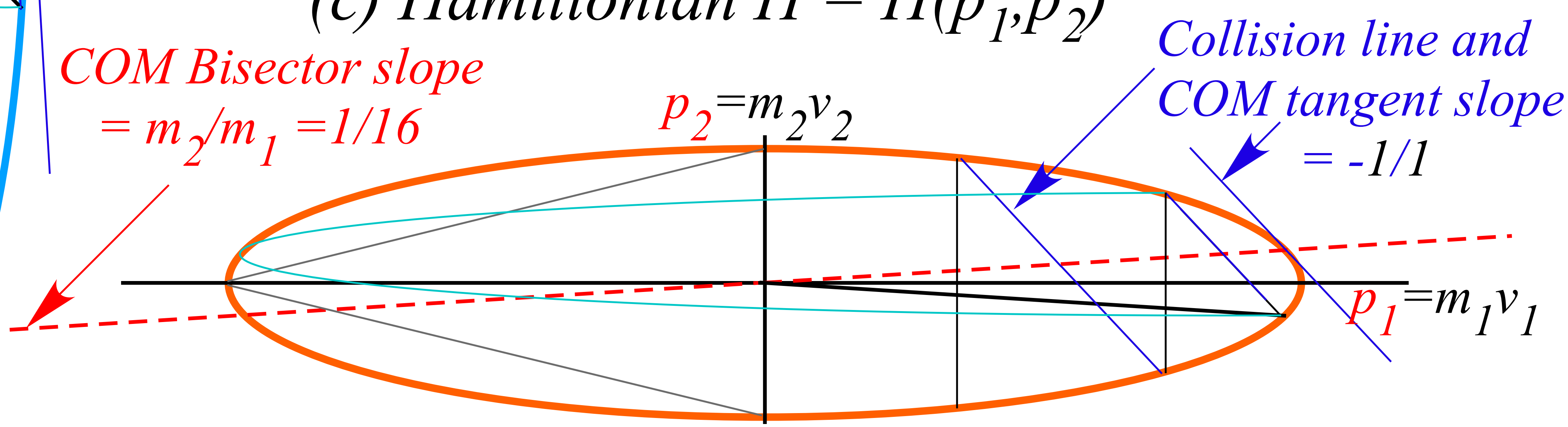
velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

Lagrangian $L(v_1,v_2) = KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

rescaled to

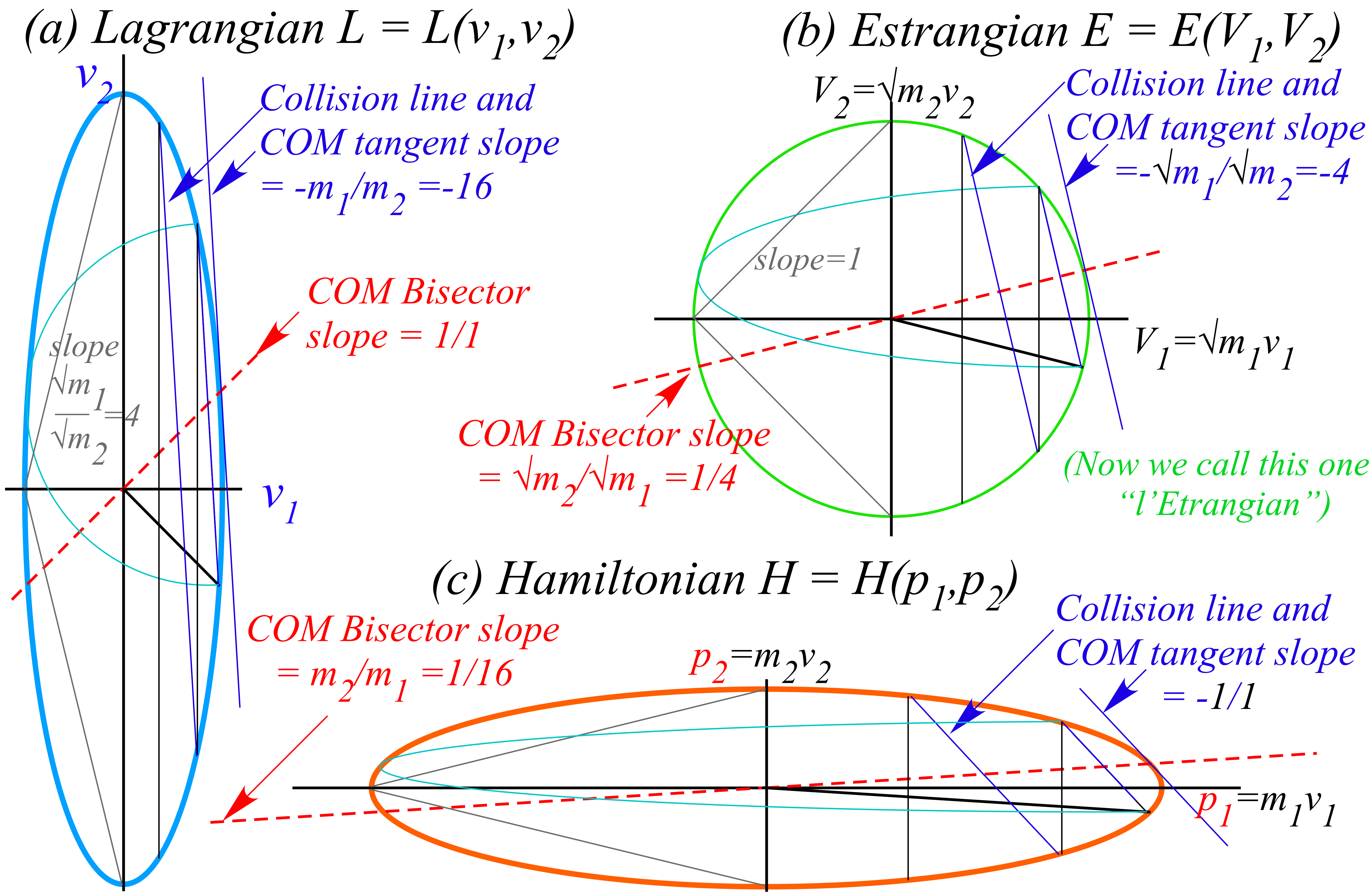
Hamiltonian $H(p_1,p_2) = KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$

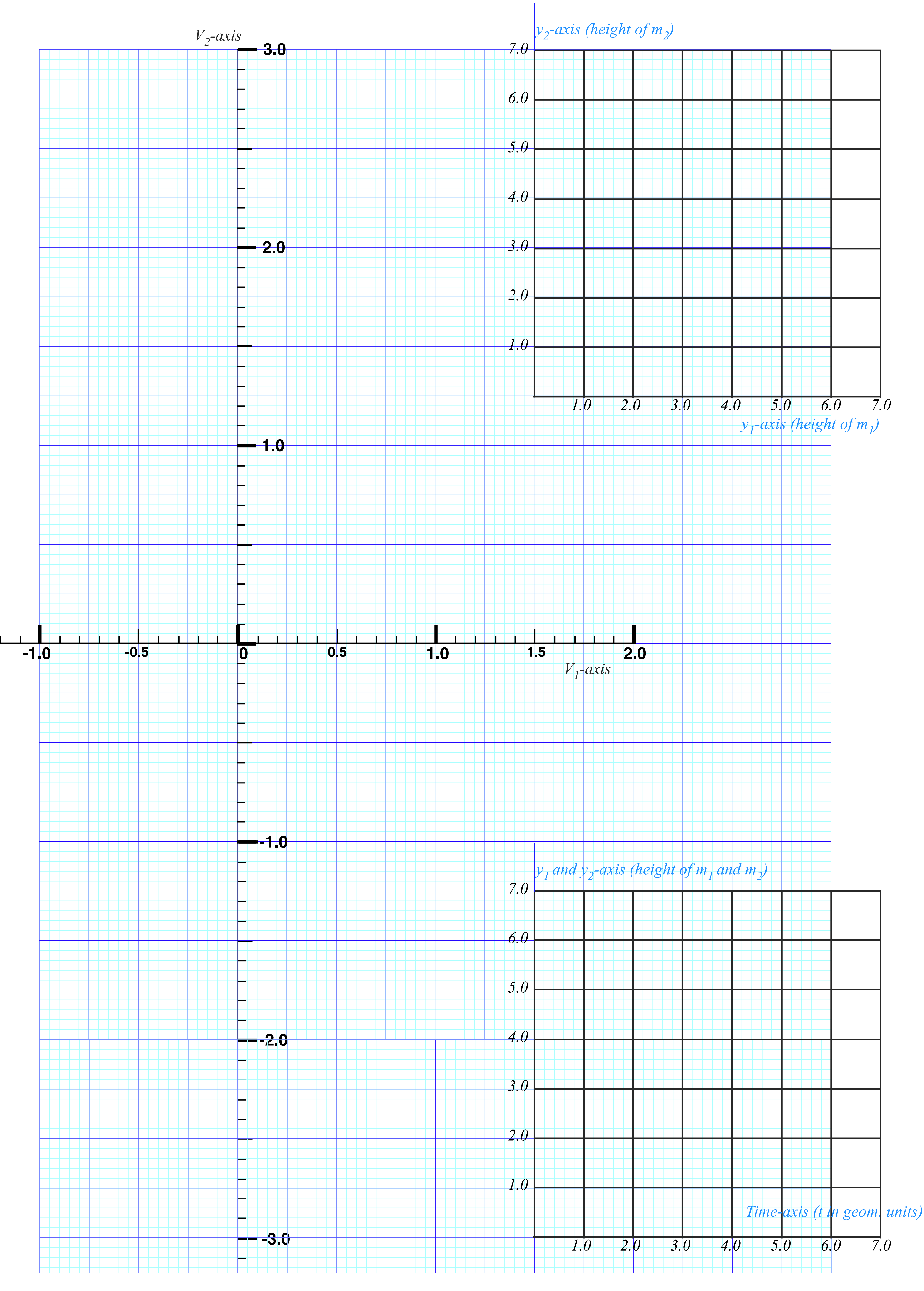
(c) Hamiltonian $H = H(p_1,p_2)$



What ellipse rescaling leads to...

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on





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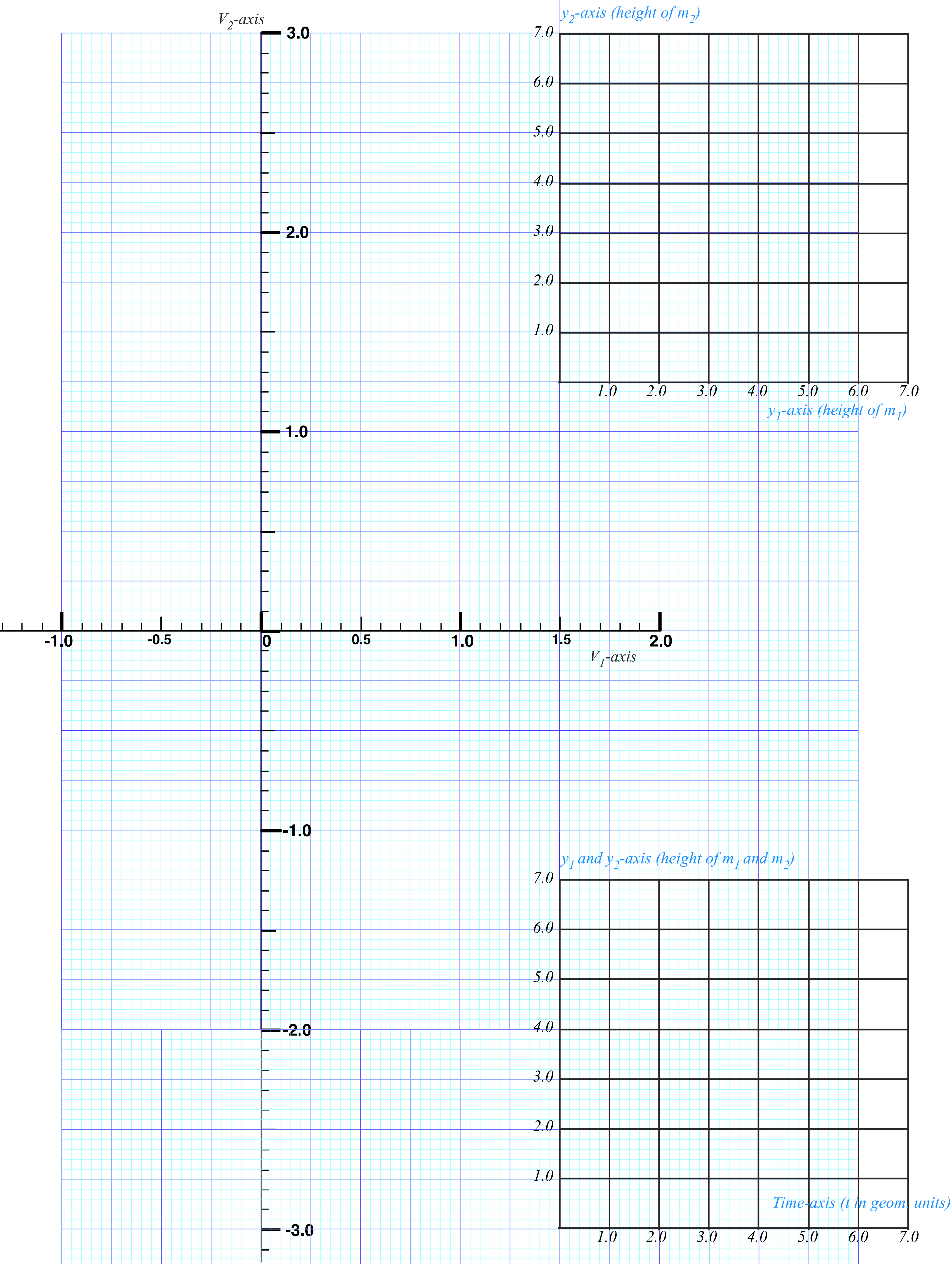
2.0
1.0
-1.0
-2.0
-3.0
-1.0
-0.5
0
0.5
1.0
1.5
2.0
V_1-axis
y_2-axis (height of m_2)
y_1-axis (height of m_1)
y_1 and y_2-axis (height of m_1 and m_2)
Time-axis (t in geom. units)
1.0
2.0
3.0
4.0
5.0
6.0
7.0


V_2-axis
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2.0
1.0
-1.0
-2.0
-3.0
-1.0
-0.5
0
0.5
1.0
1.5
2.0
V_1-axis
y_2-axis (height of m_2)
7.0
6.0
5.0
4.0
3.0
2.0
1.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
y_1-axis (height of m_1)
y_1 and y_2-axis (height of m_1 and m_2)
7.0
6.0
5.0
4.0
3.0
2.0
1.0
Time-axis (t in geom. units)
1.0
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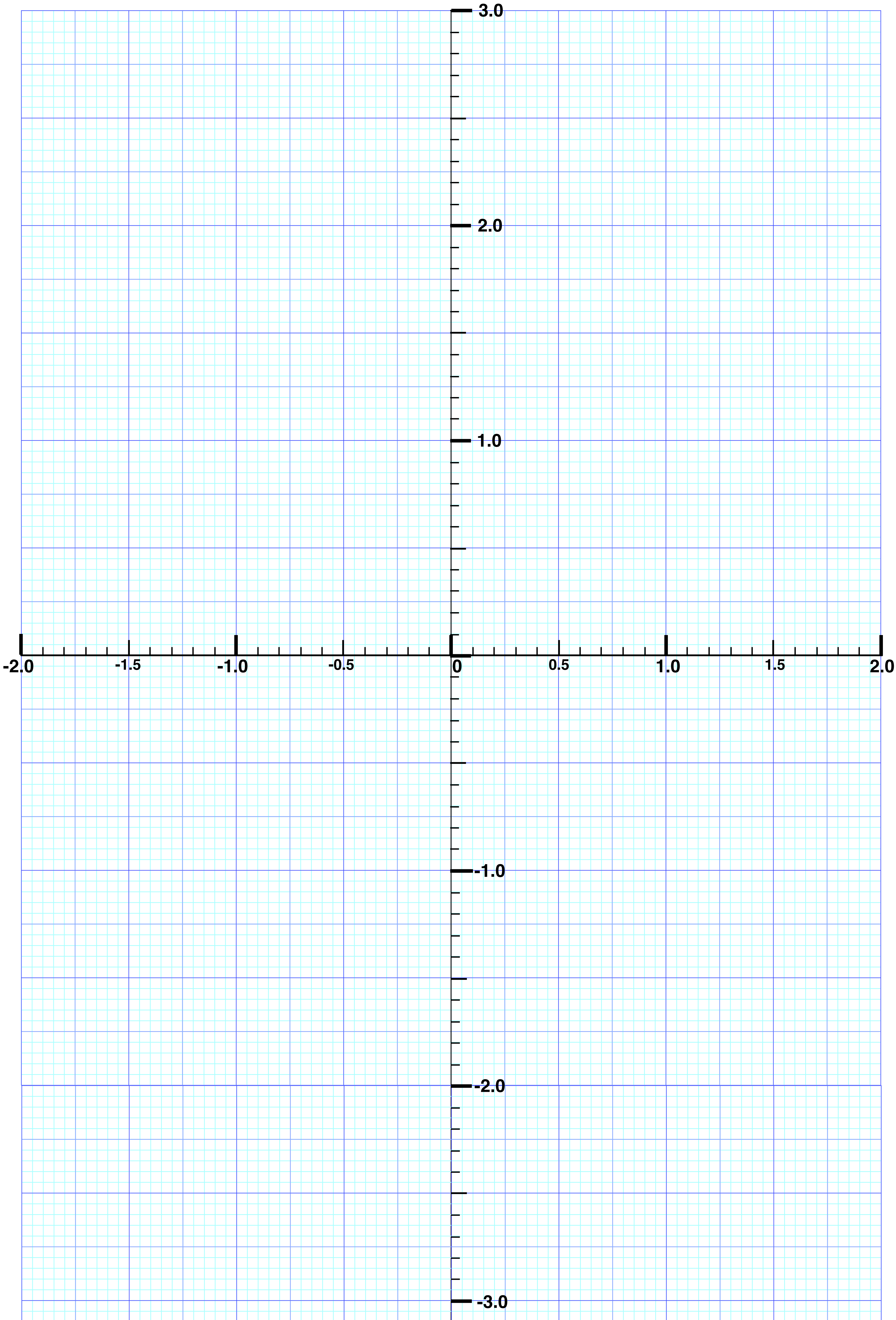

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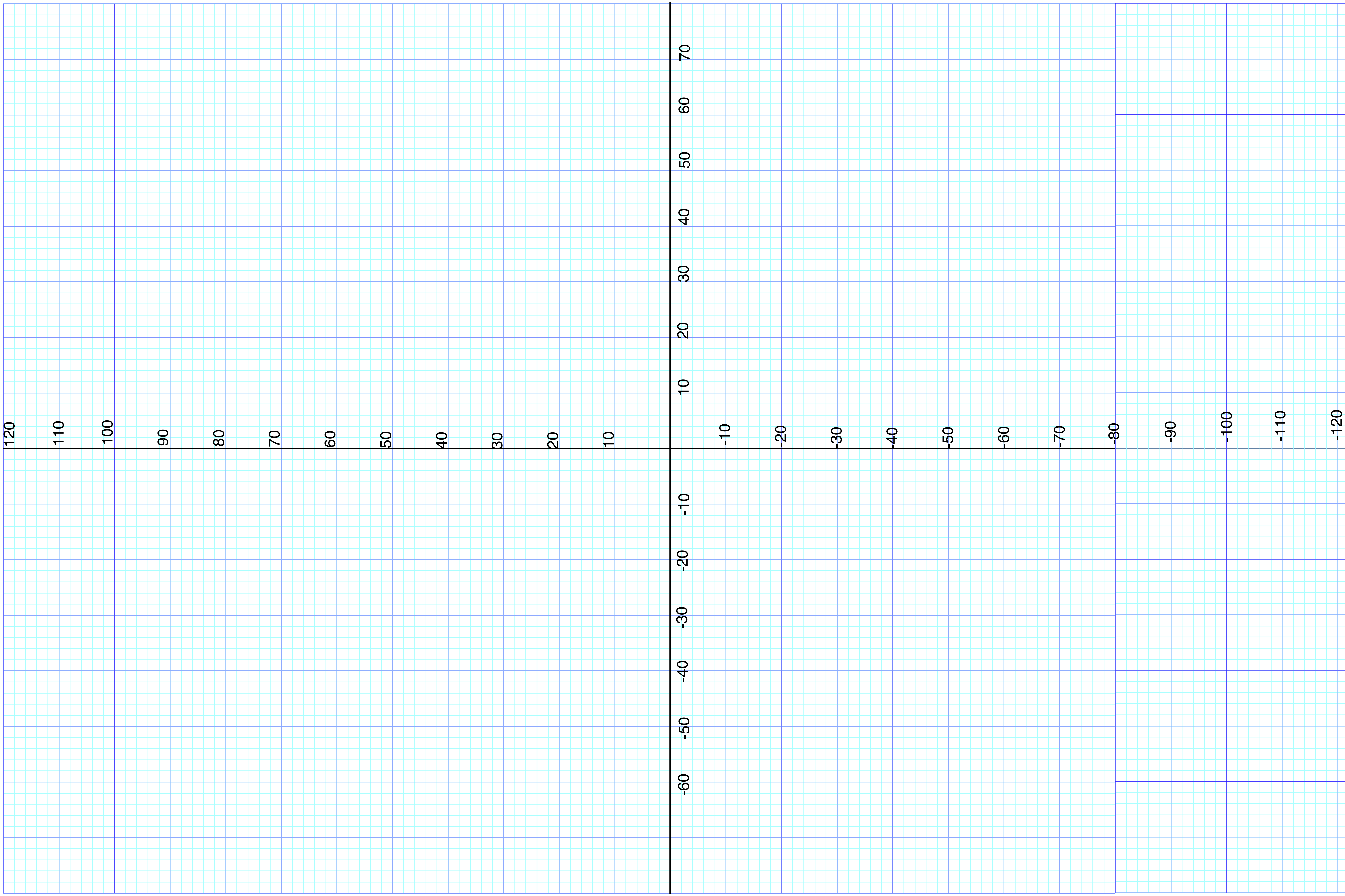
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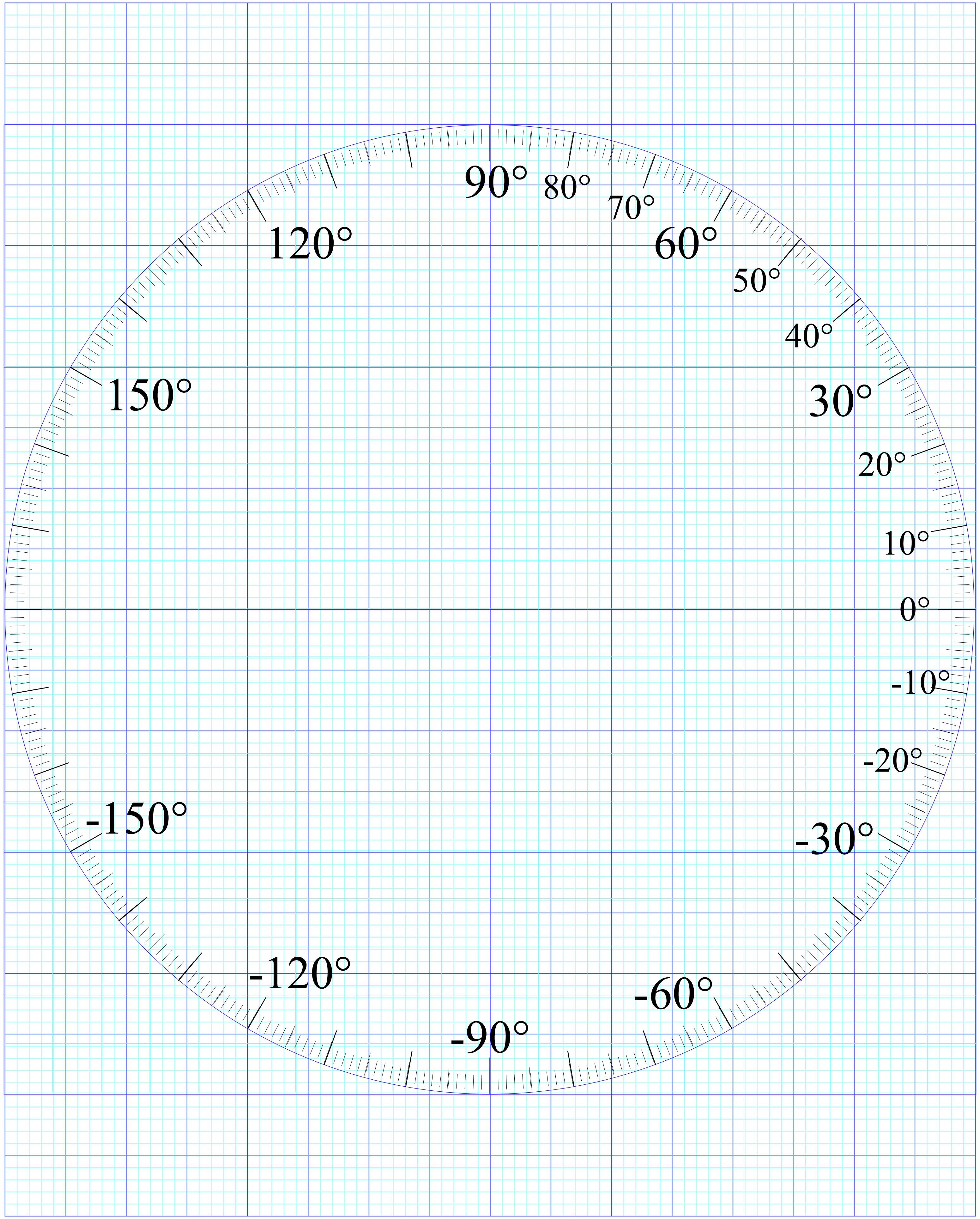
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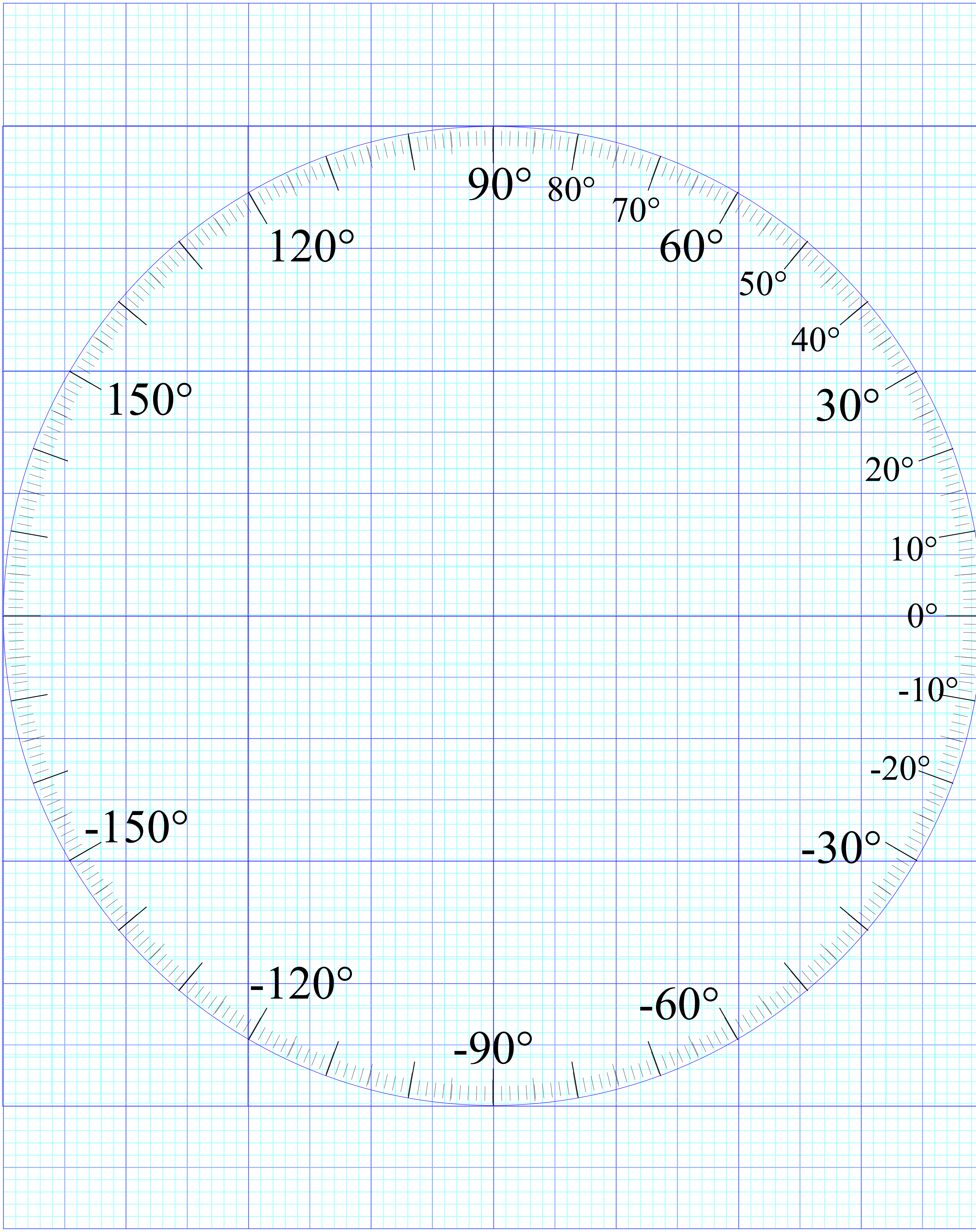

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-120°
-60°
-90°