Lecture 30
Wed. 12.05.2018

Relawavity : a novel introduction to relativistic mechanics III.

(CMwBang! Unit 8 , AMOP Ch.0,)

Review: Relawavity ρ functions and plots vs. ρ

Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

A running collection of links to course-relevant sites and articles

Physics Web Resources

Comprehensive Harter-Soft Resource Listing

UAF Physics YouTube channel

LearnIt Physics Web Applications

"Texts"

Classical Mechanics with a Bang!

Quantum Theory for the Computer Age

Principles of Symmetry, Dynamics, and Spectroscopy

Modern Physics and its Classical Foundations

Classes

2014 AMOP

2017 Group Theory for QM

2018 AMOP

2018 Adv Mechanics

Neat external material to start the class:
- AIP publications
- AJP article on superball dynamics
- AAPT summer reading

AMOP Detailed Development of Relawavity
AMOP Ch 0 Space-Time Symmetry - 2019
Seminar at Rochester Institute of Optics, Auxiliary slides, June 19, 2018
Springer AMO Handbook - Ch 32 - Harter-Reimer-2019

These *are* hot off the presses:
- Sorting ultracold atoms in a 3D optical lattice in a realization of Maxwell's demon - Kumar-Nature-Letters-2018
- Synthetic three-dimensional atomic structures assembled atom by atom - Berredo-Nature-Letters-2018

Slightly Older ones:
- Wave–particle duality of C60 molecules
- Optical vortex knots – One Photon at a Time

"*Relawavity*" and quantum basis of *Lagrangian* & *Hamiltonian* mechanics:
- 2-CW laser wave - BohrIt Web App
- Lagrangian vs Hamiltonian - RelaWavity Web App

Older Links from Lectures 14-20

http://thearmchaircritic.blogspot.com/2011/11/punkin-chunkin.html
http://www.sussexcountyonline.com/news/photos/punkinchunkin.html
Shooting-range-for-medieval-siege-weapons-Anybody-knows
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html?scenario=MontezumasRevenge
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html?scenario=SeigeOfKenilworth
The trebuchet, Chevedden, Sci Am 1995
'*Simple*' *Pendulum* Sim: https://modphys.hosted.uark.edu/markup/PendulumWeb.html
'*Cycloid*' *Pendulum*: https://modphys.hosted.uark.edu/markup/CycloidulumWeb.html
Google search on: "Satelite view of Patricia" (Images)
Physics Girl Channel - Fun with Vortex Rings in the Pool
iBall demo - Quasi-periodicity: https://youtu.be/_jntDtULxDc
https://modphys.hosted.uark.edu/markup/CoulItWeb.html?scenario=SynchrotronMotion
https://modphys.hosted.uark.edu/markup/CoulItWeb.html?scenario=SynchrotronMotion2
Mechanical Analog to EM Motion (YouTube video) - https://youtu.be/hTd5FTJ-vRk
CoulIt Web Simulation: Bound-state motion in parabolic coordinates
CoulIt Web Simulation: Bound-state motion in hyperbolic coordinates
OscillIt Web App: Simulations of various types of resonance: 18, 27, 31, 35, 38, 39
Smith Chart
http://nobelprize.org/

AnalyIt Web Application, posted 10/22/2018 in our *testing* area:
https://modphys.hosted.uark.edu/testing/markup/AnalyItBJS.html

Older Links from Lectures 21-23

Advanced Atomic and Molecular Optical Physics 2018 Class #9, pages: 5, 61
BoxIt Web Simulations
- Pure A-Type w/Cosine
- Pure B-Type w/Cosine
- Pure B-Type w/Freq ratios
- Mixed AB-Type 2:1 Freq ratio
- Pure A-Type A=4.9, B=0 ,C=0, & D=4.0
- Pure B-Type: A=4.0, B=-0.2, C=0, & D=4.0
- Pure C-Type A,D=4.055, B=0, C=0.1
- Mixed AB-Type w/Cosine
- Mixed AB Type A=4.0, BU2=0.866…, CU2=0, & D=1.0 w/Stokes & Freq rats
- Mixed AB Type A=5.086 B=-0.27 C=0 D=2.024 w/Stokes plot
- Mixed ABC Type A=4.833 B=0.2403 C=0.4162 D=4.277 w/Stokes plot
- Recent mixed ABC Type A=0.325 B=0.375 C=0.825 D=0.05 w/Stokes plot

Classical Mechanics with a Bang! 2018
- Lectures 8, 9, 23 page 93
- Text Unit 6, page=27

ColorU2 for the Web - in development
Group Theory for Quantum Mechanics - 2017 Lectures: 6, 7, 8,
and the combined 9-10
Quantum Theory for the Computer Age Unit 3 Ch.7-10, page=90
Web based 3D & XR (x∊{A,M,V}, R=Reality) https://www.babylonjs.com/
Web based 3D graphics WebGL API (Graphics Layer modeled after OpenGL)
Wiki on Pafnuty Chebyshev

continued ↘

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Classes

2014 AMOP

2017 Group Theory for QM

2018 AMOP

2018 Adv Mechanics

Repeated from previous page

Older Links from Lectures 24-27

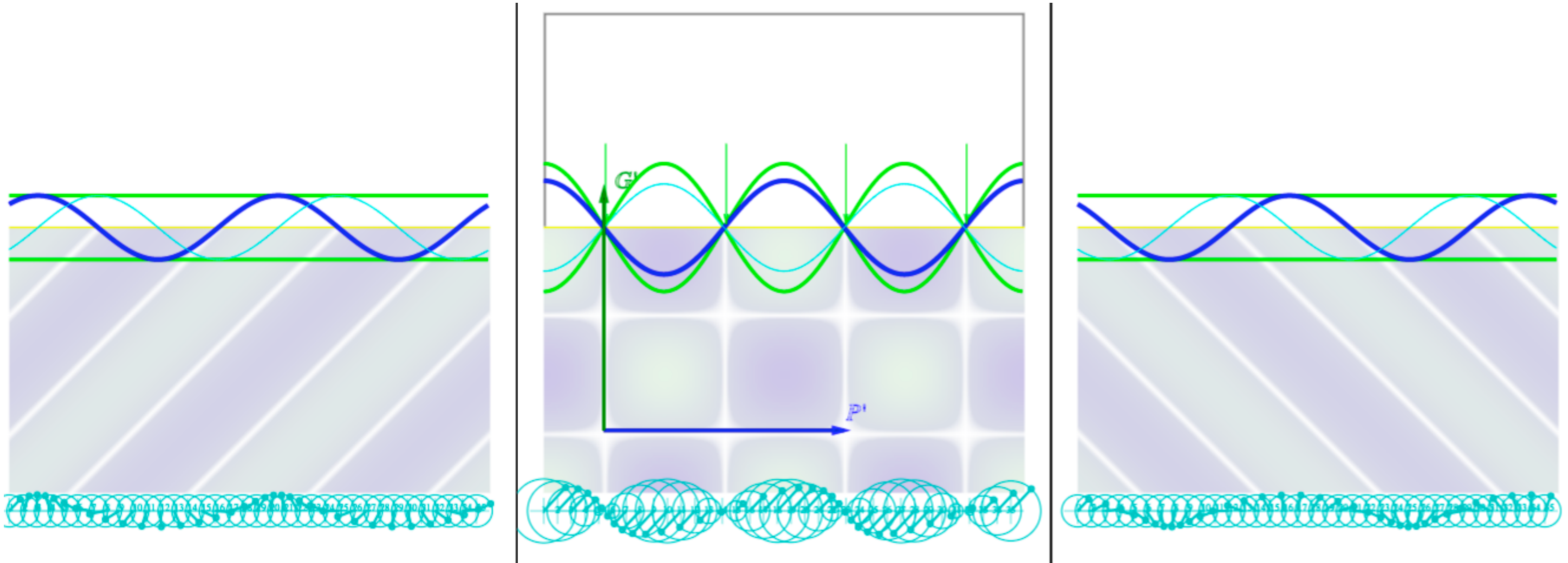
JerkIt Web App: 2-, 2+, Amp50Omega147-, Amp50Omega296, Amp50Omega602, Gap(1)
MolVibes Web App: C3vN3
WaveIt Web App:
  Dim = 3 w/Wave Components;
  Static Char Table: 6, 12, 12(b), 16, 36, 256
  Quantum Carpet with N=20: Gaussian, Boxcar
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-CPL-2015
QTCA Unit_5 Ch14 2013
Lester. R. Ford, Am. Math. Monthly 45,586(1938)
John Farey, Phil. Mag.(1816) Wolfram
Harter, J. Mol. Spec. 210, 166-182 (2001)
Harter, Li IMSS (2013)
Li, Harter, Chem.Phys.Letters (2015)
OscillatorPE Web App: IHO Scenario 2, Coulomb Scenario 3
RelaWavity Web App/Simulator/Calculator: Elliptical - IHO orbits
CoulIt Web App Simulations: p19, p32, p72, p73, p92, R=-0.375, R=+0.5, Rutherford

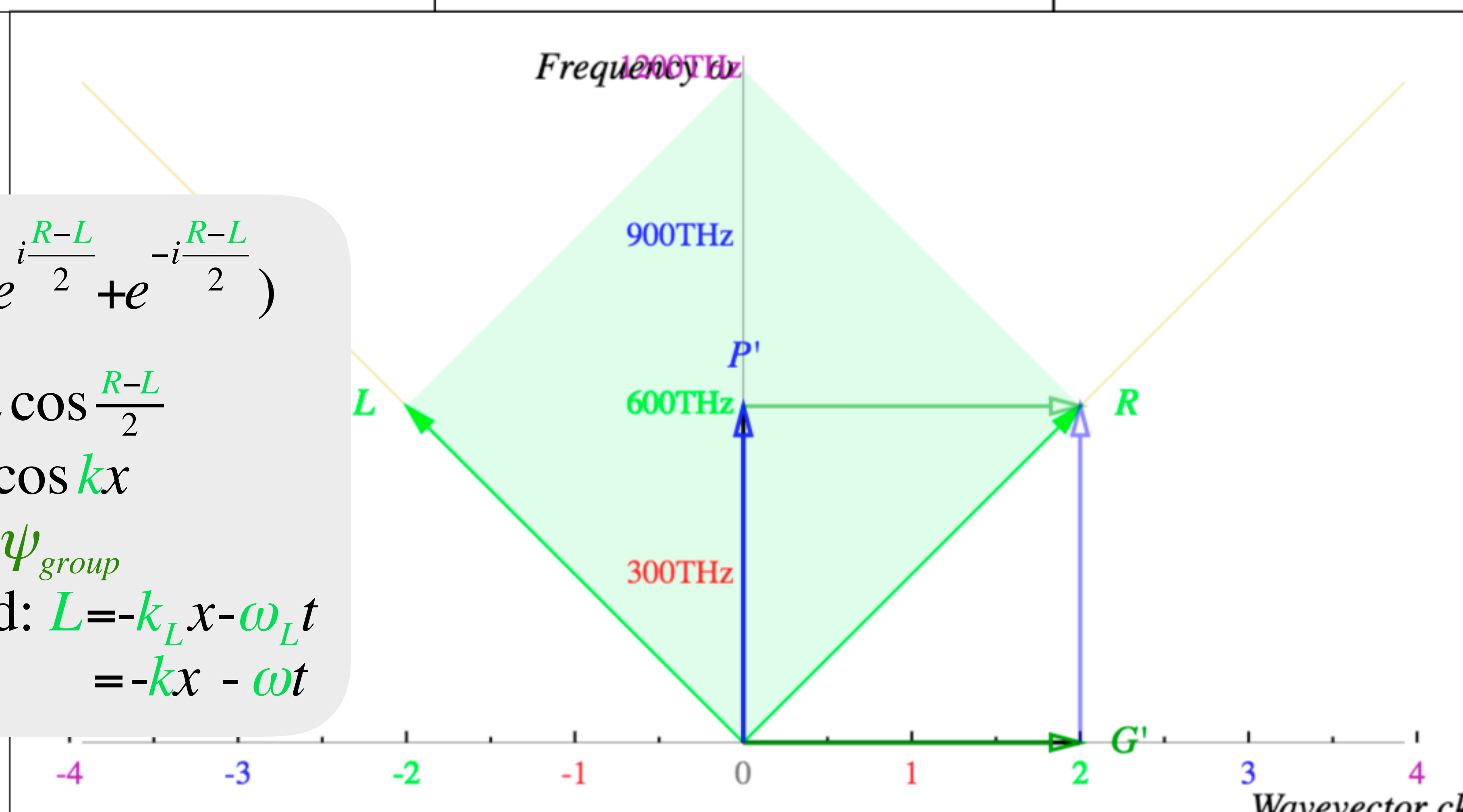
Older Links from Lectures 28

CMwBang Text 2012 Unit 6 page=5
BounceIt Web App/Scenarios:
  5002, 5003
CoulIt Web App/Scenarios:
  TwoParticleCollision_LToR, TwoParticleCollision_LToR_CM, TwoParticleOrbit_Coulomb, TwoParticleOrbit_Coulomb_CM, TwoParticleOrbit_Hooke, TwoParticleOrbit_Hooke_CM
Singular Motion of Asymetric Rotators AJP 44, 11 p1080 Harter-Kim-1976
Molecular Eigensolution Symmetry Analysis and Fine Structure - Int.J.MolSci1.4.13 Harter-Mitchell-IJMS-2013
Lenz Vector and Orbital Analog Computers - AJP 44 p348 1976
Some Geometric Aspects of Classical Coulomb Scattering AJP 40 4 p1852 1972
How Molecules do Self-NMR - Harter-Mitchell-Columbus-2009
Classical Mechanics with a Bang! - Asymmetric Top Demo
Allbookstores.com - Compare for Heller's SemiClassical Way - 0691163731
"My Bomerang Won't Come Back" (YouTube: Playlist)
Rotating Solid Bodies in Micrograverity (YouTube)
Dancing T-handle in zero-g (YouTube)

Supplemental Links for Lectures 29-30
RelaWavity

AMOP Chapter 0: Space-Time Symmetry
AMOP Detailed Development of RelaWavity
2018 Rochester Talk (Auxilary Slides)
Special Relativity and Quantum Theory by Ruler and Compass - Earlier, expanded draft
Ruler & Compass - Relawavity Exercise
2018 RelaWavity Portal Page
Pirelli Relativity Challenge Web Site:
  Clocks_12_hr, Clocks_24_hr_QT, Phasors Addition, Quantized_1
BohrIt Web App/Simulations: -130022; -30001; -30104; 30004; 30022
GuideIt Web App/Scenarios: 230; 260
RelativIt Web App/Scenarios: 22; 24; 69
RelaWavity Web App/Scenarios: 0,9; 3,6; 3,6 NoMink; 4,8; 5,6a; 6,1; 6,3a; 6,3b; 6.3c; 7,1; 7,2,1; 7,2,2; 7,2,3; 7,2,7; 8,3; 8,5; 8,6; 8,7; 8,8
2012 ModPhys Lect 35





$$e^{iR}+e^{iL}=e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}}\right)$$
$$=e^{i\frac{R+L}{2}}\,2\cos\frac{R-L}{2}$$
$$=2e^{-i\omega t}\cos kx$$
$$=\psi_{phase}\psi_{group}$$

$R=k_R x-\omega_R t$ and: $L=-k_L x-\omega_L t$

$= kx - \omega t$ $\quad$ $=-kx - \omega t$

Fig. 9 in text

[*BohrIt Web Simulation: ±600THz*]

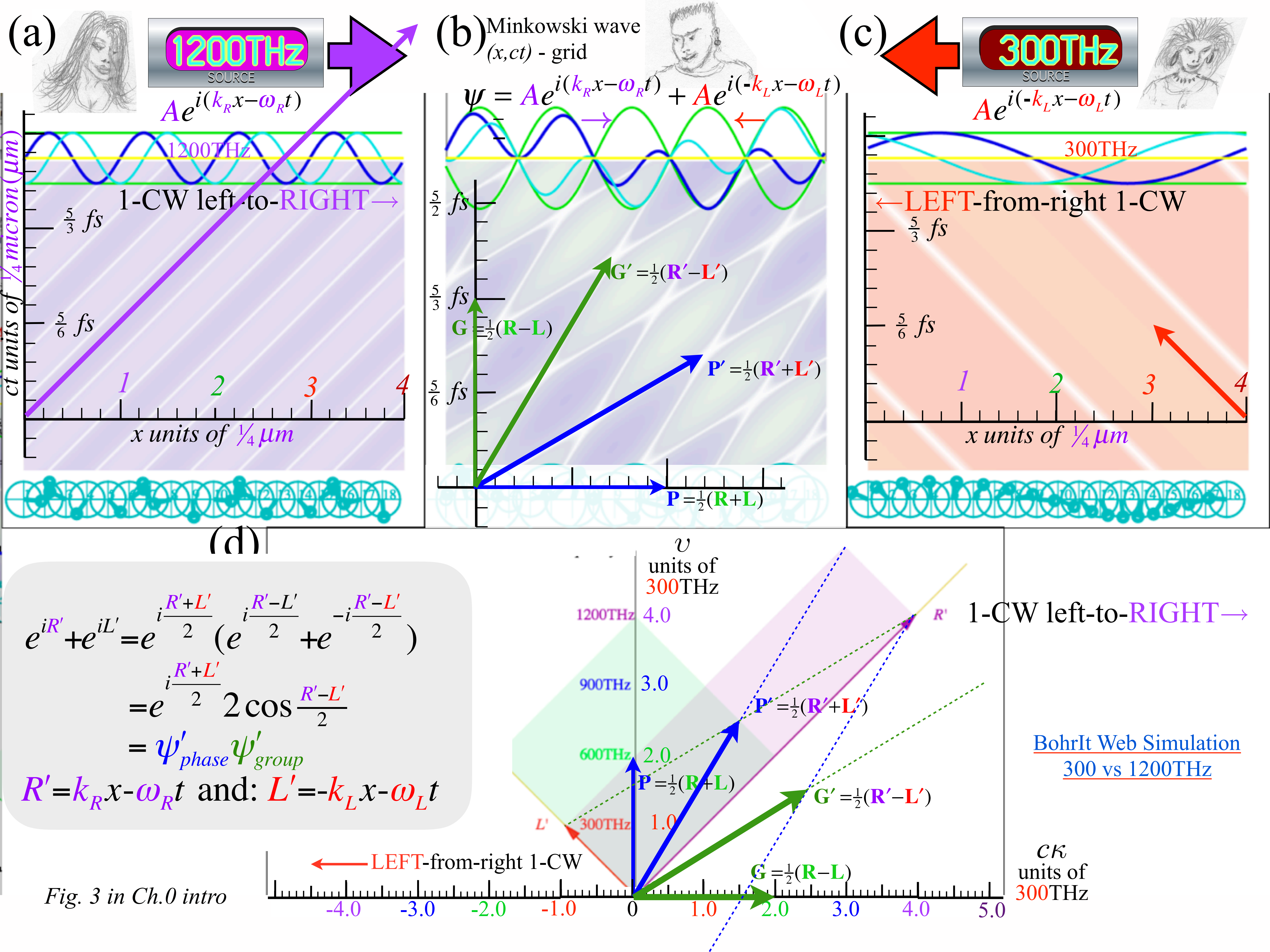
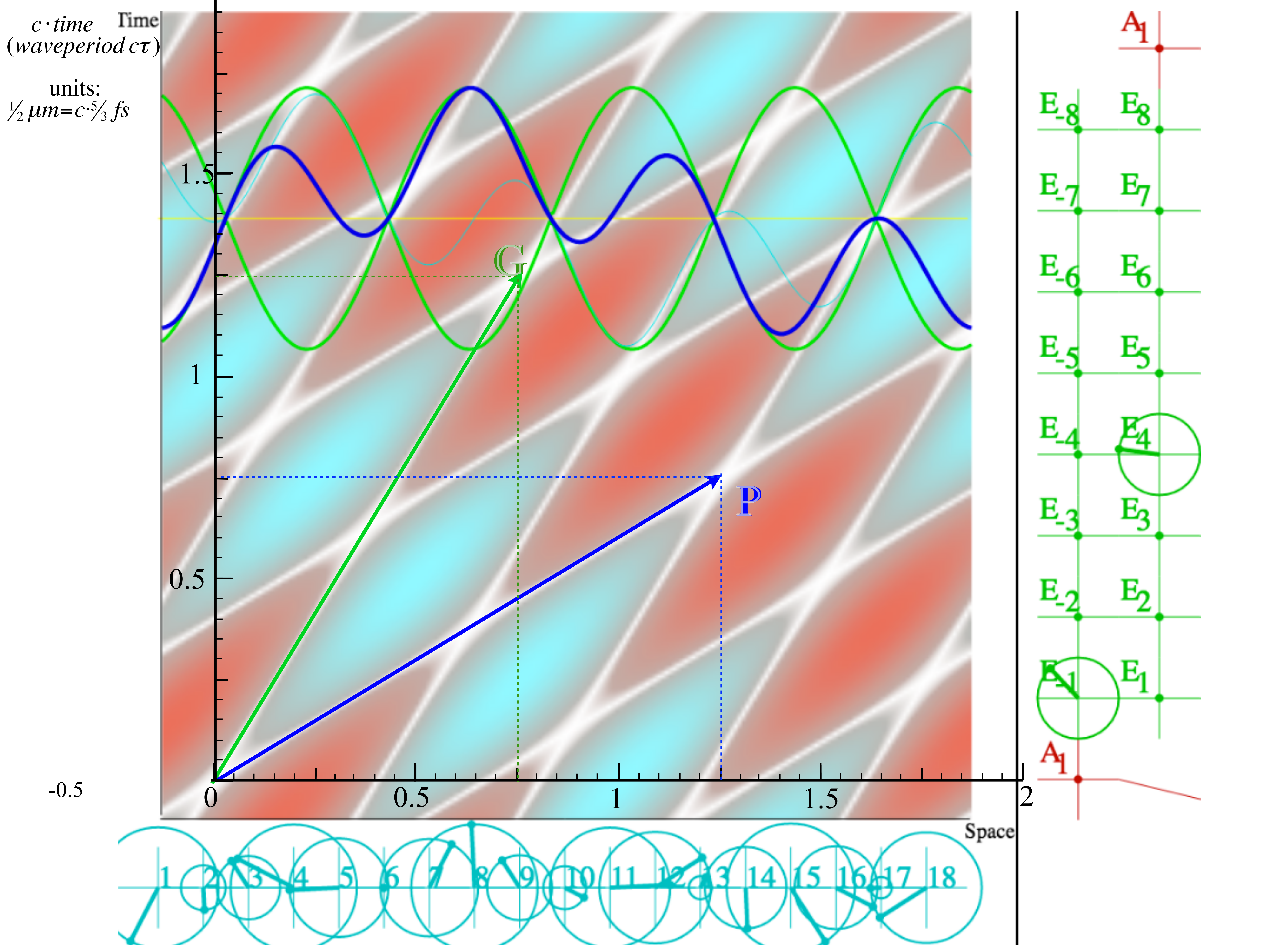
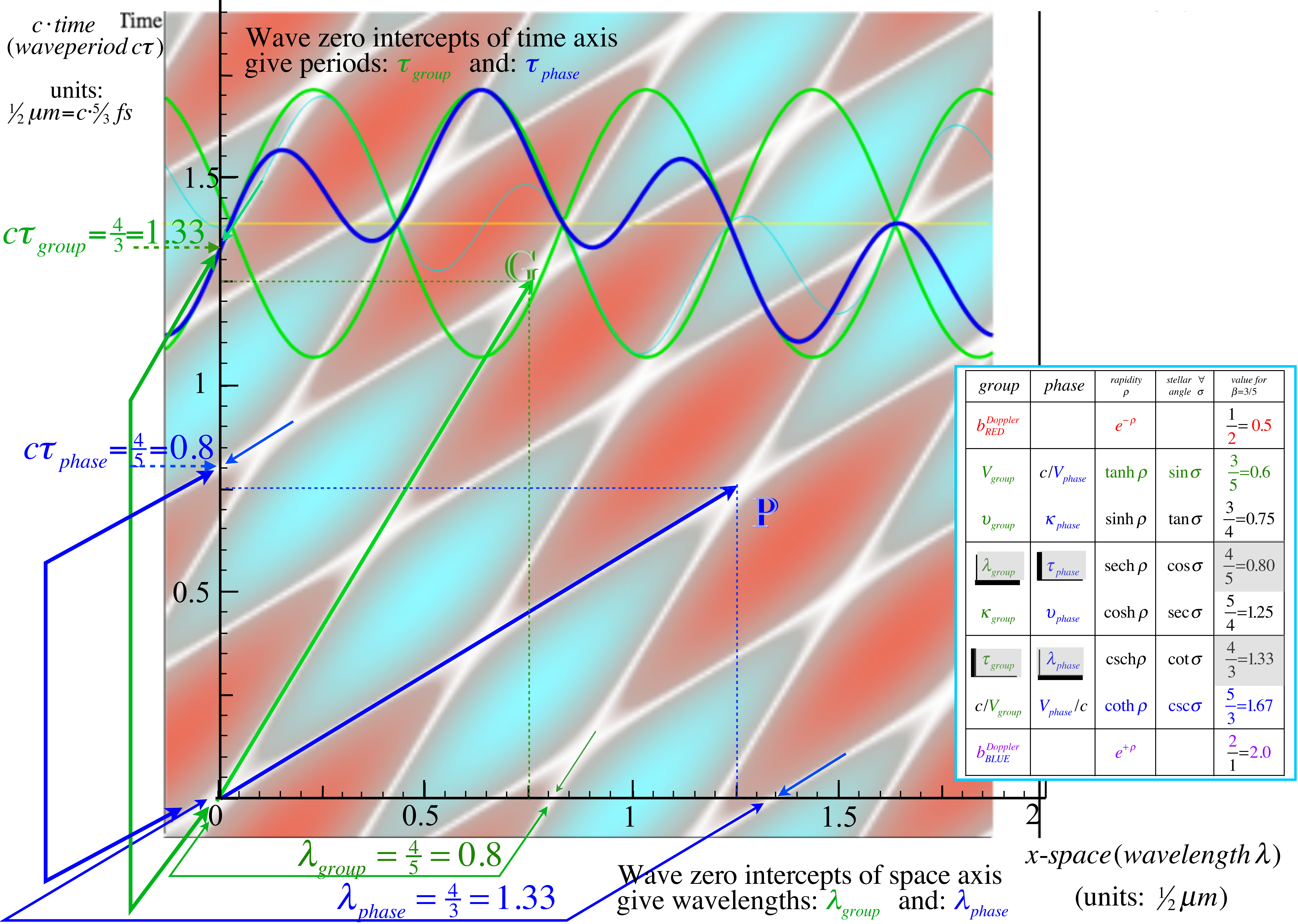


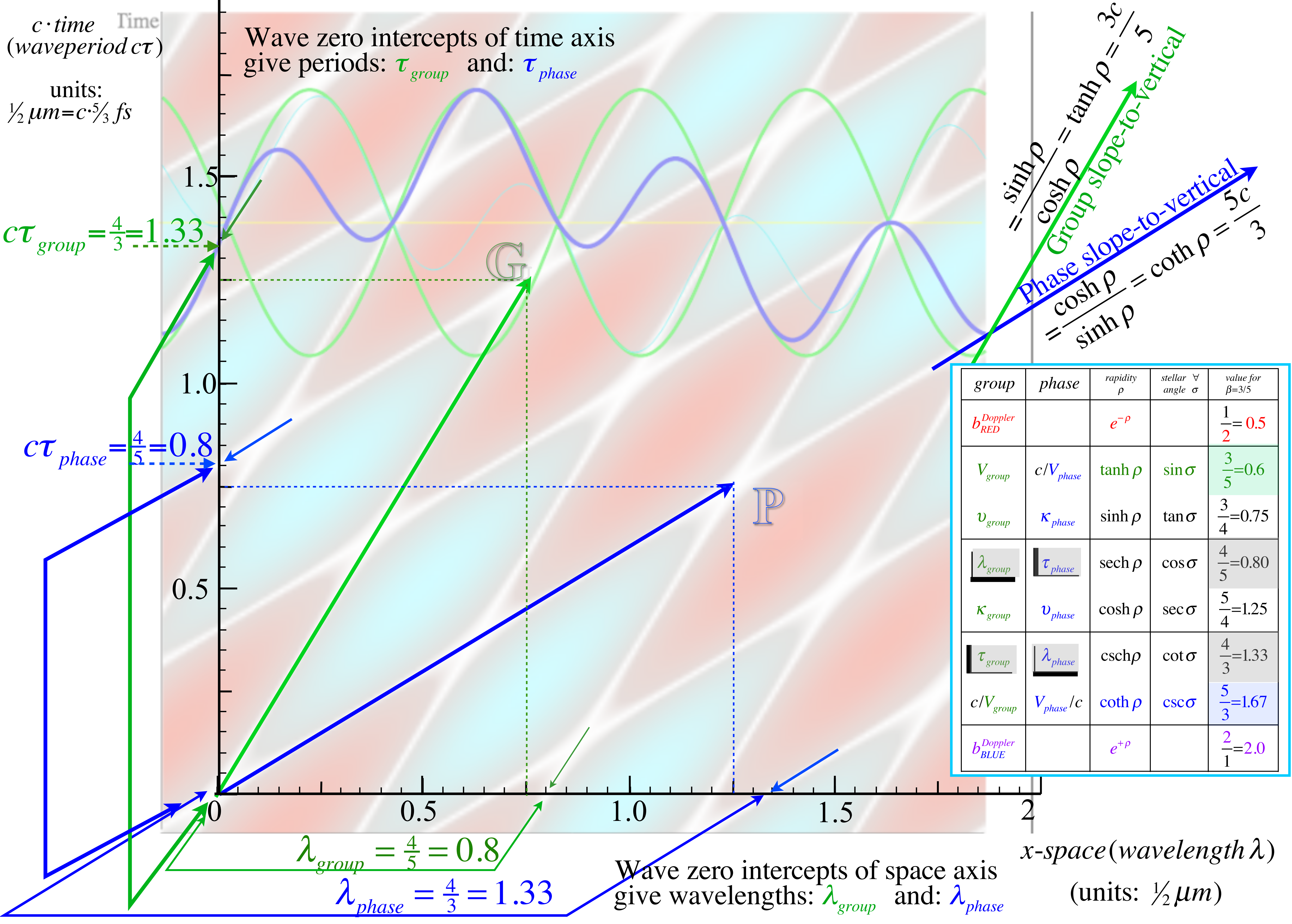
Fig. 3 in Ch.0 intro



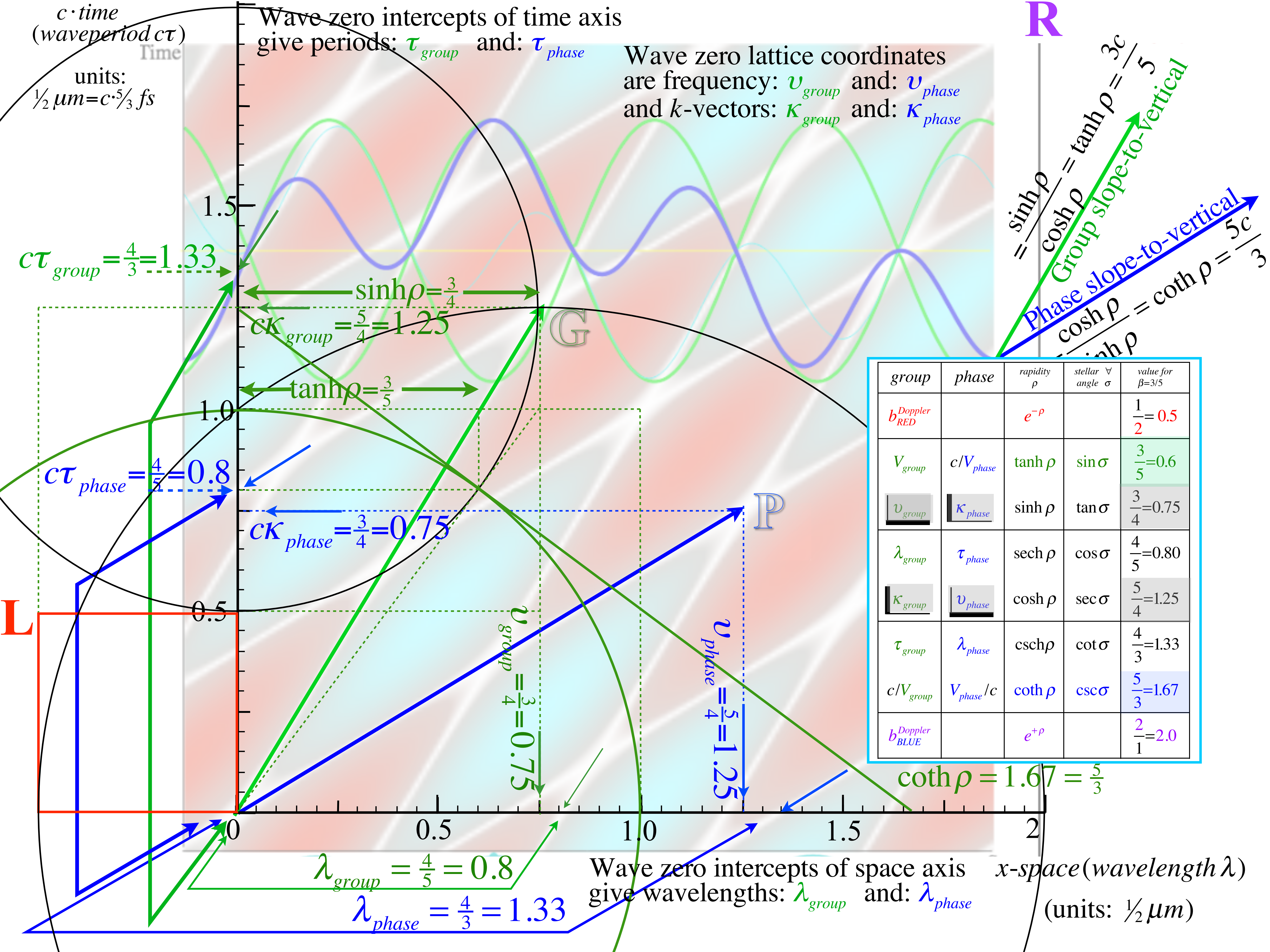
c · time
(waveperiod cτ)
units:
½ μm=c·5⁄3 fs
Time
1.5
1
0.5
-0.5
0
0.5
1
1.5
2
Space
G
P
A1
E-8 E8
E-7 E7
E-6 E6
E-5 E5
E-4 E4
E-3 E3
E-2 E2
E-1 E1
A1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18



group	*phase*	*rapidity* ρ	*stellar* ∀ *angle* σ	*value for* $\beta=3/5$
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2}=0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\operatorname{sech}\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}=1.25$
τ_{group}	λ_{phase}	$\operatorname{csch}\rho$	$\cot\sigma$	$\frac{4}{3}=1.33$
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}=1.67$
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}=2.0$



group	phase	rapidity ρ	stellar ∀ angle σ	value for $\beta=3/5$
$b^{Doppler}_{RED}$		$e^{-\rho}$		$\frac{1}{2}=0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\mathrm{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}=1.25$
τ_{group}	λ_{phase}	$\mathrm{csch}\rho$	$\cot\sigma$	$\frac{4}{3}=1.33$
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}=1.67$
$b^{Doppler}_{BLUE}$		$e^{+\rho}$		$\frac{2}{1}=2.0$



group	phase	rapidity ρ	stellar ∀ angle σ	value for $\beta=3/5$
$b^{Doppler}_{RED}$		$e^{-\rho}$		$\frac{1}{2}=0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\text{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
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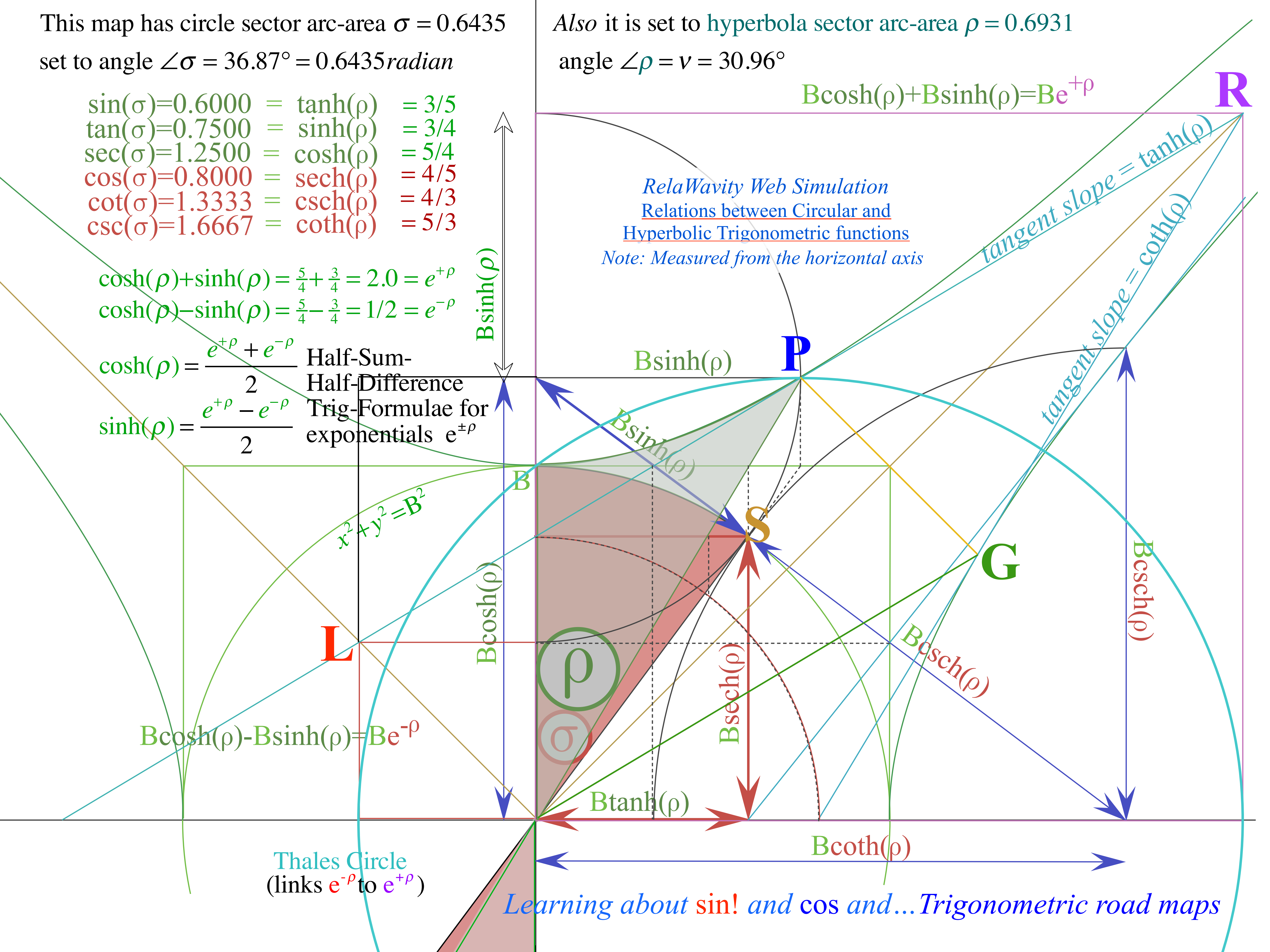

This map has circle sector arc-area σ = 0.6435
set to angle ∠σ = 36.87° = 0.6435 radian
Also it is set to hyperbola sector arc-area ρ = 0.6931
angle ∠ρ = ν = 30.96°
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
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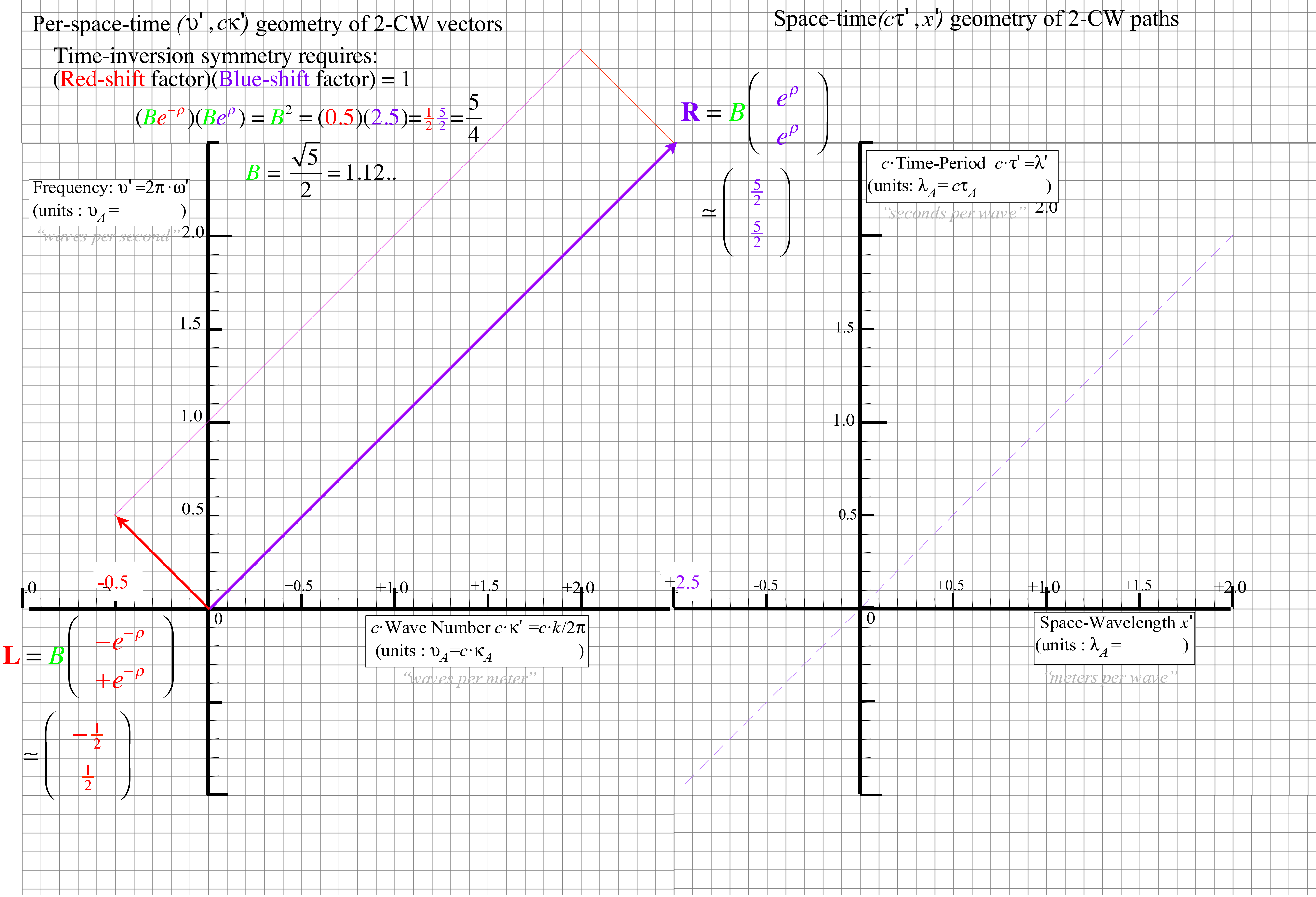
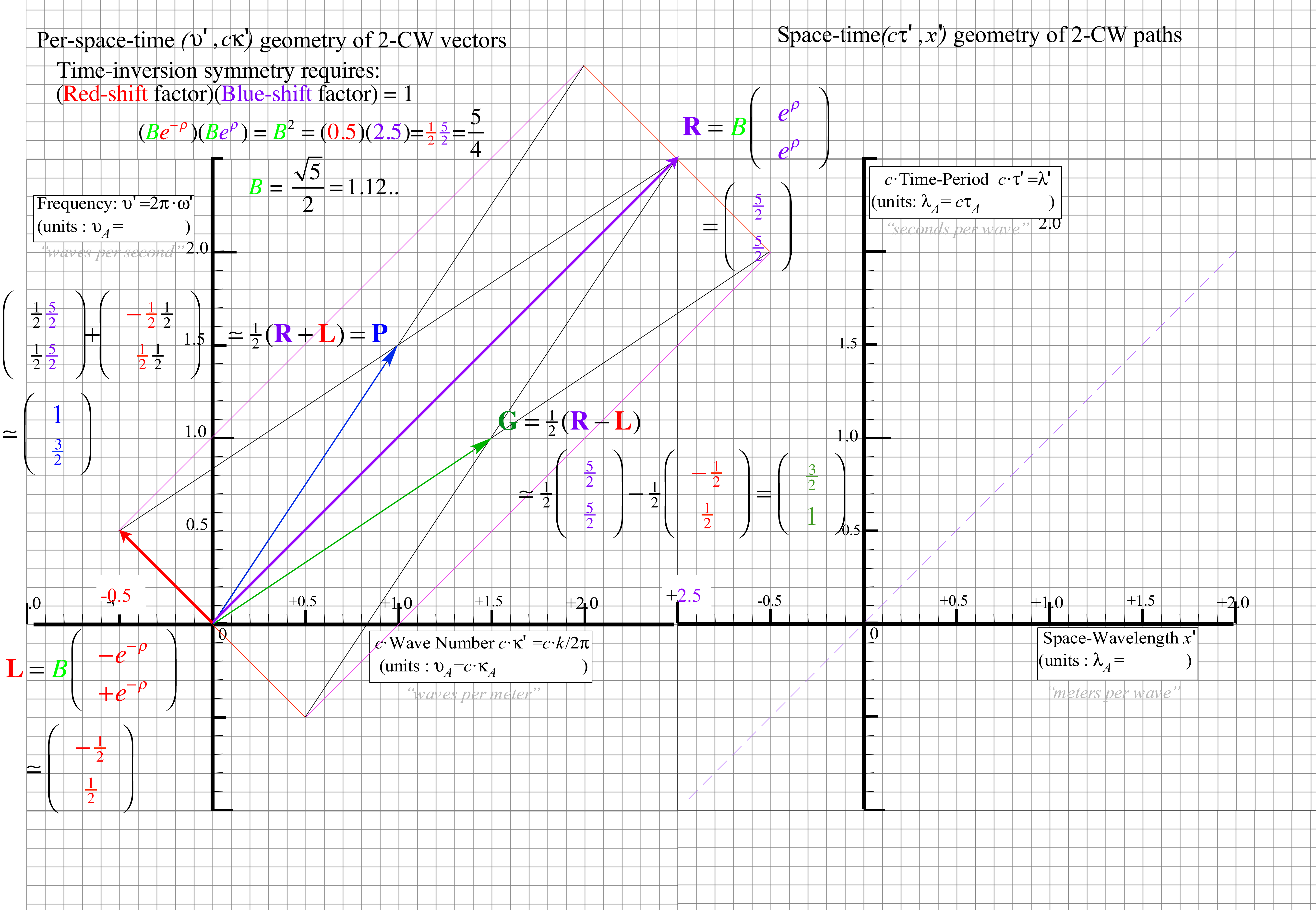
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3
cosh(ρ)+sinh(ρ) = 5/4 + 3/4 = 2.0 = e^{+ρ}
cosh(ρ)−sinh(ρ) = 5/4 − 3/4 = 1/2 = e^{−ρ}
cosh(ρ) = (e^{+ρ} + e^{−ρ})/2
sinh(ρ) = (e^{+ρ} − e^{−ρ})/2
Half-Sum-
Half-Difference
Trig-Formulae for
exponentials e^{±ρ}
Bcosh(ρ)+Bsinh(ρ)=Be^{+ρ}
R
RelaWavity Web Simulation
Relations between Circular and
Hyperbolic Trigonometric functions
Note: Measured from the horizontal axis
tangent slope = tanh(ρ)
tangent slope = coth(ρ)
Bsinh(ρ)
Bsinh(ρ)
Bsinh(ρ)
P
B
x^2+y^2=B^2
S
G
L
Bcosh(ρ)
ρ
σ
Bsech(ρ)
Bcsch(ρ)
Bcsch(ρ)
Bcosh(ρ)-Bsinh(ρ)=Be^{-ρ}
Btanh(ρ)
Bcoth(ρ)
Thales Circle
(links e^{-ρ} to e^{+ρ})
Learning about sin! and cos and…Trigonometric road maps

Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



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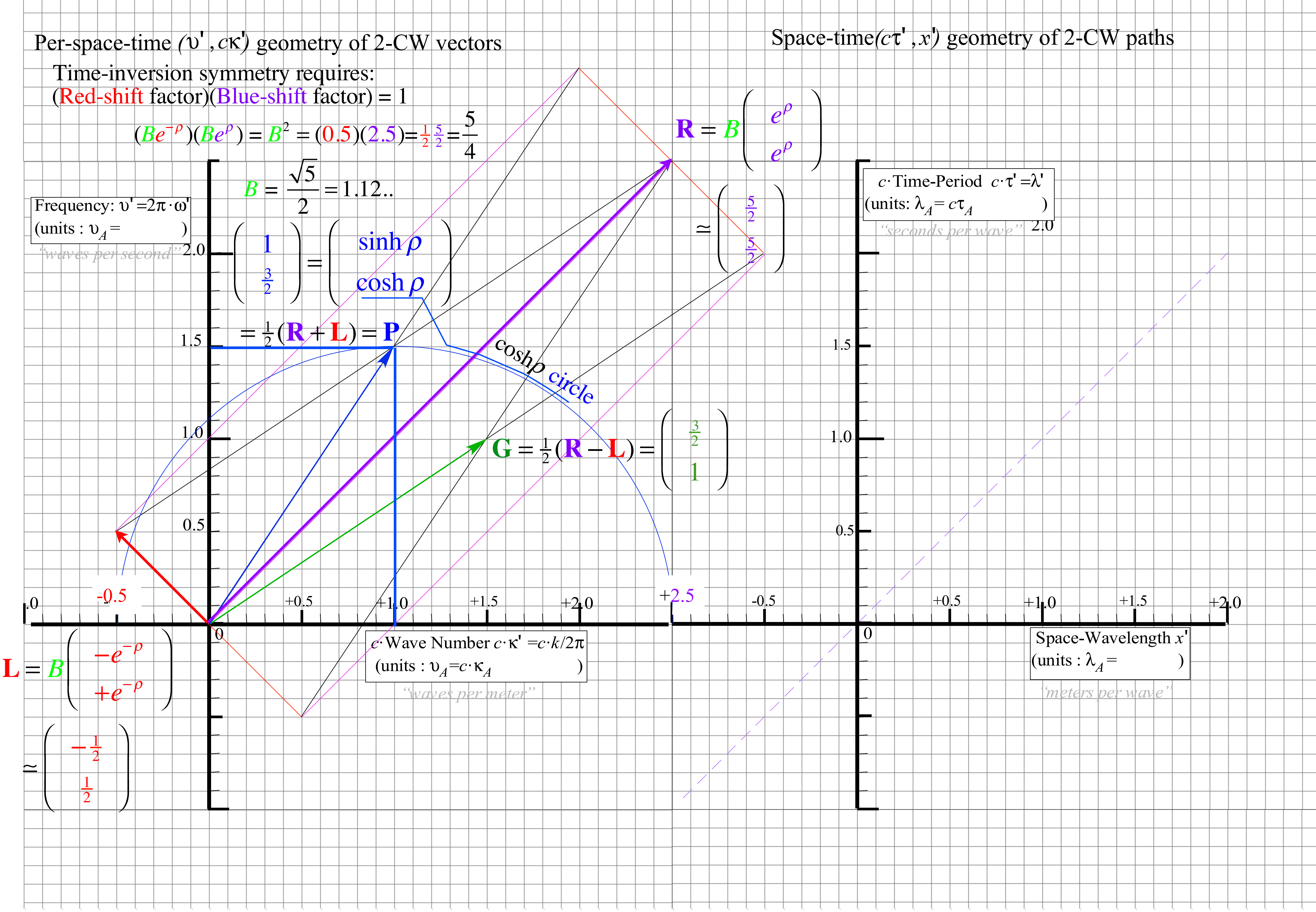
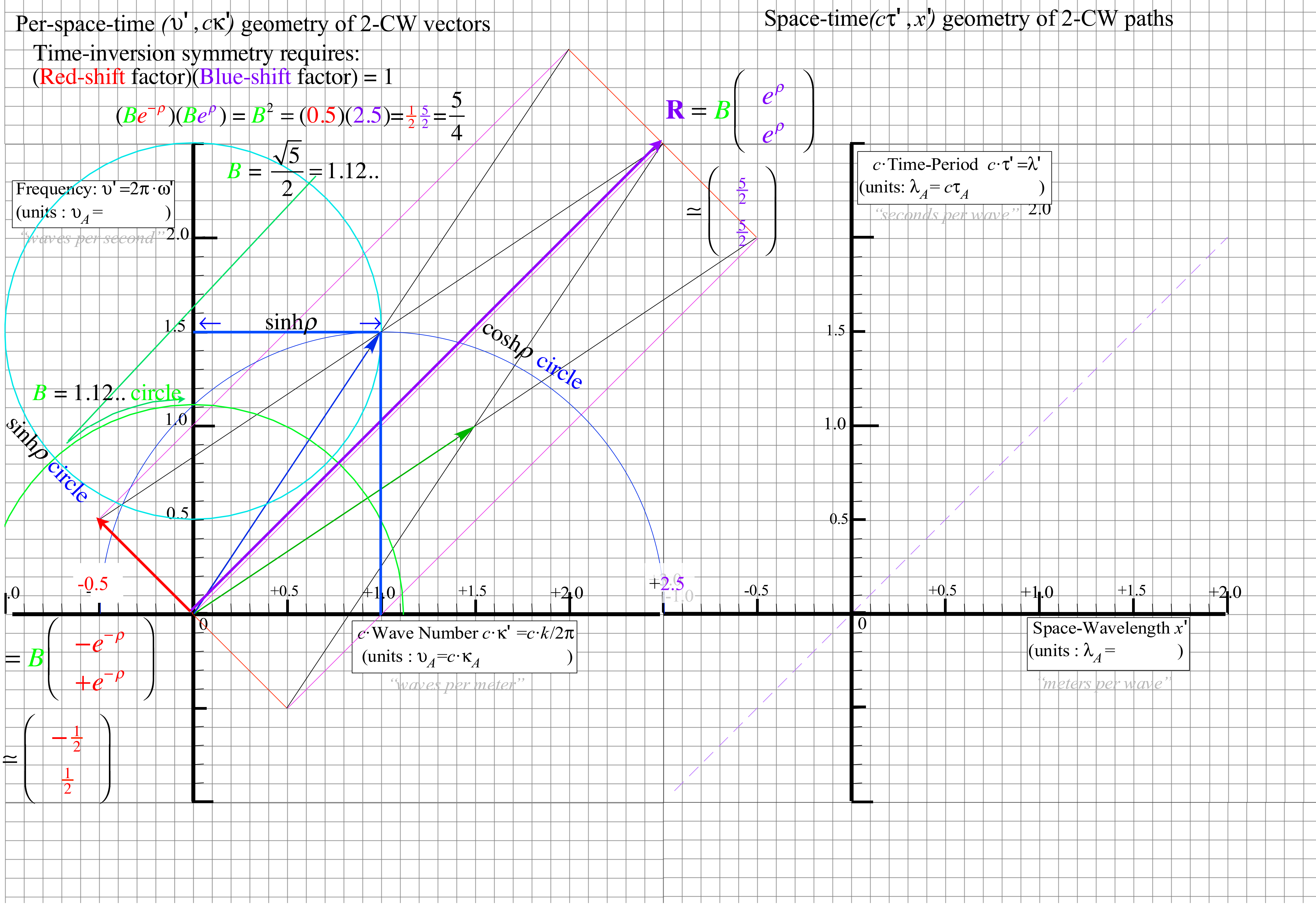
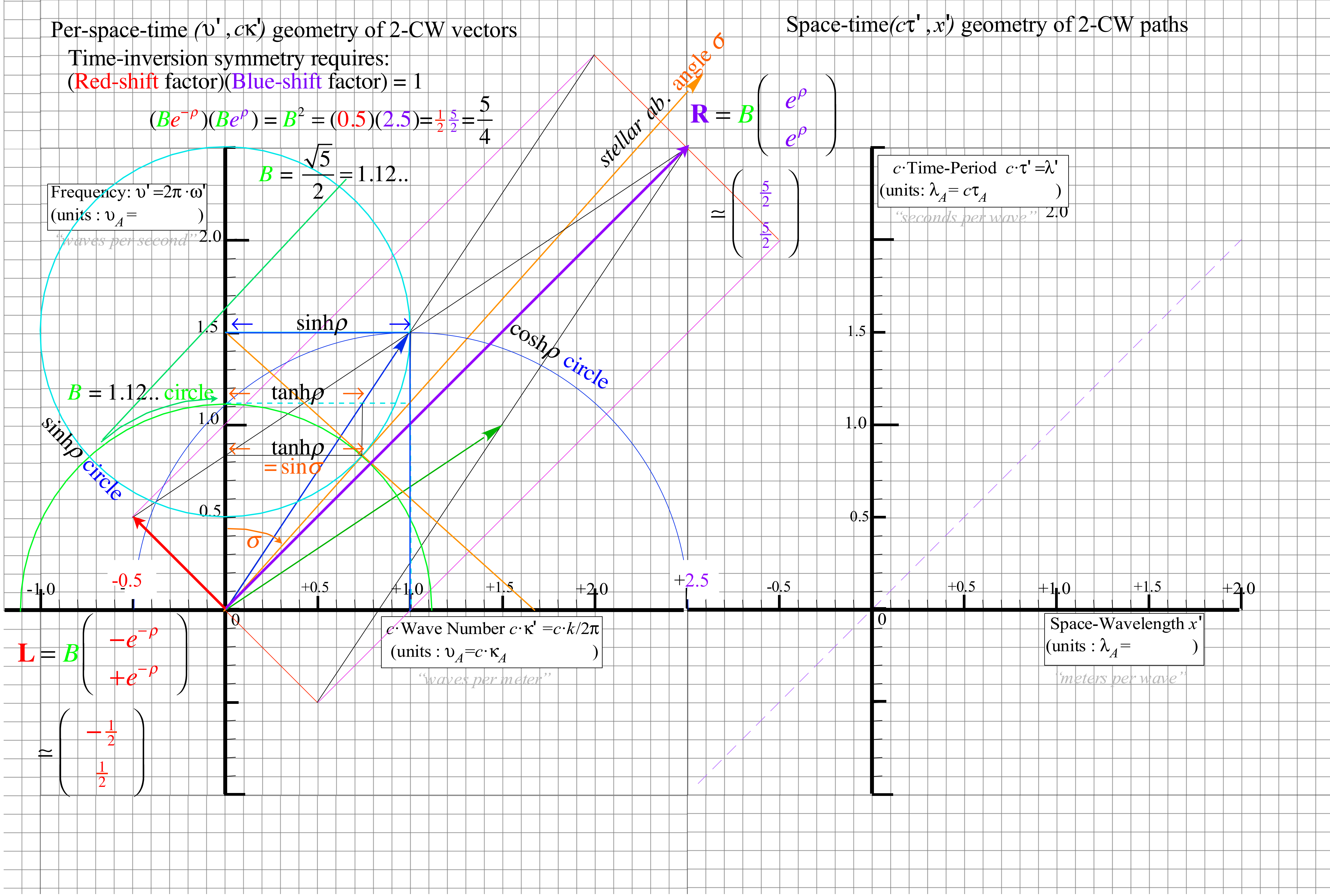


Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz



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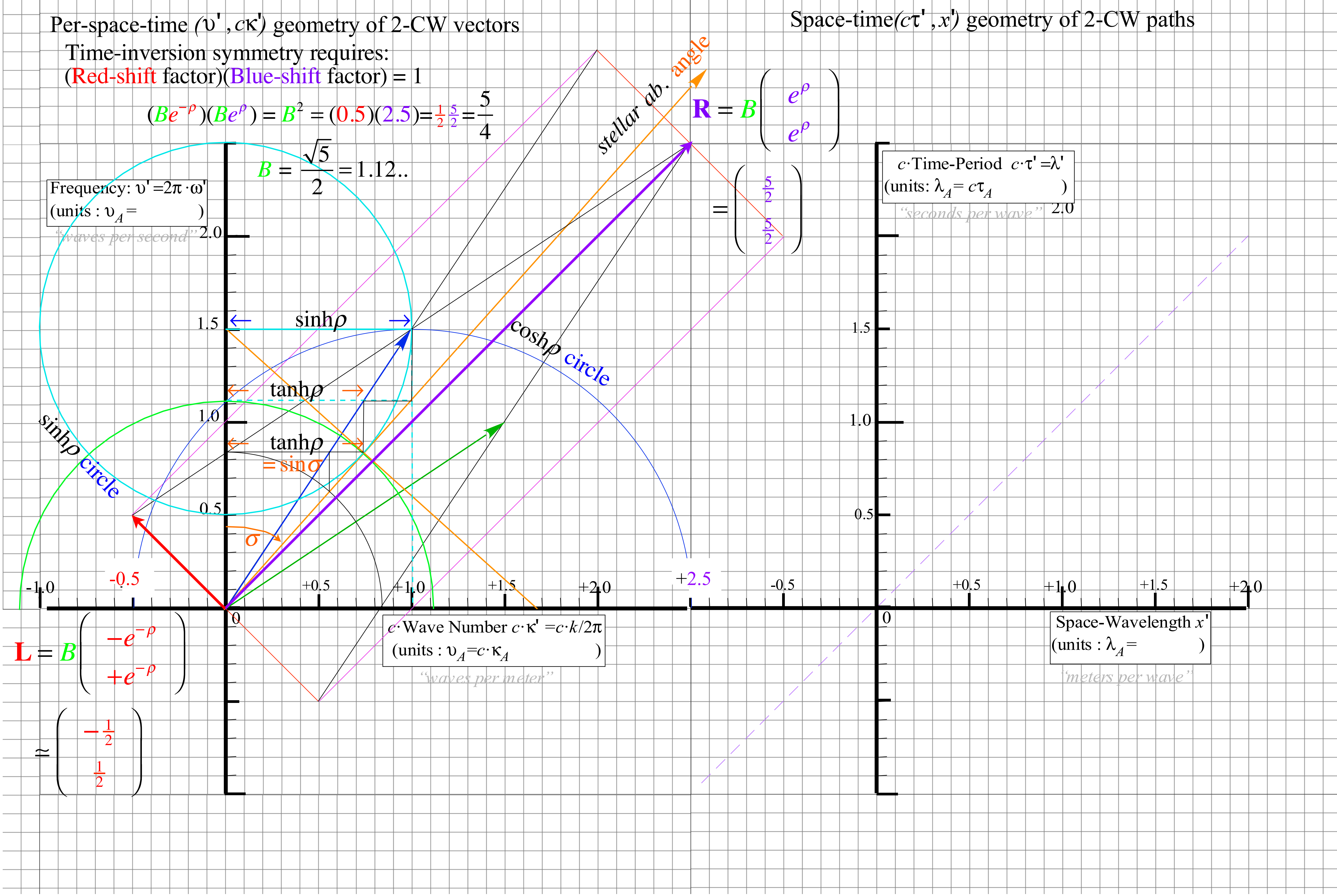
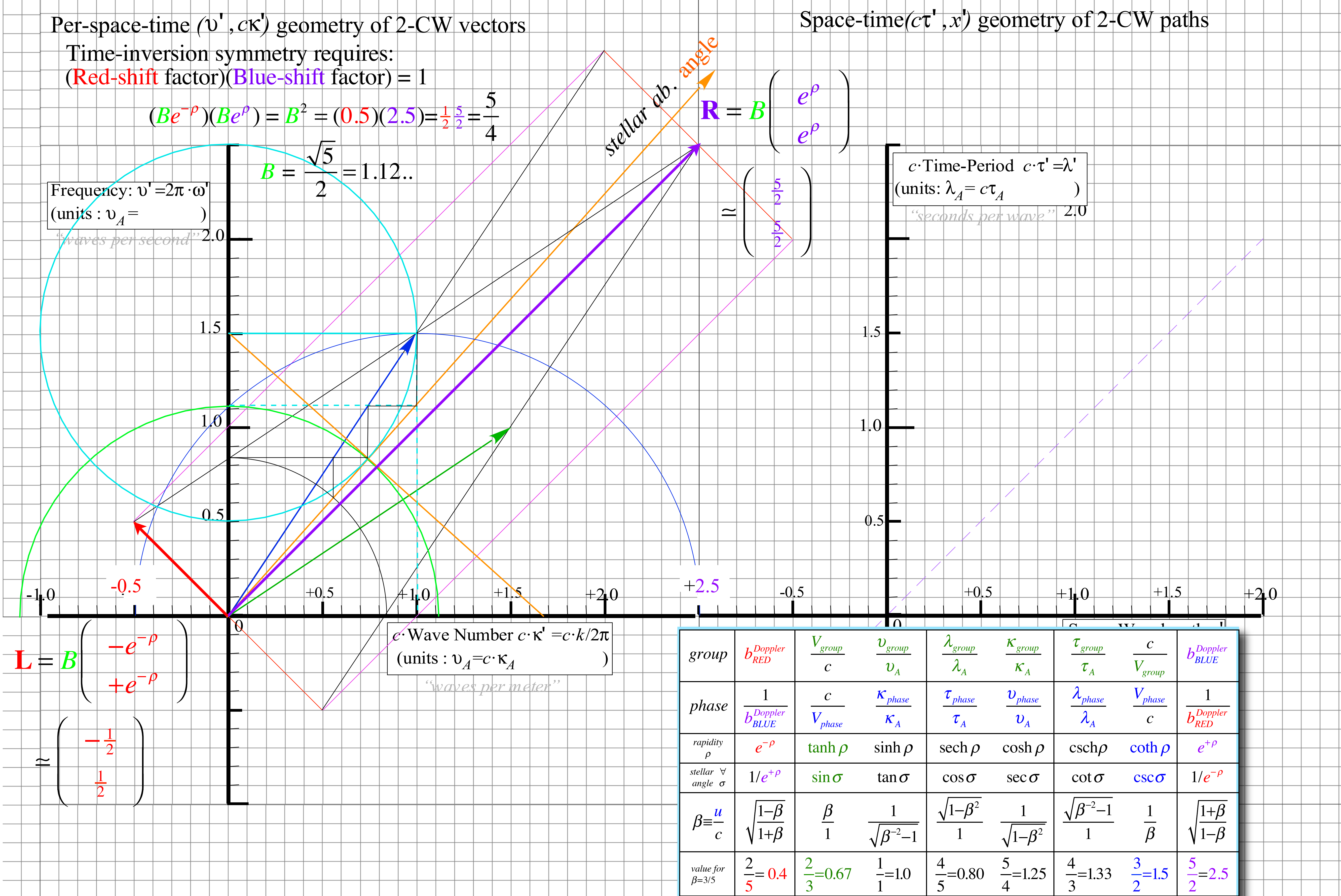
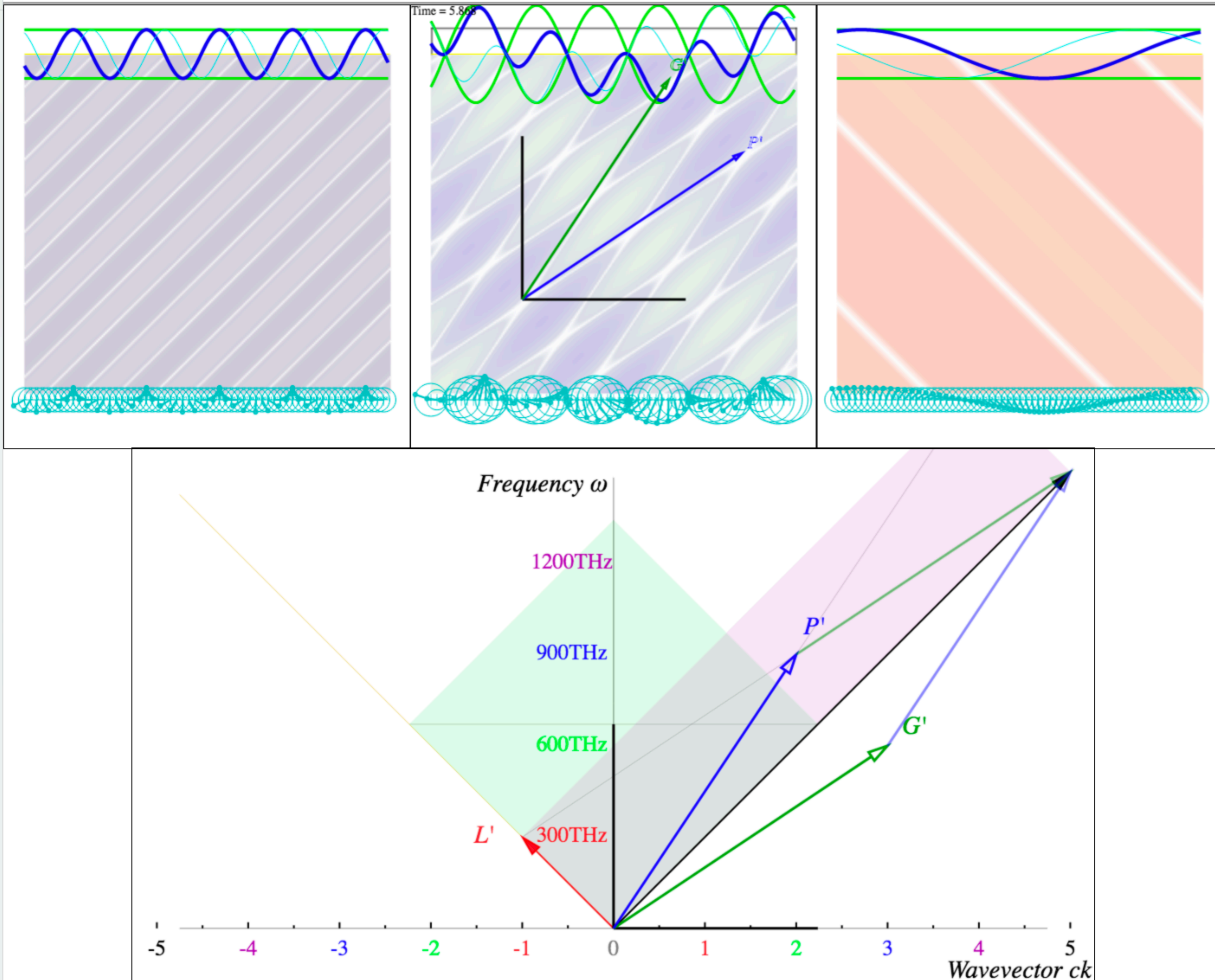


Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz

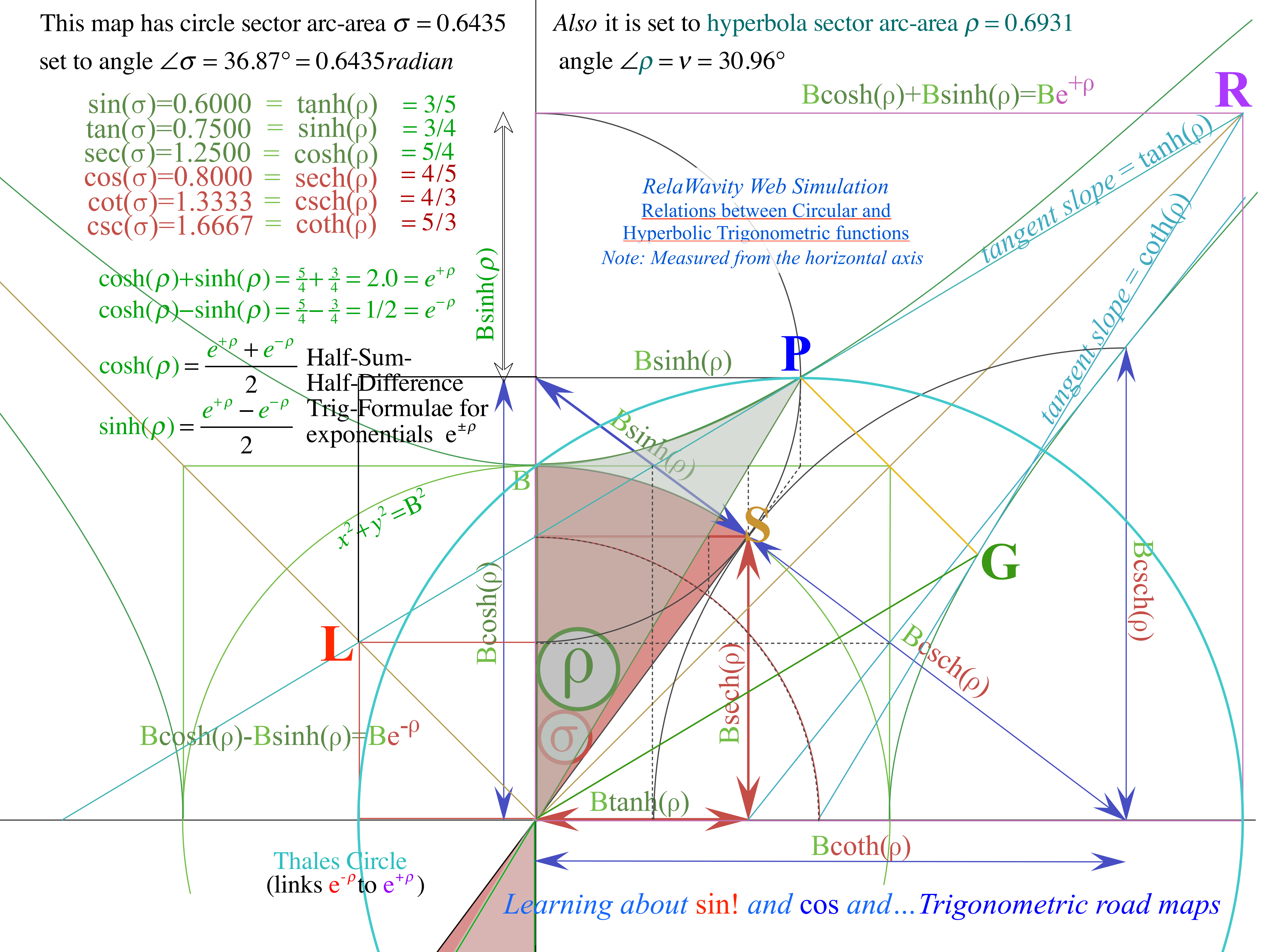


group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{2}{5}$= 0.4	$\frac{2}{3}$=0.67	$\frac{1}{1}$=1.0	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{3}{2}$=1.5	$\frac{5}{2}$=2.5

(may contain errors)



Time = 5.868
G'
P'
Frequency ω
1200THz
900THz
600THz
300THz
P'
G'
L'
-5
-4
-3
-2
-1
0
1
2
3
4
5
Wavevector ck



This map has circle sector arc-area σ = 0.6435
set to angle ∠σ = 36.87° = 0.6435 radian
Also it is set to hyperbola sector arc-area ρ = 0.6931
angle ∠ρ = ν = 30.96°
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3
cosh(ρ)+sinh(ρ) = 5/4 + 3/4 = 2.0 = e^{+ρ}
cosh(ρ)−sinh(ρ) = 5/4 − 3/4 = 1/2 = e^{−ρ}
cosh(ρ) = (e^{+ρ} + e^{−ρ})/2
sinh(ρ) = (e^{+ρ} − e^{−ρ})/2
Half-Sum-
Half-Difference
Trig-Formulae for
exponentials e^{±ρ}
RelaWavity Web Simulation
Relations between Circular and
Hyperbolic Trigonometric functions
Note: Measured from the horizontal axis
Bcosh(ρ)+Bsinh(ρ)=Be^{+ρ}
R
P
S
G
L
B
tangent slope = tanh(ρ)
tangent slope = coth(ρ)
Bsinh(ρ)
Bsinh(ρ)
Bsinh(ρ)
Bcosh(ρ)
Bcsch(ρ)
Bcsch(ρ)
Bsech(ρ)
Btanh(ρ)
Bcoth(ρ)
x^2+y^2=B^2
ρ
σ
Bcosh(ρ)-Bsinh(ρ)=Be^{-ρ}
Thales Circle
(links e^{-ρ} to e^{+ρ})
Learning about sin! and cos and...Trigonometric road maps

Fig. 11 in text *Relawavity...*

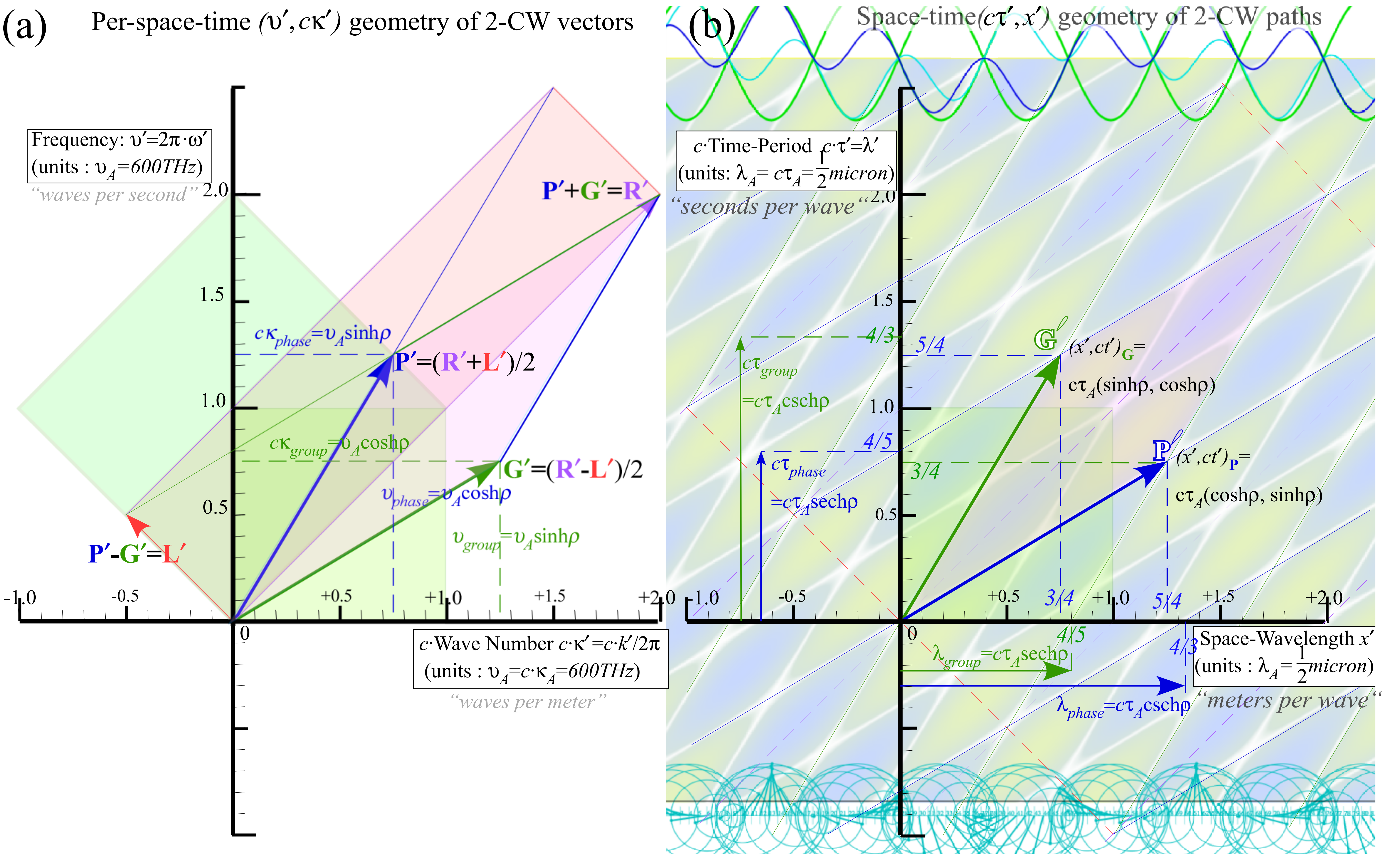
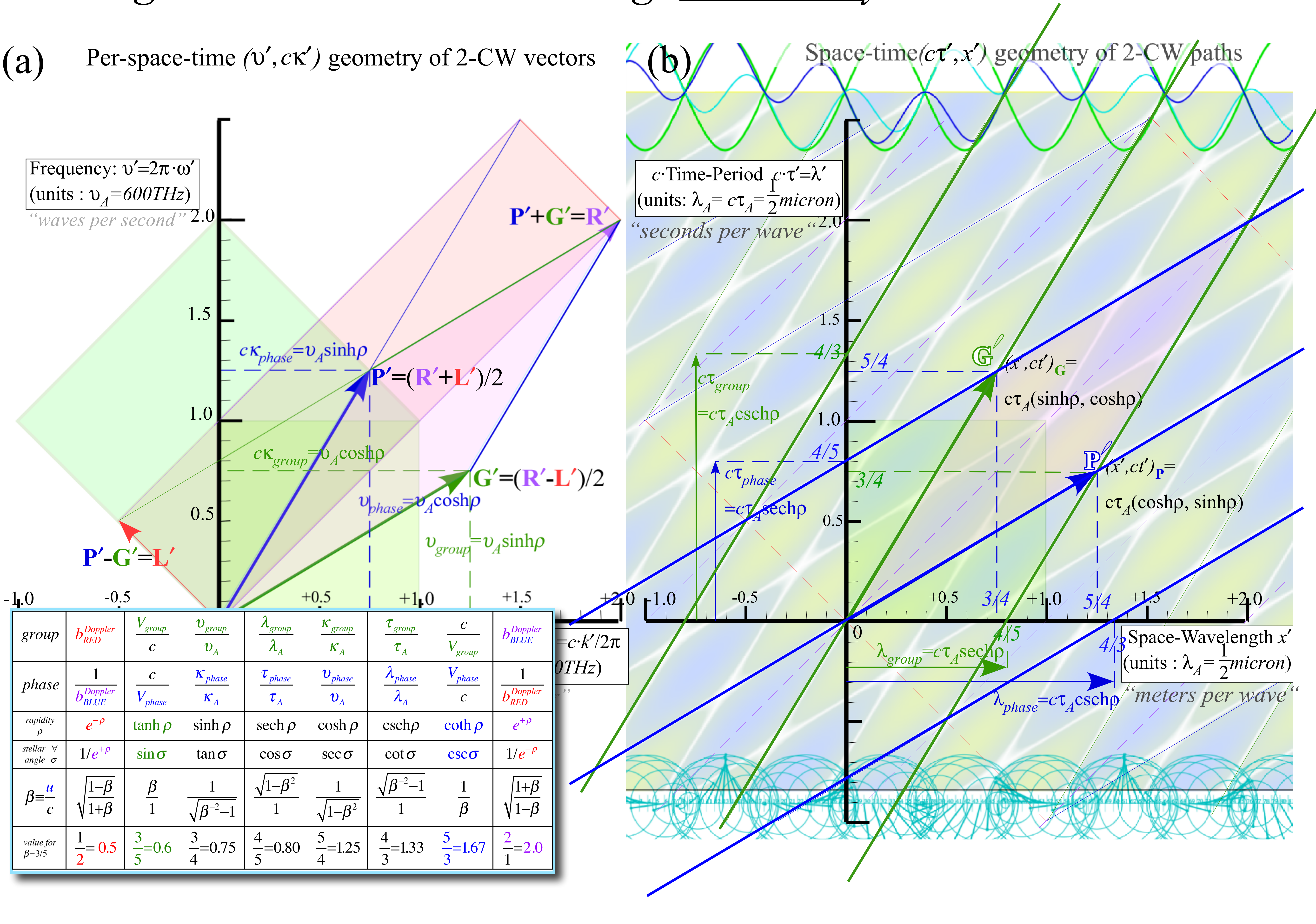


Fig. 4 in Ch.0 text introducing Relawavity...



group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Lecture 30
Wed. 12.05.2018

Review: Relawavity ρ functions and plots vs. ρ

Derive relawavity parameters and Minkowski coordinates for υ_R=2.5THz and υ_L=0.5THz

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and summary of 16 parameter functions of ρ and σ

Applications to optical waveguide, spherical waves, and accelerator radiation

➡ Derivation of relativistic quantum mechanics

What's the matter with mass? Shining some light on the Elephant in the room

Relativistic action and Lagrangian-Hamiltonian relations

Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae

Feynman diagram geometry

Compton recoil related to rocket velocity formula

Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames

Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid

Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid

Animation of mechanics and metrology of constant-g grid

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{(for } u \ll c)$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2$$

$$\sinh\rho \approx \rho$$

$$B = \upsilon_A$$

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group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation - Relativistic Terms (Short version)

Using (some) wave parameters to develop relativistic quantum theory

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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So attach scale factor h to match units.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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So attach scale factor h to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Using (some) wave parameters to develop relativistic quantum theory

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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall$ σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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RelaWavity Web Simulation - Relativistic Terms (Short version)

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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(old-fashioned notation)

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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Max Planck 1858-1947

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

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For more, visit the Pirelli Challenge Site
Quantized amplitude

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Big worry: Is not oscillator energy quadratic in frequency υ?
HO energy $= \frac{1}{2}A^2\upsilon^2$

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$			
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2} = 0.5$	$\frac{3}{5} = 0.6$	$\frac{3}{4} = 0.75$	$\frac{4}{5} = 0.80$	$\frac{5}{4} = 1.25$	$\frac{4}{3} = 1.33$	$\frac{5}{3} = 1.67$	$\frac{2}{1} = 2.0$

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$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

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So attach scale factor h *(or hN)* to match units.

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Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *...*

Need to replace h with hN *to match e.m. energy density*
$\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} = hN\upsilon_{phase}$

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

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group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$			
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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This motivates the "particle" normalization
$\int \Psi^*\Psi \, dV = N \quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

Big worry: Is not oscillator energy quadratic in frequency υ?

HO energy $= \frac{1}{2}A^2\upsilon^2$

Resolution and dirty secret: $\mathbf{E}$, N, and υ_{phase} are all frequencies!

So $\varepsilon_0 \mathbf{E} \cdot \mathbf{E} = hN\upsilon_{phase}$ *is quadratic in* υ_{phase}

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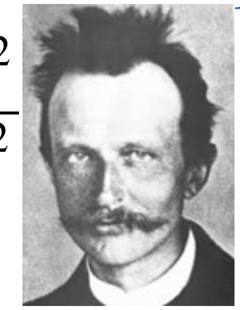

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$$ (old-fashioned notation)

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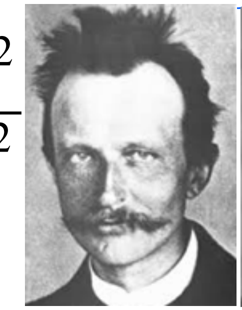

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$$cp = \frac{Muc}{\sqrt{1 - u^2/c^2}}$$

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DeBroglie (1921)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	[illegible]
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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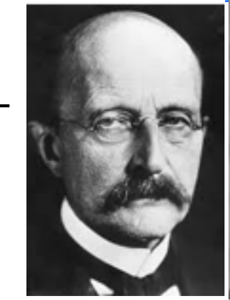

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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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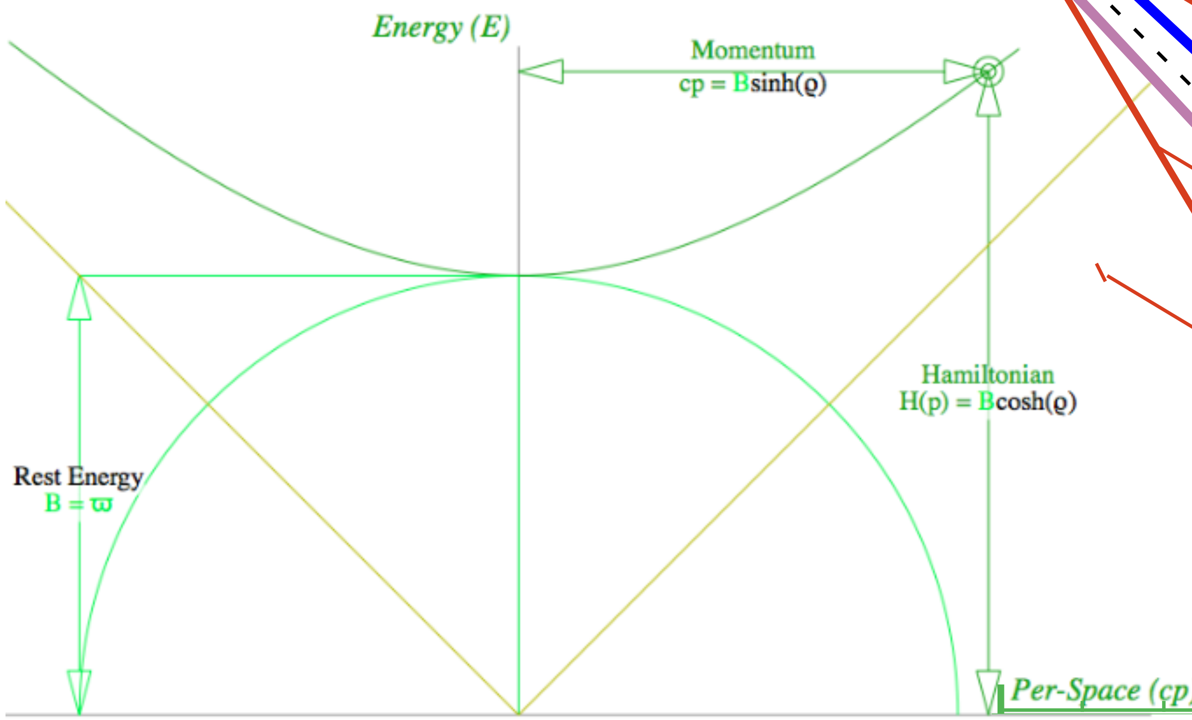
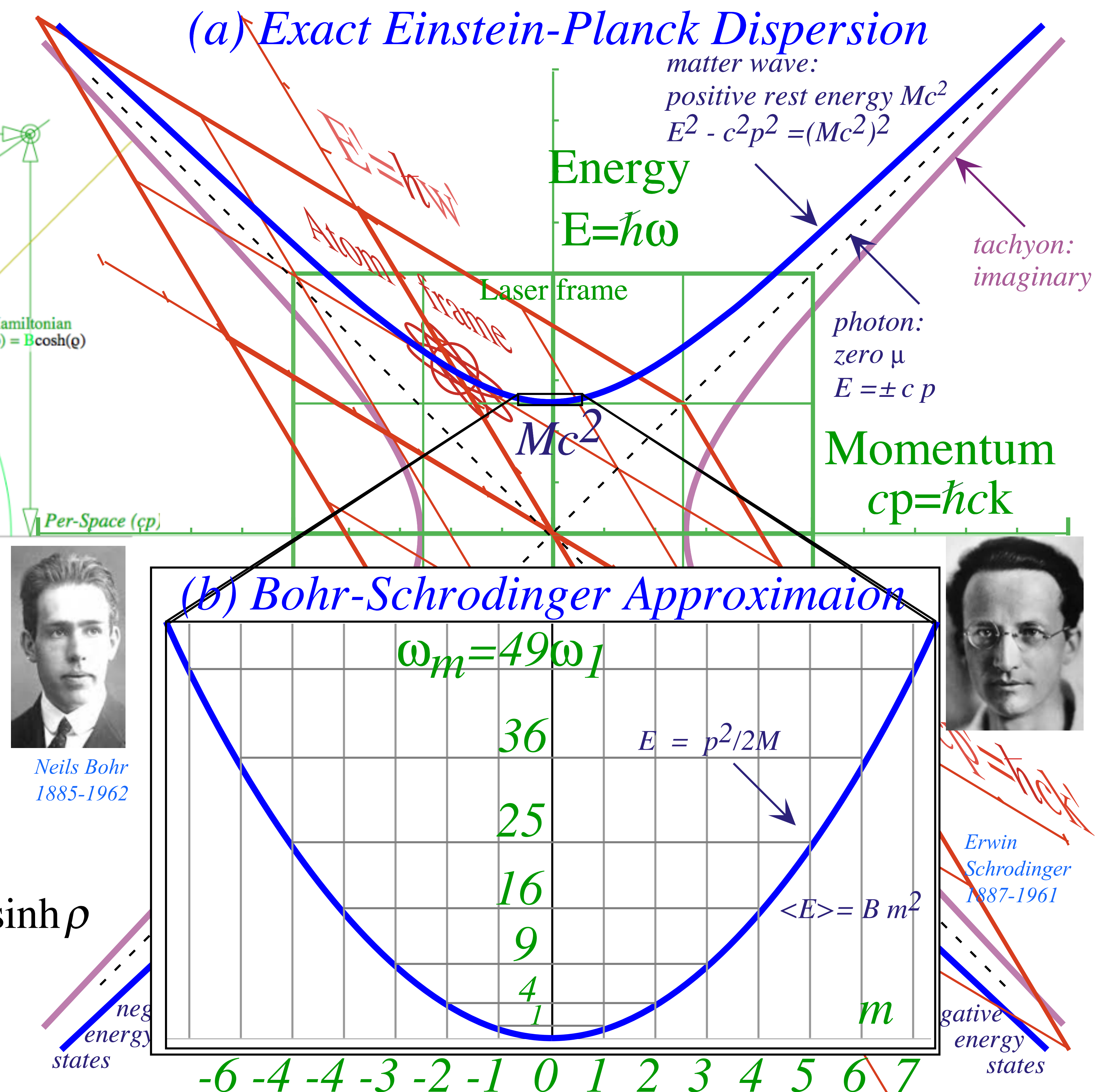
Using (some) wave coordinates for relativistic quantum theory





Neils Bohr 1885-1962

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

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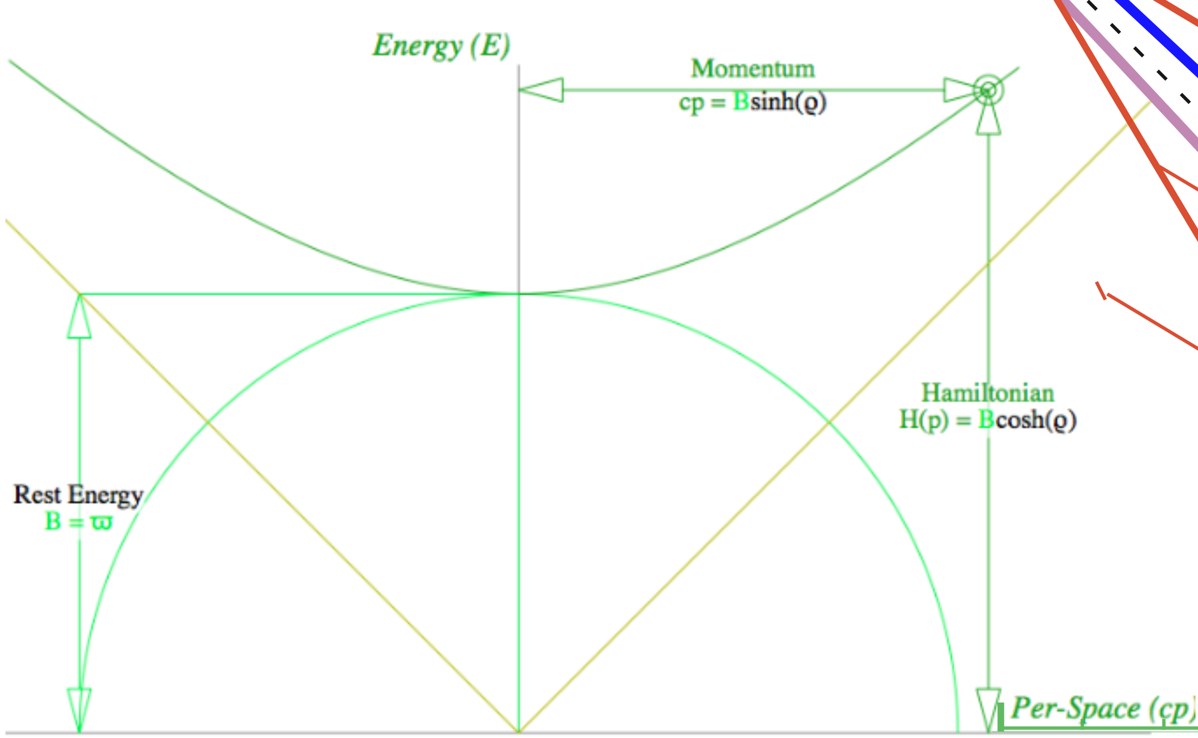

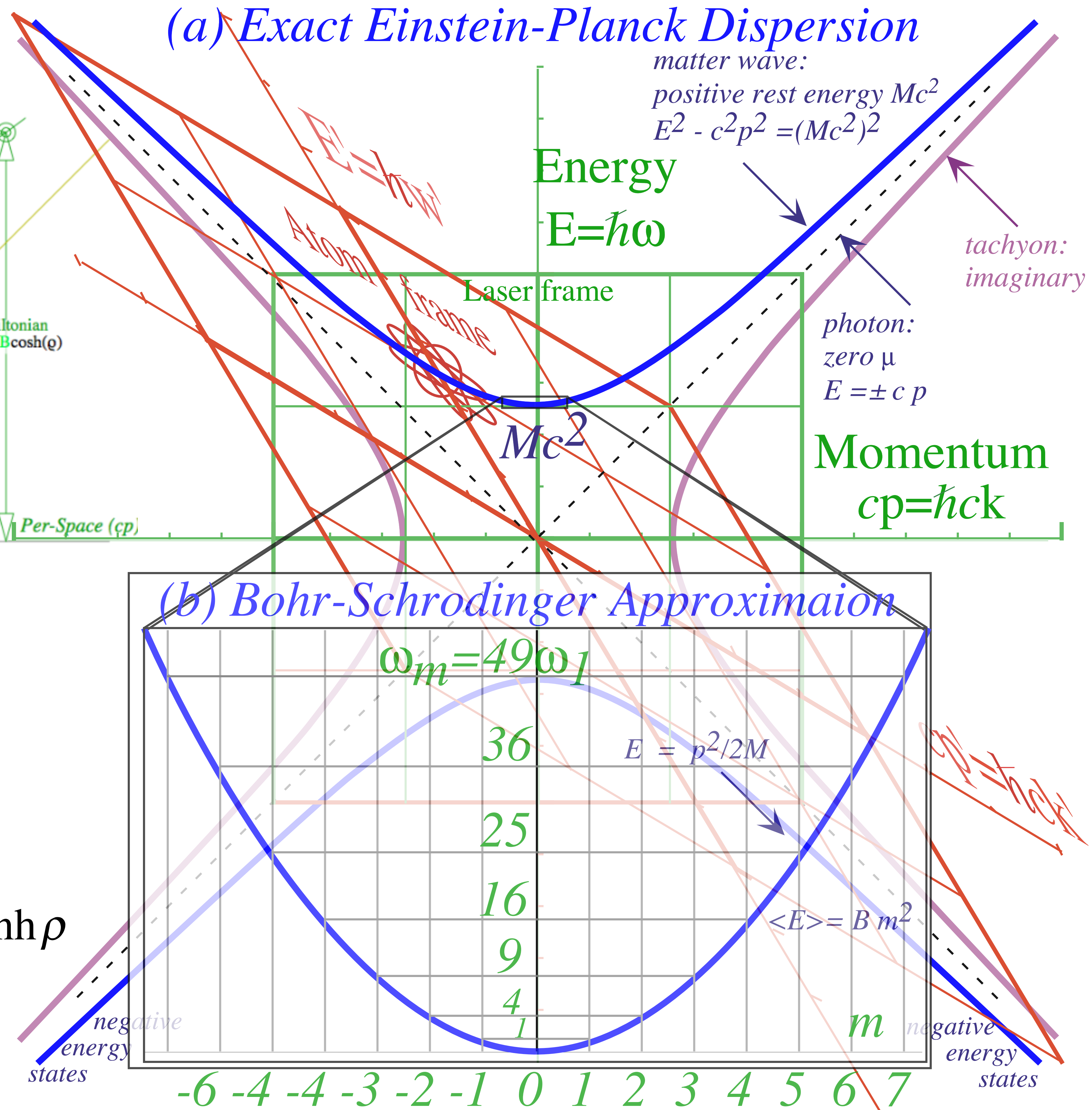

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low speed approximation

Relawavity variable tables

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b_{RED}^{Doppler}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*$^{(Lorentz)}$ τ_{phase}*-contraction*	*t-dilation*$^{(Einstein)}$ υ_{phase}*-dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b_{BLUE}^{Doppler}$

Relativistic quantum mechanics variable tables

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{\beta}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{1-\beta^2}}{\beta}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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functions		$V_{group}=c\tanh\rho$	*momentum* $cp=Mc^2\sinh\rho$	*-Lagrangian* $L=-Mc^2\operatorname{sech}\rho$	*Hamiltonian* $H=Mc^2\cosh\rho$	*DeBroglie* $\lambda=\alpha\operatorname{csch}\rho$	$V_{phase}=c\coth\rho$	

RelaWavity Web Simulation - Relativistic Terms (Expanded)

Lecture 30
Wed. 12.05.2018

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Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
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Rest Mass M_{rest} *(Einstein's mass)*

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Defines invariant hyperbola(s)

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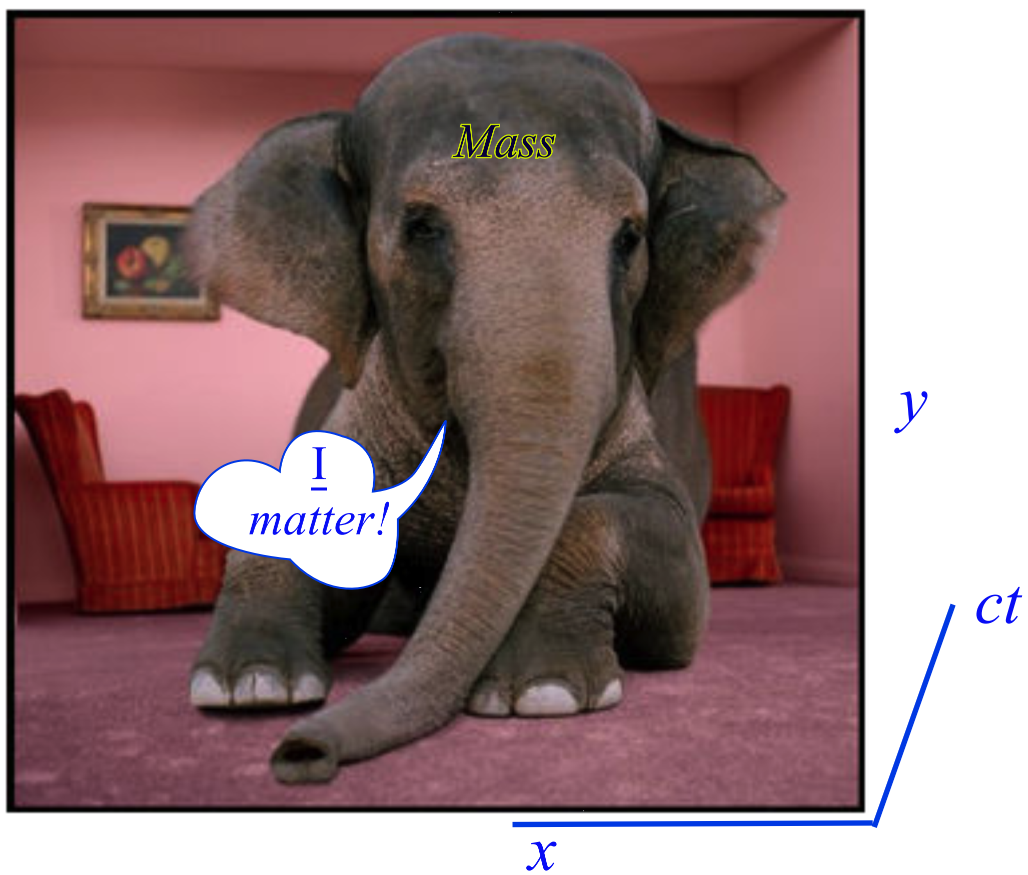
- *What's the matter with Mass?*



Shining some light on the elephant in the spacetime room

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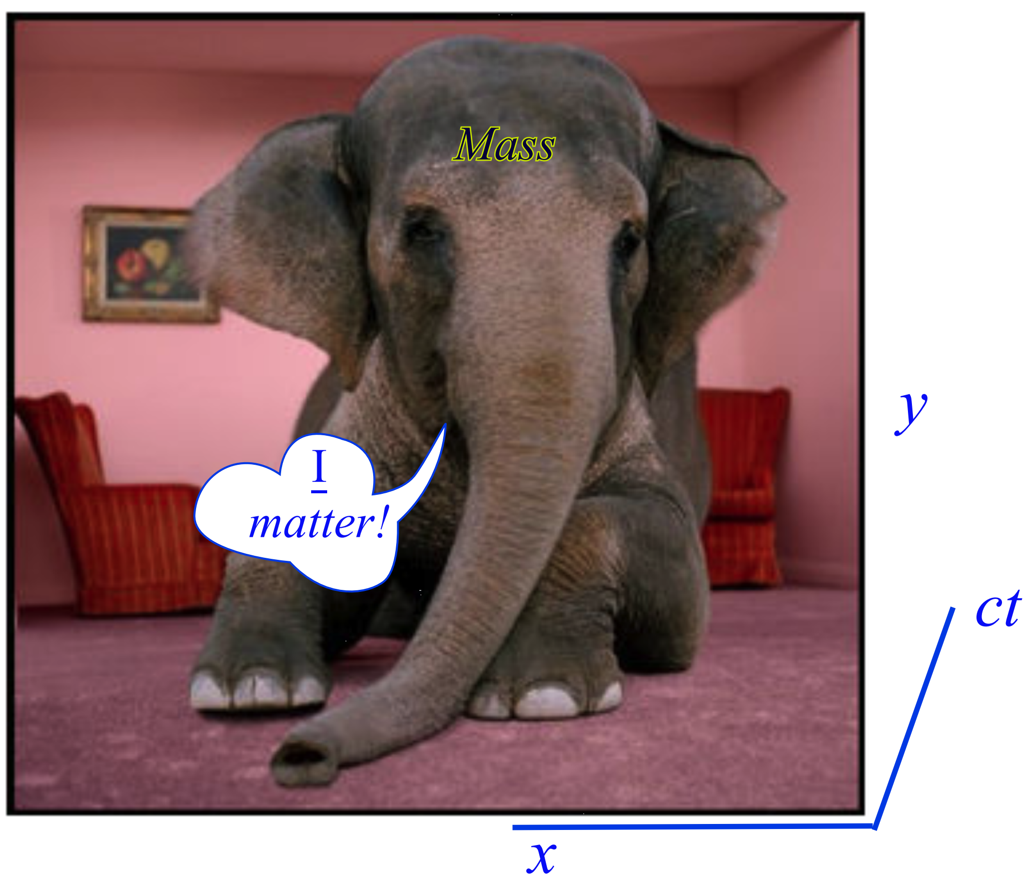

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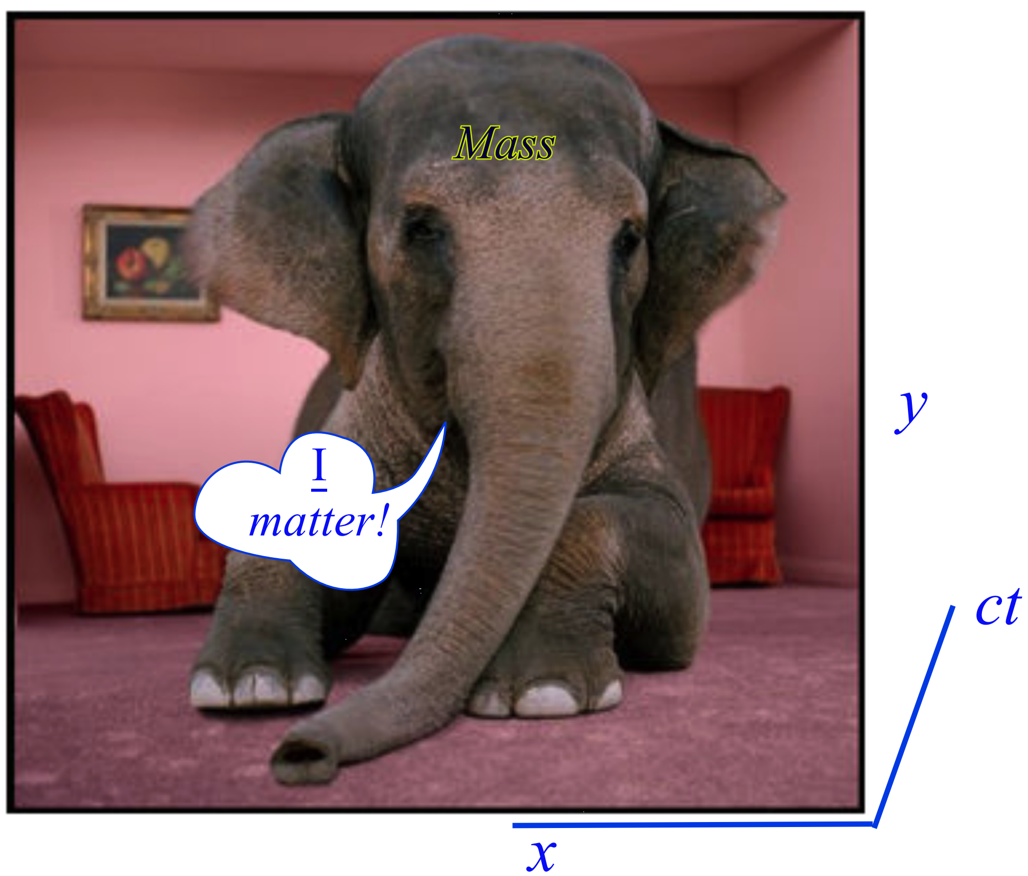

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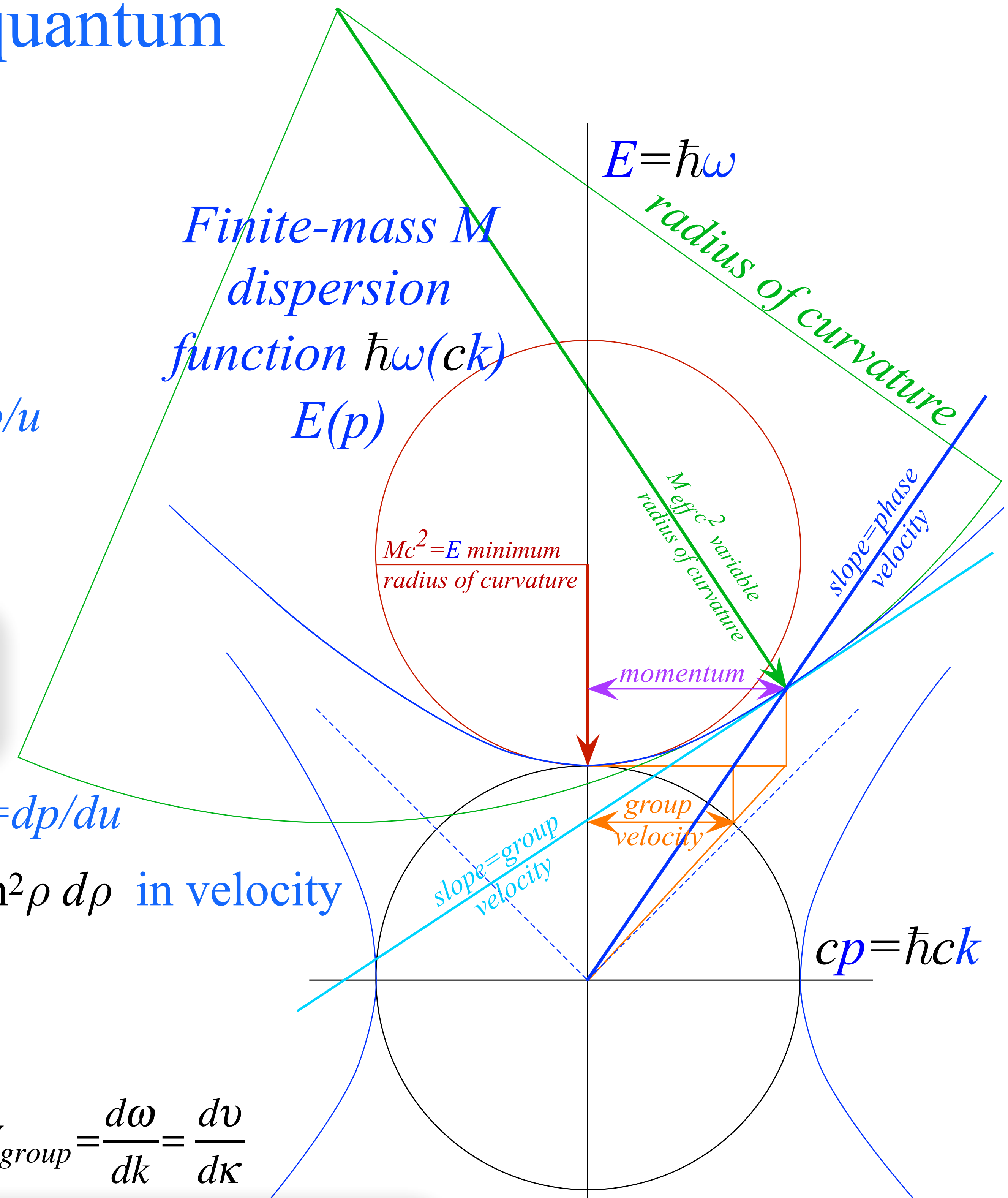
general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Definition(s) of mass for relativity/quantum



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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Effective mass is proportional to the *radius of curvature* of $\omega(k)$ *dispersion*.

Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

Rest Mass (a) γ-rest mass: $M^{\gamma}_{rest} = 0$,

Momentum Mass (b) γ-momentum mass: $M^{\gamma}_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\upsilon}{c^2}$,

Effective Mass (c) γ-effective mass: $M^{\gamma}_{eff} = \infty$.

Newton complained about *his* "corpuscles" of light having *"fits"* (going *crazy*).

(All *this* would be evidence of *triple Schizophrenia*.)

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2 \cdot 10^{-51})kg \cdot s = 4.5 \cdot 10^{-36}\, kg \quad \text{(for: } \upsilon\text{=600THz)}$$

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Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

Prior wave relations

←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
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$$p = \hbar k = Mc \sinh\rho$$

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$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

angular phasor format $\longrightarrow$

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$$
$$\hbar c k_{phase} = cp = \hbar\omega_A \sinh\rho$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

$$p = \hbar k = Mc \sinh\rho$$

$$E = \hbar\omega = Mc^2 \cosh\rho = H$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$

←linear Hz format

angular phasor→ format

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$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$

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$$\hbar \equiv \frac{h}{2\pi}$$

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Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

Prior wave relations

←linear Hz format

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
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angular phasor format→

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
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$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

Prior wave relations

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$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

$$L \text{ is}: Mc^2\frac{-1}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

Prior wave relations

←linear Hz format	angular phasor→ format
$h\upsilon_A = Mc^2 = hc\kappa_A$	$\hbar\omega_A = Mc^2 = \hbar c k_A$
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Use *Group velocity* : $u = \frac{dx}{dt} = c \tanh\rho$

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$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Note: $Mcu = Mc^2 \tanh\rho$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

Prior wave relations

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Note: $Mcu = Mc^2 \tanh\rho$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh\rho$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

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Note: $Mcu = Mc^2 \tanh\rho$

Also: $cp = Mc^2 \sinh\rho$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

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$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh\rho$$

$$= Mc^2\sqrt{1 + \sinh^2\rho} = Mc^2\sqrt{1 + (cp)^2}$$

Prior wave relations

$\hbar = h/2\pi$

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
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$$\frac{dS}{dt}=L\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv pu-H=L$$

Legendre transformation

Use *Group velocity*: $u=\frac{dx}{dt}=c\tanh\rho$

$=c\sin\sigma$

$$p=\hbar k=Mc\sinh\rho \qquad E=\hbar\omega=Mc^2\cosh\rho=H$$

$$L=pu-H=(Mc\sinh\rho)(c\tanh\rho)-Mc^2\cosh\rho$$

$$=Mc^2\frac{\sinh^2\rho-\cosh^2\rho}{\cosh\rho}=-Mc^2\operatorname{sech}\rho$$

Note: $Mcu=Mc^2\tanh\rho$

$=Mc^2\sin\sigma$

Also: $cp=Mc^2\sinh\rho$

$=\hbar ck=Mc^2\tan\sigma$

Compare *Lagrangian L*

$$\dot{S}=L=\hbar\dot{\Phi}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho=-Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S=\hbar\Phi$

Including stellar angle σ

Prior wave relations

$h\upsilon_A=Mc^2=hc\kappa_A$

$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$

$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor format→

$\hbar\omega_A=Mc^2=\hbar ck_A$

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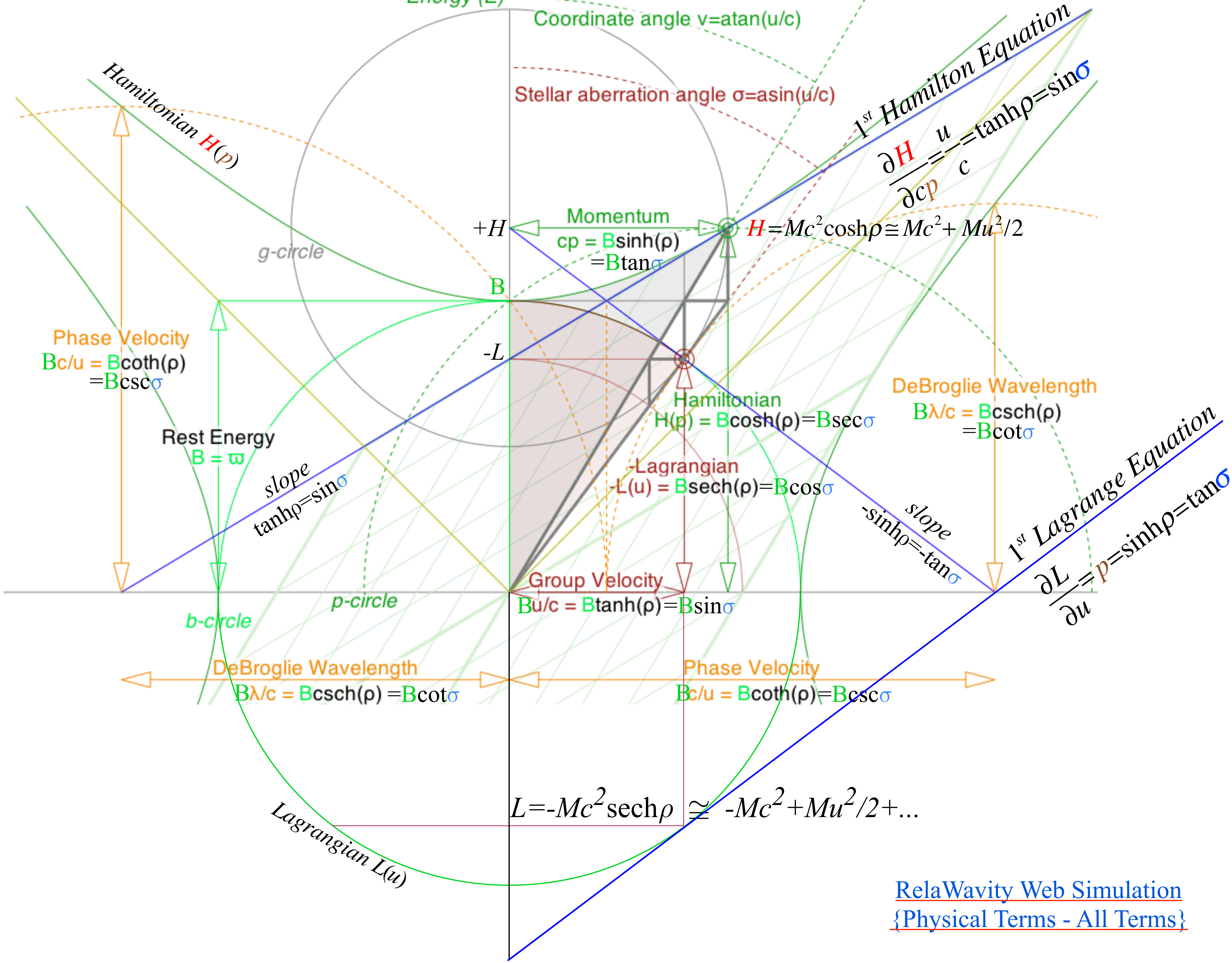
Energy (E)
Coordinate angle v=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Hamiltonian H(ρ)
1st Hamilton Equation
∂H/∂cp = u/c = tanhρ = sinσ
+H
Momentum
cp = Bsinh(ρ)
=Btanσ
H = Mc²coshρ ≅ Mc² + Mu²/2
g-circle
B
Phase Velocity
Bc/u = Bcoth(ρ)
=Bcscσ
-L
Rest Energy
B = ϖ
slope
tanhρ=sinσ
Hamiltonian
H(ρ) = Bcosh(ρ)=Bsecσ
DeBroglie Wavelength
Bλ/c = Bcsch(ρ)
=Bcotσ
-Lagrangian
-L(u) = Bsech(ρ)=Bcosσ
slope
-sinhρ=-tanσ
1st Lagrange Equation
∂L/∂u = p = sinhρ = tanσ
Group Velocity
Bu/c = Btanh(ρ)=Bsinσ
p-circle
b-circle
DeBroglie Wavelength
Bλ/c = Bcsch(ρ) =Bcotσ
Phase Velocity
Bc/u = Bcoth(ρ)=Bcscσ
L=-Mc²sechρ ≅ -Mc²+Mu²/2+...
Lagrangian L(u)
RelaWavity Web Simulation
{Physical Terms - All Terms}

Lecture 30
Wed. 12.05.2018

Review: Relawavity ρ functions and plots vs. ρ

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
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Derivation of relativistic quantum mechanics
What's the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
➔ Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
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Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
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Legendre transformation

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho = Mc^2\sec\sigma$$

$$= Mc^2\sqrt{1+\sinh^2\rho} = Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S = \hbar\Phi$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

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←linear Hz format

angular phasor format→

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Legendre transformation

$$dS \equiv L dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\, dx - H\, dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

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Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho$

$$dS \equiv L dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\, dx - H\, dt$$

Poincare Invariant action differential

$$\frac{\partial S}{\partial x} = p \qquad \frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

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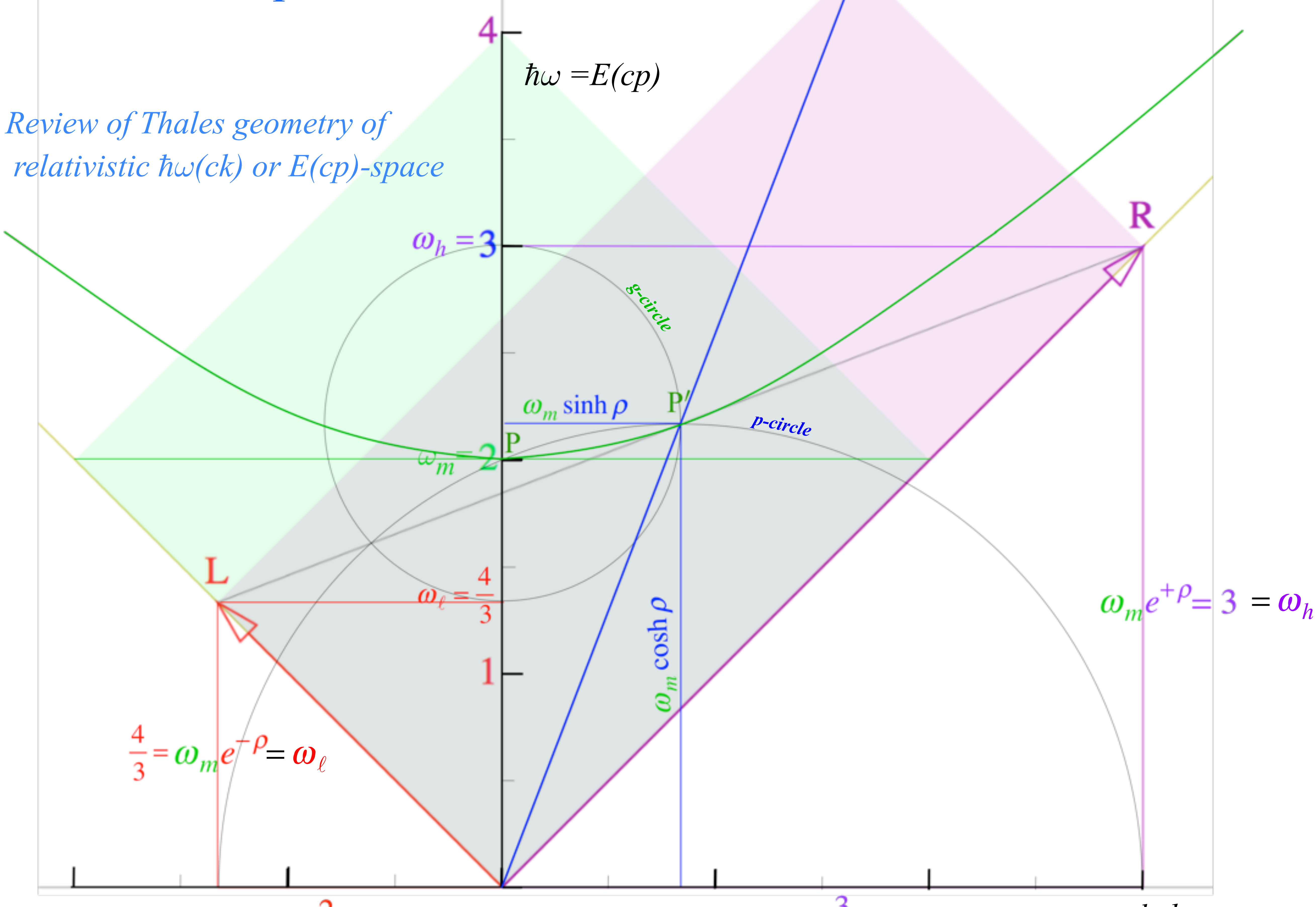
Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$

Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$-space



Doppler RED factor: $\frac{2}{3}=e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2}=e^{+\rho}$

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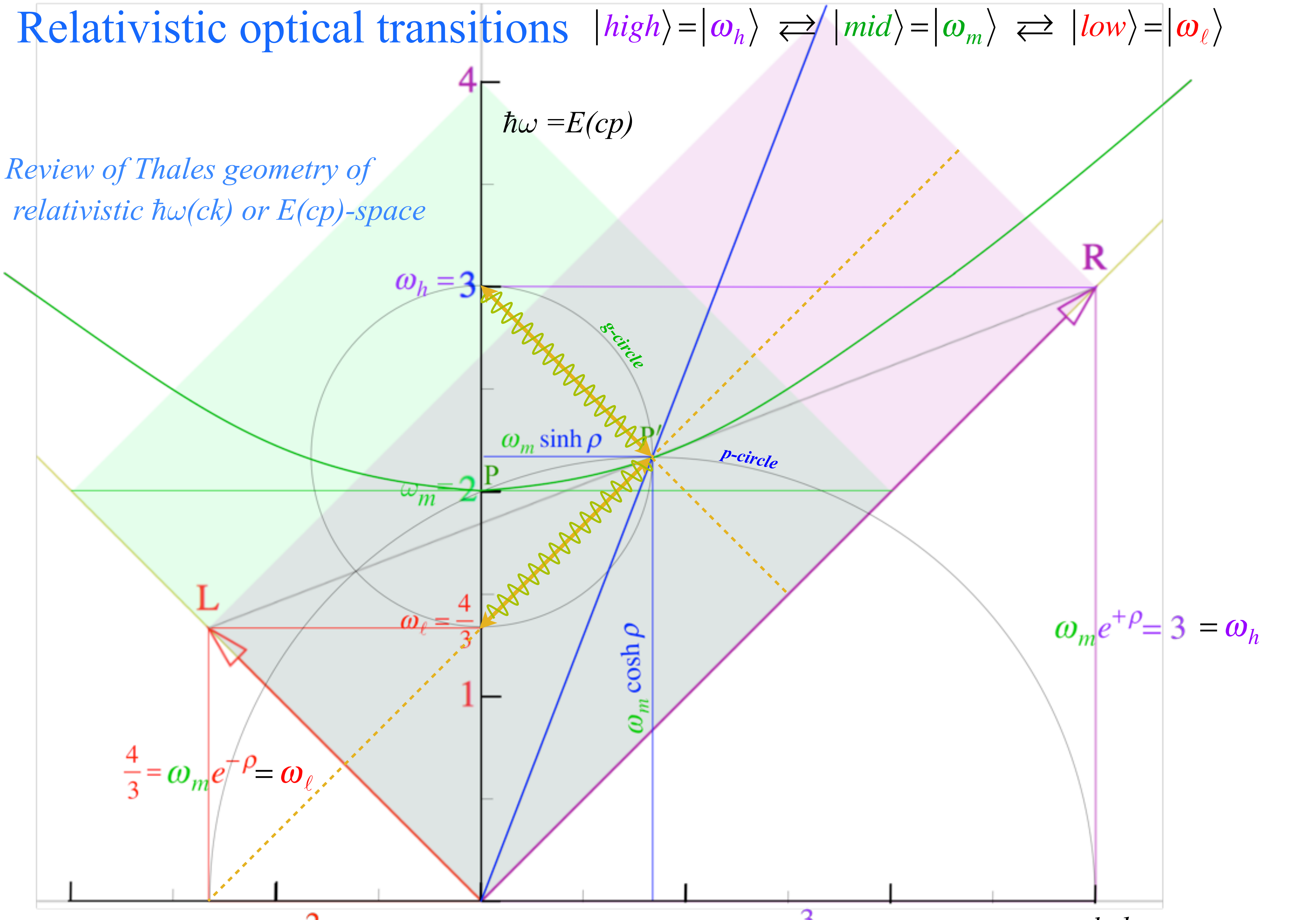

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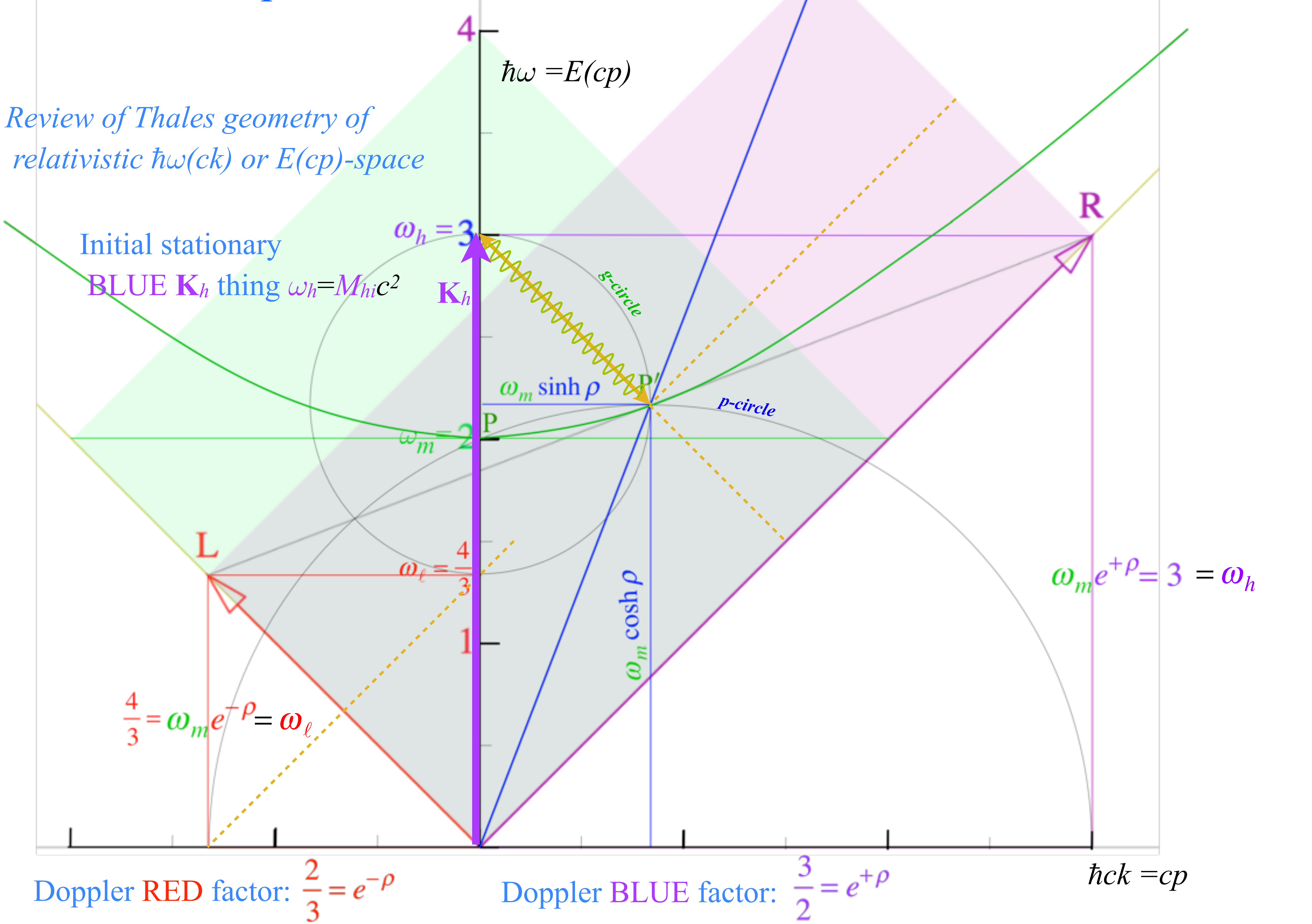

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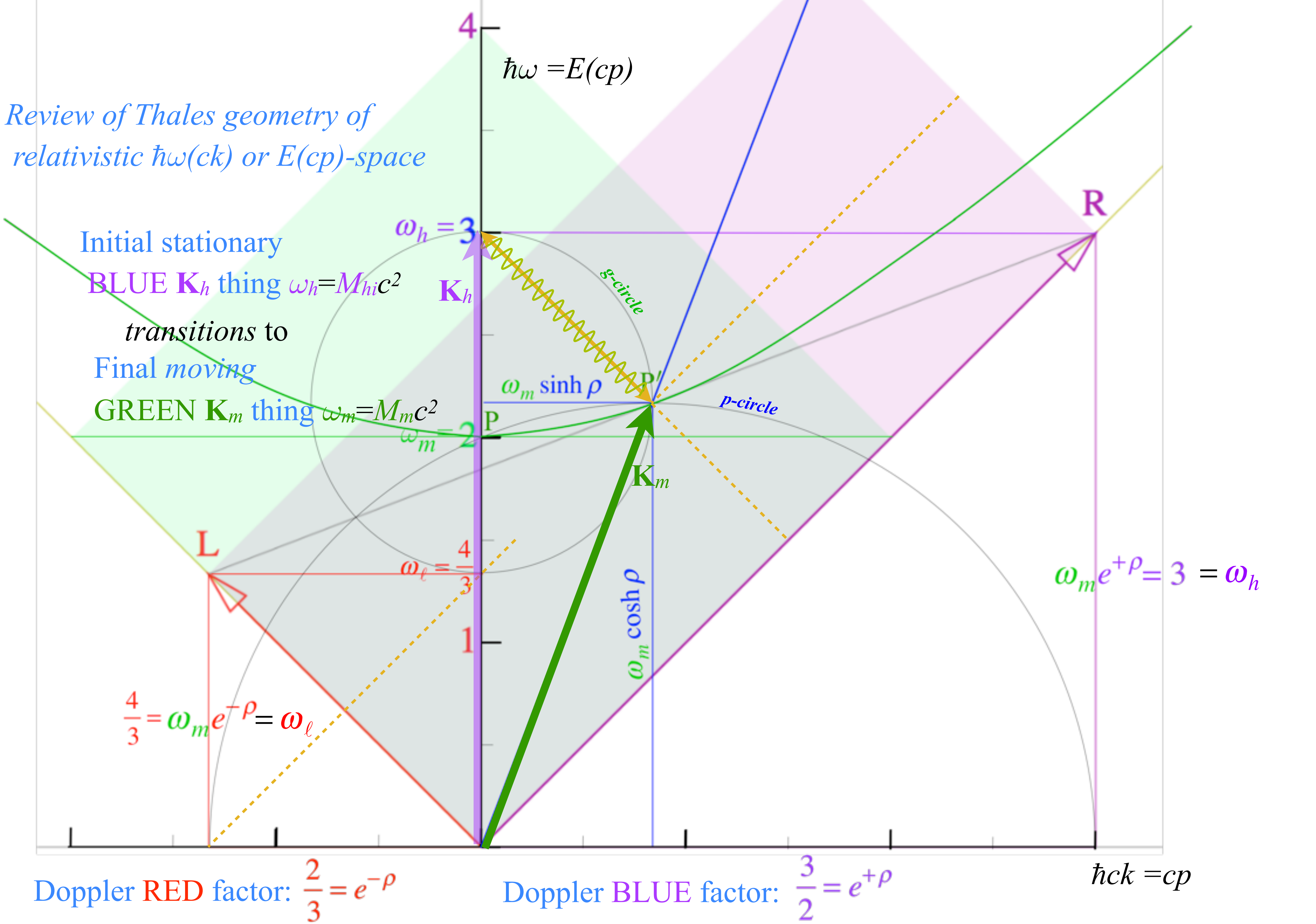

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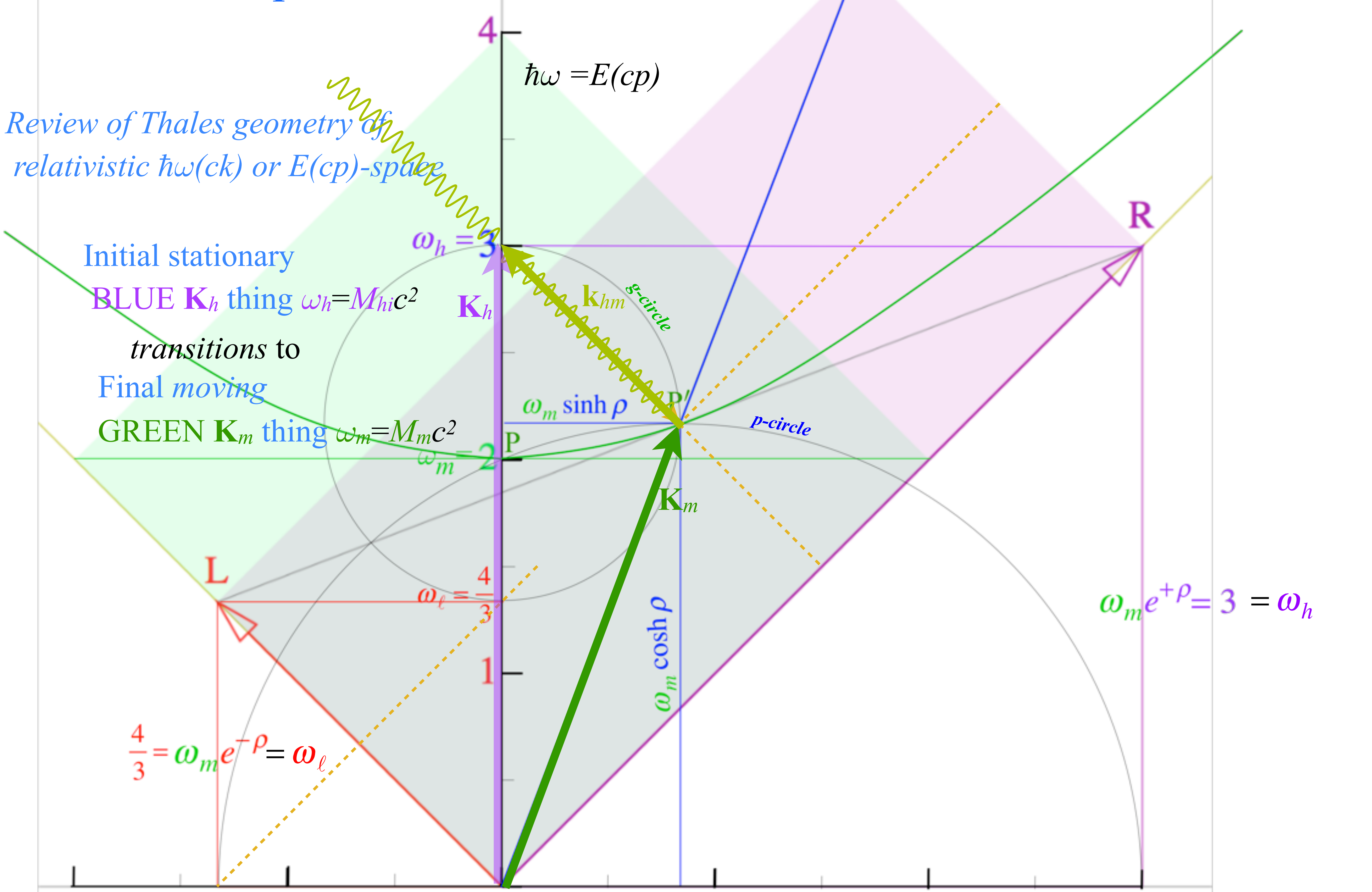

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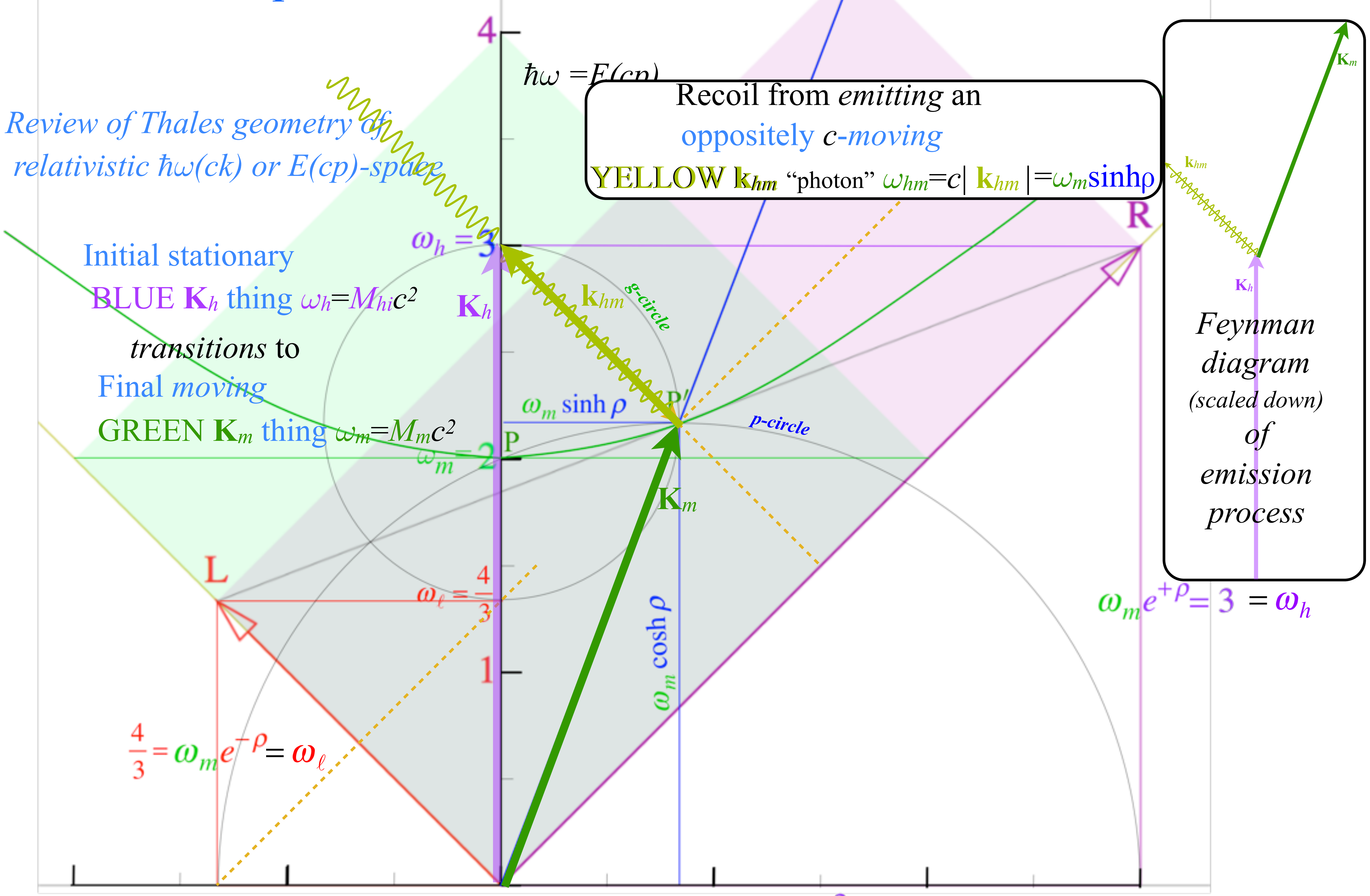
Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

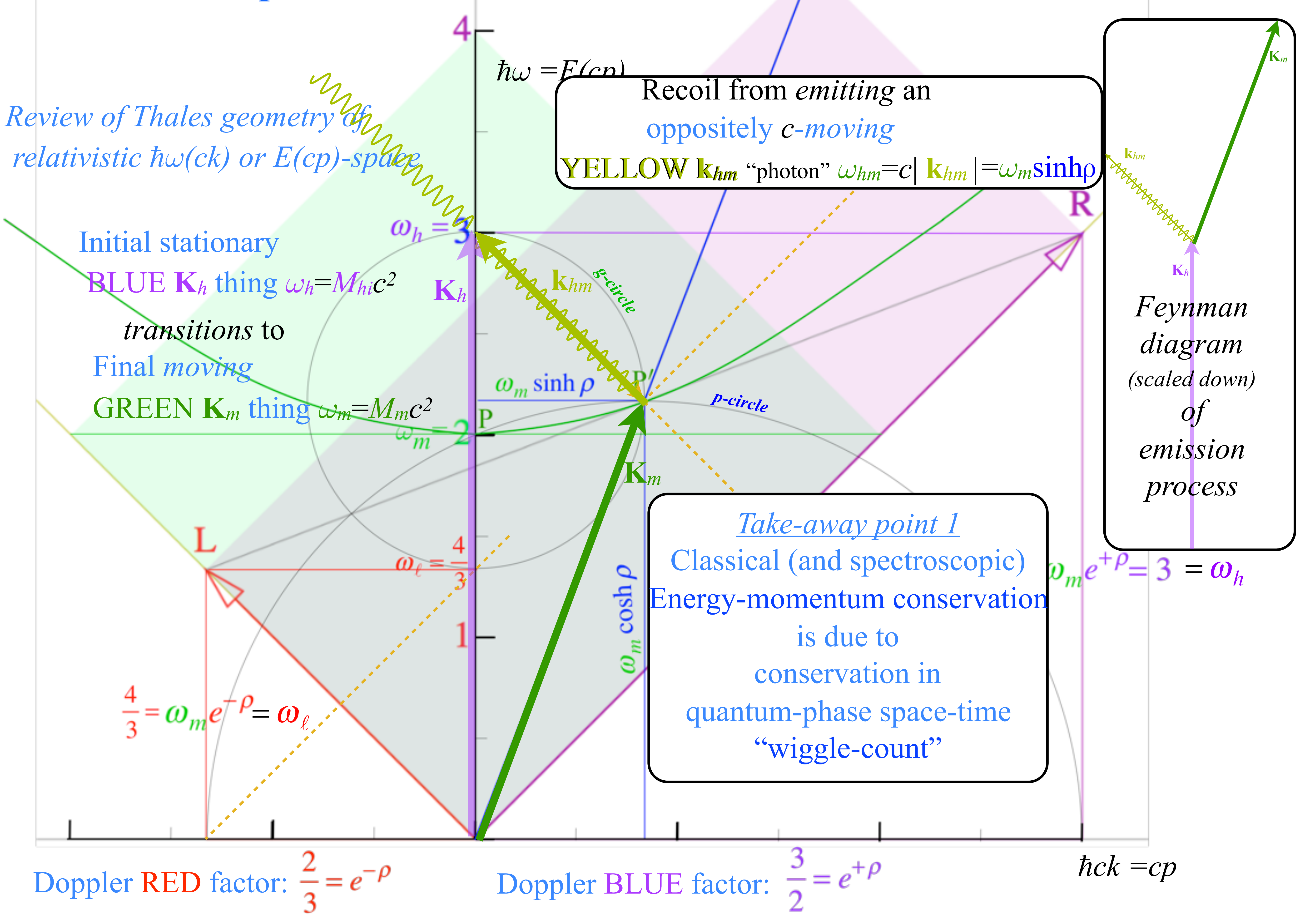
Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Lecture 30
Wed. 12.05.2018

Review: Relawavity ρ functions and plots vs. ρ

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - ➔ Compton recoil related to rocket velocity formula
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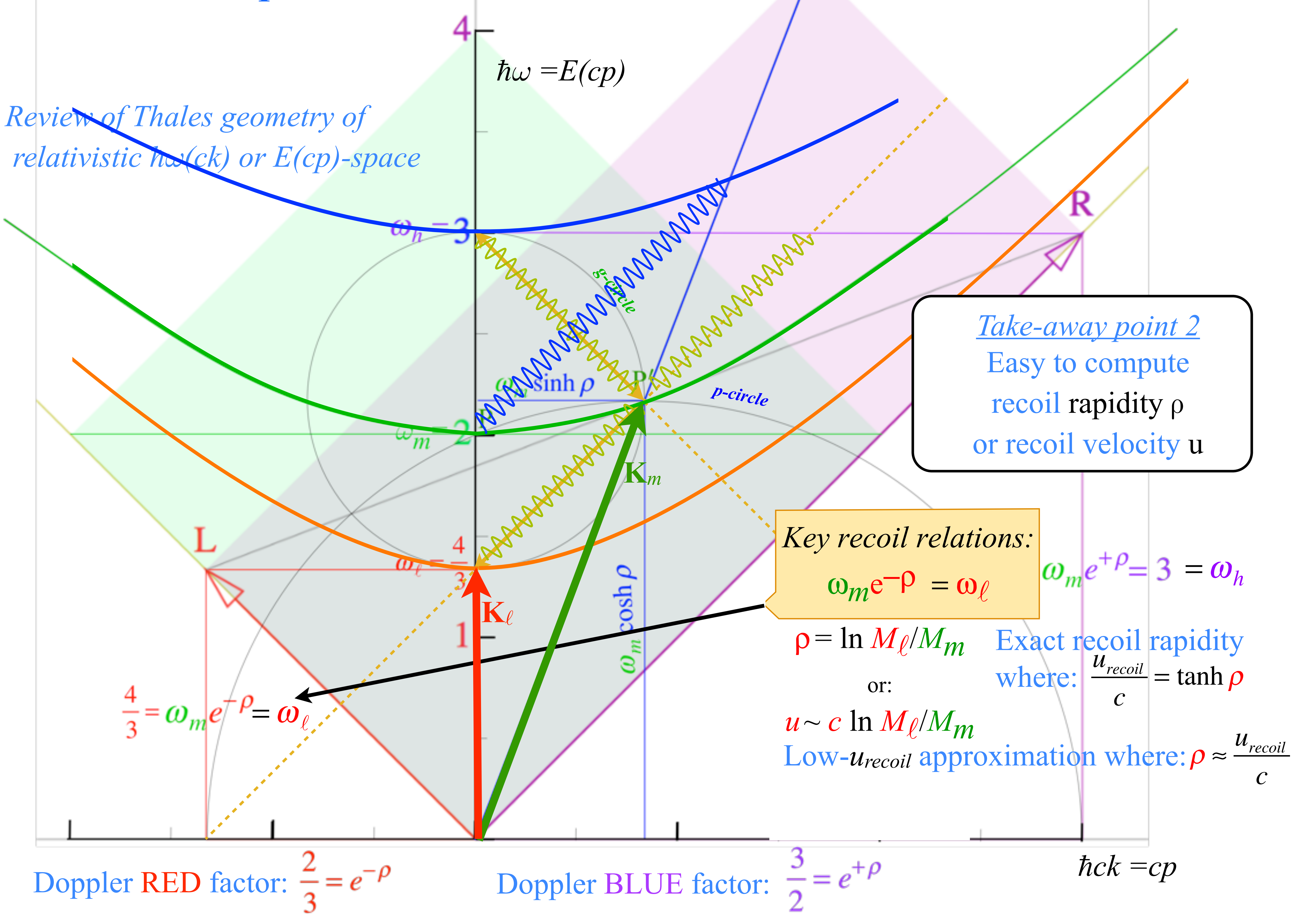
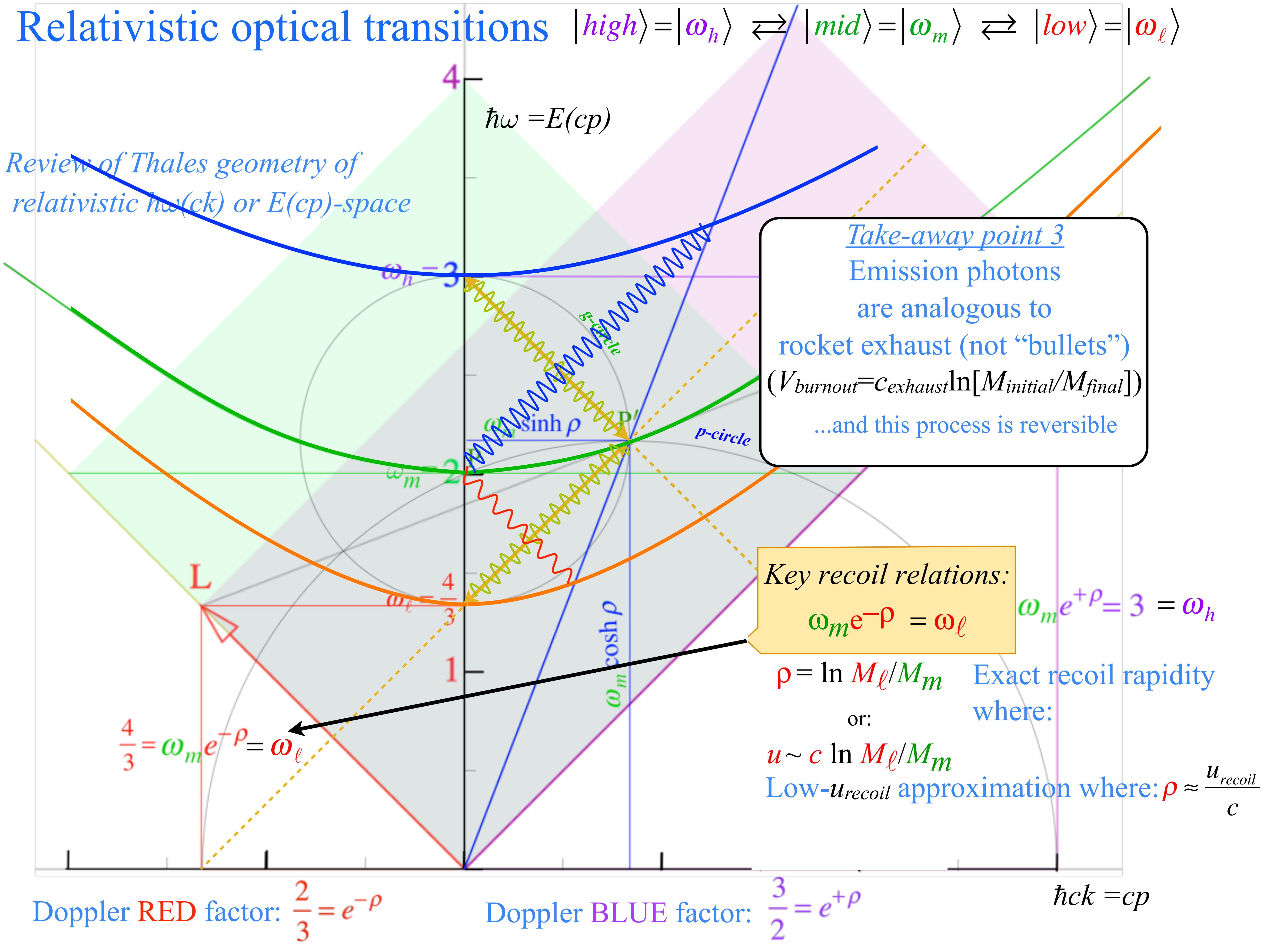
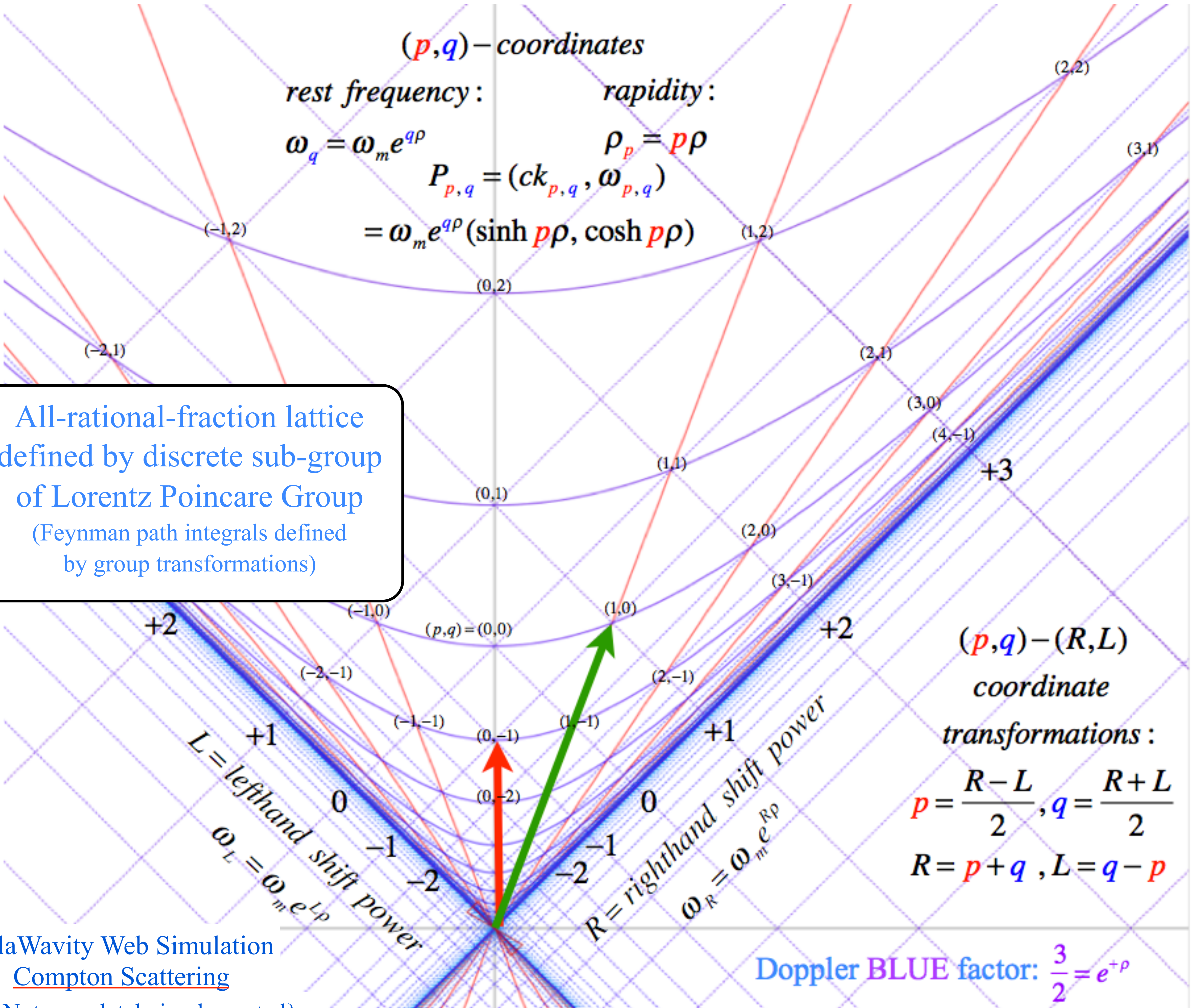




Relativistic optical transitions
$|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$
Review of Thales geometry of relativistic ħω(ck) or E(cp)-space
$\hbar\omega = E(cp)$
$\omega_h = 3$
$\omega_m = 2$
$\omega_\ell = \frac{4}{3}$
g-circle
p-circle
P'
L
$\omega_m \sinh\rho$
$\omega_m \cosh\rho$
Take-away point 3
Emission photons
are analogous to
rocket exhaust (not "bullets")
$(V_{burnout} = c_{exhaust} \ln[M_{initial}/M_{final}])$
...and this process is reversible
Key recoil relations:
$\omega_m e^{-\rho} = \omega_\ell$
$\omega_m e^{+\rho} = 3 = \omega_h$
$\rho = \ln M_\ell / M_m$
Exact recoil rapidity where:
or:
$u \sim c \ln M_\ell / M_m$
Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$
$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$
$\hbar ck = cp$
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$
Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$



RelaWavity Web Simulation

Compton Scattering

Note: Not completely implemented}

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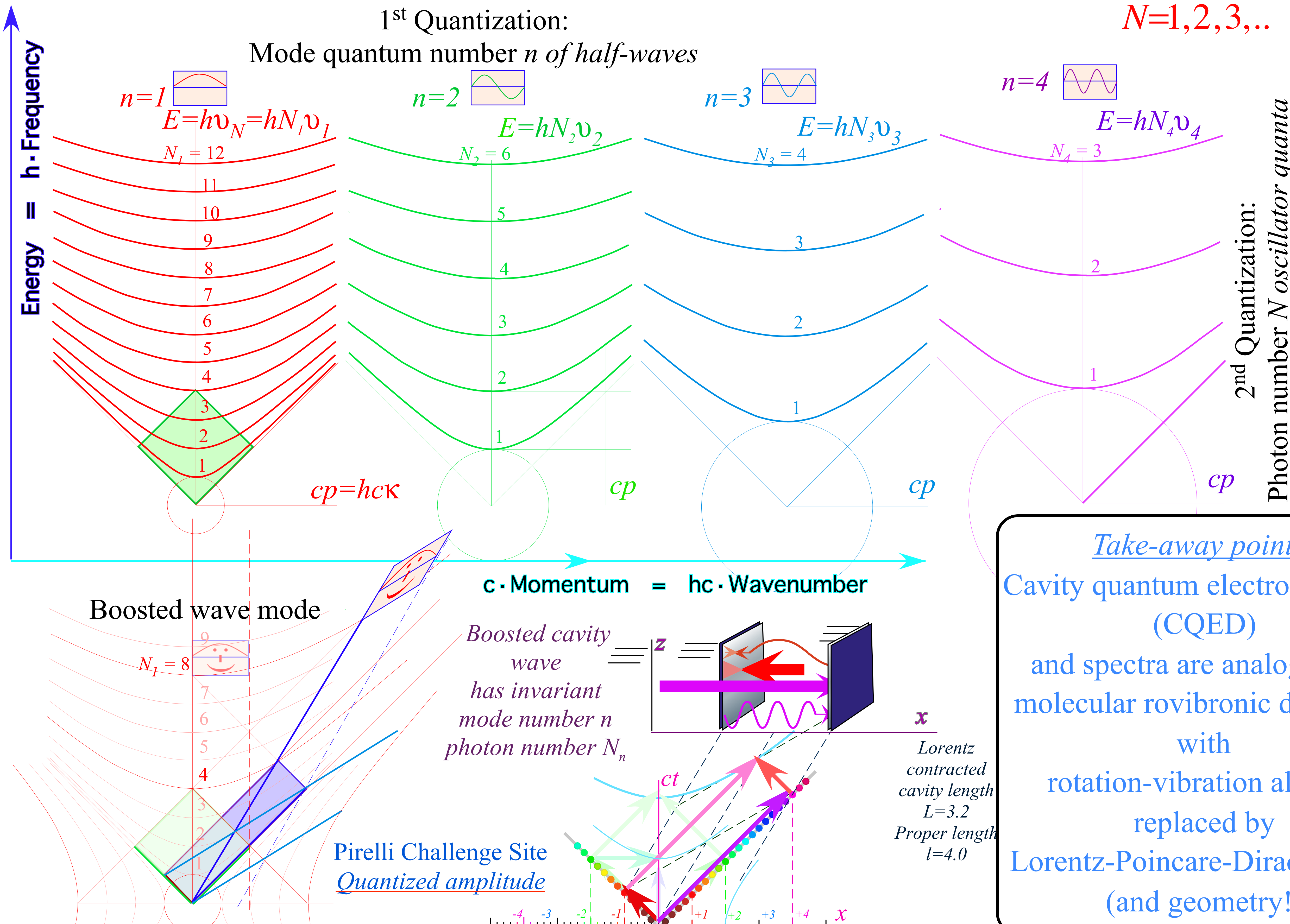
2^{nd} Quantization:

$h\upsilon$ is actually $hN\upsilon$

$(h\upsilon_{phase} = E = h\upsilon_A \cosh\rho)$ *is actually* $(hN\upsilon_{phase} = E_N = hN\upsilon_A \cosh\rho$ with *quantum* numbers$)$

$N = 1, 2, 3, ..$



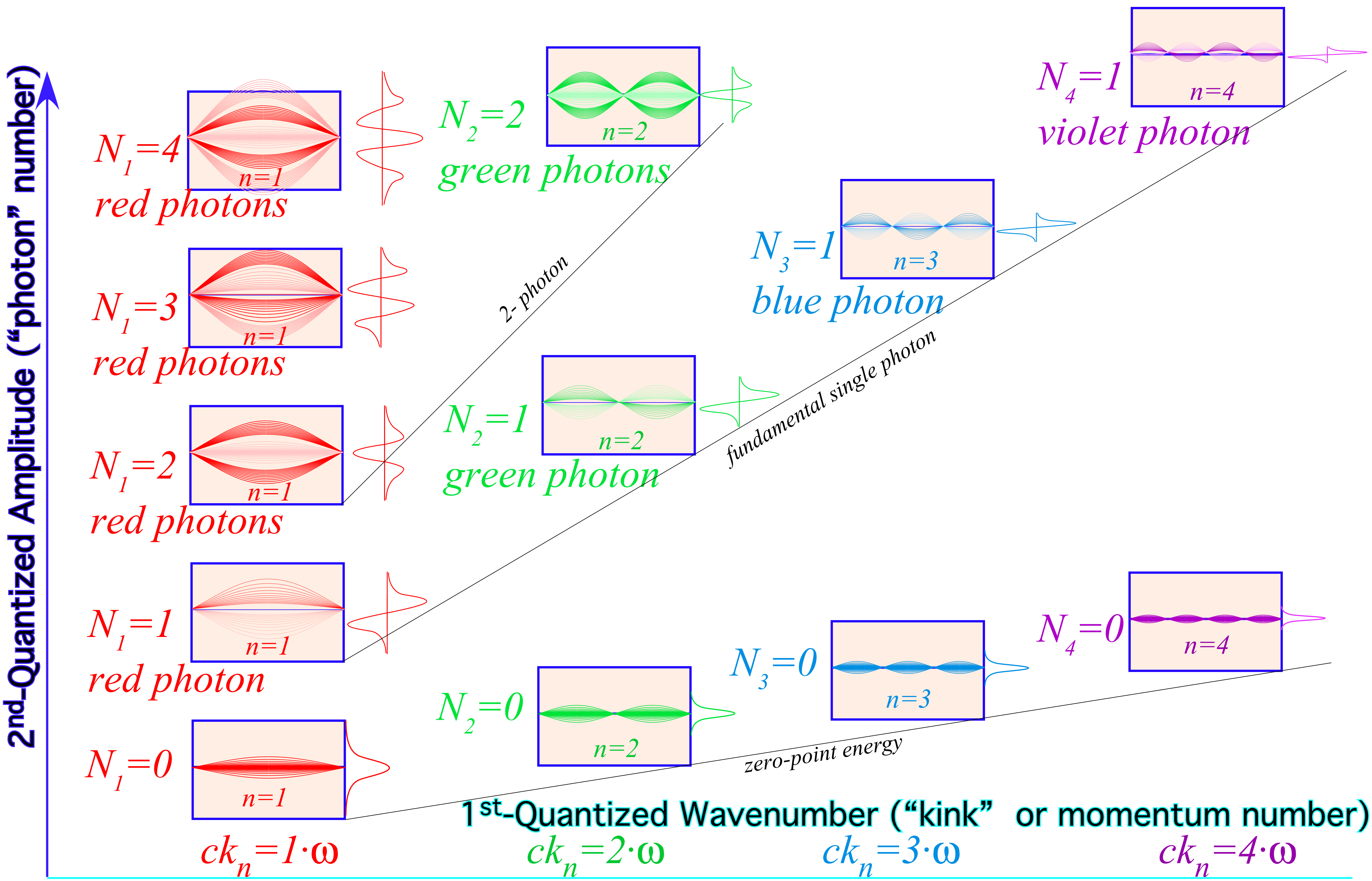
Take-away point 4

Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2nd Quantization:

$h\upsilon$ *is actually* $hN\upsilon$

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ *is actually* $(\ hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho \quad (N=1,2,..)\ )$



Lecture 30
Wed. 12.05.2018

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Acceleration by chirping laser pairs

Varying acceleration (General case)

From Lect. 35 ModPhys (2012)

Varying local acceleration $\rho = \rho(\tau)$

$$u = \frac{dx}{dt} = c\tanh(\tau)$$

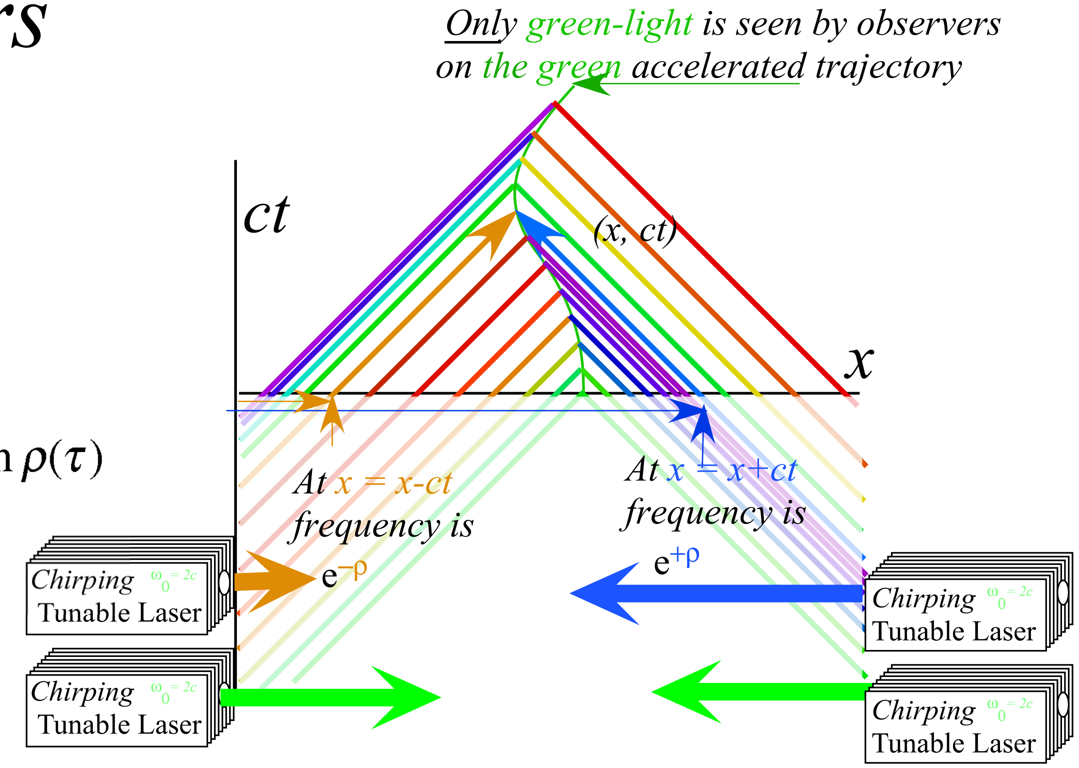
$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

$$ct = c\int \cosh\rho(\tau)\, d\tau \qquad x = c\int \sinh\rho(\tau)\, d\tau$$

Constant local acceleration $\rho = \frac{g\tau}{c}$ "*Einstein Elevator*"

$$ct = c\int \cosh\frac{g\tau}{c}\, d\tau \qquad x = c\int \sinh\frac{g\tau}{c}\, d\tau$$

$$= \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$



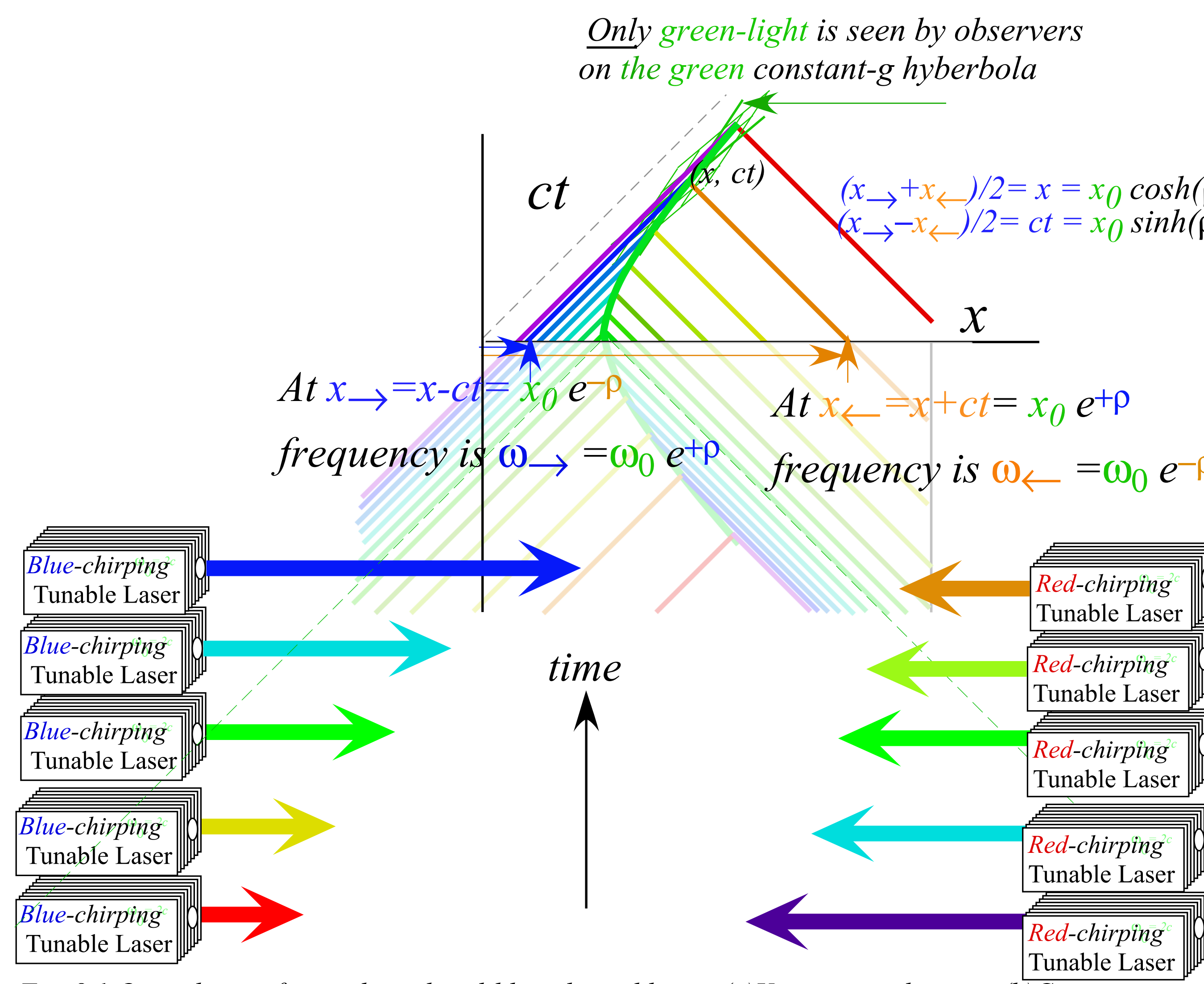


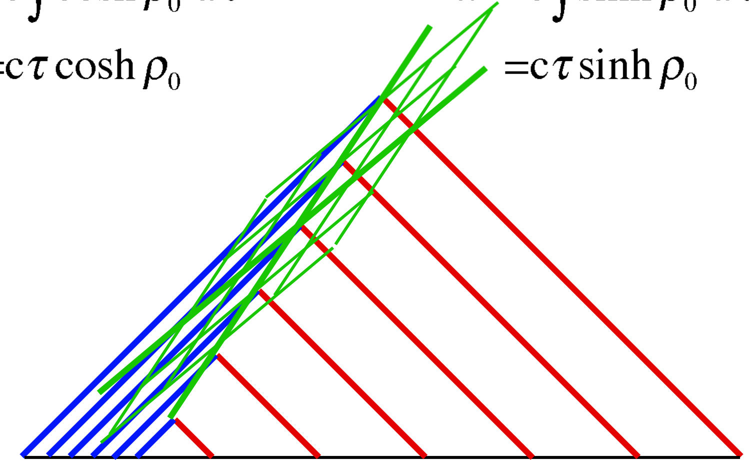
Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

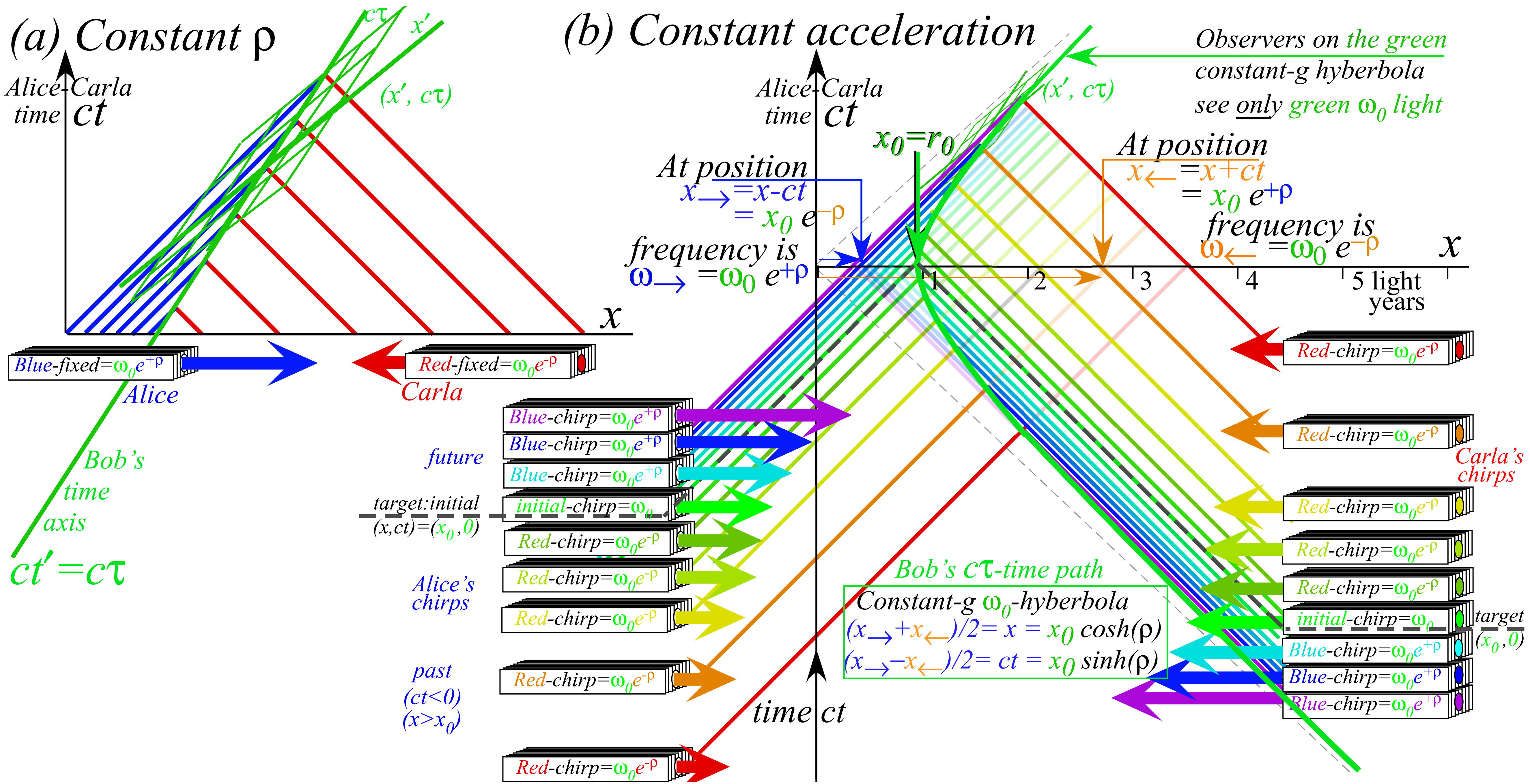
Previous examples involved constant velocity

Constant velocity $\rho = \rho_0 = const.$ "*Lorentztransformation*"

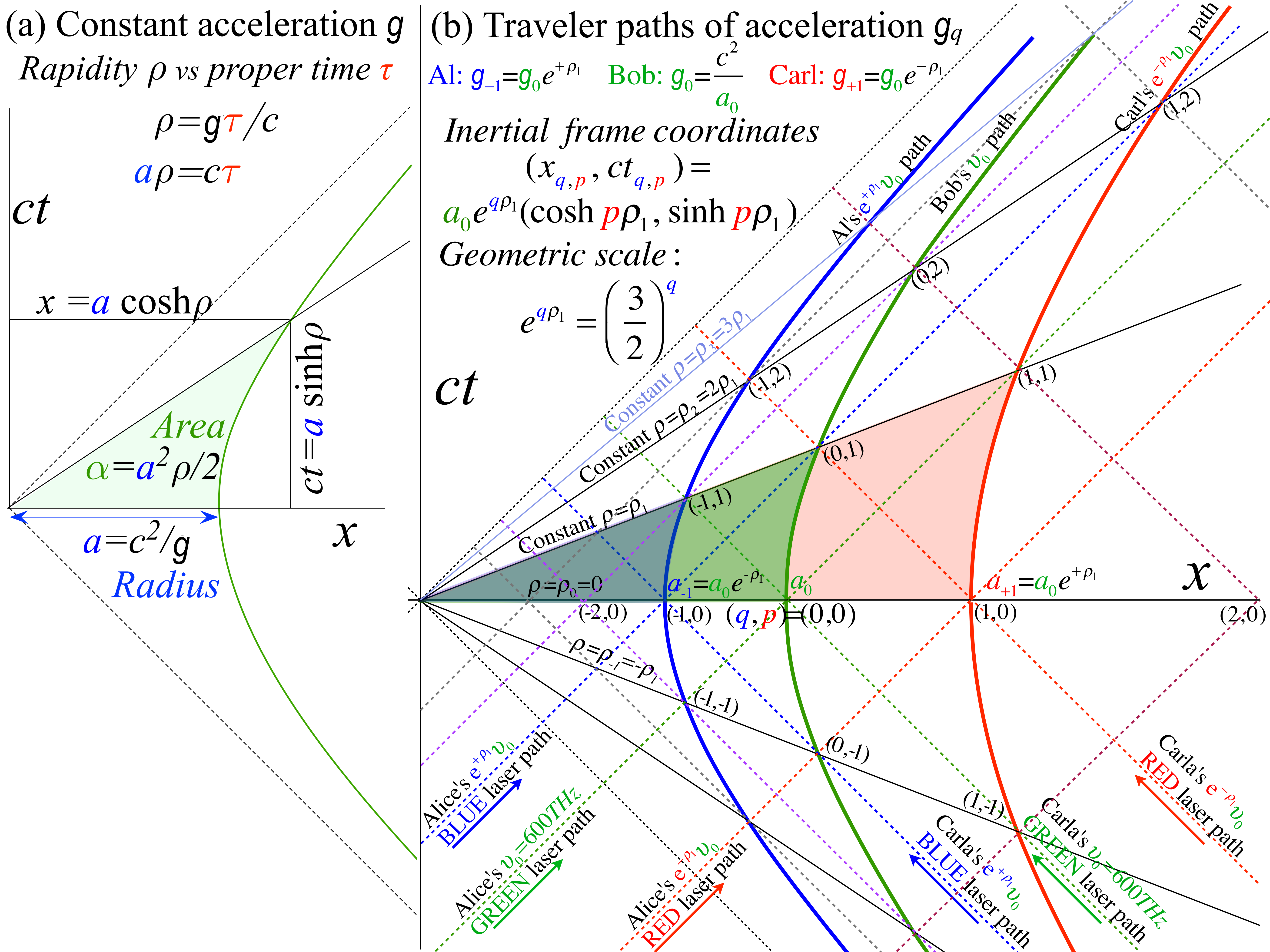
$$ct = c\int \cosh\rho_0\, d\tau \qquad x = c\int \sinh\rho_0\, d\tau$$

$$= c\tau\cosh\rho_0 \qquad = c\tau\sinh\rho_0$$





From Lect. 35 ModPhys (2012)



(a) Constant acceleration g
Rapidity ρ vs proper time τ
ρ=gτ/c
aρ=cτ
ct
x =a coshρ
ct=a sinhρ
Area
α=a²ρ/2
a=c²/g
Radius
x
(b) Traveler paths of acceleration g_q
Al: g_{-1}=g_0 e^{+ρ_1}
Bob: g_0=c²/a_0
Carl: g_{+1}=g_0 e^{-ρ_1}
Inertial frame coordinates
(x_{q,p}, ct_{q,p}) =
a_0 e^{qρ_1}(cosh pρ_1, sinh pρ_1)
Geometric scale:
e^{qρ_1} = (3/2)^q
ct
x
Al's e^{+ρ_1}υ_0 path
Bob's υ_0 path
Carl's e^{-ρ_1}υ_0 path
Constant ρ=ρ_3=3ρ_1
Constant ρ=ρ_2=2ρ_1
Constant ρ=ρ_1
ρ=ρ_0=0
ρ=ρ_{-1}=-ρ_1
a_{-1}=a_0 e^{-ρ_1}
a_0
a_{+1}=a_0 e^{+ρ_1}
(q,p)=(0,0)
(-2,0)
(-1,0)
(1,0)
(2,0)
(-1,1)
(0,1)
(1,1)
(-1,2)
(0,2)
(1,2)
(-1,-1)
(0,-1)
(1,-1)
Alice's e^{+ρ_1}υ_0 BLUE laser path
Alice's υ_0=600THz GREEN laser path
Alice's e^{-ρ_1}υ_0 RED laser path
Carla's e^{+ρ_1}υ_0 BLUE laser path
Carla's υ_0=600THz GREEN laser path
Carla's e^{-ρ_1}υ_0 RED laser path

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

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Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

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Learning about sin! *and* cos *and...*

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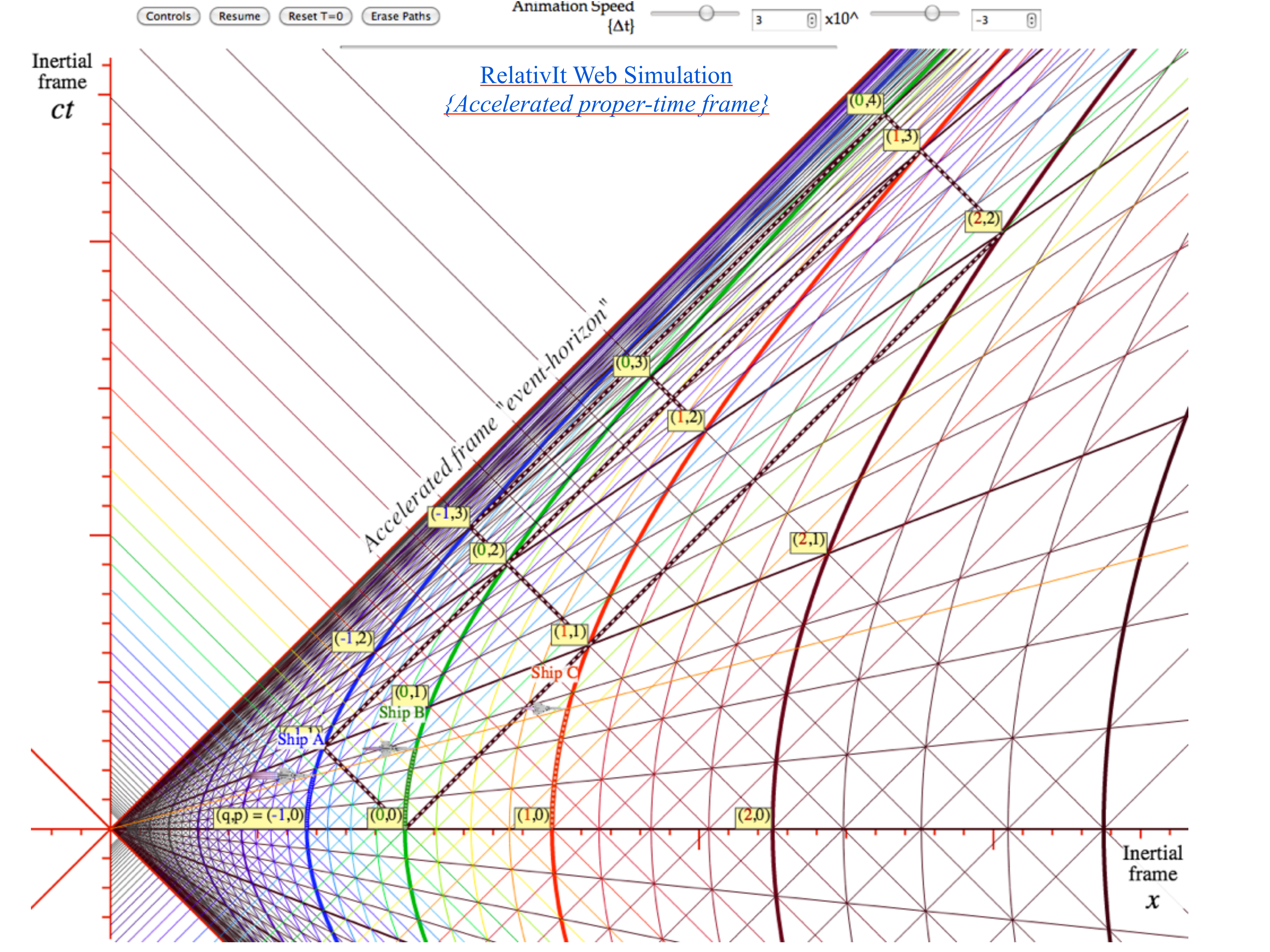
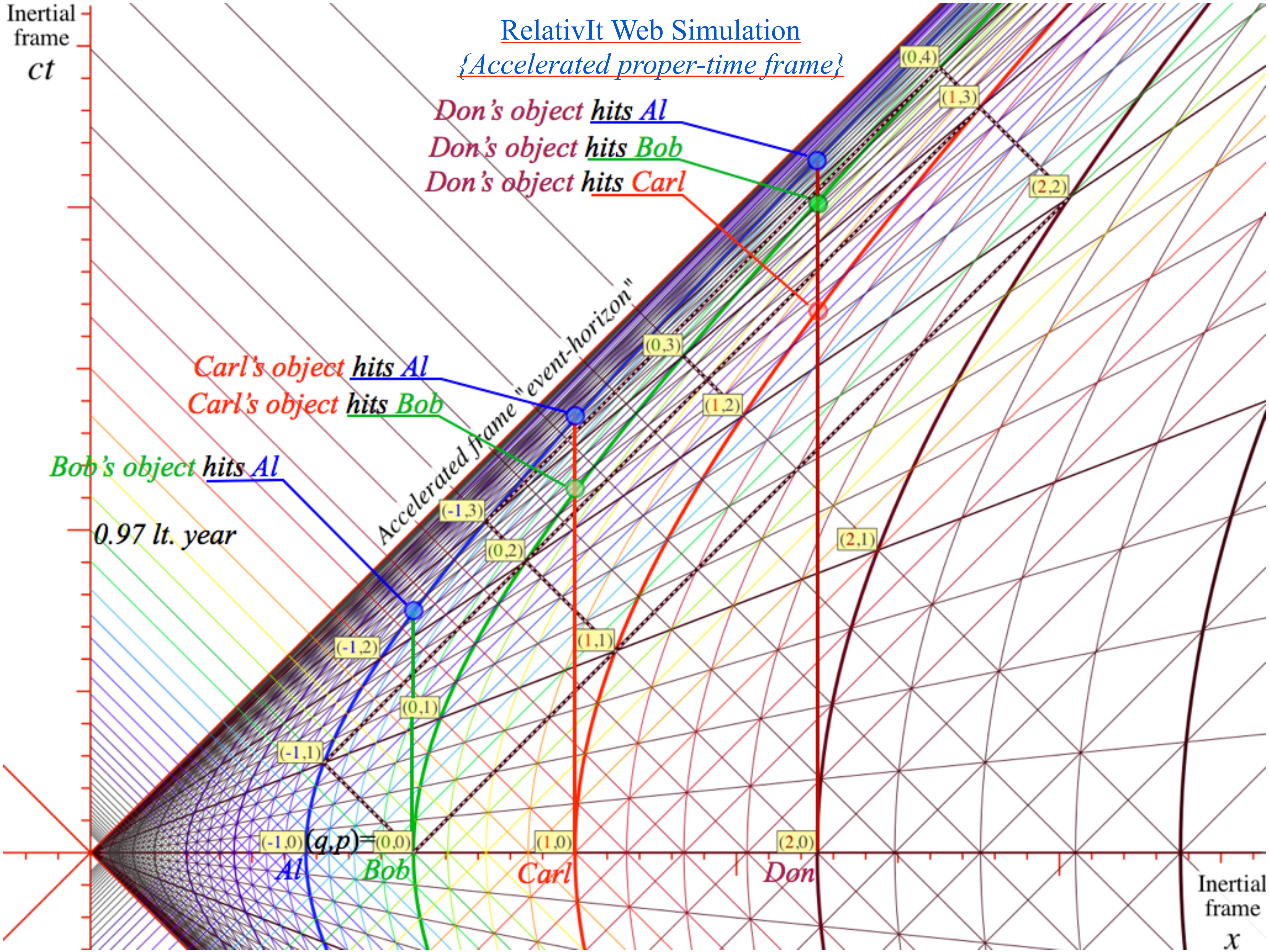
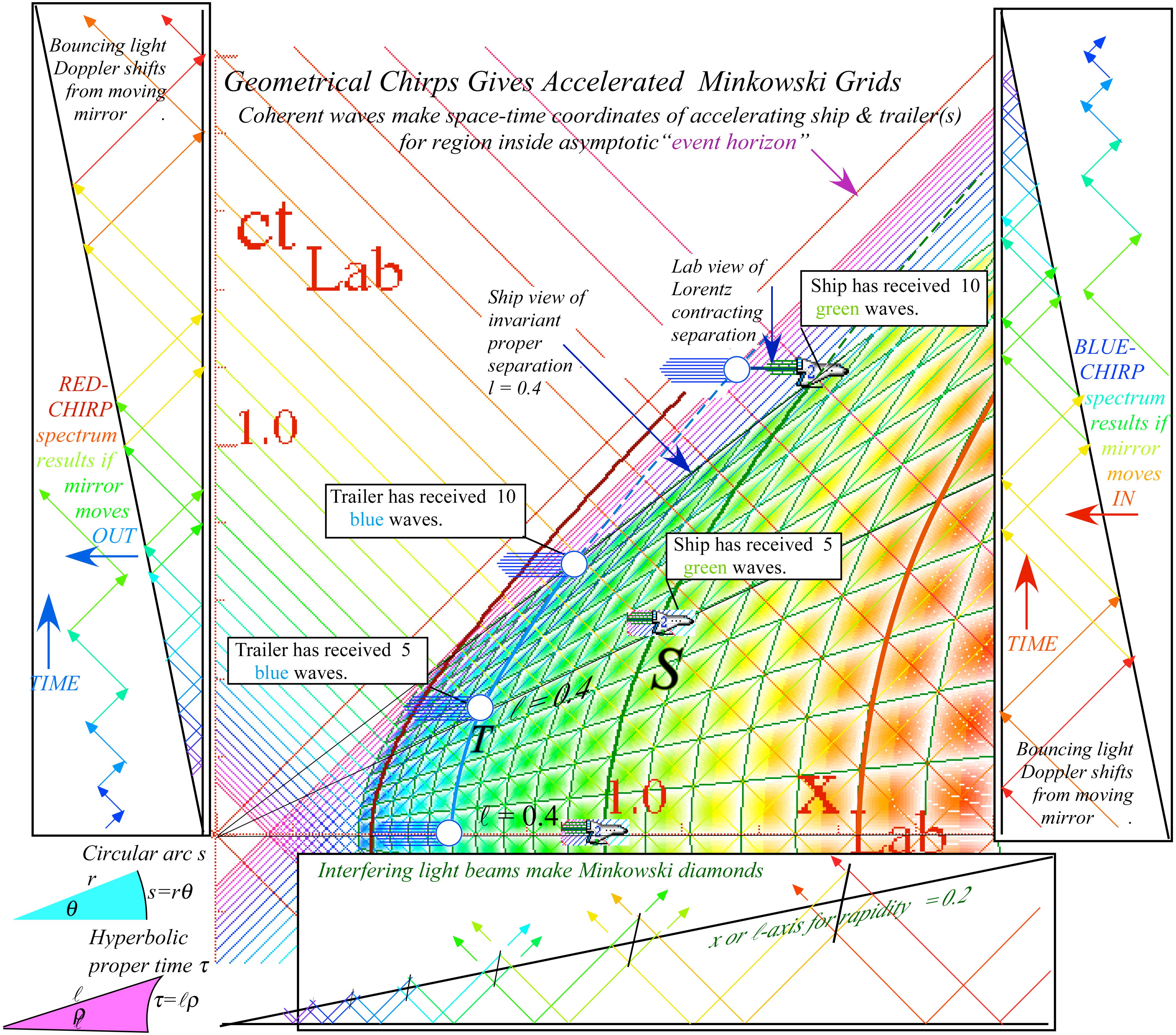


Controls
Resume
Reset T=0
Erase Paths
Animation Speed
{Δt}
3
x10^
-3
Inertial frame
ct
RelativIt Web Simulation
{Accelerated proper-time frame}
(0,4)
(1,3)
(2,2)
Accelerated frame "event-horizon"
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(-1,2)
(1,1)
Ship C
(0,1)
Ship B
Ship A
(q,p) = (-1,0)
(0,0)
(1,0)
(2,0)
Inertial frame
x

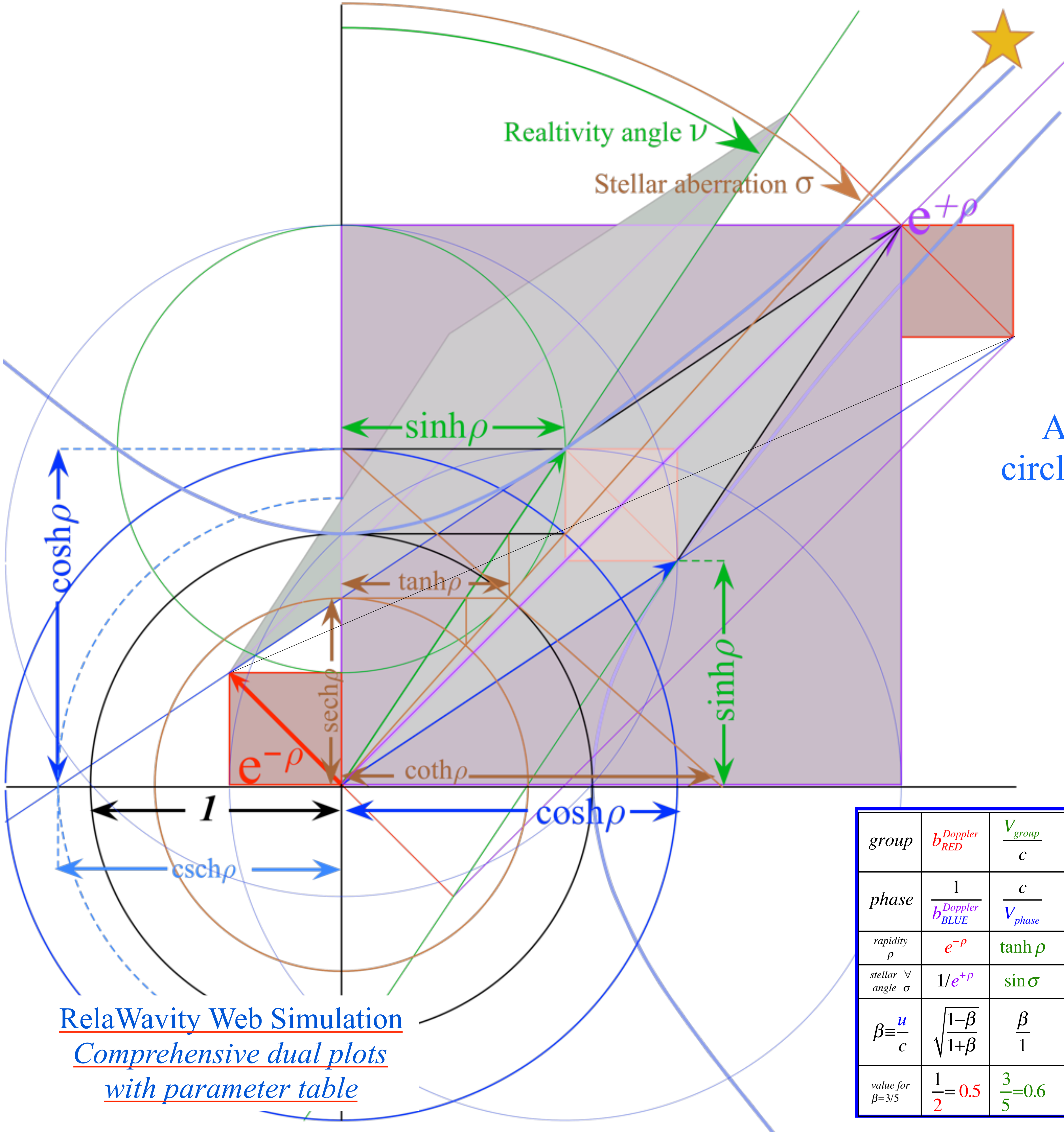


RelativIt Web Simulation
{Accelerated proper-time frame}
Inertial frame
ct
Don's object hits Al
Don's object hits Bob
Don's object hits Carl
Carl's object hits Al
Carl's object hits Bob
Bob's object hits Al
Accelerated frame "event-horizon"
0.97 lt. year
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(1,1)
(-1,2)
(0,1)
(-1,1)
(-1,0)
(q,p)=(0,0)
(1,0)
(2,0)
Al
Bob
Carl
Don
Inertial frame
x



From Lect. 35 ModPhys (2012)

Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light

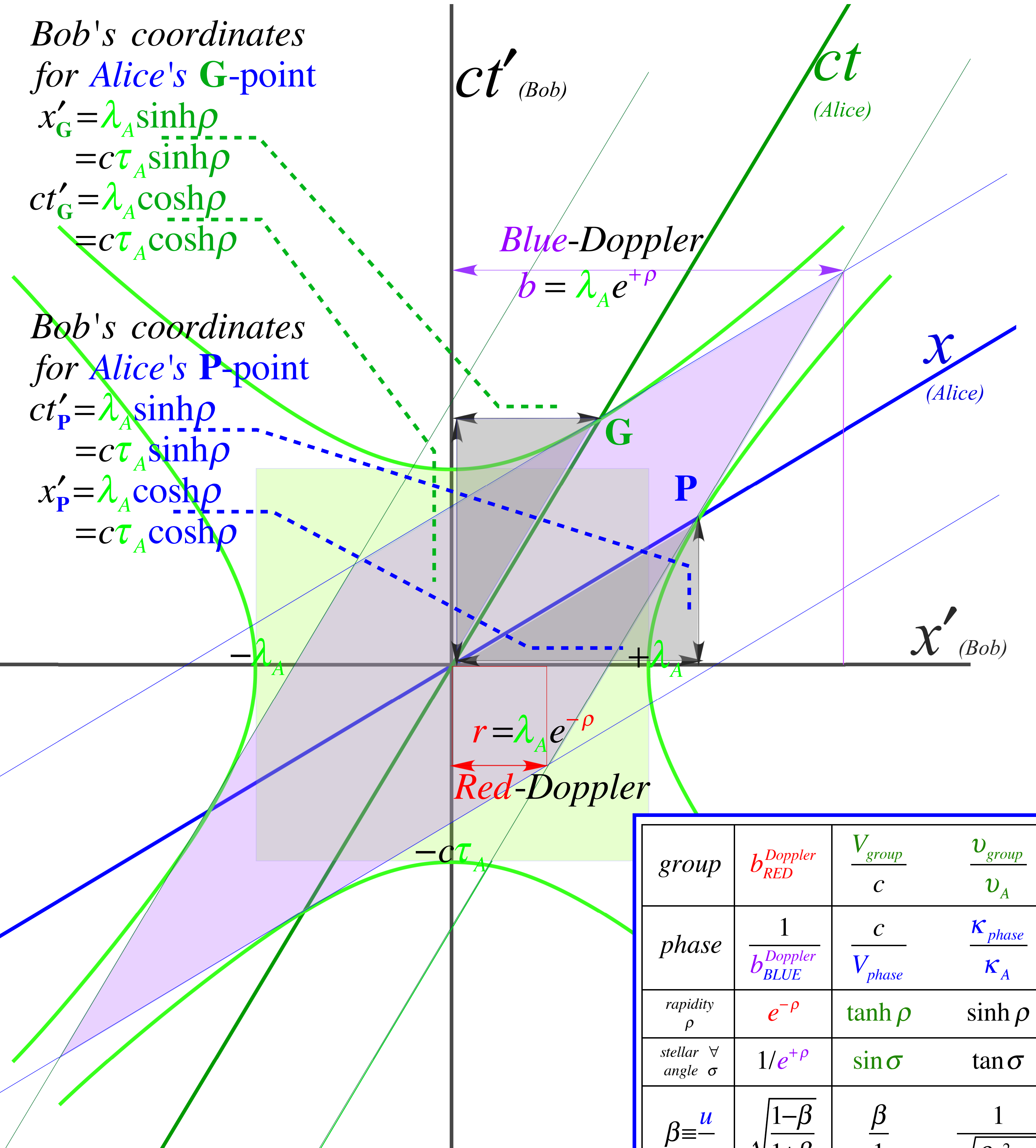


A more compact circle-based geometry

RelaWavity Web Simulation

Comprehensive dual plots with parameter table

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



Space-time parameters

$\lambda_{phase} = \lambda_A \operatorname{csch}\rho$

$\lambda_{group} = \lambda_A \operatorname{sech}\rho$

$c\tau_{phase} = c\tau_A \operatorname{sech}\rho$

$c\tau_{group} = c\tau_A \operatorname{csch}\rho$

Per-space-time parameters

$c\kappa_{phase} = c\kappa_A \sinh\rho$

$c\kappa_{group} = c\kappa_A \cosh\rho$

$\upsilon_{phase} = \upsilon_A \cosh\rho$

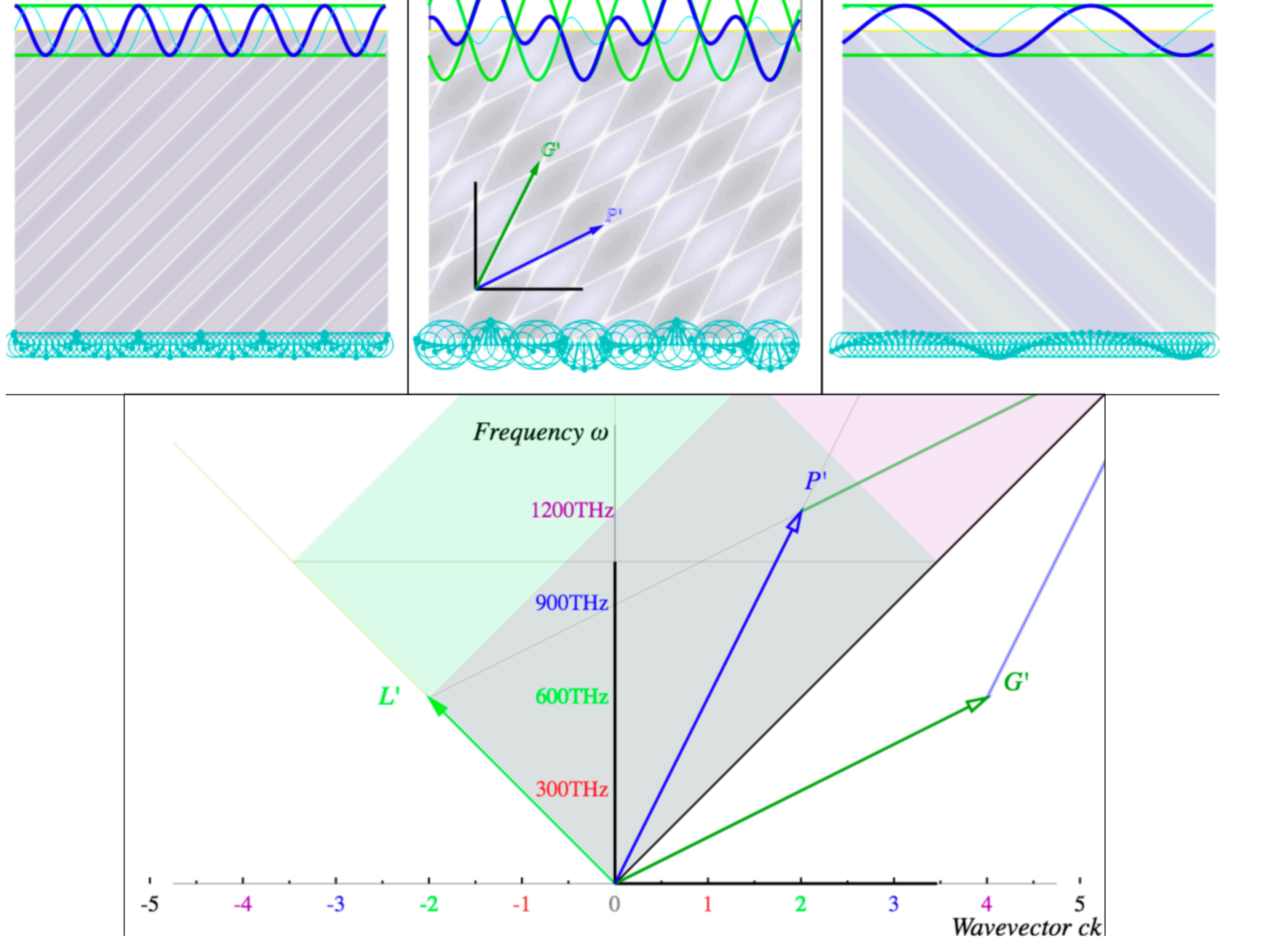
$\upsilon_{group} = \upsilon_A \sinh\rho$

RelaWavity Web Simulations

Comprehensive dual plots with parameter table

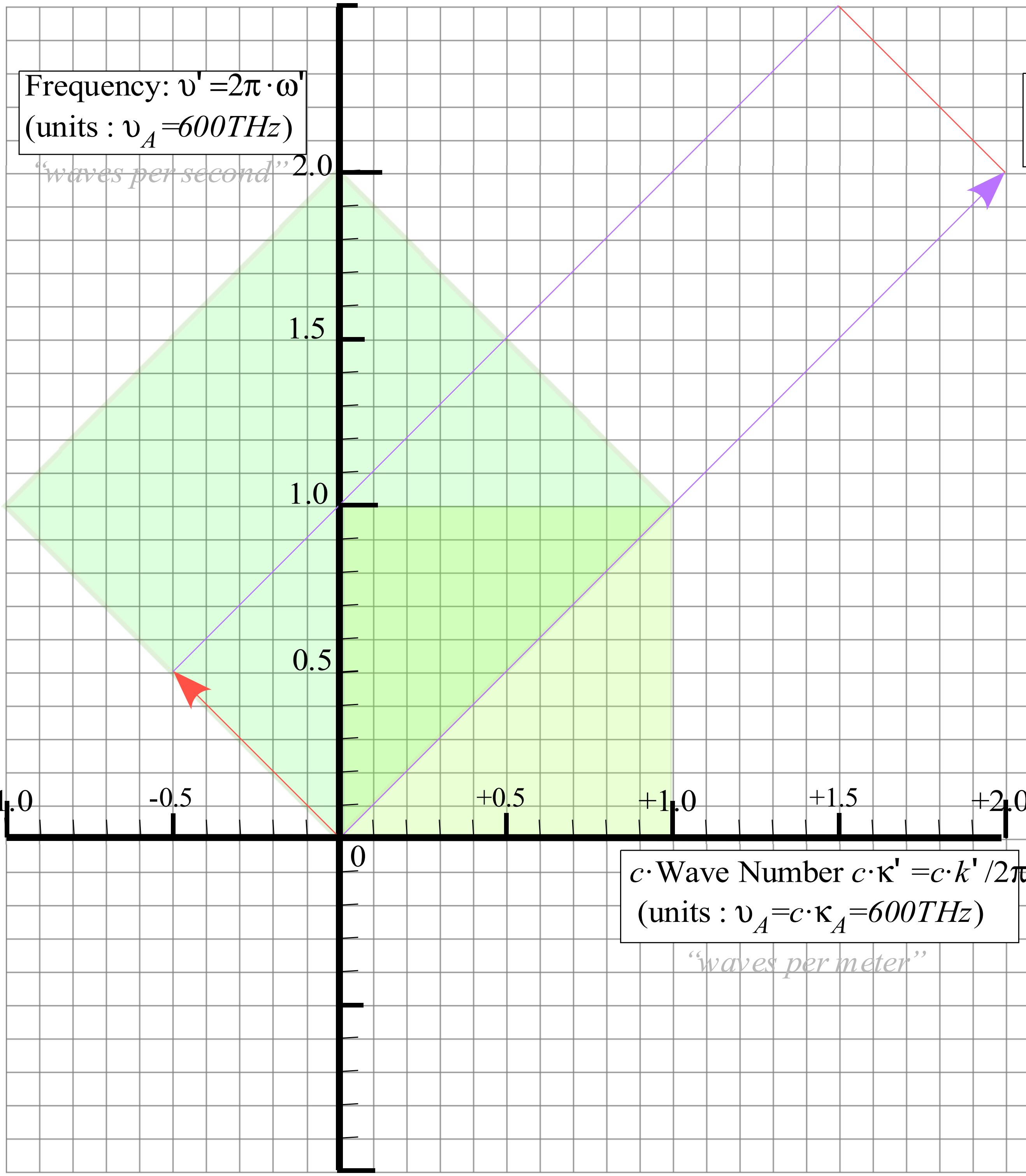
ct' vs x' with parameter table

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b^{Doppler}_{RED}$	V_{group}	past-future asymmetry (off-diagonal Lorentz-transform)	x-contraction(Lorentz) τ_{phase}-contraction	t-dilation(Einstein) υ_{phase}-dilation (on-diagonal Lorentz-transform)	inverse asymmetry	V_{phase}	$b^{Doppler}_{BLUE}$



G'
P'
Frequency ω
P'
1200THz
900THz
L'
600THz
G'
300THz
-5
-4
-3
-2
-1
0
1
2
3
4
5
Wavevector ck

Per-space-time $(\upsilon', c\kappa')$ geometry of 2-CW vectors



Space-time $(c\tau', x')$ geometry of 2-CW paths