How long does it take to get to α-Centauri in 6 months?

1. Suppose we define a velocity we will call \( v_{\text{ignorant}} \) as that speed that someone ignorant of relativity would say a spaceship had to go to get to a distant star in a given time. For example, if we ask how fast a ship would have to go to get to α-Centauri (~4 light years away) in 6 months then the "ignorant" person would say it had to go \( v_{\text{ignorant}} = 8c \), that is, eight times the speed of light. So if super-luminal travel is prohibited, then 6 months seems too short.

But the relativity expert says that there is a speed \( v_{\text{expert}} \) which will get the ship to α-Centauri in 6 months according to the ship's passengers, who, after all, are the ones really counting their time.

(a) Compute \( v_{\text{expert}} \) for α-Centauri trip and derive general algebraic relations giving \( v_{\text{expert}} \) in terms of \( v_{\text{ignorant}} \) and vice-versa.

(b) How long does it really take to get to α-Centauri in 6 months? (Lighthouse time.)

2. Consider a more realistic project discussed recently in the New York Times.

This involves sending a package that would get to α-Centauri in 16 years (its time) by blasting it up to necessary speed with powerful lasers. This one can be plotted on a Minkowski graph such as can be made using the Relawavity website. Do a plot and make an event table involving departure and arrival space-time events. Find \( v_{\text{ignorant}} \) and \( v_{\text{expert}} \).

If upon arrival the package sends a message back, when should we expect to hear from it if we had sent it out today?

3. Use the velocity \( v_{\text{ignorant}} \) defined in a preceding exercise (1.) and results concerning the “6-month” α-Centaur voyage.

(a) Relativistic momentum of particle of rest mass \( M \) can be nicely expressed using \( v_{\text{ignorant}} \). How?

(b) Redo ex.1 using Epstein space-proper time and plot it onto a protractor graph provided in class and onsite.

   Check calculation of stellar angle \( \sigma = \ldots \)° using the graph.

(b) Given the proposed journey to α-Centauri in “6 months” work up a budget estimate. How many GNPU (1GNPU = \( 10^{12} \)=$1 Trillion) will it cost to get a ship of mass \( 10^6 \) kg (1,100 tons) up to speed at a prevailing rate of power: \$0.10/ kWhr? (1 kWhr = 3600x10^3 J) Note: Don't count the rest mass energy of the ship in your cost. We may assume NASA (i.e., you, the taxpayer) has already bought that stuff. Bottom line: Cost=$

(c) If you express this as a factor of the ship rest energy, what do you get?