The following is to acquaint you with some more exponential properties and phasor views of 2D-HO

**Fun with Exponents & more of the Story of e**

1.10.1 Consider a sequence of functions, \( f_1(z) = z^z, f_2(z) = z^{f_1(z)} = z^{z^z}, f_3(z) = z^{f_2(z)} = z^{z^{z^z}}, \ldots \). The function \( f_N(z) \) has a finite limit \( f_\infty(z) \) (as \( N \) approaches infinity) if number \( z \) is small enough.

(Hints: \( z=1 \) works! But, so does \( z=\sqrt{2} \). Try solving for \( z \) and looking for a max value.)

(a) Find \( f_\infty(\sqrt{2}) = \) ____?

(b) Find an analytic expression for the limiting real \( z_{\text{max}} \) that involves the Euler constant.

\[ e=2.718281828\ldots \]

This is to introduce phasor views of 2D-HO Geometry of Phasor-to-Cartesian and vice-versa relations

1.10.2 *A day in the life of a neutron starlet*

Suppose neutron starlet orbits inside the “Sophomore-Physics-Earth” (SPE) of radius \( R_{\text{SPE}}=10 \) units. (One unit is 637 km.) Let it start at position \( r(0)=(7.0,3.0) \) with velocity \( v(0)=(6.0,8.0) \). (Let time unit be determined by setting SPE angular frequency to one \( (\omega_{\text{SPE}}=1=2\pi \upsilon_{\text{SPE}}) \).

(a) Use position-velocity-phasor graph paper to set times (phases) and amplitudes of x and y phasors.

(b) Use the phasors to locate the starlet position vector \( r(t) \) and its velocity \( v(t) \) for a complete orbit.

(c) Label times of each 12 orbit points at equal time intervals defined as follows by period \( \tau_{\text{SPE}}=1/\upsilon_{\text{SPE}}: \)

- \( t=0 \) is 12:00PM,
- \( t=\tau_{\text{SPE}}/12 \) is 1:00PM,
- \( t=2\tau_{\text{SPE}}/12 \) is 2:00PM,
- \( t=3\tau_{\text{SPE}}/12 \) is 3:00PM, and so forth.

Label velocity vector points, too.

(d) Does the starlet ever penetrate the Earth surface? If it does, how might that affect its orbit?

1.10.3 *12 days for a neutron starlet with an increasingly retarded x-phasor*

This project involves the “pincushion” graph paper for which the \( x \) and \( y \) phasors have the same unit-amplitude but their relative phase shifts so the x-phasor is retarded by \( 1/2 \) hour after each 24 hour orbit.

(a) Start by plotting an orbit starting with both phasors at 3:00 o’clock. (Should be a straight 45° line.) Then plot an orbit with the x-phasor starting at 2:30PM or 15° behind the y-phasor and maintaining that phase lag through the orbit. (Should be very narrow or eccentric ellipse.) Then plot an orbit with the x-phasor starting at 2:00PM or 30° behind the y-phasor and maintaining that phase lag through the orbit. (Should be a more rounded or less eccentric ellipse.) Continue plotting ellipses with phase lag 45°, 60°, 75°, 90°, …, 165°, and finally 180°, that last one being called “PI out of phase.” Note what is special about the 90° case.

(b) Each of the ellipses drawn in part (a) has its own major radius \( a \) and minor radius \( b \) and may be circumscribed in its own 2a-by-2b rectangle. With a ruler carefully sketch each of these rectangles.

(c) The hypotenuse of rectangular radii \( r_{\text{hyp}}(a,b) = \sqrt{a^2 + b^2} \) should be related. How?

(d) Derive the total energy of a 2D-IHO orbit and show it is proportional to \( r_{\text{hyp}}(a,b) = \sqrt{a^2 + b^2} \).

(e) Elliptical 5-by-1 surfboard rotates tangent to wall and floor. Where is its center?