Lecture 1
Tue. 1.19.2016

1st axioms and theorems of classical mechanics

(Ch. 1 and Ch. 2 of Unit 1)

Geometry of momentum conservation axiom (ala Occam's Razor)
- *Totally Inelastic "ka-runch"collisions* (begin 4:1 graph project)*
- *Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**
  - *+Intro to weighty-averages and vector notation*
- *Comments on idealization in classical models*

Geometry of Galilean translation symmetry
- *45° shift in (V_1,V_2)-space*
- *Time reversal symmetry*
- *...of COM collisions*

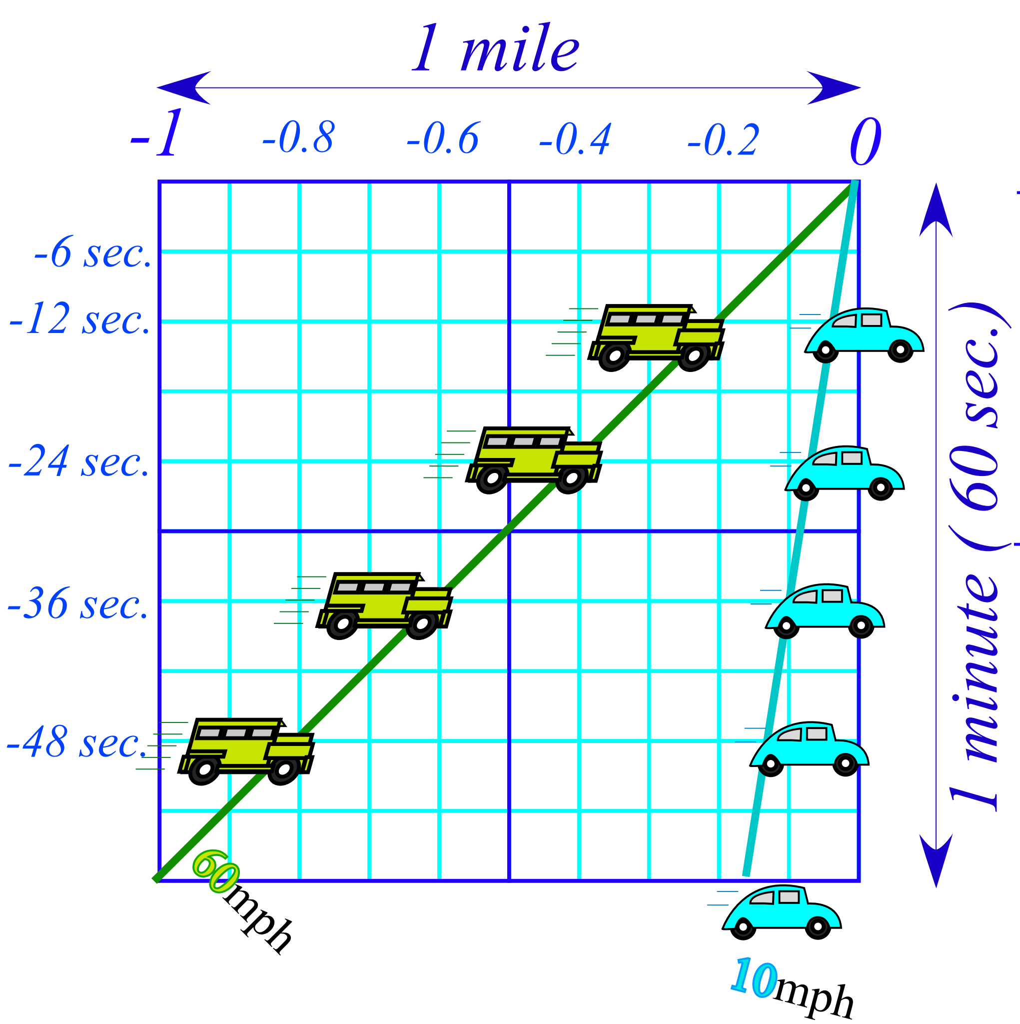
Algebra,Geometry, and Physics of momentum conservation axiom
- *Vector algebra of collisions*
- *Matrix or tensor algebra of collisions*
- *Deriving Energy Conservation Theorem*

Numerical details of collision tensor algebra

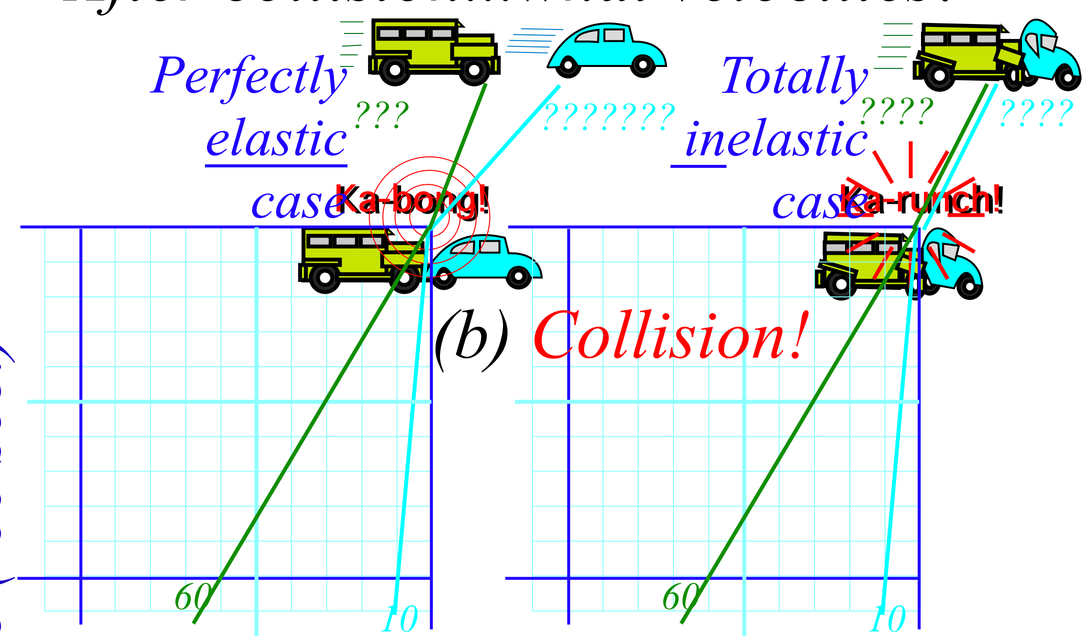
Note - Many of the underlined links throughout this lecture file link to the specific selected cases within those Web Simulators
**Launch Car Generic Collision Web Simulator* http://www.uark.edu/ua/modphys/markup/CMMotionWeb.html
**Launch Generic Superball Collision Web Simulator* http://www.uark.edu/ua/modphys/markup/BounceItWeb.html

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision.....



After collision...what velocities?



Car Simulator
Space vs Space
Elastic

Car Simulator
Space vs Space
Inelastic

Simulator
Elastic Collision
Dual Panel
Space vs Space
and
Space vs Time
(Newton)

Simulator
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Space(Minkowski)

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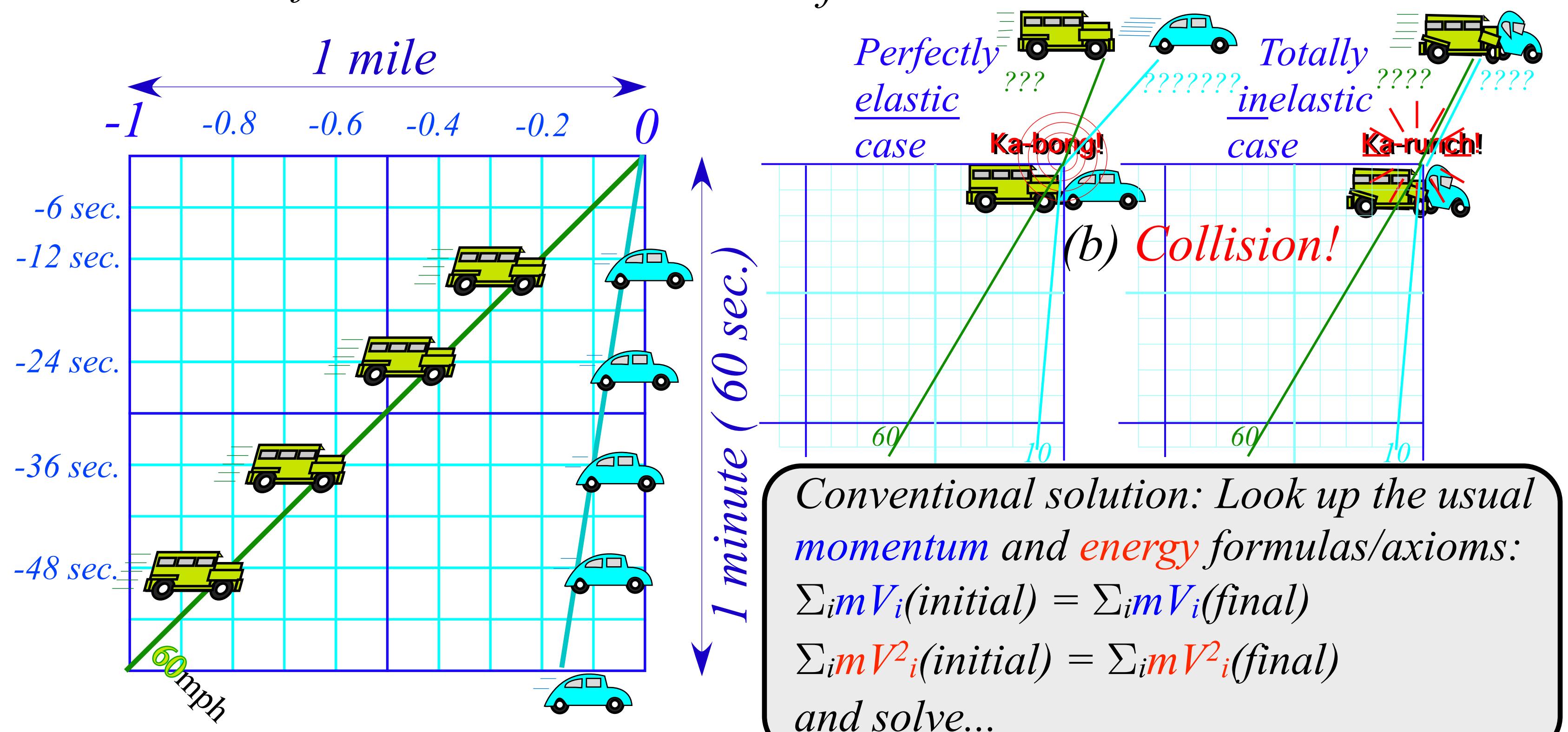
Before collision..... *After collision...what velocities?*



Conventional solution: Look up the usual momentum and energy formulas/axioms:

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and solve...

Let's see if we can solve this *easily* with just ***one*** (*or one-and-a-half*) axiom(s)

Axiom-1: All mass or masses keep their total momentum until it is changed by some outsider.

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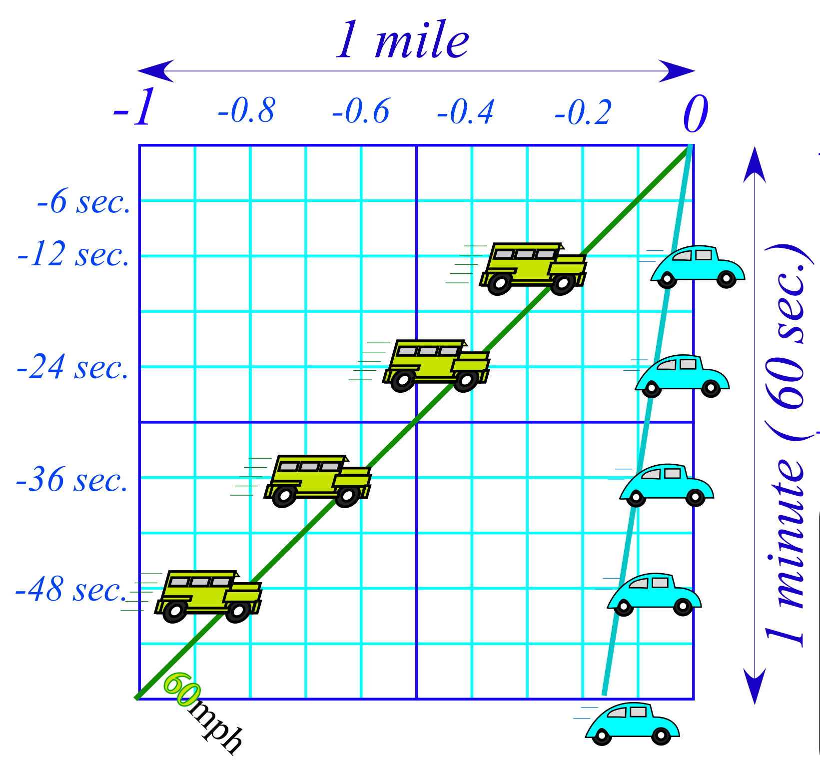
$P_{Total} = M_{SUV} V_{SUV} + M_{VW} V_{VW}$

Before collision.....



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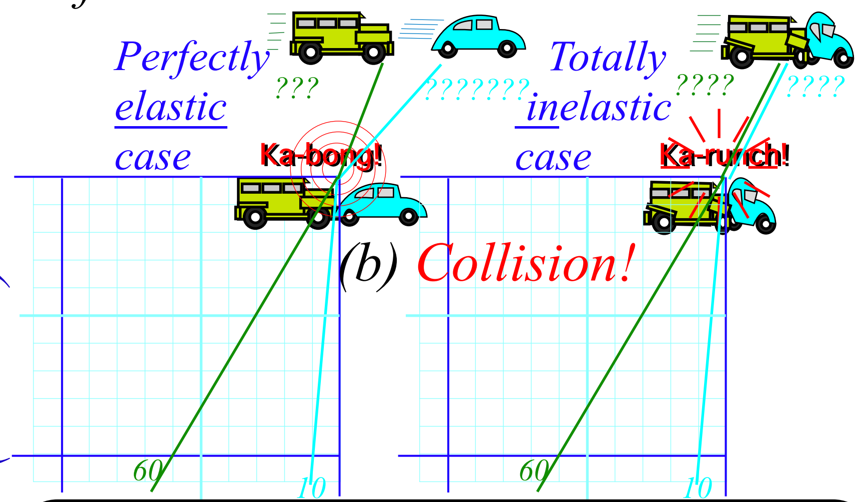

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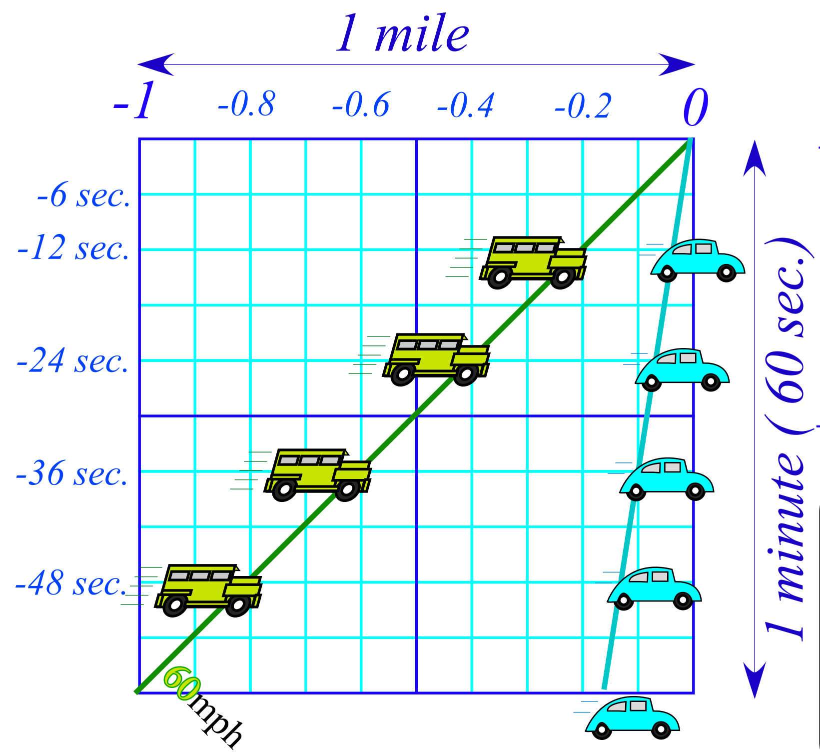

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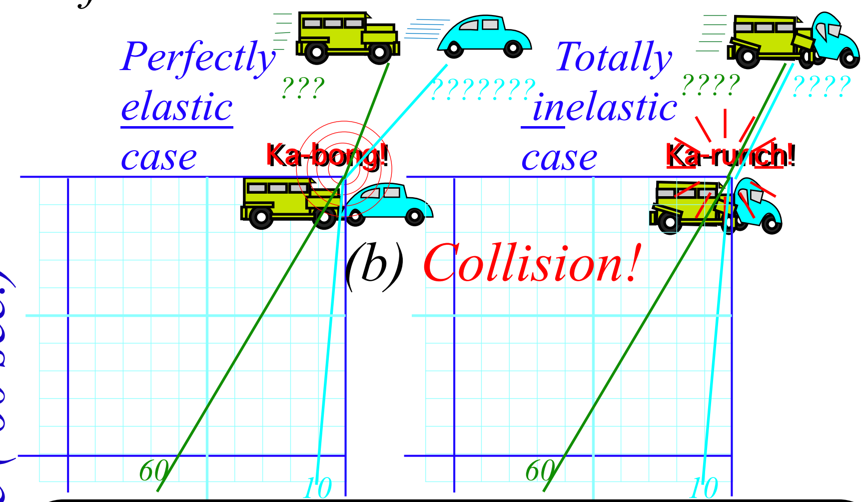

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and solve...

William of Ockham
1285-1349

Wielder of
"Occam's Razor"

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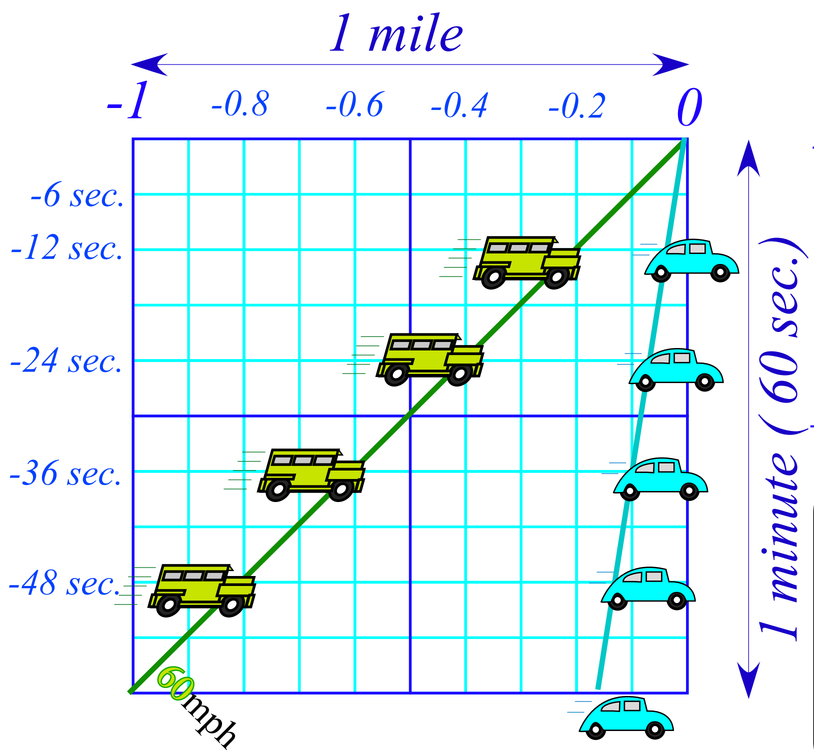
Before collision.....



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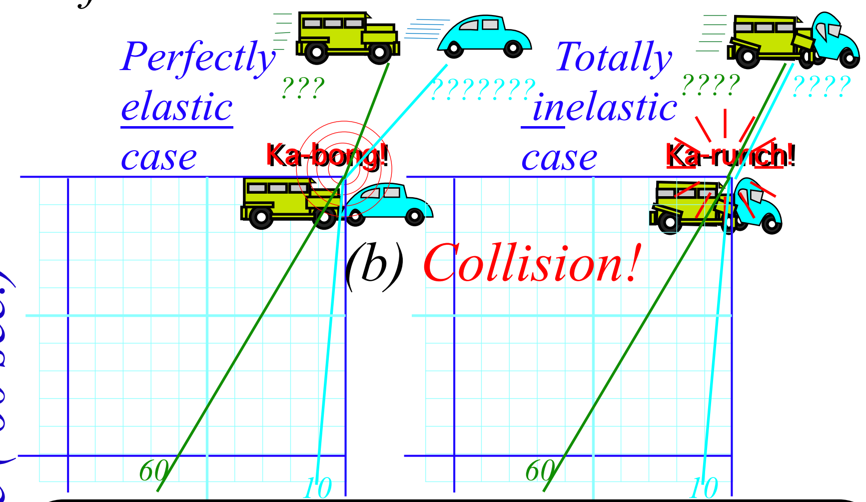

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"*Pluralitas non set poneda sine necessitate.*"
and has a number of interpretations:

1. Literally:"*Don't make pluralities of conjectures without necessity.*"
2. Logically: "*Assume less to prove more.*"
3. Practical coding advice: "*Keep it simple, make it powerful.*"

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A problem in ***space-time*** : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW) V_{SUV} and V_{VW} change violently but NOT ***total momentum***

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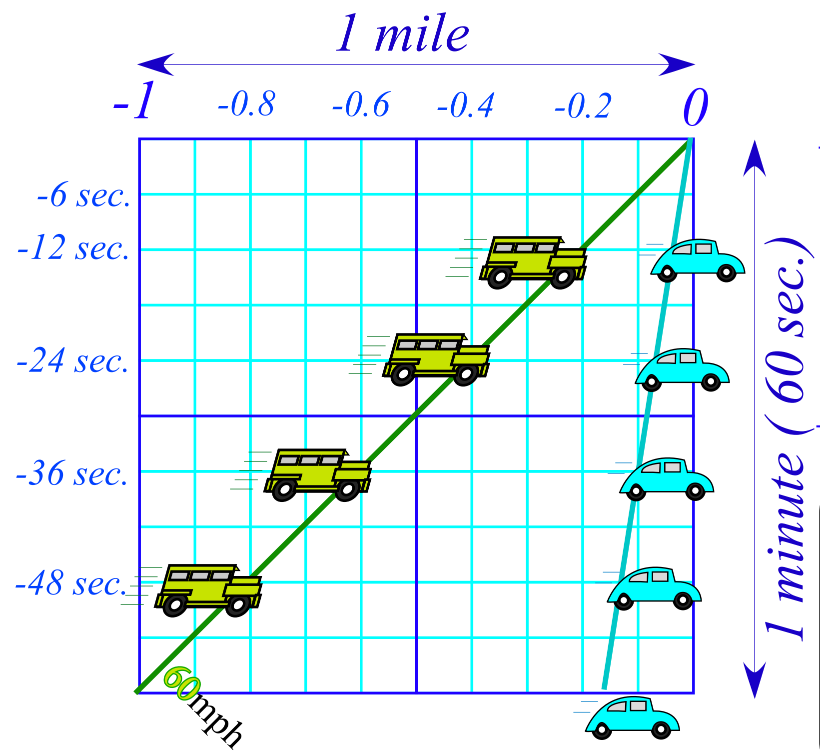
Before collision.....



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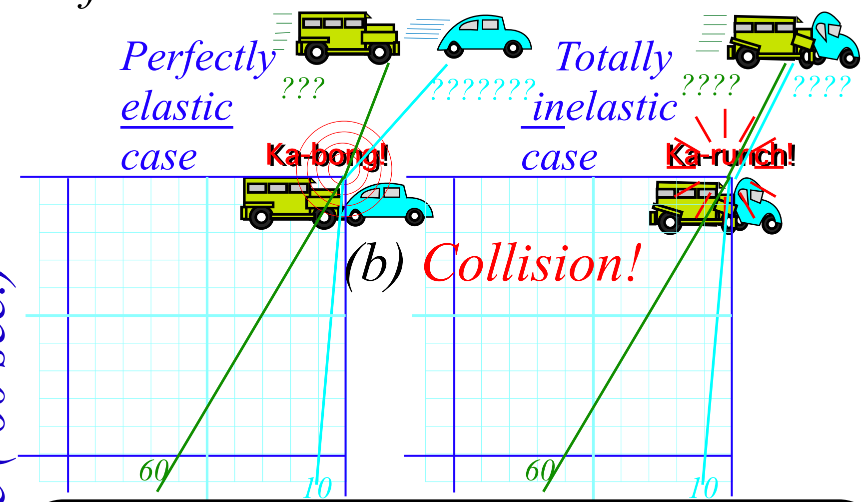

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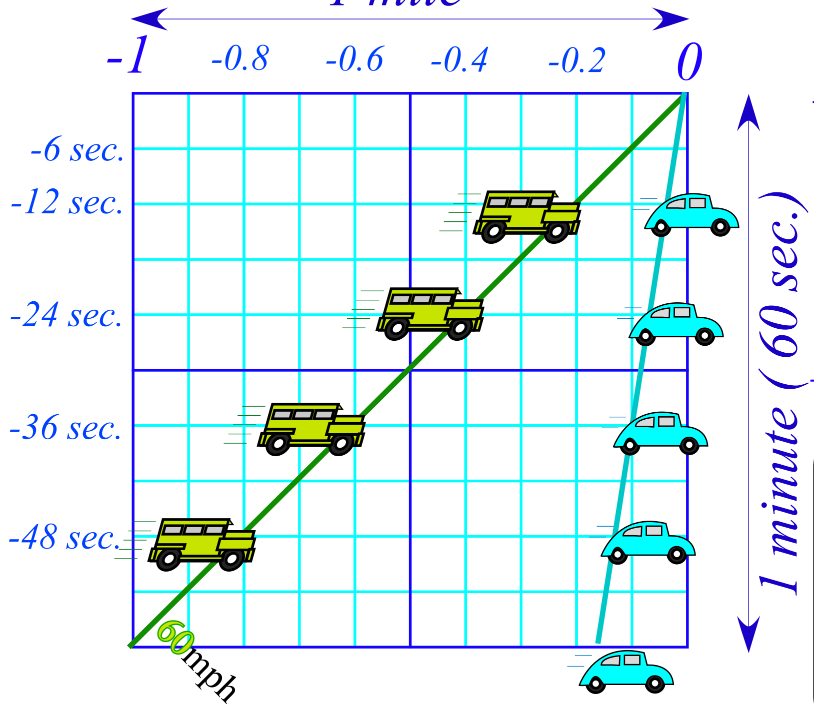

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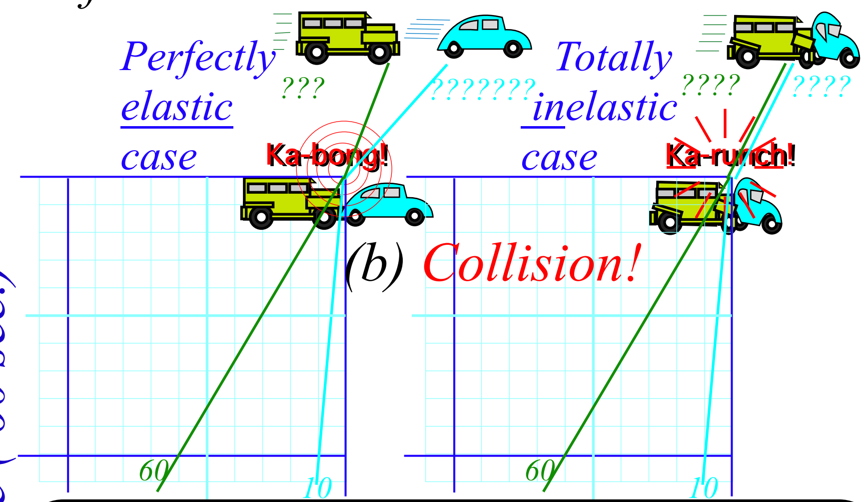

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$$4 \cdot 60 + 1 \cdot 10 = 4 \cdot ? + 1 \cdot ?? = 250$$

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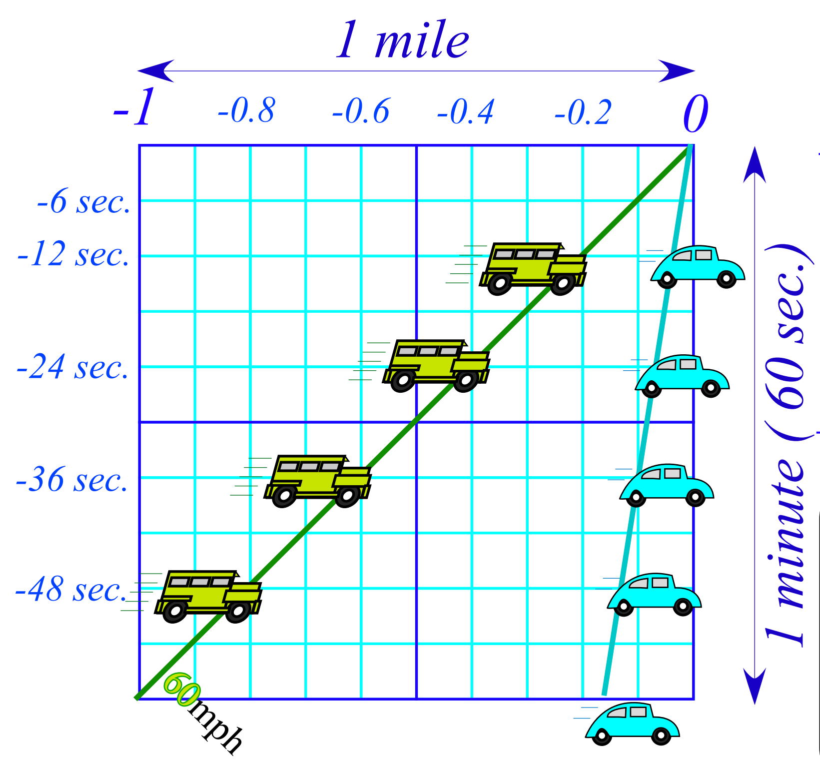

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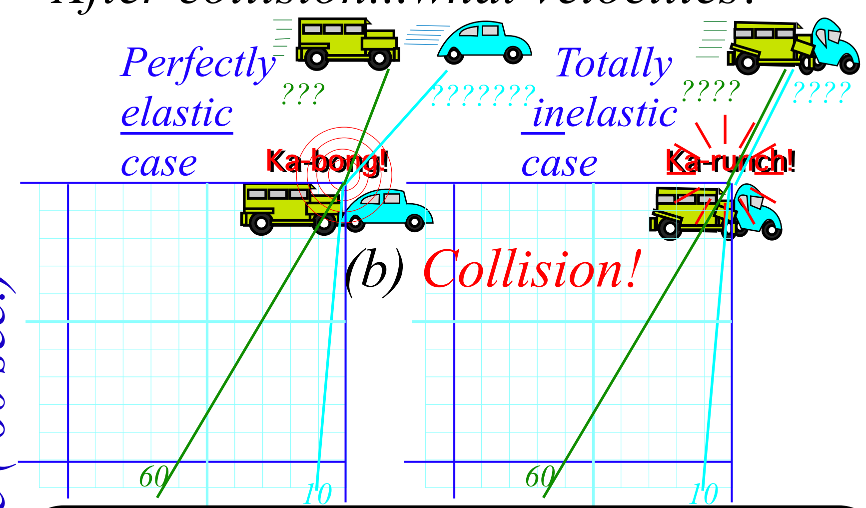

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It's a simple *Cartesian* equation

$$4 \cdot x + 1 \cdot y = 250$$

Rene Descartes
1596-1650

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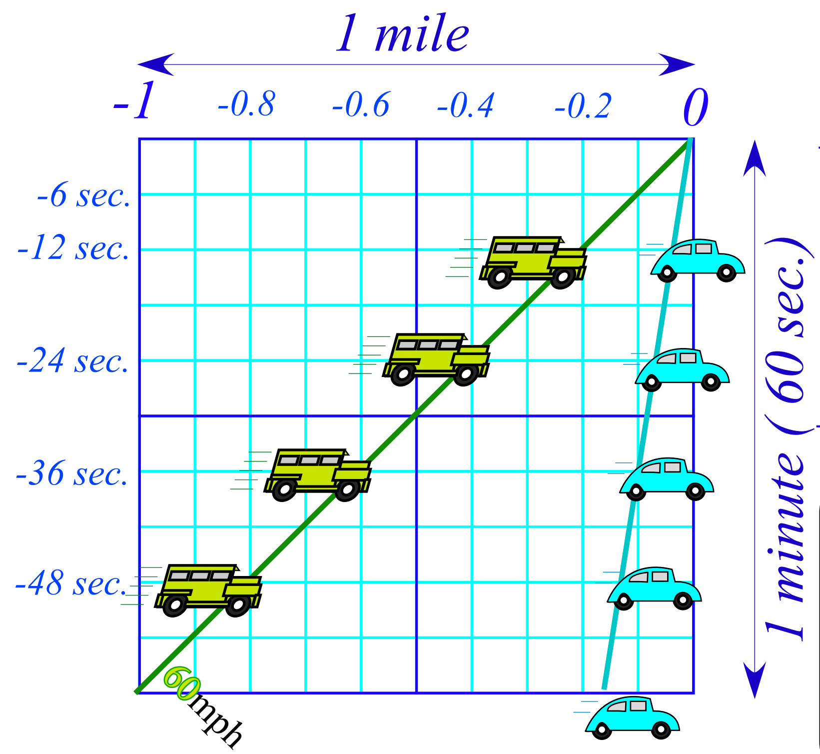

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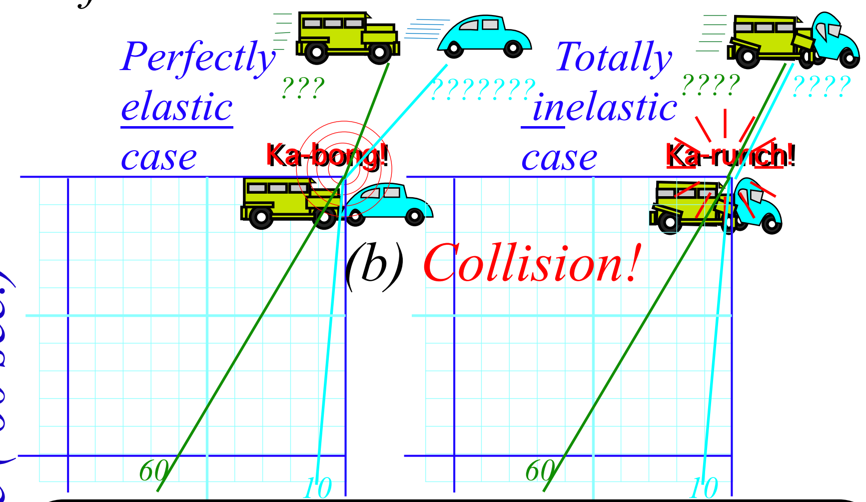

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$$4 \cdot x + 1 \cdot y = 250$$
$$y = 250 - 4 \cdot x$$

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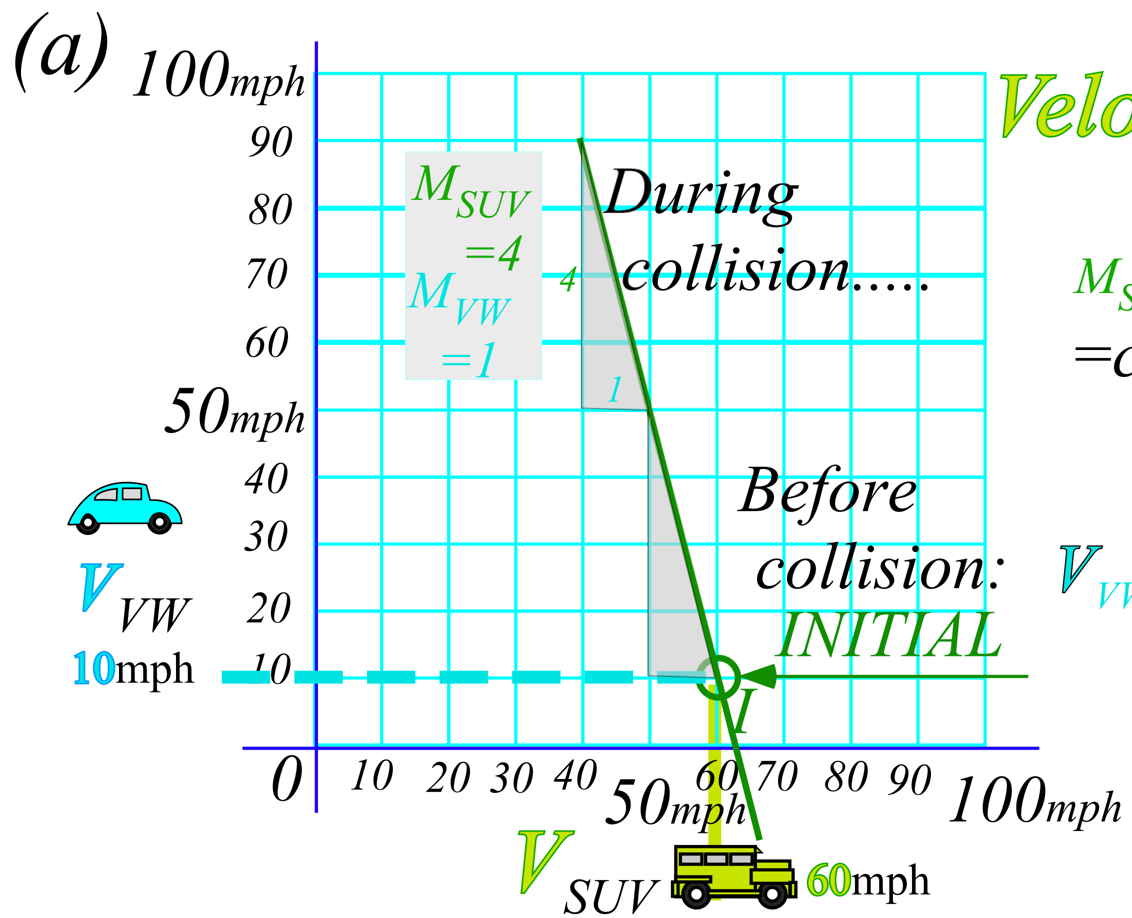
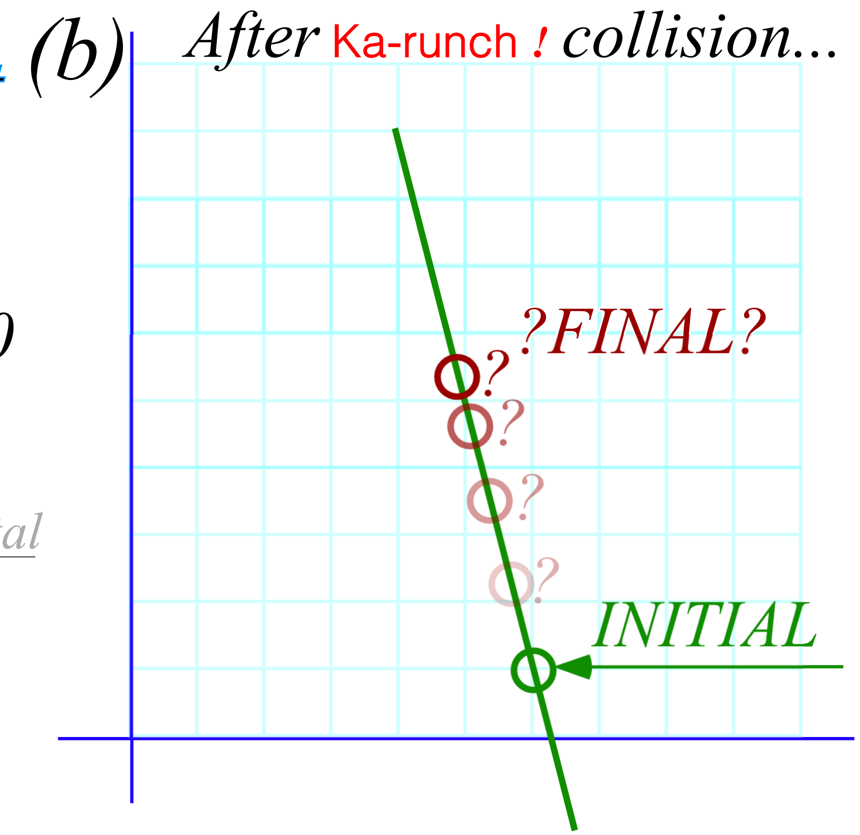


Velocity-velocity plot of Axiom-1:

$$M_{SUV}V_{SUV} + M_{VW}V_{VW}$$
$$=constant = P_{Total} = 250$$

$$V_{VW} = -\frac{M_{SUV}}{M_{VW}} V_{SUV} + \frac{P_{Total}}{M_{VW}}$$
$$= -\ 4\ V_{SUV} + 250$$



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$$4 \cdot x + 1 \cdot y = 250$$
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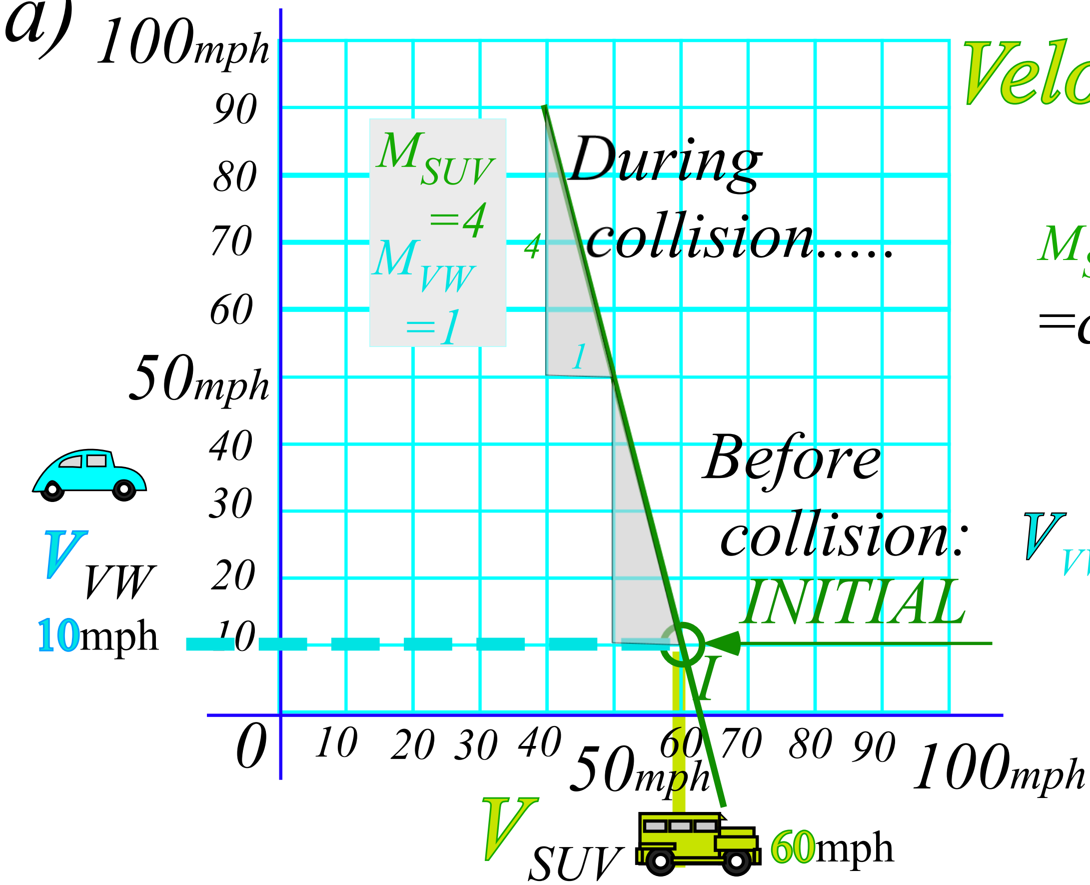
Geometry of momentum conservation axiom

→ *Totally Inelastic "ka-runch"collisions (begin 4:1 graph project)*
Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry
Comments on idealization in classical models

(a)



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$$= \text{constant} = P_{Total} = 250$$

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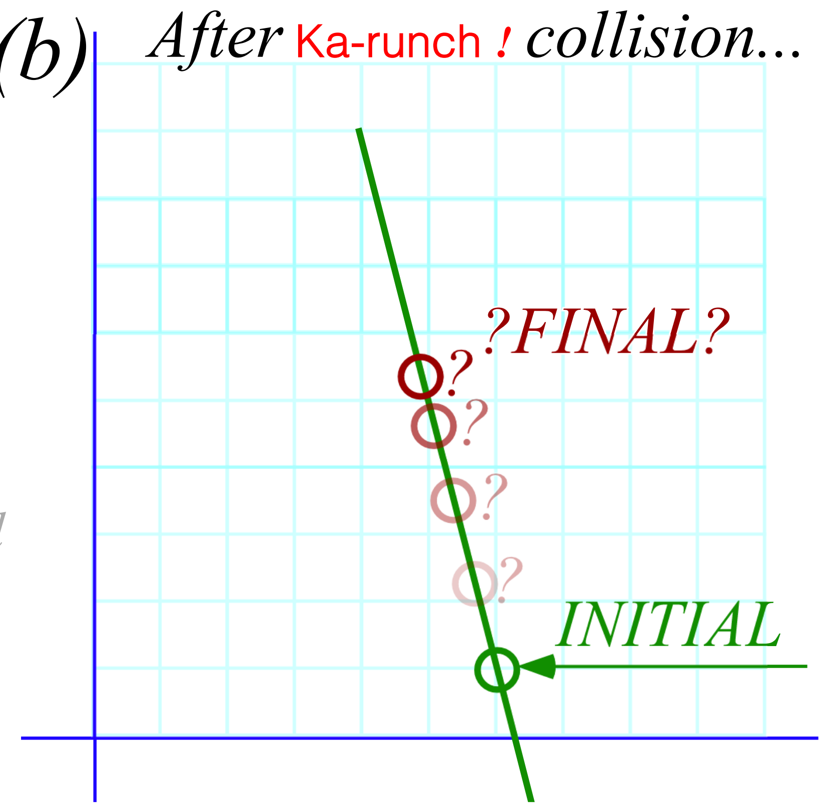
$$= -\ 4\ V_{SUV} + 250$$

(b)



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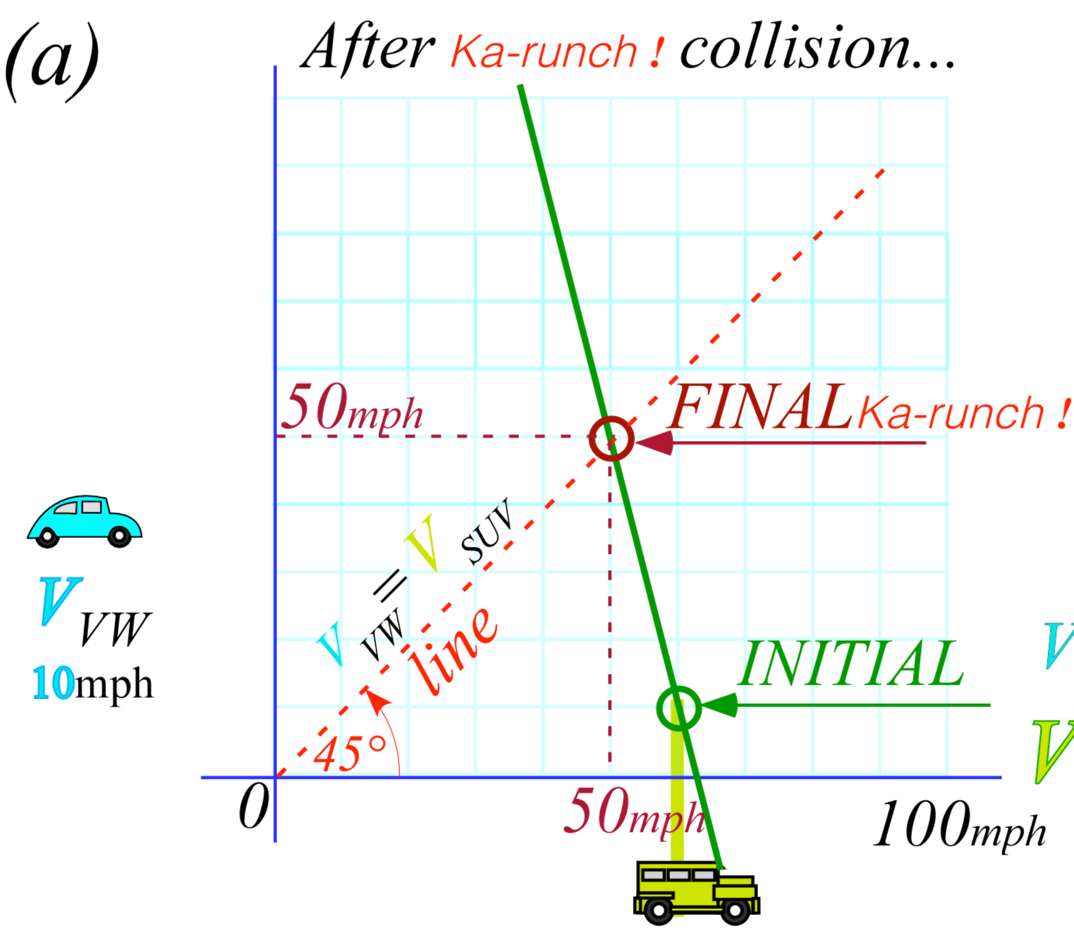

$V_{VW}^{INITIAL}$ =10mph

$V_{SUV}^{INITIAL}$ =60mph

It's a simple *Cartesian* equation...

$$4 \cdot x + 1 \cdot y = 250$$

$$y = 250 - 4 \cdot x$$

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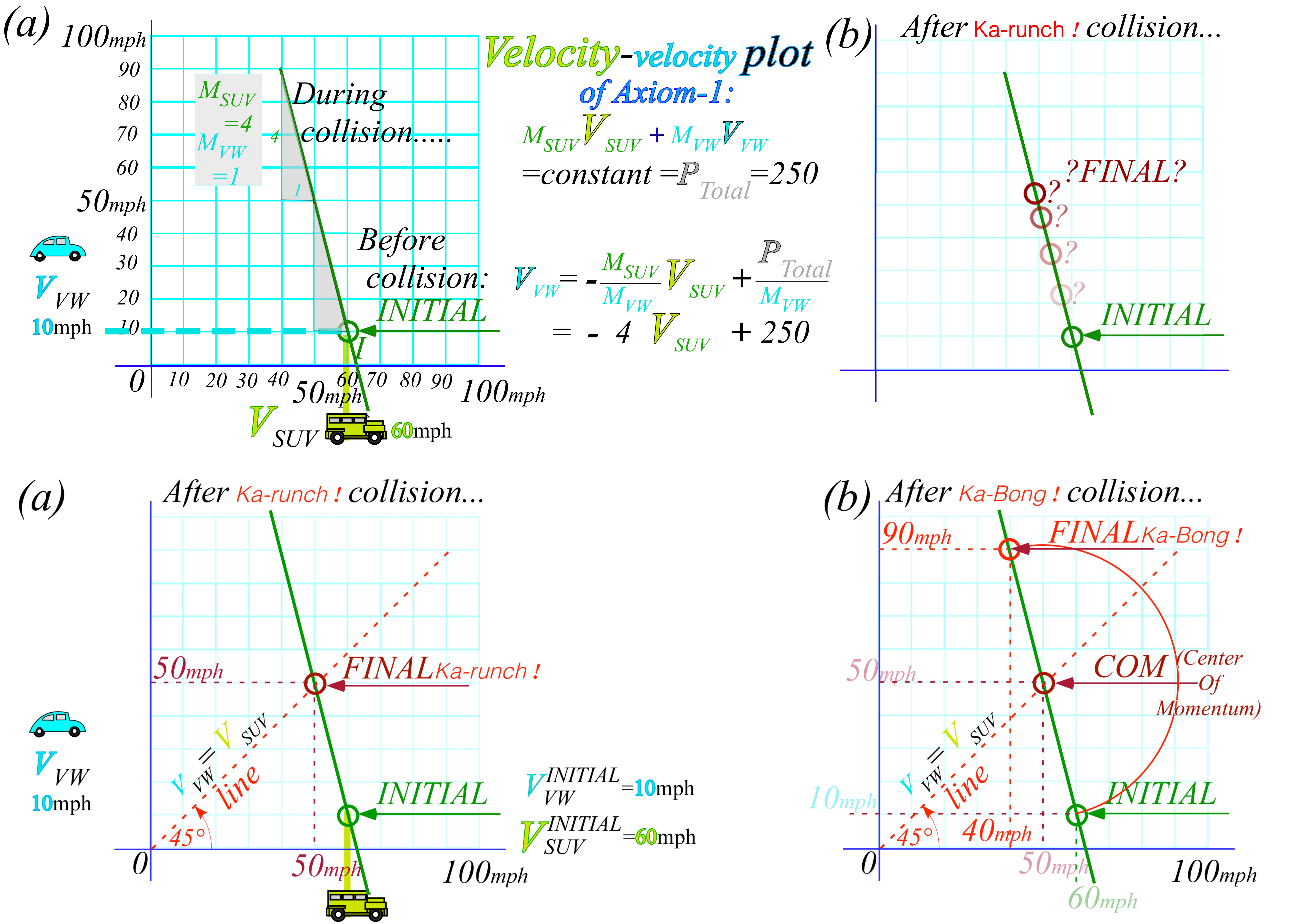

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$= -4\,V_{SUV} + 250$
(a)
100mph
50mph
0
V_{VW}
10mph
V_{SUV}
60mph
M_{SUV} =4
M_{VW} =1
During collision.....
Before collision:
INITIAL
(b) After Ka-runch ! collision...
?FINAL?
INITIAL
(a) After Ka-runch ! collision...
FINAL Ka-runch !
50mph
$V_{VW} = V_{SUV}$ line
45°
INITIAL
$V_{VW}^{INITIAL}$ =10mph
$V_{SUV}^{INITIAL}$ =60mph
100mph
(b) After Ka-Bong ! collision...
90mph
FINAL Ka-Bong !
COM (Center Of Momentum)
50mph
10mph
40mph
60mph
100mph
INITIAL

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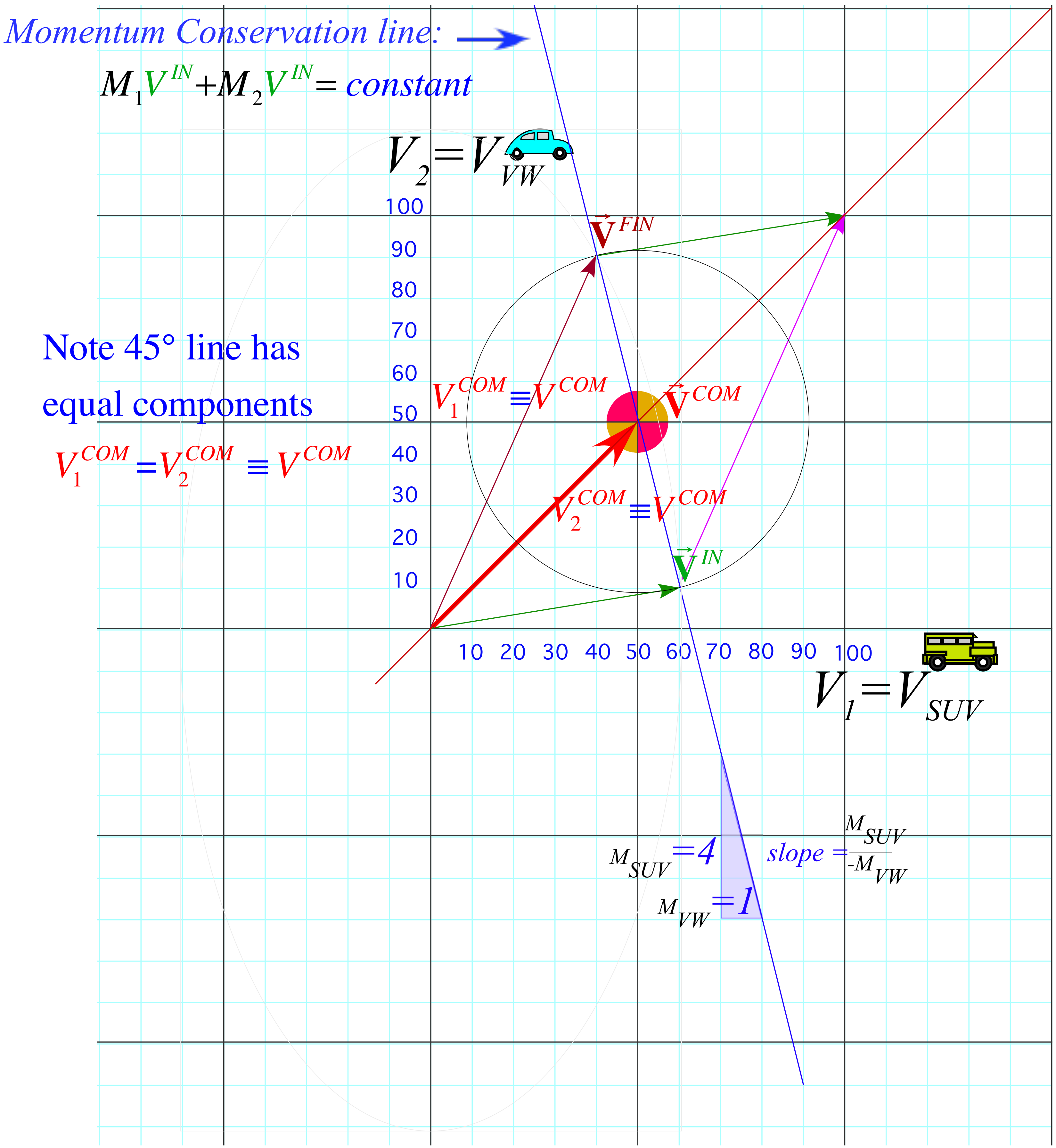
Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1V_1^{IN}+M_2V_2^{IN} = M_1V_1^{COM}+M_2V_2^{COM} = M_1V_1^{FIN}+M_2V_2^{FIN} = M_{Total}V^{COM}$$



Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{COM} + M_2 V_2^{COM} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_{Total} V^{COM}$$

Momentum Conservation line: →

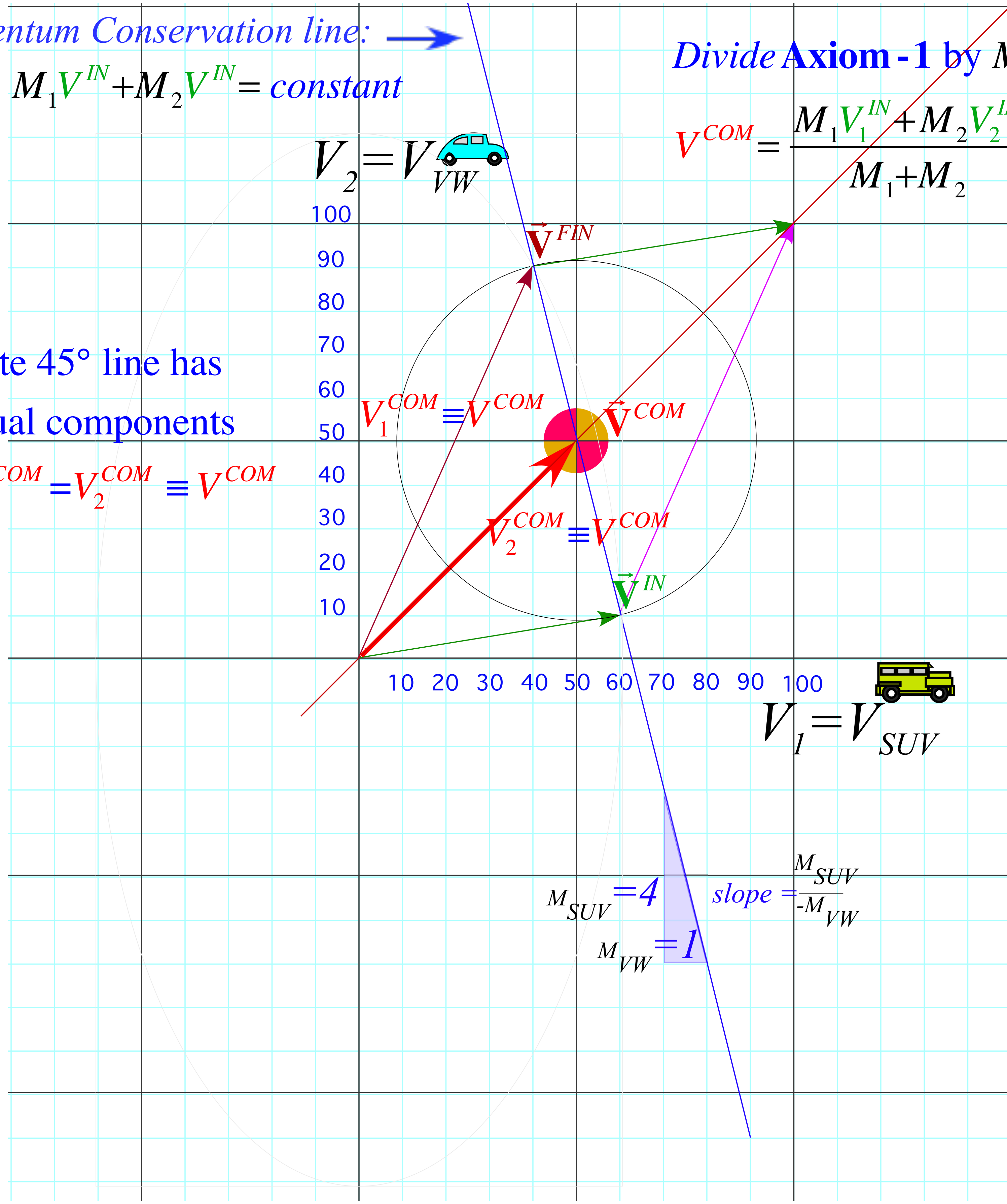
$$M_1 V^{IN} + M_2 V^{IN} = \textit{constant}$$

Divide **Axiom - 1** by $M_{Total} = (M_1+M_2)$

$$V^{COM} = \frac{M_1 V_1^{IN} + M_2 V_2^{IN}}{M_1+M_2} = \frac{M_1 V_1^{COM} + M_2 V_2^{COM}}{M_1+M_2} = \frac{M_1 V_1^{FIN} + M_2 V_2^{FIN}}{M_1+M_2} = 50$$

Note 45° line has equal components

$$V_1^{COM} = V_2^{COM} \equiv V^{COM}$$



Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM} = M_1V_1^{IN}+M_2V_2^{IN} = M_1V_1^{COM}+M_2V_2^{COM} = M_1V_1^{FIN}+M_2V_2^{FIN} = M_{Total}V^{COM}$$

Momentum Conservation line: →

$M_1V^{IN}+M_2V^{IN}=$ *constant*

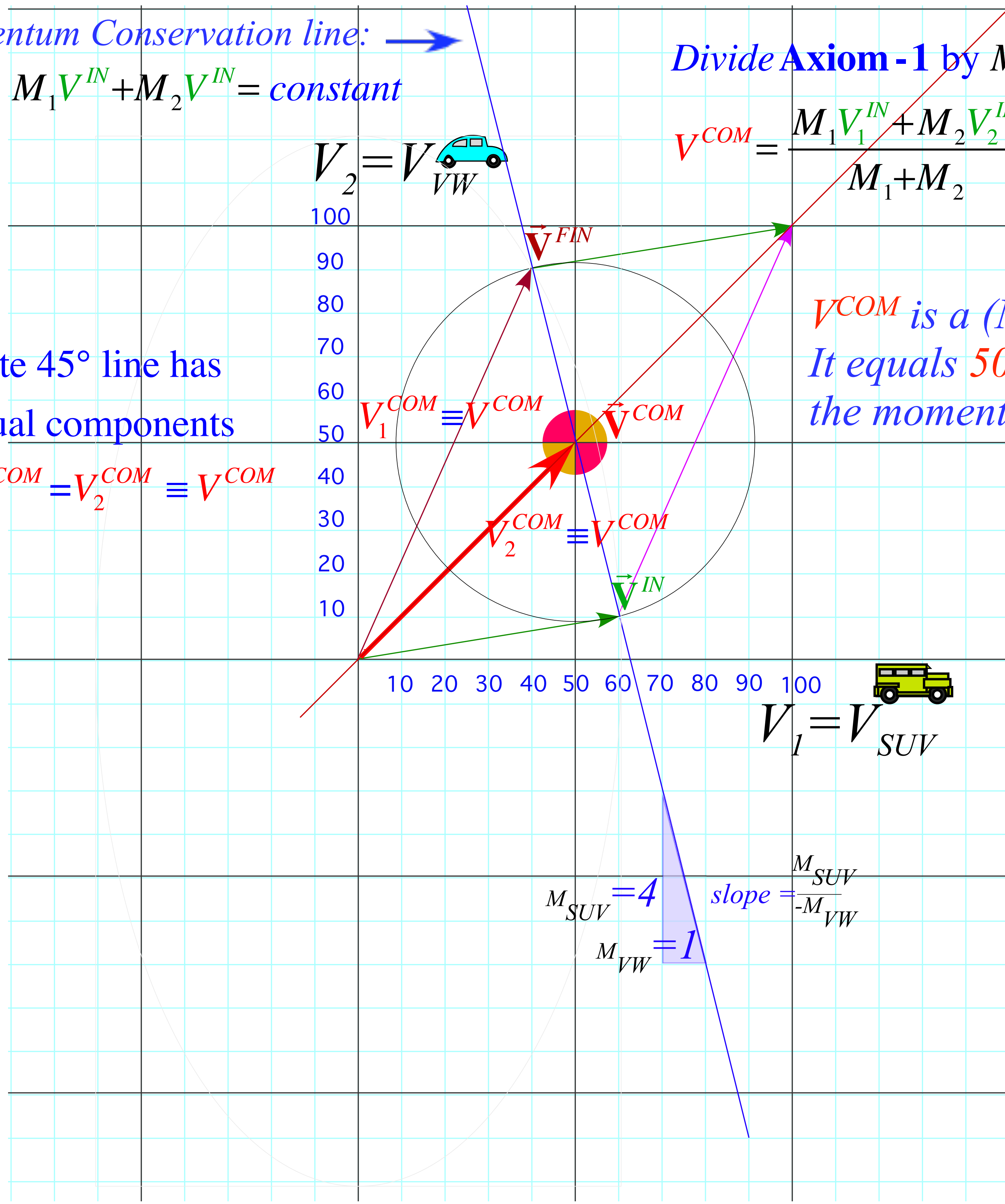
Divide **Axiom - 1** by $M_{Total}=(M_1+M_2)$

$$V^{COM}=\frac{M_1V_1^{IN}+M_2V_2^{IN}}{M_1+M_2}=\frac{M_1V_1^{COM}+M_2V_2^{COM}}{M_1+M_2}=\frac{M_1V_1^{FIN}+M_2V_2^{FIN}}{M_1+M_2}=50$$

V^{COM} is a (M_1,M_2) ***Weighted Average*** *of V_1 and V_2*
It equals 50 for every point (V_1,V_2) on the momentum line

Note 45° line has equal components

$V_1^{COM}=V_2^{COM}\equiv V^{COM}$



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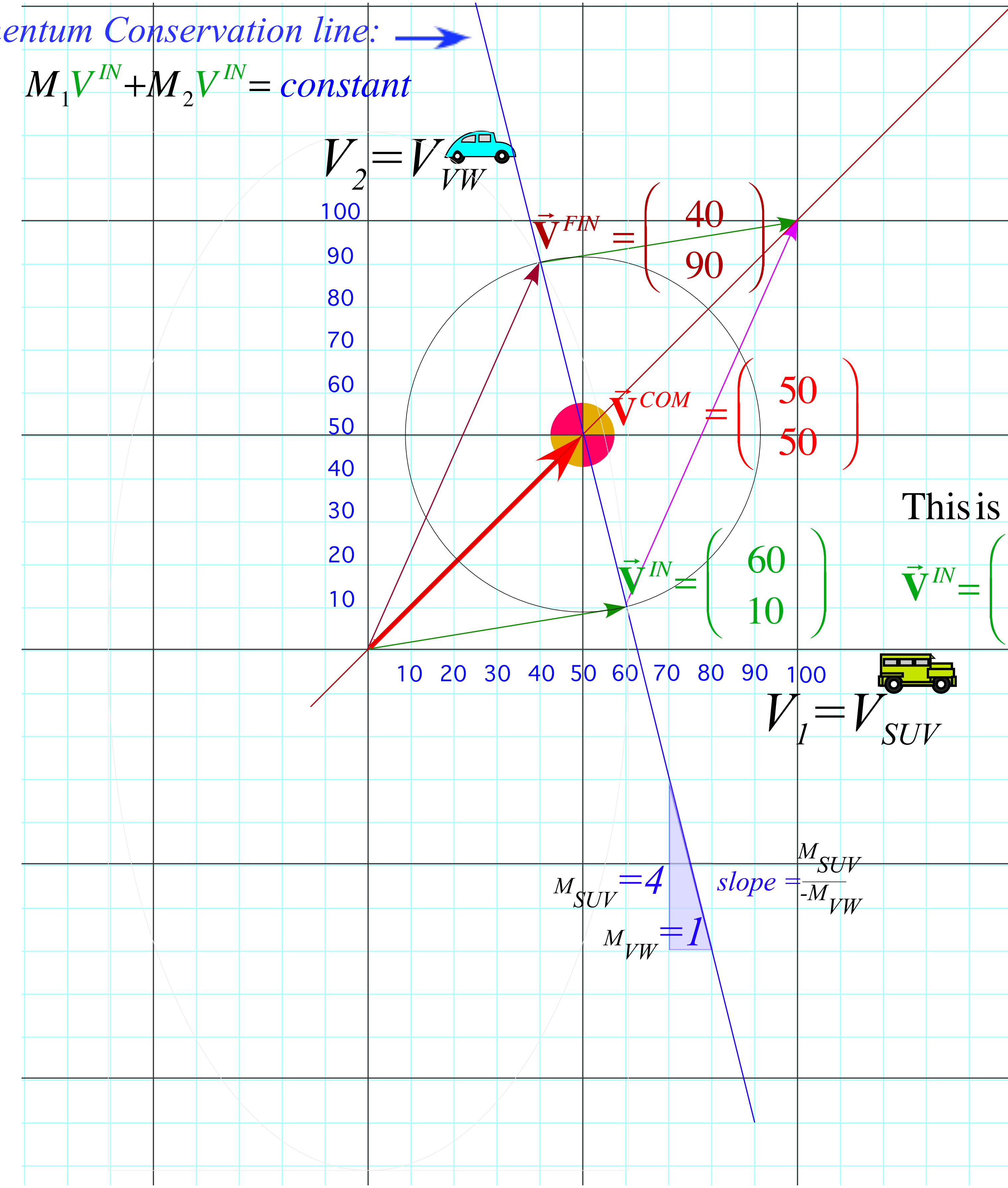
Momentum Conservation line: →

$M_1V^{IN}+M_2V^{IN}=$ *constant*



This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

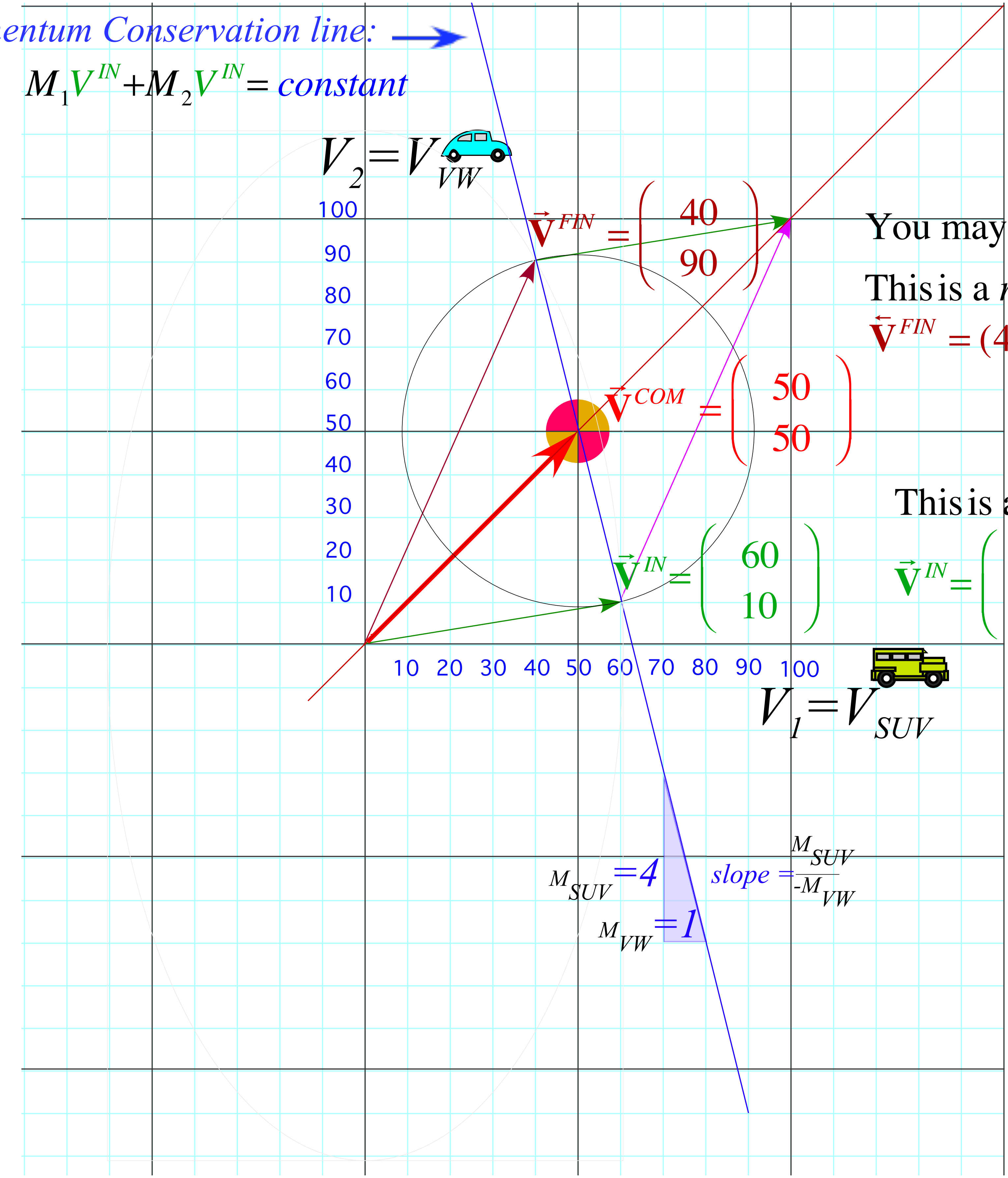
$$\vec{V}^{IN}=\begin{pmatrix}60\\10\end{pmatrix}$$

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Momentum Conservation line: →

$$M_1V^{IN}+M_2V^{IN}=\text{constant}$$



You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

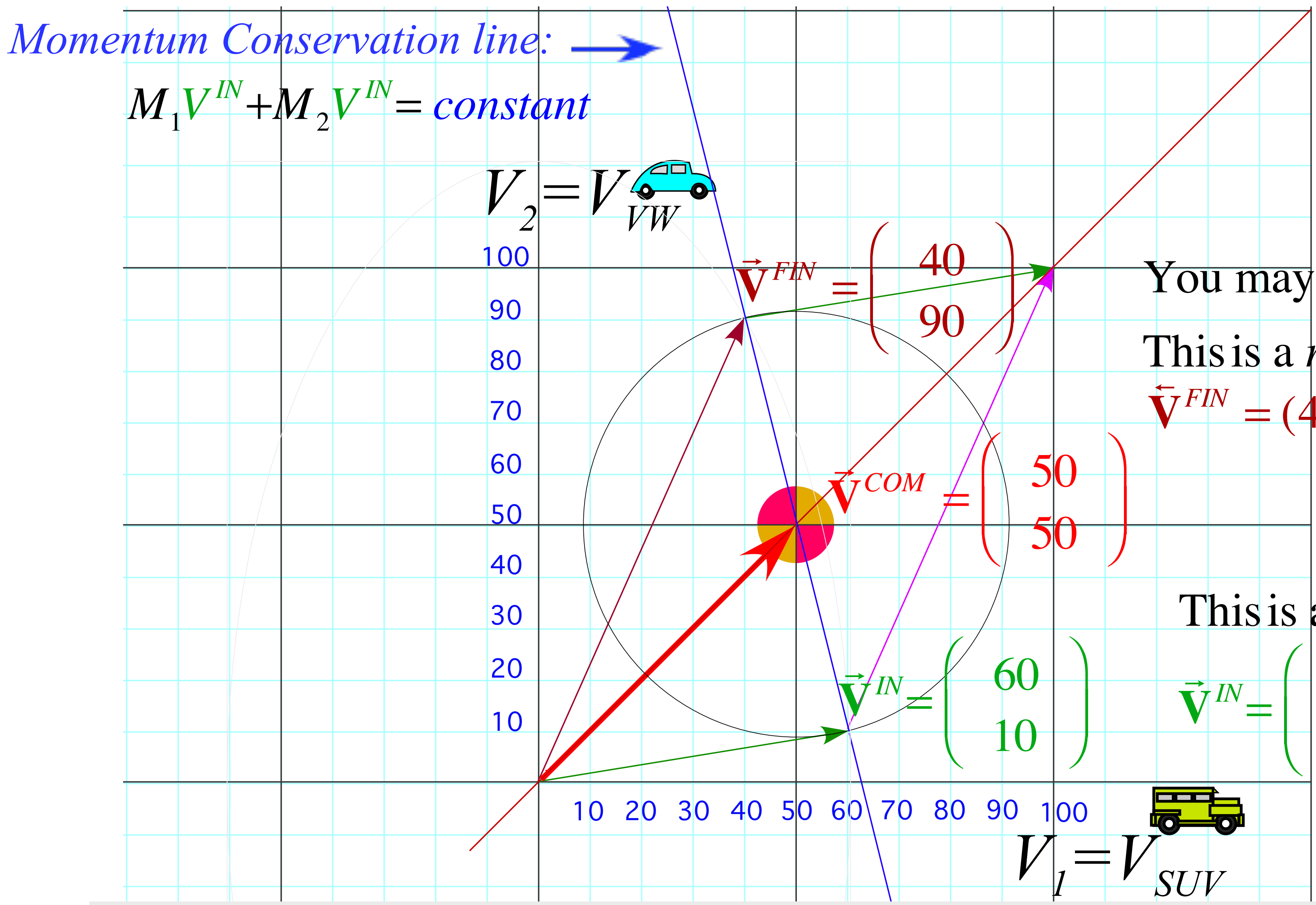
$$\vec{V}^{FIN}=(40,90)$$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$$\vec{V}^{IN}=\begin{pmatrix}60\\10\end{pmatrix}$$

Geometry of Momentum Conservation **Axiom - 1**

$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$



You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

$\vec{V}^{FIN}=(40,90)$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$\vec{V}^{IN}=\begin{pmatrix}60\\10\end{pmatrix}$

This is a *dot* product (or *scalar* product)

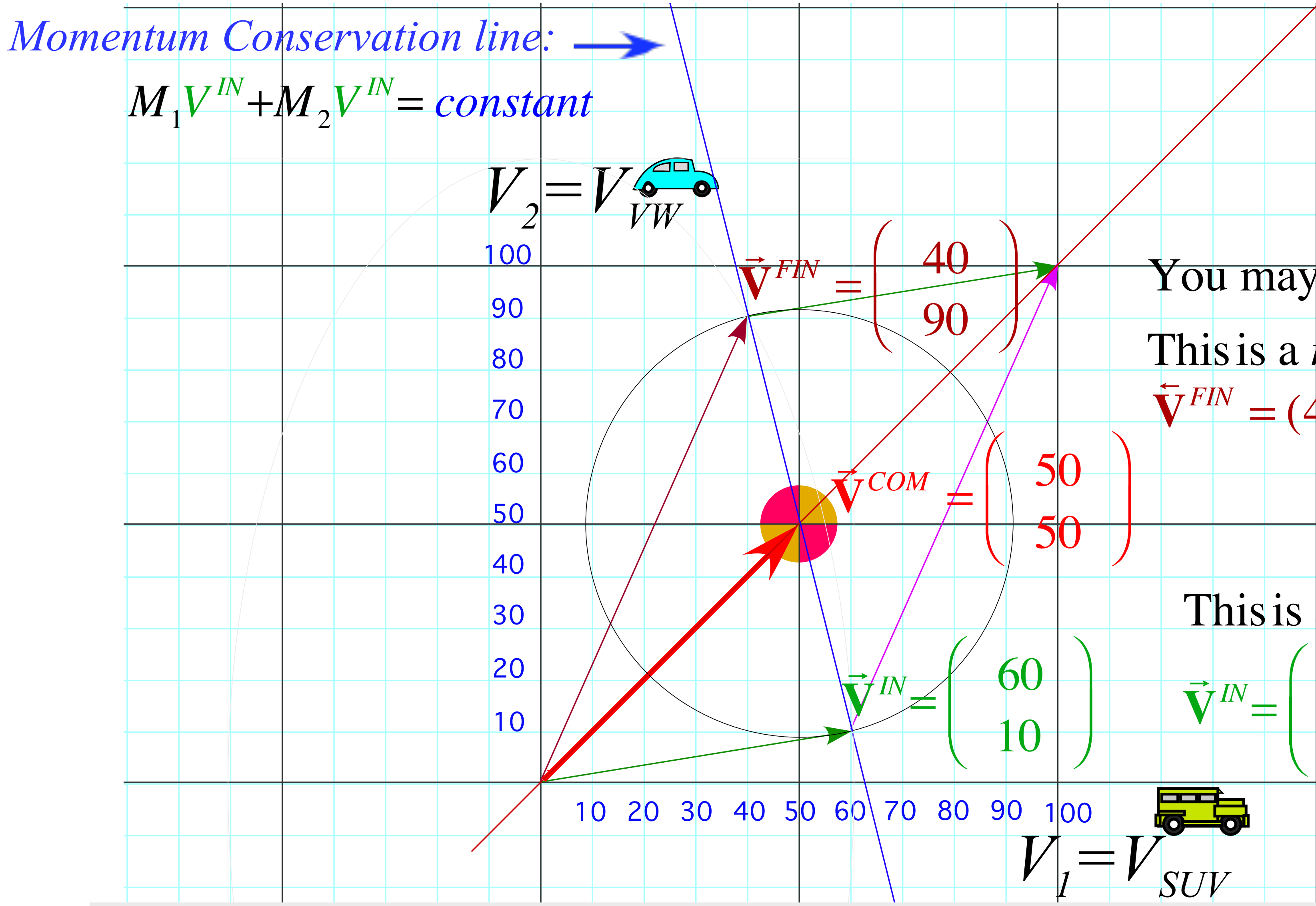
$$\vec{V}^{FIN}\bullet\vec{V}^{IN}=(40,90)\bullet\begin{pmatrix}60\\10\end{pmatrix}=\langle FIN|IN\rangle=40\ 60+90\ 10=2400+900=3300$$

of a *row-vector* $\vec{V}^{FIN}=(40,90)$ (or *bra* $\langle FIN|$)

with *column-vector* $=\begin{pmatrix}60\\10\end{pmatrix}$ (or *ket* $|IN\rangle$)

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$



You may be used to giving coordinates as (x,y).

This is a *row-vector* (or *bra* $\langle FIN|$ in QM)

$\overleftarrow{\mathbf{V}}^{FIN}=(40,90)$

This is a *column-vector* (or *ket* $|IN\rangle$ in QM)

$$\vec{\mathbf{V}}^{IN}=\begin{pmatrix}60\\10\end{pmatrix}$$

This is a *outer* product (or *tensor* product)

$$\vec{\mathbf{V}}^{IN}\otimes\overleftarrow{\mathbf{V}}^{FIN}=\begin{pmatrix}60\\10\end{pmatrix}\otimes(40,90)=|IN\rangle\langle FIN|=\begin{pmatrix}60\;40 & 60\;90\\10\;40 & 10\;90\end{pmatrix}=\begin{pmatrix}2400 & 5400\\400 & 900\end{pmatrix}$$

of a *column-vector* $=\begin{pmatrix}60\\10\end{pmatrix}$ (or *ket* $|IN\rangle$)

with a *row-vector* $\overleftarrow{\mathbf{V}}^{FIN}=(40,90)$ (or *bra* $\langle FIN|$)

Geometry of momentum conservation axiom

Totally Inelastic "ka-runch"collisions

Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry

+Intro to weighty-averages and vector notation

Comments on idealization in classical models

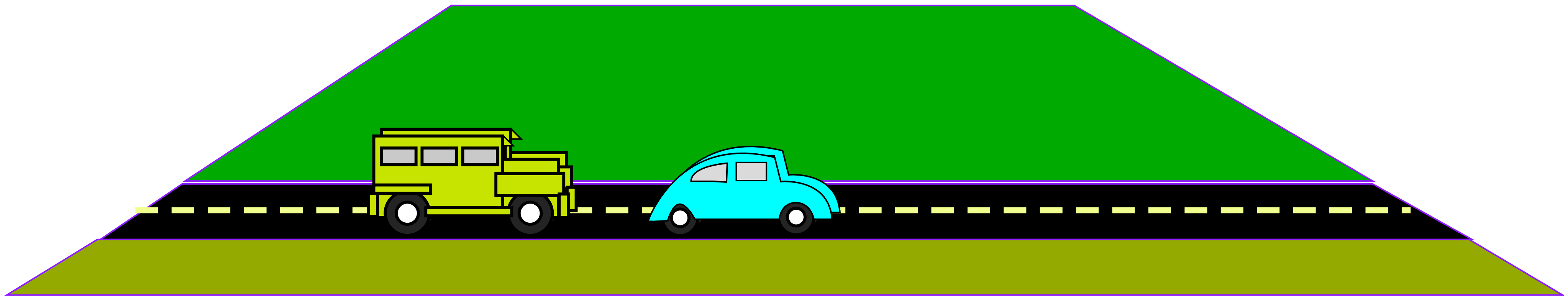
Geometry of momentum conservation axiom

*Totally Inelastic "ka-runch"collisions**

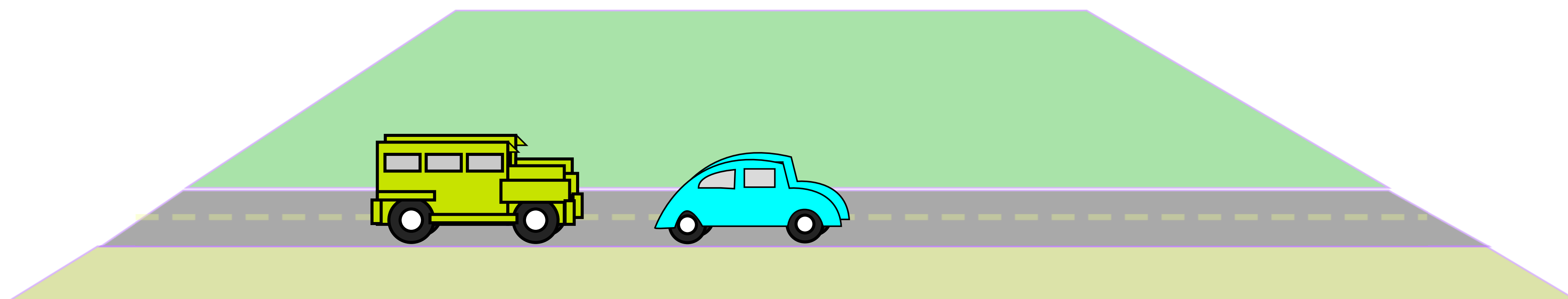
*Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**

→ *Comments on idealization in classical models*

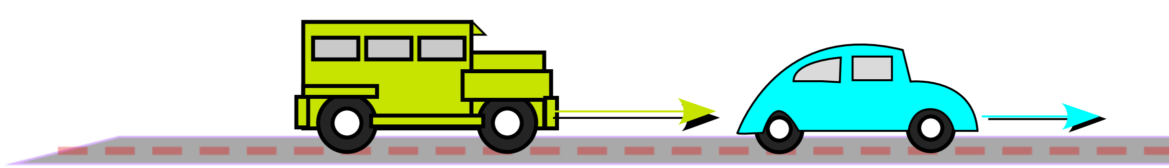
The SUV and VW *Idealized* thought experiments



Idealization 1. Ignore background.
(No rolling friction, air resistance, etc.)



Idealization 2. Make each 1-dimensional.
(Cars "constrained" to ride on frictionless rail)

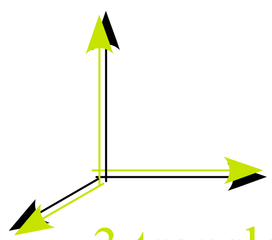


System now has
just *two* "dimensions"
or "*degrees-of-freedom*"

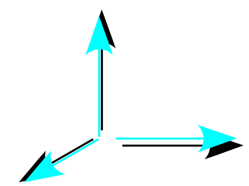
Landscape 1.1 Idealized model for collision model and thought experiments

Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)



3 translation
dimensions

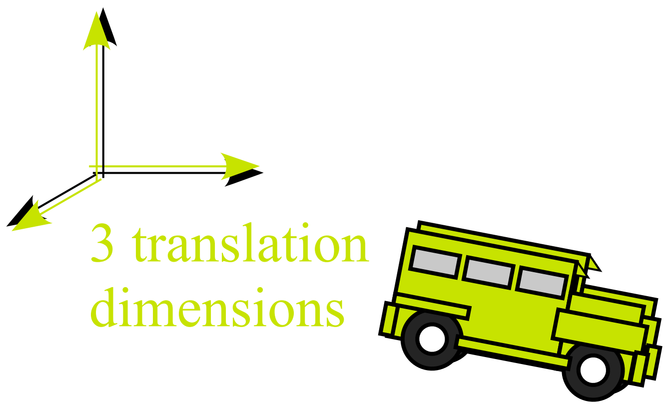


3 translation
dimensions

6 translational degrees of freedom for SUV and VW.

Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)



3 translation
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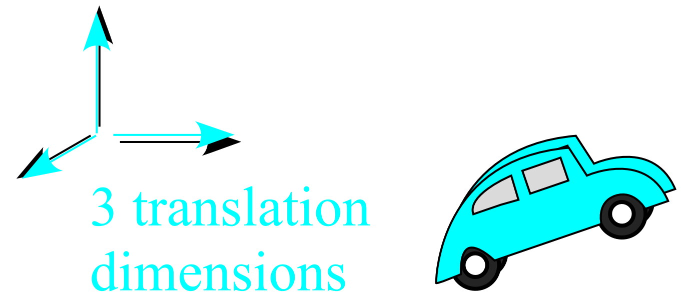
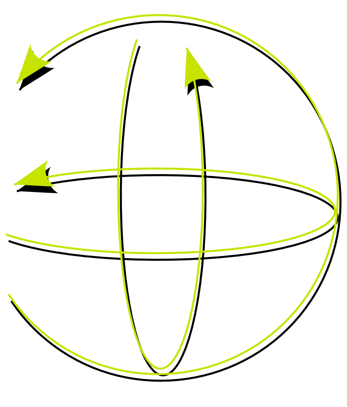


3 translation
dimensions

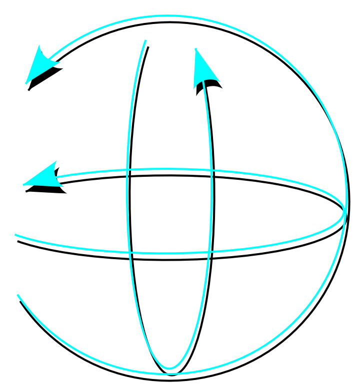
*6 translational
degrees of freedom
for SUV and VW.*

Rotation (Each body has 3 rotationaldegrees of freedom) (Intoduced in Units 3 and 7)



3 rotational
dimensions

*yaw-pitch-roll
Euler angles*



3 rotational
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*yaw-pitch-roll
Euler angles*

*6 rotational
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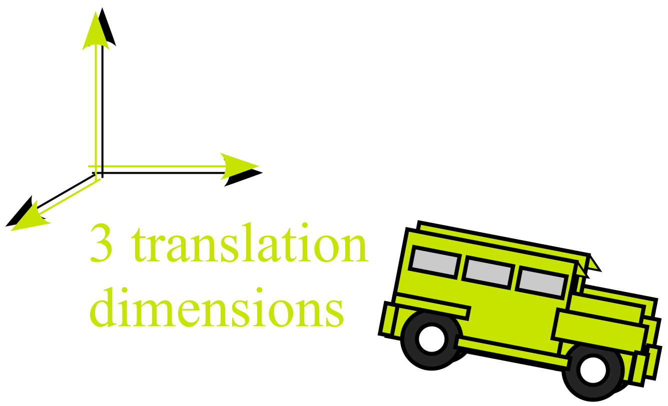
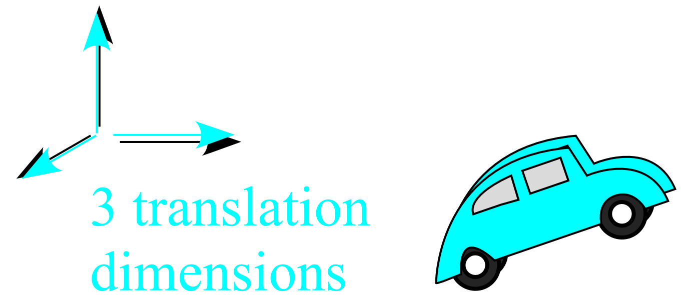


3 translation dimensions



3 translation dimensions

6 translational degrees of freedom for SUV and VW.

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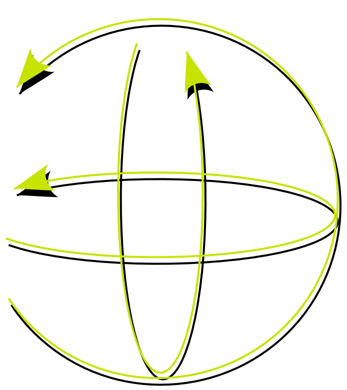
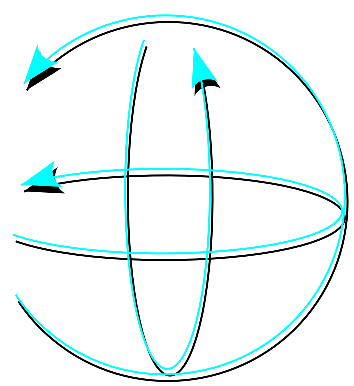


3 rotational dimensions

yaw-pitch-roll Euler angles



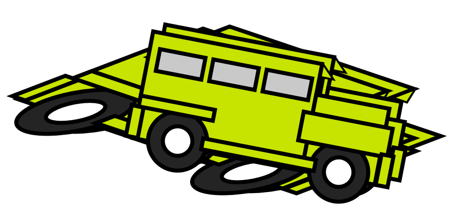
3 rotational dimensions

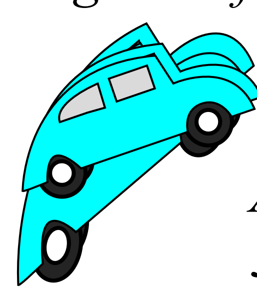
yaw-pitch-roll Euler angles

6 rotational degrees of freedom for SUV and VW.

SUV and VW system involves
12 rigid-body degrees of freedom

Vibration (Each body has <u>many</u> vibrational degrees of freedom) (Intoduced in Units 3-8)





Generalized Curvilinear Coordinates (GCC) introduced in Unit 1 Chapters 10 -12

An N-atom molecule has 3N-6 vibrational degrees of freedom

Landscape 1.2 Some idealized classical model degrees of freedom

Geometry of Galilean translation symmetry

→ *45° shift in (V_1,V_2)-space*

Time reversal symmetry

...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,... *(In some direction x,y, or z...)*

*...the rest of the world appears to be 50 mph **slower*** *(In that direction...)*

(a) Galileo transforms to COM frame

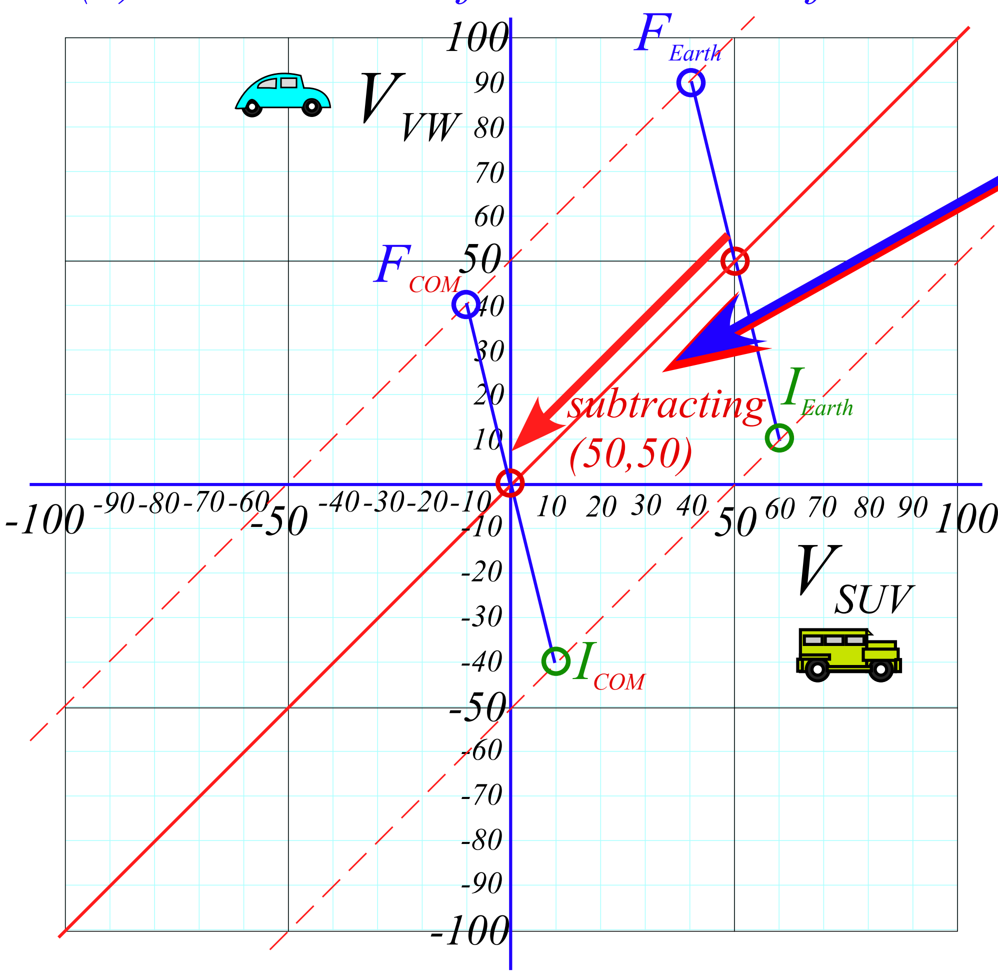
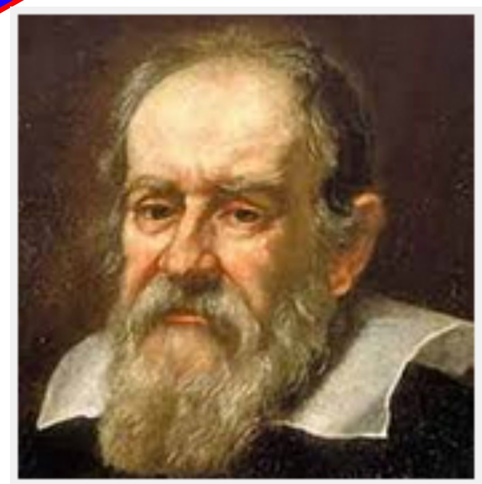


Fig.1.4a



Galileo Galilei
1564-1642

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

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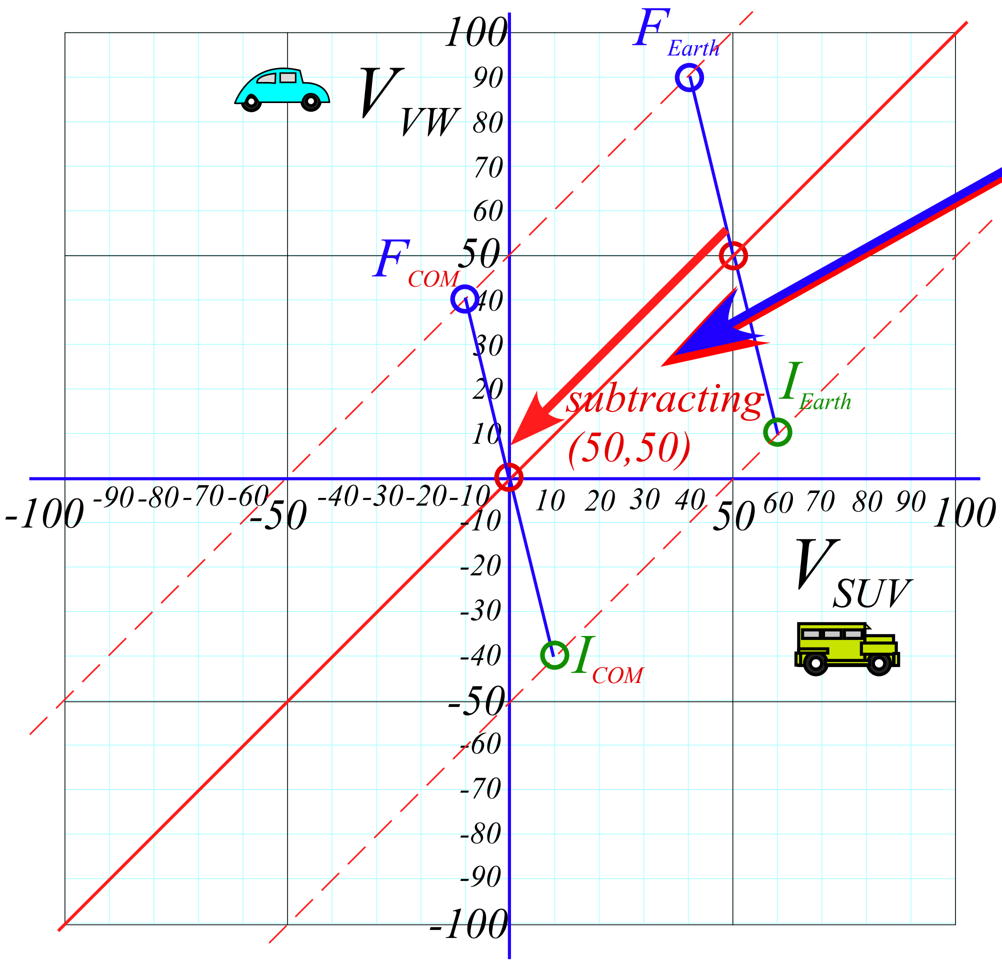

Fig.1.4a

(b) ... and to five or six other reference frames

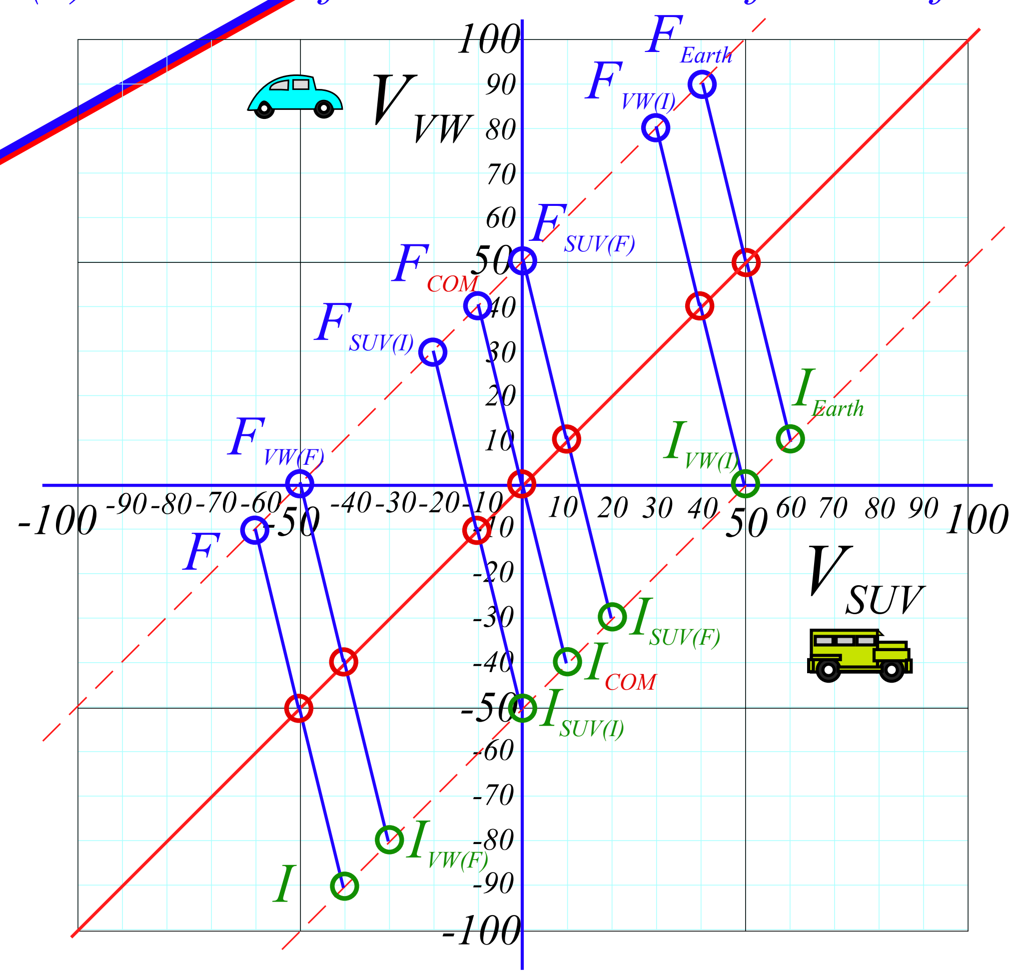

Fig.1.4b

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

→ *Time reversal symmetry*

...of COM collisions

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Final F and Initial I trade places...

Time-reversal (F-I) symmetry pairs (Four examples)

(a) Galileo transforms to COM frame

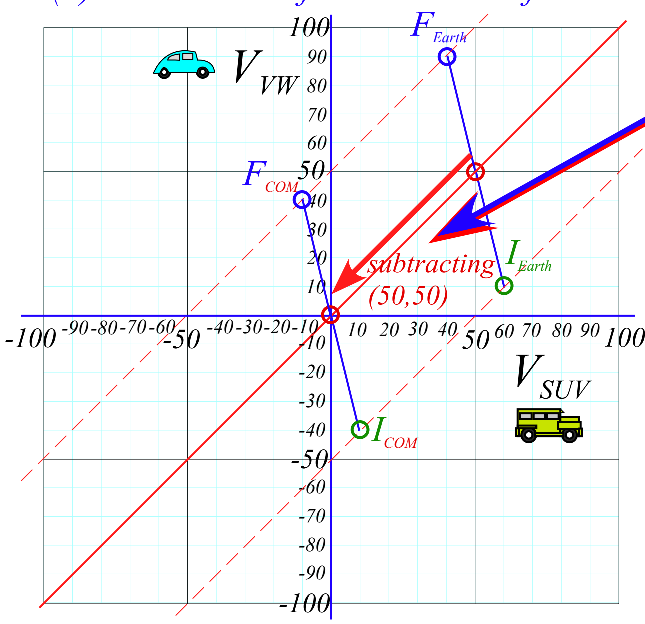


Fig.1.4a

(b) ... and to five or six other reference frames

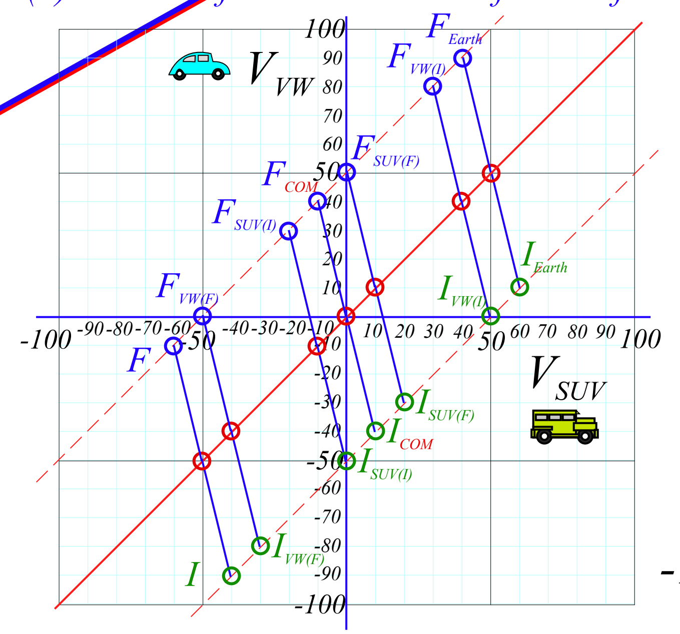
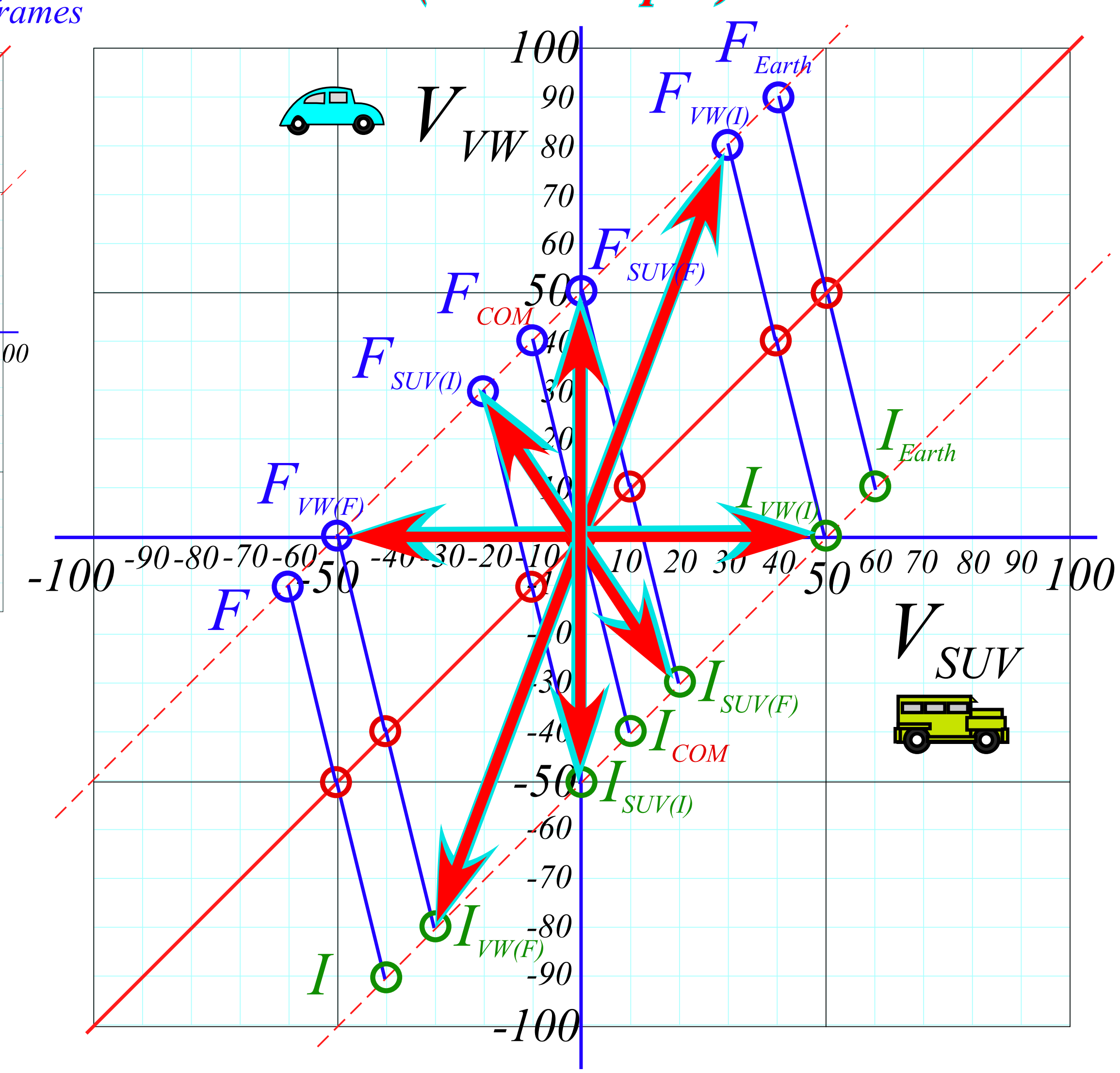


Fig.1.4b



Time-reversal means flip t to -t...
(Run a movie backwards)

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

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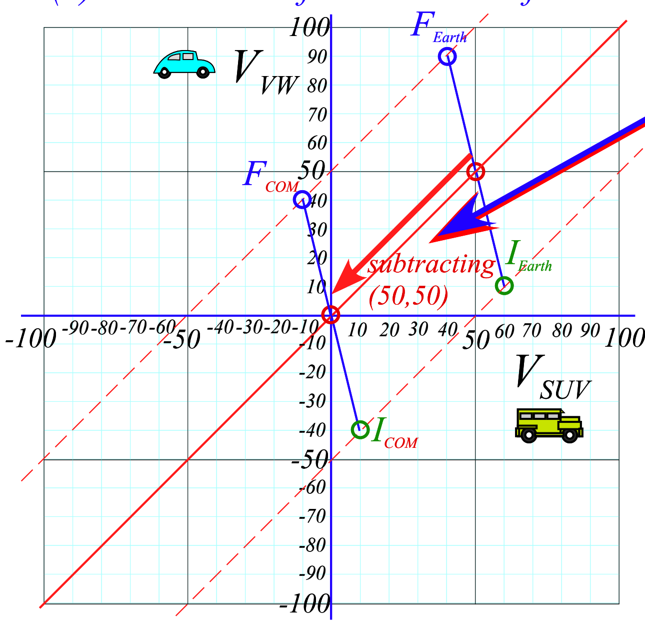


Fig.1.4a

(b) ... and to five or six other reference frames

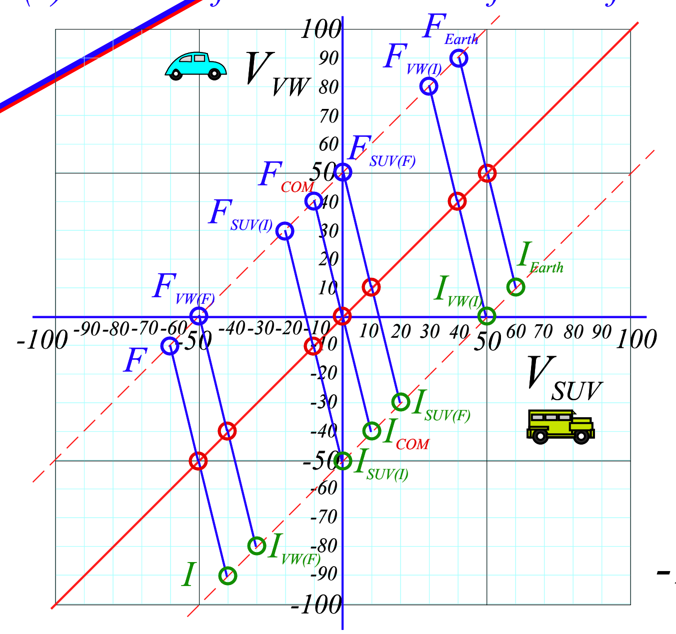
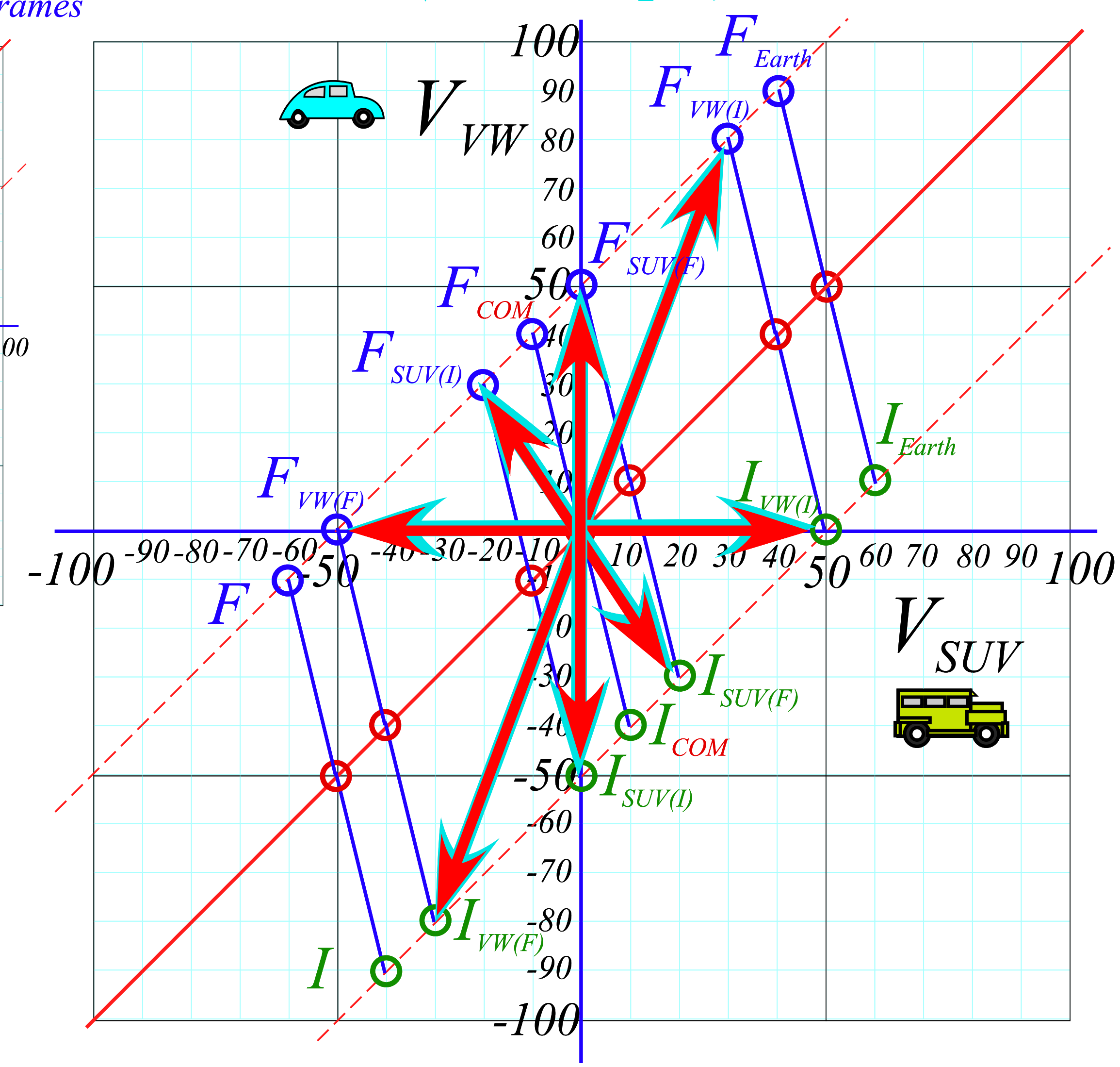


Fig.1.4b

Final F and Initial I trade places...

Time-reversal (F-I) symmetry pairs (Four examples)



Time-reversal means flip t to -t...
(Run a movie backwards)

That means you flip Velocity V to -V...
(Everything goes backwards)

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

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→ *...of COM collisions*

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If you increase your velocity by 50 mph,...

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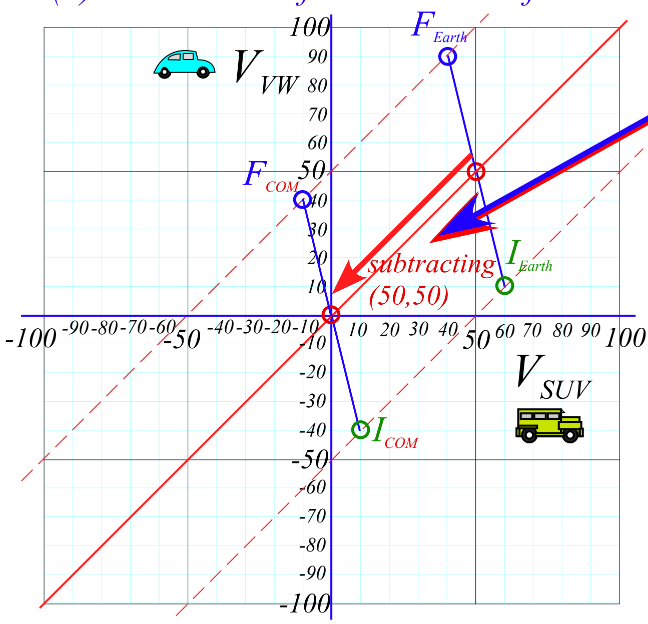
THE

COM Time-reversal symmetry pair (Just 1 case)

(a) Galileo transforms to COM frame



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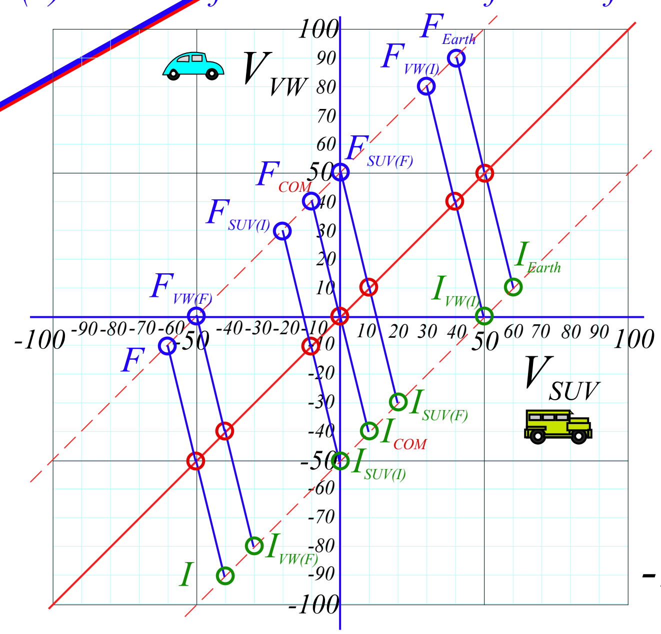

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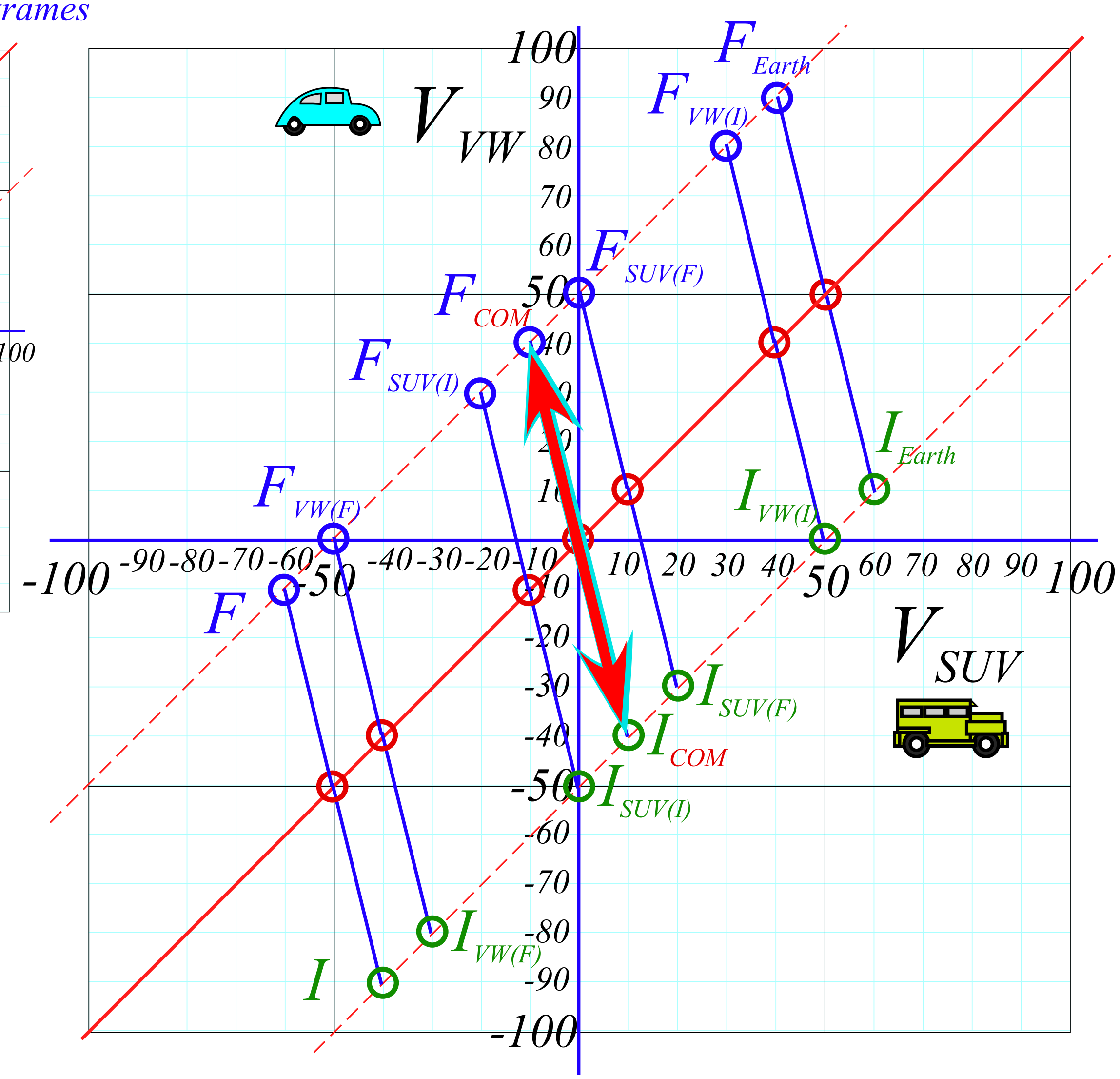

There is just one velocity frame in which the time-reversed collision looks just like the original collision

*That is the **Center-of-Momentum (COM)**-frame*

Time-reversal means flip t to -t...
(Run a movie backwards)

That means you flip Velocity V to -V...
(Everything goes backwards)

Algebra,Geometry, and Physics of momentum conservation axiom

→ *Vector algebra of collisions*
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
Energy Ellipse geometry

Quick lesson on
Gibb's notation for
dot (•) product of matrix operator **M** and column vector $\mathbf{V}^{IN}$:

$$\overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \bullet \begin{pmatrix} x^{IN} \\ y^{IN} \end{pmatrix}$$

$$= \begin{pmatrix} Ax^{IN} + By^{IN} \\ Cx^{IN} + Dy^{IN} \end{pmatrix}$$

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Quick lesson on
Dirac notation is
much simpler:

$$M|IN\rangle$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \langle x|IN\rangle \\ \langle y|IN\rangle \end{pmatrix}$$

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$$= \begin{pmatrix} Ax^{IN} + By^{IN} \\ Cx^{IN} + Dy^{IN} \end{pmatrix}$$

Quick lesson on
Dirac notation is
much simpler:

$$M|IN\rangle \quad \text{(...at first!)}$$

$$\begin{pmatrix} \langle x|M|x\rangle & \langle x|M|y\rangle \\ \langle y|M|x\rangle & \langle y|M|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|IN\rangle \\ \langle y|IN\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle x|M|x\rangle\langle x|IN\rangle + \langle x|M|y\rangle\langle y|IN\rangle \\ \langle y|M|x\rangle\langle x|IN\rangle + \langle y|M|y\rangle\langle y|IN\rangle \end{pmatrix}$$

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General **Inertia Tensor M** or inertia matrix of *2* coefficients $M_{11}=M_1$ and $M_{22}=M_2$ for 2 dimensions

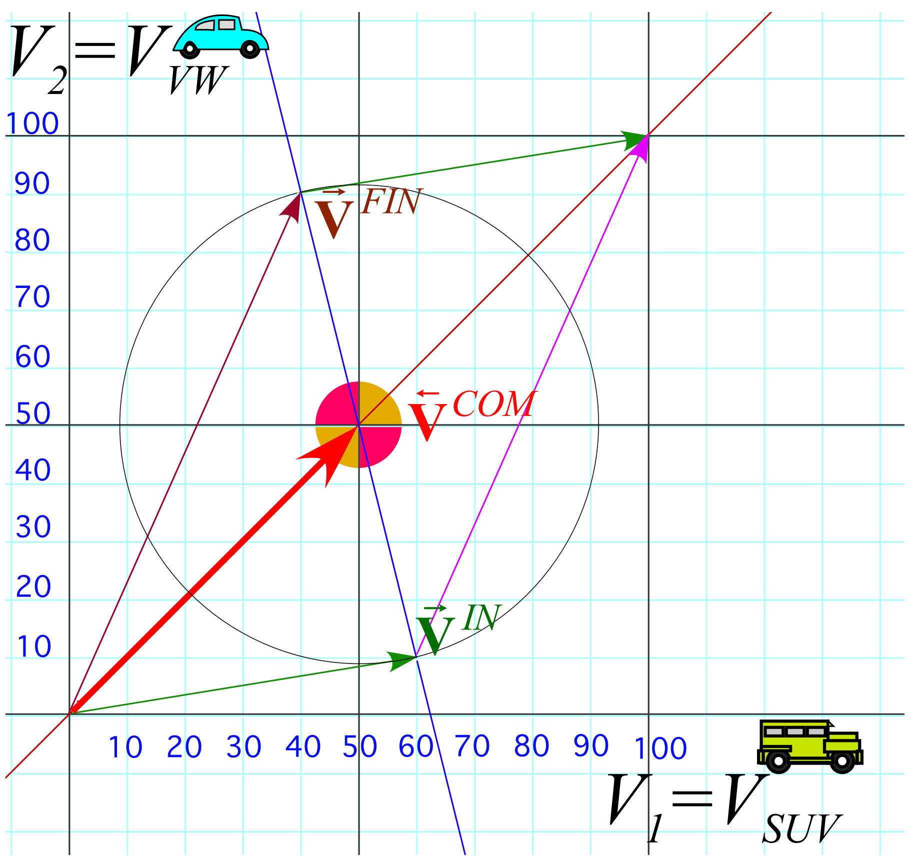
$$\left.\begin{aligned} P_1 &= M_1V_1 + M_{12}V_2 \\ P_2 &= M_{21}V_1 + M_2V_2 \end{aligned}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Generalizing the definition of *momentum*...

With 45° diagonal $\mathbf{V}^{COM}$ so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$

...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN} = M_1V^{COM} + M_2V^{COM} = (M_1+M_2)V^{COM} = M_{Total}V^{COM}$$



General Inertia Tensor M or inertia matrix of *3* coefficients M_{11}, M_{22} and M_{12}=M_{21} for 2 dimensions

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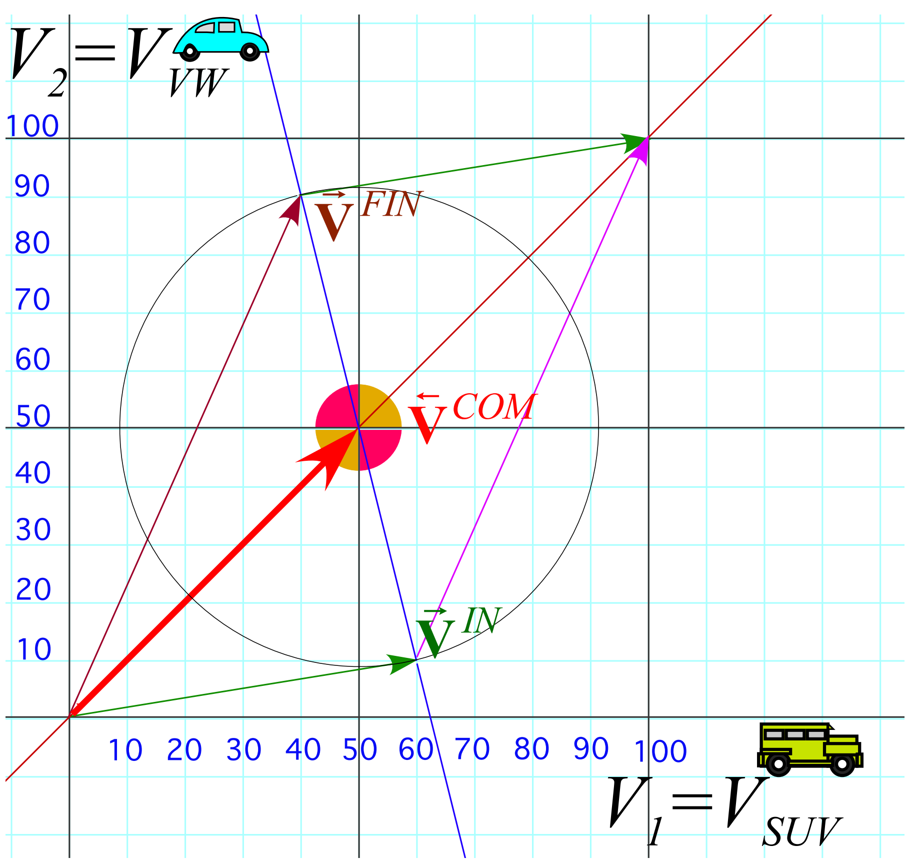
Generalizing the definition of *momentum*...some more...

With 45° diagonal $\mathbf{V}^{COM}$ so: . $V_1^{COM} = V_2^{COM} \equiv V^{COM}$

...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1V_1^{IN} + M_2V_2^{IN} = M_1V_1^{FIN} + M_2V_2^{FIN} = M_1V^{COM} + M_2V^{COM} = (M_1+M_2)V^{COM} = M_{Total}V^{COM}$$



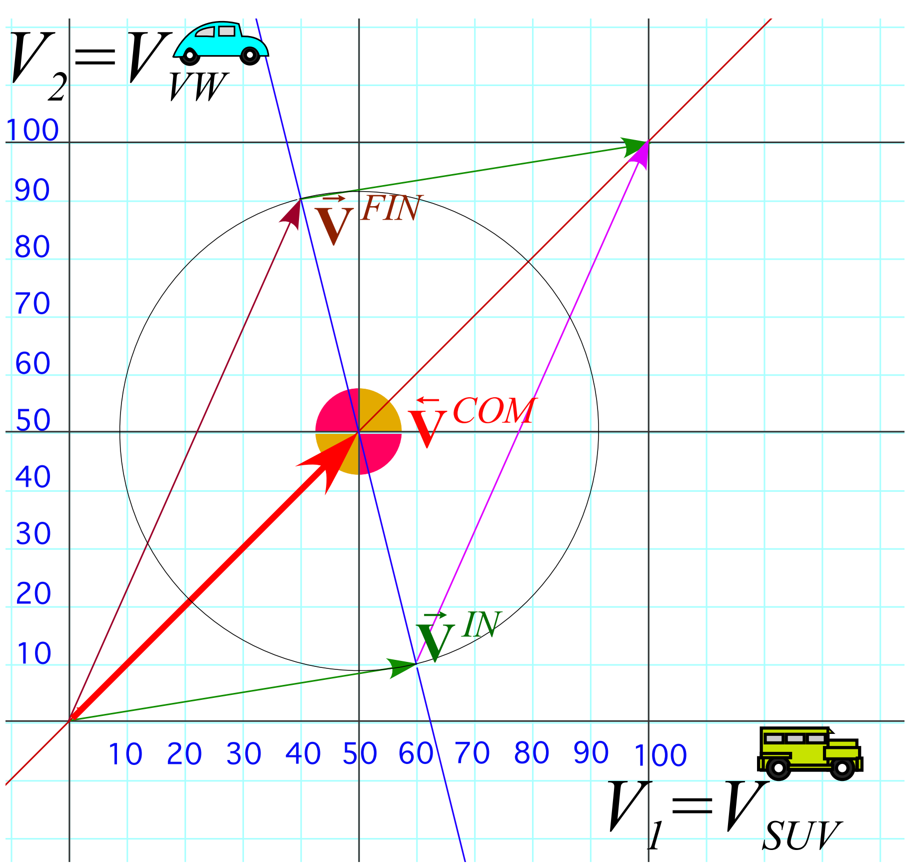
General Inertia Tensor M or inertia matrix of $(n^2+n)/2$ coefficients $M_{jk} = M_{kj}$ for dimension $n=2, 3, \ldots$

$$\left.\begin{array}{l} P_1 = M_{11}V_1 + M_{12}V_2 \cdots \\ P_2 = M_{21}V_1 + M_{22}V_2 \cdots \\ \vdots = \quad \vdots \qquad \vdots \;\; \ddots \end{array}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & \cdots \\ M_{21} & M_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \end{pmatrix}$$

Generalizing the definition of *momentum*...some more...and more

With 45°diagonal $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$ so: $V_1^{COM} = V_2^{COM} \equiv V^{COM}$ ...and 3 ways to write *Axiom-1*

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = (M_1 + M_2) V^{COM} = M_{Total} V^{COM}$$



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Generalizing the definition of *momentum*...

With 45°diagonal $\vec{\mathbf{V}}^{COM}=(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN})/2$ so: $V_1^{COM}=V_2^{COM}\equiv V^{COM}$ ...and 4 or 5 ways to write *Axiom-1*

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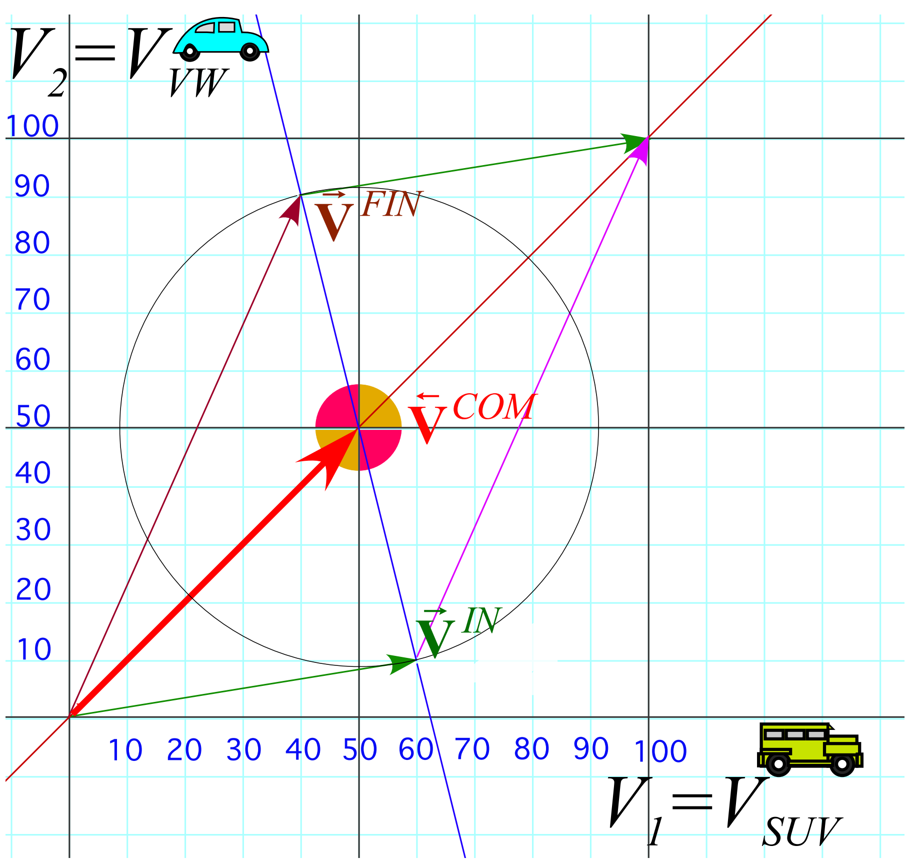

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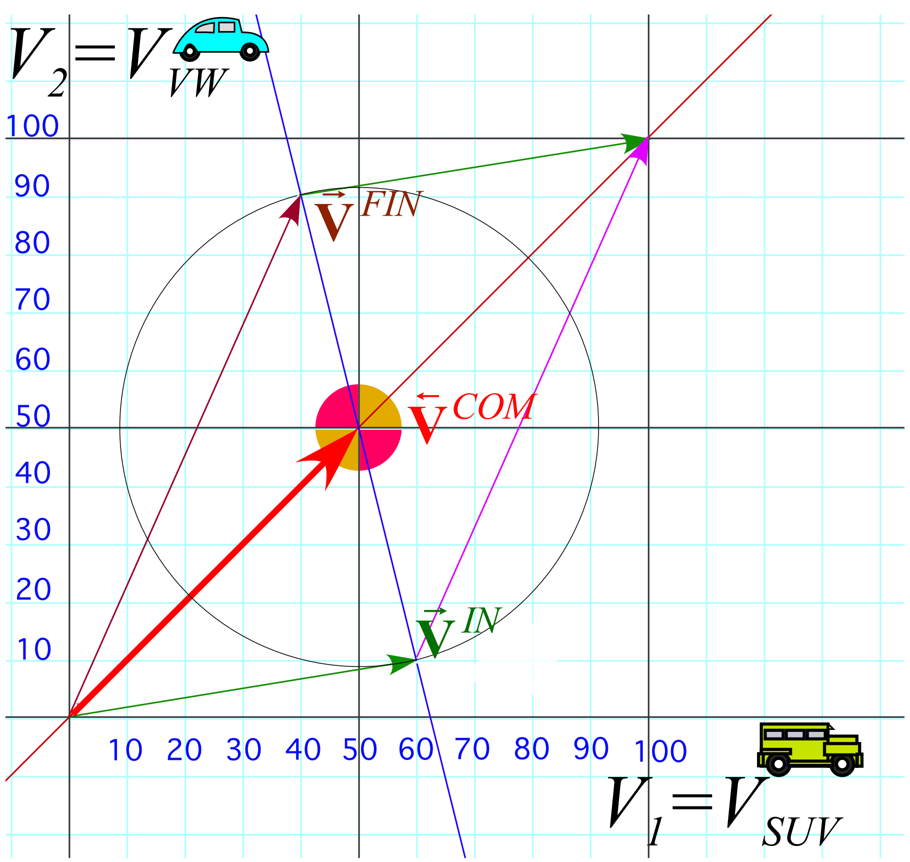

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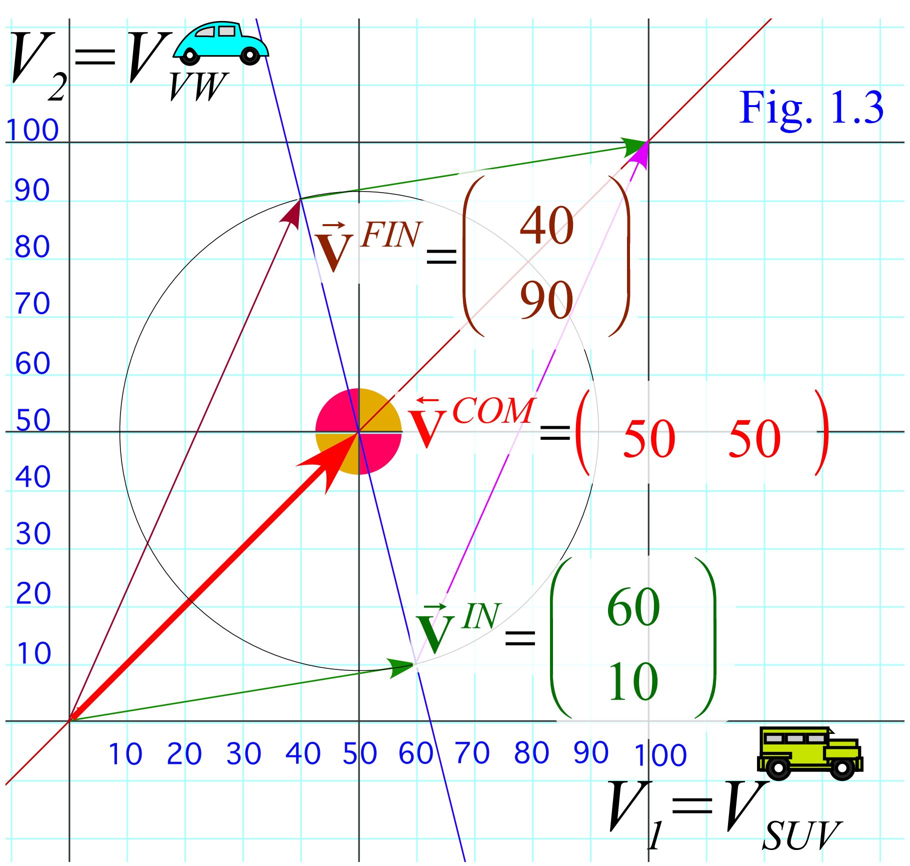

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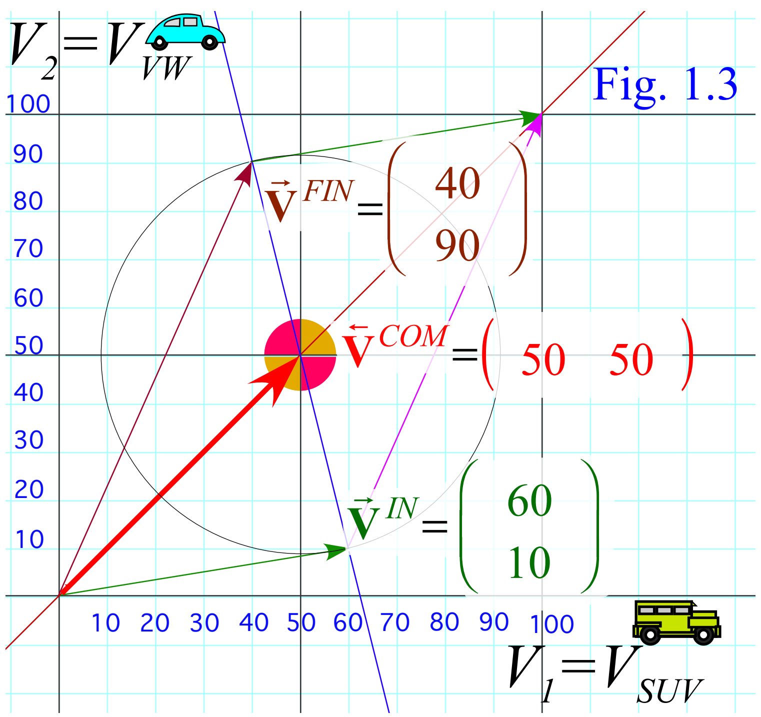

Fig. 1.3

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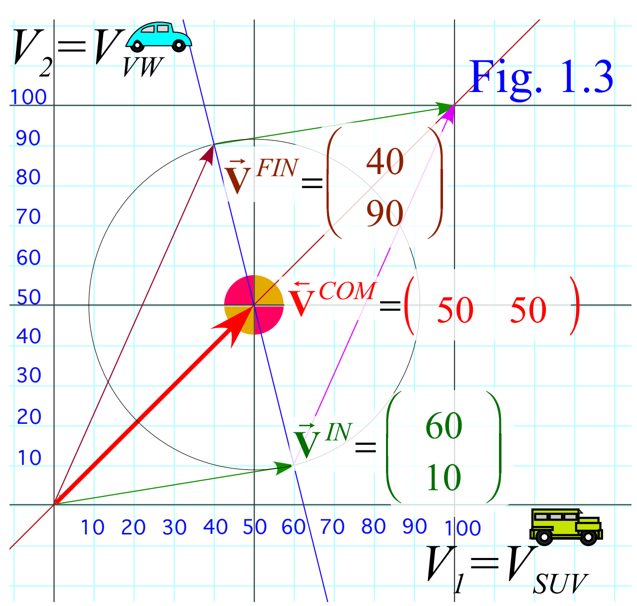
$$50 \cdot 250 = 50 \cdot 4 \cdot 60 + 50 \cdot 1 \cdot 10 \qquad = 50 \cdot 4 \cdot 40 + 50 \cdot 1 \cdot 90 \qquad = 50 \cdot 5 \cdot 50 = 12{,}500$$



Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

Matrix or tensor algebra of collisions

→ *Deriving Energy Conservation <u>Theorem</u>*

Energy Ellipse geometry

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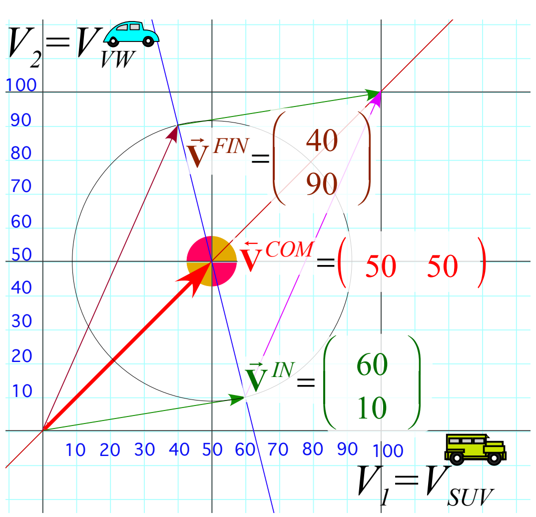

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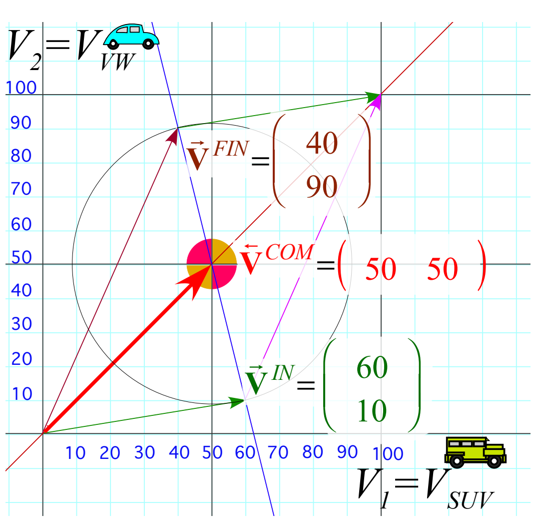

Numerical details of collision tensor algebra

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Transpose symmetry ($M_{jk} = M_{kj}$) of **M**-matrix makes 'lopsided' FIN-IN-terms equal:

$$\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

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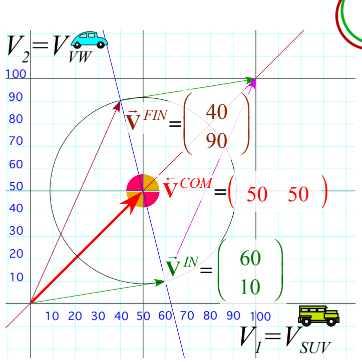

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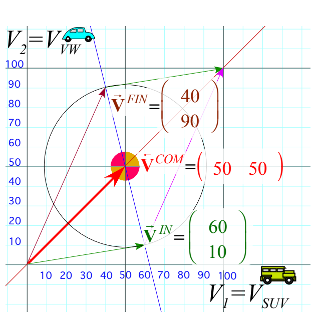
$$V^{COM}P_{Total} - \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

FIN-IN-term is subtracted to give *Conservation of* ***Kinetic Energy***

$$KE = \tfrac{1}{2}\vec{\mathbf{V}}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}} = \tfrac{1}{2}M_1V_1^2+\tfrac{1}{2}M_2V_2^2$$



$$\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

$$\begin{pmatrix} 40 & 90 \end{pmatrix}\bullet\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\bullet\begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix}\bullet\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\bullet\begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100\cdot105 = 100\cdot105 = 10{,}500$$

General **Inertia Tensor M** or inertia matrix of 2 coefficients $M_{11}=M_1$ and $M_{22}=M_2$ for 2 dimensions

$$\left.\begin{aligned} P_1 &= M_1V_1 + M_{12}V_2 \\ P_2 &= M_{21}V_1 + M_2V_2 \end{aligned}\right\} \quad denoted: \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or: \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Generalizing the definition of *momentum*...

With 45°diagonal $\vec{\mathbf{V}}^{COM}=(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN})/2$ so: $V_1^{COM}=V_2^{COM}\equiv V^{COM}$ ...and 4 or 5 ways to write ***Axiom-1***

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$$= 100\cdot125 = 100\cdot125 = 50\cdot250$$

$P_{Total}=250$ is the same at IN, FIN, and COM. Now use T-symmetry: $\vec{\mathbf{V}}^{COM}=(\vec{\mathbf{V}}^{FIN}+\vec{\mathbf{V}}^{IN})/2$ ***(Axiom-2)***

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$$50\cdot250 - \tfrac{1}{2}\cdot10{,}500 \qquad \tfrac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

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50·250 -½·10,500
12,500 - 5,250

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$$V^{COM}P_{Total} - \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

50·250 -½·10,500

12,500 - 5,250=7,250

$$\tfrac{1}{2}\vec{\mathbf{V}}^{IN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \tfrac{1}{2}\vec{\mathbf{V}}^{FIN}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$$

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$$=\tfrac{1}{2}(60\cdot 4\cdot 60+10\cdot 1\cdot 10) = \tfrac{1}{2}(40\cdot 4\cdot 40+90\cdot 1\cdot 90)$$

$$2\cdot 3600+50 = 7250 = 2\cdot 1600+\tfrac{1}{2}8100 = 7250$$

FIN-IN-term is subtracted to give *Conservation of* ***Kinetic Energy***

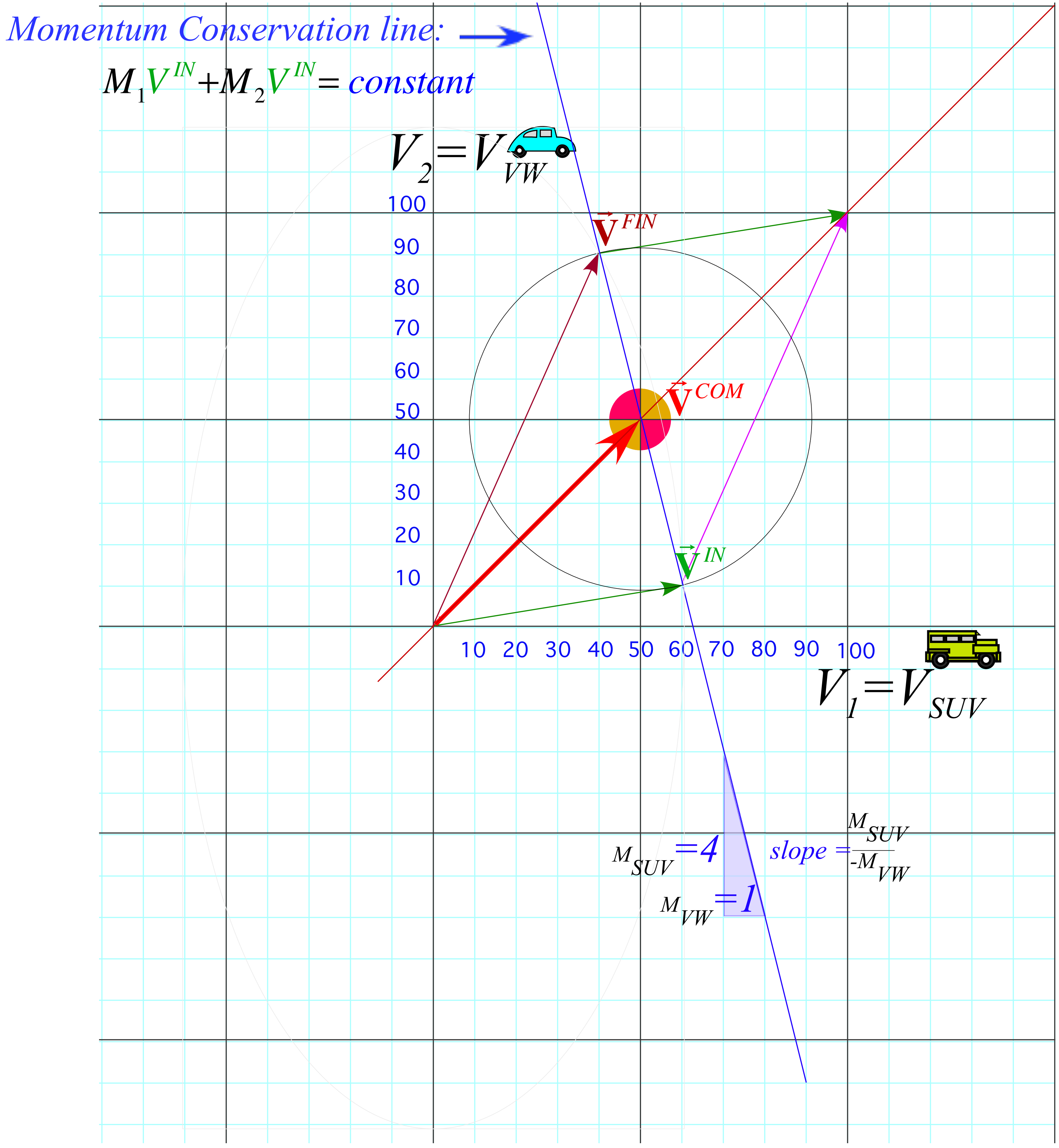
$$KE = \tfrac{1}{2}\vec{\mathbf{V}}\bullet\overleftrightarrow{\mathbf{M}}\bullet\vec{\mathbf{V}} = \tfrac{1}{2}M_1V_1^2+\tfrac{1}{2}M_2V_2^2$$

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
→ *Energy Ellipse geometry*

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$



Collision *Web Simulator*
[***Basic*** *elastic Collision*](#)
[*Dual Panel*](#)
[*Space vs Space*](#)
[*and*](#)
[*V(VW) vs. V(SUV)*](#)

BounceIt
Superball Web Simulator
[***Basic*** *elastic Collision*](#)
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$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Momentum Conservation line: →

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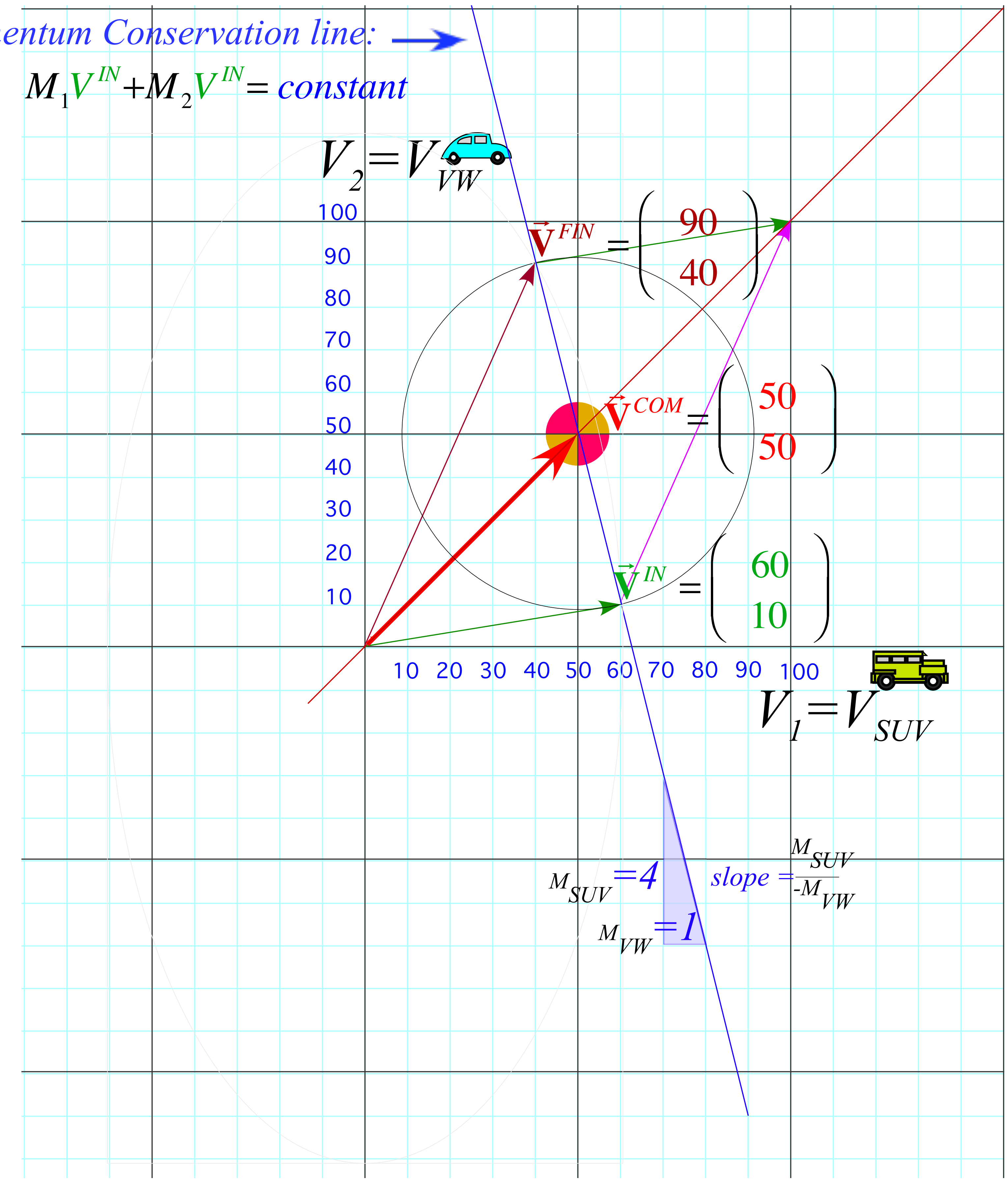

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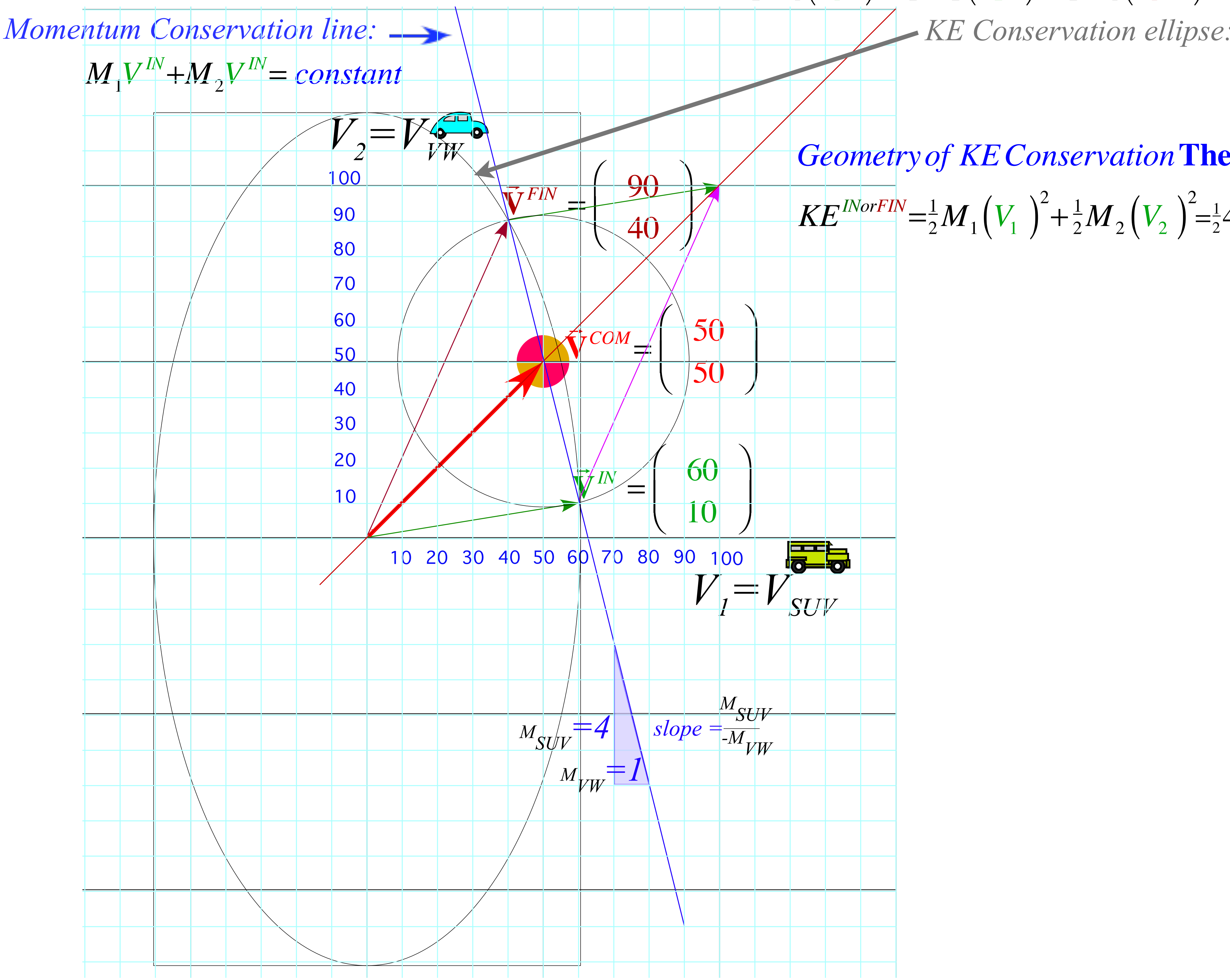
$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$



Geometry of KE Conservation **Theorem - 1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2=\tfrac{1}{2}4\left(60\right)^2+\tfrac{1}{2}1\left(10\right)^2=7{,}250$$

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

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Momentum Conservation line:

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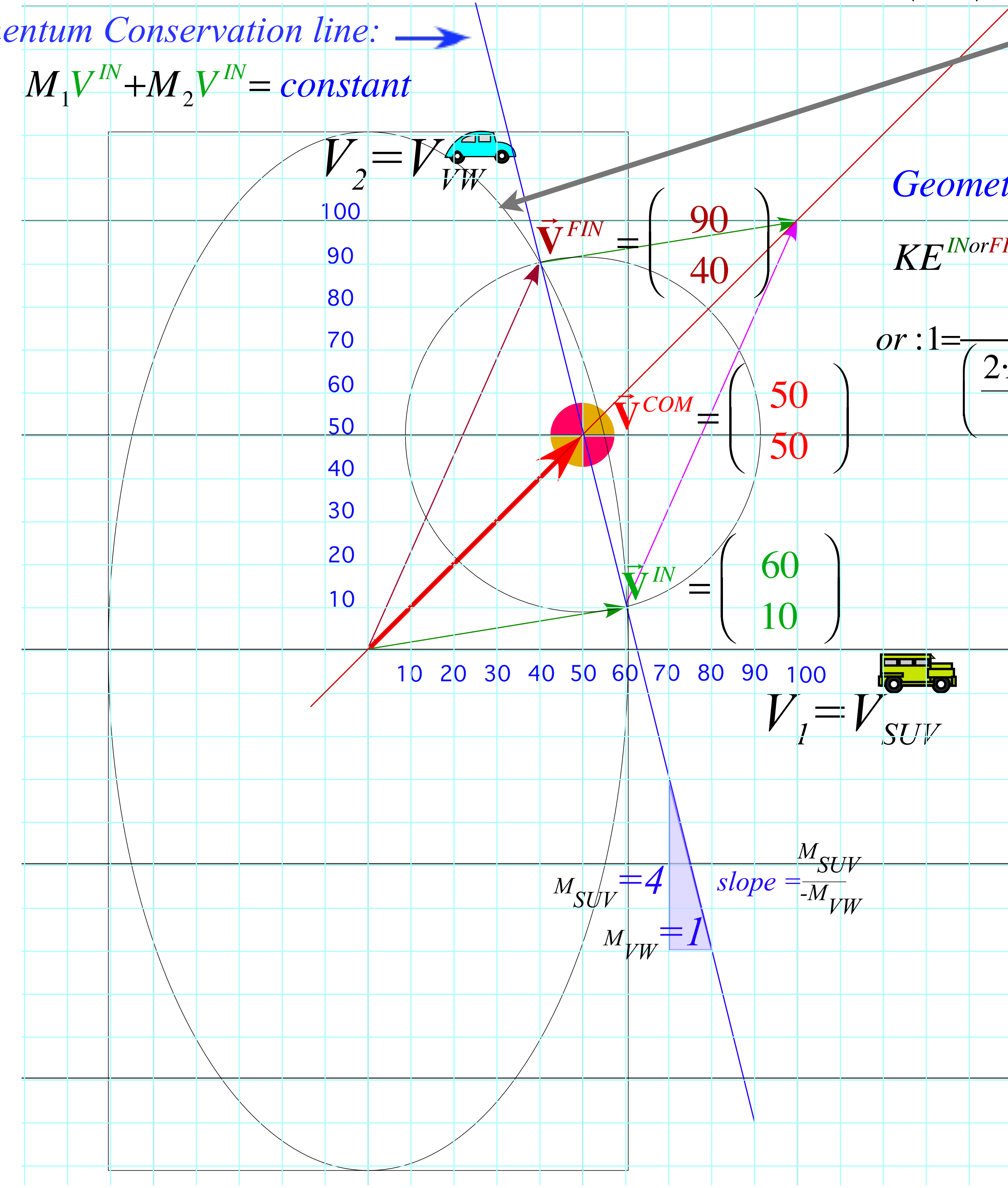
KE Conservation ellipse:



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$$or:1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

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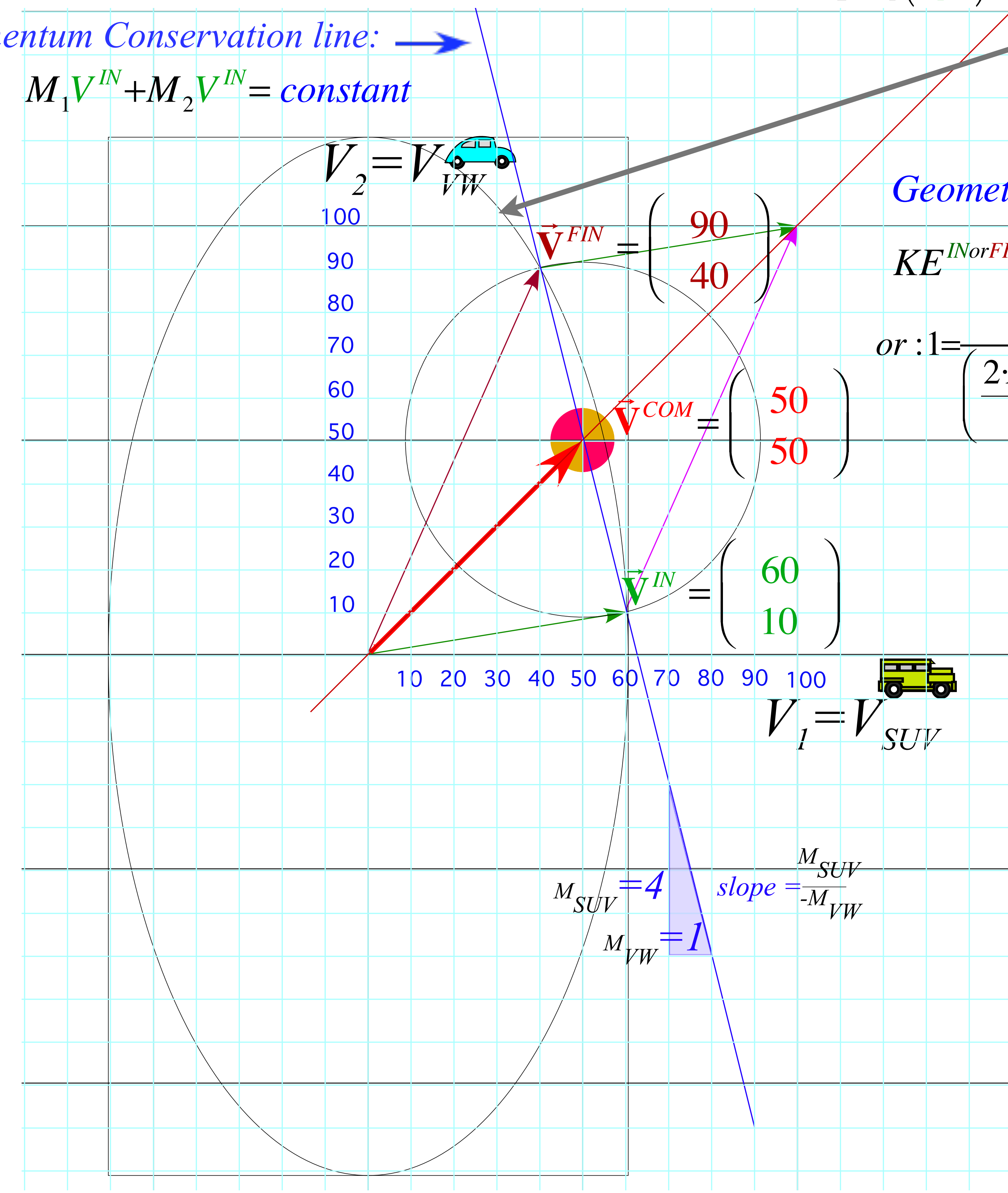

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Has the algebraic form: $1=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

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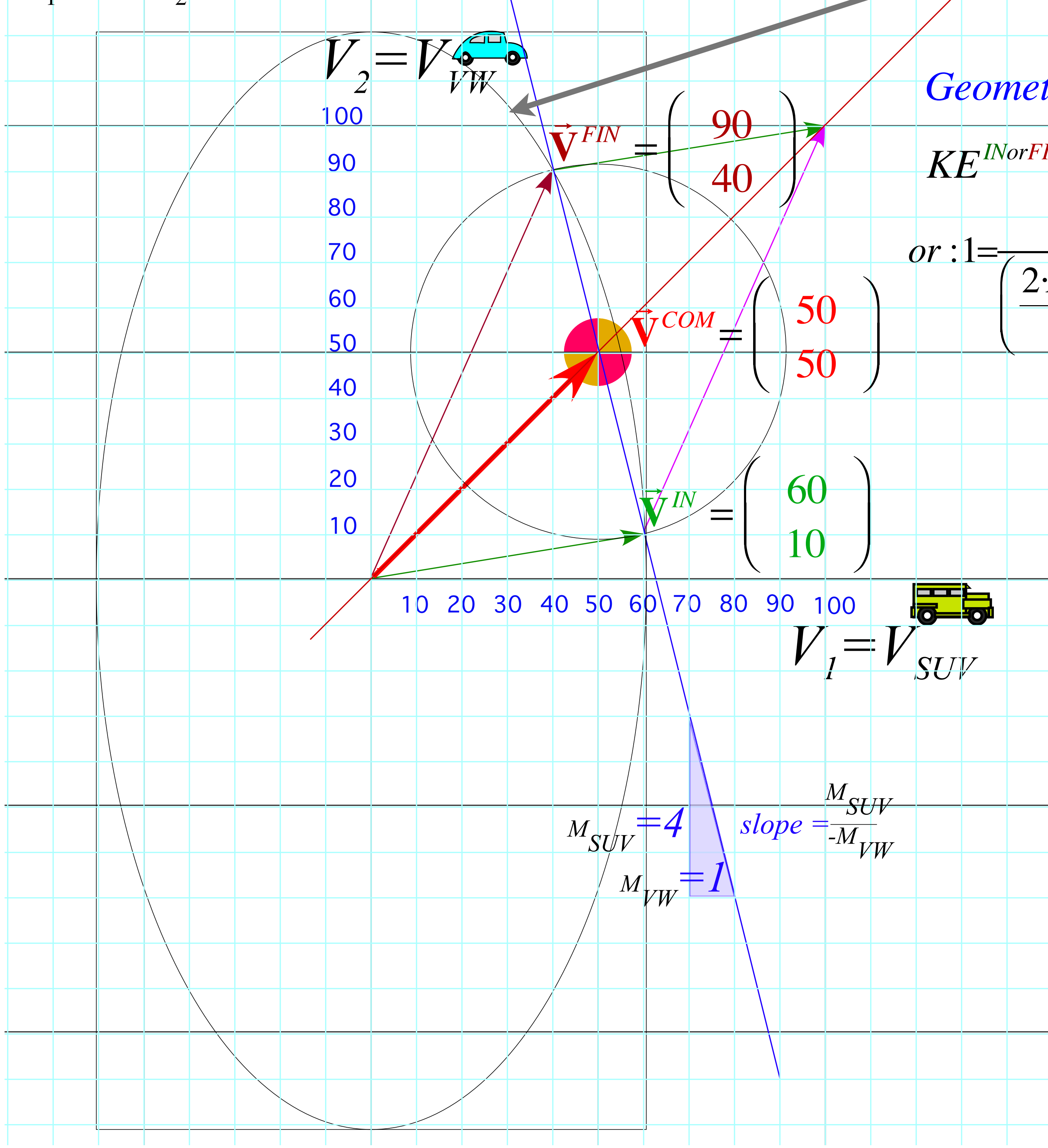

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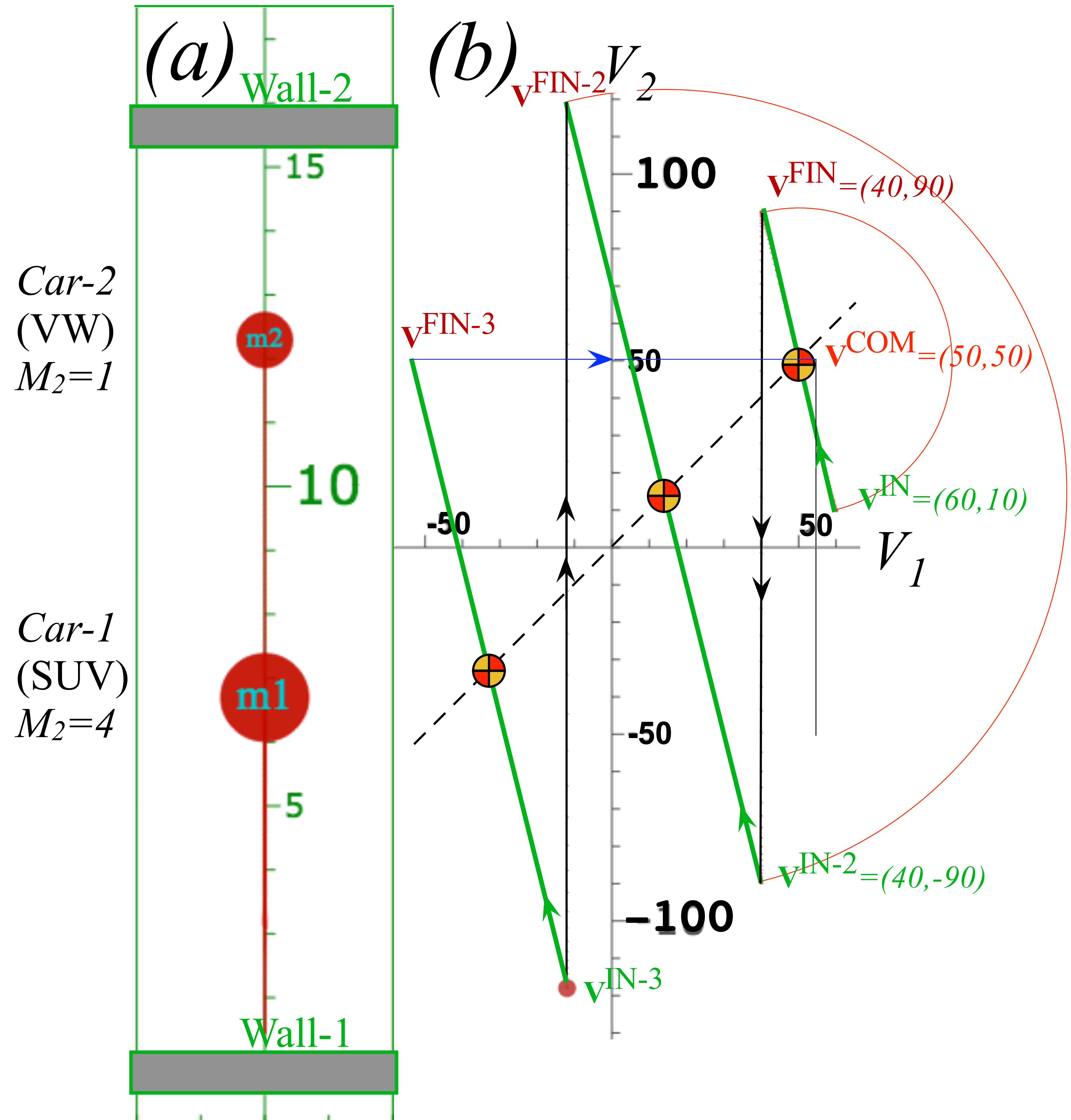
Has the algebraic form: $1=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$

$$elliptic\,radii:a=\sqrt{\frac{2KE^{INorFIN}}{M_1}}\qquad b=\sqrt{\frac{2KE^{INorFIN}}{M_2}}$$

$$=\sqrt{\frac{2\cdot 7{,}250}{4}}\qquad =\sqrt{\frac{2\cdot 7{,}250}{1}}$$

$$=60.21\qquad =120.42$$

BounceIt Simulation: frictionless 1D-track with elastic bumper cars bouncing between walls



From Fig. 2.4

BounceIt
Superball Web Simulator
***Repeated** elastic Collisions*
Dual Panel
Space vs Space
and
V(VW) vs. V(SUV)

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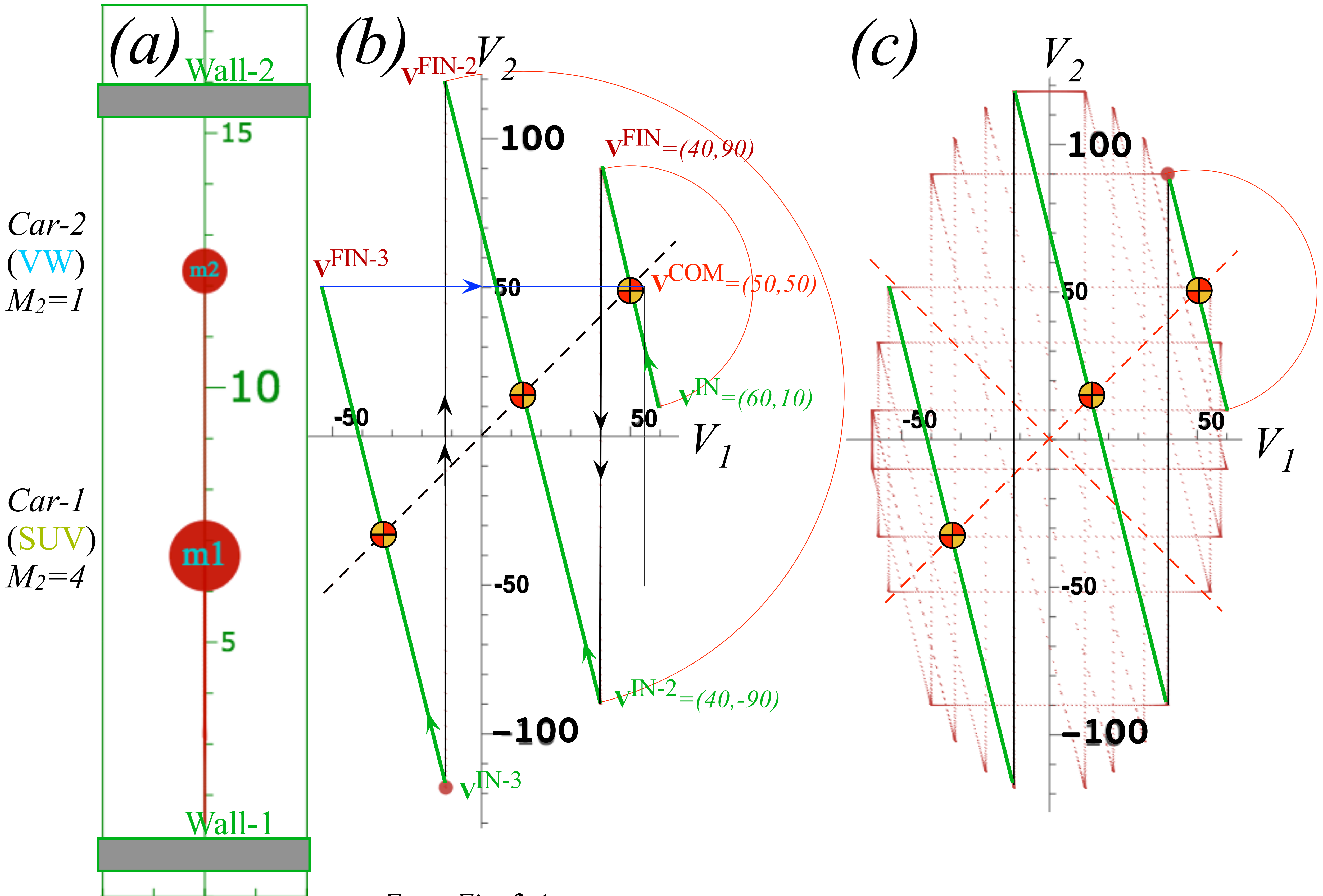


From Fig. 2.4

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \overleftarrow{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \vec{\mathbf{V}} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE^{COM} = \frac{1}{2} \vec{\mathbf{V}}^{COM} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 50 \\ 50 \end{pmatrix}$$

$$= \frac{1}{2}(50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \frac{1}{2} \cdot 2500$$

$$= 5000 + \frac{1}{2} \cdot 1250 = 6{,}250$$

Geometry of Momentum Conservation **Axiom-1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

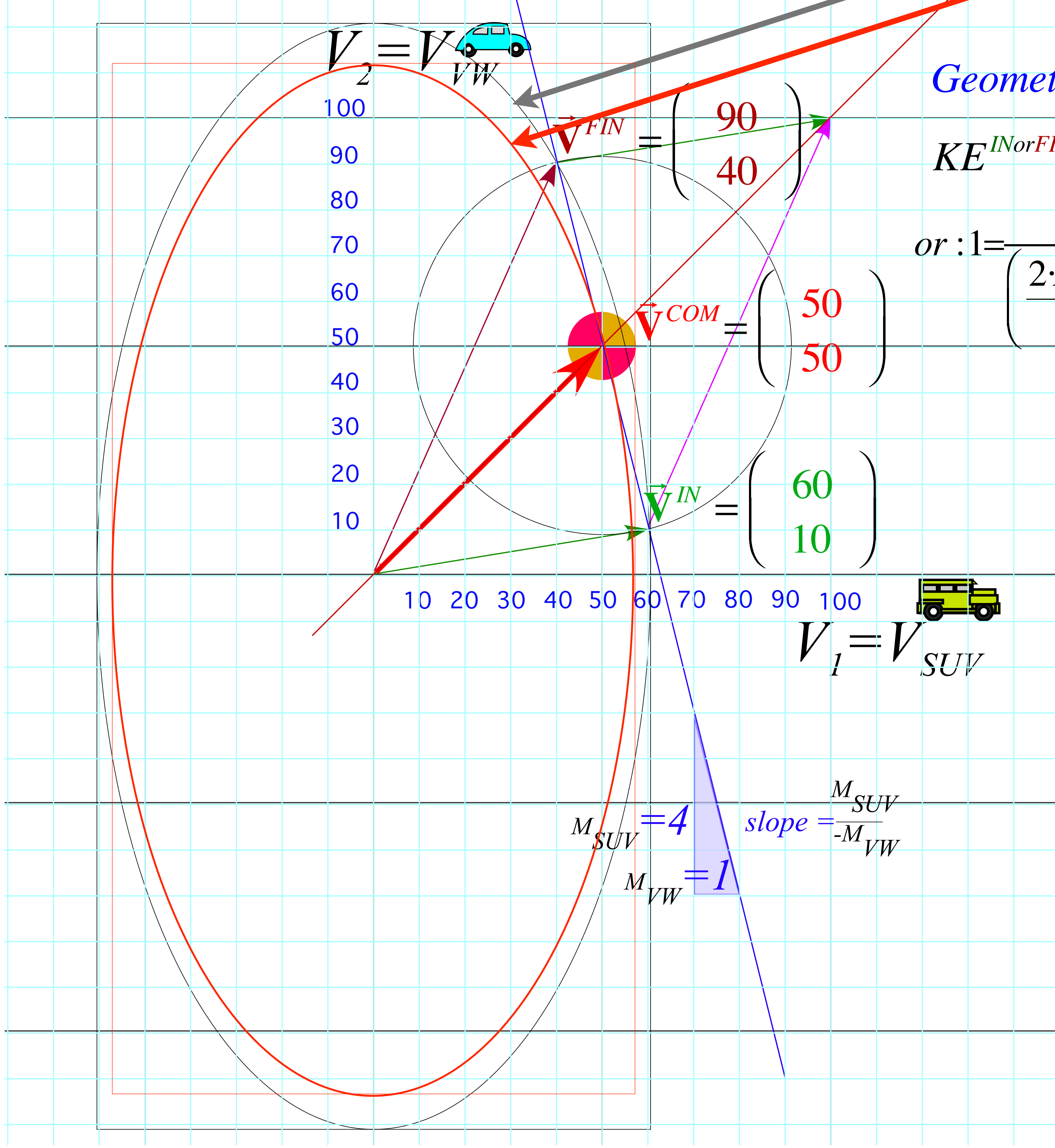
$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$

Momentum Conservation line: →

$M_1V^{IN}+M_2V^{IN}=$ *constant*

$KE^{INorFIN}$ *Conservation ellipse:*

KE^{COM} *Ka-runch ellipse:*



Geometry of KE Conservation **Theorem-1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2$$

$$or: 1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Geometry of KE^{COM} *at CenterOfMomentum*

$$KE^{COM}=\tfrac{1}{2}M_1\left(V_1^{COM}\right)^2+\tfrac{1}{2}M_2\left(V_2^{COM}\right)^2$$

$$=\tfrac{1}{2}(M_1+M_2)\left(V^{COM}\right)^2$$

elliptic radii: $a^{COM}=\sqrt{\frac{KE^{COM}}{M_1}}=\sqrt{\frac{2\cdot 6{,}250}{4}}=55.90$ $\quad b^{COM}=\sqrt{\frac{KE^{COM}}{M_2}}=\sqrt{\frac{2\cdot 6{,}250}{1}}=111.80$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\vec{V}^{FIN} \bullet \overleftrightarrow{M} \bullet \vec{V}^{IN} = \vec{V}^{IN} \bullet \overleftrightarrow{M} \bullet \vec{V}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \vec{V} \bullet \overleftrightarrow{M} \bullet \vec{V}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\vec{V}^{FIN} \bullet \overleftrightarrow{M} \bullet \vec{V}^{IN}}{2} = \frac{\vec{V}^{IN} \bullet \overleftrightarrow{M} \bullet \vec{V}^{IN}}{2} = \frac{\vec{V}^{FIN} \bullet \overleftrightarrow{M} \bullet \vec{V}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE^{COM} = V^{COM} P_{Total} - \frac{\vec{V}^{COM} \bullet \overleftrightarrow{M} \bullet \vec{V}^{COM}}{2} = \frac{1}{2} V^{COM} P_{Total} = \frac{\vec{V}^{COM} \bullet \overleftrightarrow{M} \bullet \vec{V}^{COM}}{2} = \frac{\vec{V}^{IN} \bullet \overleftrightarrow{M} \bullet \vec{V}^{IN}}{4} + \frac{\vec{V}^{FIN} \bullet \overleftrightarrow{M} \bullet \vec{V}^{IN}}{4} = \frac{1}{2} KE^{INorFIN} + \frac{\vec{V}^{FIN} \bullet \overleftrightarrow{M} \bullet \vec{V}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = 3{,}625 + 2{,}625$$

$$KE^{COM} = \tfrac{1}{2} \vec{V}^{COM} \bullet \overleftrightarrow{M} \bullet \vec{V}^{COM} = \tfrac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 50 \\ 50 \end{pmatrix}$$

$$= \tfrac{1}{2}(50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \tfrac{1}{2} \cdot 2500$$

$$= 5000 + \tfrac{1}{2} \cdot 1250 = 6{,}250$$

Transpose symmetry ($M_{jk}=M_{kj}$) of the **M**-matrix implies: $\vec{\mathbf{V}}^{FIN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix}40 & 90\end{pmatrix}\bullet\begin{pmatrix}4 & 0\\ 0 & 1\end{pmatrix}\bullet\begin{pmatrix}60\\ 10\end{pmatrix}=\begin{pmatrix}60 & 10\end{pmatrix}\bullet\begin{pmatrix}4 & 0\\ 0 & 1\end{pmatrix}\bullet\begin{pmatrix}40\\ 90\end{pmatrix}$$

$$= 100\cdot 105 = 100\cdot 105 = 10{,}500$$

With ($M_{12}=0=M_{21}$) kinetic energy $KE^{INorFIN}=\frac{1}{2}\vec{\mathbf{V}}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}$ is the same at $\mathbf{V}=\mathbf{V}^{IN}$ and $\mathbf{V}=\mathbf{V}^{FIN}$.

$$V^{COM}P_{Total}-\frac{\vec{\mathbf{V}}^{FIN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{FIN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix}60 & 10\end{pmatrix}\bullet\begin{pmatrix}4 & 0\\ 0 & 1\end{pmatrix}\bullet\begin{pmatrix}60\\ 10\end{pmatrix}}{2} = \frac{\begin{pmatrix}40 & 90\end{pmatrix}\bullet\begin{pmatrix}4 & 0\\ 0 & 1\end{pmatrix}\bullet\begin{pmatrix}40\\ 90\end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy KE^{COM} when $\mathbf{V}=\mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE^{COM}=V^{COM}P_{Total}-\frac{\vec{\mathbf{V}}^{COM}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{COM}}{2}=\frac{1}{2}V^{COM}P_{Total}=\frac{\vec{\mathbf{V}}^{COM}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{COM}}{2}=\frac{\vec{\mathbf{V}}^{IN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}}{4}+\frac{\vec{\mathbf{V}}^{FIN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}}{4}=\frac{1}{2}KE^{INorFIN}+\frac{\vec{\mathbf{V}}^{FIN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = 3{,}625 + 2{,}625$$

$$KE^{COM}=\frac{1}{2}\vec{\mathbf{V}}^{COM}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{COM}=\frac{1}{2}\begin{pmatrix}50 & 50\end{pmatrix}\bullet\begin{pmatrix}4 & 0\\ 0 & 1\end{pmatrix}\bullet\begin{pmatrix}50\\ 50\end{pmatrix}$$

$$=\frac{1}{2}(50\cdot 4\cdot 50+50\cdot 1\cdot 50)=2\cdot 2500+\frac{1}{2}\cdot 2500$$

$$=5000+\frac{1}{2}\cdot 1250 = 6{,}250$$

Introducing *Potential Energy*=*PE*

Difference is inelastic "*ka-Runch*" $KE^{INorFIN}$-KE^{COM}. For elastic "*ka-Bong*" the *1,000* is PE^{COM} of compression.

$$KE^{INorFIN}-KE^{COM}=\frac{\vec{\mathbf{V}}^{IN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}}{4}-\frac{\vec{\mathbf{V}}^{FIN}\bullet\vec{\mathbf{M}}\bullet\vec{\mathbf{V}}^{IN}}{4}$$

$$1{,}000 = 3{,}625 - 2{,}625$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE^{INorFIN} = \frac{1}{2} \vec{\mathbf{V}} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE^{INorFIN}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE^{INorFIN}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy KE^{COM} when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE^{COM} = V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{COM} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{1}{2} V^{COM} P_{Total} = \frac{\vec{\mathbf{V}}^{COM} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE^{INorFIN} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = 3{,}625 + 2{,}625$$

$$KE^{COM} = \frac{1}{2} \vec{\mathbf{V}}^{COM} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 50 \\ 50 \end{pmatrix}$$

$$= \frac{1}{2}(50 \cdot 4 \cdot 50 + 50 \cdot 1 \cdot 50) = 2 \cdot 2500 + \frac{1}{2} \cdot 2500$$

$$= 5000 + \frac{1}{2} \cdot 1250 = 6{,}250$$

Introducing
Potential Energy=PE

Difference is inelastic "*ka-Runch*" $KE^{INorFIN}$-KE^{COM}. For elastic "*ka-Bong*" the *1,000* is PE^{COM} of compression.

$$KE^{INorFIN} - KE^{COM} = \frac{\vec{\mathbf{V}}^{IN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$1{,}000 = 3{,}625 - 2{,}625$$

$$KE^{COM} = \frac{\vec{\mathbf{V}}^{IN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \vec{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$6{,}250 = 3{,}625 + 2{,}625$$

Difference $KE^{INorFIN}$-KE^{COM} = *1,000* is the same in *all* frames including *COM*-frame where $\mathbf{V}^{COM} = \mathbf{0}$.

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

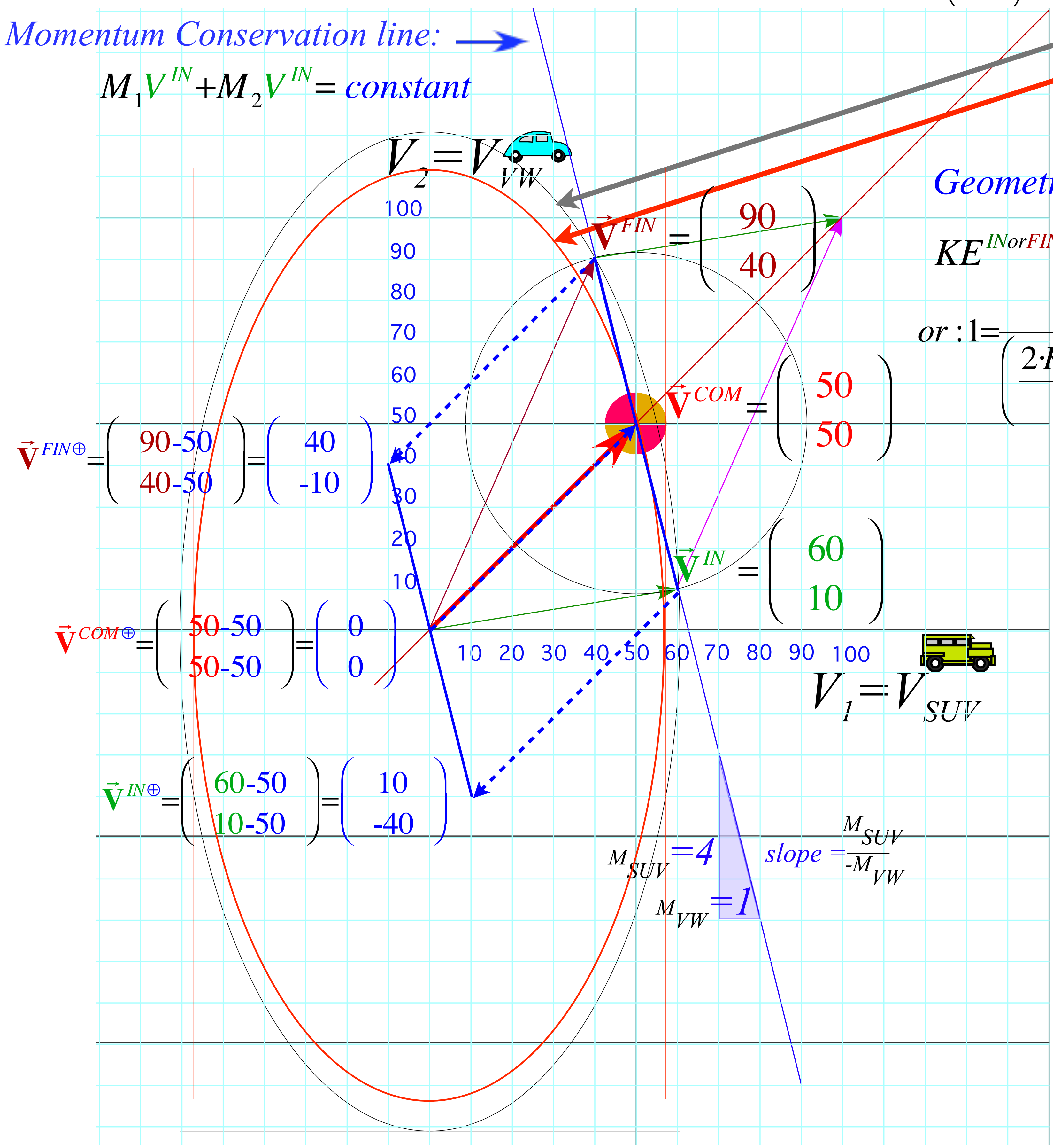
$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$

Momentum Conservation line:

$$M_1V^{IN}+M_2V^{IN}=\textit{constant}$$

$KE^{INorFIN}$ *Conservation ellipse:*

KE^{COM} *Ka-runch ellipse:*



Geometry of KE Conservation **Theorem - 1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2$$

$$or:1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$

Geometry of KE ^{COM} *at CenterOfMomentum*

$$KE^{COM}=\tfrac{1}{2}M_1\left(V_1^{COM}\right)^2+\tfrac{1}{2}M_2\left(V_2^{COM}\right)^2$$

$$=\tfrac{1}{2}\left(M_1+M_2\right)\left(V^{COM}\right)^2$$

elliptic radii: $a^{COM}=\sqrt{\dfrac{KE^{COM}}{M_1}}$ $\quad b^{COM}=\sqrt{\dfrac{KE^{COM}}{M_2}}$

$=\sqrt{\dfrac{2\cdot 6{,}250}{4}}$ $\quad =\sqrt{\dfrac{2\cdot 6{,}250}{1}}$

$=55.90$ $\quad =111.80$

Geometry of Momentum Conservation **Axiom - 1**

$$(M_1+M_2)V^{COM}=M_1V_1^{IN}+M_2V_2^{IN}=M_1V_1^{FIN}+M_2V_2^{FIN}$$

Geometry of Kinetic Energy Conservation **Axiom-2/Theorem-1**

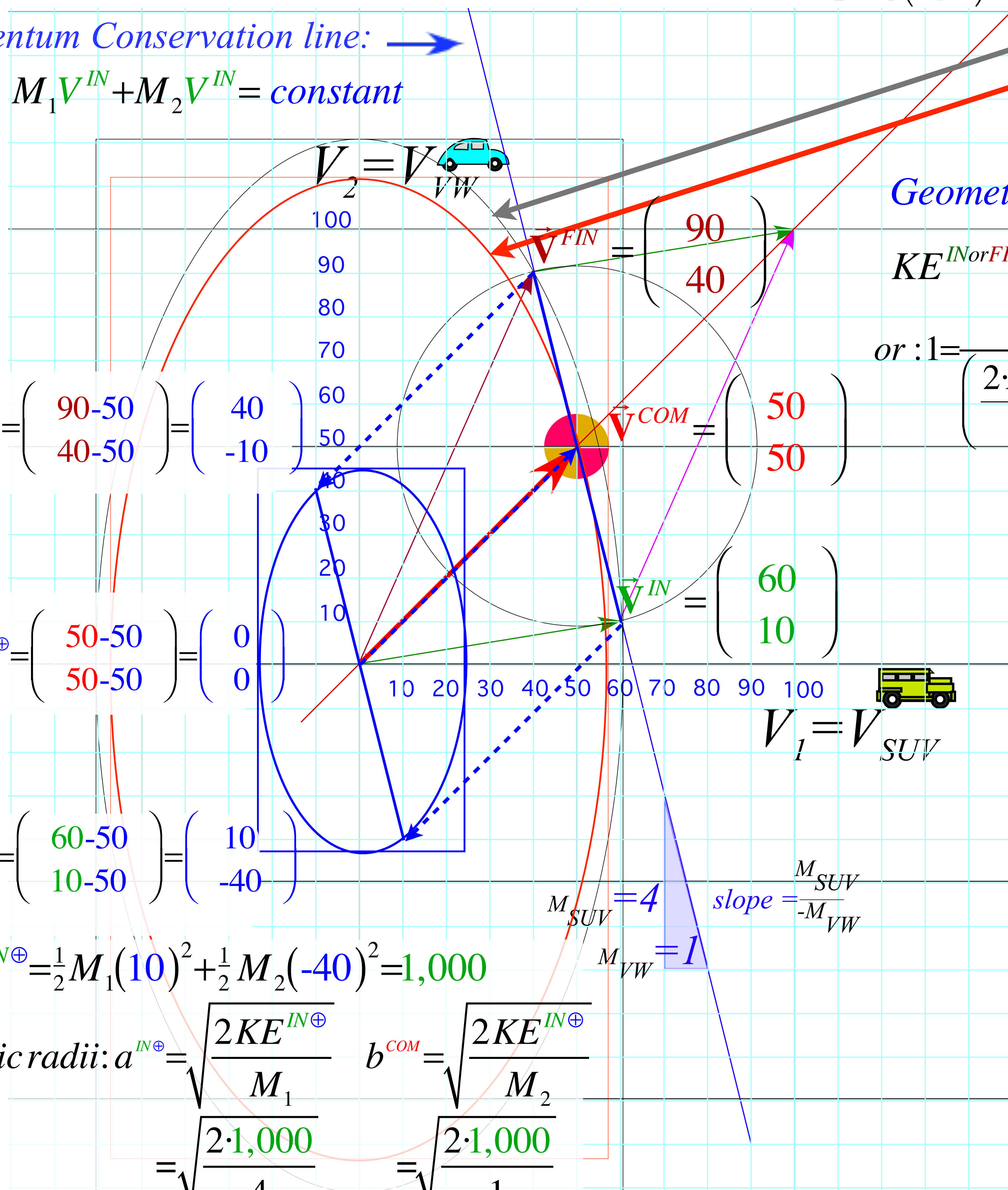
$$KE^{IN}=\tfrac{1}{2}M_1\left(V_1^{IN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{IN}\right)^2=\tfrac{1}{2}M_1\left(V_1^{FIN}\right)^2+\tfrac{1}{2}M_2\left(V_2^{FIN}\right)^2=KE^{FIN}$$

Momentum Conservation line:

$$M_1V^{IN}+M_2V^{IN}=constant$$

$KE^{INorFIN}$ *Conservation ellipse:*

KE^{COM} *Ka-runch ellipse:*



$$\vec{V}^{FIN}=\begin{pmatrix}90\\40\end{pmatrix}\qquad \vec{V}^{COM}=\begin{pmatrix}50\\50\end{pmatrix}\qquad \vec{V}^{IN}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\vec{V}^{FIN\oplus}=\begin{pmatrix}90\text{-}50\\40\text{-}50\end{pmatrix}=\begin{pmatrix}40\\-10\end{pmatrix}$$

$$\vec{V}^{COM\oplus}=\begin{pmatrix}50\text{-}50\\50\text{-}50\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$$

$$\vec{V}^{IN\oplus}=\begin{pmatrix}60\text{-}50\\10\text{-}50\end{pmatrix}=\begin{pmatrix}10\\-40\end{pmatrix}$$

$$KE^{IN\oplus}=\tfrac{1}{2}M_1(10)^2+\tfrac{1}{2}M_2(-40)^2=1{,}000$$

$$elliptic\,radii: a^{IN\oplus}=\sqrt{\frac{2KE^{IN\oplus}}{M_1}}=\sqrt{\frac{2\cdot 1{,}000}{4}}=22.36 \qquad b^{COM}=\sqrt{\frac{2KE^{IN\oplus}}{M_2}}=\sqrt{\frac{2\cdot 1{,}000}{1}}=44.72$$

Geometry of KE Conservation **Theorem - 1**

$$KE^{INorFIN}=\tfrac{1}{2}M_1\left(V_1\right)^2+\tfrac{1}{2}M_2\left(V_2\right)^2$$

$$or: 1=\frac{V_1^2}{\left(\frac{2\cdot KE^{IFN}}{M_1}\right)}+\frac{V_2^2}{\left(\frac{2\cdot KE^{IFN}}{M_2}\right)}=\frac{V_1^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_1}}\right)^2}+\frac{V_2^2}{\left(\sqrt{\frac{2\cdot KE^{IFN}}{M_2}}\right)^2}$$

Has the algebraic form: $1=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Geometry of KE^{COM} at *CenterOfMomentum*

$$KE^{COM}=\tfrac{1}{2}M_1\left(V_1^{COM}\right)^2+\tfrac{1}{2}M_2\left(V_2^{COM}\right)^2$$

$$=\tfrac{1}{2}\left(M_1+M_2\right)\left(V^{COM}\right)^2$$

$$elliptic\,radii: a^{COM}=\sqrt{\frac{KE^{COM}}{M_1}}=\sqrt{\frac{2\cdot 6{,}250}{4}}=55.90 \qquad b^{COM}=\sqrt{\frac{KE^{COM}}{M_2}}=\sqrt{\frac{2\cdot 6{,}250}{1}}=111.80$$

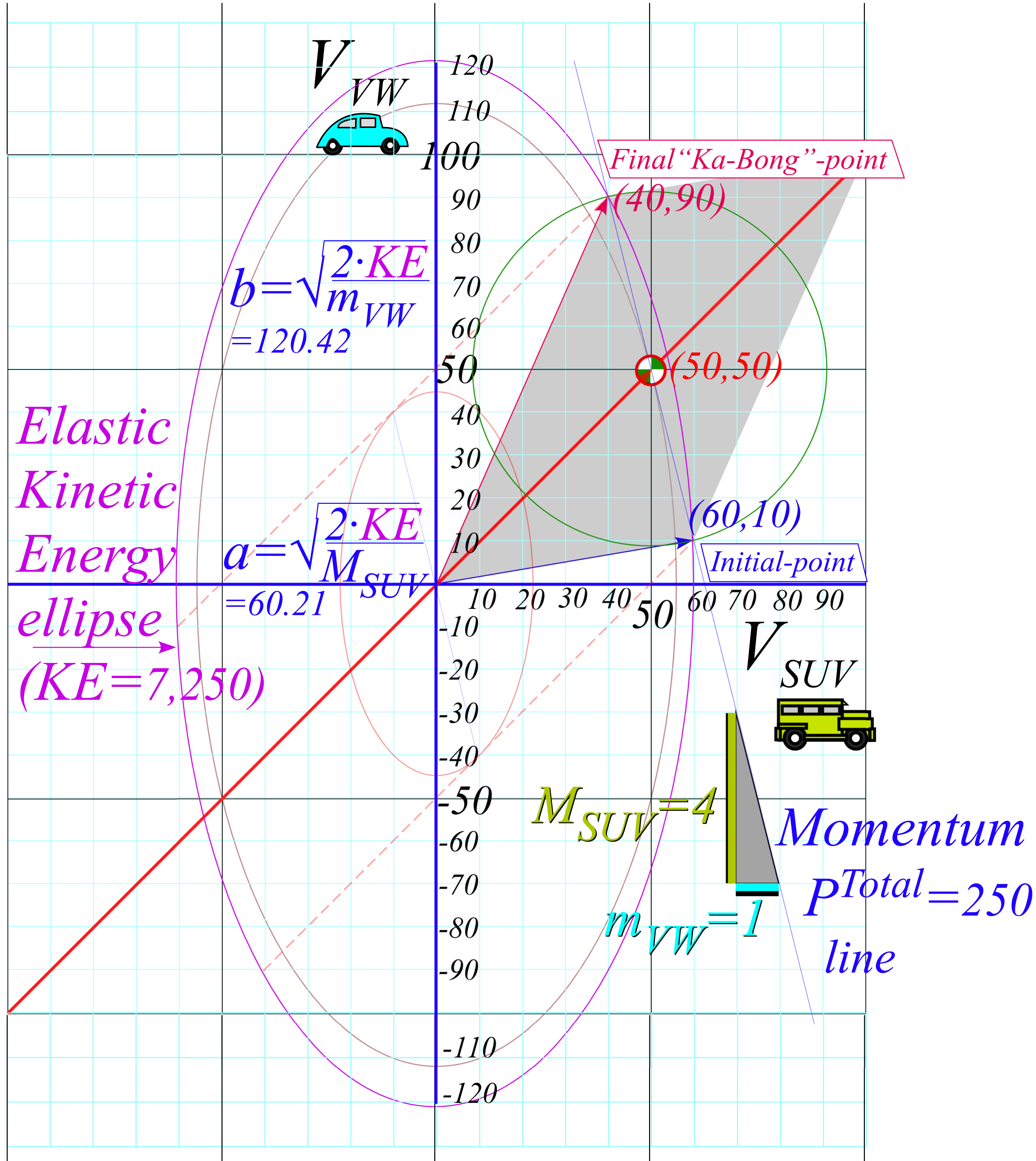


Fig. 3.1 a
in Unit 1

Developing
Conservation-of-Momentum
The key <u>axiom</u> of mechanics
leading to
Conservation-of-Energy <u>Theorem</u>

If and only if…
*there is **T-Symmetry***

Fig. 3.1

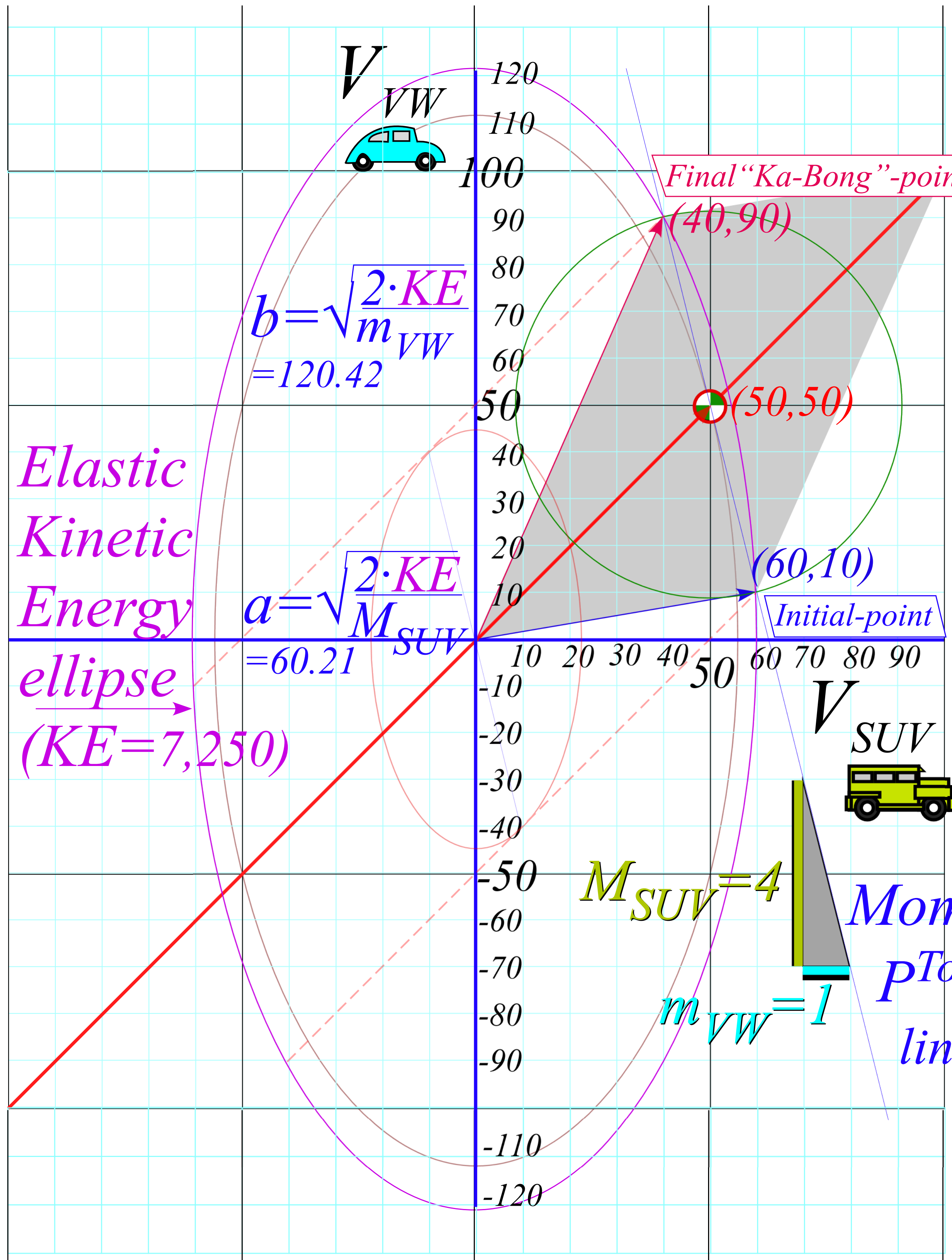


Fig. 3.1 a
in Unit 1

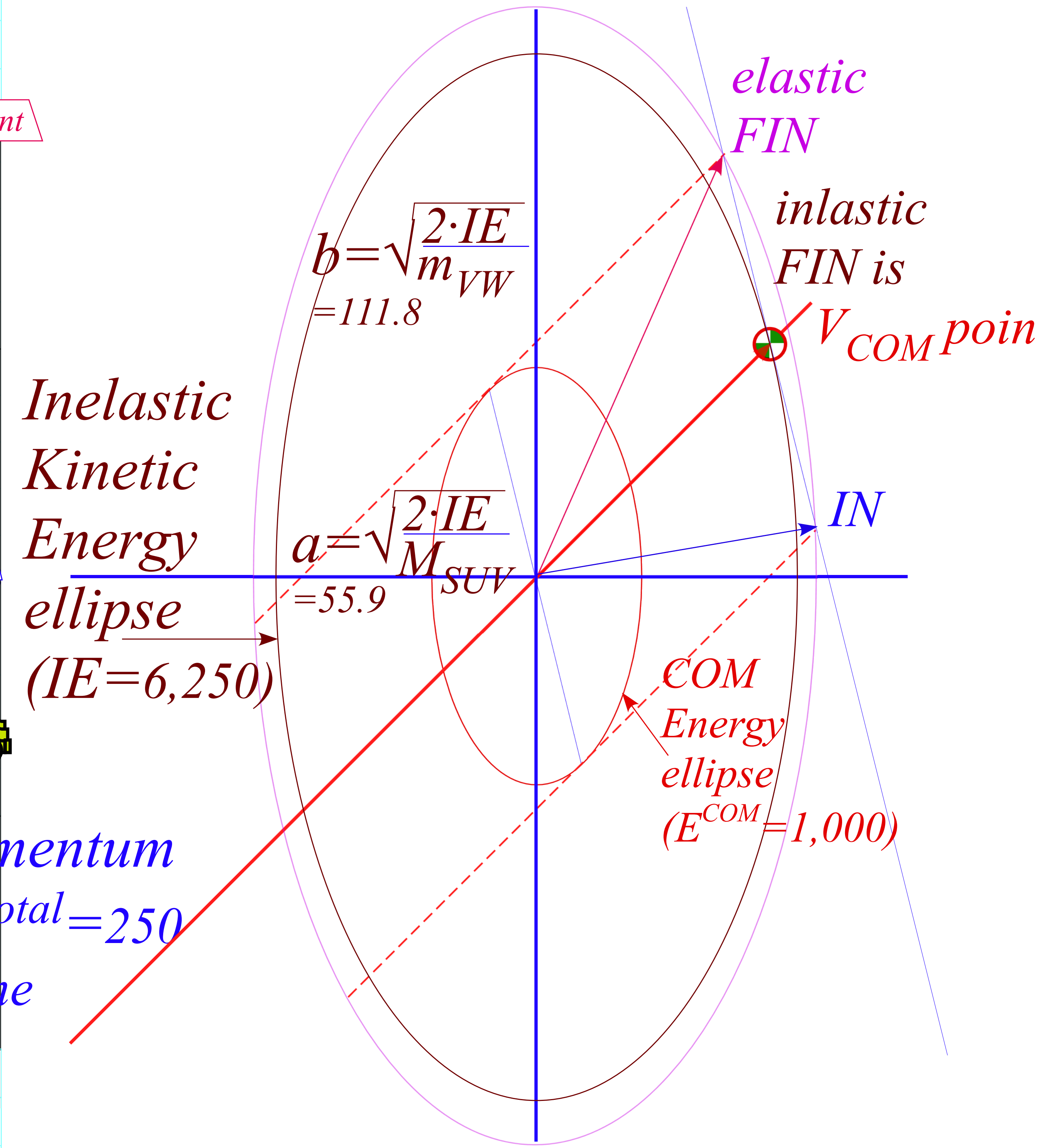


Fig. 3.1 b
in Unit 1

Fig. 3.1

As usual in physics, opposite extremes are easier to analyze than the generic "real(er) world" in between!

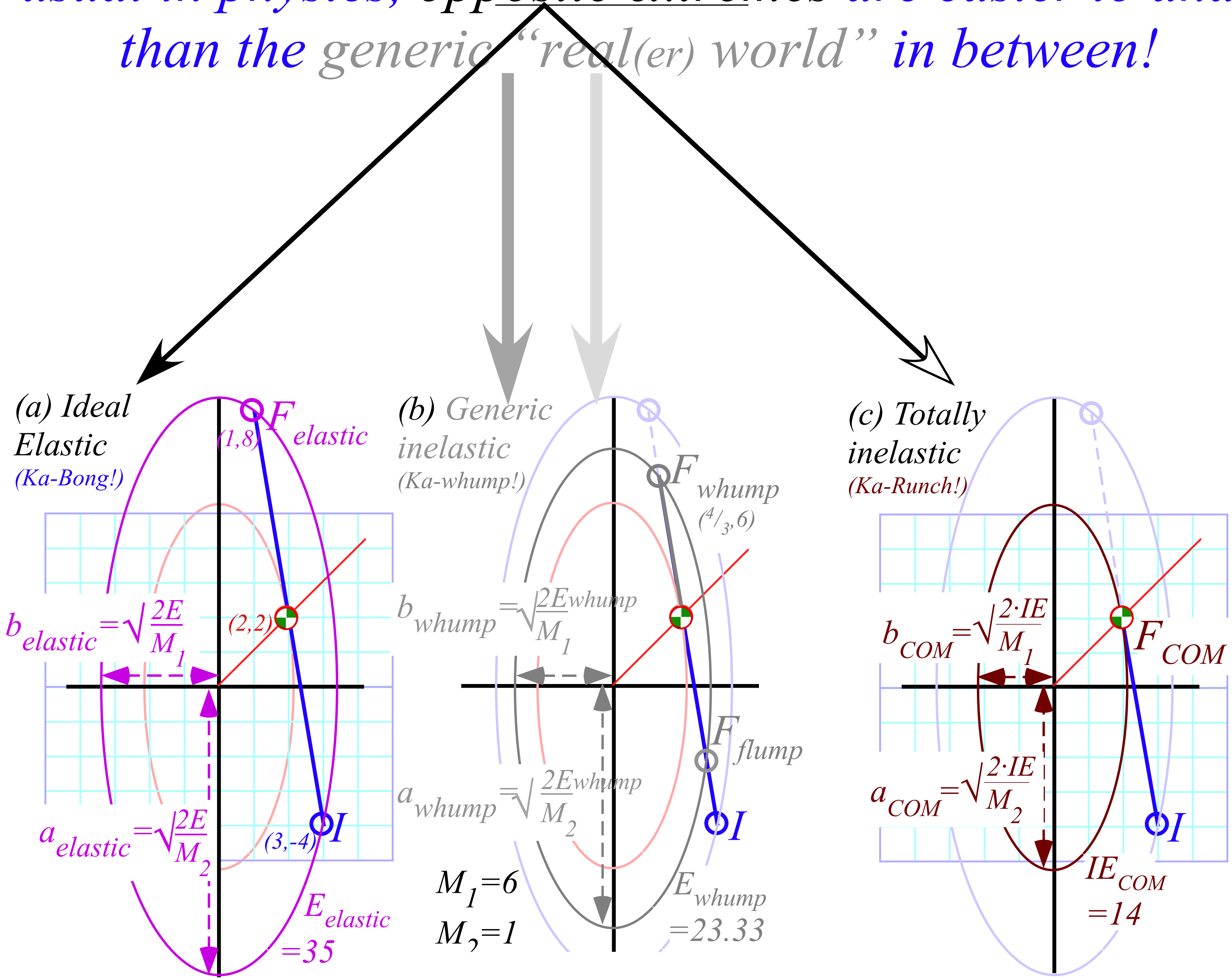


Fig. 2.3 (6-Ton SUV)

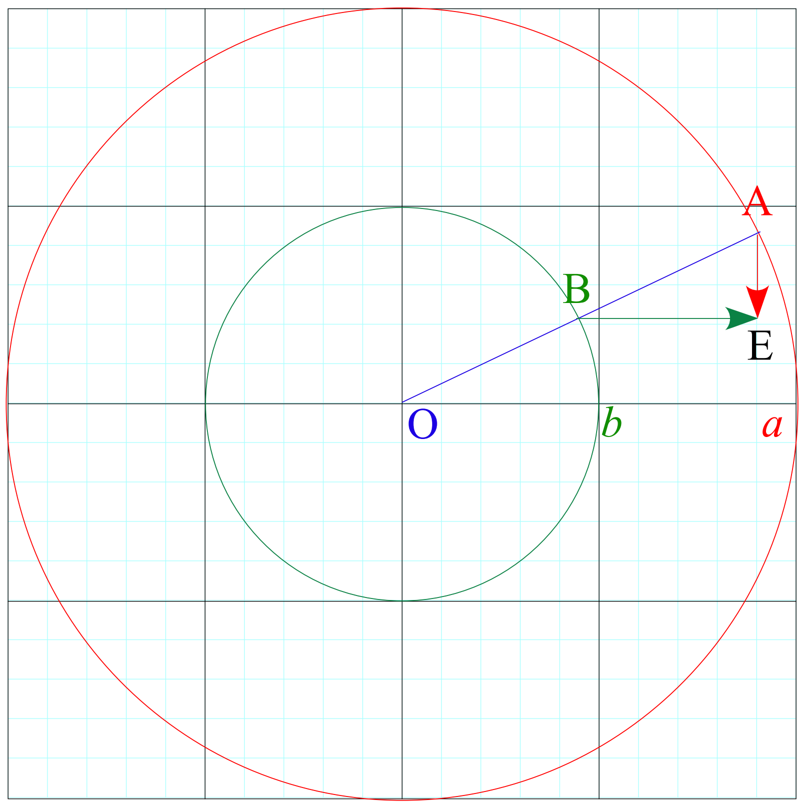
*(During Bush II era an SUV with a mass of at least **6 tons** allowed its owner to take a 100% write-off (up to $100,000) on Federal Income Tax.)*

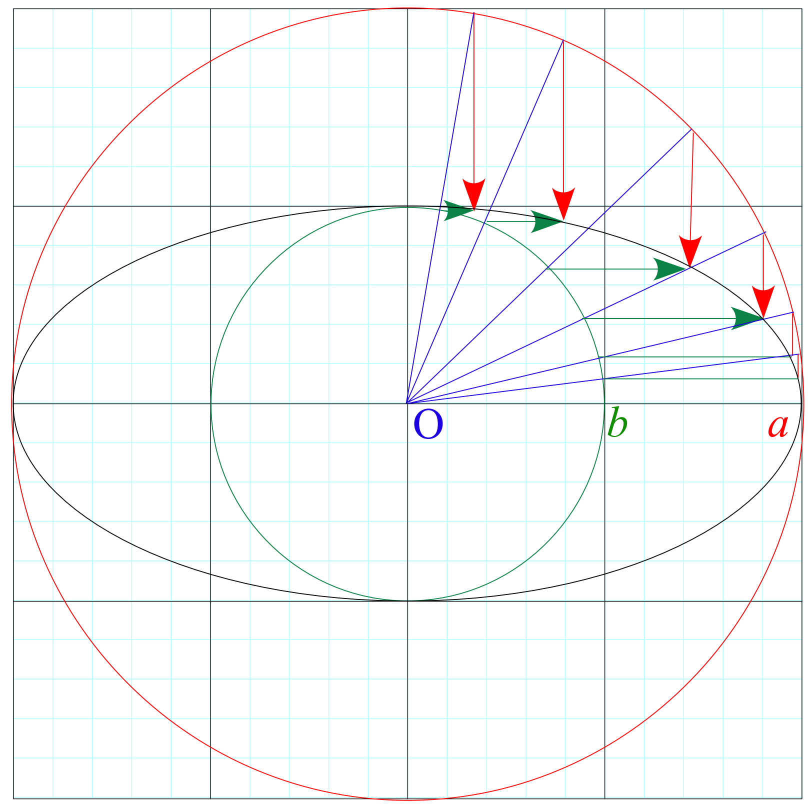
Here ***T-Symmetry*** is best

Here ***T-Symmetry*** is less

Here ***T-Symmetry*** is least

Graph paper facilitates construction of energy ellipses given the two radii a and b in KE equation.
First step: draw concentric circles of radius a and b.
Then any radial line OBA "points" to point E on the ellipse.
Ellipse point E lies at the intersection of a vertical line AE thru radial intersection A with circle a and a horizontal line BE thru radial intersection B with circle b.
Graph grid helps locate E for a radius OBA, and usually there is no need to draw AE or BE.
You can pick x and find y or else *vice-versa*.





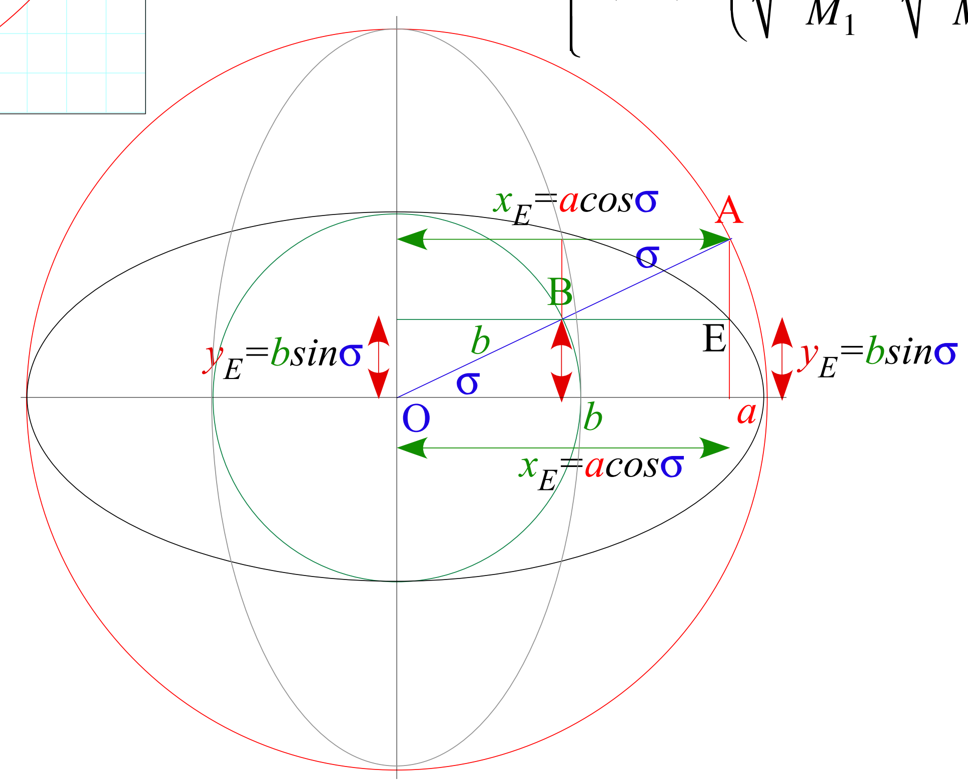
$$\frac{1}{2}M_1 \cdot V_1^2 + \frac{1}{2}M_2 \cdot V_2^2 = KE$$

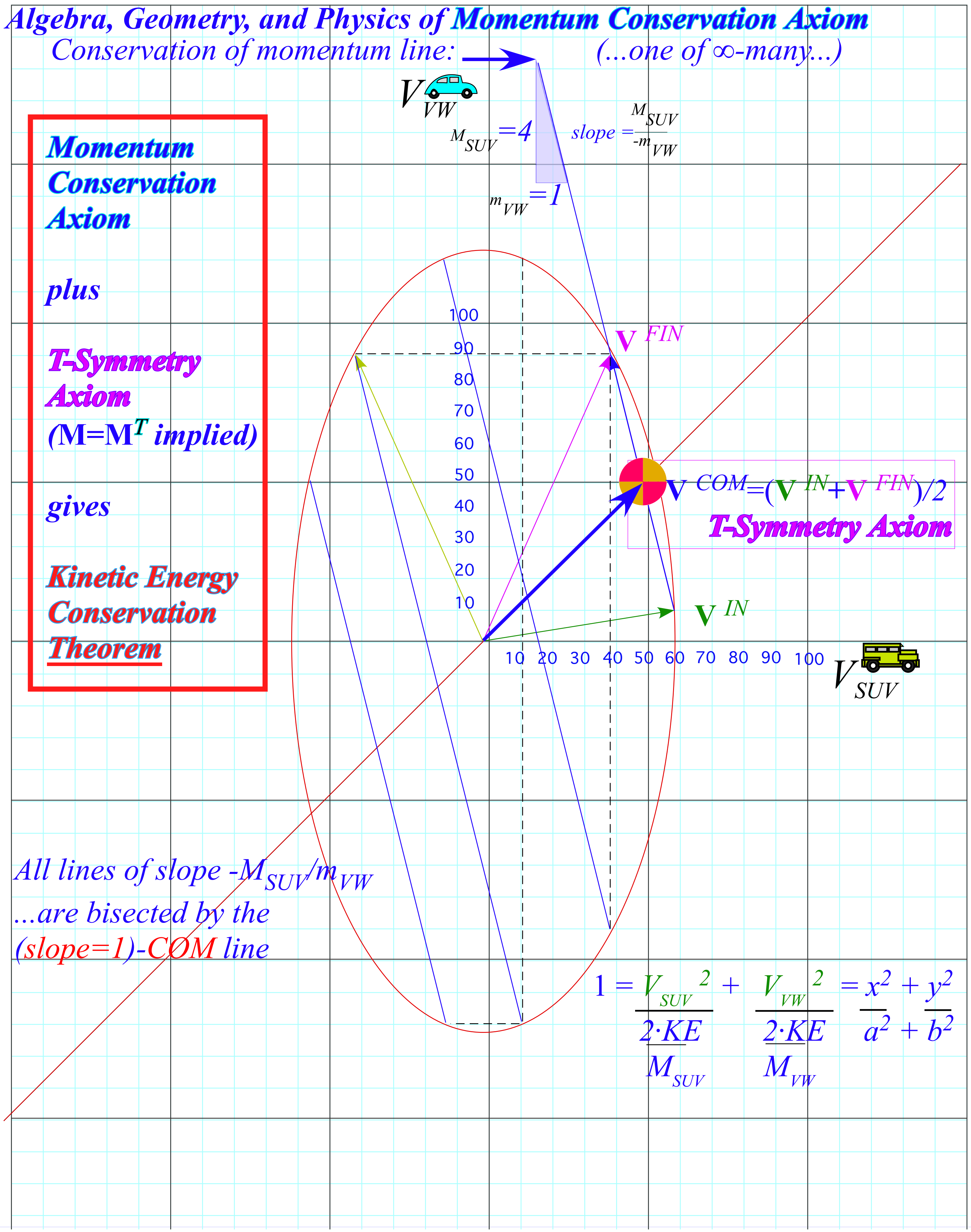
$$\frac{V_1^2}{\left(\frac{2 \cdot KE}{M_1}\right)} + \frac{V_2^2}{\left(\frac{2 \cdot KE}{M_2}\right)} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\begin{cases} (x,y) = (V_1, V_2) \\ (a,b) = \left(\sqrt{\frac{2 \cdot KE}{M_1}}, \sqrt{\frac{2 \cdot KE}{M_2}}\right) \end{cases}$$

Ellipse coordinates ($x_E = a \cdot \cos\sigma$, $y_E = b \cdot \sin\sigma$) are rescaled base and altitude ($x_r = r \cdot \cos\sigma$, $y_r = r \cdot \sin\sigma$) of Fig. 2.6.





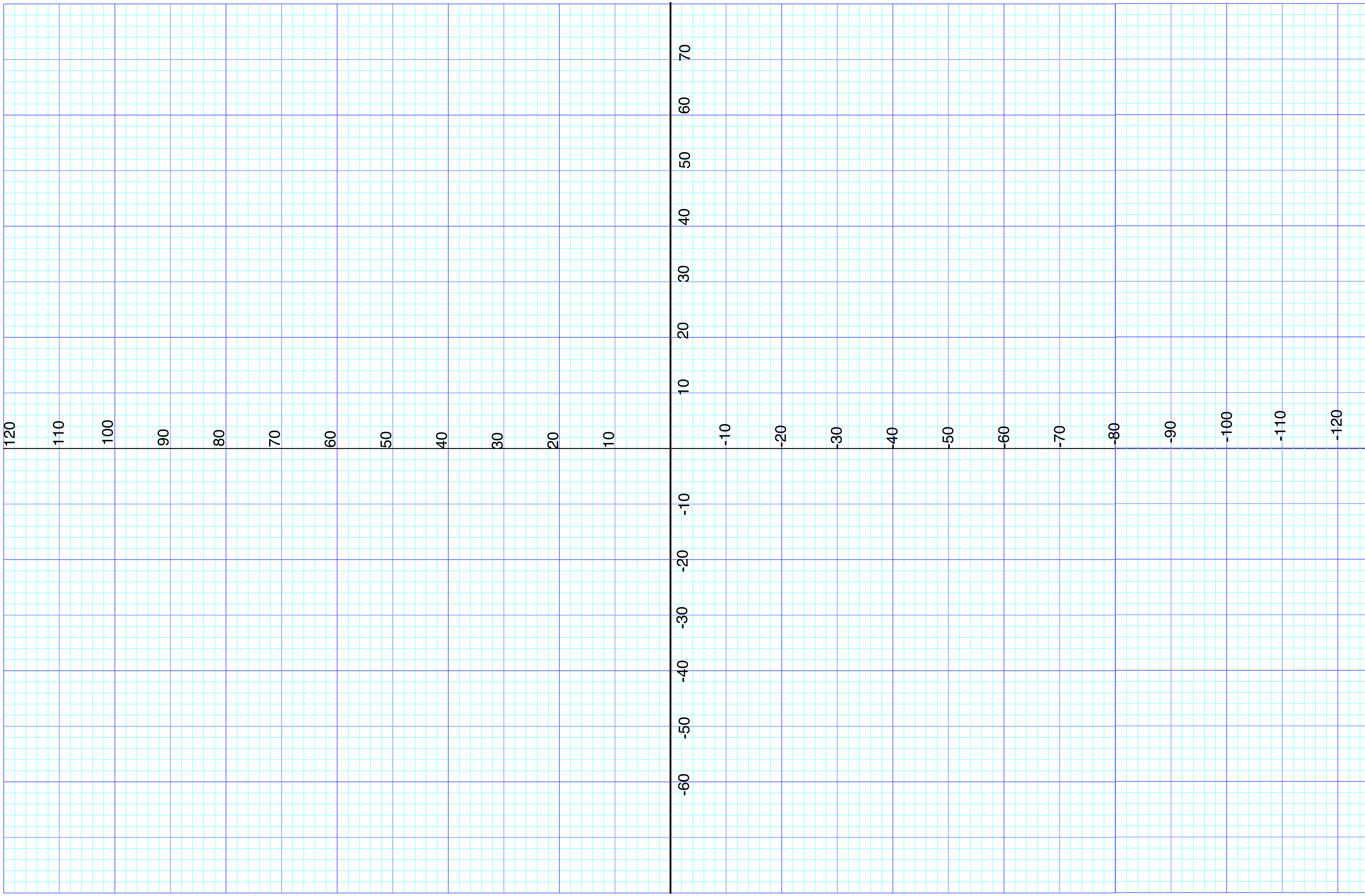
Developing
Conservation-of-Momentum
The key axiom of mechanics
leading to
Conservation-of-Energy Theorem

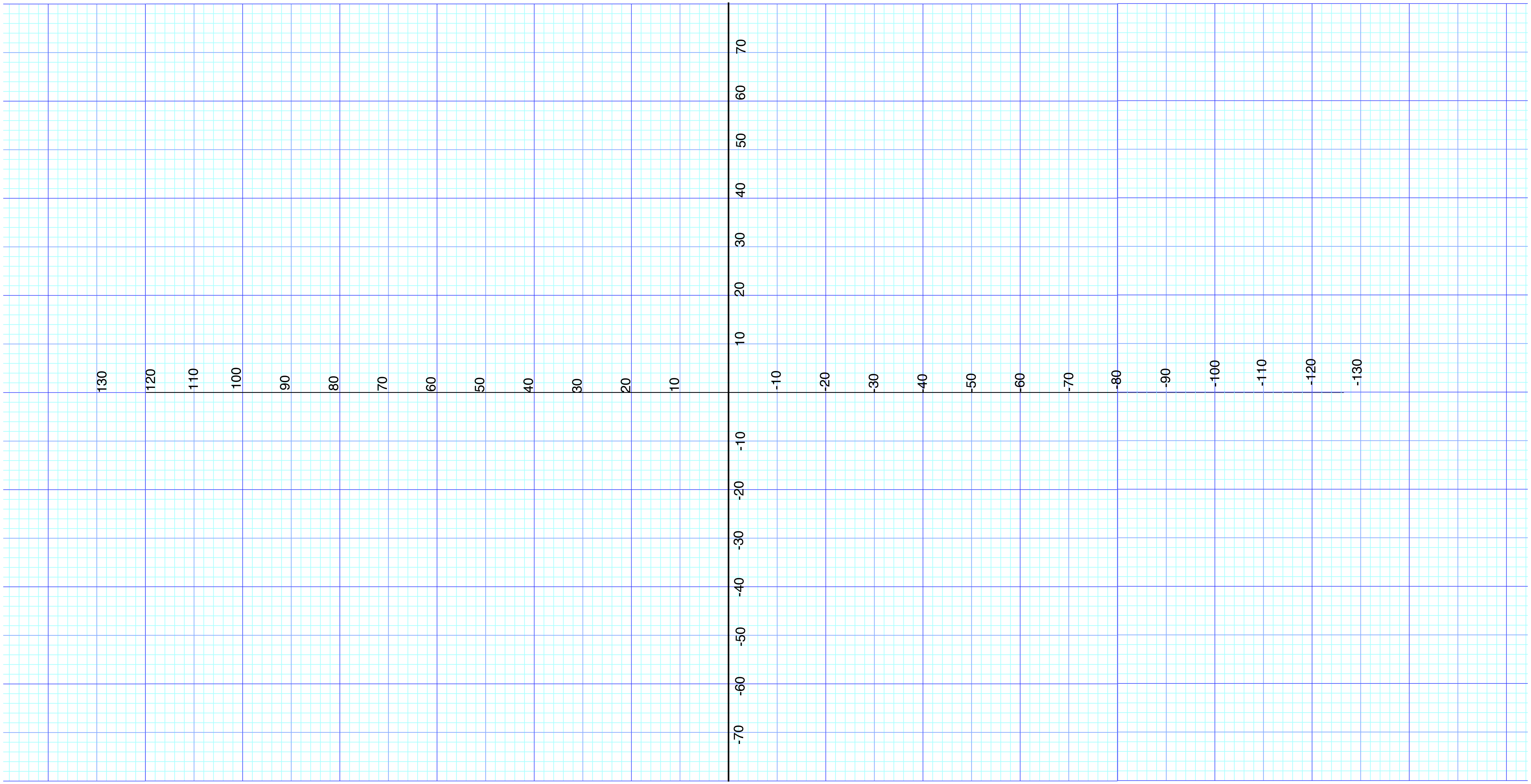
If and only if...
*there is **T-Symmetry***

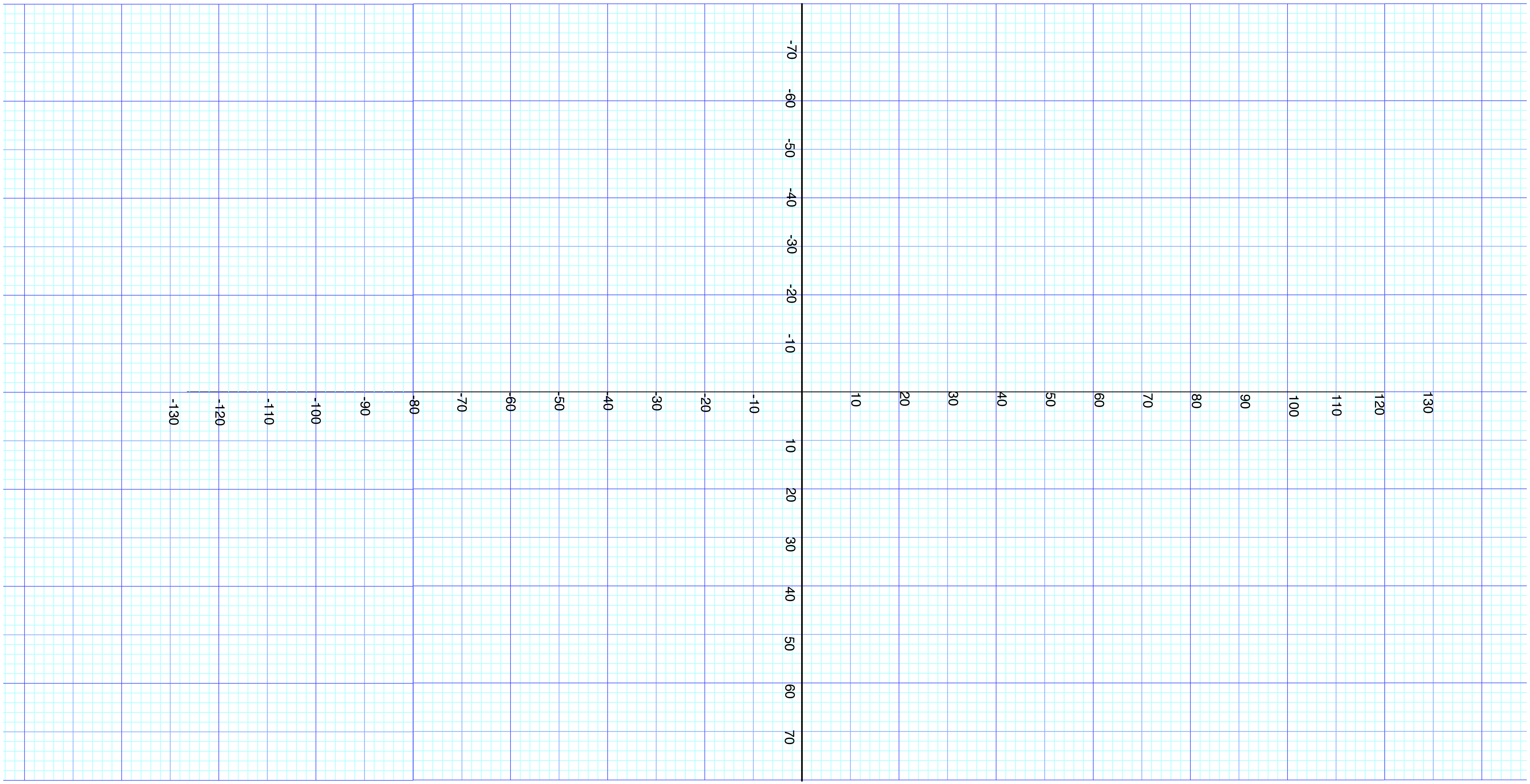
$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM} - 1/2\,\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=1/2\,\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN} = 1/2\,\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

These are equations for energy conservation ellipse:

$$KE = {}^{1}/_{2} M_{SUV} V_{SUV}^{\;2} + {}^{1}/_{2} M_{VW} V_{VW}^{\;2}$$


120 110 100 90 80 70 60 50 40 30 20 10
-10 -20 -30 -40 -50 -60 -70 -80 -90 -100 -110 -120
70 60 50 40 30 20 10
-10 -20 -30 -40 -50 -60


70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60
-70
130
120
110
100
90
80
70
60
50
40
30
20
10
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130


-70
-60
-50
-40
-30
-20
-10
-130
-120
-110
-100
-90
-80
-70
-60
-50
-40
-30
-20
-10
10
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60
70
80
90
100
110
120
130
10
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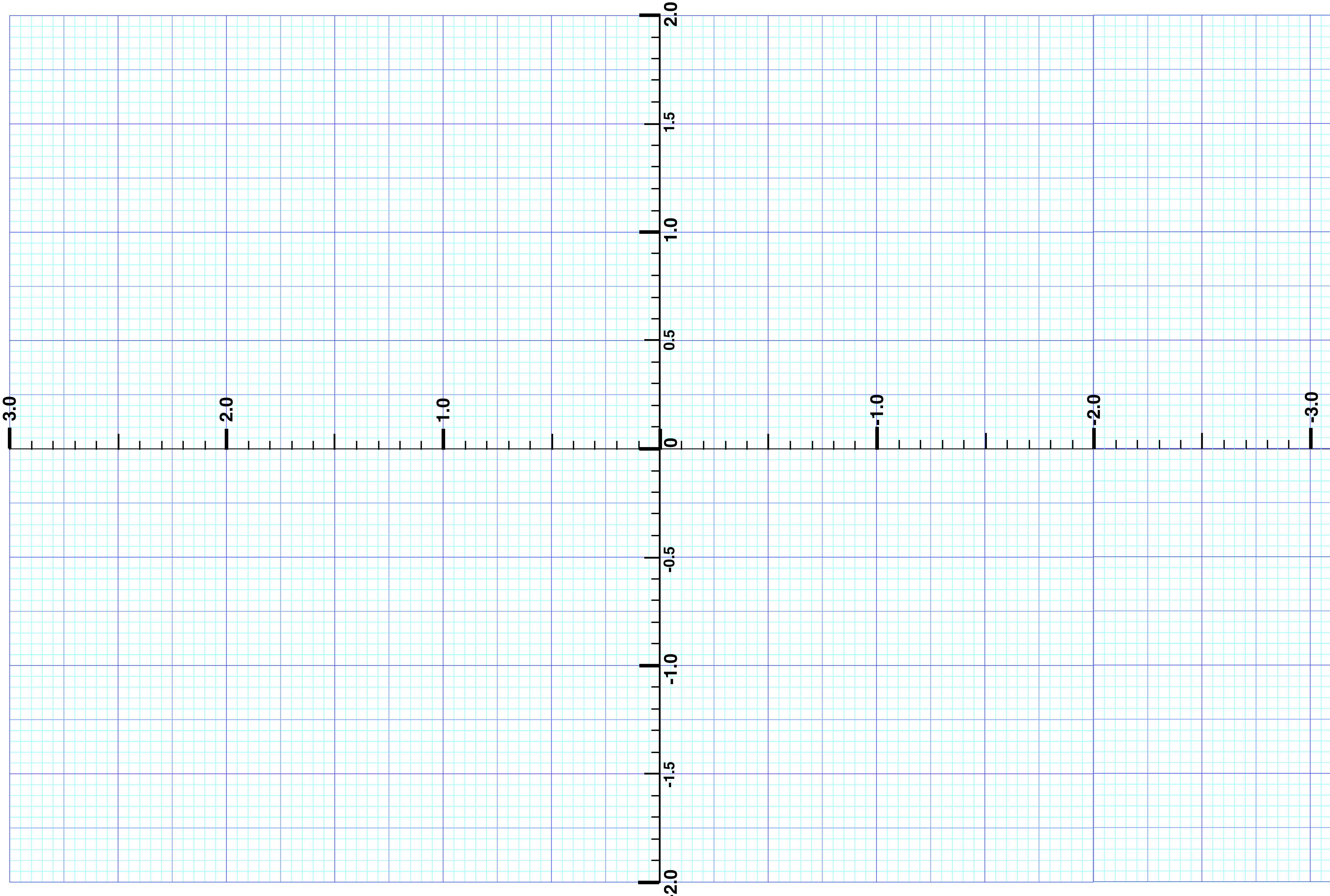
40
50
60
70


2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
3.0
2.0
1.0
-1.0
-2.0
-3.0

Note “crunch” energy $_{Elastic}KE - _{inelastic}IE = 0.21$
is the same in all frames including COM-frame.

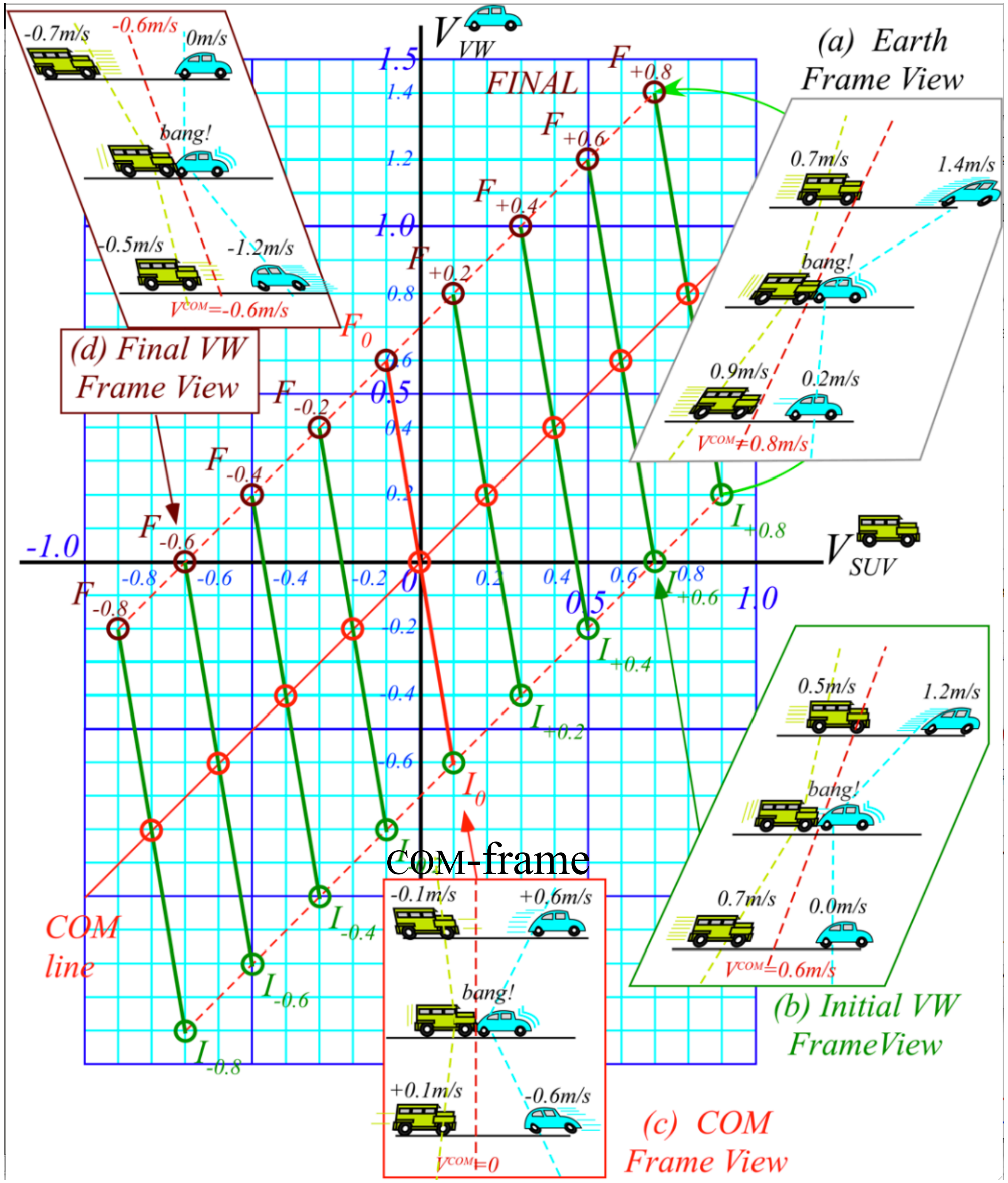


Fig. 3.4 Galilean Frame Views of collision like Fig. 2.5 or Fig. 3.1 with Bush (6:1) SUV.
(a) Earth frame view (b) Initial VW frame (VW initially fixed)
(c) COM frame view (d) Final VW frame (VW ends up fixed)

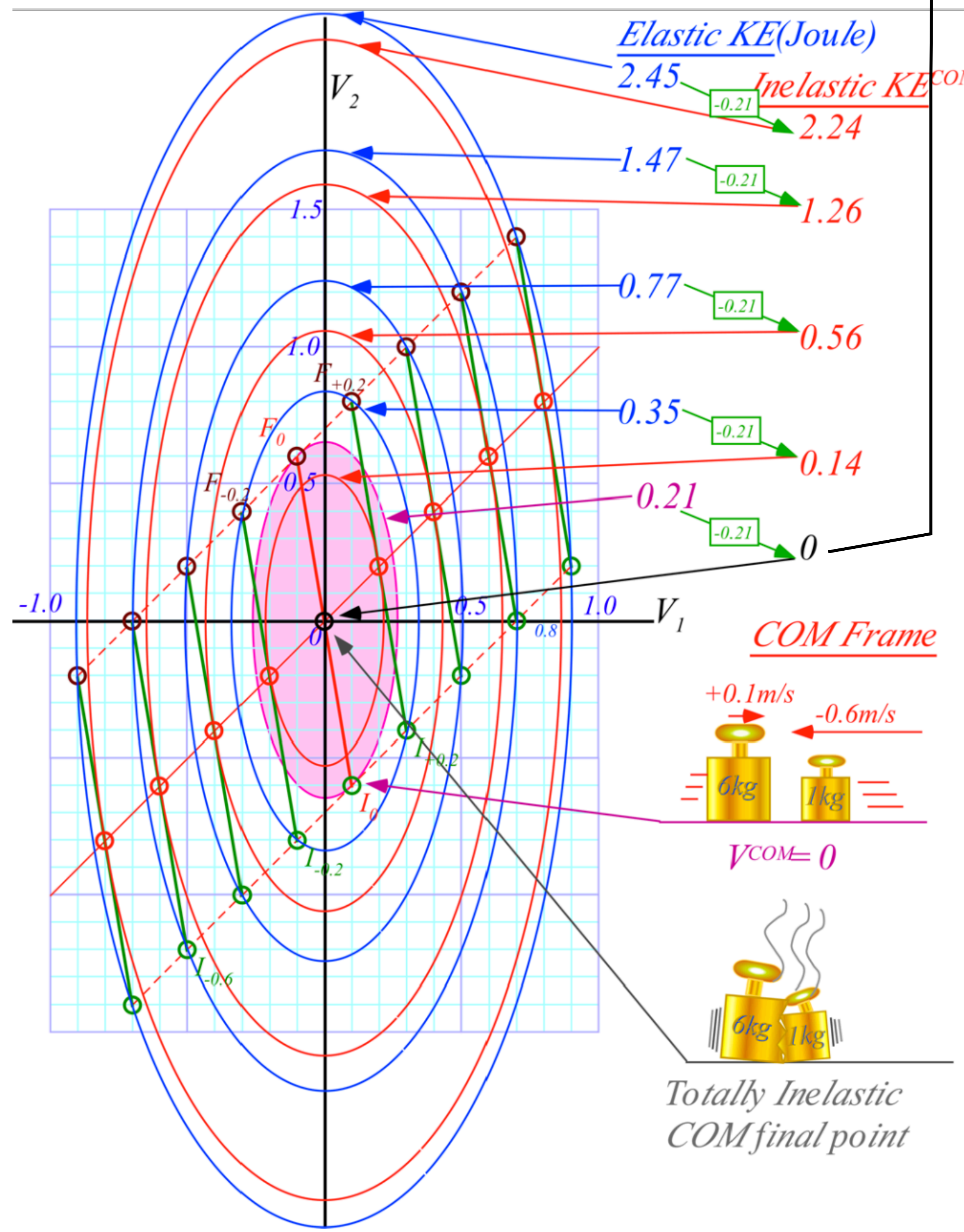


Fig. 3.5 Momentum (P=const.)-lines and energy (KE=const.)-ellipses appropriate for Fig. 3.4.