Lecture 24 Relawavity Introduction Part 2
Thursday 4.07.2016

Relawavity: Relativistic wave mechanics II. 2nd-order effects

(Unit 3 4.05.16)

➔ Review of Doppler-shift and Rapidity ρ_{AB} calculation: *Galileo's Revenge Part I Lect. 23 p.64-75*
- Relating rapidity ρ_{AB} and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

Review of ½-sum-½-difference Phase and Group factors giving relativistic space-axes and time-axes
- Colliding-CW space-time (x,ct)-graph *vs* Colliding PW space-time (R,L)-baseball diamond

Review of ½-sum-½-difference of phasor angular velocity: *Galileo's Revenge Part II* (Pirelli site)
- Elementary models: 2-comb Moire′ patterns and cosine-law constructions

Bob, Alice, and Carla combine Doppler shifted ½-sum-½-difference Phase and Group factors
- Doppler shifted Phase vector $\mathbf{P}'$ and Group vector $\mathbf{G}'$ in per-space-time
- Minkowski coordinate grid in space-time
  - Animations that compare Doppler shifted colliding CW with colliding PW

The 16 parameters of Doppler-shifted 2-CW Minkowski geometry
- Doppler shifted Phase parameters
- Doppler shifted Group parameters
  - Lorentz transformation matrix and Two Famous-Name Coefficients

Thales Mean Geometry (*Thales of Miletus 624-543 BCE*) and its role in Relawavity
- Detailed geometric construction of relawavity plot for 1-octave Doppler $(\beta_{AB}=u_{AB}/c=3/5)$

Stellar aberration and the Epstein approach to SR

Review Doppler-shift and Rapidity calculation

ALICE'S LASER GAUNTLET

Alice: Hey, Bob and Carla! Read off your Doppler shift ratios $\langle B|A\rangle$ and $\langle C|A\rangle$ to my 600THz beam.

Also, rapidity ρ_{BA} and ρ_{CA} relative to me.

Now, Carla, what's your rapidity ρ_{CB} relative to Bob?

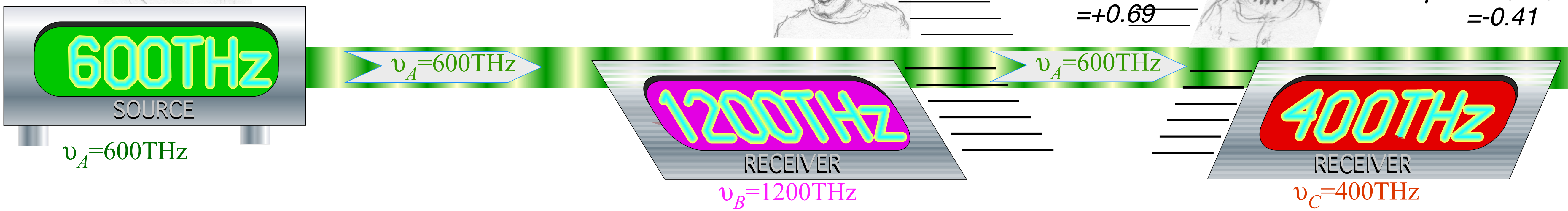
Bob: I see Doppler Blue shift to 1200THz

I got $\langle B|A\rangle$=2, and ρ_{BA} =ln(2) =+0.69

I got $\langle C|B\rangle=\langle C|A\rangle\langle A|B\rangle$=(2/3)(1/2)=1/3, and $\rho_{CB}=\rho_{CA}+\rho_{AB}$ =-1.10 We're in Splitsville!

Carla: I see Doppler Red shift to 400THz

I got $\langle C|A\rangle$=2/3, and ρ_{CA} =ln(2/3) =-0.41



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}}$$

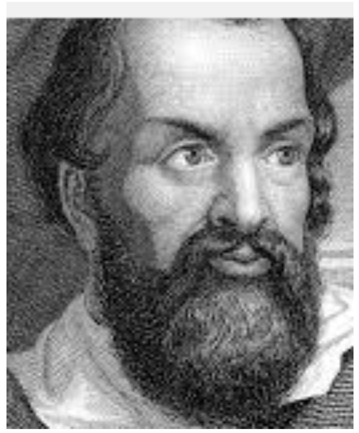
Definition of Rapidity

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A}$$

is time-reversed

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B}$$

Happy now, Galileo?



Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \log_e \langle B|A\rangle = \log_e \frac{2}{1}$$

$$\rho_{BA} = 0.69 \qquad (so: \rho_{AB} = -0.69)$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

$$\rho_{CA} = -0.41$$

Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle$$

Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} \quad \text{implies:} \quad \rho_{CB} = \rho_{CA} + \rho_{AB}$$

$$= -0.41 - 0.69 = -1.10$$

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

Review of Doppler-shift and Rapidity ρ_{AB} calculation: *Galileo's Revenge Part I Lect. 23 p.64-75*

➔ Relating rapidity ρ_{AB} and relativity velocity parameter $\beta_{AB}=u_{AB}/c$ ⬅

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More at Pirelli Challenge page: *'Un Grande Affare' - Light Meets Light*

Relating rapidity ρ_{AB} and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

Imagine Bob sees a pair of counter-propagating laser beams with wavevectors $k_R=+\omega_R/c$ and $k_L=-\omega_L/c$

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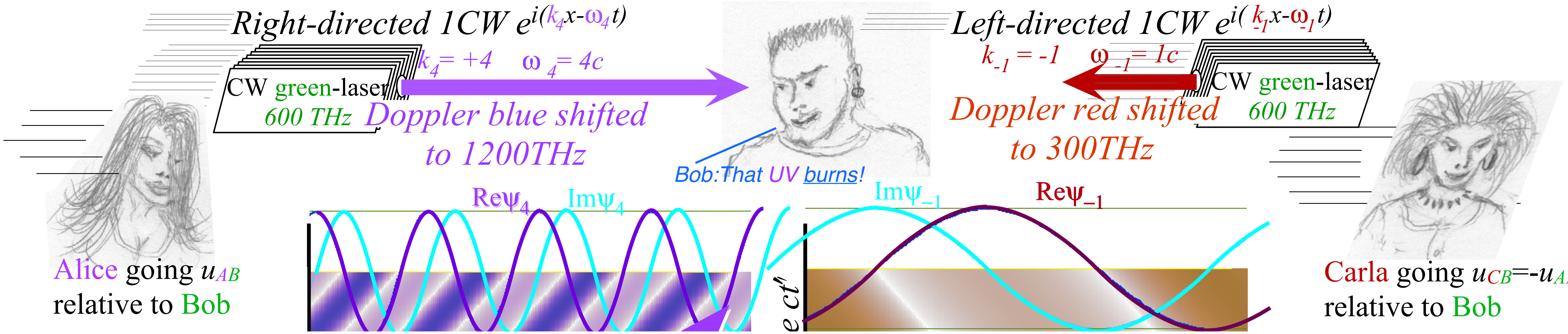

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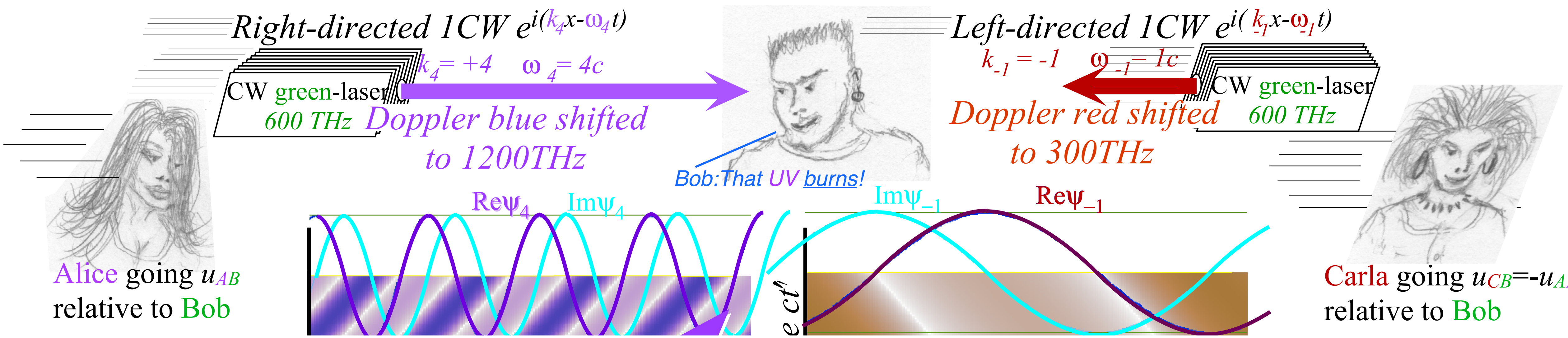

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(2.) What is that frequency ω_E ?

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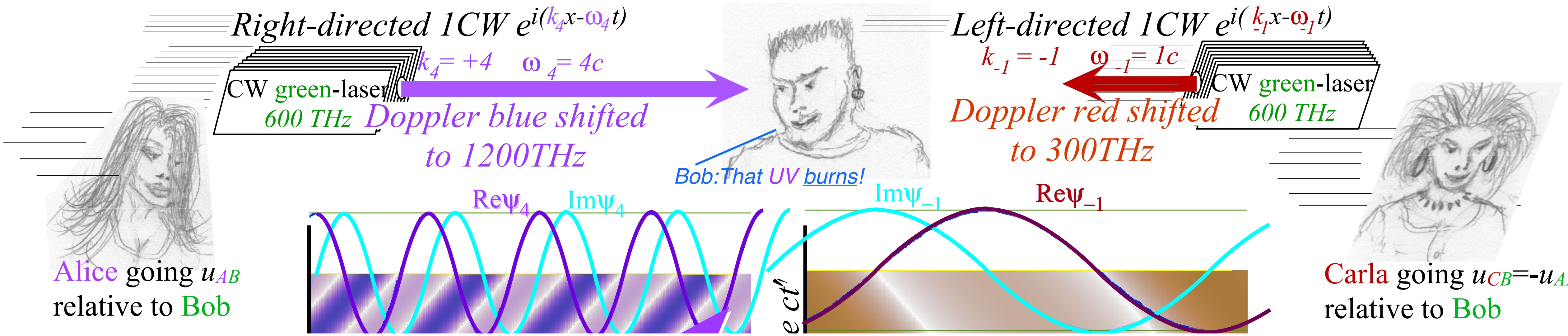

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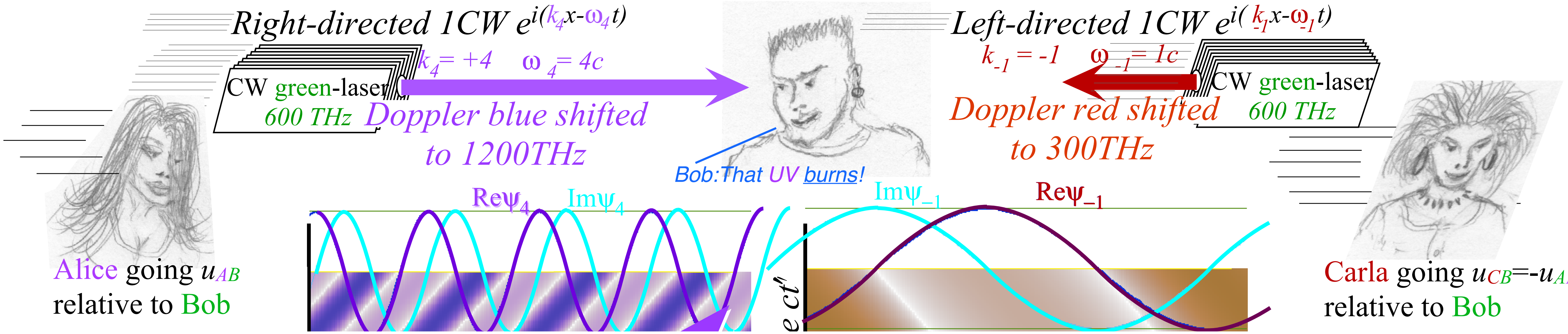

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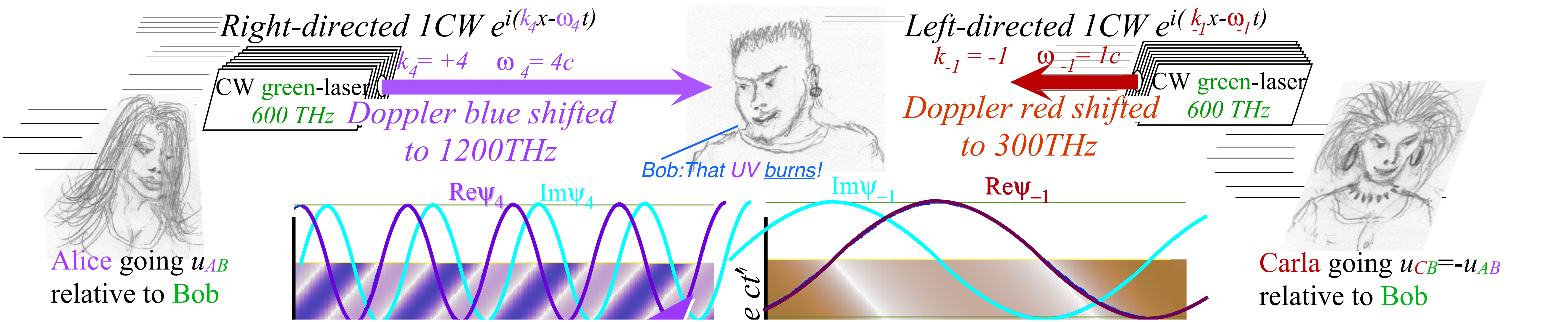

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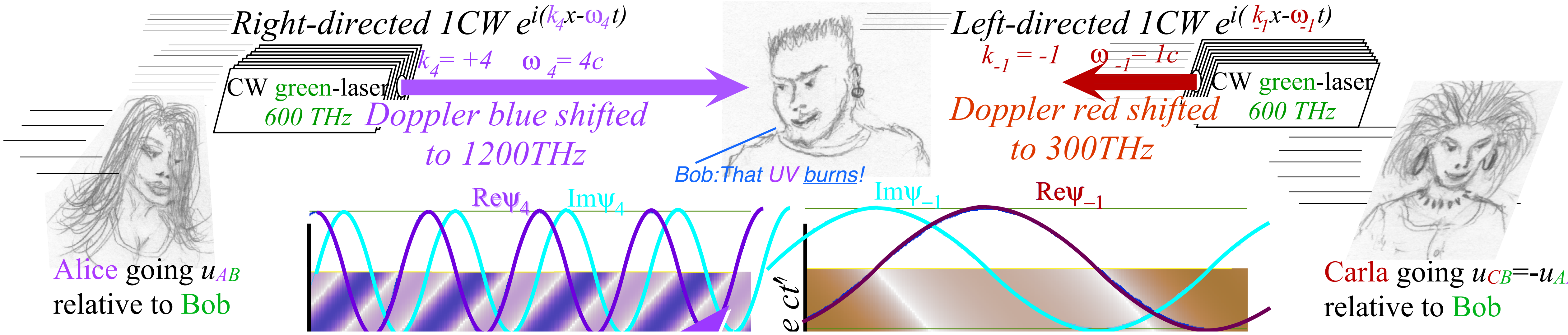

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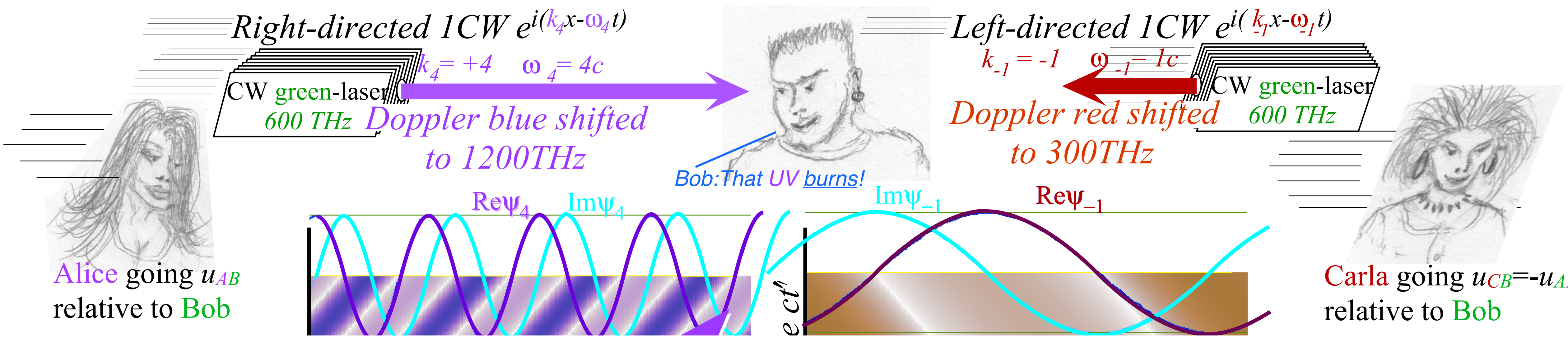

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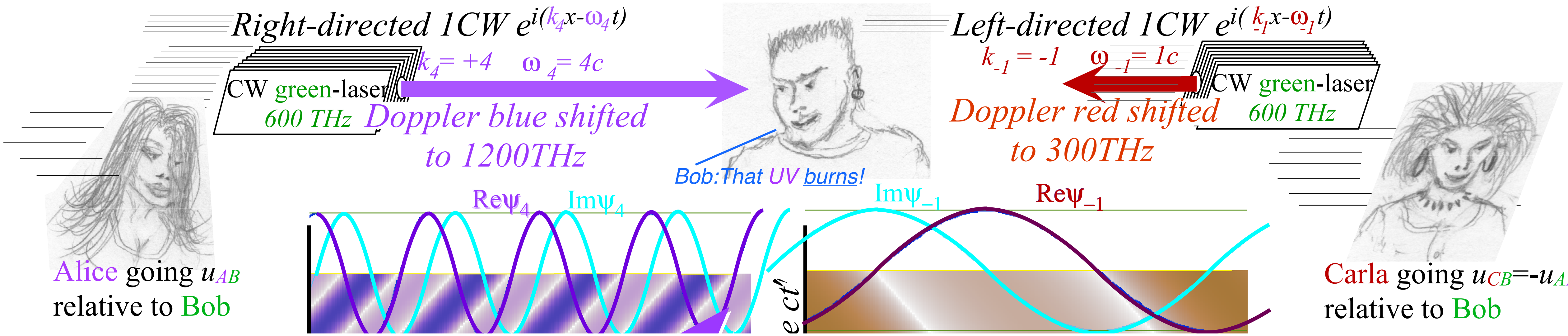

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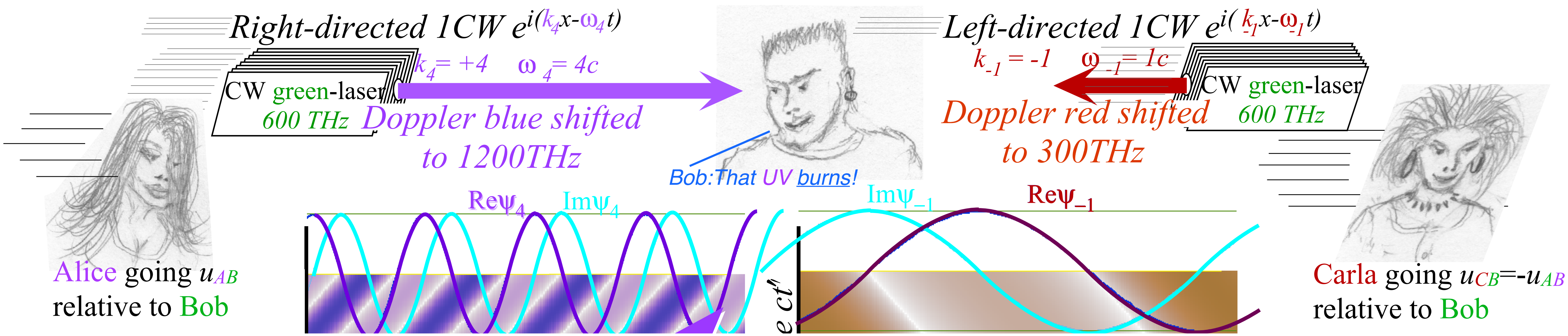

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Reply to Query (2.) in similar style:
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$$\omega_E=b\,\omega_L=\omega_R/b$$

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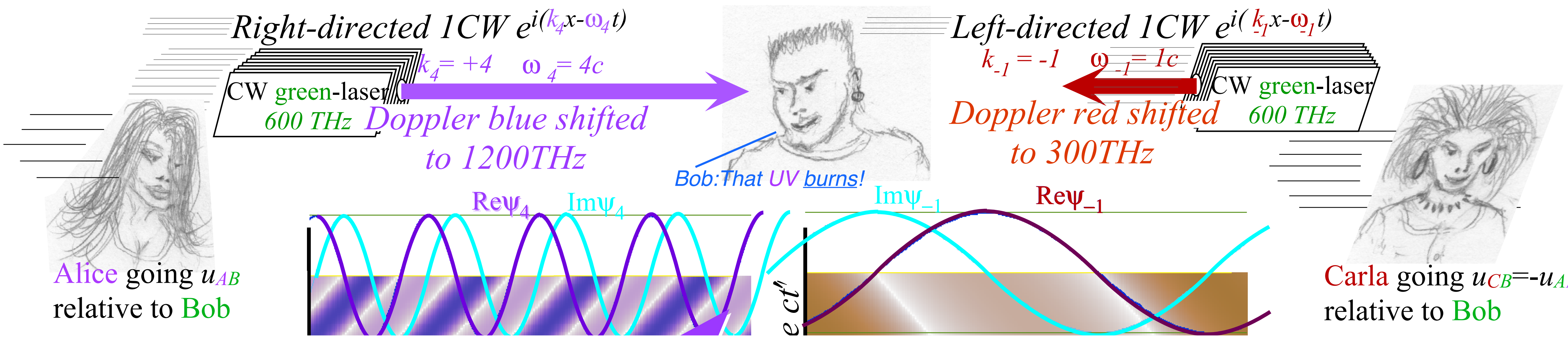

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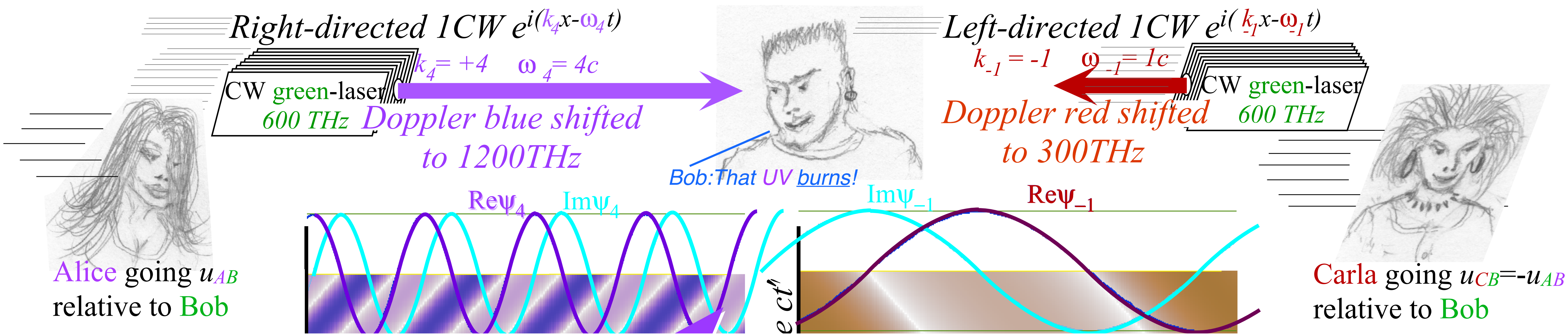

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(Geometric Mean)

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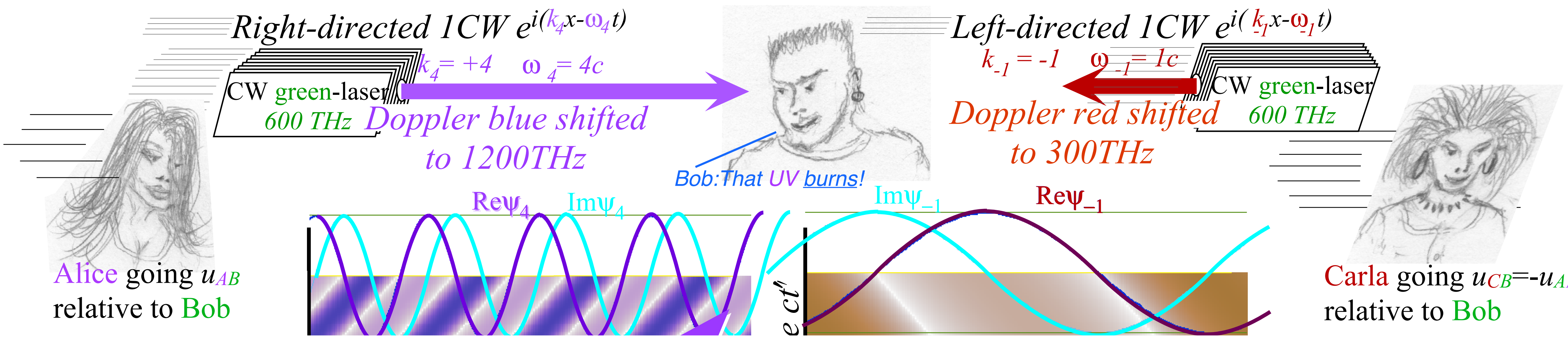

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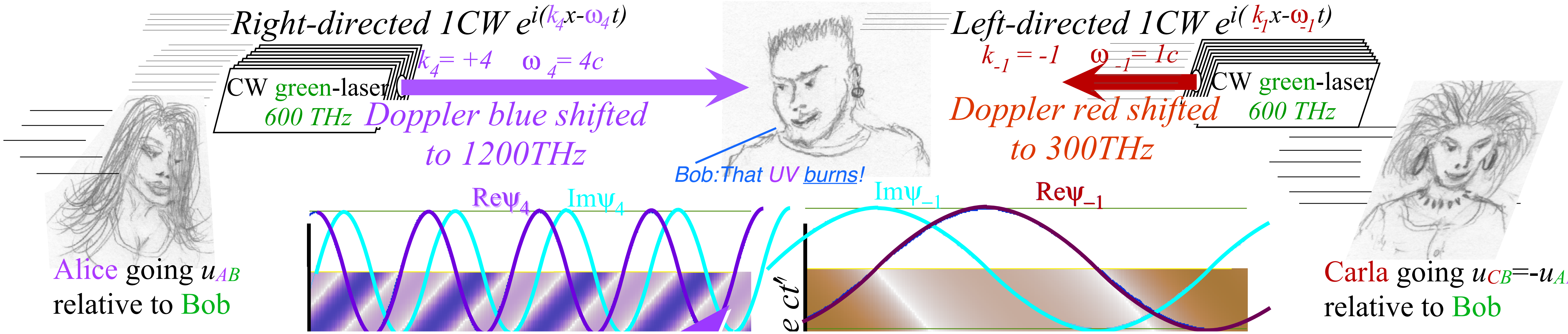

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Using *Rapidity*:
$\rho_{BA}=\log_e\langle B|A\rangle$

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Blue-shift $b=e^{\rho_{AB}}$

Red-shift $r=b^{-1}=e^{-\rho_{AB}}$

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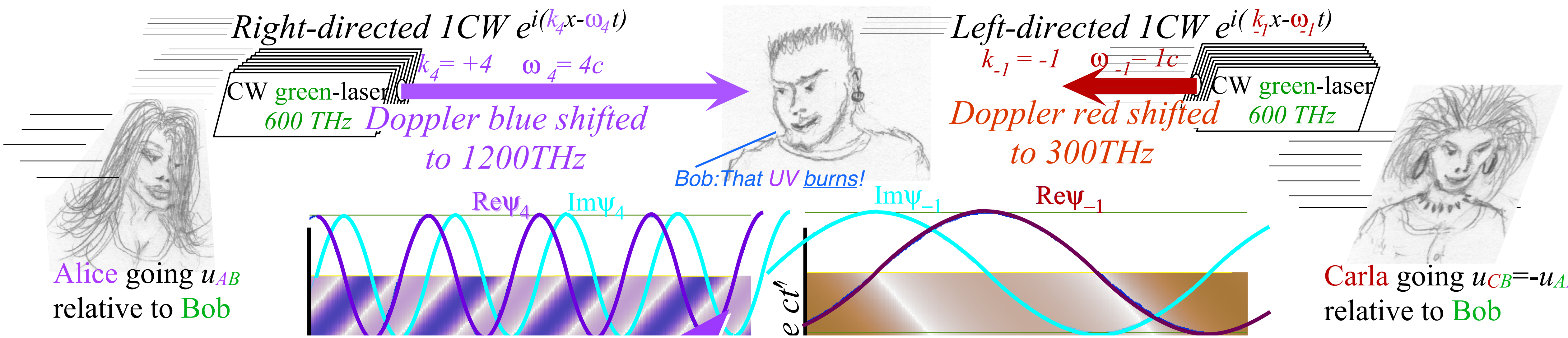

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Using *Rapidity*:
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Given: $\omega_R=e^{\rho_{AB}}\omega_{600}$

and: $\omega_L=e^{\rho_{CB}}\omega_{600}=e^{-\rho_{AB}}\omega_{600}$

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Blue-shift $b=e^{\rho_{AB}}$

Red-shift $r=b^{-1}=e^{-\rho_{AB}}$

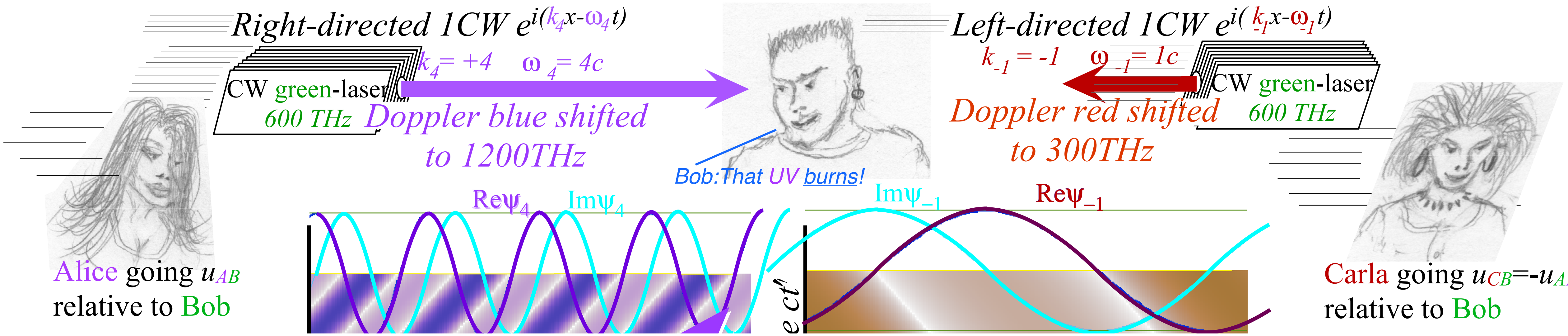
$$\omega_E=b\,\omega_L=\omega_R/b \quad\Rightarrow\quad b=\sqrt{\omega_R/\omega_L} \quad\Rightarrow\quad \omega_E=\sqrt{\omega_R\cdot\omega_L} = \sqrt{1200\cdot 300}=600THz$$

(Geometric Mean)

Relating rapidity ρ_{AB} and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

Imagine Bob sees a pair of counter-propagating laser beams with wavevectors $k_R=+\omega_R/c$ and $k_L=-\omega_L/c$

$\omega_R=\omega_A$ going left-to-right (from Alice's$_{600\text{THz}}$laser) and $\omega_L=\omega_C$ going right-to-left (from Carla's$_{600\text{THz}}$laser).



We ask two questions:

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E ?

(2.) What is that frequency ω_E ?

Reply to Query (1.) has a *Jeopardy*-style answer-by-question:

What is the beam group velocity?

$$\text{Given}: \omega_{group}=\frac{\omega_R-\omega_L}{2} \text{ and}: k_{group}=\frac{k_R-k_L}{2}$$

$$u_E=V_{group}=\frac{\omega_{group}}{k_{group}}=\frac{\omega_R-\omega_L}{k_R-k_L}=c\frac{\omega_R-\omega_L}{\omega_R+\omega_L}=c\frac{1200-300}{1200+300}=\frac{3}{5}c$$

with $k_R=+\omega_R/c$ and $k_L=-\omega_L/c$

$$\frac{u_E}{c}=\frac{u_{AB}}{c}=\frac{e^{\rho_{AB}}-e^{-\rho_{AB}}}{e^{\rho_{AB}}+e^{-\rho_{AB}}}=\frac{\sinh\rho_{AB}}{\cosh\rho_{AB}}=\tanh\rho_{AB}=\frac{3}{5}$$

Using *Rapidity*: $\rho_{AB}=\log_e\langle A|B\rangle$

Given: $\omega_R=e^{\rho_{AB}}\omega_{600}$

and: $\omega_L=e^{\rho_{CB}}\omega_{600}=e^{-\rho_{AB}}\omega_{600}$

Reply to Query (2.) in similar style:

What ω_E is blue-shift $b\omega_L$ of ω_L <u>*and*</u> red-shift ω_R/b of ω_R ?

Blue-shift $b=e^{\rho_{AB}}$

Red-shift $r=b^{-1}=e^{-\rho_{AB}}$

$$\omega_E=b\,\omega_L=\omega_R/b \quad\Rightarrow\quad b=\sqrt{\omega_R/\omega_L} \quad\Rightarrow\quad \omega_E=\sqrt{\omega_R\cdot\omega_L}=\sqrt{1200\cdot 300}=600THz$$

(Geometric Mean)

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Bob, Alice, and Carla combine Doppler shifted ½-sum-½-difference Phase and Group factors

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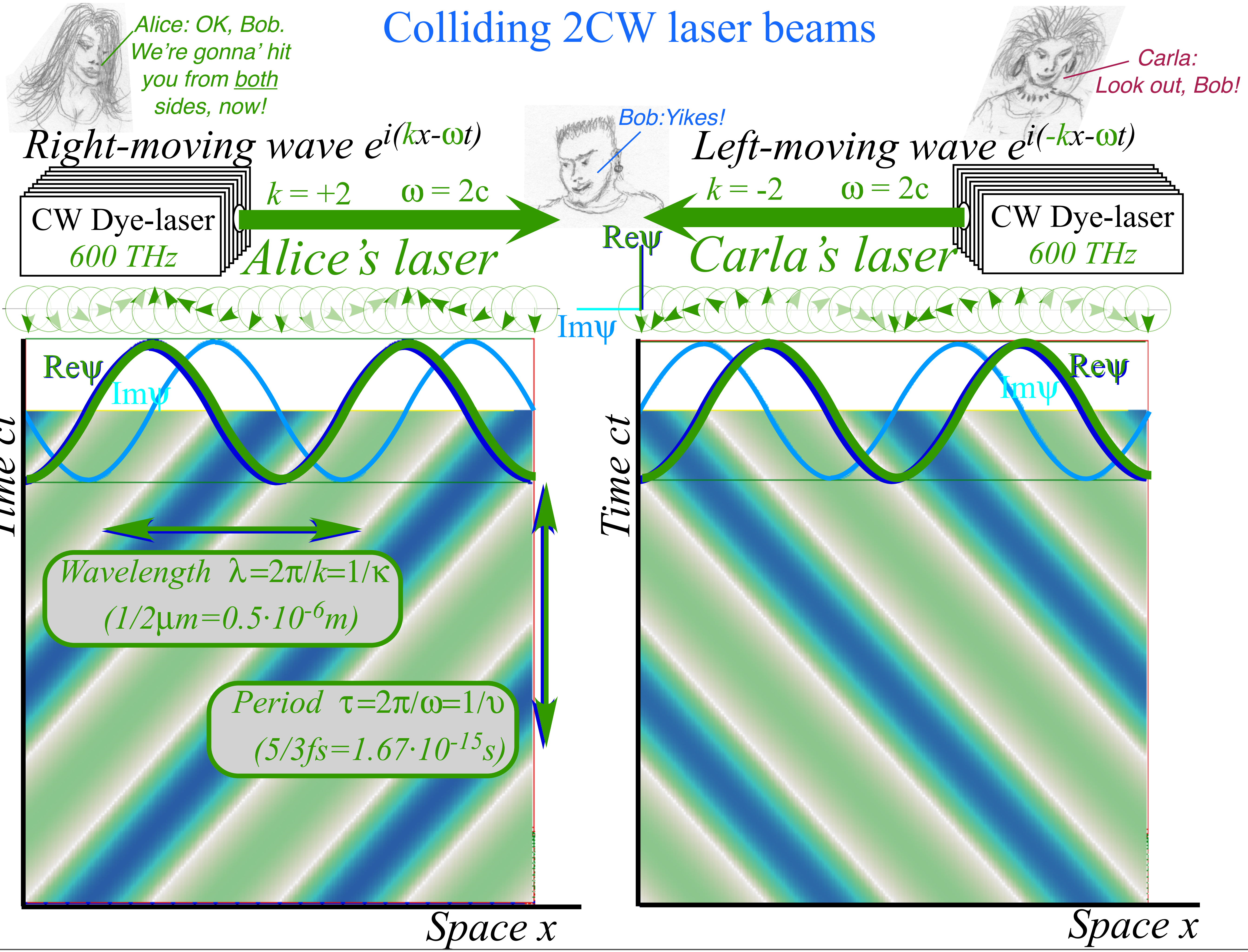
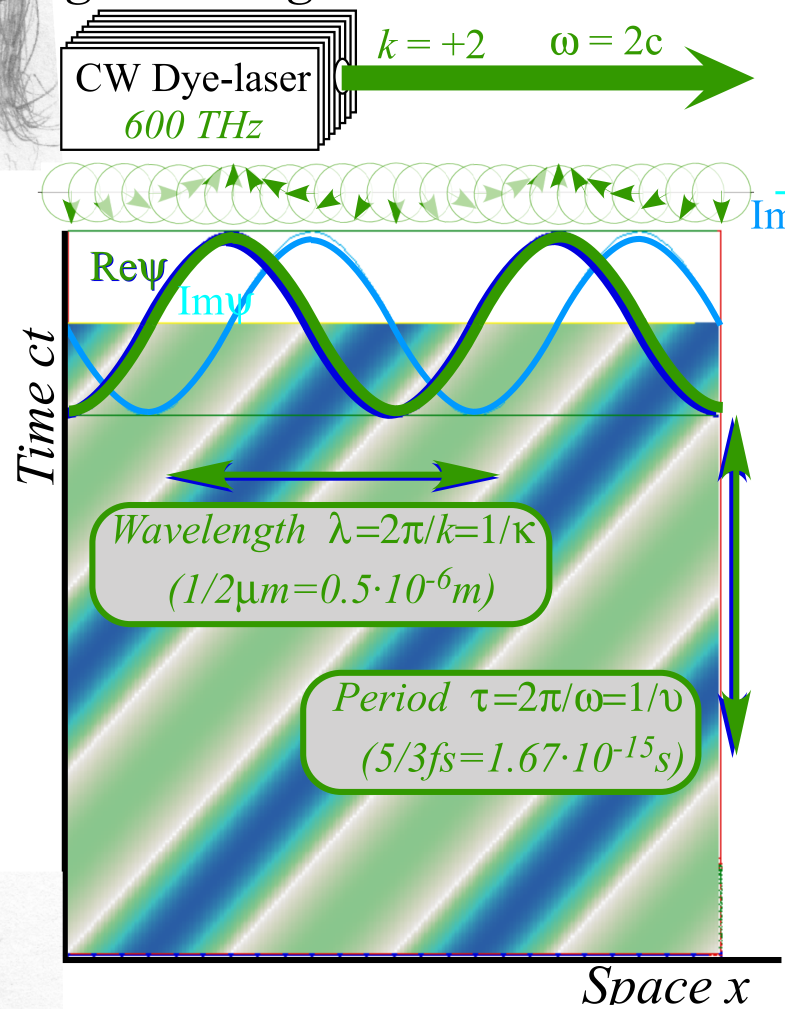
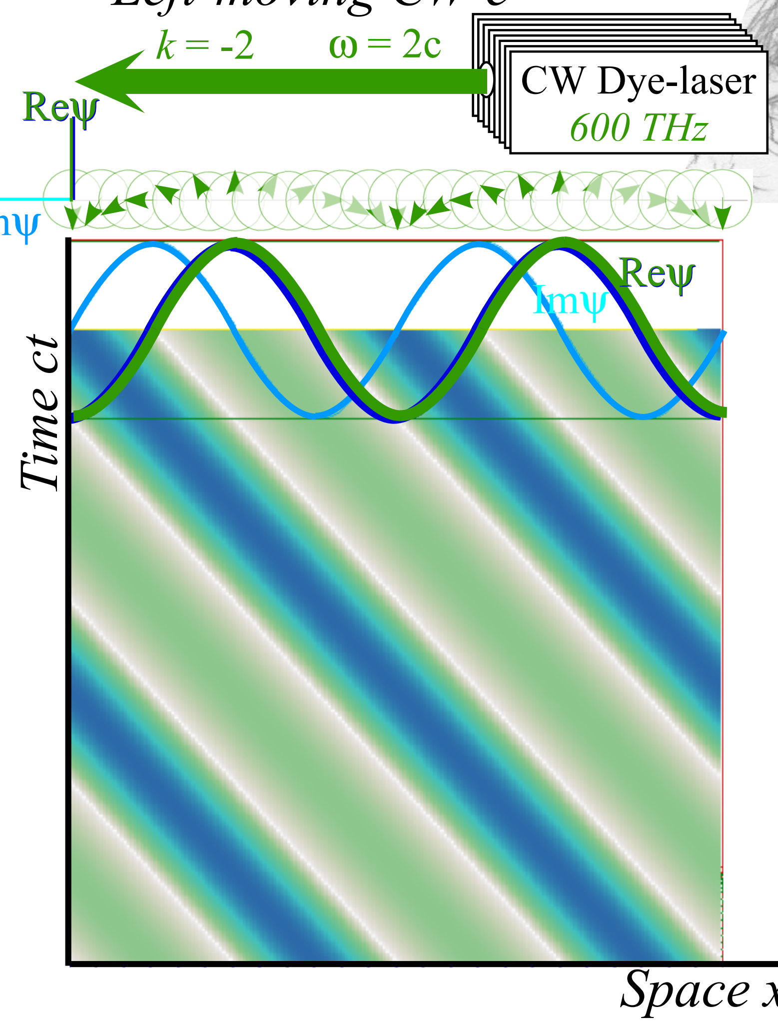
Colliding 2CW laser beams

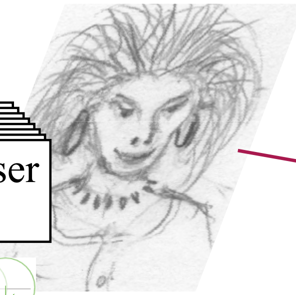


Right-moving CW $e^{i(kx-\omega t)}$



Left-moving CW $e^{i(-kx-\omega t)}$

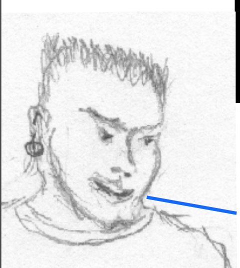




Carla:

Easy!

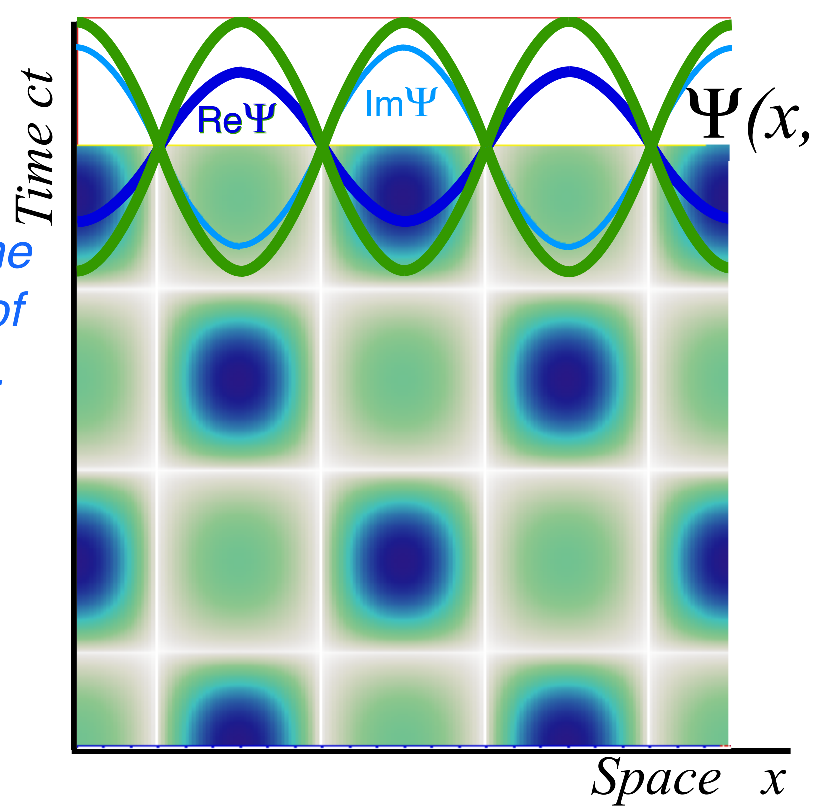
You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.



Bob:

Cool!
You guys made me a space-time graph out of real zeros.

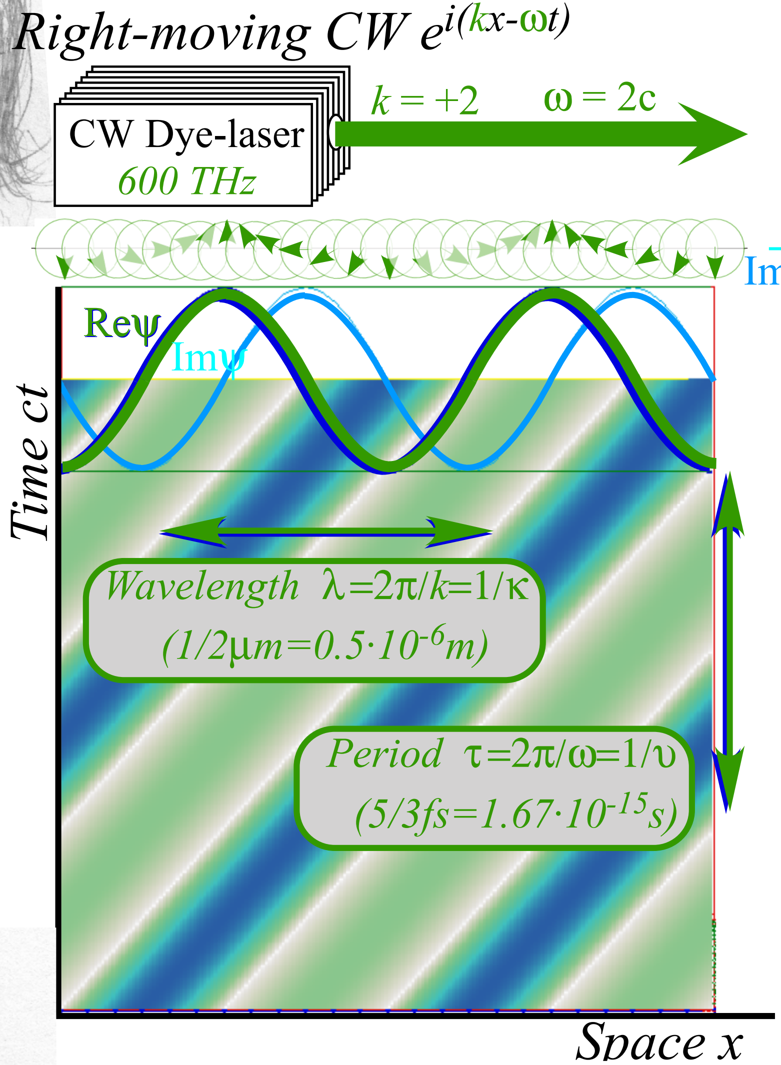
How'd it **do** that?

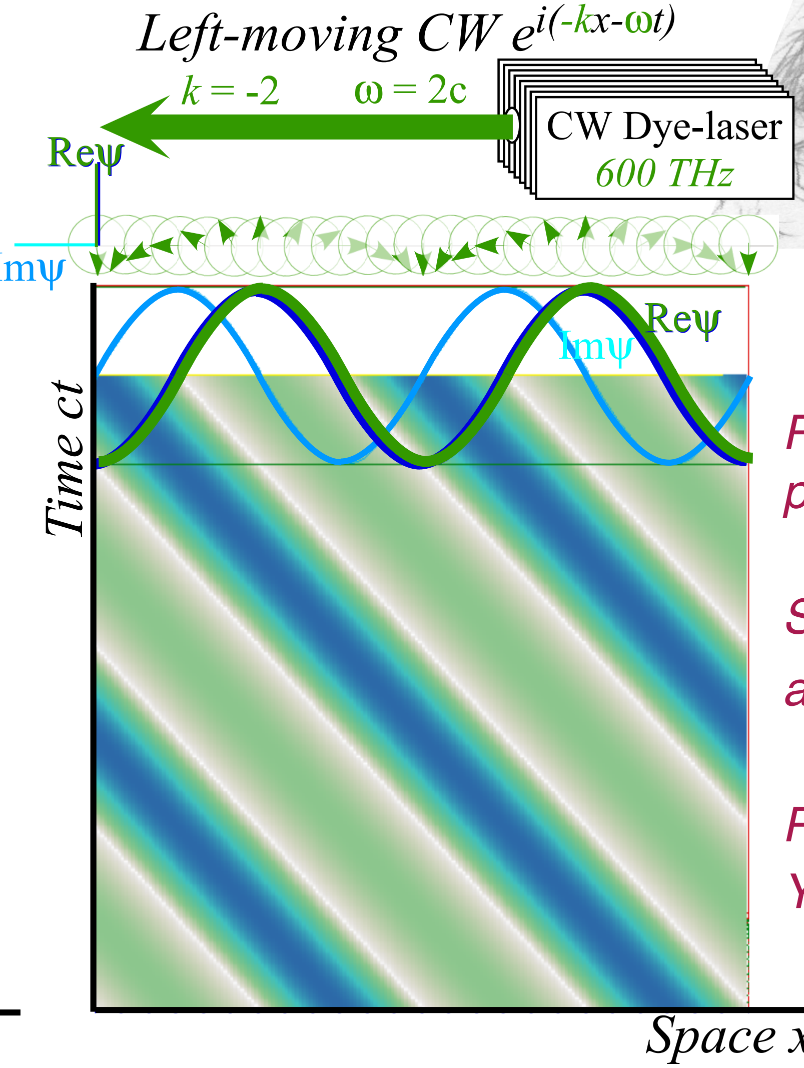


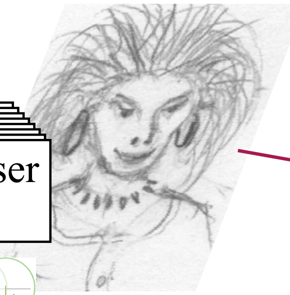
$$\Psi(x,t)=e^{ia}+e^{ib}, \quad a = kx-\omega t, \quad b = -kx-\omega t$$

[BohrIt Web Simulation
1 CW ct vs x Plot (ck = +1)
Single panel with Zero Tracers]

[BohrIt Web Simulation
2 CW ct vs x Plot (ck = ±2)
Multi-panel with Zero Tracers]







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Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives a,
and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives b.

Presto!
You factor $e^{ia}+e^{ib}$ into $e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$

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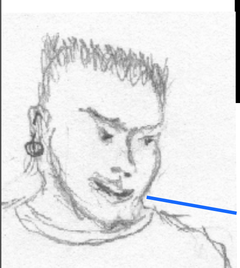
Carla 1CW phase: $b = -kx-\omega t$



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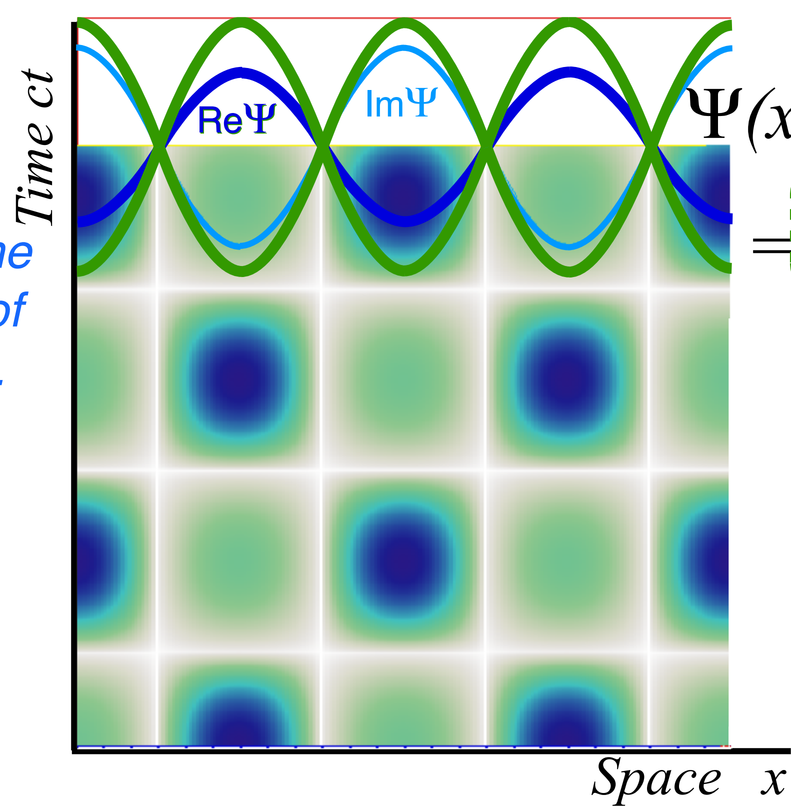
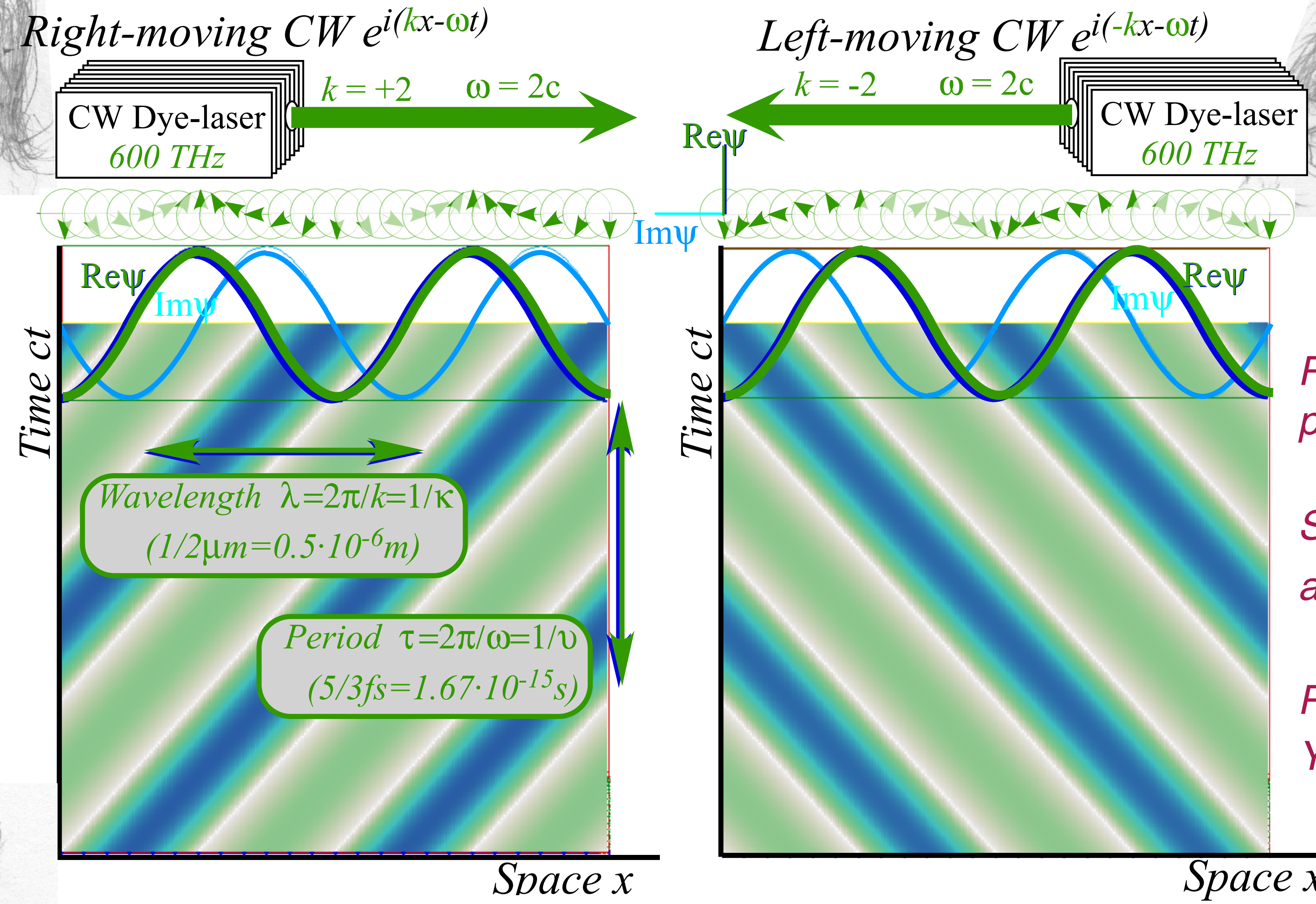
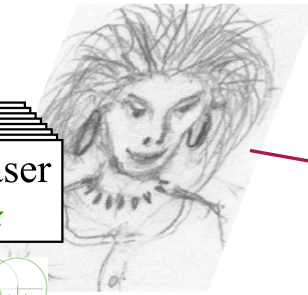


$$\Psi(x,t)=\underbrace{e^{ia}}_{kx-\omega t}+\underbrace{e^{ib}}_{-kx-\omega t}$$
$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$





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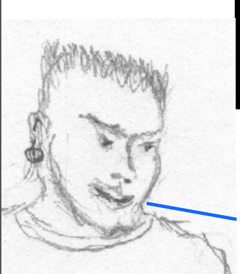

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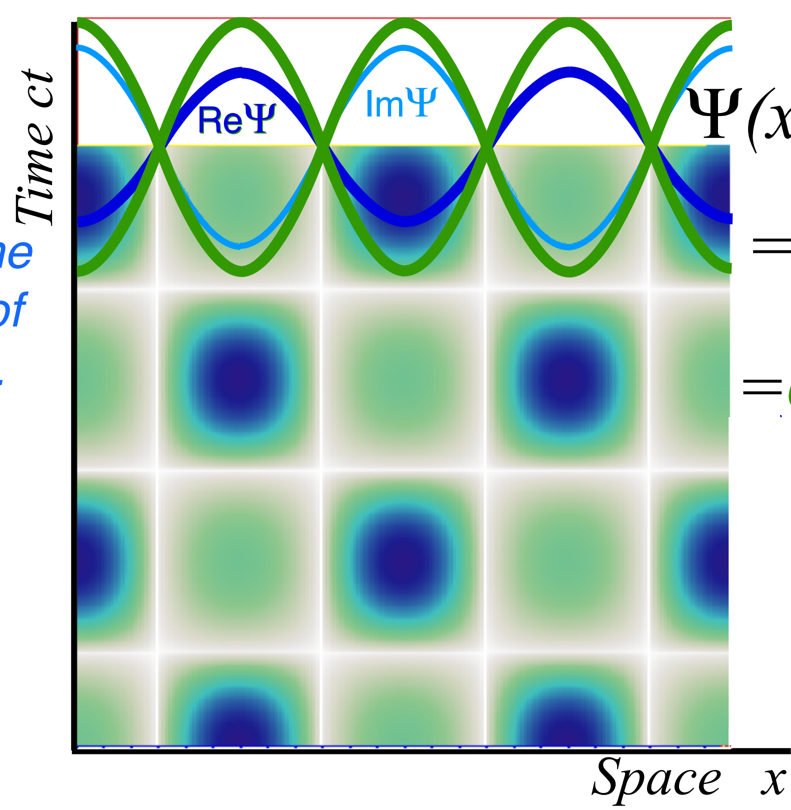
*How'd it **do** that?*



$$\Psi(x,t) = \underbrace{e^{ia}}_{kx-\omega t} + \underbrace{e^{ib}}_{-kx-\omega t}$$

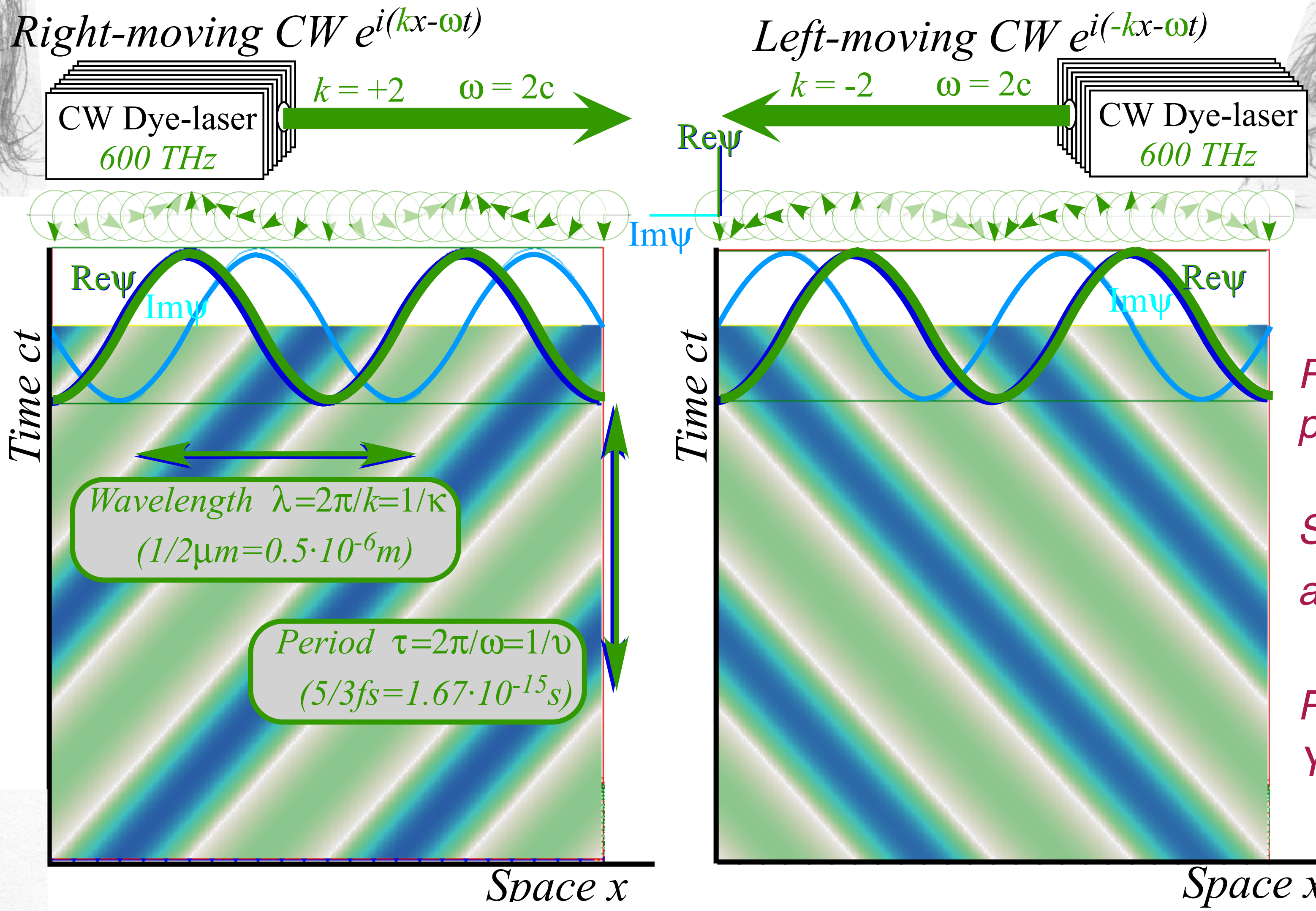
$$= e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

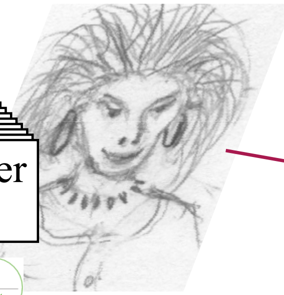
$$= e^{-i\omega t}\left(e^{ikx}+e^{-ikx}\right)$$

Bob's 2CW **Group**-*phase:* $+k = \frac{a-b}{2}$
Wave

Group wave: $e^{-ikx} + e^{-ikx} = 2\cos kx$

is *standing* wave (does not vary with time t)





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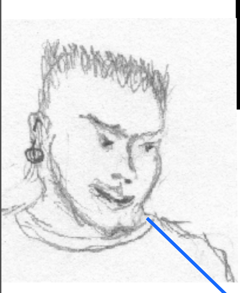
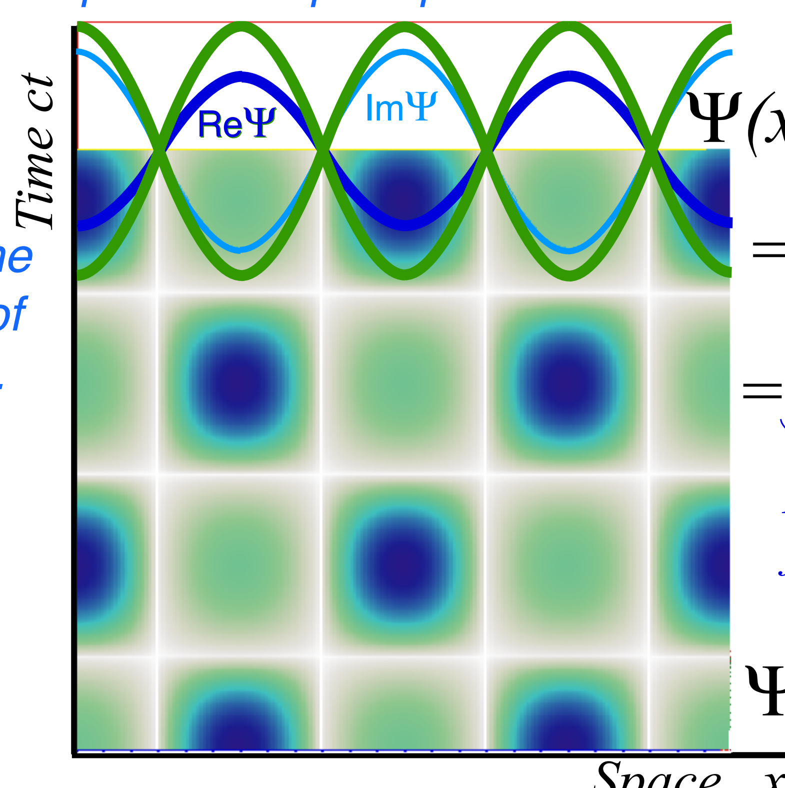


Bob: Let's plot this in per-spacetime?!

Cool!
You guys made me a space-time graph out of real zeros.

How'd it **do** that?



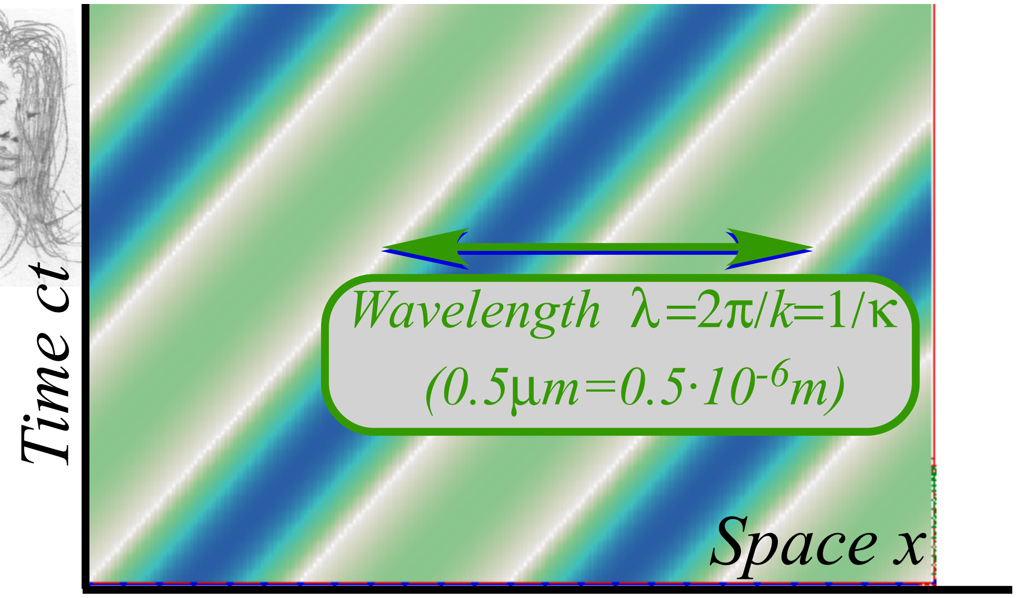
$$\Psi(x,t)=\overbrace{e^{ia}}^{kx-\omega t}+\overbrace{e^{ib}}^{-kx-\omega t}$$
$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$
$$=\underbrace{e^{-i\omega t}}_{\text{phase factor}}\underbrace{\left(e^{ikx}+e^{-ikx}\right)}_{\text{group factor}}$$
$$\Psi(x,t)=e^{-i\omega t}\,2\cos kx$$

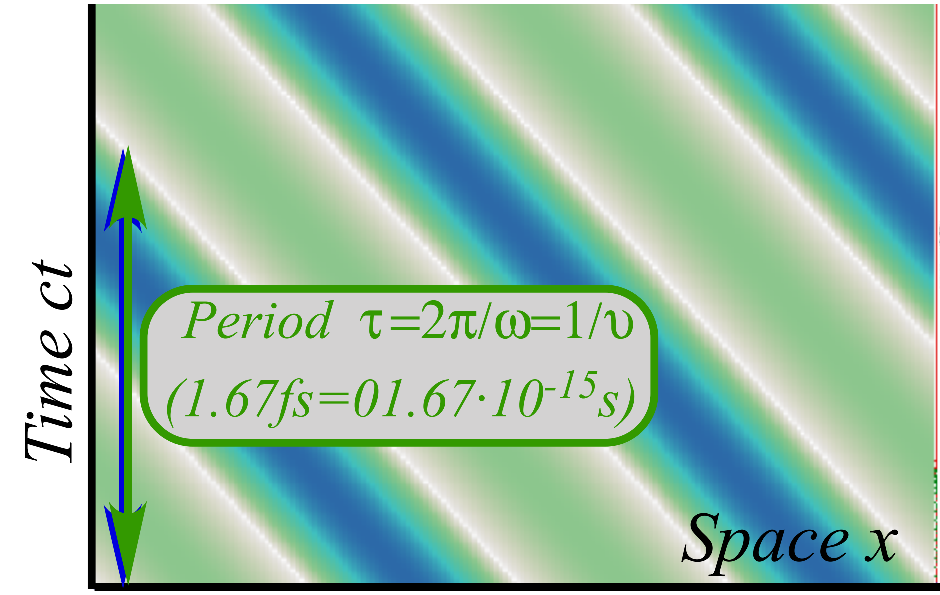
Bob's 2CW **Group**-phase: **Wave** $\quad +k=\frac{a-b}{2}$

Group wave: $e^{-ikx}+e^{-ikx}=2\cos kx$
is *standing* wave (does not vary with time t)

Bob's 2CW **Phase**-phase: **Wave** $\quad -\omega=\frac{a+b}{2}$

Phase wave real part: $\mathrm{Re}(e^{-i\omega t})=\cos(\omega t)$
is "*instanton*" wave (does not vary in space x)

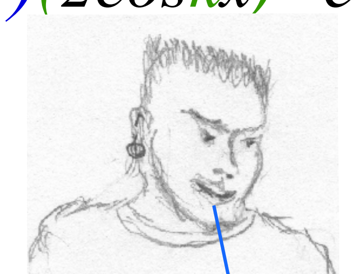




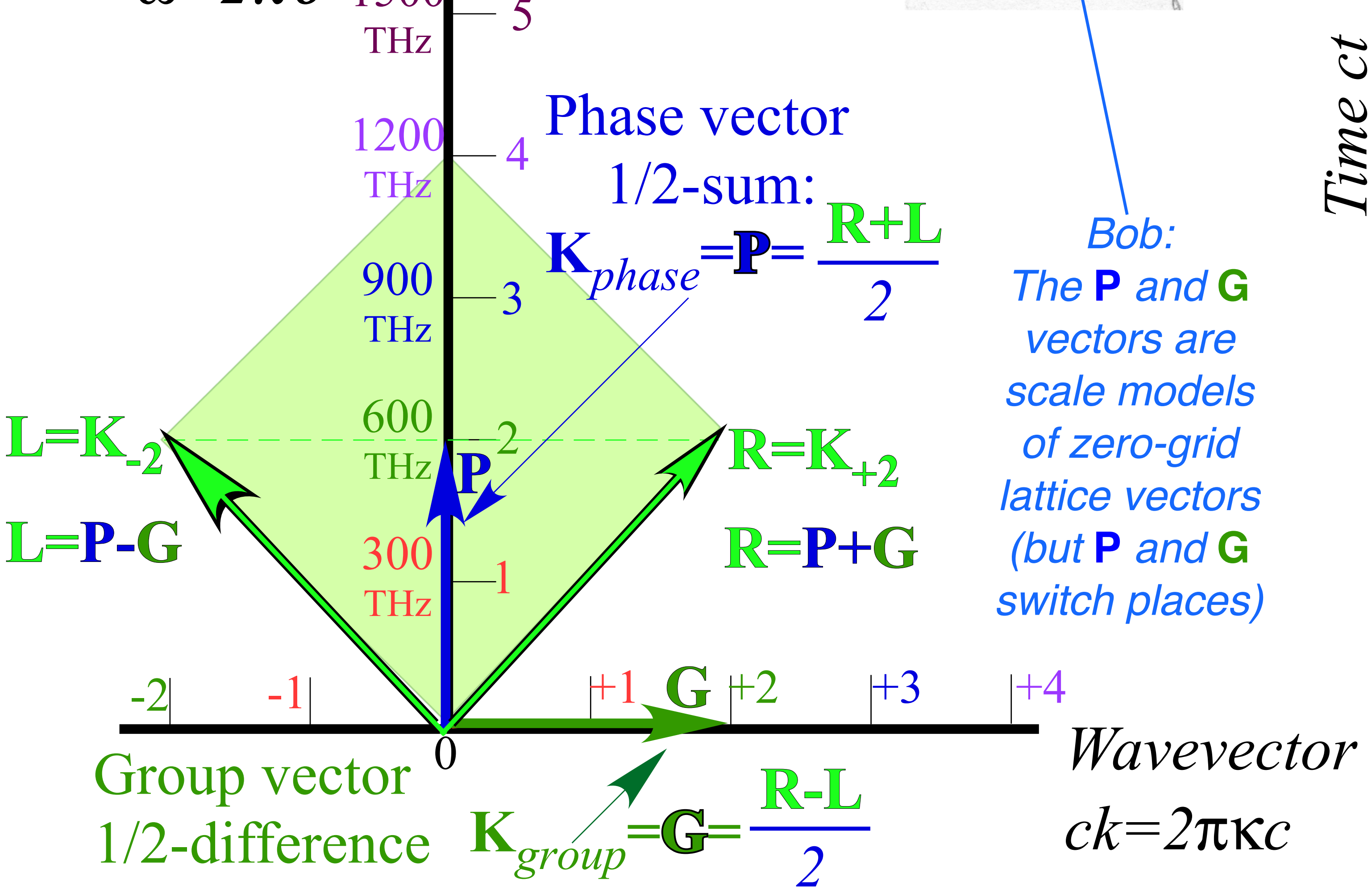
Carla:
OK, Bob!
It looks like a baseball diamond with
P at Pitcher's mound and
G at the Grandstand*.
I'm on 1st base! (**R**)

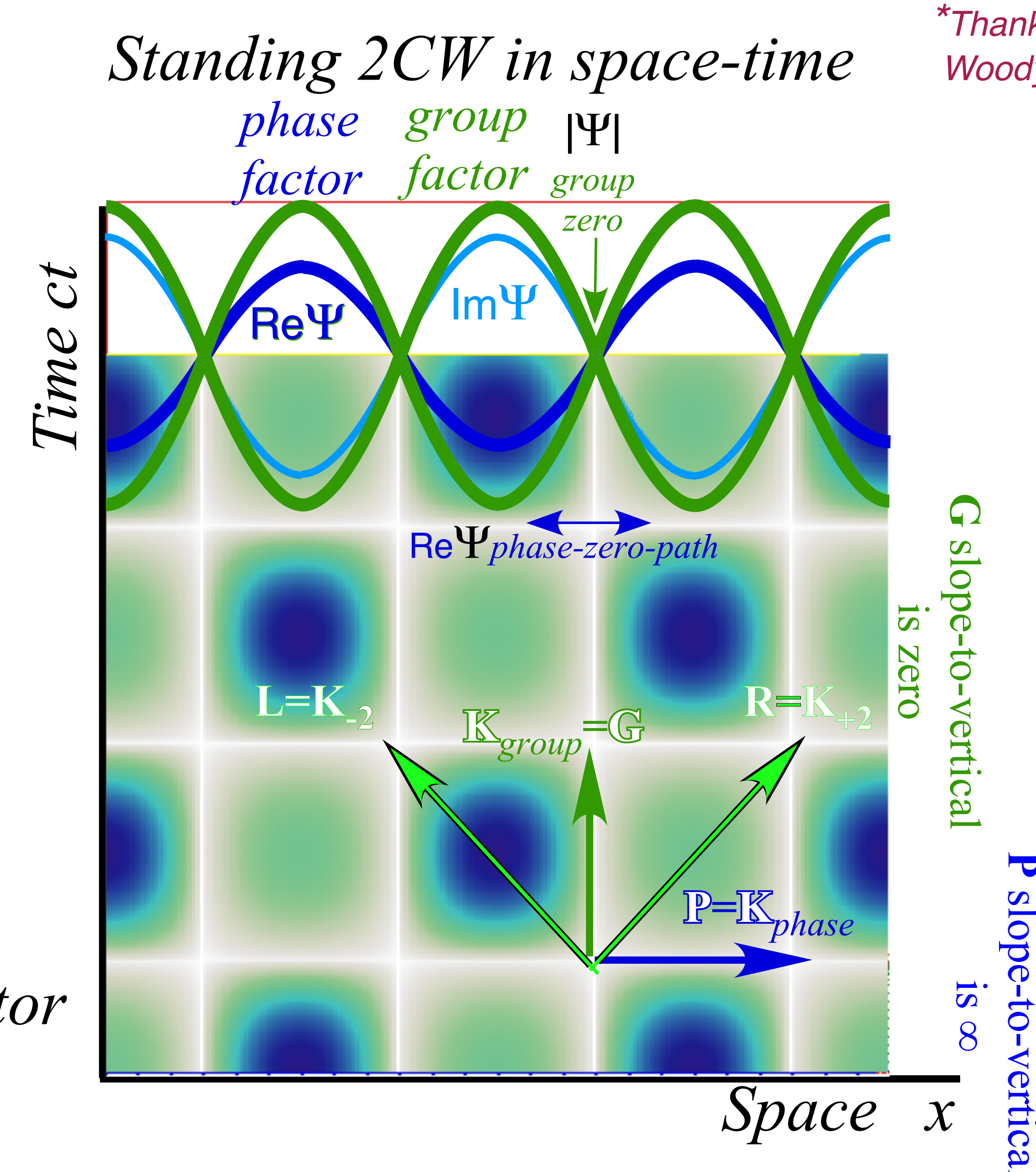
*Thanks, Woody!

$$\Psi(x,t)=(e^{-i\omega t})(2\cos kx)=e^{i(kx-\omega t)}+e^{i(-kx-\omega t)}$$



Bob:
The **P** and **G** vectors are scale models of zero-grid lattice vectors (but **P** and **G** switch places)





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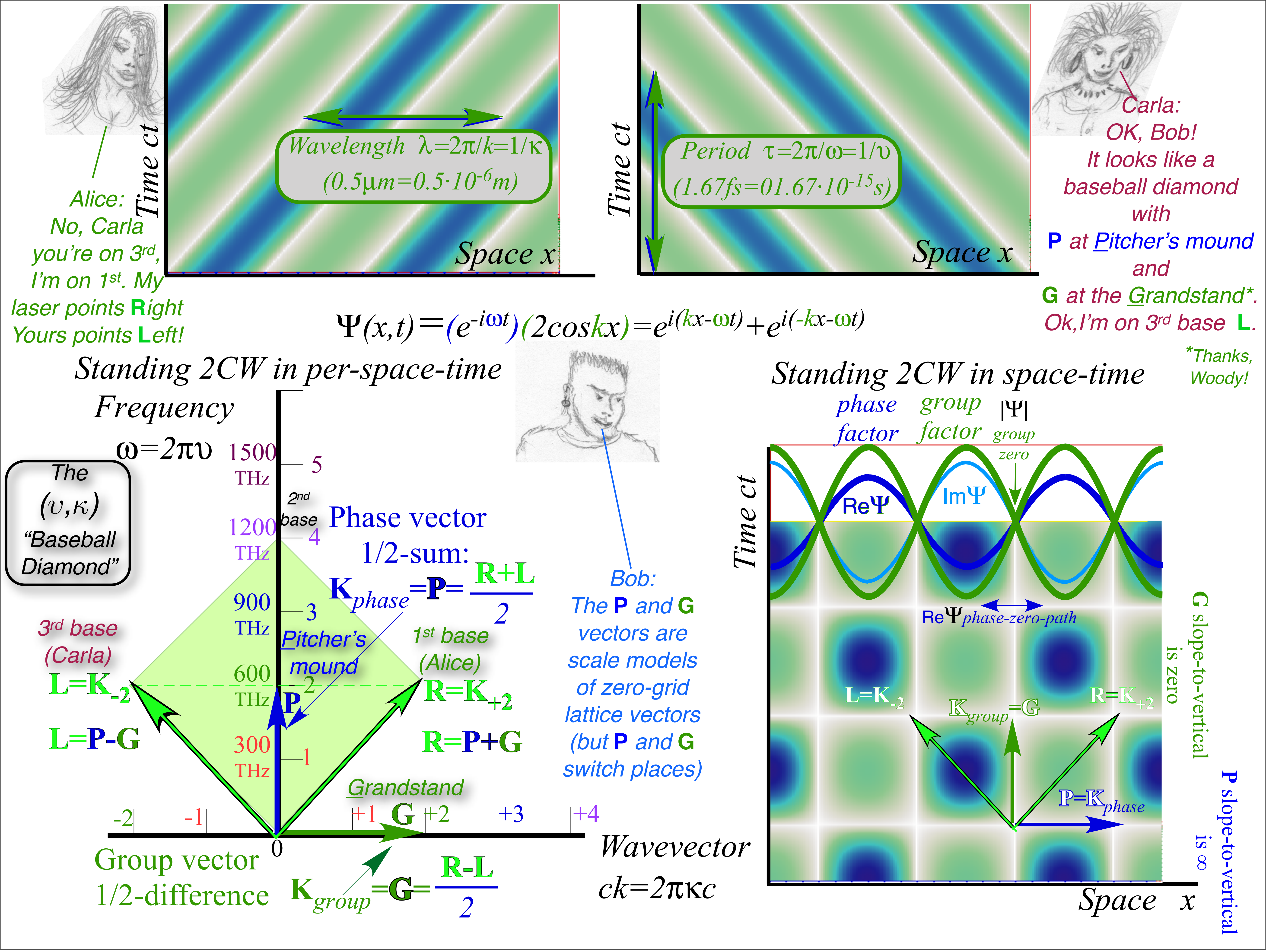
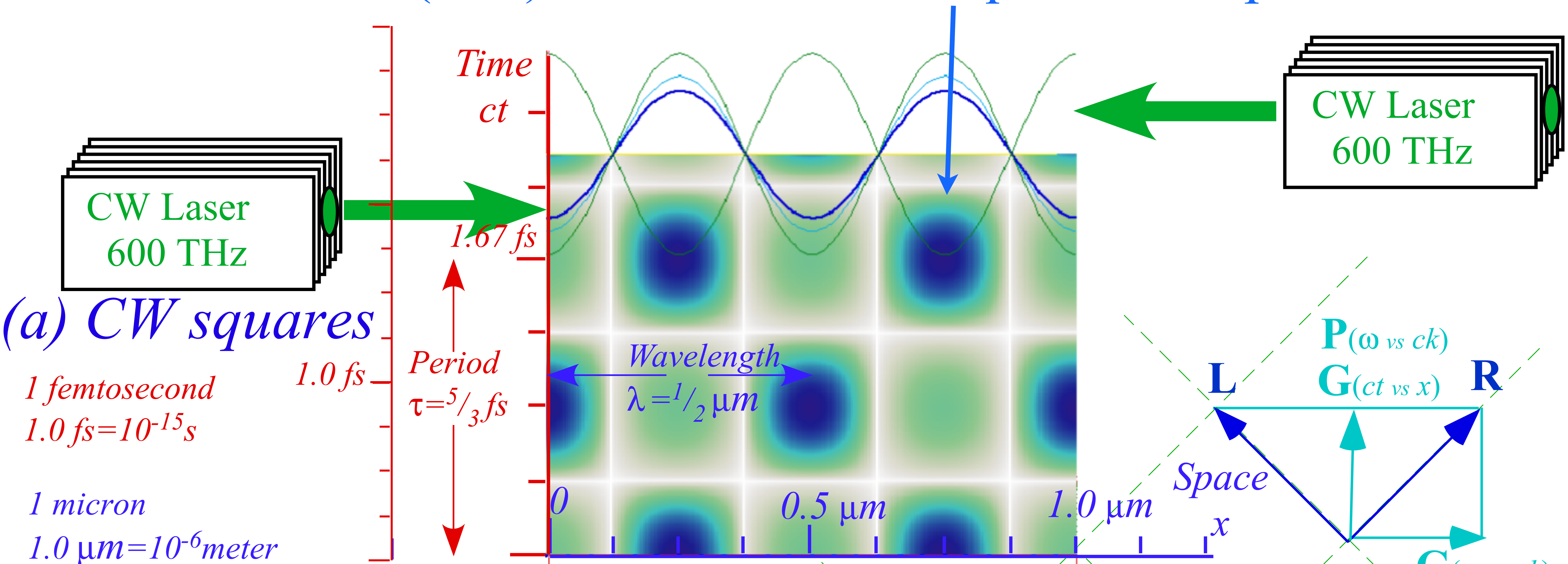
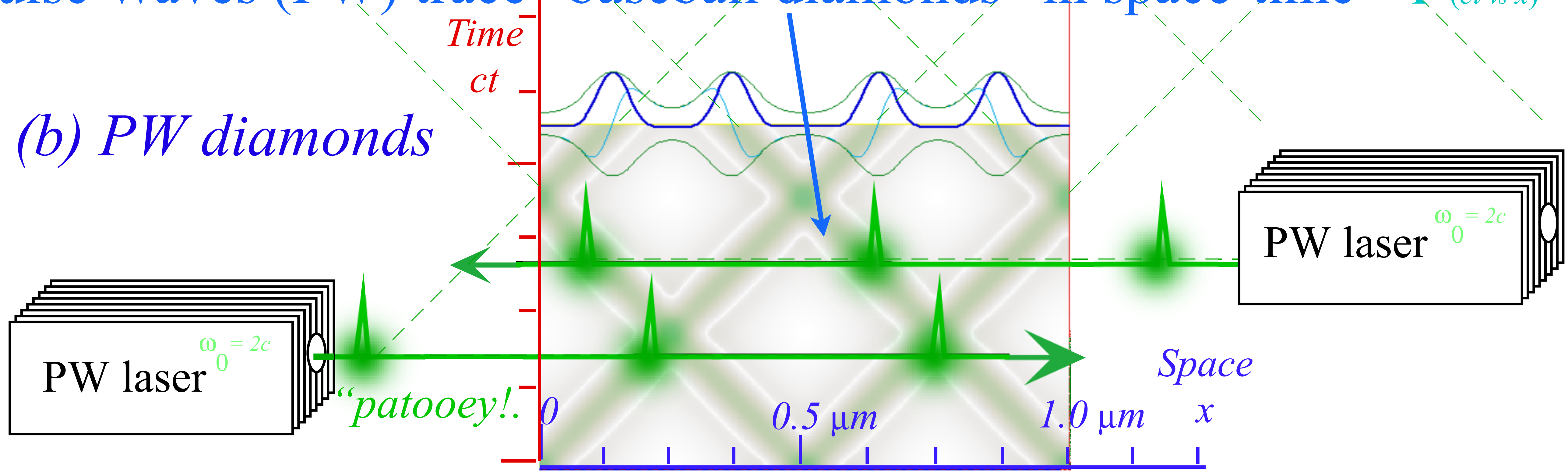


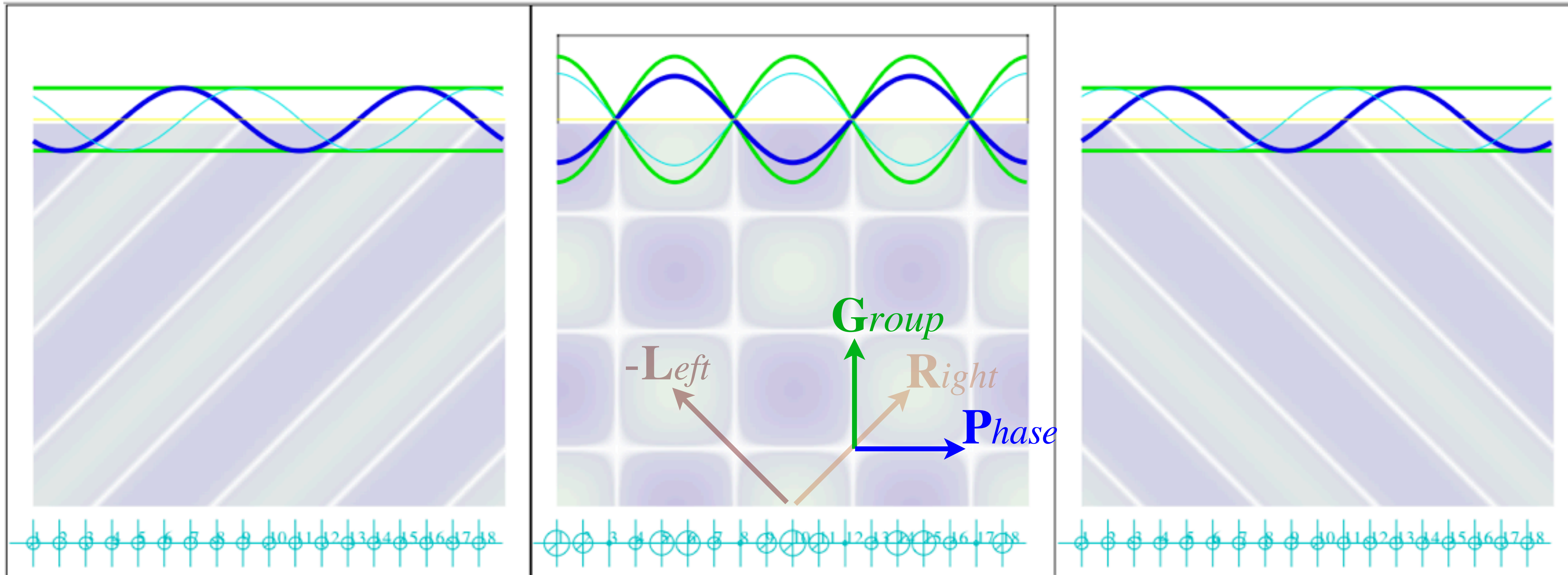
Wavelength λ=2π/k=1/κ
(0.5μm=0.5·10^-6m)
Time ct
Space x
Period τ=2π/ω=1/υ
(1.67fs=01.67·10^-15s)
Time ct
Space x
Alice:
No, Carla
you're on 3rd,
I'm on 1st. My
laser points Right
Yours points Left!
Carla:
OK, Bob!
It looks like a
baseball diamond
with
P at Pitcher's mound
and
G at the Grandstand*.
Ok,I'm on 3rd base L.
*Thanks,
Woody!
Ψ(x,t)=(e^-iωt)(2coskx)=e^i(kx-ωt)+e^i(-kx-ωt)
Standing 2CW in per-space-time
Frequency
ω=2πυ
The
(υ,κ)
"Baseball
Diamond"
1500 THz
1200 THz
900 THz
600 THz
300 THz
5
4
3
2
1
2nd base
Phase vector
1/2-sum:
K_phase=P= (R+L)/2
Pitcher's mound
3rd base
(Carla)
1st base
(Alice)
L=K_-2
L=P-G
R=K_+2
R=P+G
P
Grandstand
G
-2
-1
+1
+2
+3
+4
0
Group vector
1/2-difference
K_group=G= (R-L)/2
Wavevector
ck=2πκc
Bob:
The P and G
vectors are
scale models
of zero-grid
lattice vectors
(but P and G
switch places)
Standing 2CW in space-time
phase
factor
group
factor
|Ψ|
group
zero
ReΨ
ImΨ
Time ct
ReΨ phase-zero-path
L=K_-2
K_group=G
R=K_+2
P=K_phase
G slope-to-vertical
is zero
P slope-to-vertical
is ∞
Space x

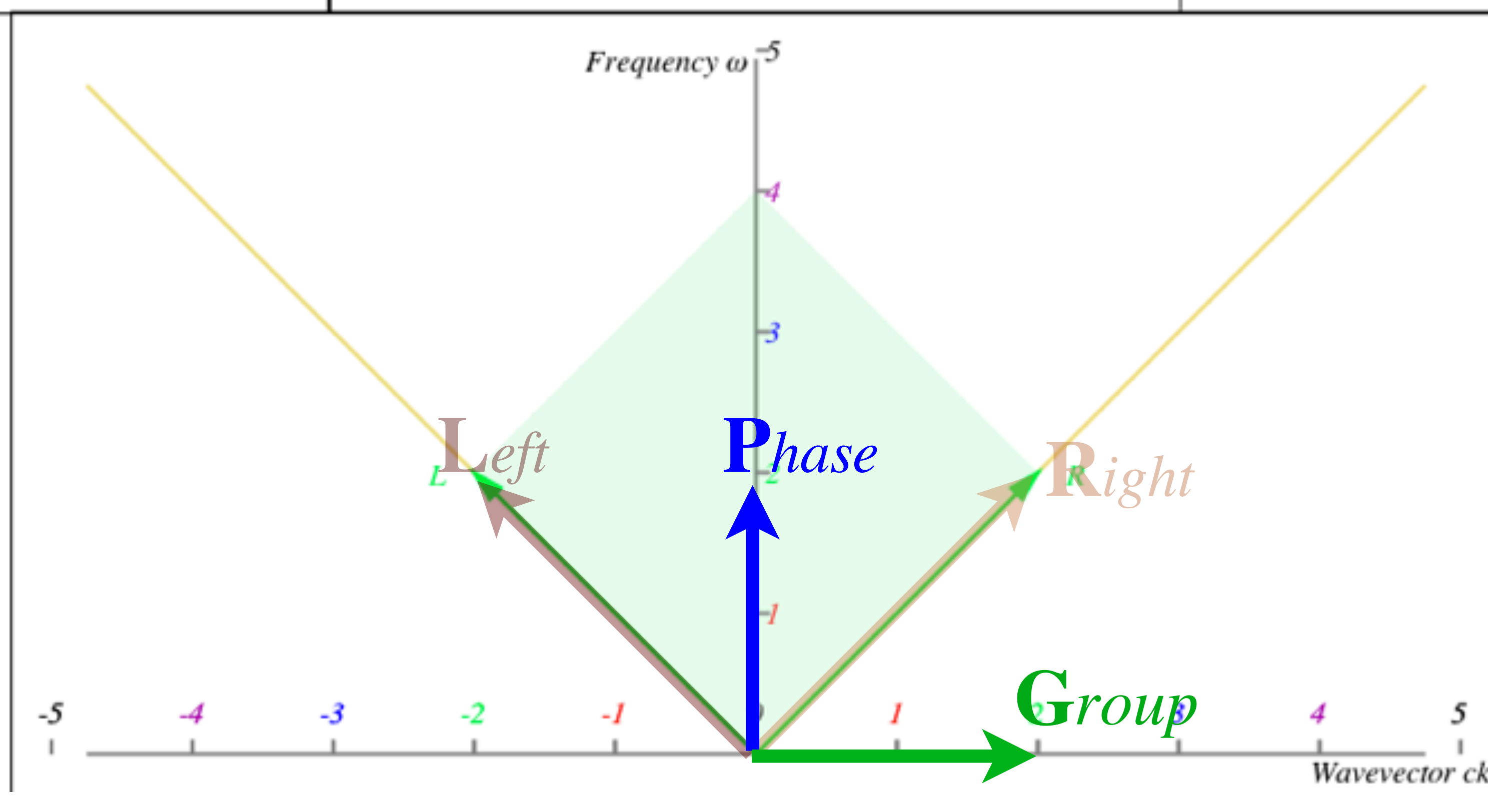
Continuous Waves (CW) trace "Cartesian squares" in space-time



Pulse Waves (PW) trace "baseball diamonds" in space-time







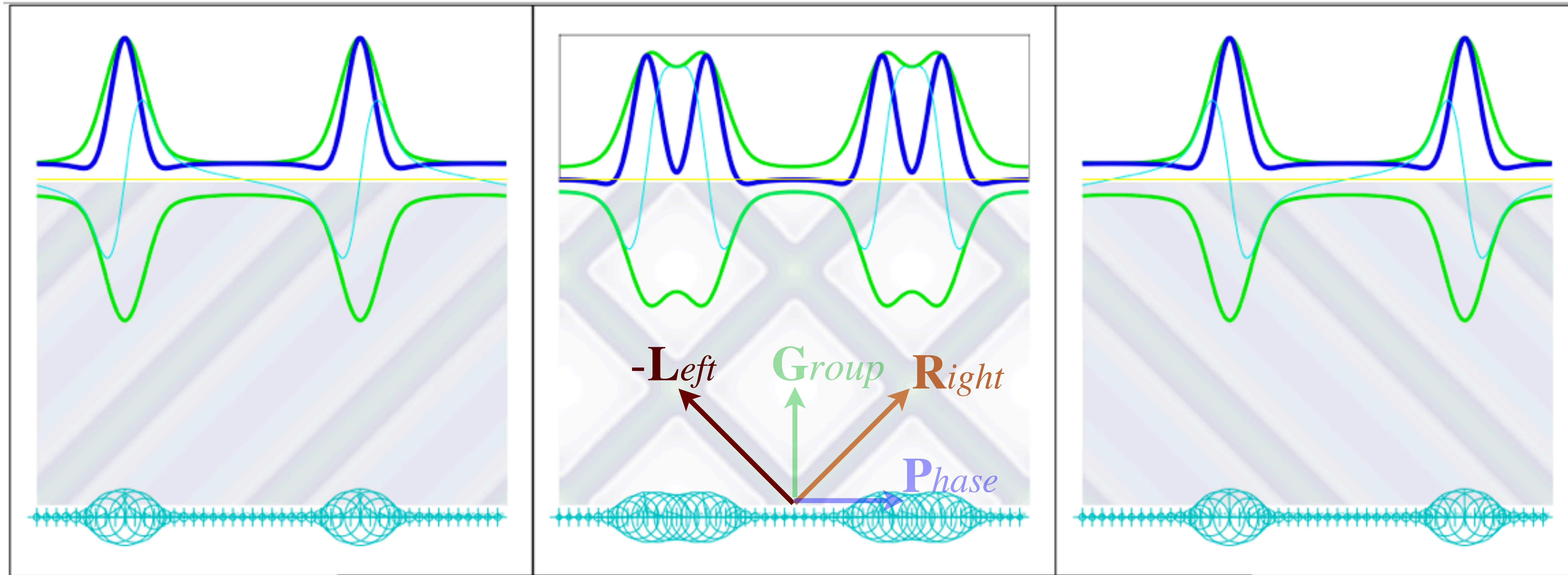
$$R = P + G$$
$$L = P - G$$

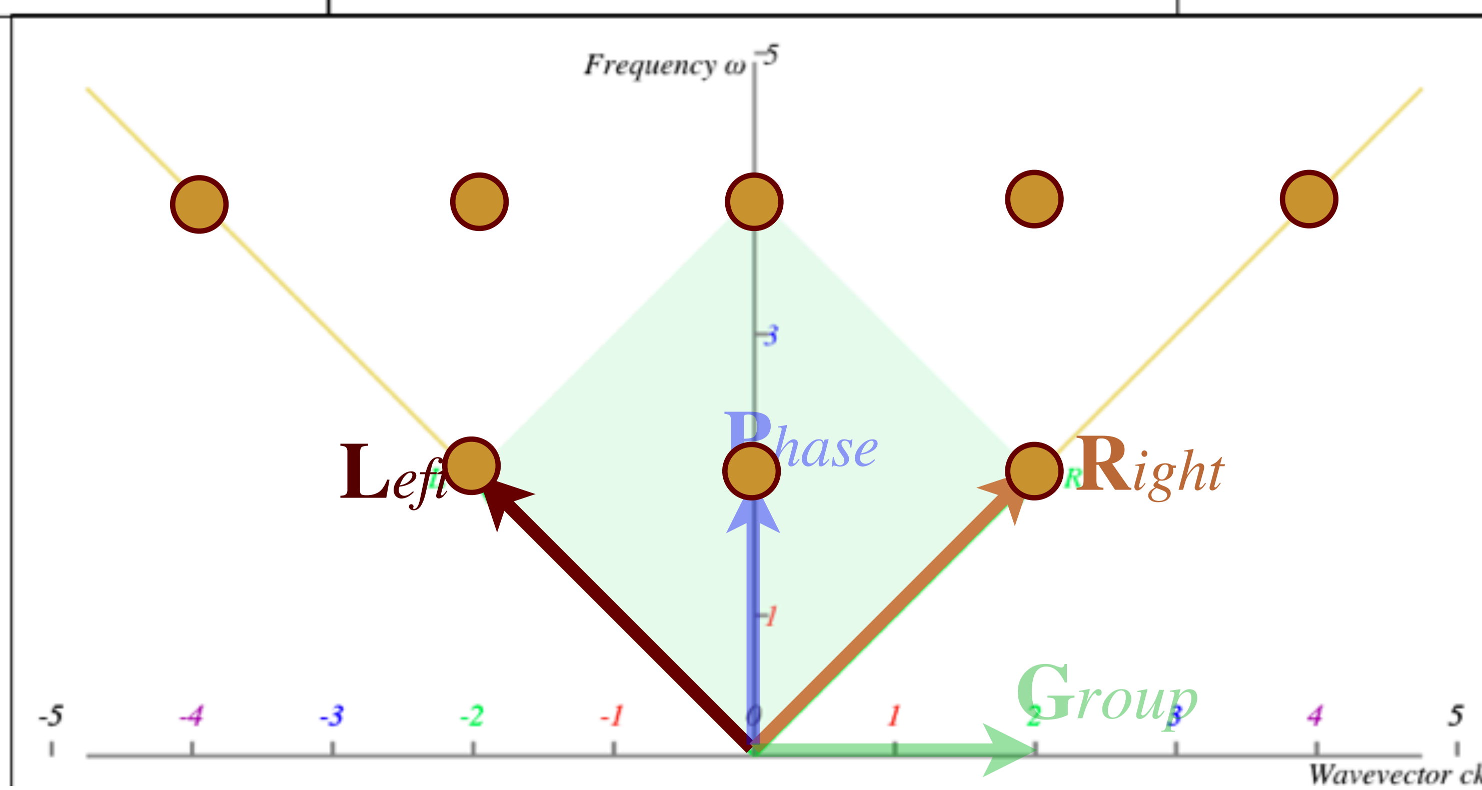
$$P = \frac{R + L}{2}$$

$$G = \frac{R - L}{2}$$

[BohrIt Web Simulation
2 CW ct vs x Plot
(ck = ±2)]

[RelaWavity Site
Phase and Group Vectors in per-Time vs per-Space]





$$\mathbf{R} = \mathbf{P} + \mathbf{G}$$
$$\mathbf{L} = \mathbf{P} - \mathbf{G}$$

$$\mathbf{P} = \frac{\mathbf{R} + \mathbf{L}}{2}$$
$$\mathbf{G} = \frac{\mathbf{R} - \mathbf{L}}{2}$$

BohrIt *Multi-Panel* Simulation
2 PW ct vs x Plot
(β = u/c = 0)

BohrIt Simulation
2 PW ct vs x Plot
(β = u/c = 0)

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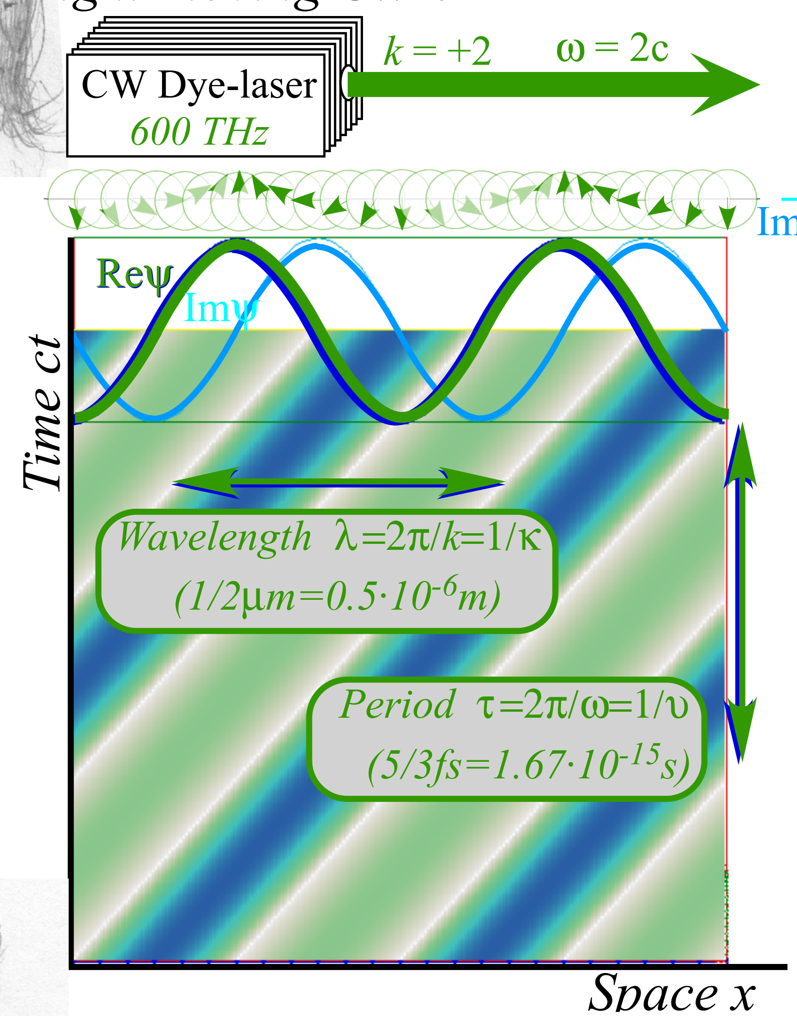

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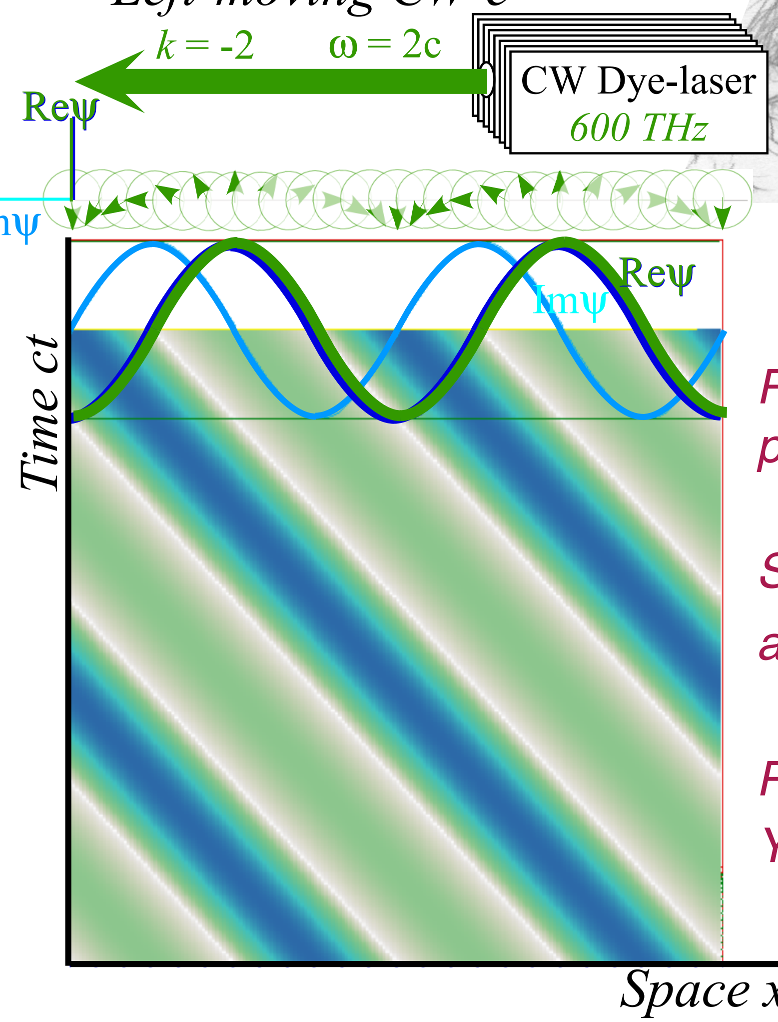

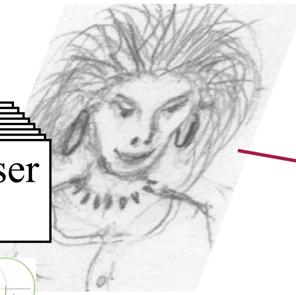

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You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.

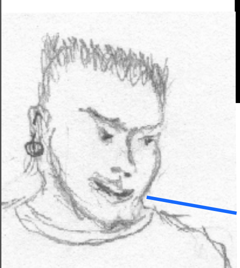
Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives a,

and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives b.

Presto!

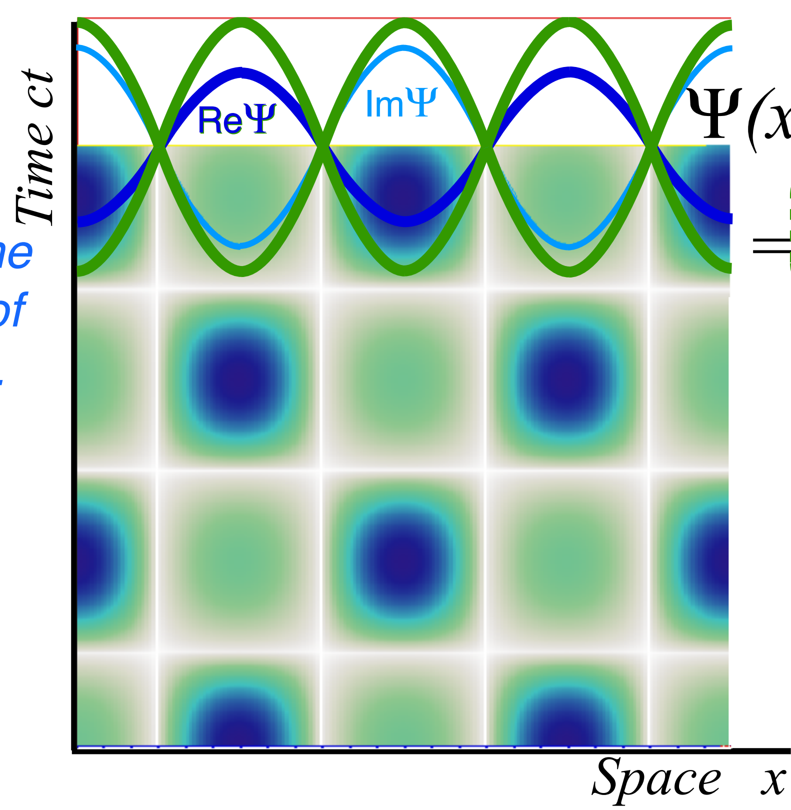
You factor $e^{ia}+e^{ib}$ into $e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$



Bob:

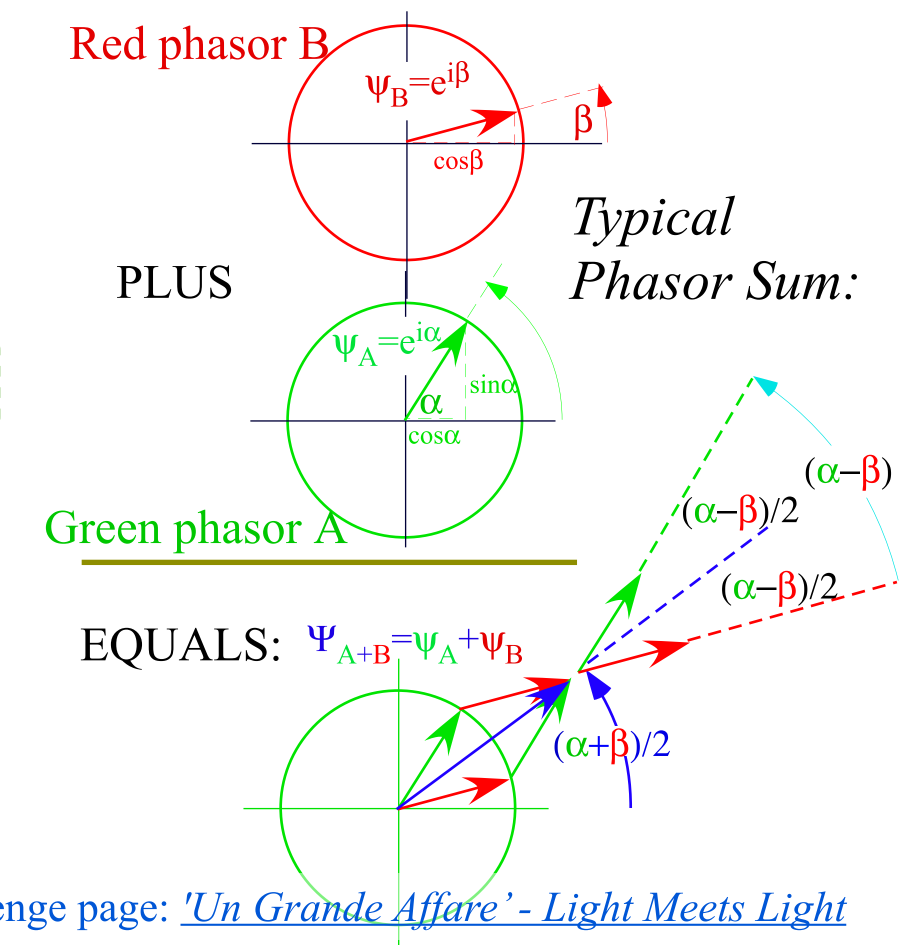
Cool!
You guys made me a space-time graph out of real zeros.

How'd it **do** that?



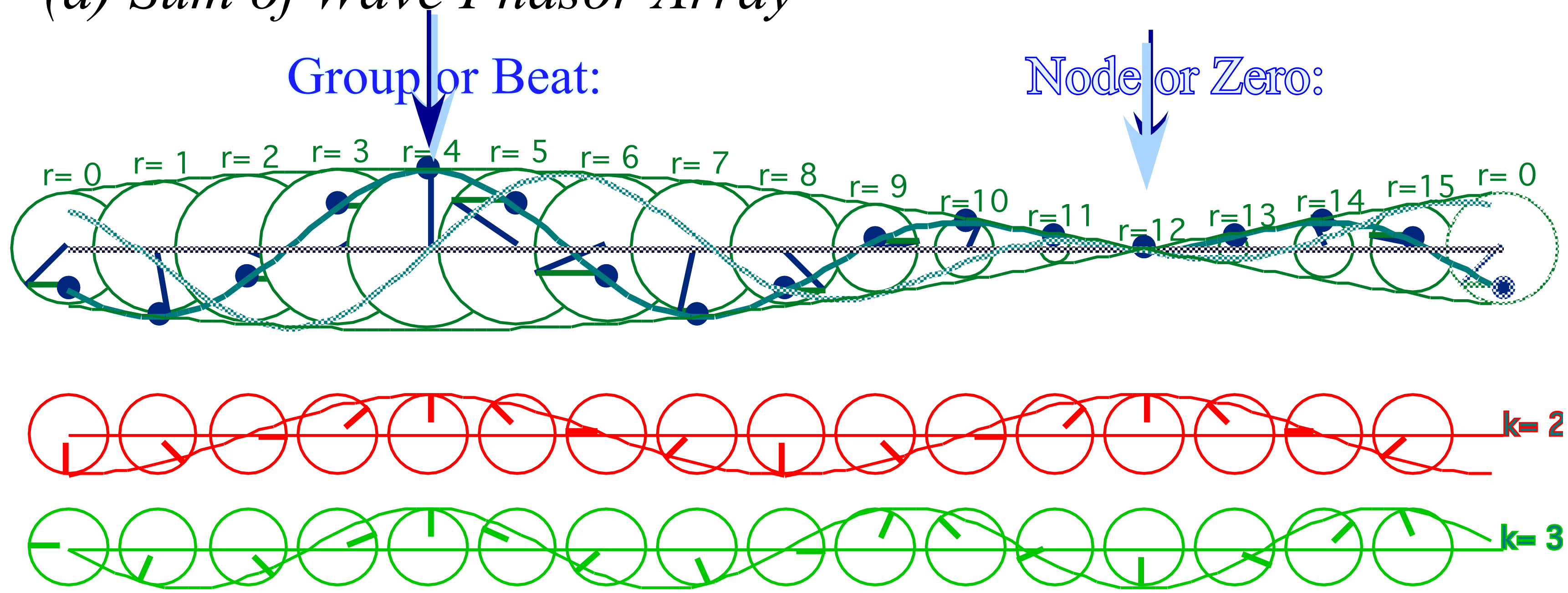
$$\Psi(x,t)=e^{ia}+e^{ib}=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

with $a = kx-\omega t$, $b = -kx-\omega t$



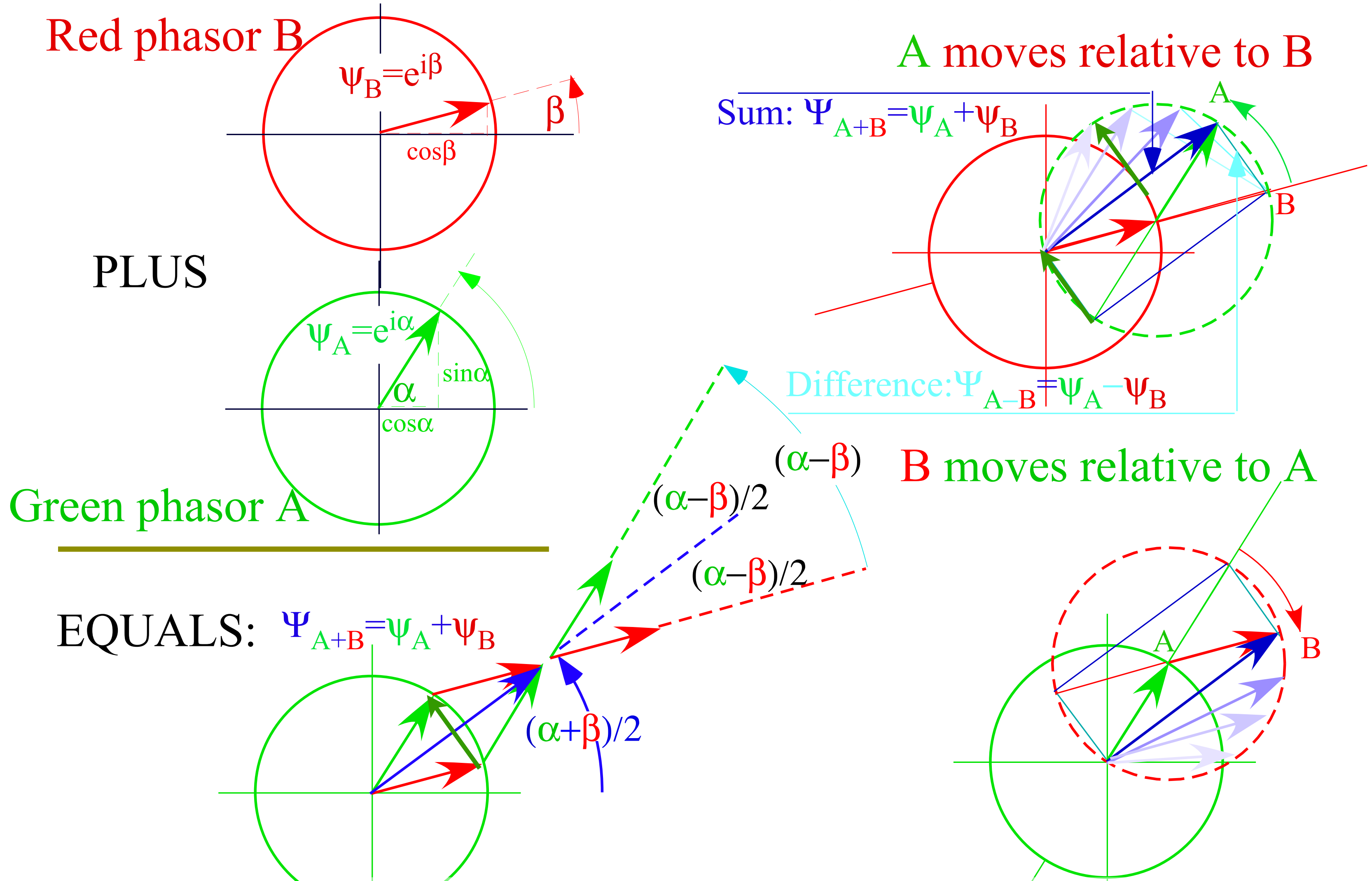
More at Pirelli Challenge page: 'Un Grande Affare' - Light Meets Light

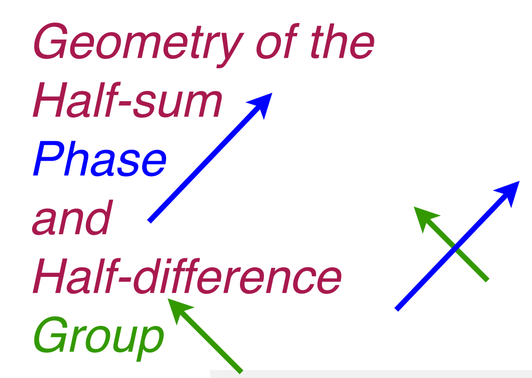
(a) Sum of Wave Phasor Array



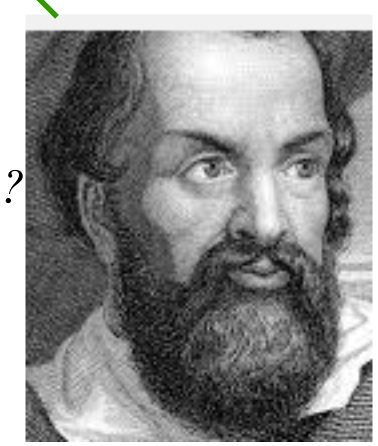
(b) Typical Phasor Sum:

(c) Phasor-relative views





Happy now?



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

More at Pirelli Challenge page: *'Un Grande Affare' - Light Meets Light*

Review of Doppler-shift and Rapidity ρ_{AB} calculation: *Galileo's Revenge Part I Lect. 23 p.64-75*

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➔ Bob, Alice, and Carla combine Doppler shifted ½-sum-½-difference Phase and Group factors ⬅

Doppler shifted Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Minkowski coordinate grid in space-time

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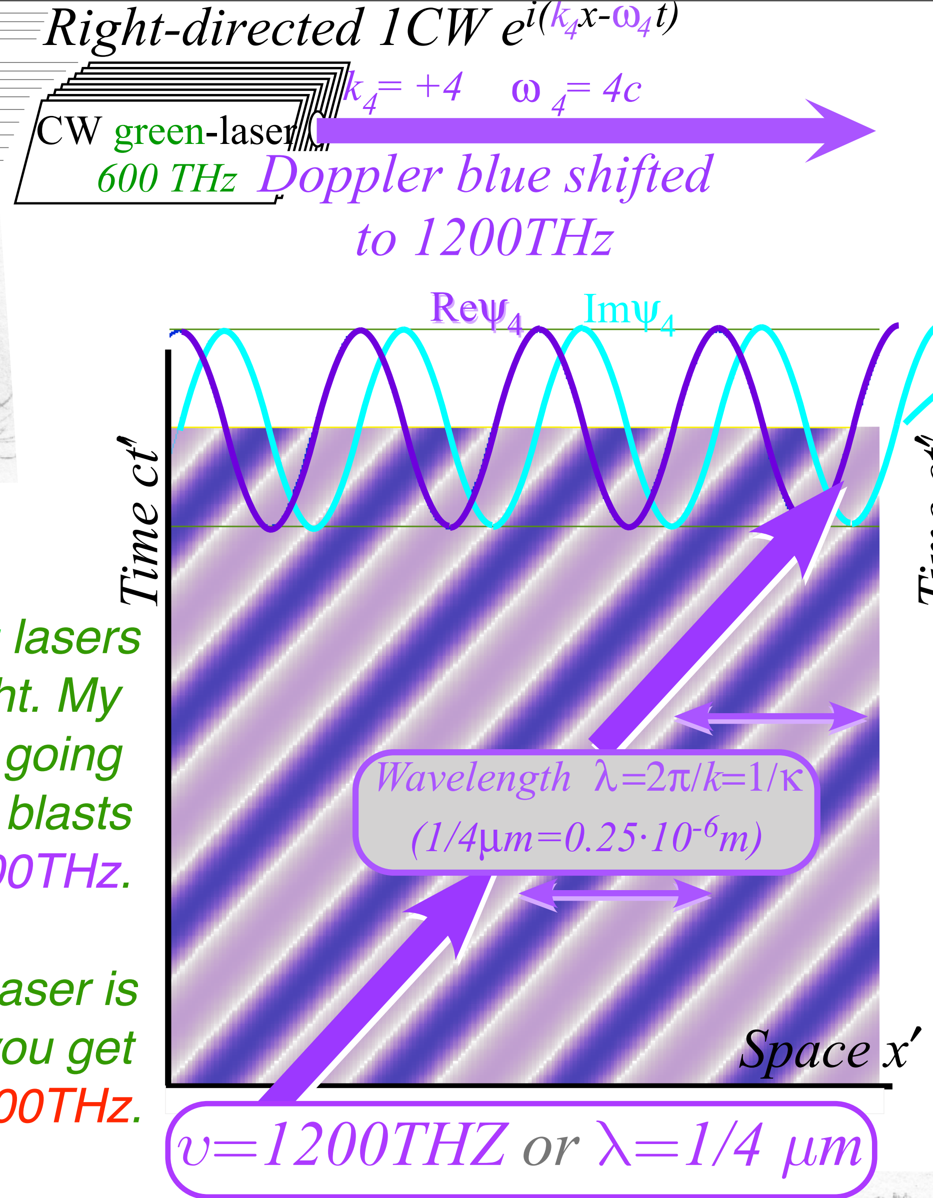
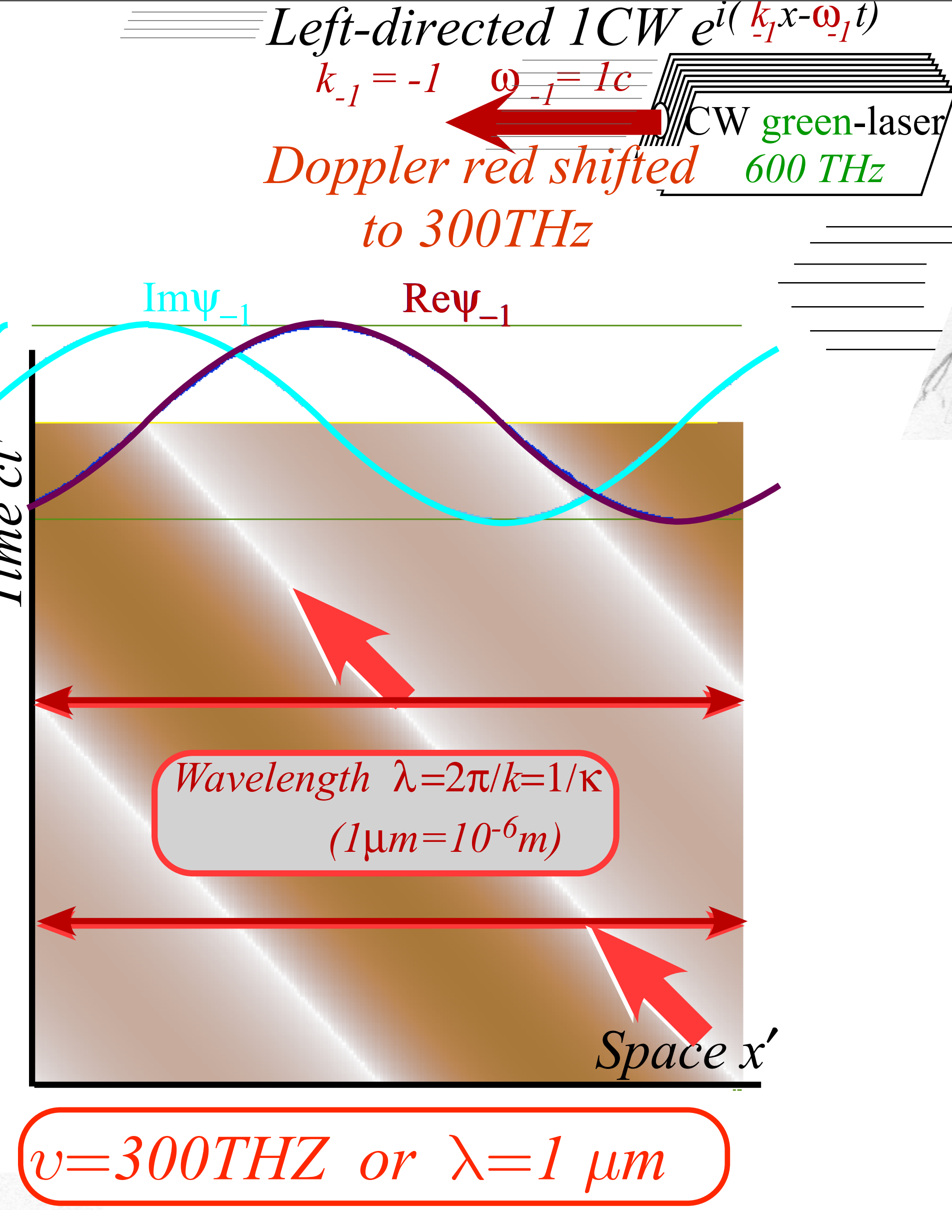
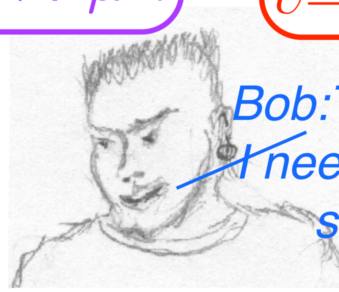
Alice:
Now our 600THz lasers move left-to-right. My 600THz laser is going so fast its beam blasts you with UV 1200THz.

Carla's 600THz laser is going away so you get a nice infrared 300THz.





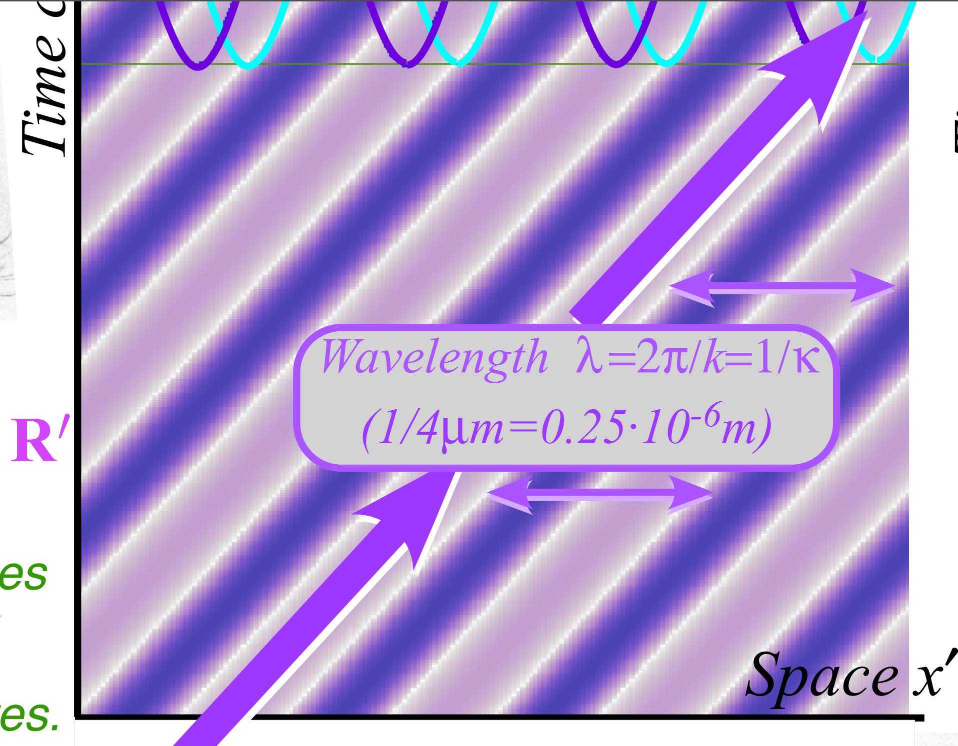


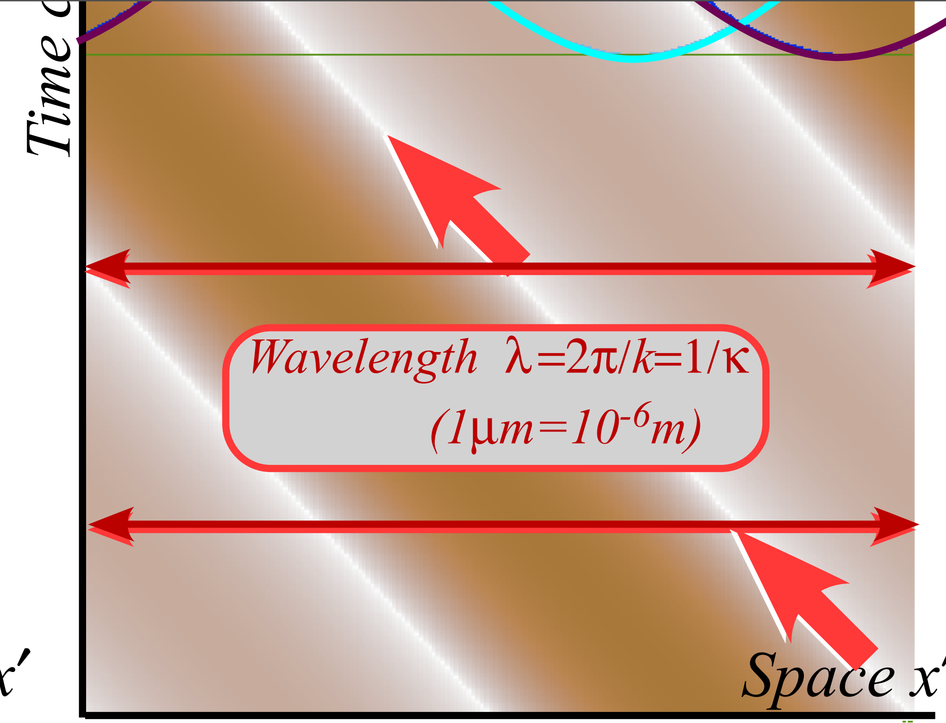
Bob: That UV burns! I need to put on my sunglasses.

BohrIt Web Simulation
1 CW ct vs x Plot
(ck = +4)

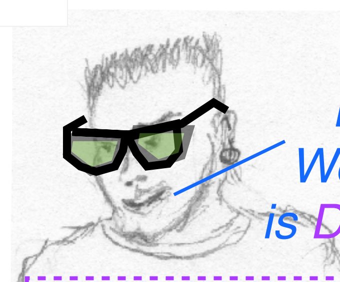
BohrIt Web Simulation
1 CW ct vs x Plot
(ck = -1)

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.

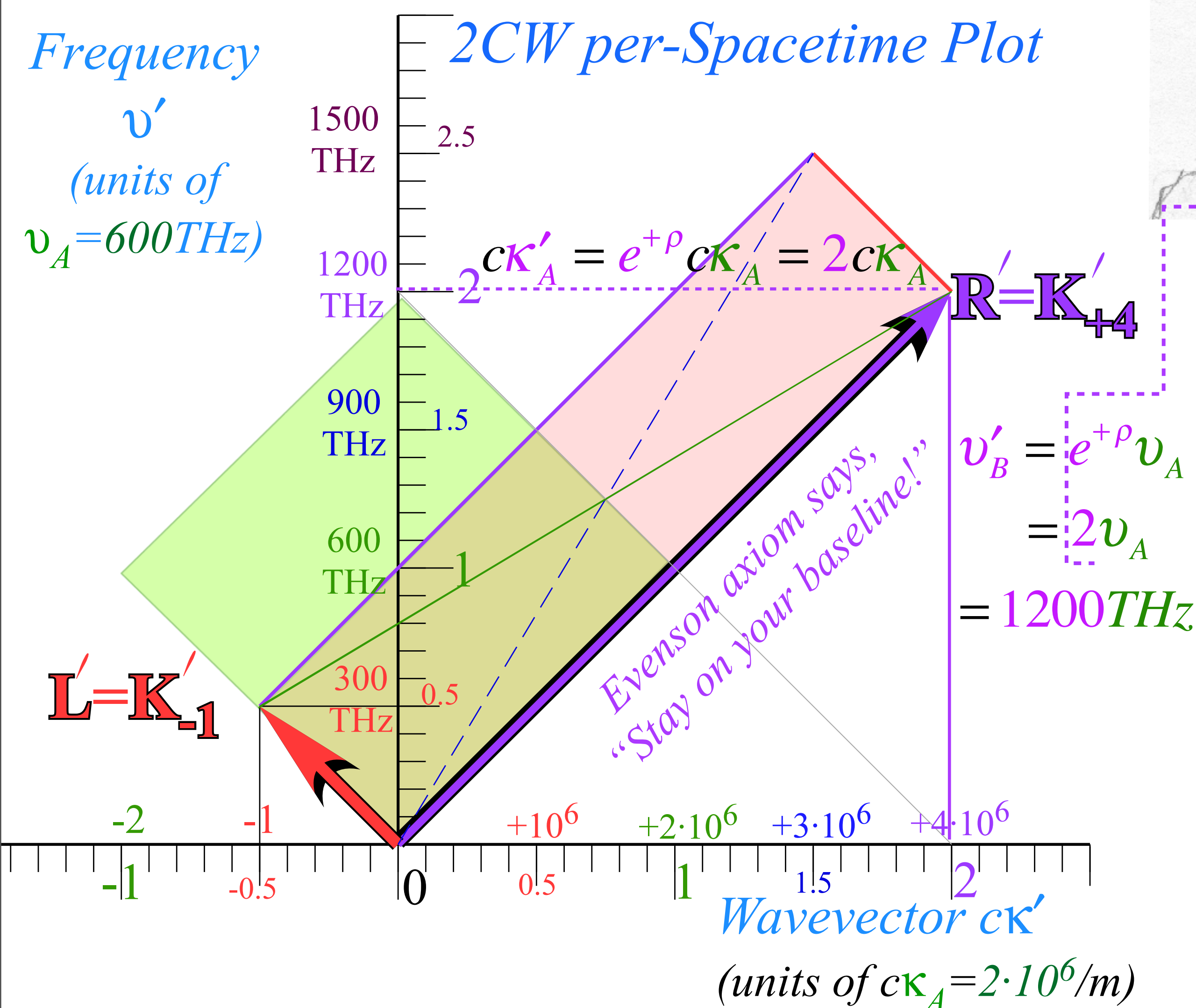


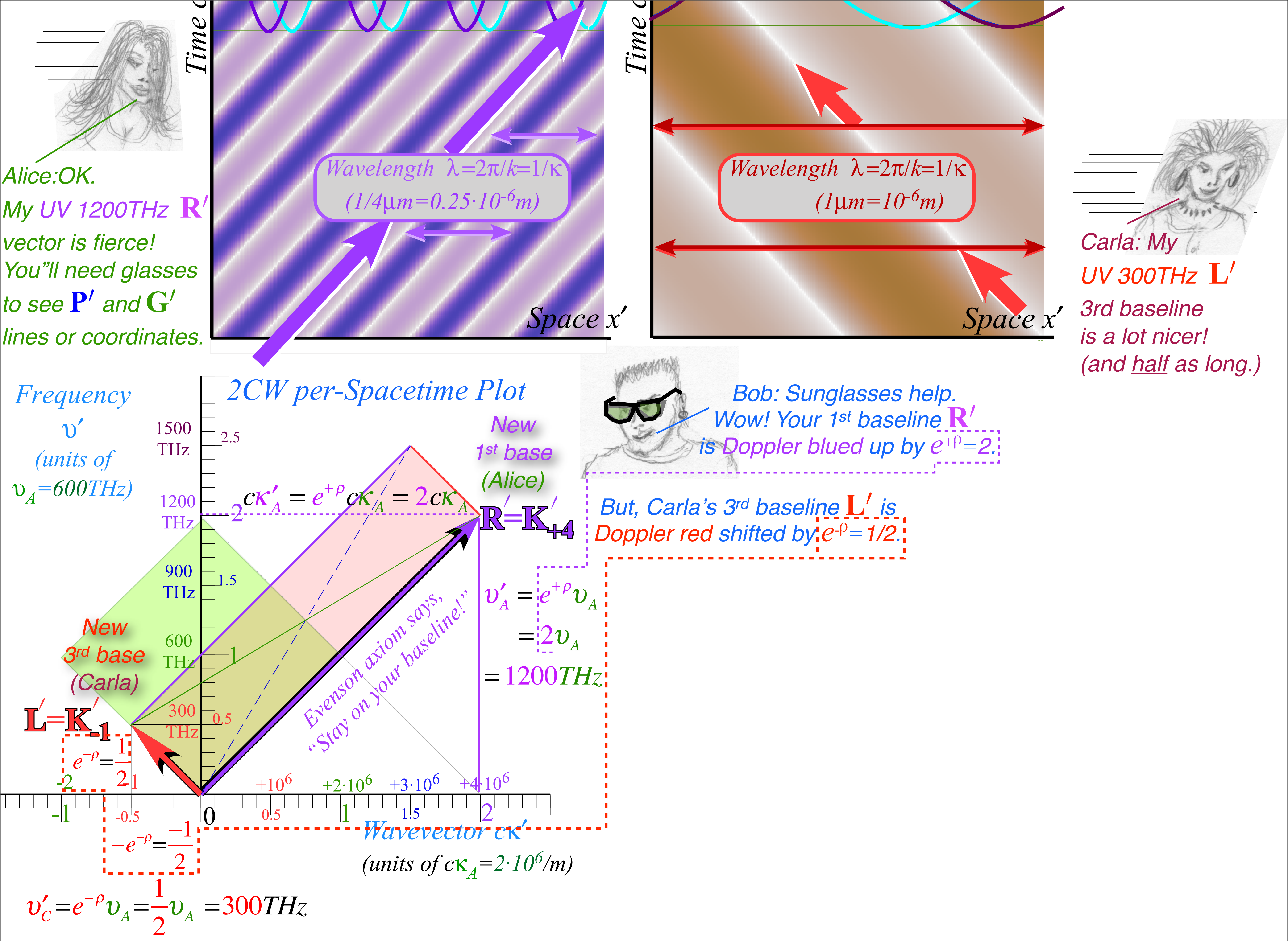


Carla: My IR 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer!



Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.




Time c
Wavelength λ=2π/k=1/κ
(1/4μm=0.25·10^-6m)
Space x′
Time c
Wavelength λ=2π/k=1/κ
(1μm=10^-6m)
Space x′
Alice:OK.
My UV 1200THz R′
vector is fierce!
You"ll need glasses
to see P′ and G′
lines or coordinates.
Carla: My
UV 300THz L′
3rd baseline
is a lot nicer!
(and half as long.)
2CW per-Spacetime Plot
Frequency
υ′
(units of
υ_A=600THz)
1500
THz
2.5
1200
THz
900
THz
1.5
600
THz
300
THz
0.5
2
1
cκ′_A = e^{+ρ} cκ′_A = 2cκ′_A
New
1st base
(Alice)
R′=K′_{+4}
Bob: Sunglasses help.
Wow! Your 1st baseline R′
is Doppler blued up by e^{+ρ}=2.
But, Carla's 3rd baseline L′ is
Doppler red shifted by e^{-ρ}=1/2.
υ′_A = e^{+ρ}υ_A
= 2υ_A
= 1200THz
New
3rd base
(Carla)
L′=K′_{-1}
e^{-ρ} = 1/2
Evenson axiom says,
"Stay on your baseline!"
+10^6
+2·10^6
+3·10^6
+4·10^6
-1
-0.5
0
0.5
1
1.5
2
Wavevector cκ′
(units of cκ_A=2·10^6/m)
−e^{-ρ} = −1/2
υ′_C = e^{-ρ}υ_A = 1/2 υ_A = 300THz

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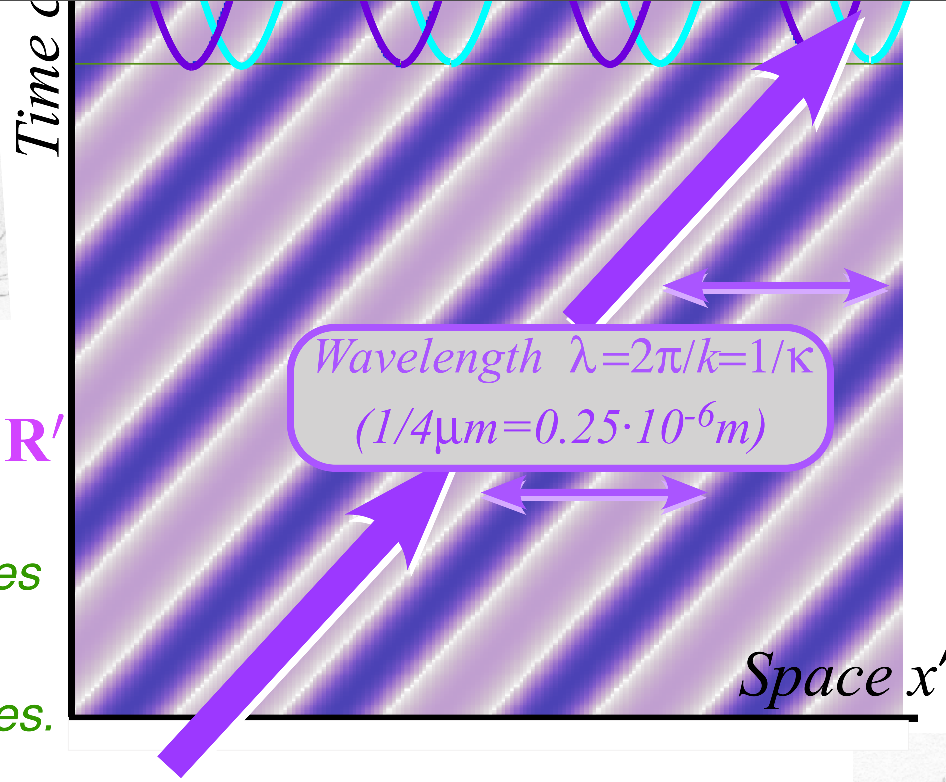
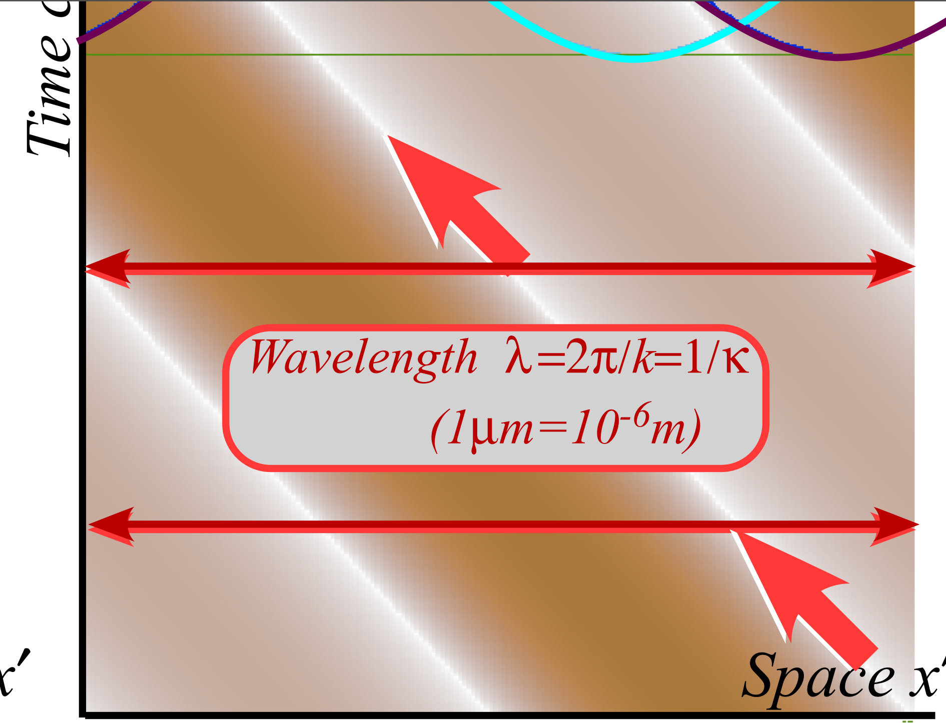
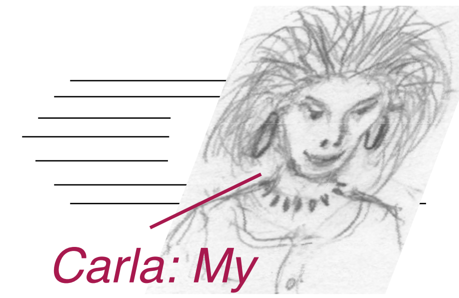
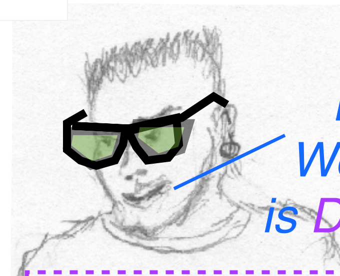
Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.







Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer!
(and half as long.)



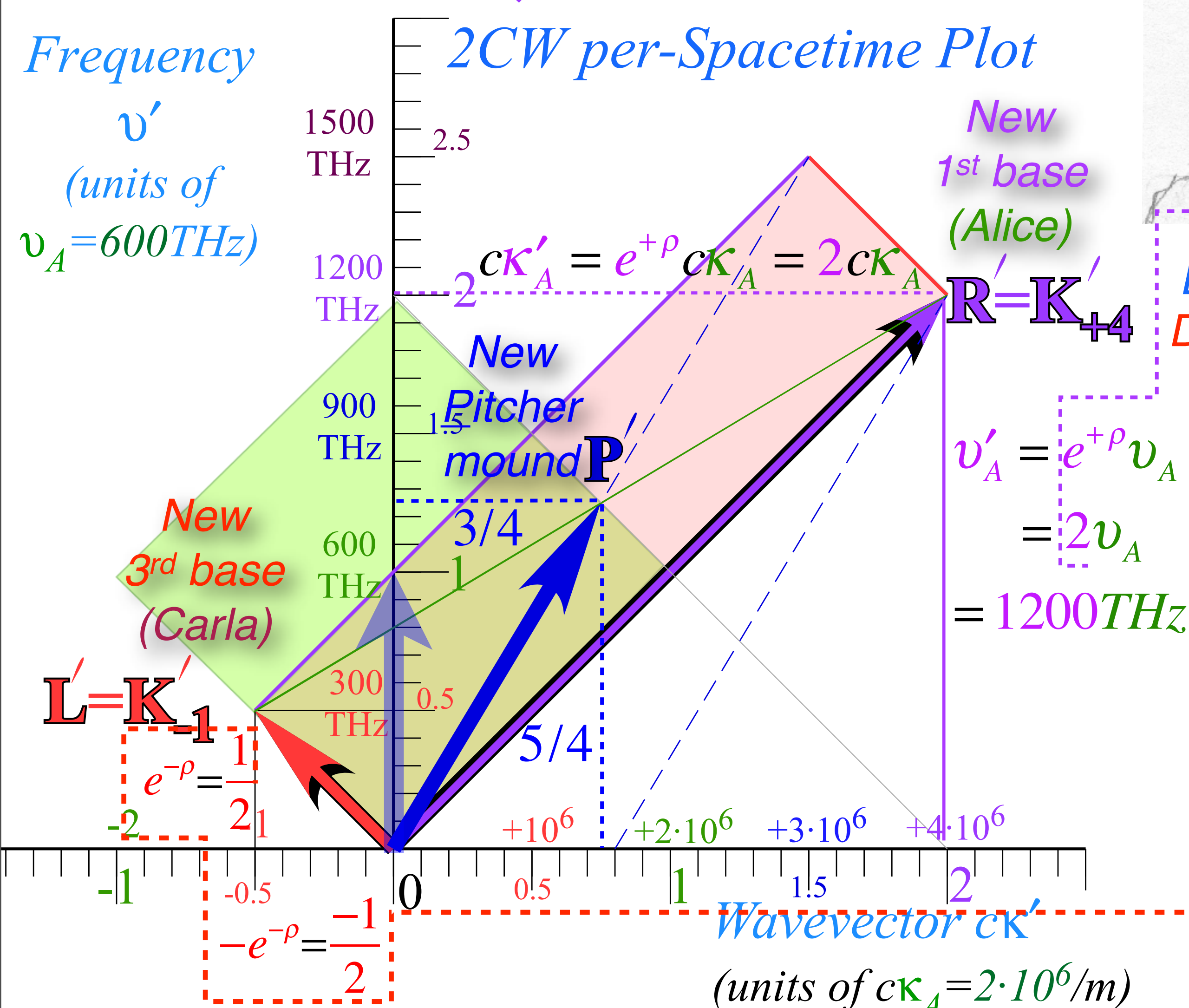
Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.

$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2-1/2}{2} \\ \frac{2+1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

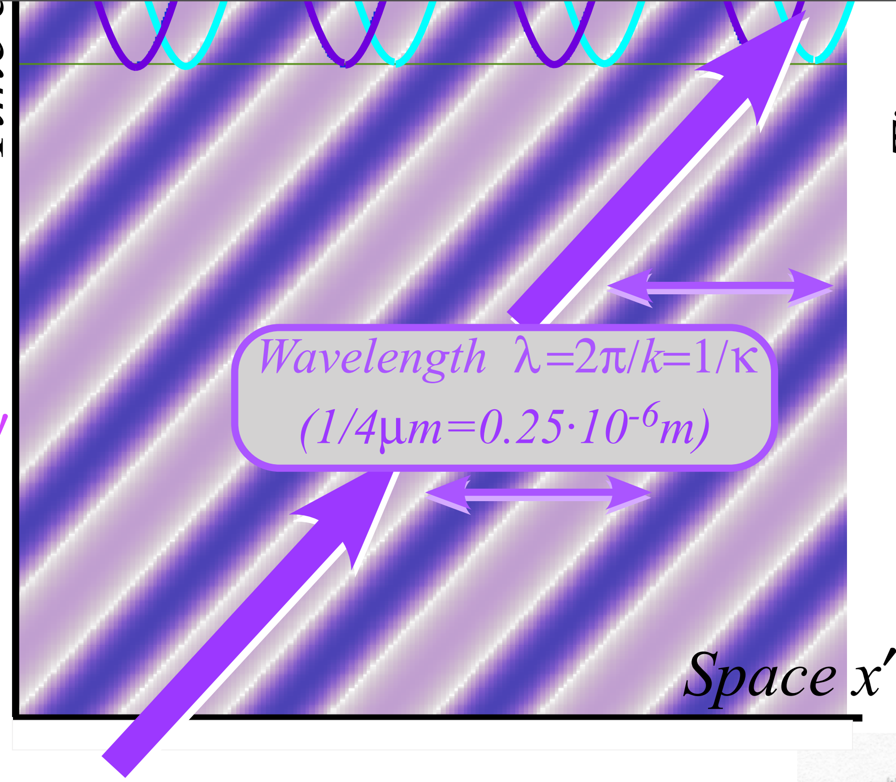


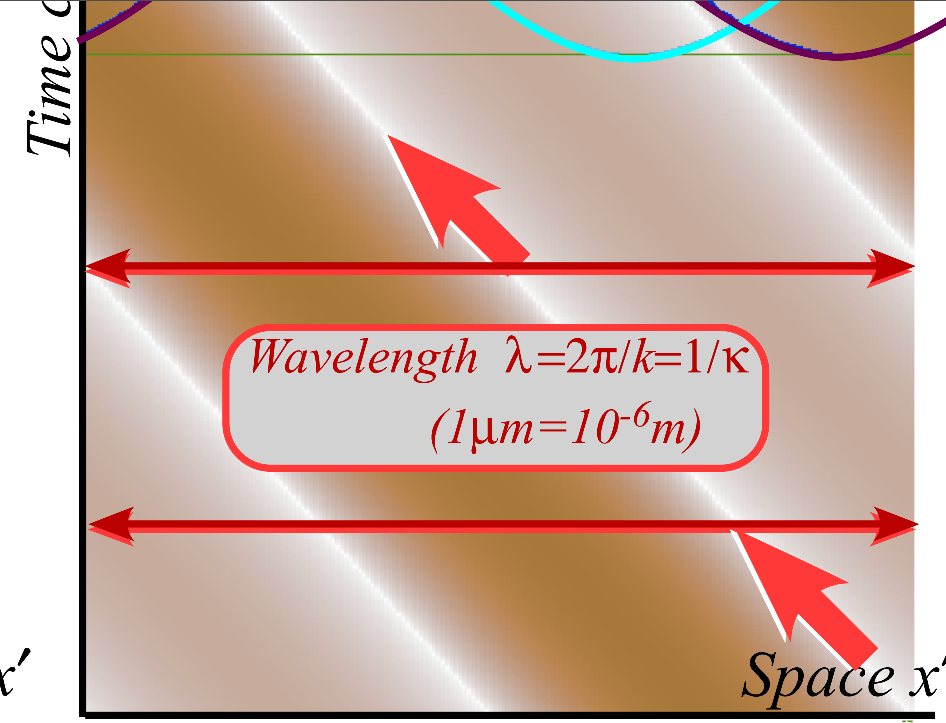
$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

RelaWavity Simulation

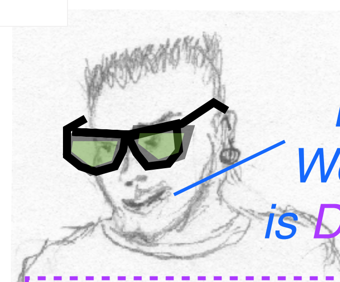
Shifted (b=2) Phase and Group Vectors in per-Time vs per-Space

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.



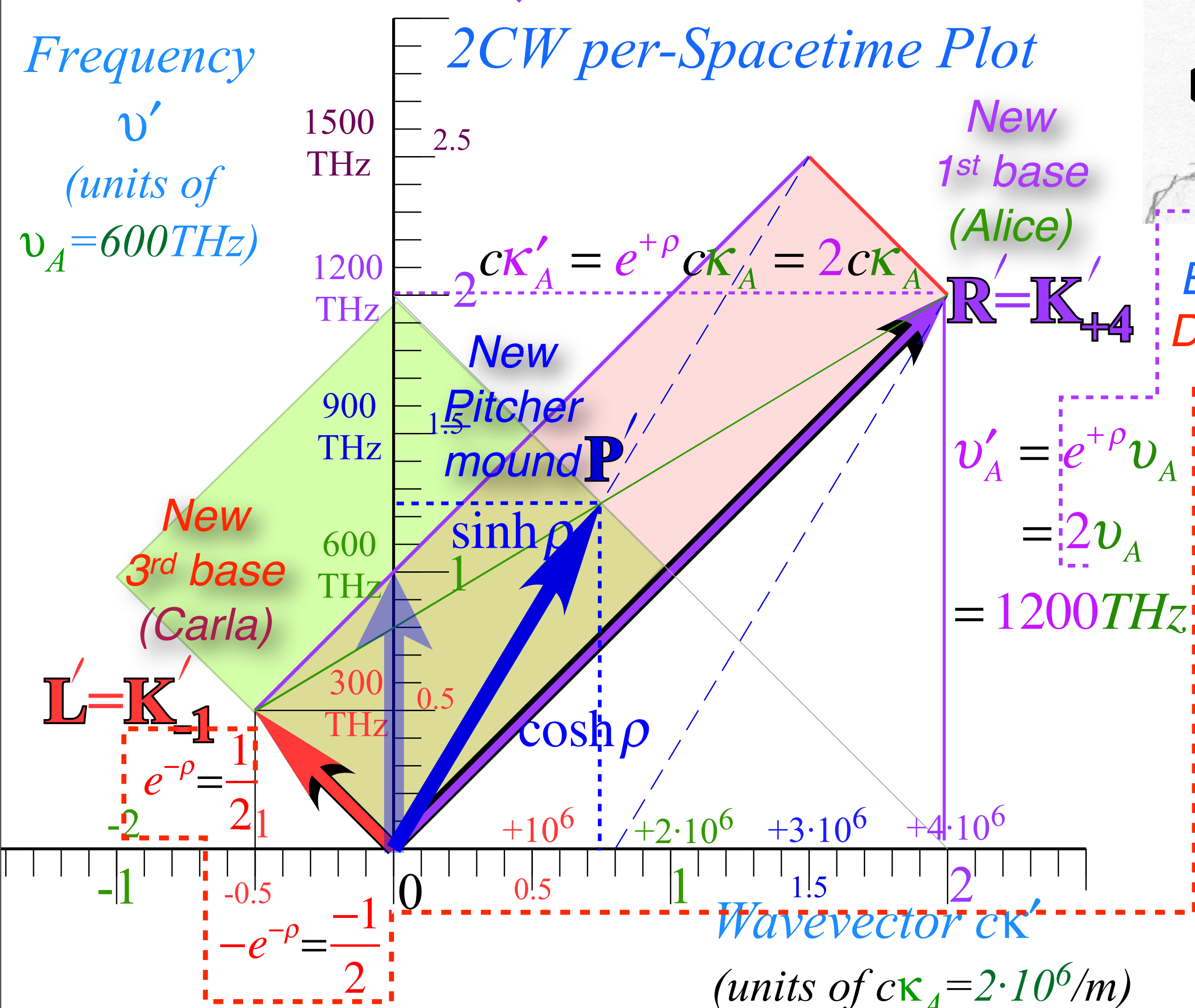


Carla: My UV 300THz $\mathbf{L}'$
3rd baseline is a lot nicer!
(and half as long.)



Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$



But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

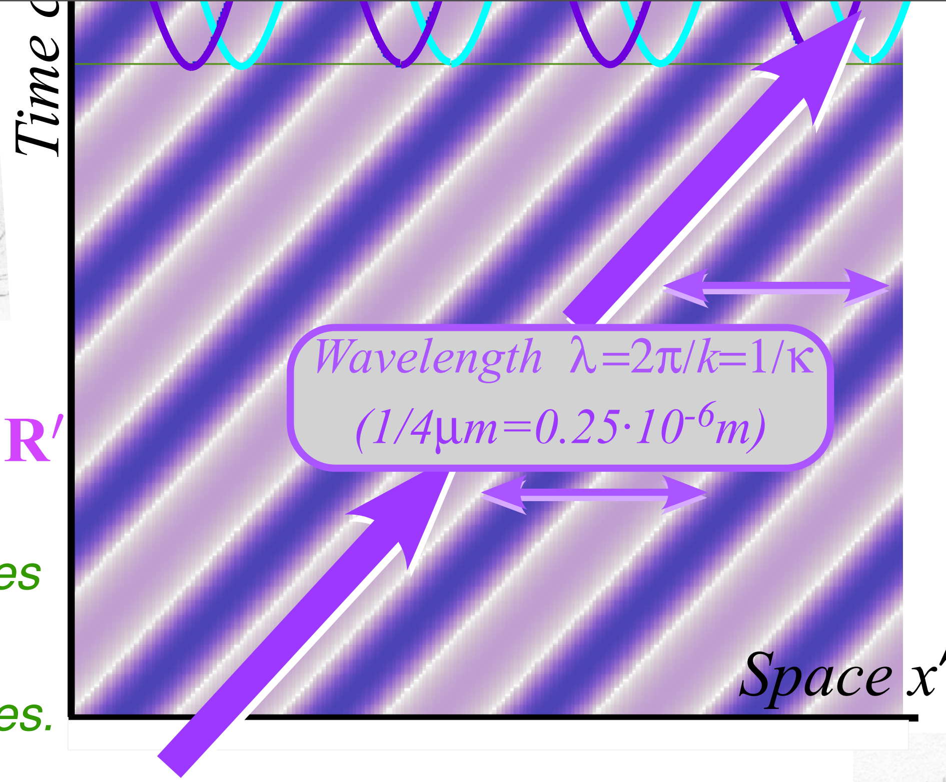
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix}$$

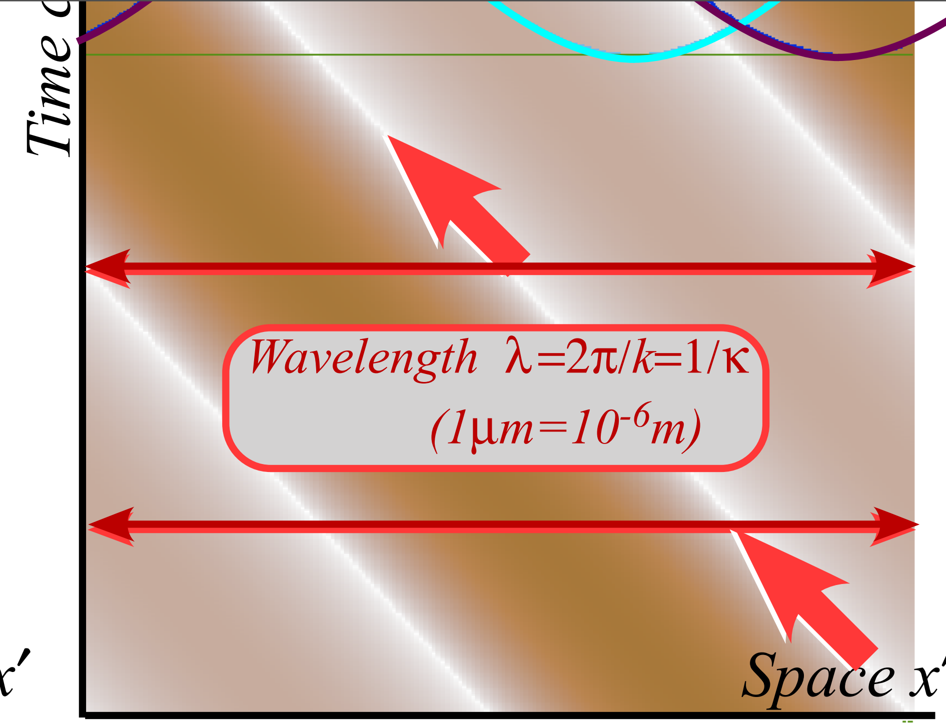
$$= \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

RelaWavity Simulation
Shifted (b=2) Phase and Group Vectors in per-Time vs per-Space

Alice:OK. My UV 1200THz $\mathbf{R}'$ vector is fierce! You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.





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Bob: Sunglasses help. Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

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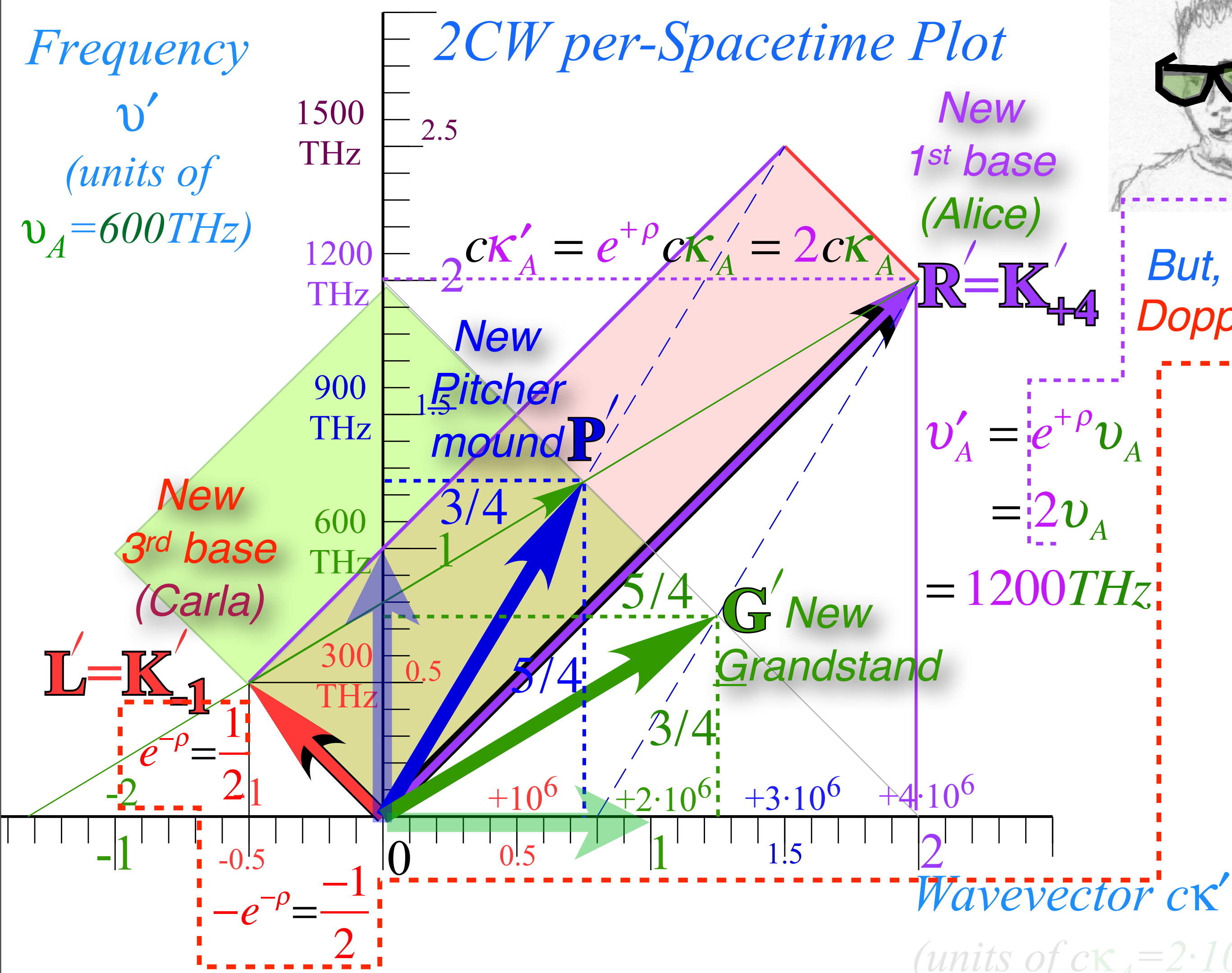
$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$



$$\upsilon'_C=e^{-\rho}\upsilon_A=\frac{1}{2}\upsilon_A=300THz$$

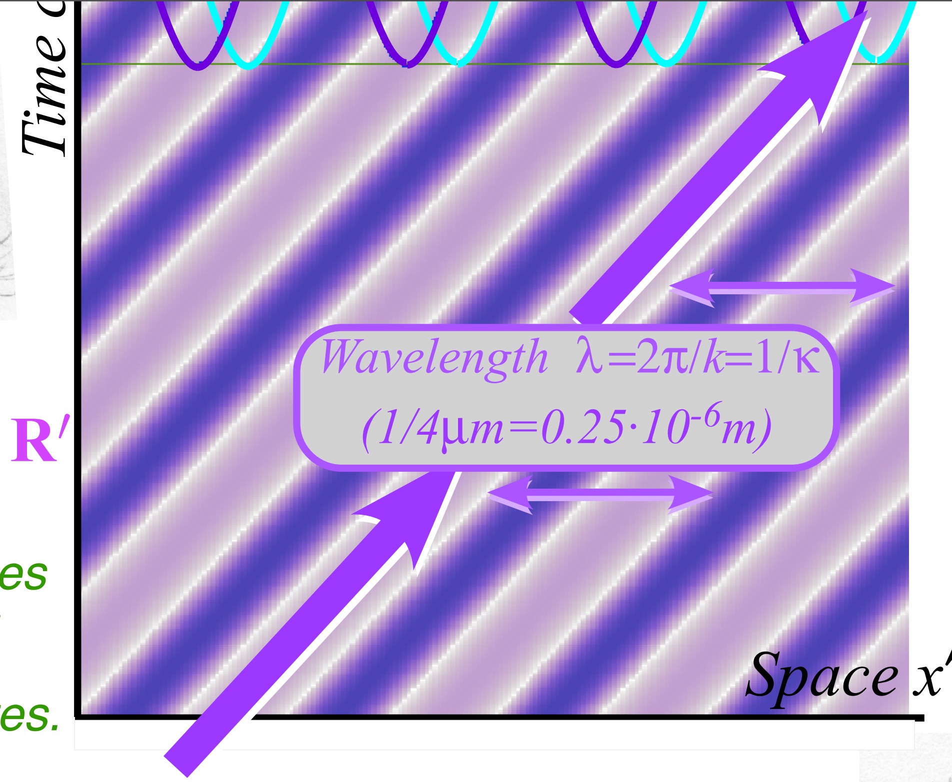
New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

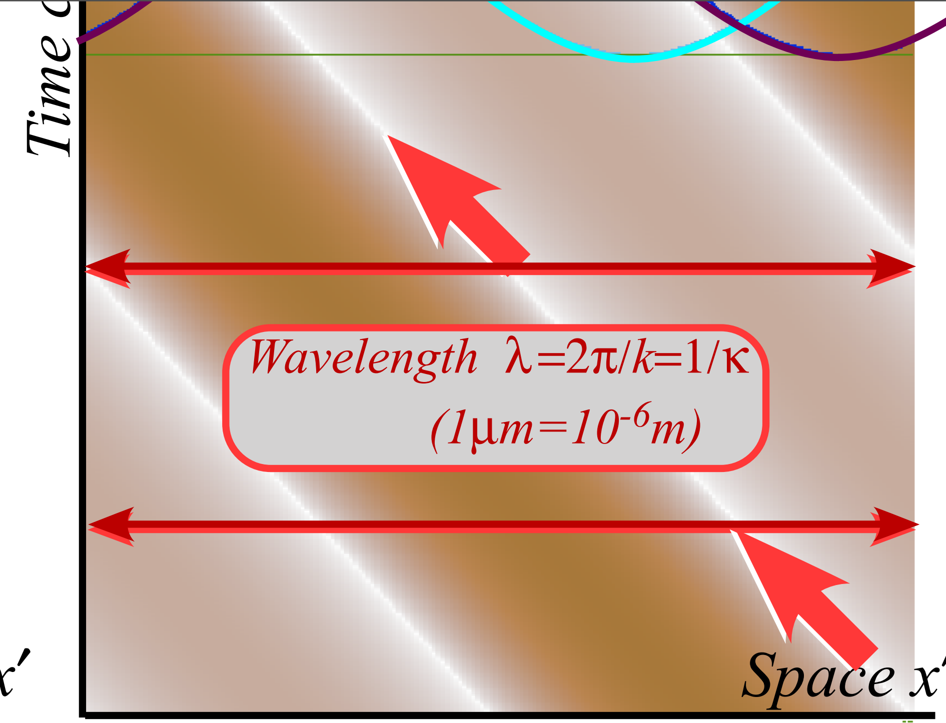
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase}\end{pmatrix}=\frac{\upsilon_A}{2}\begin{pmatrix}2\\2\end{pmatrix}+\frac{\upsilon_A}{2}\begin{pmatrix}-1/2\\+1/2\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{2-1/2}{2}\\ \frac{2+1/2}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}3/4\\5/4\end{pmatrix}$$

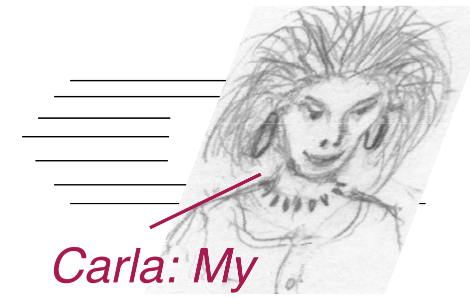
New "Grandstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase}\end{pmatrix}=\frac{\upsilon_A}{2}\begin{pmatrix}2\\2\end{pmatrix}-\frac{\upsilon_A}{2}\begin{pmatrix}-1/2\\+1/2\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{2+1/2}{2}\\ \frac{2-1/2}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}5/4\\3/4\end{pmatrix}$$

Alice: OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce! You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.







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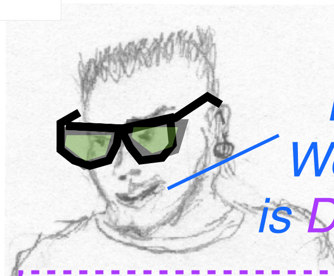

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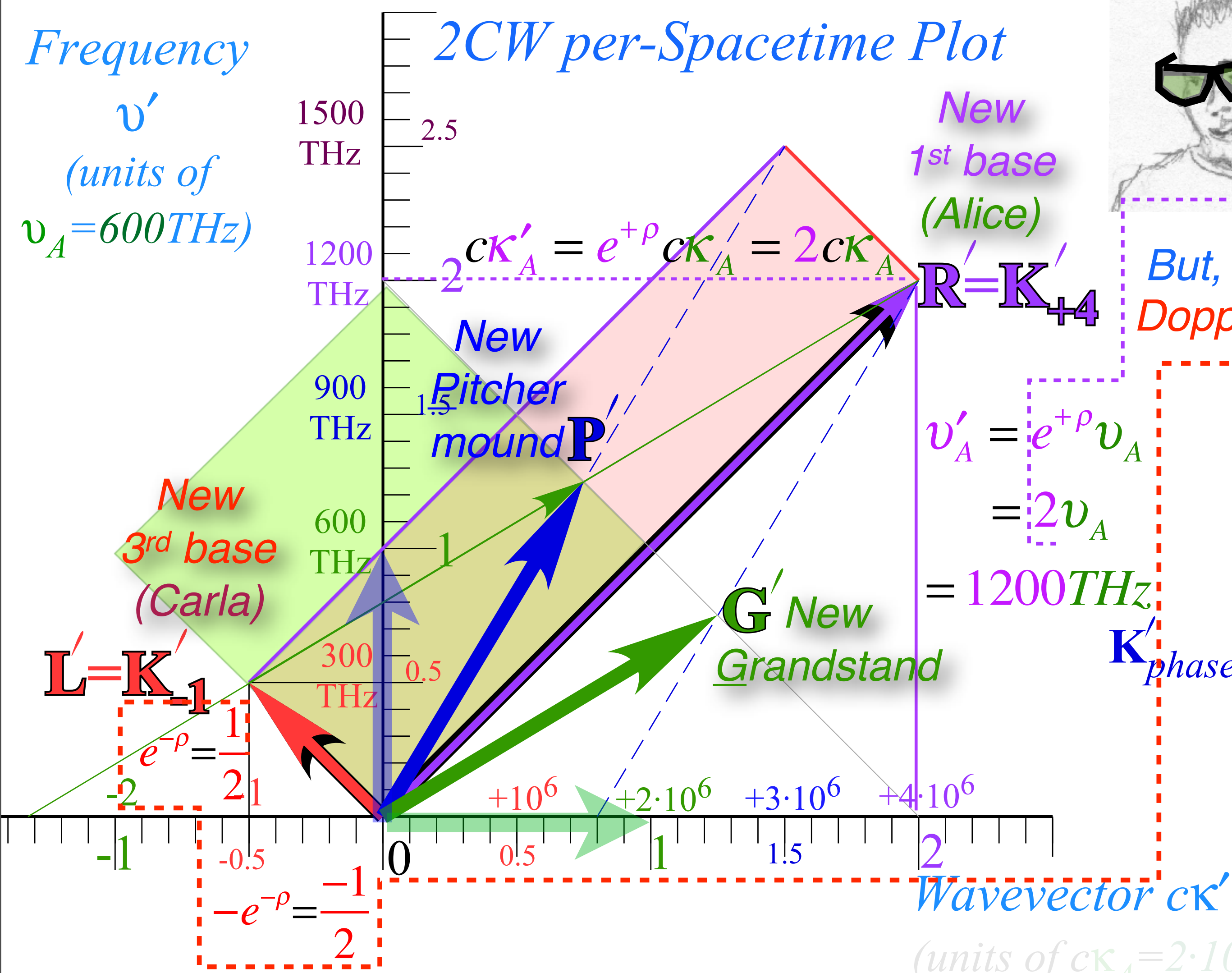

$$\upsilon'_A = e^{+\rho}\upsilon_A = 2\upsilon_A = 1200THz$$

$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix}$$

$$\mathbf{K}'_{phase} = \mathbf{P}' = \frac{\mathbf{R}+\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}$$

New "Grandstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} - \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}+e^{-\rho}}{2} \\ \frac{e^{+\rho}-e^{-\rho}}{2} \end{pmatrix}$$

Group vector $\mathbf{G}$
1/2-diff vector

$$\mathbf{K}'_{group} = \mathbf{G}' = \frac{\mathbf{R}-\mathbf{L}'}{2} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

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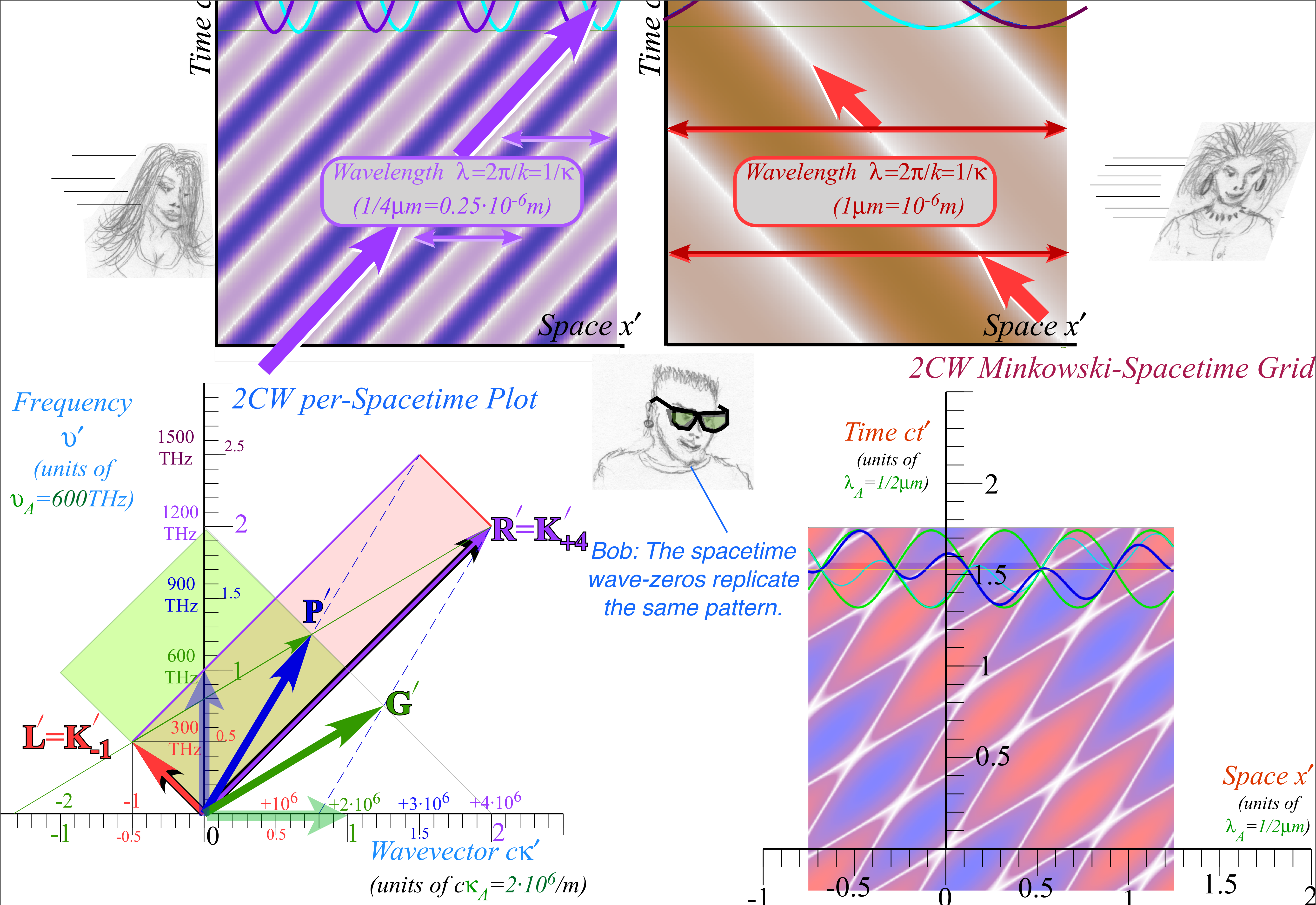
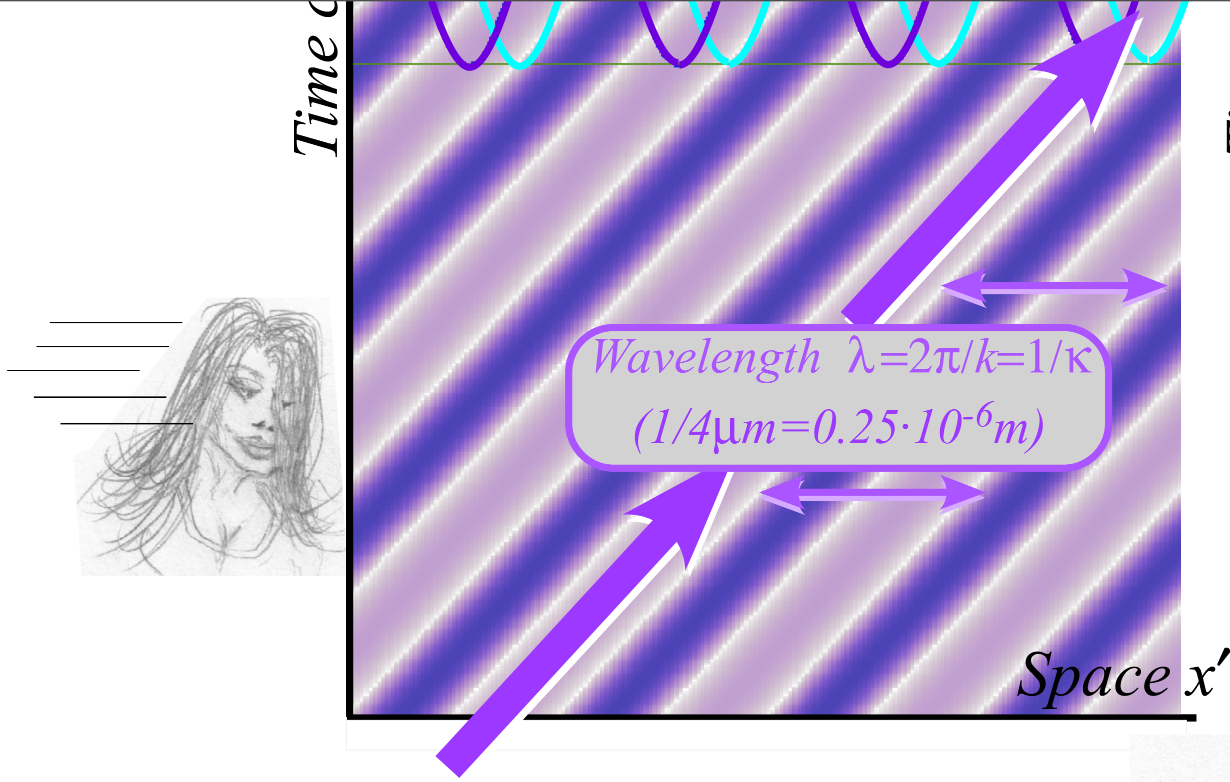


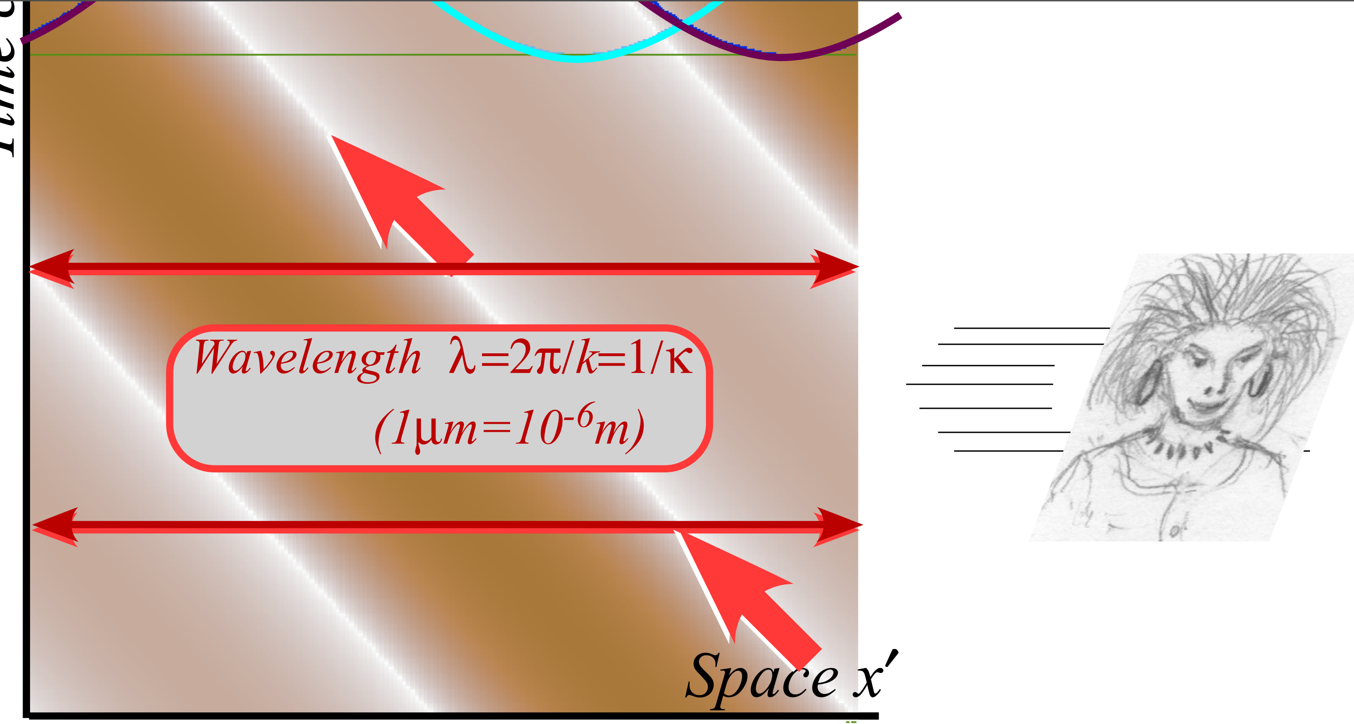
Phase vector **P**
1/2-sum vector

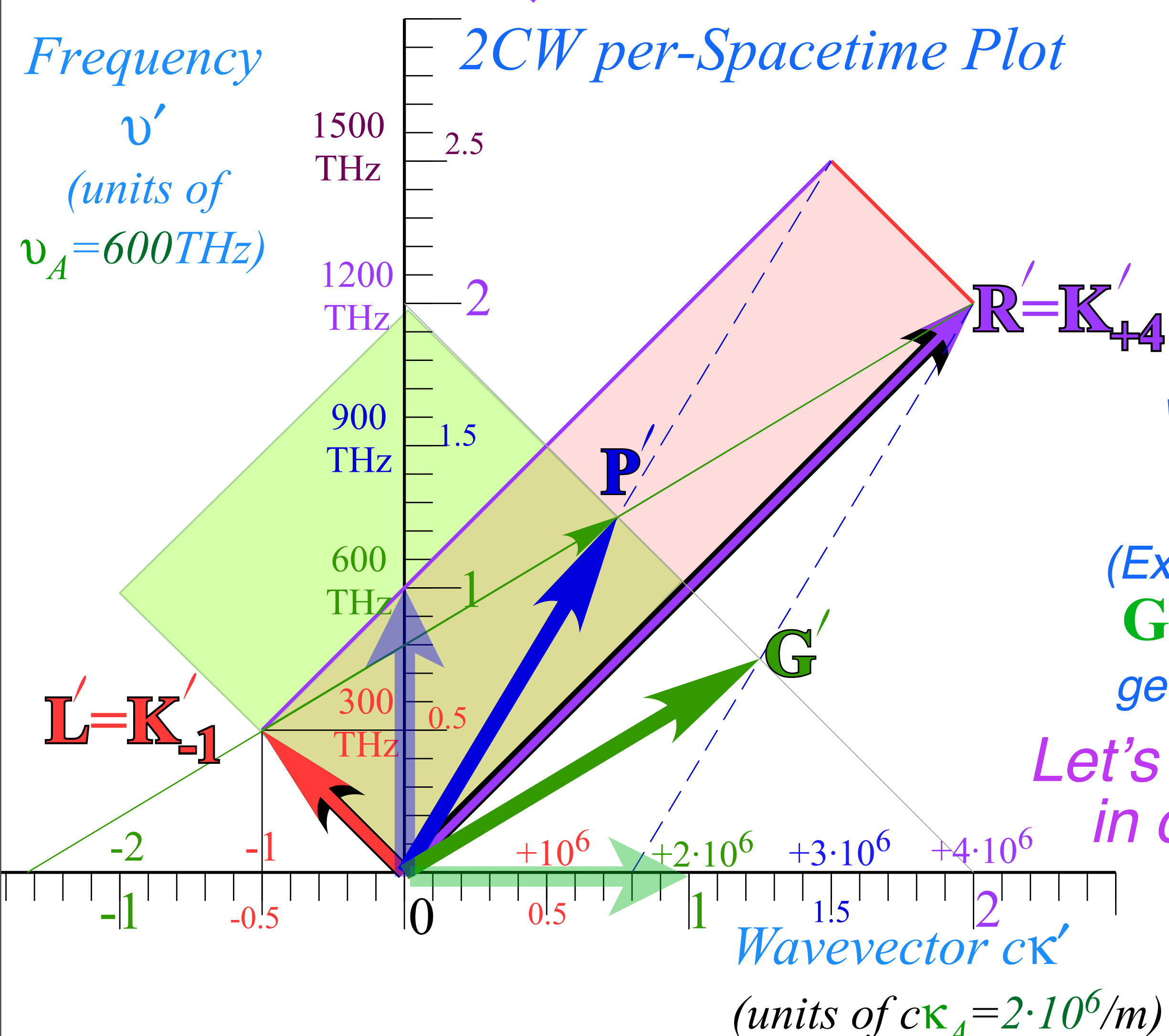
$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$

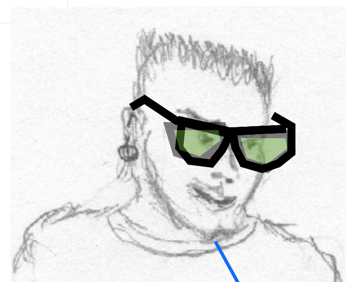
Group vector **G**
1/2-diff vector

$$\mathbf{K}'_{group}=\mathbf{G}'=\frac{\mathbf{R}'-\mathbf{L}'}{2}$$





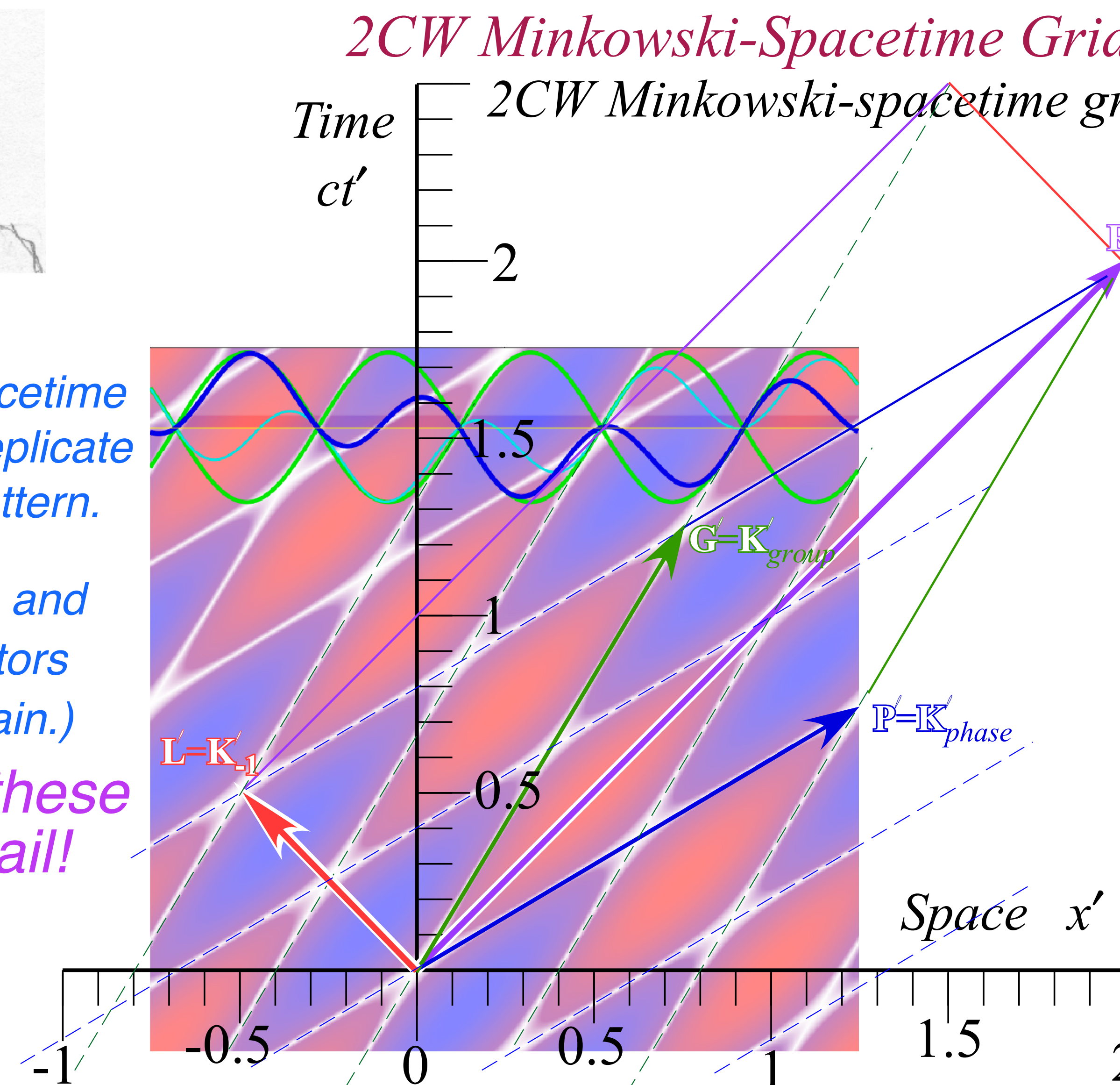




Bob: The spacetime wave-zeros replicate the same pattern.

(Except P′-phase and G′-group indicators get switched again.)

Let's *measure* these in careful detail!



Phase vector **P** 1/2-sum vector $K'_{phase}=P'=\frac{R'+L'}{2}$

Group vector **G** 1/2-diff vector $K'_{group}=G'=\frac{R'-L'}{2}$

Review of Doppler-shift and Rapidity ρ_{AB} calculation: *Galileo's Revenge Part I Lect. 23 p.64-75*

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Doppler shifted Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Minkowski coordinate grid in space-time

➔ Animations that compare Doppler shifted colliding CW with colliding PW ⬅

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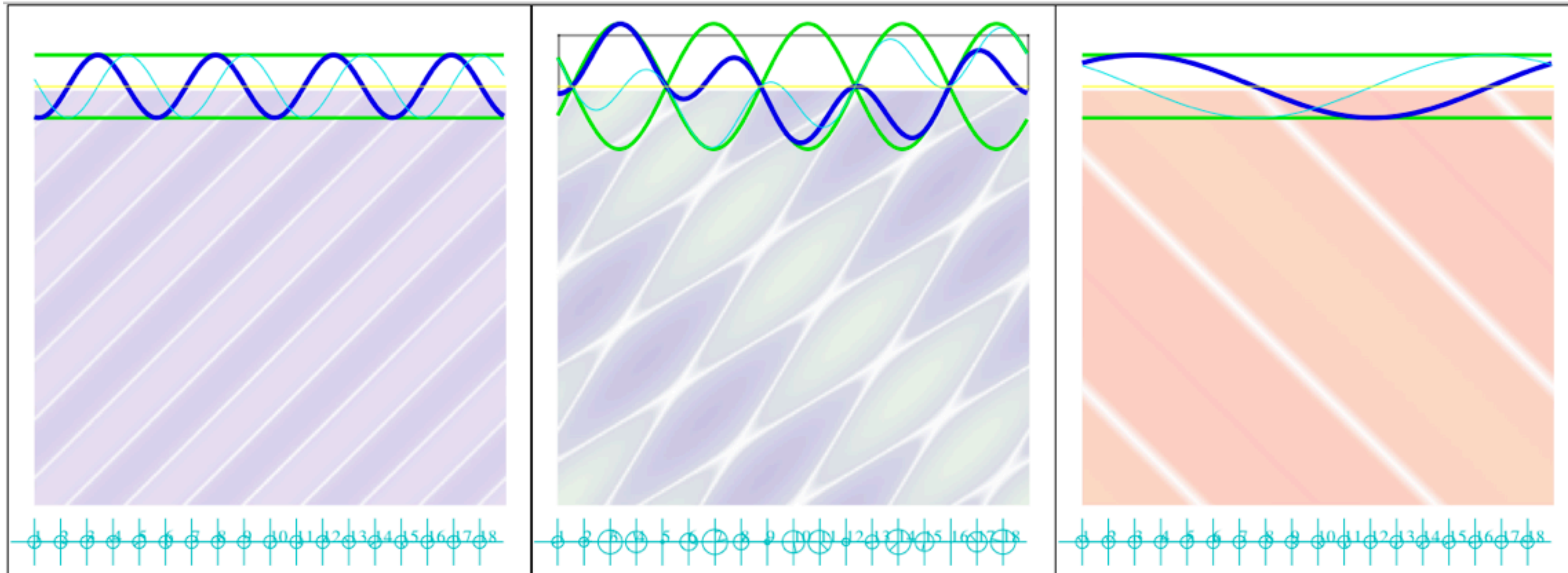
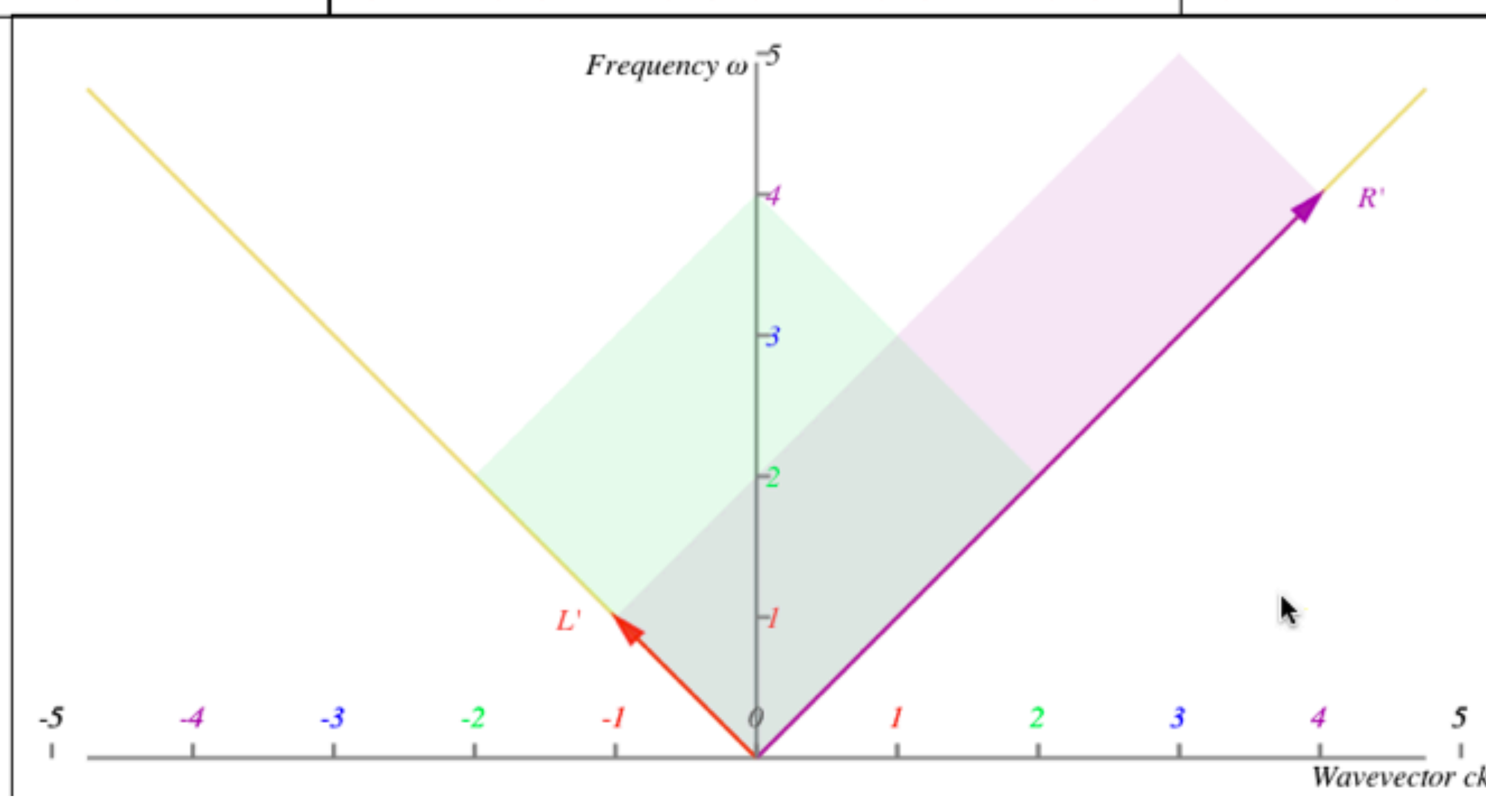
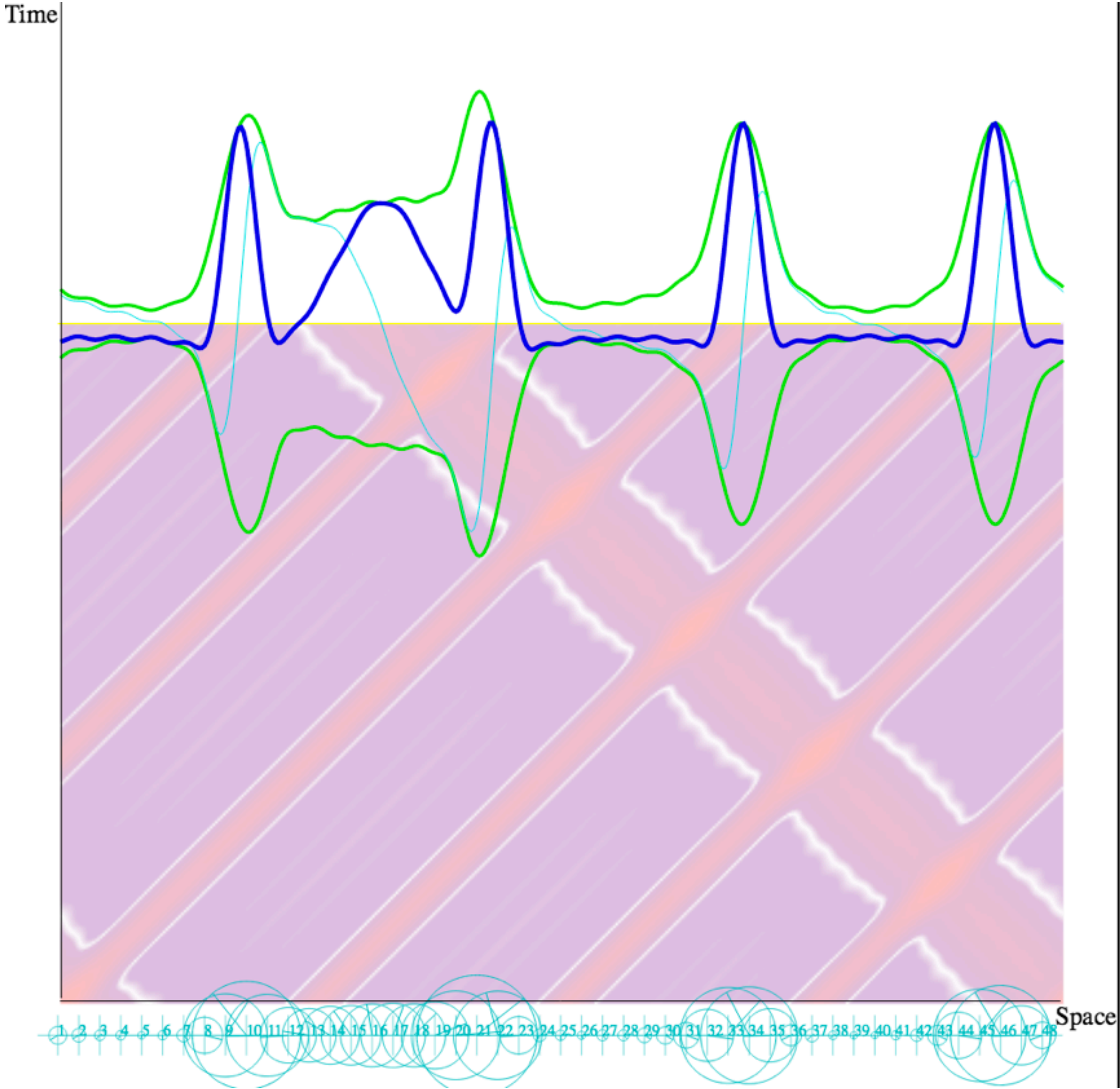
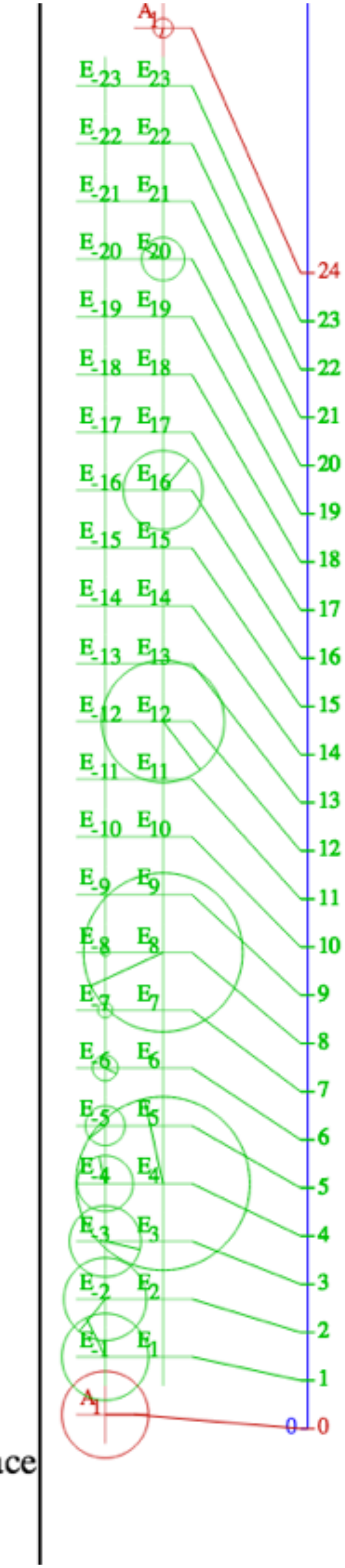




[BohrIt Web Simulation](#)
[2 CW *Minkowski* Plot (ck = -1, +4)](#)





BohrIt Web Simulation
2 PW ct vs x Plot ($\beta = u/c = 3/5$)

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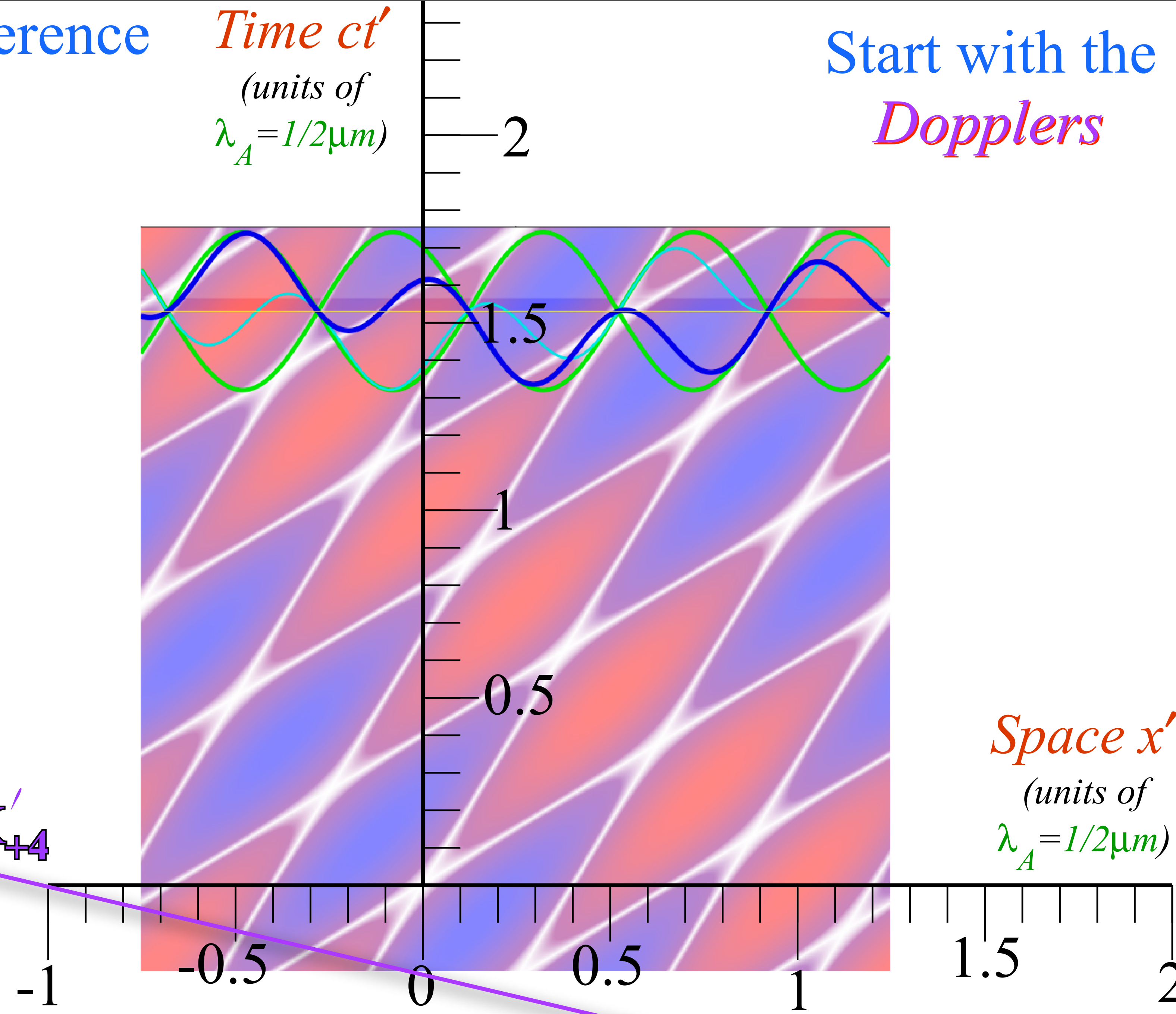
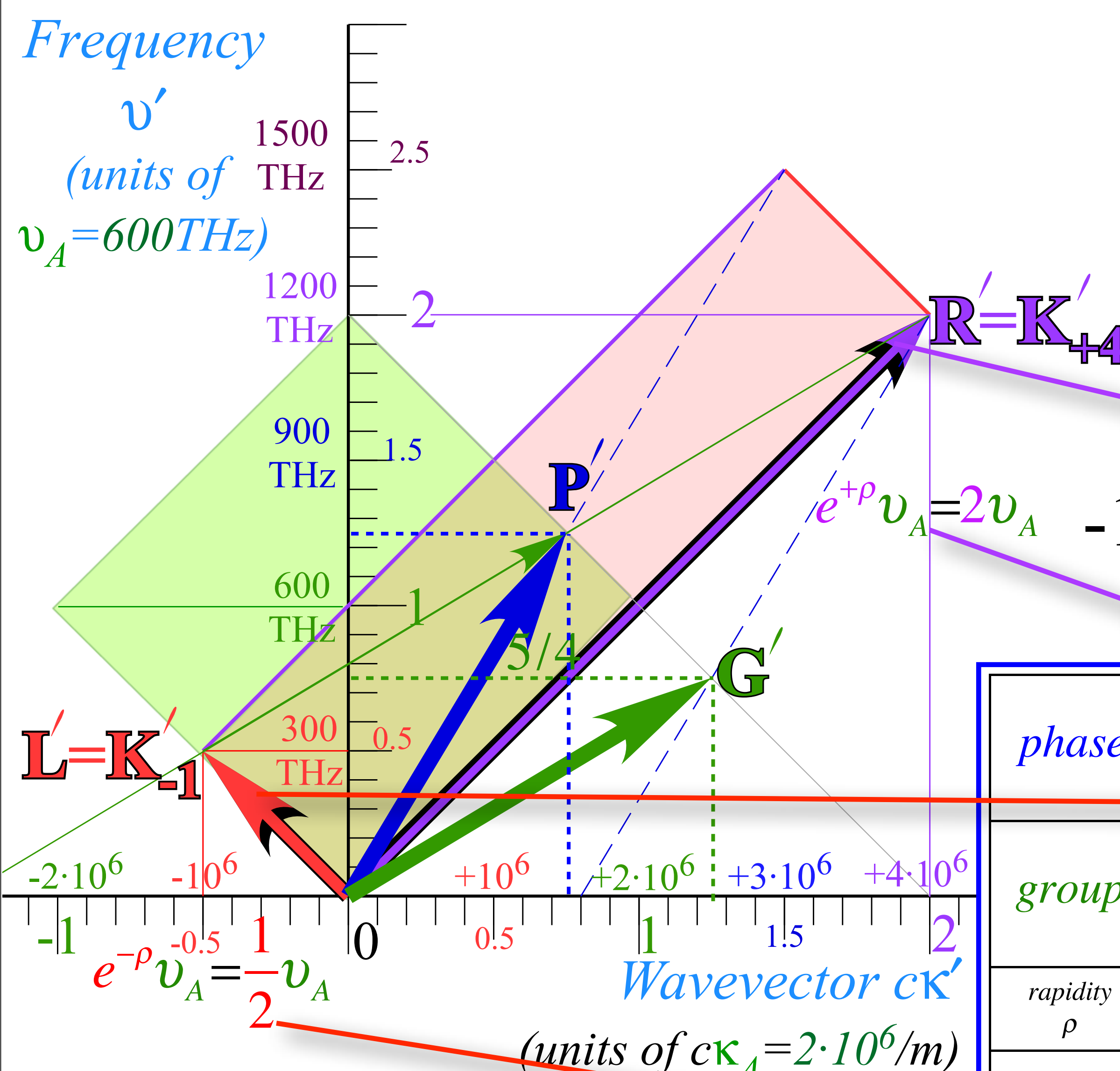
The 16 parameters of 2CW interference

Start with the *Dopplers*





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation - *16 Relativity Dimensions*

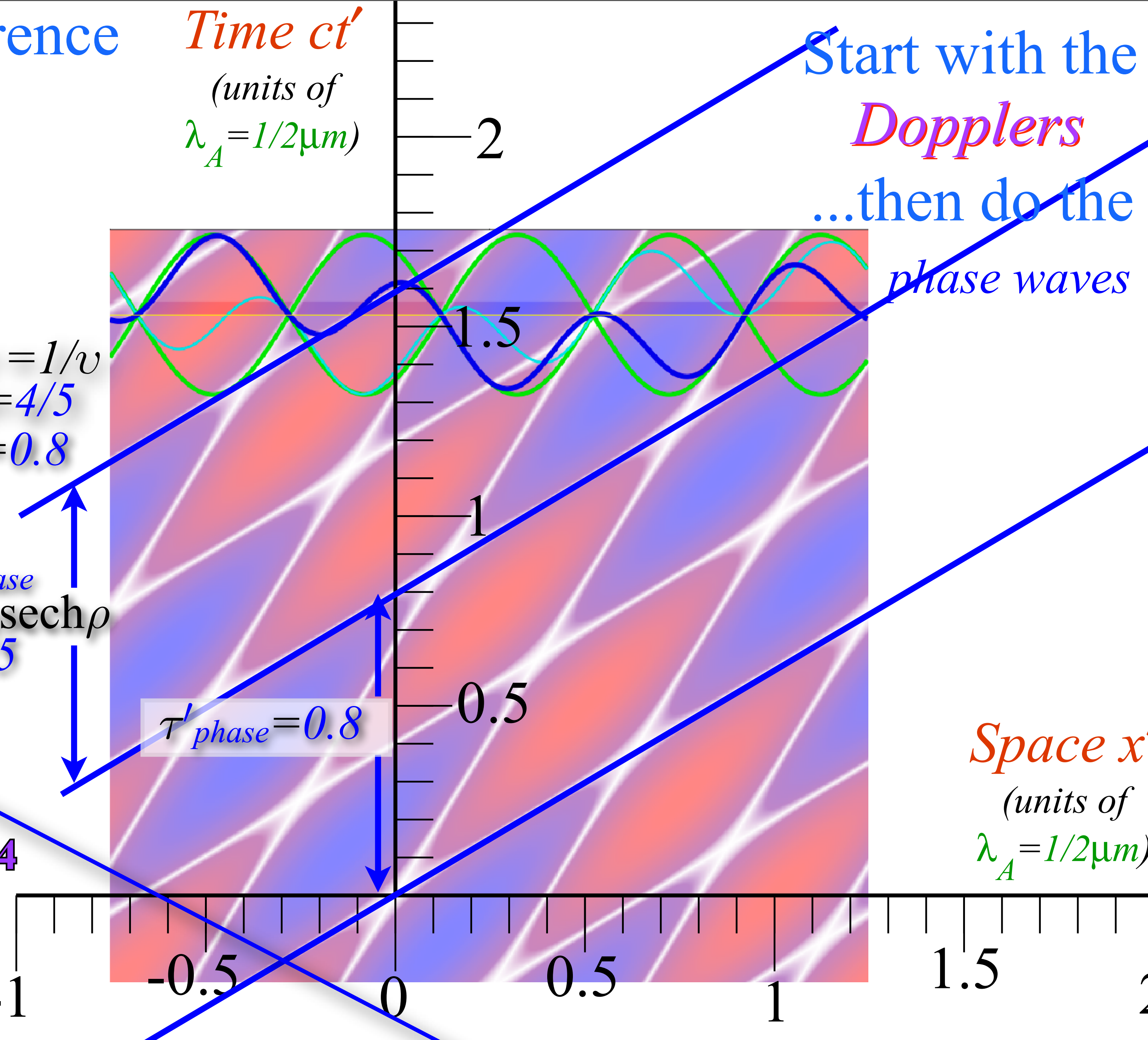
The 16 parameters of 2CW interference

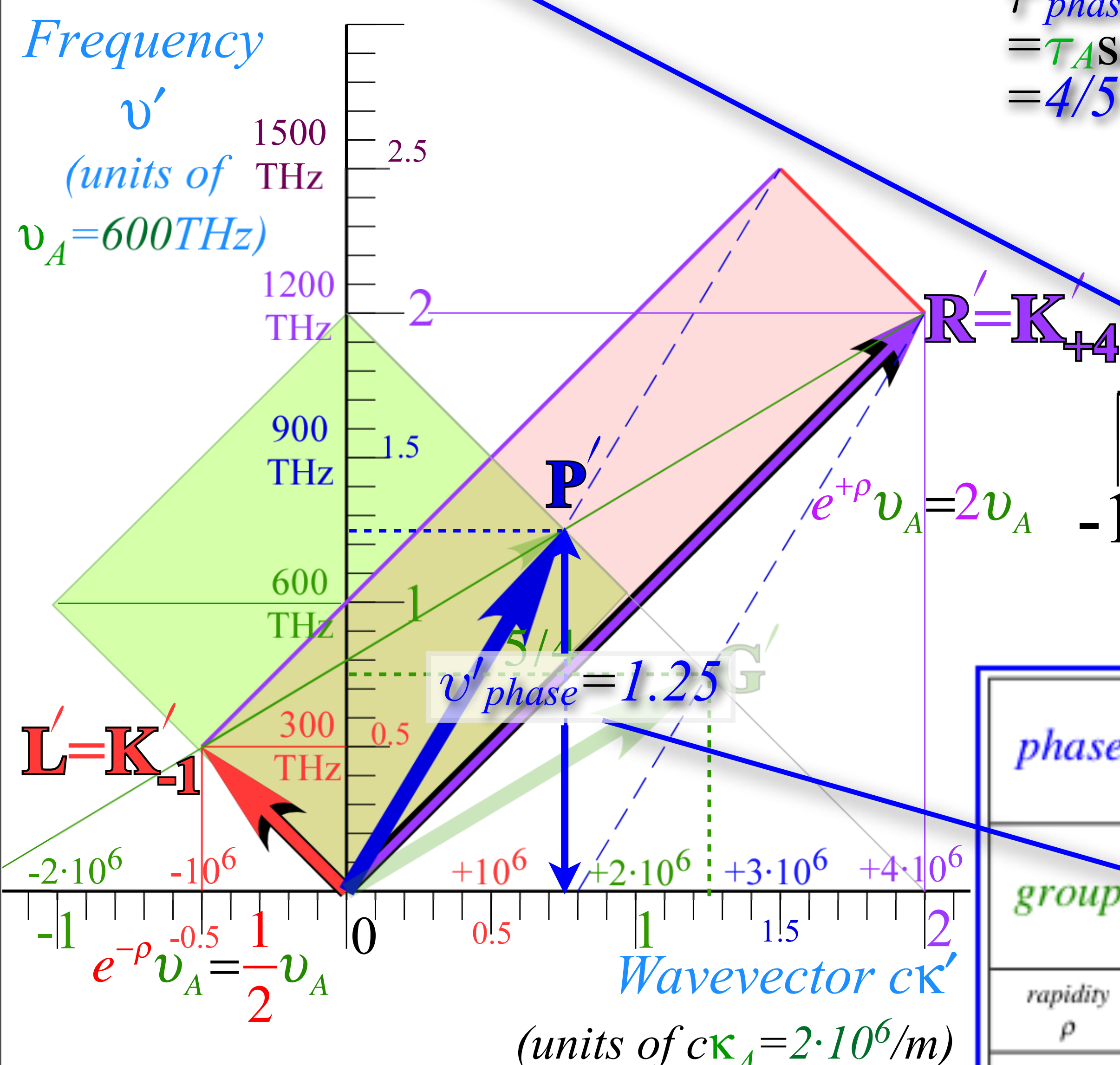
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$
$= 1.25$

flips to

Phase period $\tau = 1/\upsilon$
$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$
$= 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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The 16 parameters of 2CW interference

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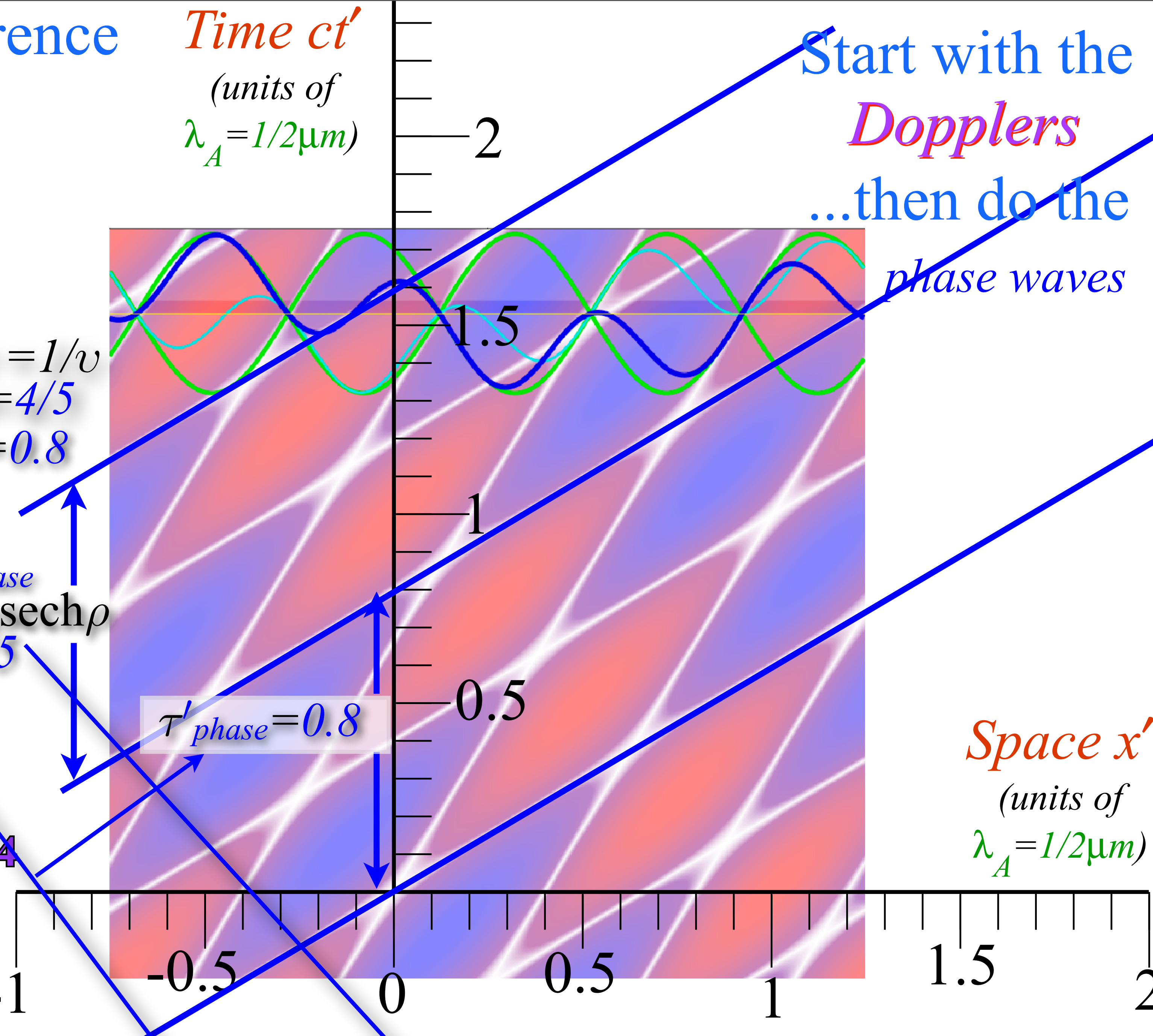
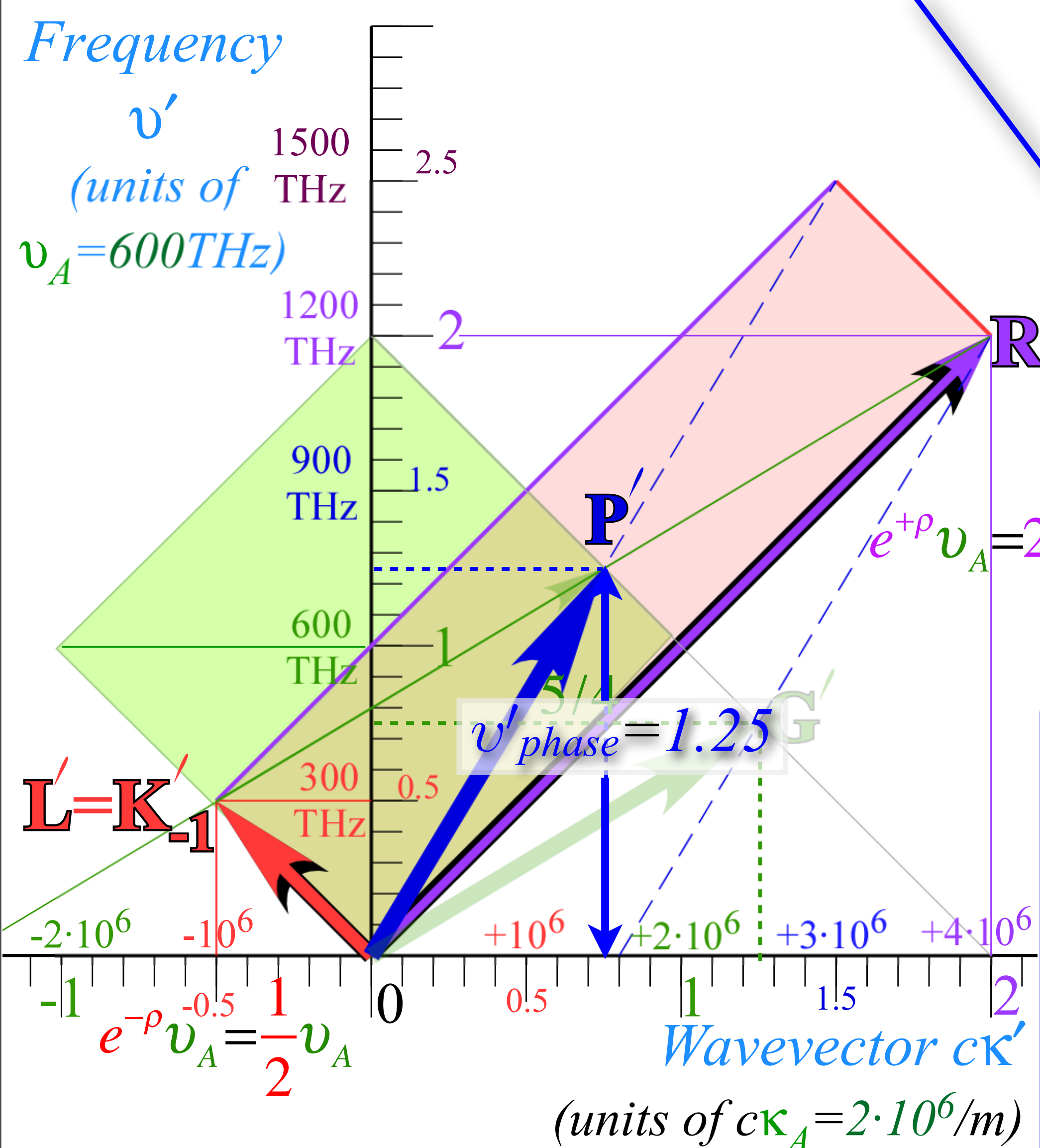
Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$
$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation - *16 Relativity Dimensions*

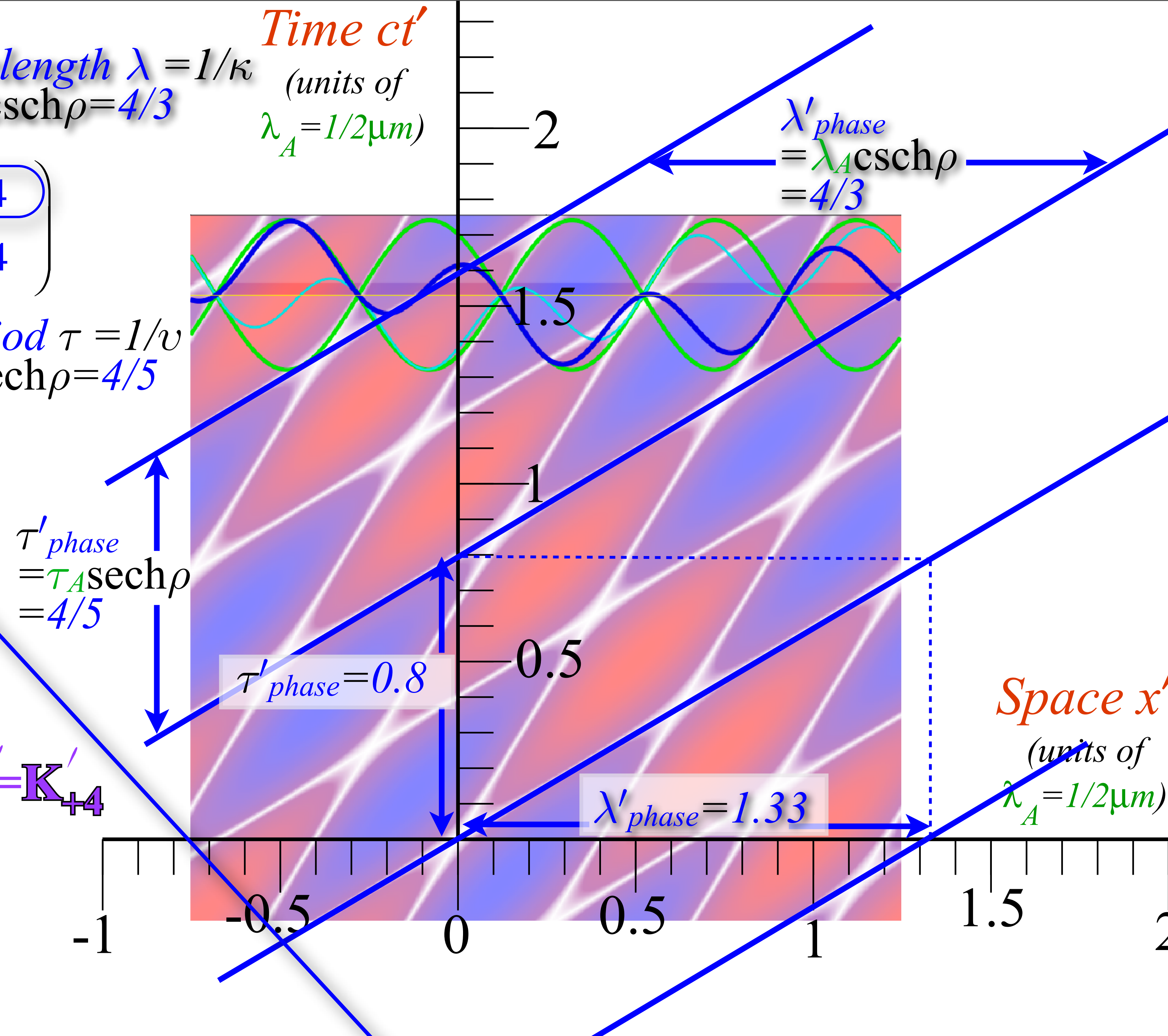
Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

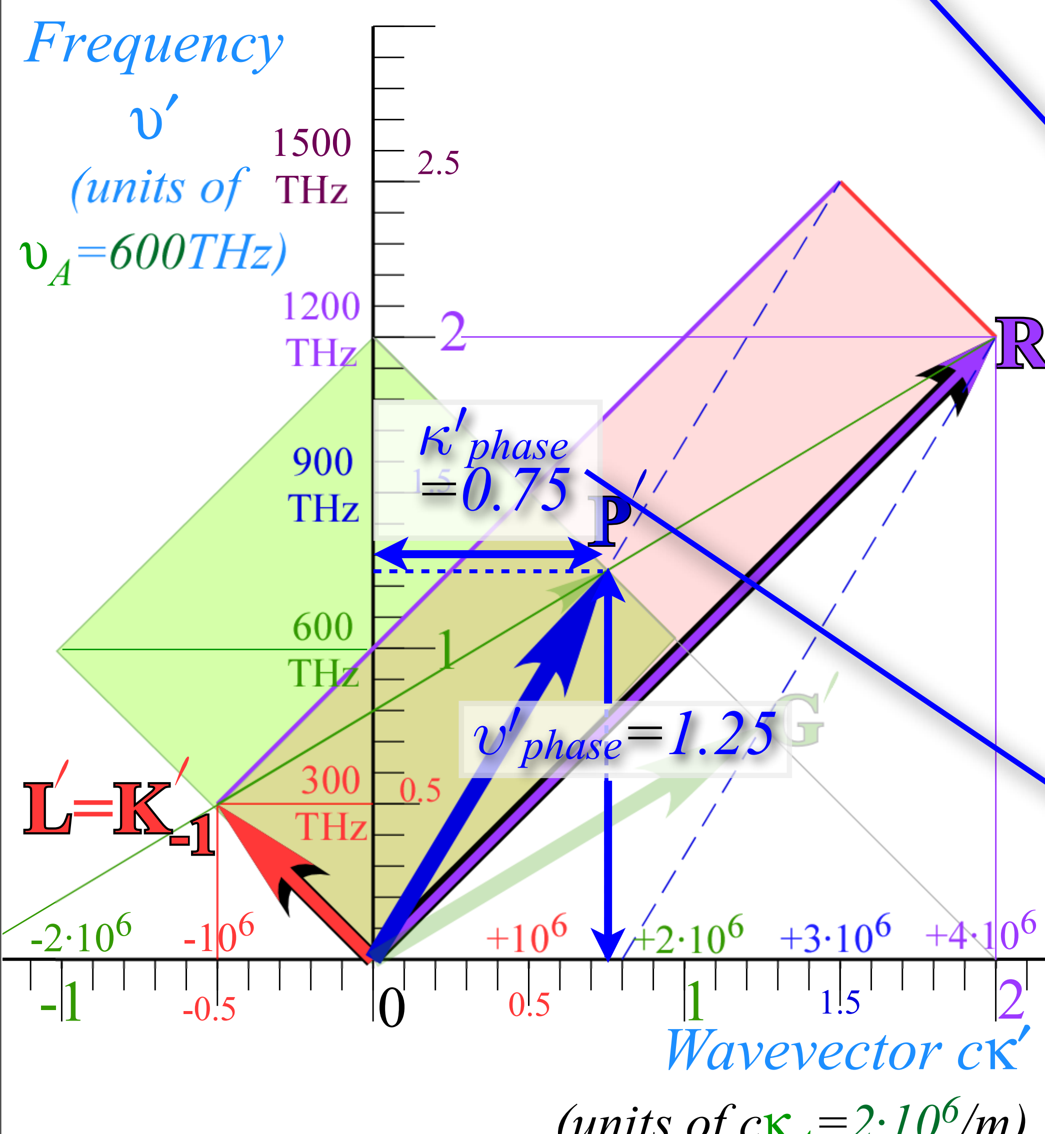
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}$=0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

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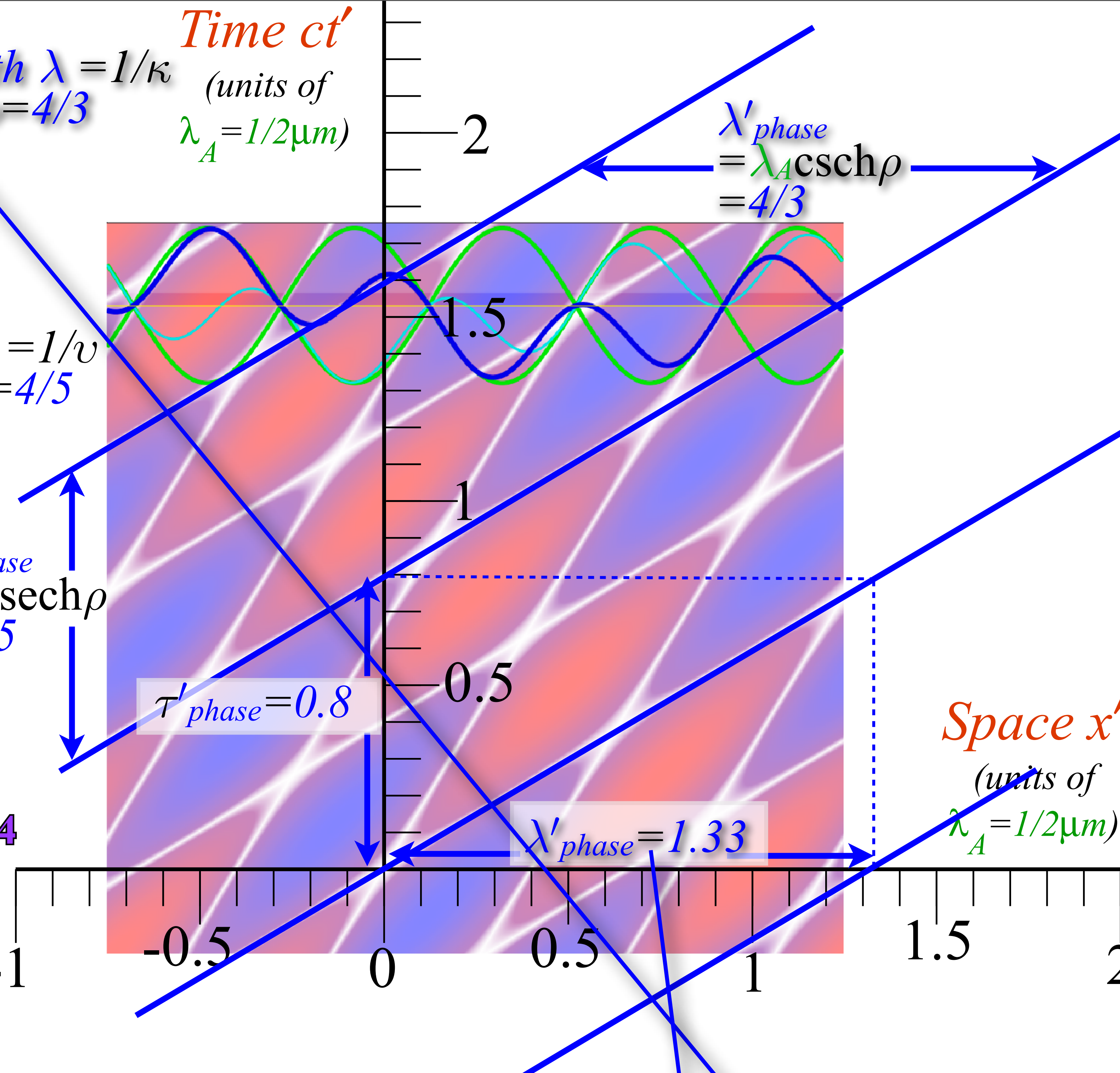
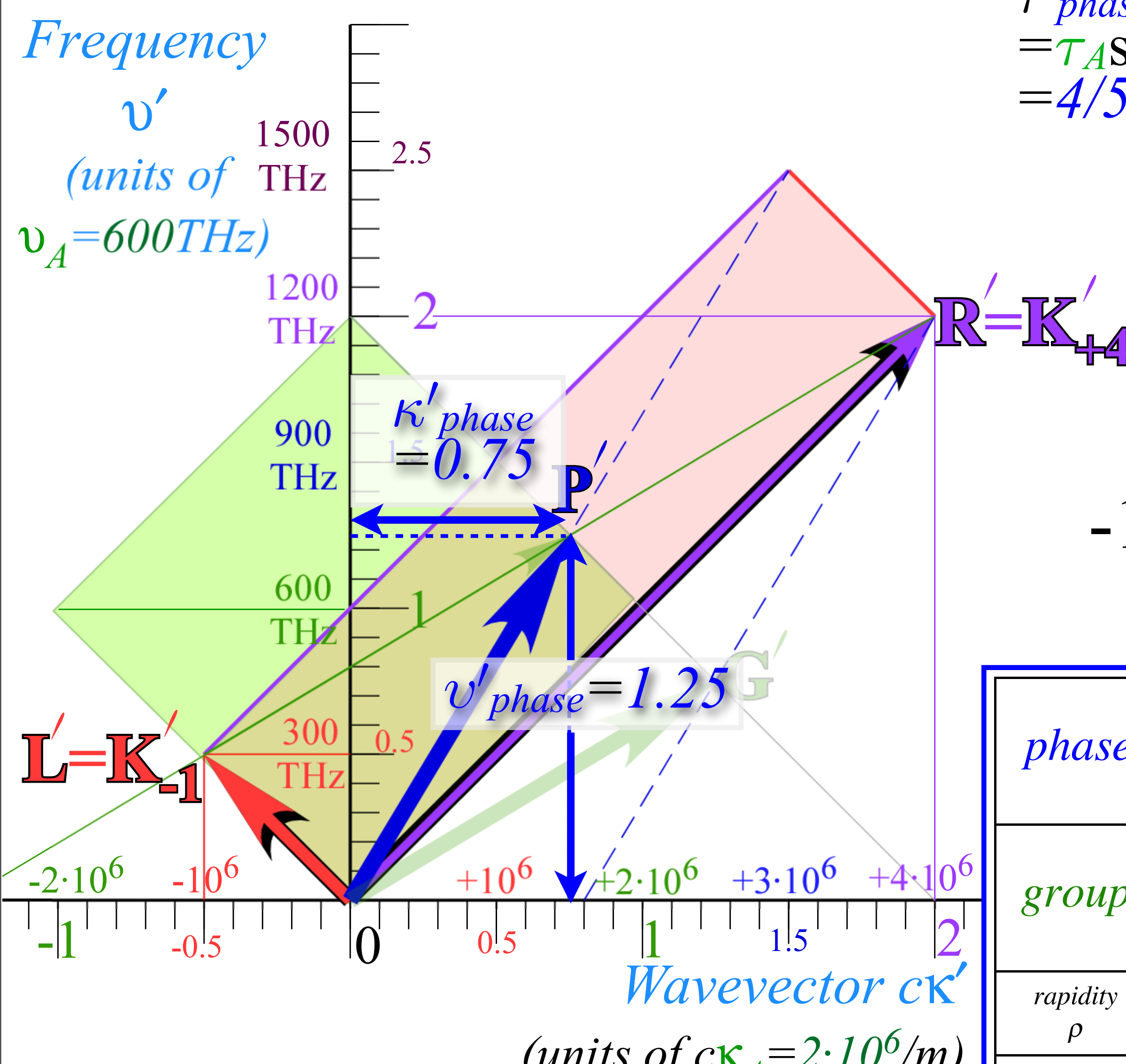
Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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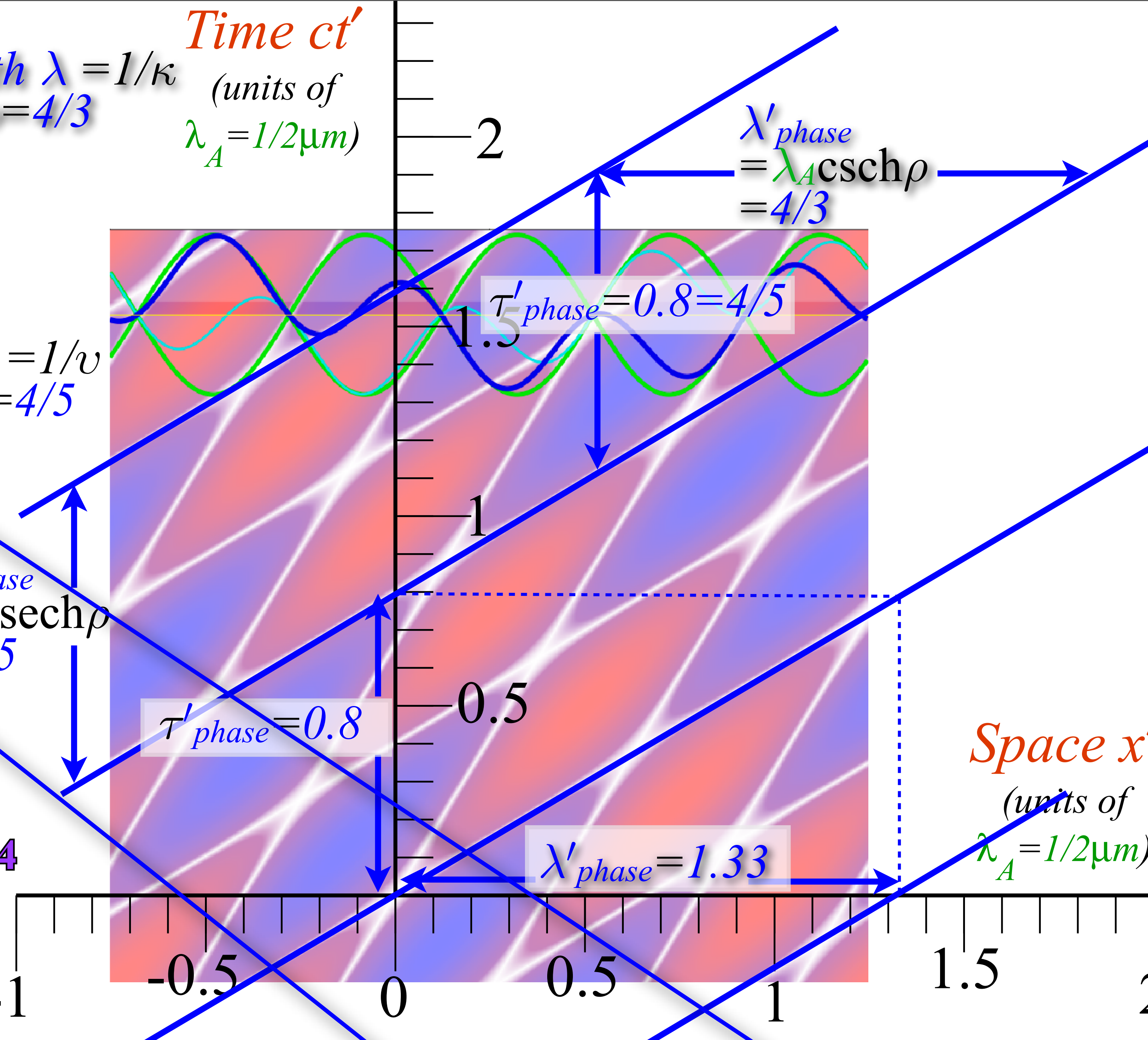
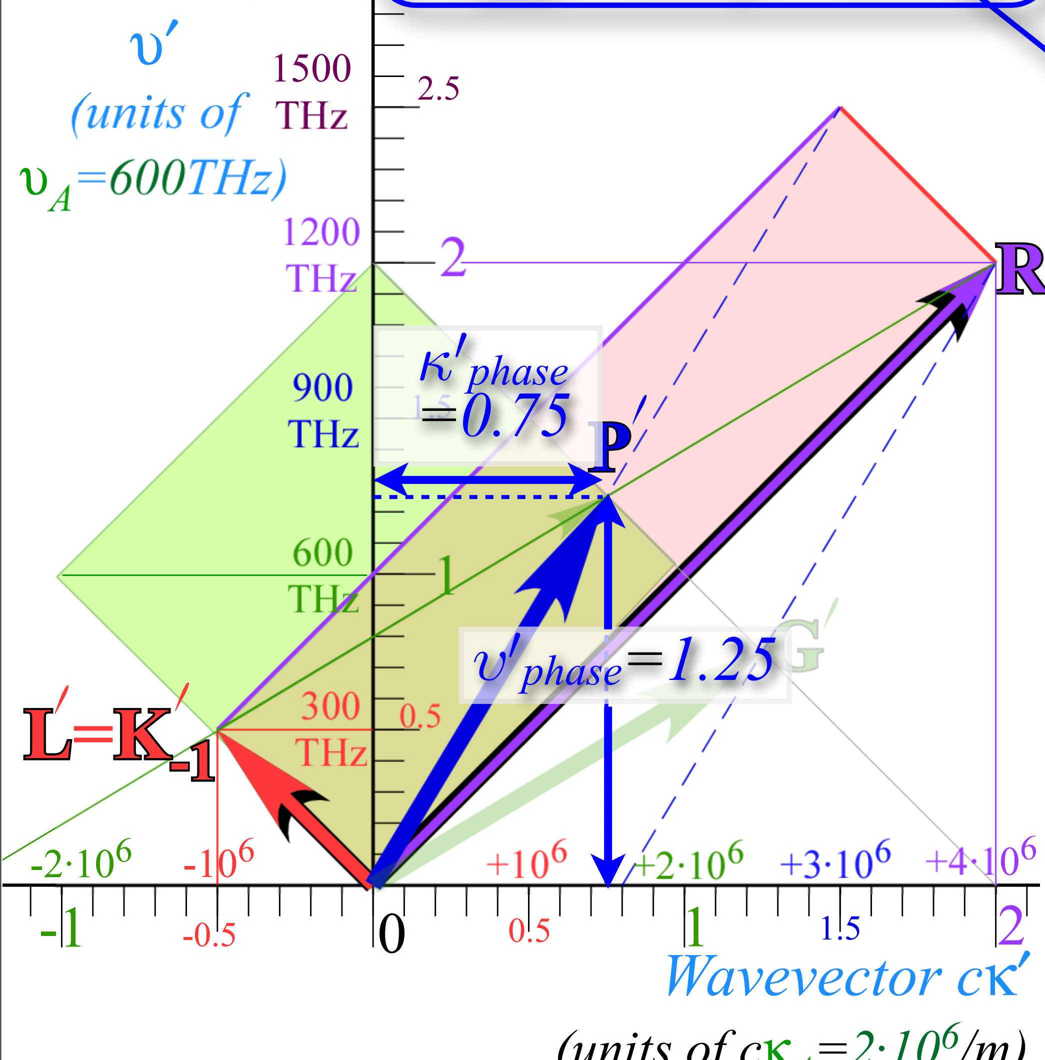
Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

$$\mathbf{P}\text{-slope} = V_{phase}/c = \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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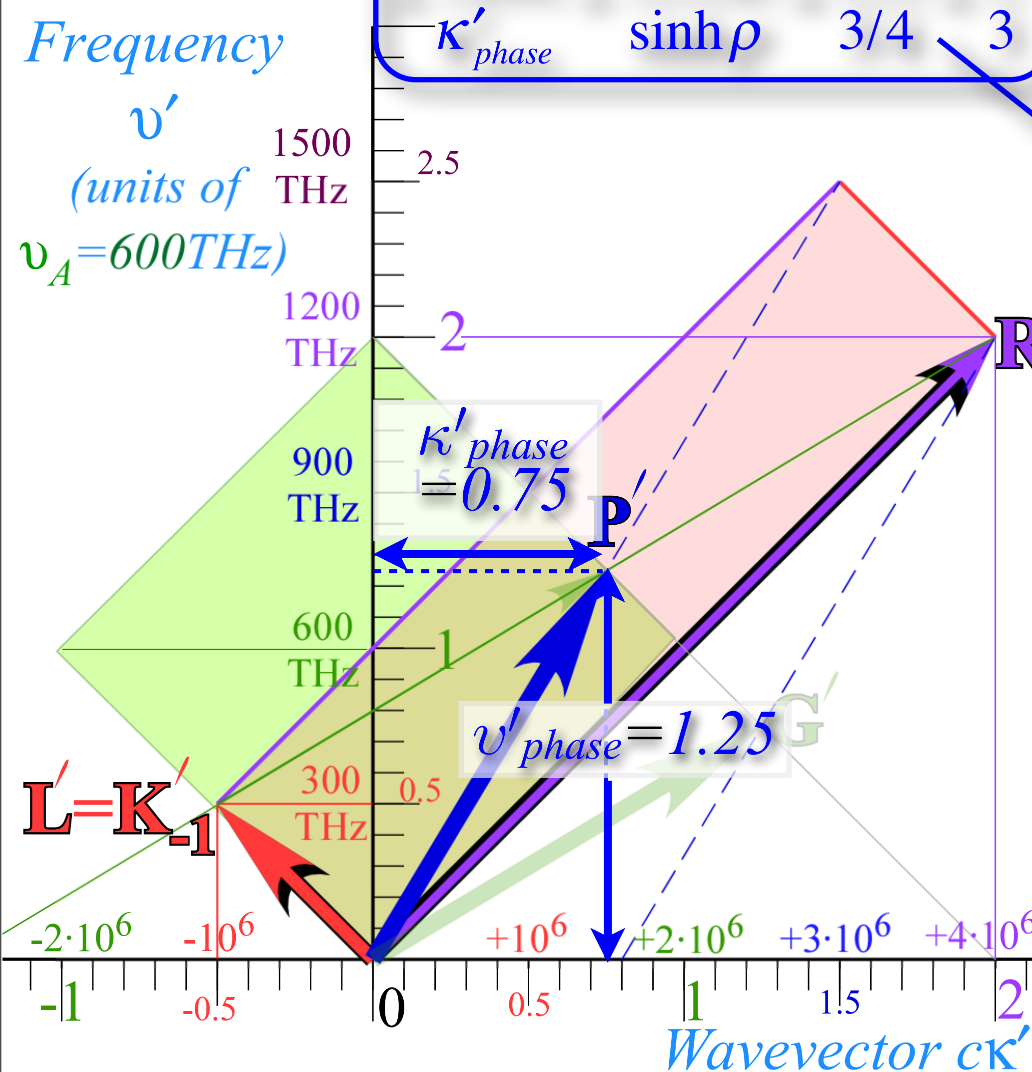
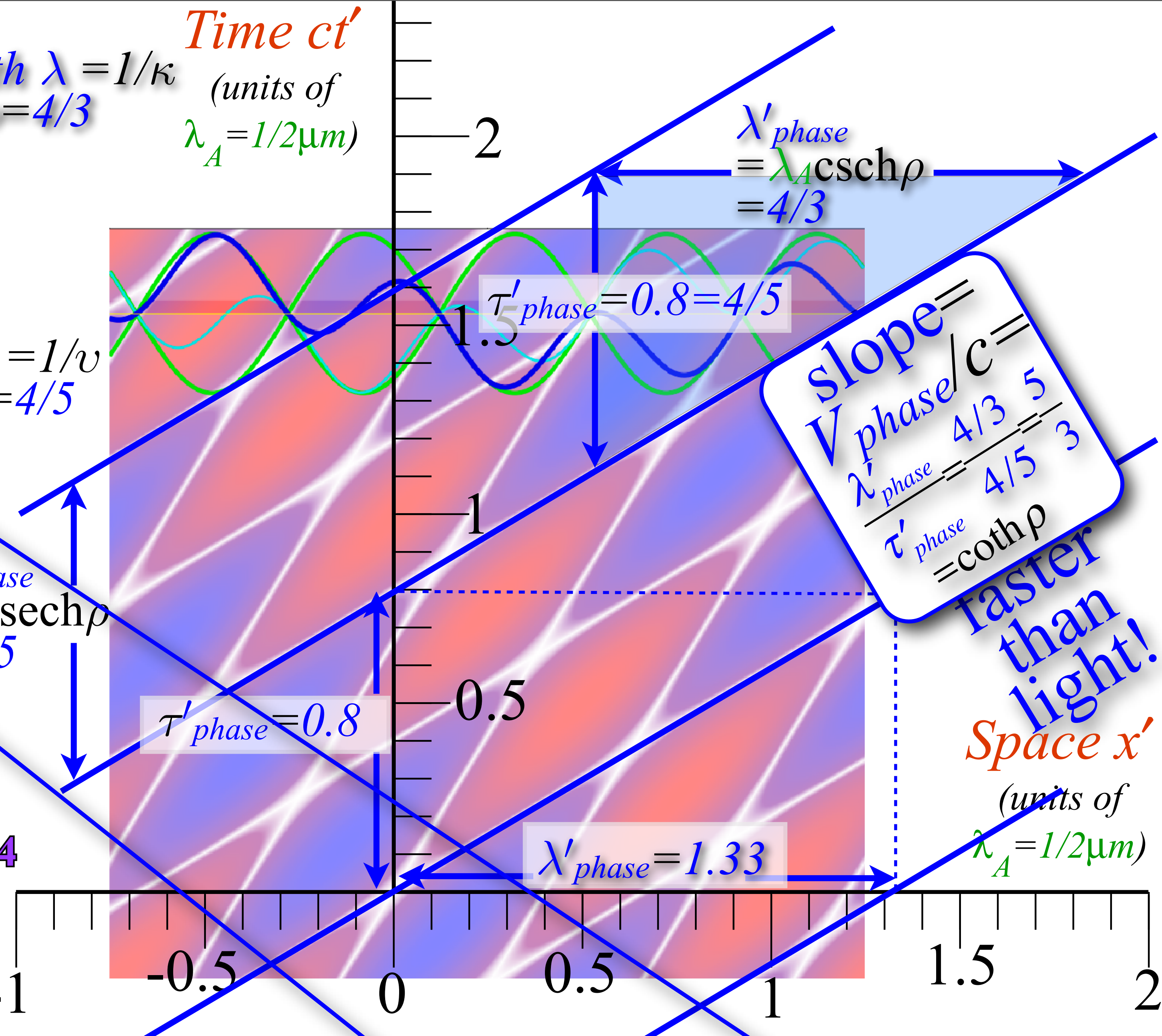
Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

P-slope $= V_{phase}/c$

$$= \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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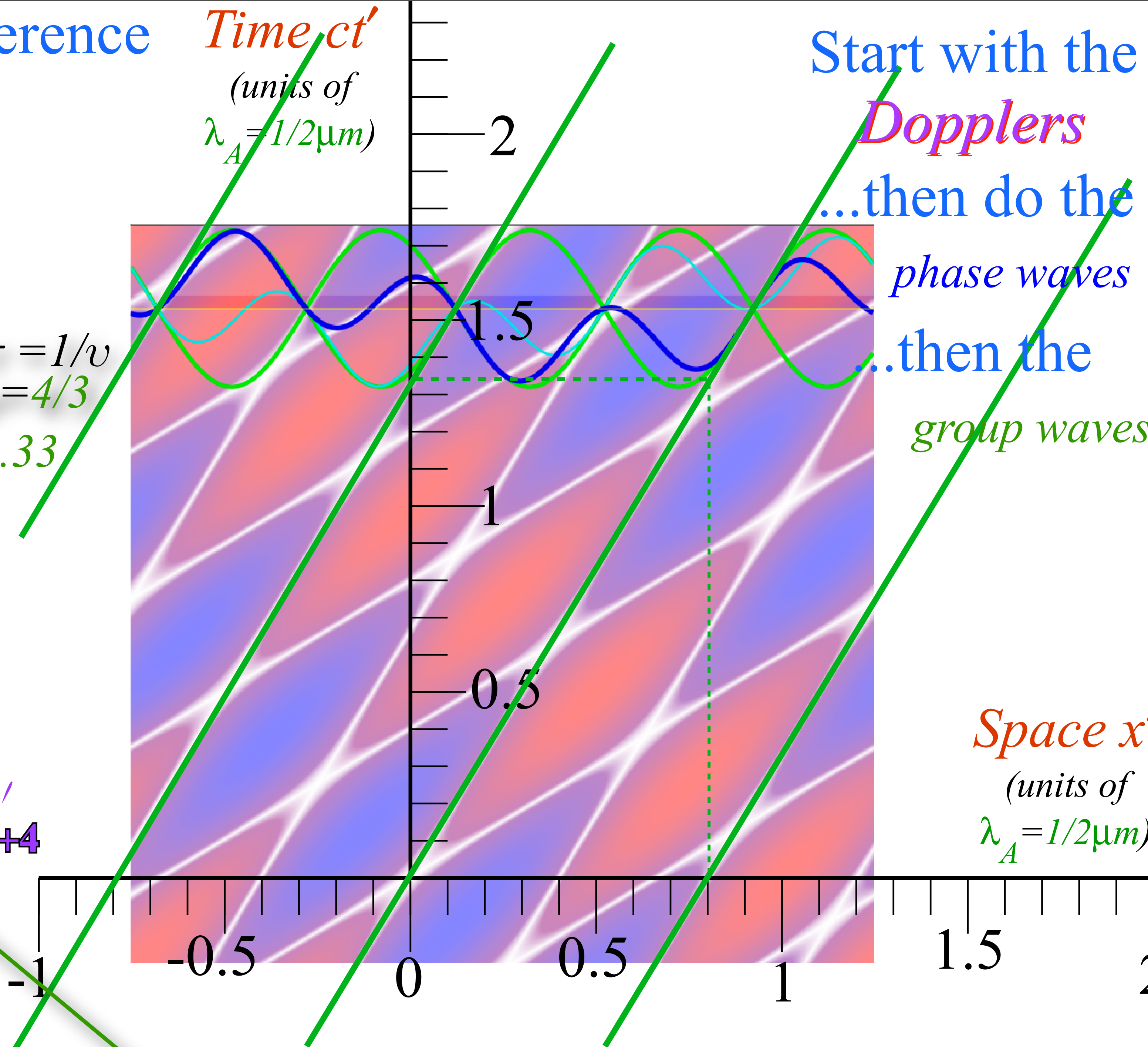
The 16 dimensions of 2CW interference

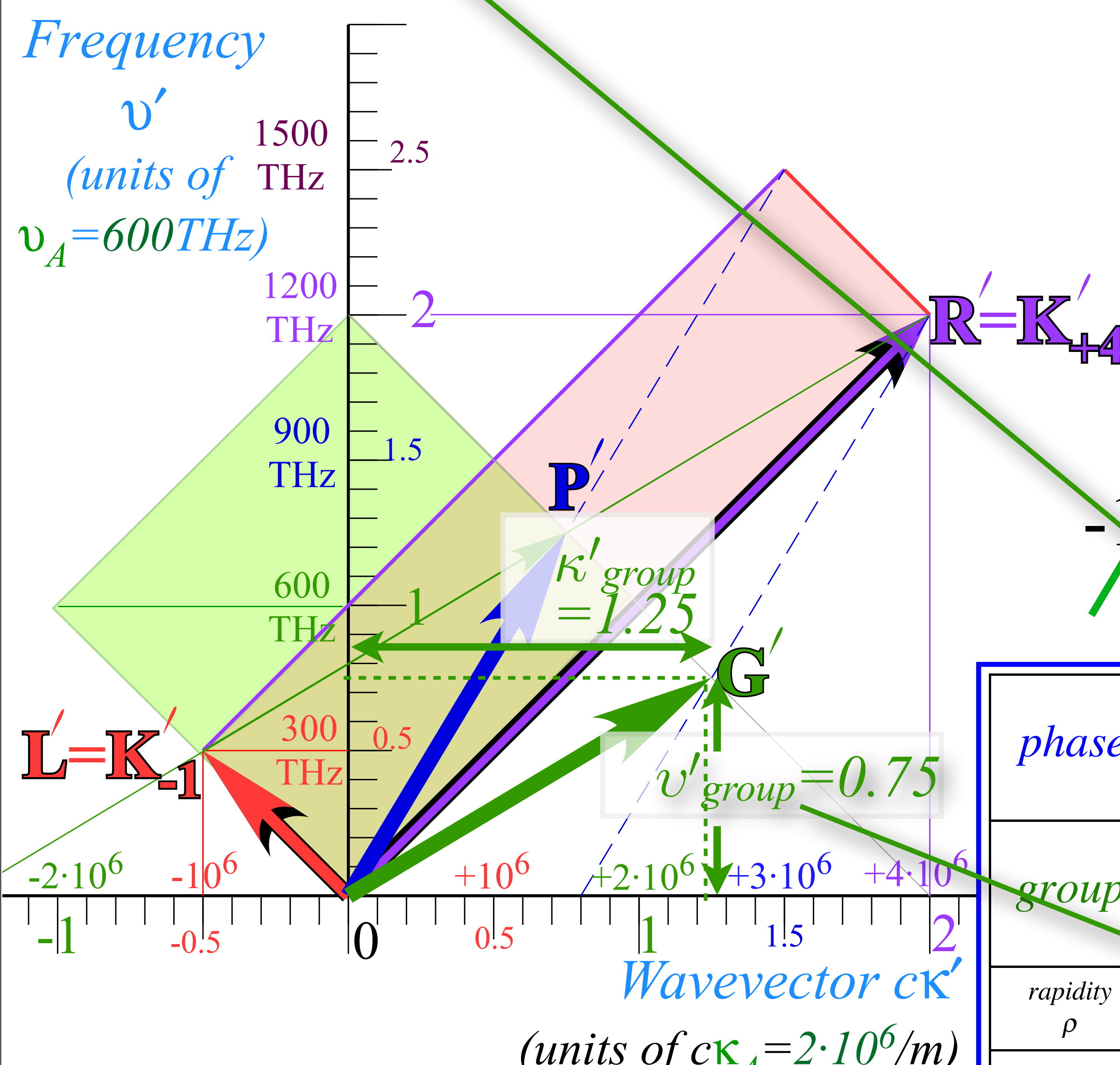
$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

flips to

Group period $\tau = 1/\upsilon$ $\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$





RelaWavity Web Simulation - *16 Relativity Dimensions*

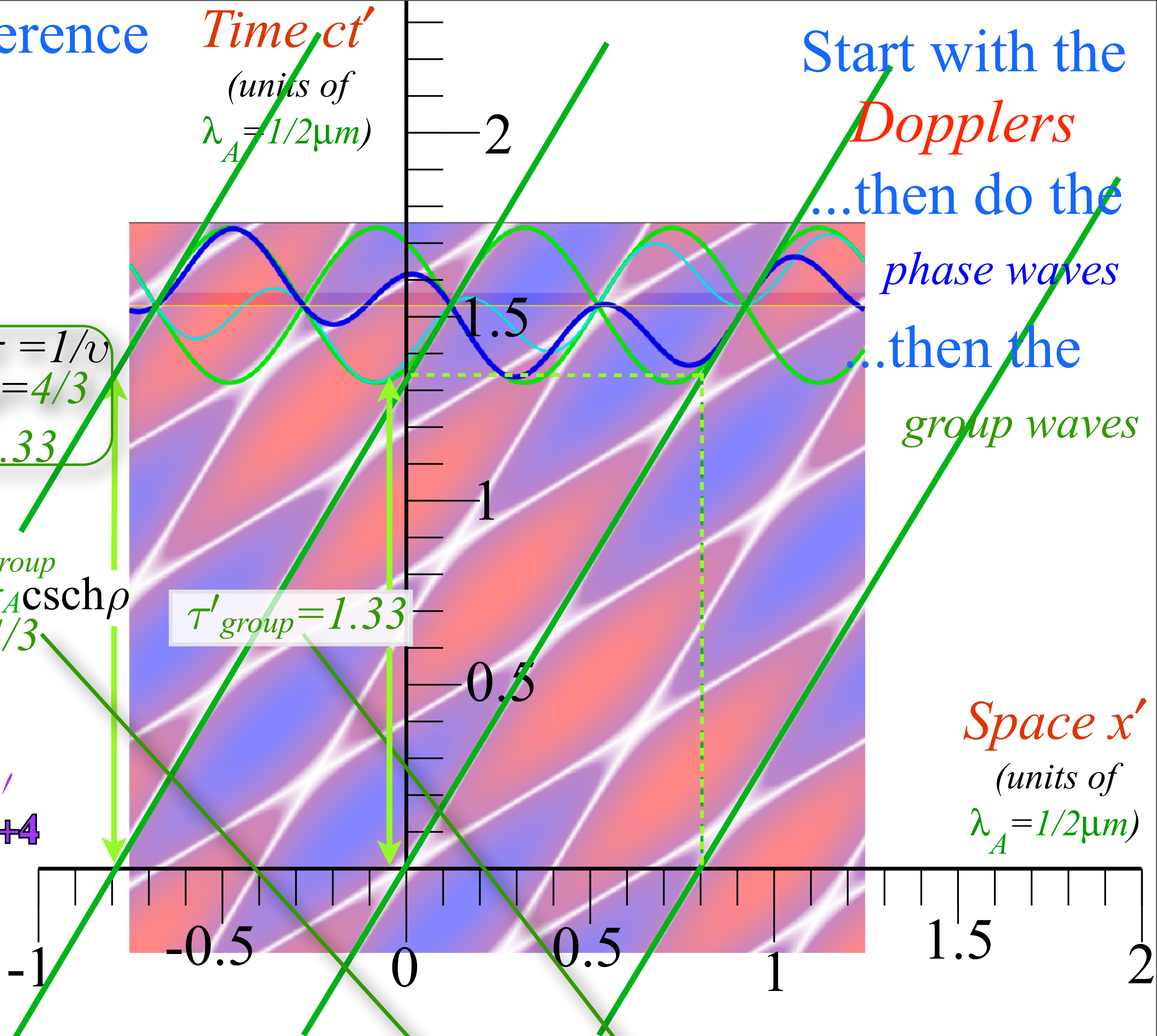
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

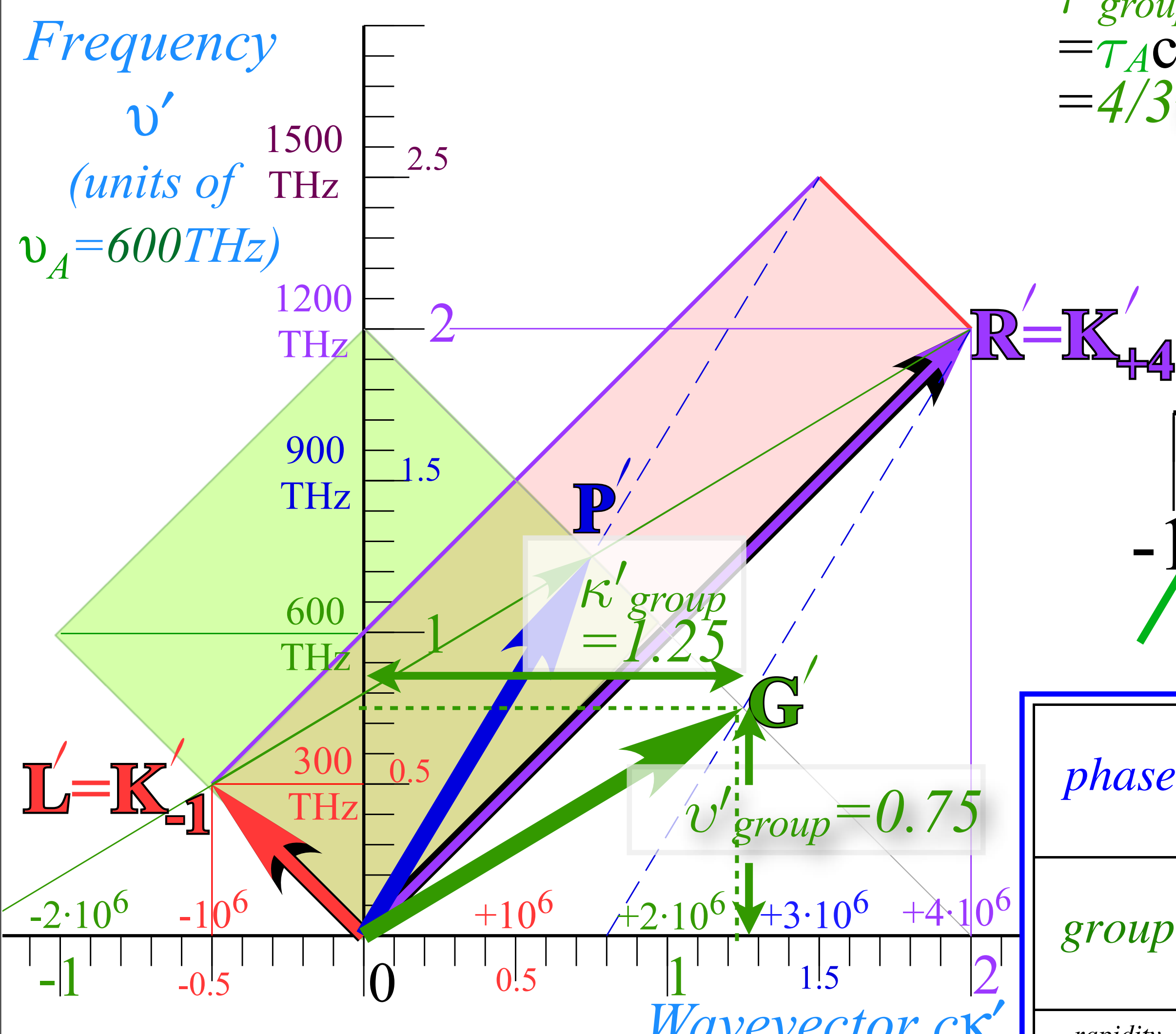
The 16 dimensions of 2CW interference

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$ flips to *Group period* $\tau = 1/\upsilon$: $\tau'_{group} = \tau_A \mathrm{csch}\rho = 4/3 = 1.33$

Start with the Dopplers ...then do the *phase waves* ...then the *group waves*





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group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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RelaWavity Web Simulation - *16 Relativity Dimensions*

Group wavenumber $\kappa'_{group}=\kappa_A\cosh\rho=5/4=1.25$

Group wavelength $\lambda=1/\kappa$

$\lambda'_{group}=\lambda_A\text{sech}\rho=4/5=0.8$

$$\mathbf{G}'=\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho \\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}5/4 \\ 3/4\end{pmatrix}$$

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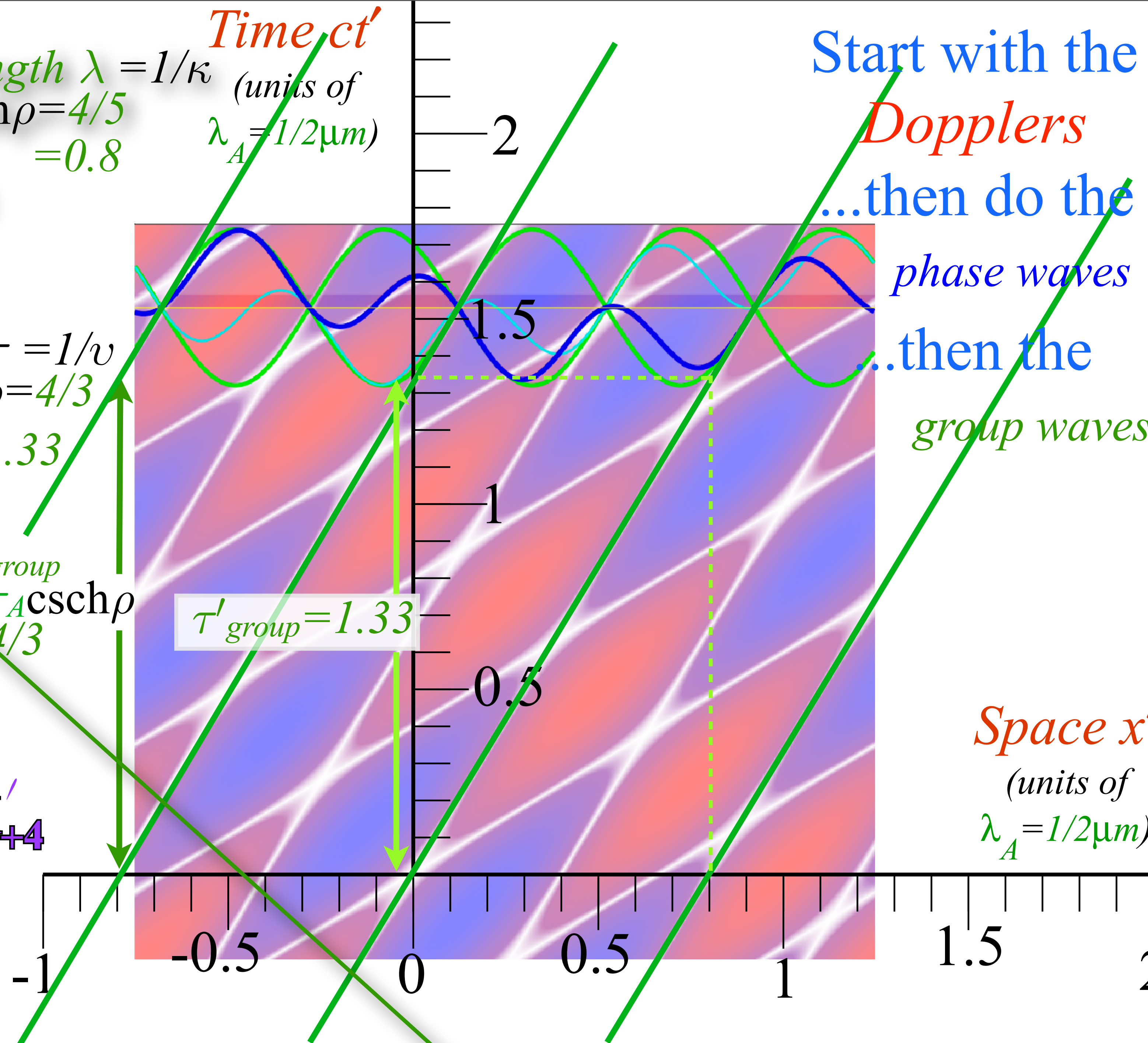

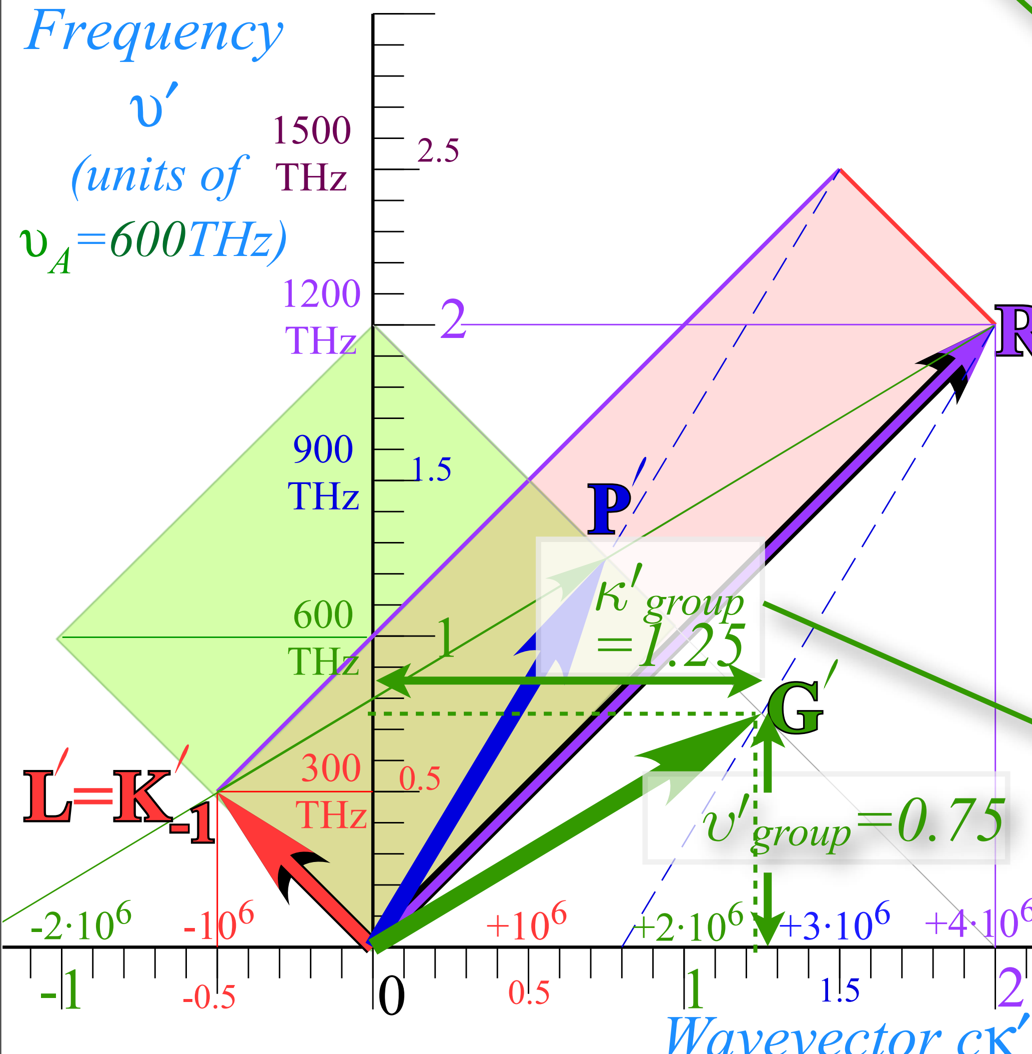

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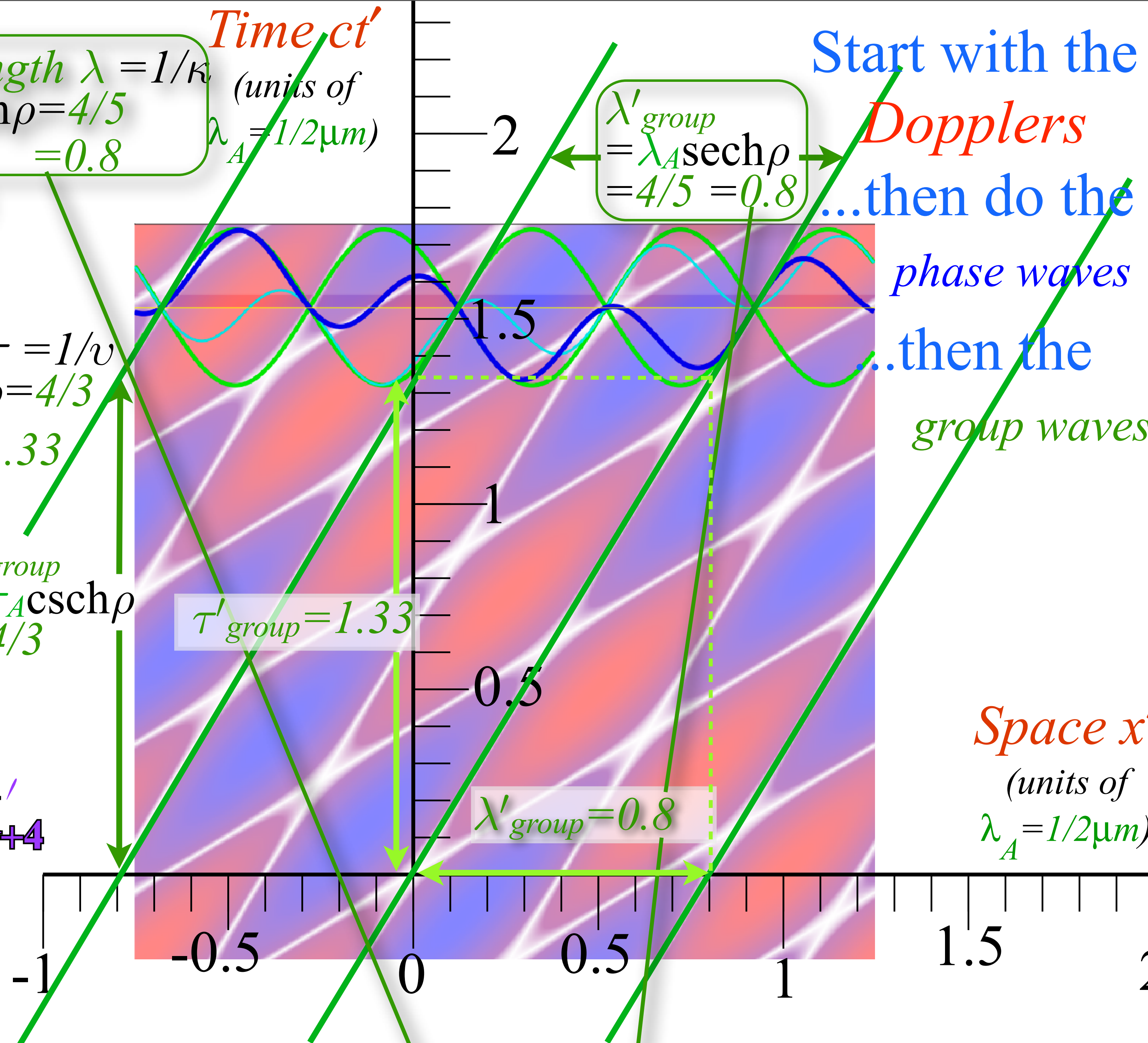

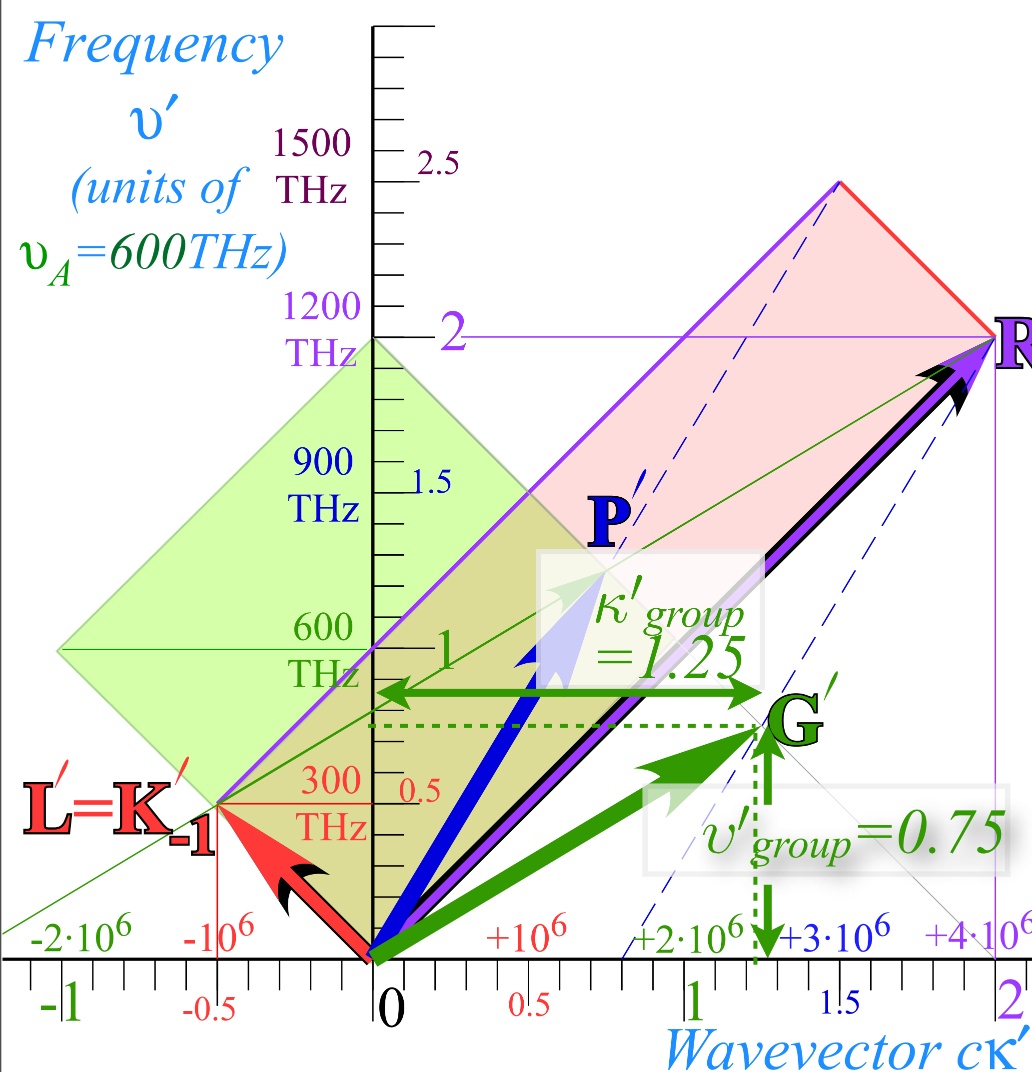

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group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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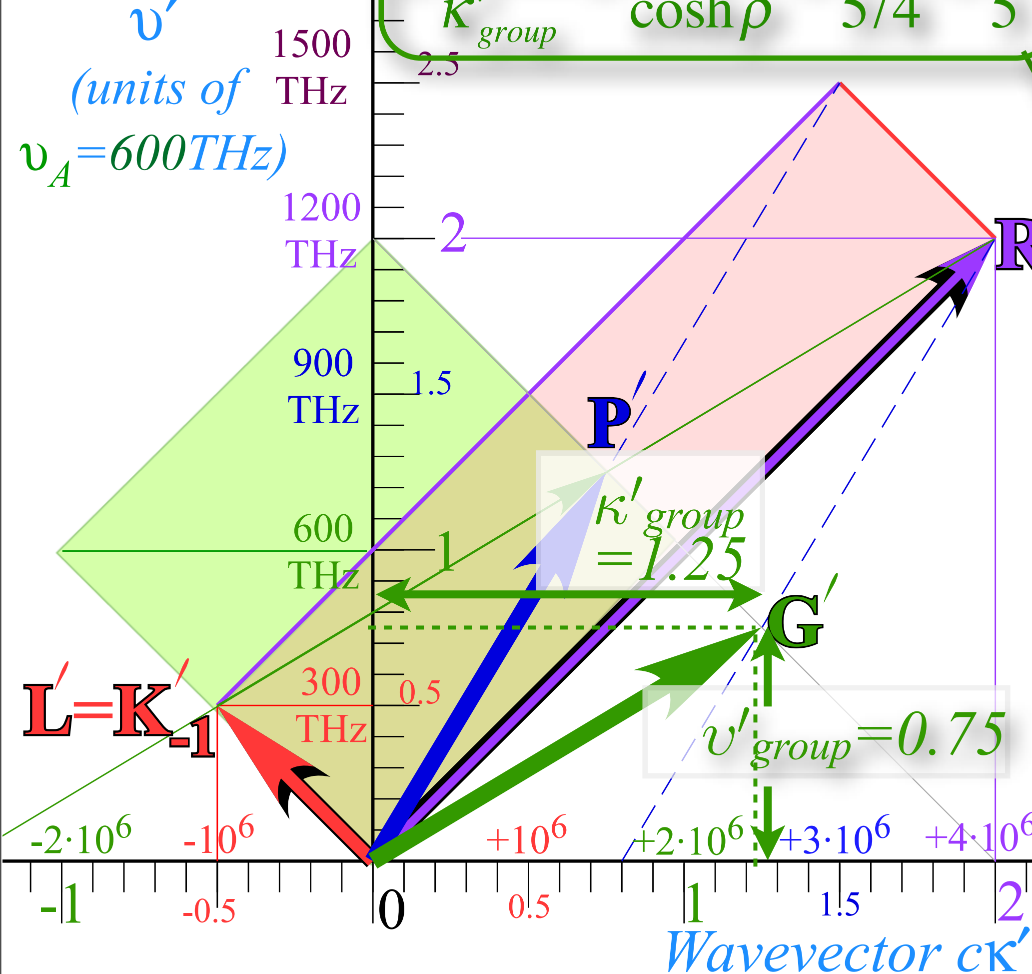
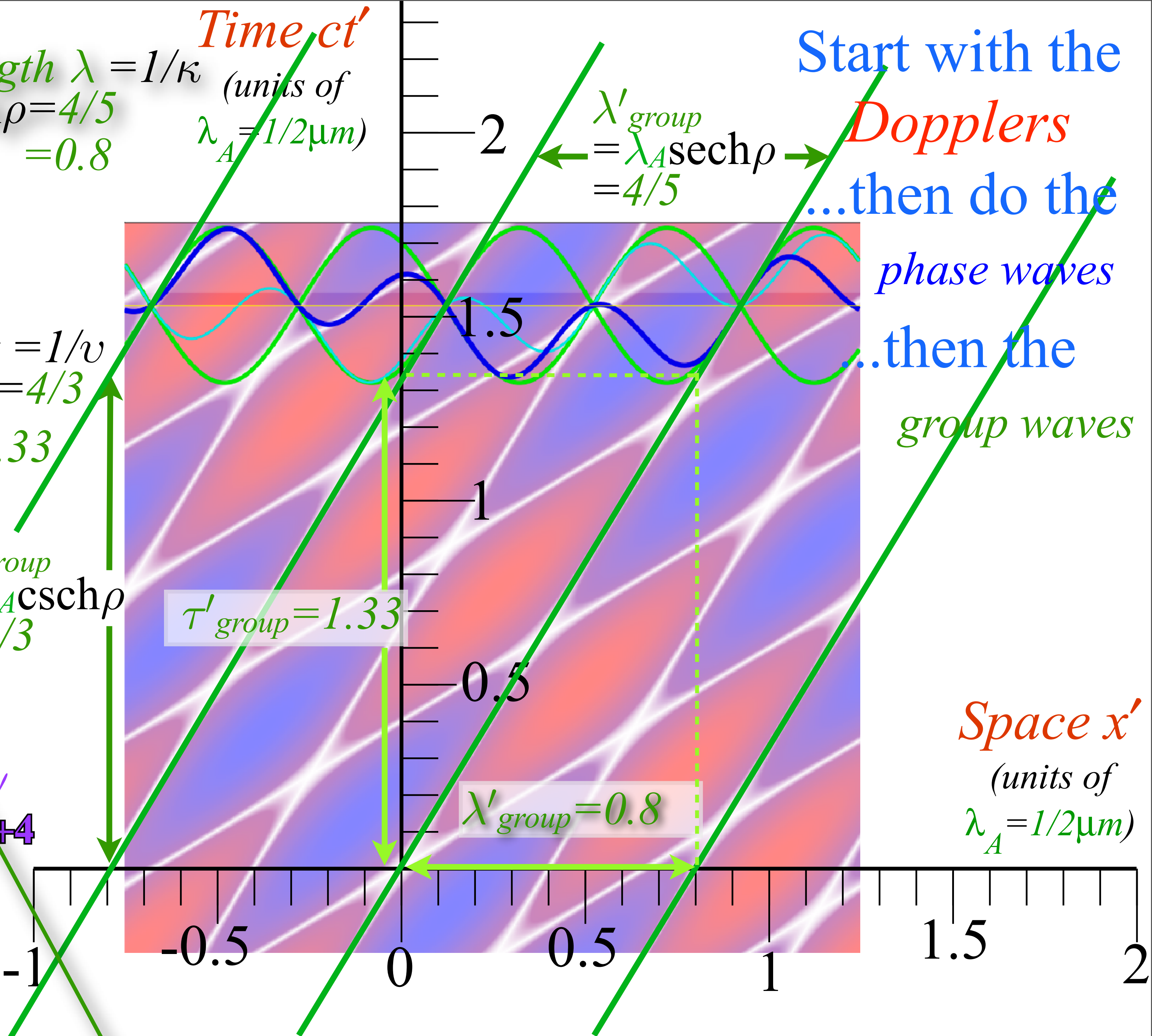
G-slope $= V_{group}/c$
$$= \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$



Wavevector $c\kappa'$
(units of $c\kappa_A = 2\cdot10^6/m$*)*



phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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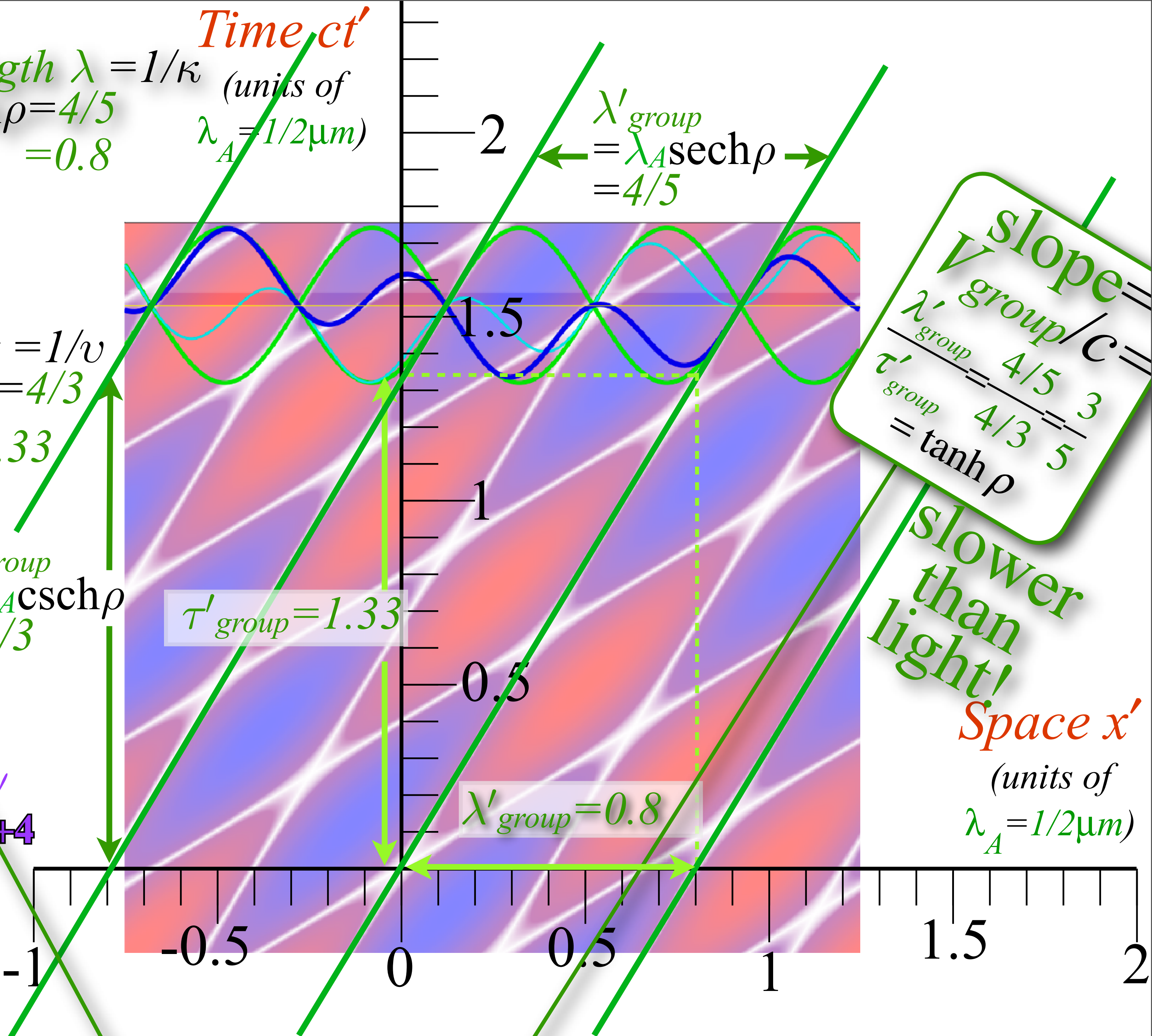
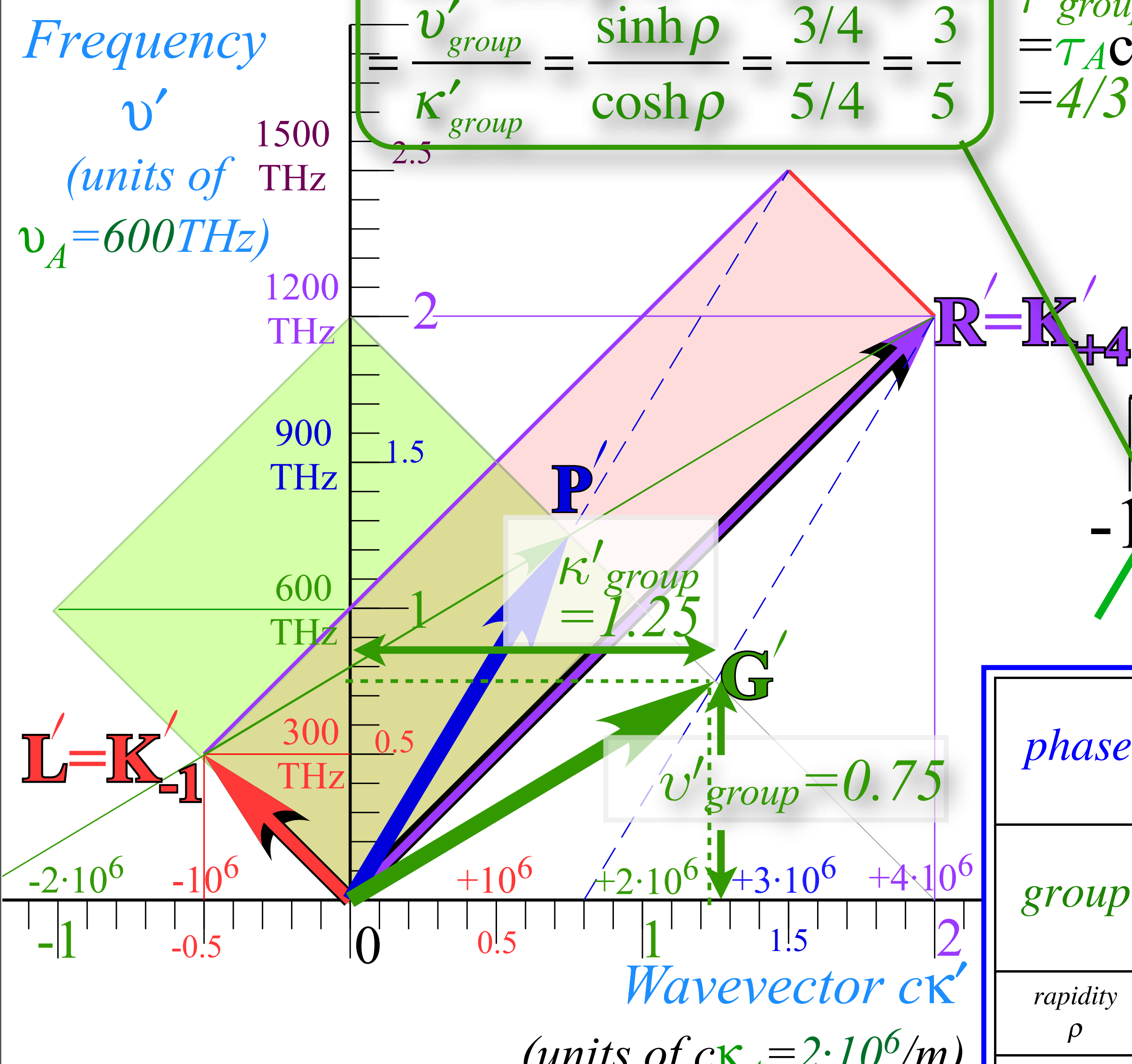
G-slope $=V_{group}/c$
$$=\frac{\upsilon'_{group}}{\kappa'_{group}}=\frac{\sinh\rho}{\cosh\rho}=\frac{3/4}{5/4}=\frac{3}{5}$$

τ'_{group}
$=\tau_A\operatorname{csch}\rho$
$=4/3$





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group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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Lorentz transformations...

write $\mathbf{G}'$ *and* $\mathbf{P}'$ *in terms of* $\mathbf{G}$ *and* $\mathbf{P}$ *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

$$\mathbf{G}' = \mathbf{G} \cosh\rho + \mathbf{P} \sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

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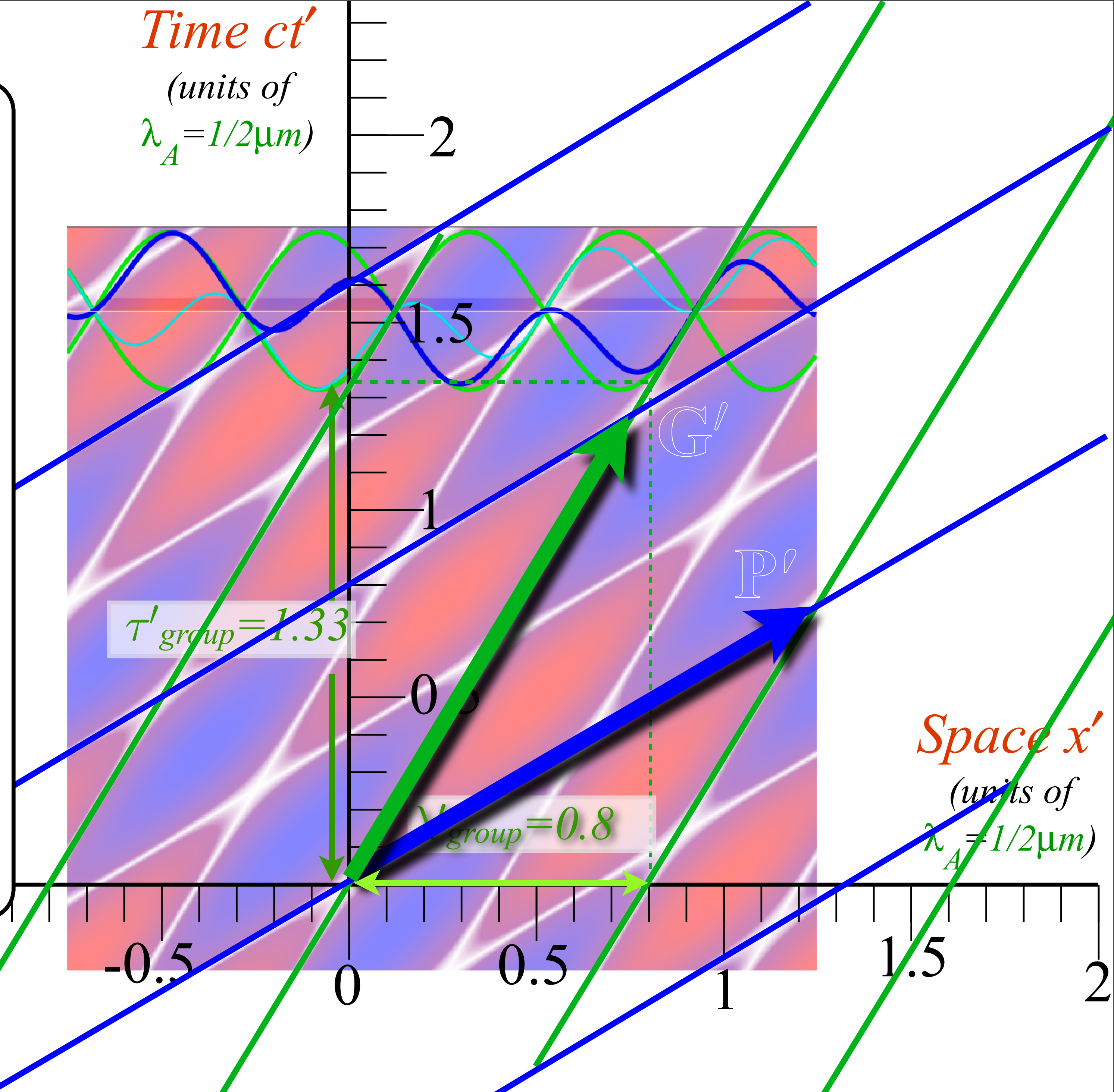

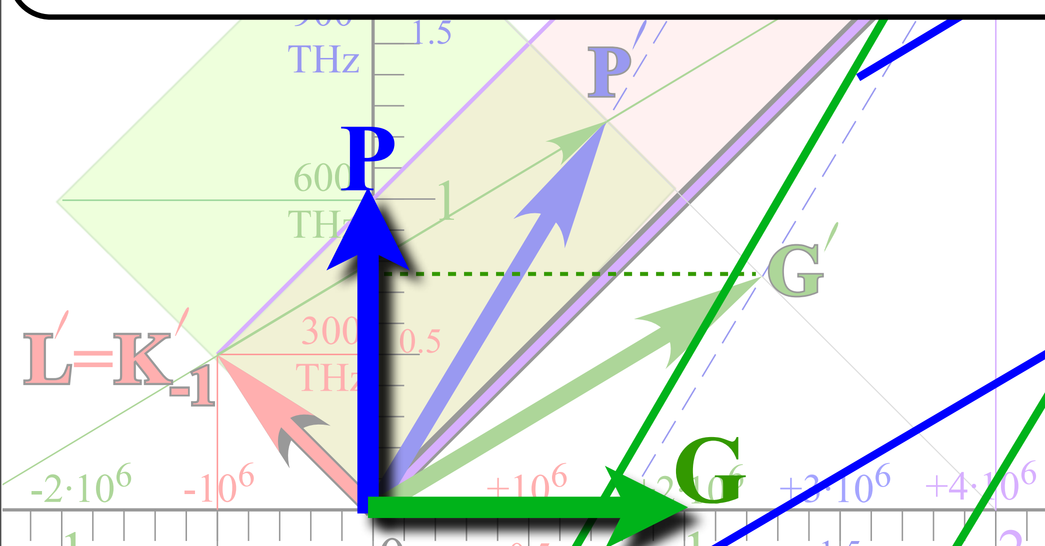

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$ Lorentz transform matrix

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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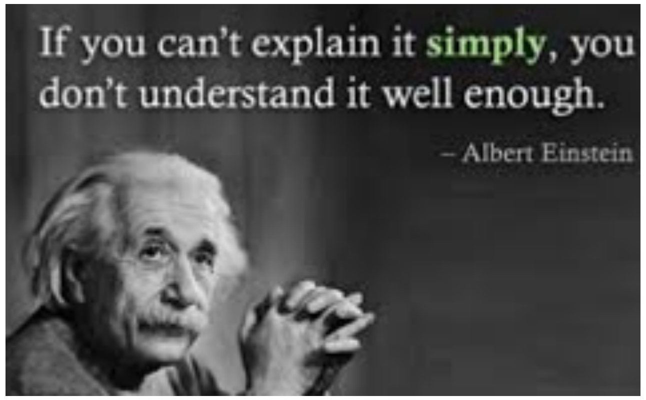
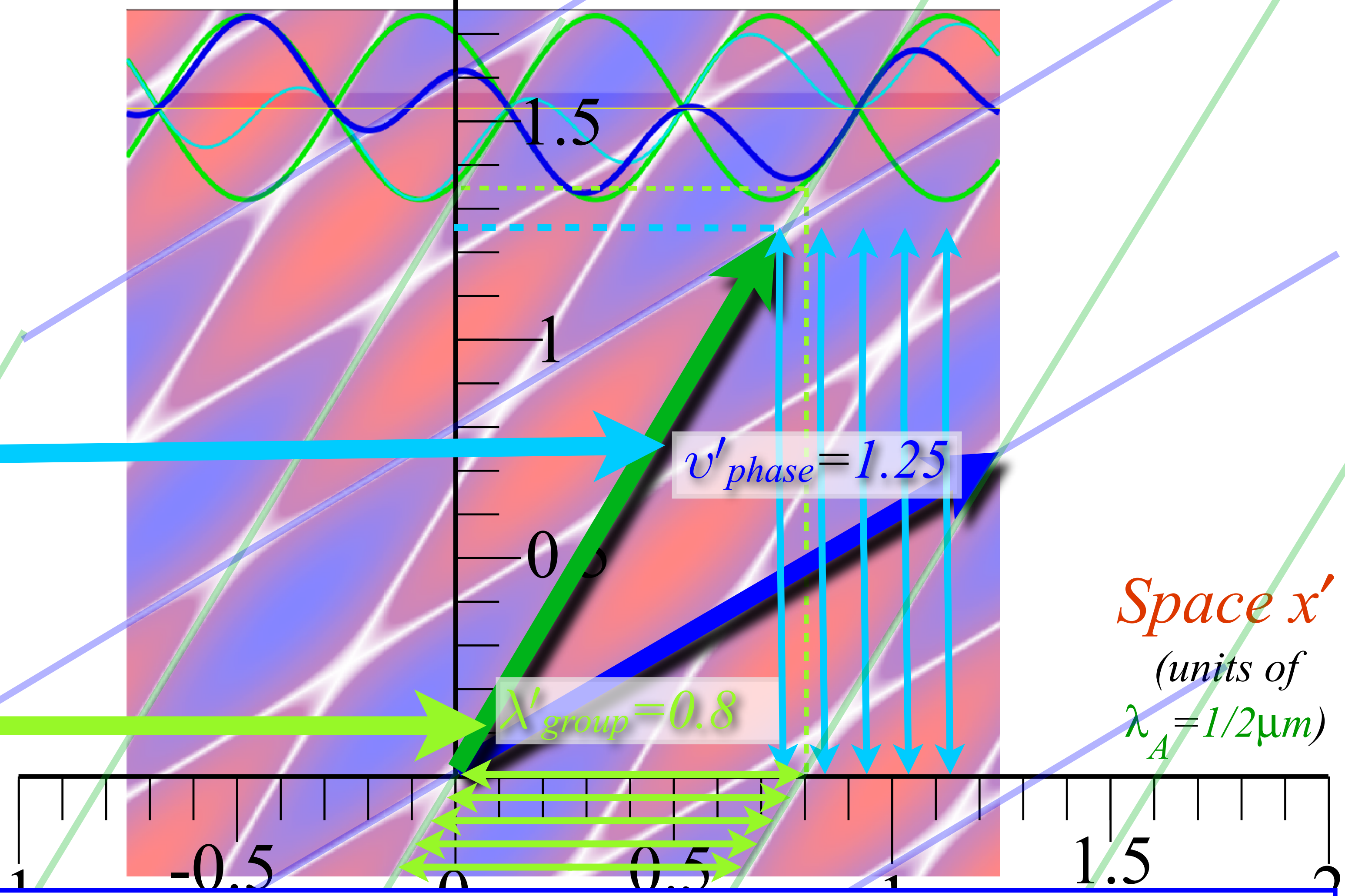
Two Famous-Name Coefficients



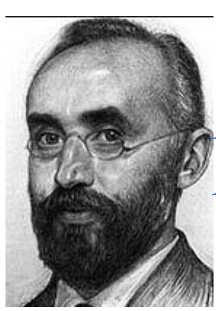
Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation** (dilated by 25% here)

This number is called a: **Lorentz length-contraction** (contracted by 20% here)



Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

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group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Relativistic *Terms (Dual plot w/expanded table)*

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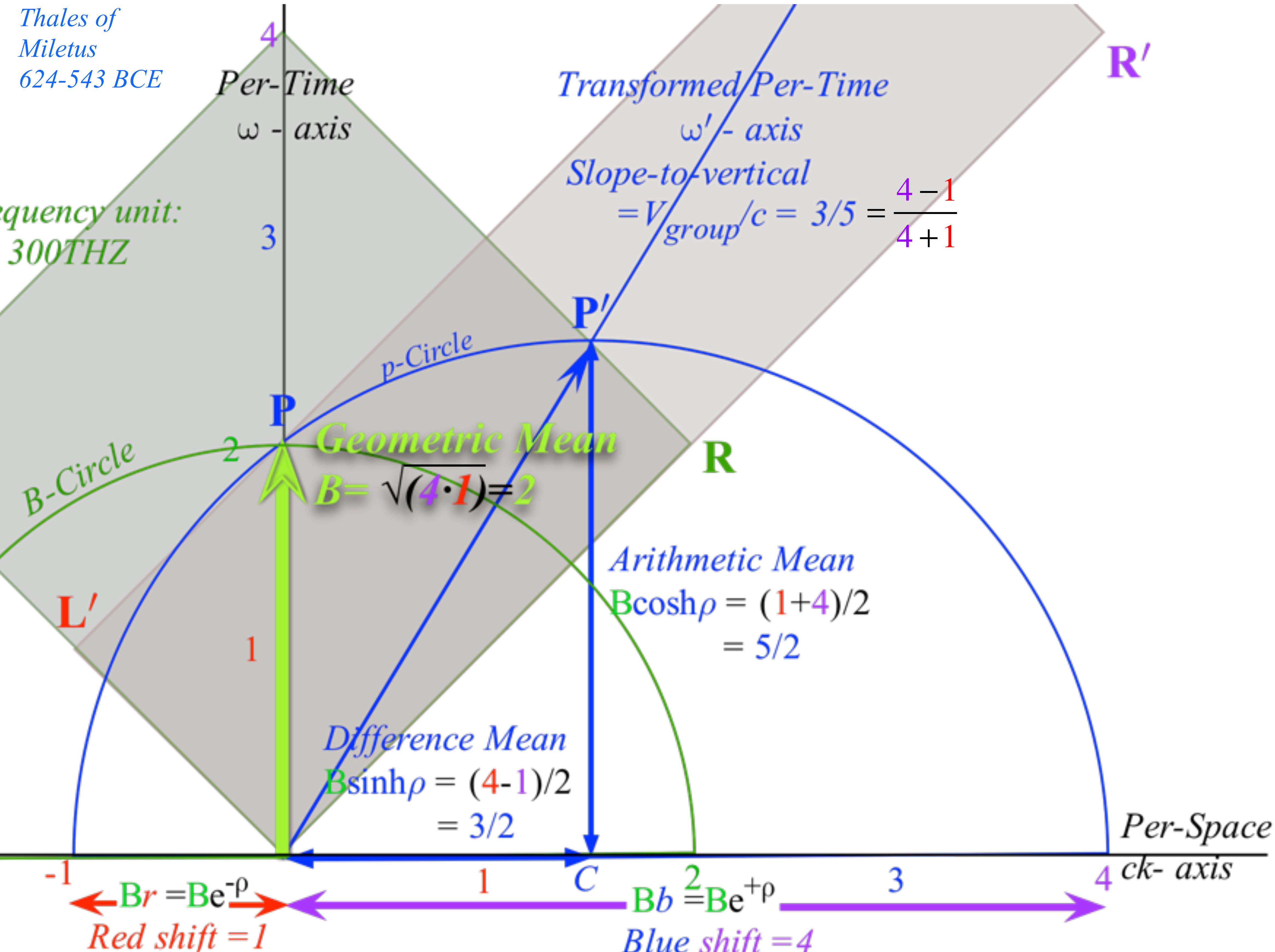
helps "Relawavity"



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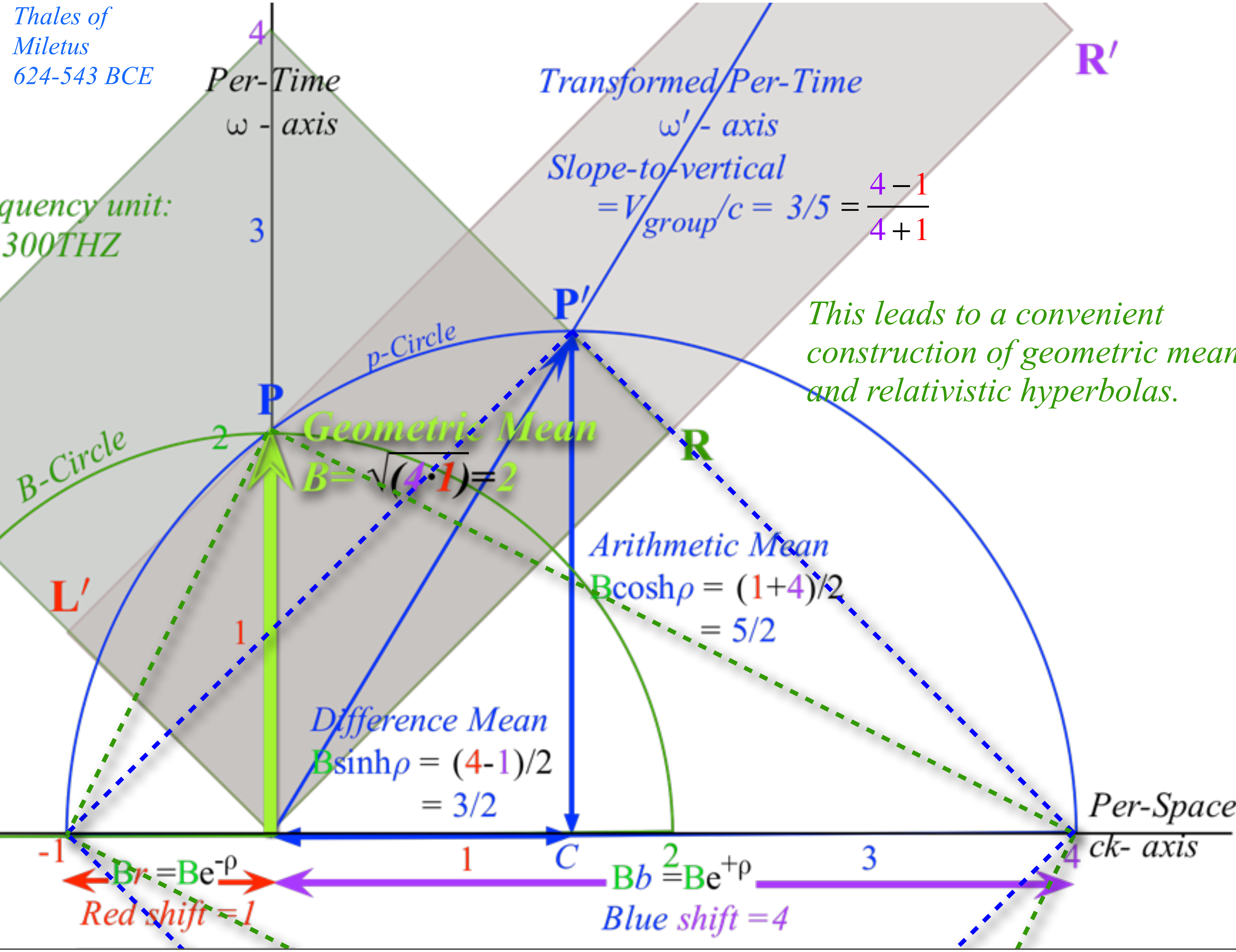

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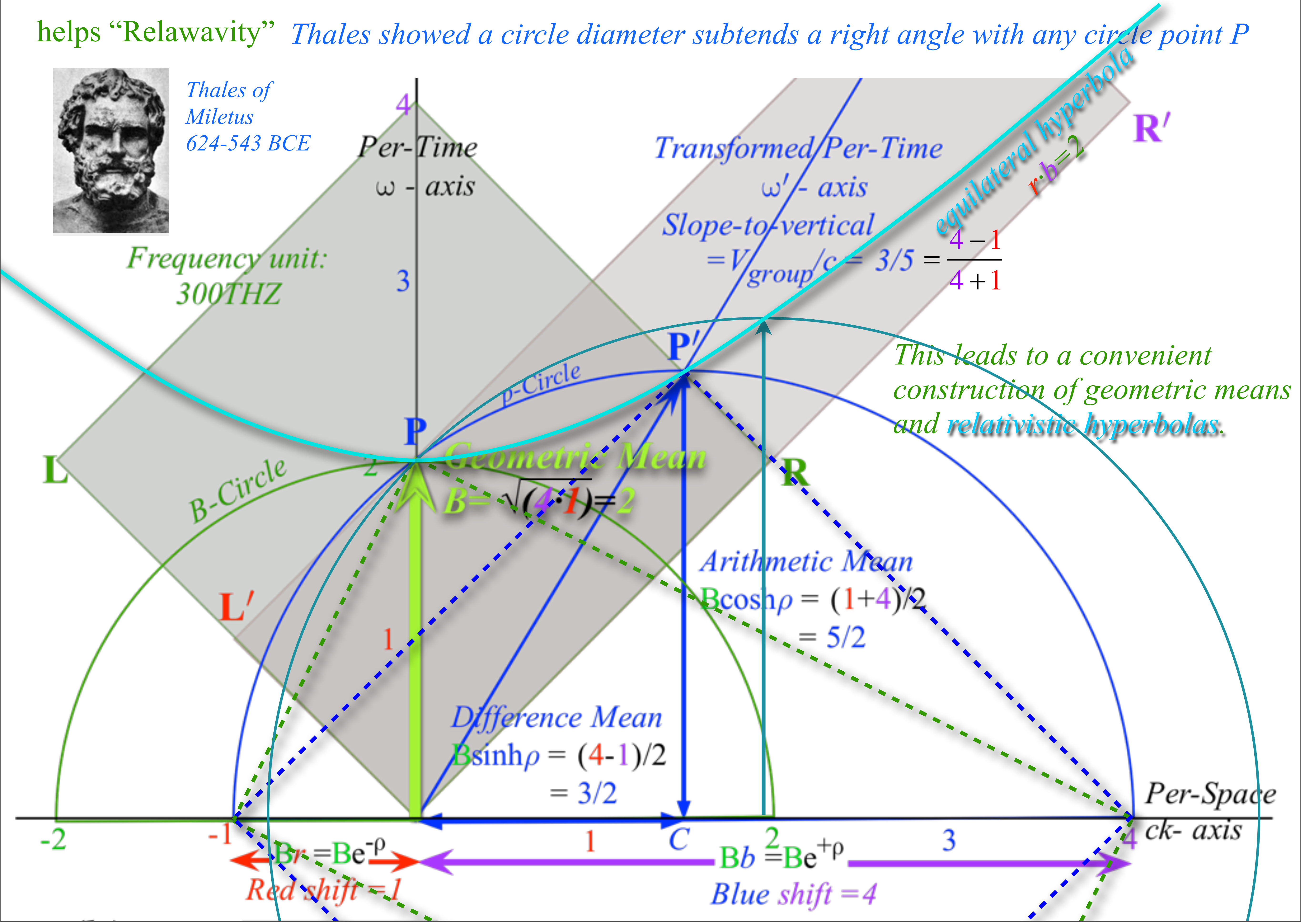

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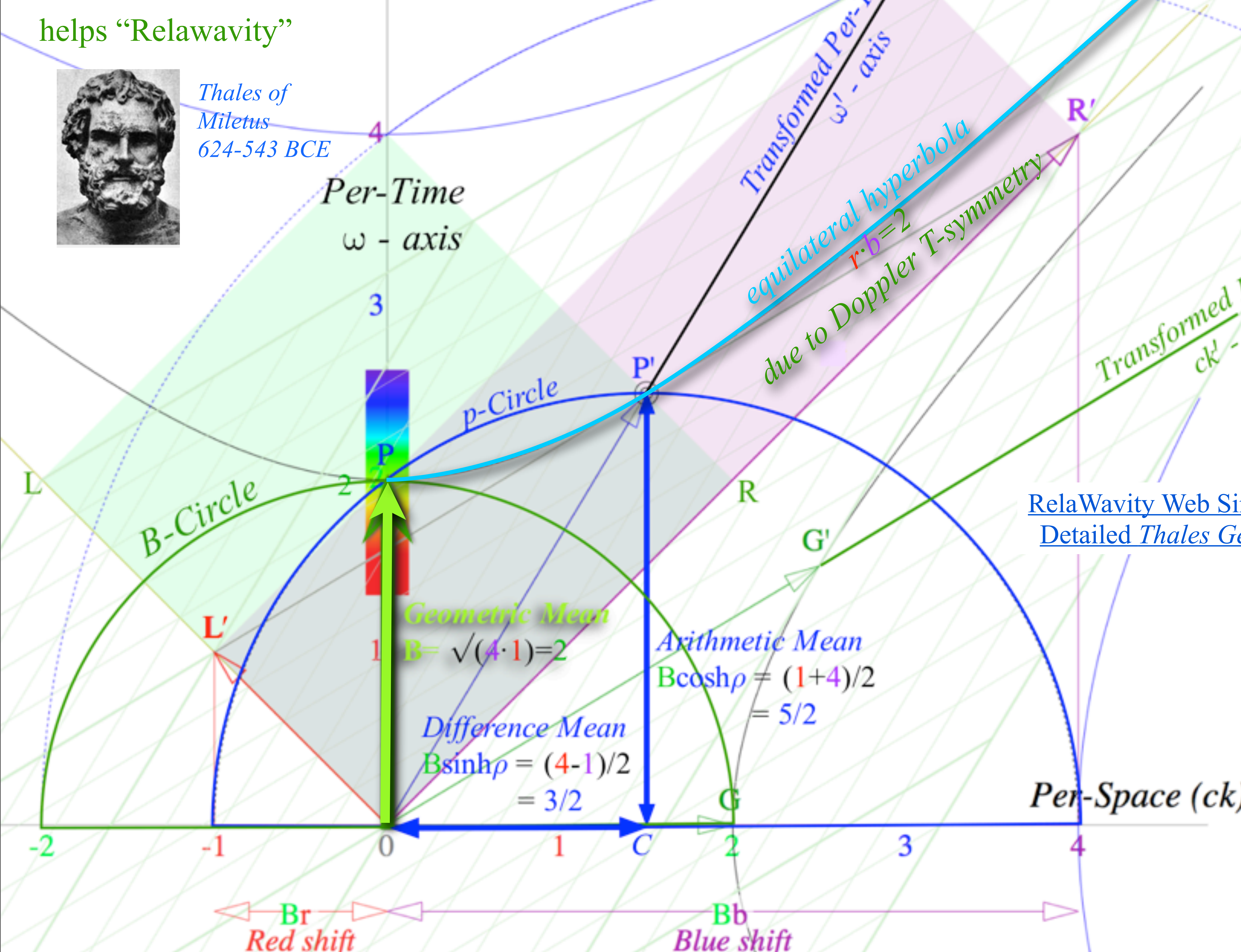

[RelaWavity Web Simulation](#)
[Detailed *Thales Geometry*](#)

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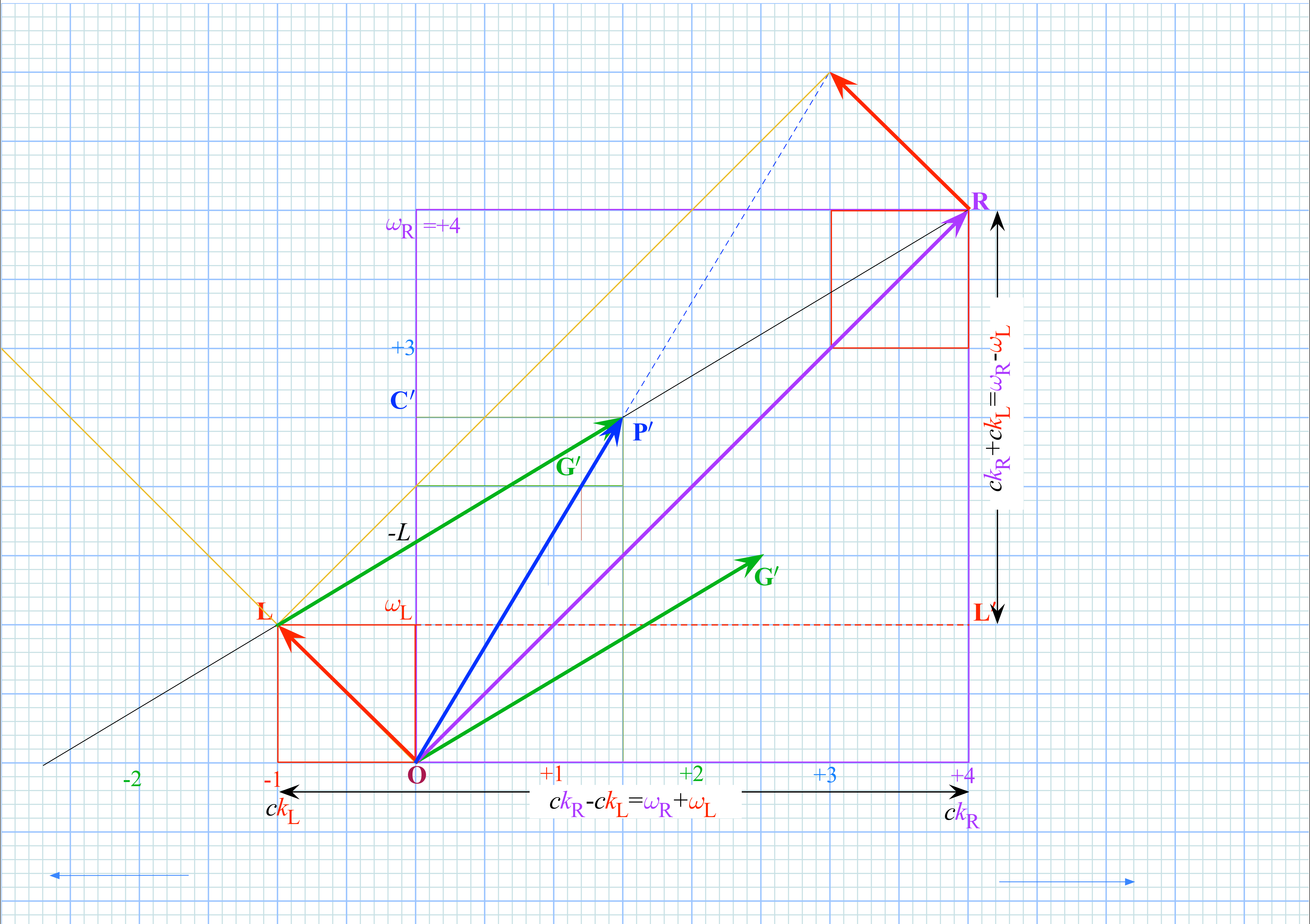
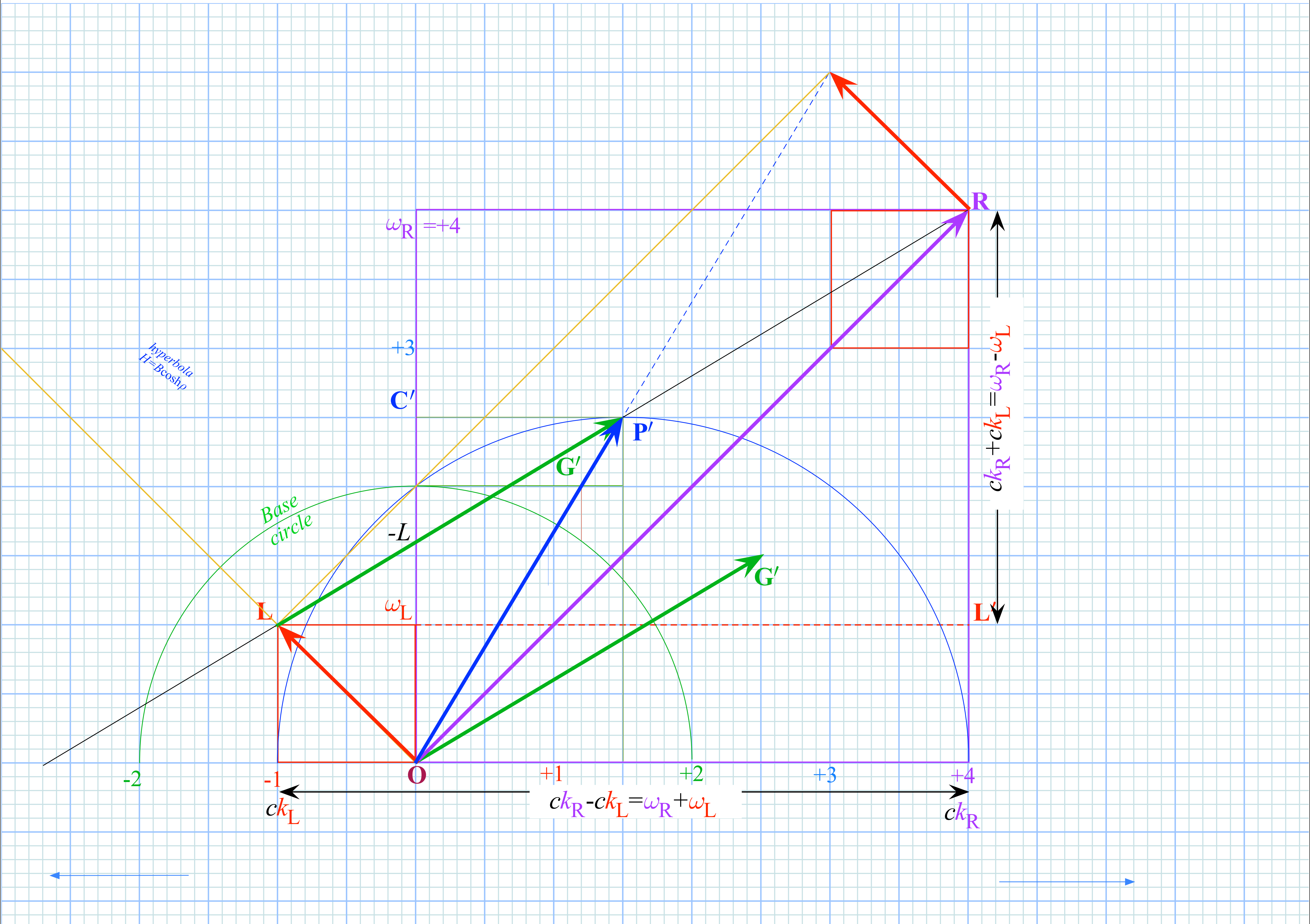
Stellar aberration and the Epstein approach to SR

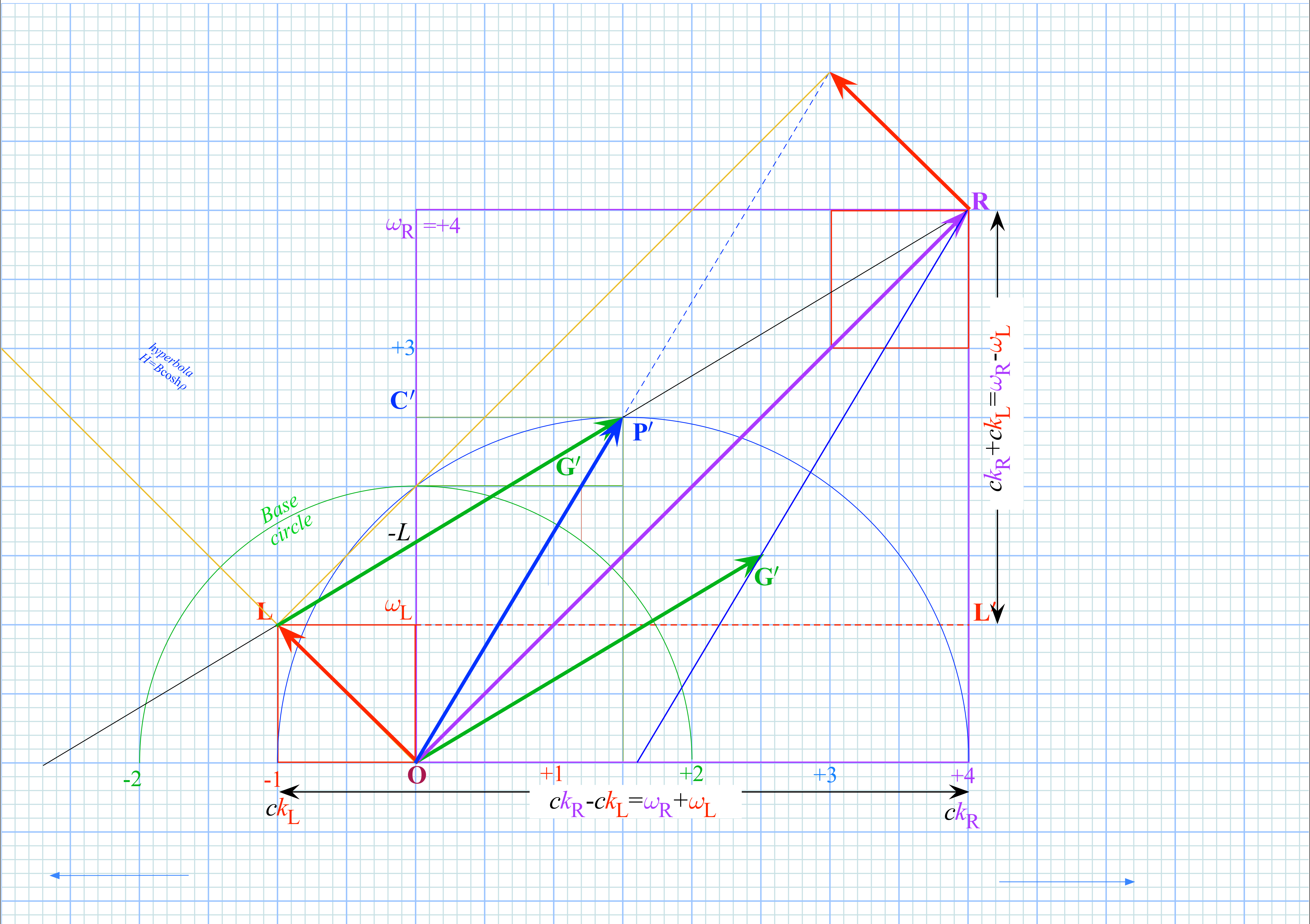
More at Pirelli Challenge page: *'Un Grande Affare' - Light Meets Light*

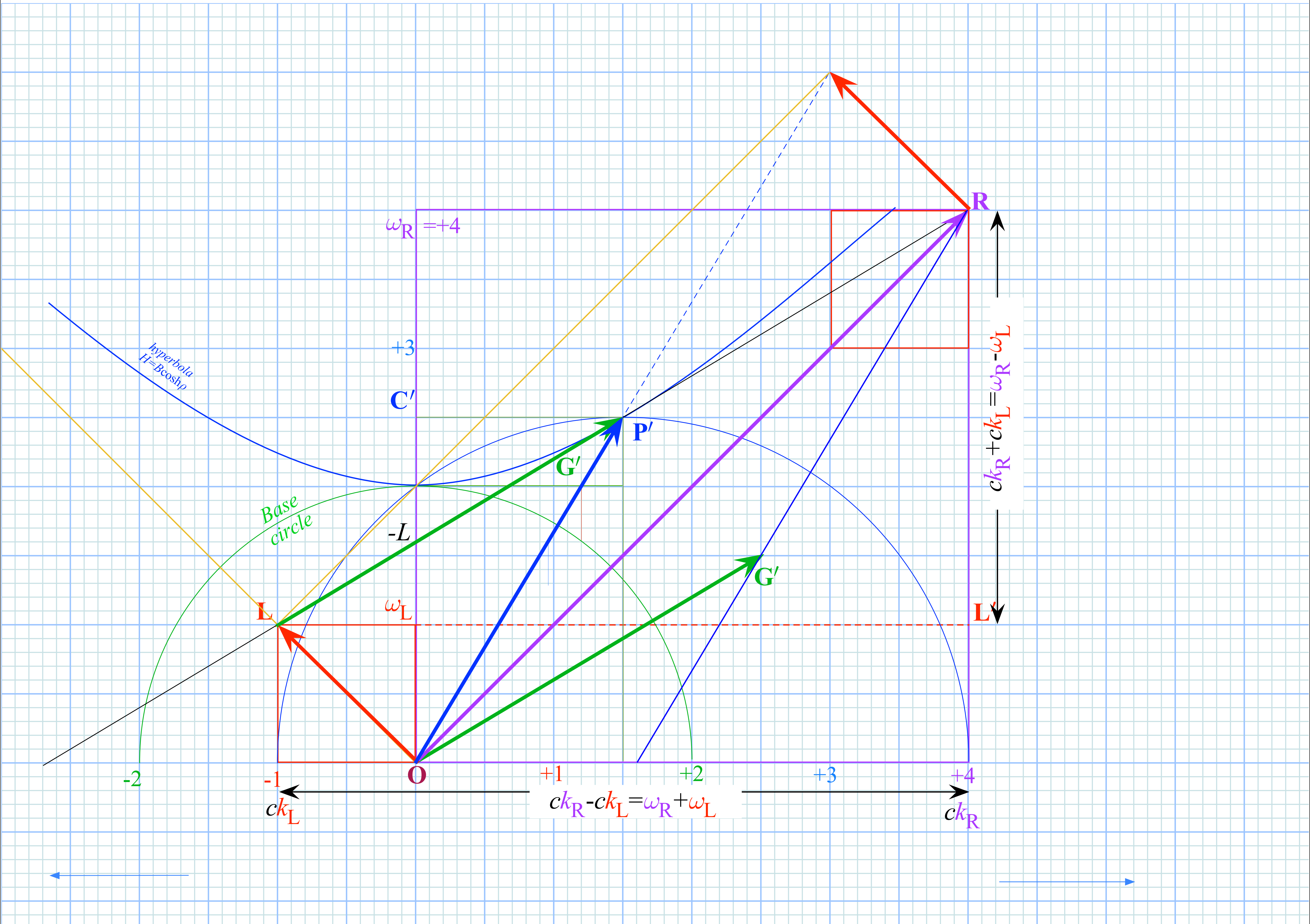


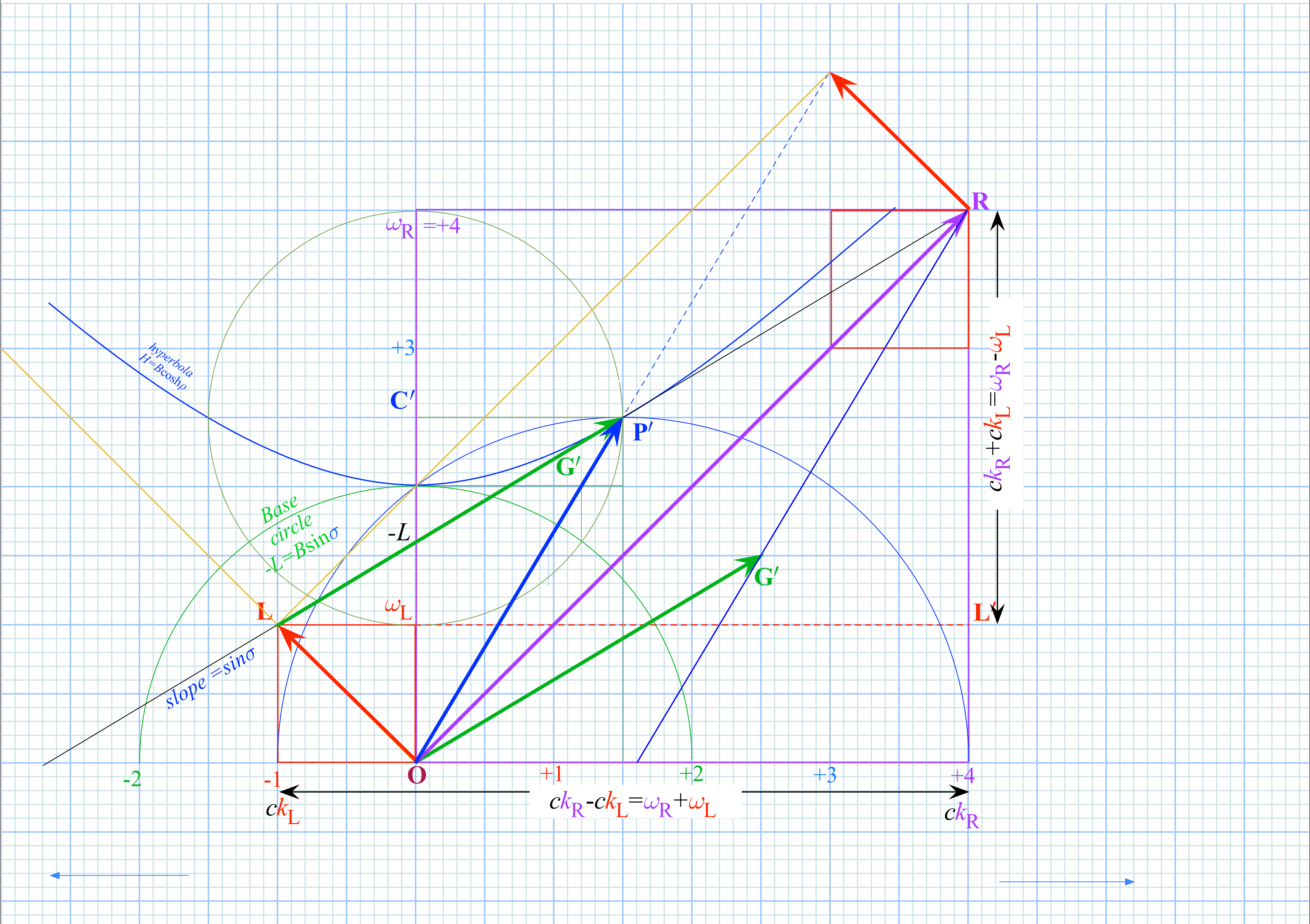
R
ω_R =+4
+3
C′
P′
G′
-L
G′
L
ω_L
L′
$ck_R + ck_L = \omega_R - \omega_L$
-2
-1
O
+1
+2
+3
+4
ck_L
$ck_R - ck_L = \omega_R + \omega_L$
ck_R

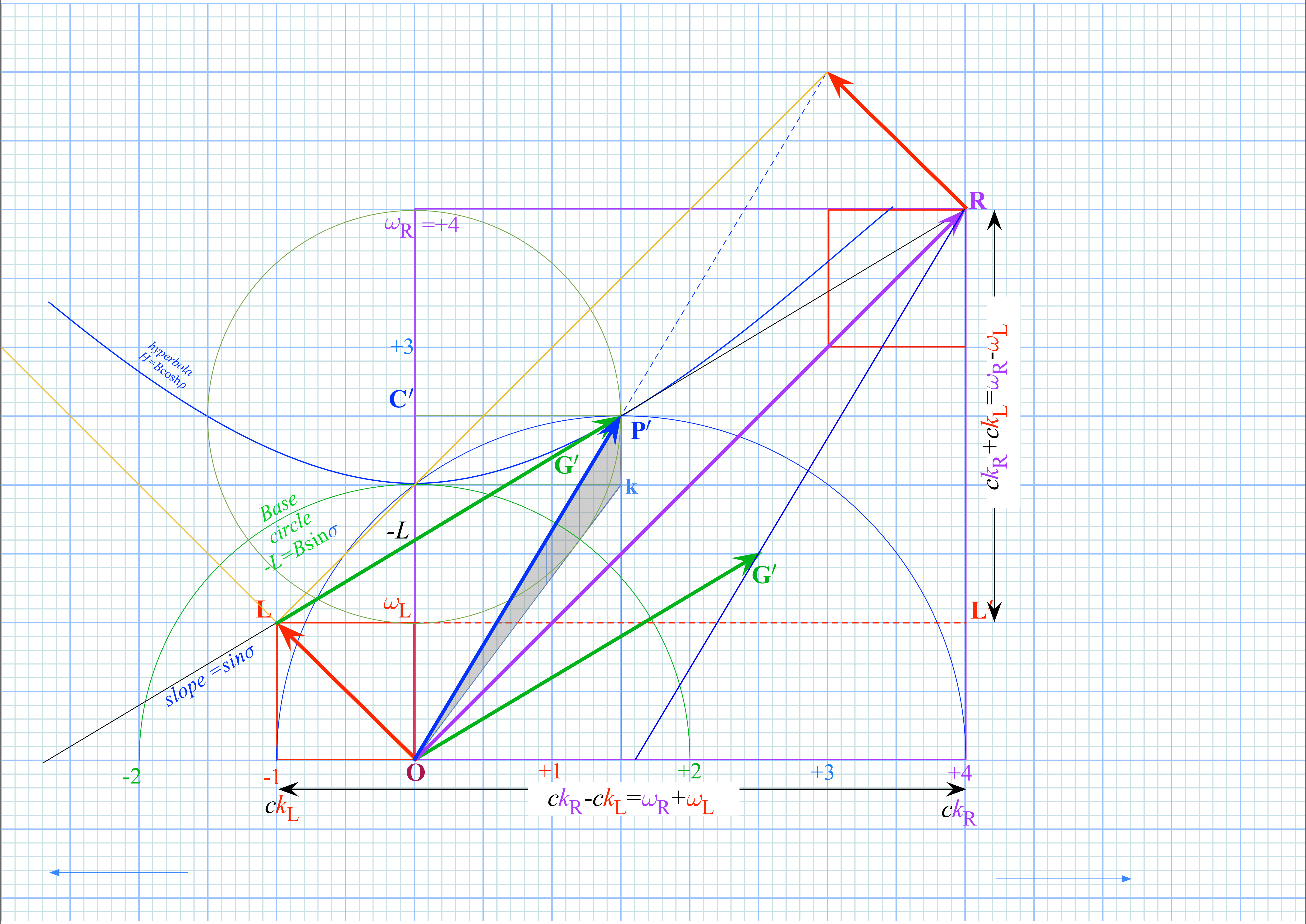


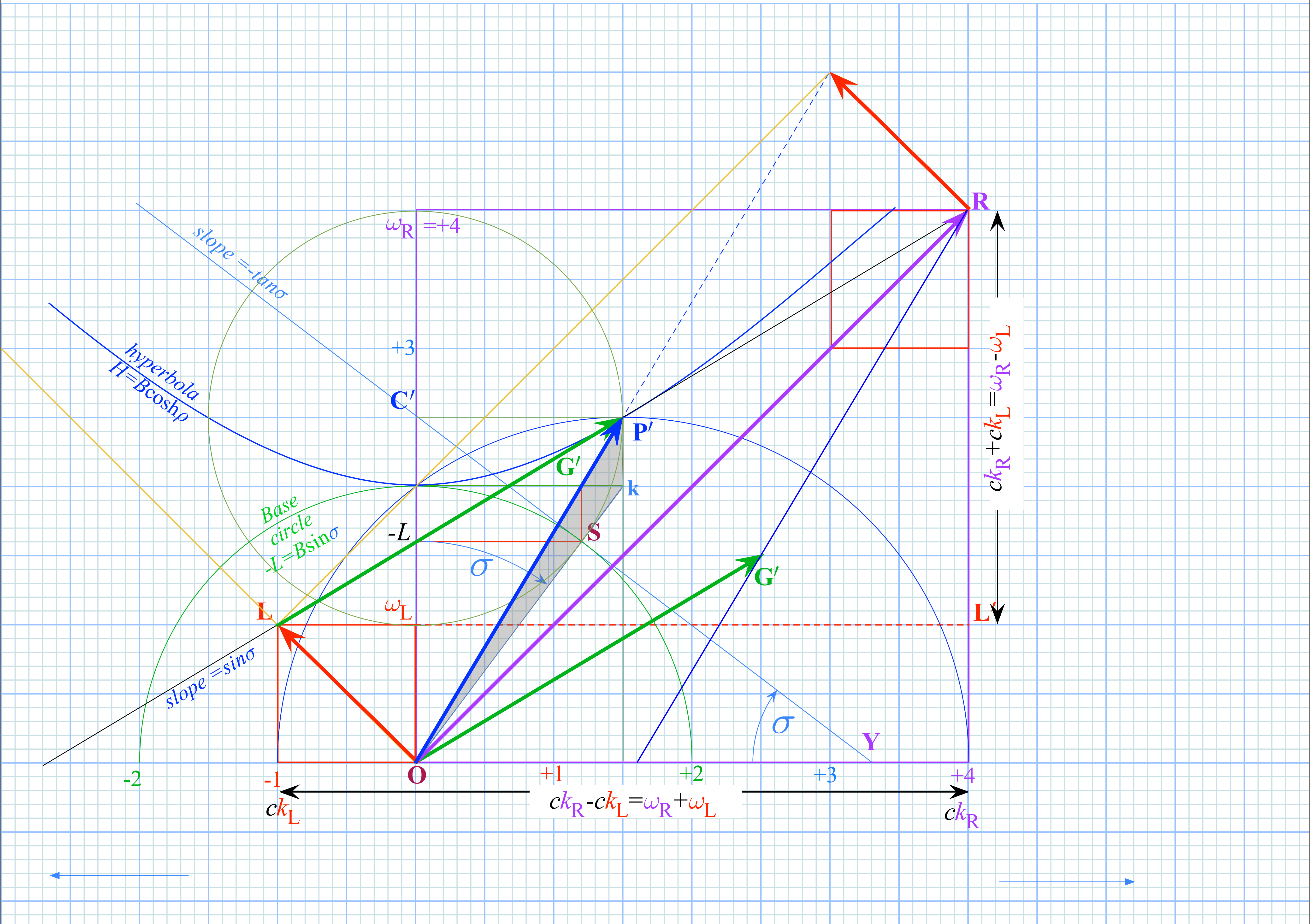
R
ω_R =+4
+3
C′
P′
G′
G′
hyperbola
H=Bcoshρ
Base
circle
-L
L
ω_L
L′
O
-2
-1
+1
+2
+3
+4
ck_L
ck_R
ck_R-ck_L=ω_R+ω_L
ck_R+ck_L=ω_R-ω_L


R
ωR =+4
+3
C′
P′
G′
G′
L
ωL
L′
-L
Base circle
hyperbola H=Bcoshρ
ckR+ckL=ωR-ωL
-2
-1
O
+1
+2
+3
+4
ckL
ckR-ckL=ωR+ωL
ckR

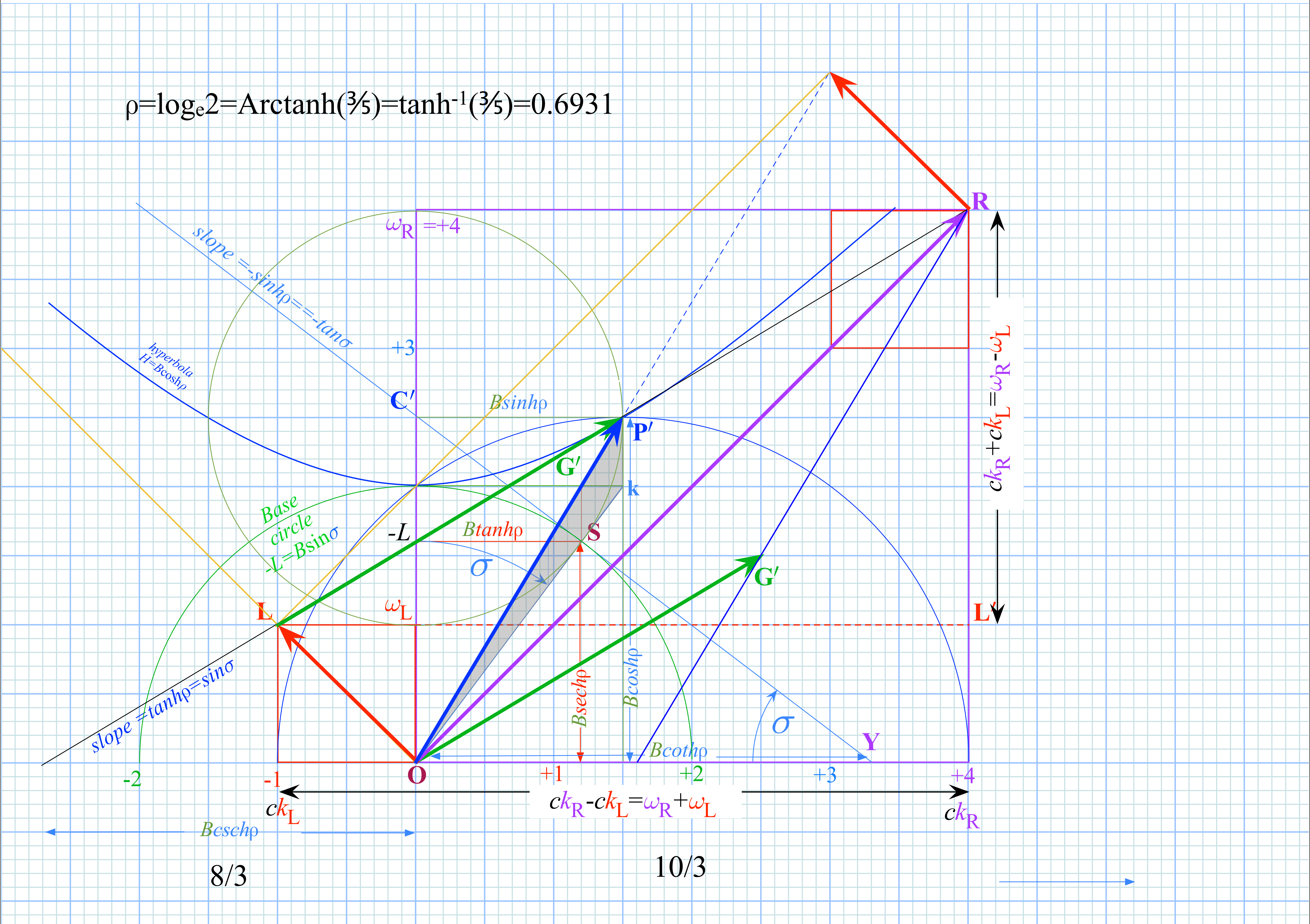

R
ω_R =+4
+3
hyperbola
H=Bcoshρ
C′
P′
G′
Base
circle
-L
G′
L
ω_L
L′
$ck_R+ck_L=\omega_R-\omega_L$
-2
-1
O
+1
+2
+3
+4
ck_L
$ck_R-ck_L=\omega_R+\omega_L$
ck_R


R
ω_R =+4
hyperbola
H=Bcoshρ
+3
C′
P′
G′
Base
circle
-L=Bsinσ
-L
ω_L
L
G′
L′
slope =sinσ
$ck_R+ck_L=\omega_R-\omega_L$
-2
-1
O
+1
+2
+3
+4
ck_L
$ck_R-ck_L=\omega_R+\omega_L$
ck_R


R
ω_R =+4
+3
C′
P′
G′
k
hyperbola
H=Bcoshρ
Base
circle
-L=Bsinσ
-L
ω_L
L
L′
G′
slope =sinσ
$ck_R+ck_L=\omega_R-\omega_L$
-2
-1
O
+1
+2
+3
+4
ck_L
$ck_R-ck_L=\omega_R+\omega_L$
ck_R



slope =-tanσ
hyperbola
H=Bcoshρ
ω_R =+4
+3
C′
P′
G′
k
Base
circle
-L=Bsinσ
-L
S
σ
G′
L
ω_L
L′
slope =sinσ
σ
Y
R
ck_R+ck_L=ω_R-ω_L
-2
-1
O
+1
+2
+3
+4
ck_L
ck_R-ck_L=ω_R+ω_L
ck_R



ρ=log_e2=Arctanh(3/5)=tanh^-1(3/5)=0.6931
slope =-sinhρ=-tanσ
hyperbola H=Bcoshρ
ω_R =+4
+3
C′
Bsinhρ
P′
G′
k
Base circle -L=Bsinσ
-L
Btanhρ
S
σ
L
ω_L
L′
R
ck_R+ck_L=ω_R-ω_L
G′
Bsechρ
Bcoshρ
slope =tanhρ=sinσ
Bcothρ
σ
Y
-2
-1
O
+1
+2
+3
+4
ck_L
ck_R-ck_L=ω_R+ω_L
ck_R
Bcschρ
8/3
10/3

Review of Doppler-shift and Rapidity ρ_{AB} calculation: *Galileo's Revenge Part I Lect. 23 p.64-75*

Relating rapidity ρ_{AB} and relativity velocity parameter $\beta_{AB}=u_{AB}/c$

Review of ½-sum-½-difference Phase and Group factors giving relativistic space-axes and time-axes

Colliding-CW space-time (x,ct)-graph *vs* Colliding PW space-time (R,L)-baseball diamond

Review of ½-sum-½-difference of phasor angular velocity: *Galileo's Revenge Part II* (Pirelli site)

Elementary models: 2-comb Moire′ patterns and cosine-law constructions

Bob, Alice, and Carla combine Doppler shifted ½-sum-½-difference Phase and Group factors

Doppler shifted Phase vector $\mathbf{P}'$ and Group vector $\mathbf{G}'$ in per-space-time

Minkowski coordinate grid in space-time

Animations that compare Doppler shifted colliding CW with colliding PW

The 16 parameters of Doppler-shifted 2-CW Minkowski geometry

Doppler shifted Phase parameters

Doppler shifted Group parameters

Lorentz transformation matrix and Two Famous-Name Coefficients

Thales Mean Geometry (*Thales of Miletus 624-543 BCE*) and its role in Relawavity

Detailed geometric construction of relawavity plot for 1-octave Doppler ($\beta_{AB}=u_{AB}/c=3/5$)

➔ **Stellar aberration and the Epstein approach to SR** ←

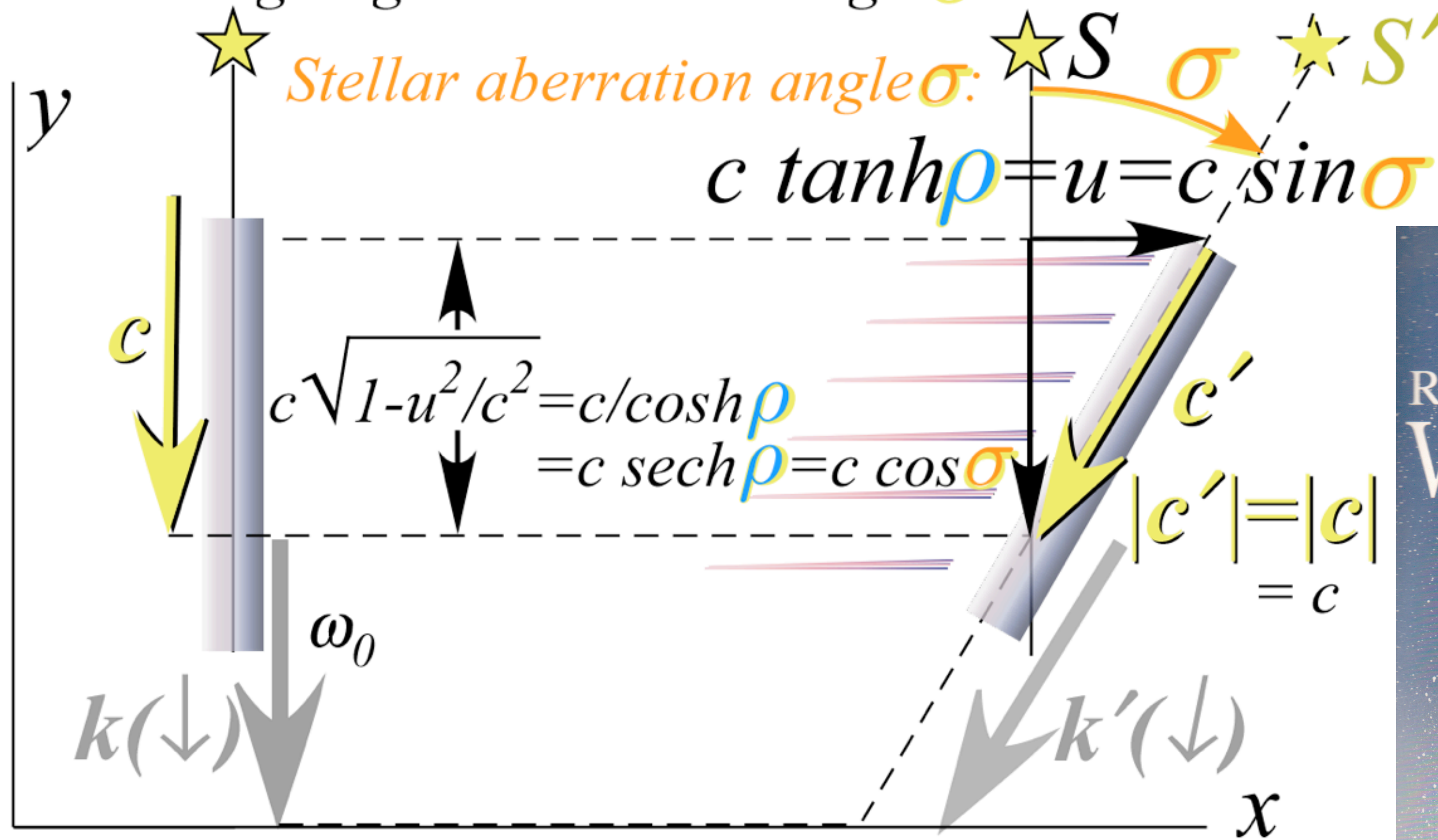
Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

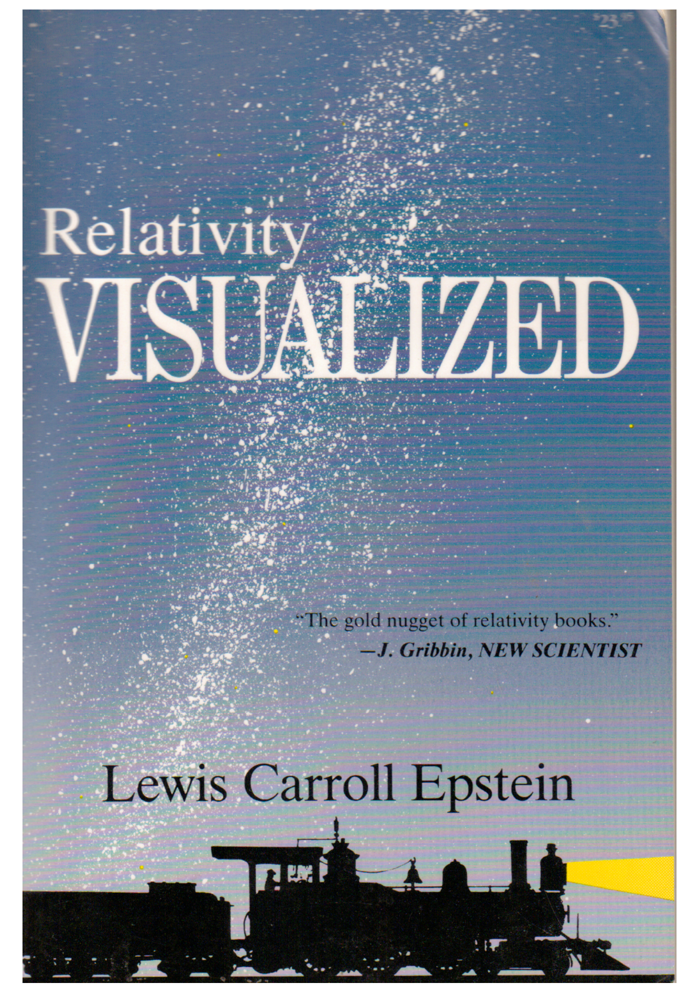
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



We used notion σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.



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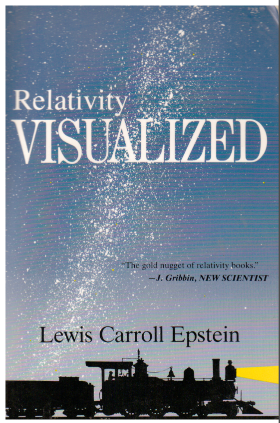
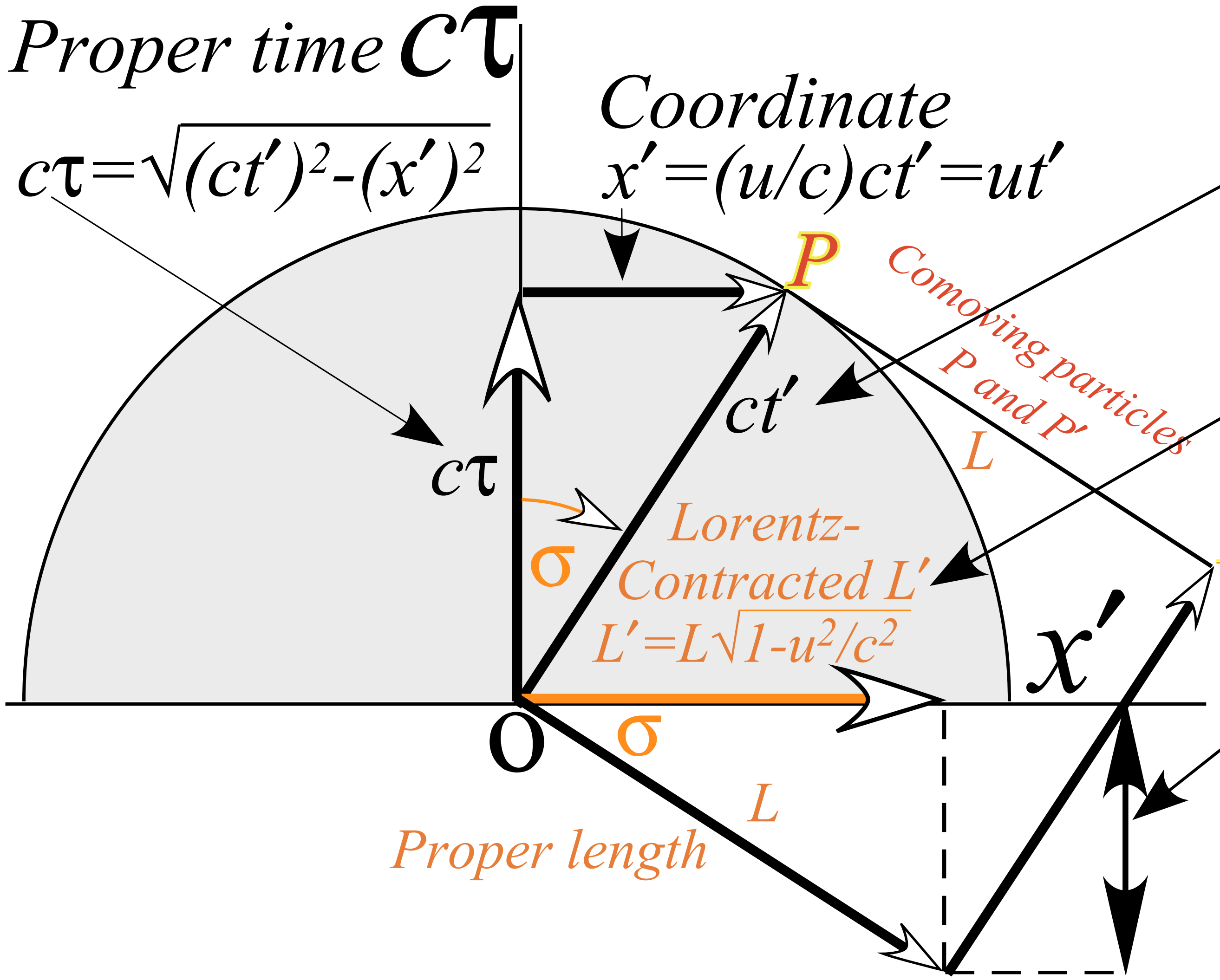


Proper time $c\tau$ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P' have speed u in (x',ct') and speed c in $(x, c\tau)$



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

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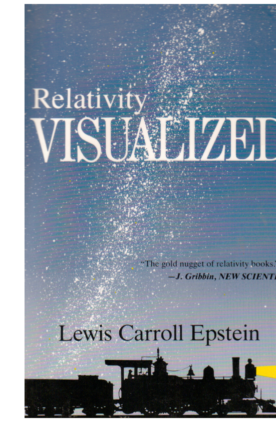
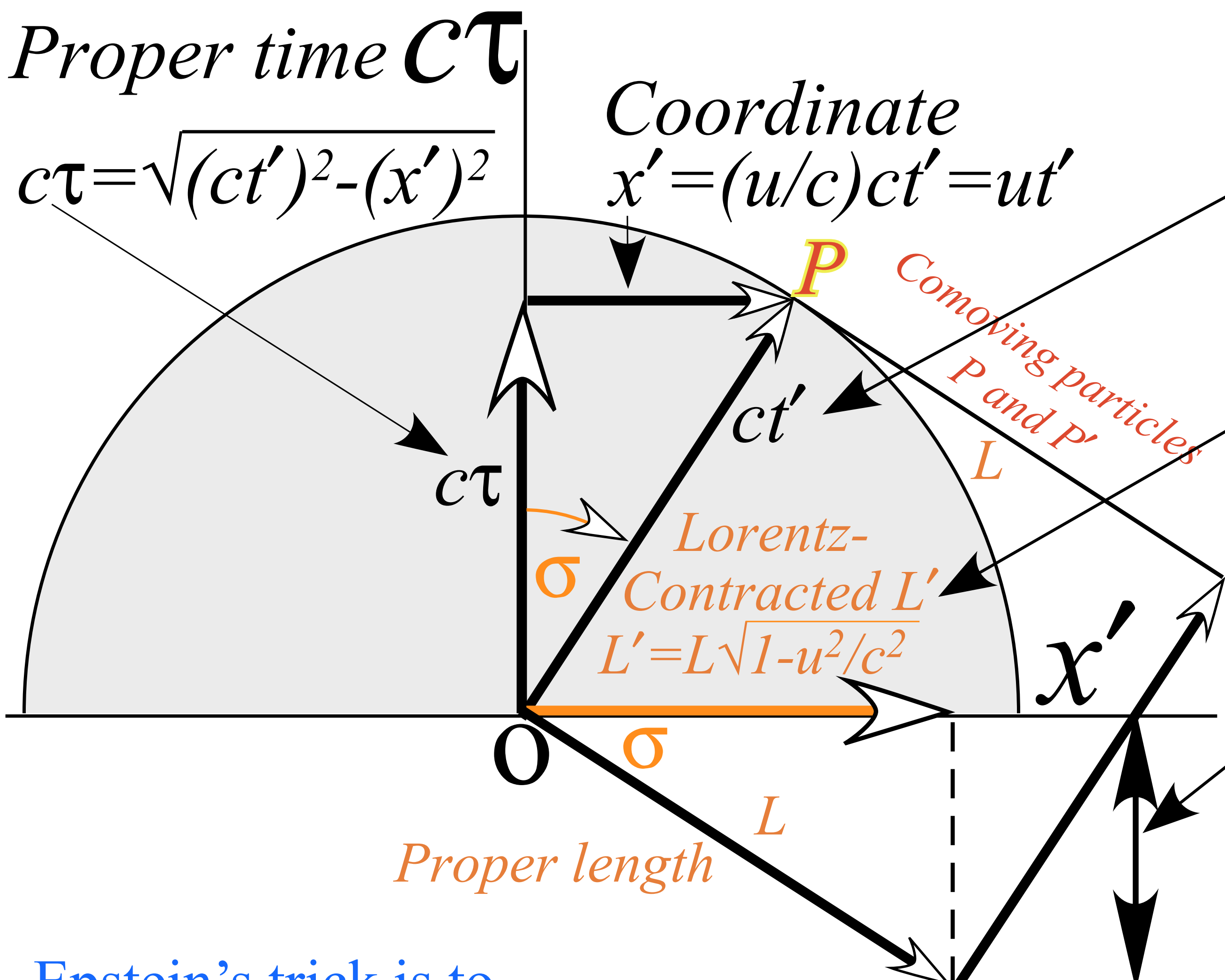


Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



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Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$

$$= L\cos\sigma\tan\sigma$$

$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

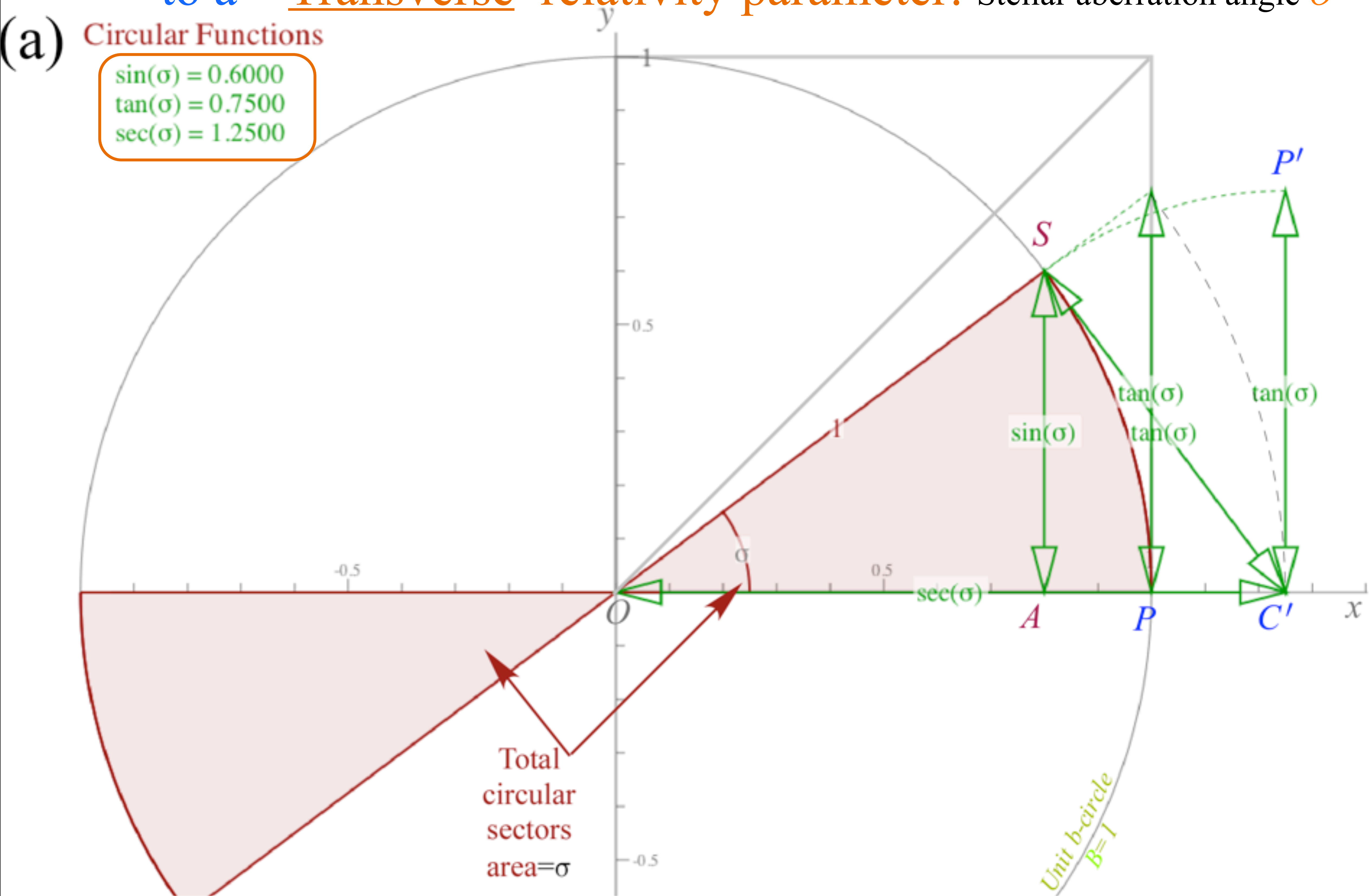
Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2 + (x')^2} = (ct')$$

Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

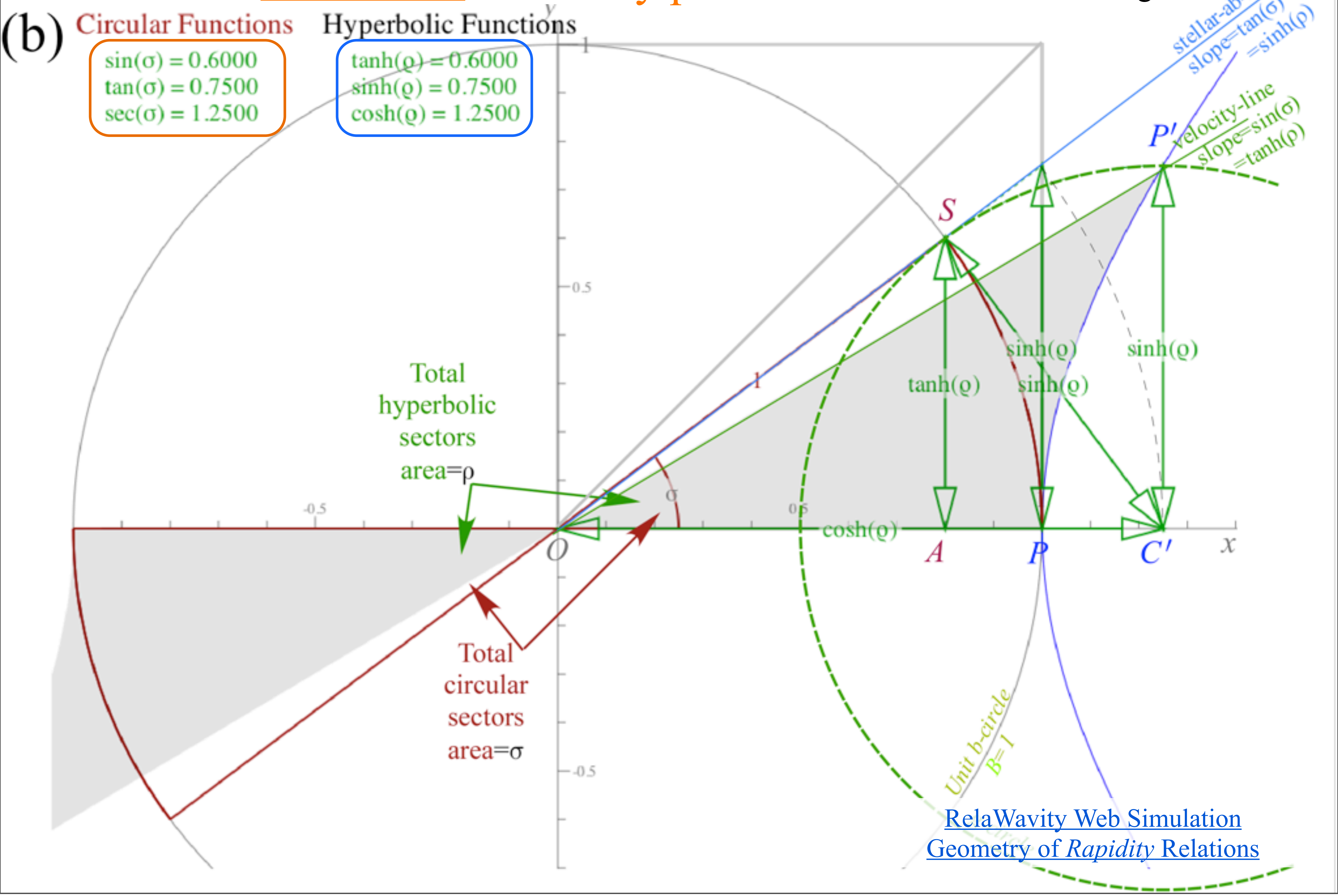
to a Transverse relativity parameter: Stellar aberration angle σ

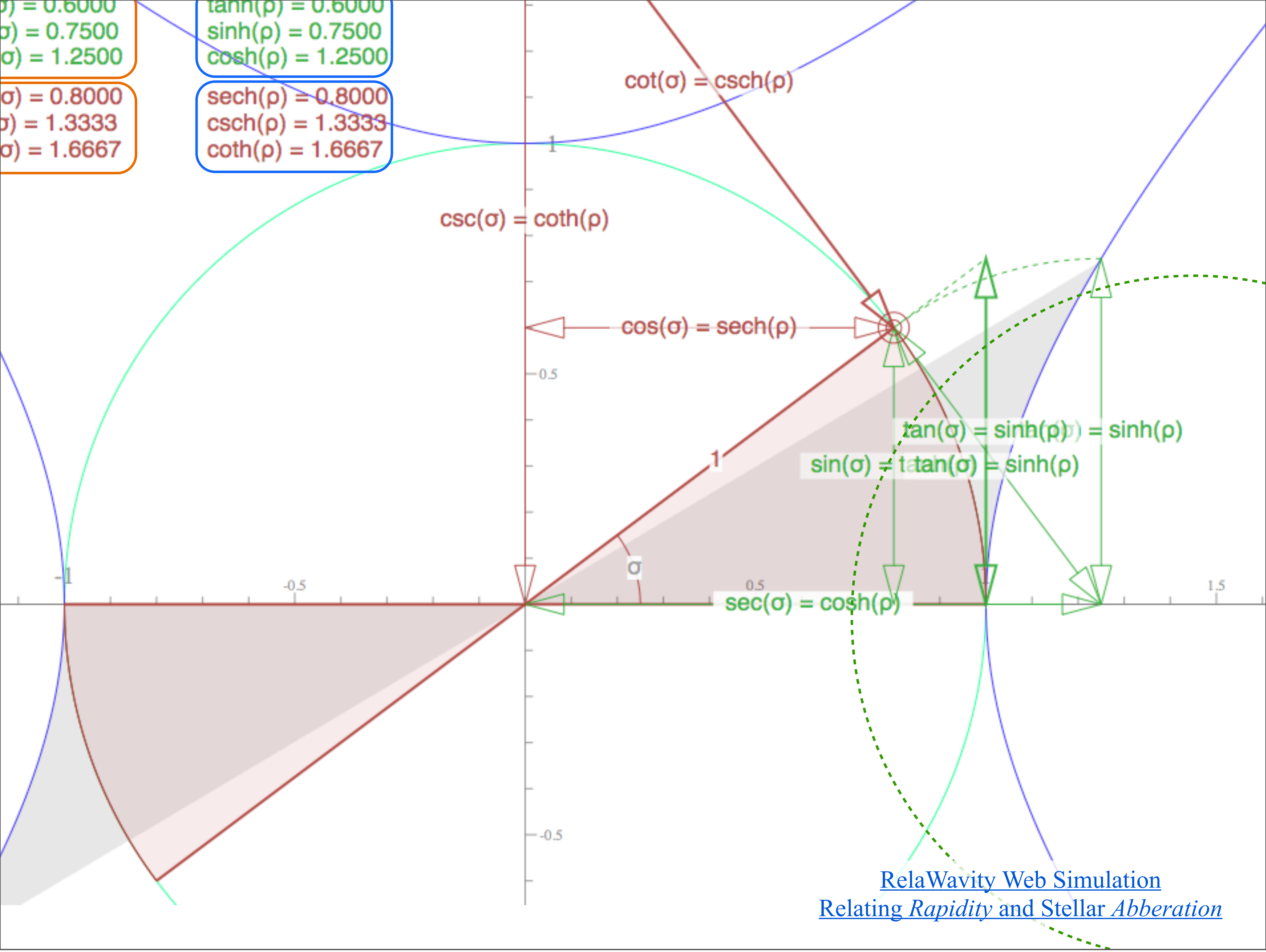


RelaWavity Web Simulation

Geometry of Stellar Aberation Angle

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)
to a Transverse relativity parameter: Stellar aberration angle σ

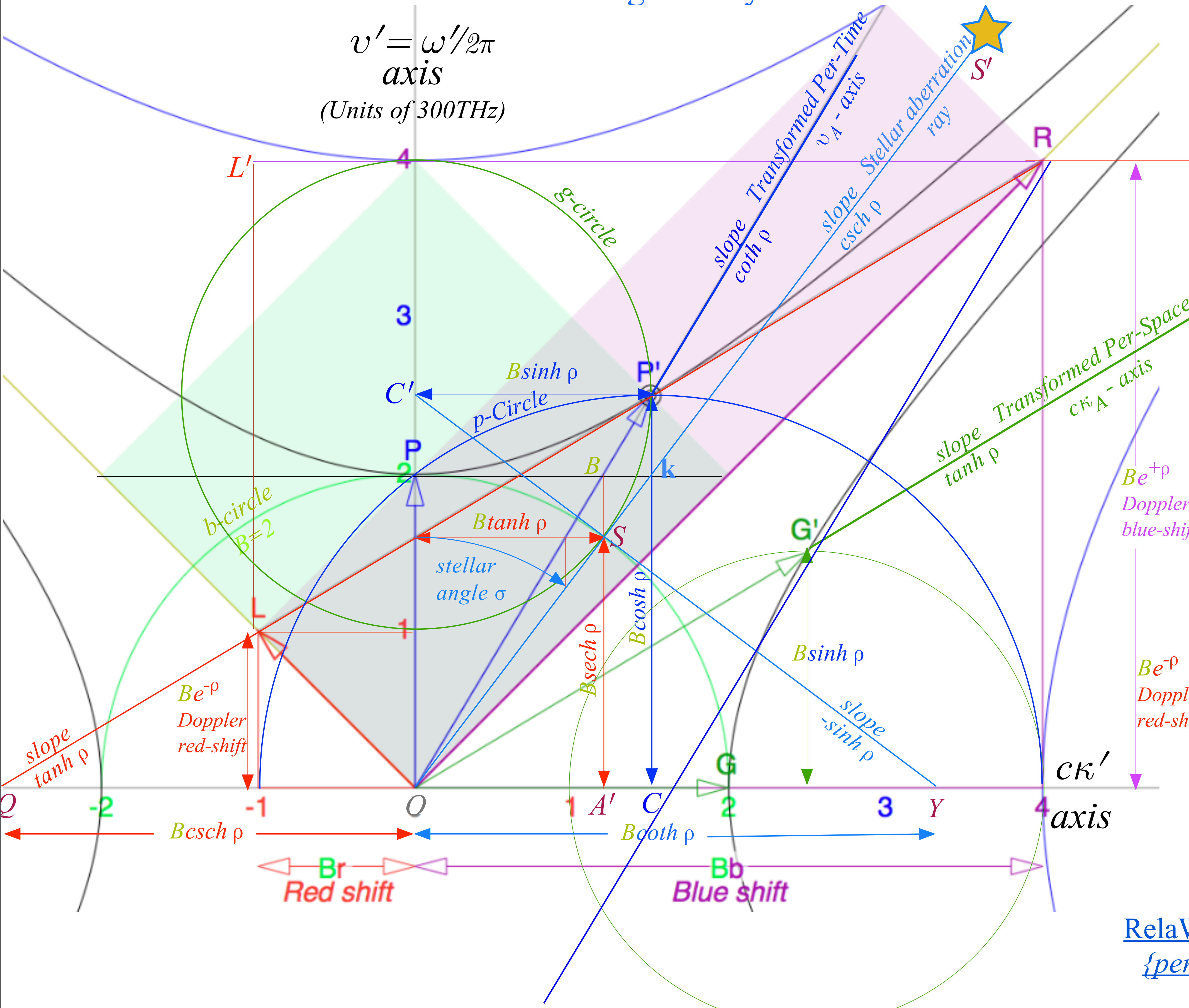




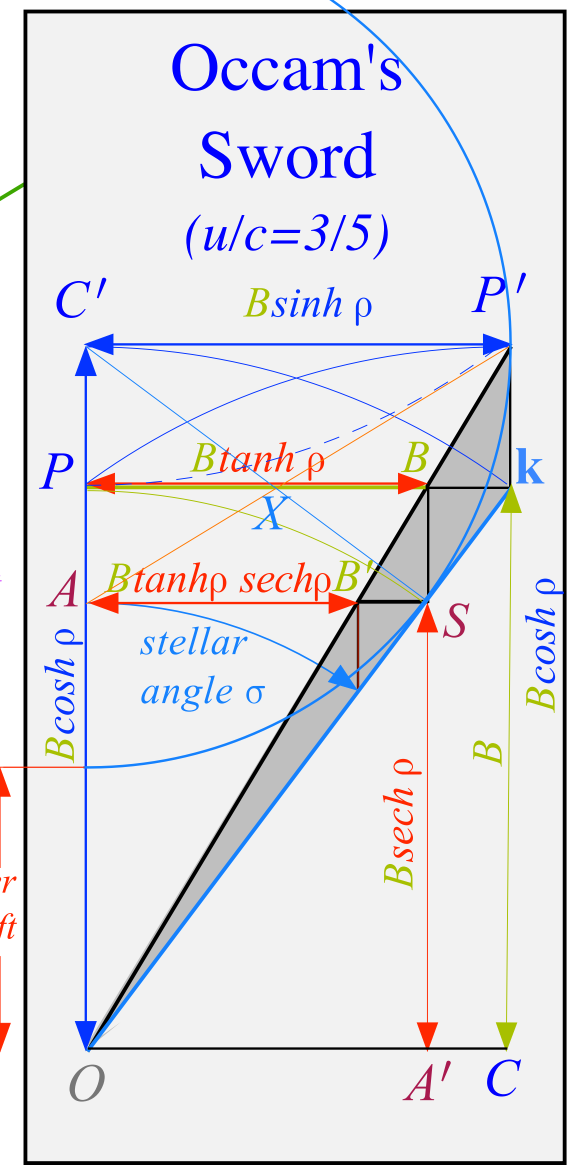
[RelaWavity Web Simulation](#)
[Relating *Rapidity* and Stellar *Abberation*](#)

Summary of optical wave parameters for relativity and QM

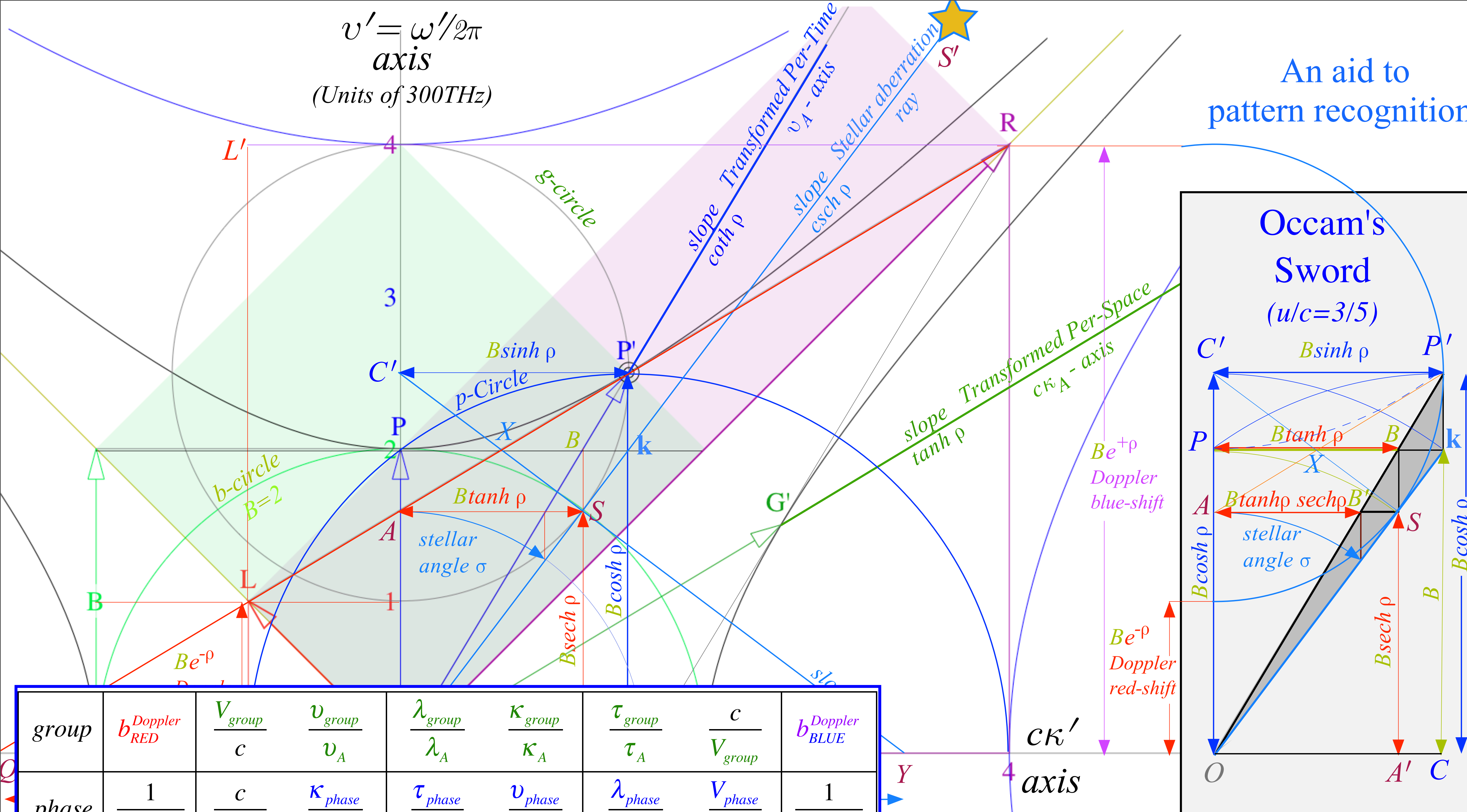
...and their geometry



An aid to pattern recognition:



RelaWavity Web Simulation {perSpace - perTime All}



An aid to pattern recognition:

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Table of 12 wave parameters (includes inverses) for relativity

...and values for $u/c=3/5$

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*