Lecture 26 Relawavity Introduction 4
Thursday 4.14.2016

Relawavity: Relativistic wave mechanics IV. Coordinate geometry

(Unit 3 4.12.16)

➔ Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = B$\cosh\rho$...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P}'$ and Group vector $\mathbf{G}'$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

➔ Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = B$\cosh\rho$...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

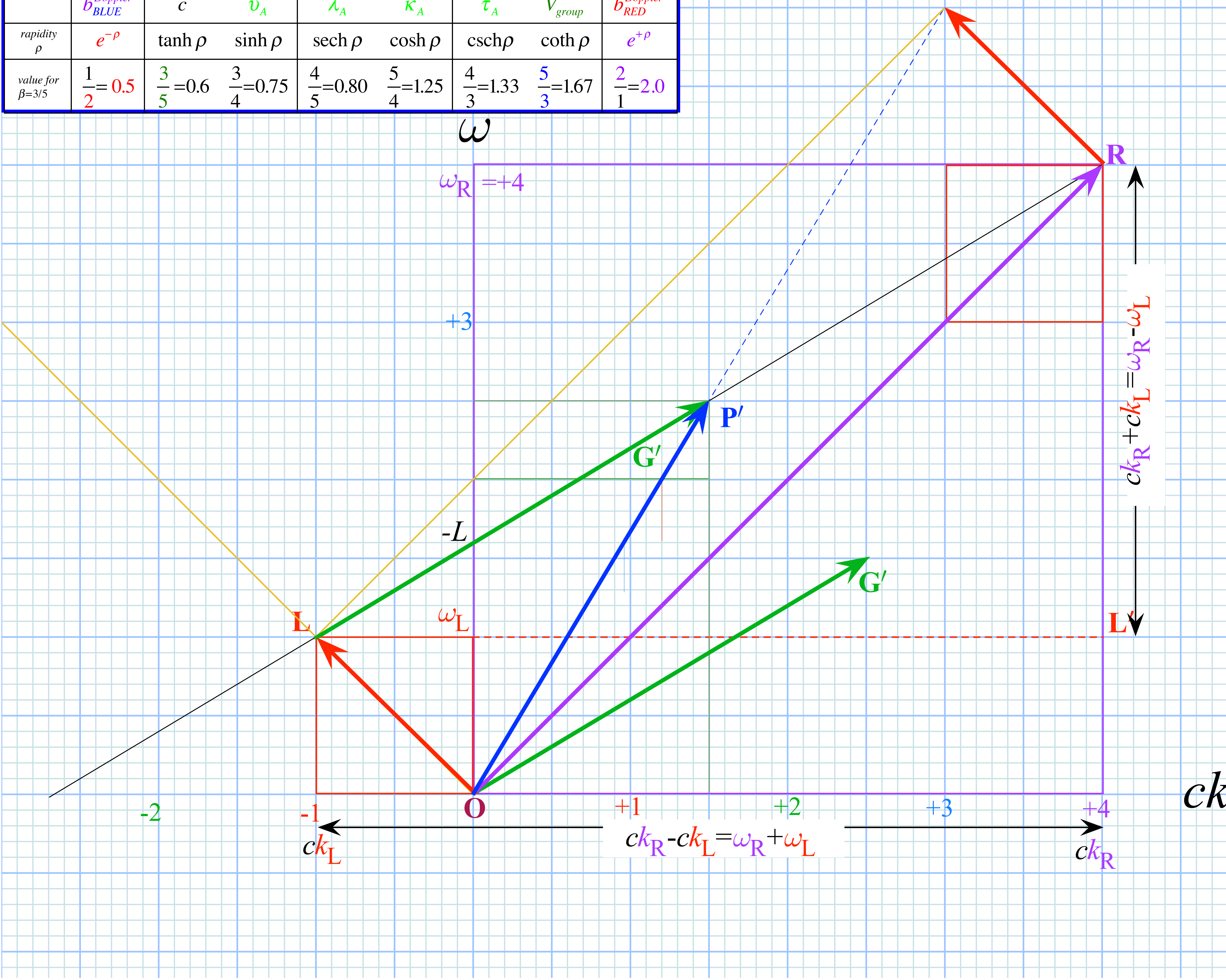
Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0



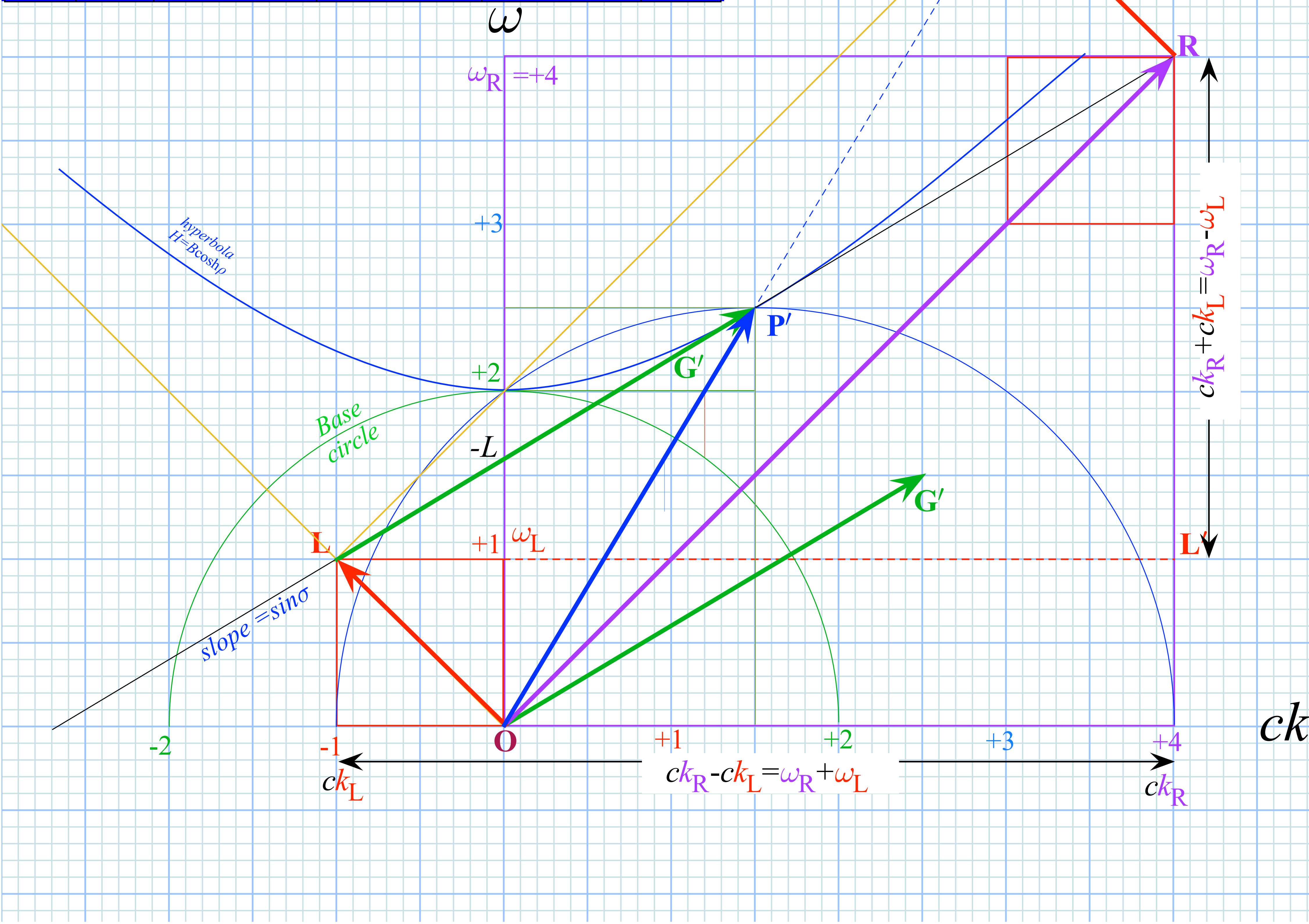
Angular 2π-factors

$k_A=2\pi\kappa_A$
$\omega_A=2\pi\upsilon_A$

$k_{phase}=2\pi\kappa_{phase}$
$\omega_{phase}=2\pi\upsilon_{phase}$

$k_{group}=2\pi\kappa_{group}$
$\omega_{group}=2\pi\upsilon_{group}$

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



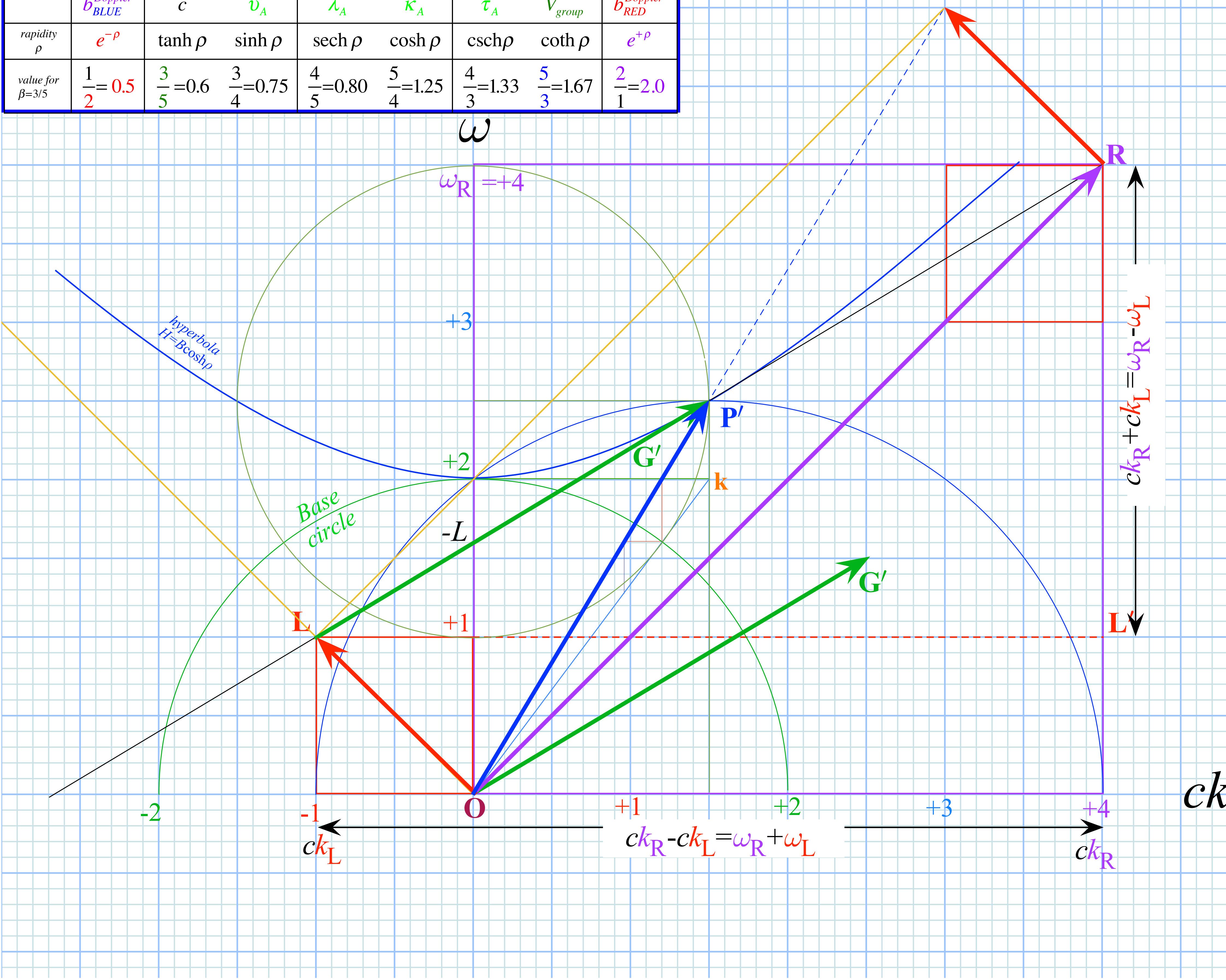
Angular 2π-factors

$k_A = 2\pi\kappa_A$
$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$
$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$
$\omega_{group} = 2\pi\upsilon_{group}$

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



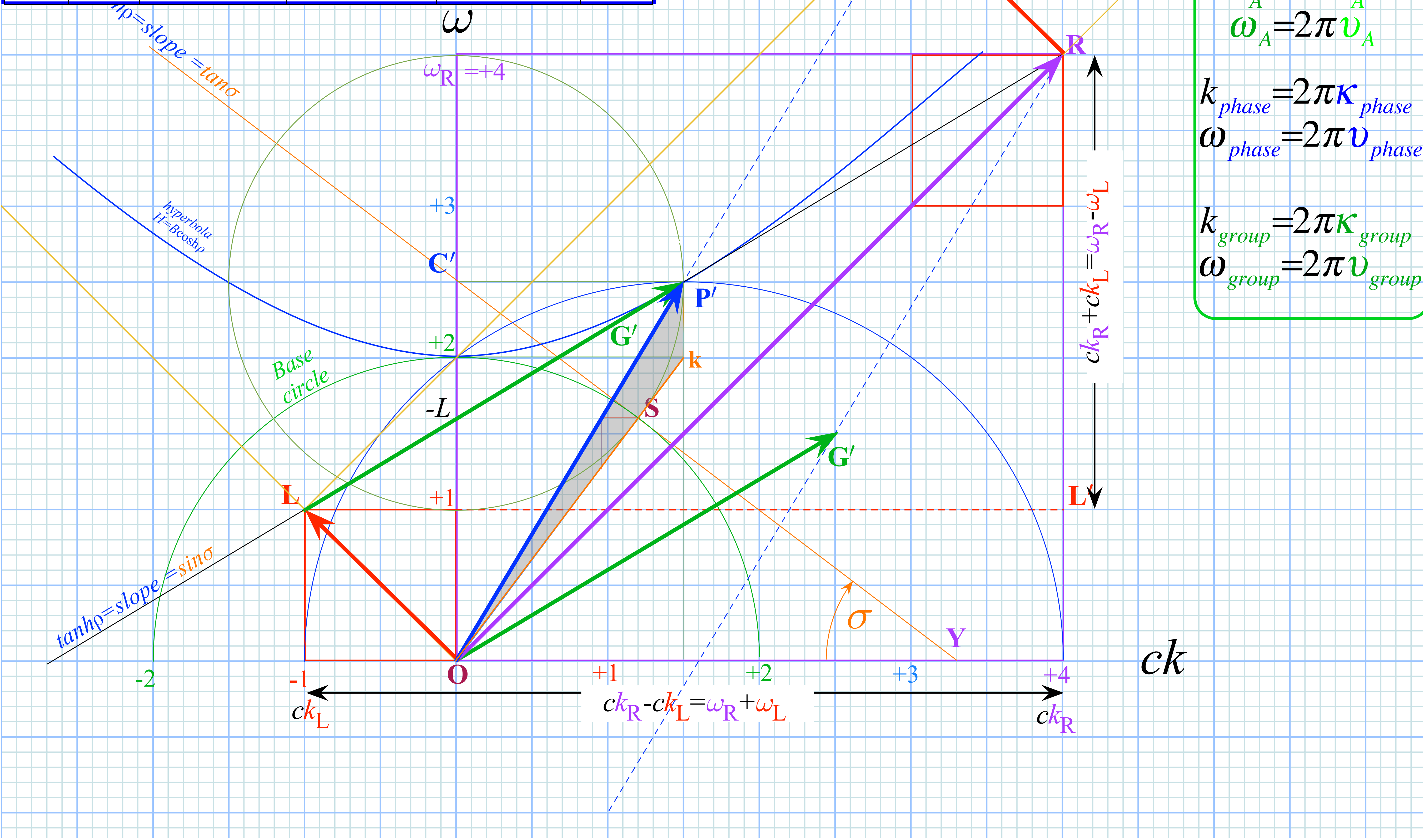
Angular 2π-factors

$k_A=2\pi\kappa_A$
$\omega_A=2\pi\upsilon_A$

$k_{phase}=2\pi\kappa_{phase}$
$\omega_{phase}=2\pi\upsilon_{phase}$

$k_{group}=2\pi\kappa_{group}$
$\omega_{group}=2\pi\upsilon_{group}$

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0



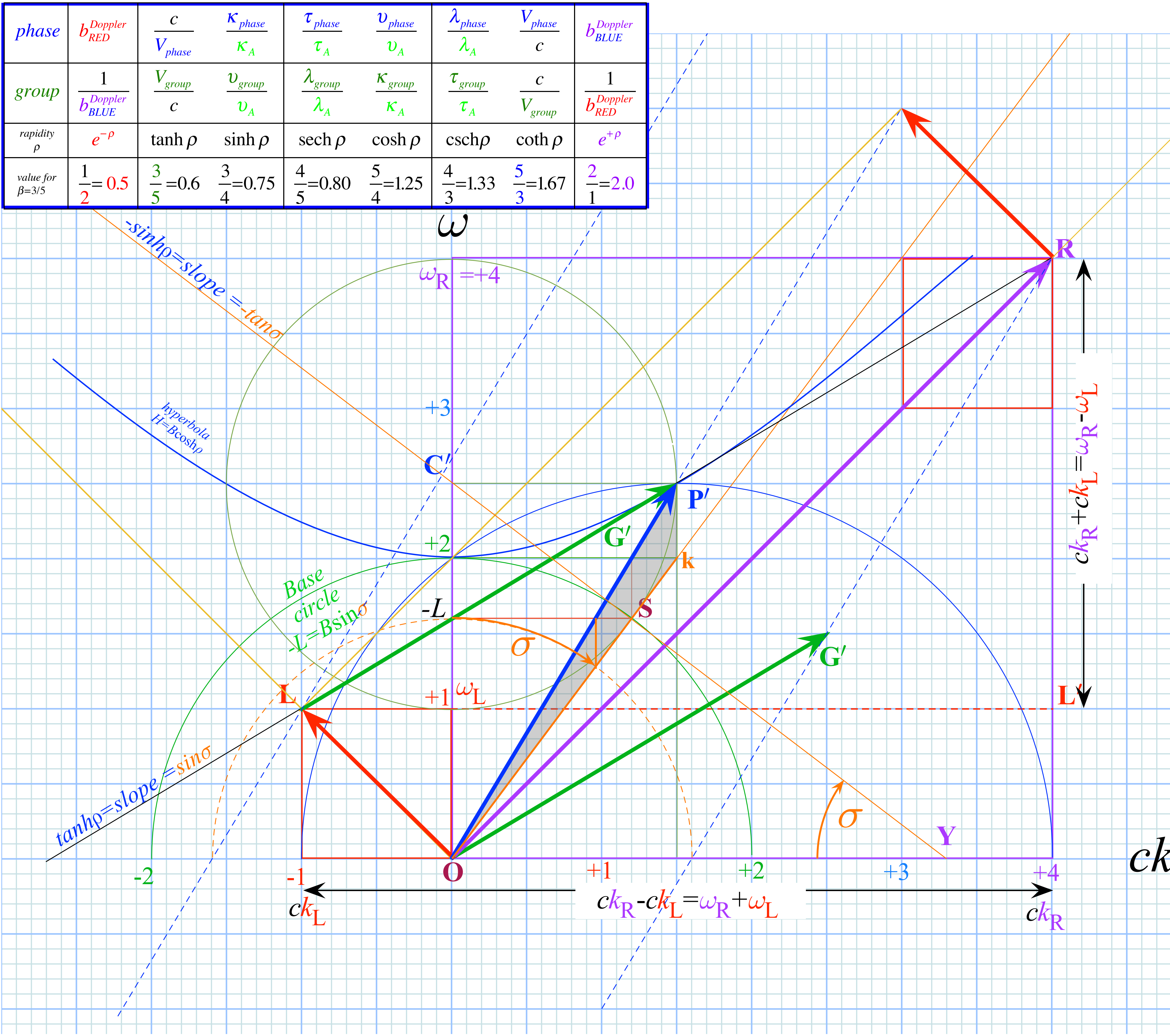
Angular 2π-factors

$k_A = 2\pi\kappa_A$
$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$
$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$
$\omega_{group} = 2\pi\upsilon_{group}$

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



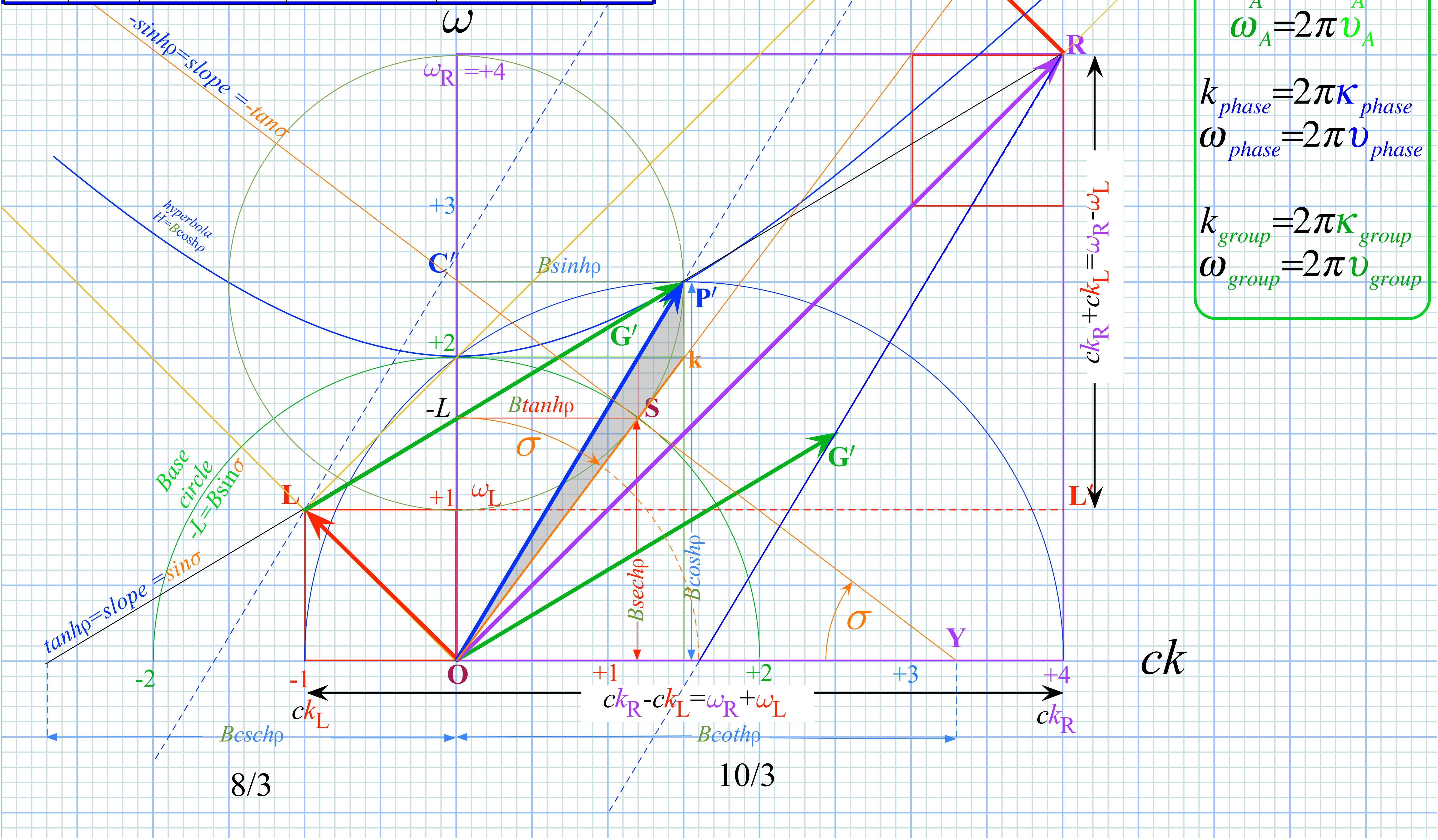
Angular 2π-factors

$k_A=2\pi\kappa_A$
$\omega_A=2\pi\upsilon_A$

$k_{phase}=2\pi\kappa_{phase}$
$\omega_{phase}=2\pi\upsilon_{phase}$

$k_{group}=2\pi\kappa_{group}$
$\omega_{group}=2\pi\upsilon_{group}$

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0



Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = B$\cosh\rho$...

➔ A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

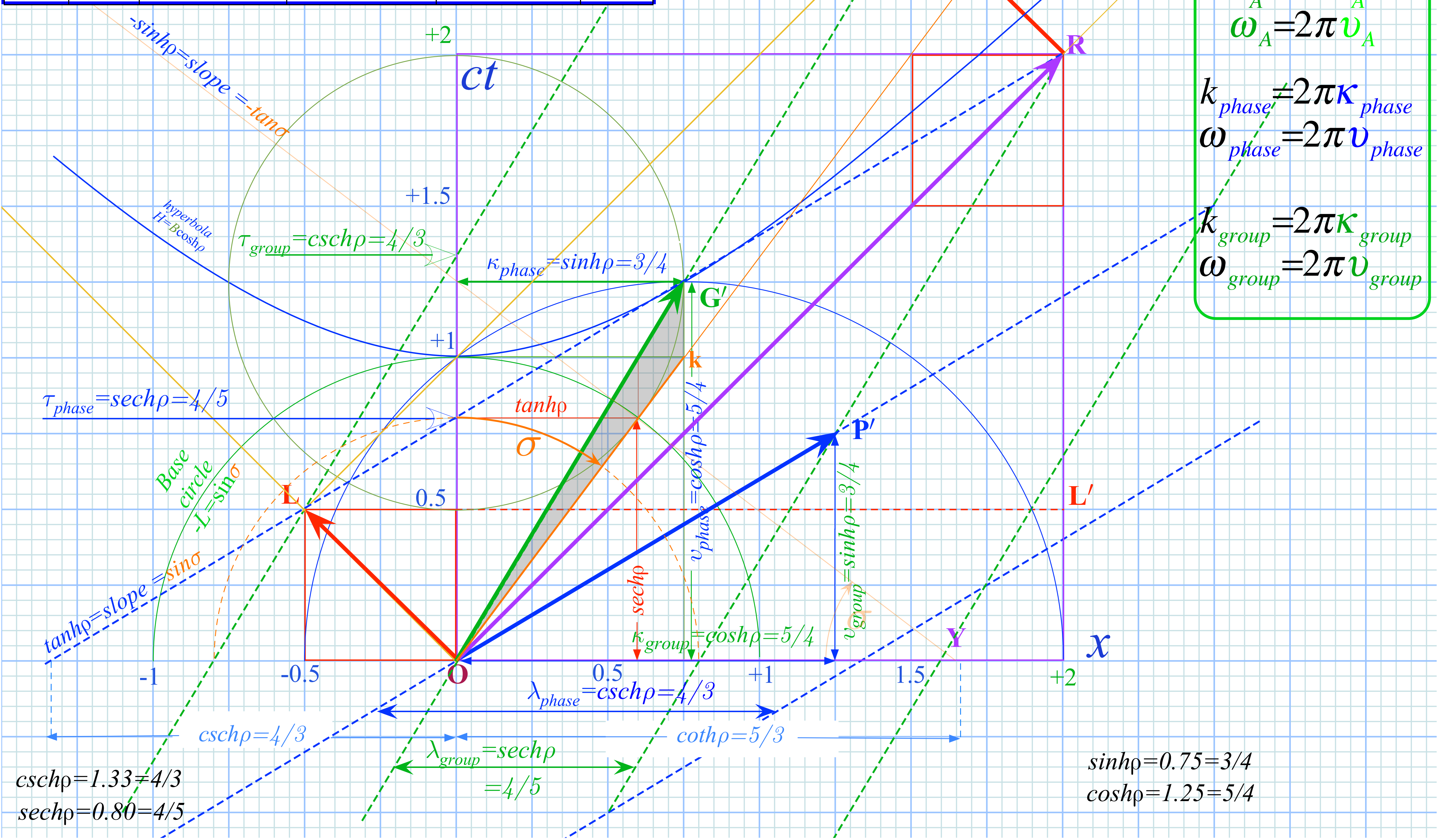
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$ =0.75	$\frac{4}{5}$ =0.80	$\frac{5}{4}$ =1.25	$\frac{4}{3}$ =1.33	$\frac{5}{3}$ =1.67	$\frac{2}{1}$ =2.0

Angular 2π-factors

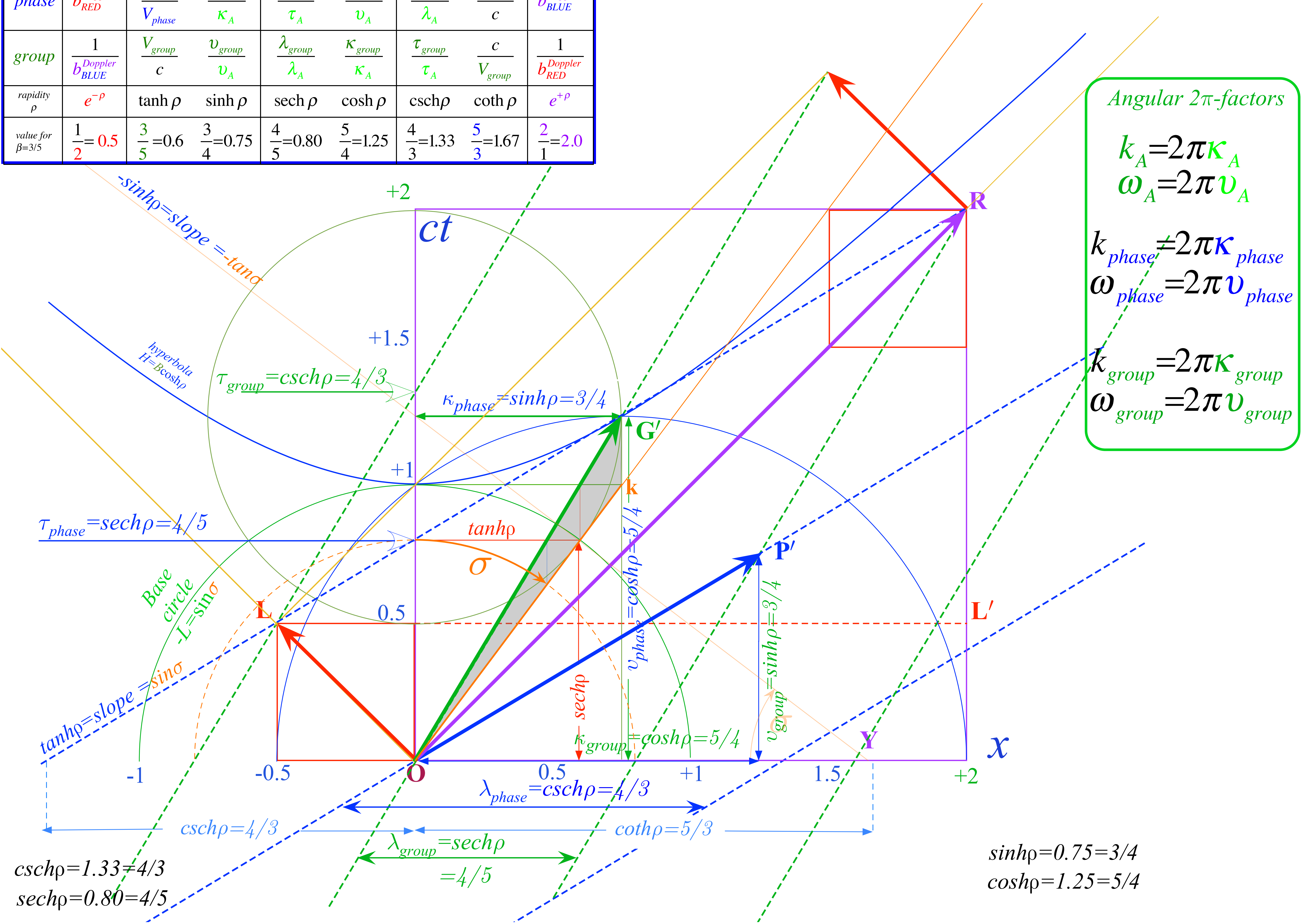
$k_A = 2\pi\kappa_A$
$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$
$\omega_{phase} = 2\pi\upsilon_{phase}$

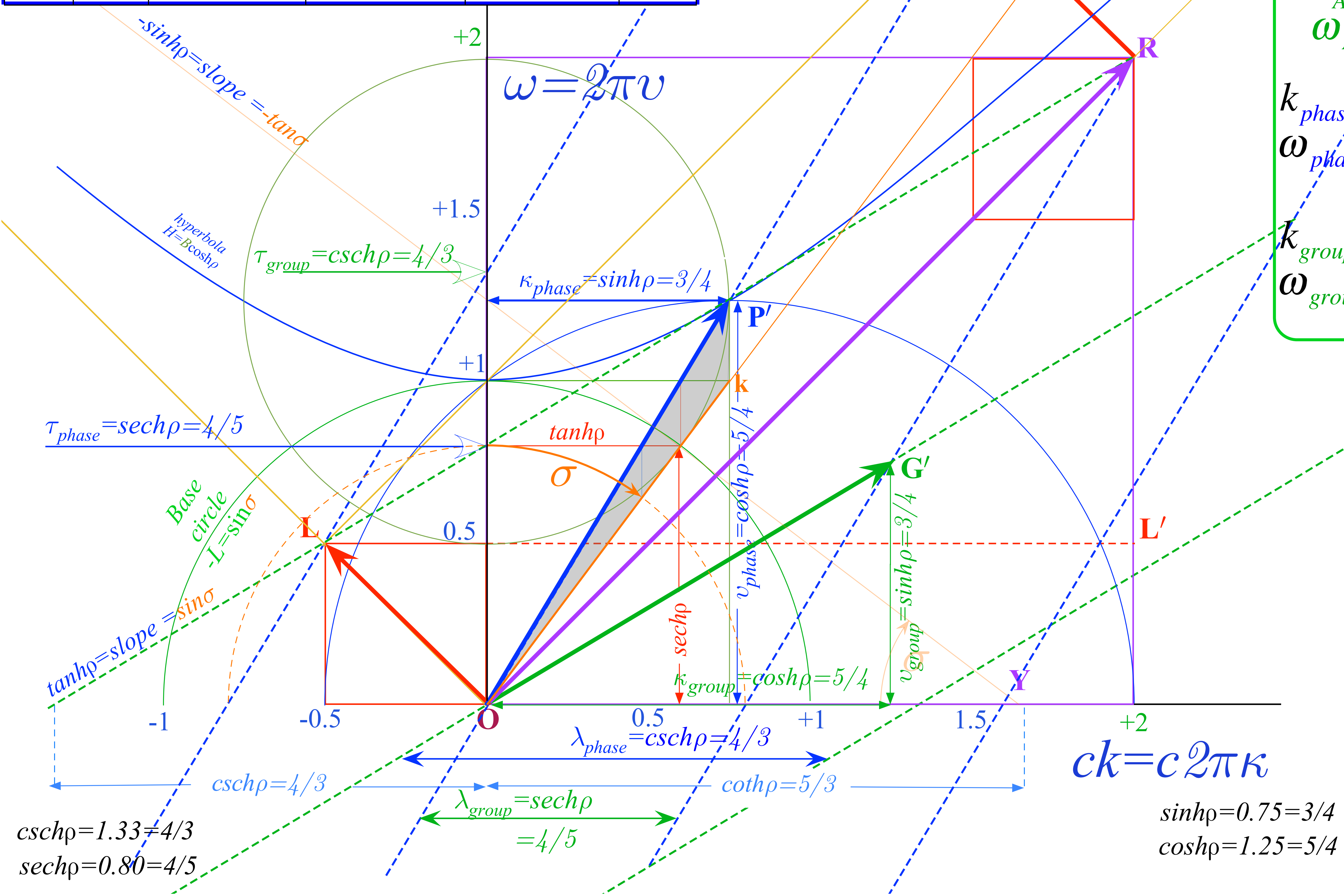
$k_{group} = 2\pi\kappa_{group}$
$\omega_{group} = 2\pi\upsilon_{group}$



phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0



Angular 2π-factors

$k_A = 2\pi\kappa_A$
$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$
$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$
$\omega_{group} = 2\pi\upsilon_{group}$

Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

➔Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

Lorentz transformations...

write $\mathbf{G}'$ *and* $\mathbf{P}'$ *in terms of* $\mathbf{G}$ *and* $\mathbf{P}$ *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

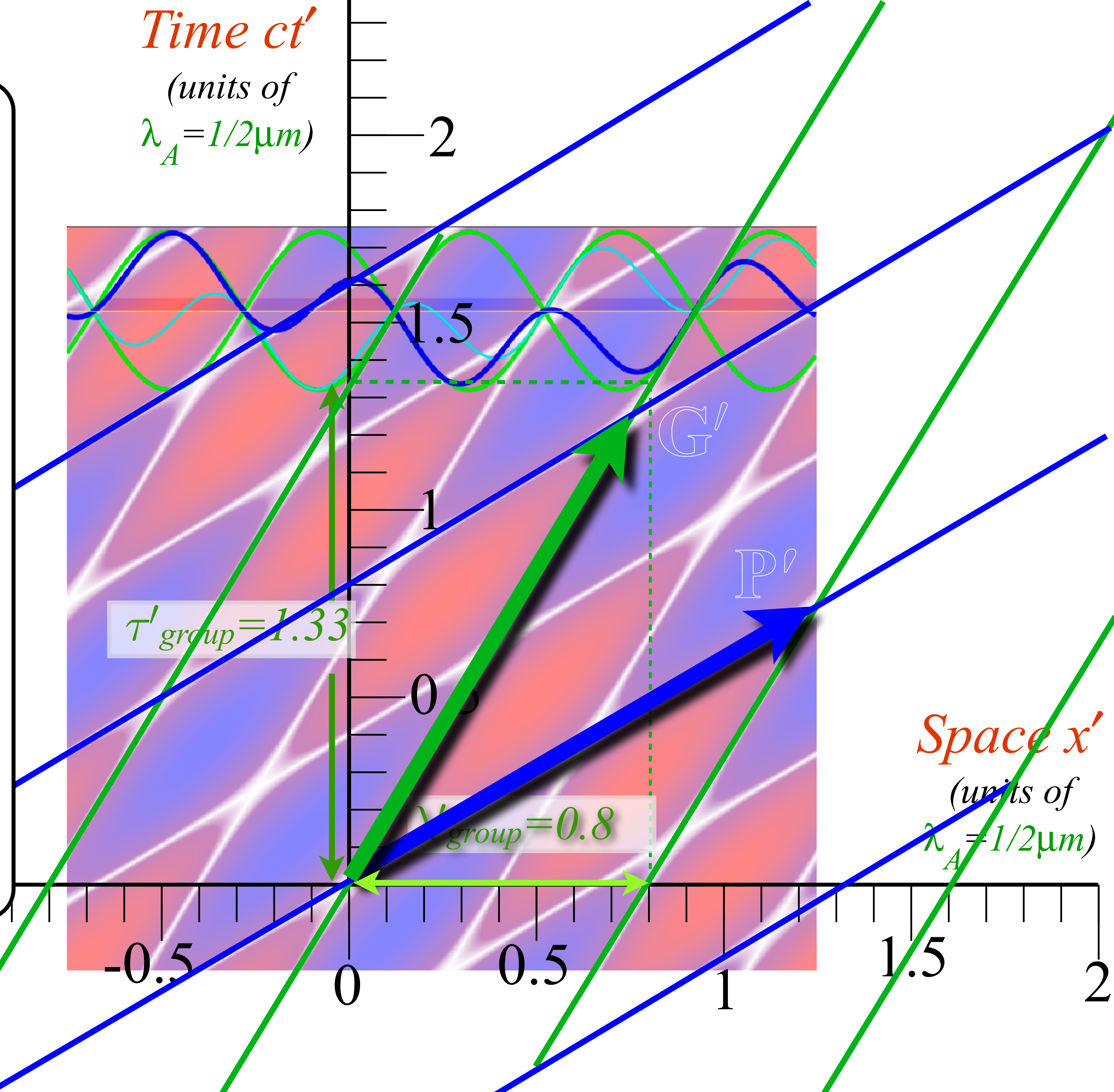
$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

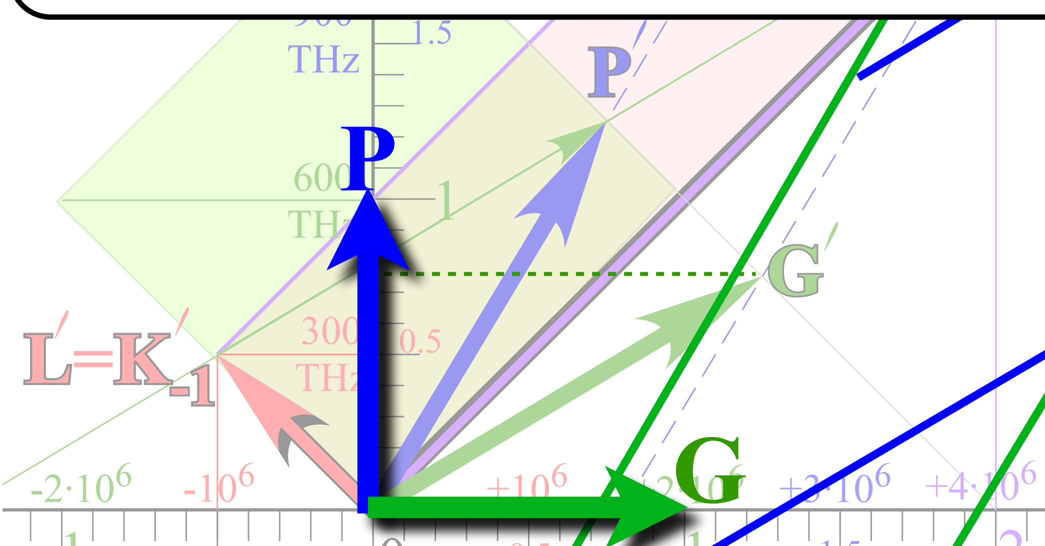
$$\mathbf{G}' = \mathbf{G} \cosh\rho + \mathbf{P} \sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sinh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cosh\rho$$

$$\mathbf{P}' = \mathbf{G} \sinh\rho + \mathbf{P} \cosh\rho$$





$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$ Lorentz transform matrix

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

➔ Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

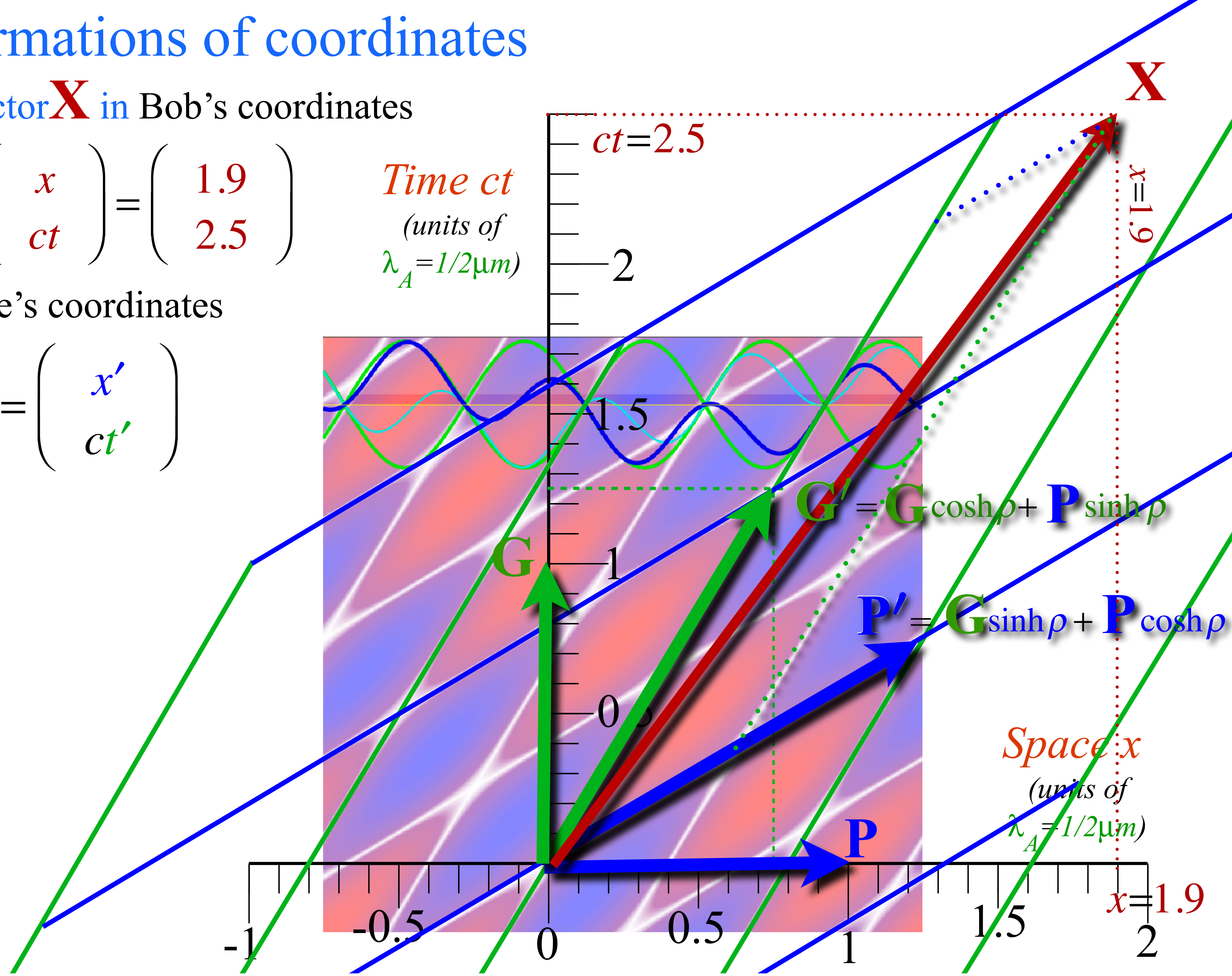
Lorentz transformations of coordinates

Space-time position vector **X** in Bob's coordinates

$$\mathbf{X} = x\mathbf{P} + ct\mathbf{G} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Same vector **X** in Alice's coordinates

$$\mathbf{X} = x'\mathbf{P}' + ct'\mathbf{G}' = \begin{pmatrix} x' \\ ct' \end{pmatrix}$$



Lorentz transform matrix for $u/c=3/5$

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$$

Lorentz transformations of coordinates

Space-time position vector **X** in Bob's coordinates

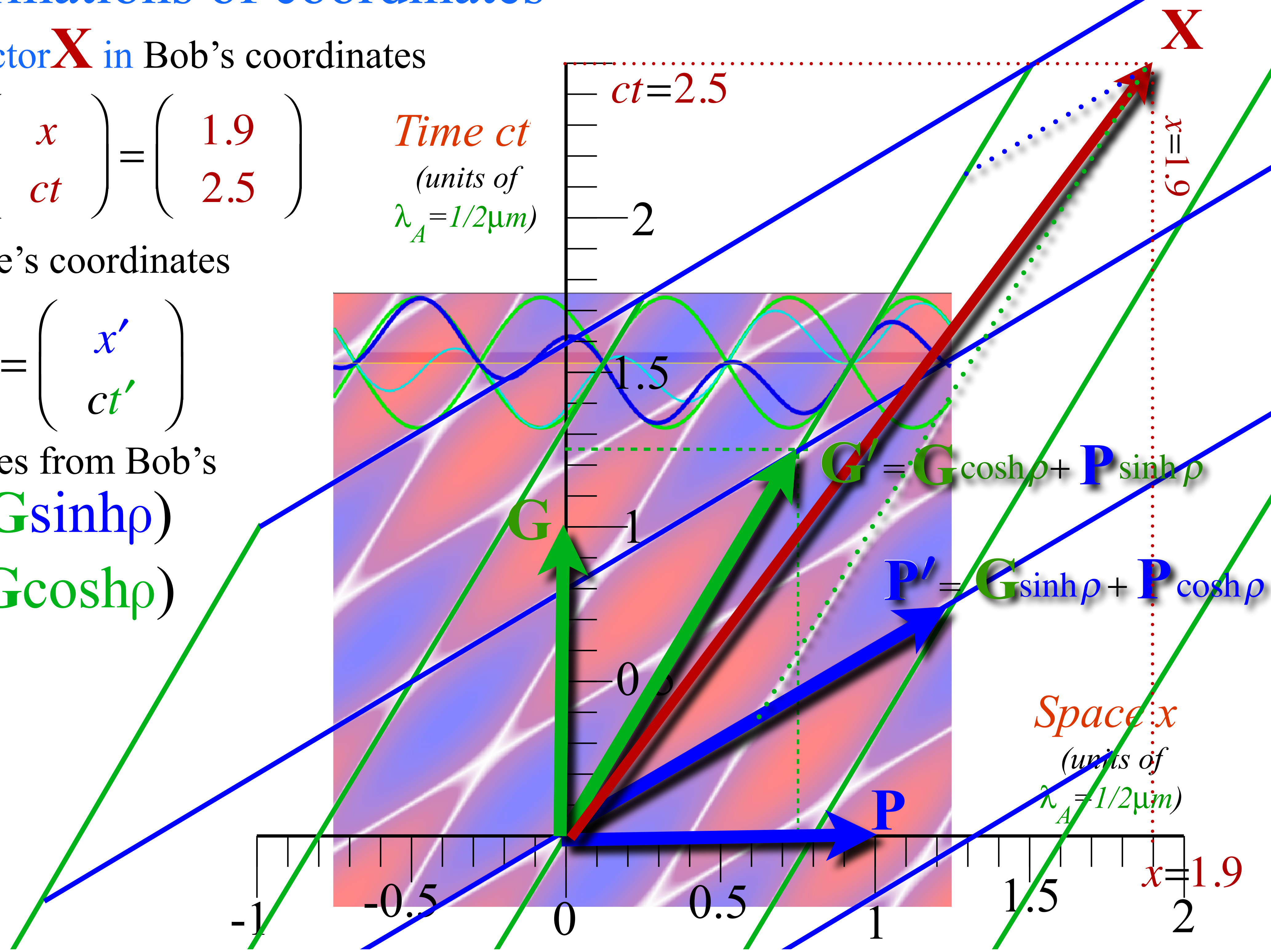
$$\mathbf{X} = x\mathbf{P} + ct\mathbf{G} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Same vector **X** in Alice's coordinates

$$\mathbf{X} = x'\mathbf{P}' + ct'\mathbf{G}' = \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Find Alice's coordinates from Bob's

$$\mathbf{X} = x'(\mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho)$$
$$+ ct'(\mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho)$$



Lorentz transform matrix for $u/c=3/5$

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$$

Lorentz transformations of coordinates

Space-time position vector **X** in Bob's coordinates

$$\mathbf{X} = x\mathbf{P} + ct\mathbf{G} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Same vector **X** in Alice's coordinates

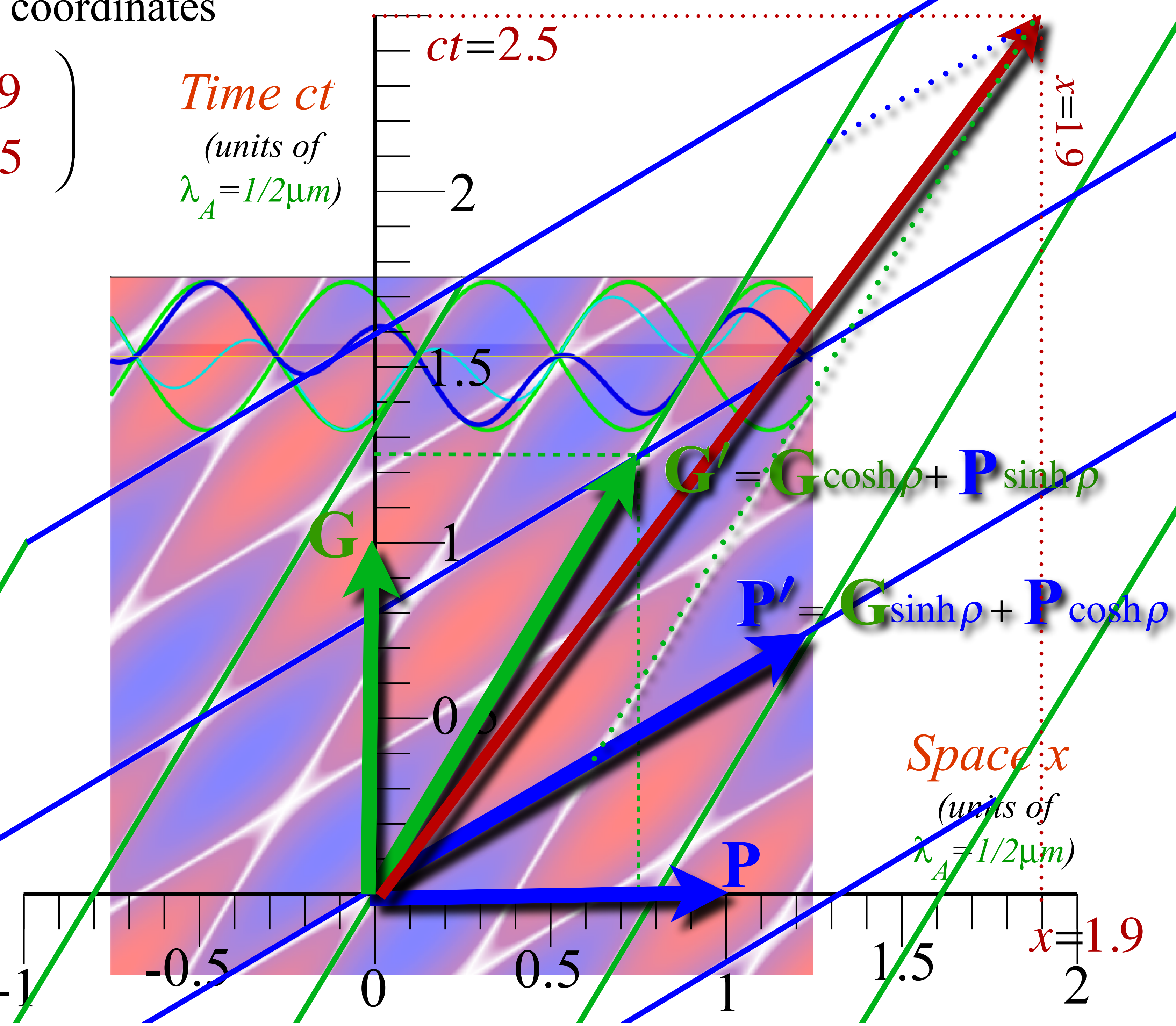
$$\mathbf{X} = x'\mathbf{P}' + ct'\mathbf{G}' = \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Find Alice's coordinates from Bob's

$$\mathbf{X} = x'(\mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho)$$
$$+ ct'(\mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho)$$

Rearrange:

$$\mathbf{X} = (x'\cosh\rho + ct'\sinh\rho)\mathbf{P}$$
$$+ (x'\sinh\rho + ct'\cosh\rho)\mathbf{G}$$



Lorentz transform matrix for $u/c=3/5$

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$$

Lorentz transformations of coordinates

Space-time position vector **X** in Bob's coordinates

$$\mathbf{X} = x\mathbf{P} + ct\mathbf{G} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Same vector **X** in Alice's coordinates

$$\mathbf{X} = x'\mathbf{P}' + ct'\mathbf{G}' = \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

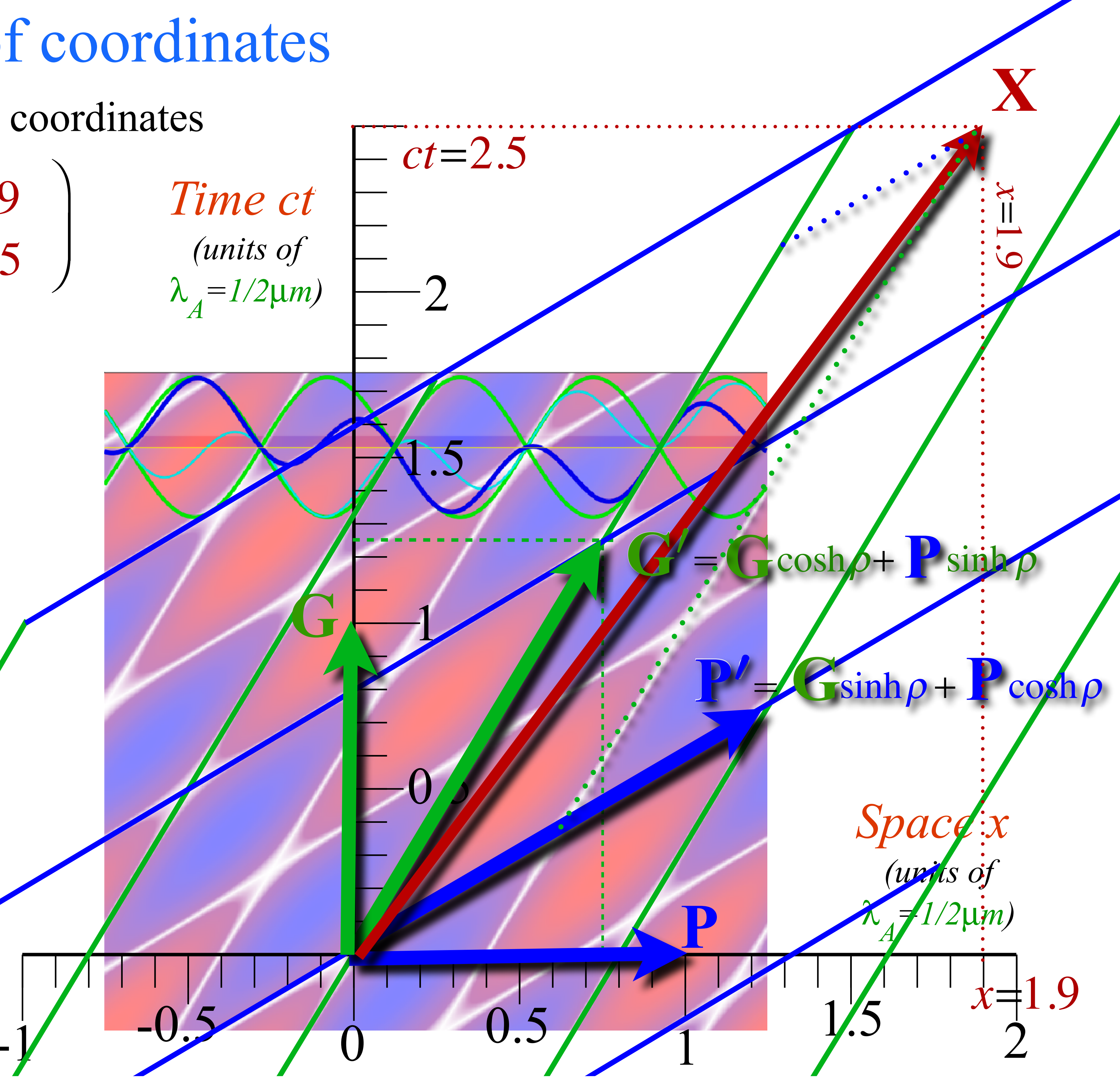
Find Alice's coordinates from Bob's

$$\mathbf{X} = x'(\mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho)$$
$$+ ct'(\mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho)$$

Rearrange:

$$\mathbf{X} = (x'\cosh\rho + ct'\sinh\rho)\mathbf{P}$$
$$+ (x'\sinh\rho + ct'\cosh\rho)\mathbf{G}$$
$$= x\mathbf{P}$$
$$+ ct\mathbf{G}$$

Put this in matrix form:



Lorentz transform matrix for $u/c=3/5$

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$$

Lorentz transformations of coordinates

Space-time position vector **X** in Bob's coordinates

$$\mathbf{X} = x\mathbf{P} + ct\mathbf{G} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Same vector **X** in Alice's coordinates

$$\mathbf{X} = x'\mathbf{P}' + ct'\mathbf{G}' = \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Find Alice's coordinates from Bob's

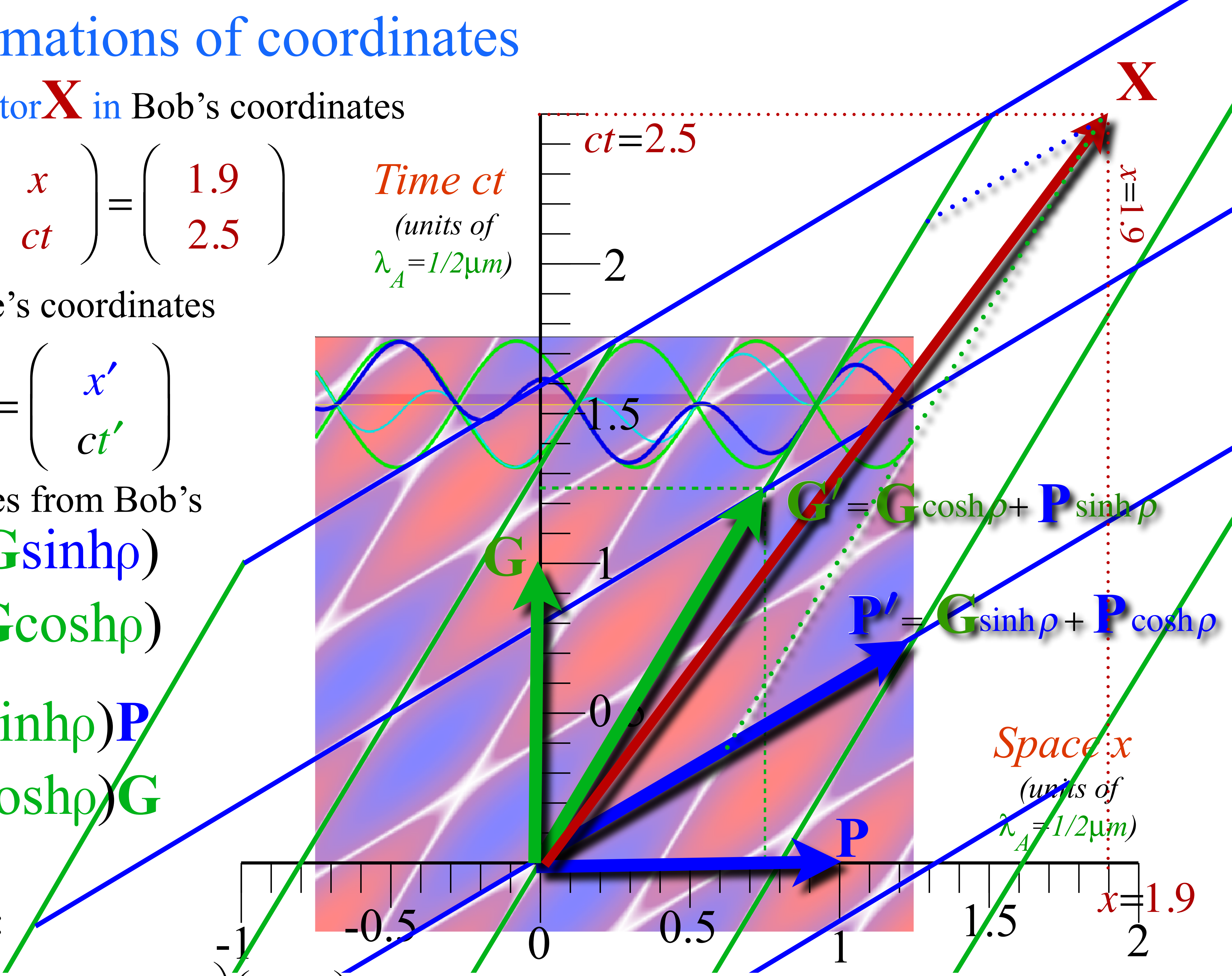
$$\mathbf{X} = x'(\mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho) + ct'(\mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho)$$

Rearrange:

$$\mathbf{X} = (x'\cosh\rho + ct'\sinh\rho)\mathbf{P} + (x'\sinh\rho + ct'\cosh\rho)\mathbf{G} = x\mathbf{P} + ct\mathbf{G}$$

Put this in matrix form:

$$\begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$



Lorentz transform matrix for $u/c=3/5$

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$$

Lorentz transformations of coordinates

Space-time position vector **X** in Bob's coordinates

$$\mathbf{X} = x\mathbf{P} + ct\mathbf{G} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Same vector **X** in Alice's coordinates

$$\mathbf{X} = x'\mathbf{P}' + ct'\mathbf{G}' = \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

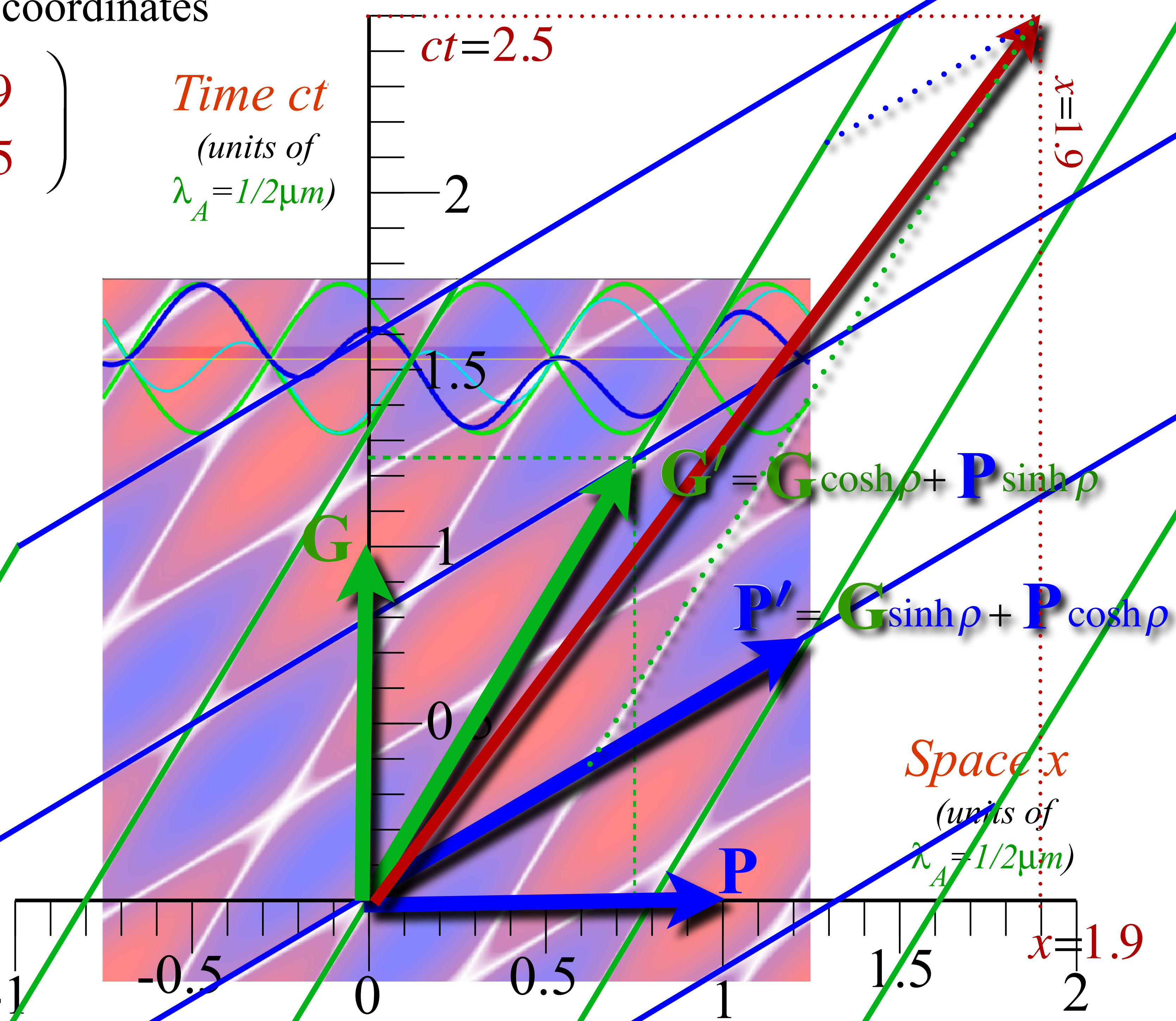
Find Alice's coordinates from Bob's

$$\mathbf{X} = x'(\mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho)$$
$$+ ct'(\mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho)$$

Rearrange:

$$\mathbf{X} = (x'\cosh\rho + ct'\sinh\rho)\mathbf{P}$$
$$+ (x'\sinh\rho + ct'\cosh\rho)\mathbf{G}$$
$$= x\mathbf{P}$$
$$+ ct\mathbf{G}$$

Put this in matrix form:



$$\begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Matrix inverse:

Set ρ to -ρ

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Lorentz transform matrix for $u/c=3/5$

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$$

Lorentz transformations of coordinates

Space-time position vector **X** in Bob's coordinates

$$\mathbf{X} = x\mathbf{P} + ct\mathbf{G} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix}$$

Same vector **X** in Alice's coordinates

$$\mathbf{X} = x'\mathbf{P}' + ct'\mathbf{G}' = \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

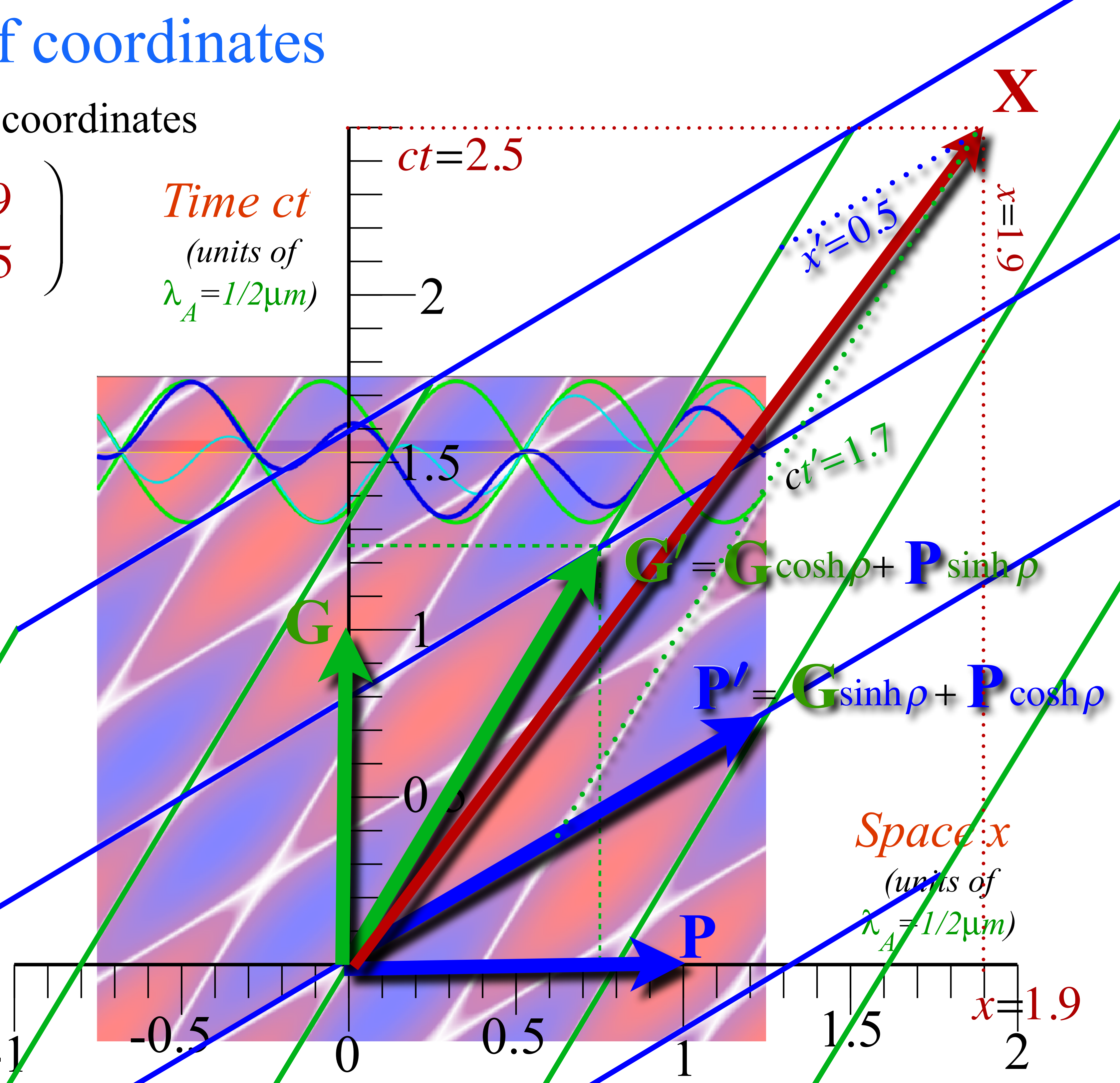
Find Alice's coordinates from Bob's

$$\mathbf{X} = x'(\mathbf{P}\cosh\rho + \mathbf{G}\sinh\rho)$$
$$+ ct'(\mathbf{P}\sinh\rho + \mathbf{G}\cosh\rho)$$

Rearrange:

$$\mathbf{X} = (x'\cosh\rho + ct'\sinh\rho)\mathbf{P}$$
$$+ (x'\sinh\rho + ct'\cosh\rho)\mathbf{G}$$
$$= x\mathbf{P}$$
$$+ ct\mathbf{G}$$

Put this in matrix form:



$$\begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix} = \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Matrix inverse:

Set ρ to -ρ

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} 1.9 \\ 2.5 \end{pmatrix} = \begin{pmatrix} \frac{5}{4}1.9 - \frac{3}{4}2.5 \\ -\frac{3}{4}1.9 + \frac{5}{4}2.5 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 1.7 \end{pmatrix}$$

Lorentz transform matrix for $u/c=3/5$

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$$

Review of geometric construction , per-space-time (ω,*ck*) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector **P′** and Group vector **G′** in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

➔ Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

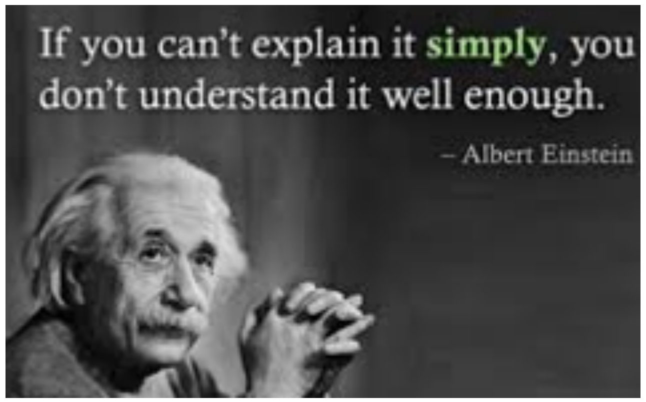
Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

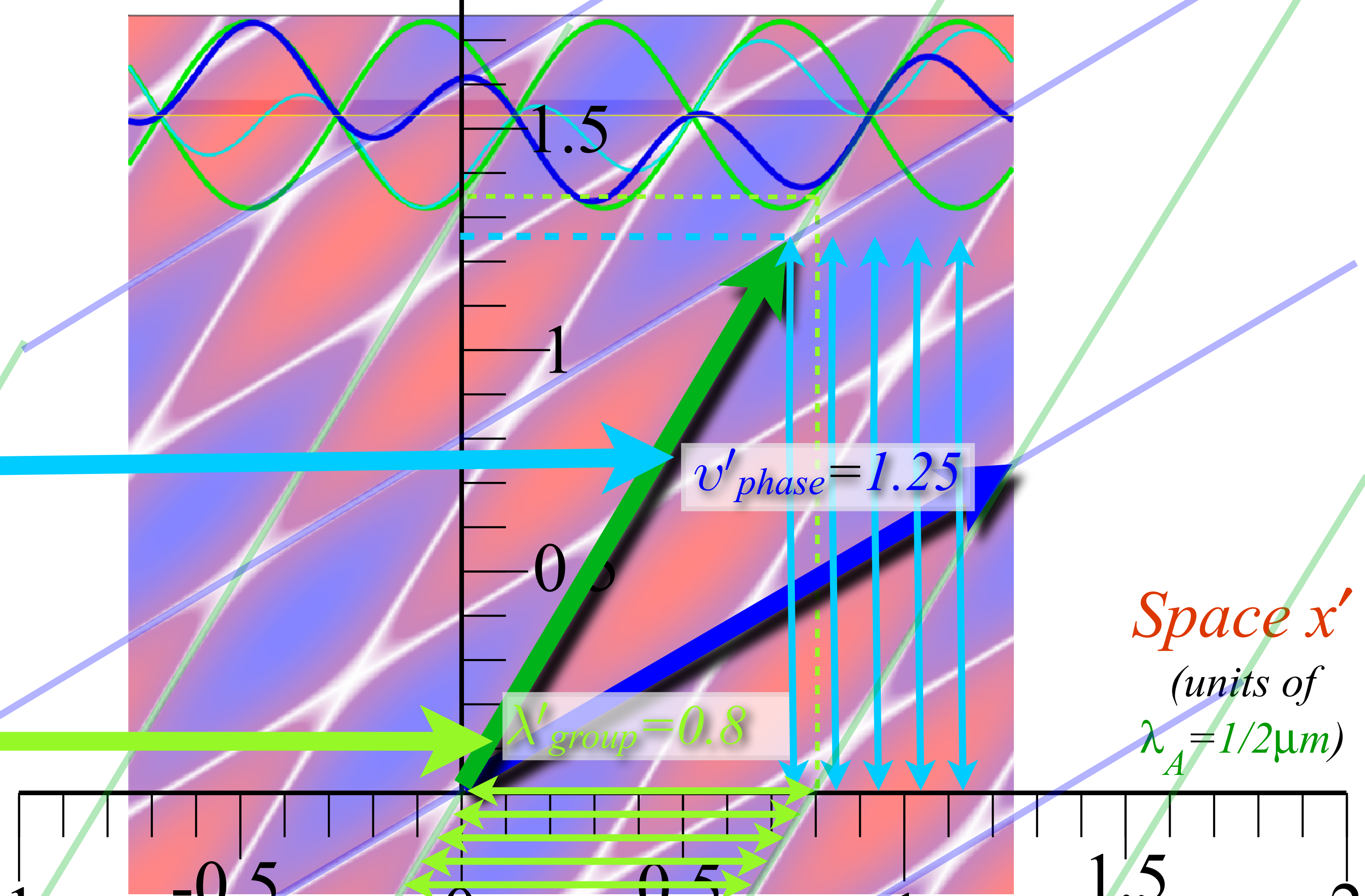
Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

Two Famous-Name Coefficients



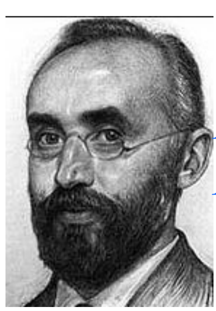
Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation** *(dilated by 25% here)*

This number is called a: **Lorentz length-contraction** *(contracted by 20% here)*



Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation
Relativistic *Terms (Dual plot w/expanded table)*

Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = B$\cosh\rho$...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

➔ Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

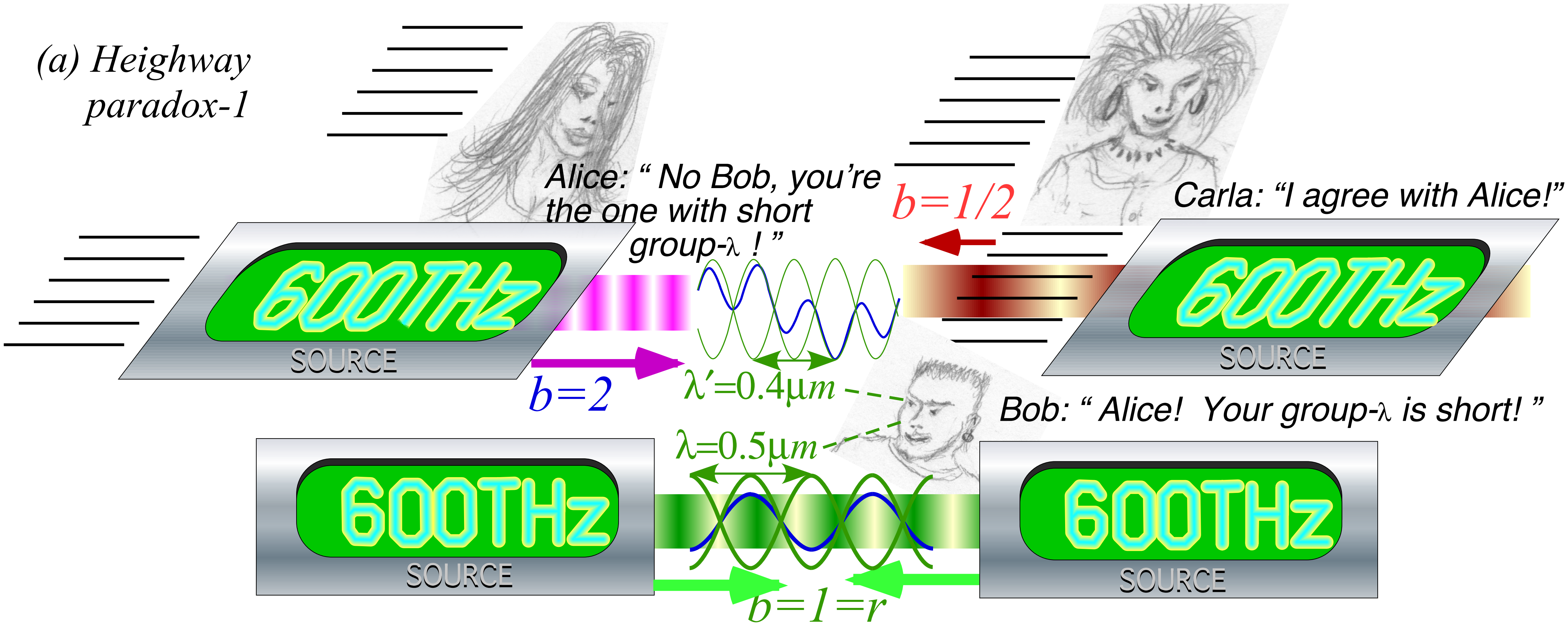
Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

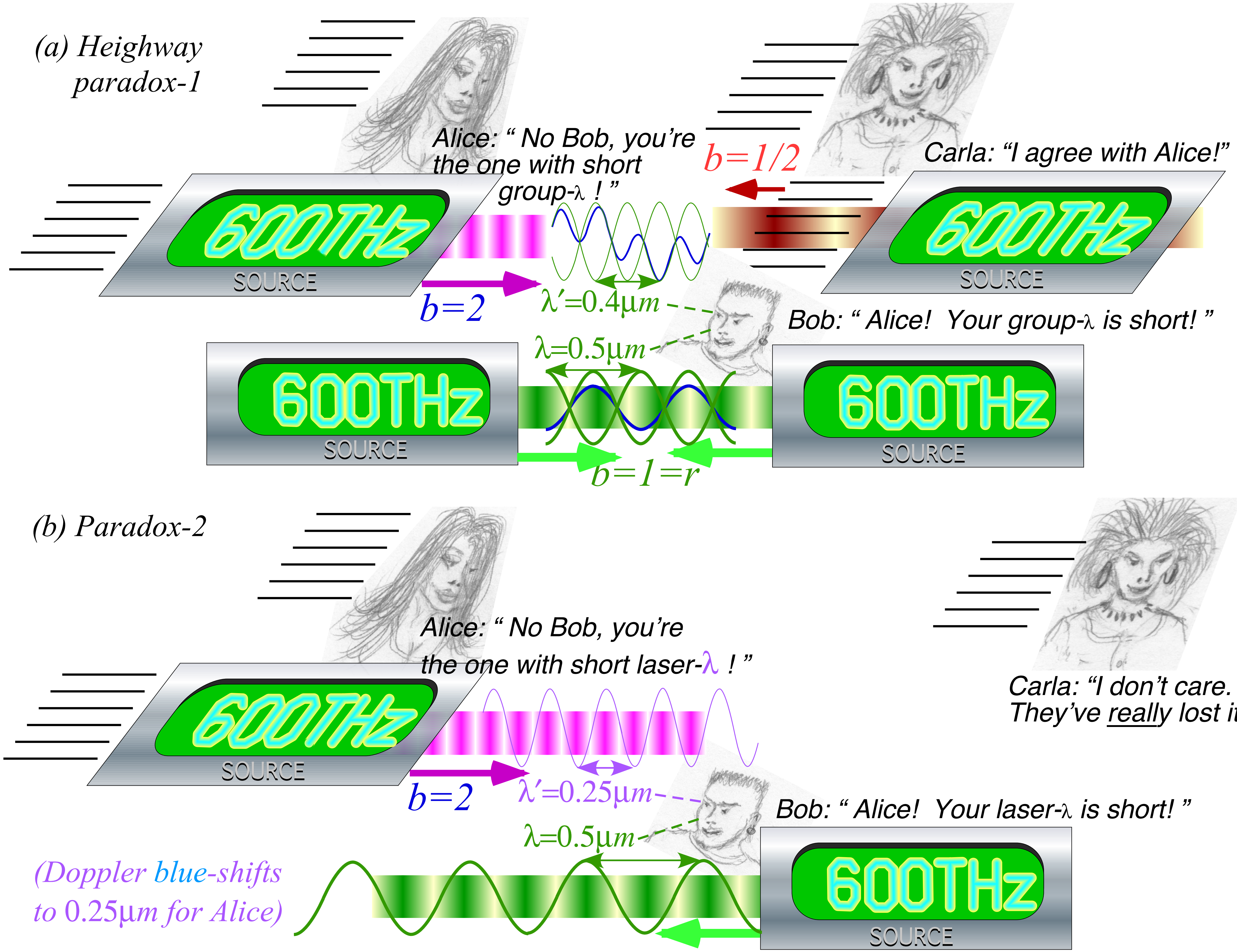
Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

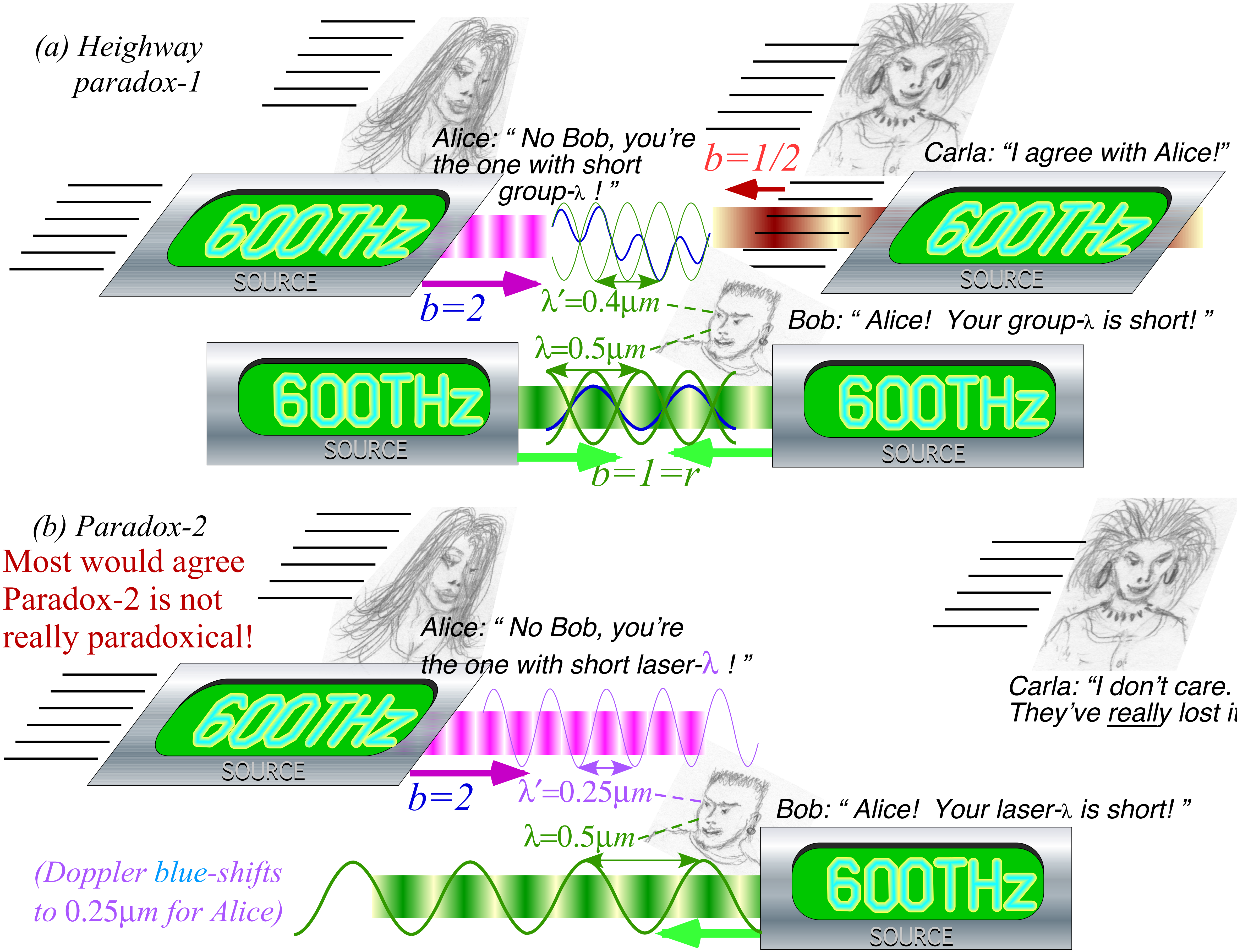
Heighway Paradoxes: A relativistic *“He said-She-said”* argument



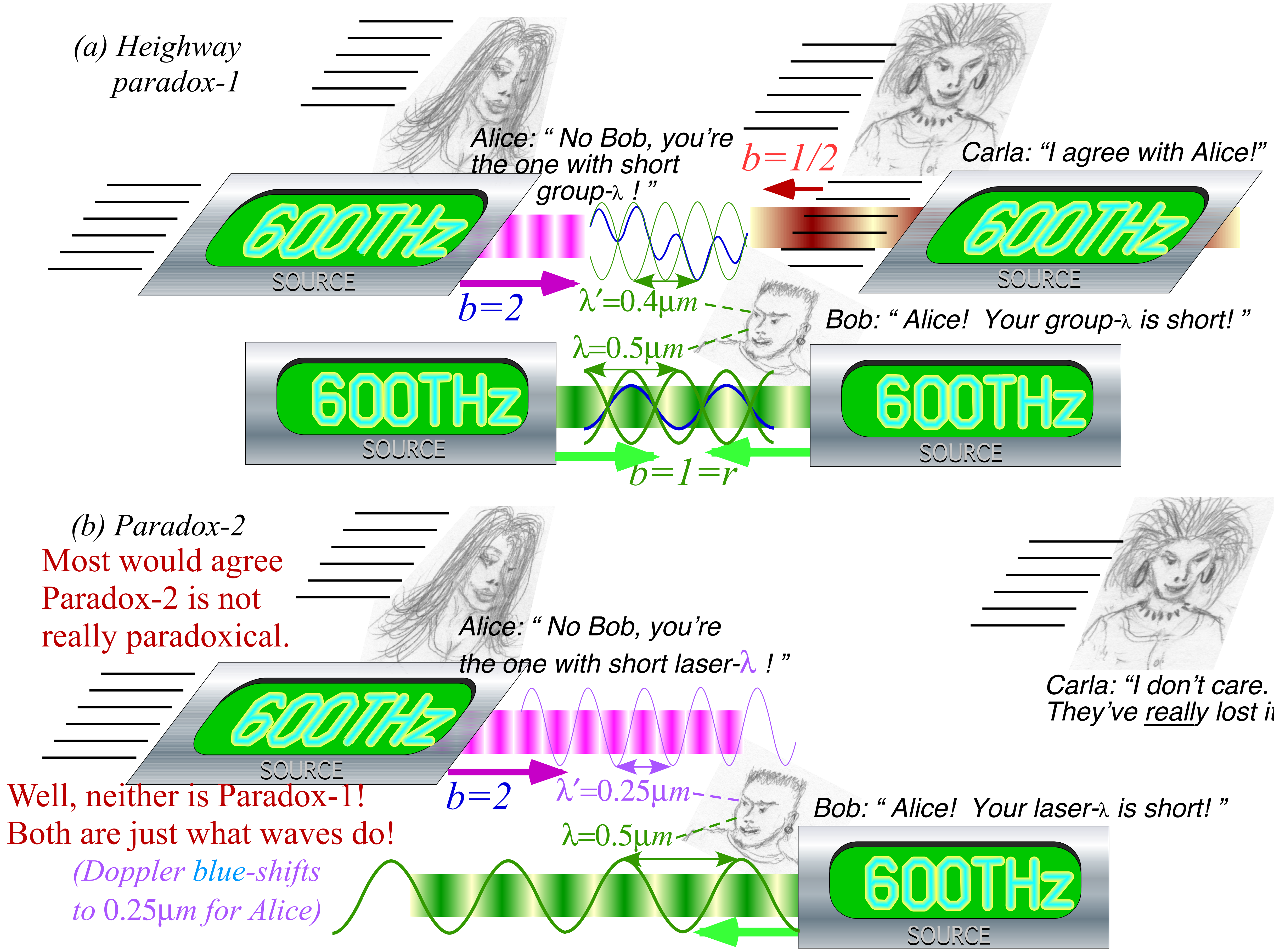
Heighway Paradoxes: A relativistic *"He said-She-said"* argument



Heighway Paradoxes: A relativistic *“He said-She-said”* argument



Heighway Paradoxes: A relativistic *"He said-She-said"* argument



Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = B$\cosh\rho$...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector **P′** and Group vector **G′** in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

➔ Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

Phase invariance...

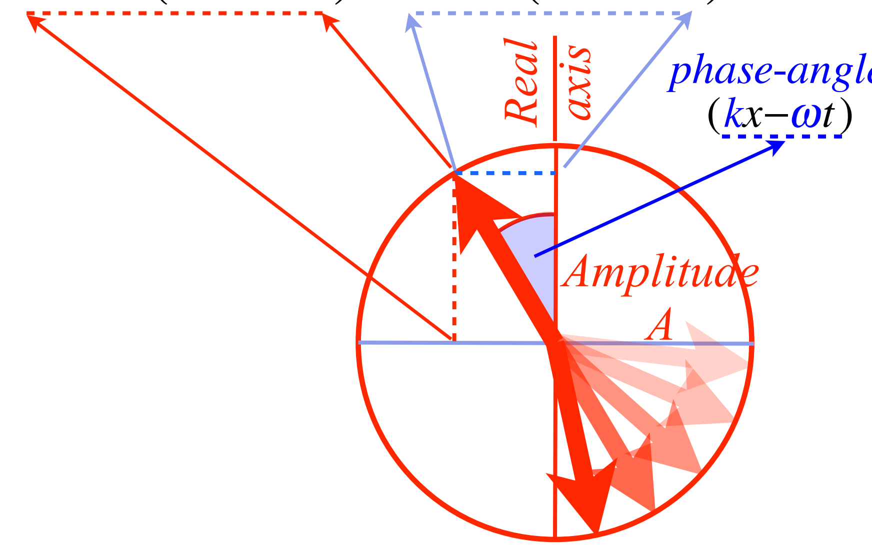
Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point <u>must equal</u> reading ϕ' by Bob in spite of <u>unequal</u> readings (x,t) and $(x',t')$for that point and <u>unequal</u> readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\ t' = k_{phase}x - \omega_{phase}\ t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\ t' = k_{group}x - \omega_{group}\ t \equiv \phi_{group}$$

$$\psi = A\cdot e^{i(kx-\omega t)}$$
$$= A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$



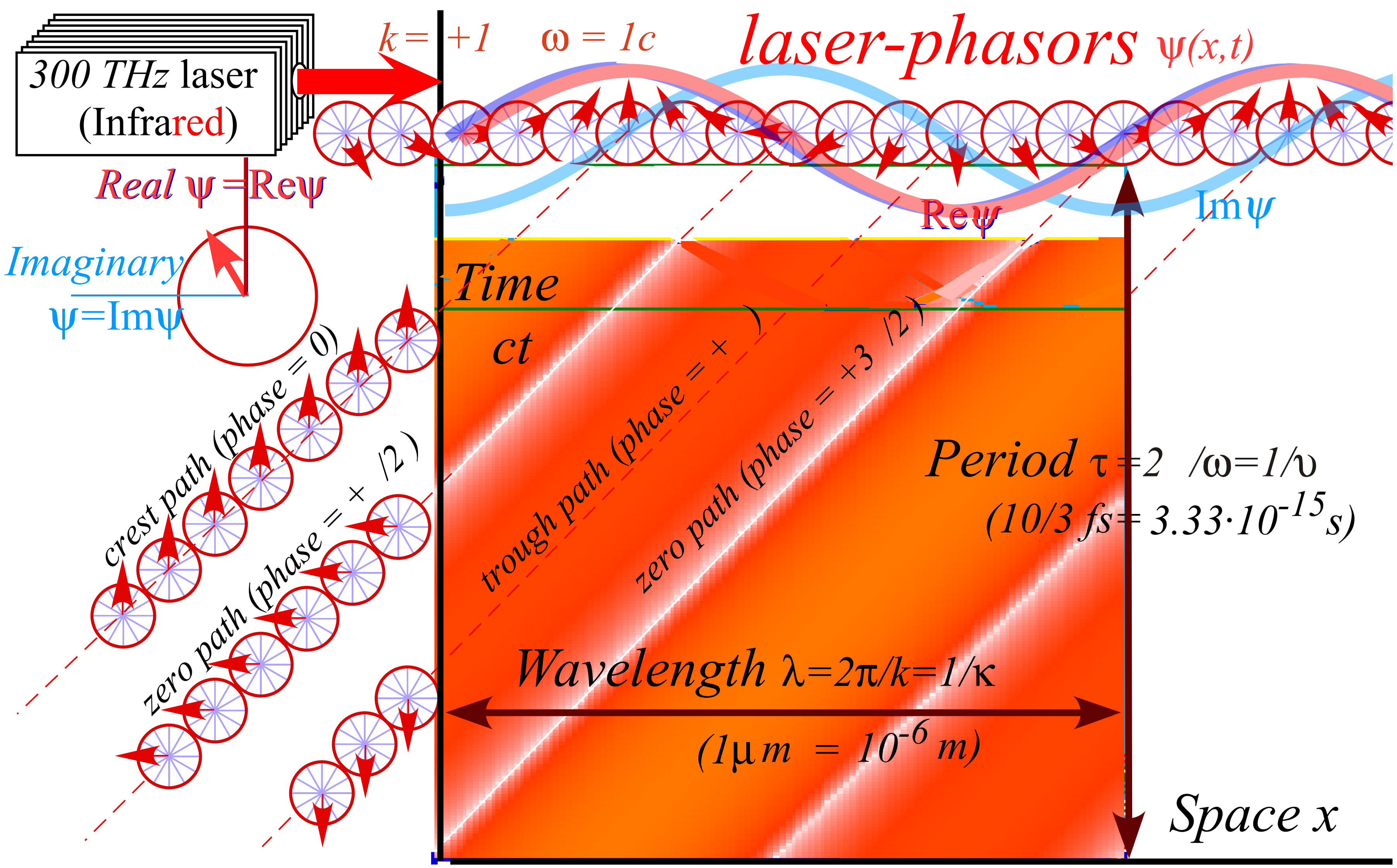


Fig. 4
Unit 3

<u>BohrIt Web Simulation</u>
<u>1 CW *ct vs x* Plot</u>
<u>*(ck = +1)*</u>

Phase invariance...

Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point <u>must equal</u> reading ϕ' by Bob in spite of <u>unequal</u> readings (x,t) and $(x',t')$for that point and <u>unequal</u> readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\ t' = k_{phase}x - \omega_{phase}\ t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\ t' = k_{group}x - \omega_{group}\ t \equiv \phi_{group}$$

...derives Lorentz transformations...

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$

Phase invariance...

Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point must equal reading ϕ' by Bob in spite of unequal readings (x,t) and (x',t') for that point and unequal readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\ t' = k_{phase}x - \omega_{phase}\ t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\ t' = k_{group}x - \omega_{group}\ t \equiv \phi_{group}$$

...derives Lorentz transformations...

Angular 2-factors

$$k_A = 2\pi\kappa_A$$
$$\omega_A = 2\pi\upsilon_A$$

$$k_{phase} = 2\pi\kappa_{phase}$$
$$\omega_{phase} = 2\pi\upsilon_{phase}$$

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$

Phase invariance...

Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point must equal reading ϕ' by Bob in spite of unequal readings (x,t) and (x',t') for that point and unequal readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\ t' = k_{phase}x - \omega_{phase}\ t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\ t' = k_{group}x - \omega_{group}\ t \equiv \phi_{group}$$

...derives Lorentz transformations...

$$k_A x' \sinh\rho_{AB} - \omega_A\ t' \cosh\rho_{AB} = 0 \cdot x - \omega_A\ t$$

since: $\sinh\rho_{AB}=0$ if $\rho_{AB}=0$

$$k'_{phase}=k_A \sinh\rho_{AB} \qquad \omega'_{phase}=\omega_A \cosh\rho_{AB} \qquad k_{phase}=0 \qquad \omega_{phase}=\omega_A$$

Angular 2-factors

$$k_A=2\pi\kappa_A$$
$$\omega_A=2\pi\upsilon_A$$

$$k_{phase}=2\pi\kappa_{phase}$$
$$\omega_{phase}=2\pi\upsilon_{phase}$$

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$

Phase invariance...

Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point must equal reading ϕ' by Bob in spite of unequal readings (x,t) and (x',t') for that point and unequal readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\, t' = k_{phase}x - \omega_{phase}\, t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\, t' = k_{group}x - \omega_{group}\, t \equiv \phi_{group}$$

...derives Lorentz transformations...

$$k_A x' \sinh\rho_{AB} - \omega_A\, t' \cosh\rho_{AB} = 0\cdot x - \omega_A\, t \qquad \text{or:} \quad ct = ct'\cosh\rho_{AB} - x'\sinh\rho_{AB}$$

using: $\omega_A / k_A = c = \upsilon_A / \kappa_A$

since: $\sinh\rho_{AB}=0$ if $\rho_{AB}=0$

$$k'_{phase} = k_A \sinh\rho_{AB} \qquad \omega'_{phase} = \omega_A \cosh\rho_{AB} \qquad k_{phase} = 0 \qquad \omega_{phase} = \omega_A$$

Angular 2π-factors

$k_A = 2\pi\kappa_A$

$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$

$\omega_{phase} = 2\pi\upsilon_{phase}$

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$

Phase invariance...

Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point must equal reading ϕ' by Bob in spite of unequal readings (x,t) and (x',t') for that point and unequal readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\, t' = k_{phase}x - \omega_{phase}\, t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\, t' = k_{group}x - \omega_{group}\, t \equiv \phi_{group}$$

...derives Lorentz transformations...

$$k_A x' \sinh\rho_{AB} - \omega_A\, t' \cosh\rho_{AB} = 0 \cdot x - \omega_A\, t \qquad \text{or:} \qquad ct = ct'\cosh\rho_{AB} - x'\sinh\rho_{AB}$$

$$k_A x' \cosh\rho_{AB} - \omega_A t' \sinh\rho_{AB} = k_A x - 0 \cdot t$$

since: $\sinh\rho_{AB}=0$ if $\rho_{AB}=0$

$$k'_{phase} = k_A \sinh\rho_{AB} \qquad \omega'_{phase} = \omega_A \cosh\rho_{AB} \qquad k_{phase} = 0 \qquad \omega_{phase} = \omega_A$$

$$k'_{group} = k_A \cosh\rho_{AB} \qquad \omega'_{group} = \omega_A \sinh\rho_{AB} \qquad k_{group} = k_A \qquad \omega_{group} = 0$$

Angular 2π-factors

$k_A = 2\pi\kappa_A$
$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$
$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$
$\omega_{group} = 2\pi\upsilon_{group}$

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$

Phase invariance...

Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point <u>must equal</u> reading ϕ' by Bob in spite of <u>unequal</u> readings (x,t) and (x',t') for that point and <u>unequal</u> readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\ t' = k_{phase}x - \omega_{phase}\ t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\ t' = k_{group}x - \omega_{group}\ t \equiv \phi_{group}$$

...derives Lorentz transformations...

$$k_A x' \sinh\rho_{AB} - \omega_A\ t' \cosh\rho_{AB} = 0 \cdot x - \omega_A\ t \qquad \text{or:} \qquad ct = ct' \cosh\rho_{AB} - x' \sinh\rho_{AB}$$

using: $\omega_A / k_A = c = \upsilon_A / \kappa_A$

$$k_A x' \cosh\rho_{AB} - \omega_A t' \sinh\rho_{AB} = k_A x - 0 \cdot t \qquad \text{or:} \qquad x = x' \cosh\rho_{AB} - ct' \sinh\rho_{AB}$$

since: $\sinh\rho_{AB} = 0$ if $\rho_{AB} = 0$

$$k'_{phase} = k_A \sinh\rho_{AB} \qquad \omega'_{phase} = \omega_A \cosh\rho_{AB} \qquad k_{phase} = 0 \qquad \omega_{phase} = \omega_A$$

$$k'_{group} = k_A \cosh\rho_{AB} \qquad \omega'_{group} = \omega_A \sinh\rho_{AB} \qquad k_{group} = k_A \qquad \omega_{group} = 0$$

Angular 2π-factors

$k_A = 2\pi\kappa_A$
$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$
$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$
$\omega_{group} = 2\pi\upsilon_{group}$

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$

Phase invariance...

Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point must equal reading ϕ' by Bob in spite of unequal readings (x,t) and (x',t') for that point and unequal readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\, t' = k_{phase}x - \omega_{phase}\, t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\, t' = k_{group}x - \omega_{group}\, t \equiv \phi_{group}$$

$$\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

...derives Lorentz transformations...

$$k_A x'\sinh\rho_{AB} - \omega_A\, t'\cosh\rho_{AB} = 0\cdot x - \omega_A\, t \qquad \text{or:} \qquad ct = ct'\cosh\rho_{AB} - x'\sinh\rho_{AB}$$

using: $\omega_A / k_A = c = \upsilon_A / \kappa_A$

$$k_A x'\cosh\rho_{AB} - \omega_A t'\sinh\rho_{AB} = k_A x - 0\cdot t \qquad \text{or:} \qquad x = x'\cosh\rho_{AB} - ct'\sinh\rho_{AB}$$

since: $\sinh\rho_{AB}=0$ if $\rho_{AB}=0$

$$k'_{phase} = k_A \sinh\rho_{AB} \qquad \omega'_{phase} = \omega_A \cosh\rho_{AB} \qquad k_{phase} = 0 \qquad \omega_{phase} = \omega_A$$

$$k'_{group} = k_A \cosh\rho_{AB} \qquad \omega'_{group} = \omega_A \sinh\rho_{AB} \qquad k_{group} = k_A \qquad \omega_{group} = 0$$

Angular 2π-factors

$k_A = 2\pi\kappa_A$

$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$

$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$

$\omega_{group} = 2\pi\upsilon_{group}$

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$

Phase invariance...

Each laser phasor sketched in Fig. 4 should be taken seriously as a gauge of time (clock) and of space (metric ruler) by giving time (wave period τ) and distance (wavelength λ).

A reading of a phase ϕ by Alice at a space-time point must equal reading ϕ' by Bob in spite of unequal readings (x,t) and (x',t') for that point and unequal readings (ω,ck) and (ω',ck') for either a laser group-wave or its phase-wave.

$$\phi'_{phase} \equiv k'_{phase}x' - \omega'_{phase}\ t' = k_{phase}x - \omega_{phase}\ t \equiv \phi_{phase}$$

$$\phi'_{group} \equiv k'_{group}x' - \omega'_{group}\ t' = k_{group}x - \omega_{group}\ t \equiv \phi_{group}$$

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & +\sinh\rho \\ +\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

$$\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

...derives Lorentz transformations...

$$k_A x' \sinh\rho_{AB} - \omega_A\ t' \cosh\rho_{AB} = 0\cdot x - \omega_A\ t \qquad \text{or:} \quad ct = ct'\cosh\rho_{AB} - x'\sinh\rho_{AB}$$

using: $\omega_A / k_A = c = \upsilon_A / \kappa_A$

$$k_A x' \cosh\rho_{AB} - \omega_A t' \sinh\rho_{AB} = k_A x - 0\cdot t \qquad \text{or:} \quad x = x'\cosh\rho_{AB} - ct'\sinh\rho_{AB}$$

since: $\sinh\rho_{AB}=0$ if $\rho_{AB}=0$

$$k'_{phase} = k_A \sinh\rho_{AB} \quad \omega'_{phase} = \omega_A \cosh\rho_{AB} \quad k_{phase} = 0 \quad \omega_{phase} = \omega_A$$

$$k'_{group} = k_A \cosh\rho_{AB} \quad \omega'_{group} = \omega_A \sinh\rho_{AB} \quad k_{group} = k_A \quad \omega_{group} = 0$$

Angular 2π-factors

$k_A = 2\pi\kappa_A$
$\omega_A = 2\pi\upsilon_A$

$k_{phase} = 2\pi\kappa_{phase}$
$\omega_{phase} = 2\pi\upsilon_{phase}$

$k_{group} = 2\pi\kappa_{group}$
$\omega_{group} = 2\pi\upsilon_{group}$

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$

Review of geometric construction , per-space-time (ω,*ck*) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

➔ Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

Another view of phasor-invariance

1CW Laser-phasor wave function

$$\psi = A\cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

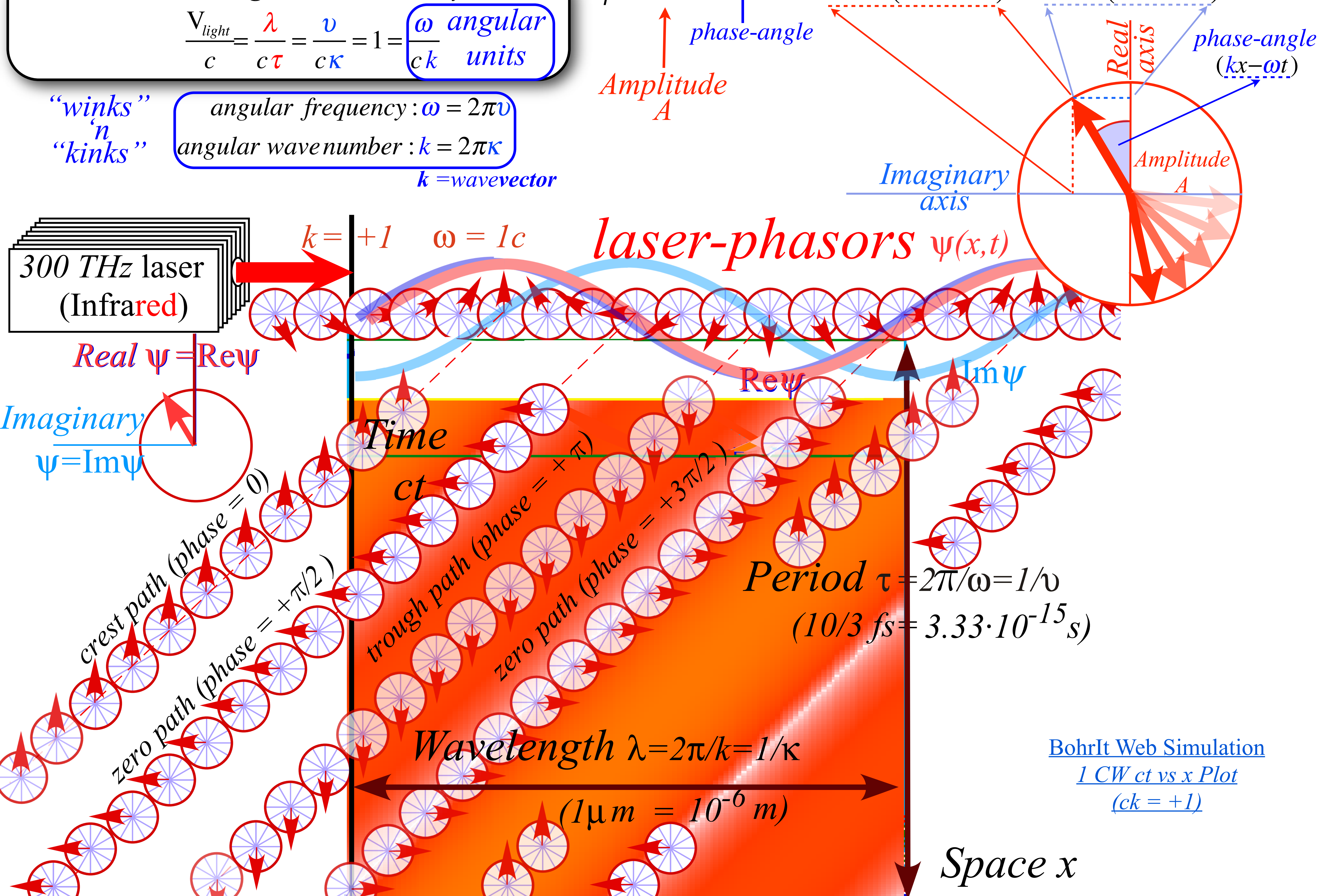
Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck}$$ *angular units*

"winks" 'n "kinks"

$$\text{angular frequency}: \omega = 2\pi\upsilon$$
$$\text{angular wavenumber}: k = 2\pi\kappa$$

k =wave**vector**



BohrIt Web Simulation
1 CW ct vs x Plot
(ck = +1)

Another view of phasor-invariance

1CW Laser-phasor wave function

Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck}$$ angular units

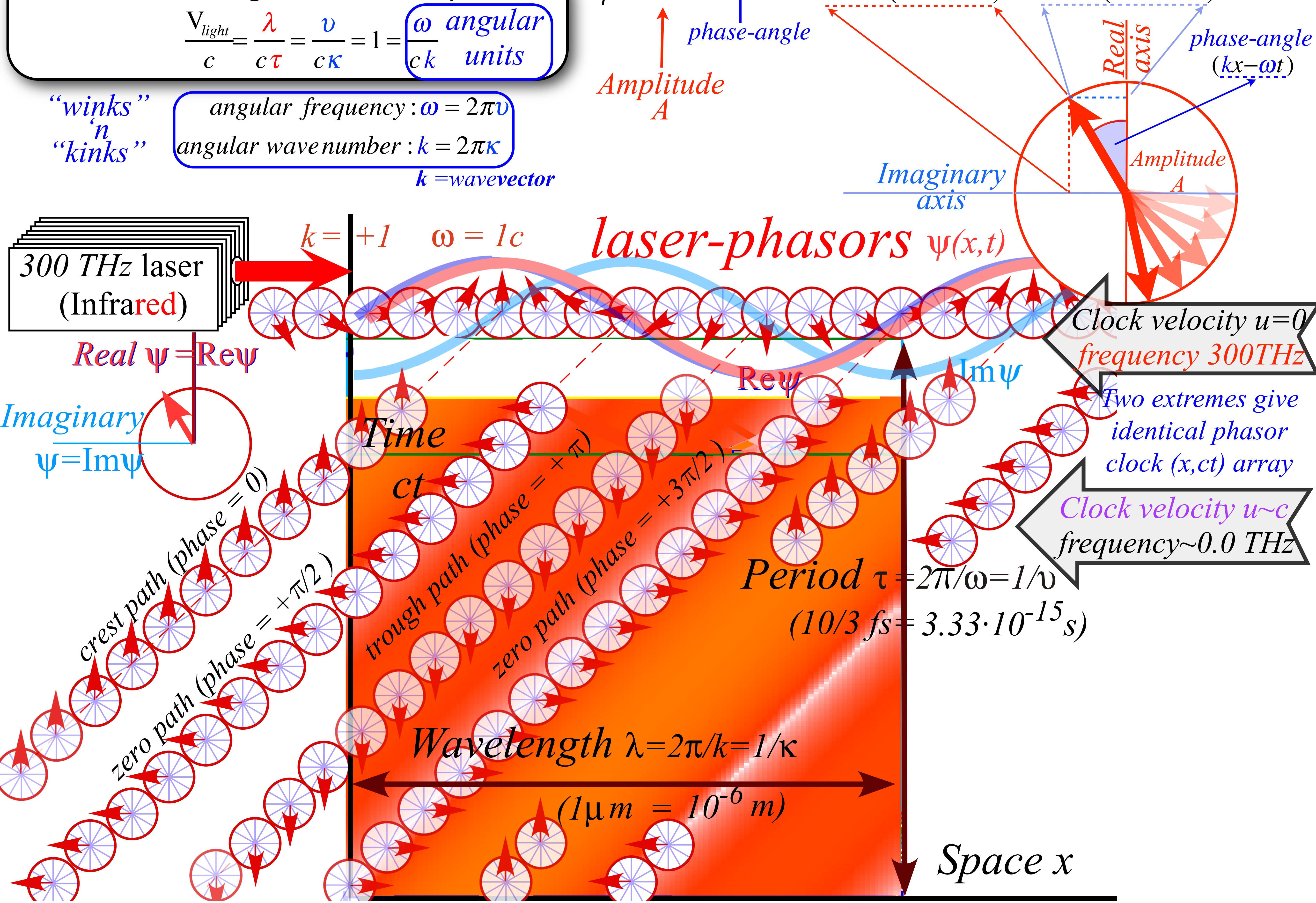
"winks" 'n "kinks"

angular frequency : $\omega = 2\pi\upsilon$

angular wavenumber : $k = 2\pi\kappa$

$\mathbf{k}$ =wave**vector**

$$\psi = A \cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$



Another view of phasor-invariance

1CW Laser-phasor wave function

Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c}=\frac{\lambda}{c\tau}=\frac{\upsilon}{c\kappa}=1=\frac{\omega}{ck}\ \textit{angular units}$$

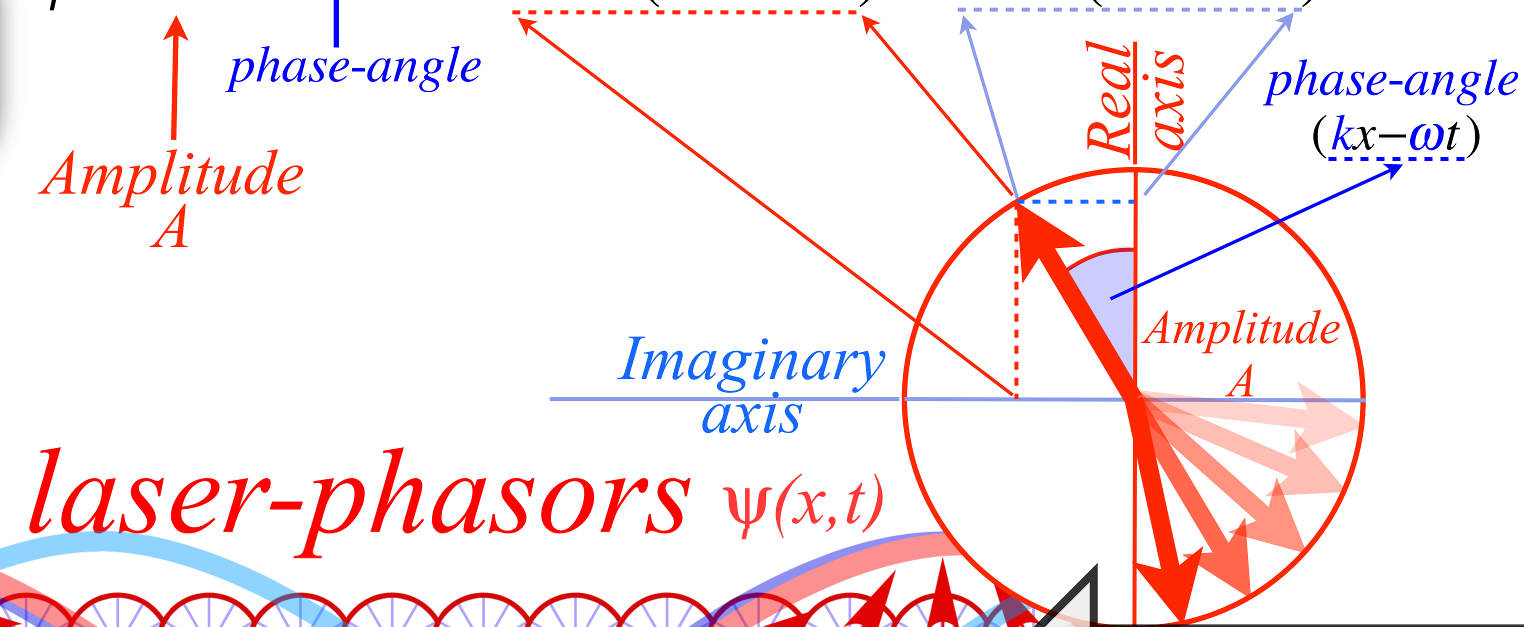
"winks" 'n "kinks"

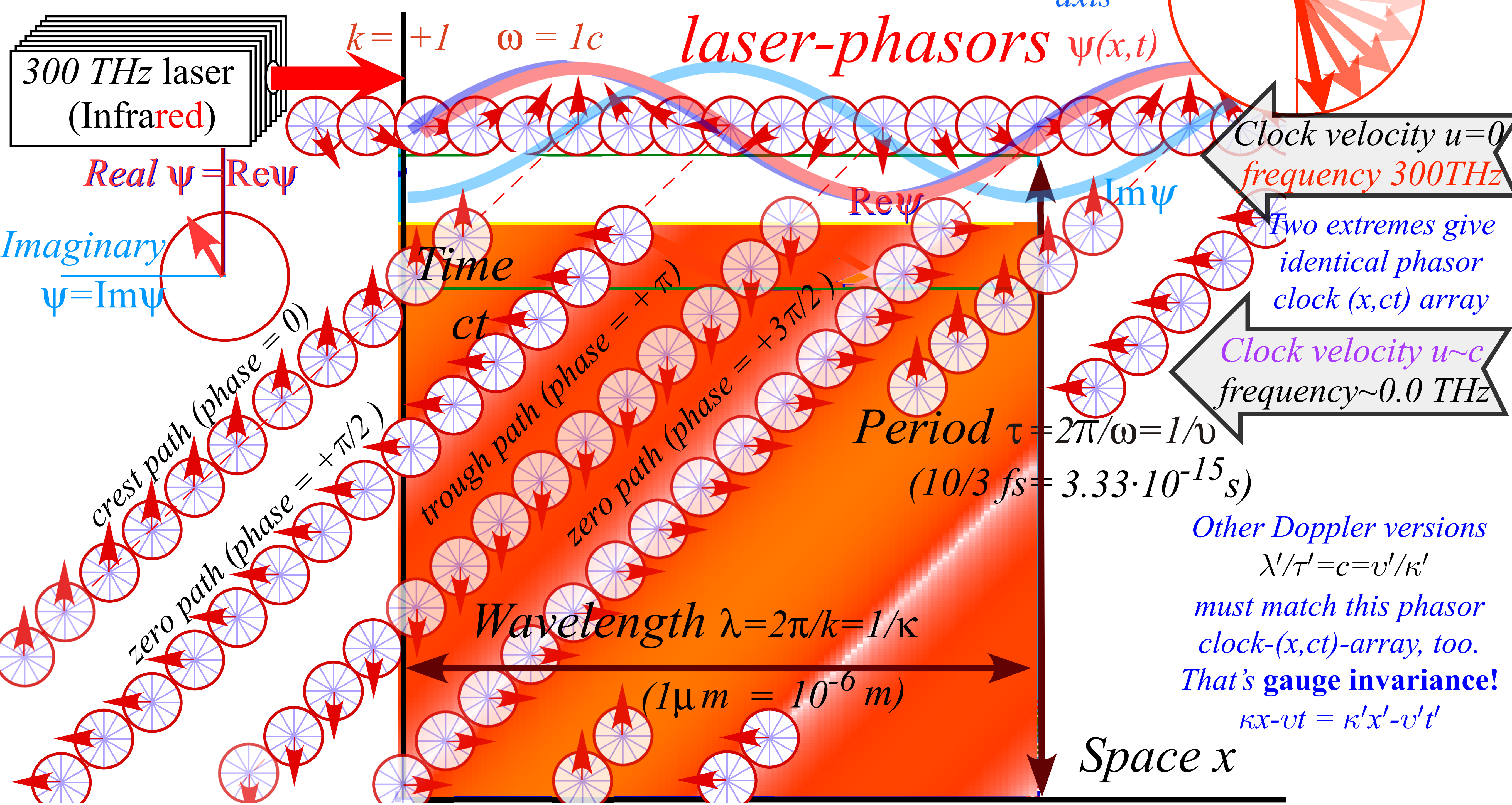
angular frequency : $\omega = 2\pi\upsilon$

angular wavenumber : $k = 2\pi\kappa$

$\boldsymbol{k}$ =wave**vector**

$$\psi = A\cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$





Review of geometric construction , per-space-time (ω,*ck*) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

➔ Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

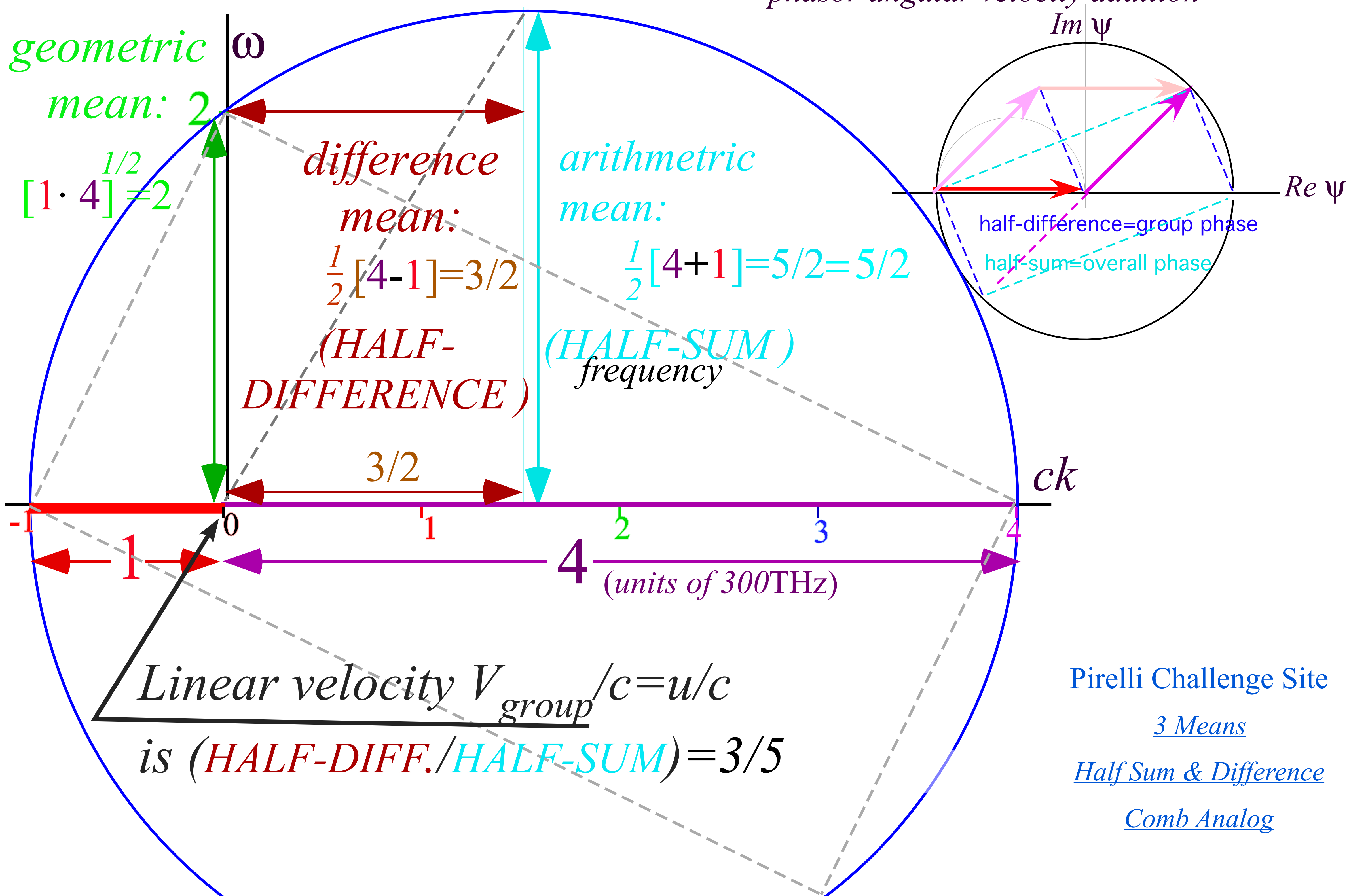
Geometry of invariant hyperbolas

Euclid's 3-means (300 BC)

Geometric "heart" of wave mechanics

Thales (580BC) rectangle-in-circle

Relates to wave interference by (Galilean) phasor angular velocity addition



Pirelli Challenge Site

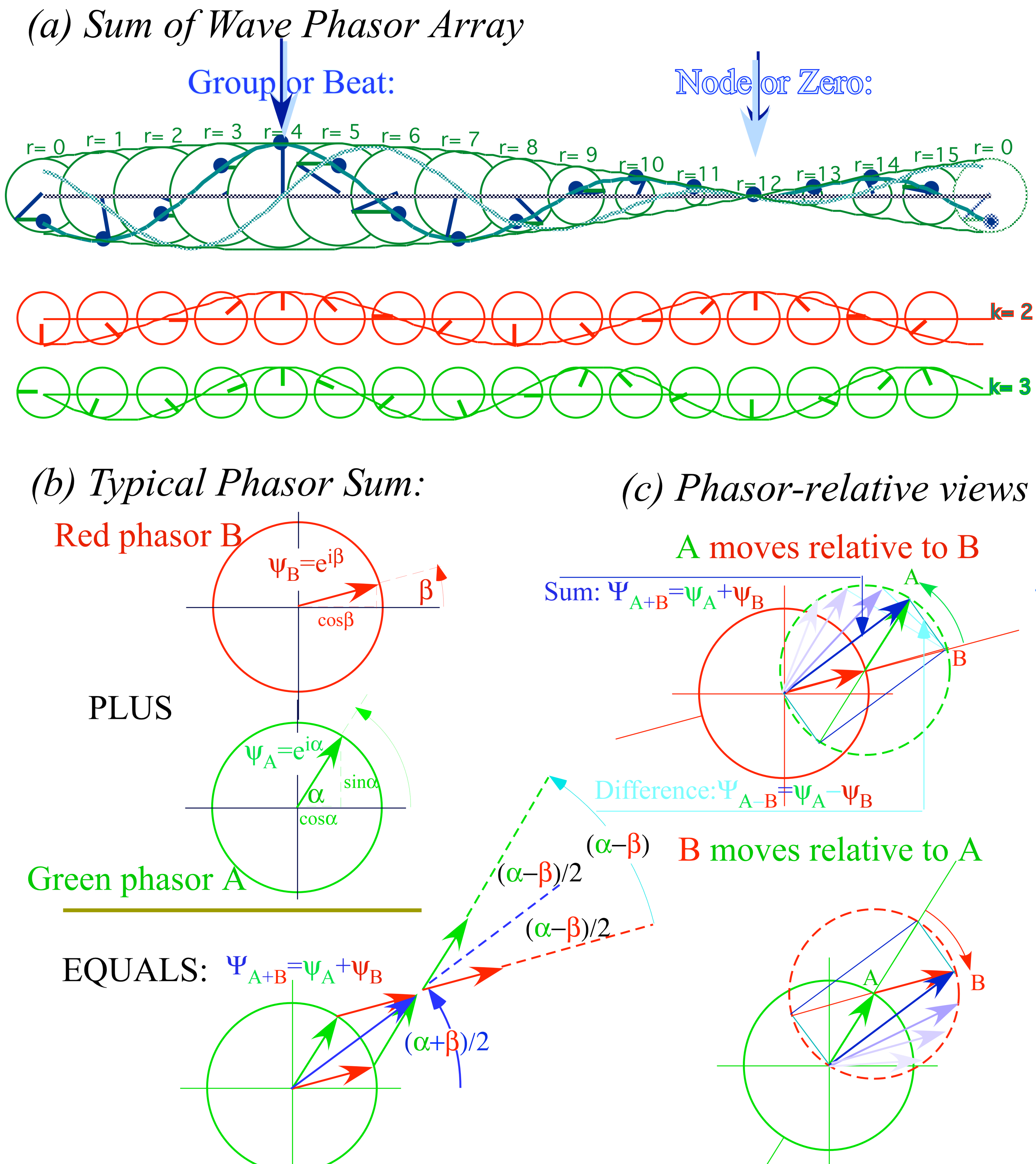
[3 Means]

[Half Sum & Difference]

[Comb Analog]

Fig. 10a Euclidian mean geometry for counter-moving waves of frequency 1 and 4. (300THz units).

Geometry of invariant hyperbolas



Galileo's revenge!

Now we use Galilean relativity to add angular velocity, that is frequency ω_a and ω_b, in phase or "gauge" space. No "c-limit" evident. (So far at 18-fig. precision.)

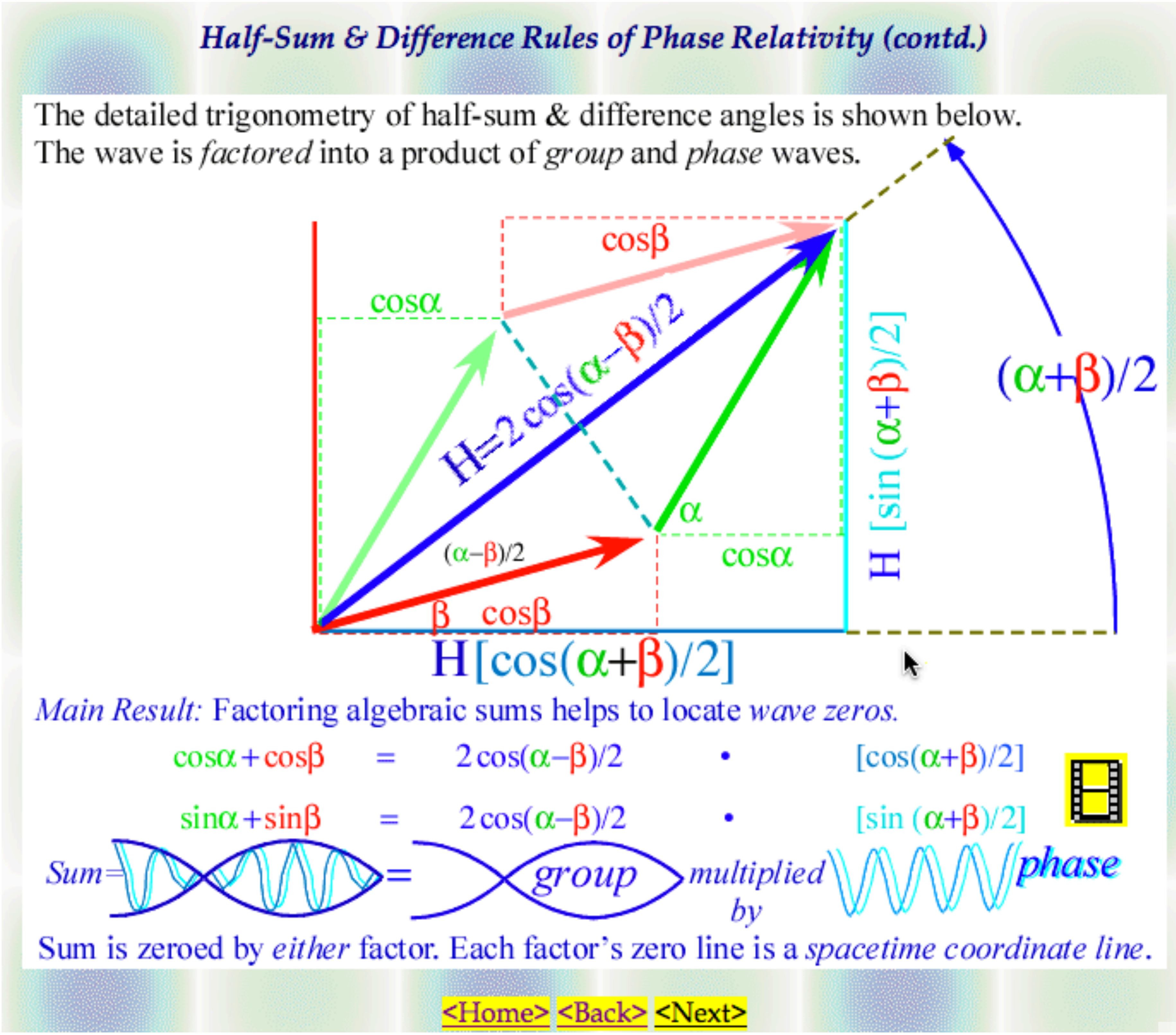
Pirelli Challenge Site

3 Means

Half Sum & Difference Rules

Wave phasor addition. (a) Each phasor in a wave array is a sum (b) of two component phasors.

Geometry of invariant hyperbolas



Pirelli Challenge Site

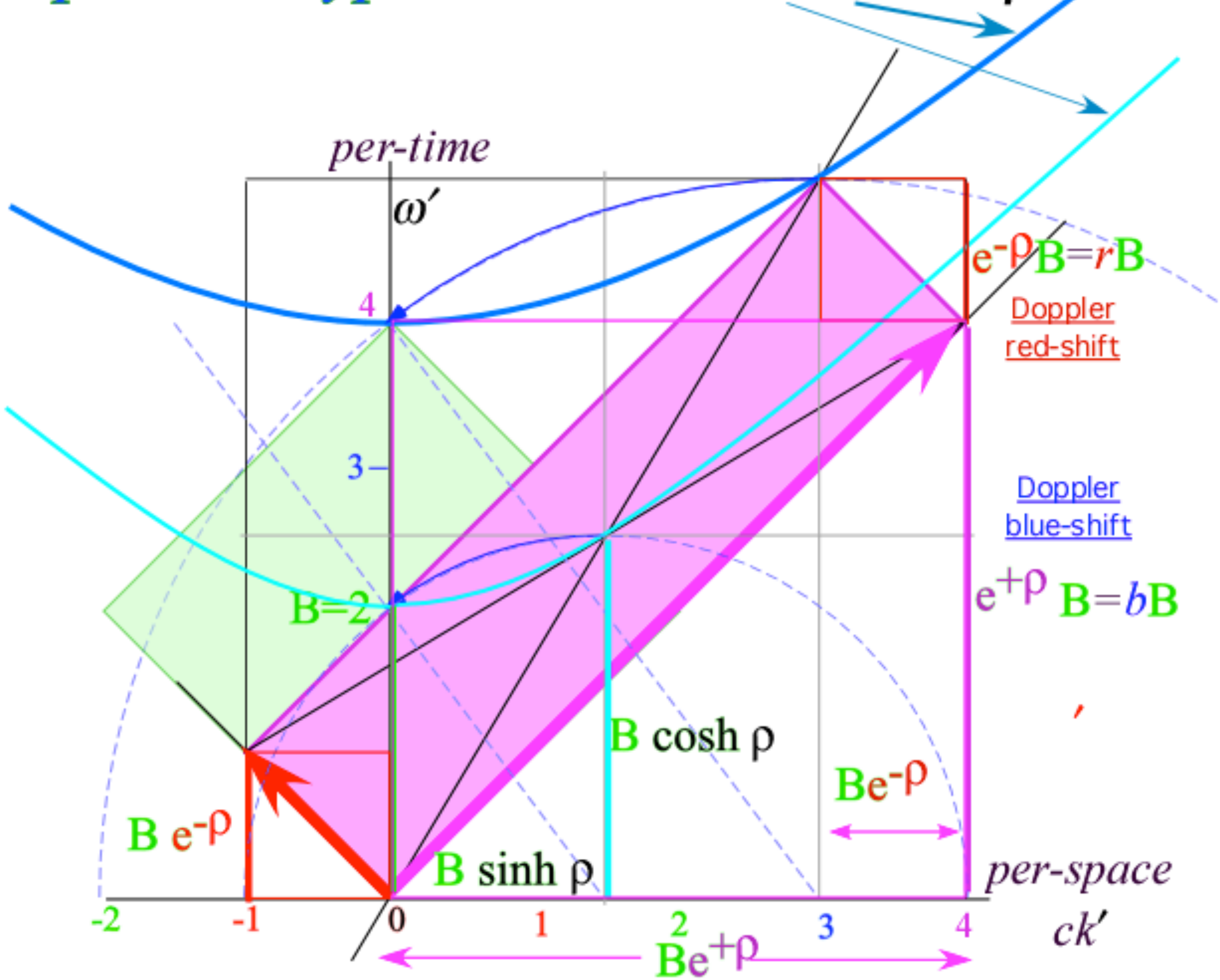
Half Sum & Difference Rules

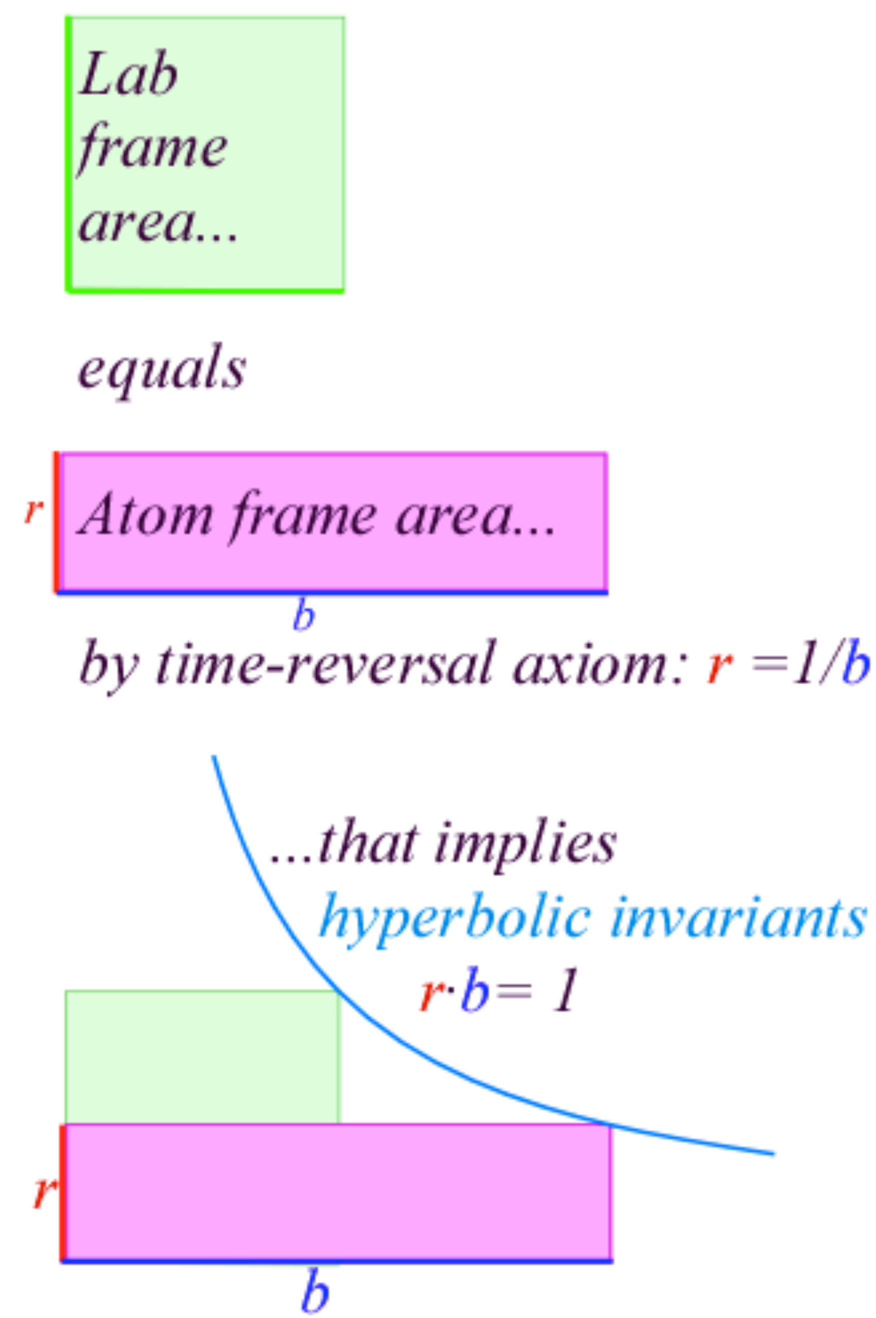
Play Animation

Geometry of invariant hyperbolas

Euclidian wave geometry with time-reversal symmetry imply

dispersion hyperbolas: $\omega = nB\cosh\rho$





$B\sinh\rho = (Be^{+\rho} - Be^{-\rho})/2$ $\qquad$ $B\cosh\rho = (Be^{+\rho} + Be^{-\rho})/2$

Time $r=1/b$ symmetry shows geometry of 2-CW grid transformation that leaves hyperbolas invariant.

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=3l5&minkGridPosCells=2

Geometry of invariant hyperbolas

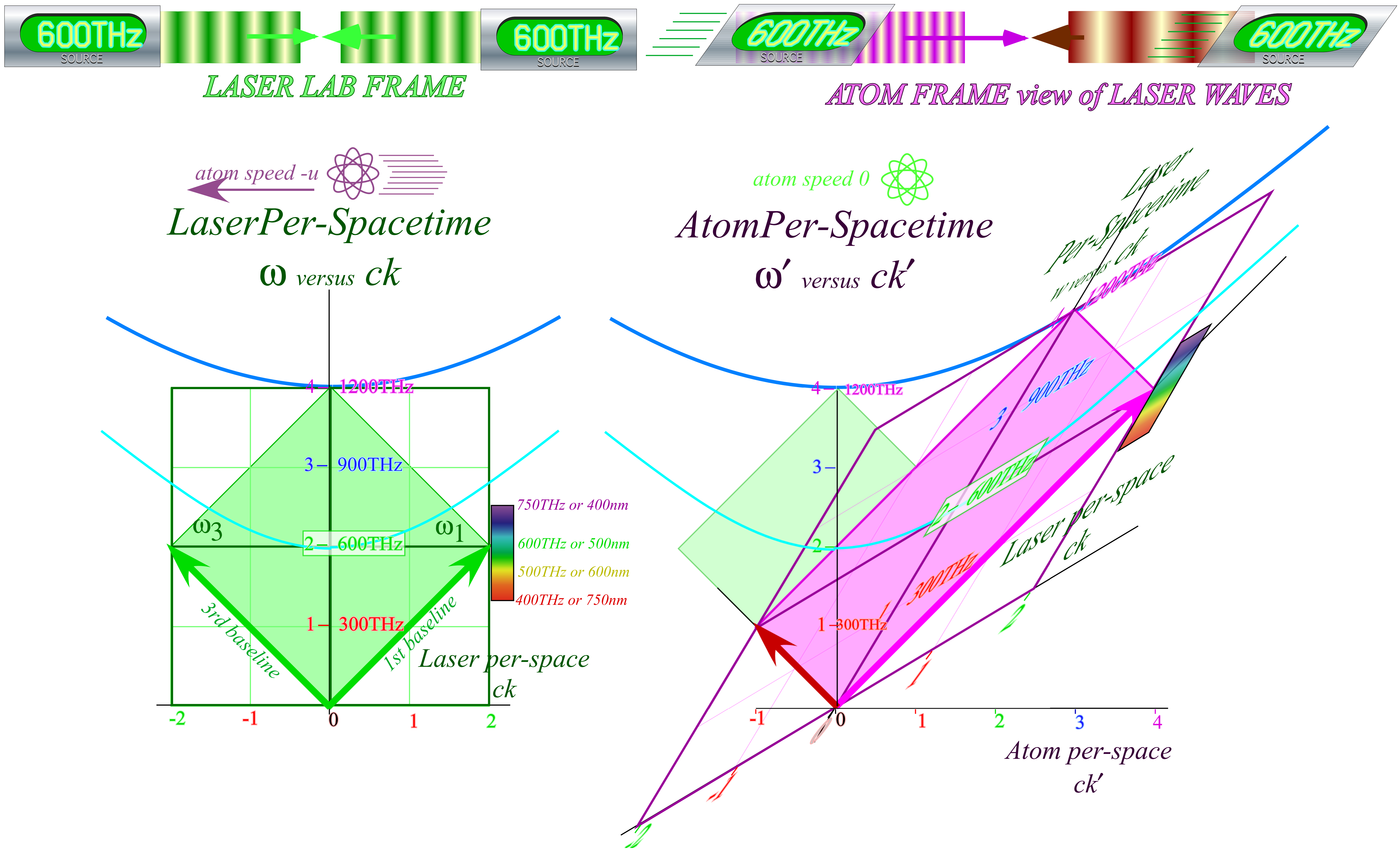


Fig.10b Laser (Alice-Carla) and Atom frame (Bob) views of 2-CW grid shows hyperbola invariance.

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=3l5&minkGridPosCells=2

Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

➔ Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

Algebra of invariant hyperbolas: Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 *is called "proper frequency" or rate of "aging"*

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 *is called "proper time" or "age":*

Algebra of invariant hyperbolas: Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1-\frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1-\frac{k'^2}{(c\omega')^2}}$$

$$= \omega\sqrt{1-\frac{u^2}{c^2}} = \omega'\sqrt{1-\frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1-\frac{x^2}{(ct)^2}} = t'\sqrt{1-\frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1-\frac{u^2}{c^2}} = t'\sqrt{1-\frac{u'^2}{c^2}}$$

Algebra of invariant hyperbolas: Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1 - \frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1 - \frac{k'^2}{(c\omega')^2}}$$

$$= \omega\sqrt{1 - \frac{u^2}{c^2}} = \omega'\sqrt{1 - \frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1 - \frac{x^2}{(ct)^2}} = t'\sqrt{1 - \frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1 - \frac{u^2}{c^2}} = t'\sqrt{1 - \frac{u'^2}{c^2}}$$

The "grand-daddy-of 'em all" invariant

Phase invariance:

$$\Phi_0 = k\cdot x - \omega\cdot t = k'\cdot x' - \omega'\cdot t'$$

Proof: ?

Algebra of invariant hyperbolas: Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1 - \frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1 - \frac{k'^2}{(c\omega')^2}}$$

$$= \omega\sqrt{1 - \frac{u^2}{c^2}} = \omega'\sqrt{1 - \frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1 - \frac{x^2}{(ct)^2}} = t'\sqrt{1 - \frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1 - \frac{u^2}{c^2}} = t'\sqrt{1 - \frac{u'^2}{c^2}}$$

The "grand-daddy-of 'em all" invariant

Phase invariance:

$$\Phi_0 = k\cdot x - \omega\cdot t = k'\cdot x' - \omega'\cdot t'$$

Proof: $ck' \cdot x' - \omega' \cdot ct'$

	$x\cdot\cosh$	$ct\cdot\sinh$		$x\cdot\sinh$	$ct\cdot\cosh$
$ck\cdot\cosh$	$ck\cdot x\cdot\cosh^2$	$ck\cdot ct\cdot\cosh\cdot\sinh$	$ck\cdot\sinh$	$ck\cdot x\cdot\sinh^2$	$ck\cdot ct\cdot\sinh\cdot\cosh$
$\omega\cdot\sinh$	$\omega\cdot x\cdot\sinh\cdot\cosh$	$\omega\cdot ct\cdot\sinh^2$	$\omega\cdot\cosh$	$\omega\cdot x\cdot\cosh\cdot\sinh$	$\omega\cdot ct\cdot\cosh^2$

Algebra of invariant hyperbolas: Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1 - \frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1 - \frac{k'^2}{(c\omega')^2}} = \omega\sqrt{1 - \frac{u^2}{c^2}} = \omega'\sqrt{1 - \frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1 - \frac{x^2}{(ct)^2}} = t'\sqrt{1 - \frac{x'^2}{(ct')^2}} = t\sqrt{1 - \frac{u^2}{c^2}} = t'\sqrt{1 - \frac{u'^2}{c^2}}$$

The "grand-daddy-of 'em all" invariant

Phase invariance:

$$\Phi_0 = k \cdot x - \omega \cdot t = k' \cdot x' - \omega' \cdot t'$$

Proof: $ck' \cdot x' \quad - \quad \omega' \cdot ct'$

	$x \cdot \cosh$	$ct \cdot \sinh$		$x \cdot \sinh$	$ct \cdot \cosh$
$ck \cdot \cosh$	$ck \cdot x \cdot \cosh^2$	~~$ck \cdot ct \cdot \cosh \cdot \sinh$~~	$ck \cdot \sinh$	$ck \cdot x \cdot \sinh^2$	~~$ck \cdot ct \cdot \sinh \cdot \cosh$~~
$\omega \cdot \sinh$	~~$\omega \cdot x \cdot \sinh \cdot \cosh$~~	$\omega \cdot ct \cdot \sinh^2$	$\omega \cdot \cosh$	~~$\omega \cdot x \cdot \cosh \cdot \sinh$~~	$\omega \cdot ct \cdot \cosh^2$

$$ck \cdot x \cdot \cosh^2 - ck \cdot x \cdot \sinh^2 = ck \cdot x$$

$$\omega \cdot ct \cdot \sinh^2 - \omega \cdot ct \cdot \cosh^2 = -\omega \cdot ct$$

Review of geometric construction , per-space-time (ω,*ck*) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector **P′** and Group vector **G′** in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

➡ A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

A politically incorrect analogy of rotation to Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

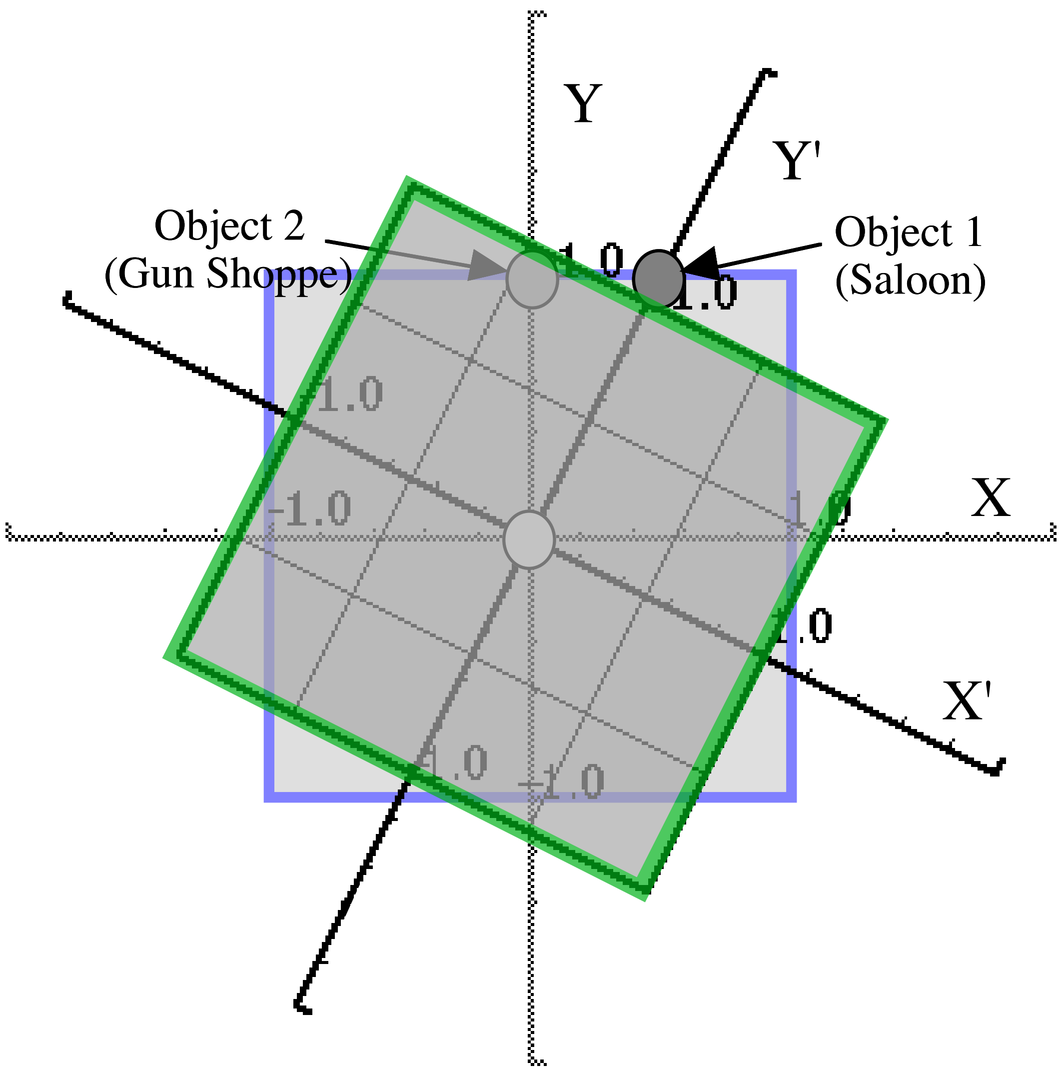
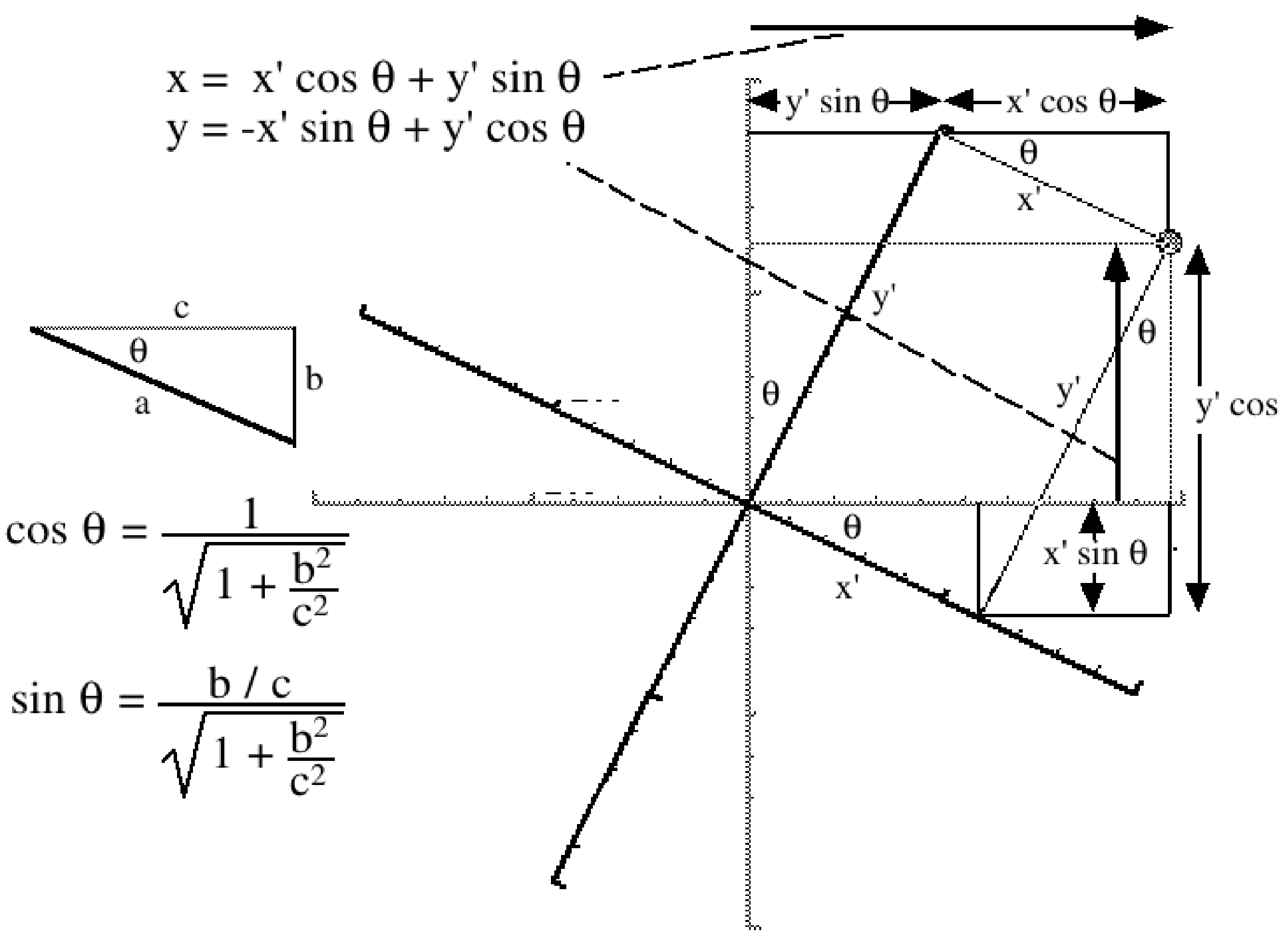


Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.



$$\cos\theta = \frac{1}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$\sin\theta = \frac{b/c}{\sqrt{1+\frac{b^2}{c^2}}}$$

Object 0: Town Square.	Object 1: Saloon.	Object 2: Gun Shoppe.
(US surveyor) $x = 0$, $y = 0$	$x = 0.5$, $y = 1.0$	$x = 0$, $y = 1.0$
(2nd surveyor) $x' = 0$, $y' = 0$	$x' = 0$, $y' = 1.1$	$x' = -0.45$, $y' = 0.89$

$$x' = x\cos\theta - y\sin\theta = \frac{x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{-(b/c)y}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$y' = x\sin\theta + y\cos\theta = \frac{(b/c)x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{y}{\sqrt{1+\frac{b^2}{c^2}}}$$

A politically incorrect analogy of rotation to Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

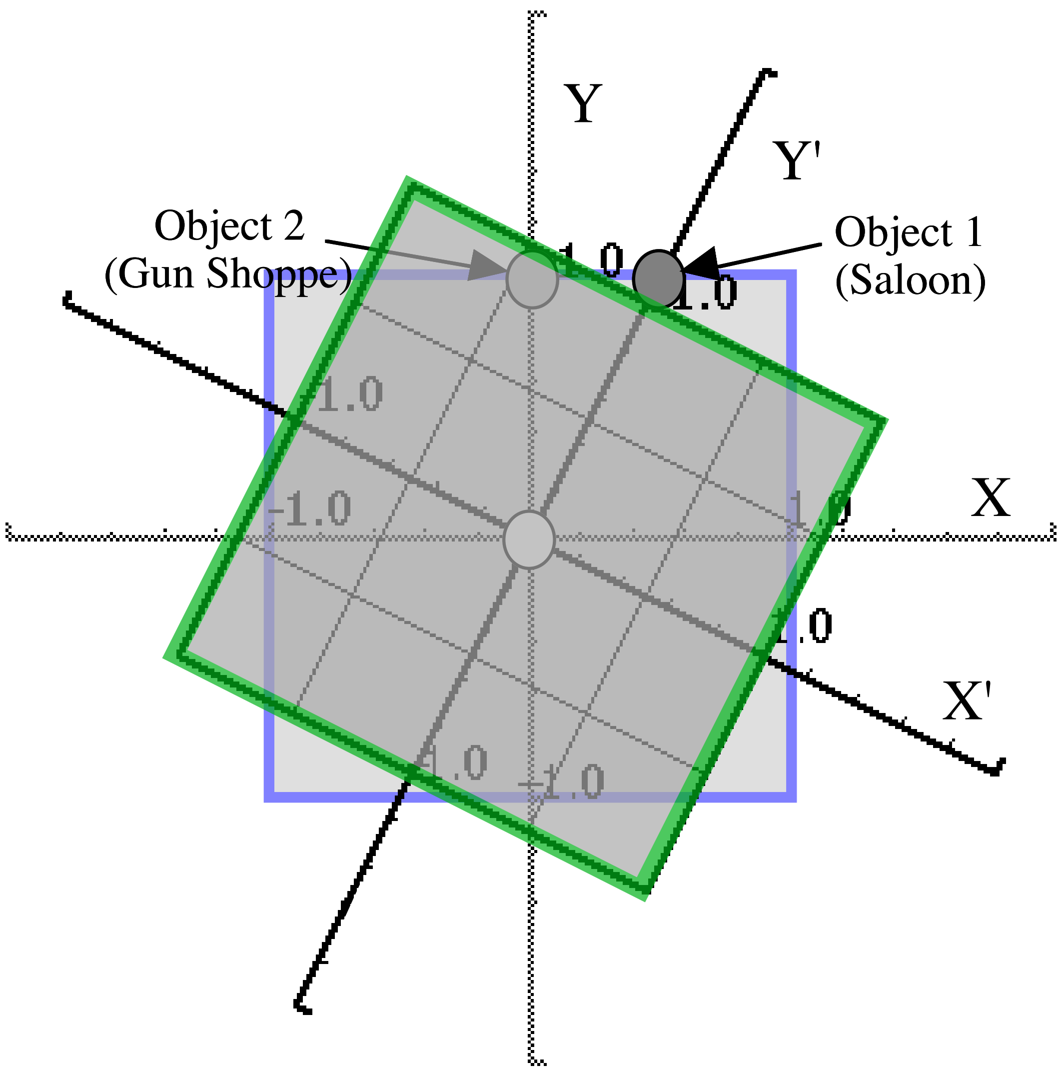
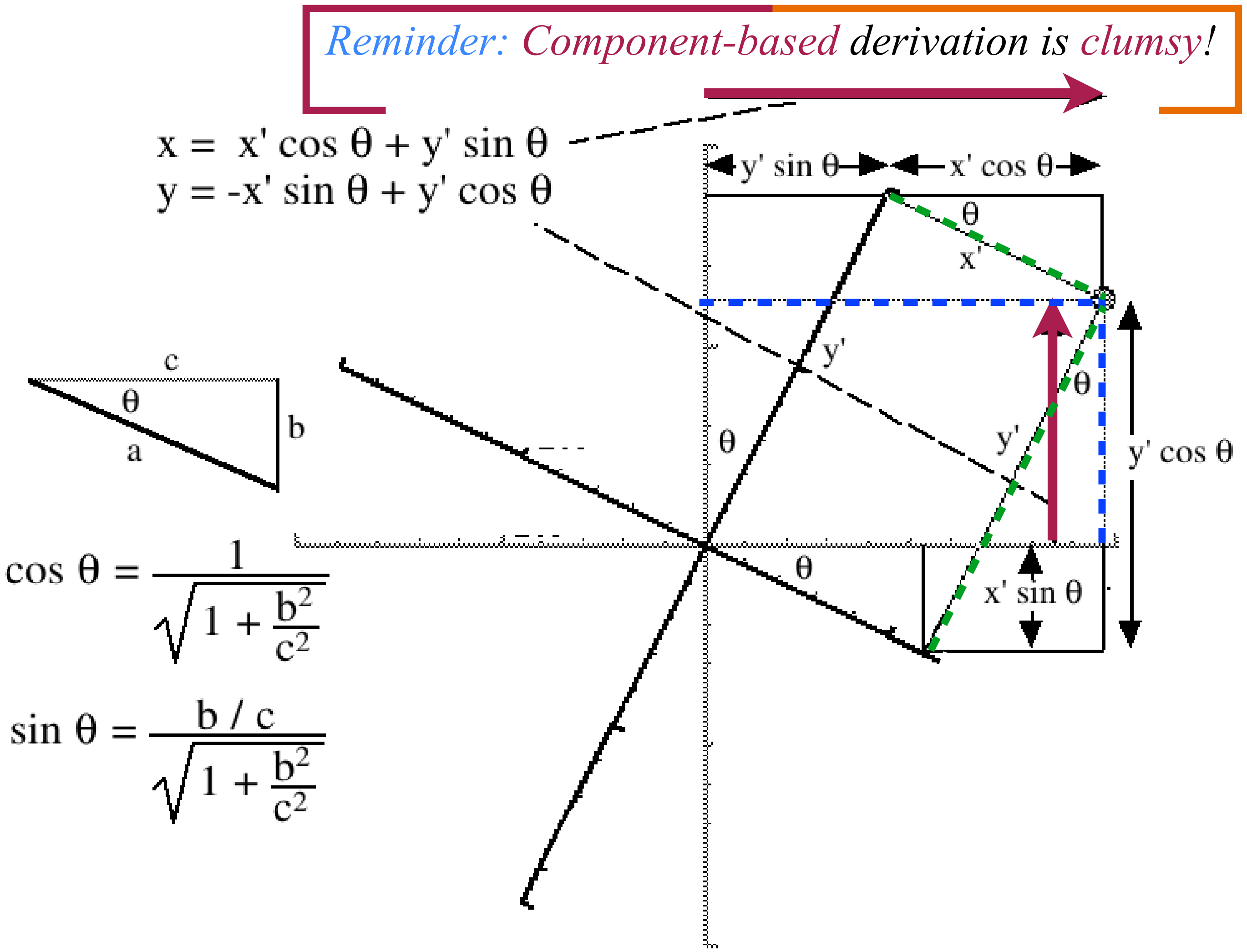


Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.



Object 0: Town Square.	Object 1: Saloon.	Object 2: Gun Shoppe.
(US surveyor) $x = 0$, $y = 0$	$x = 0.5$, $y = 1.0$	$x = 0$, $y = 1.0$
(2nd surveyor) $x' = 0$, $y' = 0$	$x' = 0$, $y' = 1.1$	$x' = -0.45$, $y' = 0.89$

A politically incorrect analogy of rotation to Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

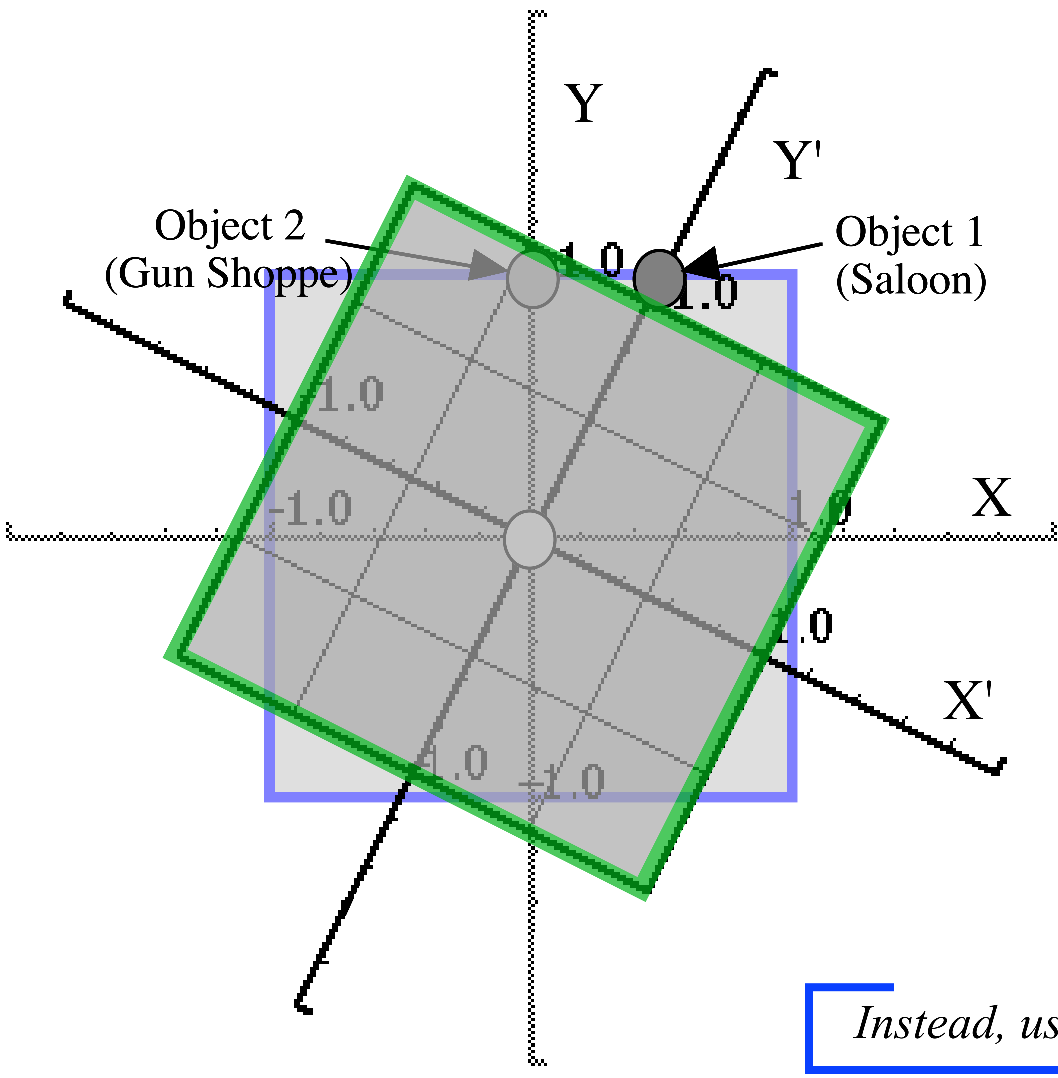
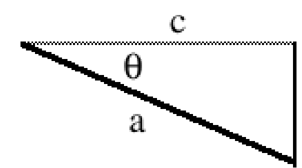


Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

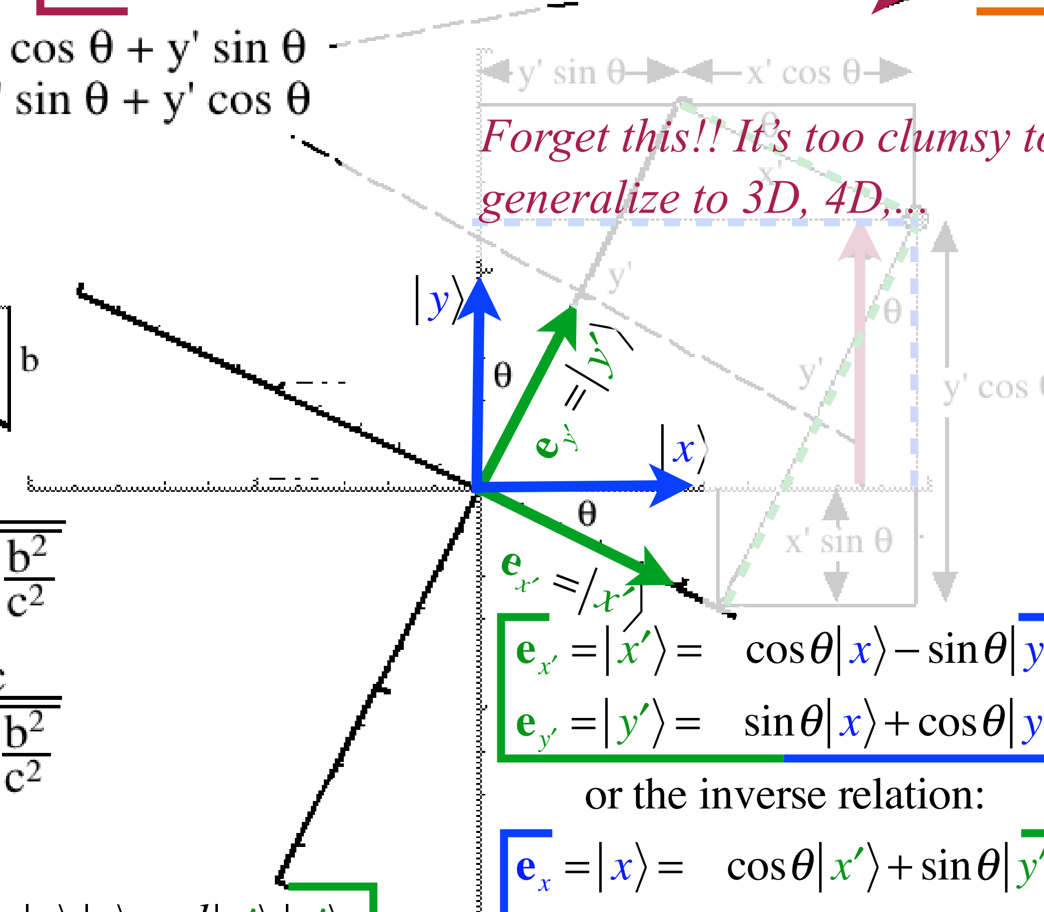
Reminder: Component-based derivation is *clumsy!*

x = x' cos θ + y' sin θ
y = -x' sin θ + y' cos θ



$$\cos\theta = \frac{1}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$\sin\theta = \frac{b/c}{\sqrt{1+\frac{b^2}{c^2}}}$$



$\mathbf{e}_{x'} = |x'\rangle = \cos\theta|x\rangle - \sin\theta|y\rangle$

$\mathbf{e}_{y'} = |y'\rangle = \sin\theta|x\rangle + \cos\theta|y\rangle$

or the inverse relation:

$\mathbf{e}_{x} = |x\rangle = \cos\theta|x'\rangle + \sin\theta|y'\rangle$

$\mathbf{e}_{y} = |y\rangle = -\sin\theta|x'\rangle + \cos\theta|y'\rangle$

Instead, use Dirac unit vectors $|x\rangle, |y\rangle$ *and* $|x'\rangle, |y'\rangle$

Object 0: Town Square.	Object 1: Saloon.	Object 2: Gun Shoppe.
(US surveyor) $x = 0$, $y = 0$	$x = 0.5$, $y = 1.0$	$x = 0$, $y = 1.0$
(2nd surveyor) $x' = 0$, $y' = 0$	$x' = 0$, $y' = 1.1$	$x' = -0.45$, $y' = 0.89$

A politically incorrect analogy of rotation to Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

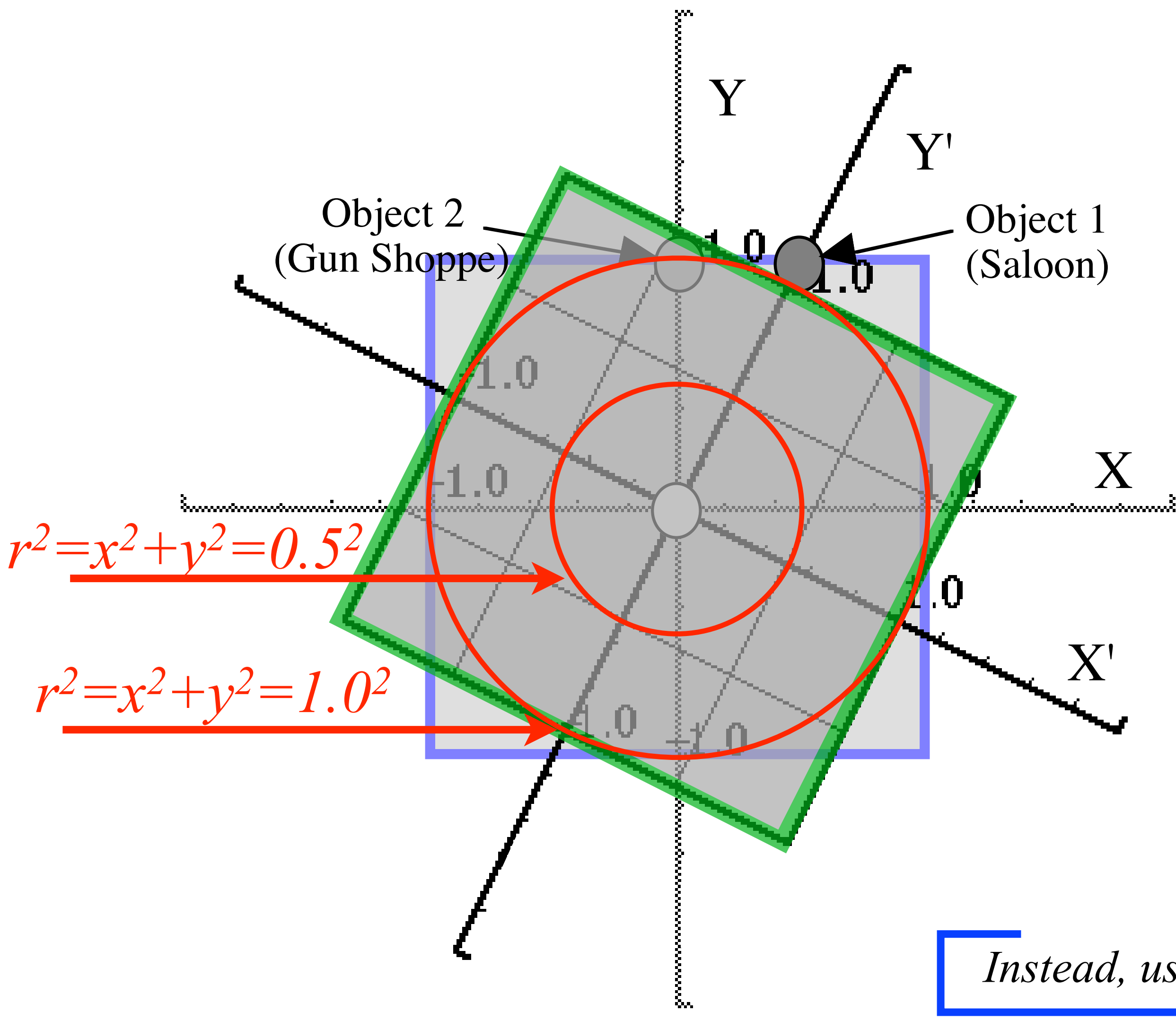
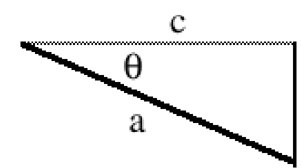


Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

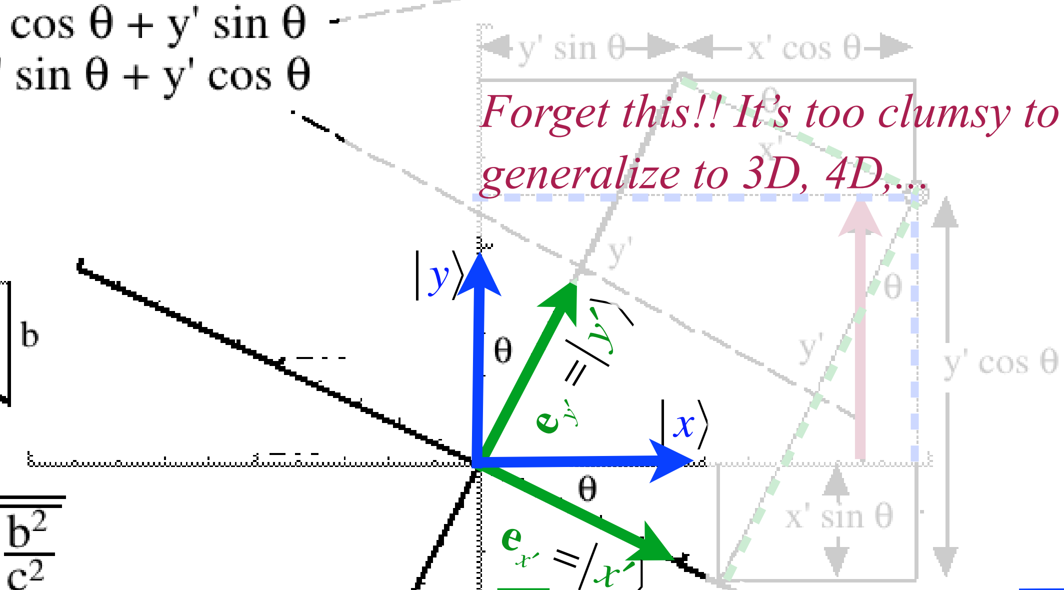
Reminder: Component-based derivation is clumsy!

$$x = x' \cos\theta + y' \sin\theta$$
$$y = -x' \sin\theta + y' \cos\theta$$



$$\cos\theta = \frac{1}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$\sin\theta = \frac{b/c}{\sqrt{1+\frac{b^2}{c^2}}}$$



$$\mathbf{e}_{x'}=|x'\rangle = \cos\theta|x\rangle - \sin\theta|y\rangle$$
$$\mathbf{e}_{y'}=|y'\rangle = \sin\theta|x\rangle + \cos\theta|y\rangle$$

or the inverse relation:

$$\mathbf{e}_{x}=|x\rangle = \cos\theta|x'\rangle + \sin\theta|y'\rangle$$
$$\mathbf{e}_{y}=|y\rangle = -\sin\theta|x'\rangle + \cos\theta|y'\rangle$$

Instead, use Dirac unit vectors $|x\rangle, |y\rangle$ and $|x'\rangle, |y'\rangle$

Object 0: Town Square.	Object 1: Saloon.	Object 2: Gun Shoppe.
(US surveyor) $x = 0$, $y = 0$	$x = 0.5$, $y = 1.0$	$x = 0$, $y = 1.0$
(2nd surveyor) $x' = 0$, $y' = 0$	$x' = 0$, $y' = 1.1$	$x' = -0.45$, $y' = 0.89$

You may apply (Jacobian) transform matrix:

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

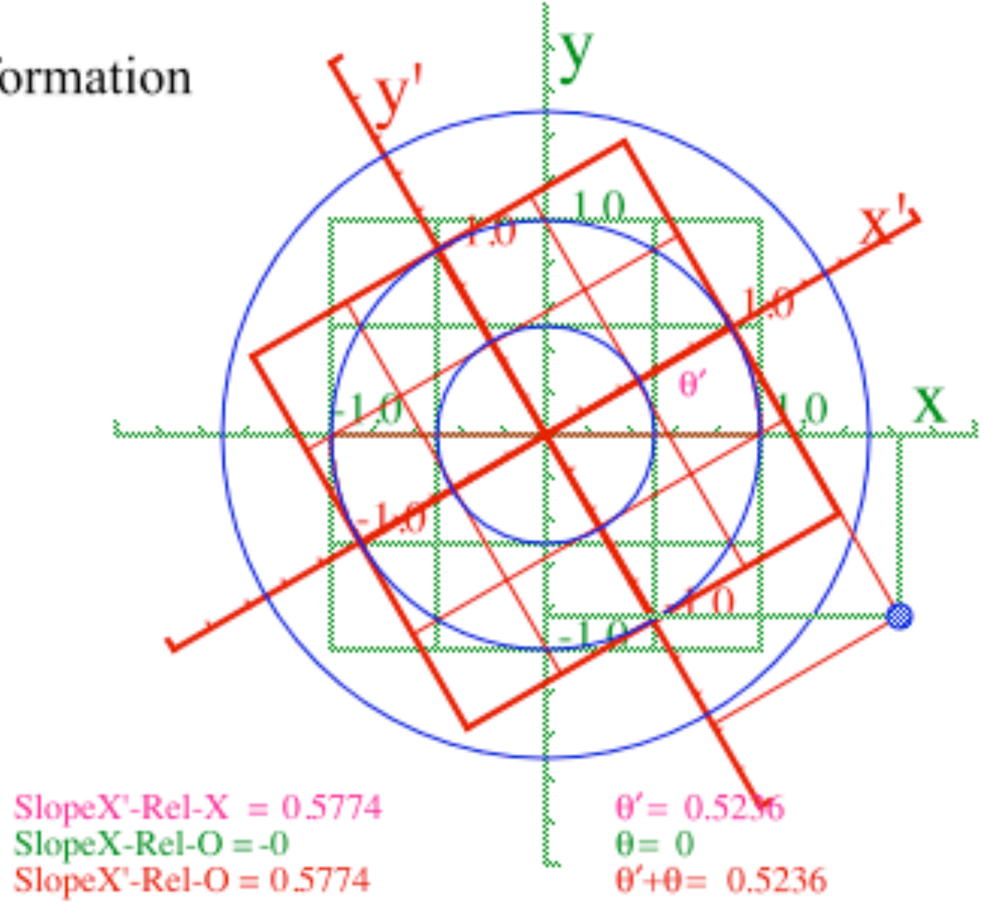
or the inverse (Kajobian) transformation:

$$\begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

to *any* vector $\mathbf{V}=|V\rangle = |x\rangle\langle x|V\rangle + |y\rangle\langle y|V\rangle$
$= |x'\rangle\langle x'|V\rangle + |y'\rangle\langle y'|V\rangle$

Circular invariants $r^2=x^2+y^2$

(a) Rotation Transformation
and Invariants



$x= 1.65$

$y= -0.85$

$x^2+y^2= 3.43$

$x'= 1.00$

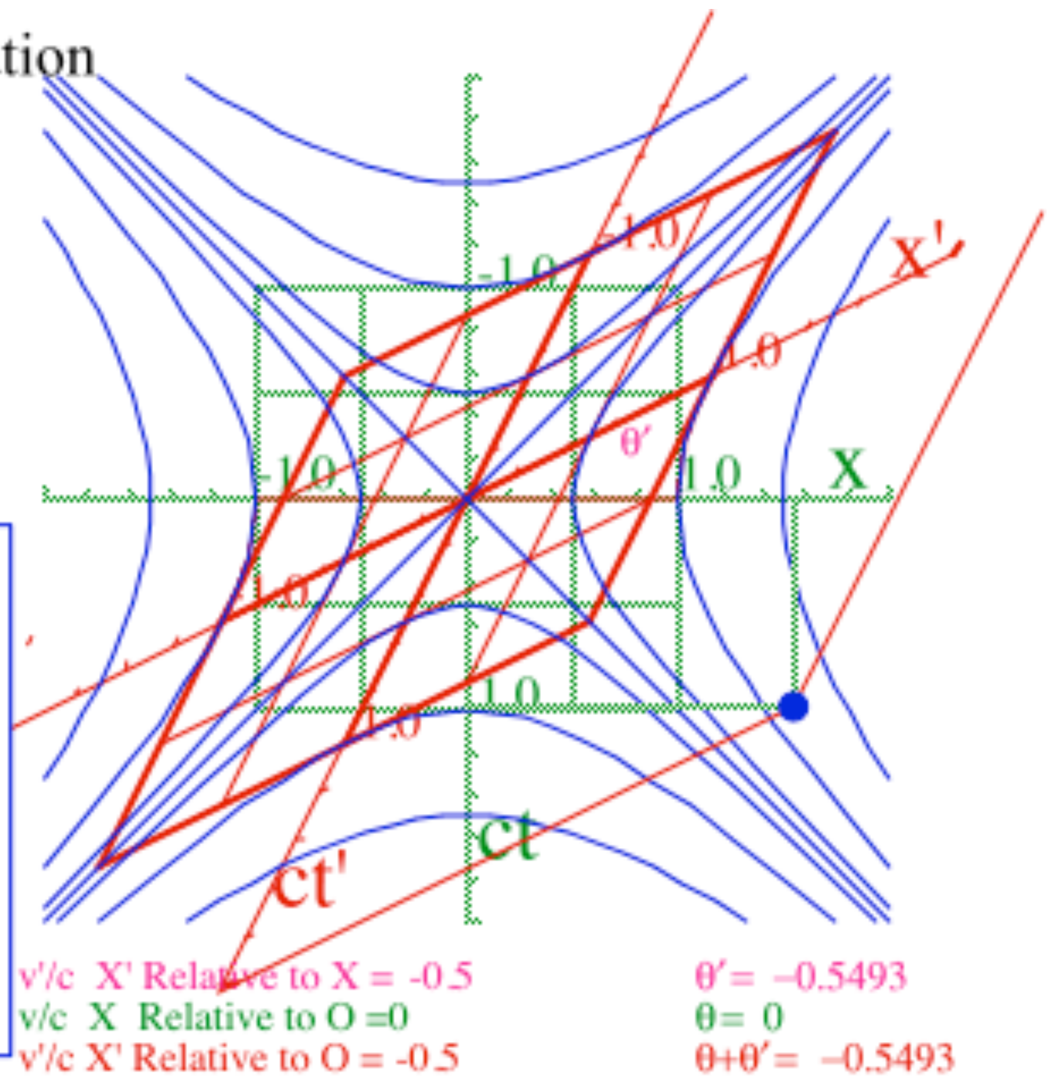
$y'= -1.56$

$x'^2+y'^2= 3.43$

$$x' = x\cos\theta - y\sin\theta = \frac{x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{-(b/c)y}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$y' = x\sin\theta + y\cos\theta = \frac{(b/c)x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{y}{\sqrt{1+\frac{b^2}{c^2}}}$$

(b) Lorentz Transformation
and Invariants



$x = 1.5453$

$ct = 0.9819$

$x^2-(ct)^2= 1.42$

$x' = 2.3512$

$ct' = 2.0260$

$x'^2-(ct')^2= 1.42$

$$x' = \frac{x}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{\frac{v}{c}ct}{\sqrt{1-\frac{v^2}{c^2}}} = x\cosh\rho + y\sinh\rho$$

$$ct' = \frac{\frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{ct}{\sqrt{1-\frac{v^2}{c^2}}} = x\sinh\rho + y\cosh\rho$$

Review of geometric construction , per-space-time (ω,*ck*) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector **P′** and Group vector **G′** in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

➔ Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

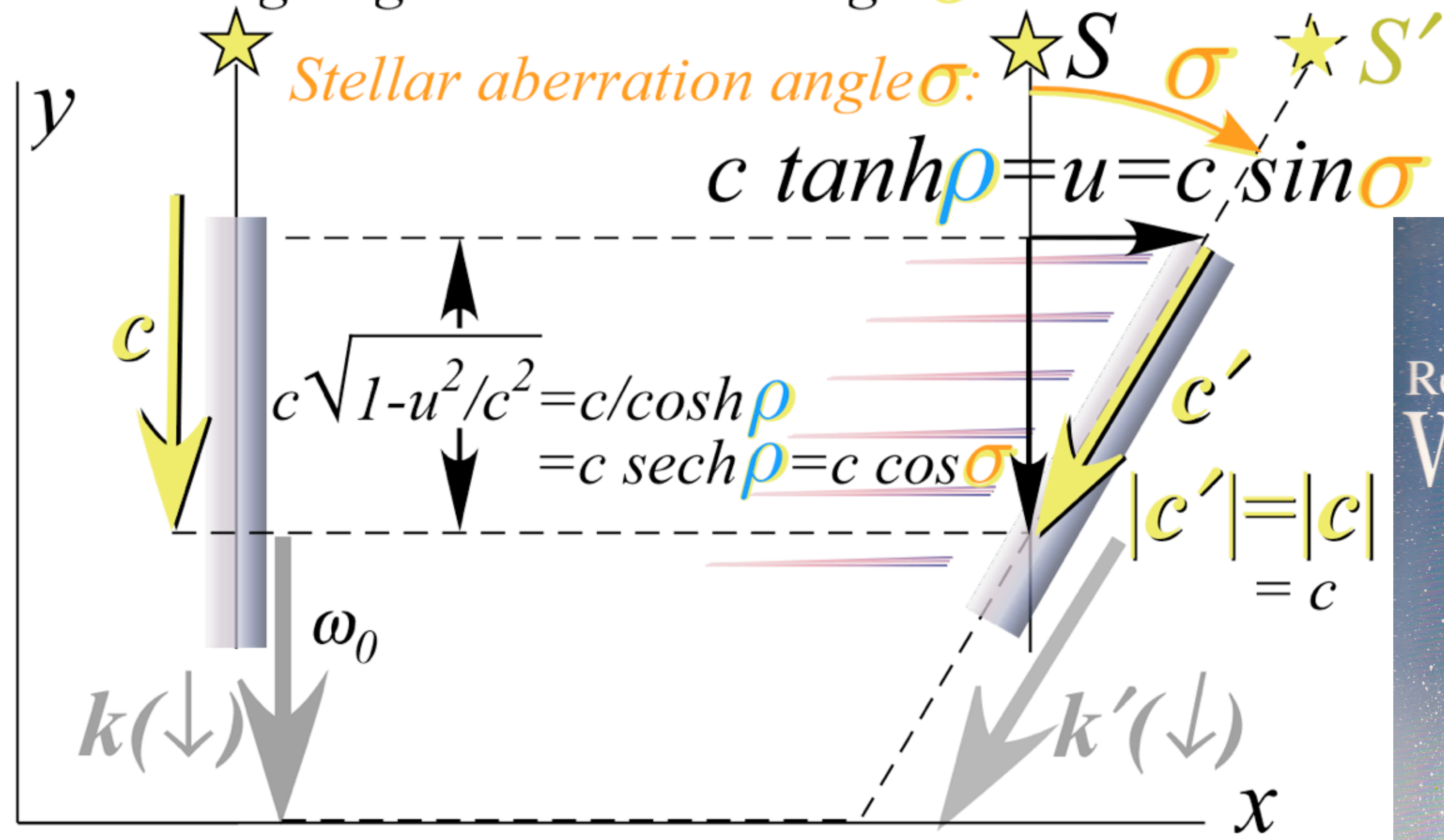
Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

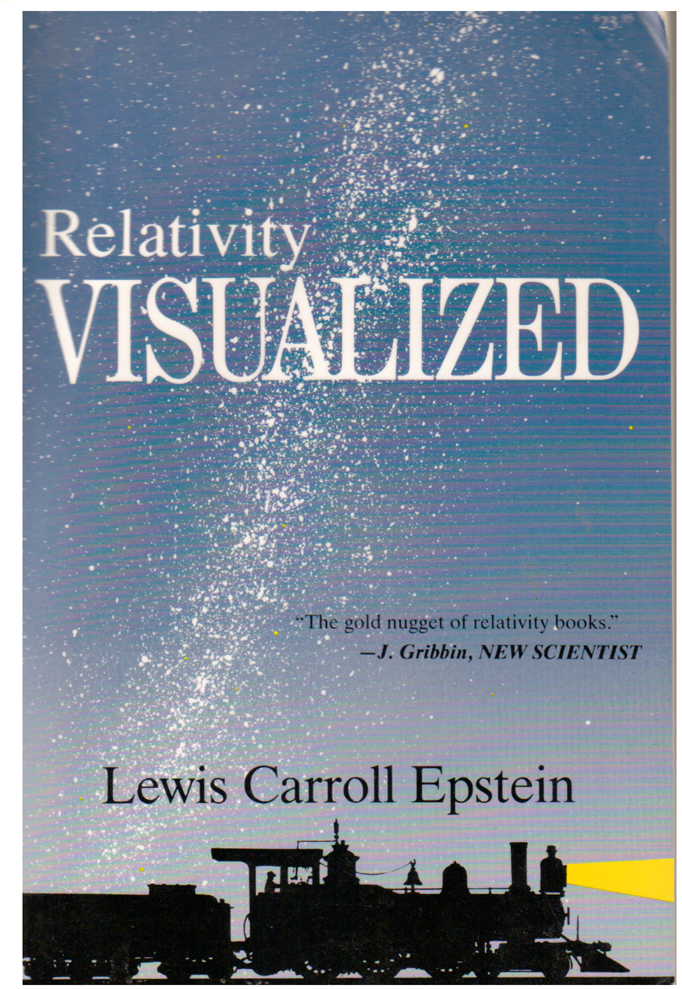
Relating Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)
to a Transverse relativity parameter*: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.
*Observer going **u** sees star at angle σ in **u** direction.*

We used notion σ for stellar-ab-angle, (a "flipped-out" ρ).





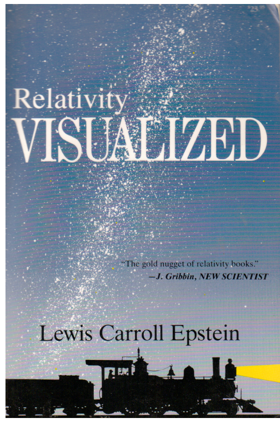
Epstein seemed uninterested in ρ analysis or in relation of σ and ρ.

AllBookstores.com
Purchase at: amazon

Relating Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

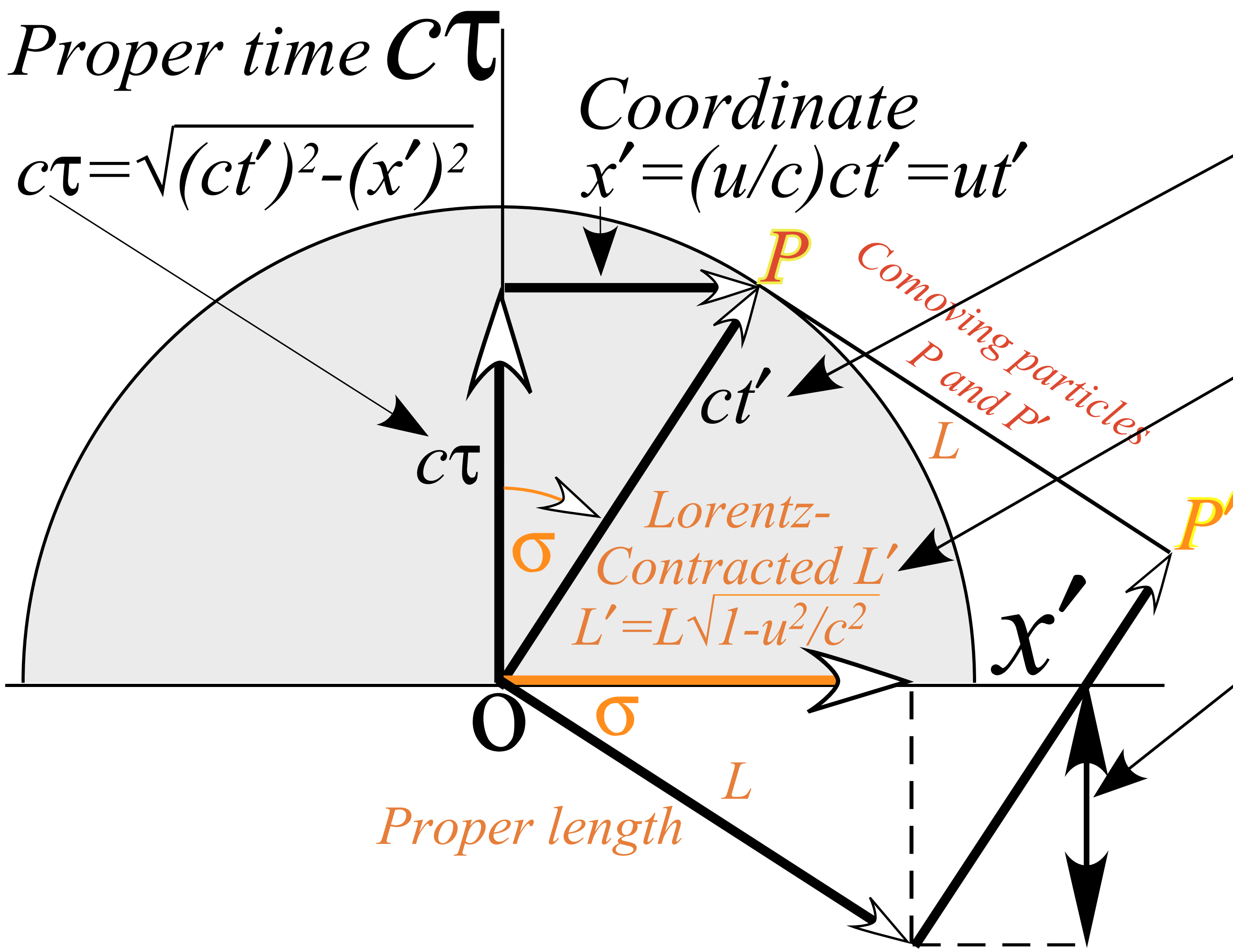
to Transverse relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

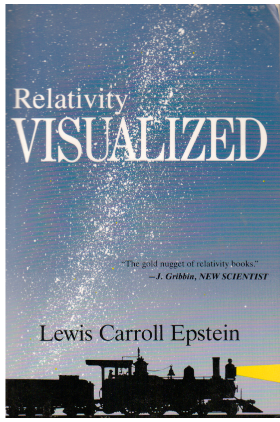
$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

"Relativity Visualized" can be purchased at:

Relating Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

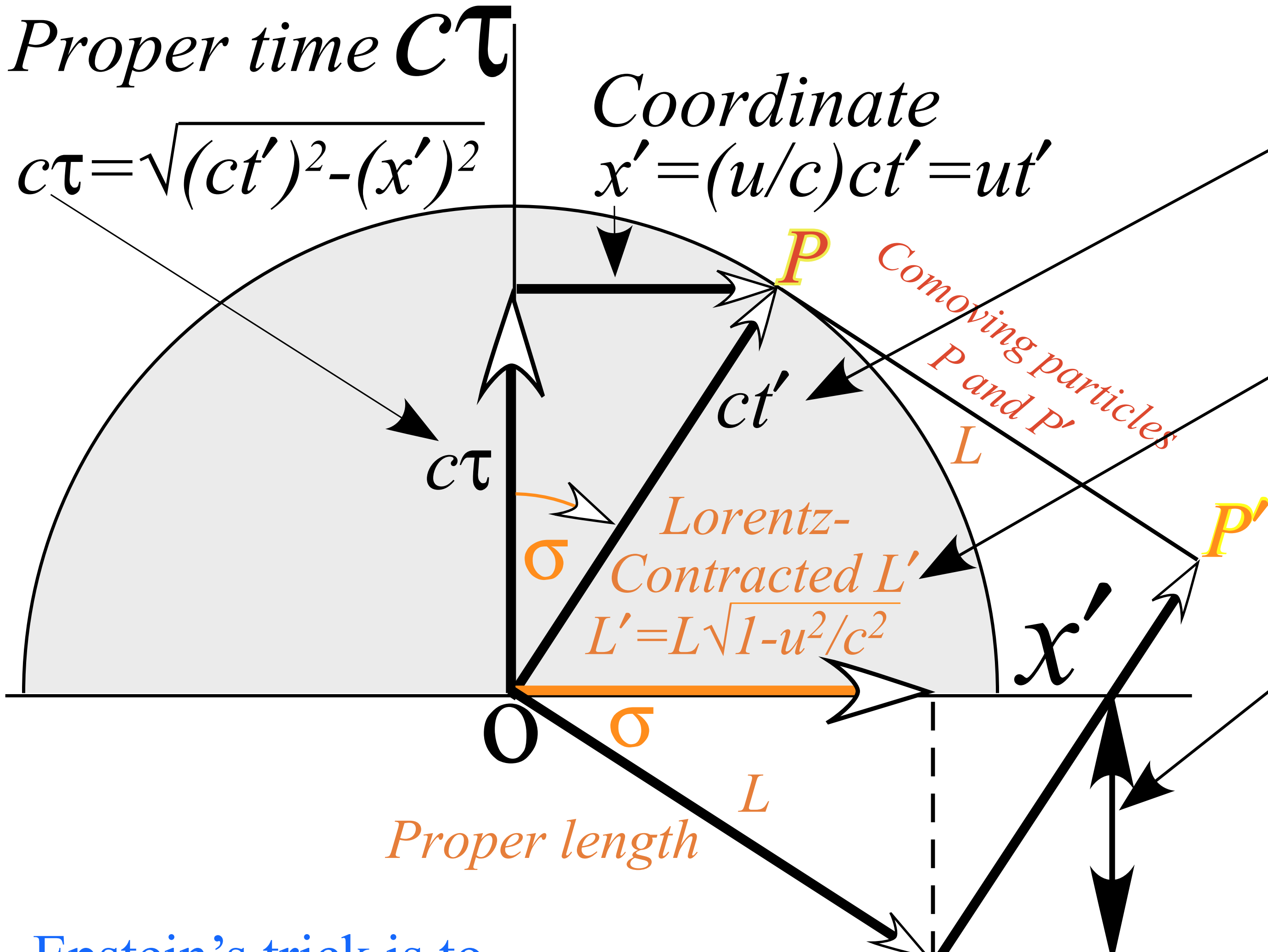
to Transverse relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

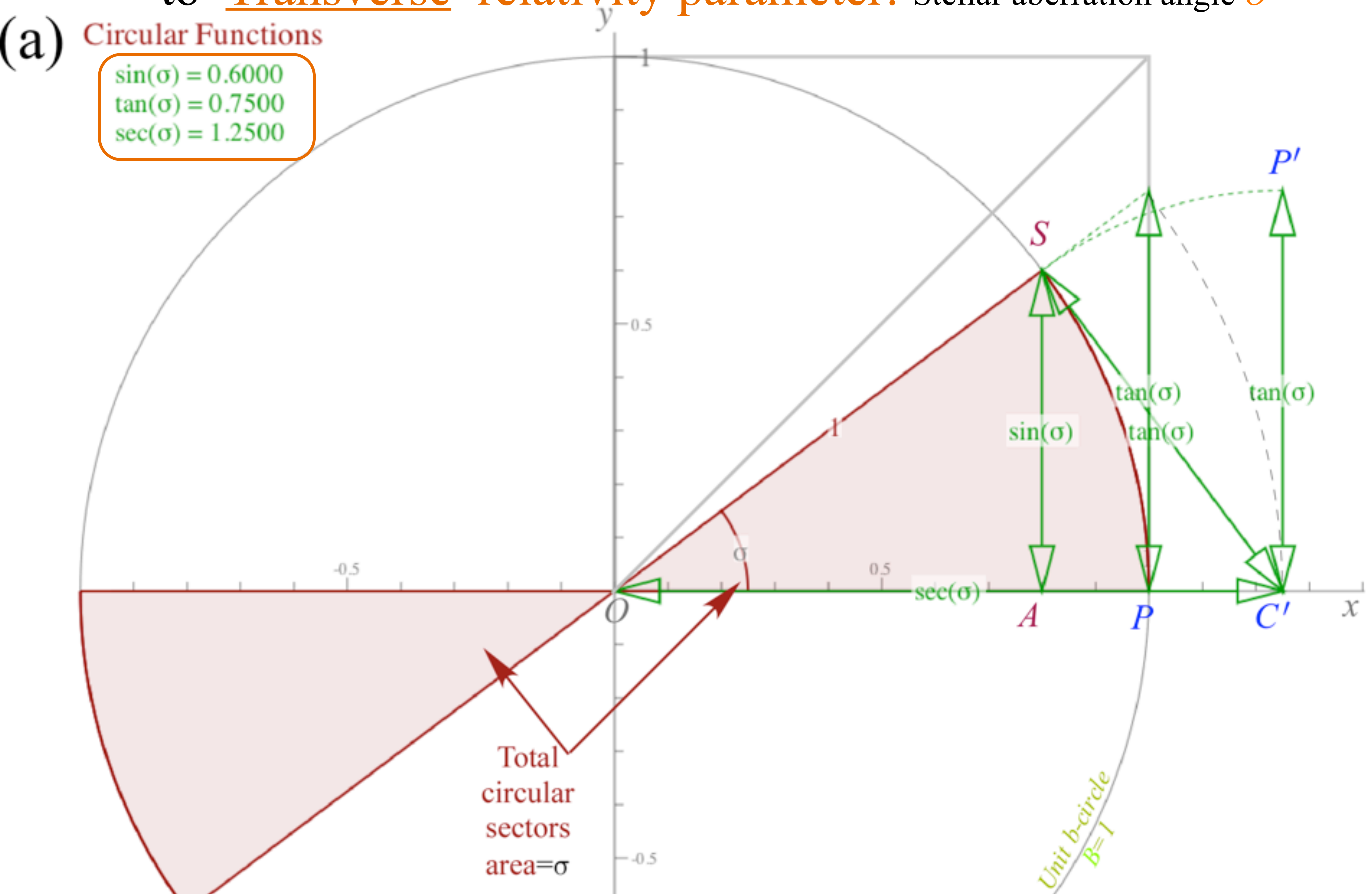
➔ Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

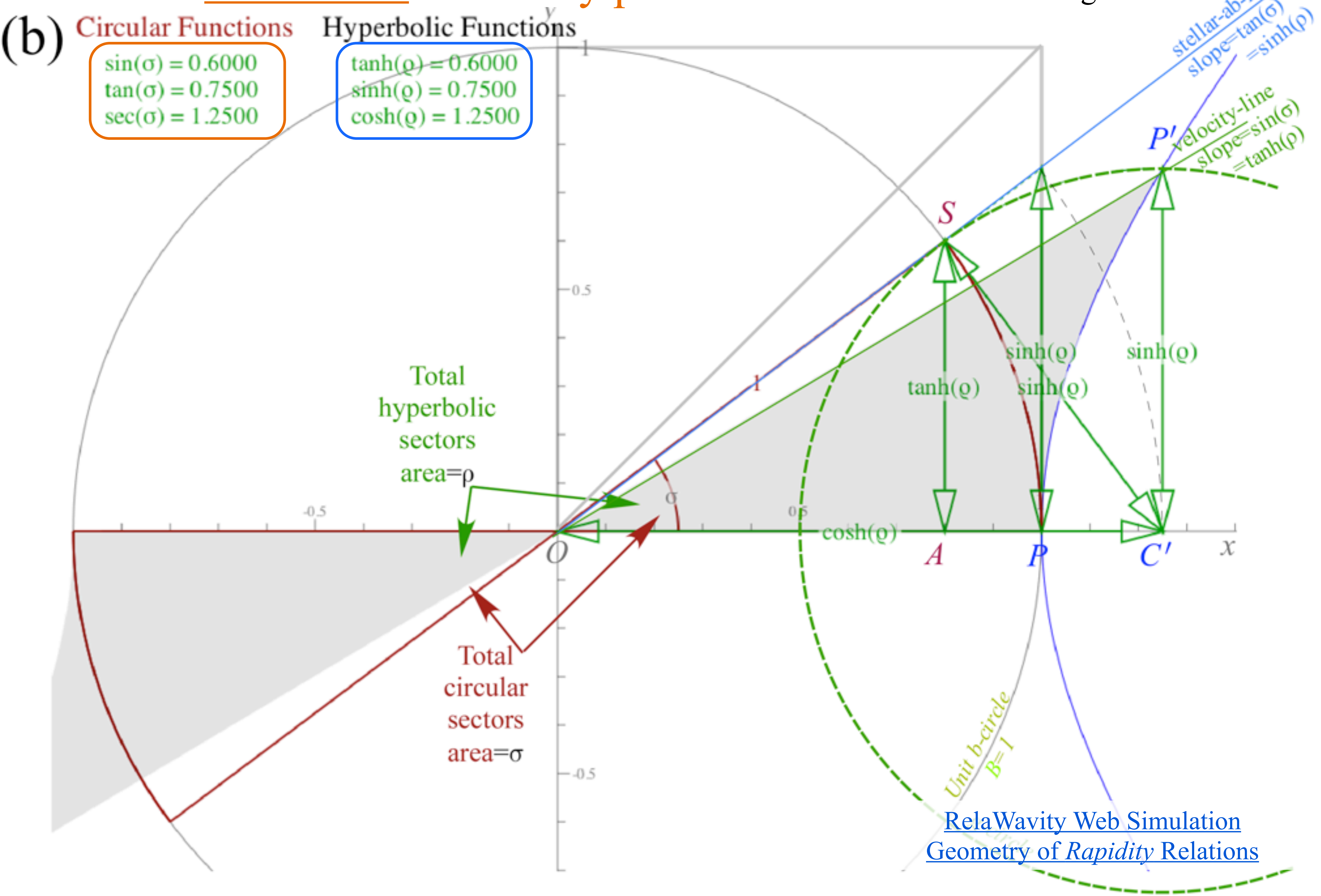
to Transverse relativity parameter: Stellar aberration angle σ



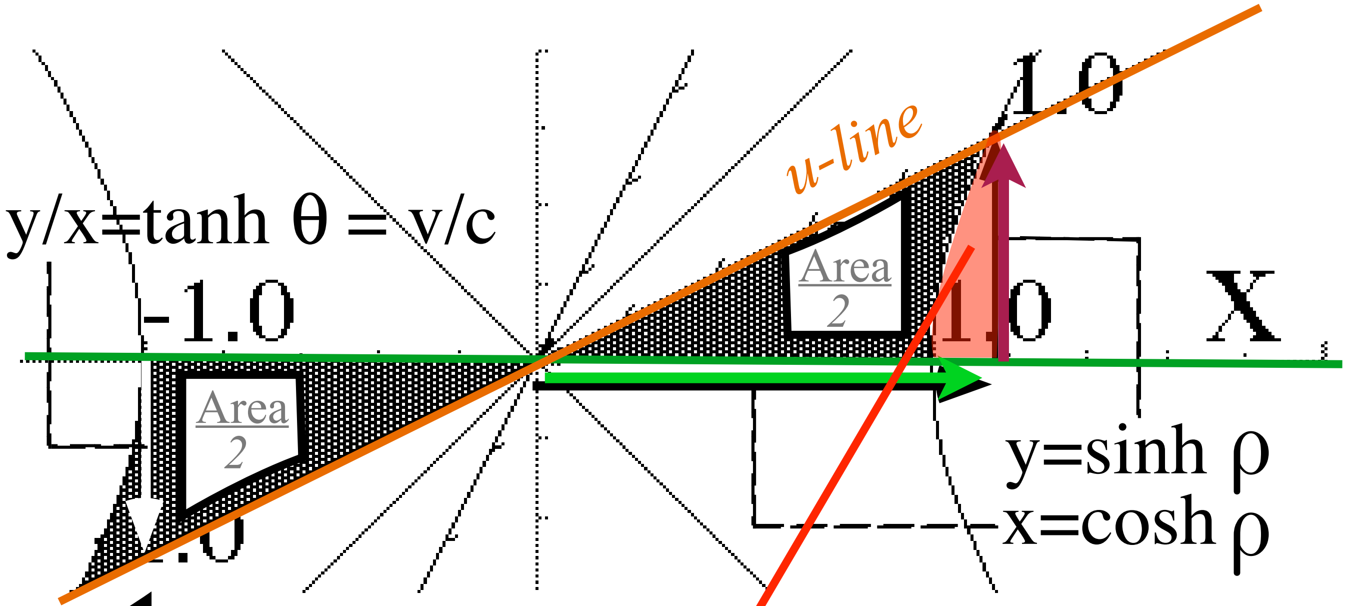
RelaWavity Web Simulation

Geometry of Stellar Aberation Angle

Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift) to Transverse relativity parameter: Stellar aberration angle σ



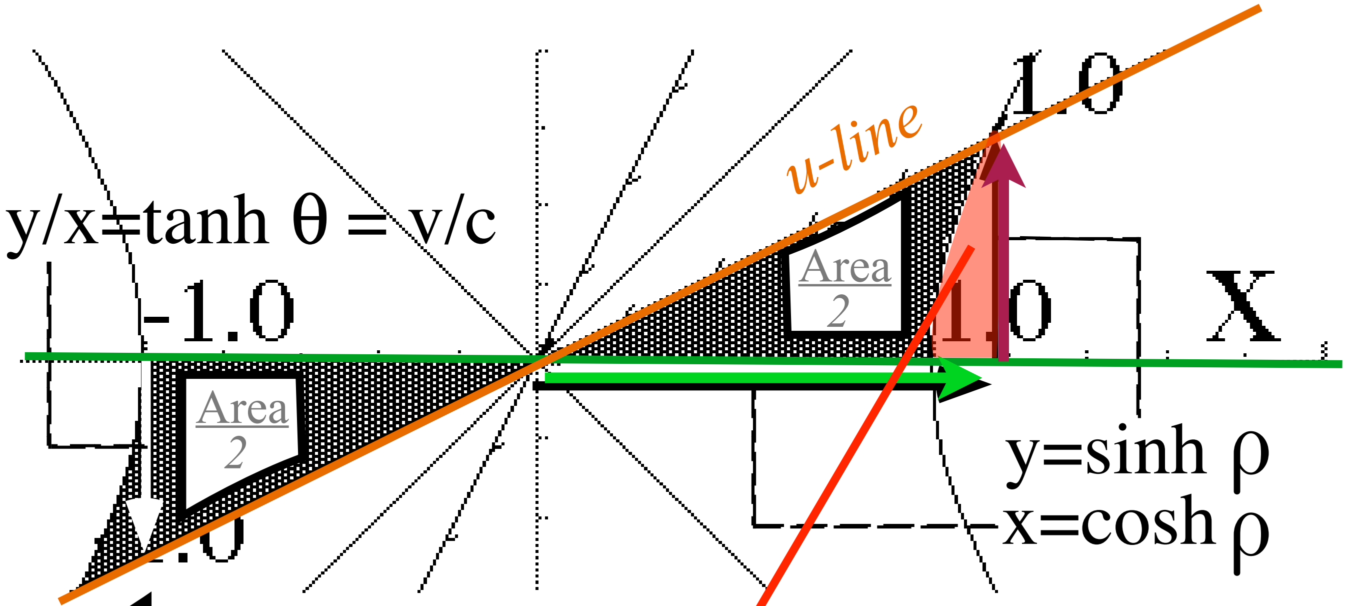
The straight scoop on "angle" and "rapidity" (They're area!)



$$\frac{Area}{2} = \frac{1}{2}\,\textit{base}\cdot\textit{altitude} - \text{area under curve} = \frac{1}{2}xy - \int y\,dx$$

The "Area" being calculated is the *total* **Gray Area** *between hyperbola pairs, X axis, and sloping u-line*

The straight scoop on "angle" and "rapidity" (They're area!)



The "Area" being calculated is the *total* **Gray Area** *between hyperbola pairs, X axis, and sloping u-line*

$$\frac{Area}{2} = \frac{1}{2} base \cdot altitude - \text{area under curve} = \frac{1}{2} xy - \int y\, dx$$

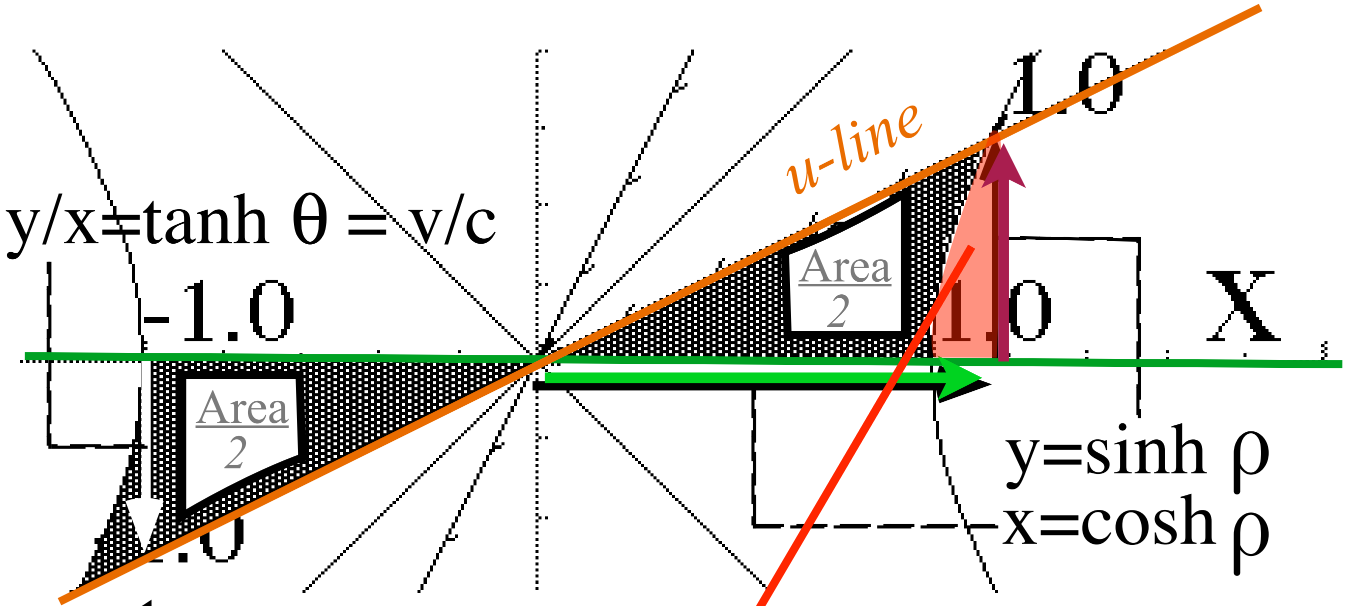
$$\frac{Area}{2} = \frac{1}{2} \sinh\rho \cosh\rho - \int \sinh\rho\, d(\cosh\rho)$$

Useful hyperbolic identities

$$\sinh^2\rho = \left(\frac{e^{\rho} - e^{-\rho}}{2}\right)^2 = \frac{1}{4}\left(e^{2\rho} + e^{-2\rho} - 2\right) = \frac{\cosh 2\rho - 1}{2}$$

$$\sinh\rho \cosh\rho = \left(\frac{e^{\rho} - e^{-\rho}}{2}\right)\left(\frac{e^{\rho} + e^{-\rho}}{2}\right) = \frac{1}{4}\left(e^{2\rho} - e^{-2\rho}\right) = \frac{1}{2}\sinh 2\rho$$

The straight scoop on "angle" and "rapidity" (They're area!)



The "Area" being calculated is the *total* **Gray Area** *between hyperbola pairs, X axis, and sloping u-line*

$$\frac{Area}{2} = \frac{1}{2}\, base \cdot altitude - \text{area under curve} = \frac{1}{2}xy - \int y\,dx$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int \sinh\rho\, d(\cosh\rho)$$

Useful hyperbolic identities

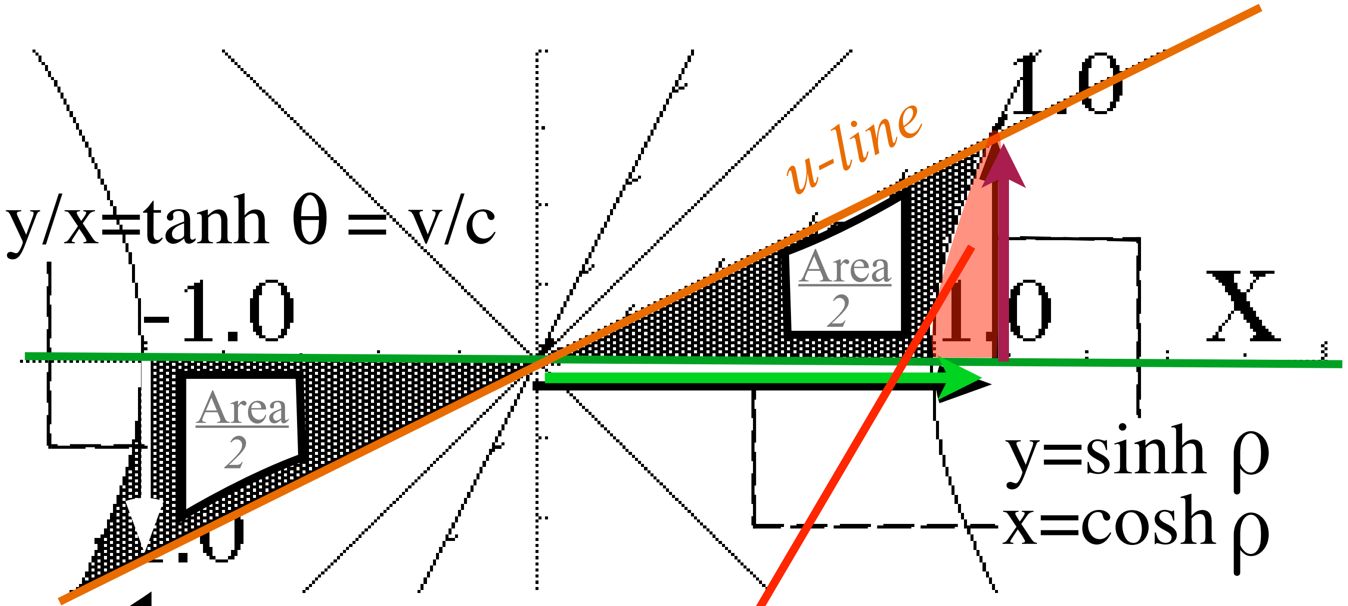
$$\sinh^2\rho = \left(\frac{e^{\rho} - e^{-\rho}}{2}\right)^2 = \frac{1}{4}\left(e^{2\rho} + e^{-2\rho} - 2\right) = \frac{\cosh 2\rho - 1}{2}$$

$$\sinh\theta\cosh\theta = \left(\frac{e^{\theta} - e^{-\theta}}{2}\right)\left(\frac{e^{\theta} + e^{-\theta}}{2}\right) = \frac{1}{4}\left(e^{2\theta} - e^{-2\theta}\right) = \frac{1}{2}\sinh 2\theta$$

$$\int \cosh a\rho\, d\rho = \frac{1}{a}\sinh a\rho$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int \sinh^2\rho\, d\rho = \frac{1}{4}\sinh 2\rho - \int \frac{\cosh 2\rho - 1}{2}\, d\rho$$

The straight scoop on "angle" and "rapidity" (They're area!)



The "Area" being calculated is the *total* **Gray Area** *between hyperbola pairs, X axis, and sloping u-line*

$$\frac{Area}{2} = \frac{1}{2} base \cdot altitude - \text{area under curve} = \frac{1}{2} xy - \int y\, dx$$

$$\frac{Area}{2} = \frac{1}{2} \sinh\rho \cosh\rho - \int \sinh\rho \, d(\cosh\rho)$$

Useful hyperbolic identities

$$\sinh^2\rho = \left(\frac{e^{\rho} - e^{-\rho}}{2}\right)^2 = \frac{1}{4}\left(e^{2\rho} + e^{-2\rho} - 2\right) = \frac{\cosh 2\rho - 1}{2}$$

$$\sinh\rho \cosh\rho = \left(\frac{e^{\rho} - e^{-\rho}}{2}\right)\left(\frac{e^{\rho} + e^{-\rho}}{2}\right) = \frac{1}{4}\left(e^{2\rho} - e^{-2\rho}\right) = \frac{1}{2}\sinh 2\rho$$

$$\int \cosh a\theta \, d\theta = \frac{1}{a}\sinh a\theta$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int \sinh^2\rho \, d\rho = \frac{1}{4}\sinh 2\rho - \int \frac{\cosh 2\rho - 1}{2} d\rho$$

$$= \frac{1}{4}\sinh 2\rho - \frac{1}{4}\sinh 2\rho + \int \frac{1}{2} d\rho$$

$$= \frac{\rho}{2}$$

Amazing result: ***Area*** $= \rho$ *is rapidity*

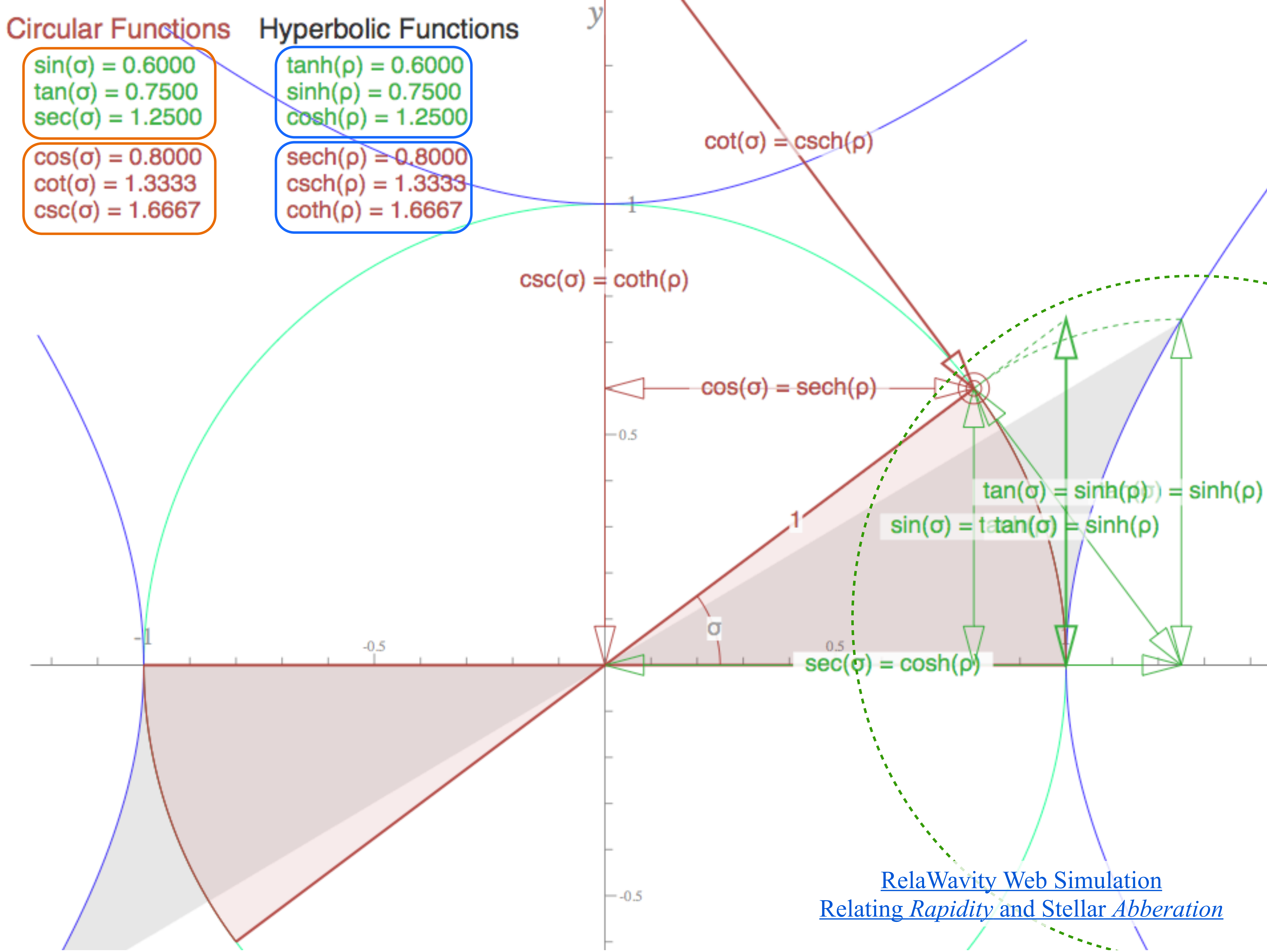
Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = B$\cosh\rho$...
A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P}'$ and Group vector $\mathbf{G}'$ in per-space-time
Lorentz matrix transformation of *(x,ct)* space-time coordinates
Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation
Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa
Another view of phas*or*-invariance
Geometry of invariant hyperbolas
Algebra of invariant hyperbolas
Proper time τ_0 and proper frequency ω_0
A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ
Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area
➔ Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).



RelaWavity Web Simulation
Relating *Rapidity* and Stellar *Abberation*

Review of geometric construction , per-space-time (ω,*ck*) dispersion hyperbola $\omega = B\cosh\rho$...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

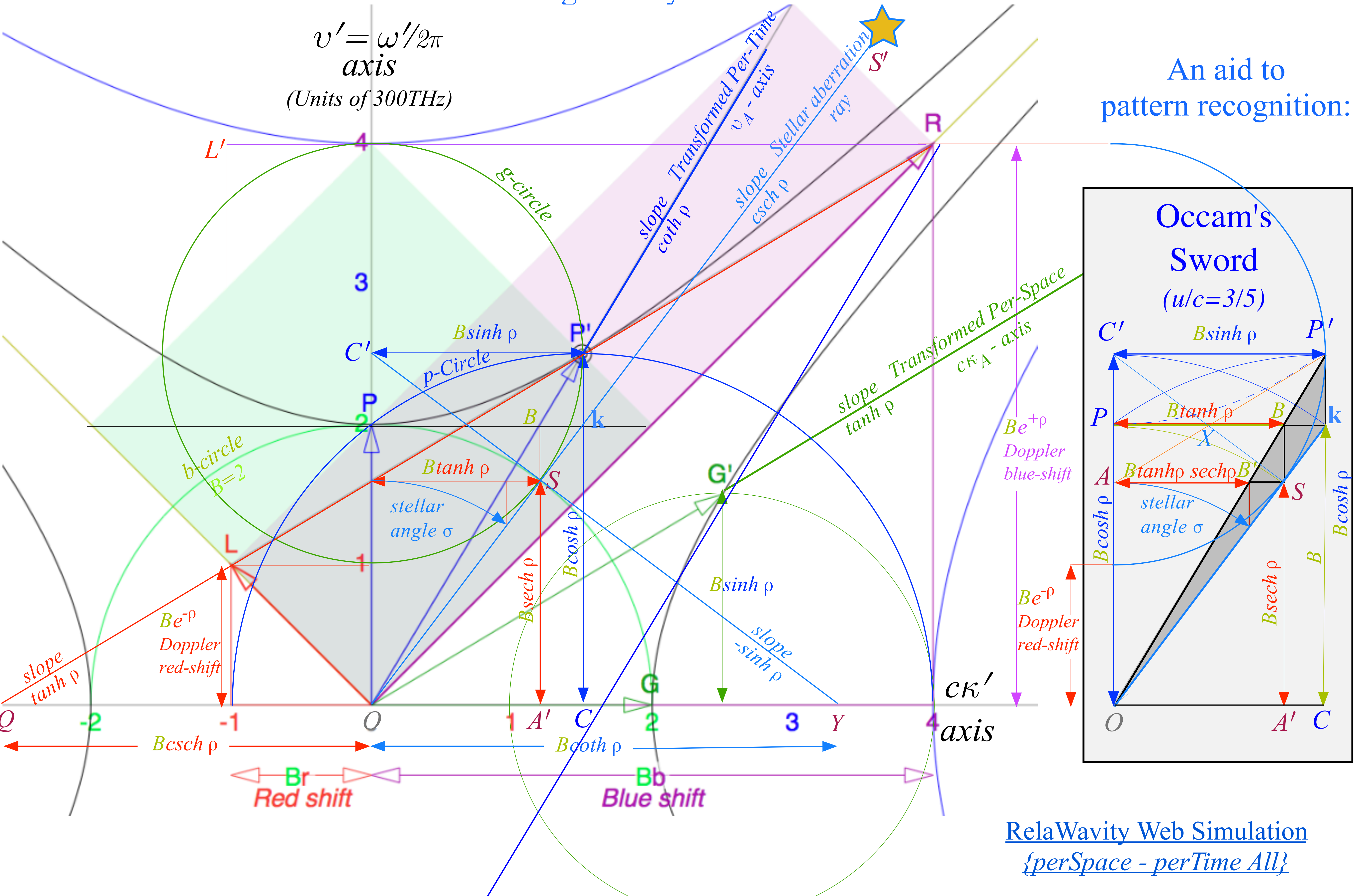
➔ Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

Summary of optical wave parameters for relativity and QM

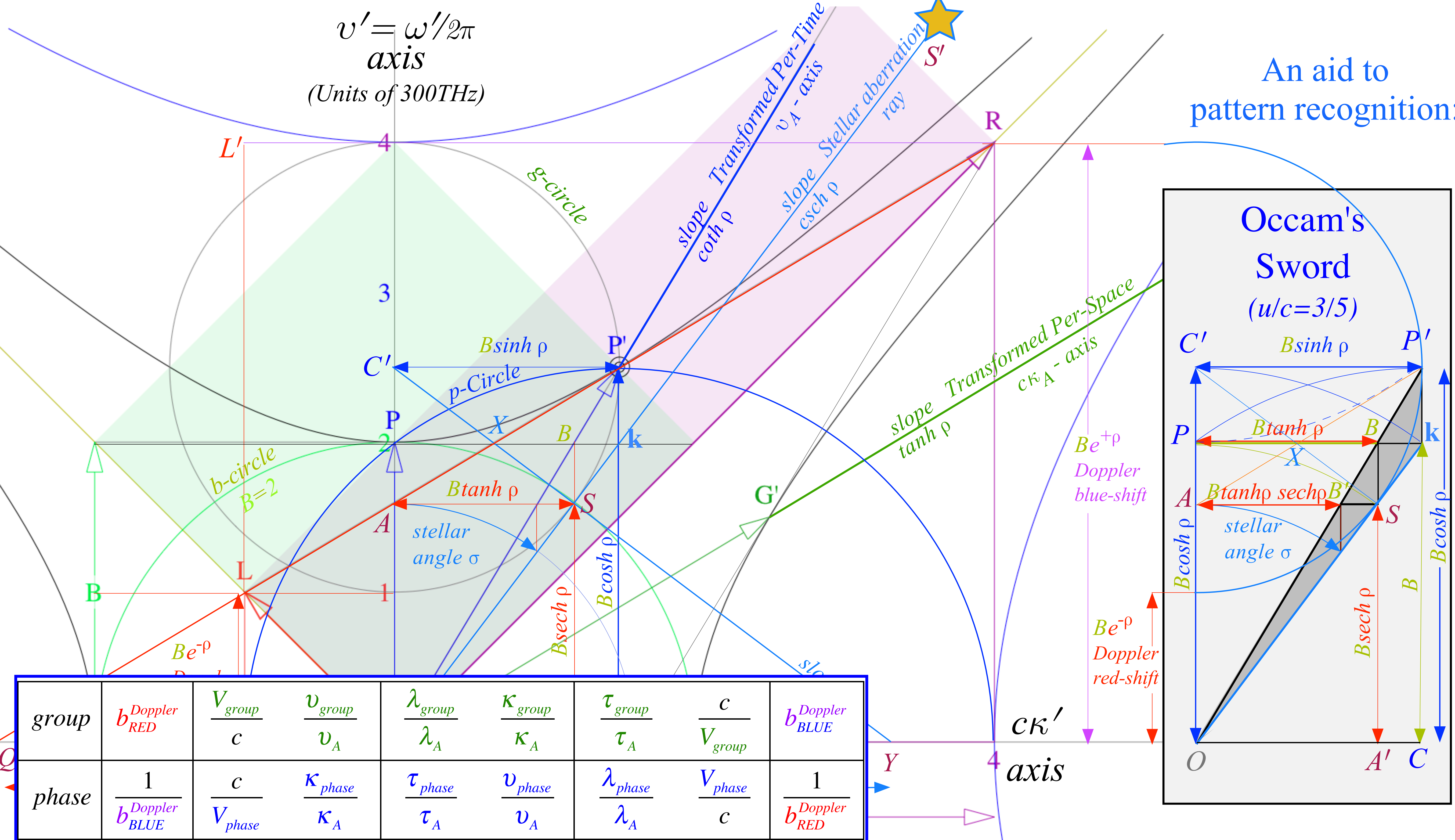
...and their geometry

An aid to pattern recognition:



RelaWavity Web Simulation
{perSpace - perTime All}

An aid to pattern recognition:



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Table of 12 wave parameters (includes inverses) for relativity

...and values for u/c=3/5

RelaWavity Web Simulation
Relativistic *Terms (Dual plot w/expanded table)*

Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = B$\cosh\rho$...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector **P′** and Group vector **G′** in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

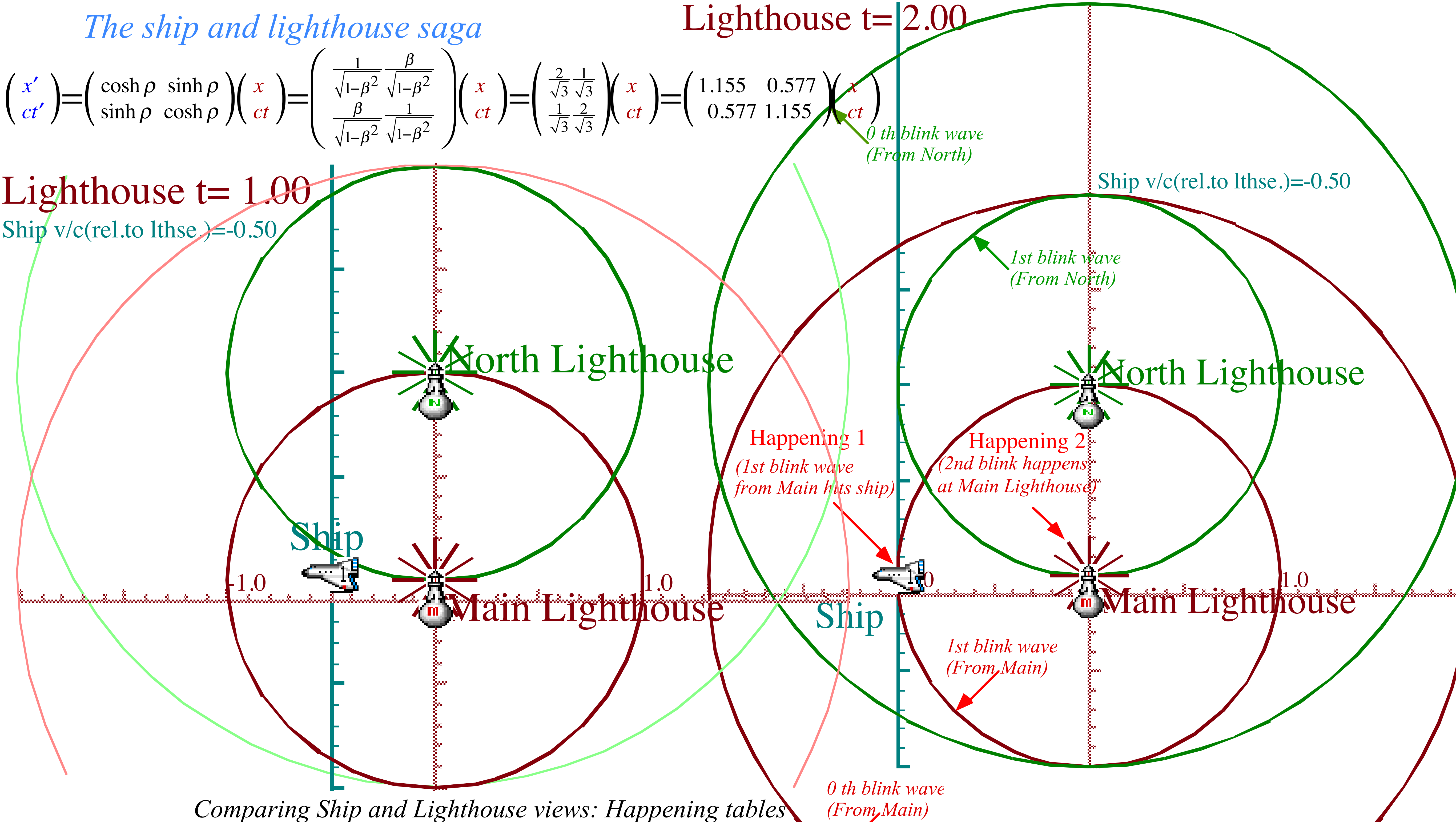
Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

➔ Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

The ship and lighthouse saga

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix}$$



Comparing Ship and Lighthouse views: Happening tables

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\,c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\,\Delta$ $t' = 2\Delta = 2.30$

RelativIt Web Simulation
Relativistic Events in
Main Lighthouse's Frame

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

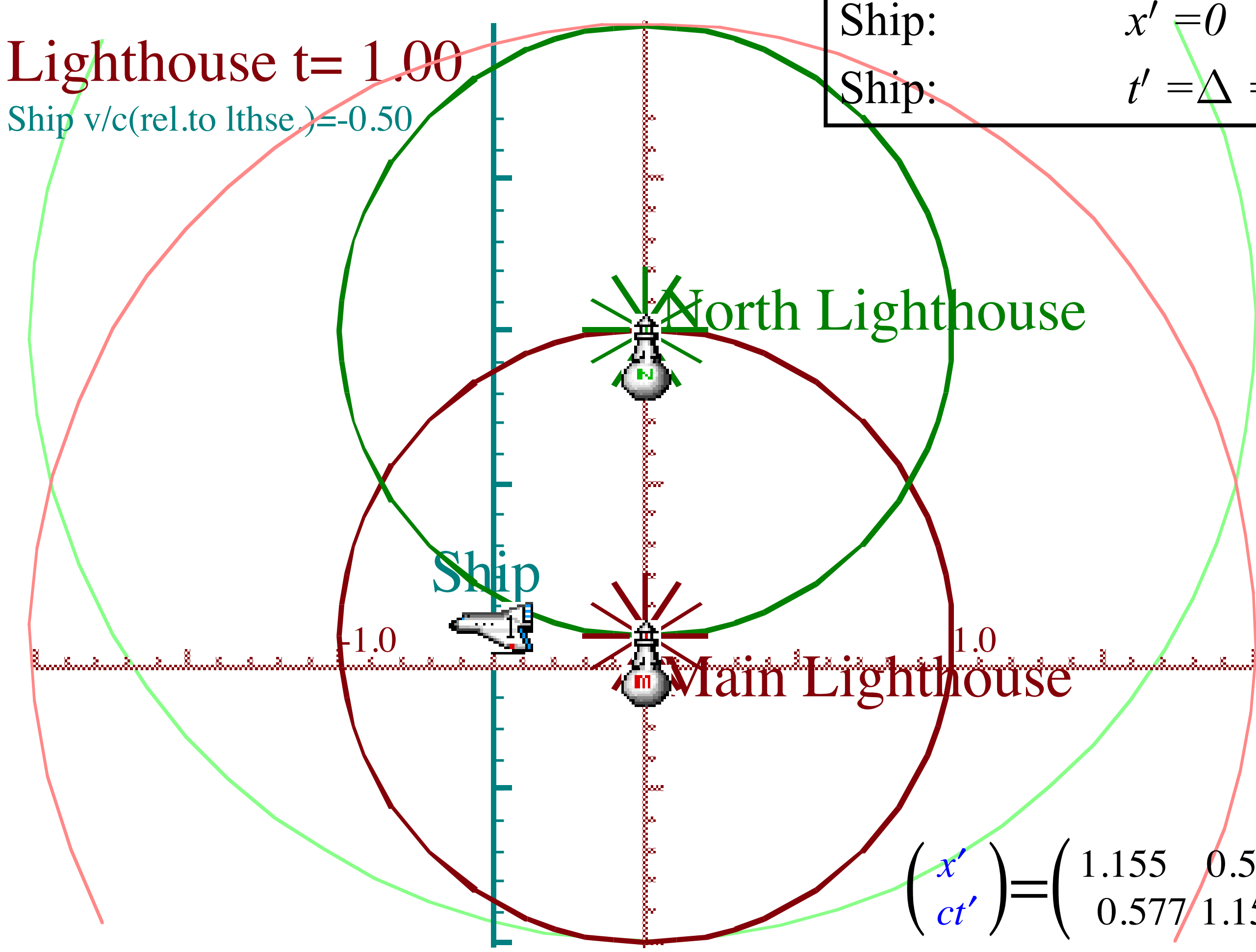
The ship and lighthouse saga

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

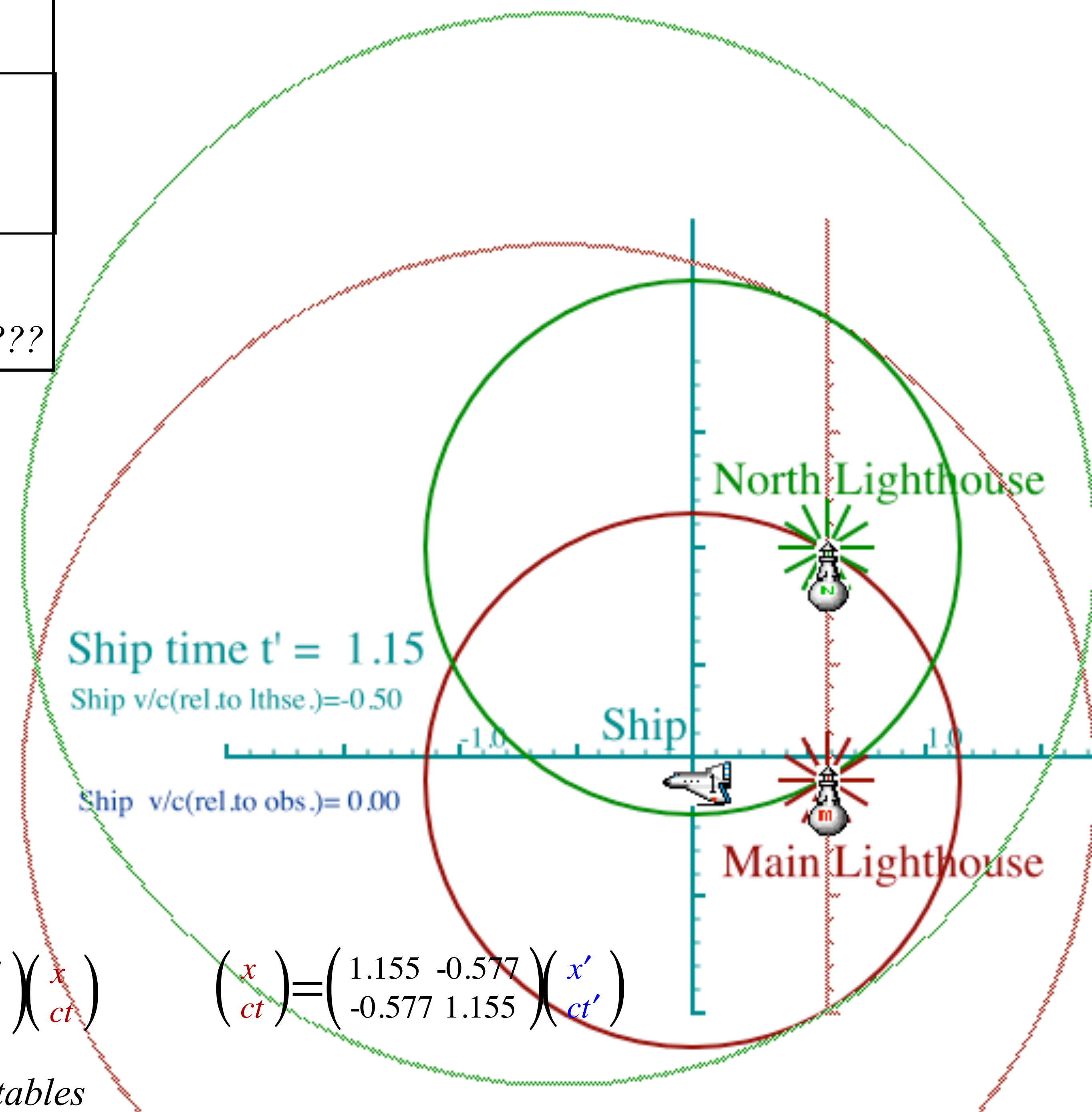
$$= \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

Ship Time $t' = \Delta = ??$



$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$



$$\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & -0.577 \\ -0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Comparing Ship and Lighthouse views: Happening tables

RelativIt Web Simulation
Relativistic Events in
Main Lighthouse's Frame

	Happening 0: Ship passes Main Lighthouse.		Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space)		$x = 0$	$x = -1.00\,c$	$x = 0$
(Lighthouse time)		$t = 0$	$t = 2.00$	$t = 2.00$
(Ship space)		$x' = 0$	$x' = 0$	$x' = c\,\Delta$
(Ship time)		$t' = 0$	$t' = 1.75$	$t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

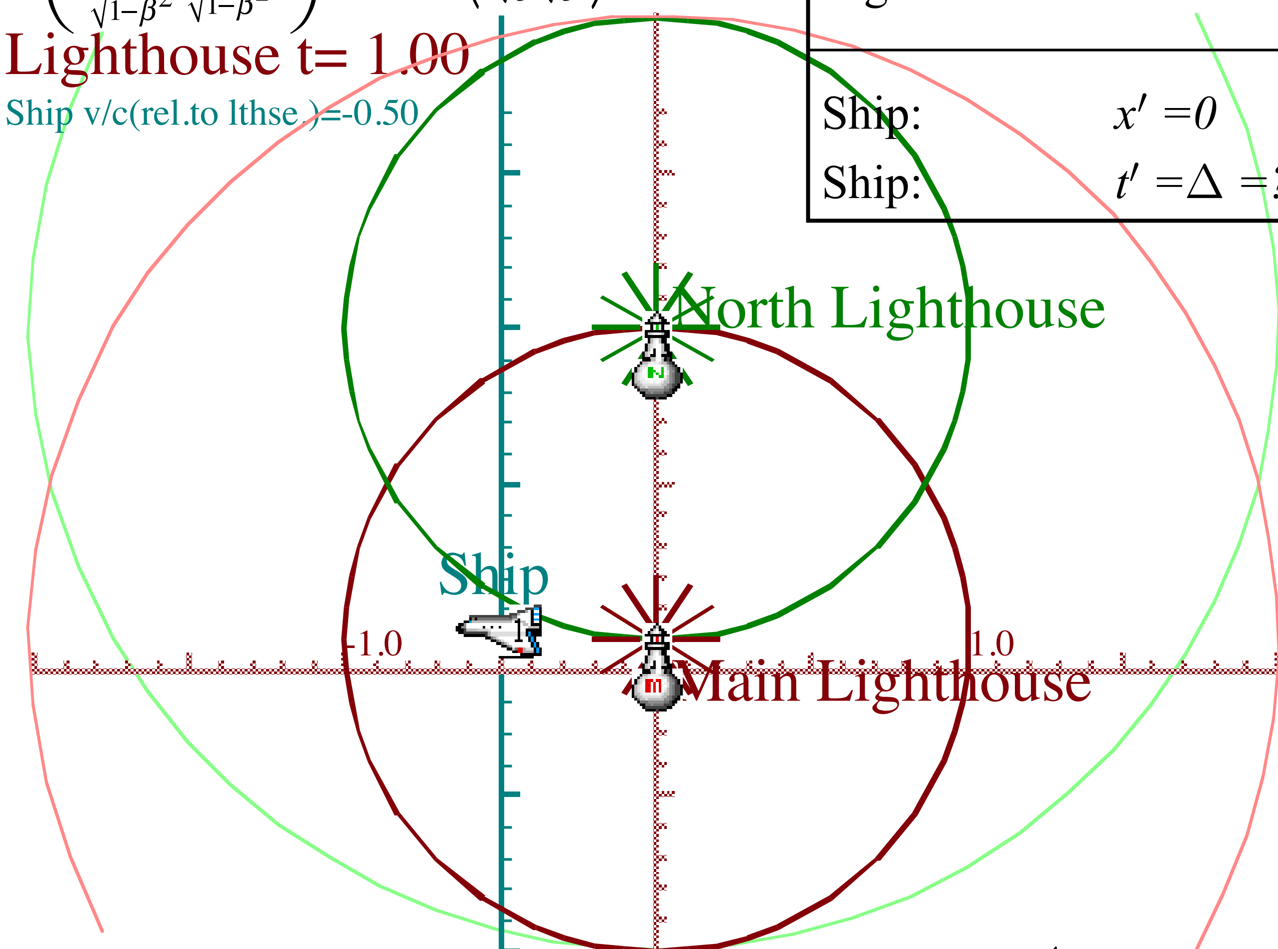
The ship and lighthouse saga

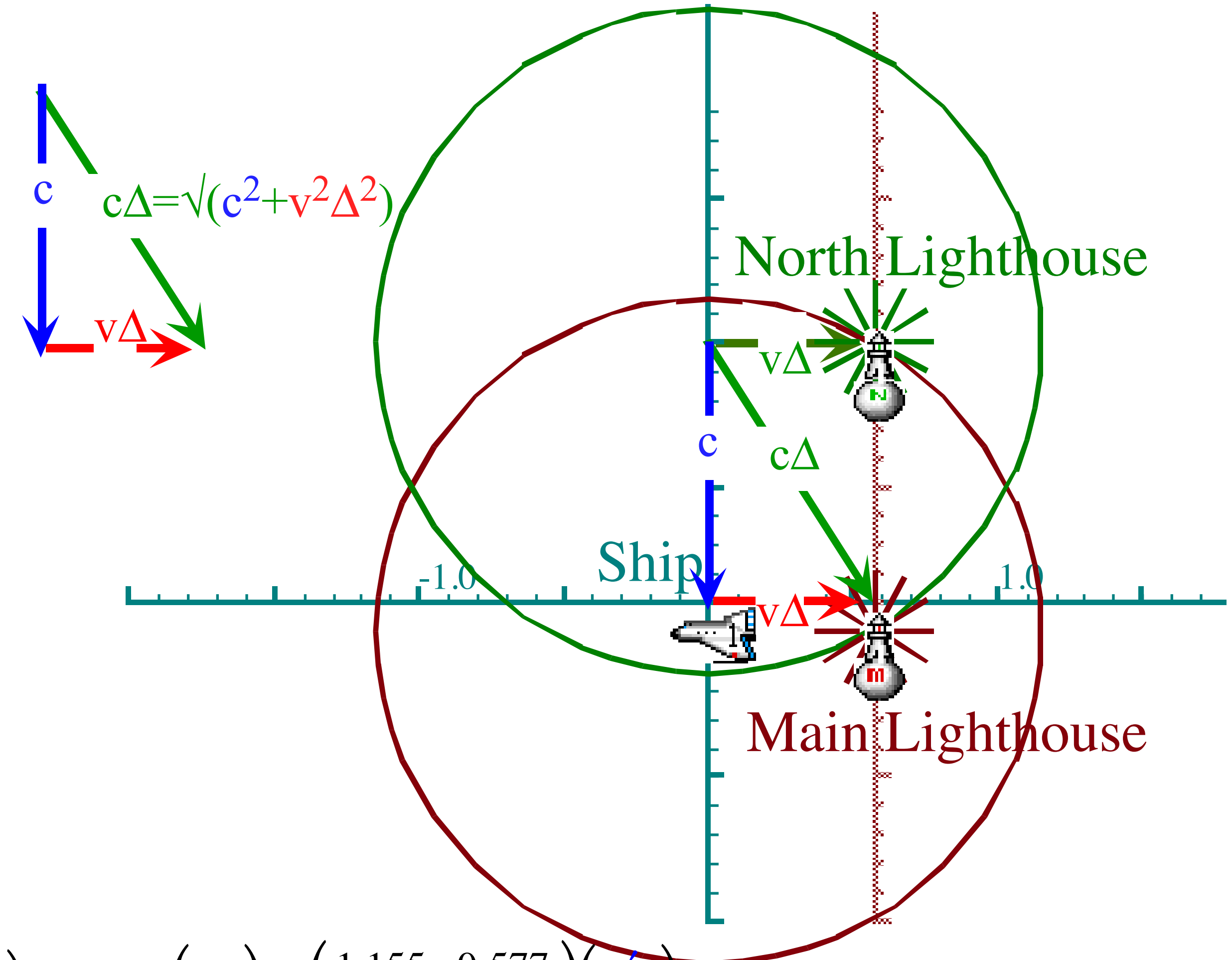
$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

Ship Time $t' = \Delta = ???$





$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & -0.577 \\ -0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Comparing Ship and Lighthouse views: Happening tables

[RelativIt Web Simulation](#)
Relativistic Events in Main Lighthouse's Frame

	Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space)	$x = 0$	$x = -1.00\,c$	$x = 0$
(Lighthouse time)	$t = 0$	$t = 2.00$	$t = 2.00$
(Ship space)	$x' = 0$	$x' = 0$	$x' = c\,\Delta$
(Ship time)	$t' = 0$	$t' = 1.75$	$t' = 2\Delta = 2.30$

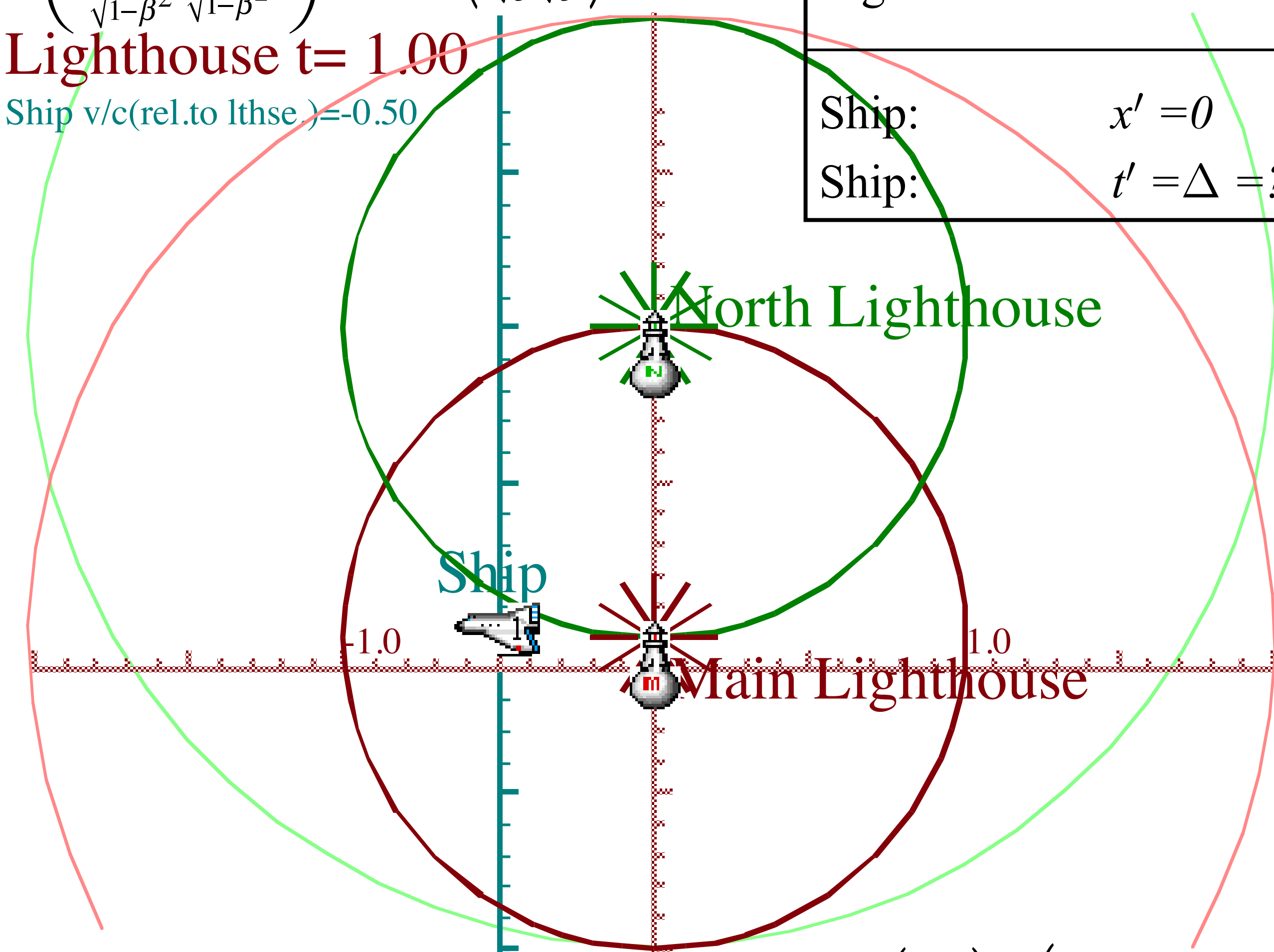
Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

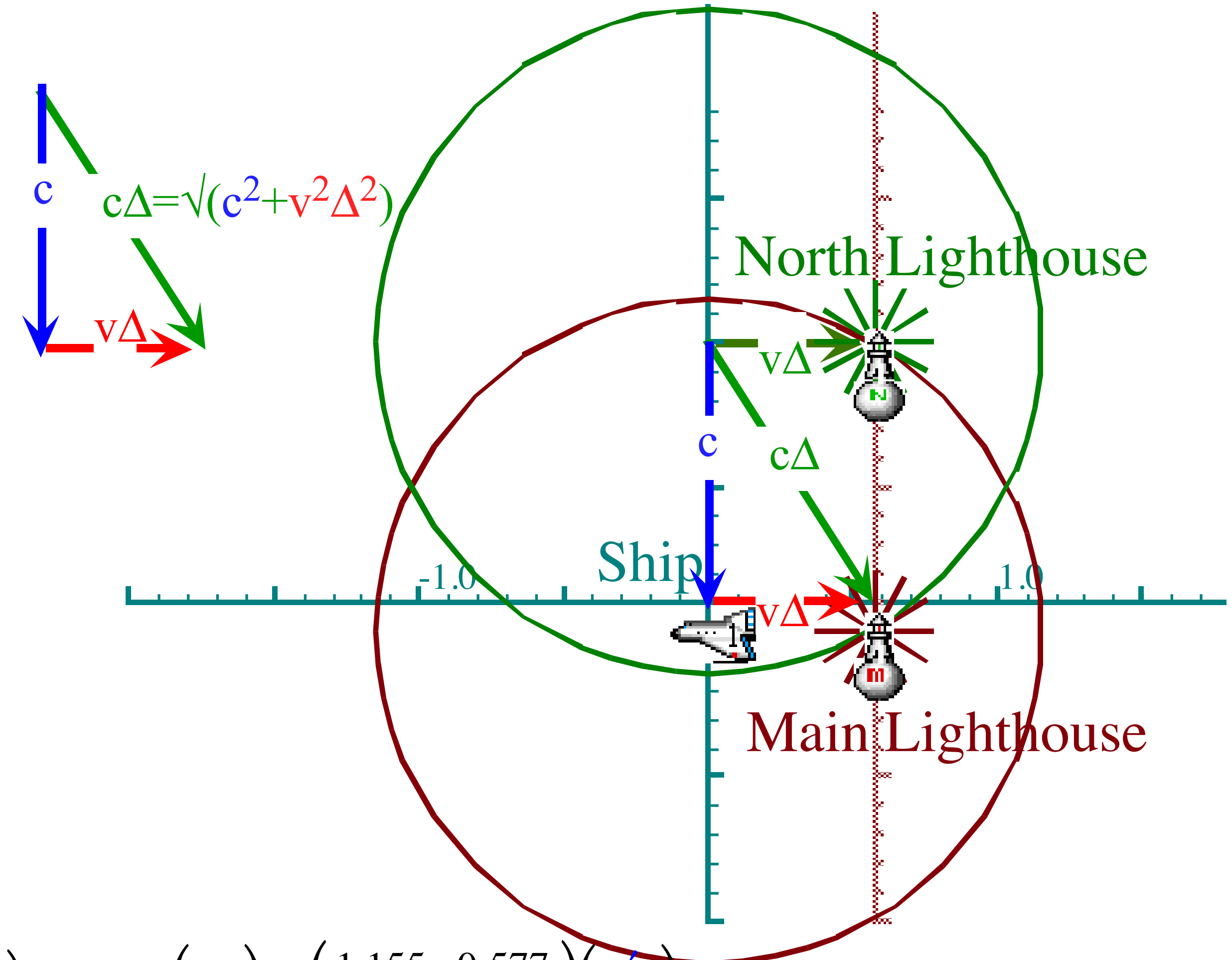


$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

Ship Time $t' = \Delta = ???$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$



$$\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & -0.577 \\ -0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Comparing Ship and Lighthouse views: Happening tables

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\,c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\,\Delta$ $t' = 2\Delta = 2.30$

[RelativIt Web Simulation](#)
Relativistic Events in Main Lighthouse's Frame

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

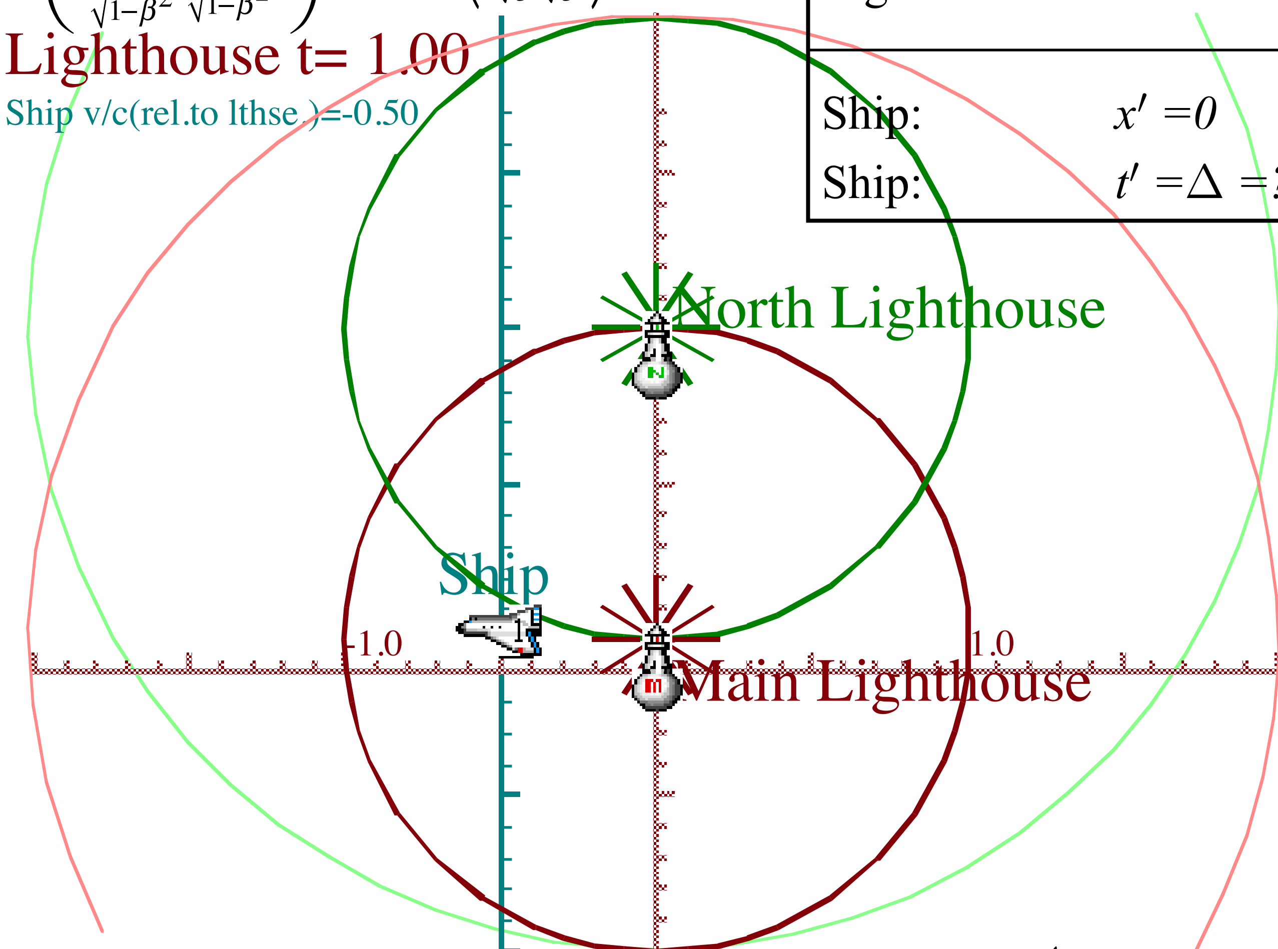
Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = ???$

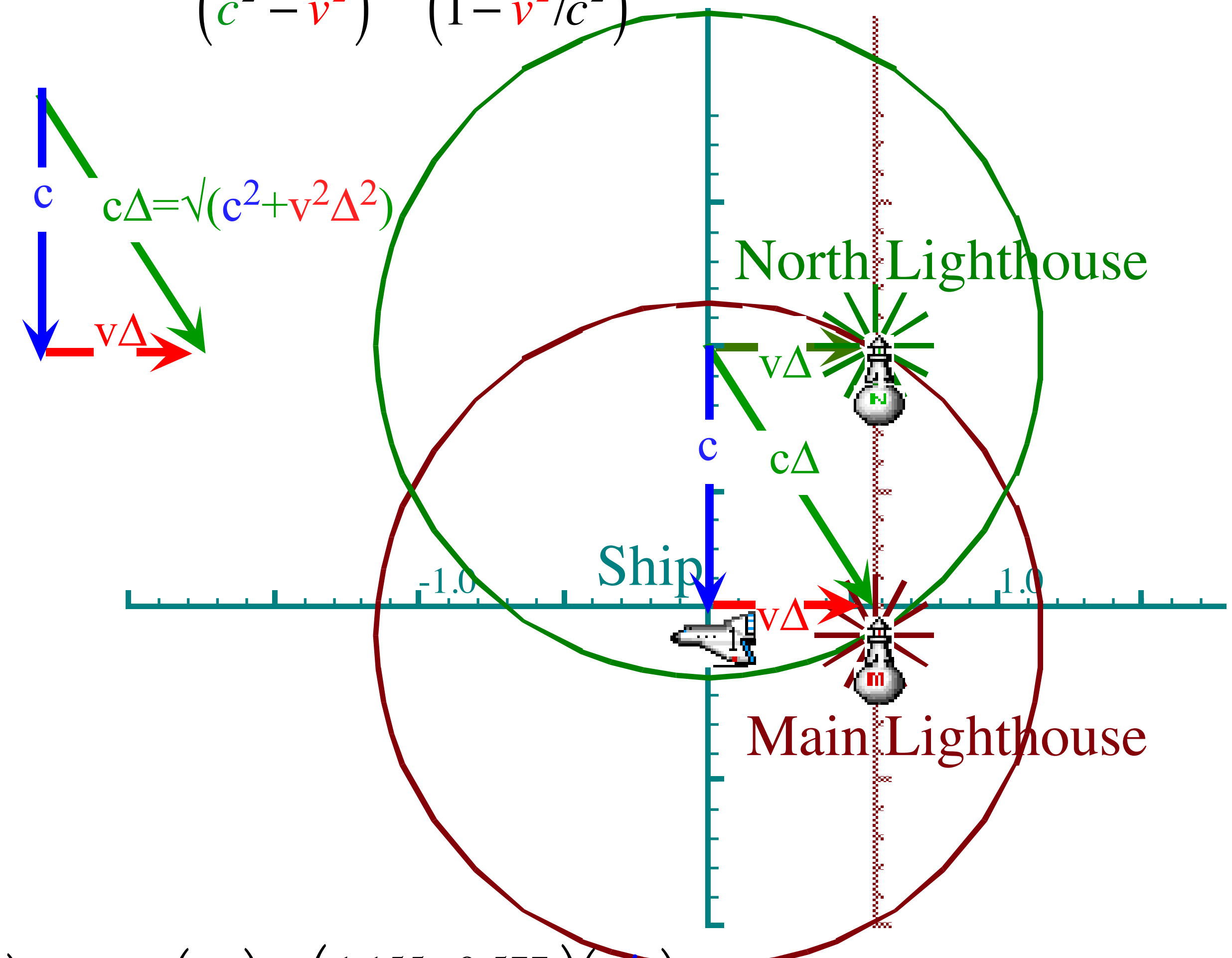
Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$\left(c^2 - v^2\right)\Delta^2 = c^2$$

$$\Delta^2 = \frac{c^2}{\left(c^2 - v^2\right)} = \frac{1}{\left(1 - v^2/c^2\right)}$$





$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & -0.577 \\ -0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Comparing Ship and Lighthouse views: Happening tables

[RelativIt Web Simulation](#)
Relativistic Events in
Main Lighthouse's Frame

	Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space)	$x = 0$	$x = -1.00\,c$	$x = 0$
(Lighthouse time)	$t = 0$	$t = 2.00$	$t = 2.00$
(Ship space)	$x' = 0$	$x' = 0$	$x' = c\,\Delta$
(Ship time)	$t' = 0$	$t' = 1.75$	$t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

The ship and lighthouse saga

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

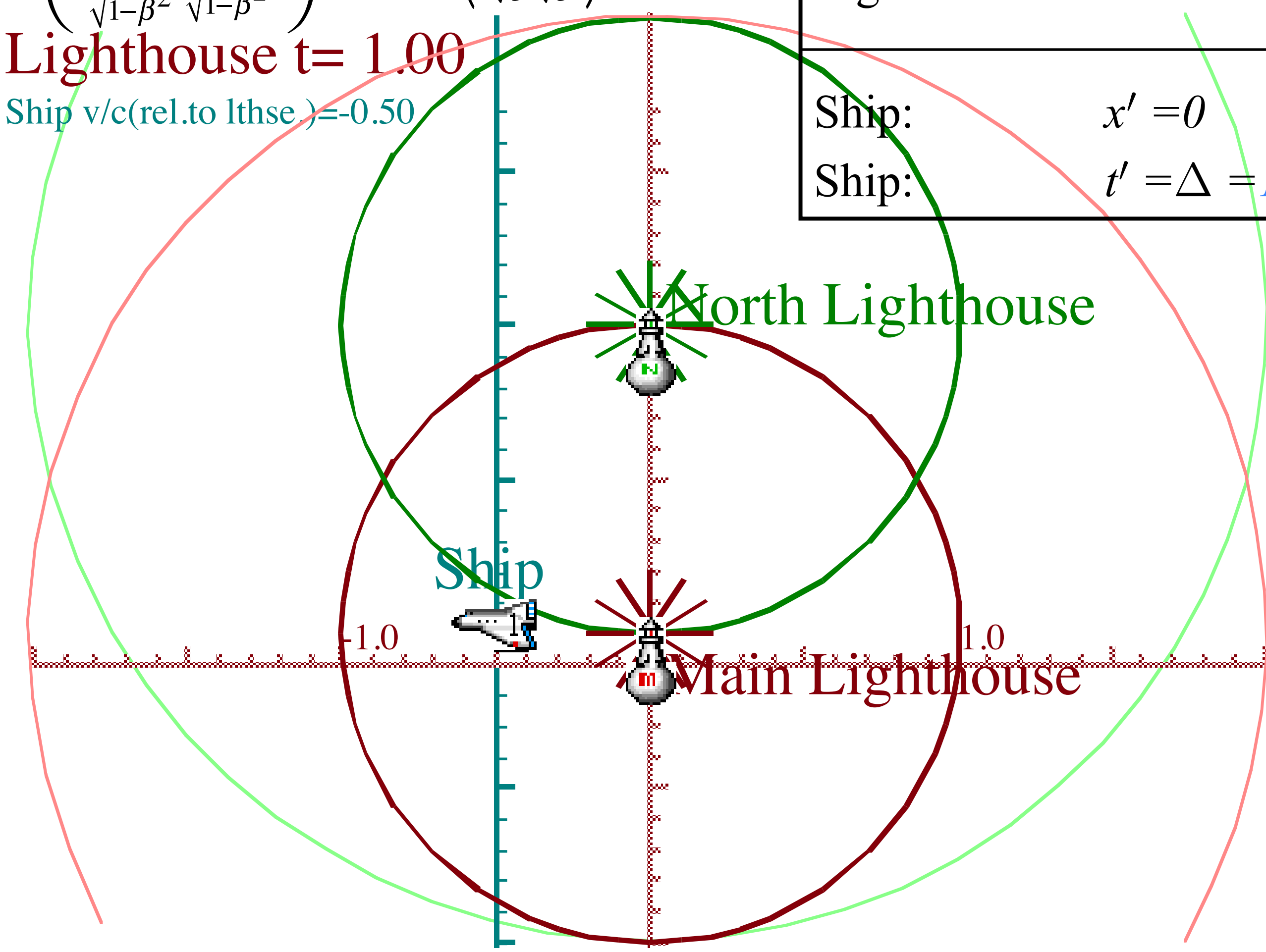
Happening 0.5: Main Lite blinks first time.	
Lighthouse:	$x = 0$
Lighthouse:	$t = 1.00$
Ship:	$x' = 0$
Ship:	$t' = \Delta = 1.15$

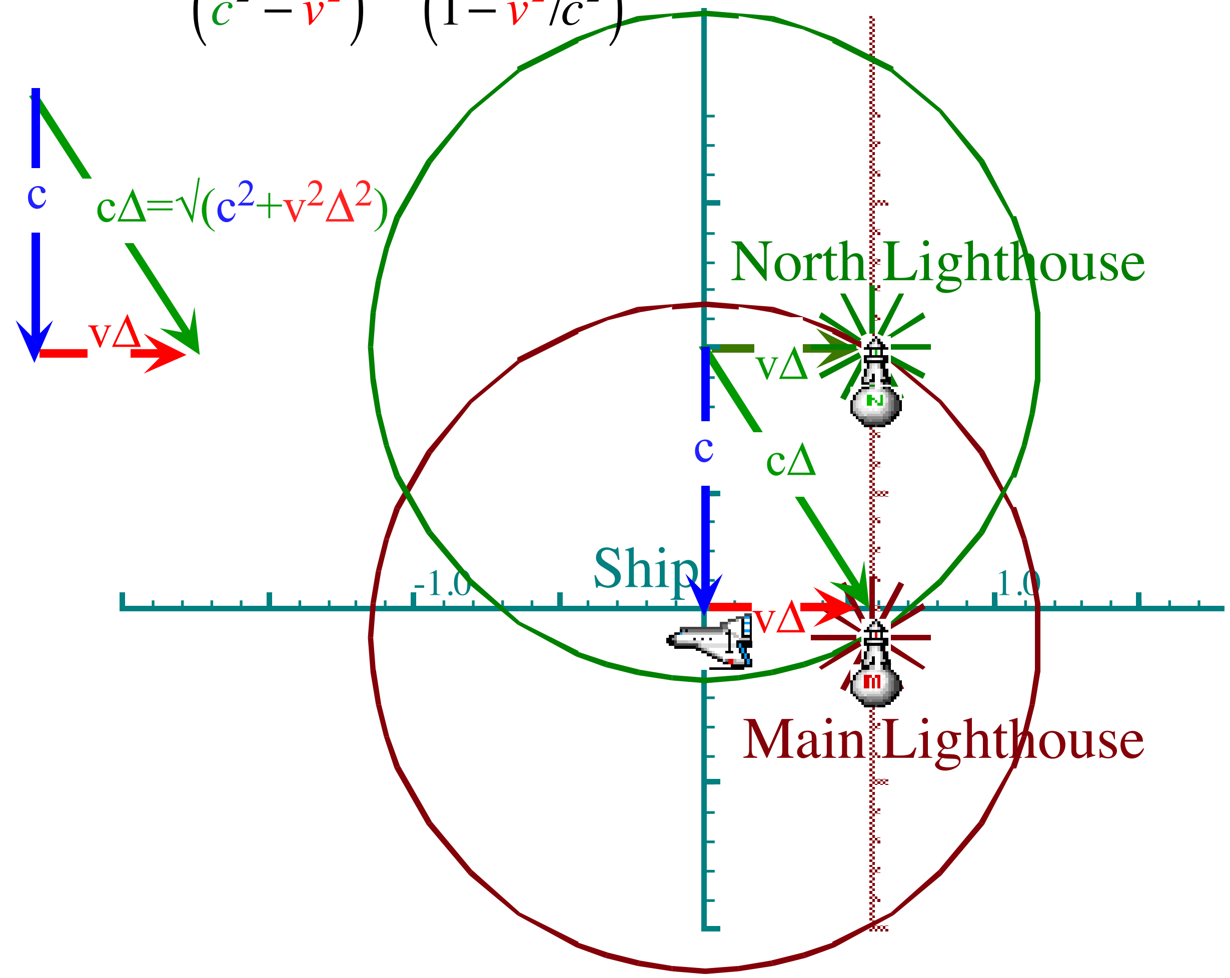
Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho = 1.15$

$$c^2\Delta^2 = c^2 + v^2\Delta^2$$

$$(c^2 - v^2)\Delta^2 = c^2$$

$$\Delta^2 = \frac{c^2}{(c^2 - v^2)} = \frac{1}{(1 - v^2/c^2)}$$





$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & -0.577 \\ -0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

For u/c=1/2

$\Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15..$

Comparing Ship and Lighthouse views: Happening tables

RelativIt Web Simulation

Relativistic Events in

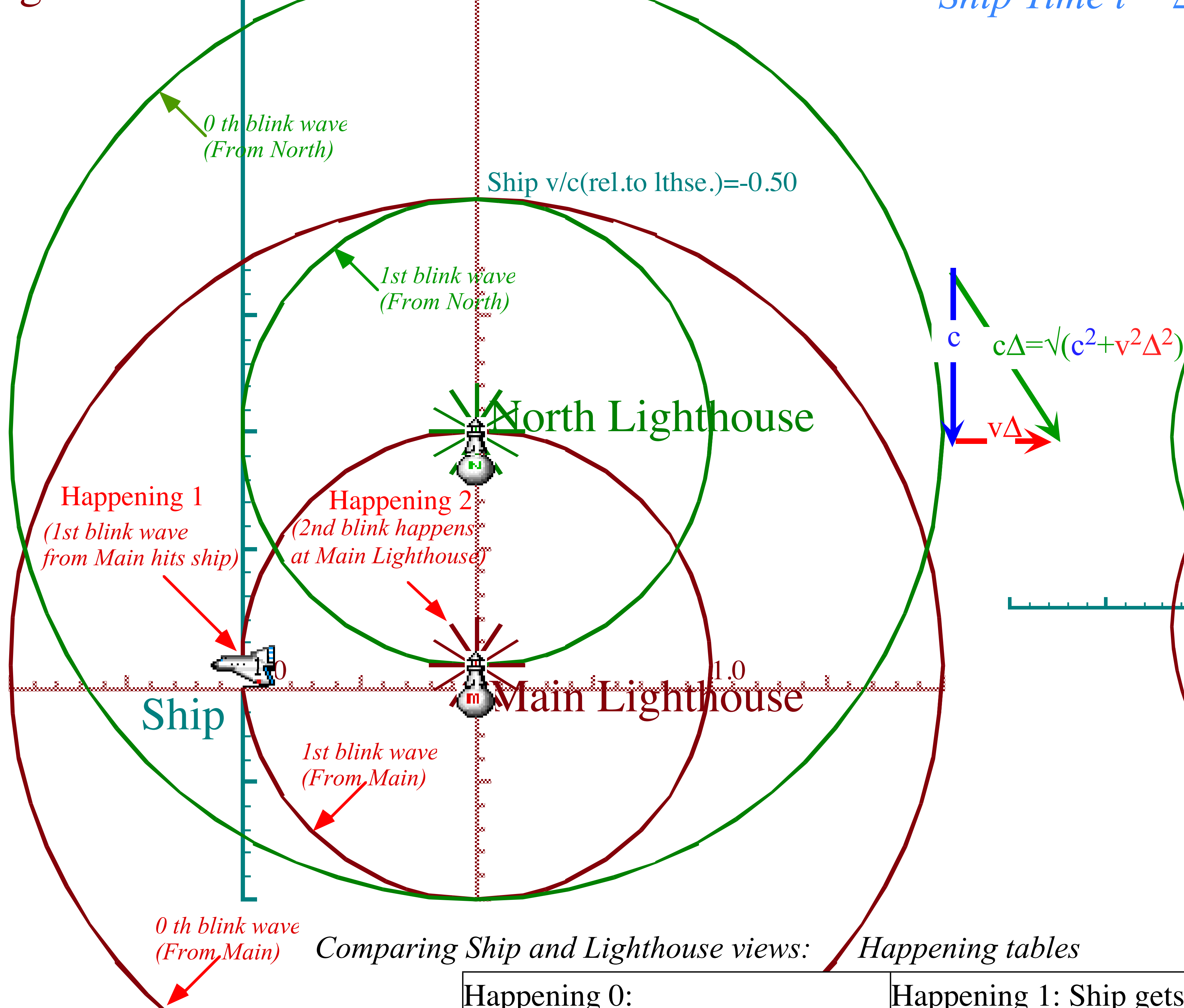
Main Lighthouse's Frame

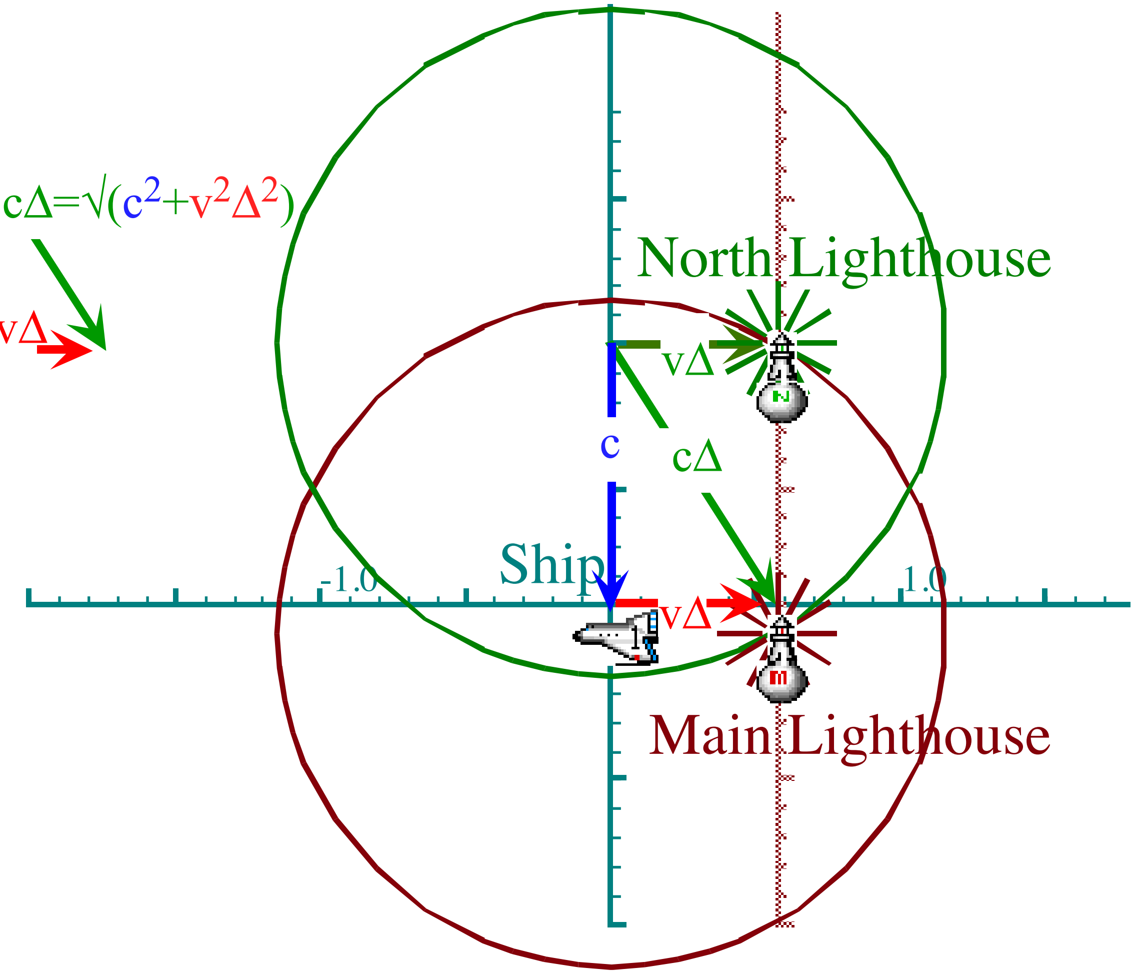
	Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space)	$x = 0$	$x = -1.00\,c$	$x = 0$
(Lighthouse time)	$t = 0$	$t = 2.00$	$t = 2.00$
(Ship space)	$x' = 0$	$x' = 0$	$x' = c\,\Delta$
(Ship time)	$t' = 0$	$t' = 1.75$	$t' = 2\Delta = 2.30$

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

Lighthouse t= 2.00

Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho = 1.15$





For u/c=1/2

$\Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15..$

Comparing Ship and Lighthouse views: Happening tables

Happening 0: Ship passes Main Lighthouse.	Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $t = 0$	$x = -1.00\,c$ $t = 2.00$	$x = 0$ $t = 2.00$
(Ship space) $x' = 0$ (Ship time) $t' = 0$	$x' = 0$ $t' = 1.75$	$x' = c\,\Delta$ $t' = 2\Delta = 2.30$

RelativIt Web Simulation

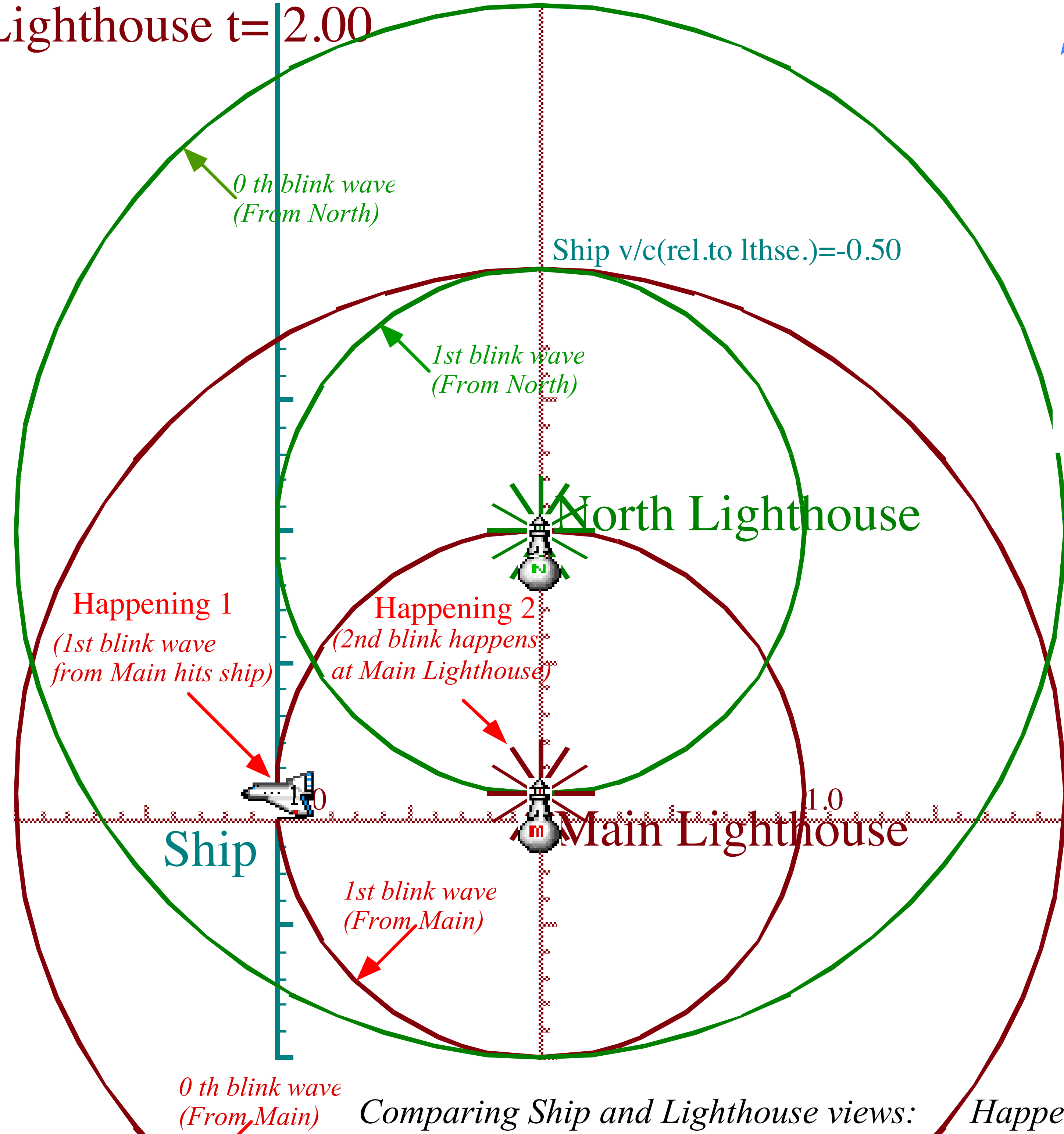
Relativistic Events in

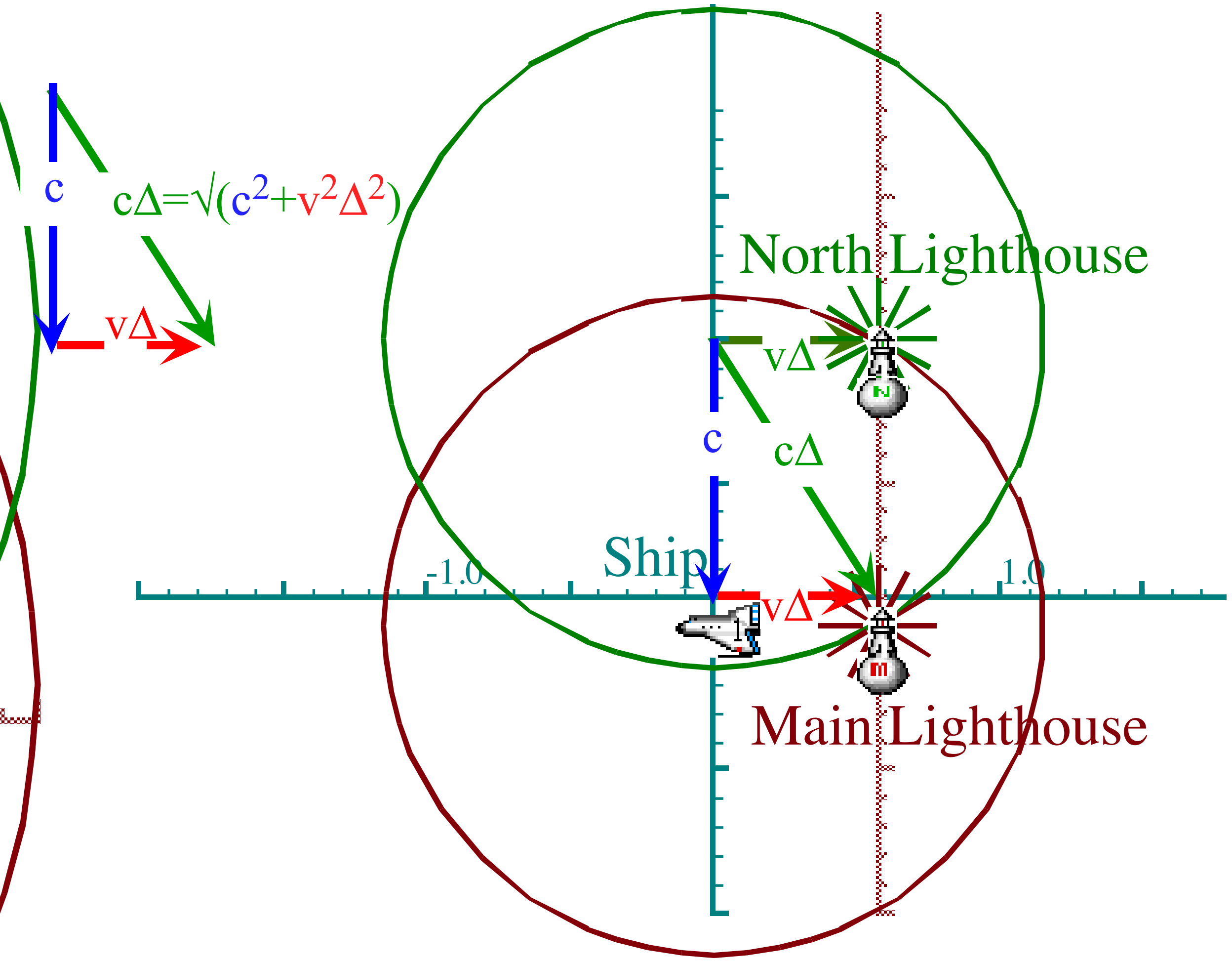
Main Lighthouse's Frame

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

Lighthouse t= 2.00

Ship Time $t' = \Delta = 1/\sqrt{(1-v^2/c^2)} = \cosh\rho = 1.15$





For u/c=1/2

$\Delta = 1/\sqrt{(1-1/4)} = 2/\sqrt{3} = 1.15..$

Comparing Ship and Lighthouse views: Happening tables

Happening 0: Ship passes Main Lighthouse.		Happening 1: Ship gets hit by first blink from Main Lighthouse.	Happening 2: Main Lighthouse blinks second time.
(Lighthouse space) (Lighthouse time)	$x = 0$ $t = 0$	$x = -vc/(c-v)$ $t = c/(c-v)$	$x = 0$ $t = 2.00$
(Ship space) (Ship time)	$x' = 0$ $t' = 0$	$x' = 0$ $t' = (v+c)\Delta/c$	$x' = 2v\Delta$ $t' = 2\Delta$

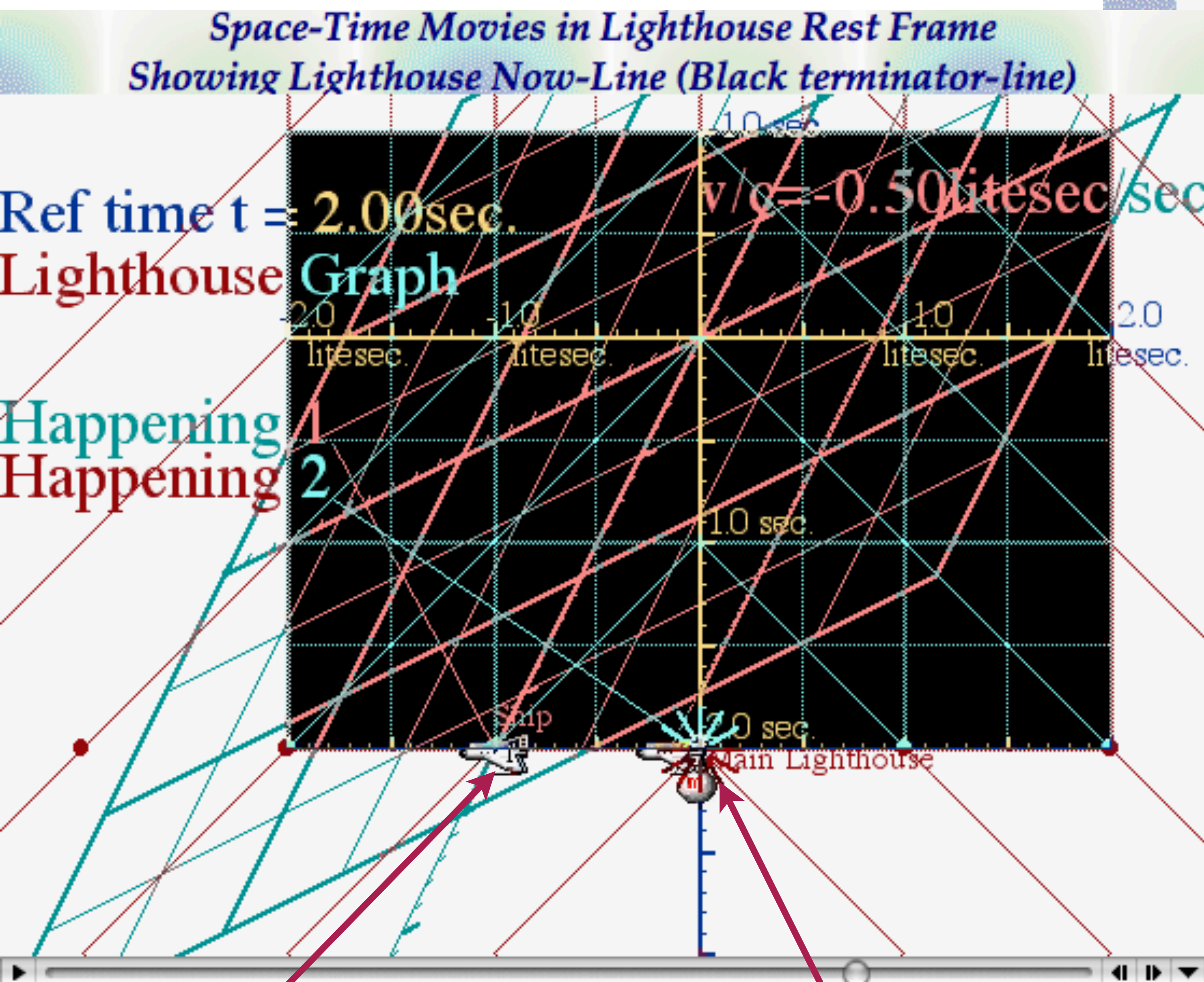
RelativIt Web Simulation
Relativistic Events in
Main Lighthouse's Frame

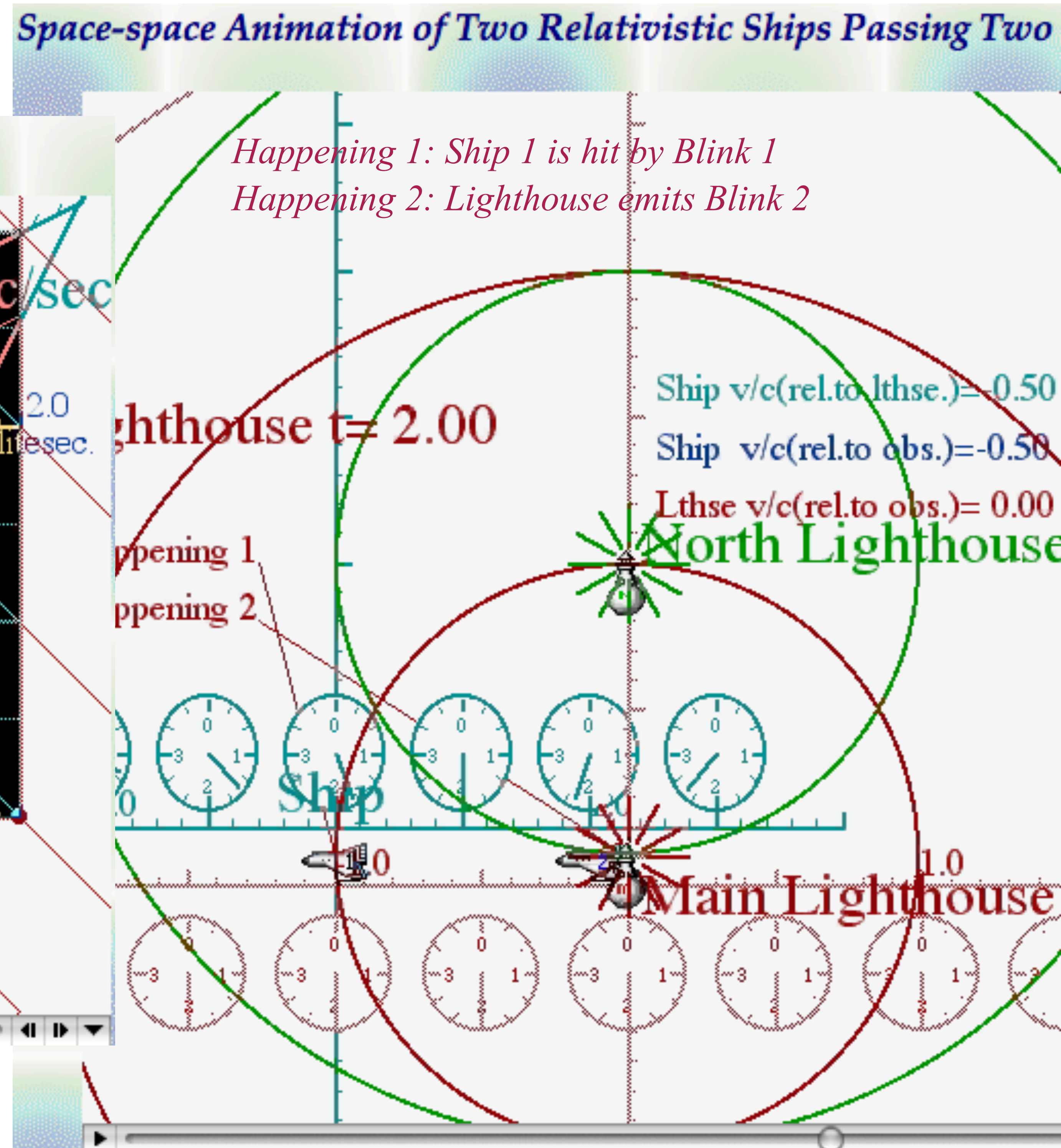
RelativIt Web Simulation
Relativistic Events in
Ship's Space-Time Frame

Fig. 2.A.3 Happening 1 (1st blink hits ship) and 2 (2nd blink at Main) both happen at t=2.

Lecture 26 ends here

Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at t=2.00sec.





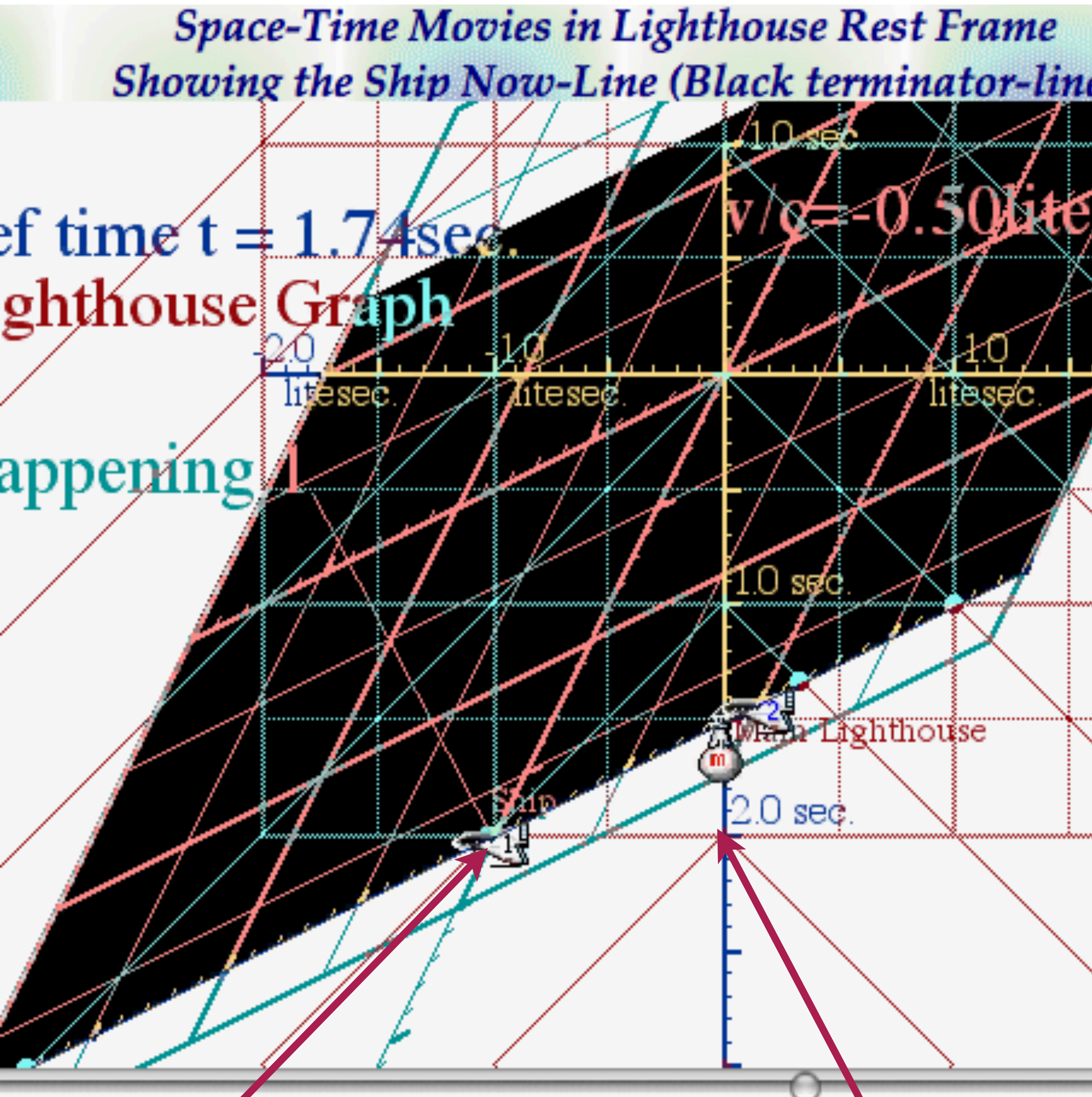
Happening 1

Happening 2

www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php

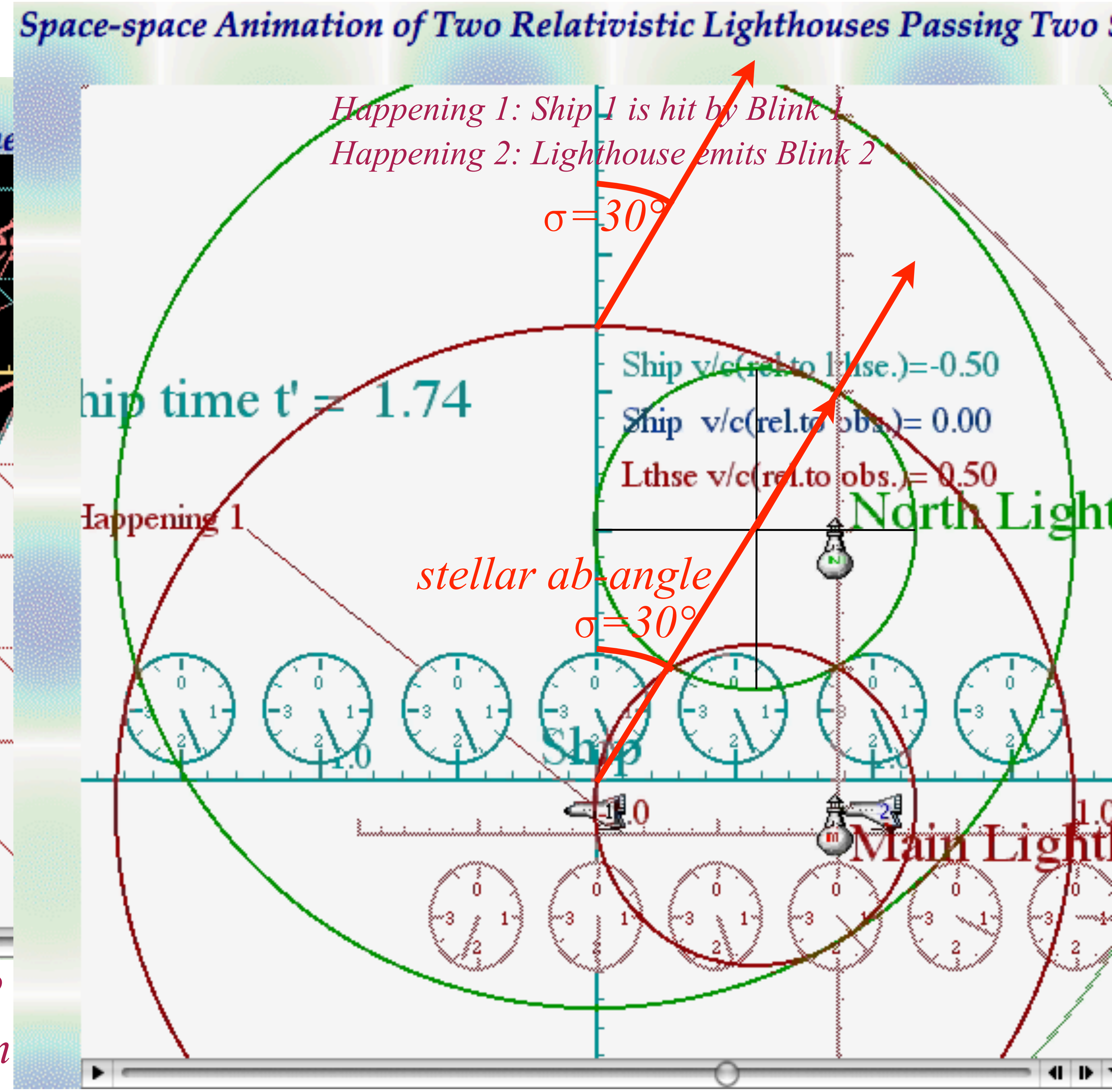
How Minkowski's space-time graphs help visualize relativity (Here:r=atanh(1/2)=0.549,

Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at t=2.00sec.
...but, in Ship frame Happening 1 is at t′=1.74 and Happening 2 is at t′=2.30sec.



Happening 1

Happening 2 won't happen 'til t=2.00

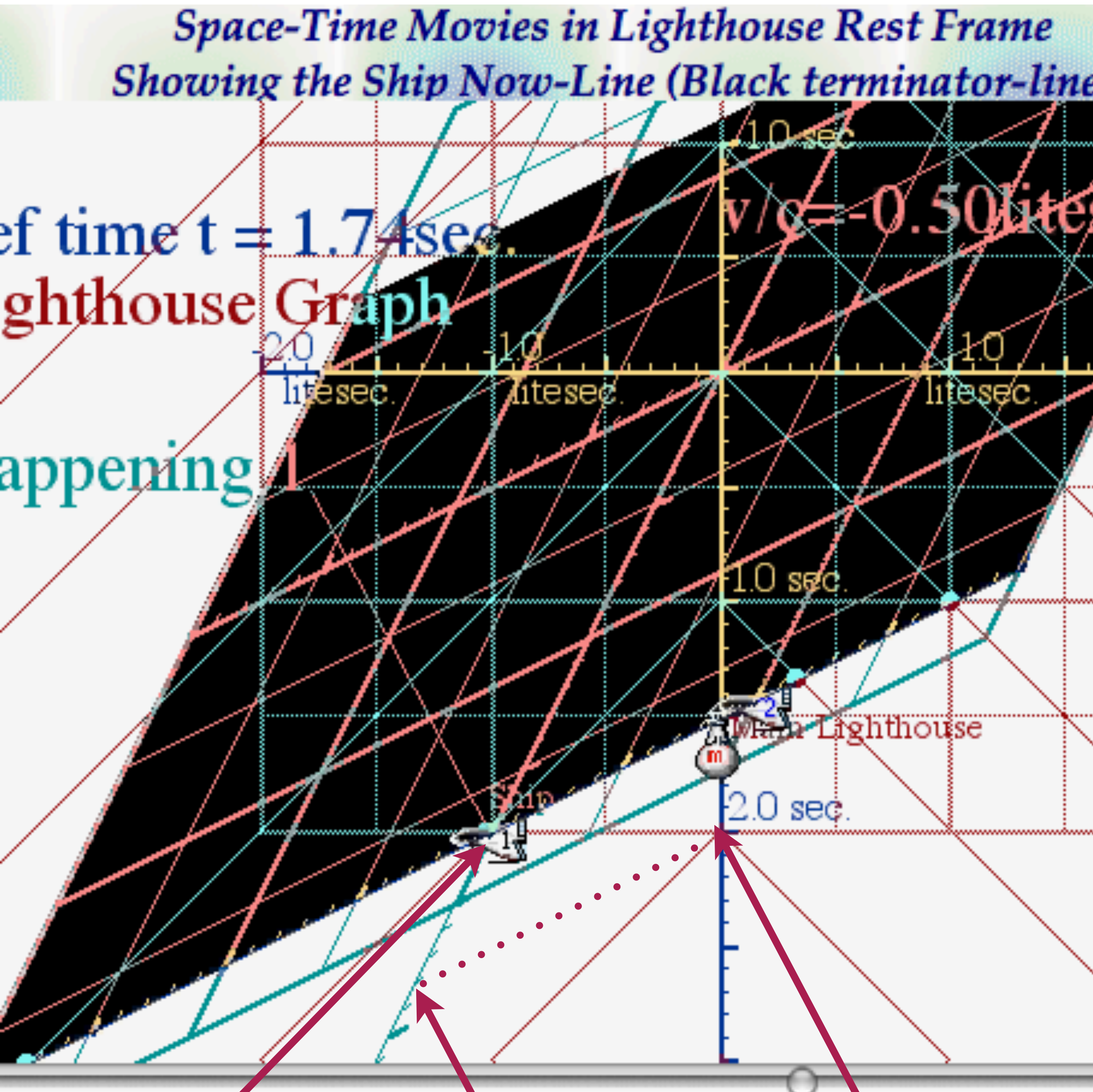


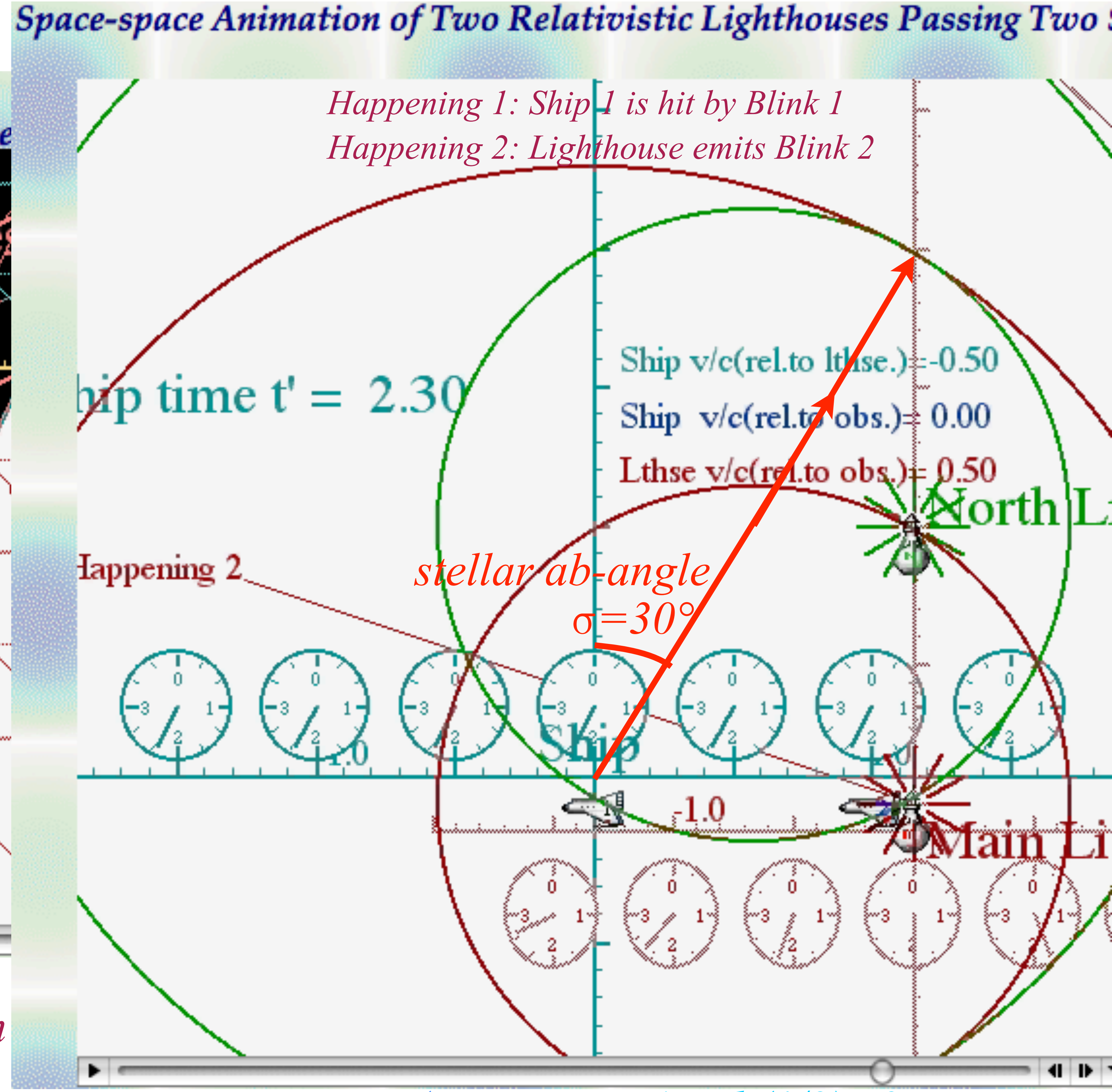
(Here: ρ=Atanh(1/2)=0.55,
and: σ=Asin(1/2)=0.52 or 30°)

www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php

Note that in Lighthouse frame Happening 1 is simultaneous with Happening 2 at t=2.00sec.

...but, in Ship frame Happening 1 is at t'=1.74 and Happening 2 is at t'=2.30sec.





Happening 1

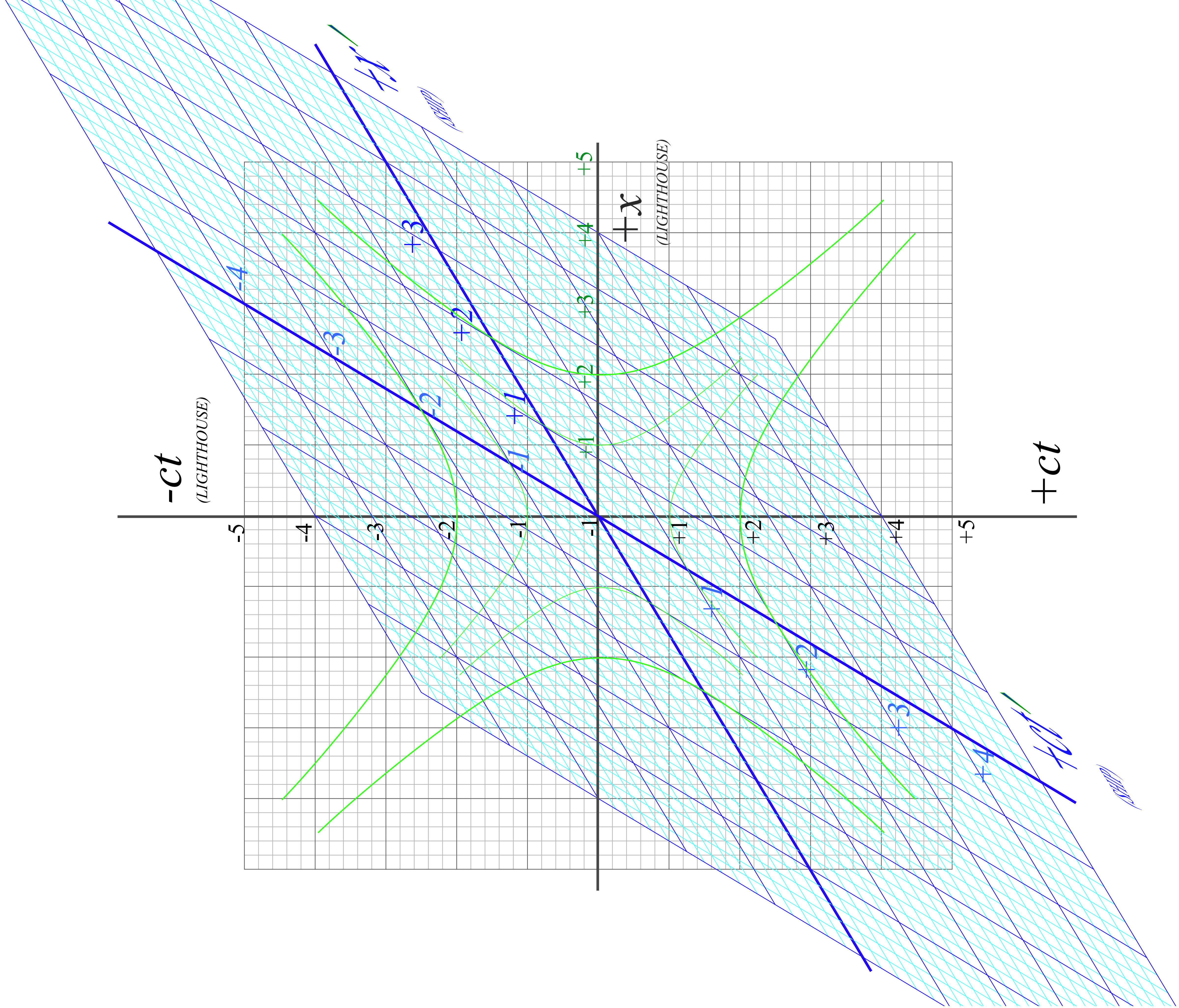
Happening 2 won't happen 'til t=2.00

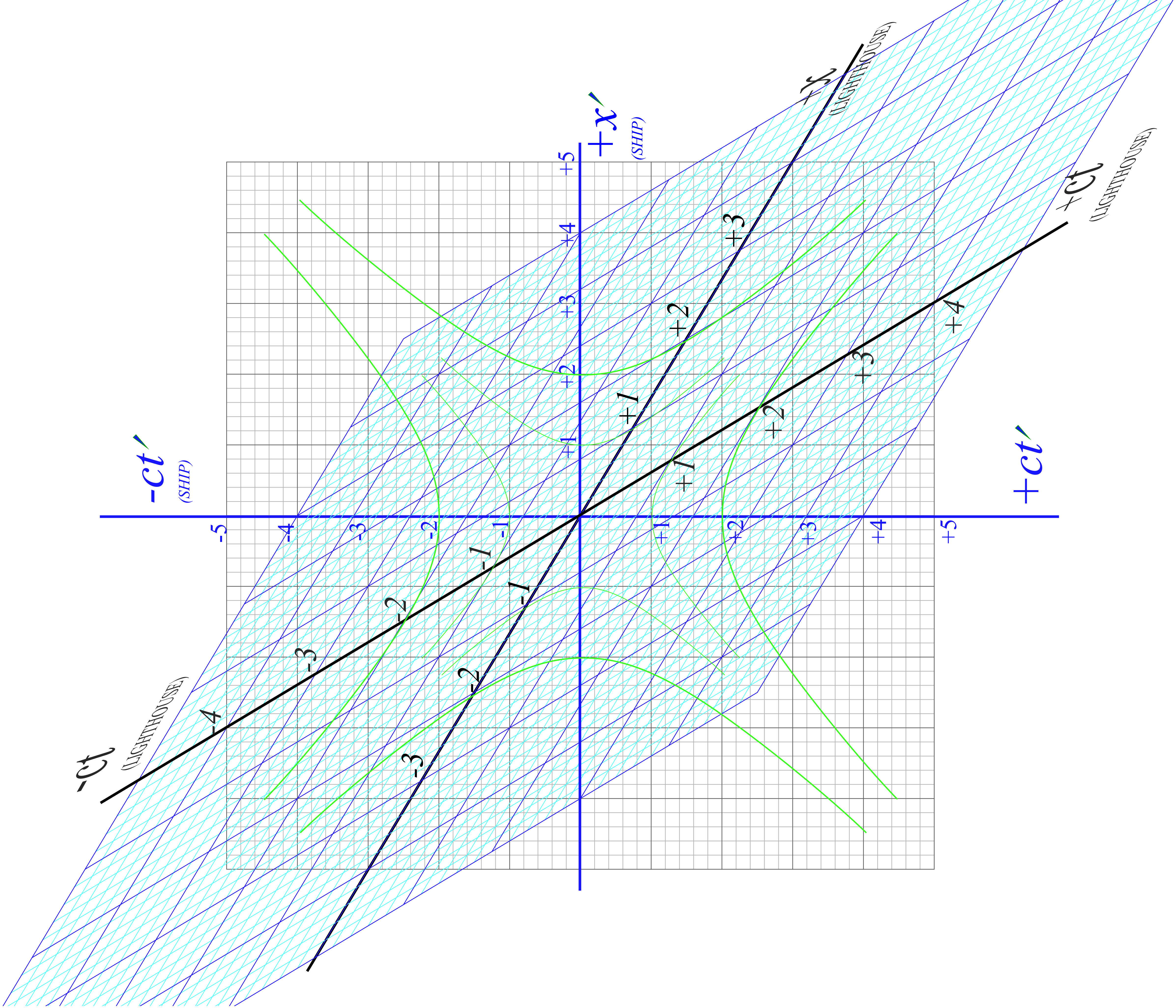
That is t'=2.30 ship time

(Here: ρ=Atanh(1/2)=0.55,

and: σ=Asin(1/2)=0.52 or 30°)

www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php


+x
(LIGHTHOUSE)
-ct
(LIGHTHOUSE)
+ct
+x'
(SHIP)
+ct'
(SHIP)
-5
-4
-3
-2
-1
+1
+2
+3
+4
+5


+x′
(SHIP)
-ct′
(SHIP)
+ct′
+x
(LIGHTHOUSE)
+ct
(LIGHTHOUSE)
-ct
(LIGHTHOUSE)
-5
-4
-3
-2
-1
+1
+2
+3
+4
+5


+10
+9
+8
+7
+6
+5
+4
+3
+2
+1
-3
-2
-1
O
+1
+2
+3
+4
+5
+6
+7
+8
+9
+10