Lecture 29 Relawavity-Applications 3
Tuesday 4.26.2016

Relawavity: Relativistic wave mechanics VI. Space-time geometry

(Unit 3 p.28-42 - 4.26.16)

➔ Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)
and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion
Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ
Review of proper time relations and basis of Epstein's cosmic speedometer
Epstein vs Einstein-Minkowski geometry of relativity
Einstein time dilation
Lorentz space contraction
Time-simultaneity-breaking
Velocity addition

Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

➔ Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

Lorentz space contraction

Time-simultaneity-breaking

Velocity addition

Twin-paradox resolution in space-proper-time

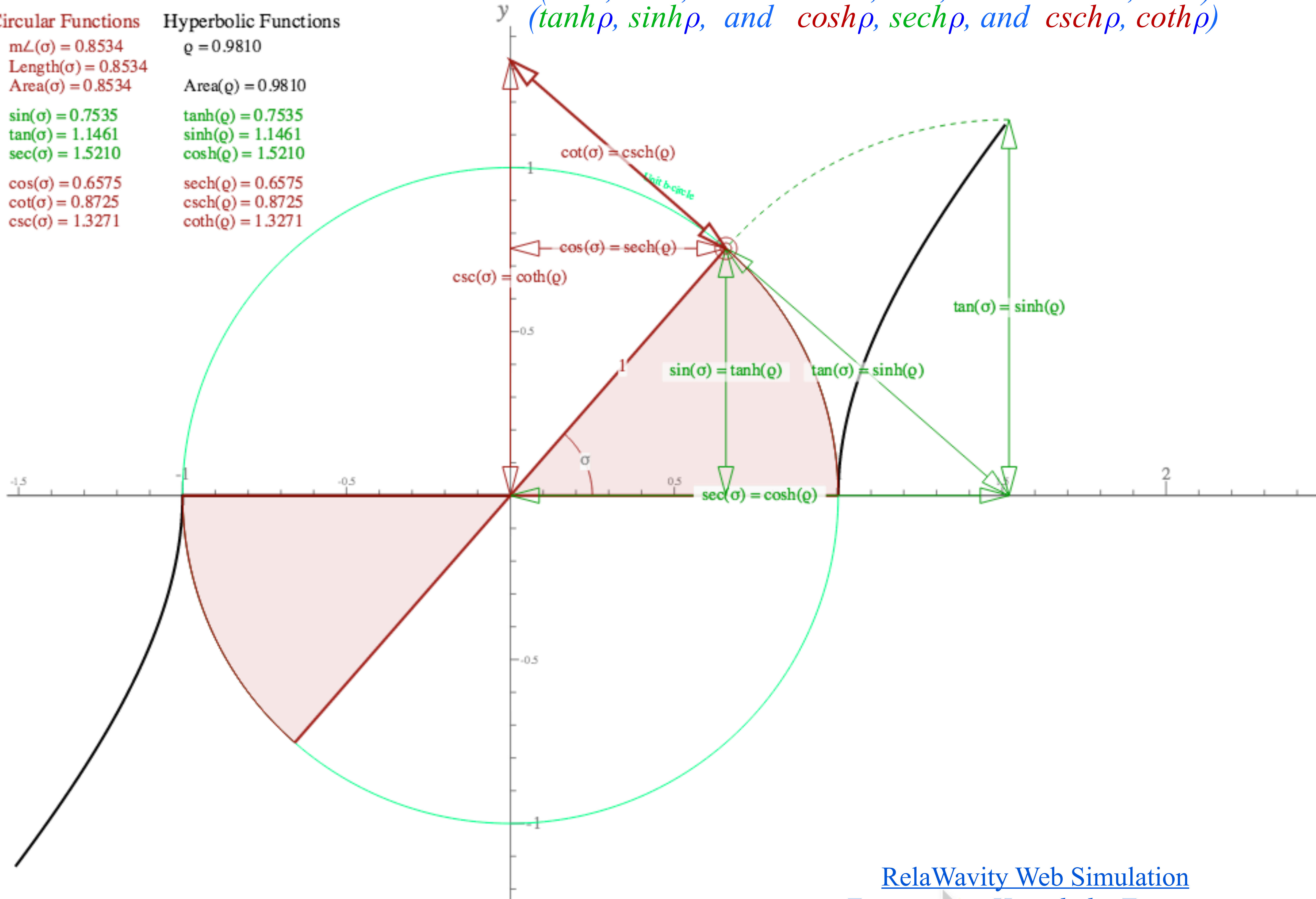
Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

A Hyper-trigonometry *Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)*

(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)

(tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ)

Circular Functions	Hyperbolic Functions
m∠(σ) = 0.8534	ϱ = 0.9810
Length(σ) = 0.8534	
Area(σ) = 0.8534	Area(ϱ) = 0.9810
sin(σ) = 0.7535	tanh(ϱ) = 0.7535
tan(σ) = 1.1461	sinh(ϱ) = 1.1461
sec(σ) = 1.5210	cosh(ϱ) = 1.5210
cos(σ) = 0.6575	sech(ϱ) = 0.6575
cot(σ) = 0.8725	csch(ϱ) = 0.8725
csc(σ) = 1.3271	coth(ϱ) = 1.3271



RelaWavity Web Simulation

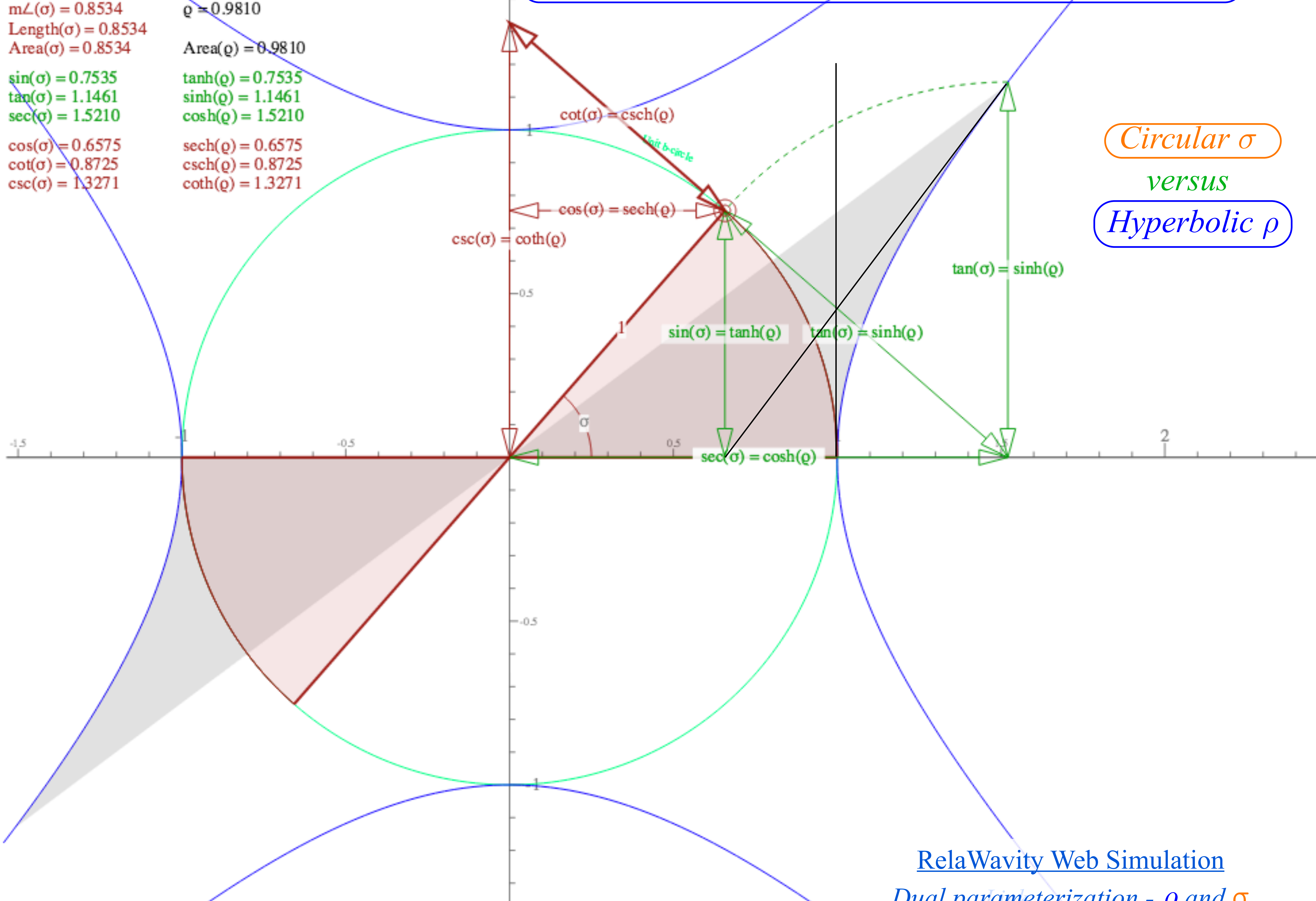
Transition to Hyperbolic Functions

Same Link

A Hyper-trigonometry *Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)*

(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)

(tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ)



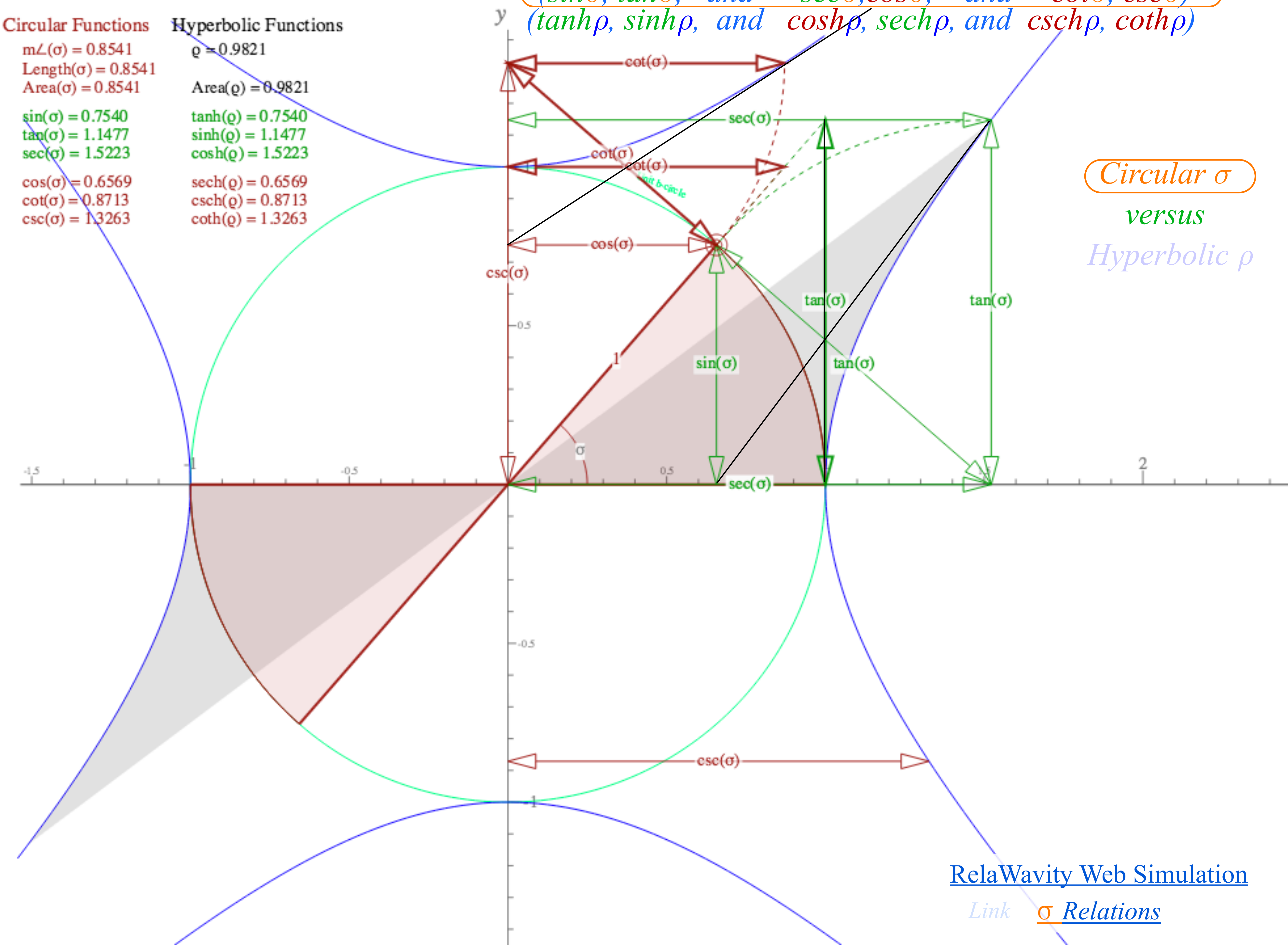
[RelaWavity Web Simulation](#)

[*Dual parameterization - ρ and σ*](#)

A Hyper-trigonometry *Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)*

(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)

(tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ)



Circular σ

versus

Hyperbolic ρ

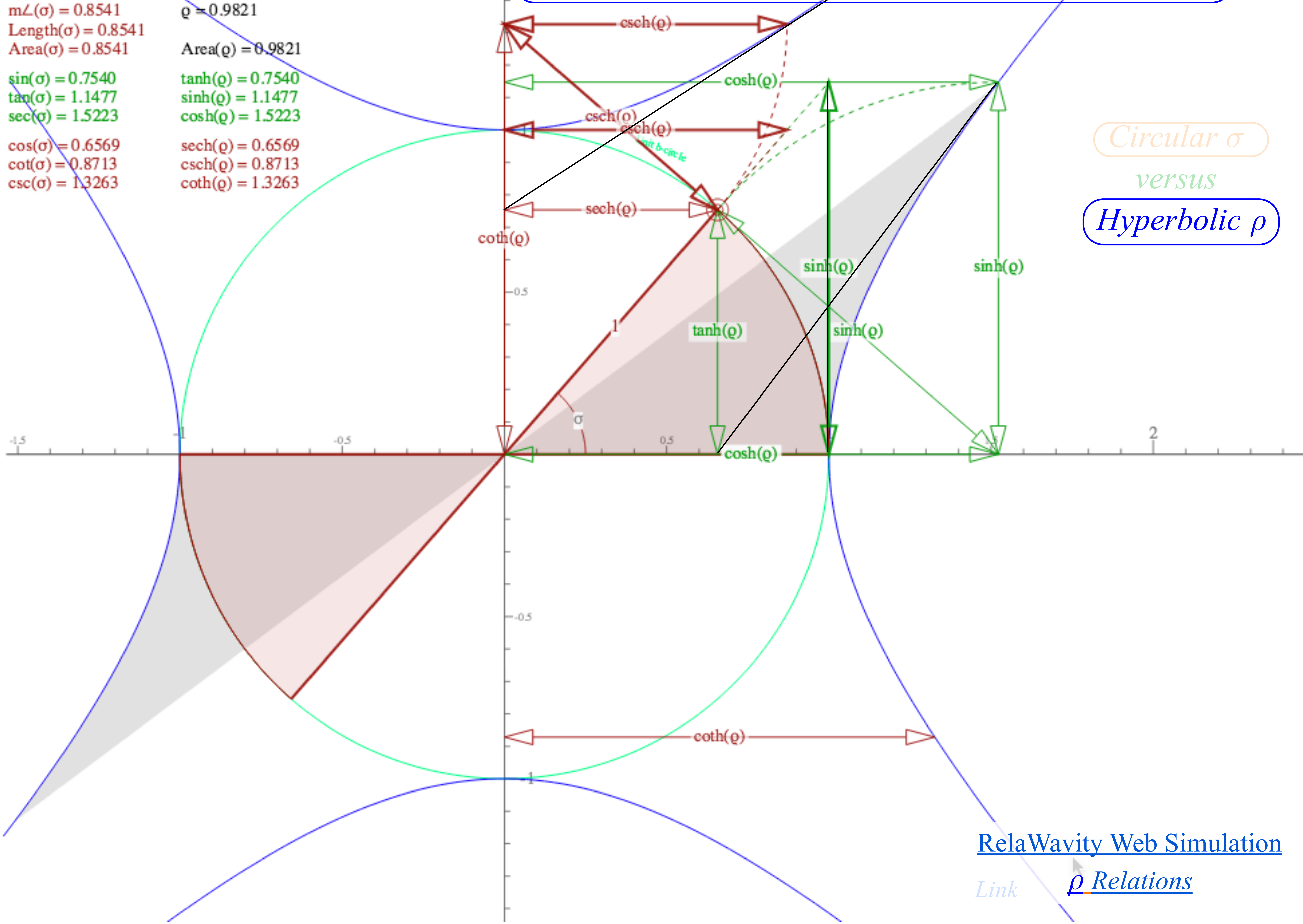
RelaWavity Web Simulation

Link *σ Relations*

A Hyper-trigonometry *Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)*

(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)

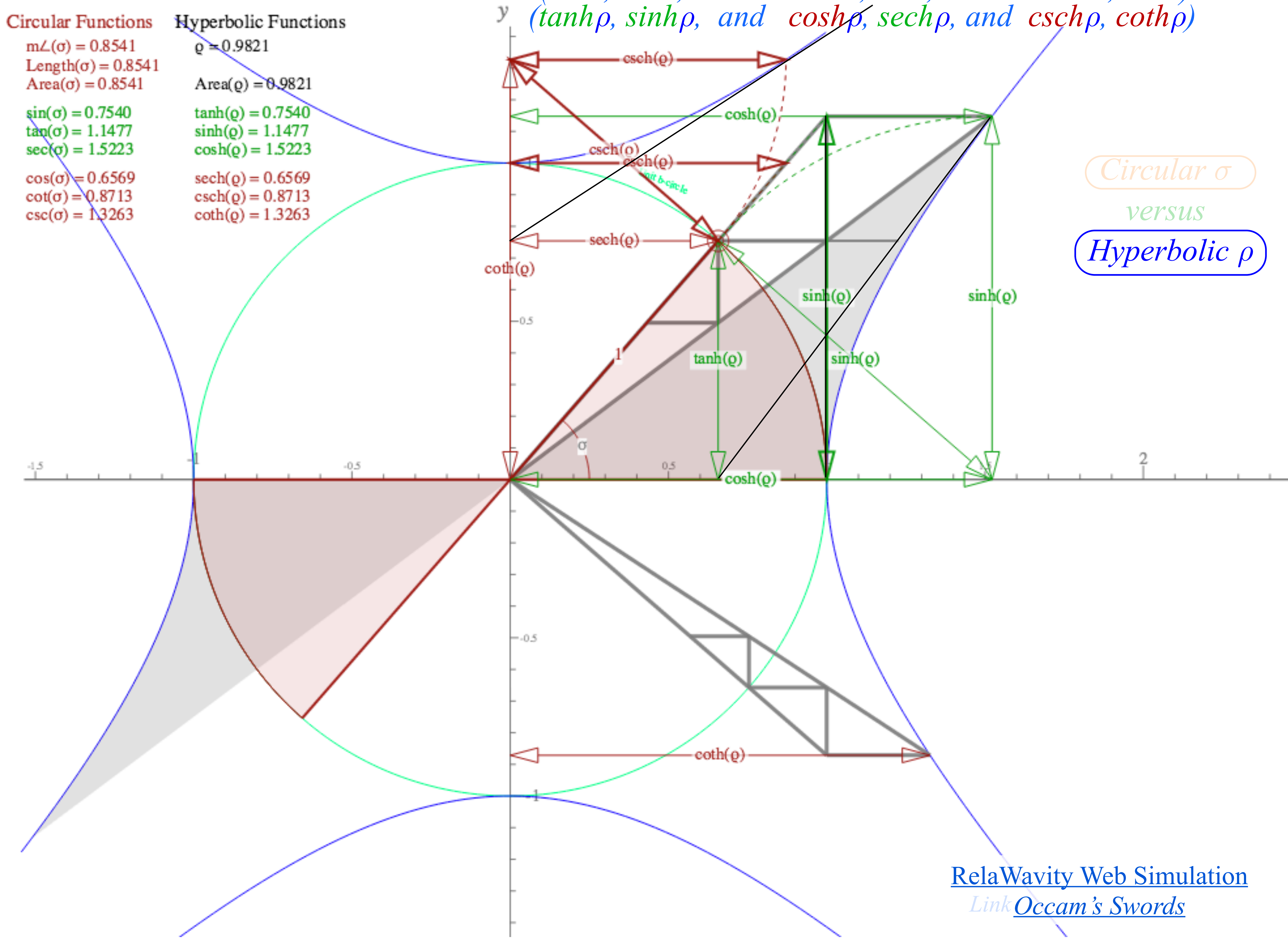
(tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ)



A Hyper-trigonometry *Hyper-(sine, tangent, and secant, cosine, and cotangent, cosecant)*

(sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ)

(tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ)



[RelaWavity Web Simulation](#)

Link [Occam's Swords](#)

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)
and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

➔ Review of "Occam-sword" geometry and wave parameters for phase and group motion
Wave parameter symmetry points

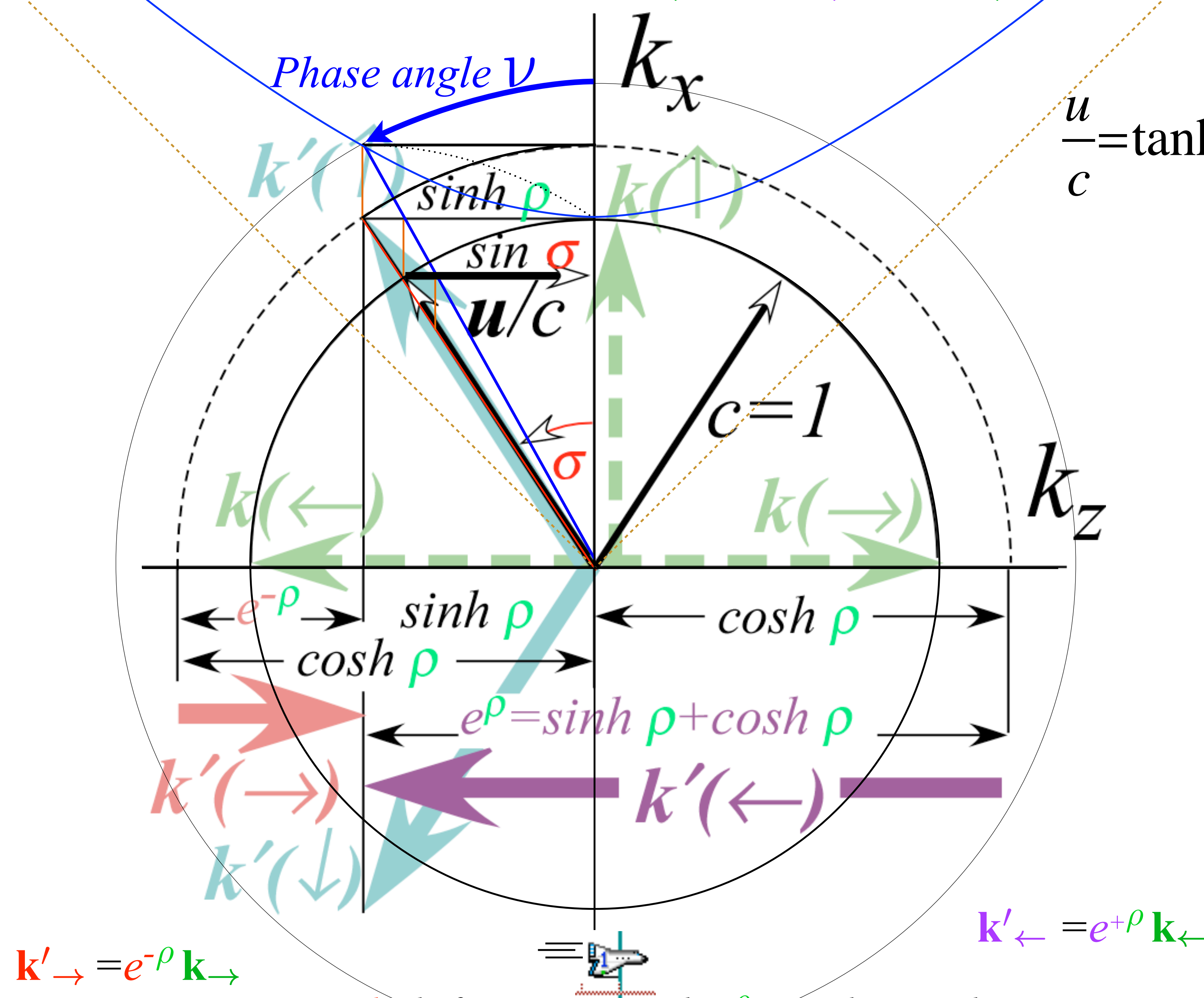
Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ
Review of proper time relations and basis of Epstein's cosmic speedometer
Epstein vs Einstein-Minkowski geometry of relativity
Einstein time dilation
Lorentz space contraction
Time-simultaneity-breaking
Velocity addition
Twin-paradox resolution in space-proper-time
Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

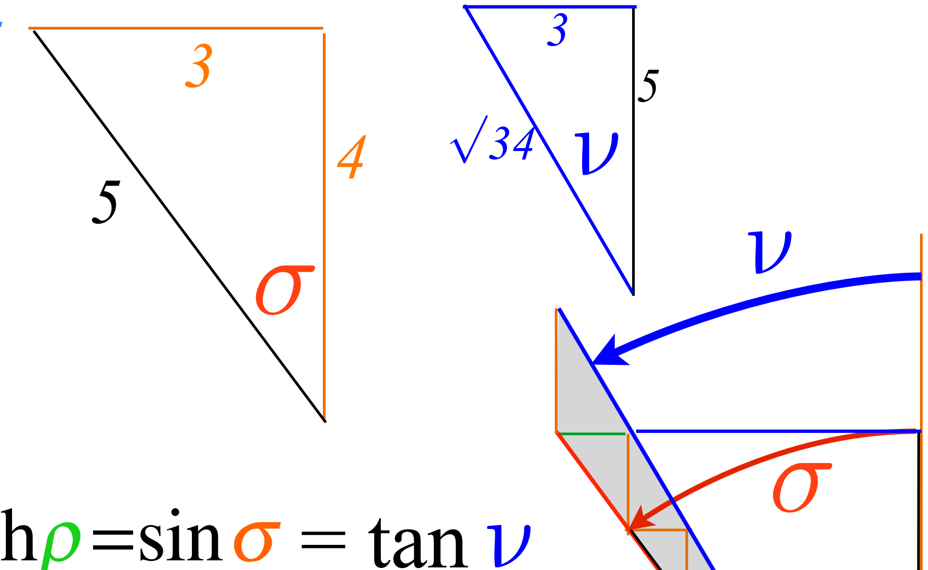
Pattern recognition: Occam's Sword for $u/c=3/5$

Cosmic speedometer of Epstein is based upon geometry of

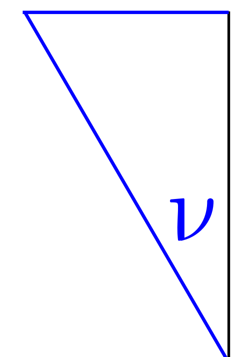
Doppler shifts of colliding CW $\mathbf{k}_{\rightarrow}$ *into* $\mathbf{k}'_{\rightarrow} = e^{-\rho}\,\mathbf{k}_{\rightarrow}$

and $\mathbf{k}_{\leftarrow}$ *into* $\mathbf{k}'_{\leftarrow} = e^{+\rho}\,\mathbf{k}_{\leftarrow}$



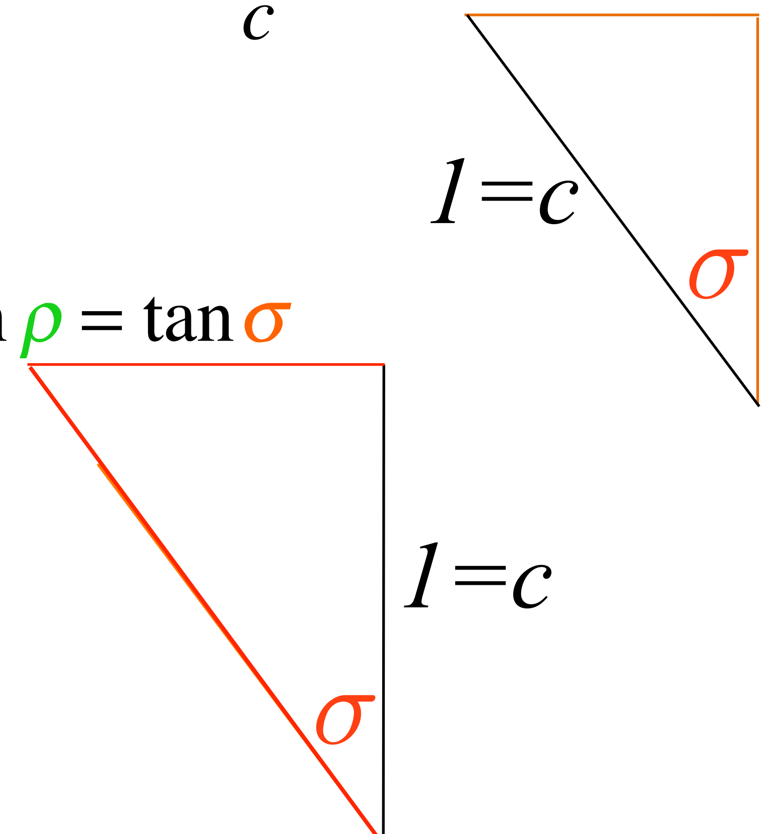


$$\frac{u}{c} = \tanh\rho = \sin\sigma = \tan\nu$$



$$\frac{u}{c} = \tanh\rho = \sin\sigma$$

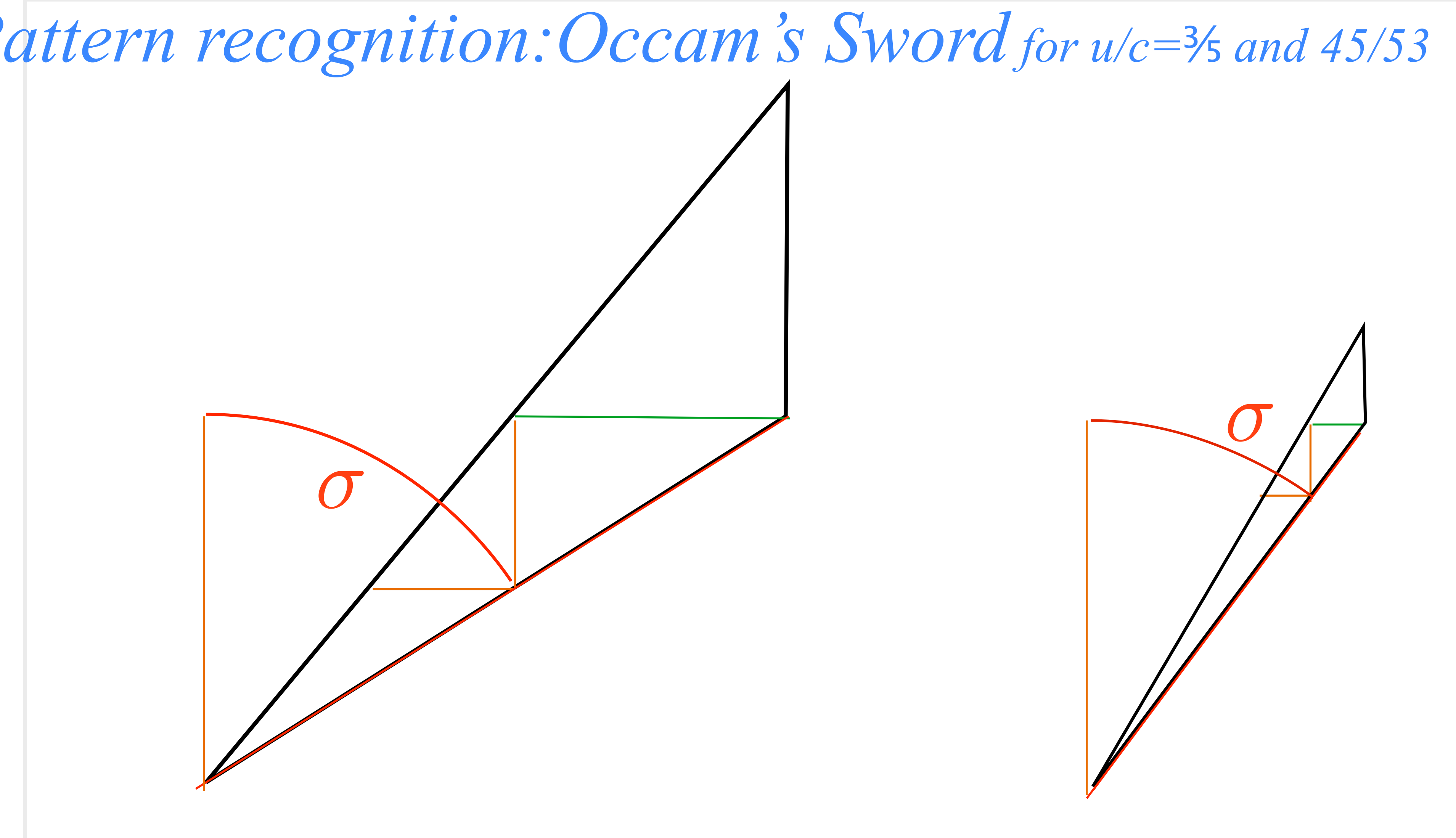
$$\sinh\rho = \tan\sigma$$

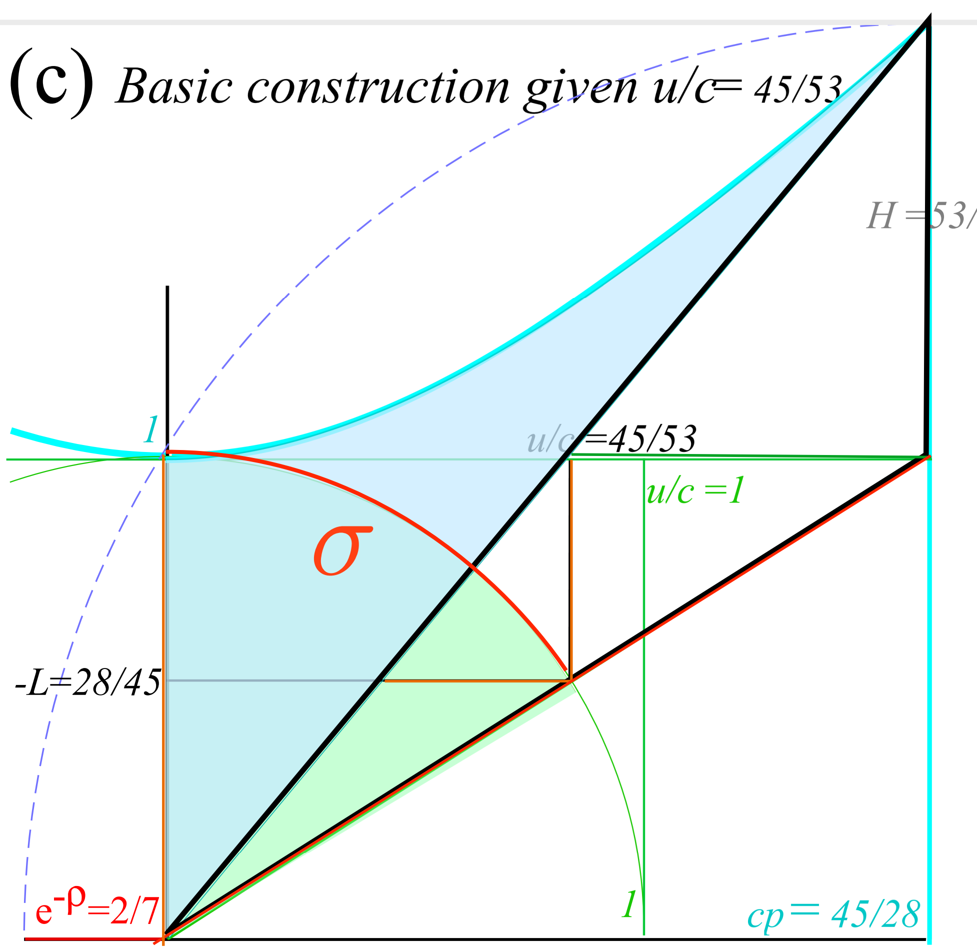


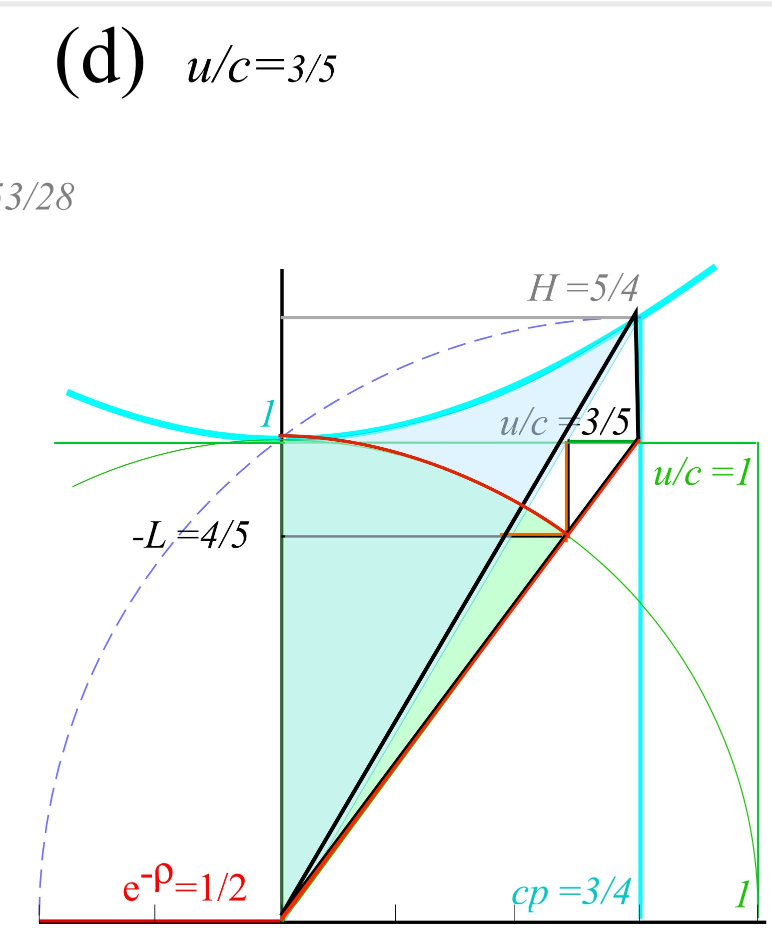
Red-shift exponential $e^{-\rho} = \cosh\rho - \sinh\rho$

Blue-shift exponential $e^{+\rho} = \cosh\rho + \sinh\rho$

Pattern recognition: Occam's Sword *for u/c=⅗ and 45/53*





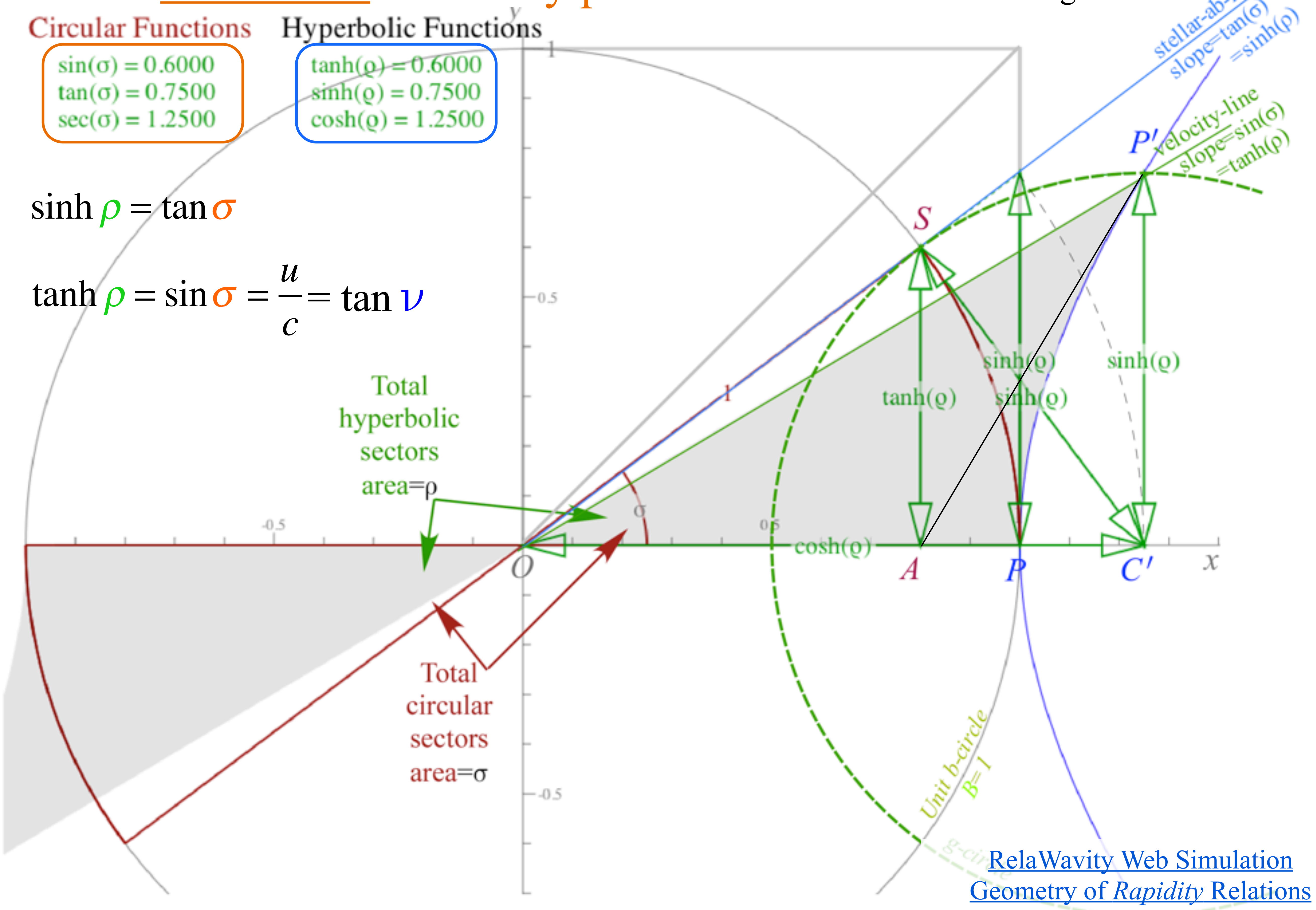


Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to Transverse relativity parameter: Stellar aberration angle σ

$$\sinh\rho = \tan\sigma$$

$$\tanh\rho = \sin\sigma = \frac{u}{c} = \tan\nu$$



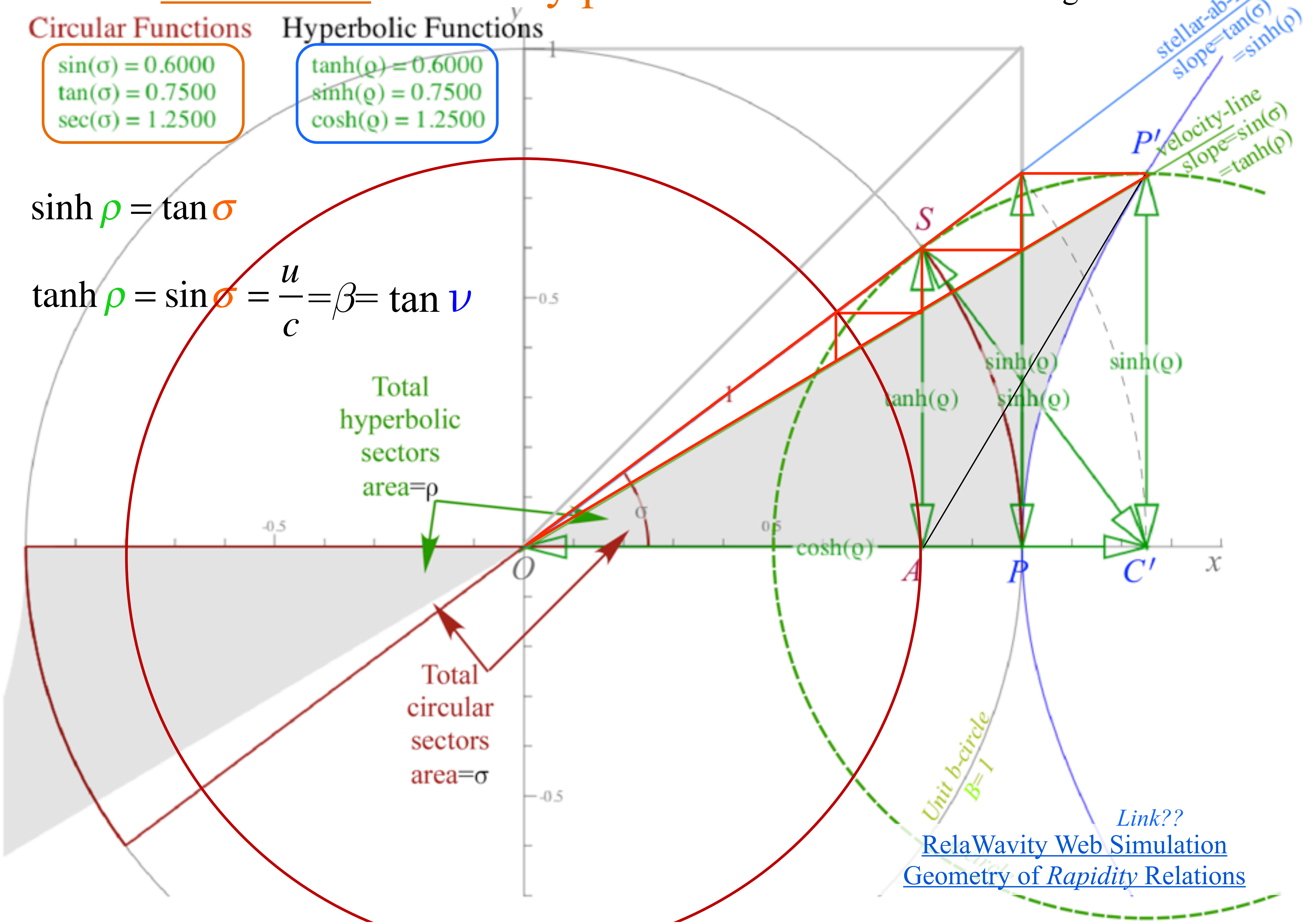
RelaWavity Web Simulation
Geometry of *Rapidity* Relations

Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to Transverse relativity parameter: Stellar aberration angle σ

$$\sinh \rho = \tan \sigma$$

$$\tanh \rho = \sin \sigma = \frac{u}{c} = \beta = \tan \nu$$



Link??

[RelaWavity Web Simulation](#)

[Geometry of *Rapidity* Relations](#)

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

➔ Review of "Occam-sword" geometry and wave parameters for phase and group motion ➔

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

Lorentz space contraction

Time-simultaneity-breaking

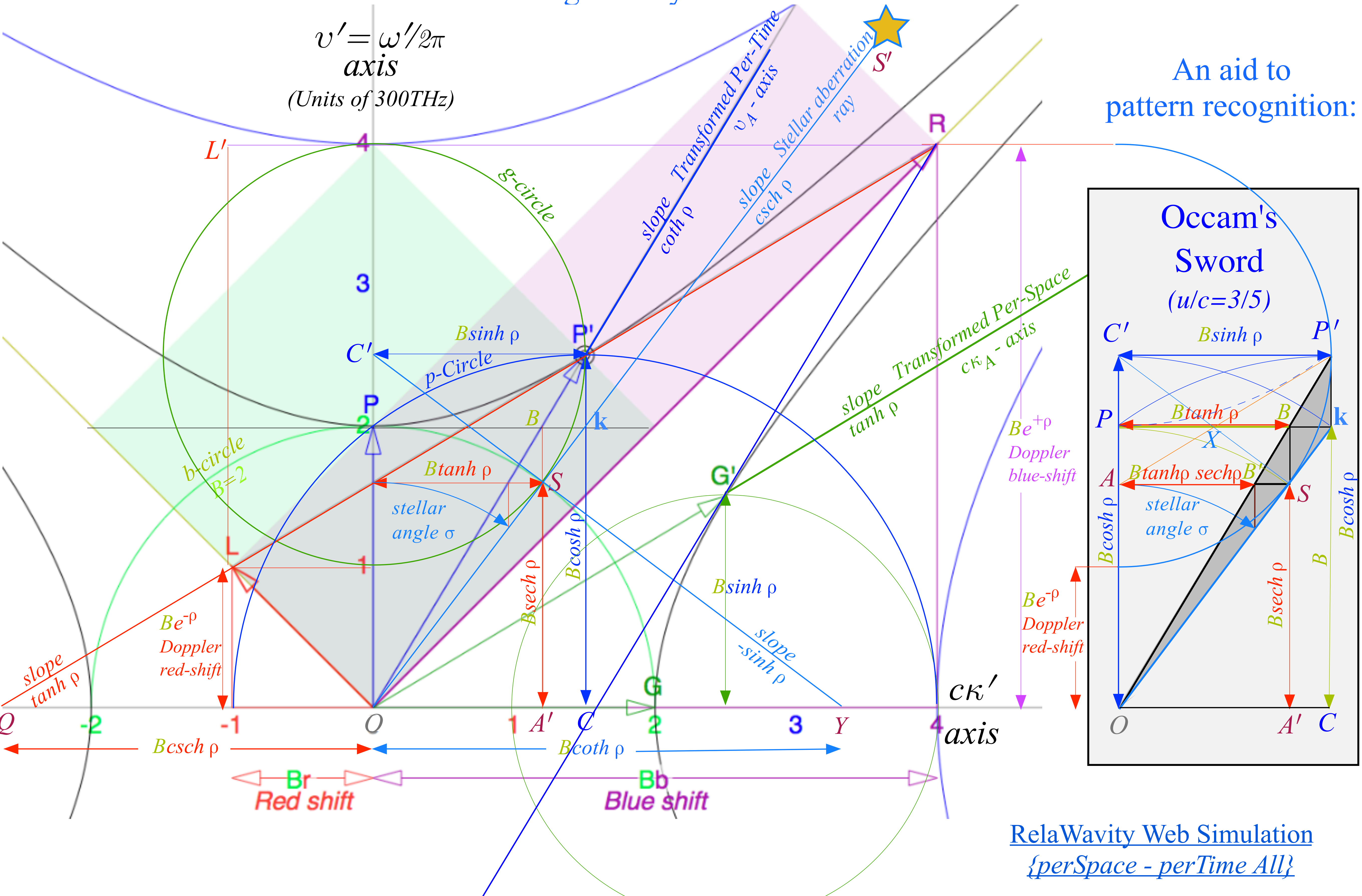
Velocity addition

Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) =(\omega,ck_x,ck_y,ck_z)$ transformation

Summary of optical wave parameters for relativity and QM

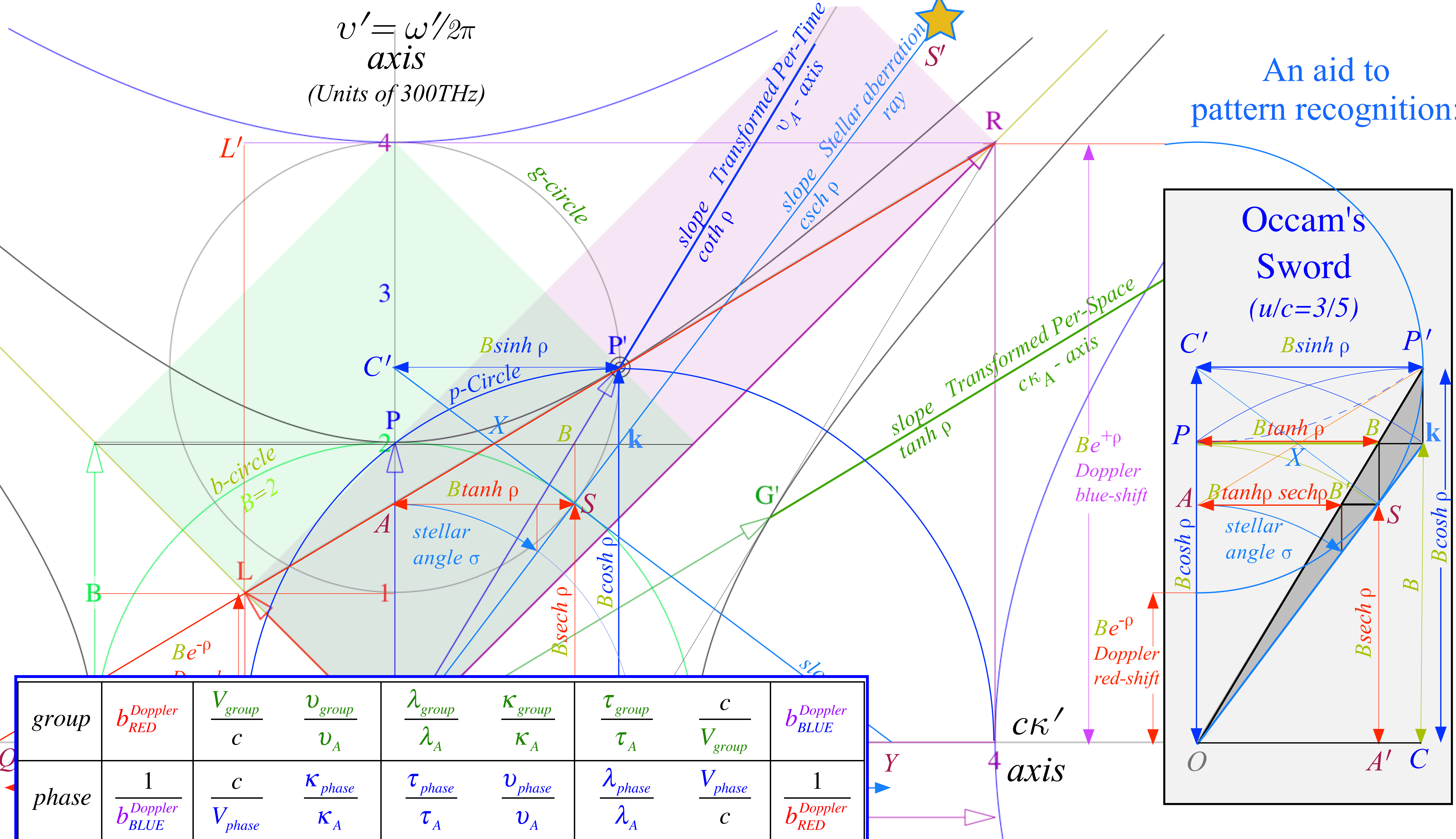
...and their geometry



[RelaWavity Web Simulation {perSpace - perTime All}]

Link??

An aid to pattern recognition:



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Table of 12 wave parameters (includes inverses) for relativity

...and 8 values for *u/c=3/5*

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

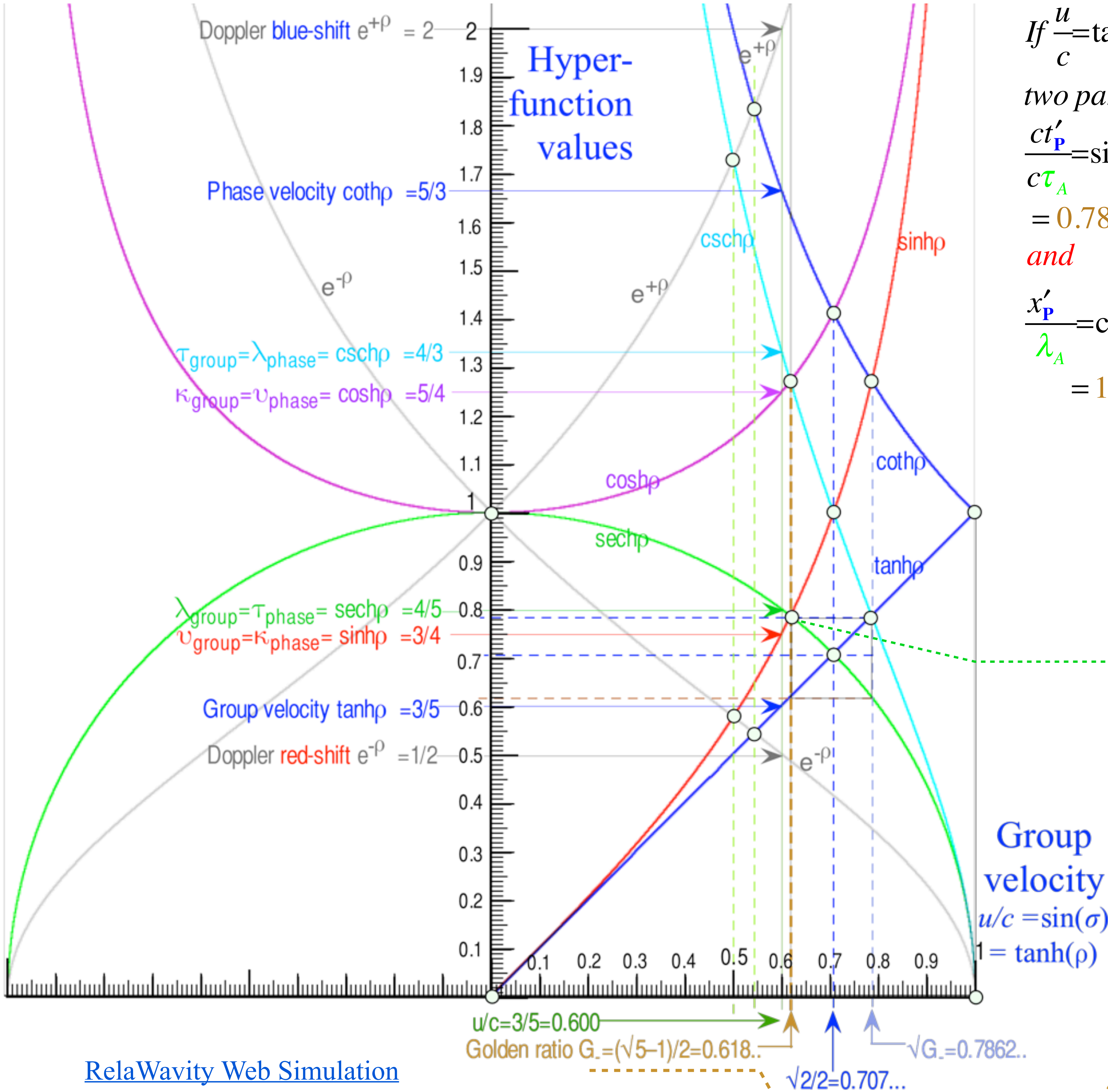
Lorentz space contraction

Time-simultaneity-breaking

Velocity addition

Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation



$If\ \frac{u}{c} = \tanh\rho = 0.618...(Golden\text{-}Mean\ G\_)$

two parameters become exactly equal :

$$\frac{ct'_P}{c\tau_A} = \sinh\rho = \frac{\lambda_{group}}{\lambda_A} = \frac{\tau_{phase}}{\tau_A} = \text{sech}\,\rho$$

$$= 0.786.. = \sqrt{G\_} \qquad = 0.786..$$

and

$$\frac{x'_P}{\lambda_A} = \cosh\rho = \frac{\lambda_{phase}}{\lambda_A} = \frac{\tau_{group}}{\tau_A} = \text{csch}\,\rho$$

$$= 1.272.. = 1/\sqrt{G\_} \qquad = 1.272..$$

Solve :

$\text{sech}\,\rho = \sinh\rho$

or:

$\sinh\rho\cosh\rho = 1$

or:

$\sinh 2\rho = 2$

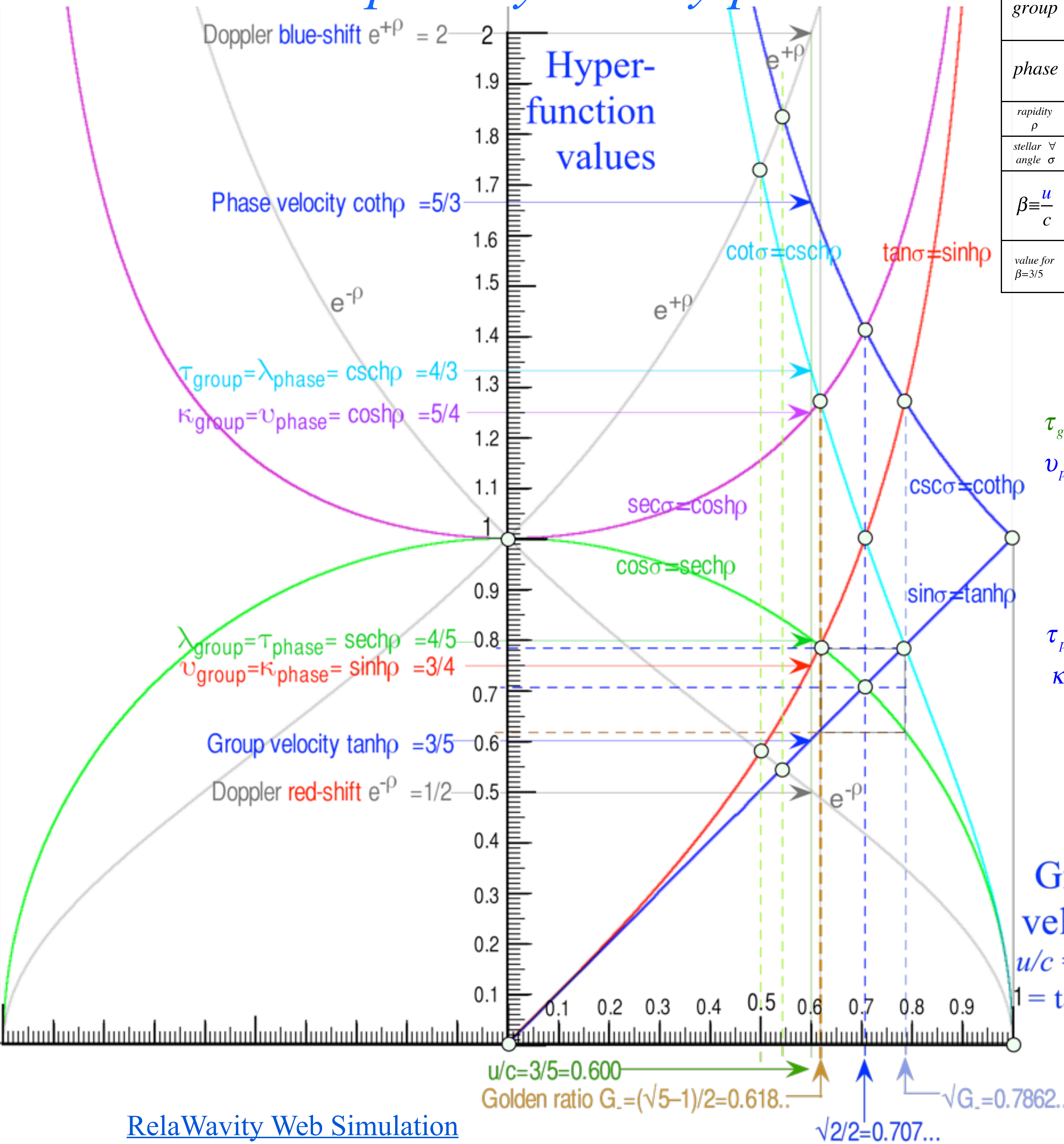
$\rho = \frac{1}{2}\sinh^{-1}2 = 0.7218...$

$\tanh\rho = 0.618... = \frac{\sqrt{5}-1}{2}$

[RelaWavity Web Simulation](#)
[Relativistic *Terms (Dual plot w/expanded table)*](#)

Parameter-space symmetry points

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



$V_{phase}=\coth\rho$

$\upsilon_{phase}=\kappa_{group}=\cosh\rho$

cross at $\frac{u}{c}=\frac{\sqrt{2}}{2}$

$\tau_{group}=\lambda_{phase}=\text{csch}\rho$

$\upsilon_{phase}=\kappa_{group}=\cosh\rho$

cross at $\frac{u}{c}=\frac{\sqrt{5}-1}{2}$

$V_{phase}=\coth\rho$

$\kappa_{phase}=\upsilon_{group}=\sinh\rho$

cross at $\frac{u}{c}=\sqrt{\frac{\sqrt{5}-1}{2}}$

$\tau_{group}=\lambda_{phase}=\text{csch}\rho$

$\kappa_{phase}=\upsilon_{group}=\sinh\rho$

cross at $\frac{u}{c}=\frac{\sqrt{2}}{2}$

$\tau_{phase}=\lambda_{group}=\text{sech}\rho$

$\kappa_{phase}=\upsilon_{group}=\sinh\rho$

cross at $\frac{u}{c}=\frac{\sqrt{5}-1}{2}$

$\tau_{group}=\lambda_{phase}=\text{csch}\rho$

$V_{group}=\tanh\rho$

cross at $\frac{u}{c}=\sqrt{\frac{\sqrt{5}-1}{2}}$

$\tau_{phase}=\lambda_{group}=\text{sech}\rho$

$V_{group}=\tanh\rho$

cross at $\frac{u}{c}=\frac{\sqrt{2}}{2}$

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*

A neo-liberal trigonometry lesson (*sine*, *tangent*, and *secant*) functions of angular sector area σ

Complimentary functions (... *cosine,and cotangent, cosecant*)

Hyper-trigonometry of (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

Functions of hyper-angular sector area ρ related to functions of σ

Each circular trig function has a hyperbolic "country-cousin" function

...and big-party fun was had by all!

Pattern recognition aids and "Occam-sword" geometry

Relating velocity parameters β*=u/c* to *rapidity* ρ to **k**-*angle* σ to *u/c-angle* ν

Relating wave dimensional parameters of phase wave and group wave

Parameter-space symmetry points

➡ Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein geometry for relativistic parameters

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

➤ Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ ➤

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

Lorentz space contraction

Time-simultaneity-breaking

Velocity addition

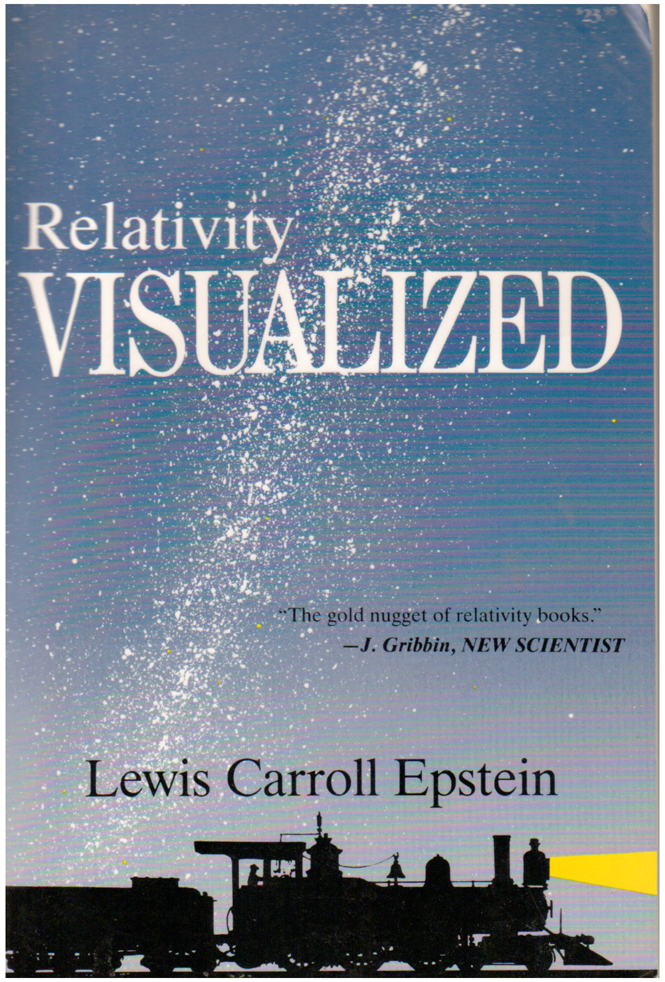
Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)
to a Transverse relativity parameter: Stellar aberration angle σ*

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

We use the notion σ for stellar-aberration-angle (a "flipped-over" ρ).



Epstein seemed resistant to ρ analysis or relations between σ and ρ.

Purchase at:

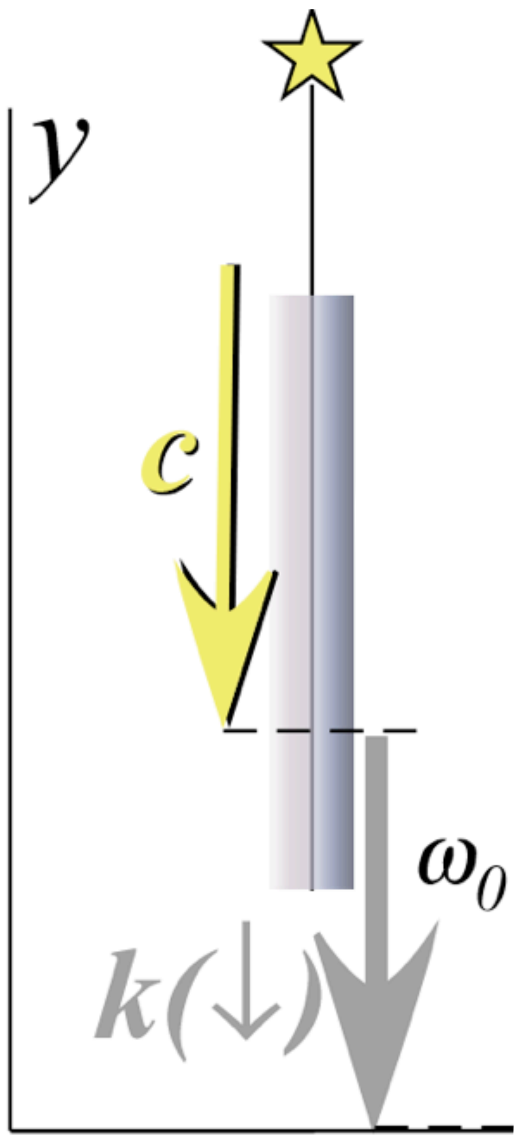
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

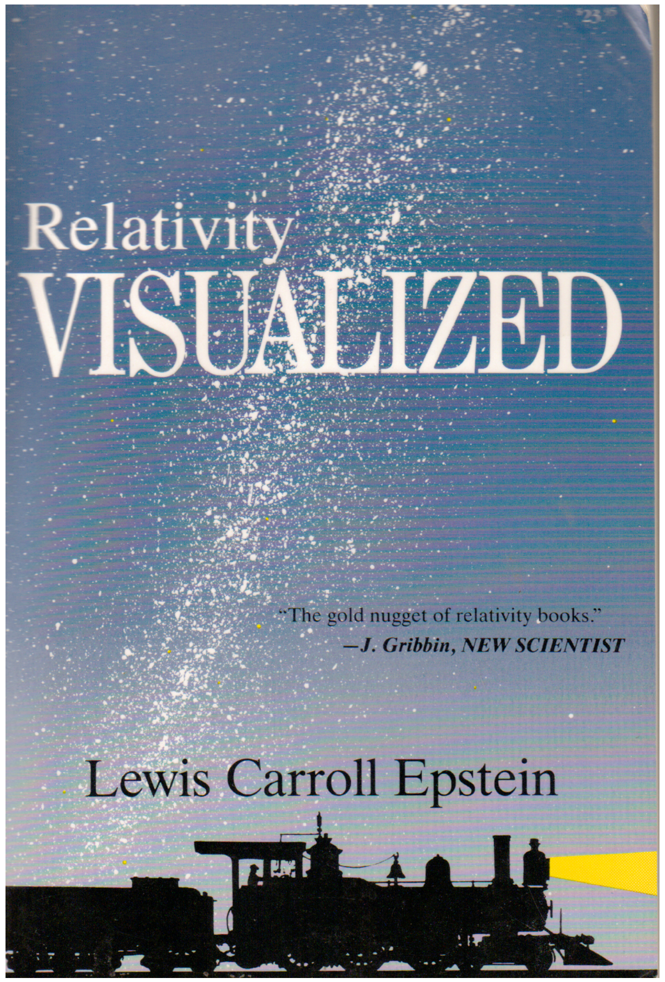
to a Transverse relativity parameter: Stellar aberration angle σ*

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

We use the notion σ for stellar-aberration-angle (a "flipped-over" ρ).





AllBookstores.com

Purchase at: amazon

Epstein seemed resistant to ρ analysis or relations between σ and ρ.

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

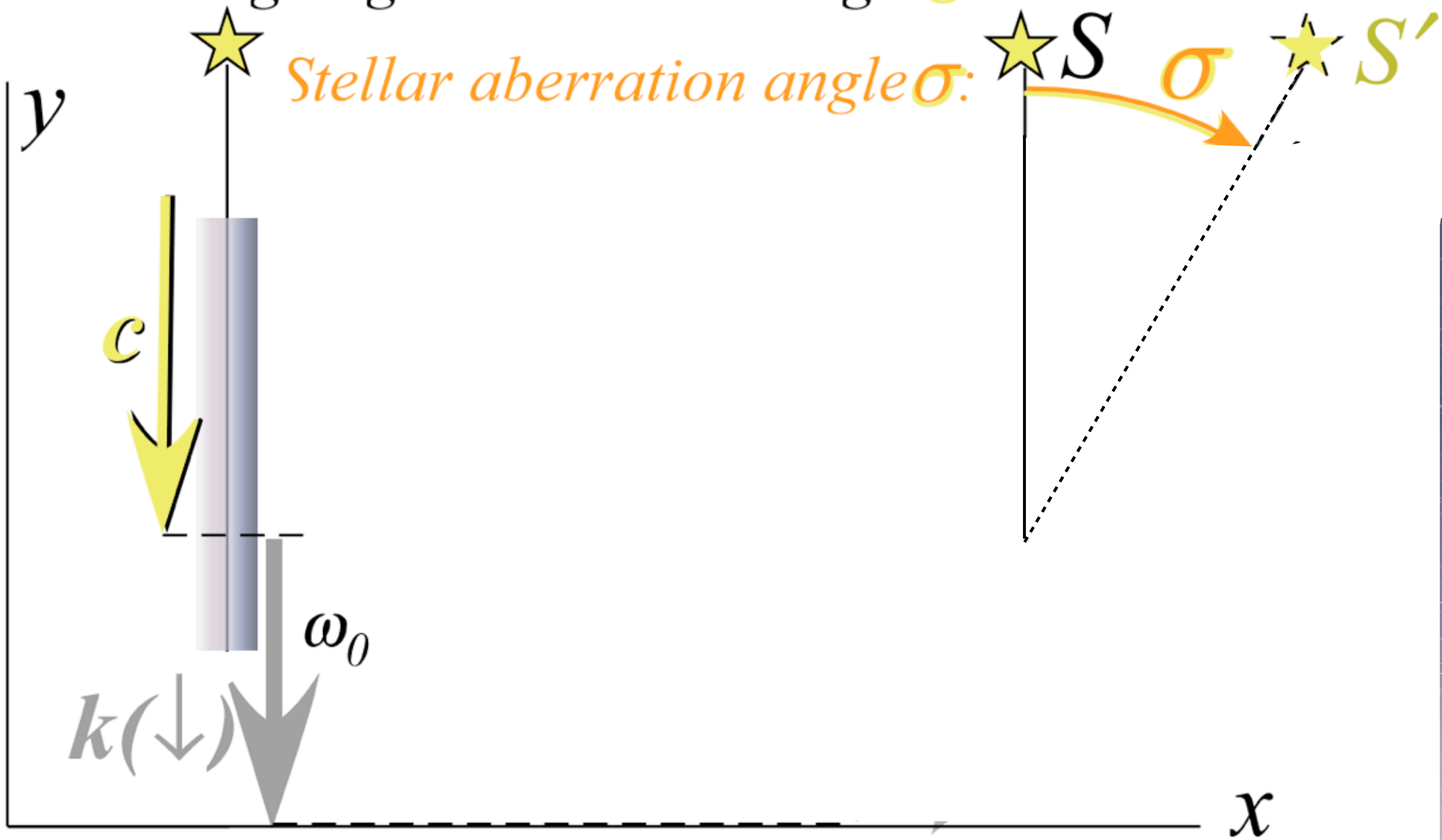
to a Transverse relativity parameter: Stellar aberration angle σ*

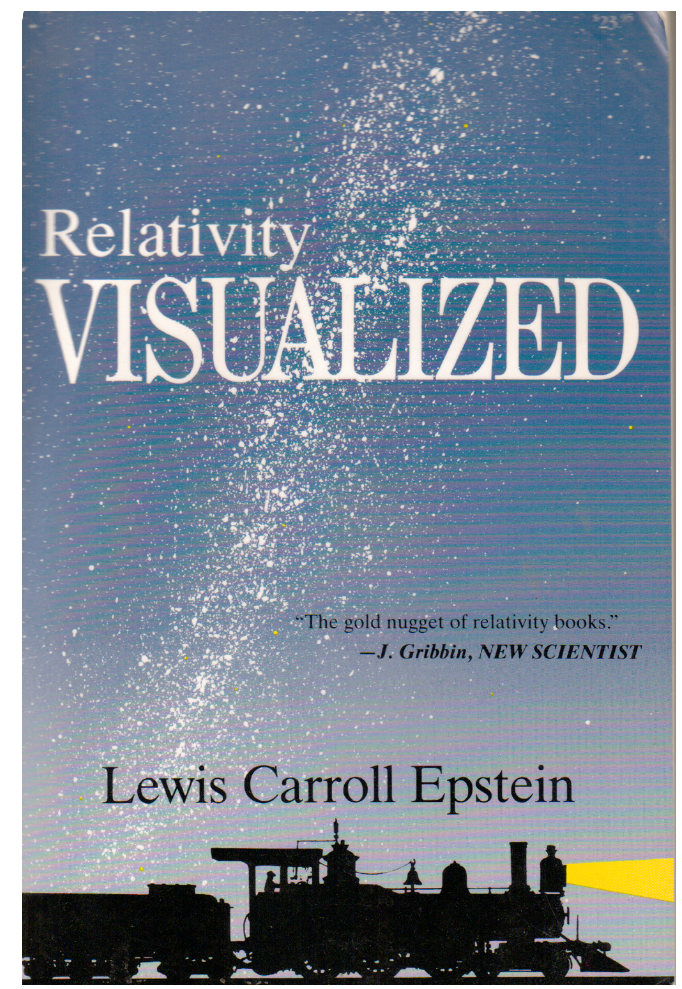
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

We use the notion σ for stellar-aberration-angle (a "flipped-over" ρ).

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*





Epstein seemed resistant to ρ analysis or relations between σ and ρ.

AllBookstores.com

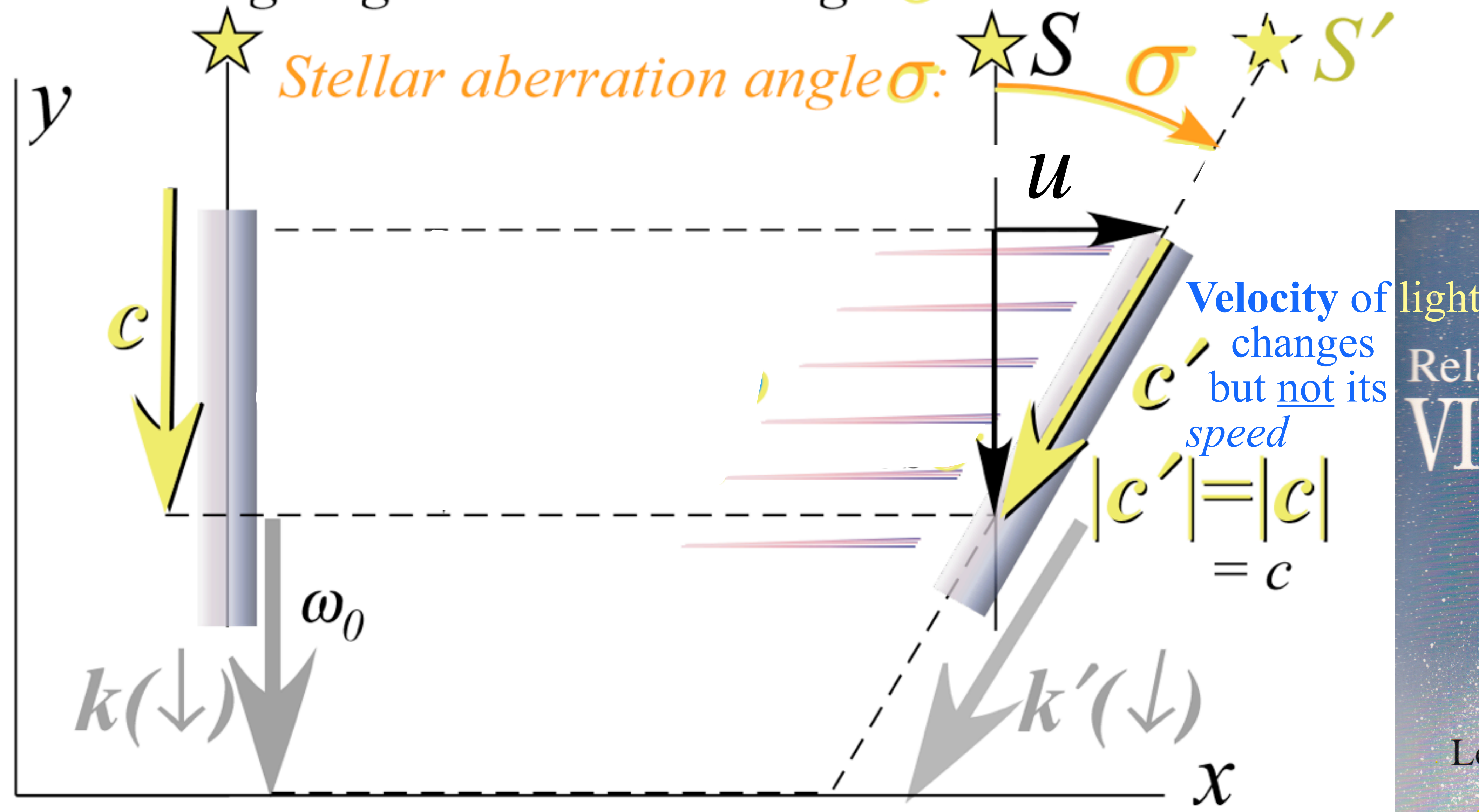
Purchase at: amazon

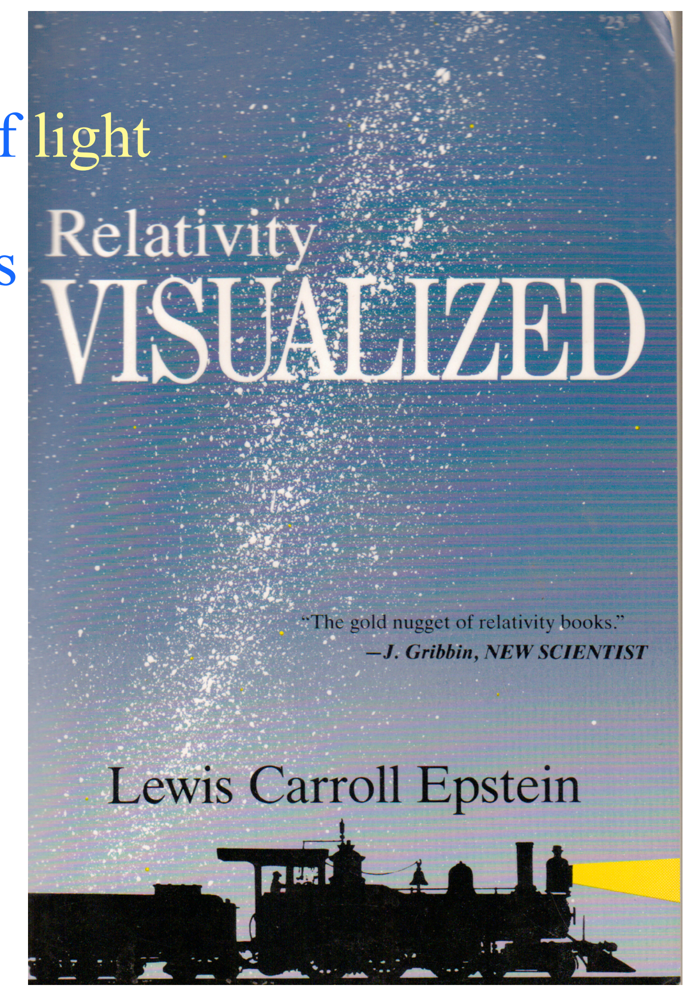
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift) to a Transverse relativity parameter: Stellar aberration angle σ*

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.
*Observer going **u** sees star at angle σ in **u** direction.*

We use the notion σ for stellar-aberration-angle (a "flipped-over" ρ).





Epstein seemed resistant to ρ analysis or relations between σ and ρ.

AllBookstores.com
Purchase at: amazon

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

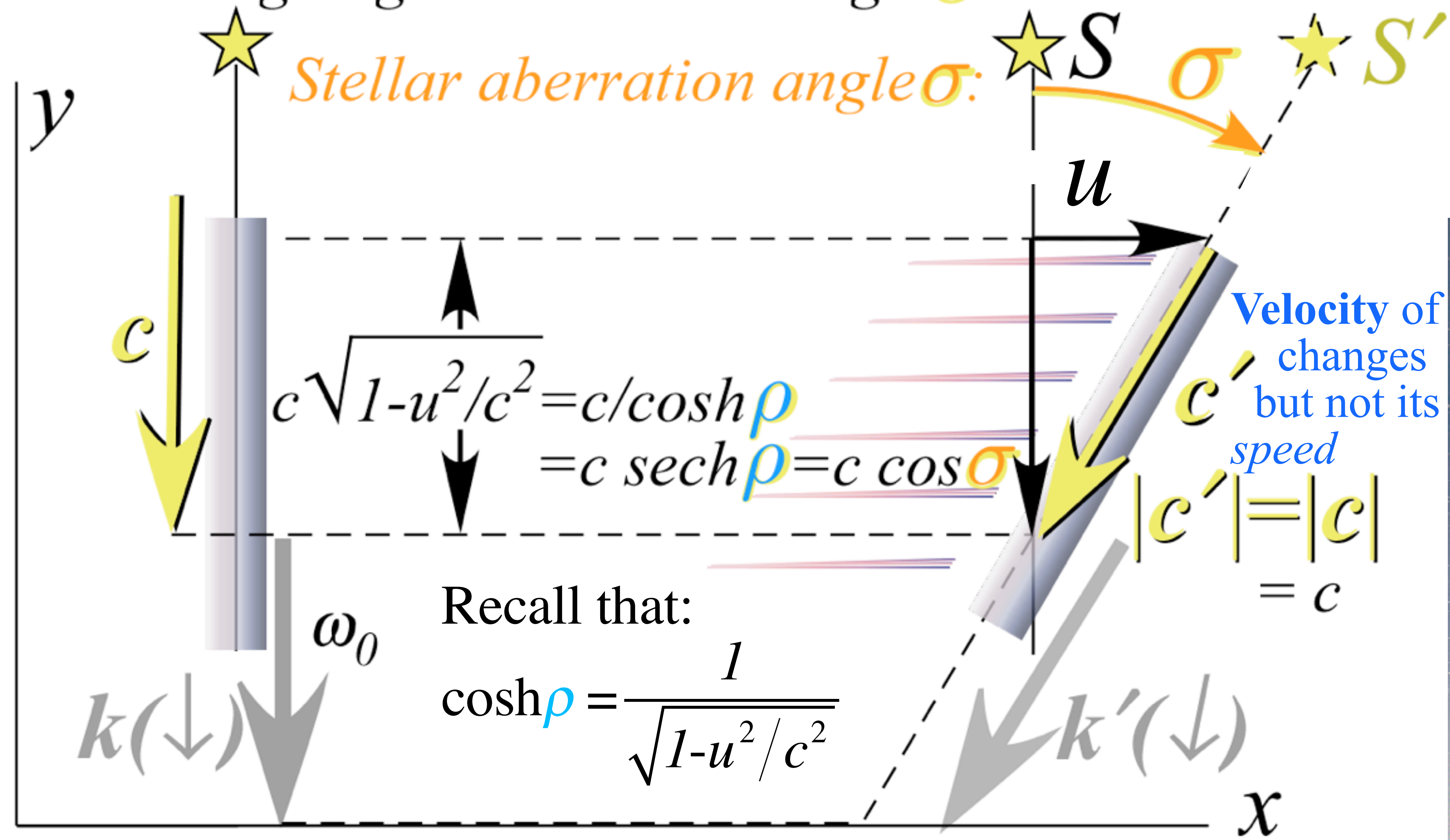
to a Transverse relativity parameter: Stellar aberration angle σ*

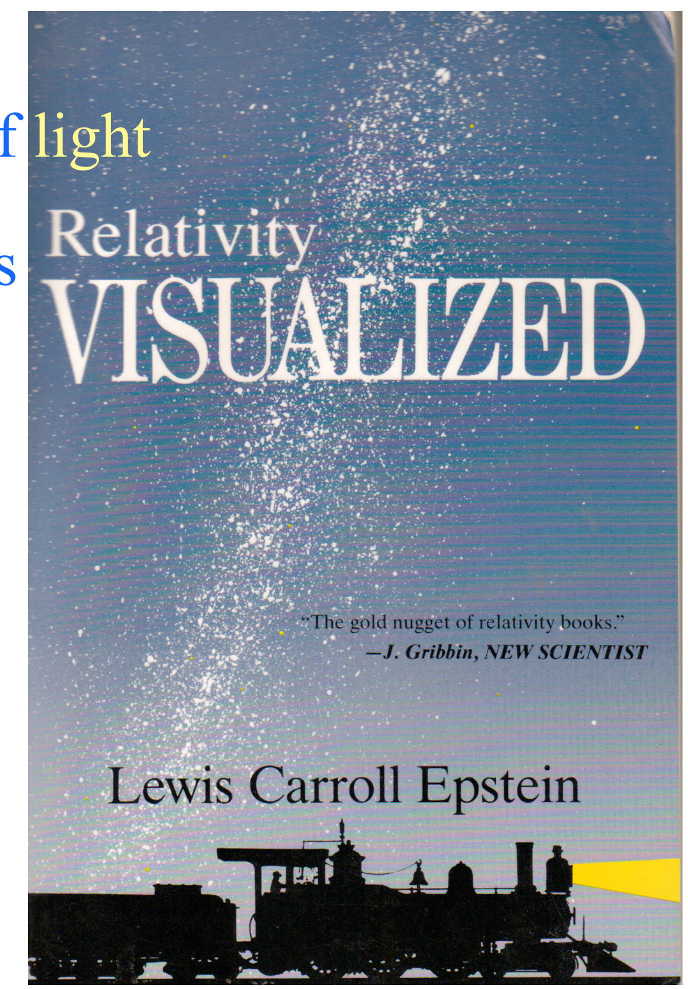
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

Observer going ***u*** *sees star at angle σ in* ***u*** *direction.*

We use the notion σ for stellar-aberration-angle (a "flipped-over" ρ).





Epstein seemed resistant to ρ analysis or relations between σ and ρ.

Comparing <u>Longitudinal</u> relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

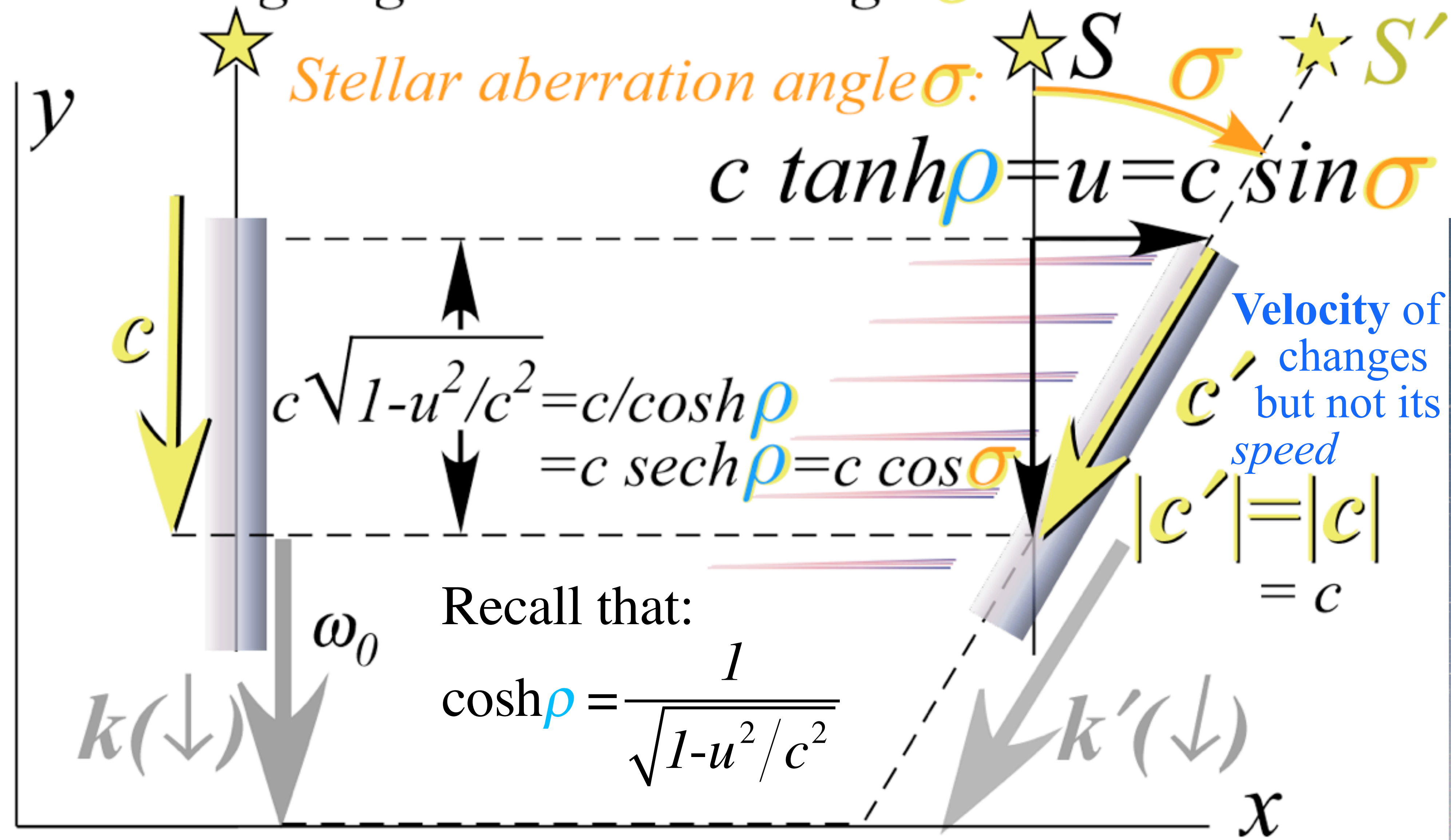
to a <u>Transverse</u> relativity parameter: Stellar aberration angle σ*

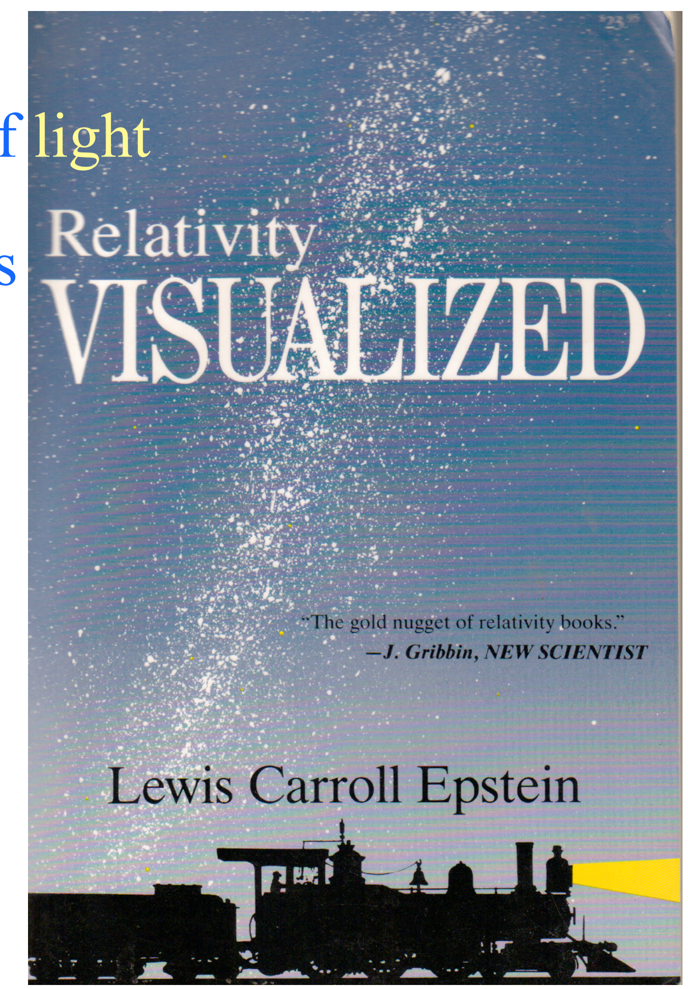
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

We use the notion σ for stellar-aberration-angle (a "flipped-over" ρ).

Observer fixed below star sees it directly overhead.

Observer going ***u*** *sees star at angle* σ *in* ***u*** *direction.*





AllBookstores.com

Purchase at: amazon

Epstein seemed resistant to ρ analysis or relations between σ and ρ.

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

➔ Review of proper time relations and basis of Epstein's cosmic speedometer ←

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

Lorentz space contraction

Time-simultaneity-breaking

Velocity addition

Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) =(\omega,ck_x,ck_y,ck_z)$ transformation

Review of Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

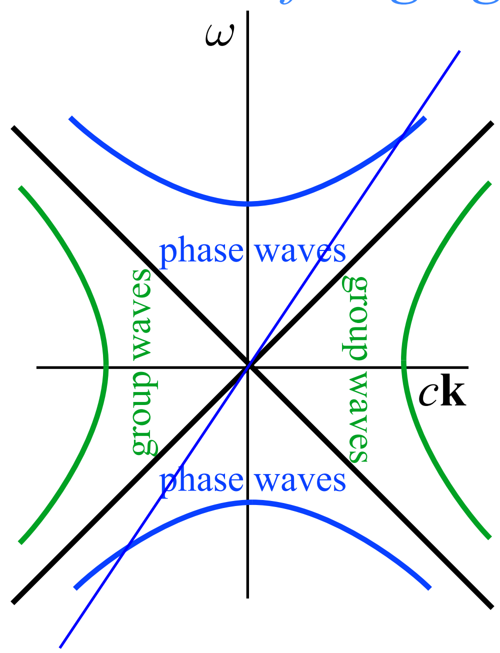
$$\omega_0 = \omega\sqrt{1-\frac{c^2k^2}{\omega^2}} = \omega'\sqrt{1-\frac{c^2k'^2}{\omega'^2}}$$

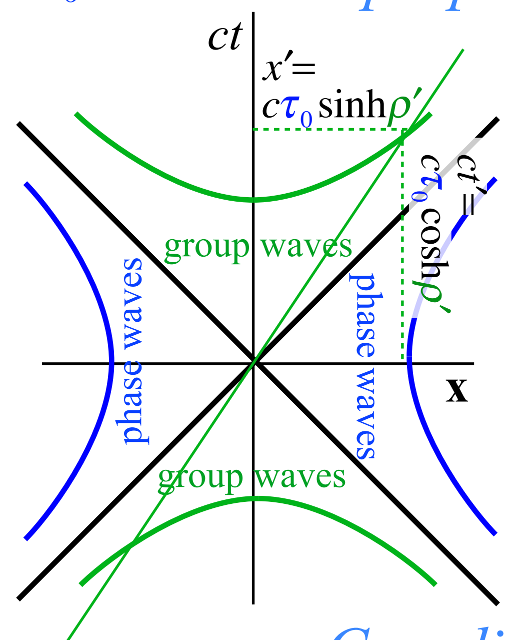
$$= \omega\sqrt{1-\frac{c^2}{V_{phase}^2}} = \omega'\sqrt{1-\frac{c^2}{V'^2_{phase}}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":





$$\tau_0 = t\sqrt{1-\frac{x^2}{(ct)^2}} = t'\sqrt{1-\frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1-\frac{u^2}{c^2}} = t'\sqrt{1-\frac{u'^2}{c^2}}$$

Coordinate time t dilates as u grows and is greater than τ_0

$$\omega = \frac{\omega_0}{\sqrt{1-\frac{k^2}{(c\omega)^2}}} = \frac{\omega_0}{\sqrt{1-\frac{c^2}{V_{phase}^2}}} \qquad \omega' = \frac{\omega_0}{\sqrt{1-\frac{k'^2}{(c\omega')^2}}} = \frac{\omega_0}{\sqrt{1-\frac{c^2}{V'^2_{phase}}}}$$

$$t = \frac{\tau_0}{\sqrt{1-\frac{x^2}{(ct)^2}}} \qquad t' = \frac{\tau_0}{\sqrt{1-\frac{x'^2}{(ct')^2}}}$$

$$t = \frac{\tau_0}{\sqrt{1-\frac{u^2}{c^2}}} = \tau_0\cosh\rho \qquad t' = \frac{\tau_0}{\sqrt{1-\frac{u'^2}{c^2}}} = \tau_0\cosh\rho'$$

Review of Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

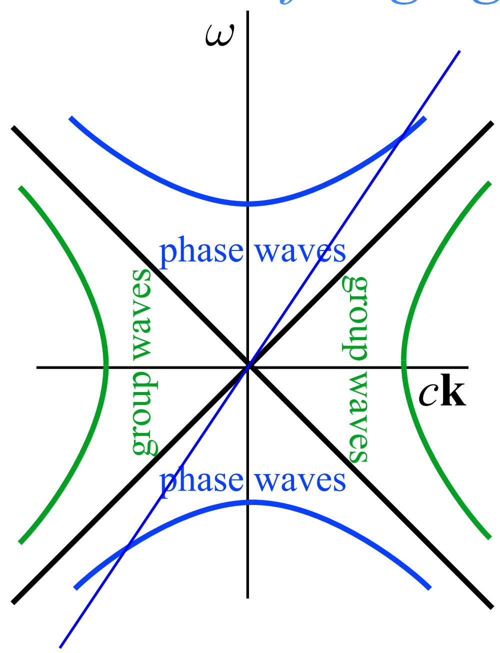
Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1-\frac{c^2k^2}{\omega^2}} = \omega'\sqrt{1-\frac{c^2k'^2}{\omega'^2}}$$

$$= \omega\sqrt{1-\frac{c^2}{V_{phase}^2}} = \omega'\sqrt{1-\frac{c^2}{V'^2_{phase}}}$$

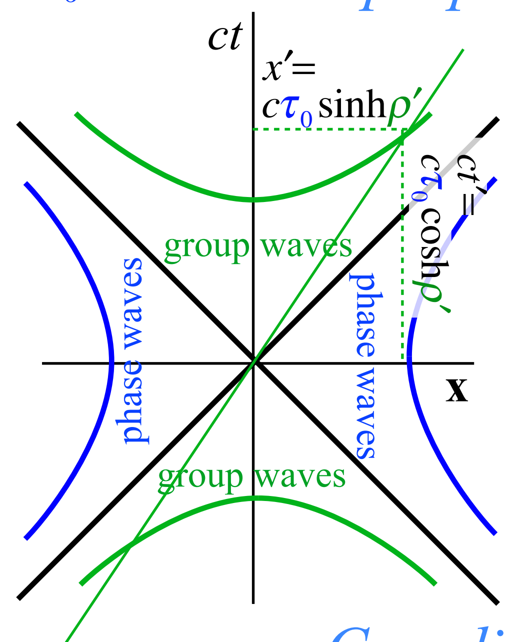


$$\omega = \frac{\omega_0}{\sqrt{1-\frac{k^2}{(c\omega)^2}}} = \frac{\omega_0}{\sqrt{1-\frac{c^2}{V_{phase}^2}}} \qquad \omega' = \frac{\omega_0}{\sqrt{1-\frac{k'^2}{(c\omega')^2}}} = \frac{\omega_0}{\sqrt{1-\frac{c^2}{V'^2_{phase}}}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":



$$\tau_0 = t\sqrt{1-\frac{x^2}{(ct)^2}} = t'\sqrt{1-\frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1-\frac{u^2}{c^2}} = t'\sqrt{1-\frac{u'^2}{c^2}}$$

Coordinate time t dilates as u grows and is greater than τ_0

$$t = \frac{\tau_0}{\sqrt{1-\frac{x^2}{(ct)^2}}} \qquad t' = \frac{\tau_0}{\sqrt{1-\frac{x'^2}{(ct')^2}}}$$

$$t = \frac{\tau_0}{\sqrt{1-\frac{u^2}{c^2}}} = \tau_0\cosh\rho \qquad t' = \frac{\tau_0}{\sqrt{1-\frac{u'^2}{c^2}}} = \tau_0\cosh\rho'$$

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

➔ Epstein vs Einstein-Minkowski geometry of relativity ←

Einstein time dilation

Lorentz space contraction

Time-simultaneity-breaking

Velocity addition

Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

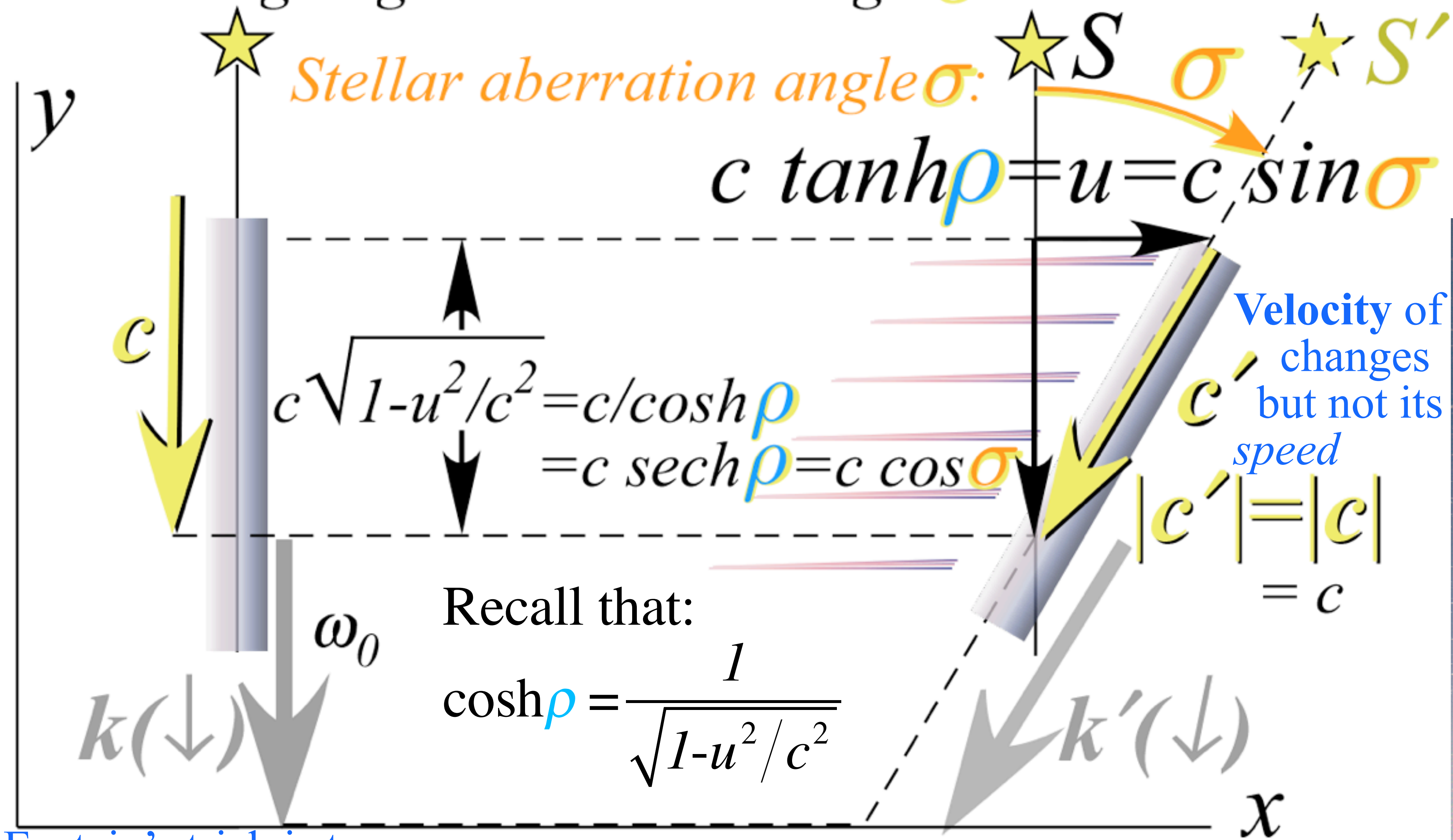
to a Transverse relativity parameter: Stellar aberration angle σ*

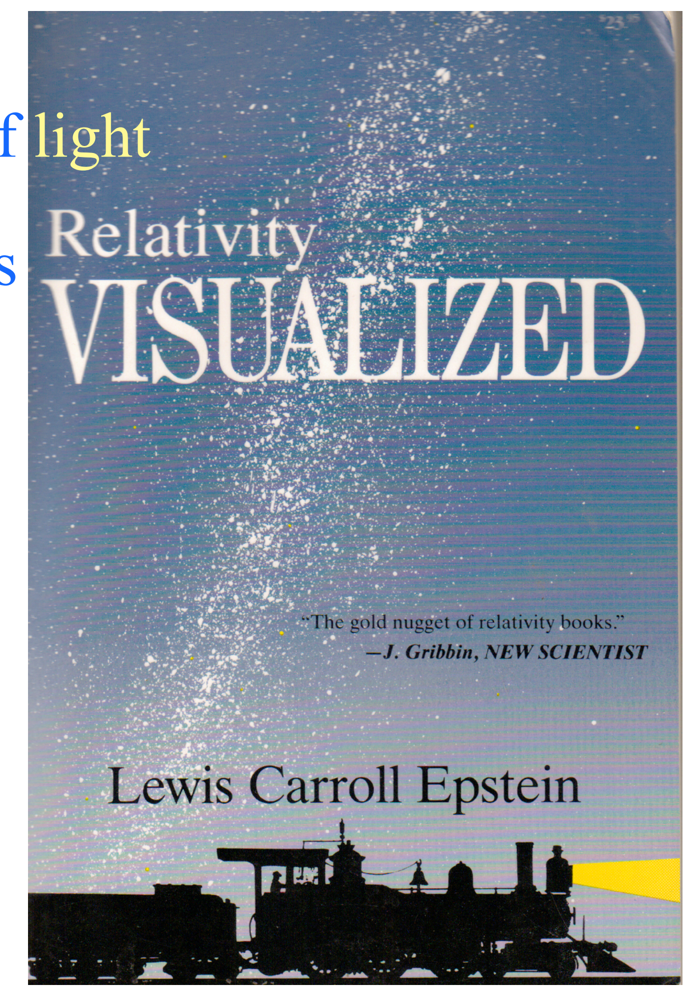
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*

We use the notion σ for stellar-aberration-angle (a "flipped-over" ρ).





Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ (for Proper time) into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Comparing <u>Longitudinal</u> relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

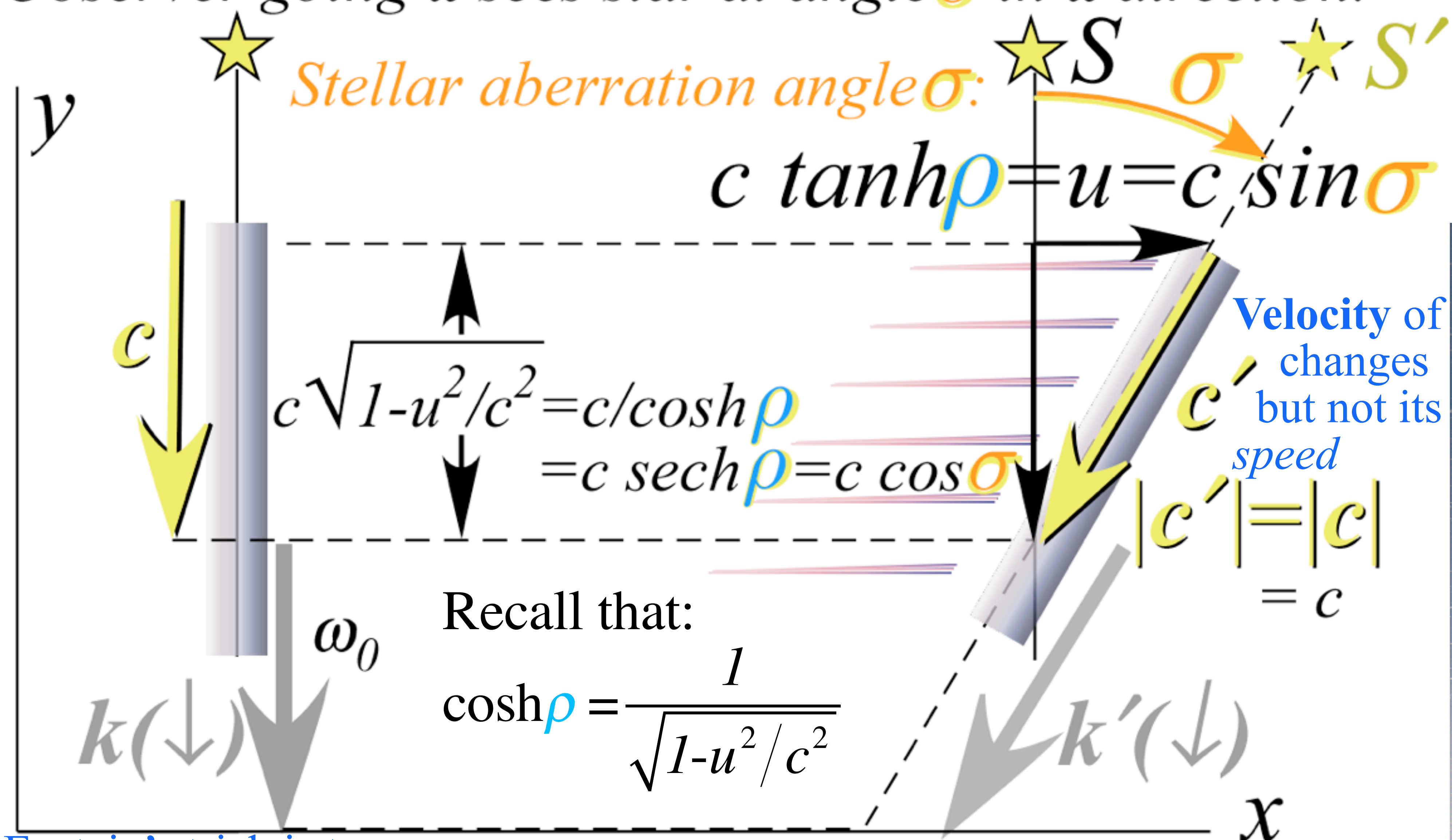
to a <u>Transverse</u> relativity parameter: Stellar aberration angle σ*

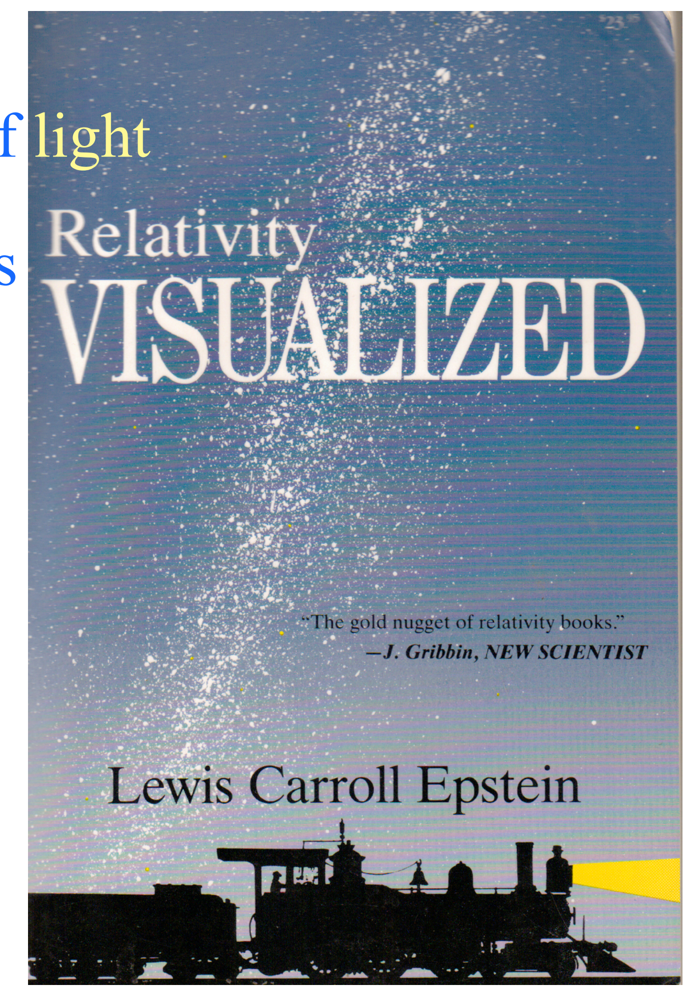
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*

We use the notion σ for stellar-aberration-angle (a "flipped-over" ρ).





Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ (for Proper time)

into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Then he imagines everything (and everybody) always goes speed c through $(x', c\tau)$ space!

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity ←

→ Einstein time dilation

Lorentz space contraction

Time-simultaneity-breaking

Velocity addition

Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

Epstein's space-proper-time (x,cτ) plots

("c-tau" plots) *Time contraction-dilation revisited*

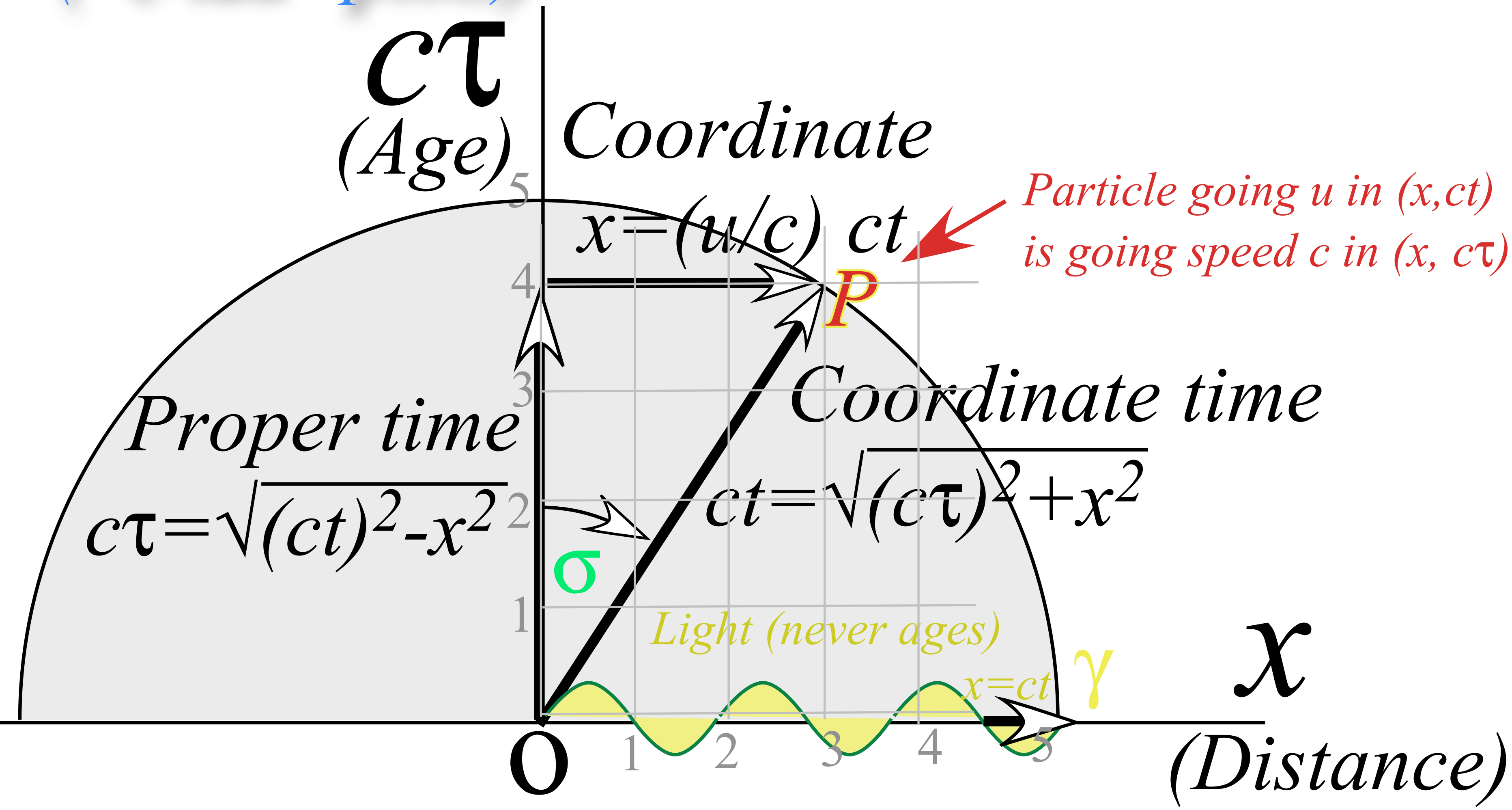


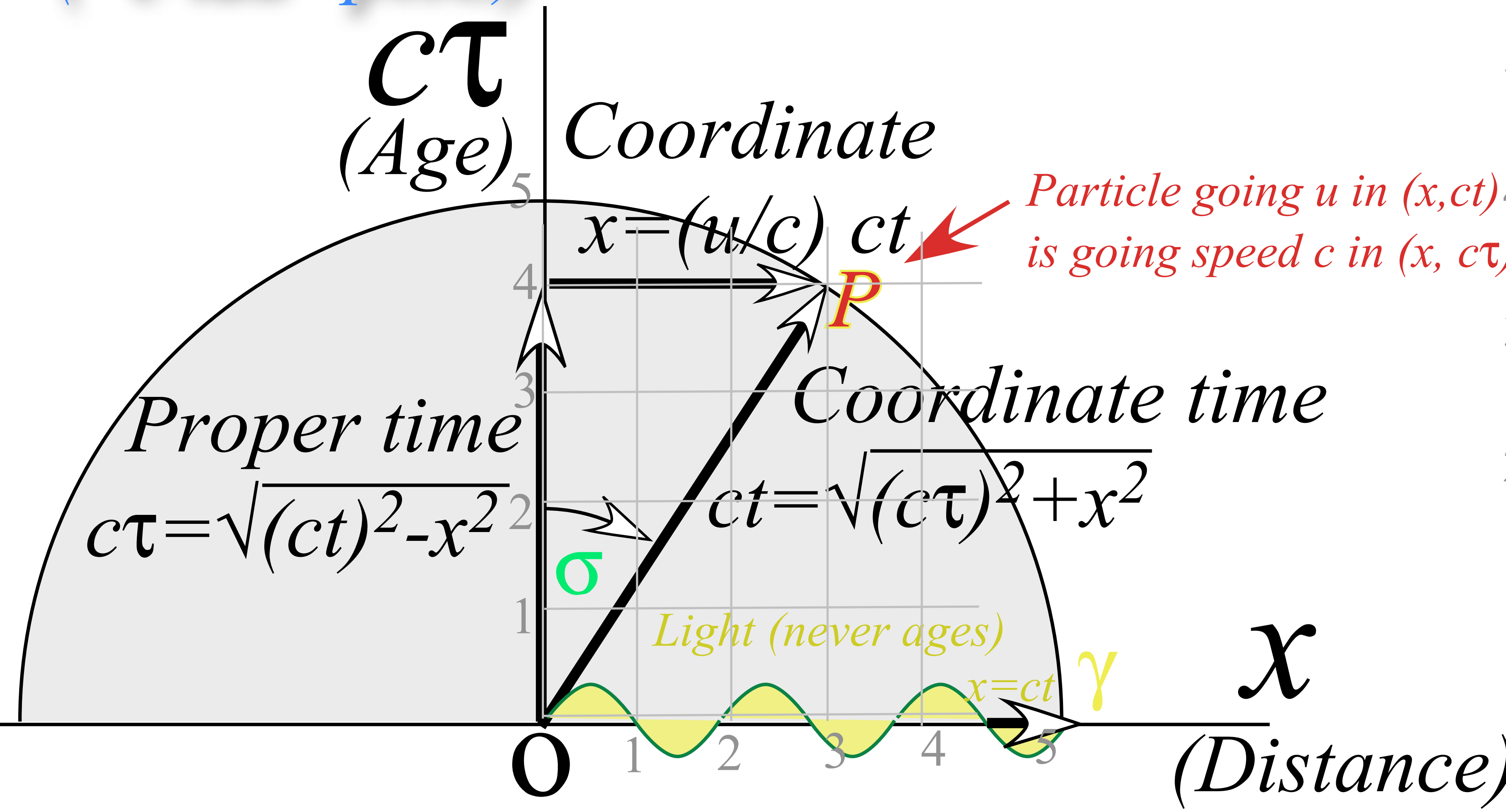
Fig.12 (1st-part) Space-proper-time plot makes all objects move at speed c along their cosmic speedometer.

Epstein (x,cτ) plot

(for u/c=3/5)

Dual View Space-Space and Space-properTime
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=600
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=601
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=602

Epstein's space-proper-time (x,cτ) plots
("c-tau" plots)
Time contraction-dilation revisited



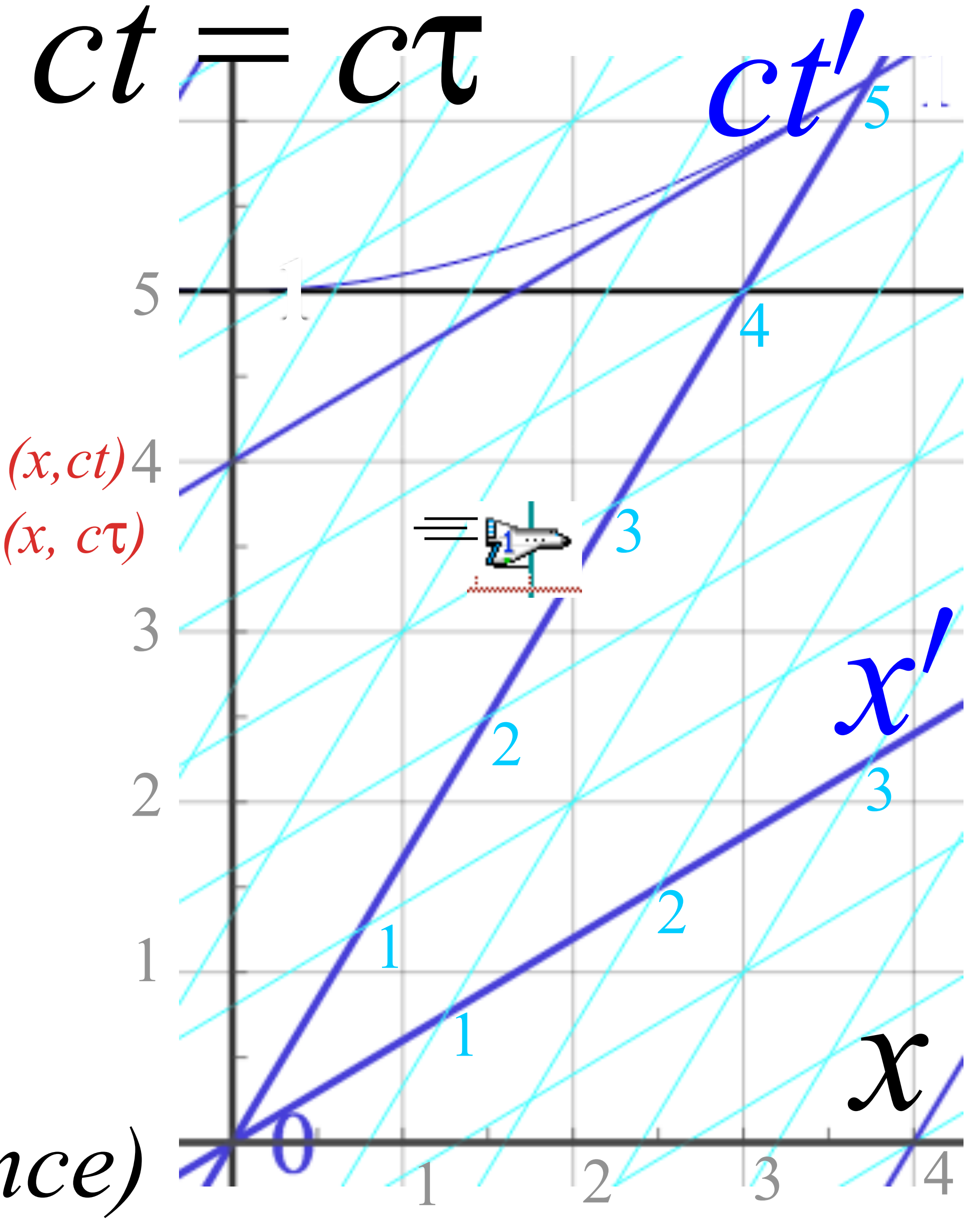


Fig.12 (1st-part) Space-proper-time plot makes all objects move at speed c along their cosmic speedometer.

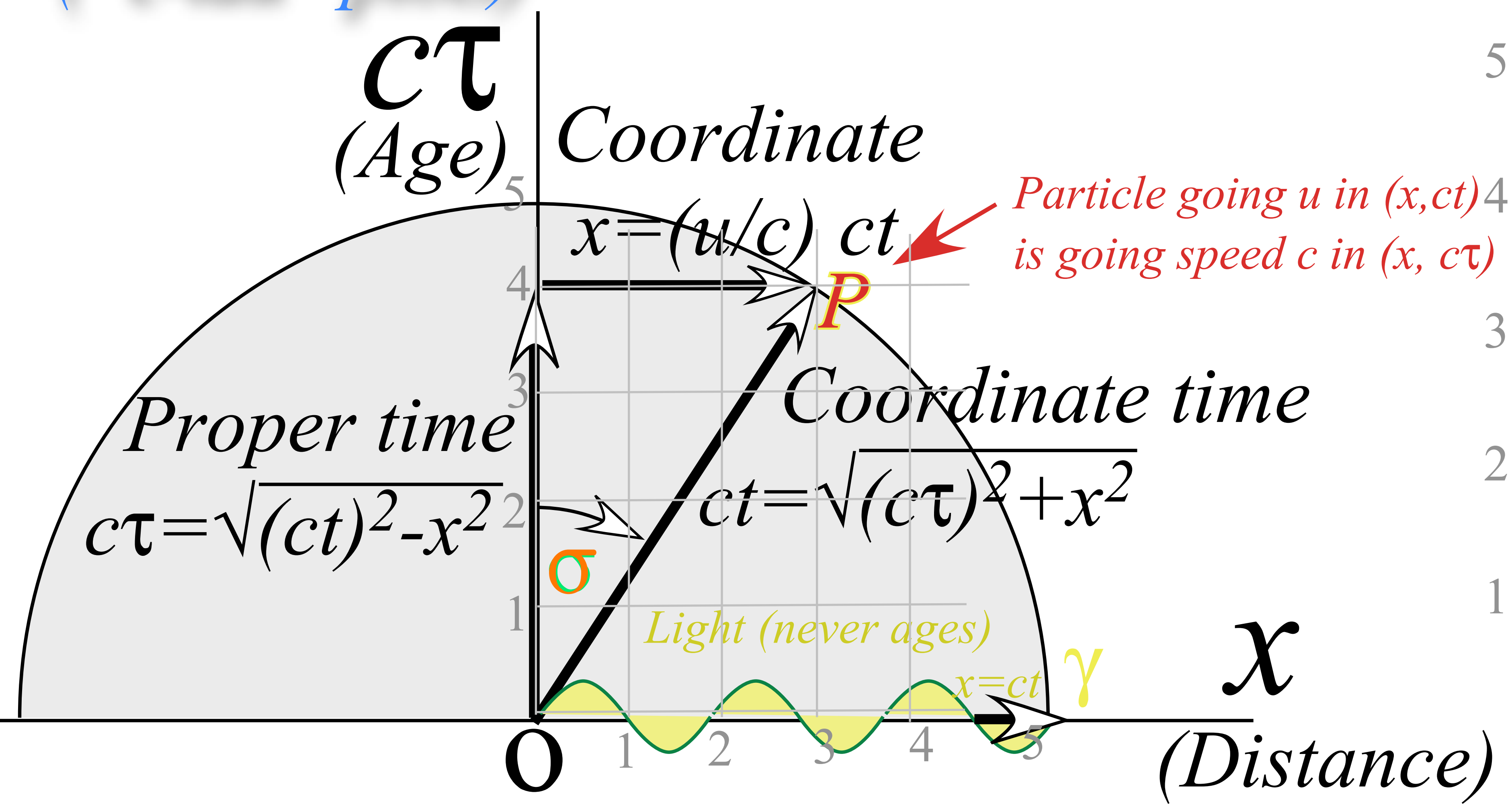
Compare Epstein (x,cτ) plot to Einstein-Minkowsky (x,ct) on (x′,ct′)

(for u/c=3/5)

Dual View Space-Space and Space-properTime
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=600
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=601
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=602

Epstein's space-proper-time $(x, c\tau)$ plots ("c-tau" plots)

Time contraction-dilation revisited



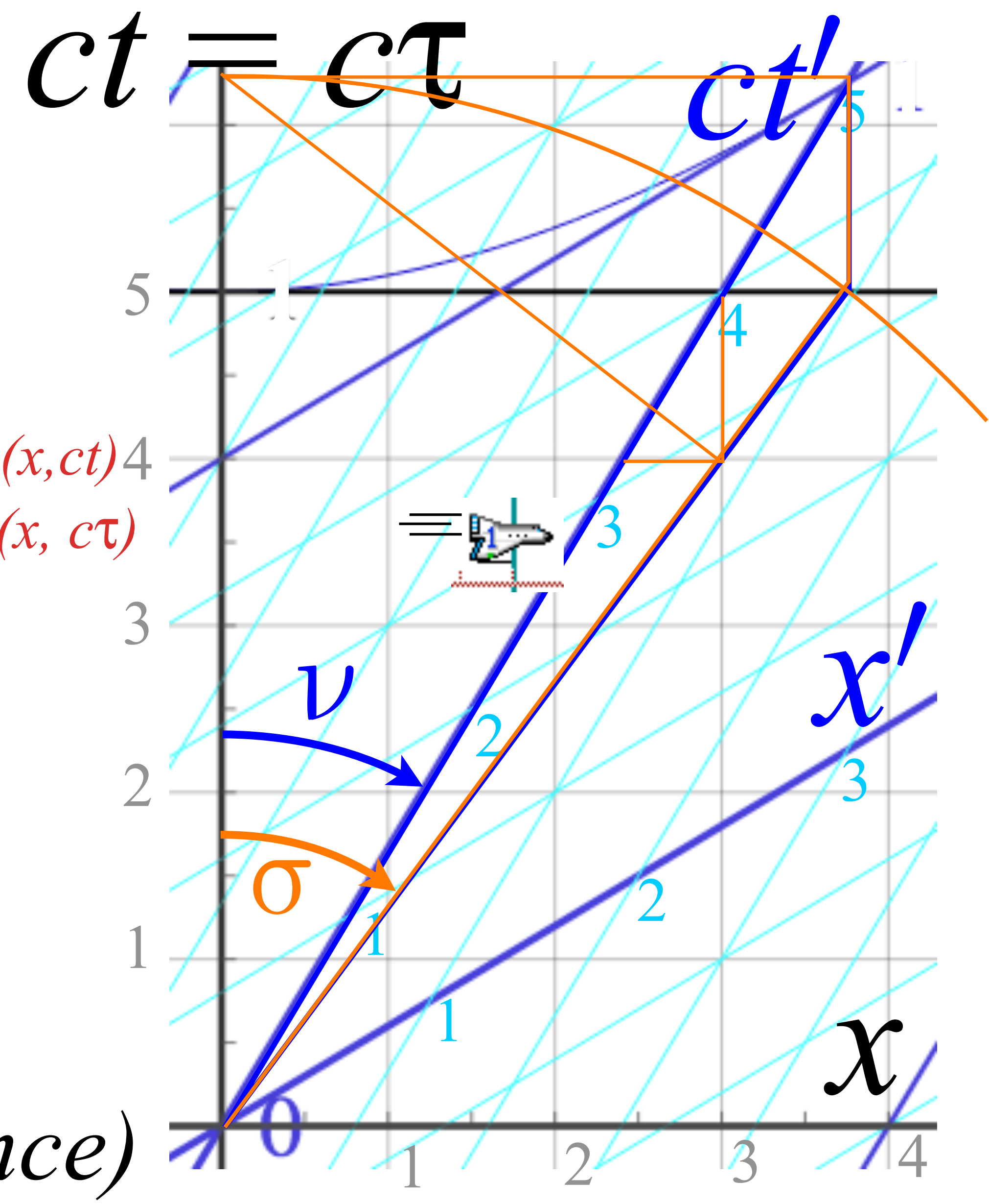


Fig.12 (1st-part) Space-proper-time plot makes all objects move at speed c along their cosmic speedometer.

Compare Epstein $(x,c\tau)$ plot to Einstein-Minkowsky (x,ct) on (x',ct')

(for $u/c=3/5$)

Dual View Space-Space and Space-properTime
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=600
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=601
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=602

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

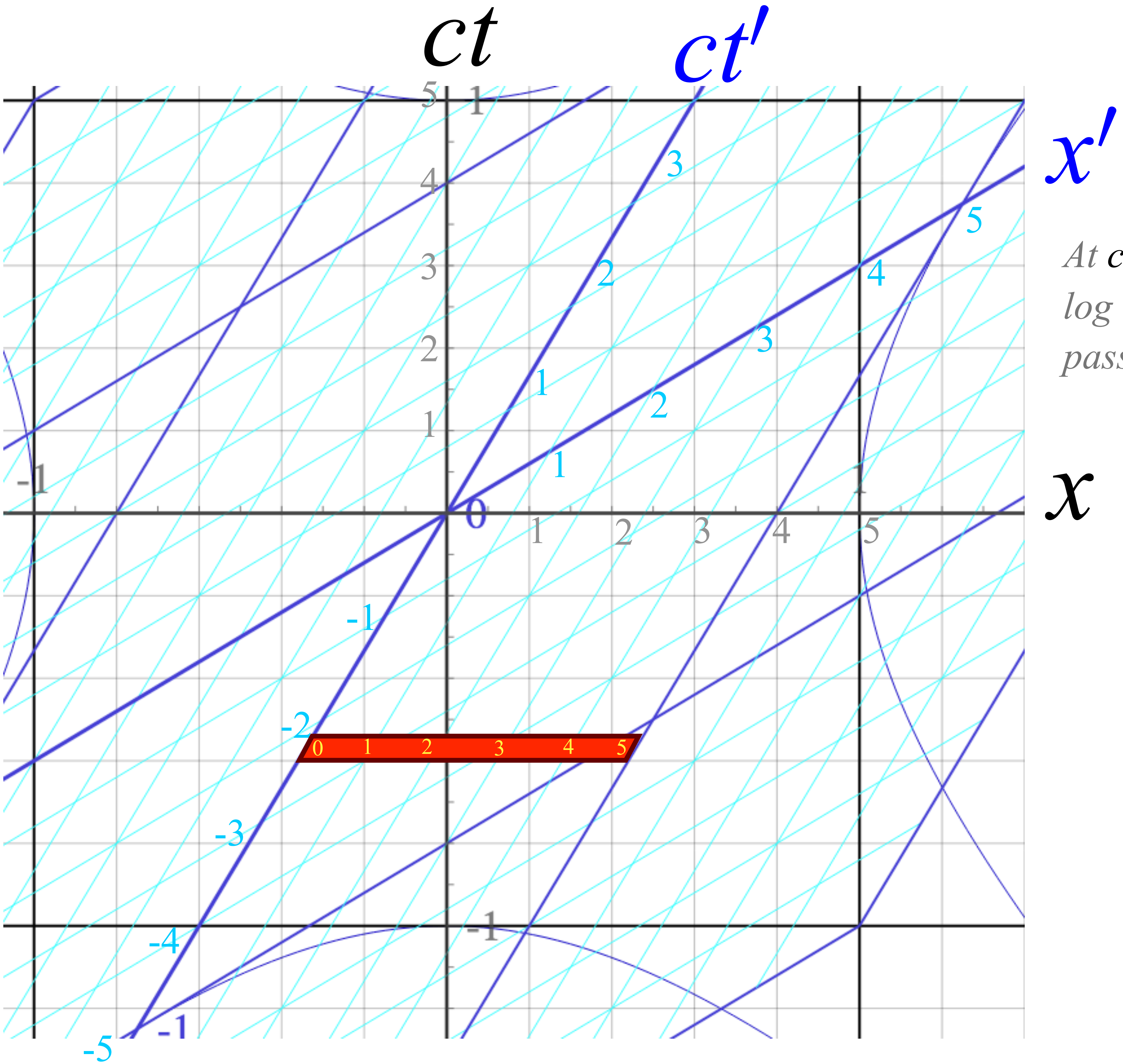
➔ Lorentz space contraction

Time-simultaneity-breaking ←

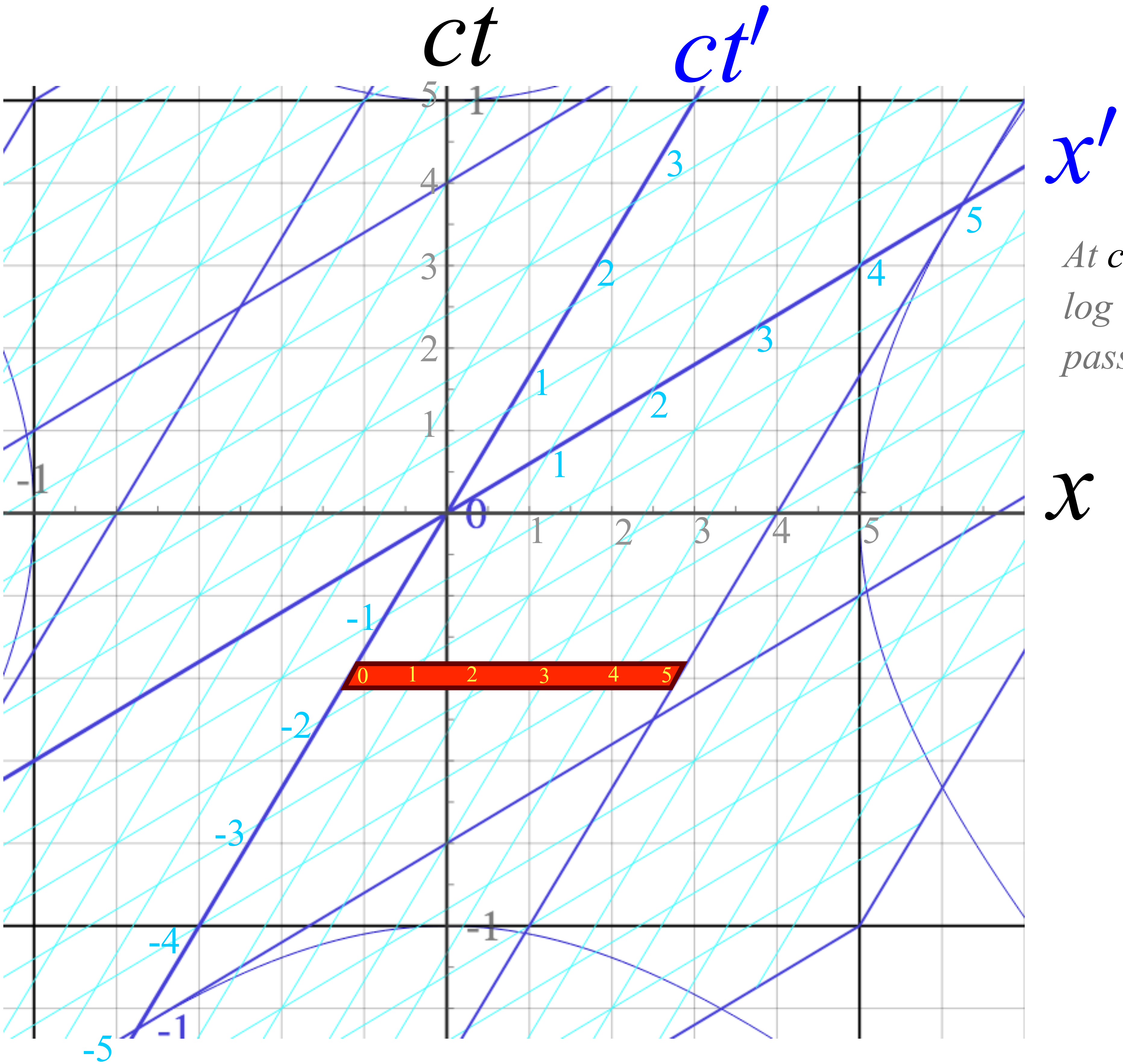
Velocity addition

Twin-paradox resolution in space-proper-time

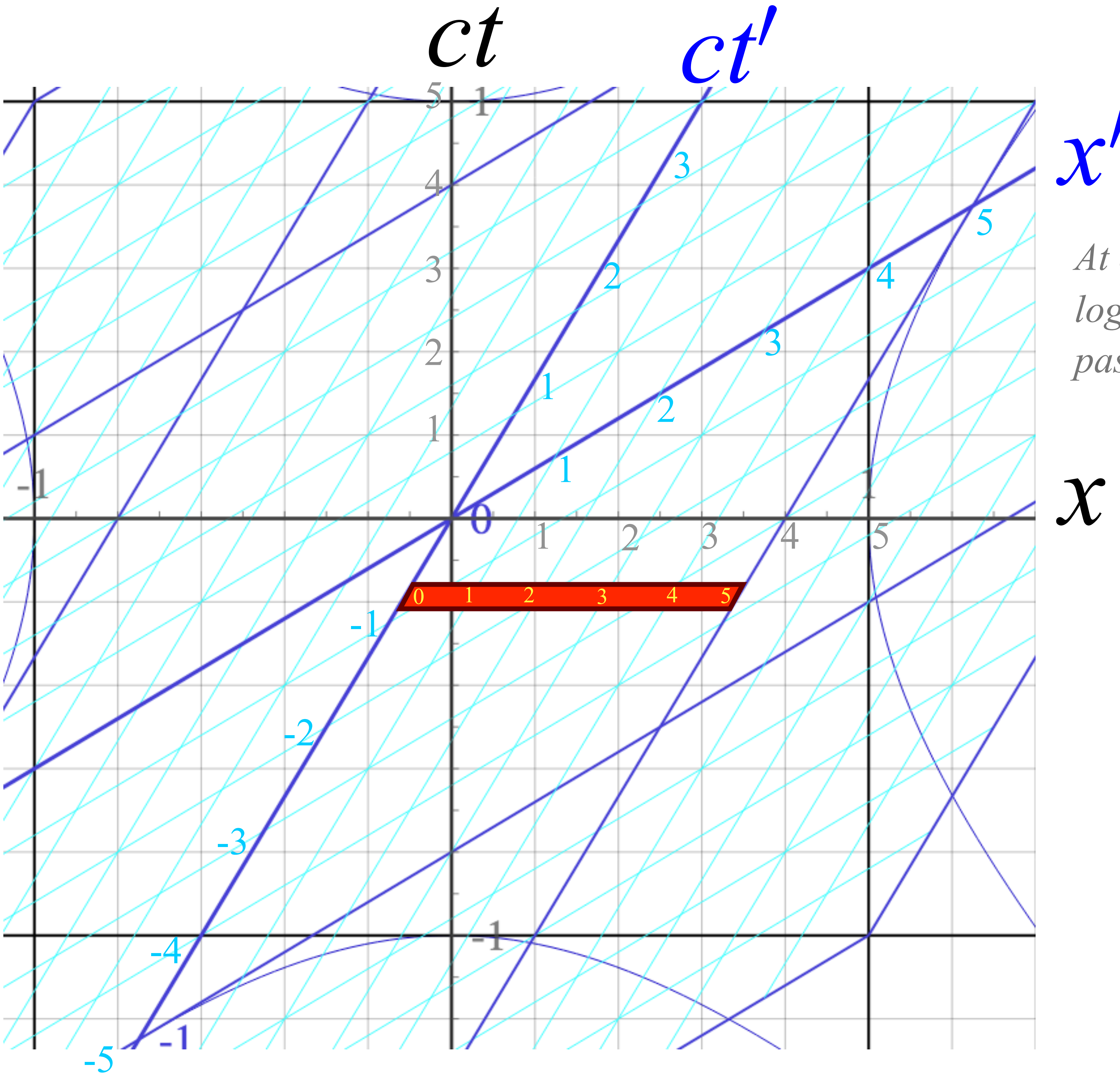
Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) =(\omega,ck_x,ck_y,ck_z)$ transformation



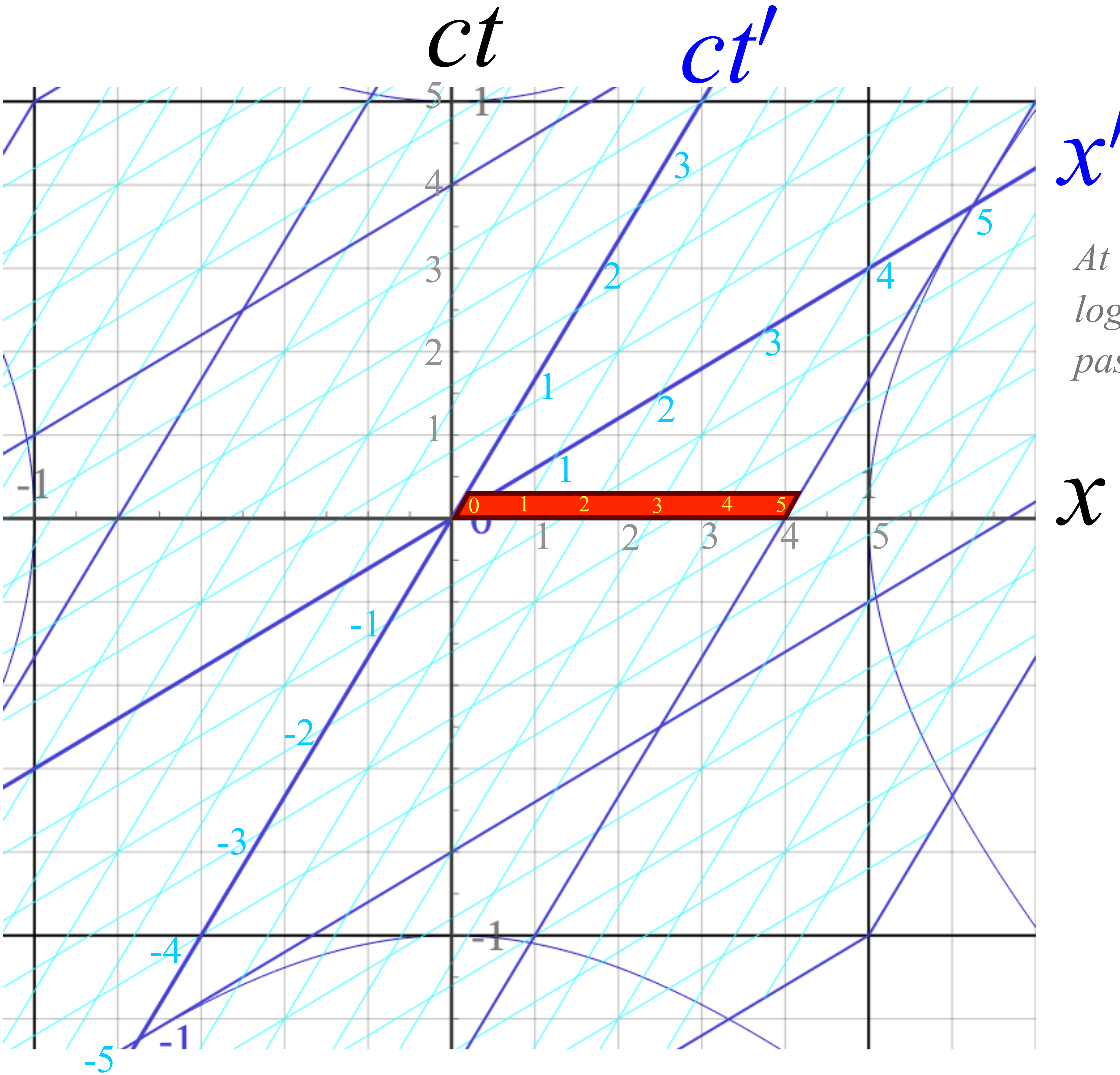
At ct=0 Lighthouse will log the Ship's tail (x′=0) passing Main Light (x=0)



At ct=0 Lighthouse will log the Ship's tail (x′=0) passing Main Light (x=0)

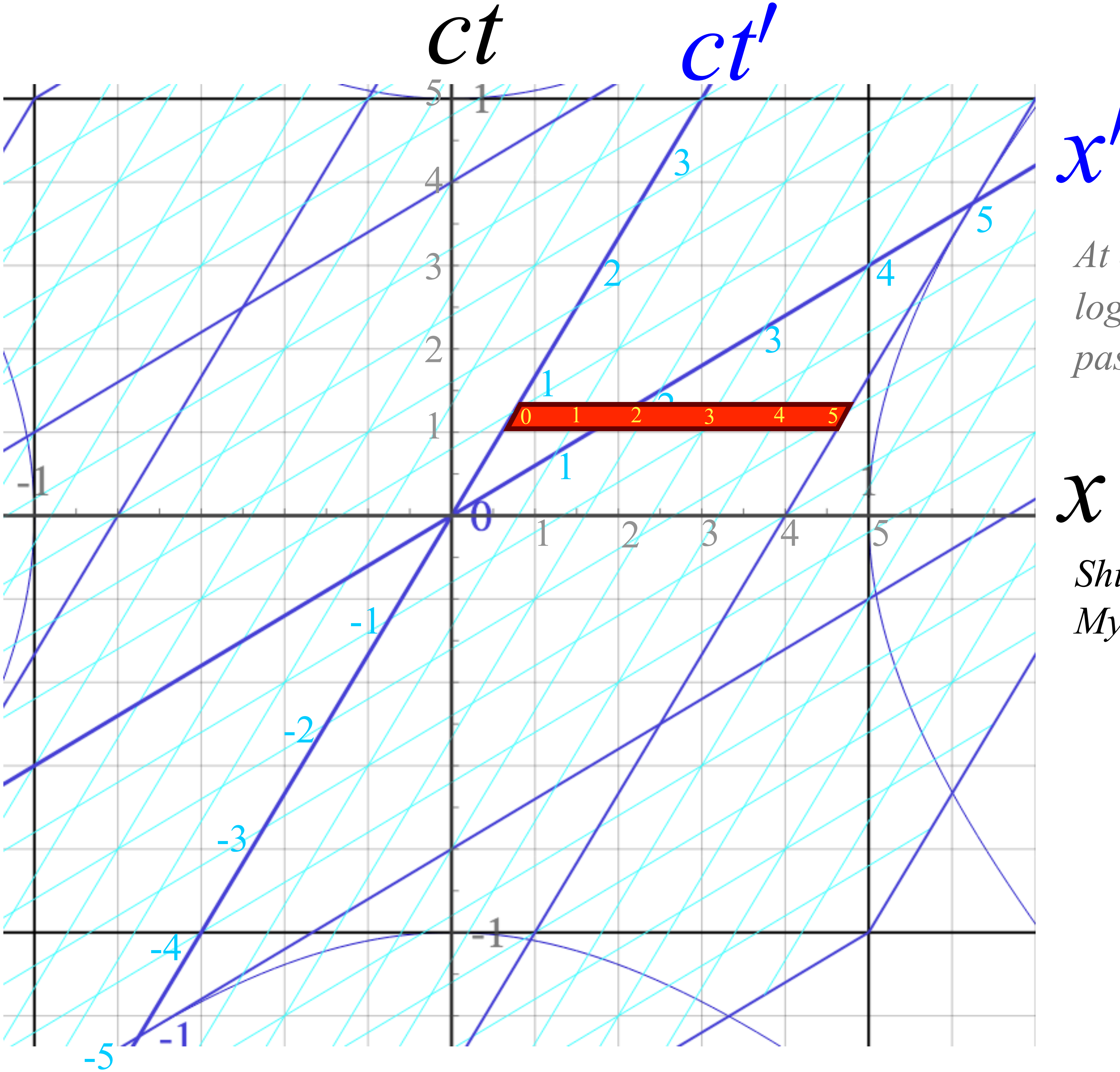


At ct=0 Lighthouse will log the Ship's tail (x′=0) passing Main Light (x=0)



At ct=0 Lighthouse will log the Ship's tail (x′=0) passing Main Light (x=0)

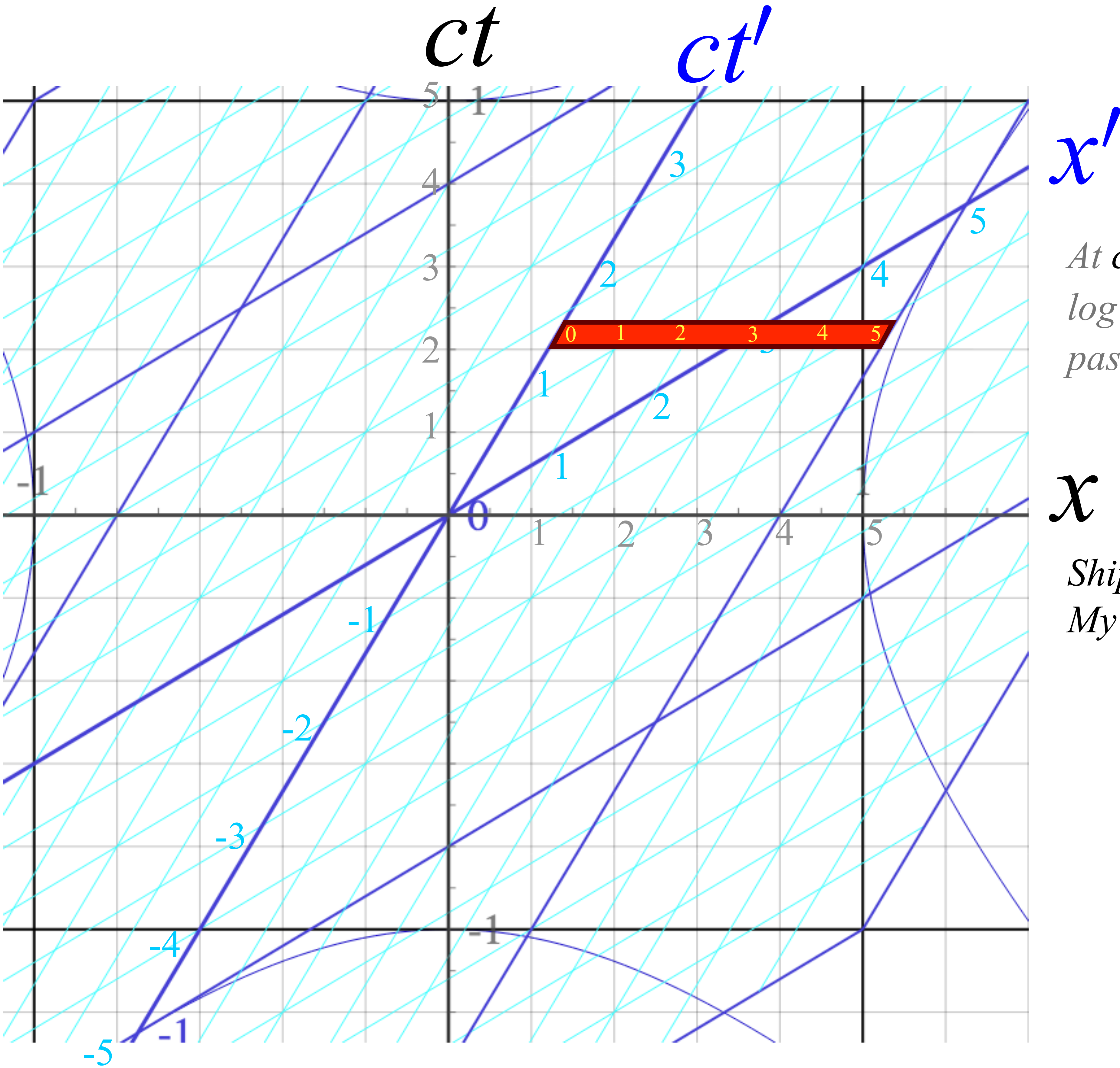
Lighthouse recorded length is (x=0) to (x=4)



At ct=0 Lighthouse will log the Ship's tail (x′=0) passing Main Light (x=0)

Lighthouse recorded length is (x=0) to (x=4)

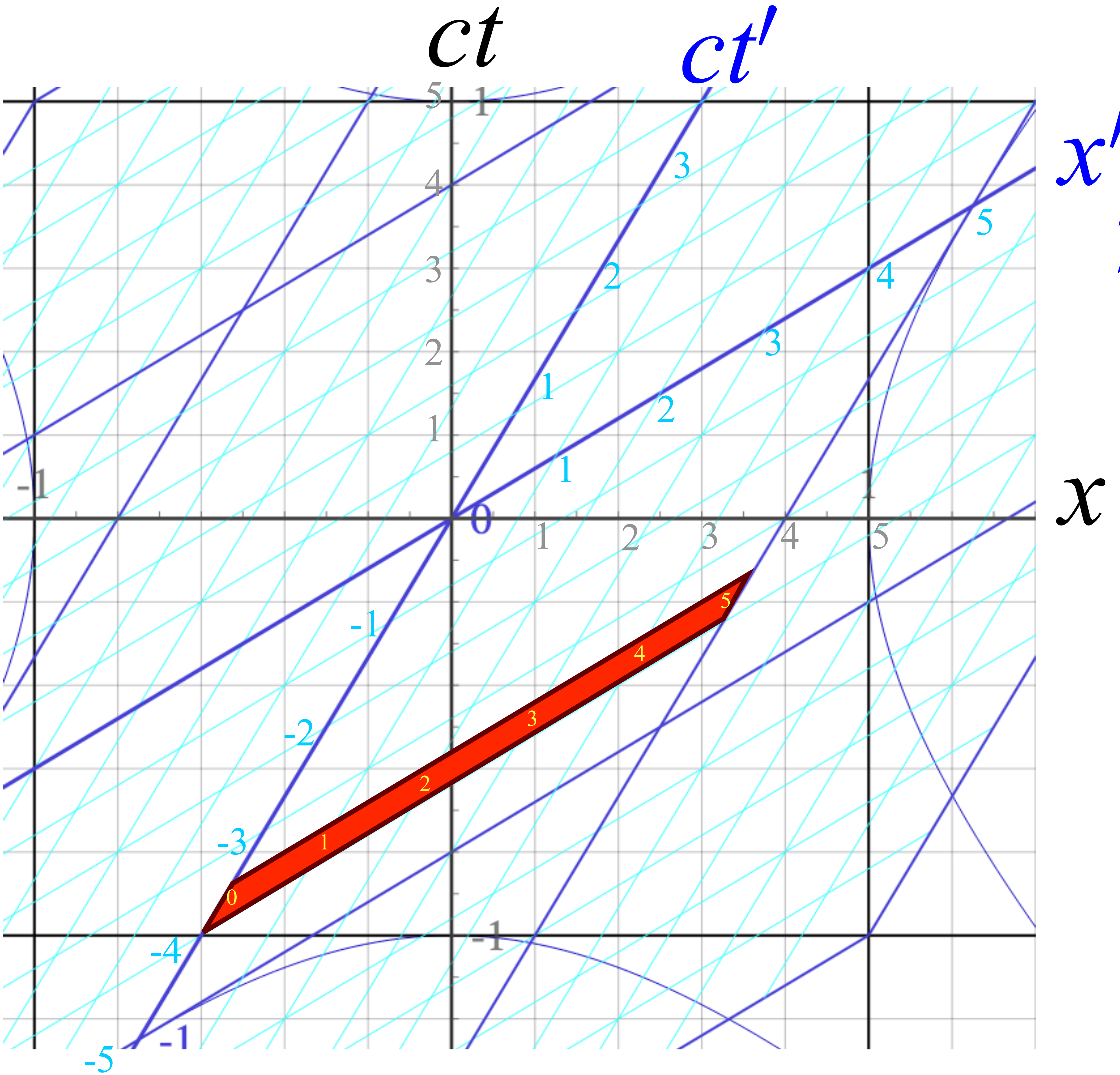
*Ship says "No way, Jose!… My length is **5**…!"*



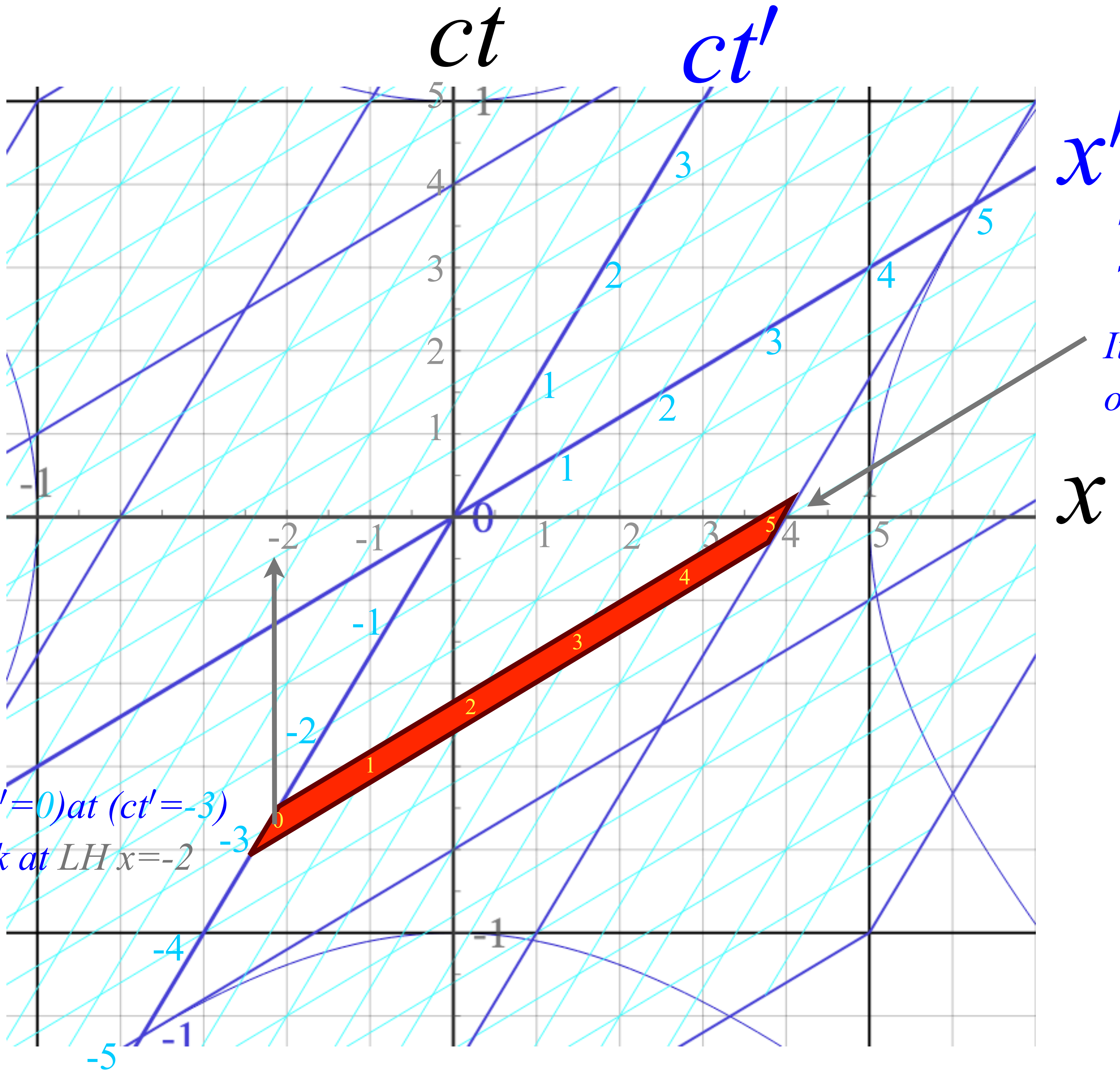
At ct=0 Lighthouse will log the Ship's tail (x′=0) passing Main Light (x=0)

Lighthouse recorded length is (x=0) to (x=4)

Ship says "No way, Jose!... My length is ***5****...!"*



Ship says Lighthouse did sloppy measurement.

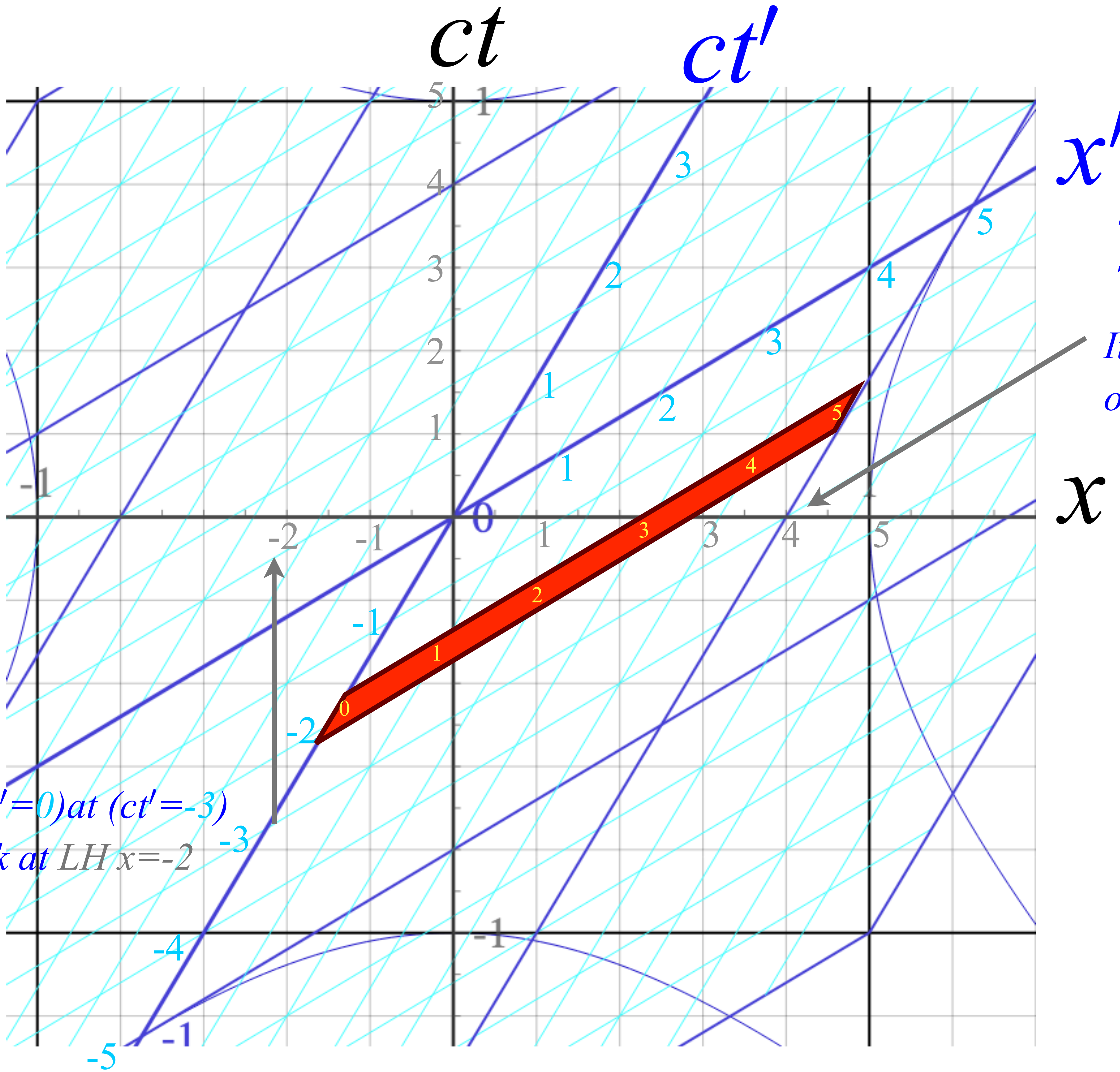


Ship says Lighthouse did sloppy measurement.

It first recorded x=4 for our nose (x'=5) at (ct'=-3)

Then we moved forward.

Tail (x'=0) at (ct'=-3) is back at LH x=-2

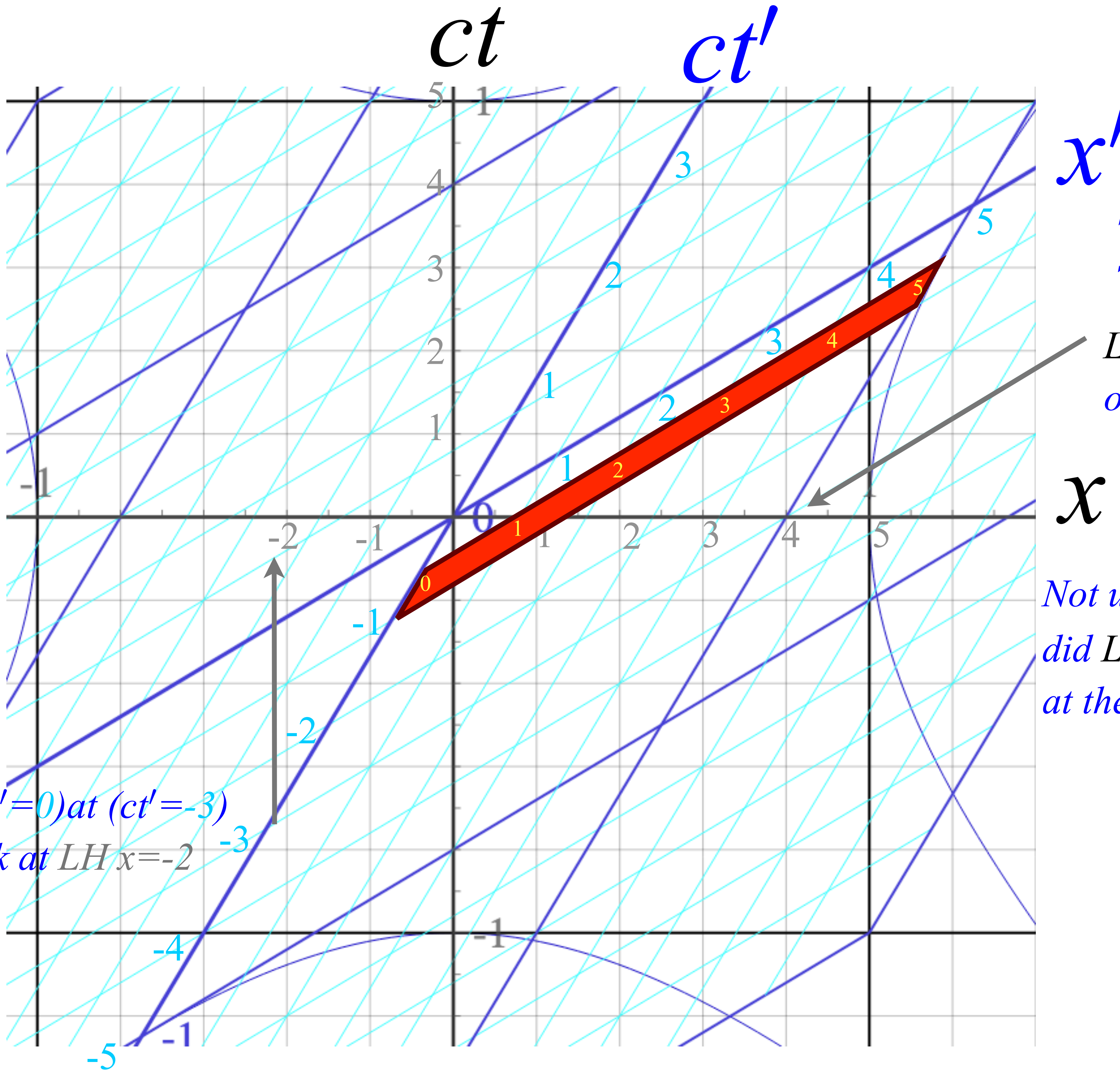


Ship says Lighthouse did sloppy measurement.

It first recorded x=4 for our nose (x′=5)at (ct′=-3)

Then we moved forward.

Tail (x′=0)at (ct′=-3) is back at LH x=-2



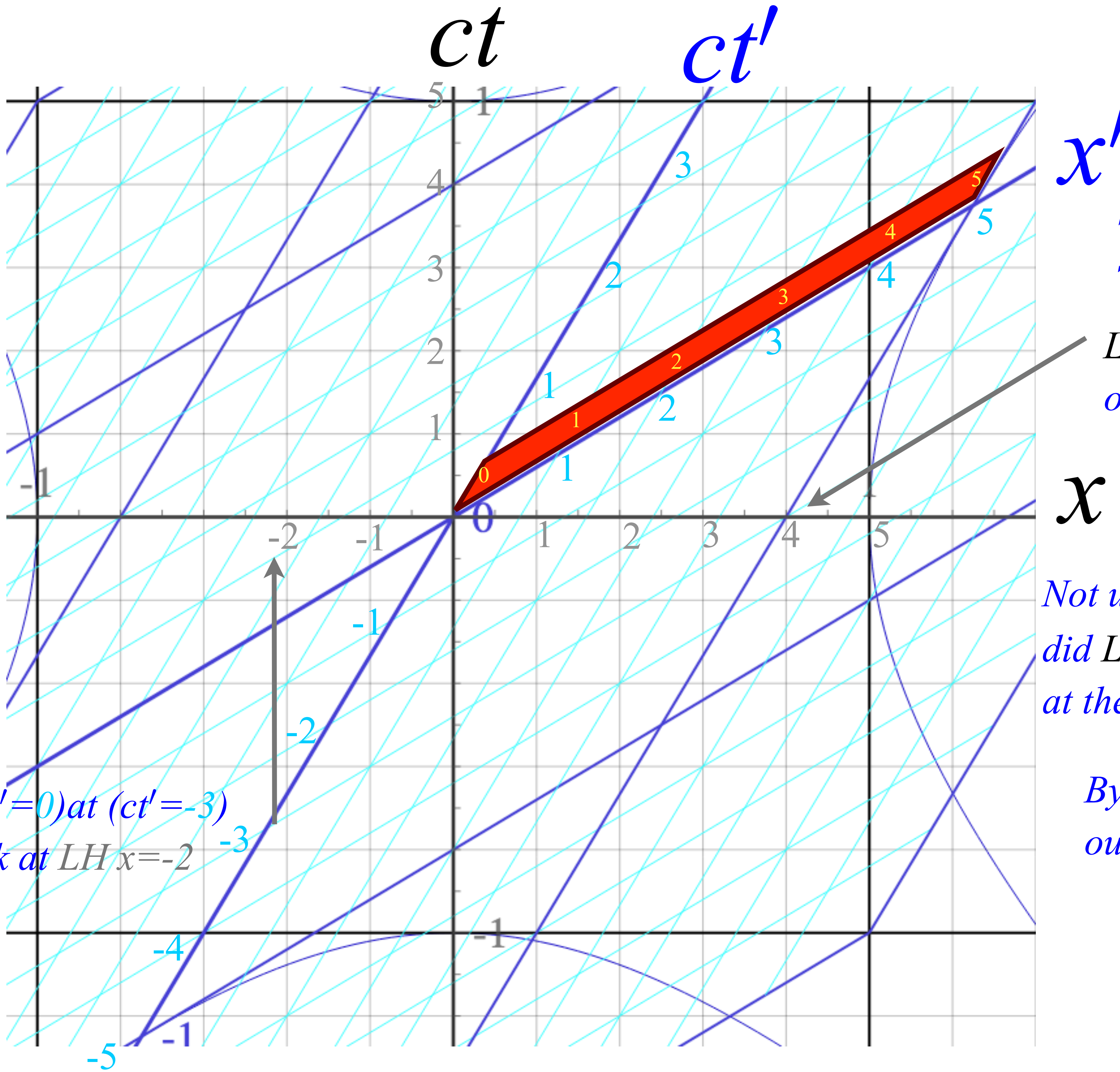
Ship says Lighthouse *did sloppy measurement.*

LH *first recorded* x=4 *for our nose (x′=5)at (ct′=-3)*

Then we moved forward.

Not until 3 sec. later (ct′=0) did LH *record our tail (x′=0) at their x=0*

Tail (x′=0)at (ct′=-3) is back at LH x=-2



Ship says Lighthouse did sloppy measurement.

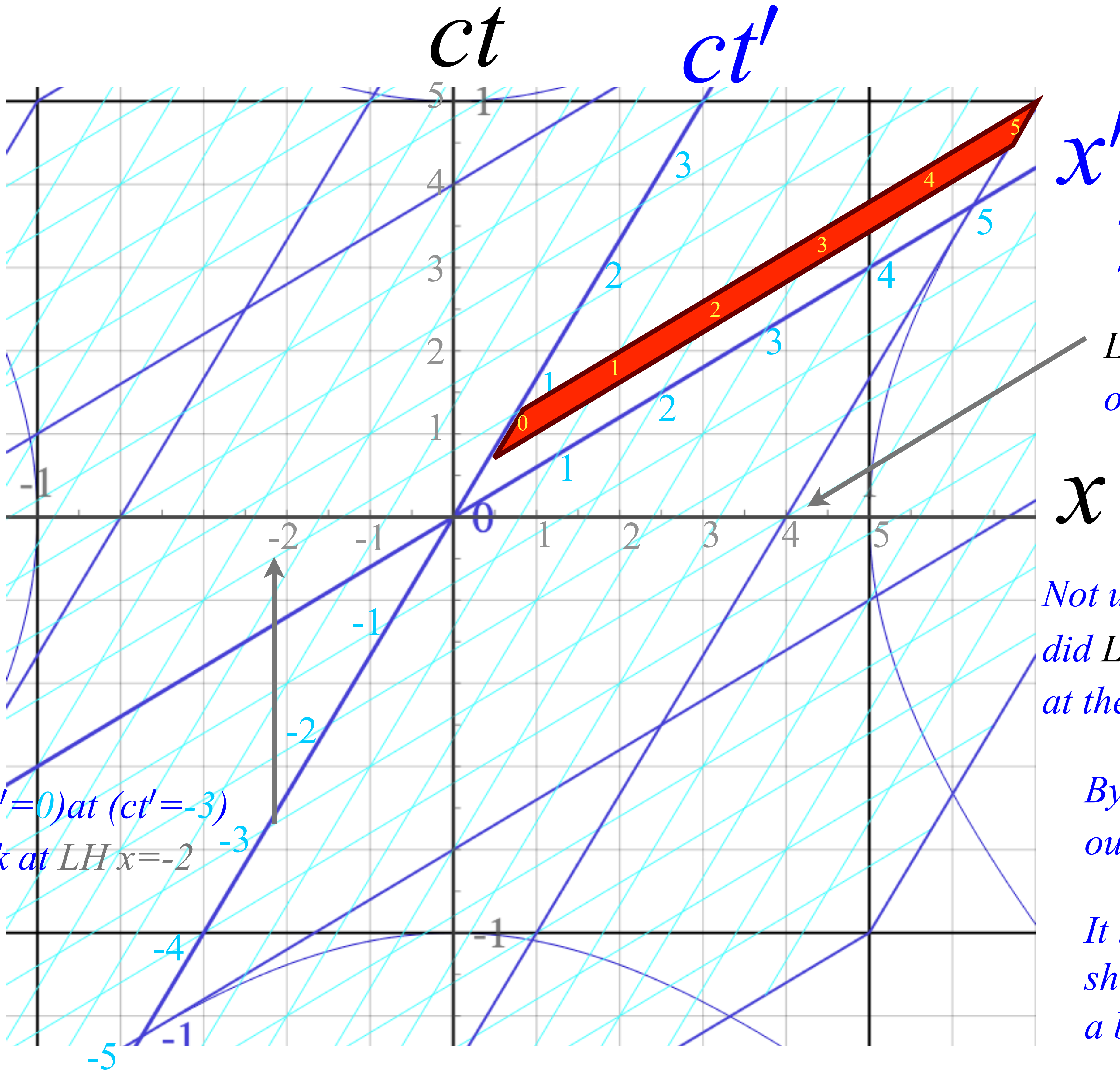
LH *first recorded* $x=4$ *for our nose* ($x'=5$)*at* ($ct'=-3$)

Then we moved forward.

Not until 3 sec. later ($ct'=0$)
did LH *record our tail* ($x'=0$)
at their $x=0$

By then our nose ($x'=5$) *is out past* LH ($x=6$) *mark.*

Tail ($x'=0$)*at* ($ct'=-3$)
is back at LH $x=-2$



Ship says Lighthouse did sloppy measurement.

LH *first recorded* $x=4$ *for our nose* ($x'=5$)*at* ($ct'=-3$)

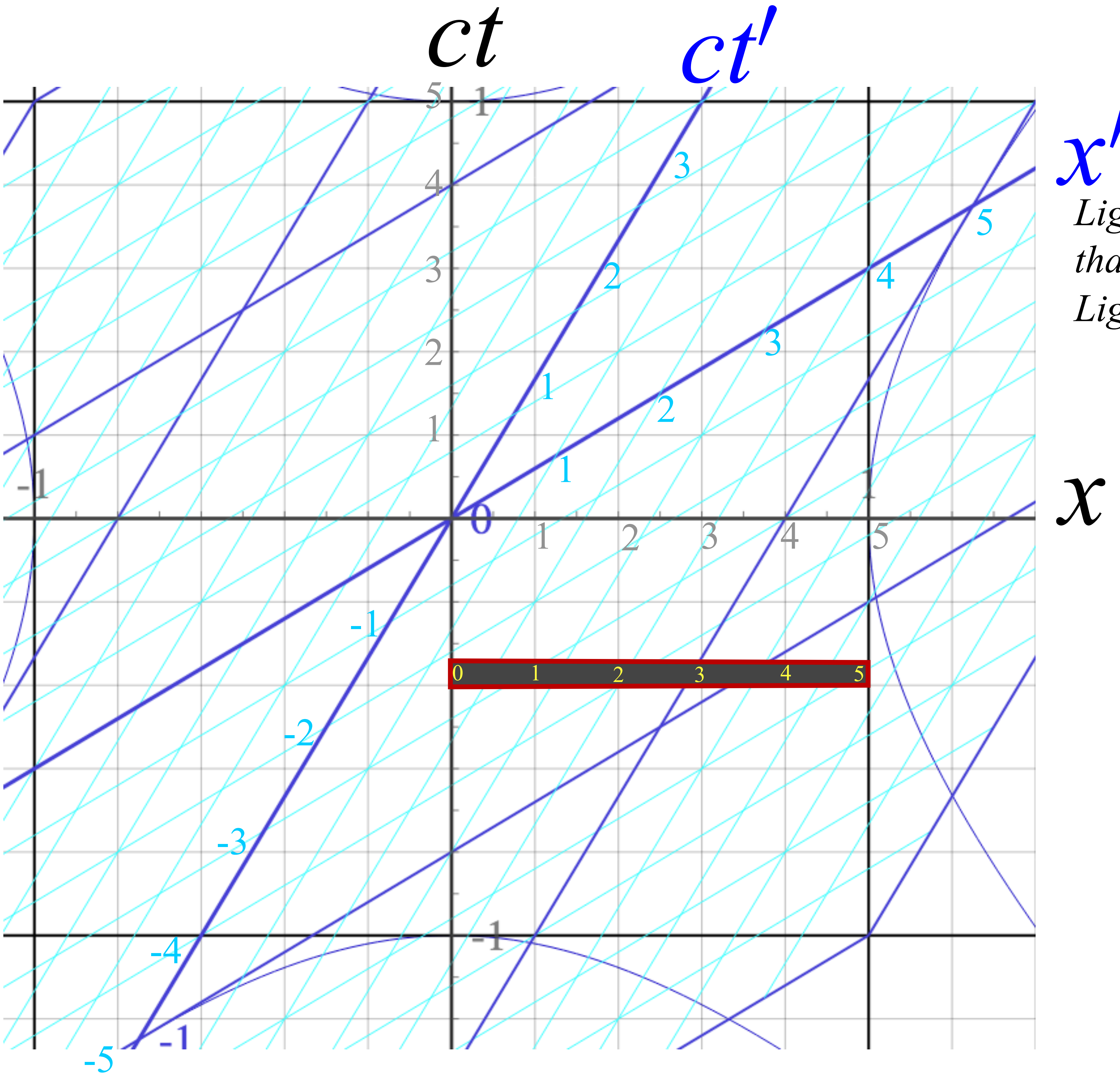
Then we moved forward.

Not until 3 sec. later ($ct'=0$) *did* LH *record our tail* ($x'=0$) *at their* $x=0$

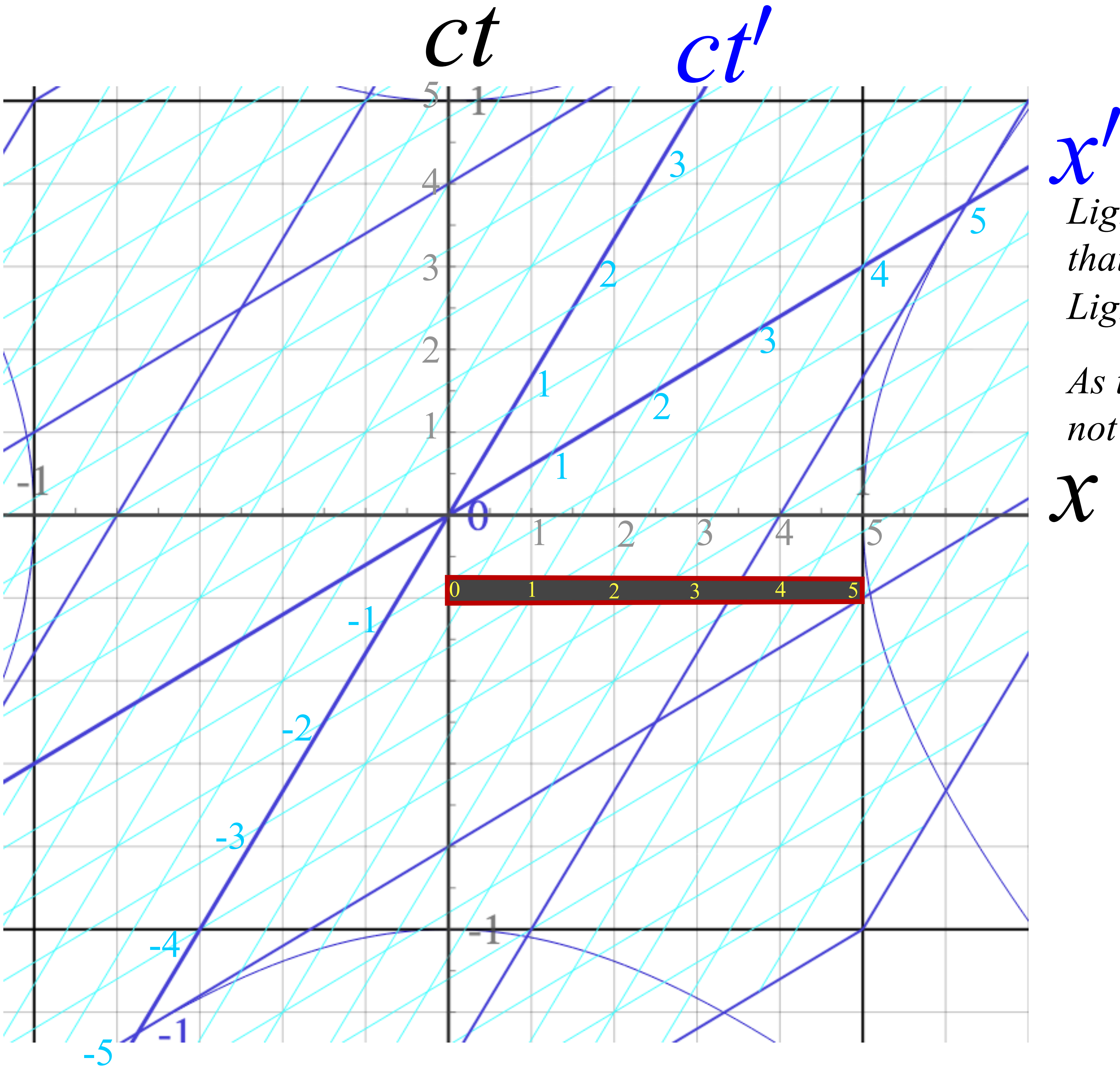
By then our nose ($x'=5$) *is out past* LH ($x=6$) *mark.*

It looks like our ship is not shorter than yours but quite a bit longer!

Tail ($x'=0$)*at* ($ct'=-3$) *is back at* LH $x=-2$

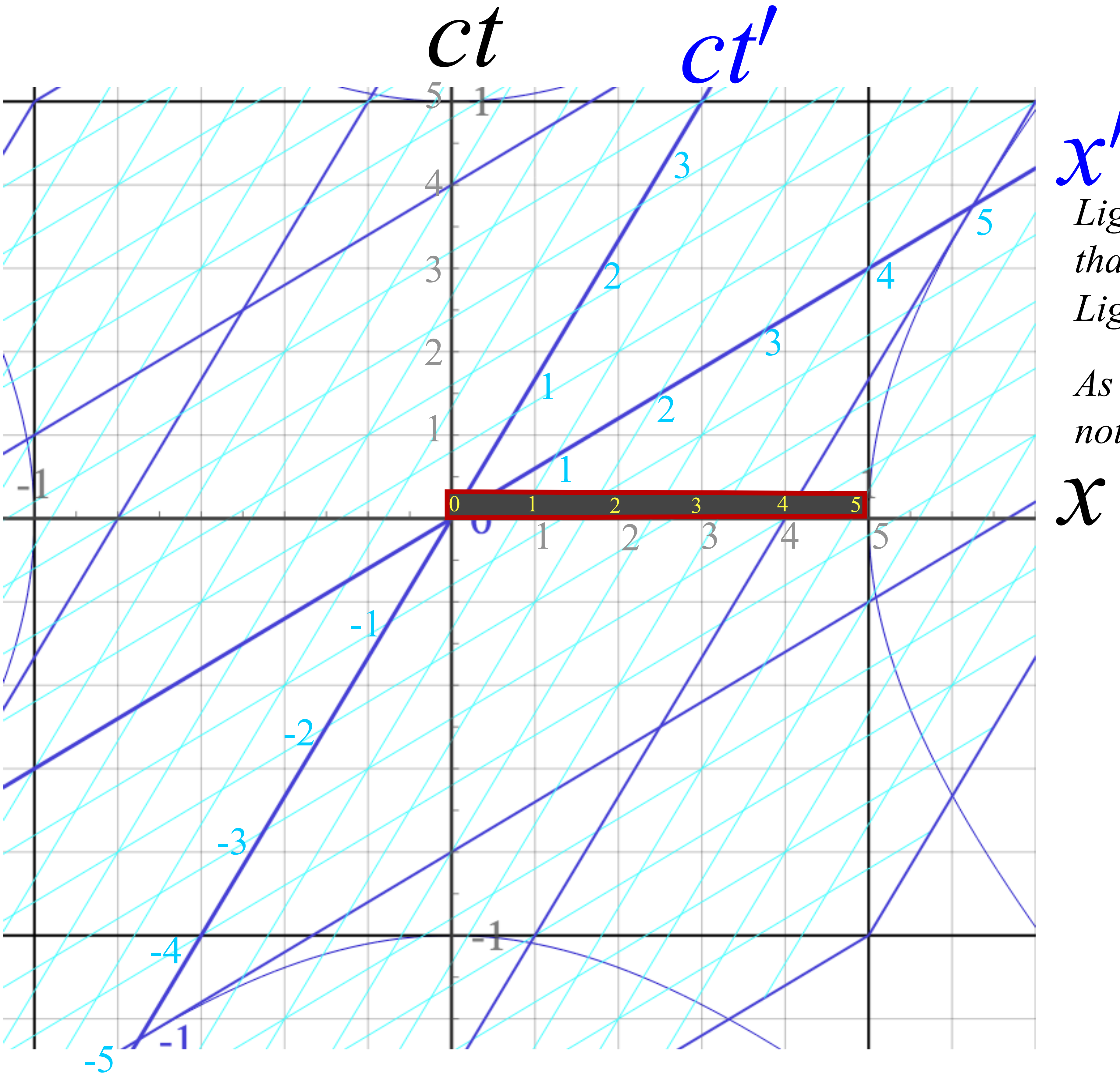


Lighthouse has an L=5 ship that sits with its tail at the Lighthouse (x=0).



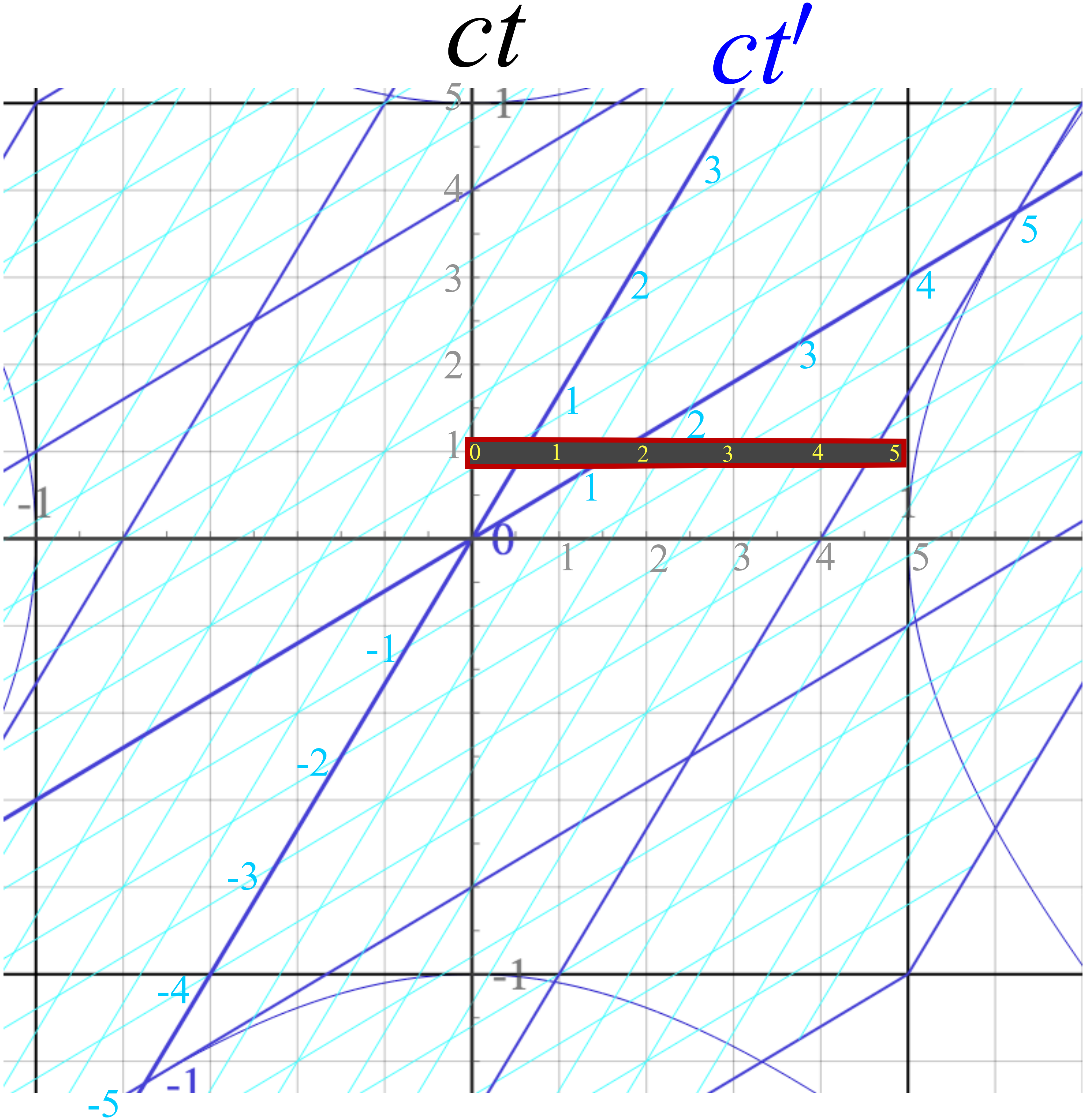
Lighthouse has an L=5 ship that sits with its tail at the Lighthouse (x=0).

As time goes on its ship does not move relative to LH



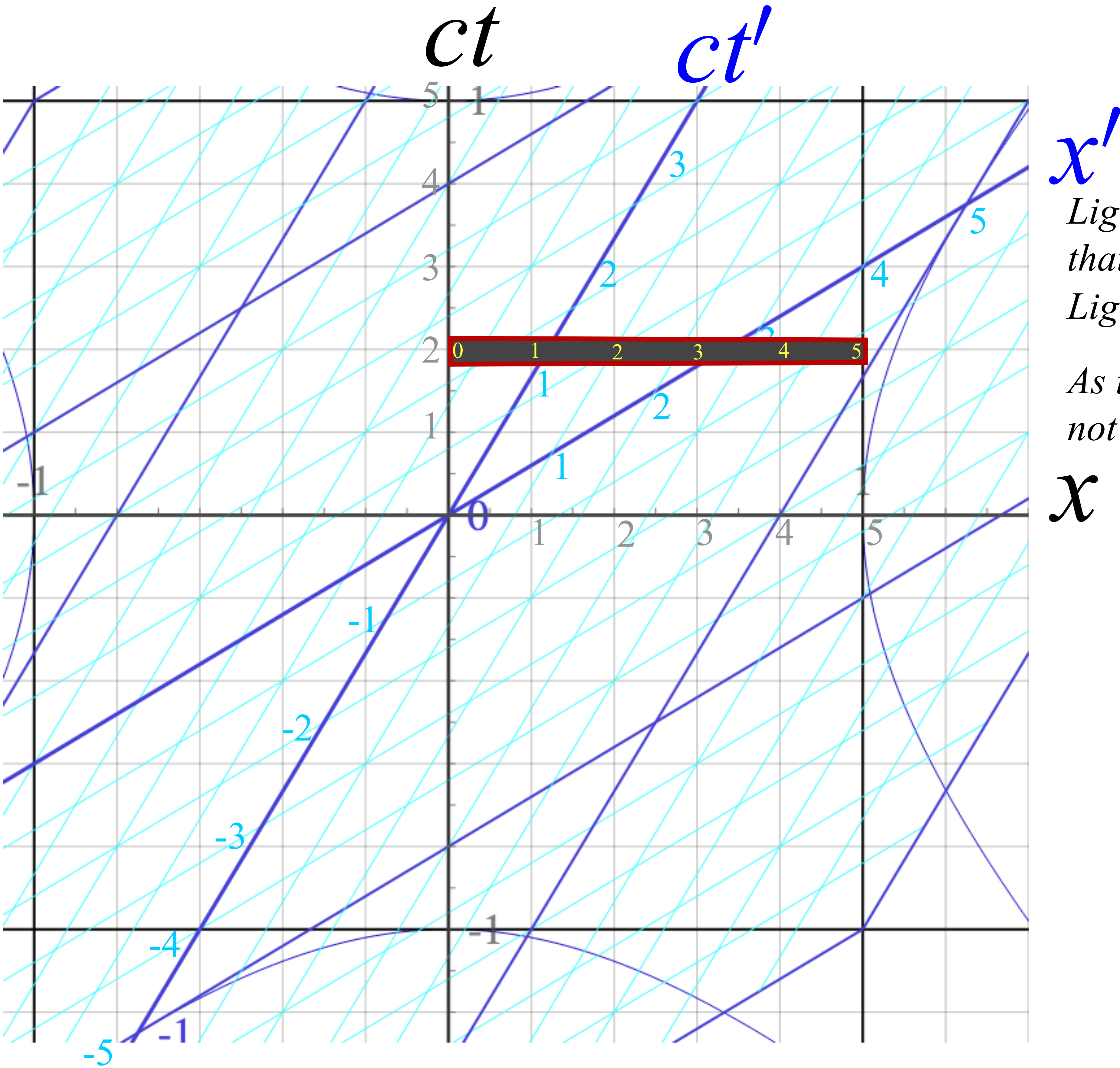
Lighthouse has an L=5 ship that sits with its tail at the Lighthouse (x=0).

As time goes on its ship does not move relative to LH



Lighthouse has an L=5 ship that sits with its tail at the Lighthouse (x=0).

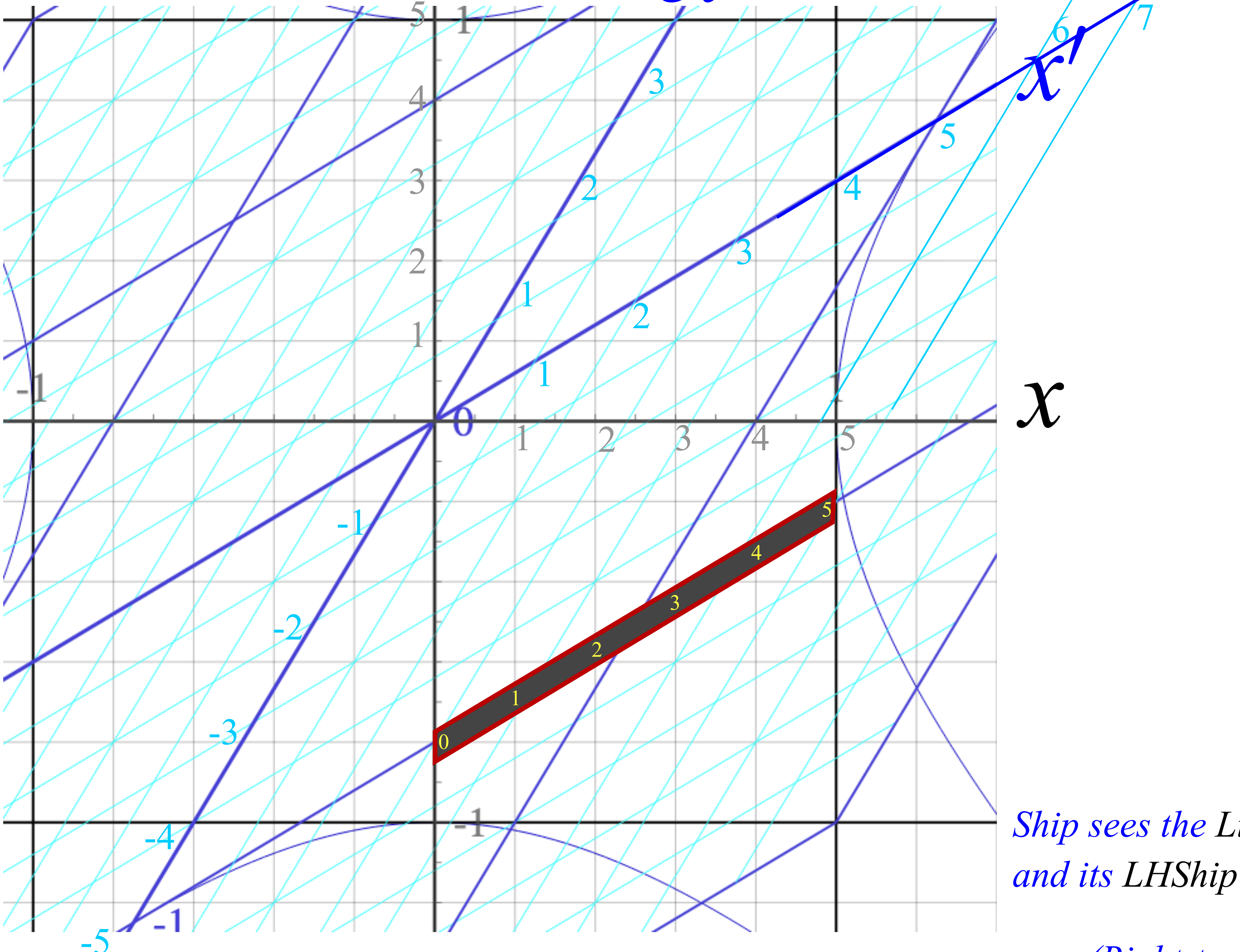
As time goes on its ship does not move relative to LH



Lighthouse has an L=5 ship that sits with its tail at the Lighthouse (x=0).

As time goes on its ship does not move relative to LH

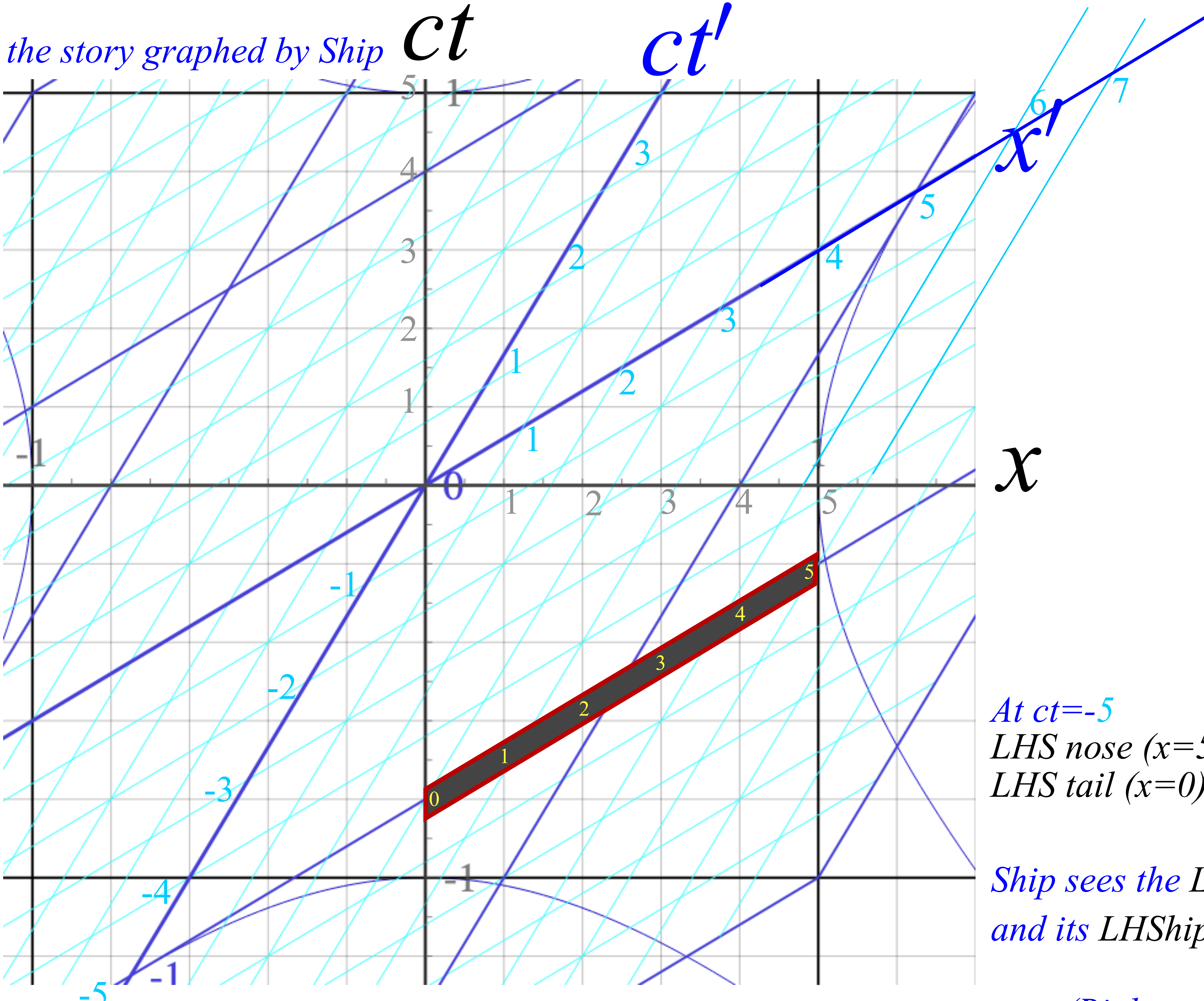
Now the story graphed by Ship



Ship sees the Lighthouse
and its LHShip *moving at -3⁄5c.*

(Right-to-Left)

Now the story graphed by Ship

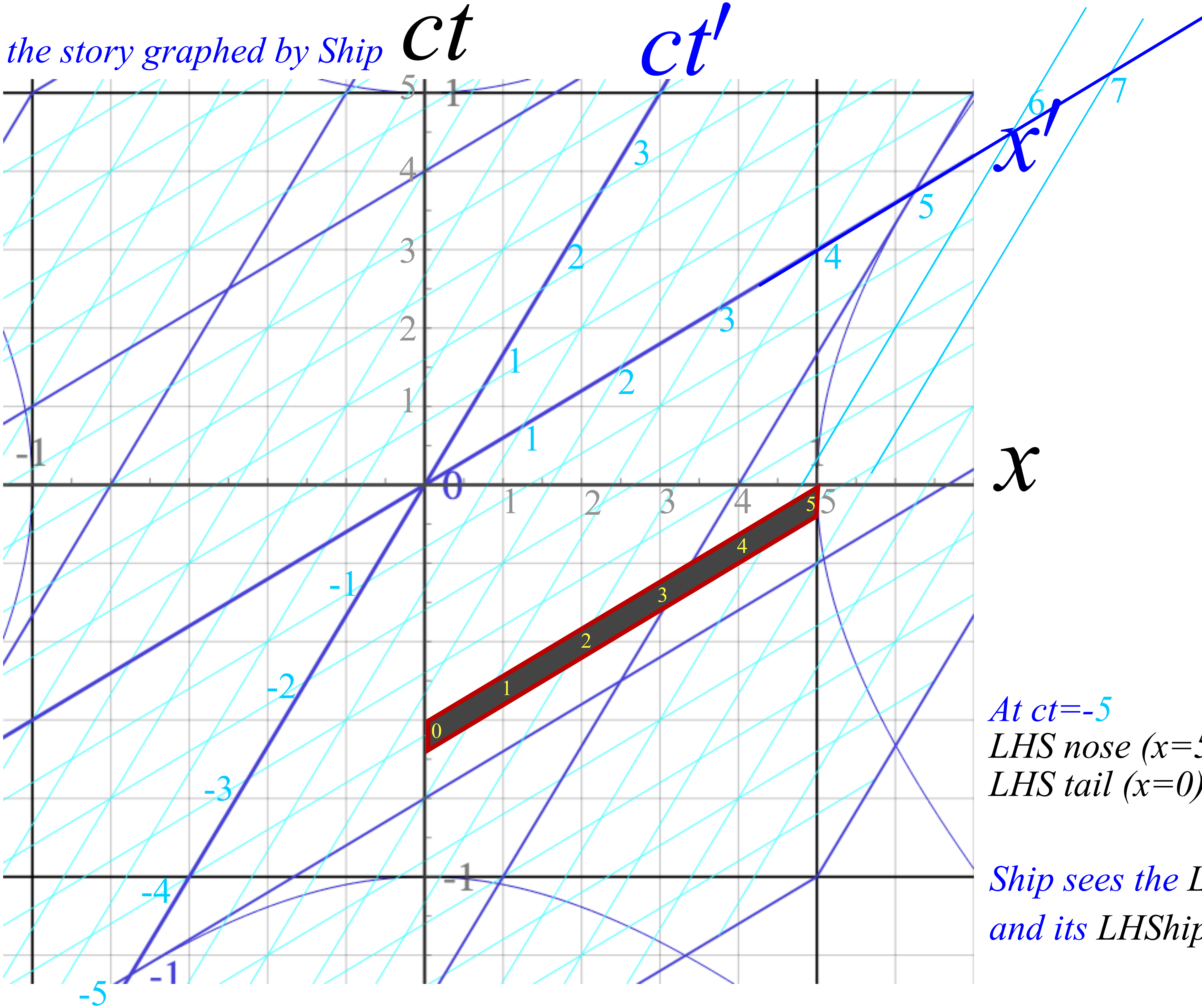


At ct=-5
LHS nose (x=5) passes x′=7
LHS tail (x=0) passes x′=3

Ship sees the Lighthouse
and its LHShip *moving at -3⁄5c.*

(Right-to-Left)

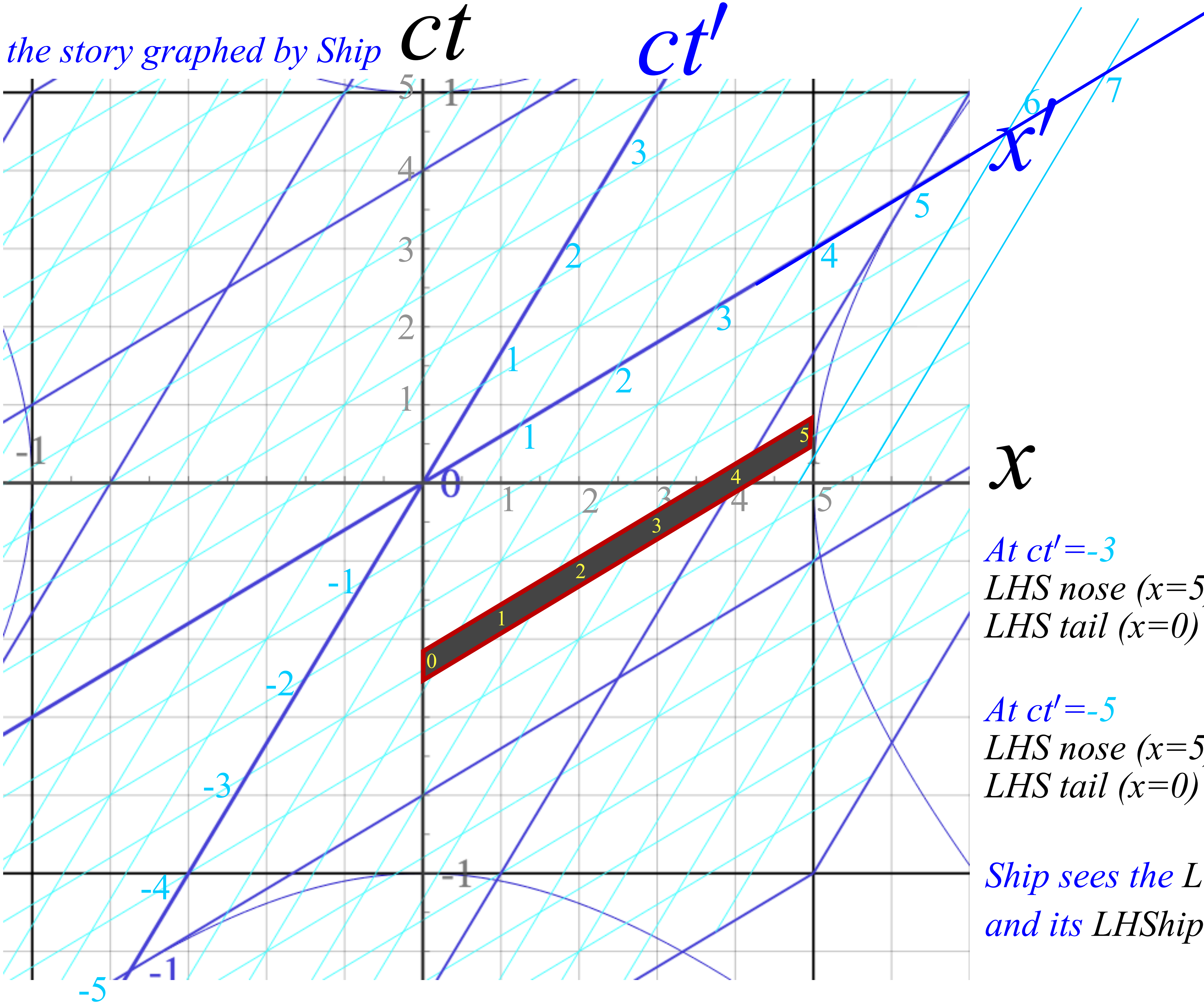
Now the story graphed by Ship



At $ct=-5$
LHS nose $(x=5)$ *passes* $x'=7$
LHS tail $(x=0)$ *passes* $x'=3$

Ship sees the Lighthouse and its LHShip moving at $-\frac{3}{5}c$.

Now the story graphed by Ship

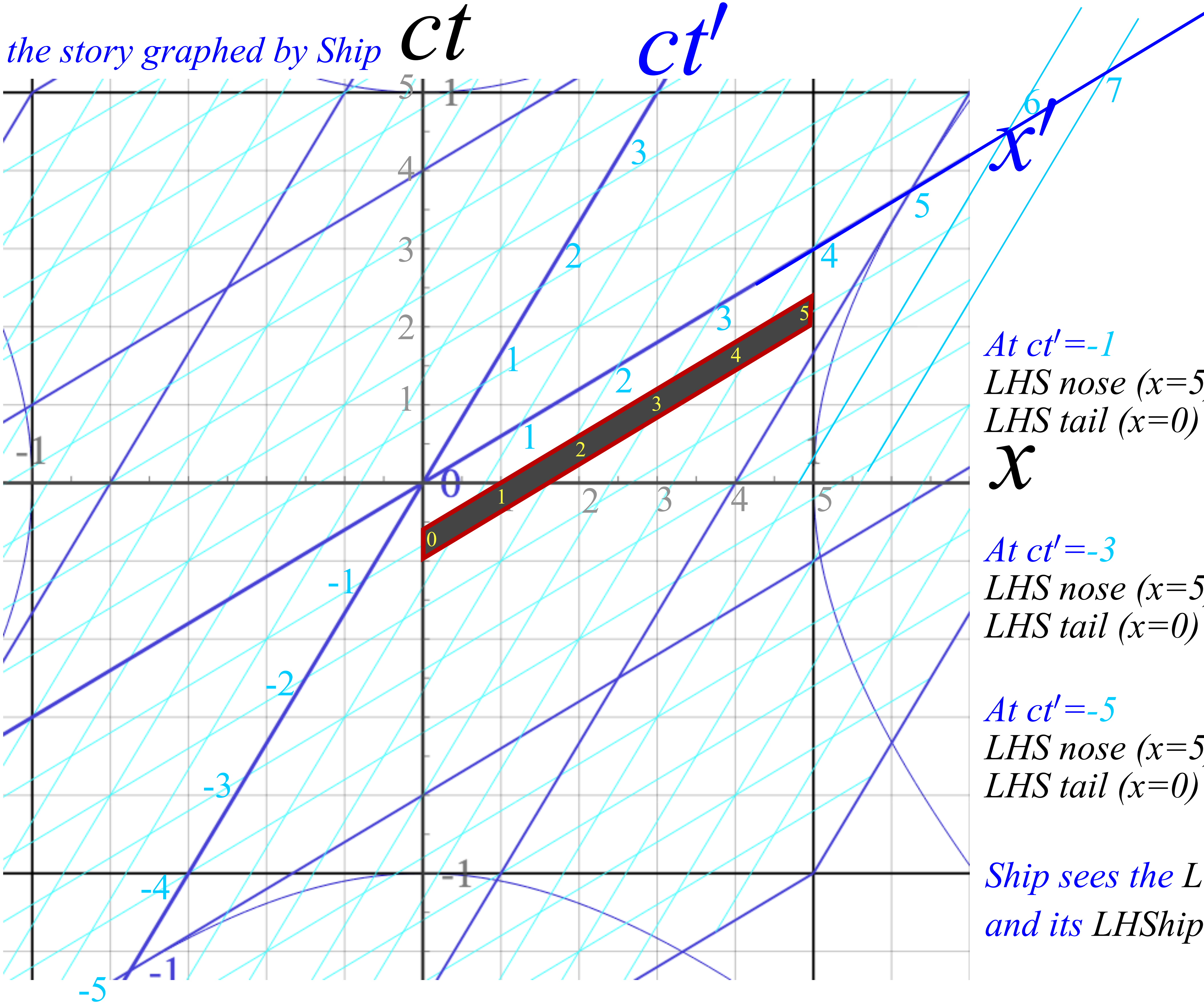


At ct'=-3
LHS nose (x=5) passes x'=6
LHS tail (x=0) passes x'=2

At ct'=-5
LHS nose (x=5) passes x'=7
LHS tail (x=0) passes x'=3

Ship sees the Lighthouse
and its LHShip moving at -⅗c.

Now the story graphed by Ship



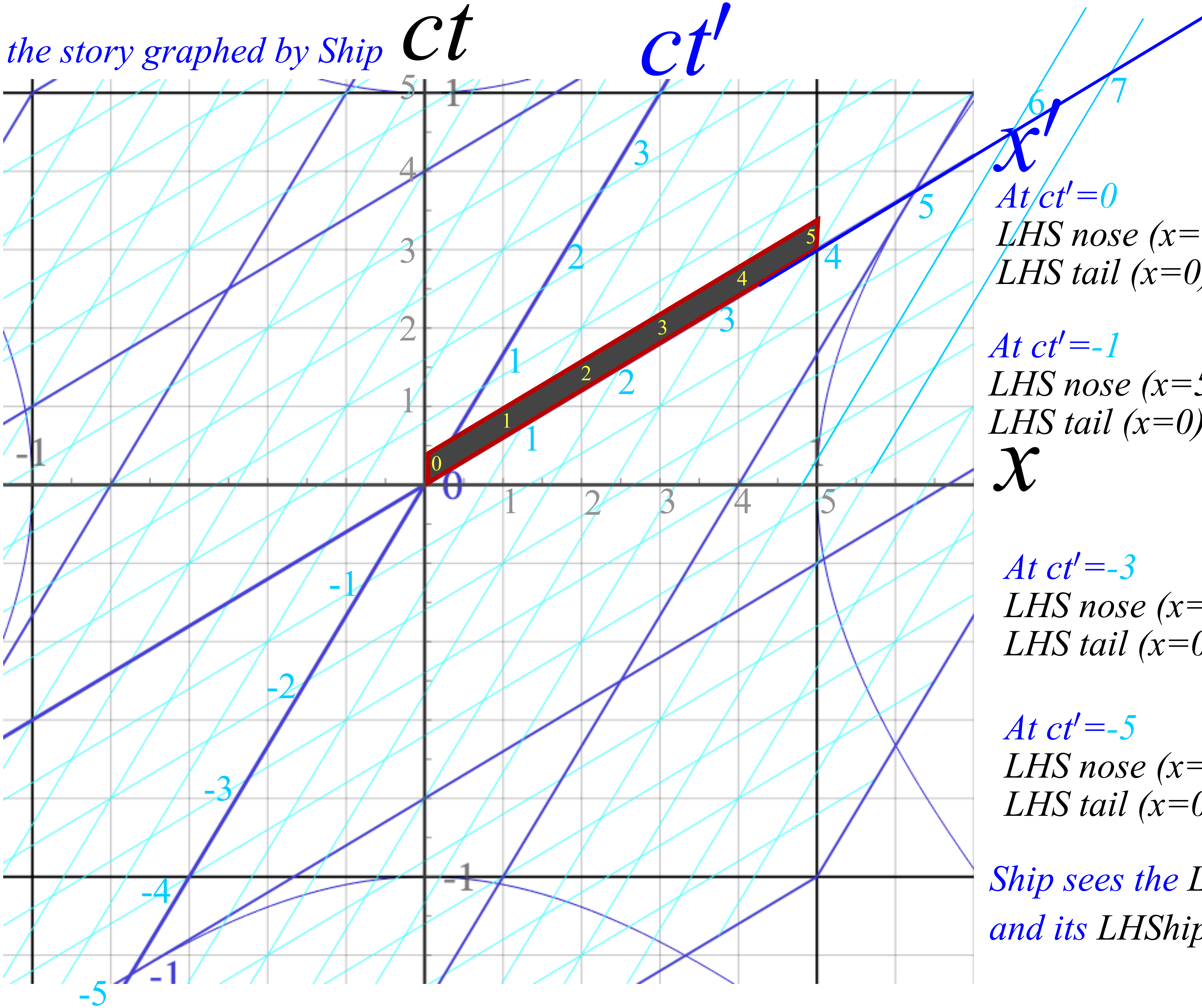
At ct'*=-1*
LHS nose (x=5) *passes* x'*=4.5*
LHS tail (x=0) *passes* x'*=0.5*

At ct'*=-3*
LHS nose (x=5) *passes* x'*=6*
LHS tail (x=0) *passes* x'*=2*

At ct'*=-5*
LHS nose (x=5) *passes* x'*=7*
LHS tail (x=0) *passes* x'*=3*

Ship sees the Lighthouse
and its LHShip *moving at -⅗c.*

Now the story graphed by Ship



At ct′=0
LHS nose (x=5) *passes x′=4*
LHS tail (x=0) *passes x′=0*

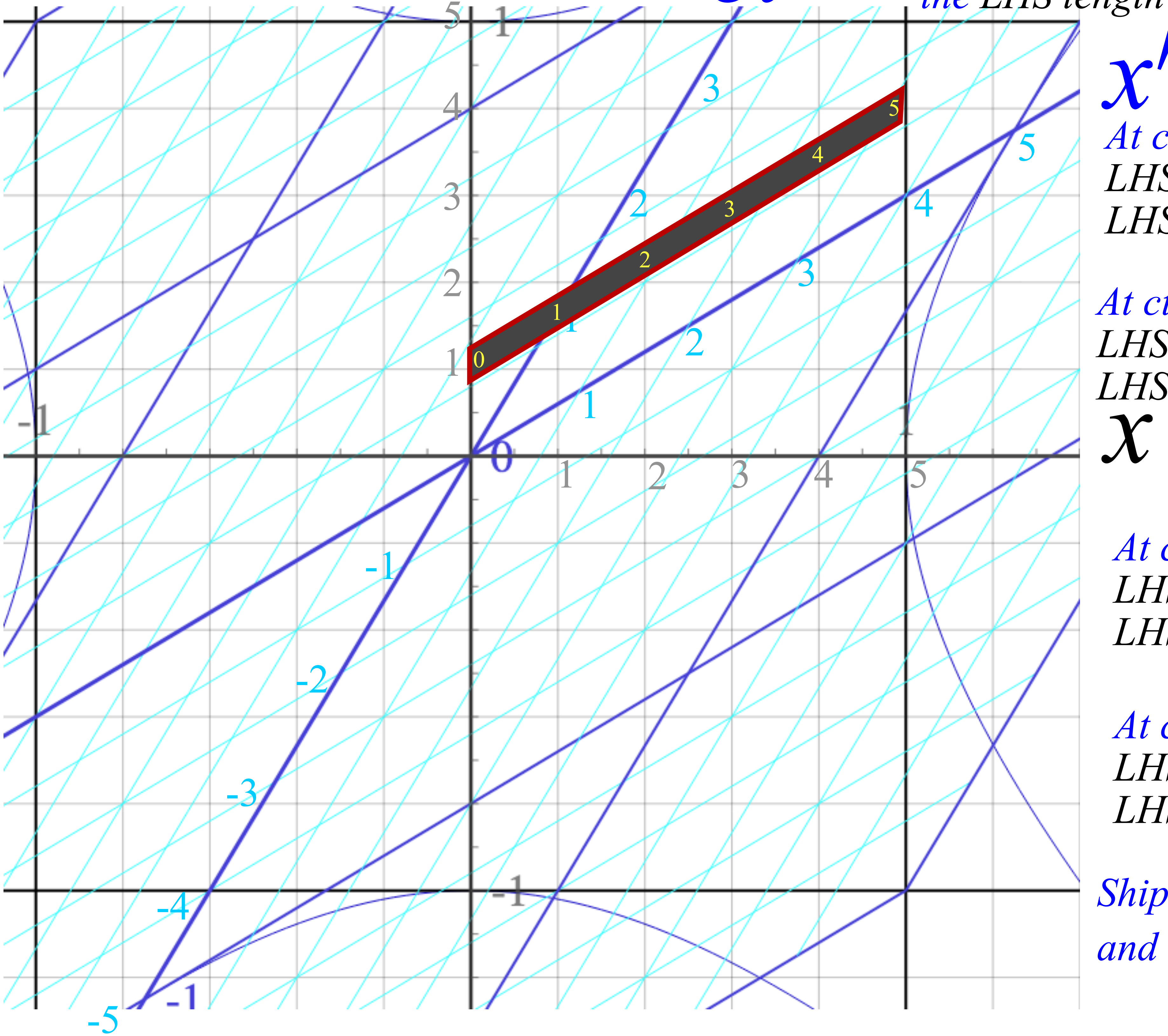
At ct′=-1
LHS nose (x=5) *passes x′=4.5*
LHS tail (x=0) *passes x′=0.5*

At ct′=-3
LHS nose (x=5) *passes x′=6*
LHS tail (x=0) *passes x′=2*

At ct′=-5
LHS nose (x=5) *passes x′=7*
LHS tail (x=0) *passes x′=3*

Ship sees the Lighthouse *and its* LHShip *moving at -⅗c.*

Now the story graphed by Ship



From the Ship's graph it appears that the LHS length is only 4 and not 5

At $ct'=0$
LHS nose ($x=5$) *passes* $x'=4$
LHS tail ($x=0$) *passes* $x'=0$

At $ct'=-1$
LHS nose ($x=5$) *passes* $x'=4.5$
LHS tail ($x=0$) *passes* $x'=0.5$

At $ct'=-3$
LHS nose ($x=5$) *passes* $x'=6$
LHS tail ($x=0$) *passes* $x'=2$

At $ct'=-5$
LHS nose ($x=5$) *passes* $x'=7$
LHS tail ($x=0$) *passes* $x'=3$

Ship sees the Lighthouse *and its* LHShip *moving at* -⅗c.

Review of hyper-trigonometry (*tanhρ, sinhρ, and coshρ, sechρ, and cschρ, cothρ*)

and co-trigonometry (*sinσ, tanσ, and secσ, cosσ, and cotσ, cscσ*)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

➔ Lorentz space contraction

Time-simultaneity-breaking ←

Velocity addition

Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Length contraction-dilation revisited

A cute Epstein feature is that Lorentz-Fitzgerald contraction of a proper length L to $L'=L\sqrt{1-u^2/c^2}$ is simply rotational projection onto the x-axis of a length L rotated by σ.

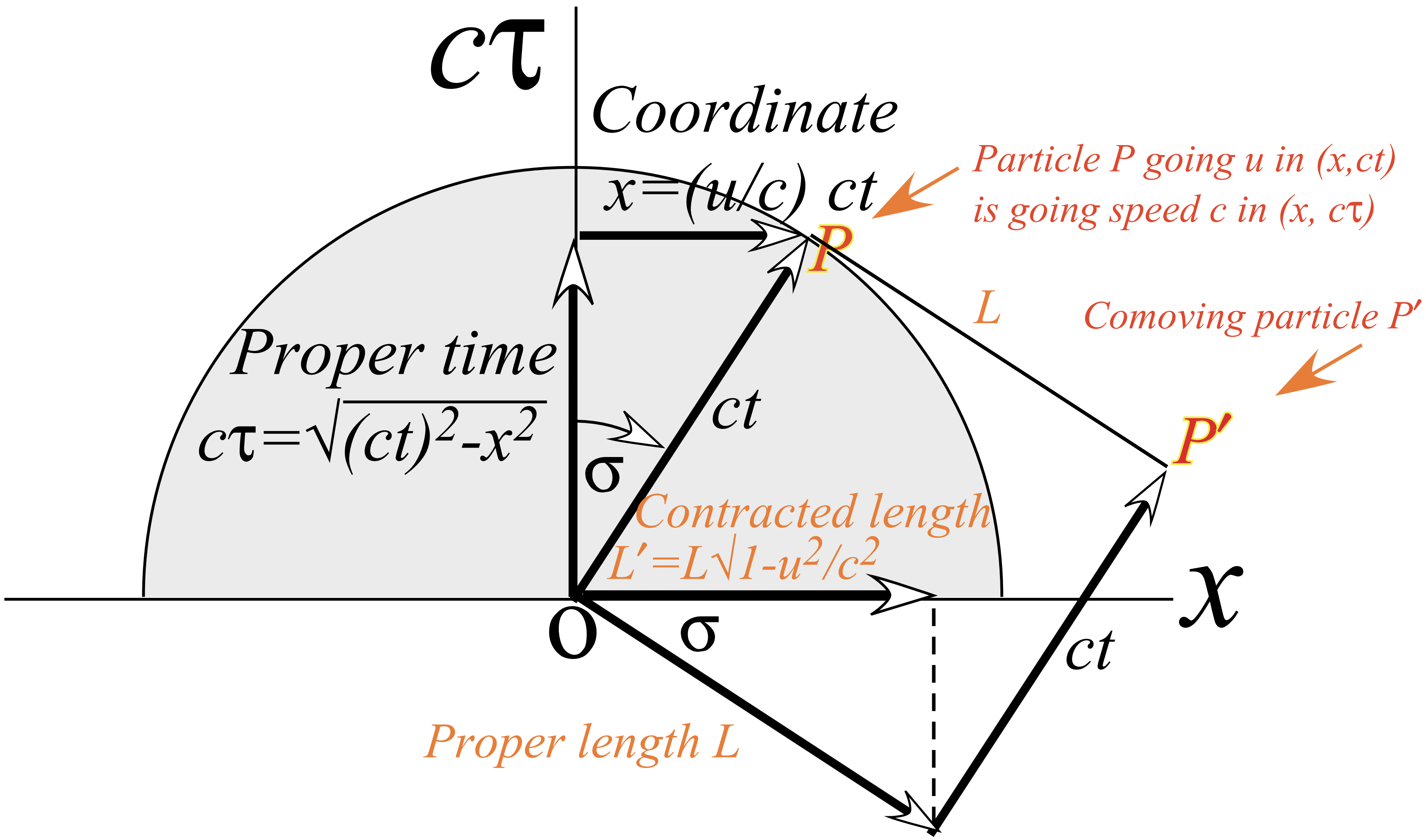


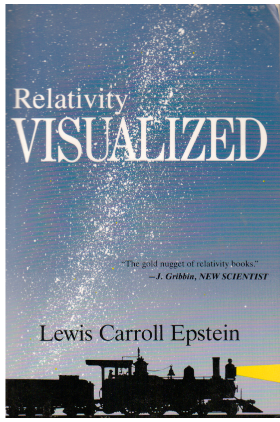
Fig.12 (2nd-part) Space-proper-time plot makes all objects move at speed c along their cosmic speedometer.

Dual View Space-Space and Space-properTime
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=600
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=601
Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=602

Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

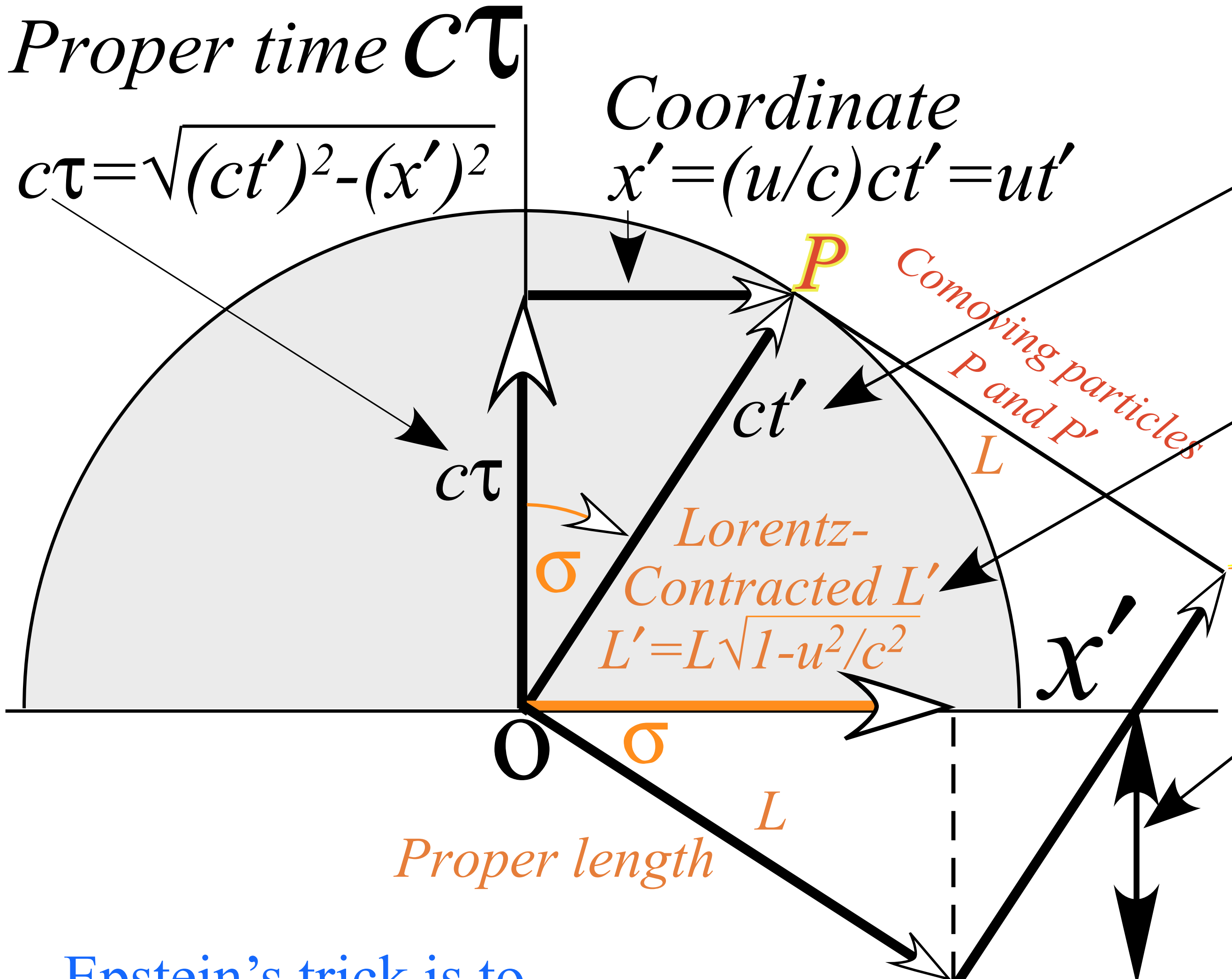
to Transverse relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - *(L. C. Epstein's "Cosmic Speedometer")*

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$

Lorentz length contraction:

$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$

Proper Time asimultaneity:

$c\,\Delta\tau = L' \sinh\rho = L\cos\sigma \sinh\rho$

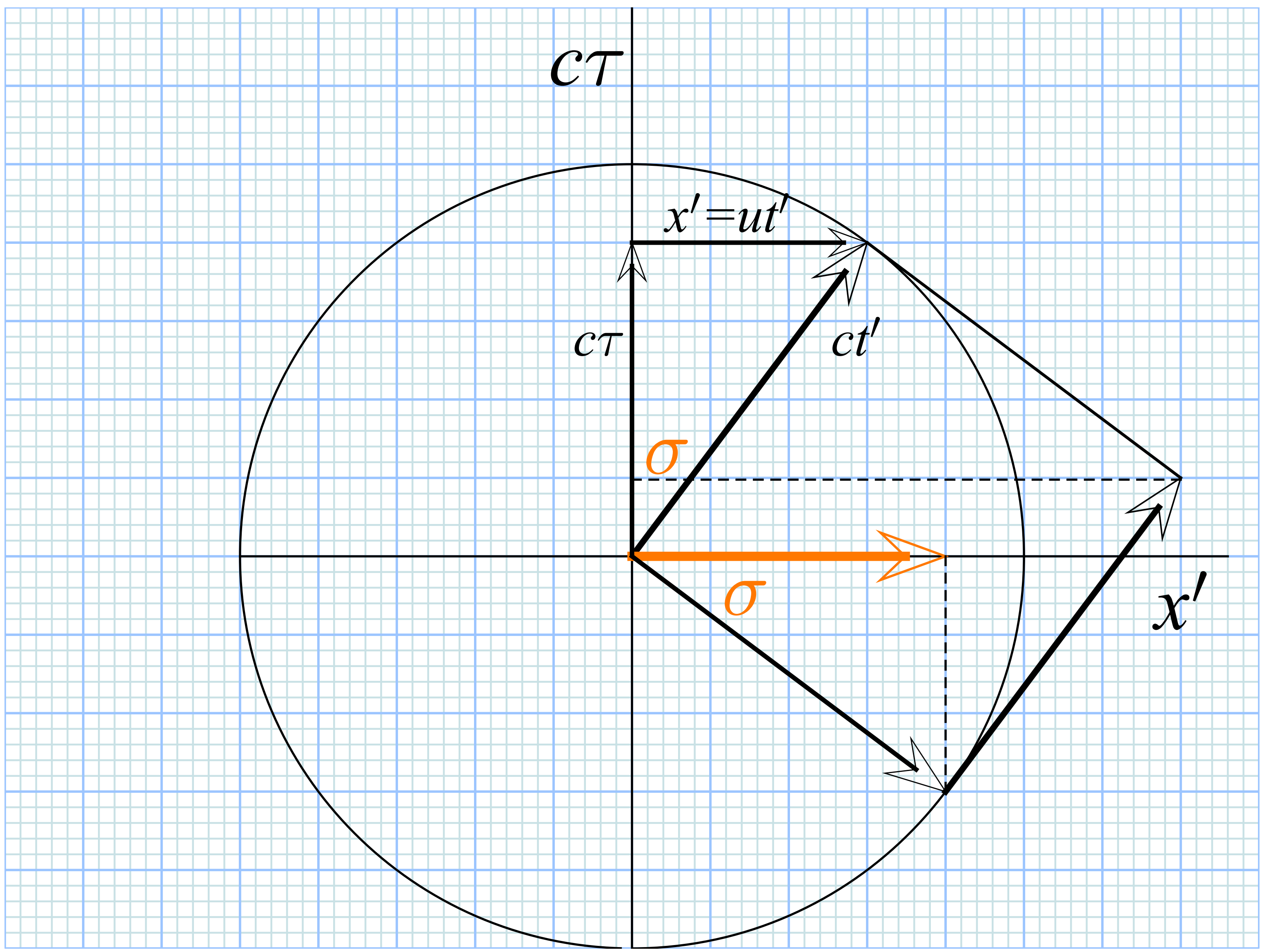
$= L\cos\sigma\tan\sigma$

$= L\sin\sigma \quad = L\tanh\rho = L\,u/c$

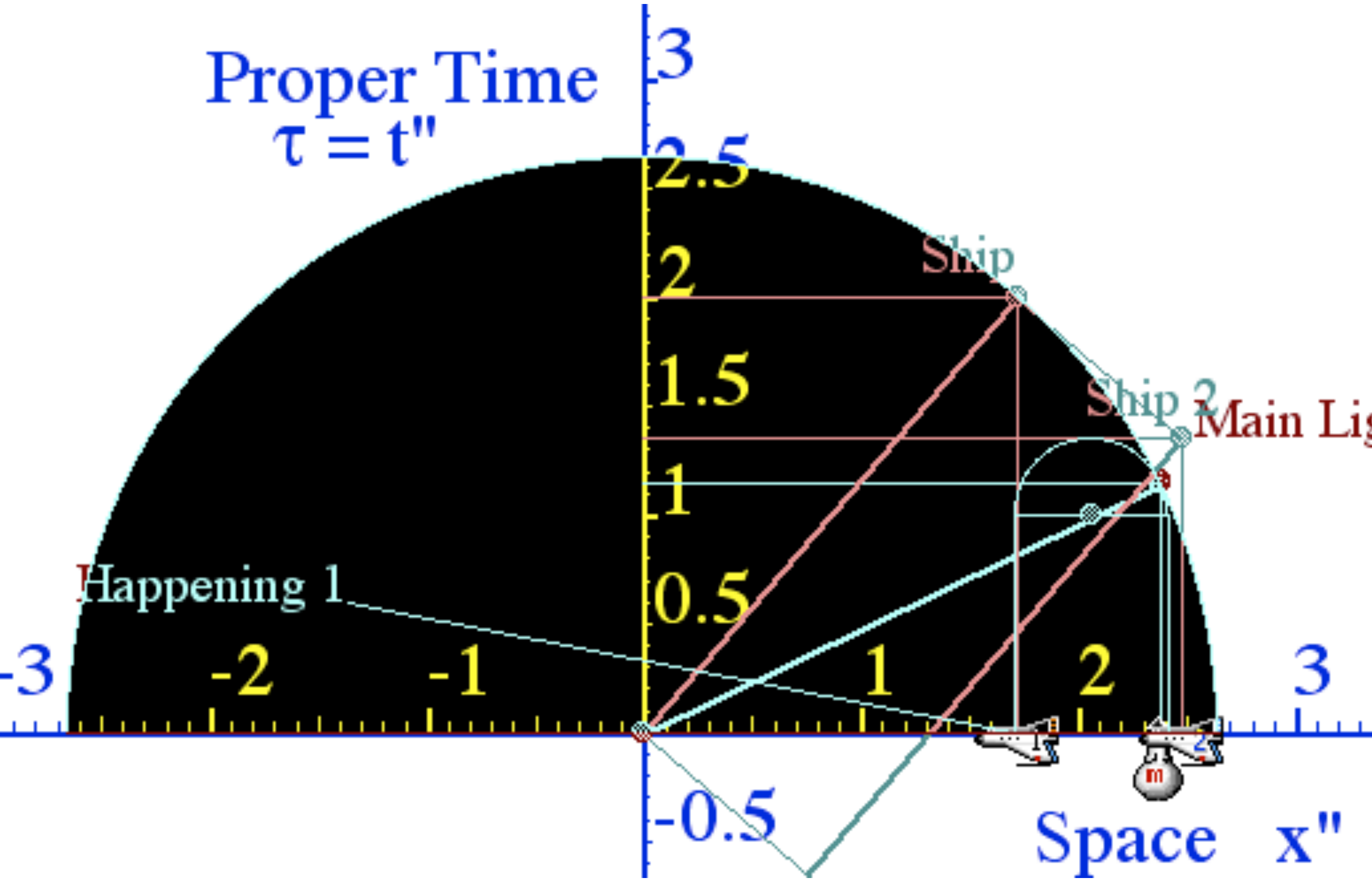
(for Proper time)

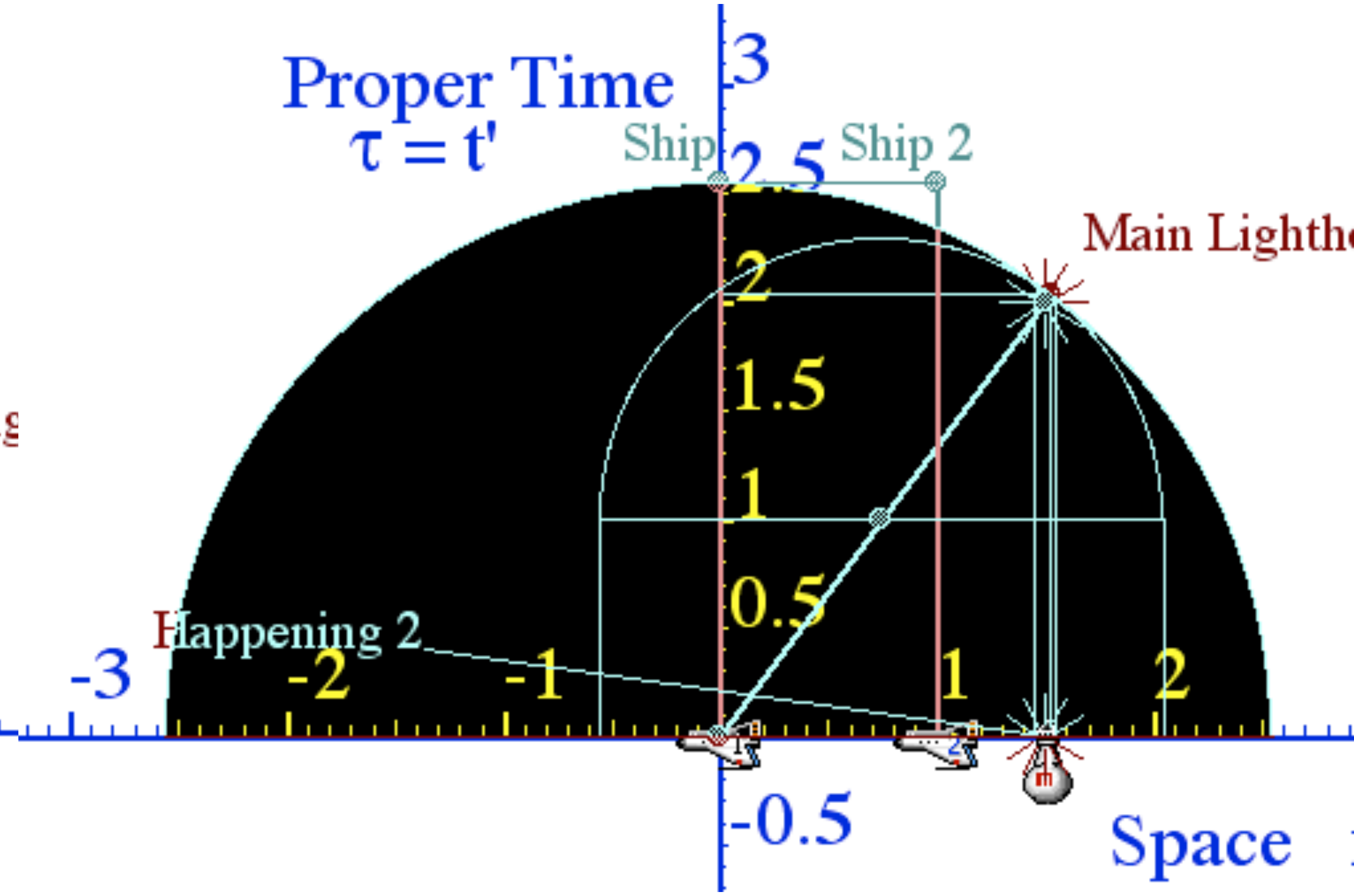
Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

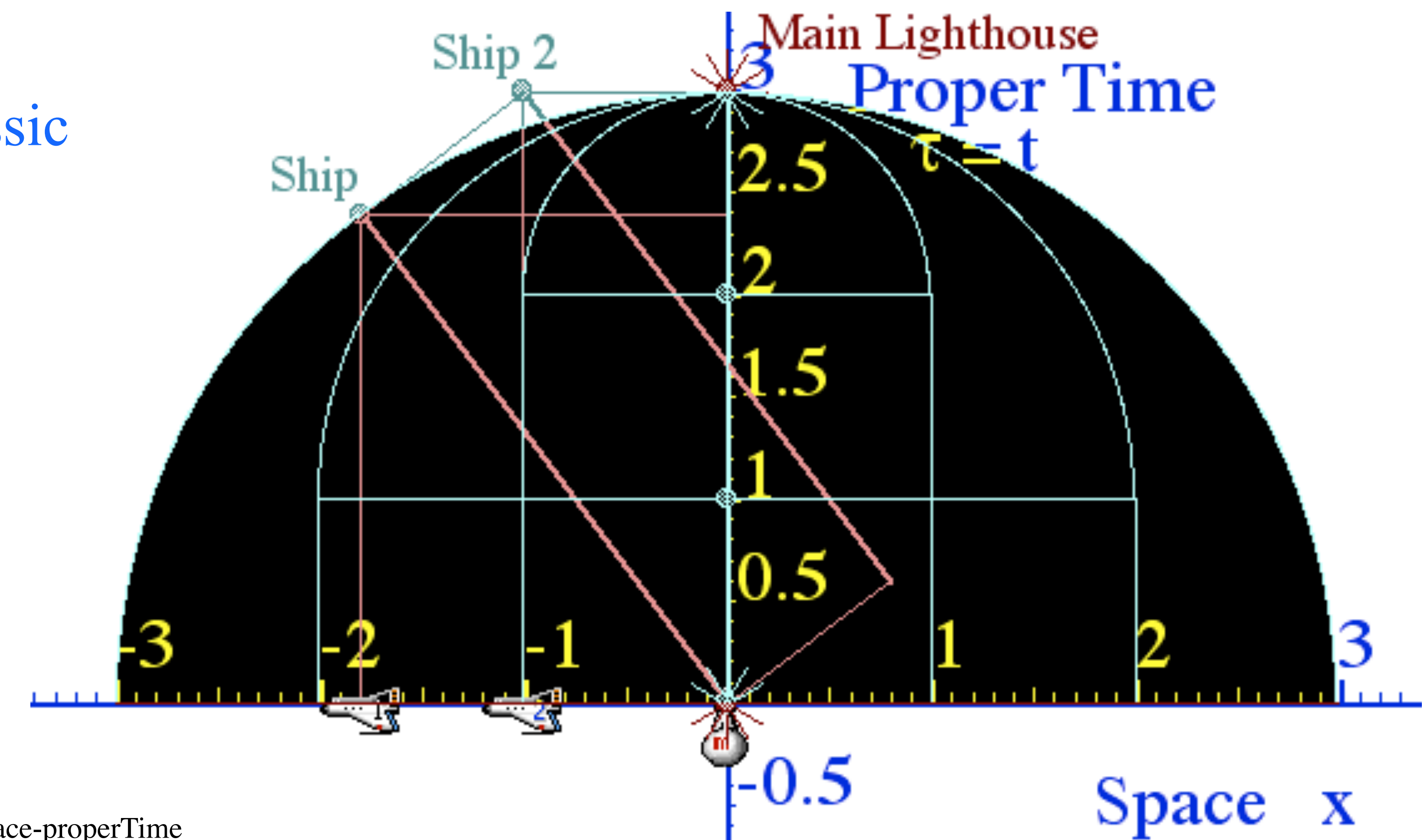


$c\tau$
$x'=ut'$
$c\tau$
ct'
σ
σ
x'





2005 Mac Classic simulations



2016 Web-App simulations

Dual View Space-Space and Space-properTime

Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=600

Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=601

Link: http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=602

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

Lorentz space contraction

Time-simultaneity-breaking

➔ Velocity addition (Galileo's Revenge) ⬅

Twin-paradox resolution in space-proper-time

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

Relativistic Velocity addition: Galileo's Revenge

Recall rapidity (logarithm of Doppler factor : $\rho_{AB} = \ln(A|B)$). *It adds like Galilean velocity.*

Also, recall that hyper-tangent of rapidity is (group u or classical u) velocity: $\frac{u}{c} = \tanh \rho = \beta$

Relativistic Velocity addition: Galileo's Revenge

Recall rapidity (logarithm of Doppler factor : $\rho_{AB} = \ln(A|B)$). *It adds like Galilean velocity.*

Also, recall that hyper-tangent of rapidity is (group u or classical u) velocity: $\frac{u}{c} = \tanh\rho = \beta$

Suppose someone going with rapidity ρ_a=2 toward you. (That is u_a/c=tanh(2)=0.964...)
Suppose they throw something at you with rapidity ρ_b=3. (That is u_b/c=tanh(3)=0.995...)
Then you will see it approach you with rapidity ρ_{a+b}=2+3 =5. (That is u_{a+b}/c=tanh(5)=0.99909.)

Relativistic Velocity addition: Galileo's Revenge

Recall rapidity (logarithm of Doppler factor : $\rho_{AB} = \ln(A|B)$). *It adds like Galilean velocity.*

Also, recall that hyper-tangent of rapidity is (group u or classical u) velocity: $\frac{u}{c} = \tanh\rho = \beta$

Suppose someone going with rapidity ρ_a=2 toward you. (That is u_a/c=tanh(2)=0.964...)
Suppose they throw something at you with rapidity ρ_b=3. (That is u_b/c=tanh(3)=0.995...)
Then you will see it approach you with rapidity ρ_{a+b}=2+3 =5. (That is u_{a+b}/c=tanh(5)=0.99909.)

Lower speed example:
Suppose someone going with rapidity ρ_a=.02 toward you. (That is u_a/c=tanh(.02)=0.0199973.)
Suppose they throw something at you with rapidity ρ_b=.03. (That is u_b/c=tanh(.03)=0.0299910.)
Then you will see it approach you with rapidity ρ_{a+b}=.02+.03 =.05. (u_{a+b}/c=tanh(.05)=0.049958375.)

Note: $u_a/c+u_b/c$=0.04988337 *Too High!*

Relativistic Velocity addition: Galileo's Revenge

Recall rapidity (logarithm of Doppler factor : $\rho_{AB} = \ln(A|B)$). *It adds like Galilean velocity.*

Also, recall that hyper-tangent of rapidity is (group u or classical u) velocity: $\frac{u}{c} = \tanh\rho = \beta$

Suppose someone going with rapidity ρ_a=2 toward you. (That is u_a/c=tanh(2)=0.964...)
Suppose they throw something at you with rapidity ρ_b=3. (That is u_b/c=tanh(3)=0.995...)
Then you will see it approach you with rapidity ρ_{a+b}=2+3 =5. (That is u_{a+b}/c=tanh(5)=0.99909.)

Lower speed example:
Suppose someone going with rapidity ρ_a=.02 toward you. (That is u_a/c=tanh(.02)=0.0199973.)
Suppose they throw something at you with rapidity ρ_b=.03. (That is u_b/c=tanh(.03)=0.0299910.)
Then you will see it approach you with rapidity ρ_{a+b}=.02+.03 =.05. (u_{a+b}/c=tanh(.05)=0.049958375.)

Note: $u_a/c+u_b/c$=0.049988337 *Too High!*

Velocity Addition involves $\frac{u_{a+b}}{c} = \beta_{ab} = \tanh(\rho_a+\rho_b) = \frac{\tanh\rho_a+\tanh\rho_b}{1+\tanh\rho_a\tanh\rho_b}$

So velocity in units of light-speed c is added as follows: $\beta_{a+b} = \frac{\beta_a+\beta_b}{1+\beta_a\beta_b}$

Relativistic Velocity addition: Galileo's Revenge

Recall rapidity (logarithm of Doppler factor : $\rho_{AB} = \ln(A|B)$). *It adds like Galilean velocity.*

Also, recall that hyper-tangent of rapidity is (group u or classical u) velocity: $\frac{u}{c} = \tanh\rho = \beta$

Suppose someone going with rapidity ρ_a=2 toward you. (That is u_a/c=tanh(2)=0.964...)
Suppose they throw something at you with rapidity ρ_b=3. (That is u_b/c=tanh(3)=0.995...)
Then you will see it approach you with rapidity ρ_{a+b}=2+3 =5. (That is u_{a+b}/c=tanh(5)=0.99909.)

Lower speed example:
Suppose someone going with rapidity ρ_a=.02 toward you. (That is u_a/c=tanh(.02)=0.0199973.)
Suppose they throw something at you with rapidity ρ_b=.03. (That is u_b/c=tanh(.03)=0.0299910.)
Then you will see it approach you with rapidity ρ_{a+b}=.02+.03 =.05. (u_{a+b}/c=tanh(.05)=0.049958375.)

Note: $u_a/c+u_b/c$=0.049988337 *Too High!*

Velocity Addition involves $\frac{u_{a+b}}{c} = \beta_{ab} = \tanh(\rho_a+\rho_b) = \frac{\tanh\rho_a + \tanh\rho_b}{1+\tanh\rho_a \tanh\rho_b}$

So velocity in units of light-speed c is added as follows: $\beta_{a+b} = \frac{\beta_a + \beta_b}{1+\beta_a\beta_b}$

No longer is ½ plus ½ equal to 1. Instead it is: $\beta_{\frac{1}{2}+\frac{1}{2}} = \frac{\frac{1}{2}+\frac{1}{2}}{1+\frac{1}{2}\frac{1}{2}} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$

Relativistic Velocity addition: Galileo's Revenge

Recall rapidity (logarithm of Doppler factor : $\rho_{AB} = \ln(A|B)$). *It adds like Galilean velocity.*

Also, recall that hyper-tangent of rapidity is (group u or classical u) velocity: $\frac{u}{c} = \tanh\rho = \beta$

Suppose someone going with rapidity ρ_a=2 toward you. (That is u_a/c=tanh(2)=0.964...)
Suppose they throw something at you with rapidity ρ_b=3. (That is u_b/c=tanh(3)=0.995...)
Then you will see it approach you with rapidity ρ_{a+b}=2+3 =5. (That is u_{a+b}/c=tanh(5)=0.99909.)

Lower speed example:
Suppose someone going with rapidity ρ_a=.02 toward you. (That is u_a/c=tanh(.02)=0.0199973.)
Suppose they throw something at you with rapidity ρ_b=.03. (That is u_b/c=tanh(.03)=0.0299910.)
Then you will see it approach you with rapidity ρ_{a+b}=.02+.03 =.05. (u_{a+b}/c=tanh(.05)=0.049958375.)

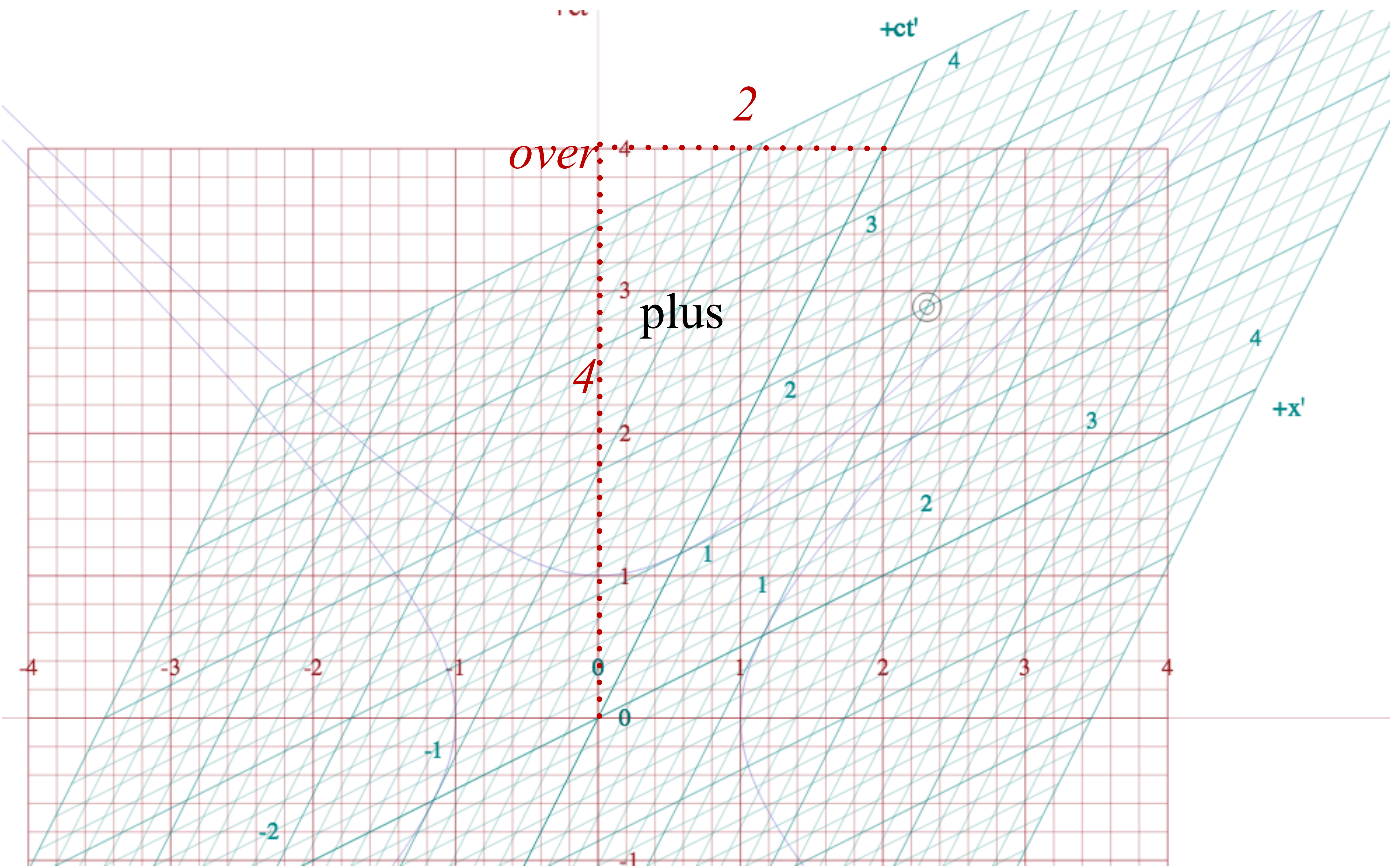
Note: $u_a/c+u_b/c$=0.049988337 *Too High!*

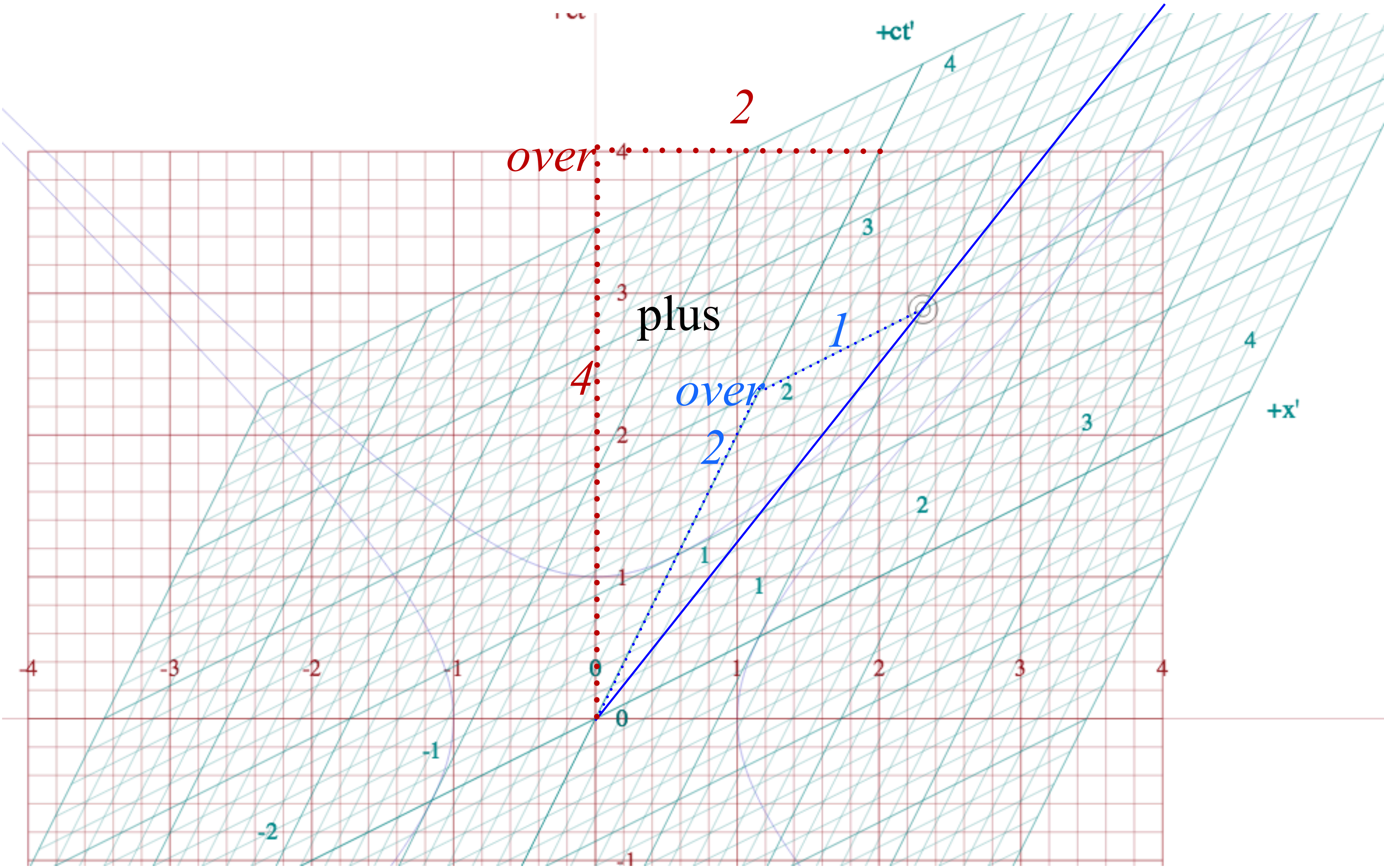
Velocity Addition involves $\frac{u_{a+b}}{c} = \beta_{ab} = \tanh(\rho_a + \rho_b) = \frac{\tanh\rho_a + \tanh\rho_b}{1 + \tanh\rho_a \tanh\rho_b}$

So velocity in units of light-speed c is added as follows: $\beta_{a+b} = \frac{\beta_a + \beta_b}{1 + \beta_a \beta_b}$

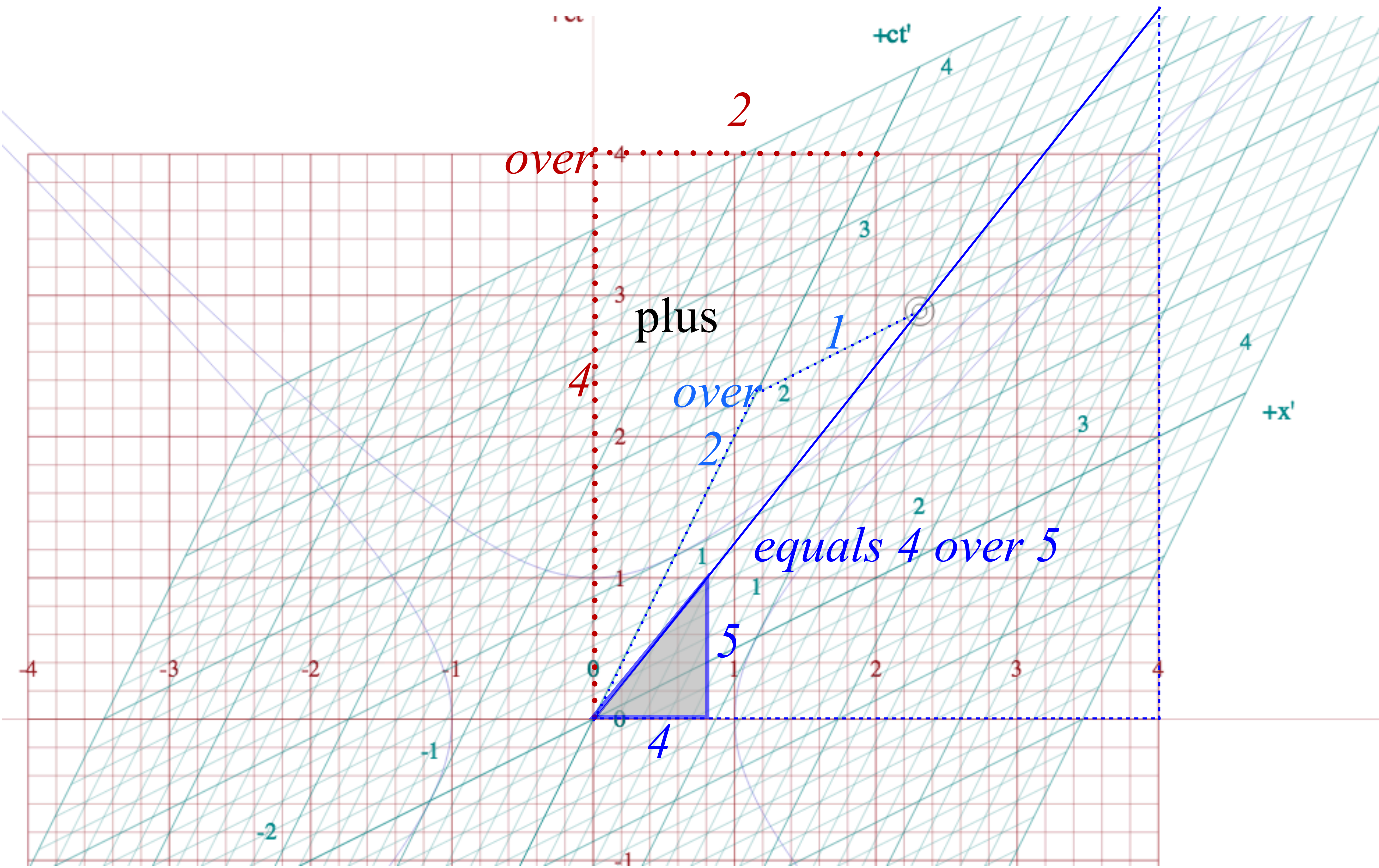
No longer is ½ plus ½ equal to 1. Instead it is: $\beta_{\frac{1}{2}+\frac{1}{2}} = \frac{\frac{1}{2} + \frac{1}{2}}{1 + \frac{1}{2}\frac{1}{2}} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$

Try out this sum-rule using Minkowski graph paper.


+ct'
+x'
2
over
4
plus

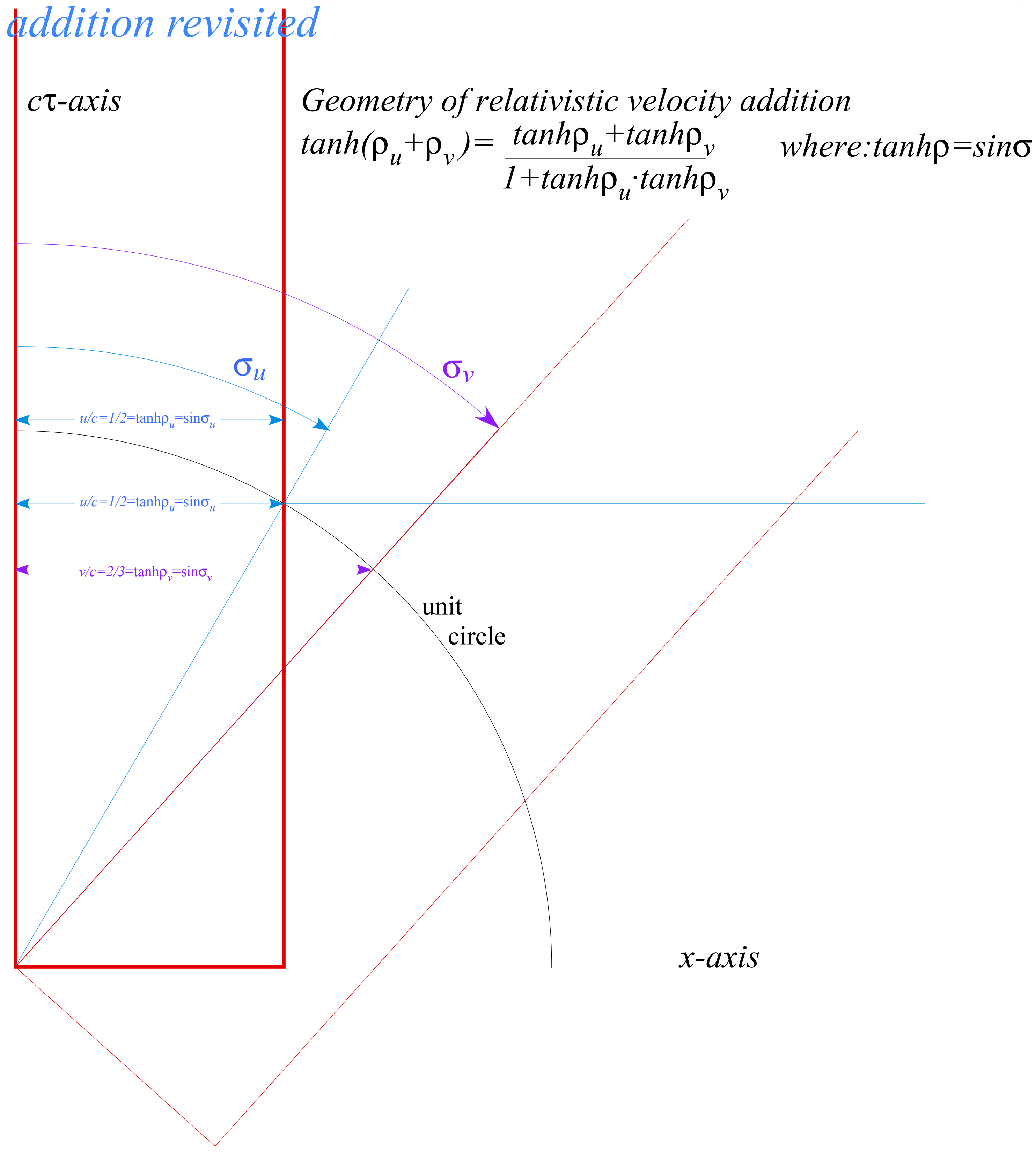


+ct'
2
over
4
plus
1
over
2
+x'


+ct'
2
over
plus
4
1
over
2
equals 4 over 5
5
4
+x'
-4
-3
-2
-1
0
1
2
3
4
-1
-2

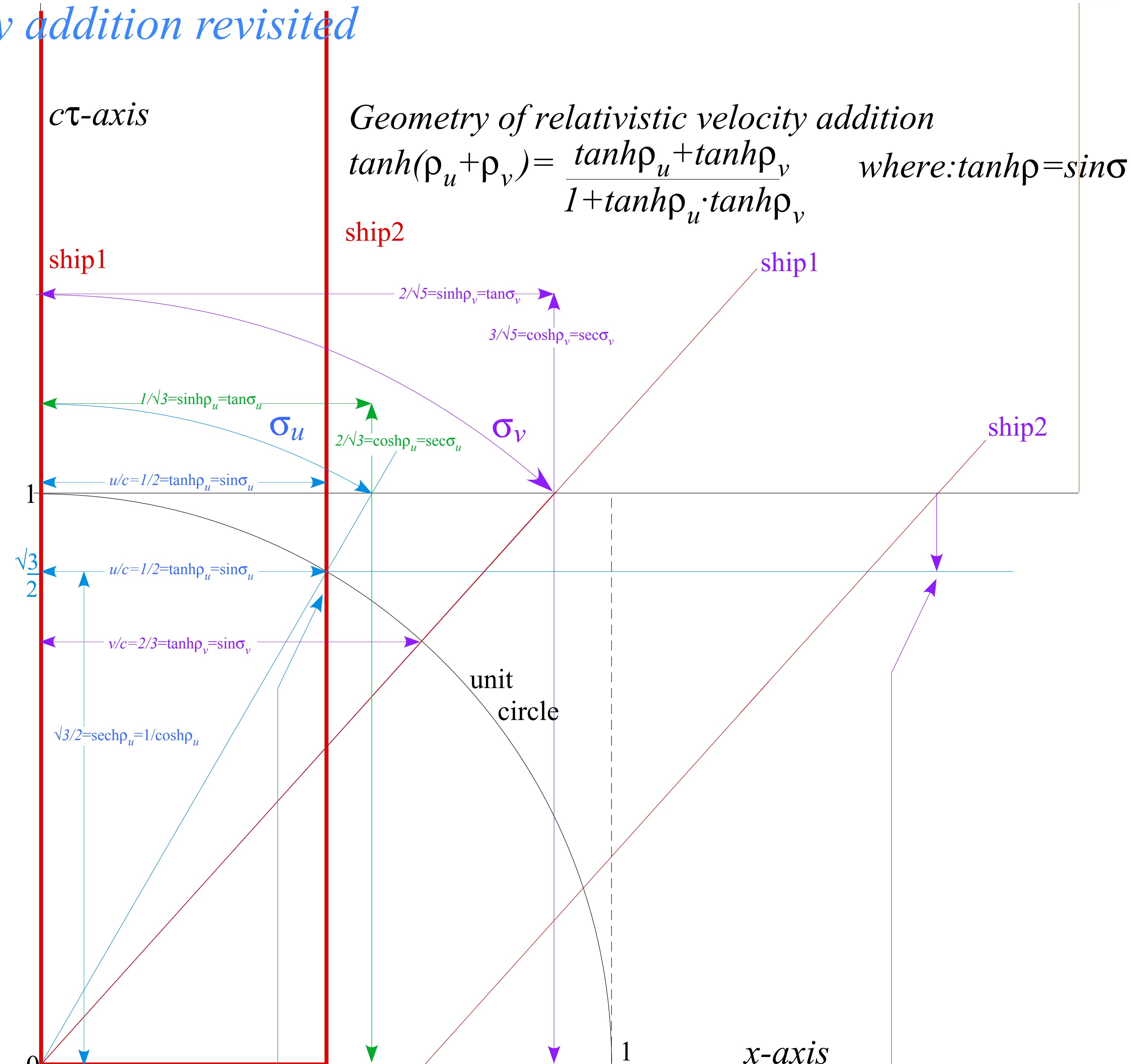
Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Velocity addition revisited



Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Velocity addition revisited

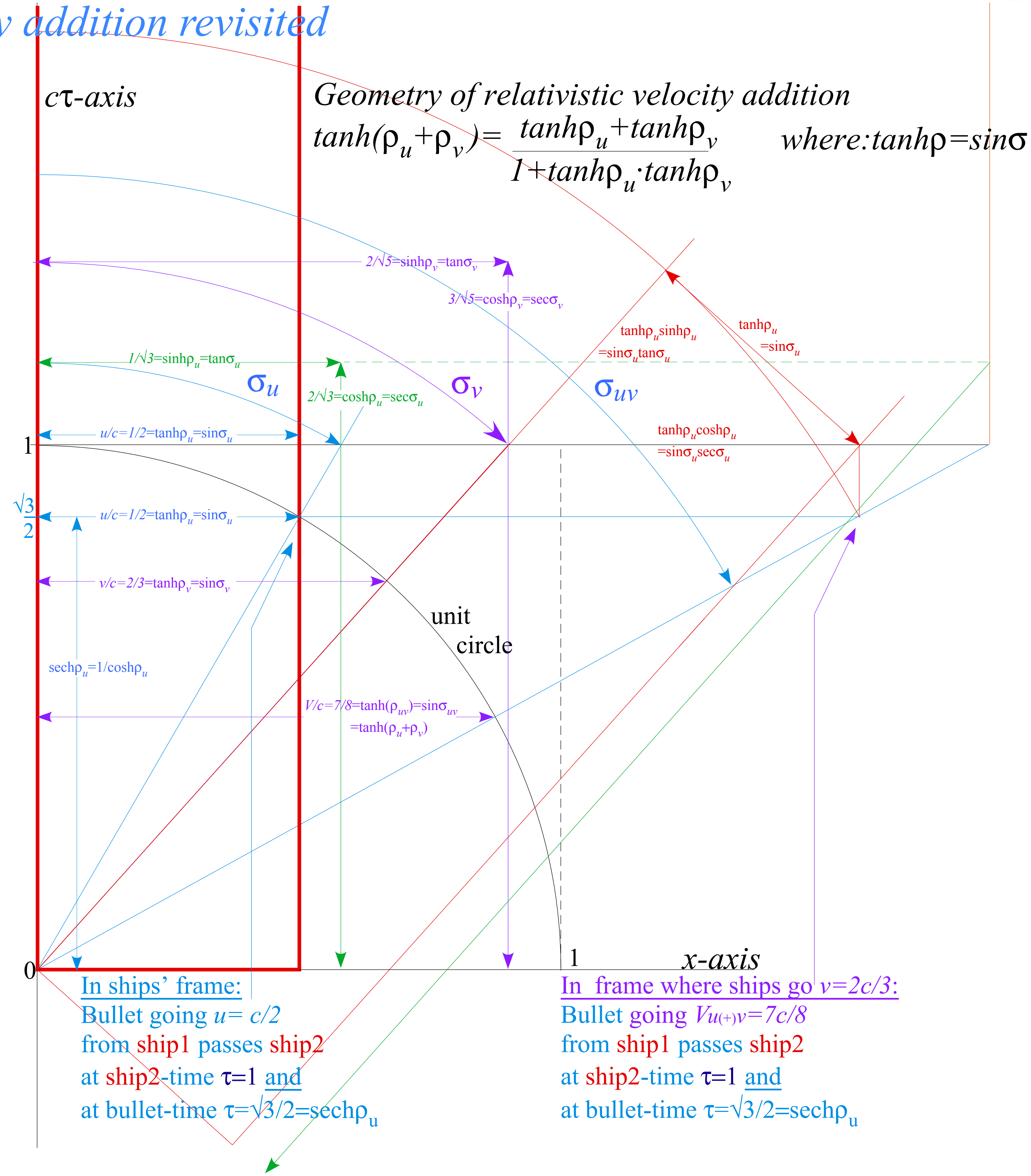


In ships' frame:
Bullet going $u= c/2$
from ship1 passes ship2
at ship2-time $\tau=1$ and
at bullet-time $\tau=\sqrt{3}/2=\mathrm{sech}\rho_u$

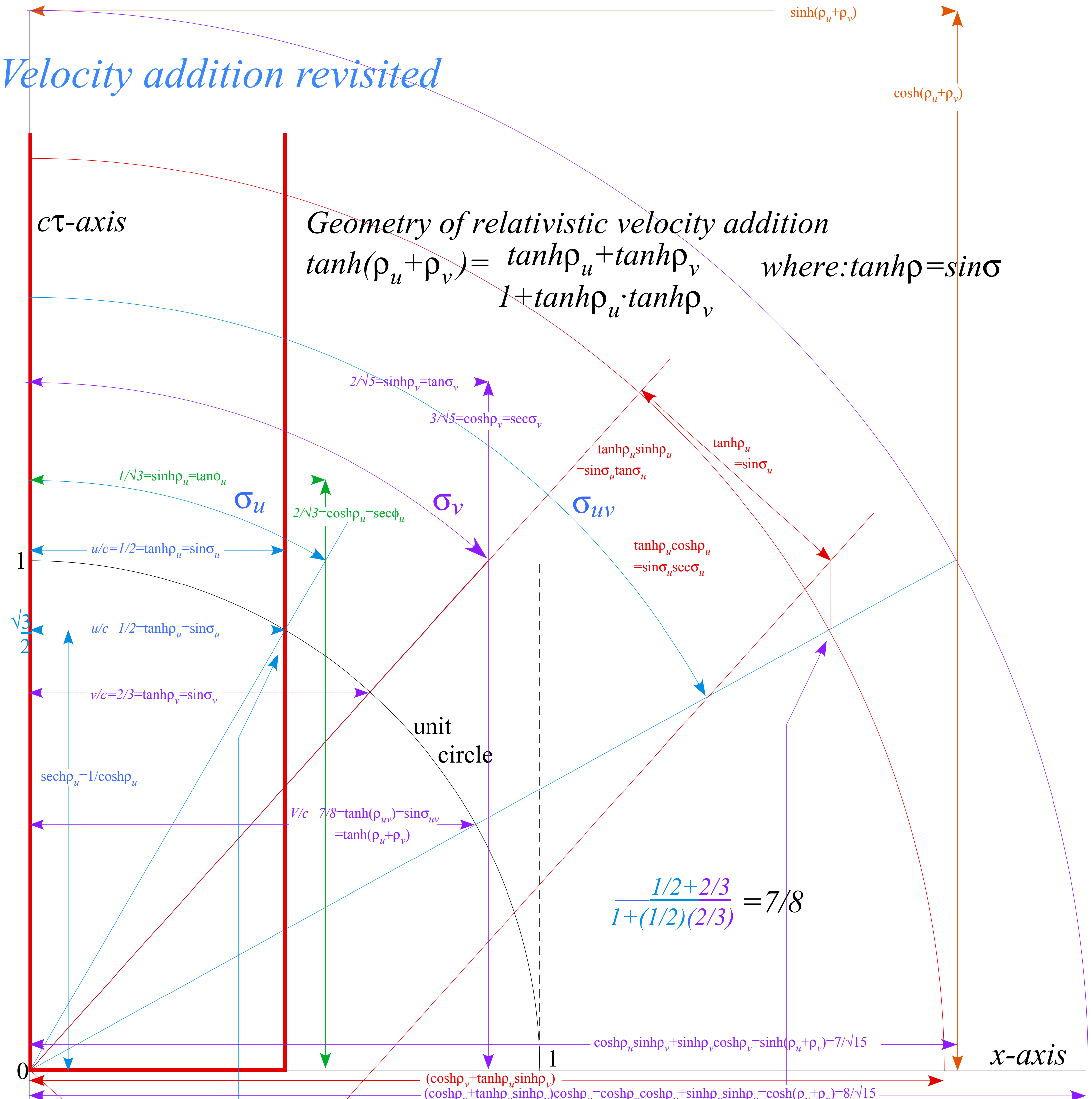
In frame where ships go $v=2c/3$:
Bullet going $V_{u(+)v}= ??$
from ship1 passes ship2
at ship2-time $\tau=1$ and
at bullet-time $\tau=\sqrt{3}/2=\mathrm{sech}\rho_u$

Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Velocity addition revisited

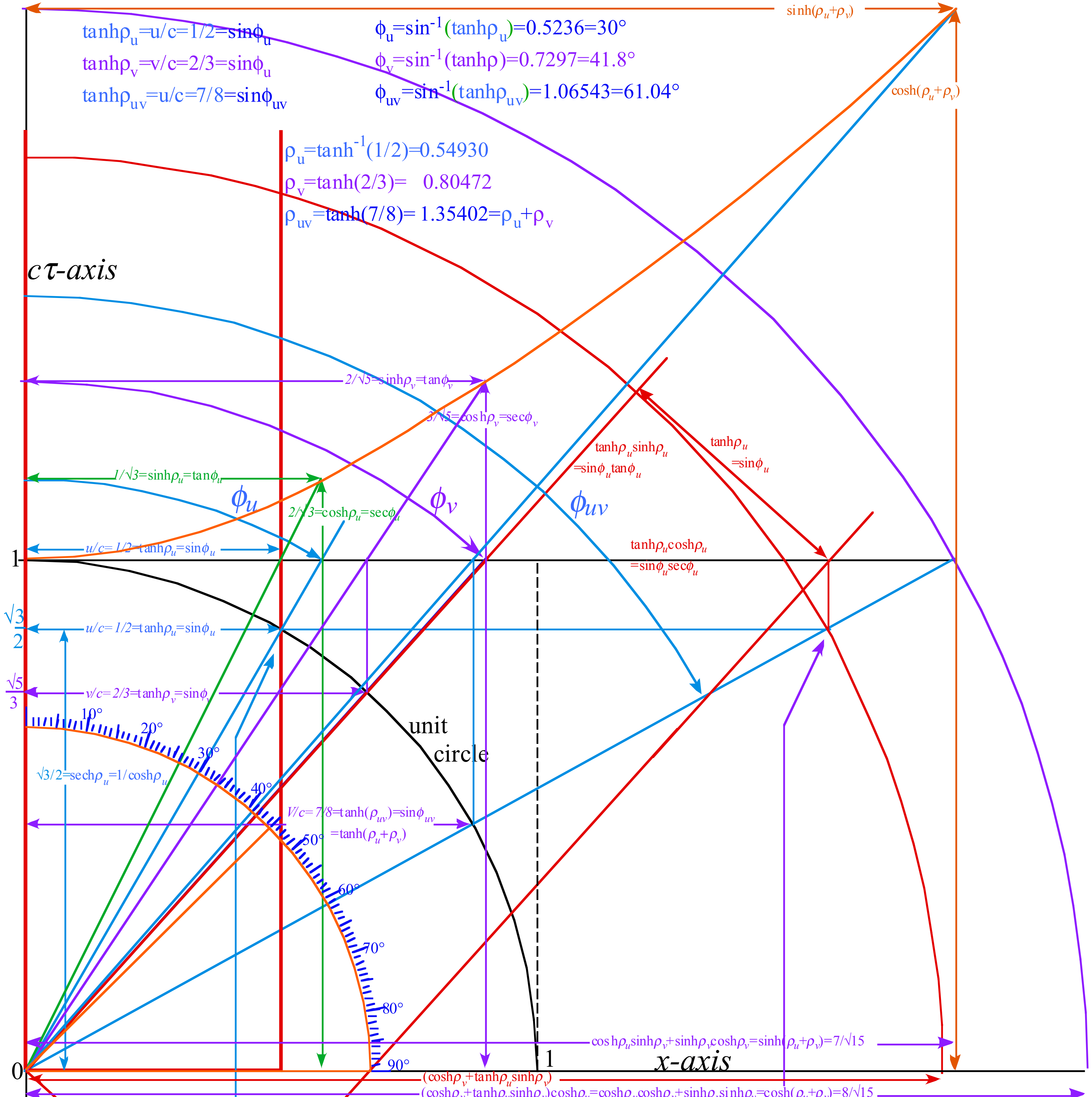


Velocity addition revisited



<u>In ships' frame:</u>
Bullet going $u= c/2$
from ship1 passes ship2
at ship2-time $\tau=1$ <u>and</u>
at bullet-time $\tau=\sqrt{3}/2=\text{sech}\rho_u$

<u>In frame where ships go $v=2c/3$:</u>
Bullet going $V_{u(+)v}=7c/8$
from ship1 passes ship2
at ship2-time $\tau=1$ <u>and</u>
at bullet-time $\tau=\sqrt{3}/2=\text{sech}\rho_u$



In ships' frame:
Bullet going $u=c/2$
from ship1 passes ship2
at ship2-time $\tau=1$ and
at bullet-time $\tau=\sqrt{3}/2=\mathrm{sech}\rho_u$

In frame where ships go $v=2c/3$:
Bullet going $V_{u(+)v}=7c/8$
from ship1 passes ship2
at ship2-time $\tau=1$ and
at bullet-time $\tau=\sqrt{3}/2=\mathrm{sech}\rho_u$

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

Lorentz space contraction

Time-simultaneity-breaking

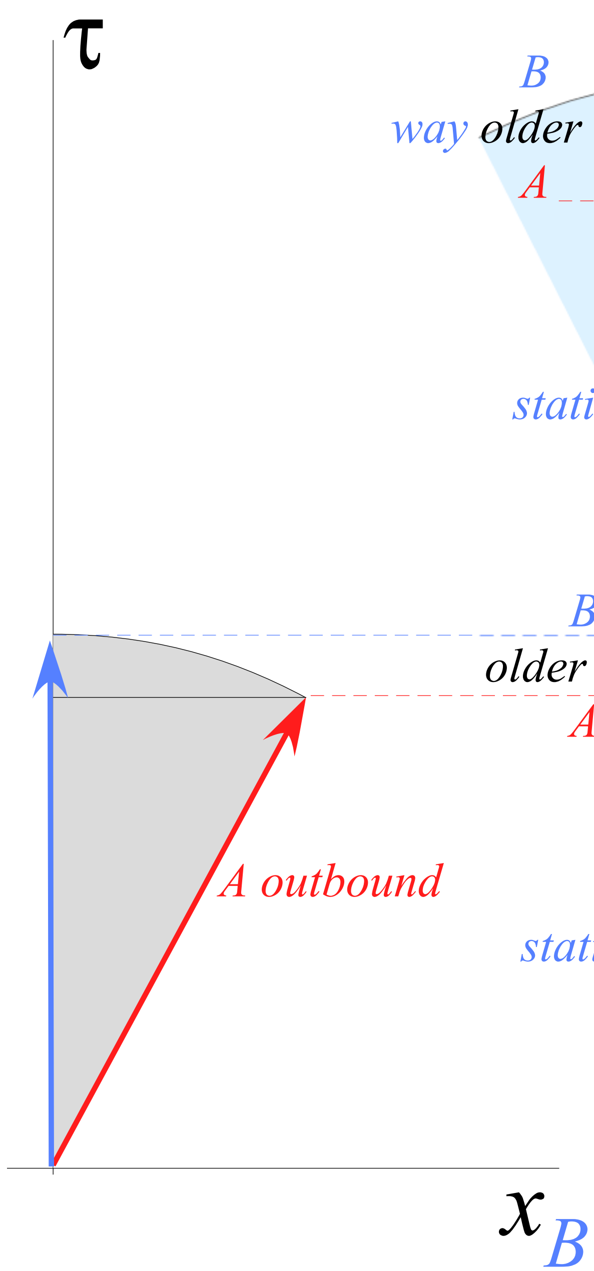
Velocity addition

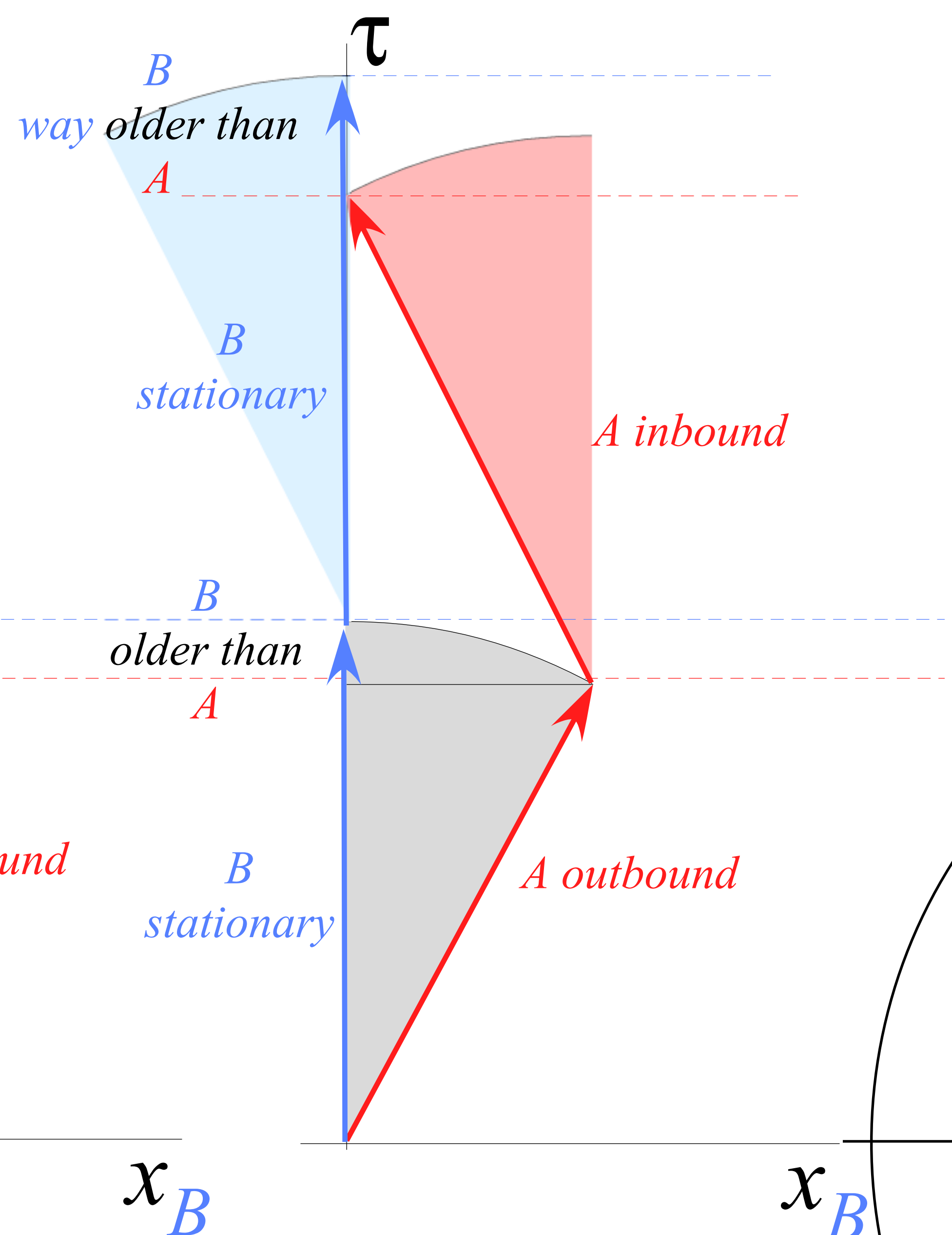
➔ Twin-paradox resolution in space-proper-time ⬅

Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

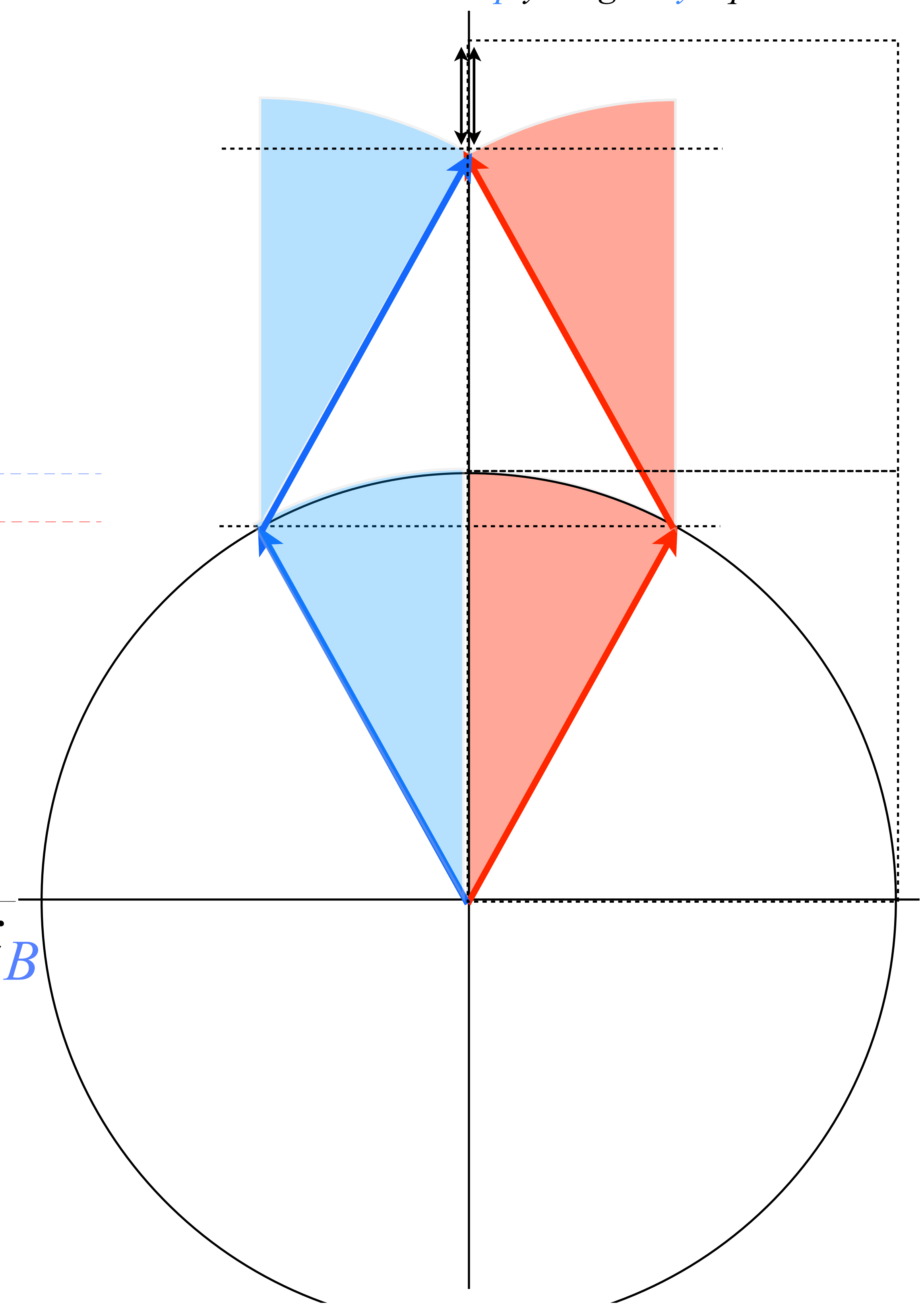
Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Twin-paradox revisited



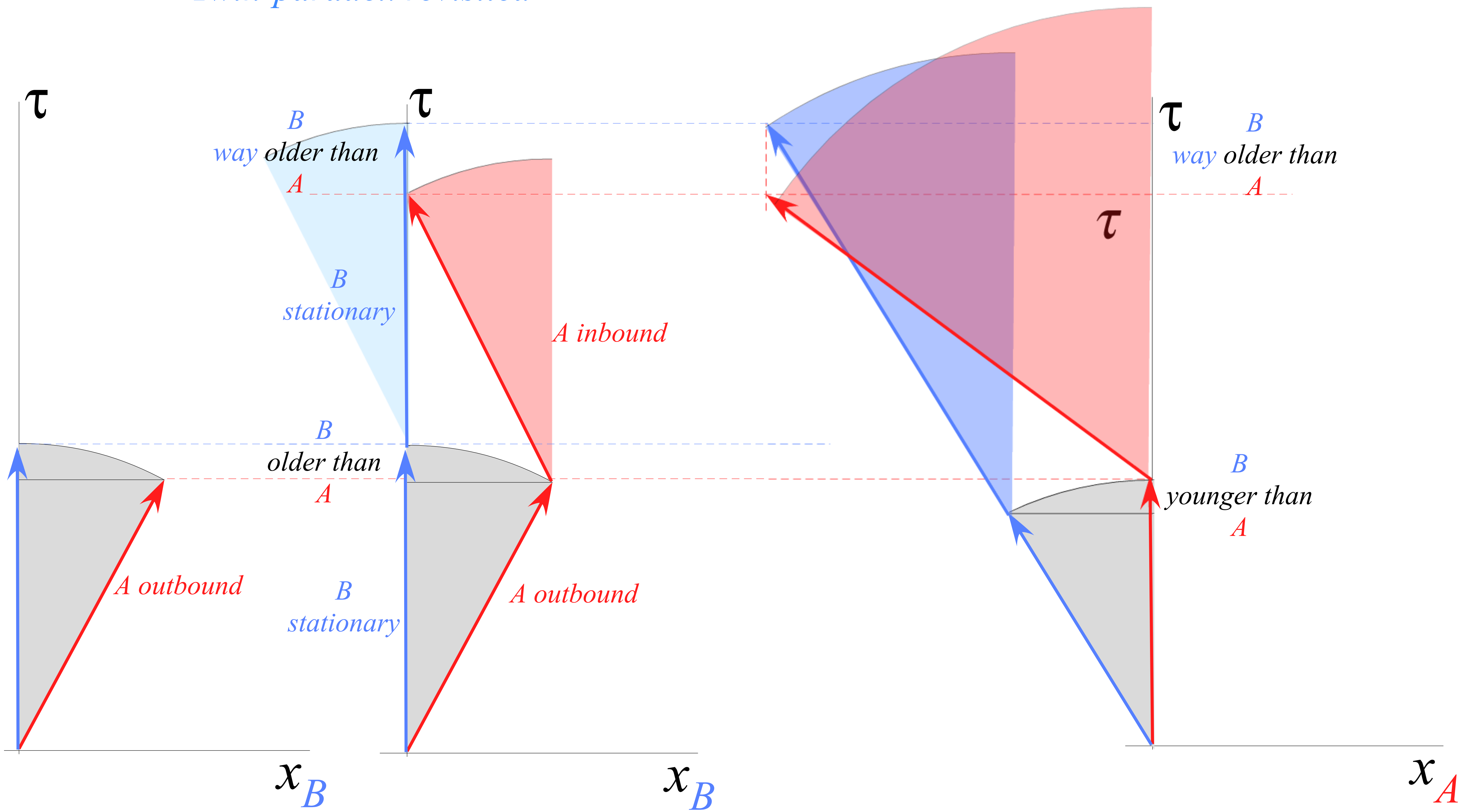


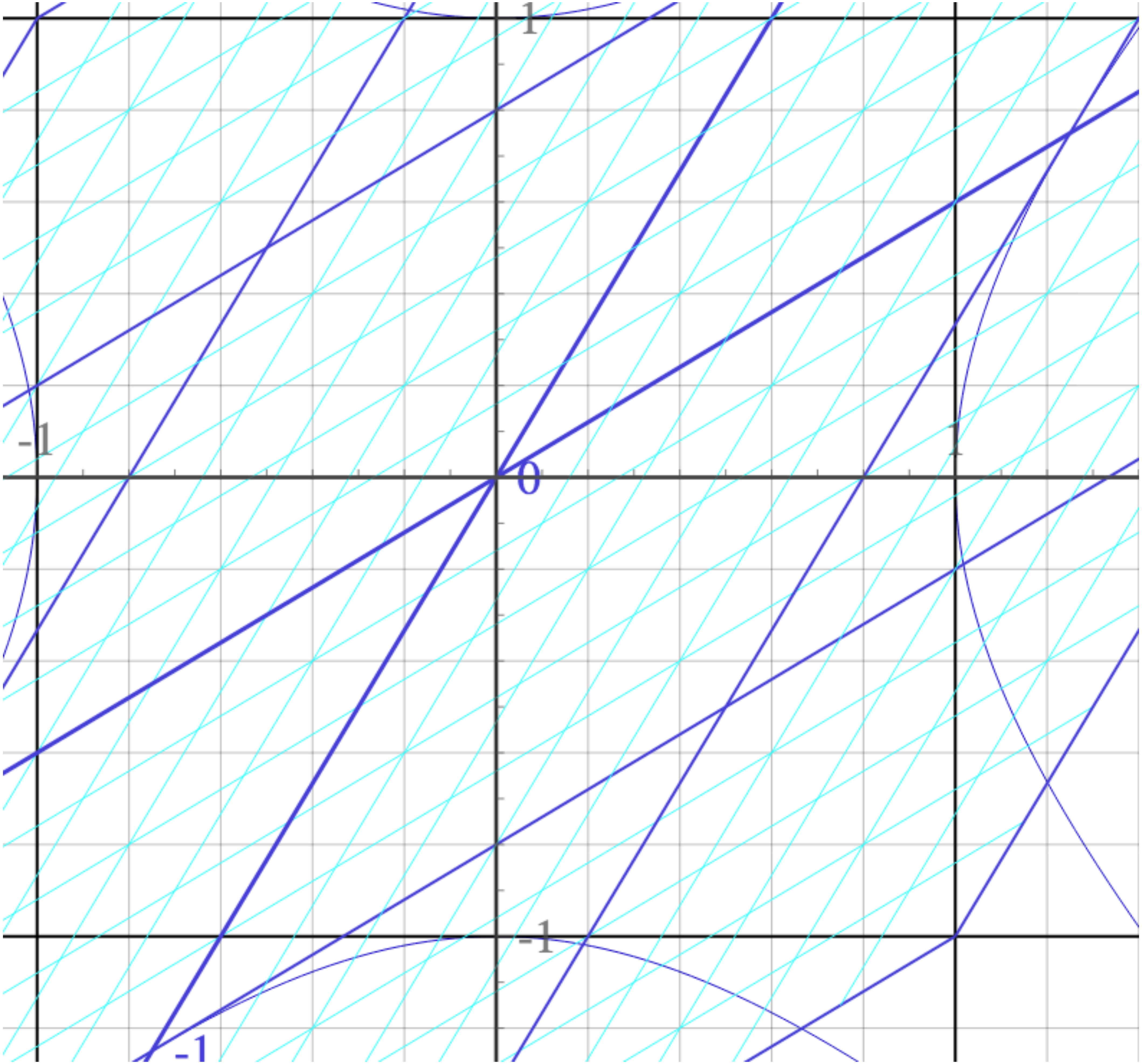
Both A and B end up younger by equal amounts

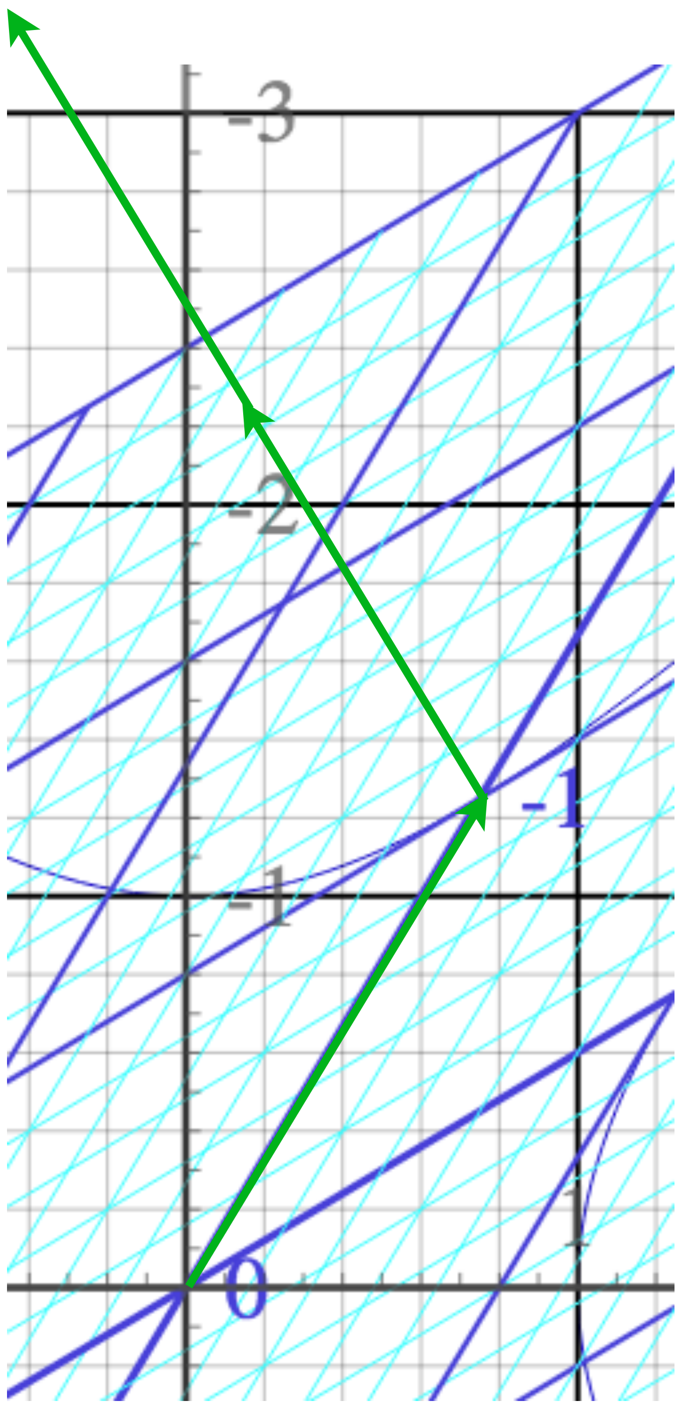


Epstein's space-proper-time (x,cτ) plots ("c-tau" plots)

Twin-paradox revisited




1
-1
0
1
-1
-1



-3
-2
-1
0
-1
1

Review of hyper-trigonometry (*tanh*ρ, *sinh*ρ, *and* *cosh*ρ, *sech*ρ, *and* *csch*ρ, *coth*ρ)

and co-trigonometry (*sin*σ, *tan*σ, *and* *sec*σ, *cos*σ, *and* *cot*σ, *csc*σ)

Review of "Occam-sword" geometry and wave parameters for phase and group motion

Wave parameter symmetry points

Yet another view: Epstein's space-proper-time approach to SR and stellar aberration **k**-angle σ

Review of proper time relations and basis of Epstein's cosmic speedometer

Epstein vs Einstein-Minkowski geometry of relativity

Einstein time dilation

Lorentz space contraction

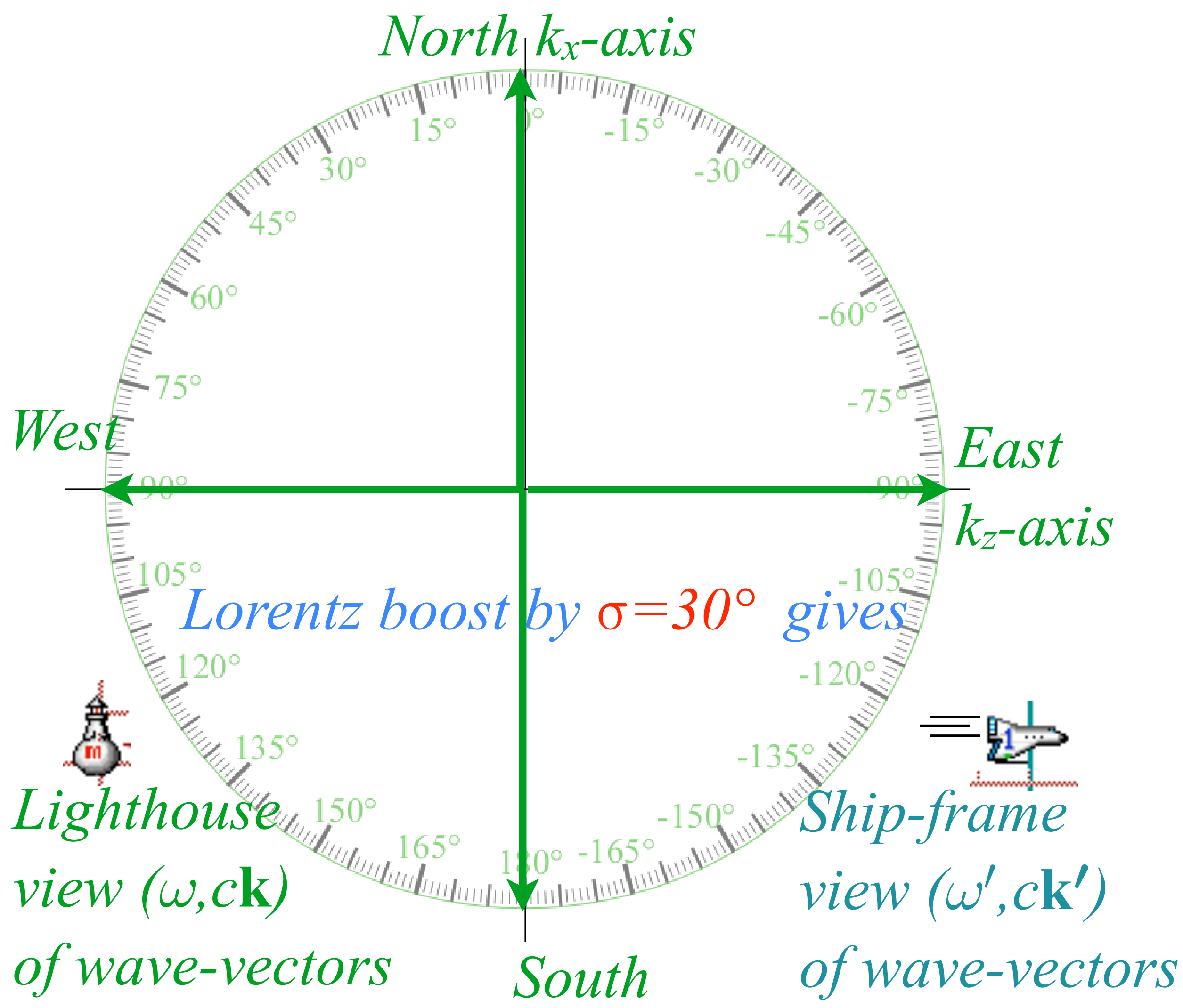
Time-simultaneity-breaking

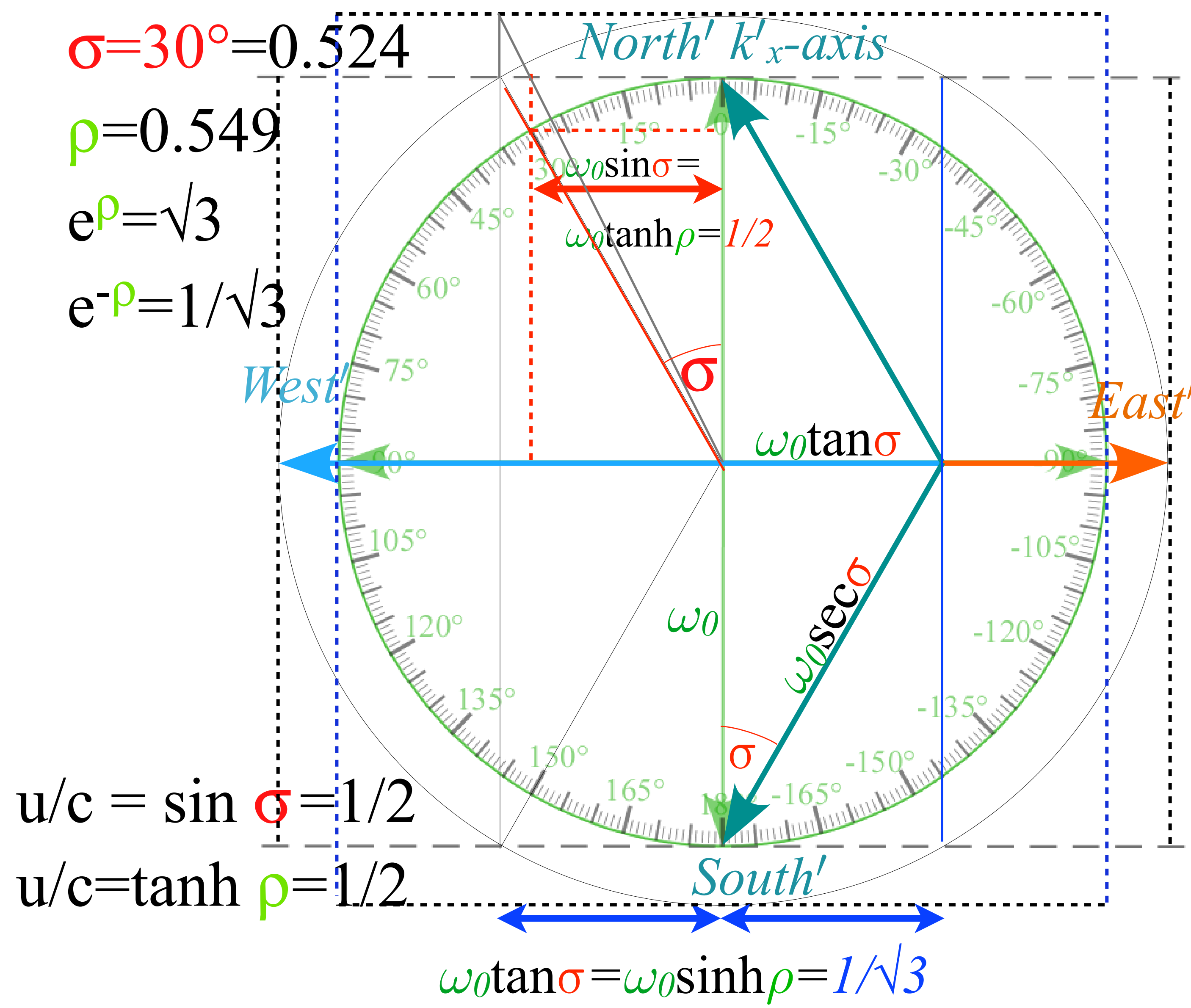
Velocity addition

Twin-paradox resolution in space-proper-time ←

→ Spectral details of per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) =(\omega,ck_x,ck_y,ck_z)$ transformation

Spectral details of Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

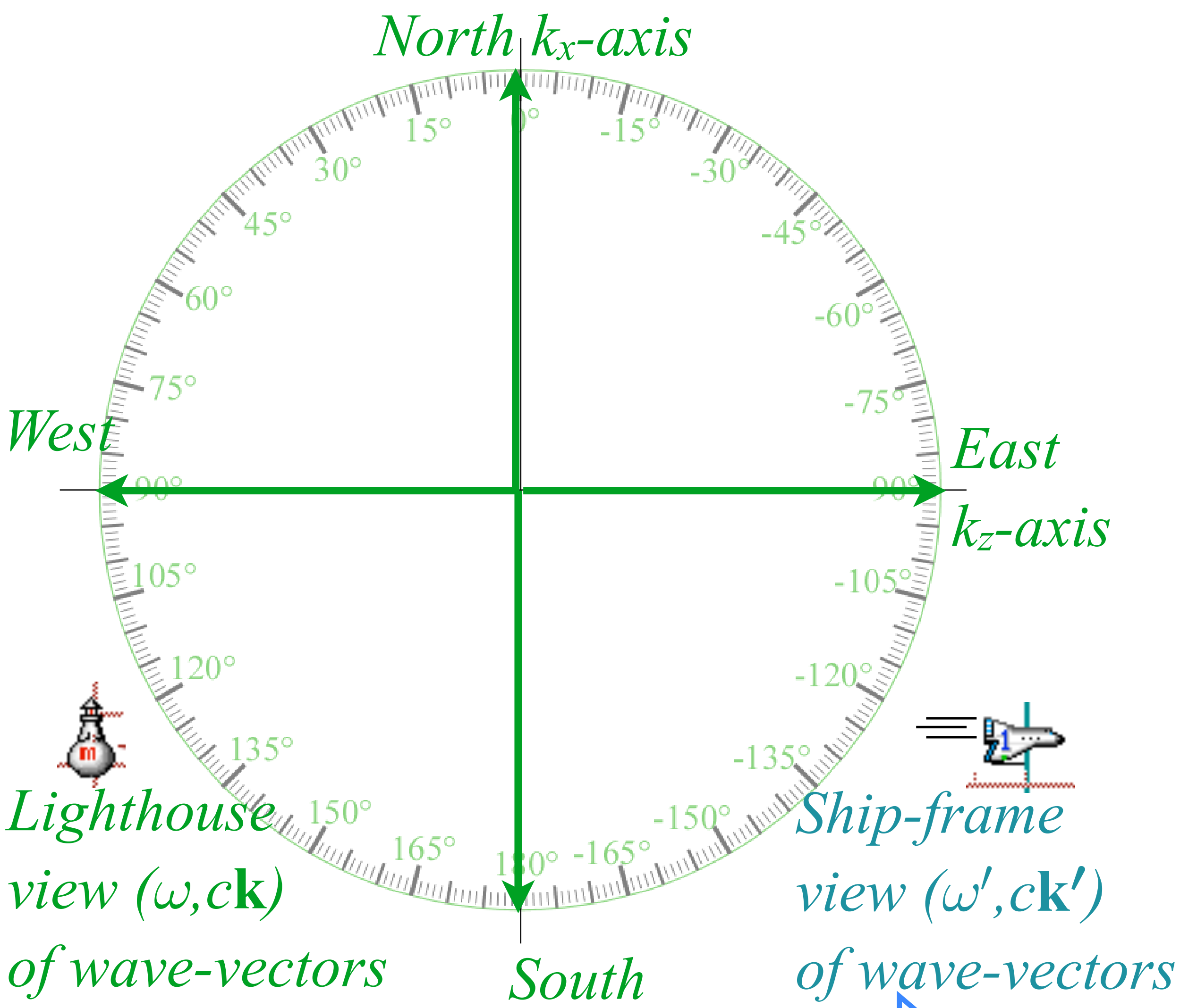


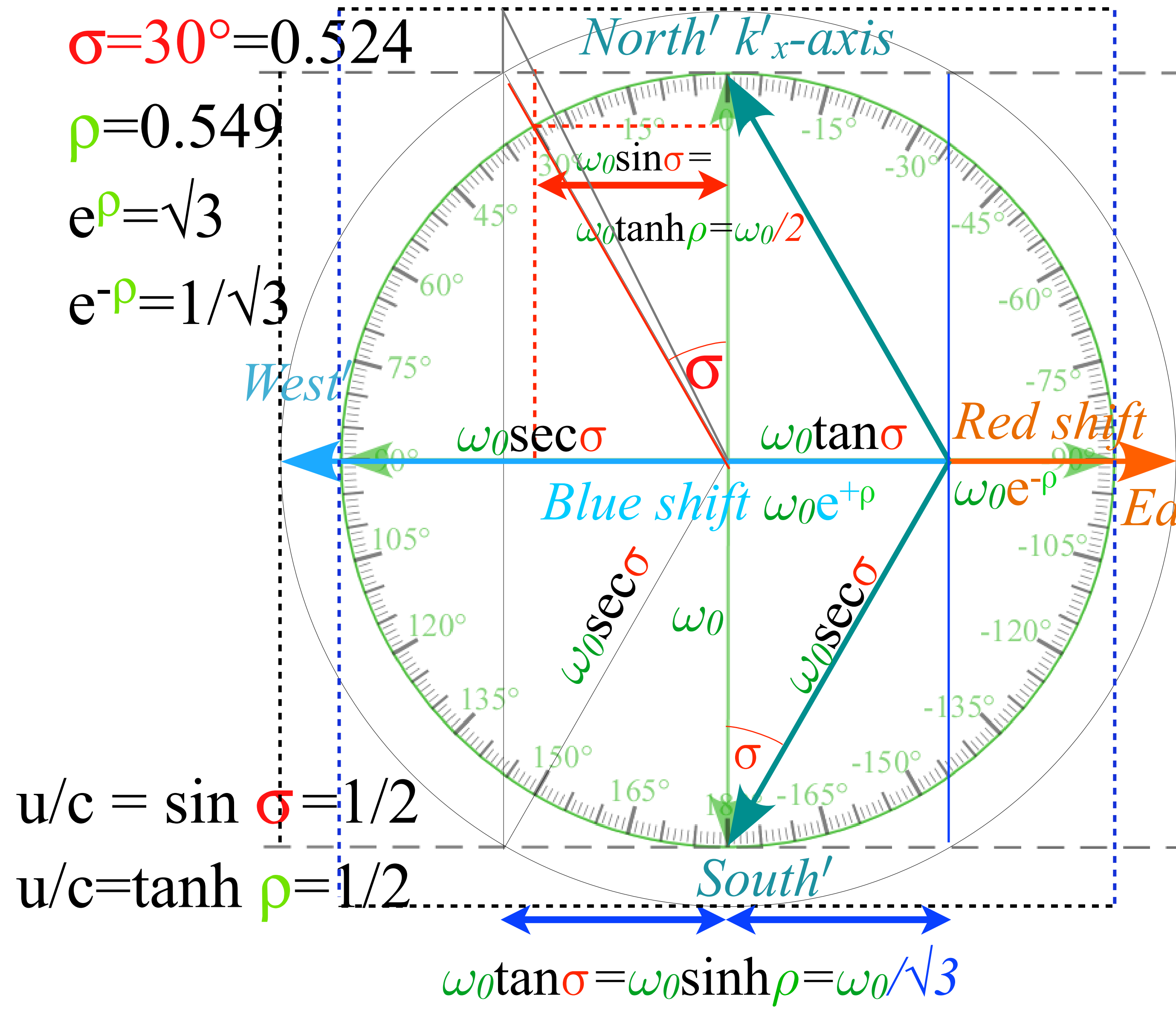


South starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{z\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

$+\rho_z$-rapidity ship frame sees starlight Lorentz transformed to : $\left(\omega'_{\downarrow}, ck'_{x\downarrow}, ck'_{y\downarrow}, ck'_{z\downarrow}\right) = \left(\omega_0 \cosh\rho_z, -\omega_0, 0, -\omega_0 \sinh\rho_z\right)$

$$\begin{pmatrix} \omega'_{\downarrow} \\ ck'_{x\downarrow} \\ ck'_{y\downarrow} \\ ck'_{z\downarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_{\downarrow} \\ ck_{x\downarrow} \\ ck_{y\downarrow} \\ ck_{z\downarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ -\omega_0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_0 \cosh\rho_z \\ -\omega_0 \\ 0 \\ -\omega_0 \sinh\rho_z \end{pmatrix} = \begin{pmatrix} \omega_0 \sec\sigma \\ -\omega_0 \\ 0 \\ -\omega_0 \tan\sigma \end{pmatrix}$$





Lorentz boost by $\sigma=30°$ or $e^{+\rho}=\sqrt{3}$

For ship going $u=c\tanh\rho$ along z-axis

West starlight $(\omega_0,0,0,-\omega_0)$ is *blue shifted by* $e^{+\rho}=\cosh\rho+\sinh\rho$

$$\begin{pmatrix}\omega'_{\leftarrow}\\ ck'_{x\leftarrow}\\ ck'_{y\leftarrow}\\ ck'_{z\leftarrow}\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\\ 0\\ 0\\ -\sinh\rho_z-\cosh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0 e^{+\rho_z}\\ 0\\ 0\\ -\omega_0 e^{+\rho_z}\end{pmatrix}$$

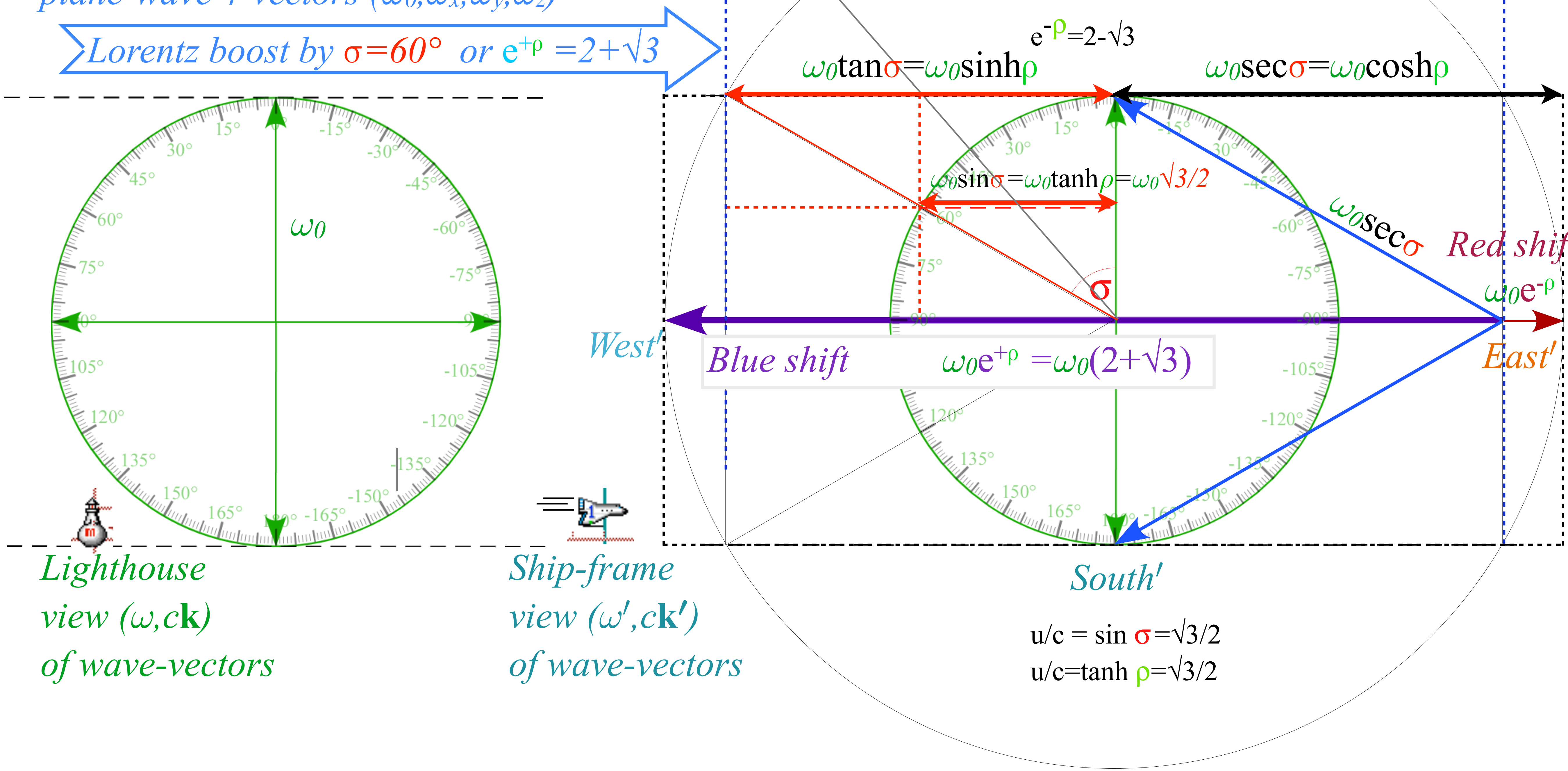
and *East* starlight $(\omega_0,0,0,+\omega_0)$ is *red shifted by* $e^{-\rho}=\cosh\rho-\sinh\rho$

$$\begin{pmatrix}\omega'_{\rightarrow}\\ ck'_{x\rightarrow}\\ ck'_{y\rightarrow}\\ ck'_{z\rightarrow}\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z-\sinh\rho_z\\ 0\\ 0\\ -\sinh\rho_z+\cosh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0 e^{-\rho_z}\\ 0\\ 0\\ -\omega_0 e^{-\rho_z}\end{pmatrix}$$

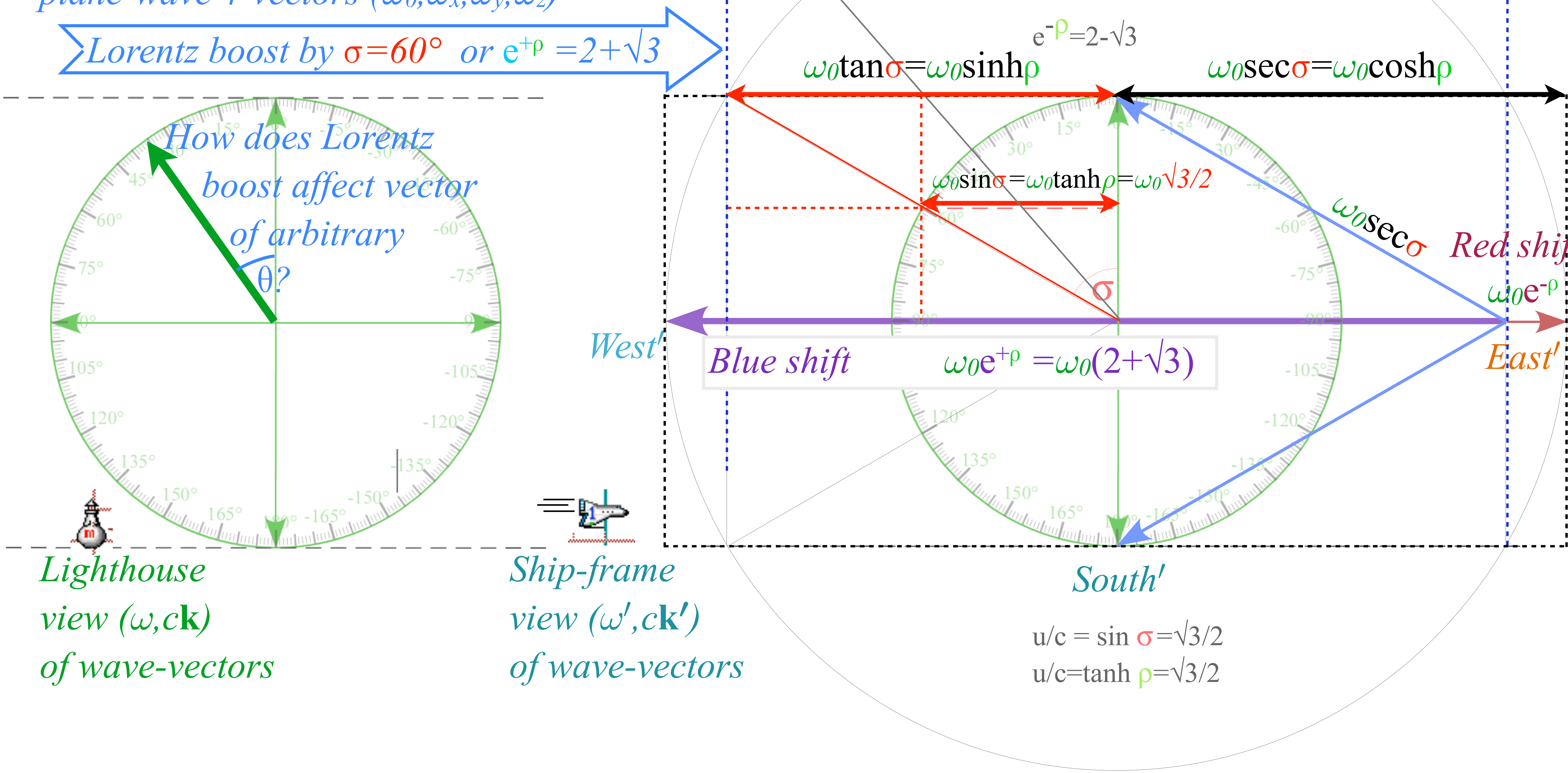
Blue shift factor is $e^{+\rho}=\cosh\rho+\sinh\rho=\sec\sigma+\tan\sigma$

Red shift factor is $e^{-\rho}=\cosh\rho-\sinh\rho=\sec\sigma-\tan\sigma$

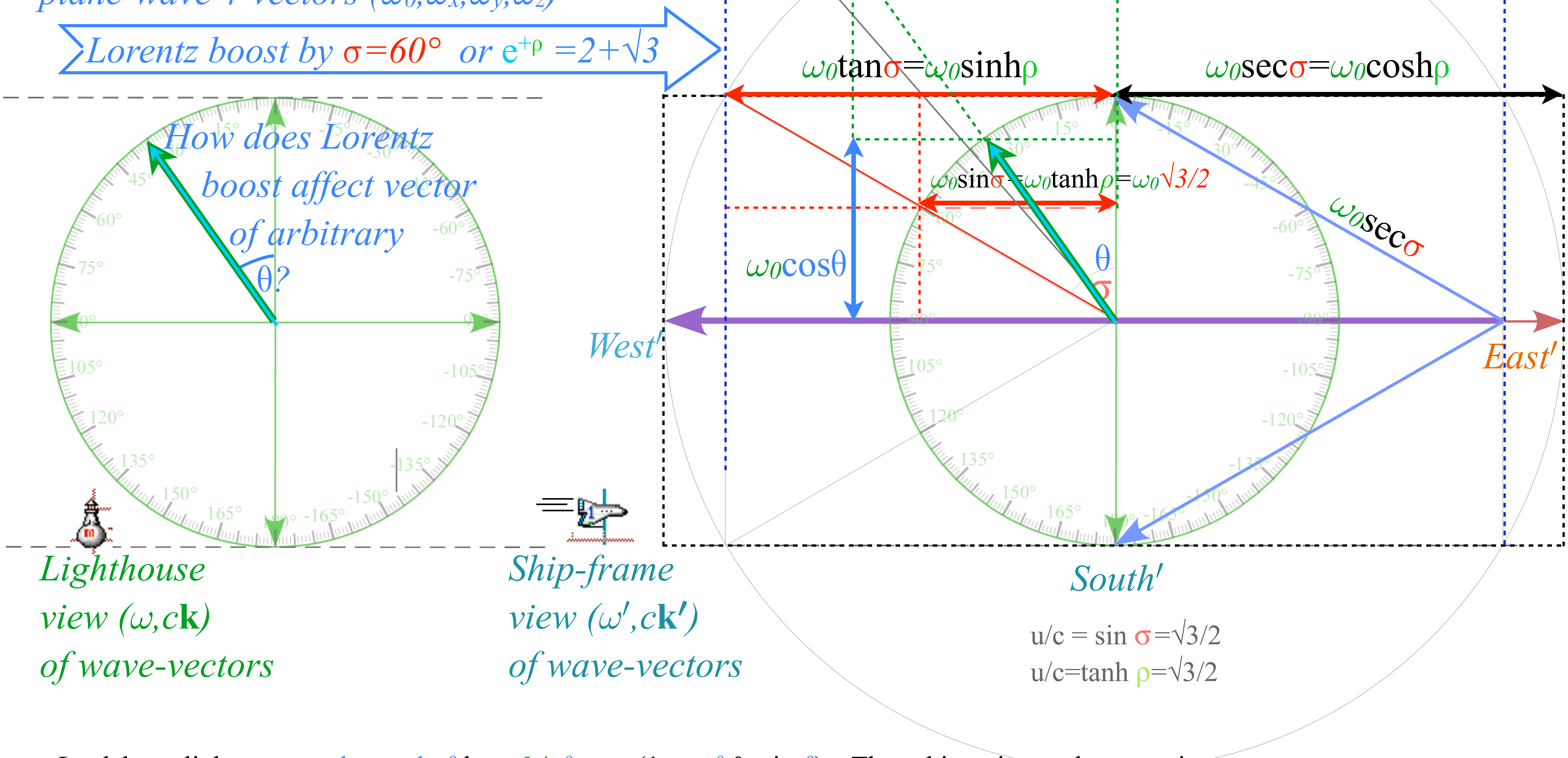
Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0, \omega_x, \omega_y, \omega_z)$



Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0, \omega_x, \omega_y, \omega_z)$



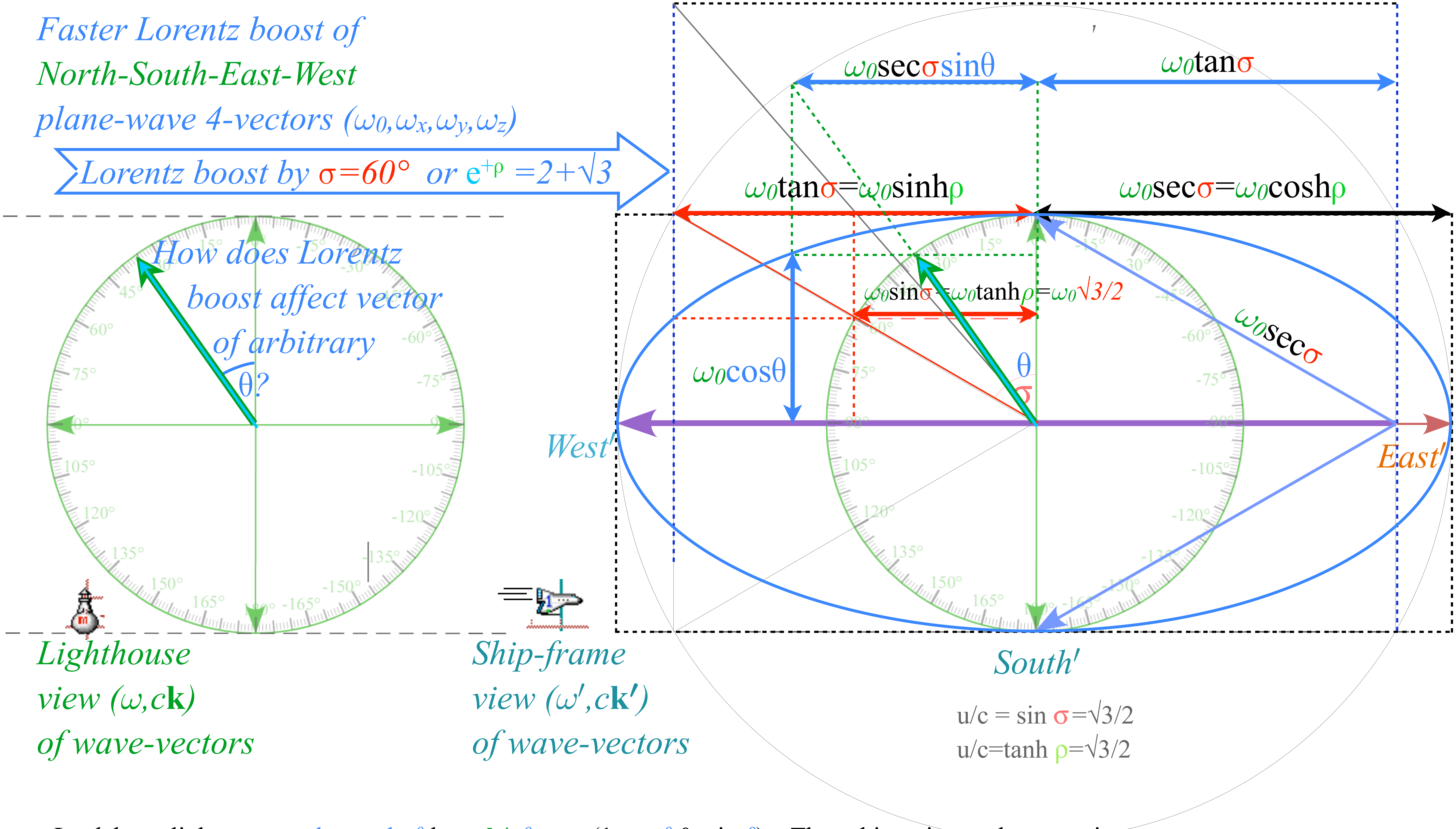
Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix}=\omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$

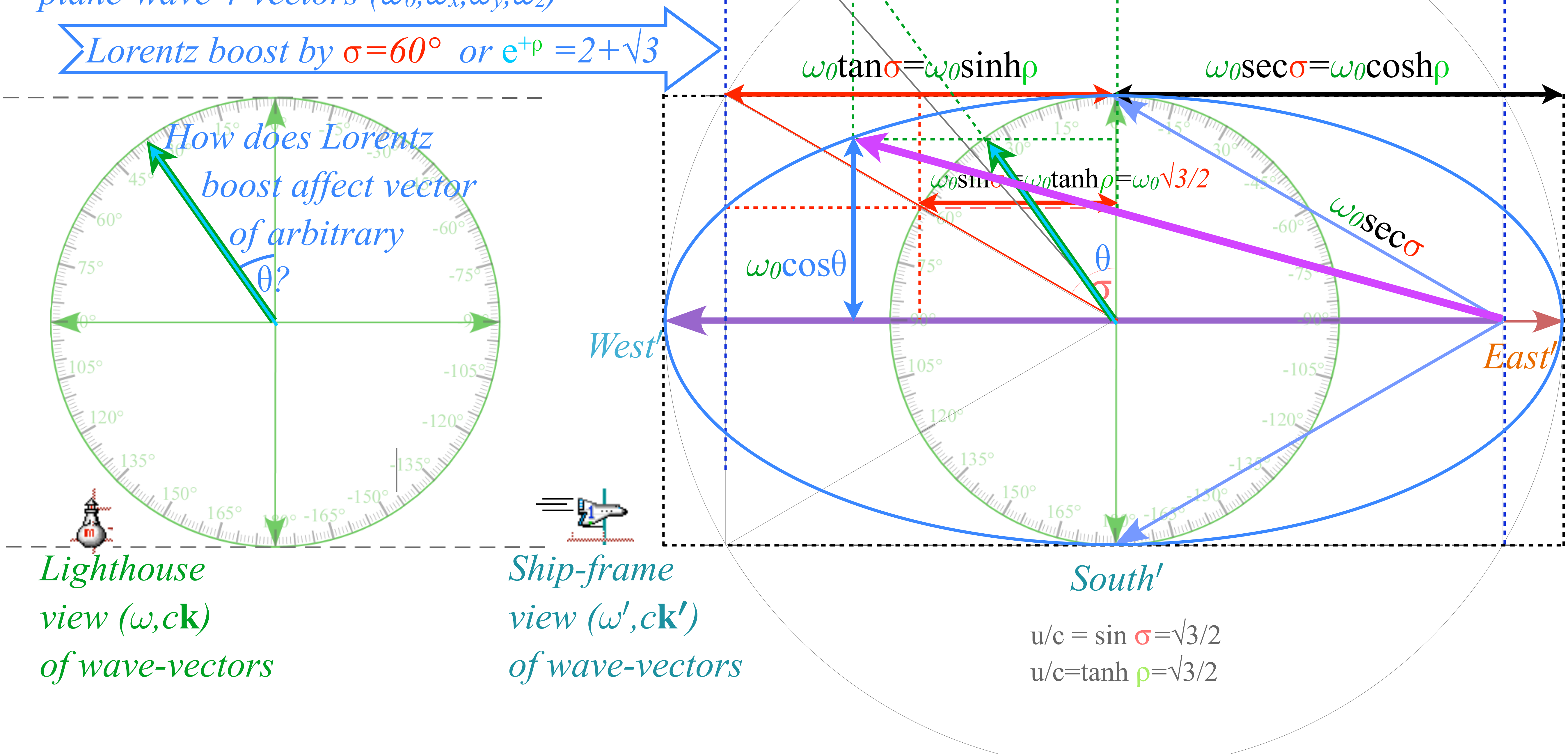
Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\!\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix} = \begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix} = \omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix} = \omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$

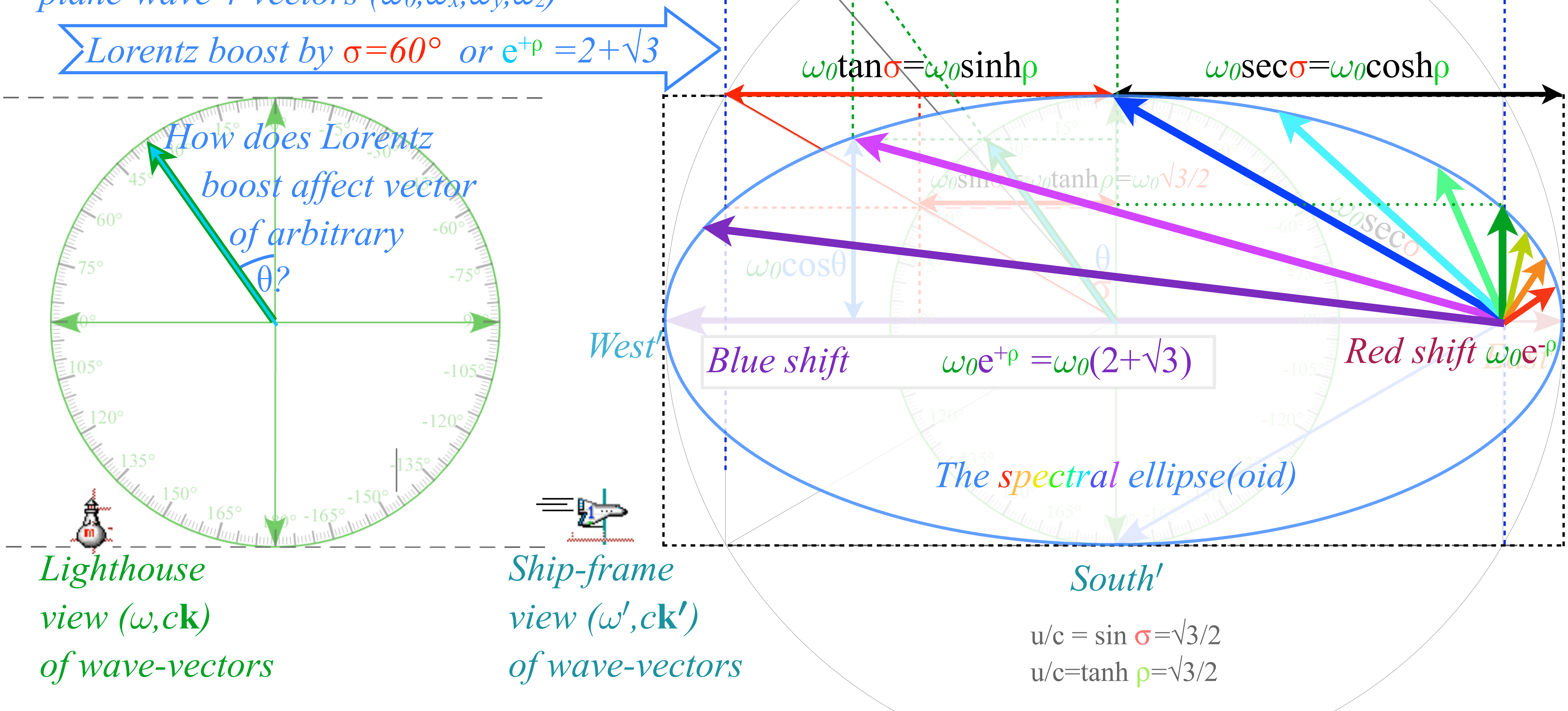
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix}\omega'_{\uparrow\theta}\\ ck'_{x\uparrow\theta}\\ ck'_{y\uparrow\theta}\\ ck'_{z\uparrow\theta}\end{pmatrix} = \begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ \omega_0\cos\theta\\ 0\\ -\omega_0\sin\theta\end{pmatrix} = \omega_0\begin{pmatrix}\cosh\rho_z+\sinh\rho_z\sin\theta\\ \cos\theta\\ 0\\ -\sinh\rho_z-\cosh\rho_z\sin\theta\end{pmatrix} = \omega_0\begin{pmatrix}\sec\sigma+\tan\sigma\sin\theta\\ \cos\theta\\ 0\\ -\tan\sigma-\sec\sigma\sin\theta\end{pmatrix}$$

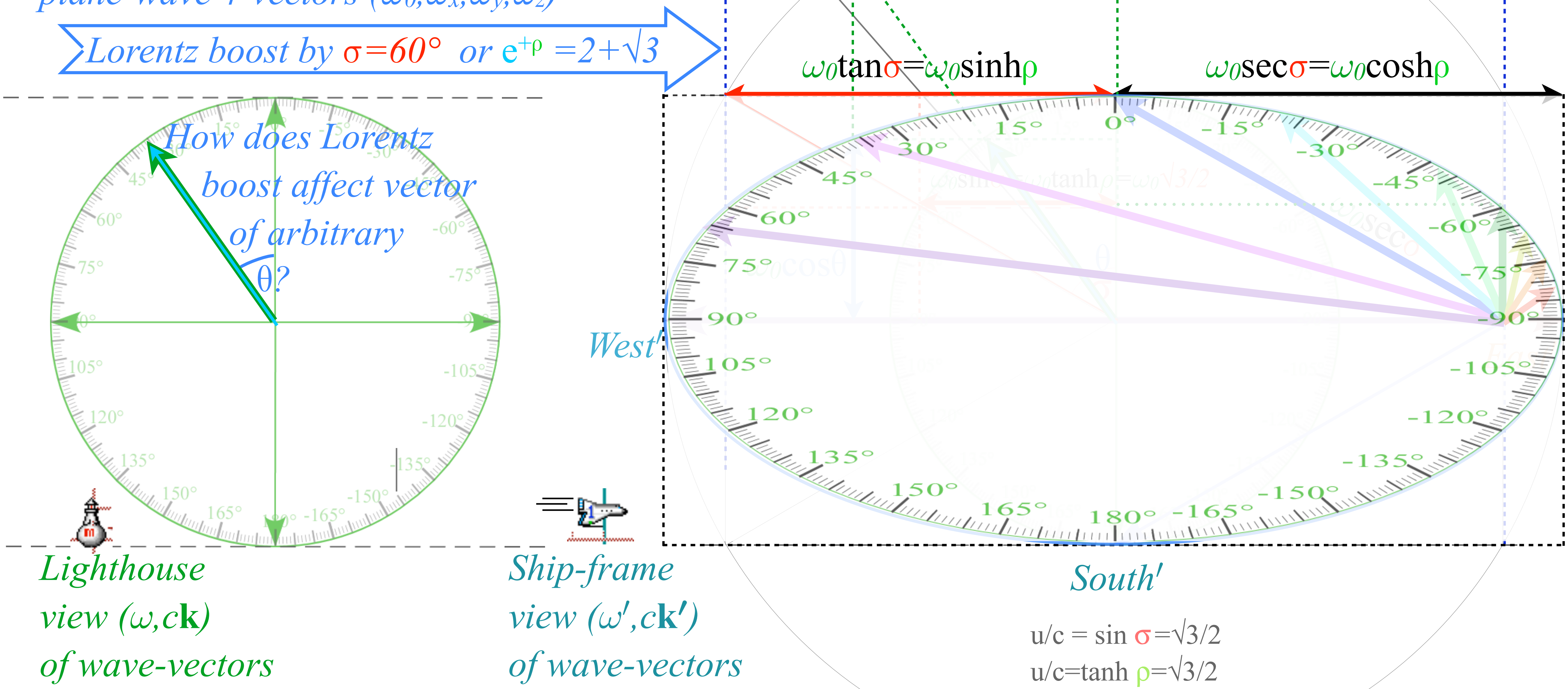
Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0, \omega_x, \omega_y, \omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0(1, \cos\theta, 0, -\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

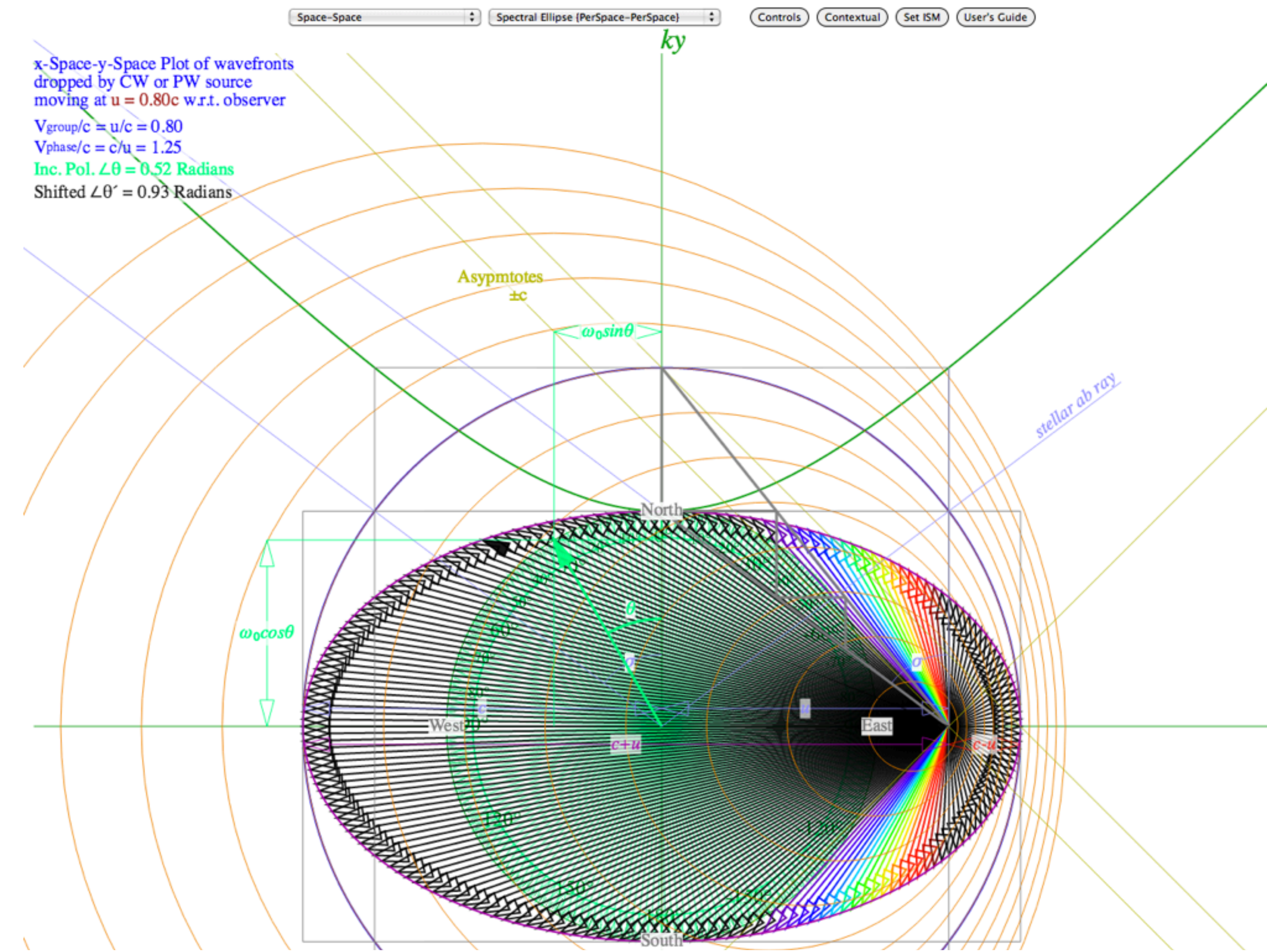
$$\begin{pmatrix} \omega'_{\uparrow\theta} \\ ck'_{x\uparrow\theta} \\ ck'_{y\uparrow\theta} \\ ck'_{z\uparrow\theta} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ \omega_0\cos\theta \\ 0 \\ -\omega_0\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z \sin\theta \\ \cos\theta \\ 0 \\ -\sinh\rho_z - \cosh\rho_z \sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \sec\sigma + \tan\sigma\sin\theta \\ \cos\theta \\ 0 \\ -\tan\sigma - \sec\sigma\sin\theta \end{pmatrix}$$

Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0, \omega_x, \omega_y, \omega_z)$



Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0(1,\cos\theta,0,-\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix} \omega'_{\uparrow\theta} \\ ck'_{x\uparrow\theta} \\ ck'_{y\uparrow\theta} \\ ck'_{z\uparrow\theta} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ \omega_0\cos\theta \\ 0 \\ -\omega_0\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z\sin\theta \\ \cos\theta \\ 0 \\ -\sinh\rho_z - \cosh\rho_z\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \sec\sigma + \tan\sigma\sin\theta \\ \cos\theta \\ 0 \\ -\tan\sigma - \sec\sigma\sin\theta \end{pmatrix}$$


Space-Space
Spectral Ellipse (PerSpace-PerSpace)
Controls
Contextual
Set ISM
User's Guide
ky
x-Space-y-Space Plot of wavefronts
dropped by CW or PW source
moving at u = 0.80c w.r.t. observer
Vgroup/c = u/c = 0.80
Vphase/c = c/u = 1.25
Inc. Pol. ∠θ = 0.52 Radians
Shifted ∠θ′ = 0.93 Radians
Asypmtotes
±c
ω0sinθ
stellar ab ray
North
ω0cosθ
θ
σ
West
East
c+u
c-u
u
c
South
120°
150°

Light-cone-sections are hyperbolas

Main Lighthouse on (x=0,y=0) time line

North Lighthouse-1 on (x=0,y=1) time line

Main Lighthouse blinks trace x=±ct "V"-lines thru each time tic

North Lighthouse-1 blinks trace $x^2-(ct)^2=1$ hyperbolas thru each tic

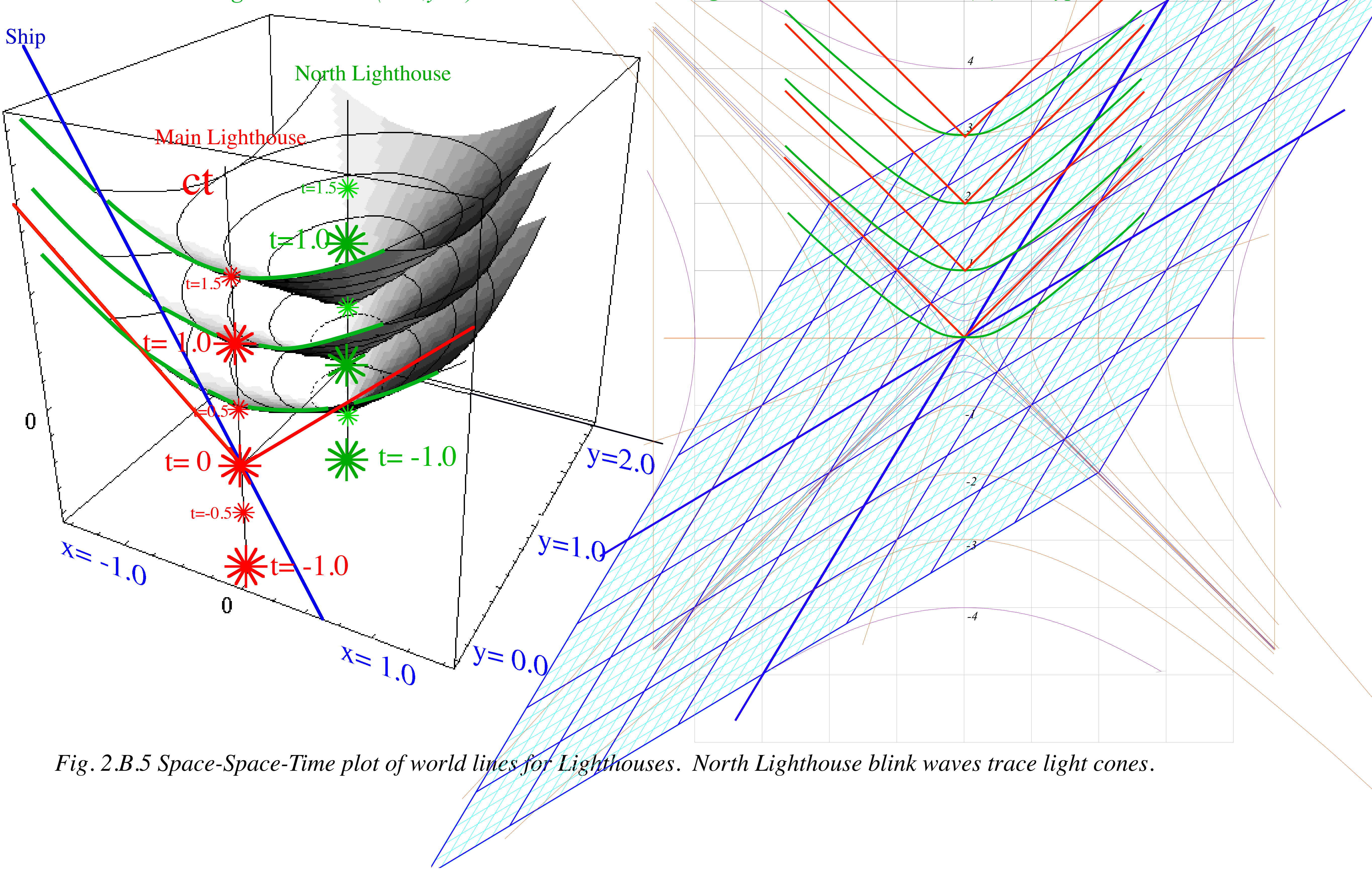
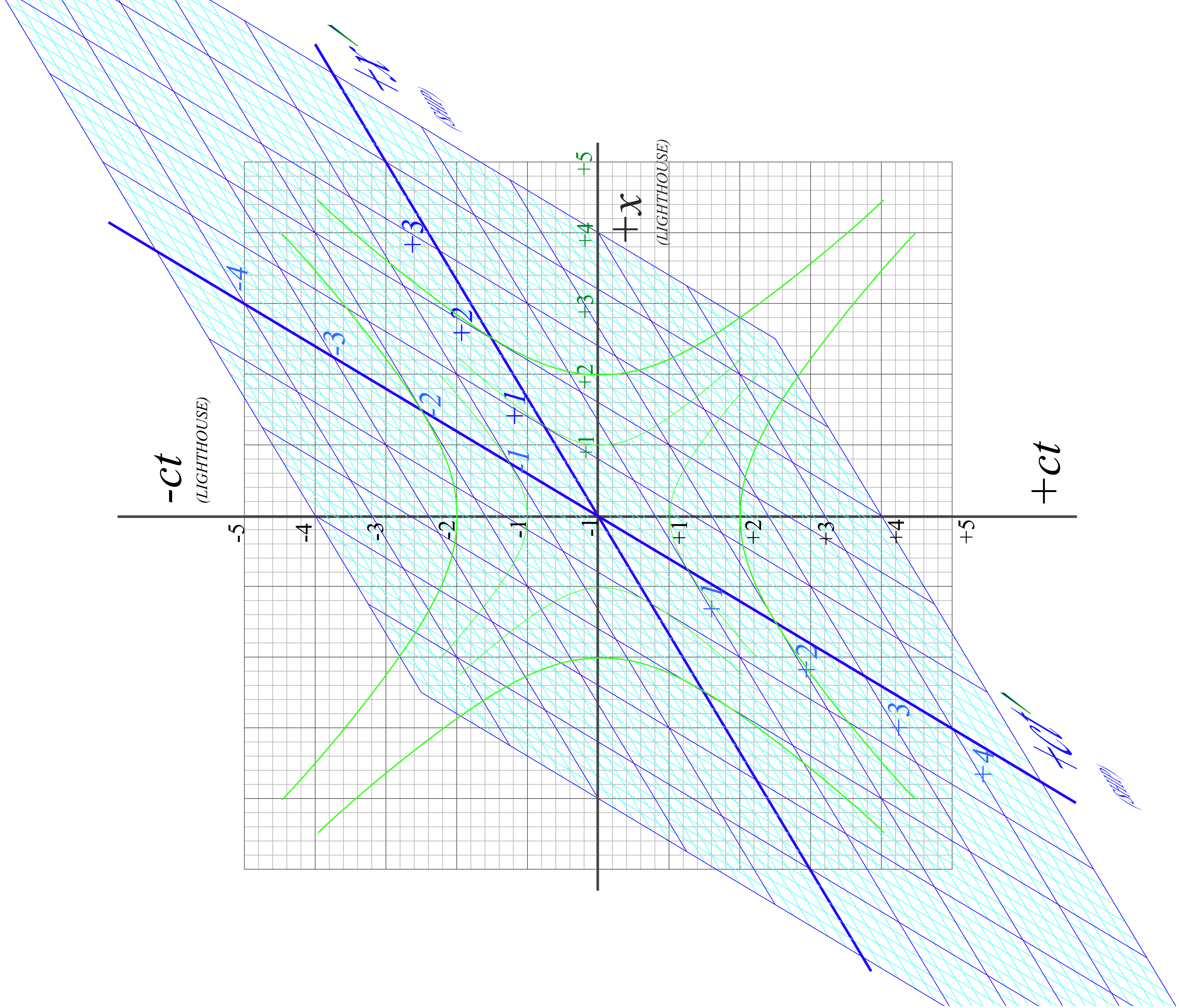
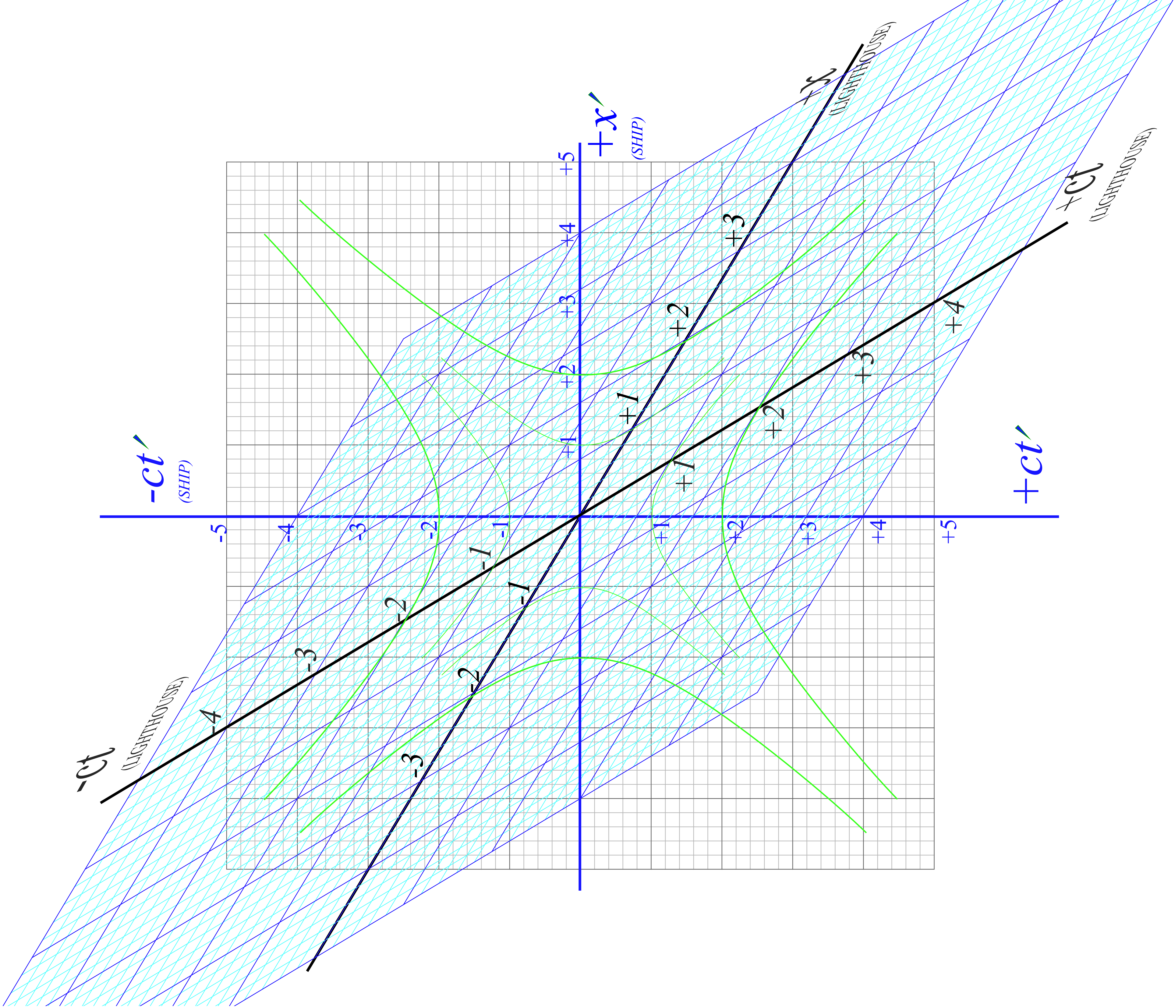


Fig. 2.B.5 Space-Space-Time plot of world lines for Lighthouses. North Lighthouse blink waves trace light cones.


+ct
-ct
(LIGHTHOUSE)
+x
(LIGHTHOUSE)
+ct'
(SHIP)
+x'
(SHIP)


+ct′
-ct′
(SHIP)
+x′
(SHIP)
+x
(LIGHTHOUSE)
+ct
(LIGHTHOUSE)
-ct
(LIGHTHOUSE)
-5
-4
-3
-2
-1
+1
+2
+3
+4
+5