Lecture 32 Relawavity-Dynamics
Thursday 5.05.2016

Relawavity: Spectroscopy, transitions, and acceleration

(Unit 3 p.45-61 - 4.26.16)

Relativity relates charge, current, and magnetic fields
- Geometric derivation of magnetic constant μ_0 from electric ε_0

Lorentz-Poincare symmetry and energy-momentum spectral conservation rules
- Review of 2nd-quantization "photon" number N and 1st-quantization wavenumber $\kappa = m$
  - Sketches of atomic and molecular spectroscopy

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Recoils shifts
    - Compton recoil related to rocket velocity formula
      - Geometric transition coordinate grids

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes g-acceleration grid
  - Analysis of constant-g grid compared to zero-g Minkowsi frame
    - Animation of mechanics and metrology of constant-g grid

Xtra stuff: *Some numerology: Which is bigger...H-atom or an electron? What' spin?*
Space-Space waves gone mad

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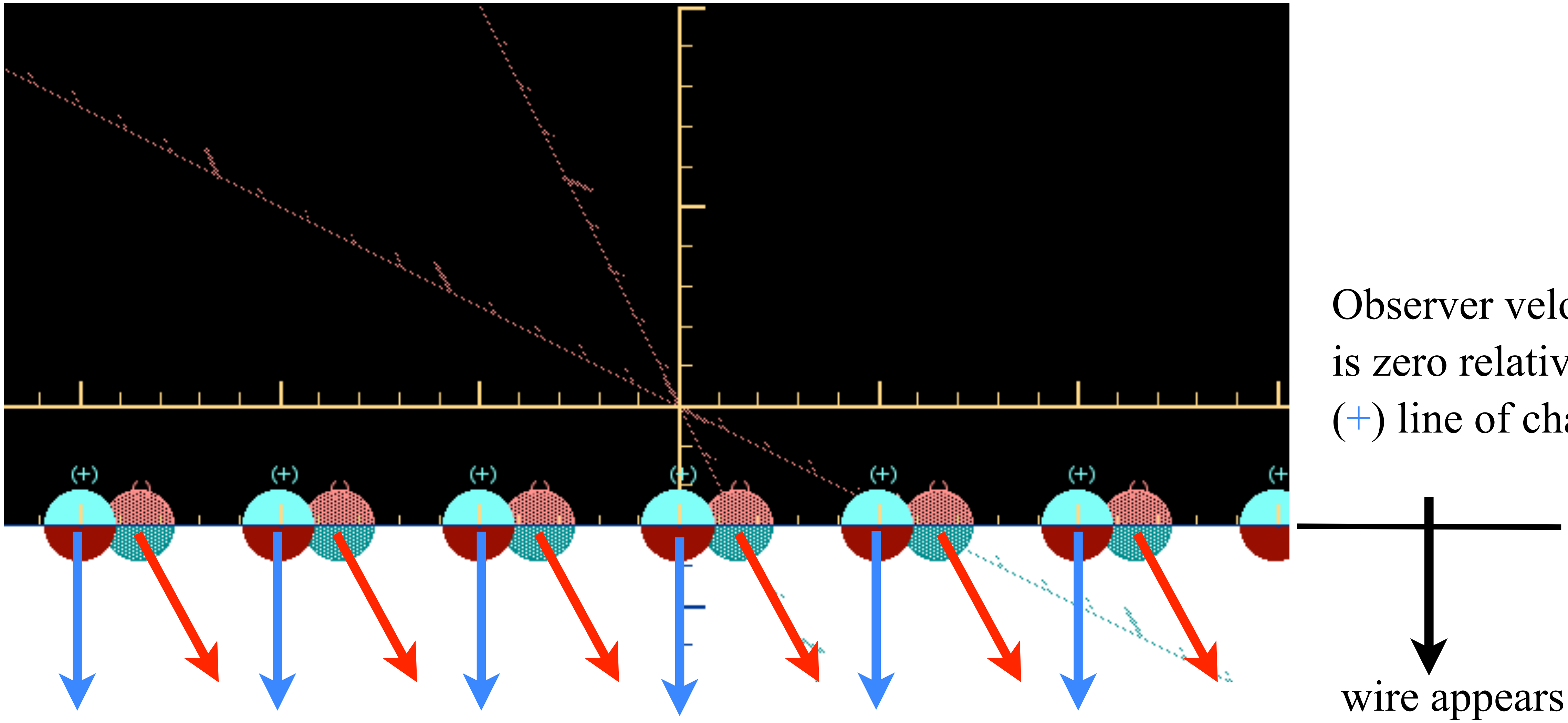
Relativistic effects on charge, current, and Maxwell Fields



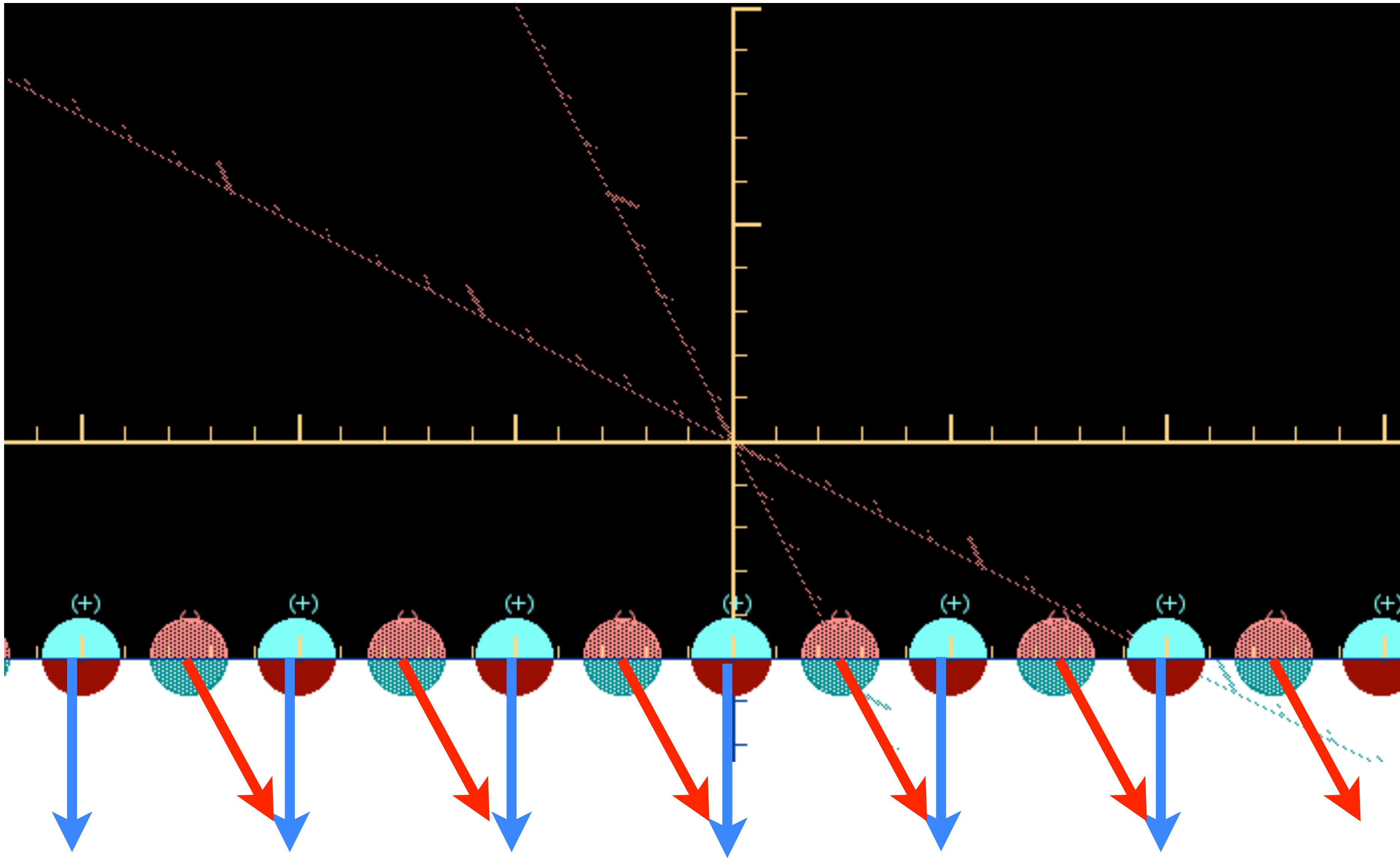
Observer velocity is zero relative to (+) line of charge

wire appears neutral

(+) Charge fixed (-) Charge moving to right (*Negative current density)*

(+) Charge density is Equal to the (-) Charge density

Relativistic effects on charge, current, and Maxwell Fields



Observer velocity is zero relative to (+) line of charge

wire appears neutral

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

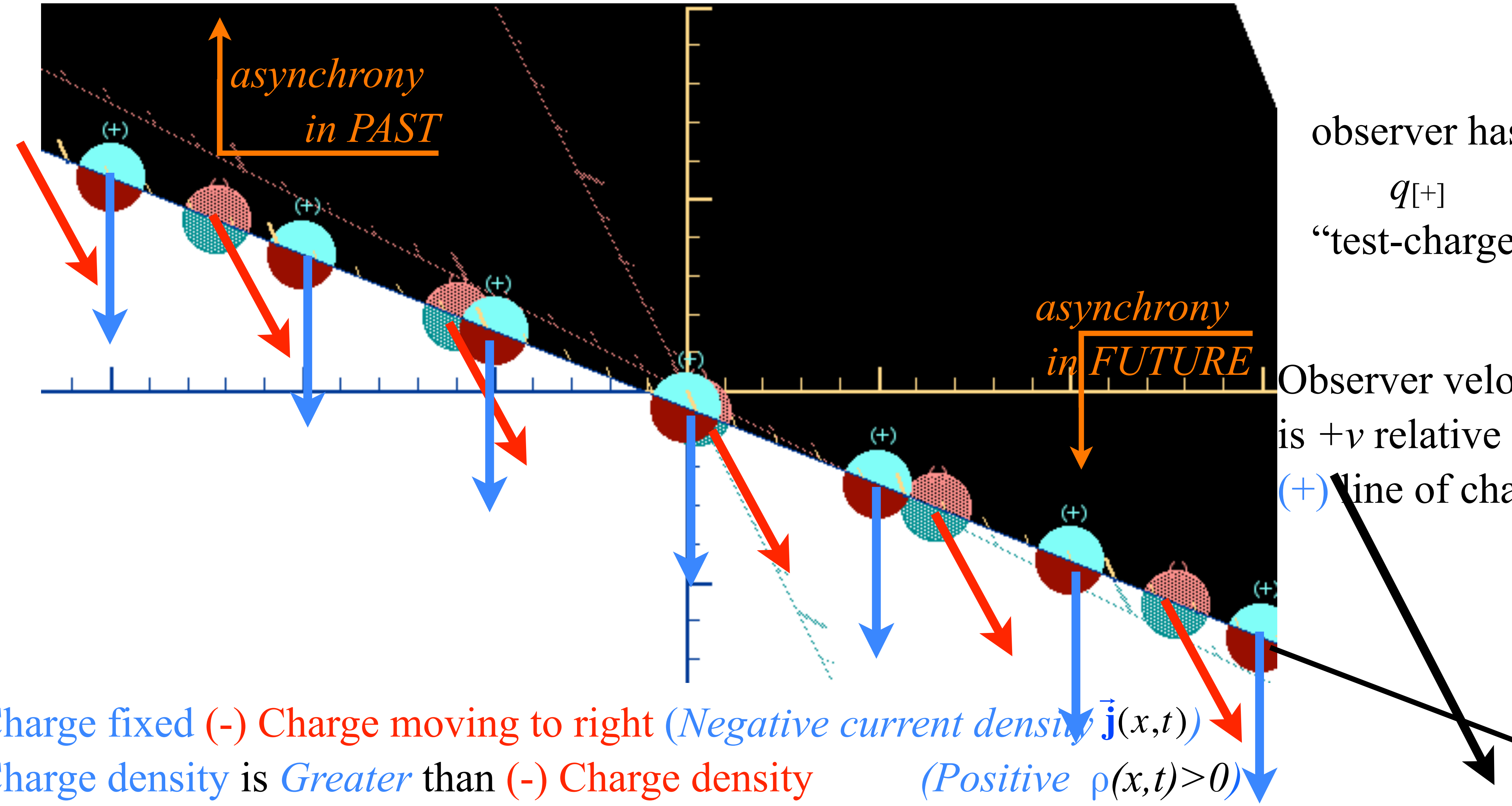
(+) Charge density is Equal to the (-) Charge density (*Zero* $\rho(x,t)=0$)

Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony

Asynchrony due to off-diagonal $\sinh\rho$ (a 1st-order effect)

$$\textit{in Lorentz tranform}: \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$$



observer has $q_{[+]}$ "test-charge"

Observer velocity is $+v$ relative to (+) line of charge

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

(+) Charge density is *Greater* than (-) Charge density (*Positive* $\rho(x,t)>0$)

wire appears postive (+) (repulsive to observer $q_{[+]}$)

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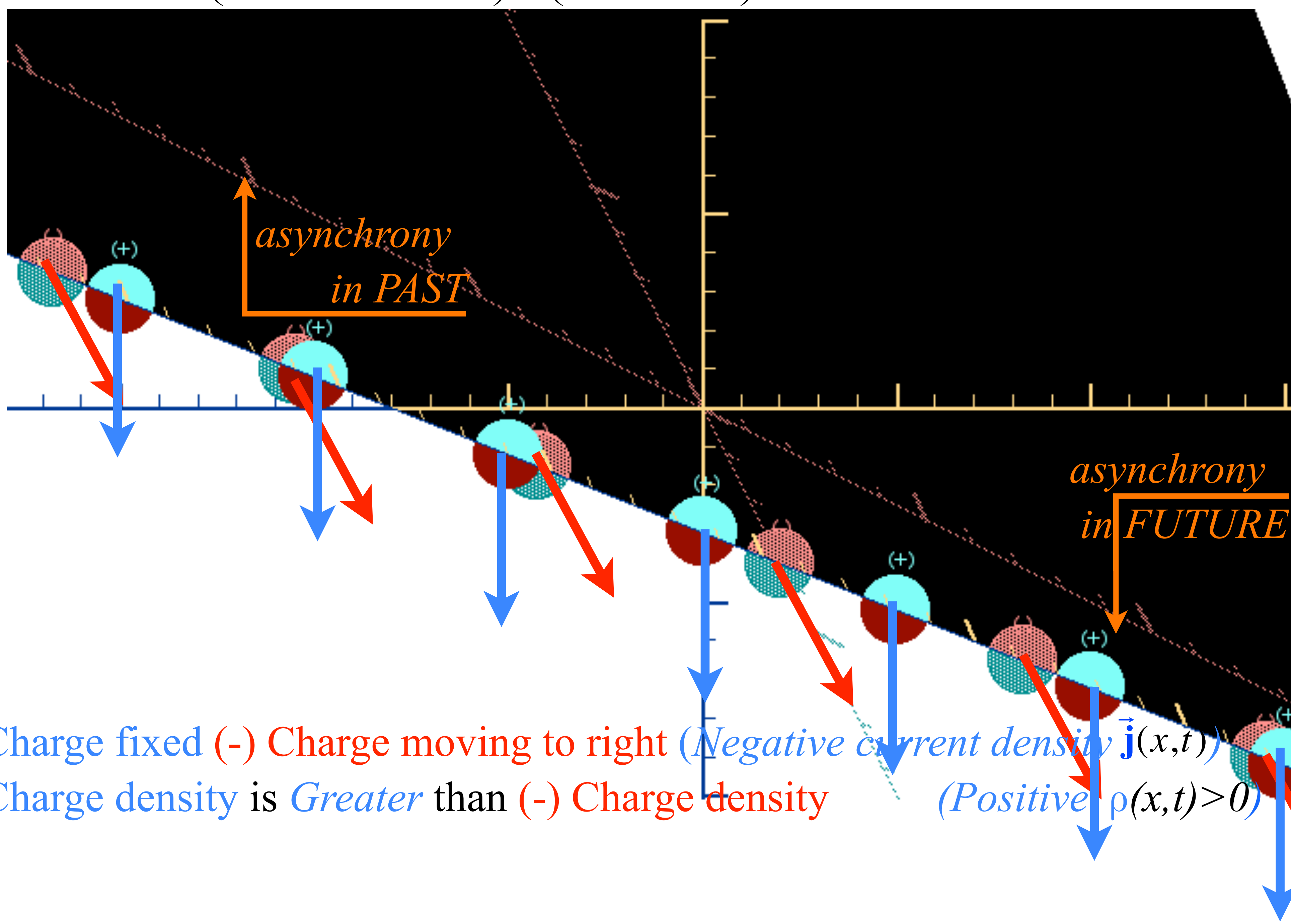

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wire appears positive (+) (repulsive to observer $q_{[+]}$)

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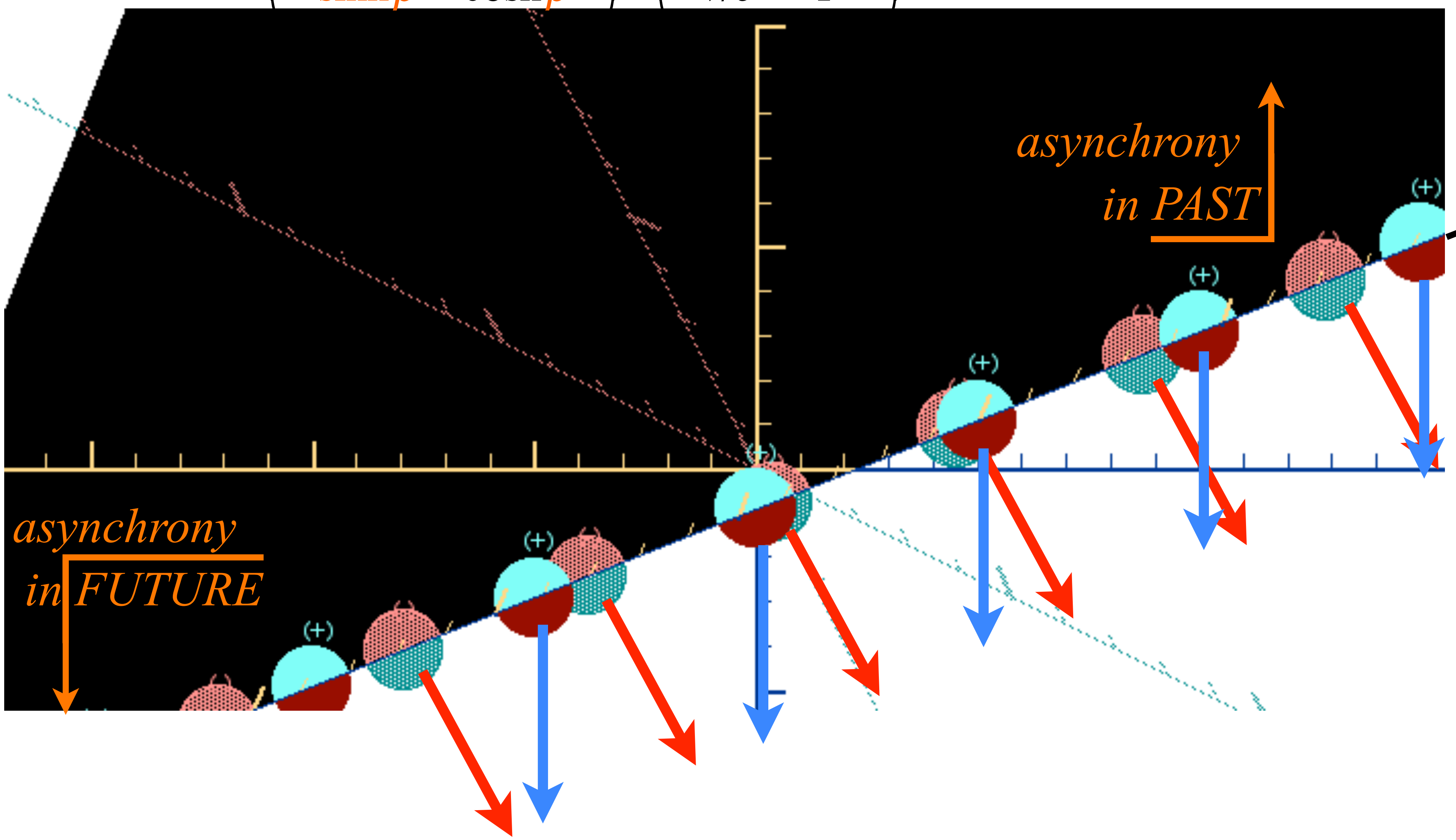
observer has $q_{[+]}$ "test-charge"

Observer velocity is -*v* relative to (+) line of charge



wire appears negative (-) (attractive to observer $q_{[+]}$)

(+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{\mathbf{j}}(x,t)$)

(+) Charge density is *Less* than (-) Charge density (*Negative* $\rho(x,t)<0$)

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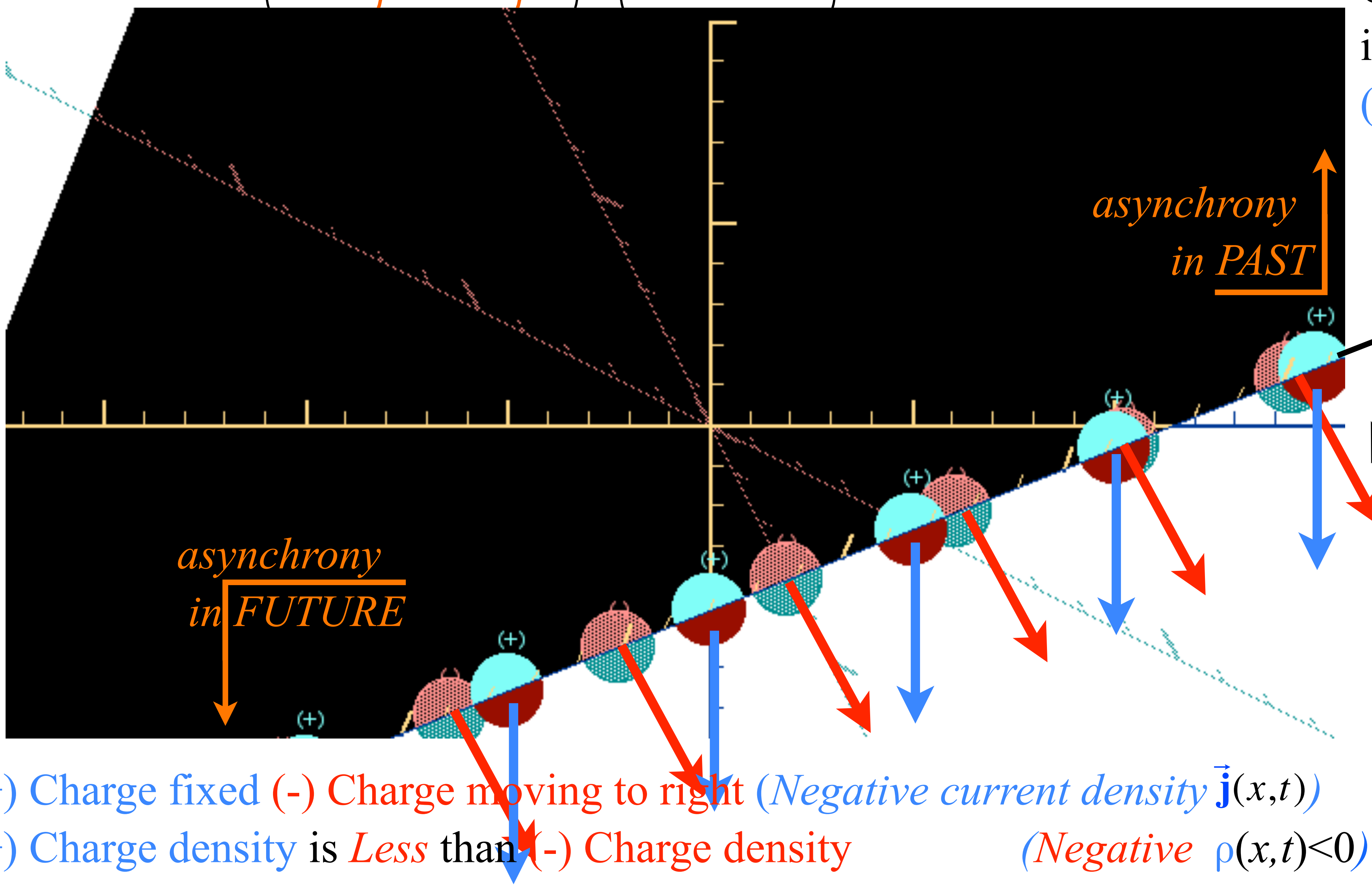
observer has $q_{[+]}$ "test-charge"

Observer velocity is -v relative to (+) line of charge



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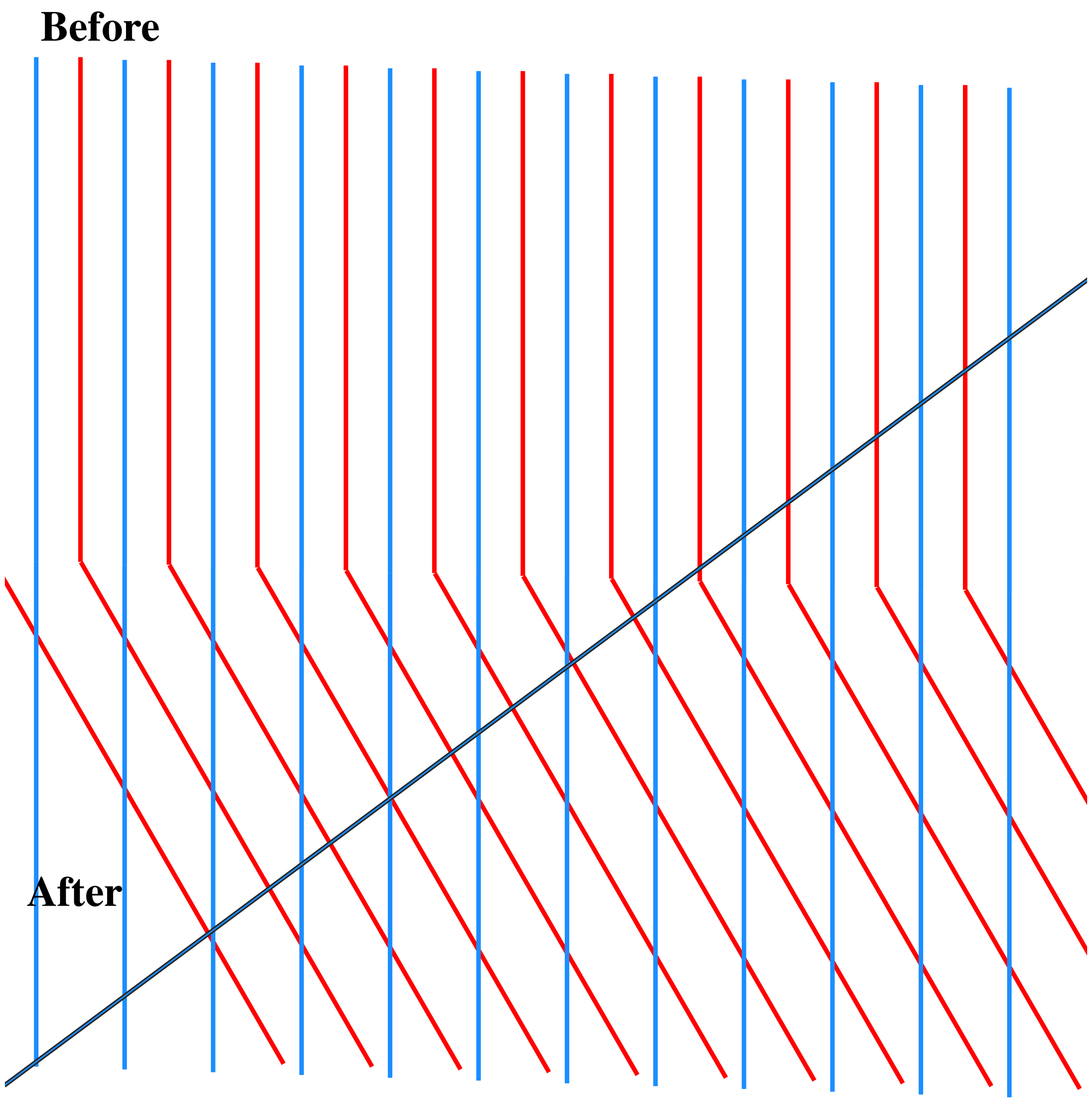
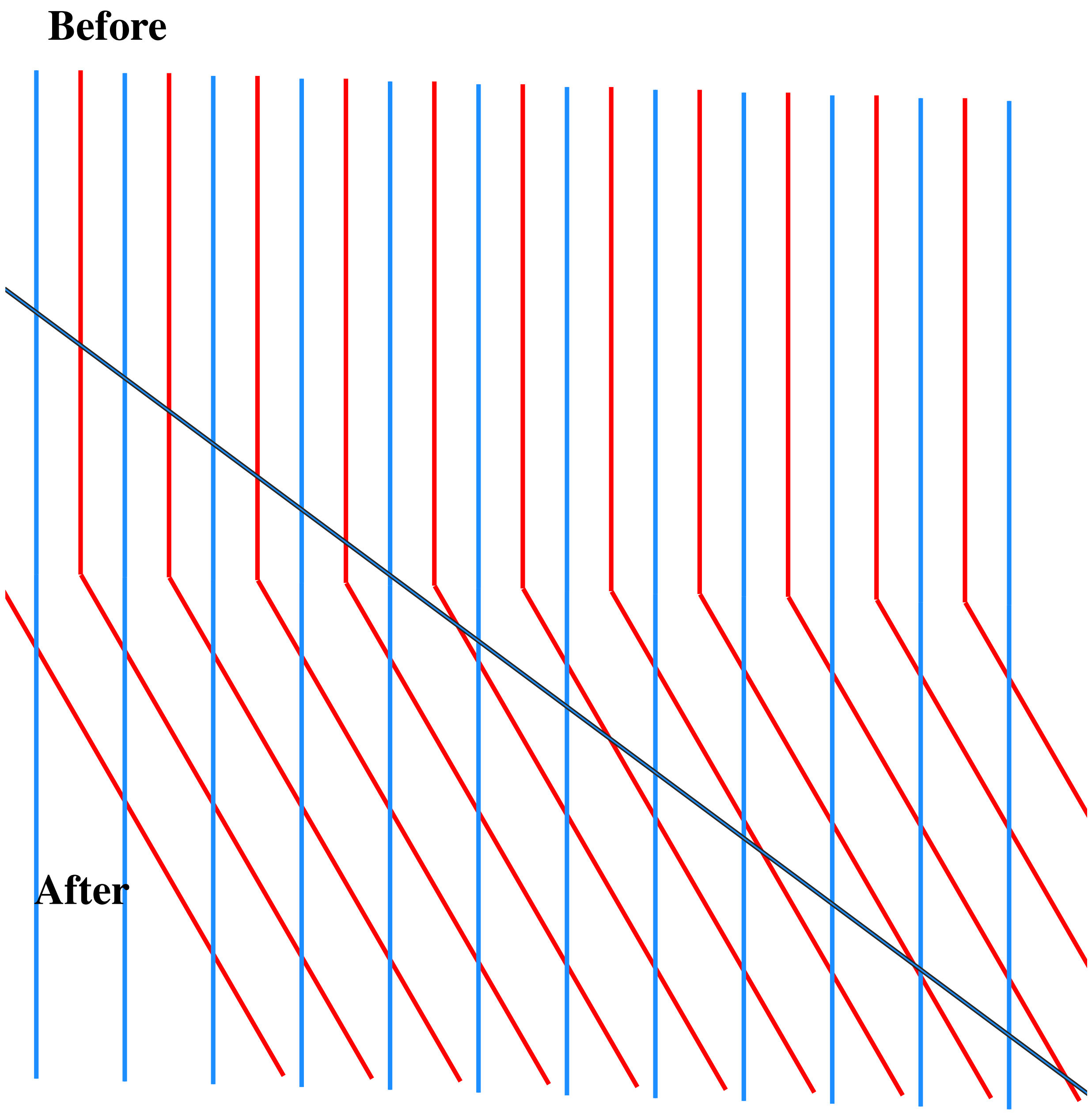
Simple 1st-order relativistic geometry of magnetism





If Black is moving to Left

Before red starts moving to right

Black sees same number of red and blue

After red starts moving to right

Black sees more red than blue

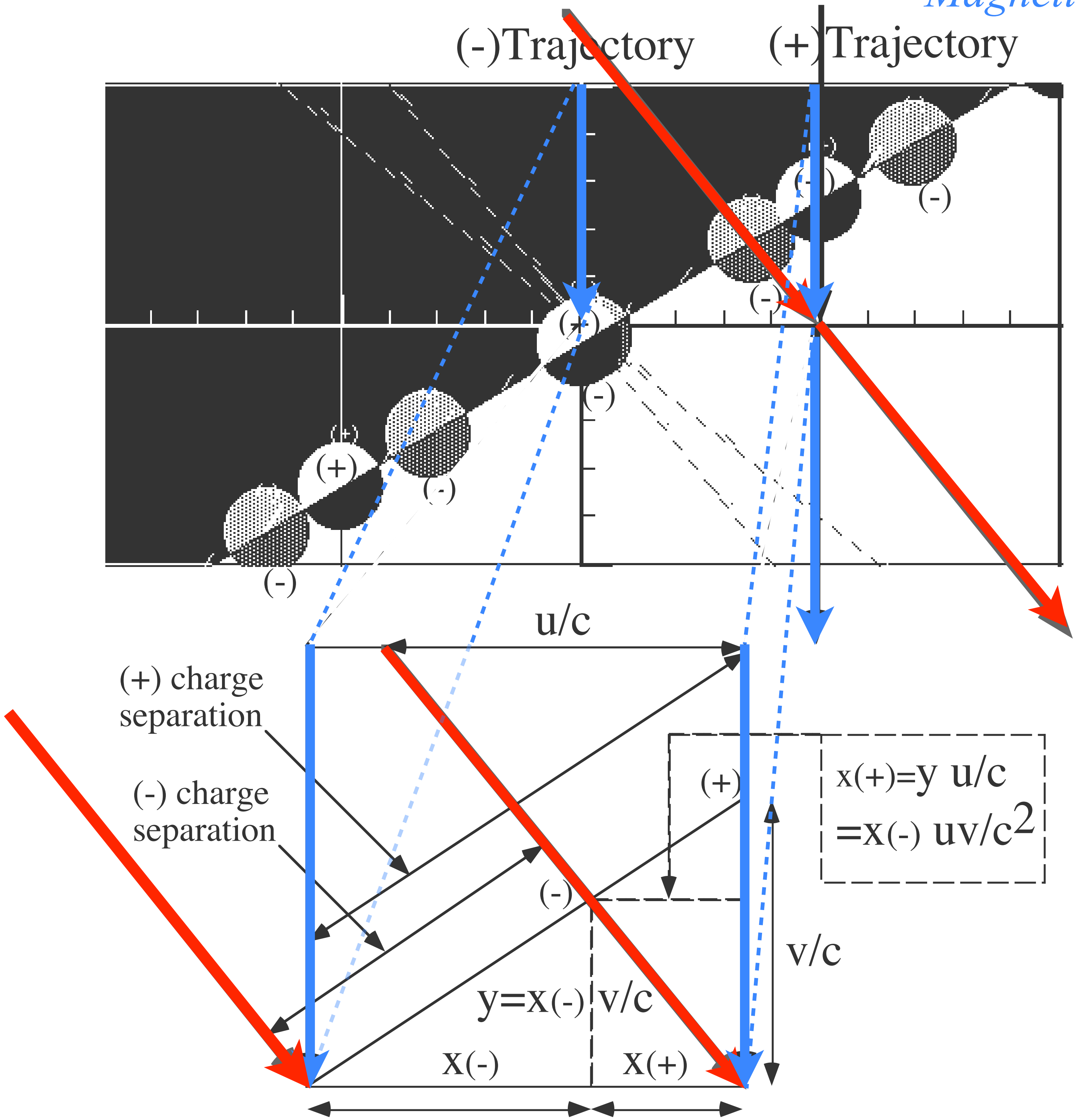
If Black is moving to Right

Before red starts moving to right

Black sees same number of red and blue

After red starts moving to right

Black sees more blue than red



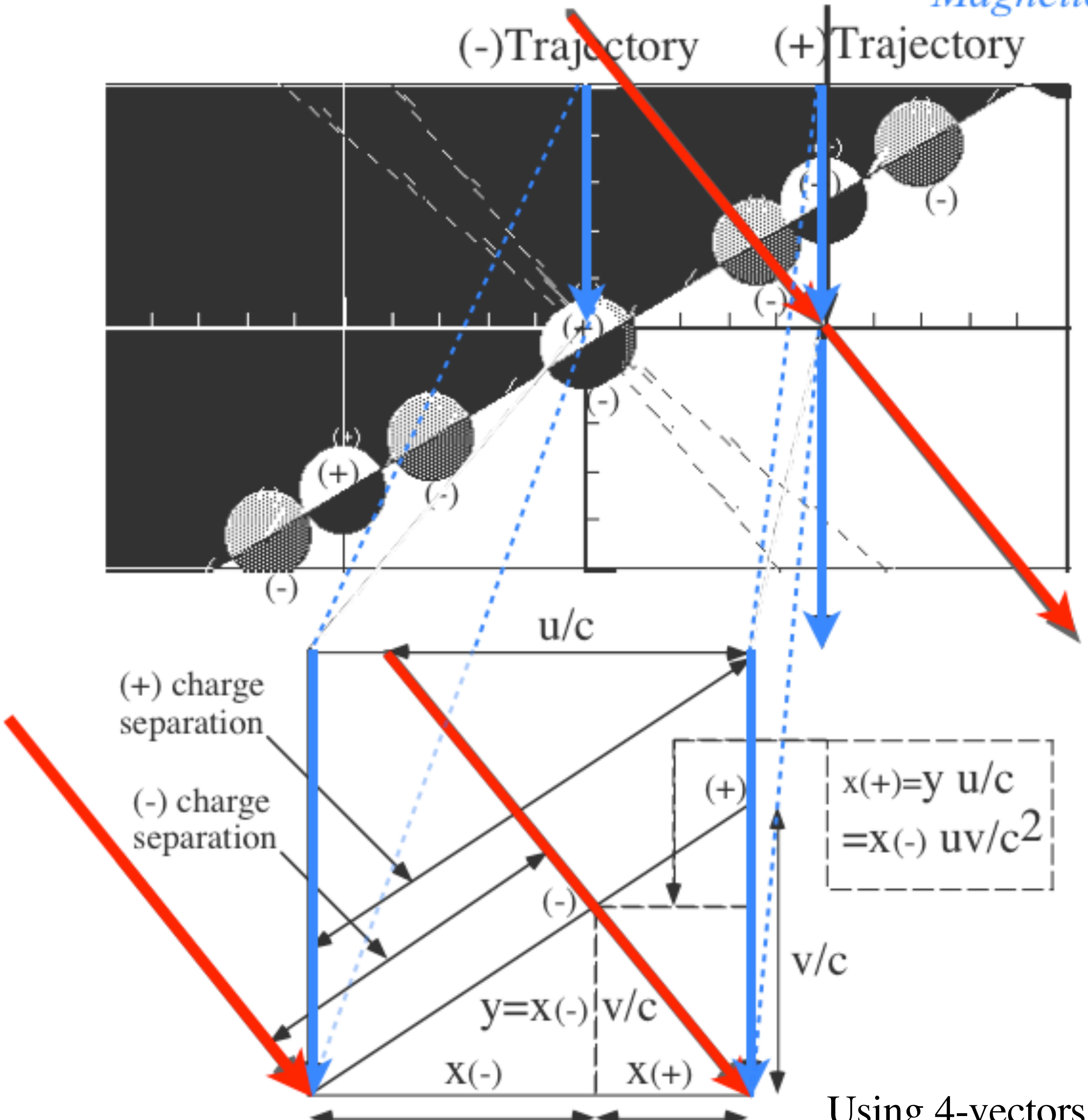
$$\frac{\rho(-)}{\rho(+)} = \frac{(+)\text{ charge separation}}{(-)\text{ charge separation}} = \frac{x(+)+x(-)}{x(-)}$$

$$\frac{\rho(-)}{\rho(+)} = \frac{x(+)}{x(-)} + 1 = \frac{uv}{c^2} + 1$$

$$\rho(+) - \rho(-) = \rho(+)\left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2}\rho(+)$$

Unit square: (u/c) /1 = x(+)/y

(v/c) /1 = y/x(-)



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$$\rho(+) - \rho(-) = \rho(+)\left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2}\rho(+)$$

Using 4-vectors to EL Transform (charge-current)=$(c\rho, \mathbf{j})$

$$\begin{pmatrix} c\rho' \\ j_{x'} \\ j_{y'} \\ j_{z'} \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho & \cdot & \cdot \\ \sinh\rho & \cosh\rho & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{pmatrix} c\rho \\ j_x \\ j_y \\ j_z \end{pmatrix}$$

Magnetic B-field is relativistic sinhρ *1st order-effect*

The electric force field **E** of a charged line varies inversely with radius. The Gauss formula for force in mks units :

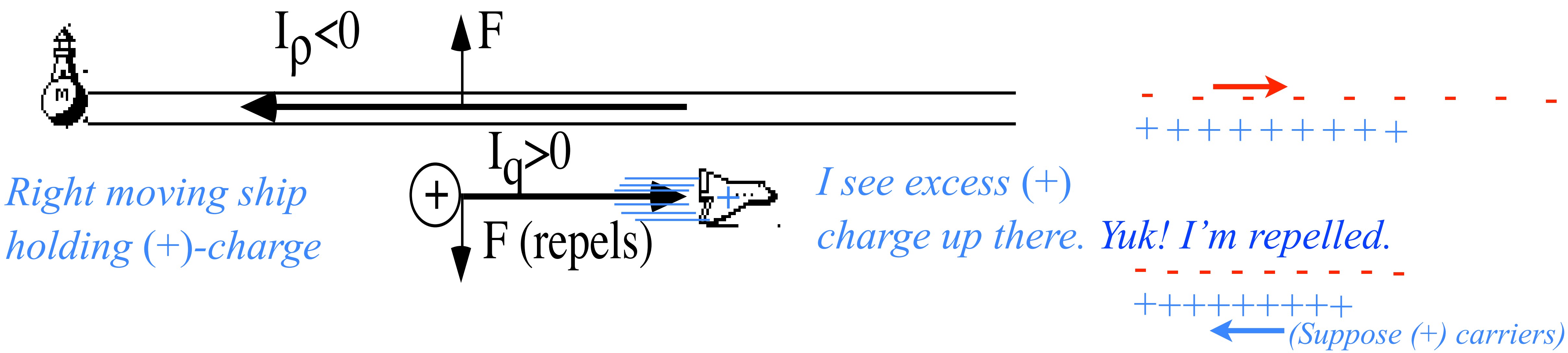
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2\rho}{r}\right] \quad , \quad \text{where:} \quad \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\frac{N\cdot m^2}{Coul.}$$

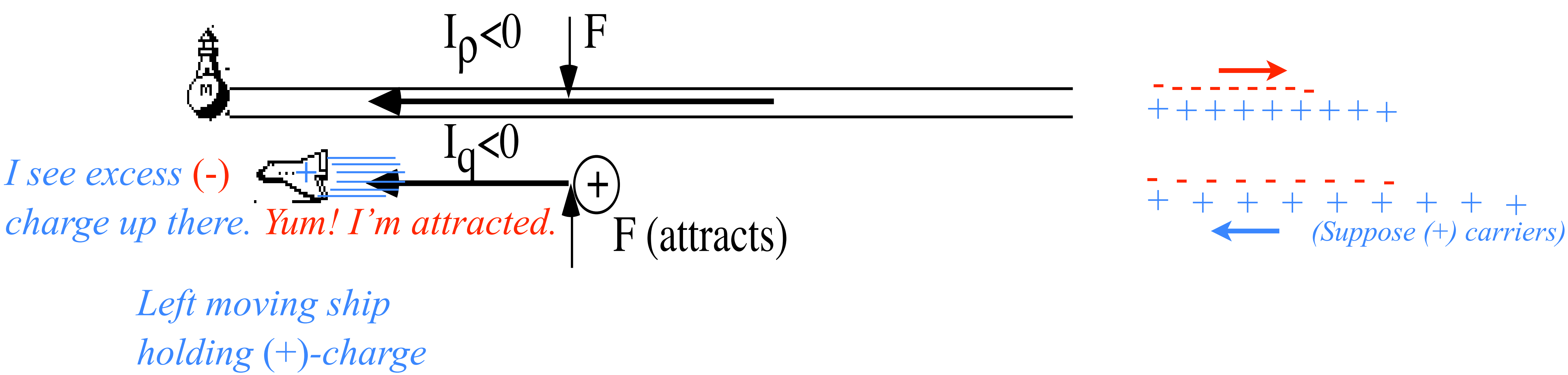
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2}{r}\left(-\frac{uv}{c^2}\rho(+)\right)\right] = -\frac{2\,qv\,\rho(+)u}{4\pi\varepsilon_0 c^2\,r} = -2\times10^{-7}\,\frac{I_q\,I_\rho}{r}$$

$1/4\pi\varepsilon_0 = 9\cdot10^9$

$c^2 = 9\cdot10^{-16}$

$1/(4\pi\varepsilon_0 c^2) = 10^{-7}$





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*Lorentz-Poincare symmetry and (E,c**P**) spectral conservation rules*

Evenson axiom has symmetry based on isotropy of space-time and invariance to translation operation $\mathbf{T}(\vec{\delta},\tau)$ having plane wave eigenfunctions $\left\langle x,t \middle| \psi_{\mathbf{k},\omega} \right\rangle$ and roots-of-unity eigenvalues.

$$\mathbf{T}(\vec{\delta},\tau)\left|\psi_{\mathbf{k},\omega}\right\rangle = e^{i(\mathbf{k}\bullet\vec{\delta}-\omega\cdot\tau)}\left|\psi_{\mathbf{k},\omega}\right\rangle \quad \left(\begin{smallmatrix}\textit{eigen-ket}\\ \textit{relation}\end{smallmatrix}\right) \qquad \left\langle\psi_{\mathbf{k},\omega}\right|\mathbf{T}^{\dagger}(\vec{\delta},\tau) = \left\langle\psi_{\mathbf{k},\omega}\right| e^{-i(\mathbf{k}\bullet\vec{\delta}-\omega\cdot\tau)} \quad \left(\begin{smallmatrix}\textit{eigen-bra}\\ \textit{relation}\end{smallmatrix}\right)$$

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This also applies to N-particle product states $\Psi_{\mathbf{K},\Omega}=\psi_{\mathbf{k}_1,\omega_1}\psi_{\mathbf{k}_2,\omega_2}\cdots\psi_{\mathbf{k}_N,\omega_N}$ where exponents add (k,ω)-values of each constituent in $\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2+\ldots$ and $\Omega=\omega_1+\omega_2+\ldots$ so $\mathbf{T}(\vec{\delta},\tau)$-eigenvalues have the form $e^{i(\mathbf{K}\bullet\vec{\delta}-\Omega\cdot\tau)}$.

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Matrix $\langle\Psi'_{\mathbf{K}',\Omega'}|\mathbf{U}|\Psi_{\mathbf{K},\Omega}\rangle$ of $\mathbf{T}$-symmetric evolution operator $\mathbf{U}$ (Hamiltonian $\mathbf{H}$ or $\mathbf{U}=e^{-i\mathbf{H}t}$) is tested by assuming symmetry $\mathbf{T}^{\dagger}(\vec{\delta},\tau)\mathbf{U}\mathbf{T}(\vec{\delta},\tau)=\mathbf{U}$ or commutivity $\mathbf{UT}=\mathbf{TU}$ for *all* $\vec{\delta}$ and τ.

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Exponents must cancel for all (δ,τ)

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That's TOTAL momentum ($\mathbf{P}=\hbar\mathbf{K}$) and energy ($E=\hbar\Omega$) conservation!

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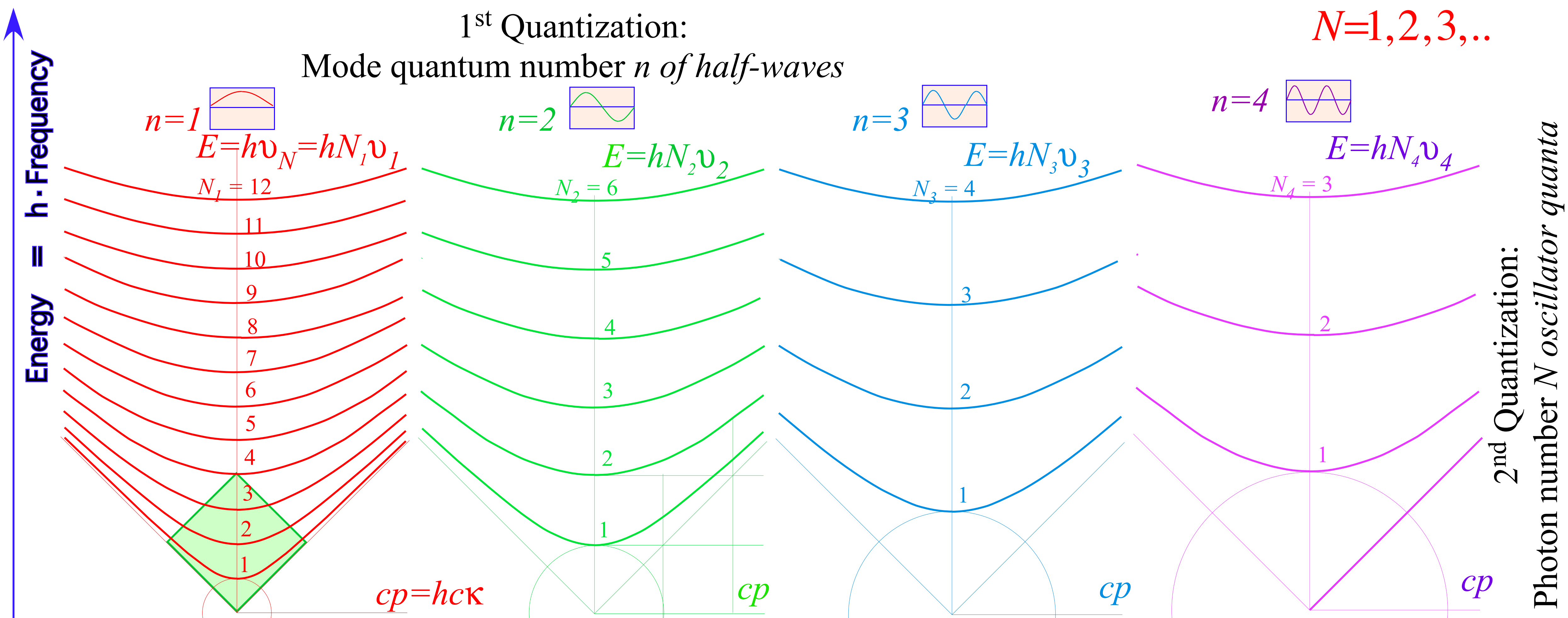
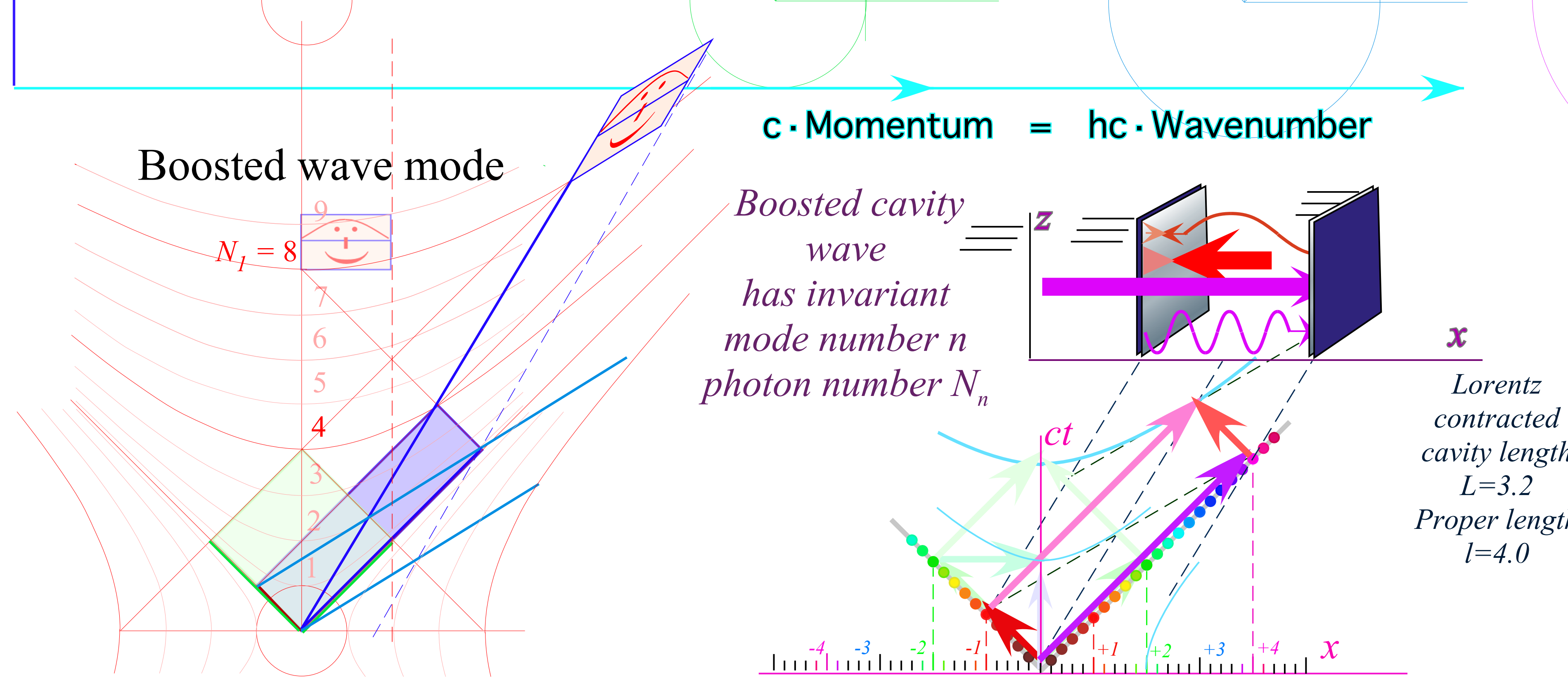
2nd Quantization: $h\upsilon$ *implies* $hN\upsilon$

($h\upsilon_{phase}=E=h\upsilon_A \cosh\rho$) *implies* ($hN\upsilon_{phase}=E_N=hN\upsilon_A \cosh\rho$ with *quantum* numbers)

N=1,2,3,..

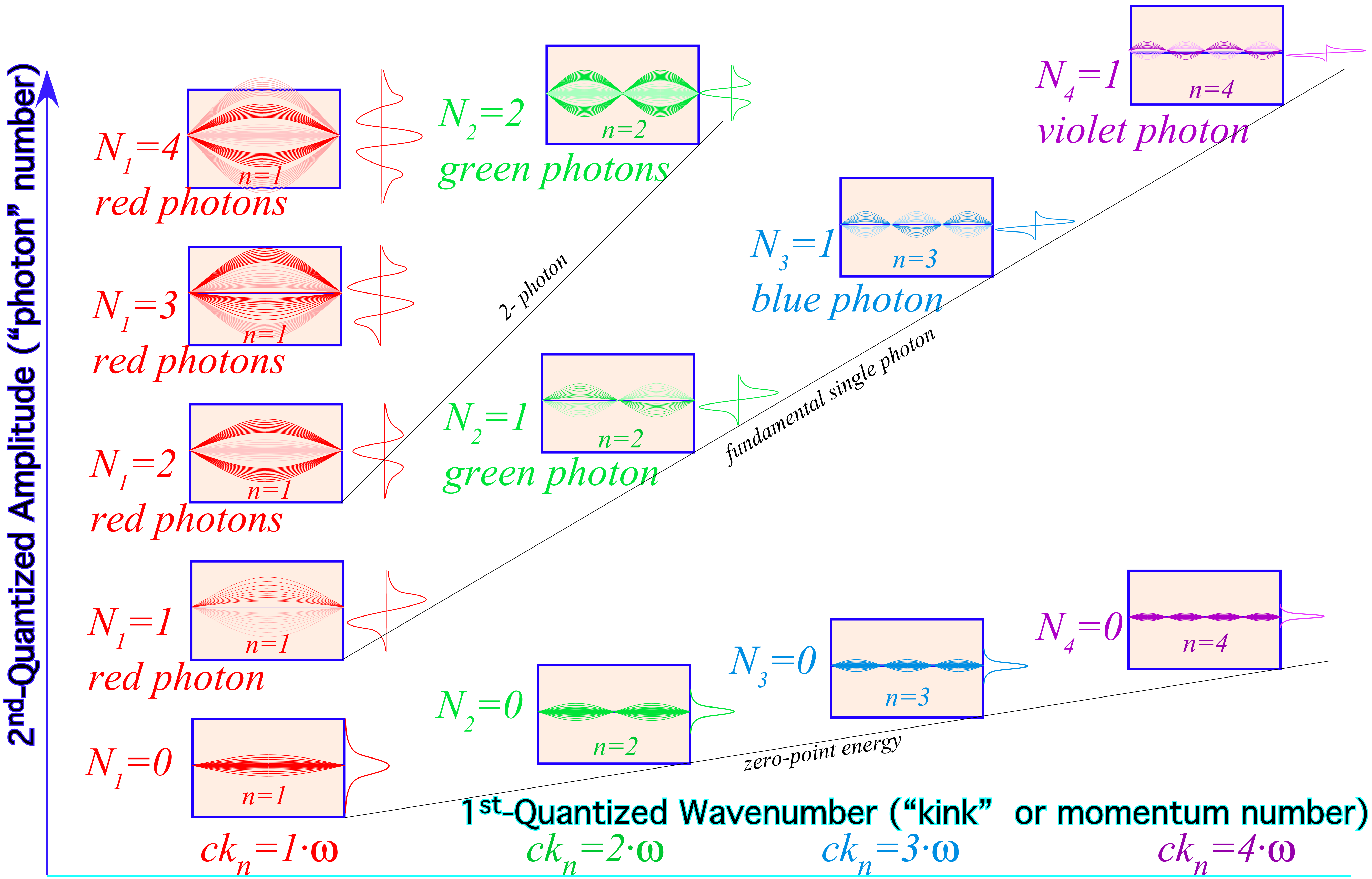




Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2nd Quantization: $h\upsilon$ implies $hN\upsilon$

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ implies $(h N\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho \quad (N=1,2,..))$



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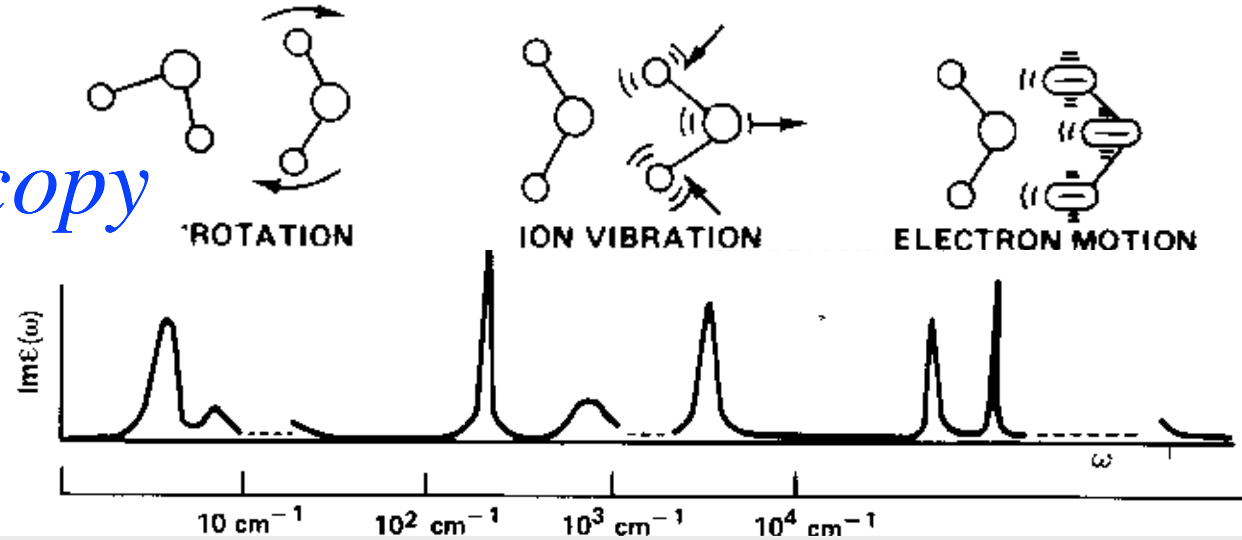
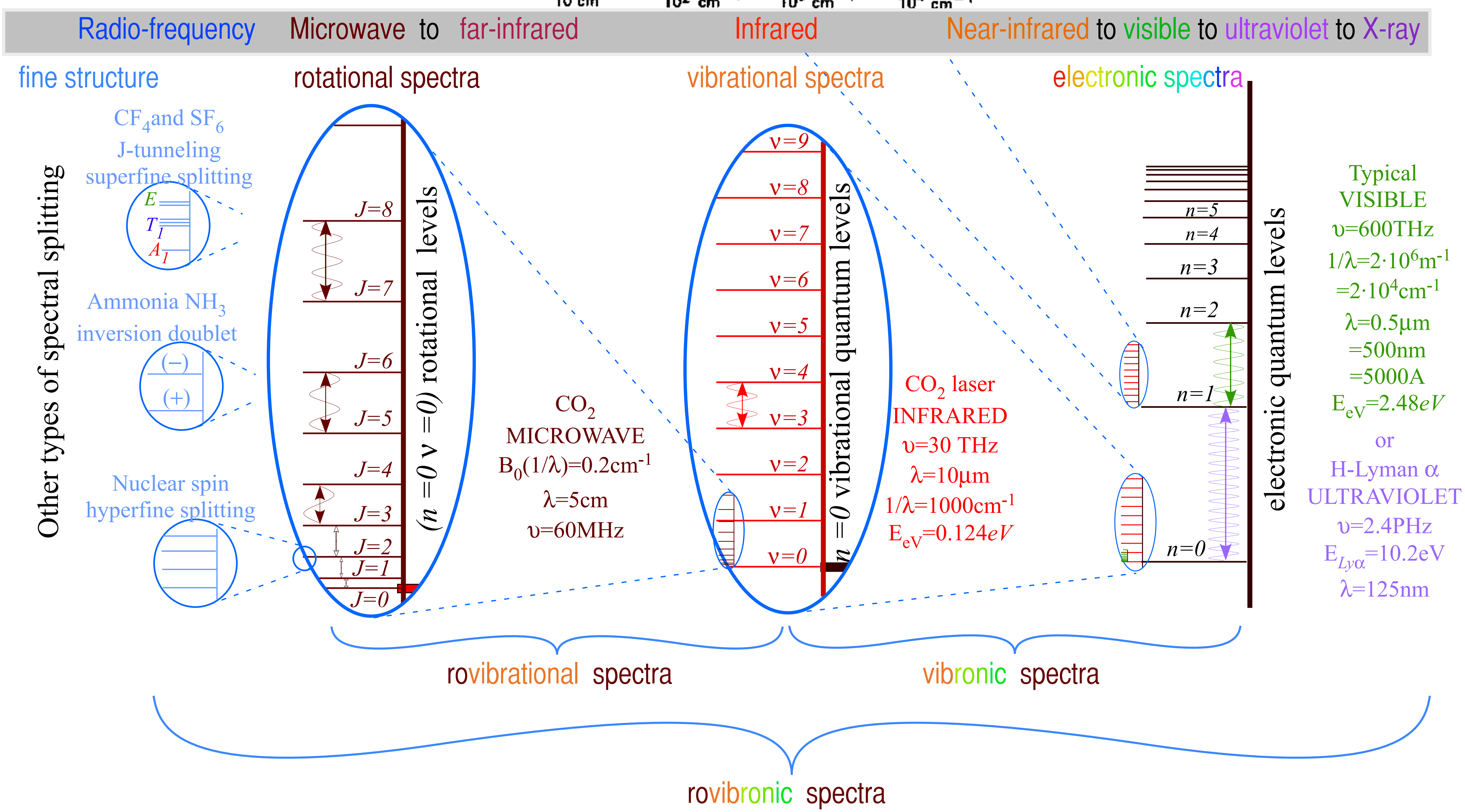
A sketch of modern molecular spectroscopy



From Fig. 6.5.5.
Principles of Symmetry, Dynamics, and Spectroscopy
W. G. Harter, Wiley Interscience, NY (1993)

The frequency hierarchy



Spectral Quantities

<u>Frequency υ</u>
<u>Hertz(sec^{-1})</u>
THz $10^{12}s^{-1}$
GHz $10^{9}s^{-1}$
MHz $10^{6}s^{-1}$
kHz $10^{3}s^{-1}$

<u>Wavelength λ</u>
<u>meters(m)</u>
fm $10^{-15}m$
pm $10^{-12}m$
nm $10^{-9}m$
μm $10^{-6}m$
mm $10^{-3}m$
cm $10^{-2}m$
km $10^{3}m$

<u>Wavenumber</u>
<u>per meter(m^{-1})</u>
cm^{-1} $10^{2}m^{-1}$

<u>Energy $eh\upsilon$</u>
<u>electonVolts</u>
<u>(eV)</u>

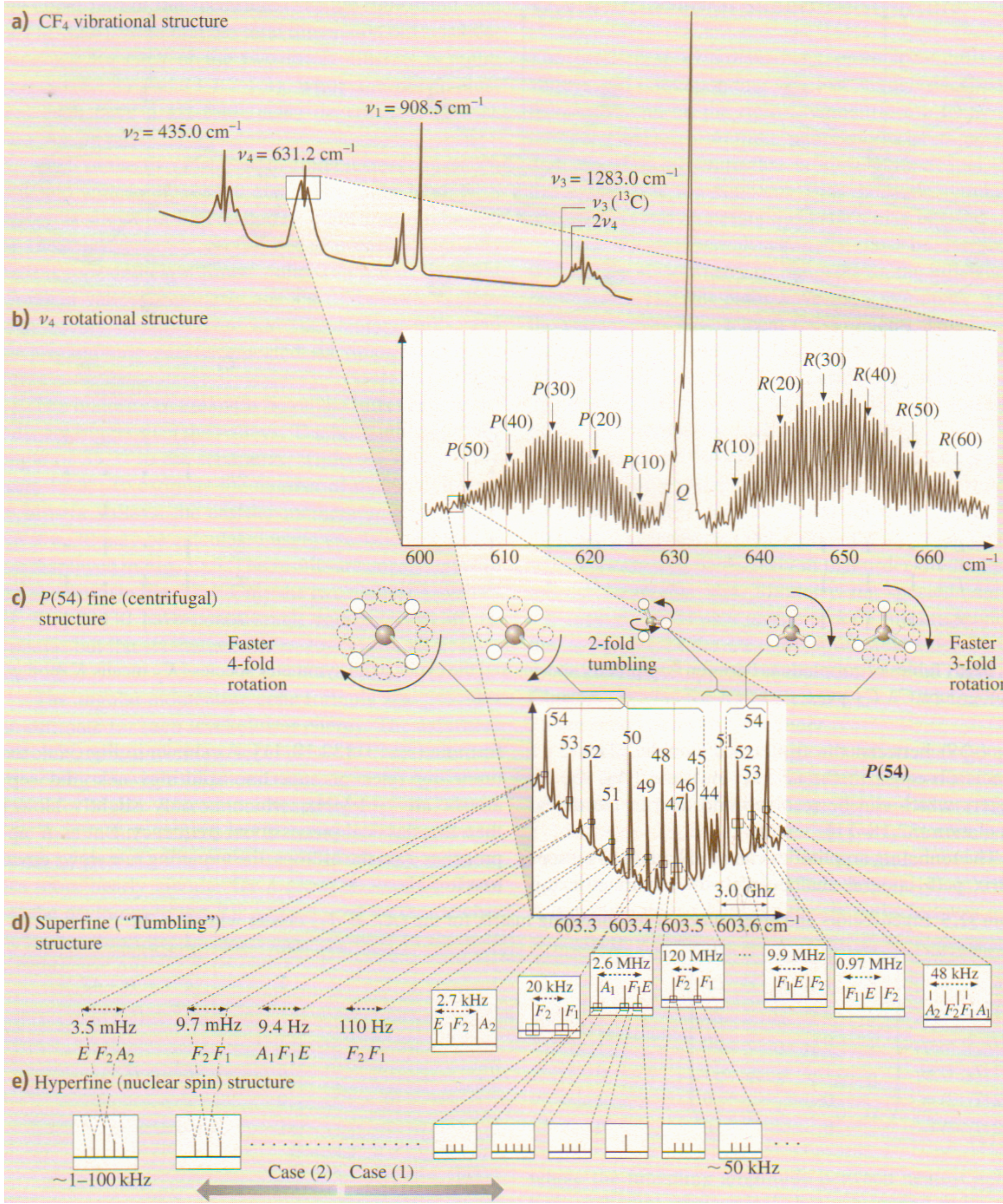
Example of frequency hierarchy for 16μm spectra of CF_4 (Freon-14)

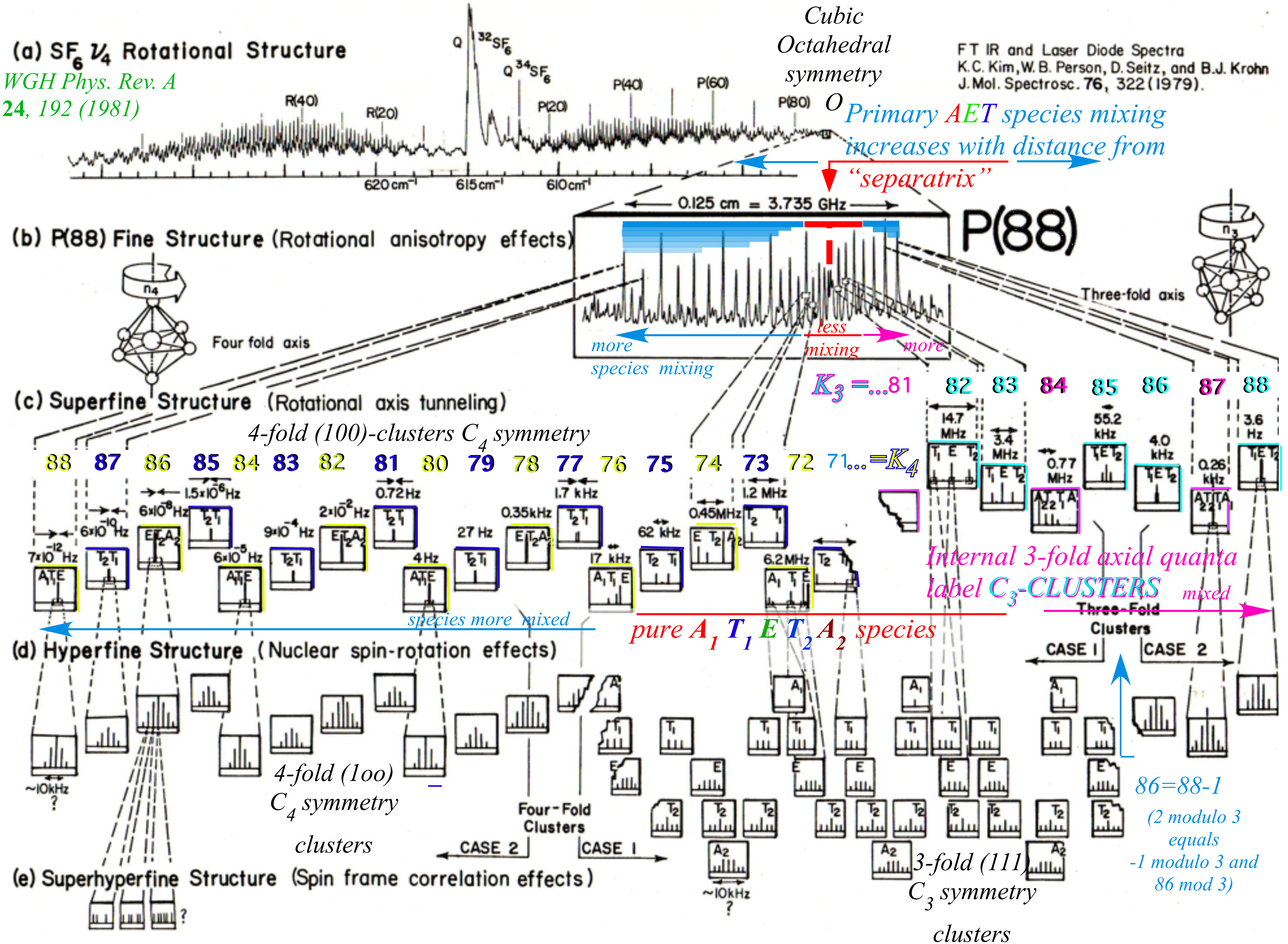
W.G.Harter

Fig. 32.7

Springer Handbook of Atomic, Molecular, & Optical Physics

Gordon Drake Editor (2005)




(a) SF6 ν4 Rotational Structure
WGH Phys. Rev. A 24, 192 (1981)
Cubic Octahedral symmetry O
FT IR and Laser Diode Spectra
K.C. Kim, W.B. Person, D. Seitz, and B.J. Krohn
J. Mol. Spectrosc. 76, 322 (1979).
Primary AET species mixing increases with distance from "separatrix"
R(40) R(20) Q 32SF6 Q 34SF6 P(20) P(40) P(60) P(80)
620 cm-1 615 cm-1 610 cm-1
(b) P(88) Fine Structure (Rotational anisotropy effects)
0.125 cm = 3.735 GHz
P(88)
Four fold axis
Three-fold axis
more species mixing
less mixing
more
(c) Superfine Structure (Rotational axis tunneling)
4-fold (100)-clusters C4 symmetry
K3 =...81 82 83 84 85 86 87 88
88 87 86 85 84 83 82 81 80 79 78 77 76 75 74 73 72 71...=K4
Internal 3-fold axial quanta label C3-CLUSTERS
mixed
species more mixed
pure A1 T1 E T2 A2 species
Three-Fold Clusters
CASE 1 CASE 2
(d) Hyperfine Structure (Nuclear spin-rotation effects)
4-fold (1oo) C4 symmetry clusters
Four-Fold Clusters
CASE 2 CASE 1
3-fold (111) C3 symmetry clusters
86=88-1
(2 modulo 3 equals -1 modulo 3 and 86 mod 3)
(e) Superhyperfine Structure (Spin frame correlation effects)

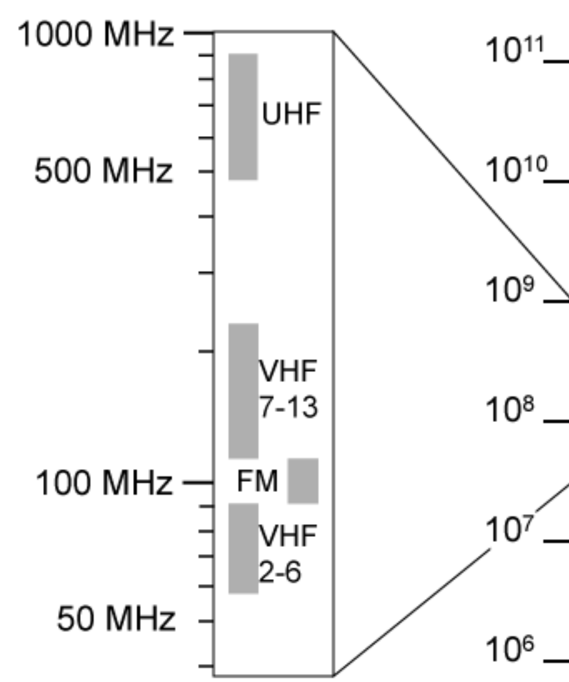
Units of frequency (Hz), wavelength (m), and energy (eV)

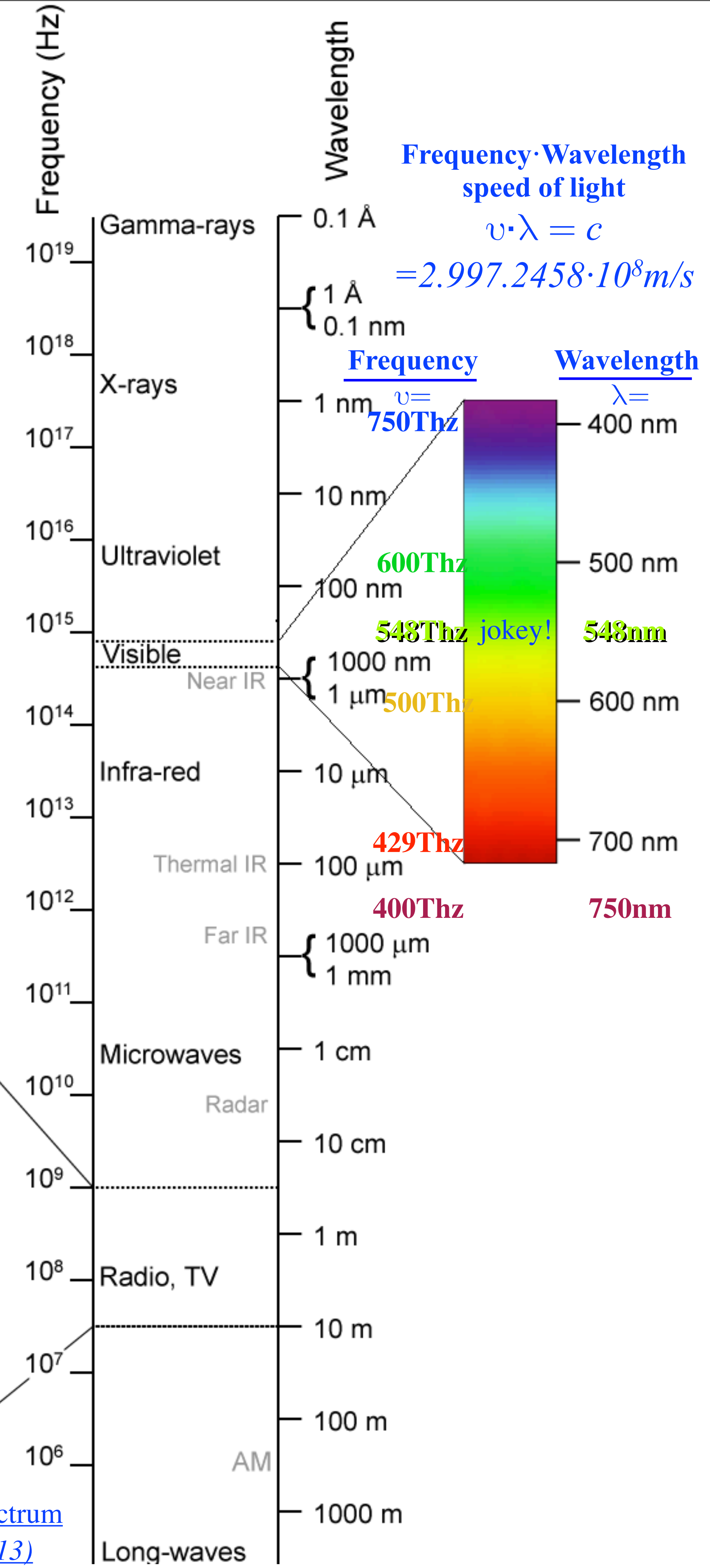
CLASS	FREQUENCY	WAVELENGTH	ENERGY
γ	300 EHz	1 pm	1.24 MeV
HX	30 EHz	10 pm	124 keV
	3 EHz	100 pm	12.4 keV
SX	300 PHz	1 nm	1.24 keV
EUV	30 PHz	10 nm	124 eV
NUV	3 PHz	100 nm	12.4 eV
NIR	300 THz	1 μm	1.24 eV
MIR	30 THz	10 μm	124 meV
FIR	3 THz	100 μm	12.4 meV
EHF	300 GHz	1 mm	1.24 meV
SHF	30 GHz	1 cm	124 μeV
UHF	3 GHz	1 dm	12.4 μeV
VHF	300 MHz	1 m	1.24 μeV
HF	30 MHz	10 m	124 neV
MF	3 MHz	100 m	12.4 neV
LF	300 kHz	1 km	1.24 neV
VLF	30 kHz	10 km	124 peV
VF/ULF	3 kHz	100 km	12.4 peV
SLF	300 Hz	1 Mm	1.24 peV
ELF	30 Hz	10 Mm	124 feV
	3 Hz	100 Mm	12.4 feV

From: Electromagnetic Spectrum
Wikepedia Commons (2013)

Exa: 10^{18}
Peta: 10^{15}
Tera: 10^{12}
Giga: 10^{9}
Mega: 10^{6}
kilo: 10^{3}

milli: 10^{-3}
micro: 10^{-6}
nano: 10^{-9}
pico: 10^{-12}
femto: 10^{-15}
atto: 10^{-18}





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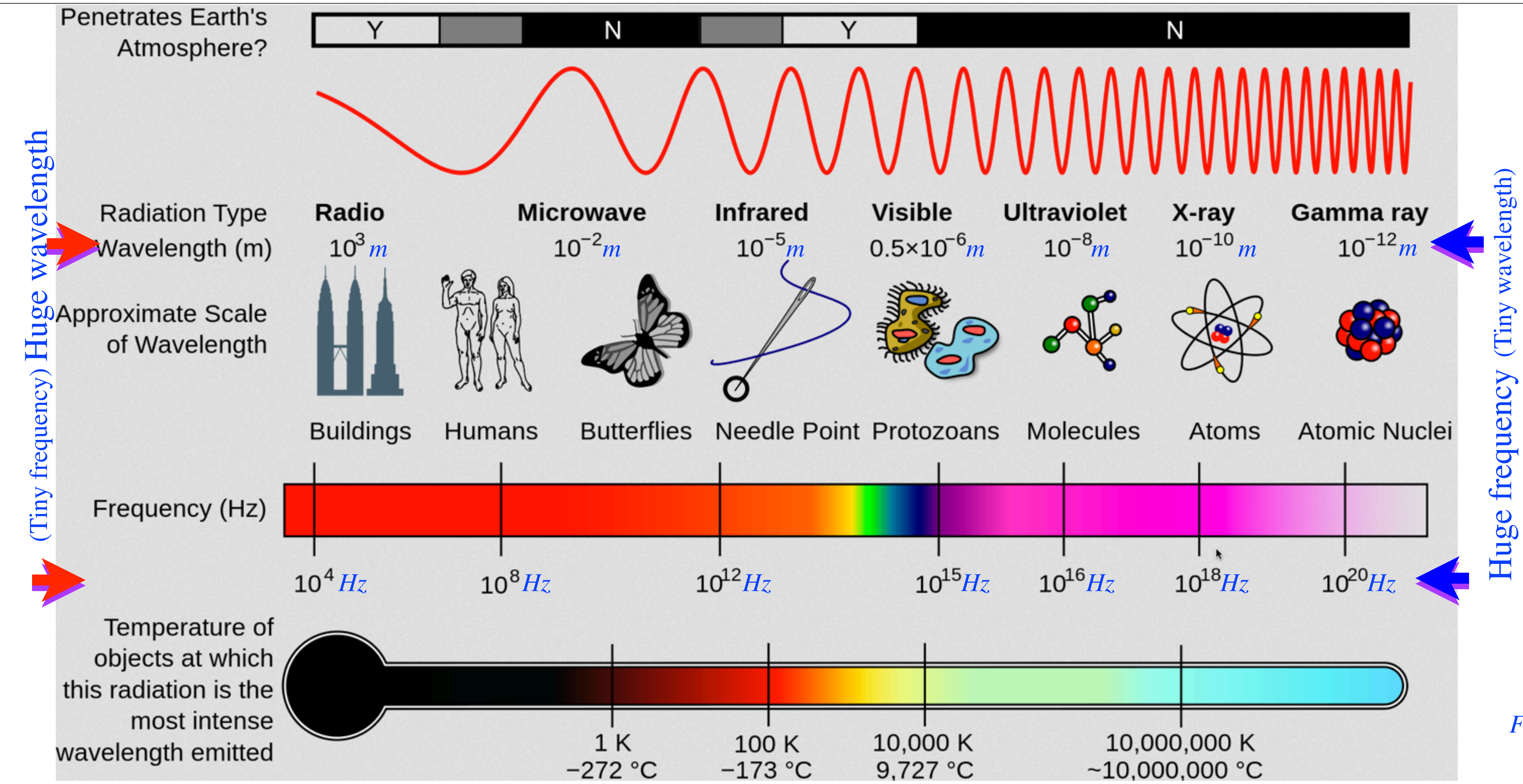

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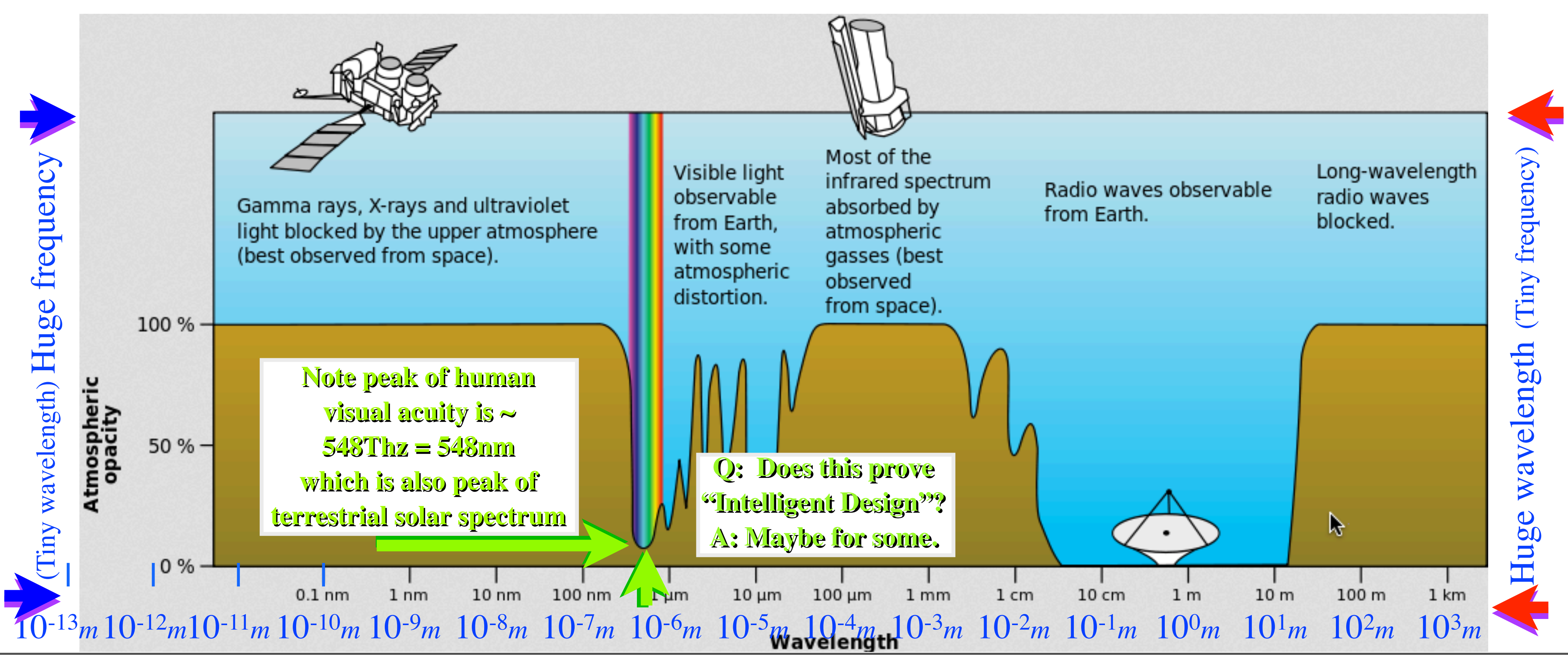
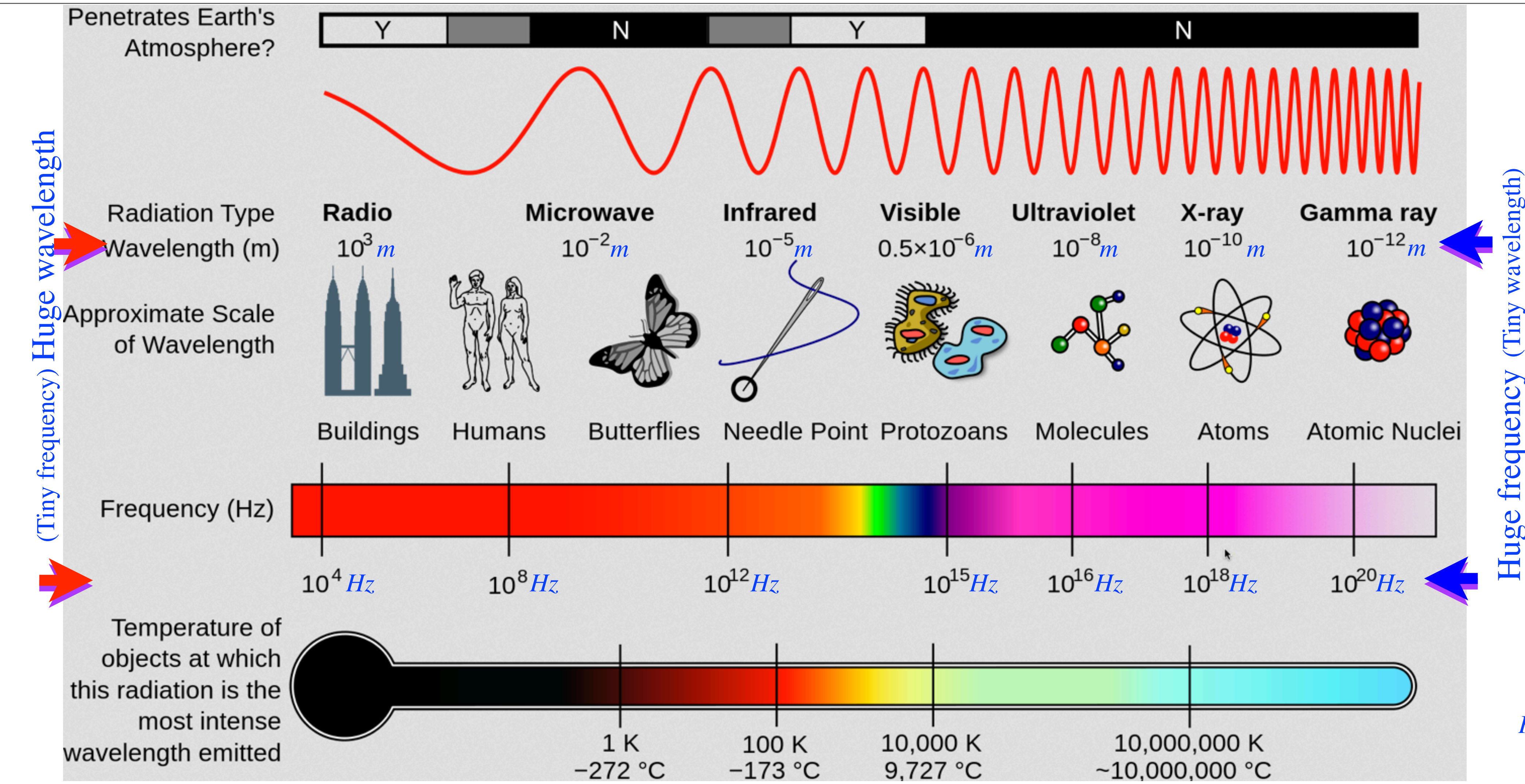


Spectral windows in Earth atmosphere

From: Electromagnetic Spectrum
Wikepedia Commons (2013)



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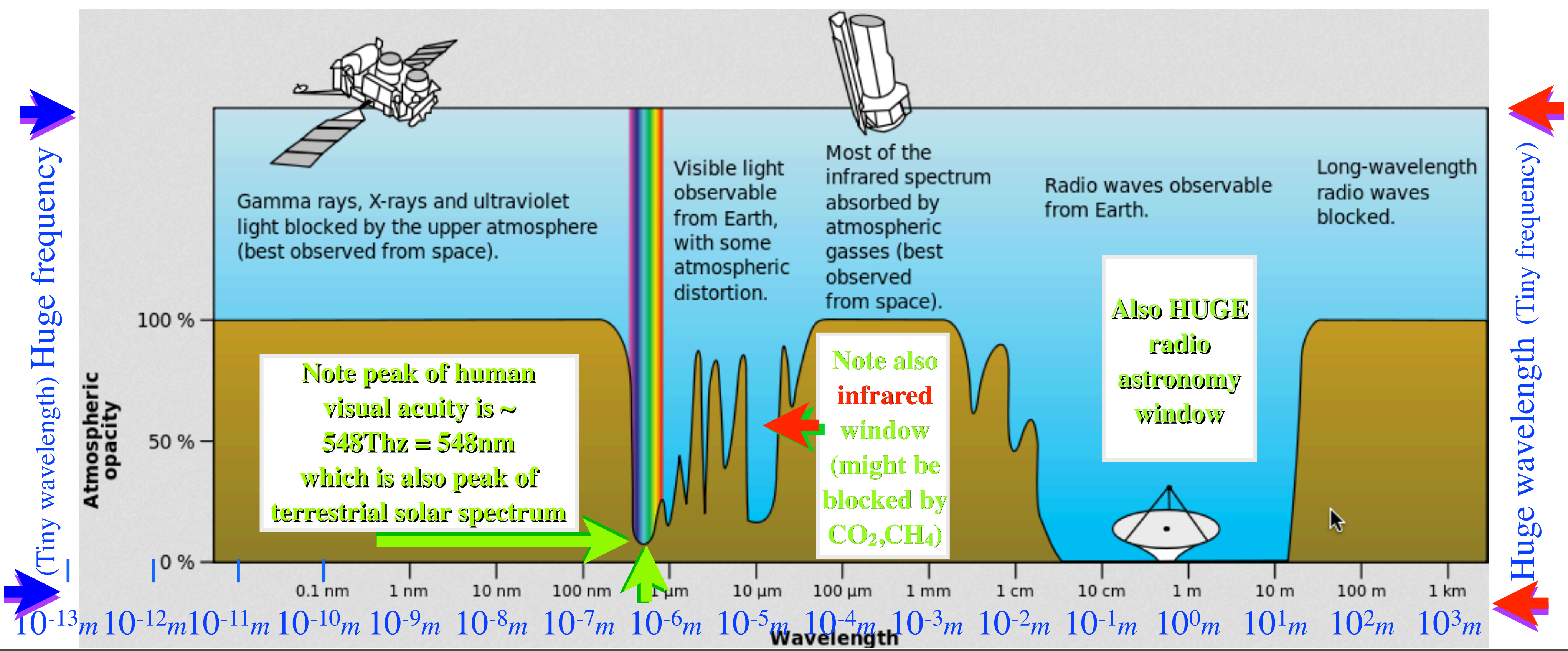

Spectral windows in Earth atmosphere

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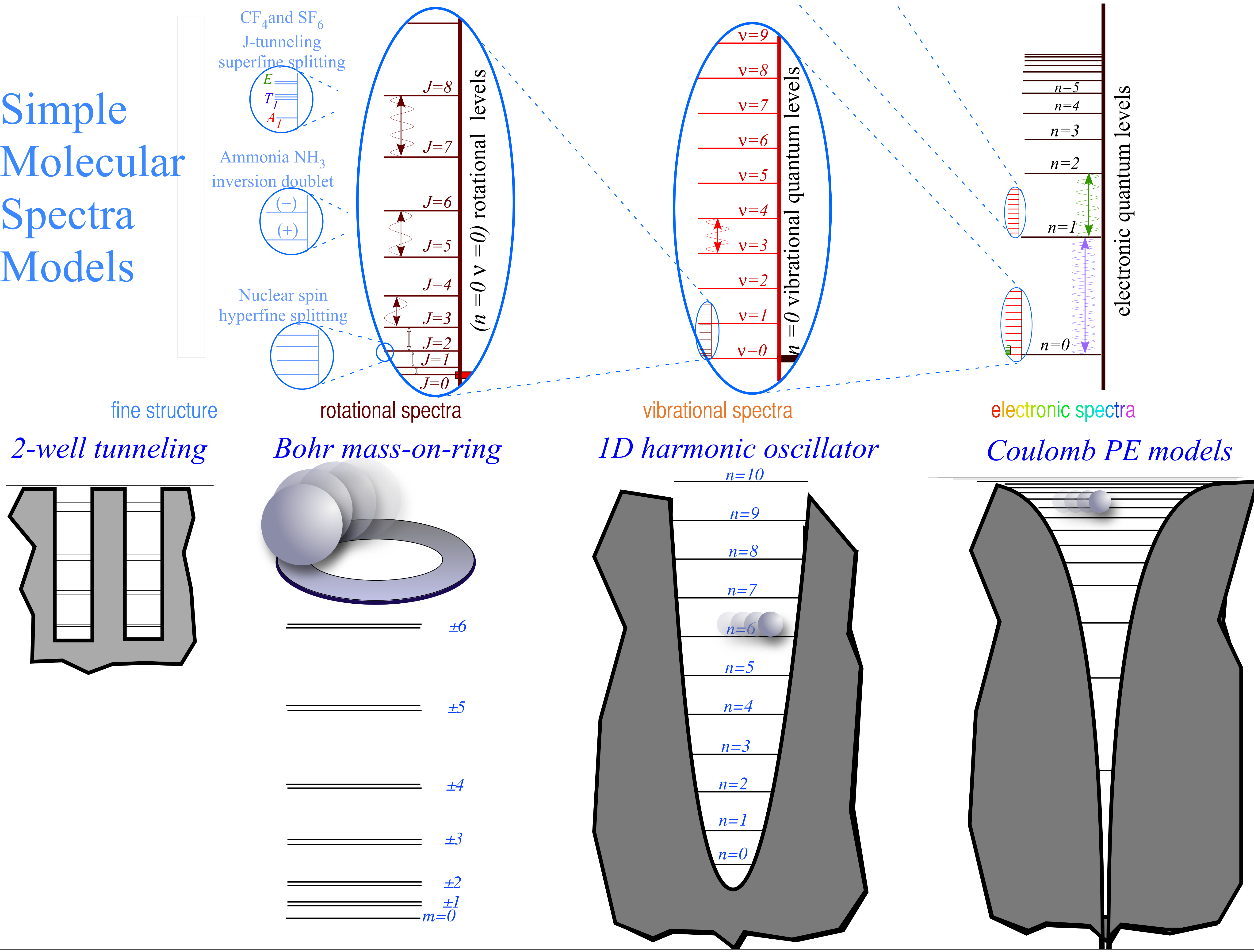
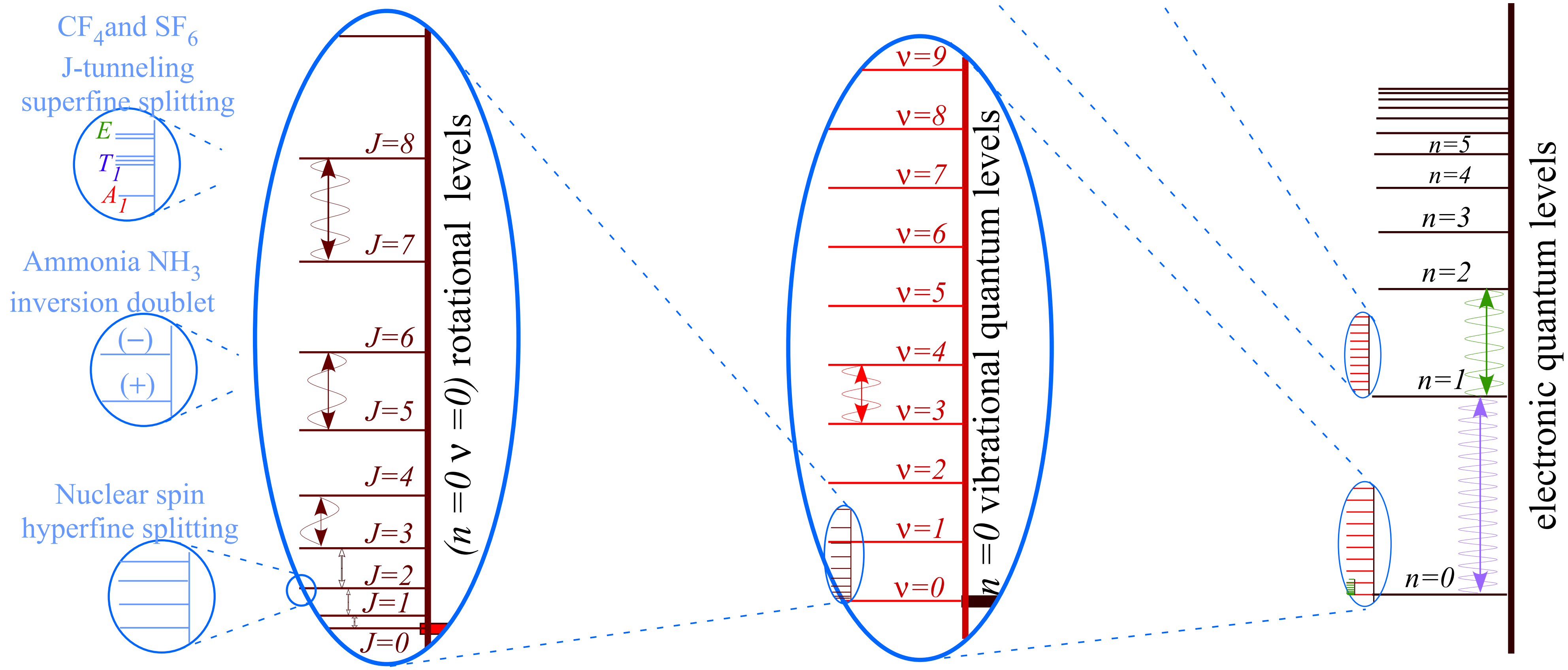
Simple Molecular Spectra Models



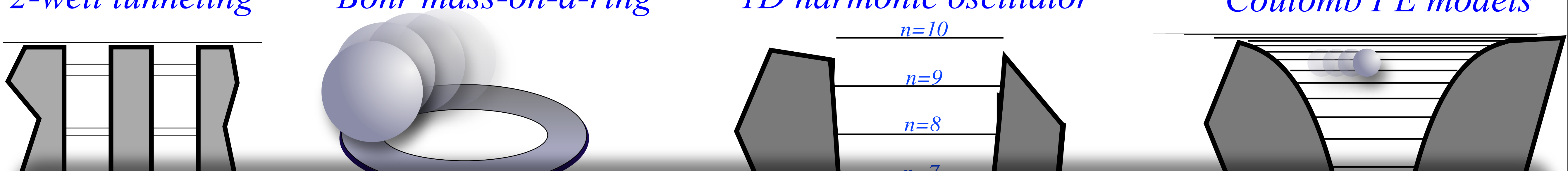
More Advanced Molecular Spectra Models

(Use symmetry group theory)



fine structure | rotational spectra | vibrational spectra | electronic spectra

2-well tunneling | *Bohr mass-on-a-ring* | *1D harmonic oscillator* | *Coulomb PE models*

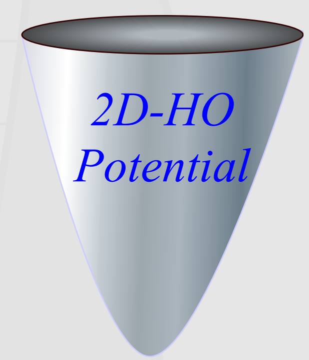


2-state U(2)-spin and quasi-spin tunneling models

3D R(3)-rotor and D-function lab-body wave models

2D harmonic oscillator and U(2) 2nd quantization

*U(m)*Sn analysis of multi-electron states*



Rotational Energy Surface (RES) analysis of rovibronic tensor spectra

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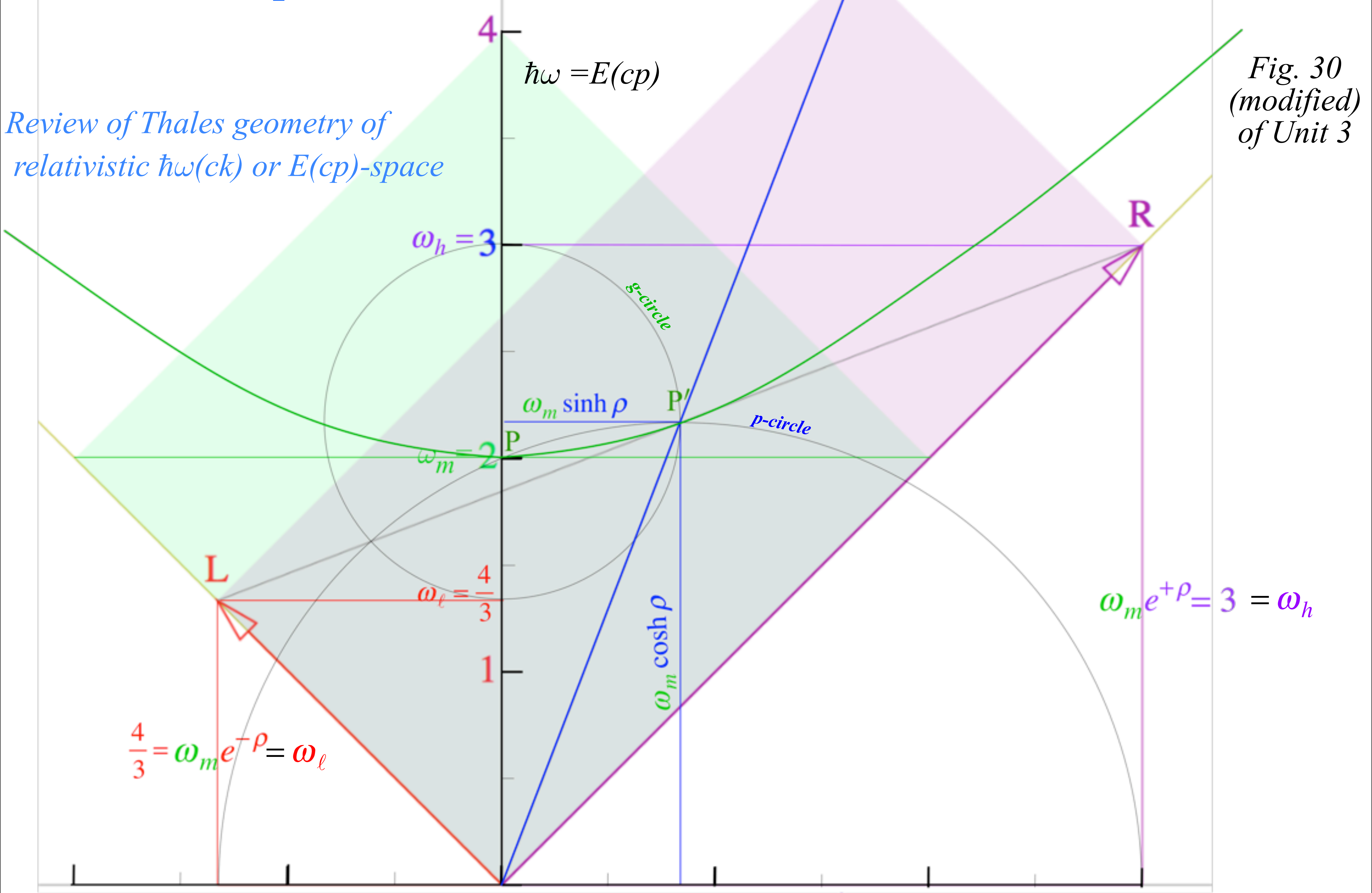

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

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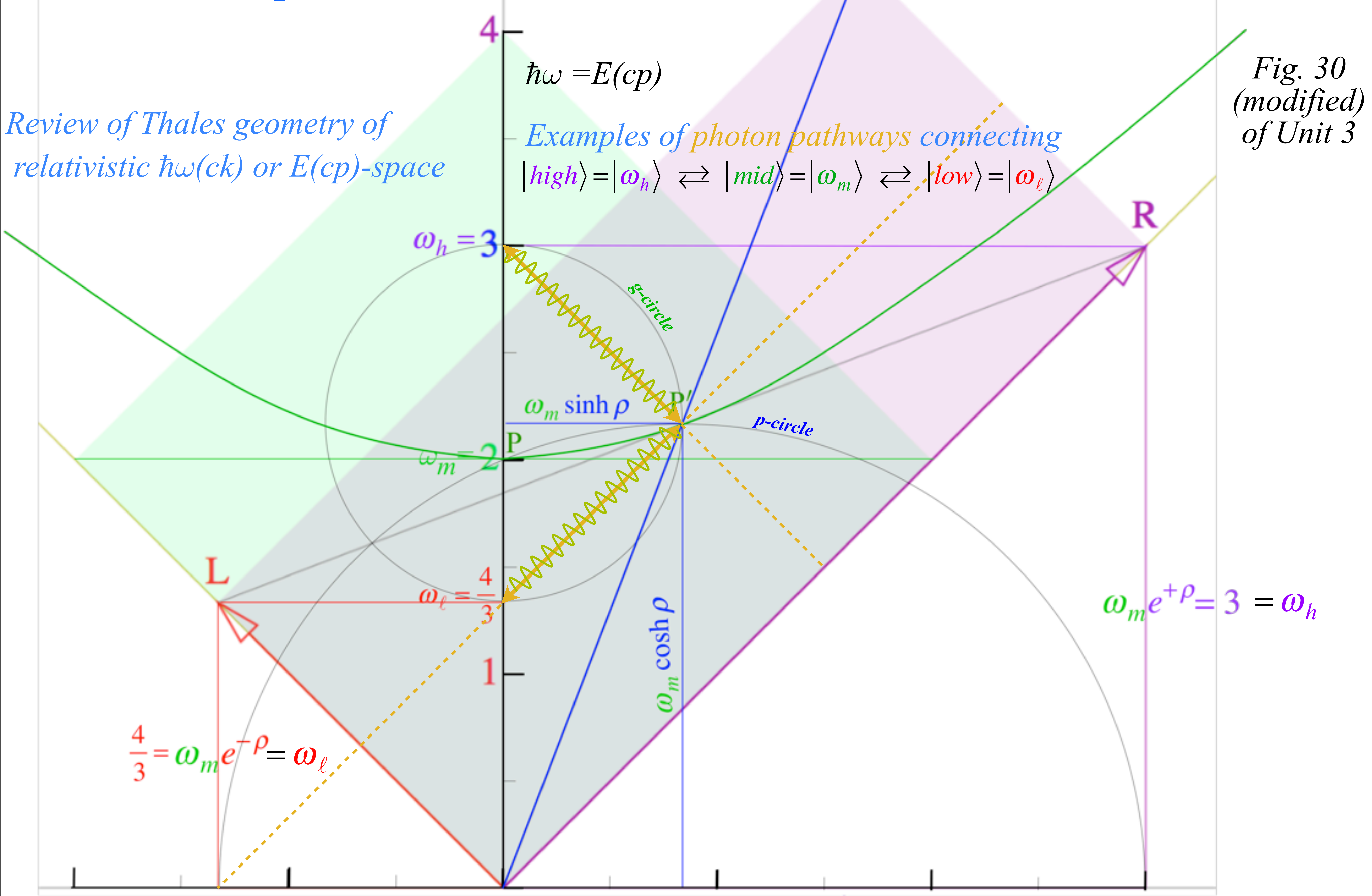

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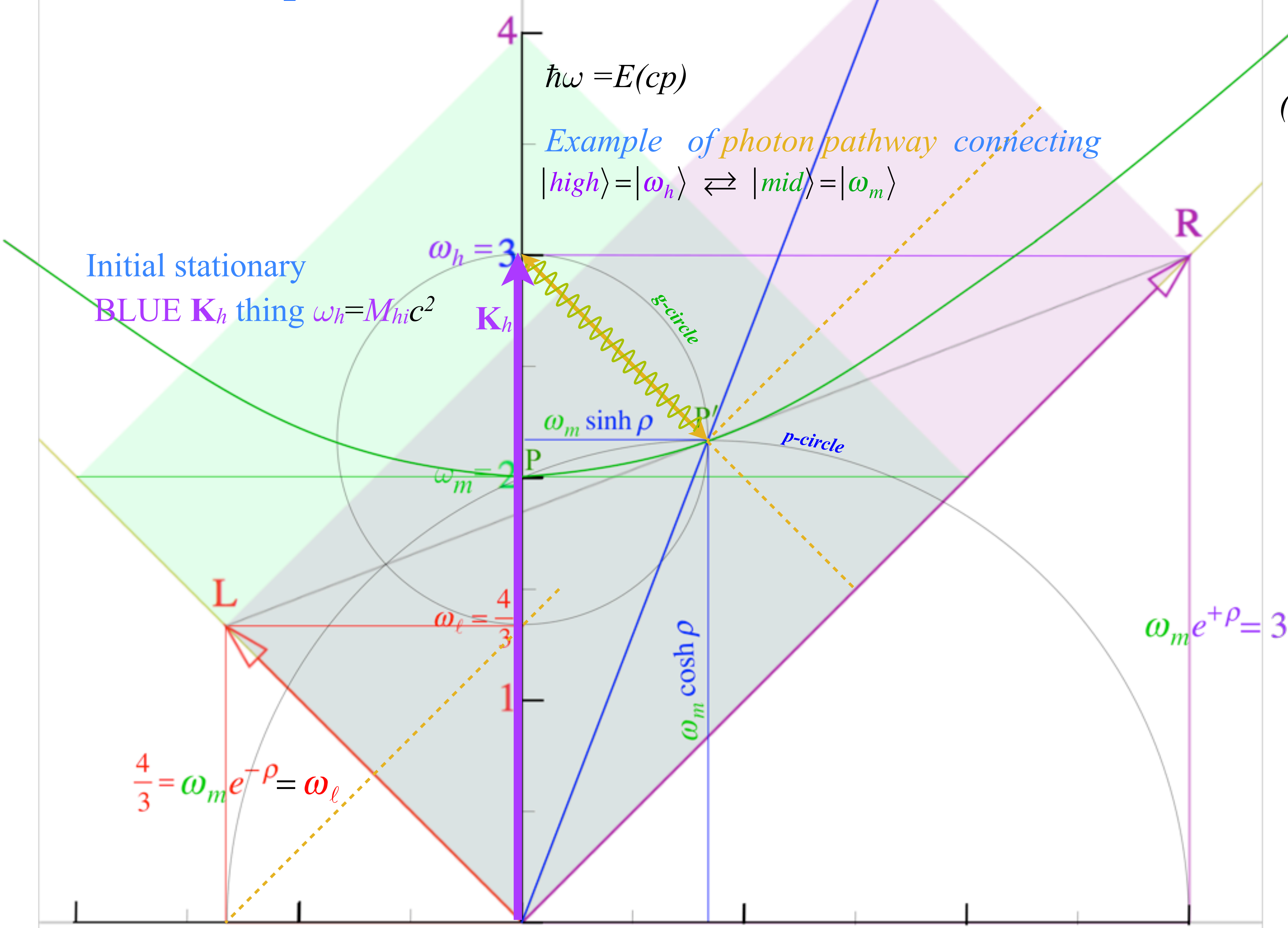

Fig. 30 (modified) of Unit 3

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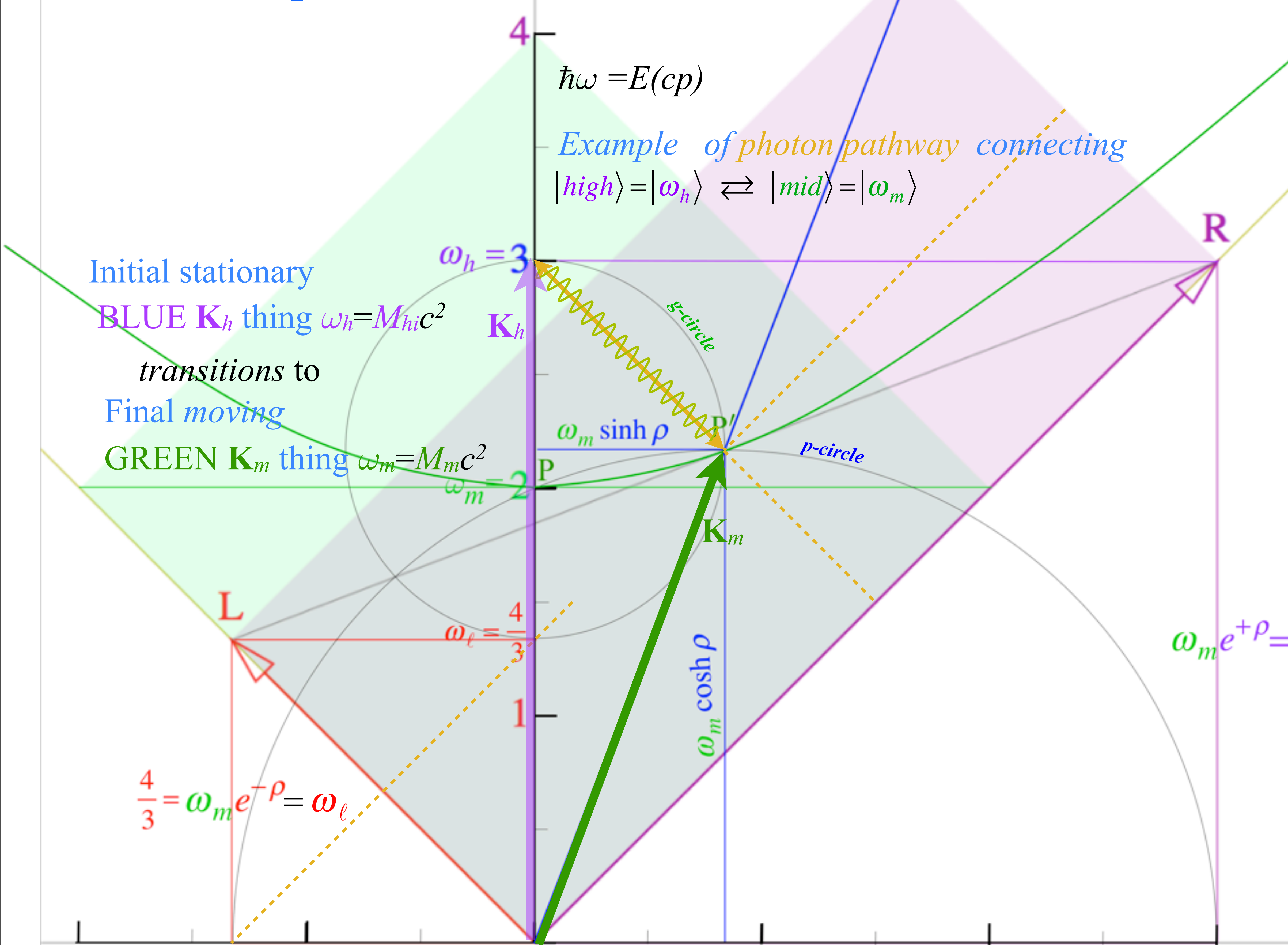

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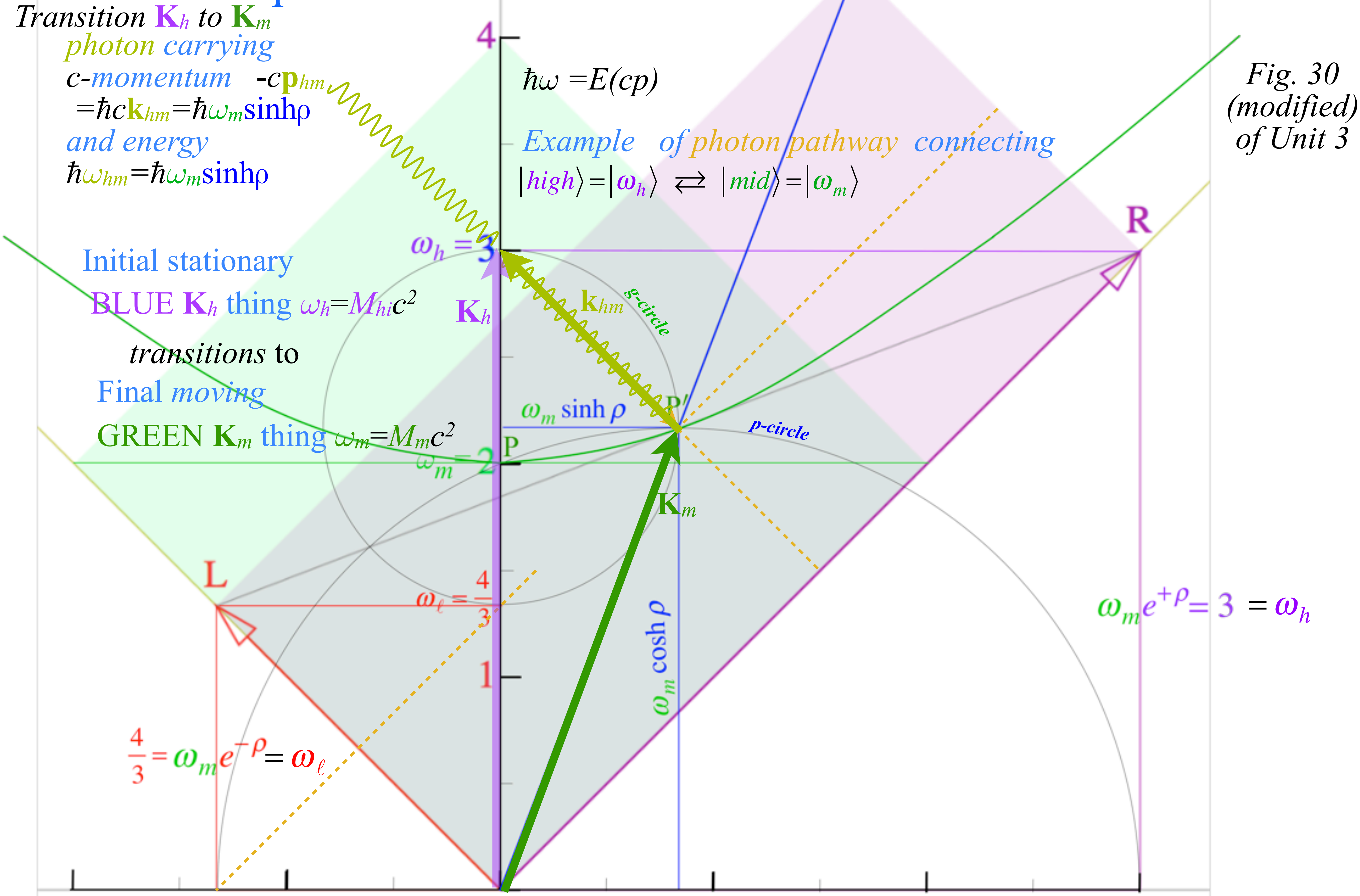

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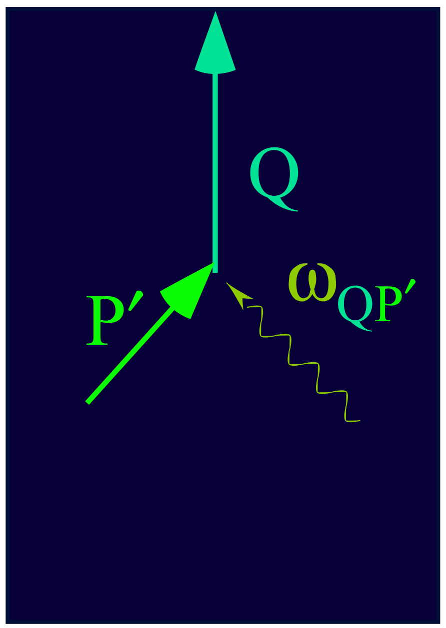
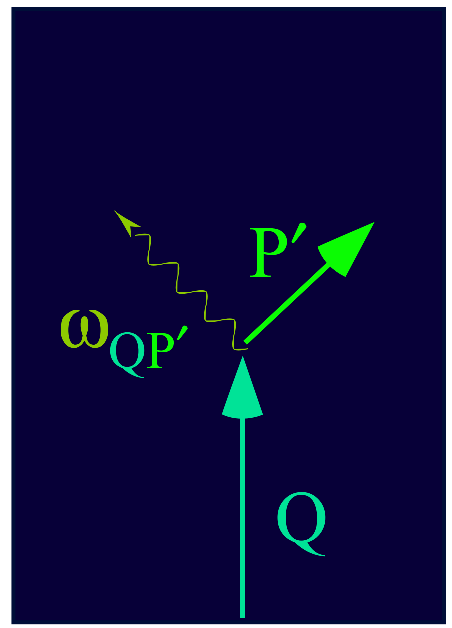
Fundamental light-matter processes:

Absorption A *Emission E* *AE Together*

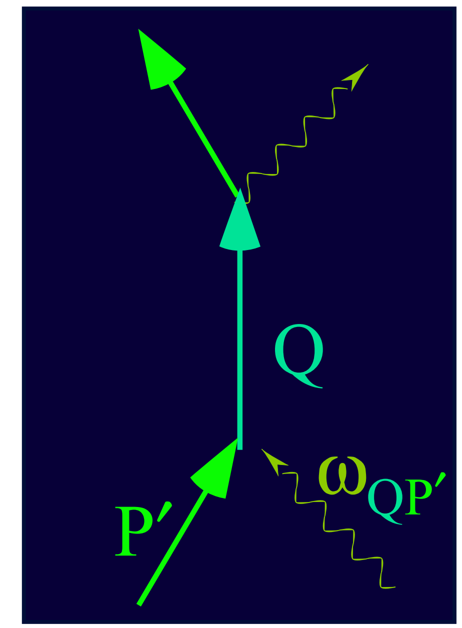
(Compton Scattering)



1-photon processes



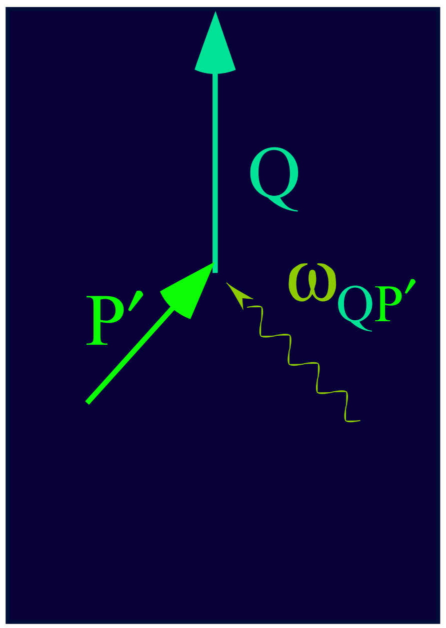
2-photon process



Modified from Mod. Phys. Lect. 33

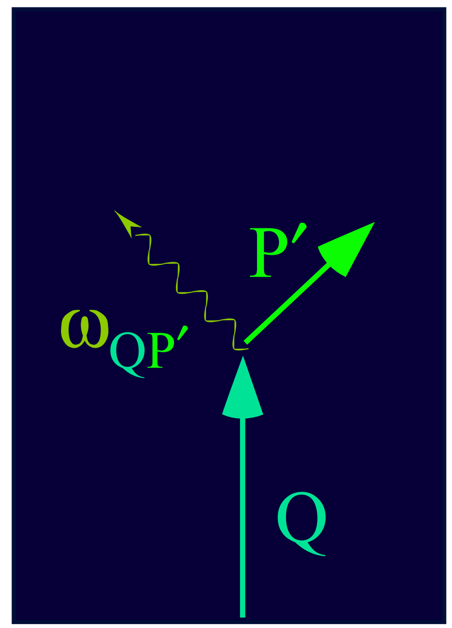
Fundamental light-matter processes:

Absorption A

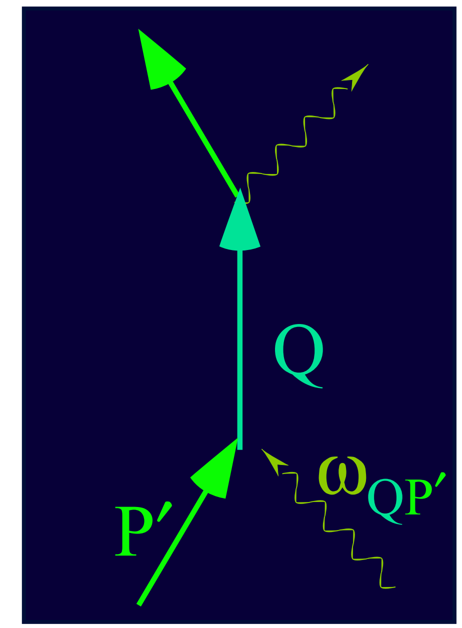


1-photon processes

Emission E

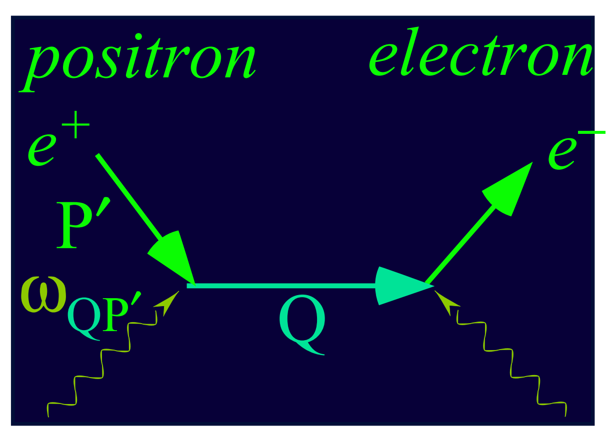


AE Together
(Compton Scattering)

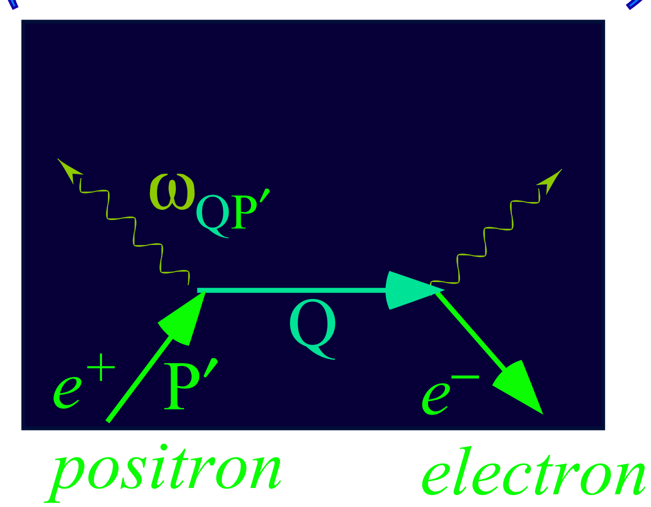


2-photon processes

"Exotic" processes: AA Together
(Pair-Creation)

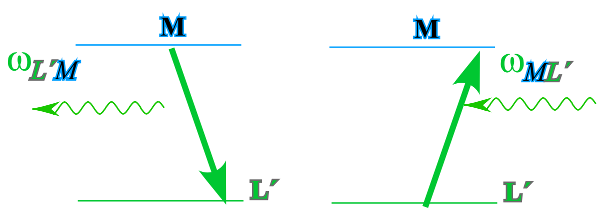


EE Together
(Pair-Annhilation)

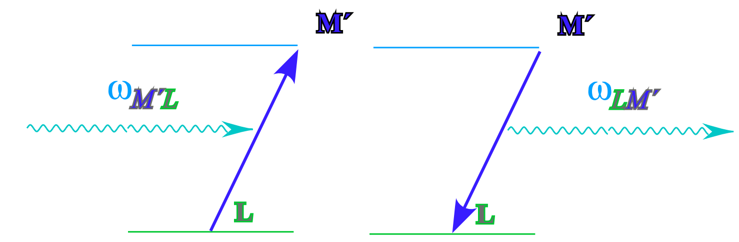


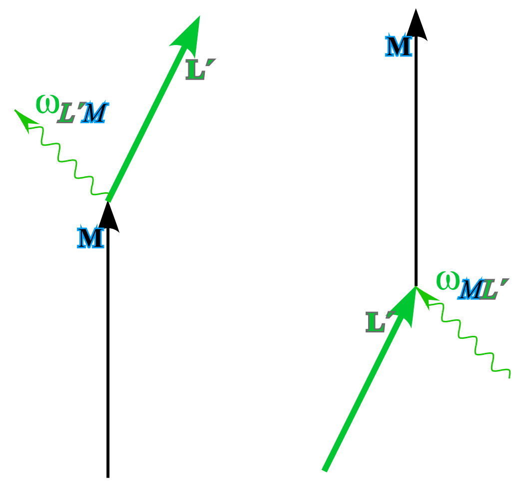
Modified from Mod. Phys. Lect. 33

Wave geometry of 1-photon transitions and Compton recoil



Grotian 2-level diagrams

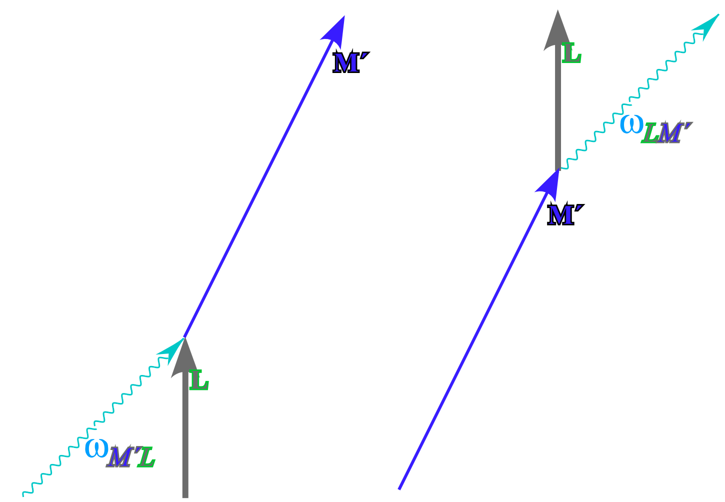




M-to-L′ emission

L′-to-M absorption

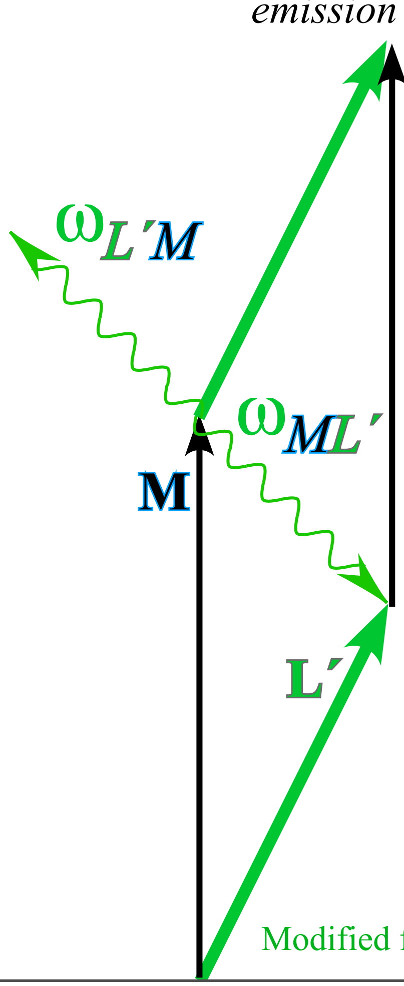
Feynman (ω,ck) diagrams (1-photon)



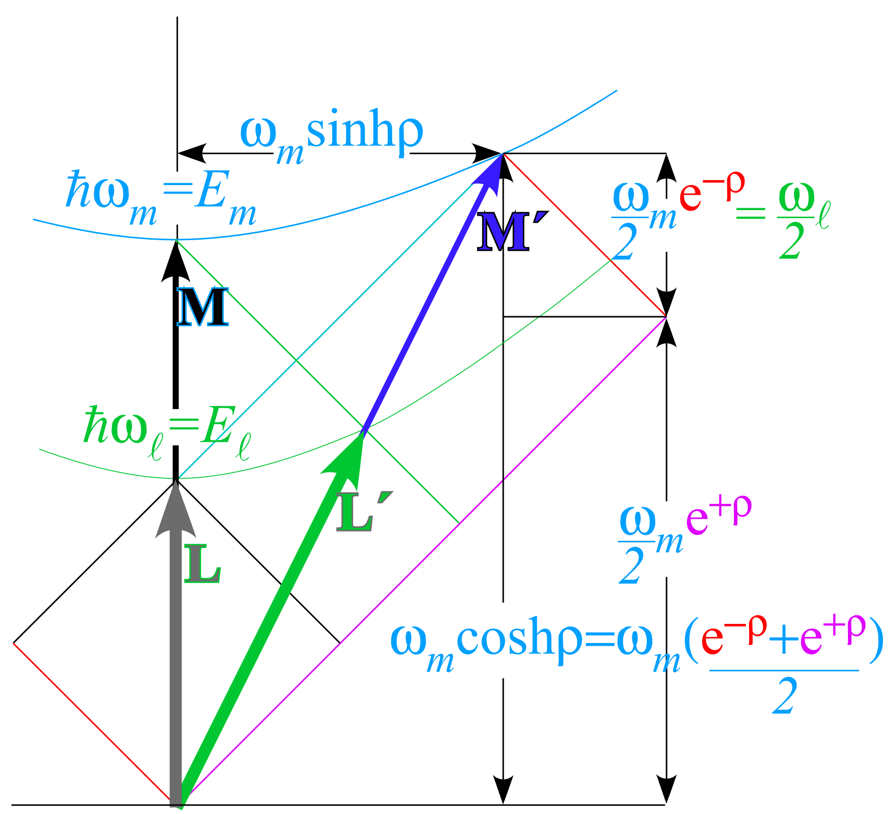
L-to-M′ absorption

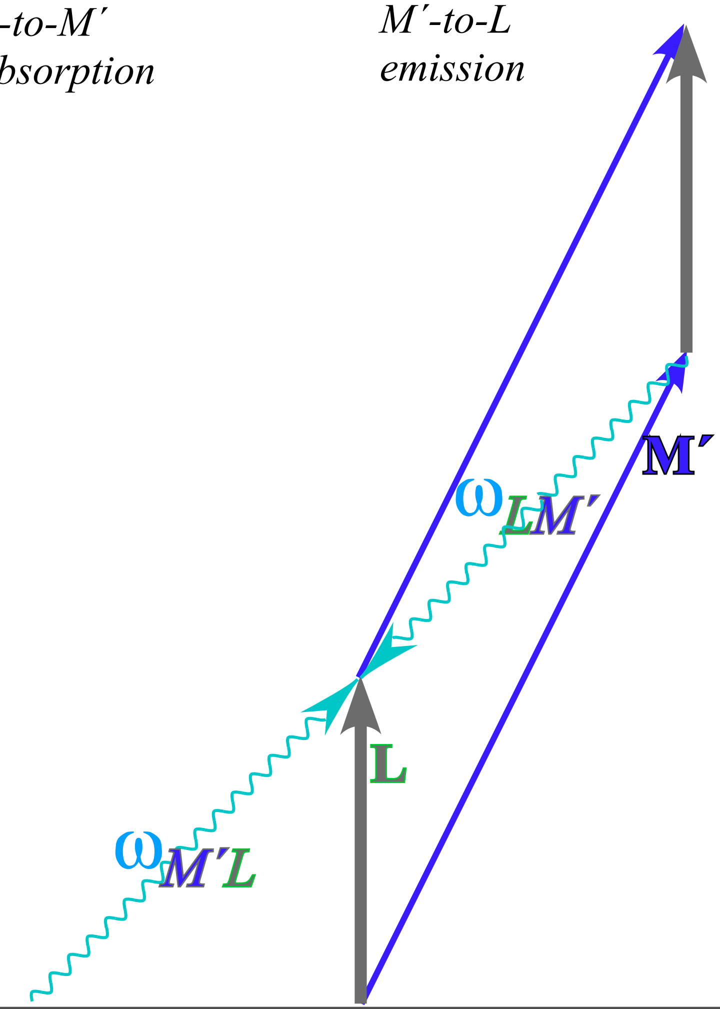
M′-to-L emission

2-Level (ω,ck) "baseball" diamonds



(ω,ck) vector sum (energy-momentum conservation)





Modified from Mod. Phys. Lect. 33

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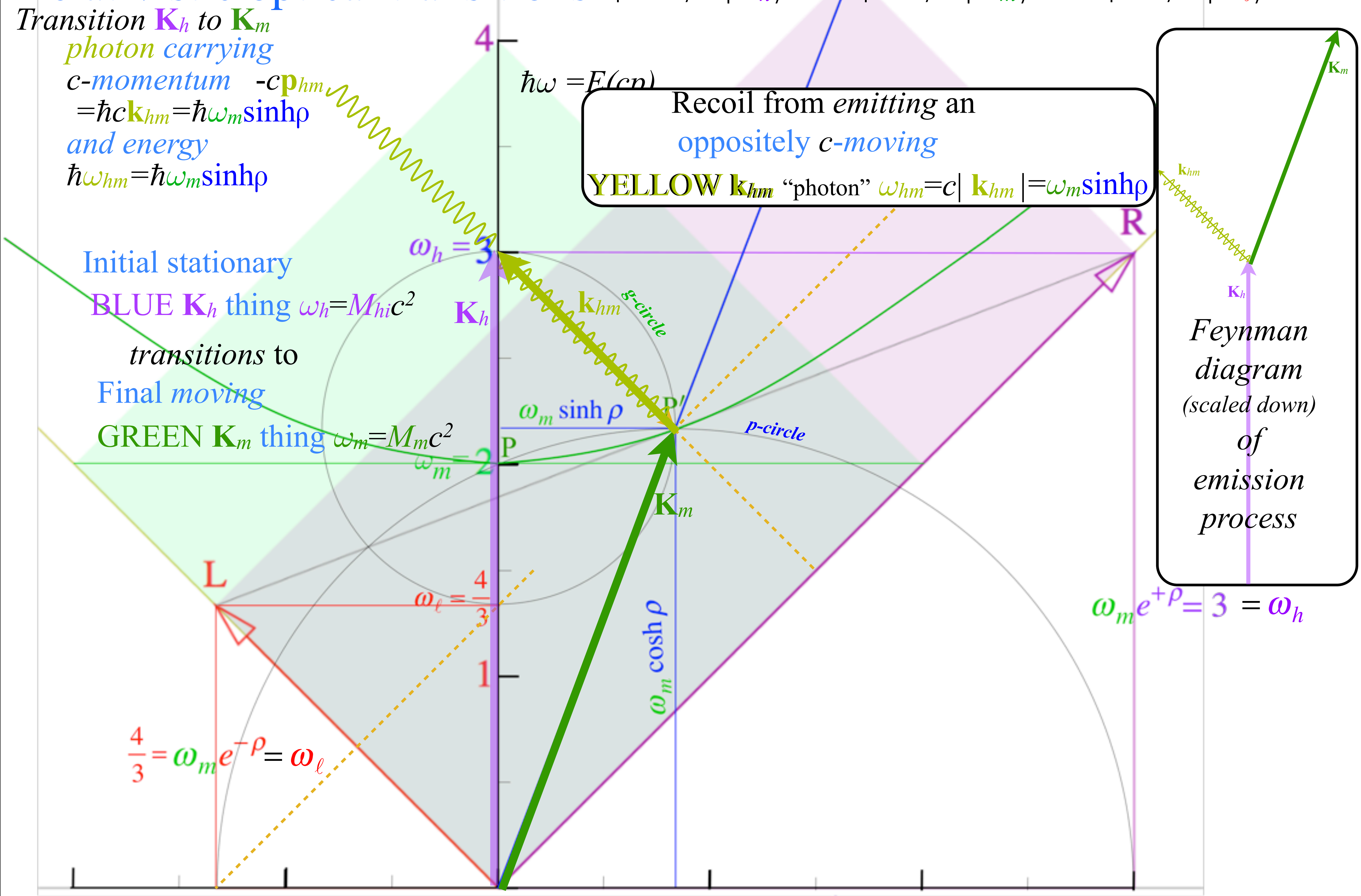

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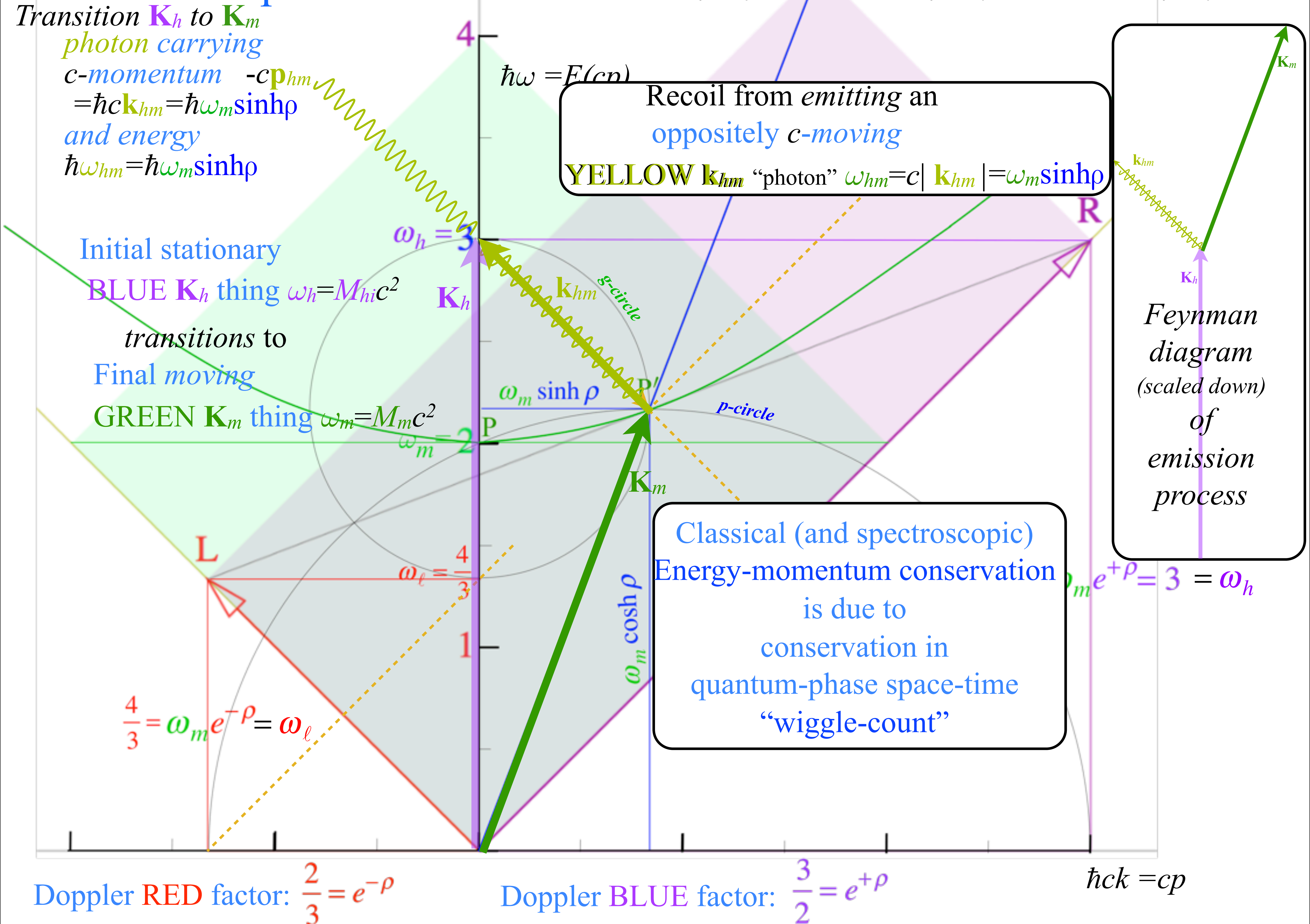


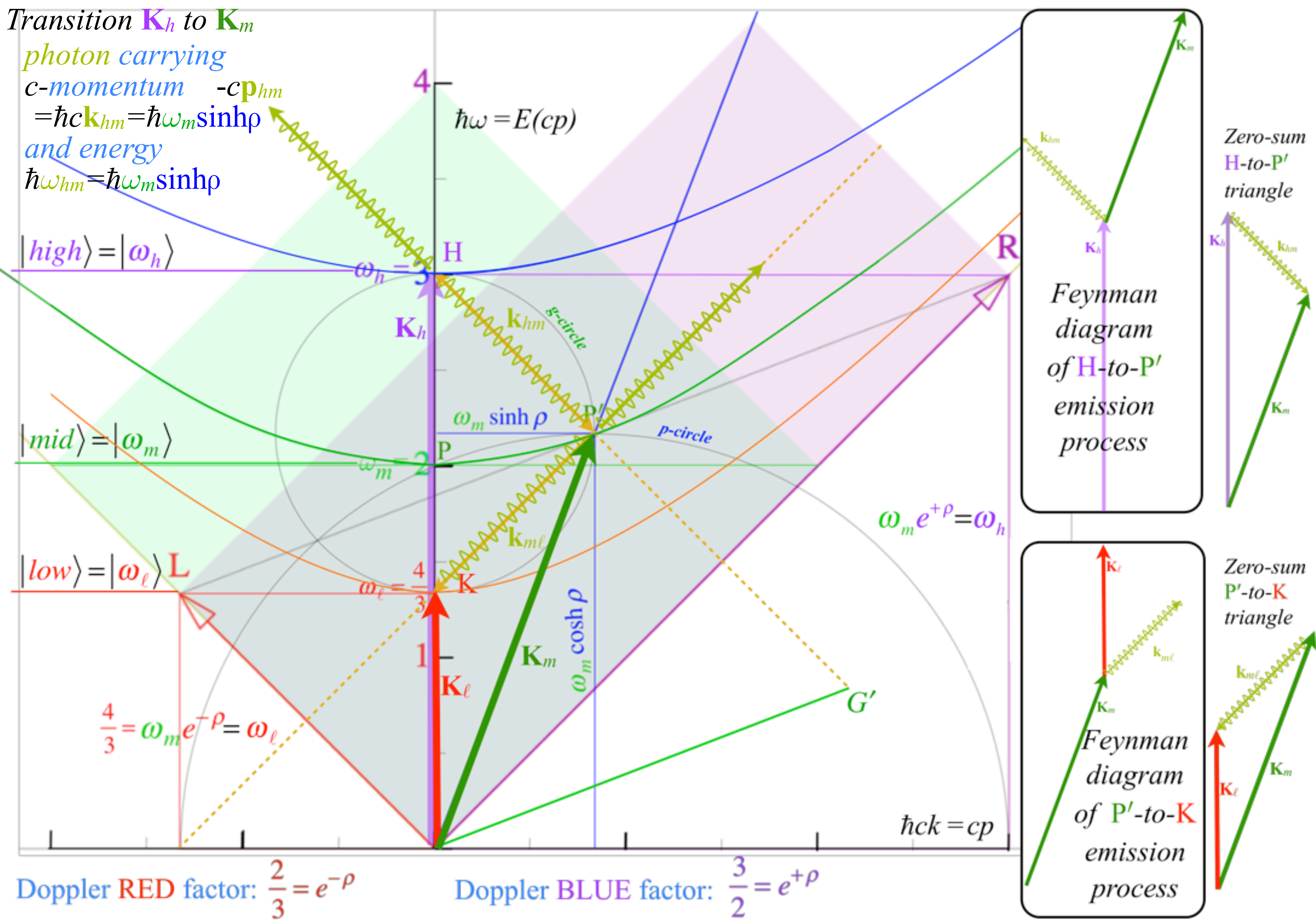
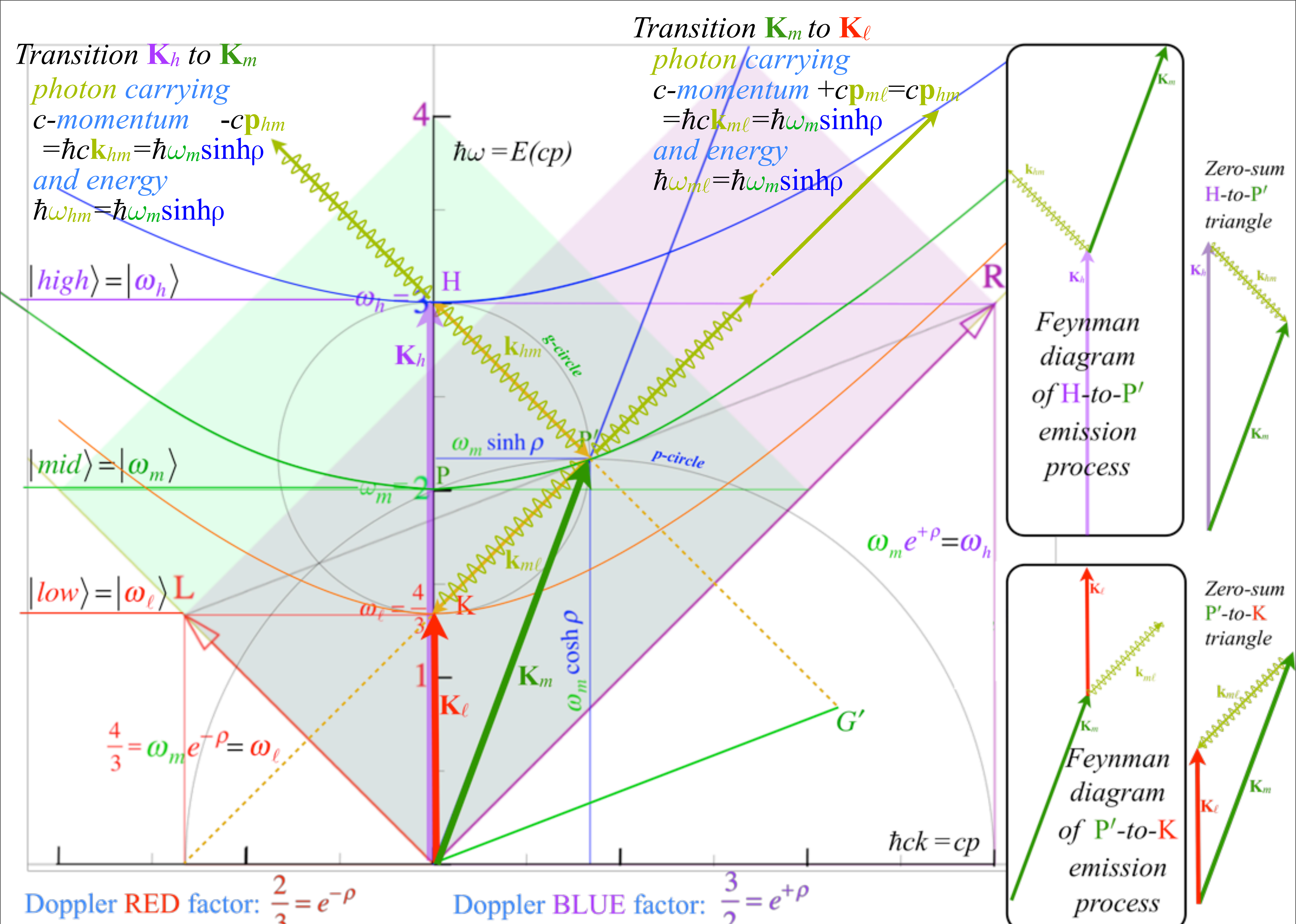


Fig. 31 (modified) of Unit 3



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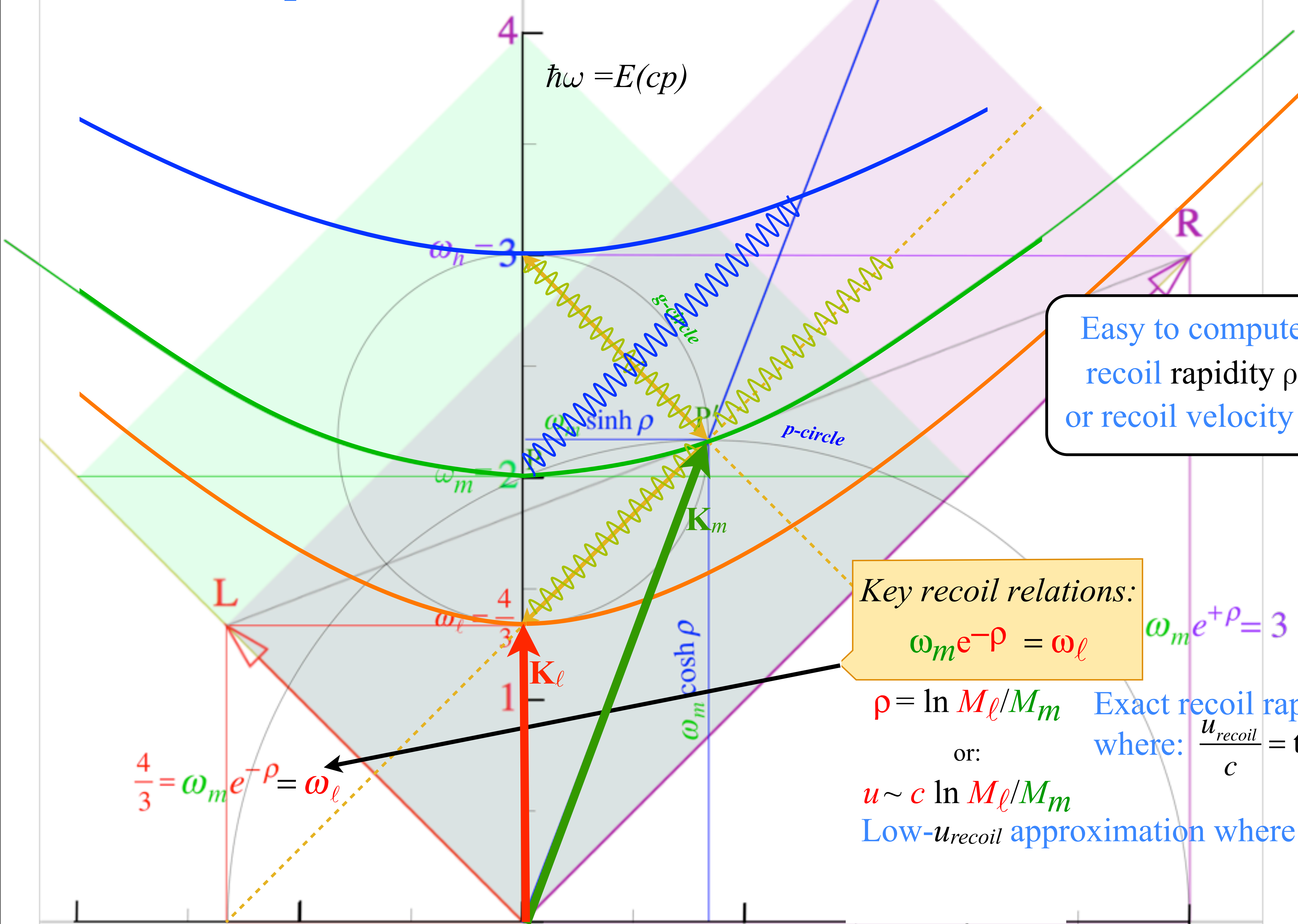

Easy to compute recoil rapidity ρ or recoil velocity u

Key recoil relations:

$\omega_m e^{-\rho} = \omega_\ell$

$\rho = \ln M_\ell / M_m$

or:

$u \sim c \ln M_\ell / M_m$

Exact recoil rapidity where: $\frac{u_{recoil}}{c} = \tanh \rho$

Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

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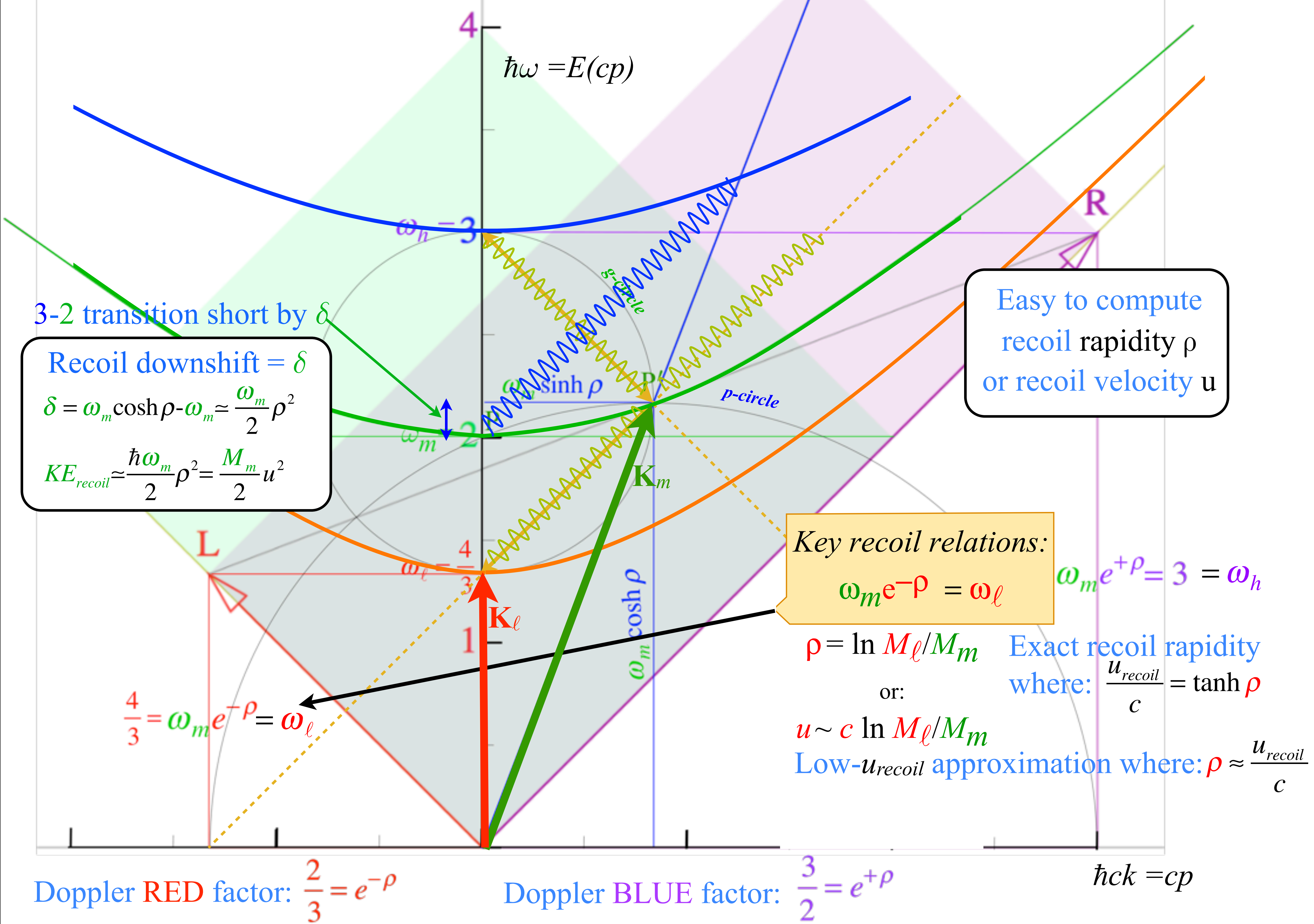

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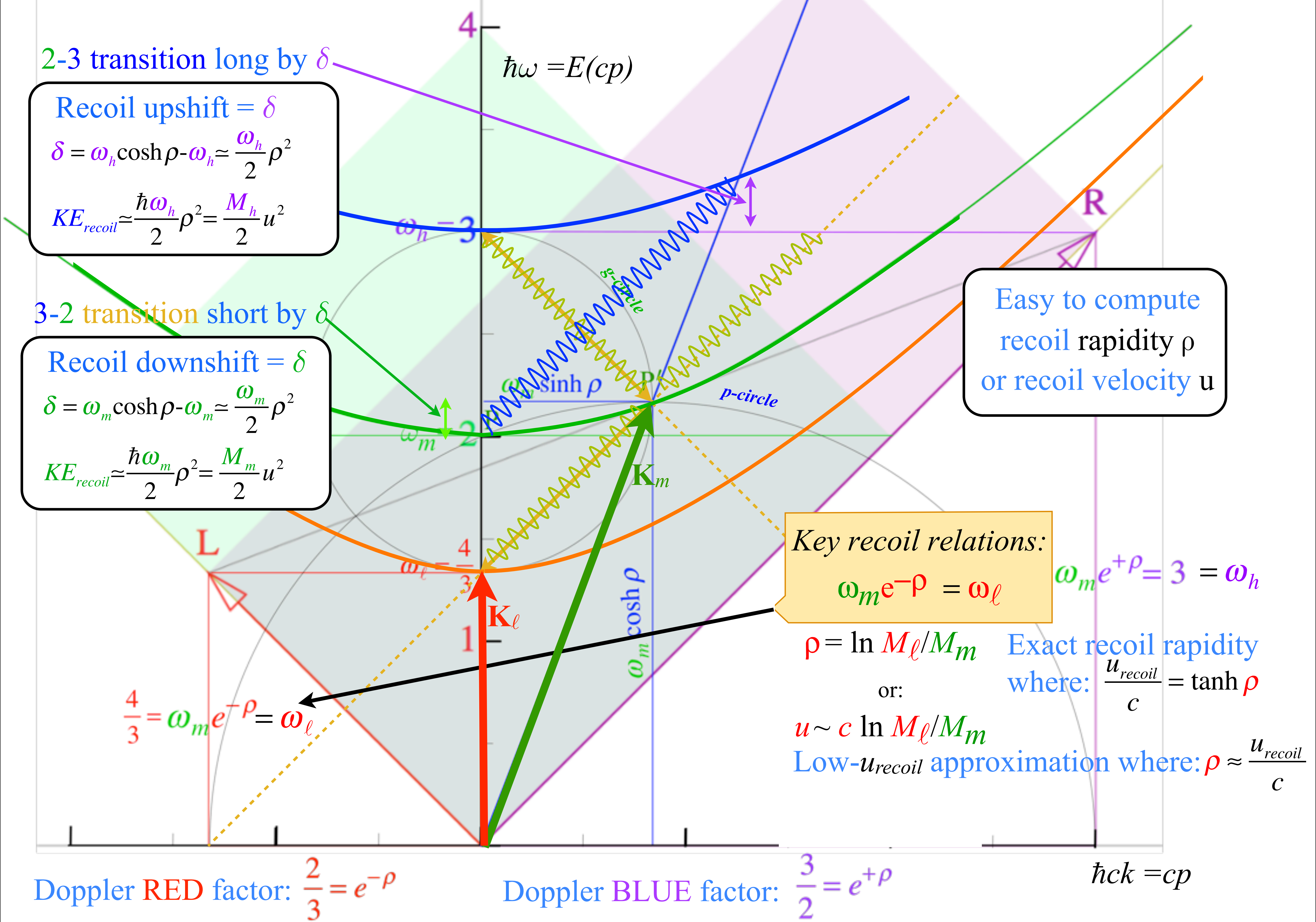

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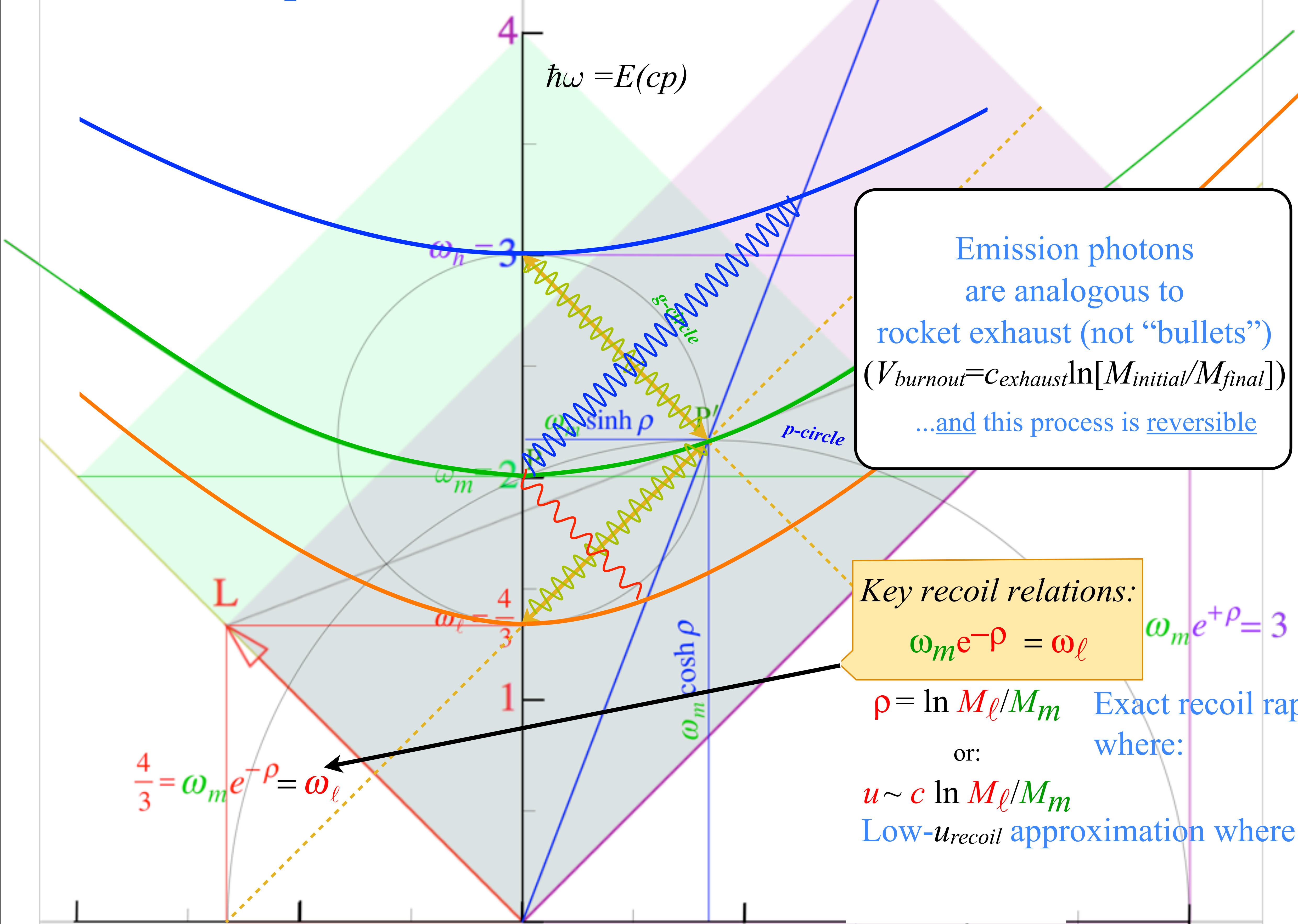


Emission photons
are analogous to
rocket exhaust (not "bullets")
($V_{burnout} = c_{exhaust} \ln[M_{initial}/M_{final}]$)
...and this process is reversible

Key recoil relations:

$\omega_m e^{-\rho} = \omega_\ell$

$\rho = \ln M_\ell / M_m$ Exact recoil rapidity where:

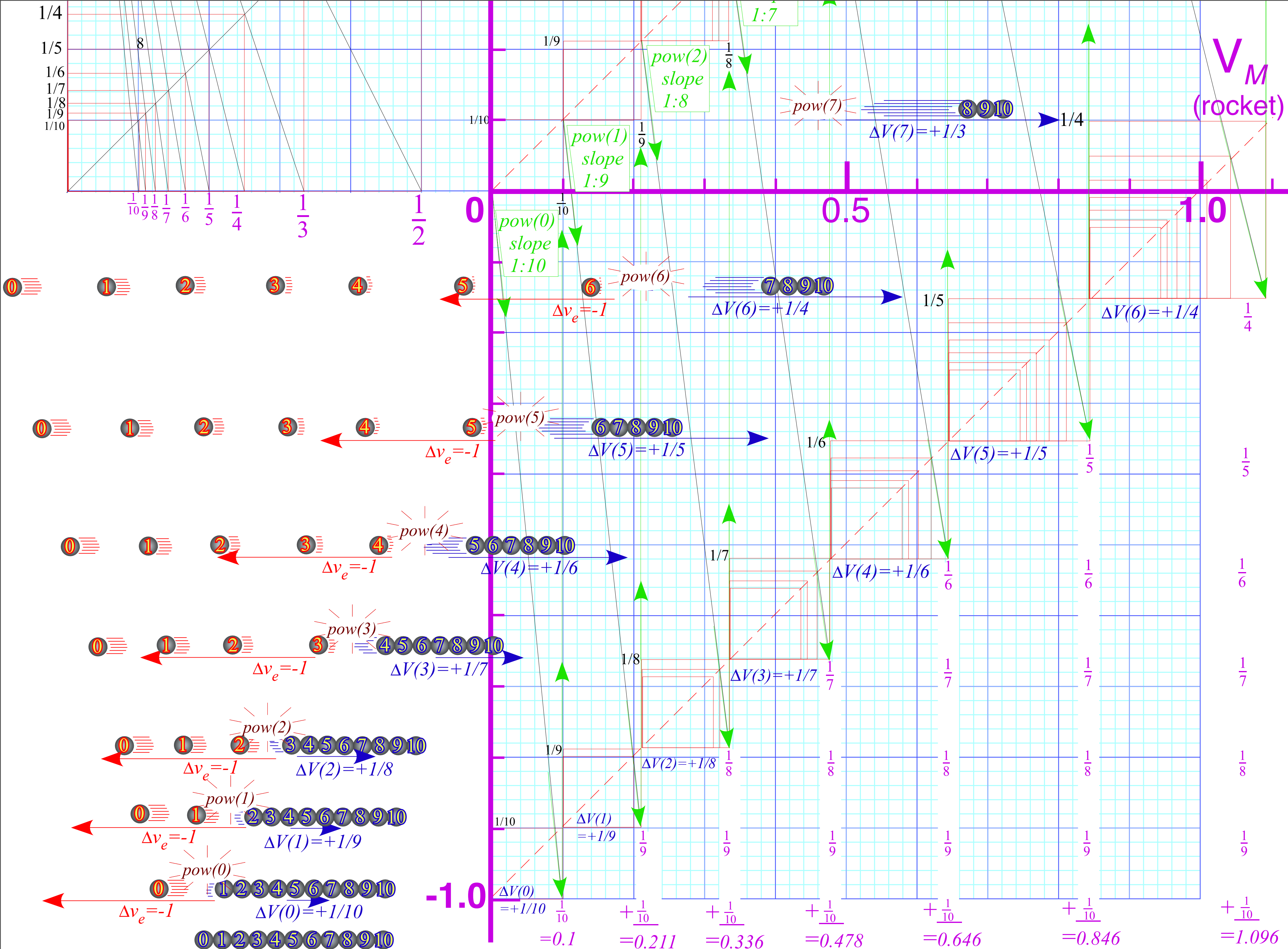
or:

$u \sim c \ln M_\ell / M_m$

Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$



Unit 1
Fig. 7.8a-b

[Rocket Science! from Lecture 10 p.85]

0^{th}: $V(0)=1/10=0.1$ 1^{st}: $V(1)=1/10+1/9=0.211$ 2^{nd}: $V(2)=1/10+1/9+1/8=0.336$

3^{rd}: $V(3)=V(2)+1/7=0.478$ 4^{th}: $V(4)=V(3)+1/6=0.646$ 5^{th}: $V(5)=V(4)+1/5=0.846$

6^{th}: $V(6)=V(5)+1/4=1.096$ 7^{th}: $V(7)= V(6)+1/3=1.429$ 8^{th}: $V(8)=V(7)+1/2=1.929$

v_e known as "Specific Impulse"

By calculus: $M\cdot\Delta V=-v_e\cdot\Delta M$ or: $dV=-v_e\dfrac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV=-v_e\int_{M_{IN}}^{M_{FIN}}\dfrac{dM}{M}$

The Rocket Equation: $V_{FIN}-V_{IN}=-v_e\left[\ln M_{FIN}-\ln M_{IN}\right]=v_e\left[\ln\dfrac{M_{IN}}{M_{FIN}}\right]$

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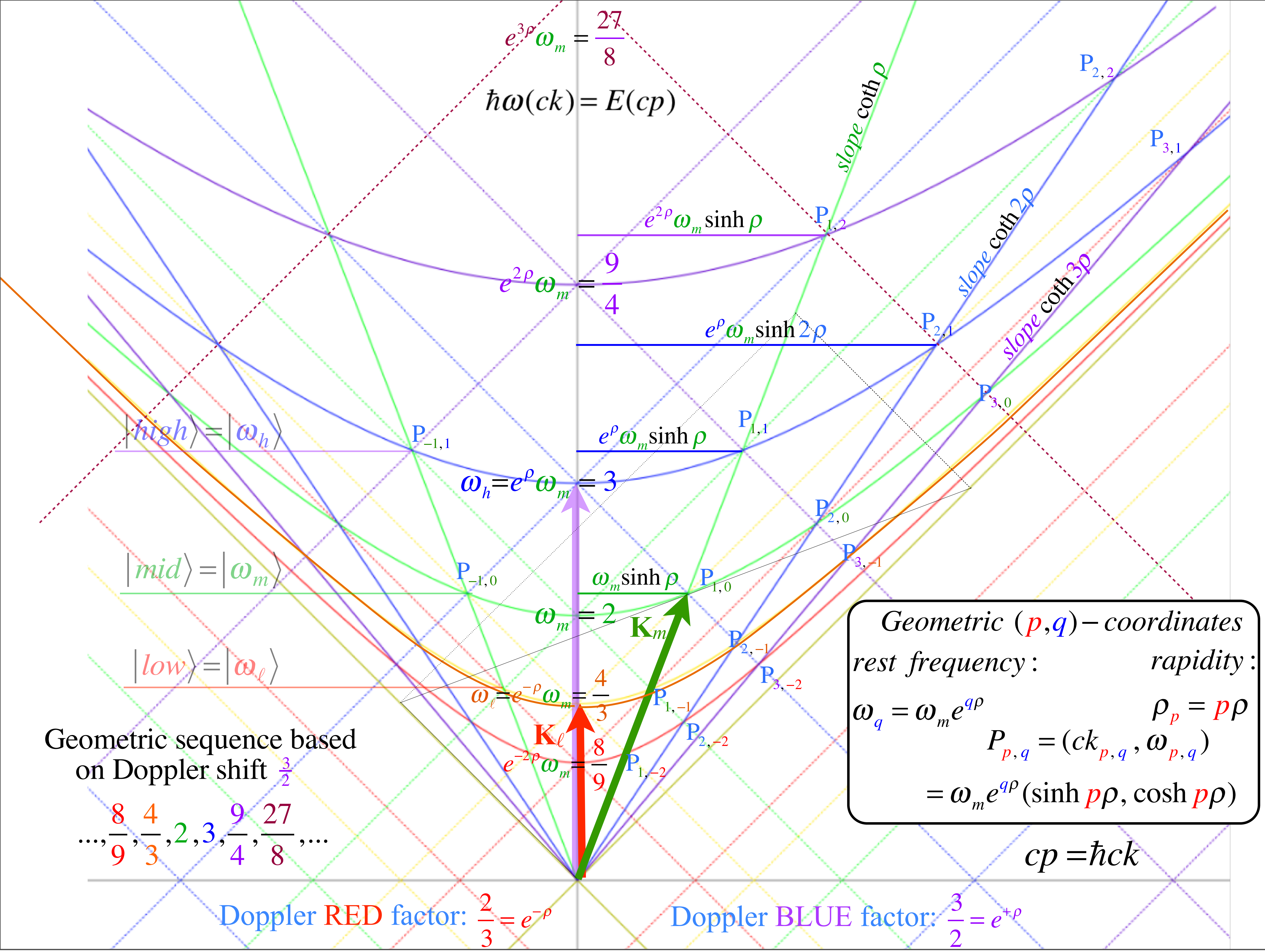
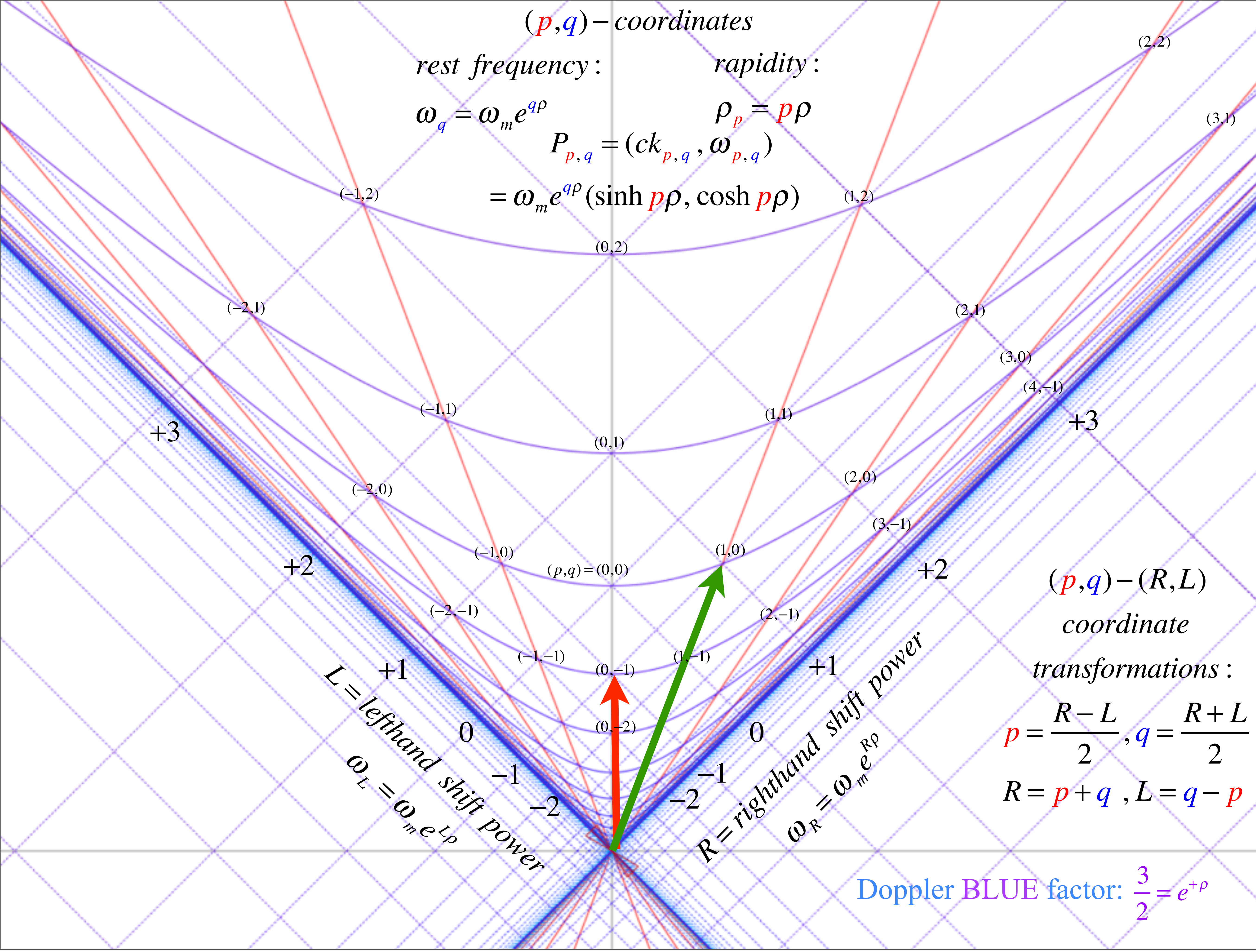
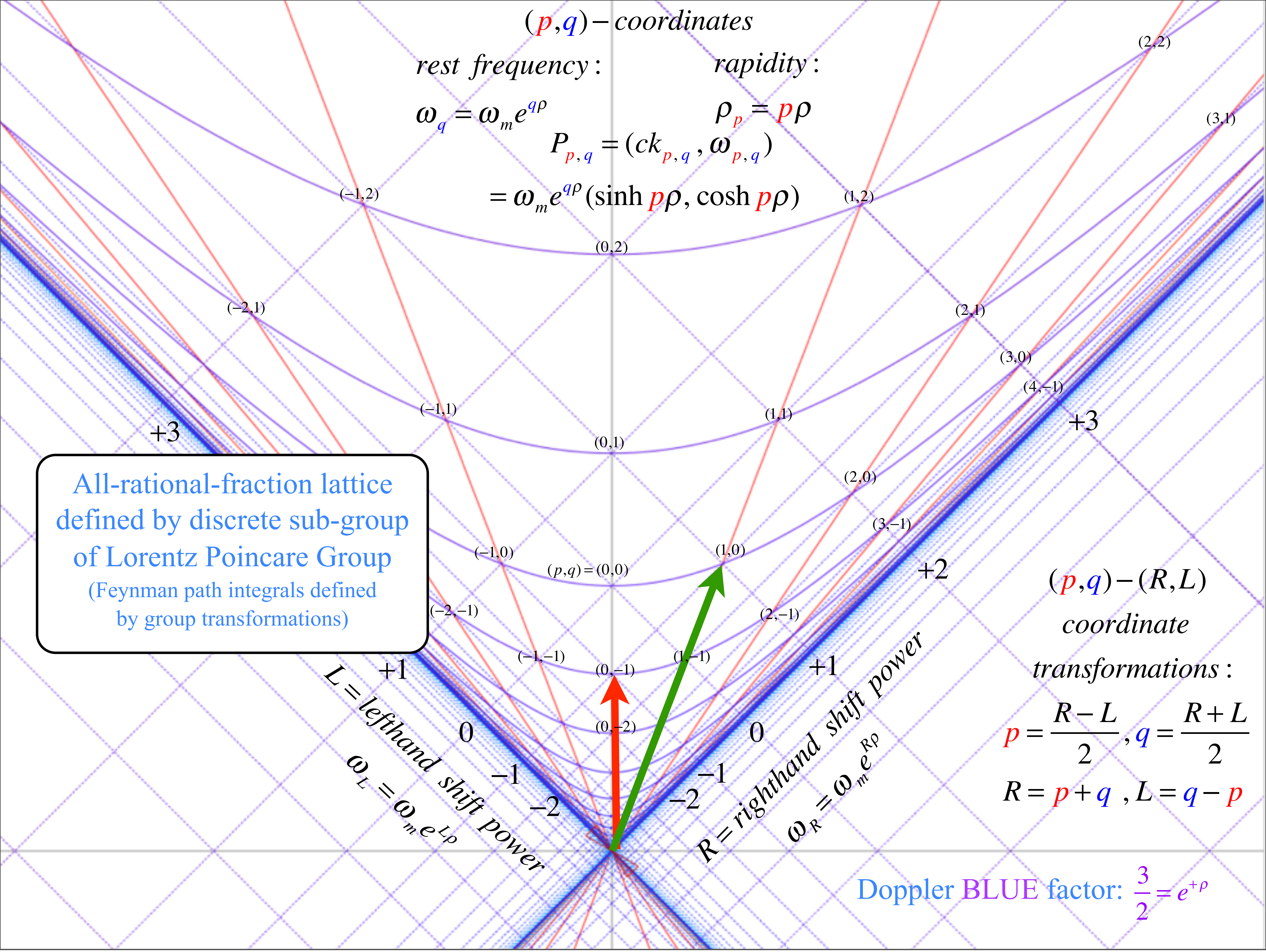


e^{3ρ}ω_m = 27/8
ħω(ck) = E(cp)
slope coth ρ
slope coth 2ρ
slope coth 3ρ
e^{2ρ}ω_m sinh ρ
e^{2ρ}ω_m = 9/4
e^{ρ}ω_m sinh 2ρ
e^{ρ}ω_m sinh ρ
|high⟩ = |ω_h⟩
ω_h = e^{ρ}ω_m = 3
|mid⟩ = |ω_m⟩
ω_m sinh ρ
ω_m = 2
|low⟩ = |ω_ℓ⟩
ω_ℓ = e^{−ρ}ω_m = 4/3
e^{−2ρ}ω_m = 8/9
K_m
K_ℓ
P_{2,2} P_{3,1} P_{1,2} P_{2,1} P_{3,0} P_{1,1} P_{−1,1} P_{2,0} P_{3,−1} P_{−1,0} P_{1,0} P_{2,−1} P_{3,−2} P_{1,−1} P_{2,−2} P_{1,−2}
Geometric sequence based on Doppler shift 3/2
..., 8/9, 4/3, 2, 3, 9/4, 27/8, ...
Geometric (p,q)−coordinates
rest frequency:
rapidity:
ω_q = ω_m e^{qρ}
ρ_p = pρ
P_{p,q} = (ck_{p,q}, ω_{p,q}) = ω_m e^{qρ}(sinh pρ, cosh pρ)
cp = ħck
Doppler RED factor: 2/3 = e^{−ρ}
Doppler BLUE factor: 3/2 = e^{+ρ}


(p,q)−coordinates
rest frequency:
$\omega_q = \omega_m e^{q\rho}$
rapidity:
$\rho_p = p\rho$
$P_{p,q} = (ck_{p,q}, \omega_{p,q})$
$= \omega_m e^{q\rho}(\sinh p\rho, \cosh p\rho)$
(−1,2)
(0,2)
(1,2)
(2,2)
(3,1)
(−2,1)
(2,1)
(3,0)
(4,−1)
(−1,1)
(0,1)
(1,1)
(−2,0)
(2,0)
(3,−1)
(−1,0)
(p,q)=(0,0)
(1,0)
(−2,−1)
(2,−1)
(−1,−1)
(0,−1)
(1,−1)
(0,−2)
+3
+2
+1
0
−1
−2
L = lefthand shift power
$\omega_L = \omega_m e^{L\rho}$
R = righthand shift power
$\omega_R = \omega_m e^{R\rho}$
(p,q)−(R,L) coordinate transformations:
$p = \frac{R-L}{2}, q = \frac{R+L}{2}$
$R = p+q , L = q-p$
Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$


(p,q) − coordinates
rest frequency :
$\omega_q = \omega_m e^{q\rho}$
rapidity :
$\rho_p = p\rho$
$P_{p,q} = (ck_{p,q}, \omega_{p,q})$
$= \omega_m e^{q\rho}(\sinh p\rho, \cosh p\rho)$
(2,2)
(3,1)
(−1,2)
(1,2)
(0,2)
(−2,1)
(2,1)
(3,0)
(4,−1)
(−1,1)
(1,1)
(0,1)
+3
+3
All-rational-fraction lattice
defined by discrete sub-group
of Lorentz Poincare Group
(Feynman path integrals defined
by group transformations)
(2,0)
(3,−1)
(−1,0)
(1,0)
(p,q) = (0,0)
+2
(−2,−1)
(2,−1)
(−1,−1)
(1,−1)
(0,−1)
+1
+1
(0,−2)
0
0
−1
−1
−2
−2
L = lefthand shift power
$\omega_L = \omega_m e^{L\rho}$
R = righthand shift power
$\omega_R = \omega_m e^{R\rho}$
(p,q) − (R,L)
coordinate
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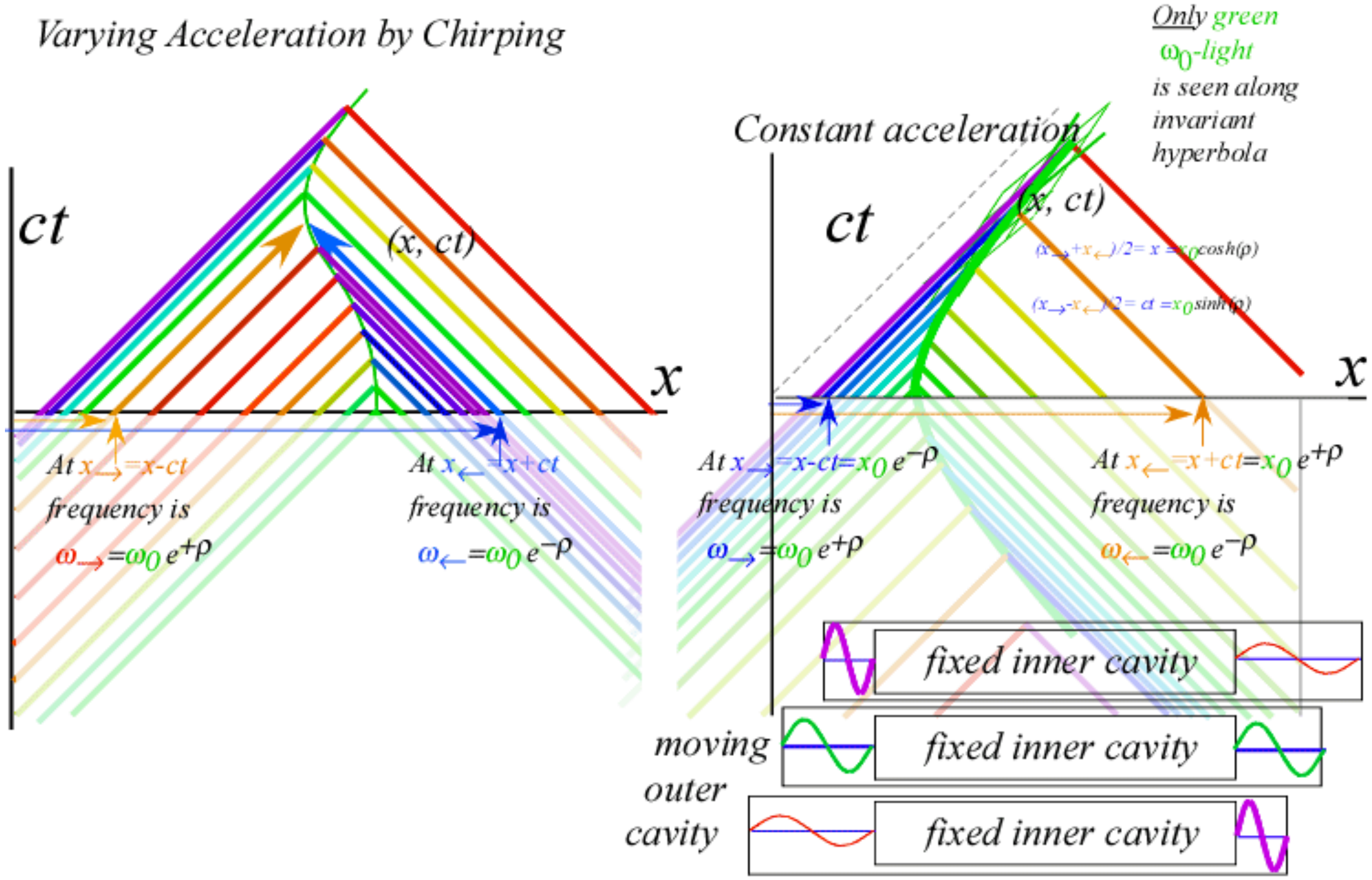
Einstein Elevators Made by Chirped 2-CW Light



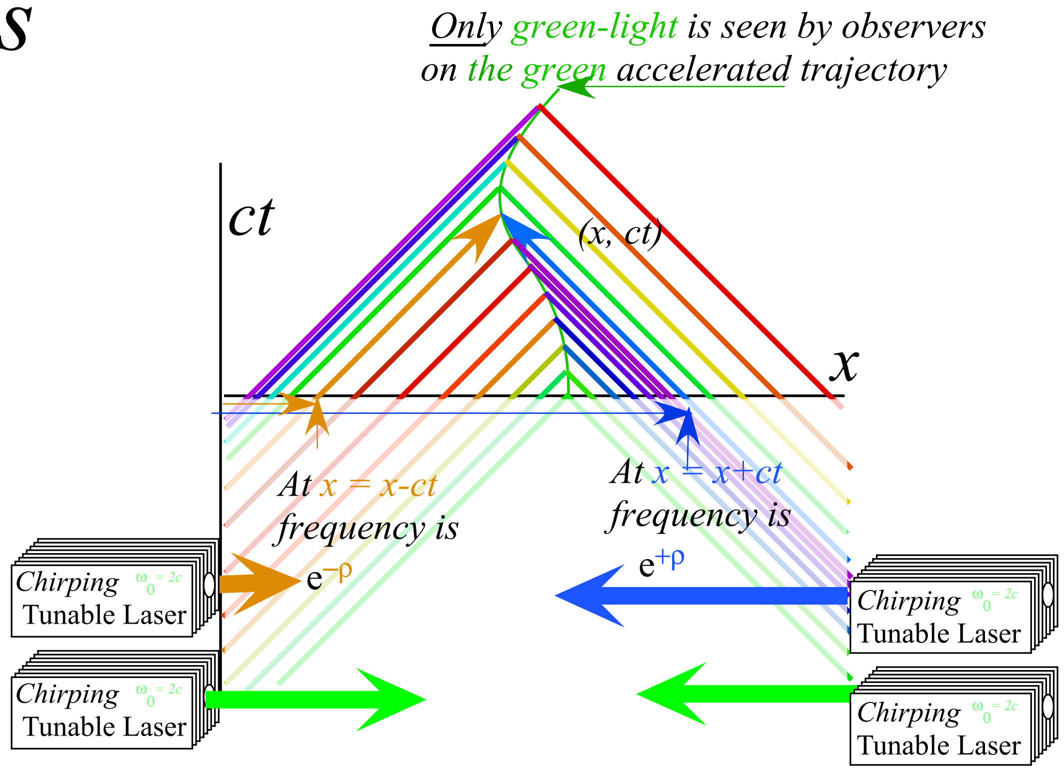
Wave frames of varying acceleration

Acceleration by chirping laser pairs

Varying acceleration (General case)

Varying local acceleration $\rho=\rho(\tau)$ Lab time dt *vs* proper time $d\tau$

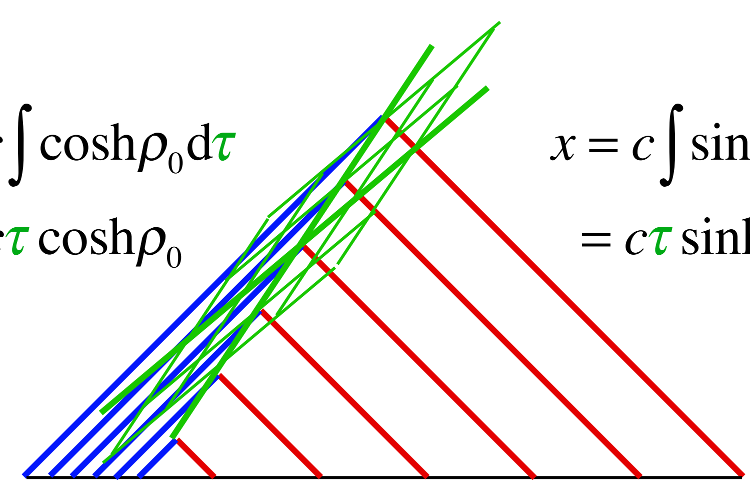
$$u = \frac{dx}{dt} = c\tanh\rho(\tau) \qquad dt = d\tau\cosh\rho(\tau) = \frac{d\tau}{\sqrt{1-u^2/c^2}}$$



Previous examples involved constant velocity

Constant velocity $\rho=\rho_0$ "Lorentz-Transformation"

$$ct = c\int\cosh\rho_0 d\tau = c\tau\cosh\rho_0 \qquad x = c\int\sinh\rho_0 d\tau = c\tau\sinh\rho_0$$



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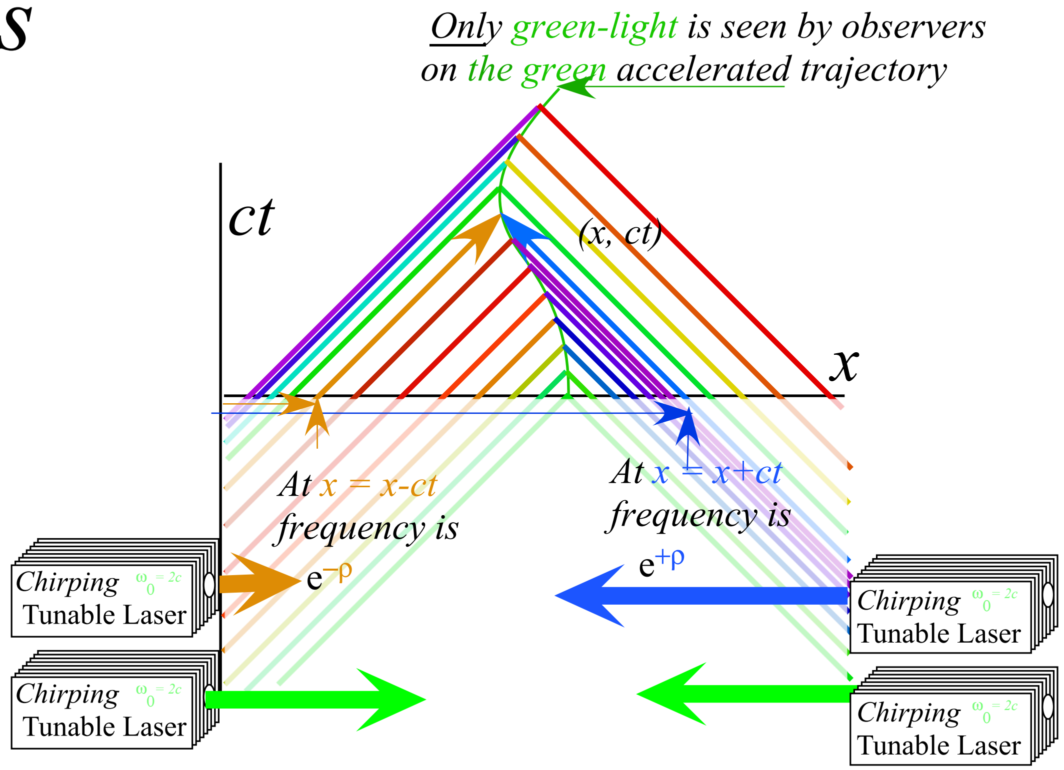
$$u=\frac{dx}{dt}=c\tanh\rho(\tau) \qquad dt=d\tau\cosh\rho(\tau)=\frac{d\tau}{\sqrt{1-u^2/c^2}}$$

$$\frac{dt}{d\tau}=\cosh\rho(\tau) \qquad \frac{dx}{d\tau}=\frac{dx}{dt}\frac{dt}{d\tau}=c\tanh\rho(\tau)\cosh\rho(\tau)$$

$$ct=c\int\cosh\rho(\tau)d\tau \qquad x=c\int\sinh\rho(\tau)d\tau$$



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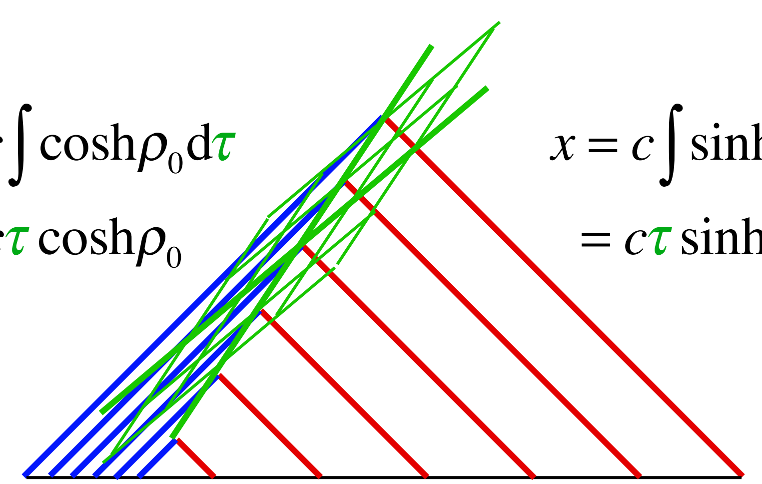

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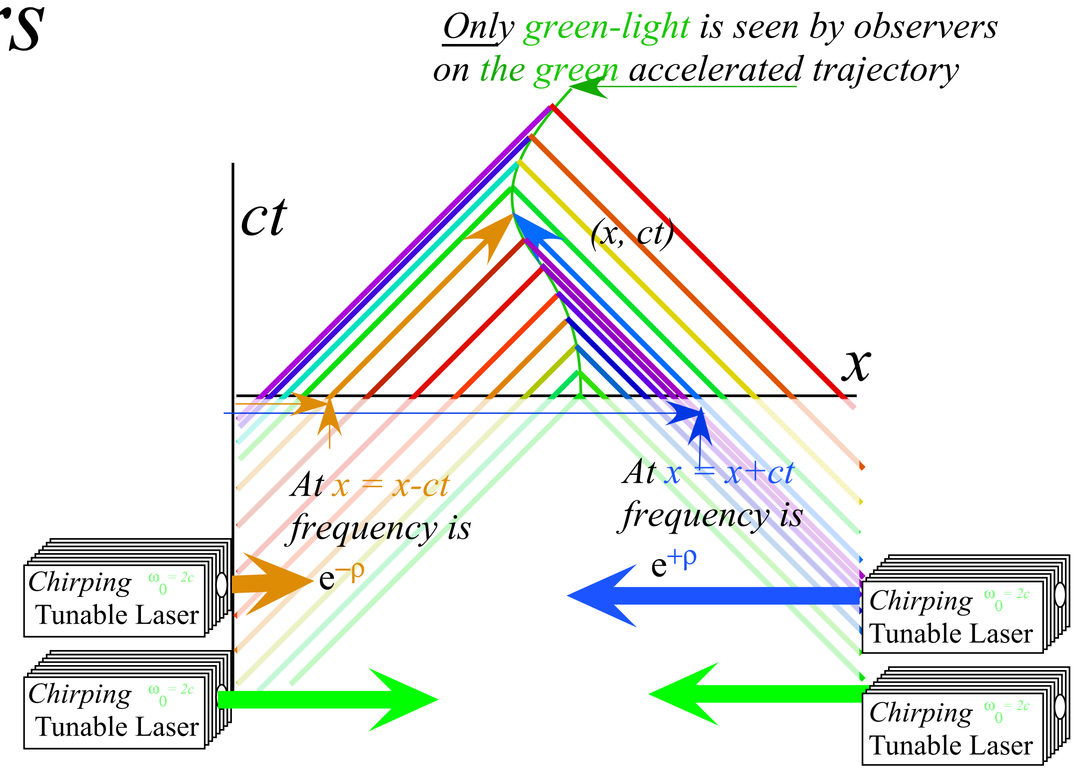
$$\frac{dt}{d\tau}=\cosh\rho(\tau) \qquad \frac{dx}{d\tau}=\frac{dx}{dt}\frac{dt}{d\tau}=c\tanh\rho(\tau)\cosh\rho(\tau)$$

$$ct=c\int\cosh\rho(\tau)\mathrm{d}\tau \qquad x=c\int\sinh\rho(\tau)\mathrm{d}\tau$$



Constant local acceleration $\rho=\frac{g\tau}{c}$ "Einstein-Elevator"

$$ct=c\int\cosh\frac{g\tau}{c}\mathrm{d}\tau \qquad x=c\int\sinh\frac{g\tau}{c}\mathrm{d}\tau$$

$$=\frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad x==\frac{c^2}{g}\cosh\frac{g\tau}{c}$$

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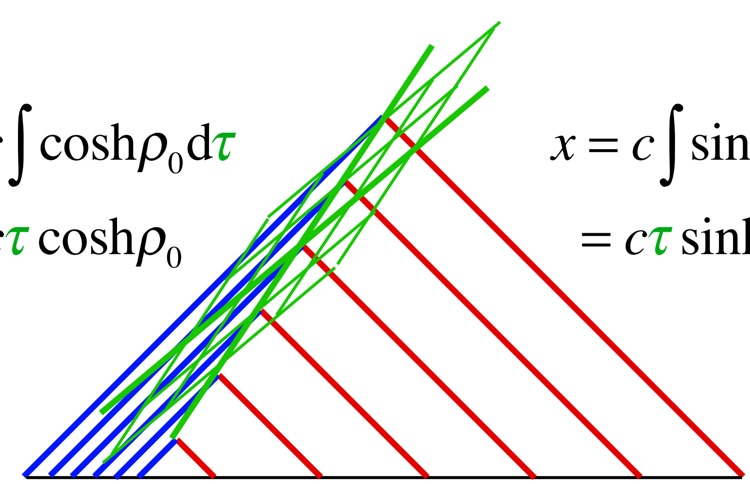

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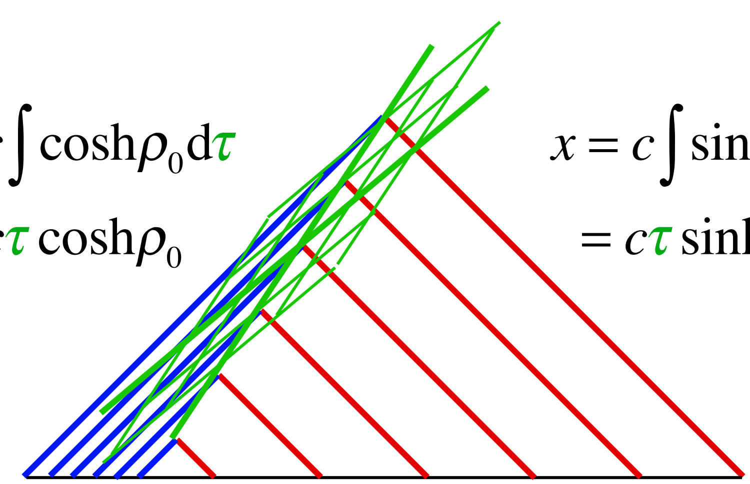

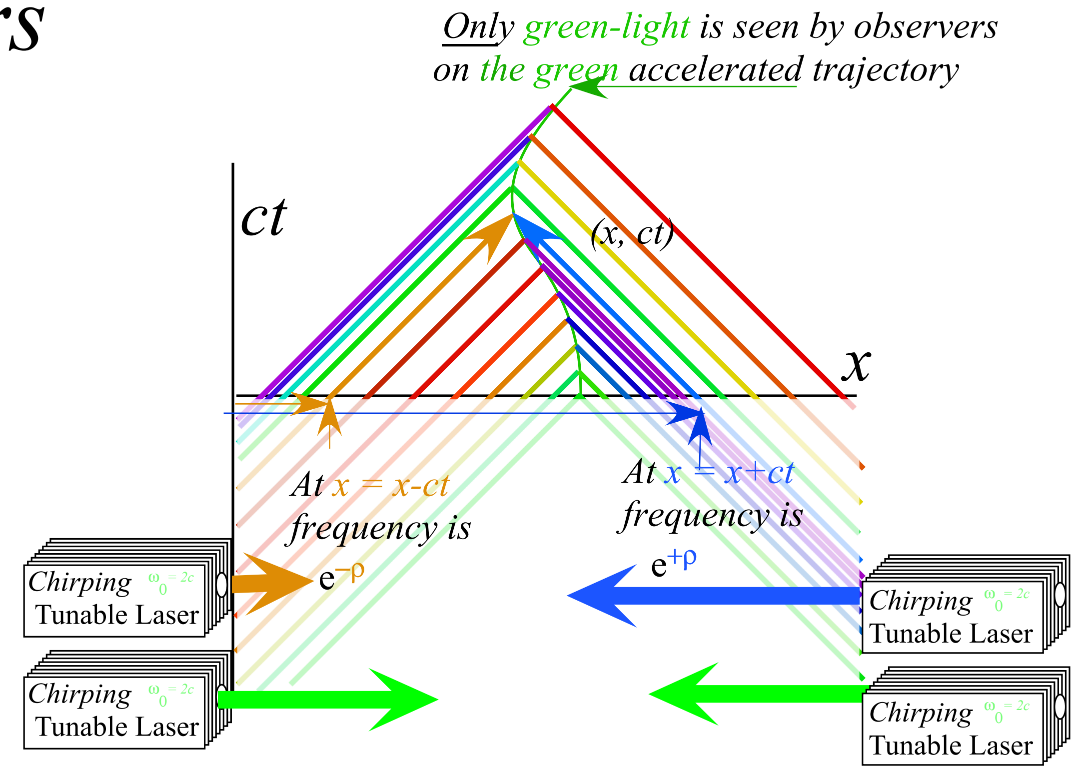

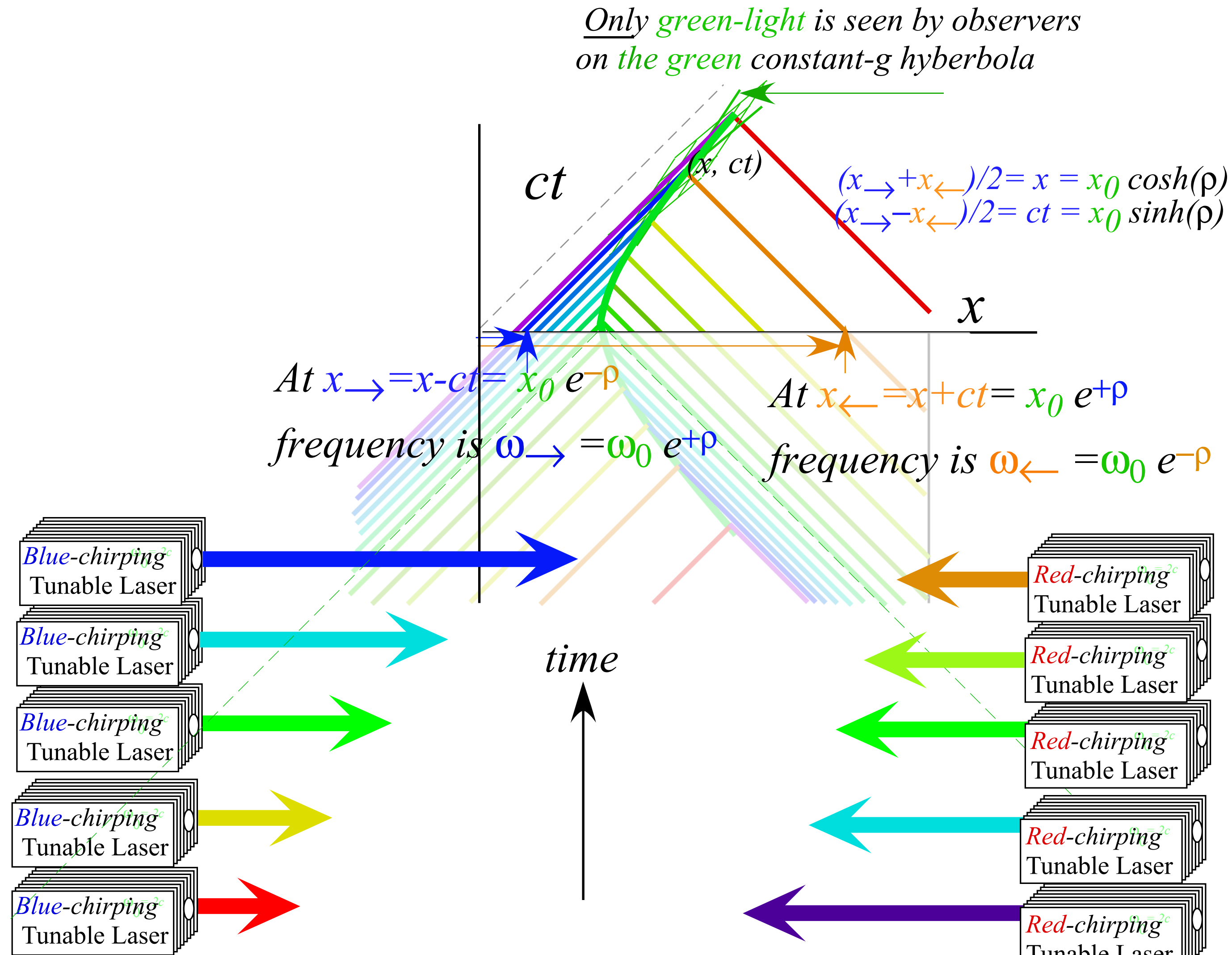

Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

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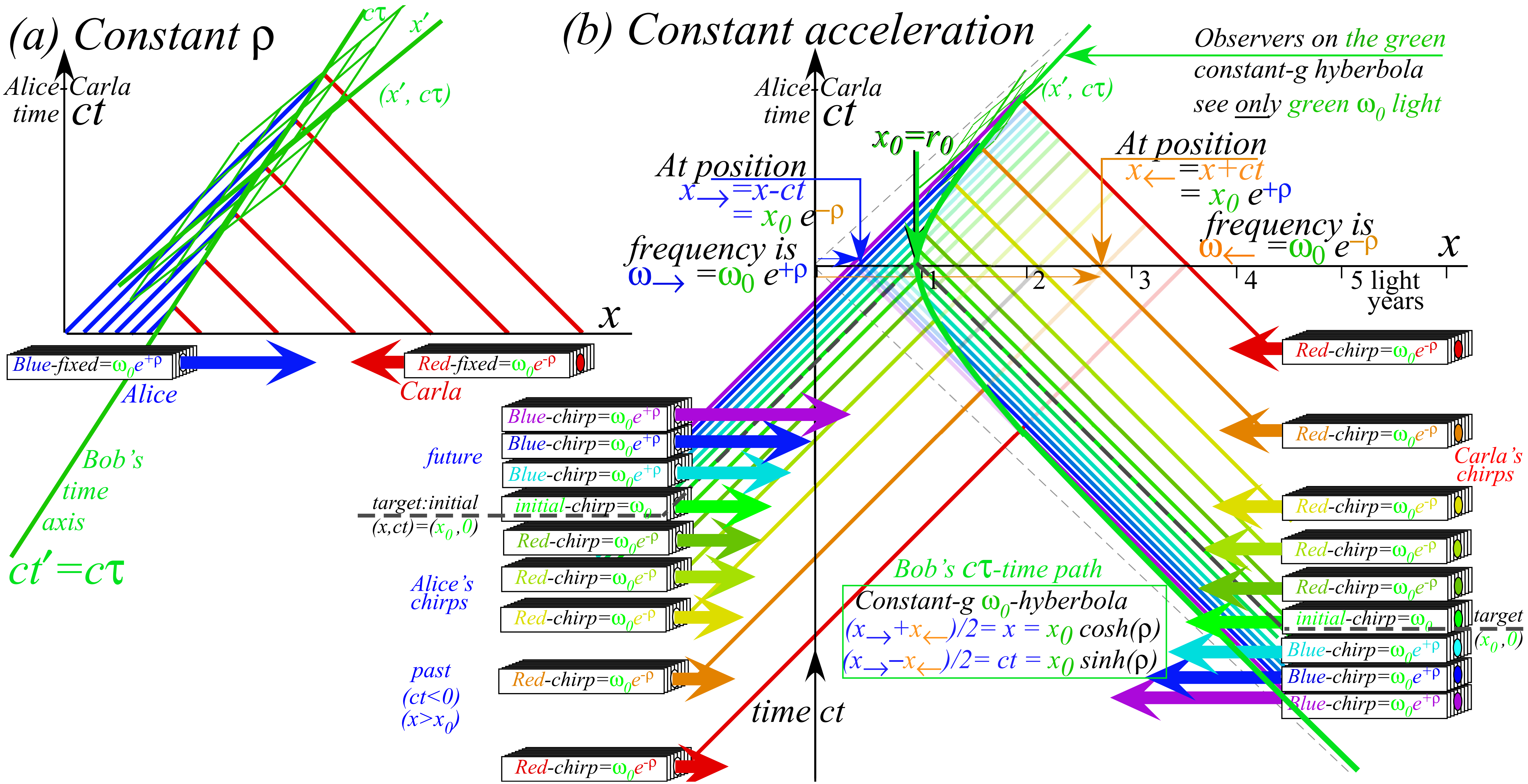
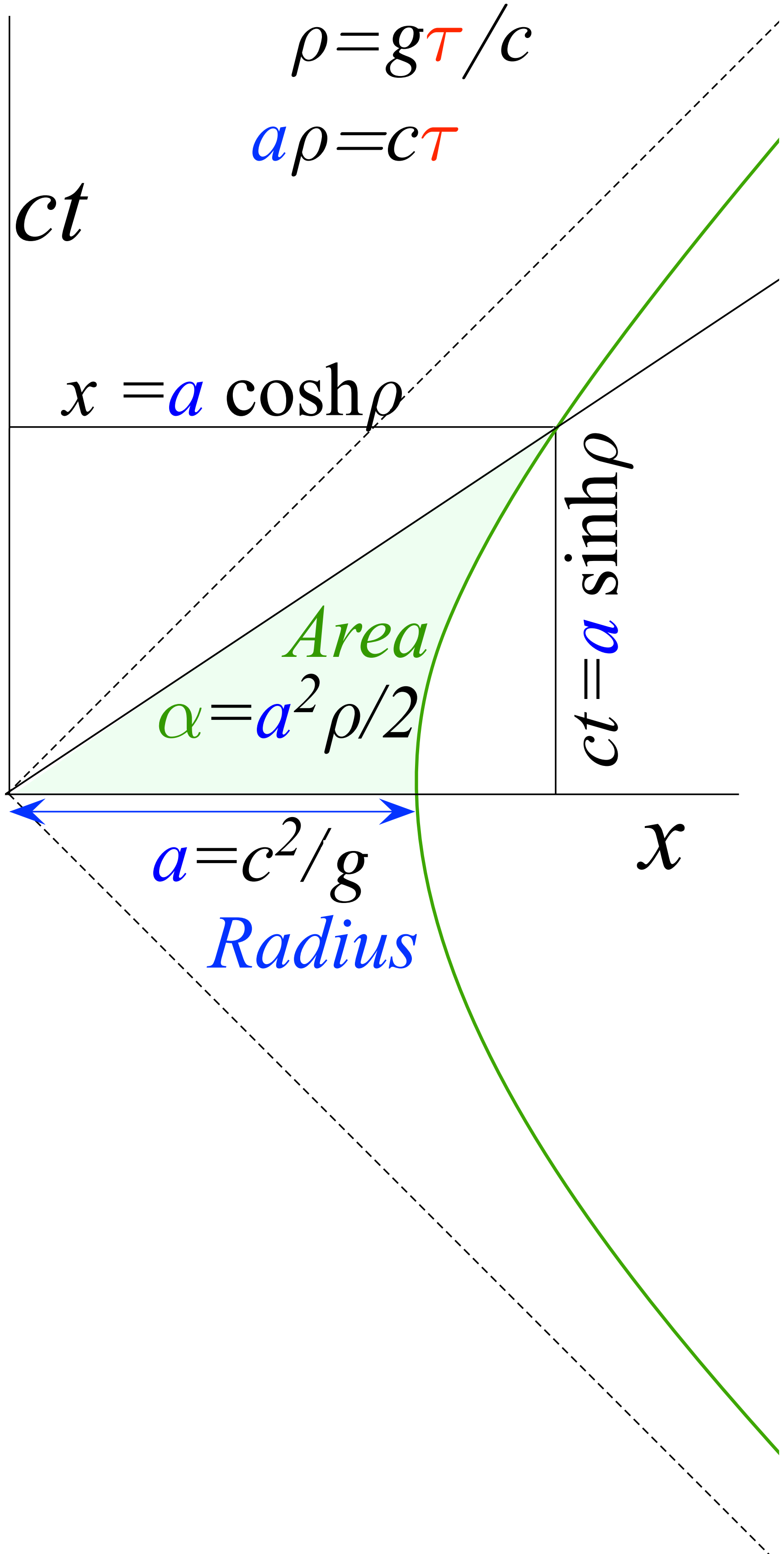


(a) Constant ρ
Alice-Carla time ct
x
(x′, cτ)
Blue-fixed=ω0e+ρ
Red-fixed=ω0e-ρ
Alice
Carla
Bob's time axis
ct′=cτ
(b) Constant acceleration
Observers on the green constant-g hyberbola see only green ω0 light
Alice-Carla time ct
x0=r0
At position x→=x-ct = x0 e−ρ
frequency is ω→ =ω0 e+ρ
At position x←=x+ct = x0 e+ρ
frequency is ω← =ω0 e−ρ
1 2 3 4 5 light years
Blue-chirp=ω0e+ρ
initial-chirp=ω0
Red-chirp=ω0e-ρ
future
target:initial (x,ct)=(x0,0)
Alice's chirps
past (ct<0) (x>x0)
Carla's chirps
target (x0,0)
Bob's cτ-time path
Constant-g ω0-hyberbola
(x→+x←)/2= x = x0 cosh(ρ)
(x→−x←)/2= ct = x0 sinh(ρ)
time ct

(a) Constant acceleration g

Rapidity ρ vs proper time τ



for: $\rho c = g\tau$ or: $\rho = \frac{g\tau}{c}$

$$ct = c\int \cosh\left(\frac{g\tau}{c}\right)d\tau = \frac{c^2}{g}\sinh\left(\frac{g\tau}{c}\right) = a\sinh\rho$$

$$x = c\int \sinh\left(\frac{g\tau}{c}\right)d\tau = \frac{c^2}{g}\cosh\left(\frac{g\tau}{c}\right) = a\cosh\rho$$

(a) Constant acceleration g

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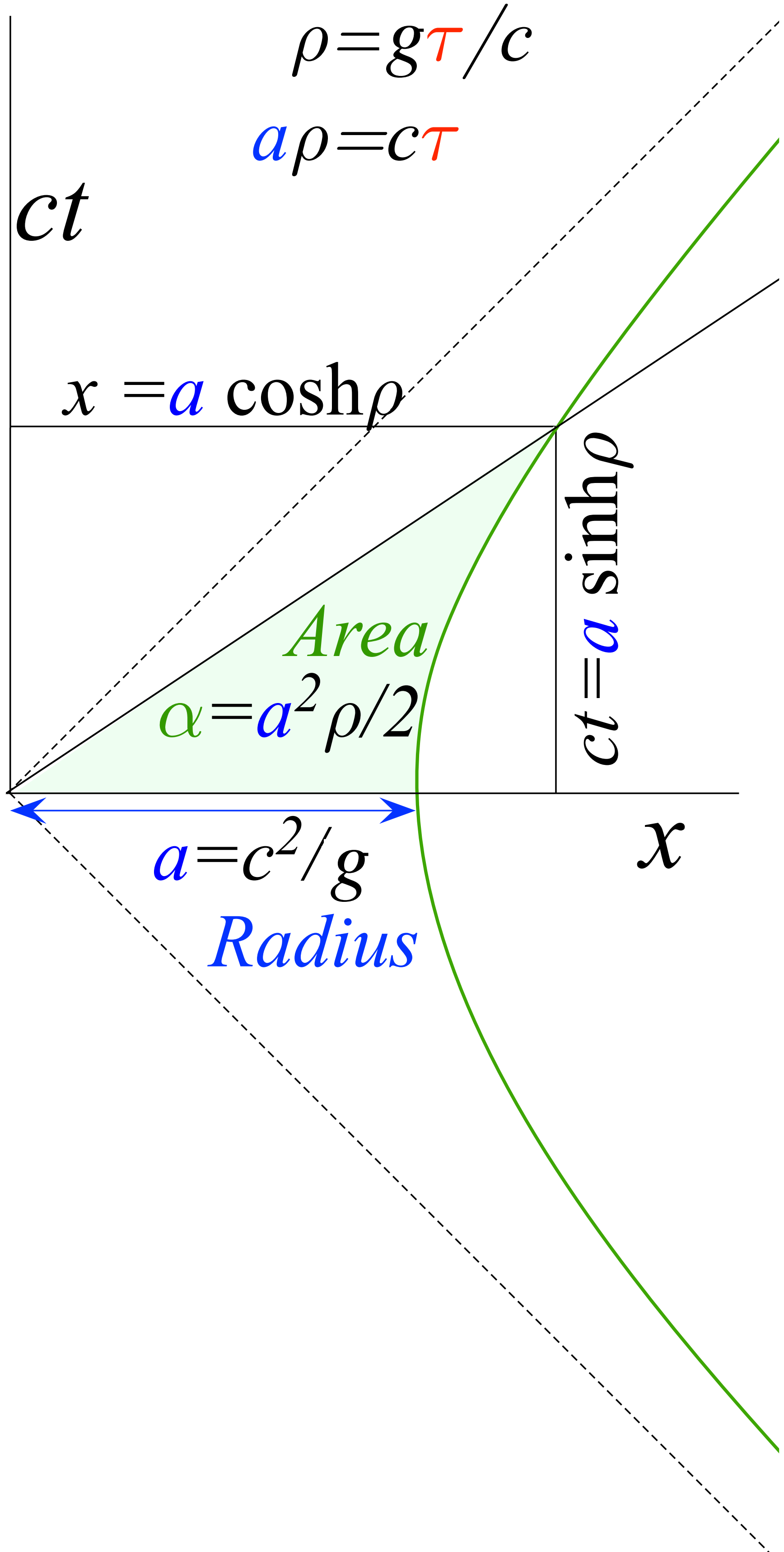

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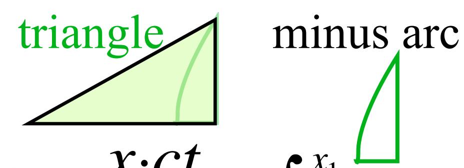
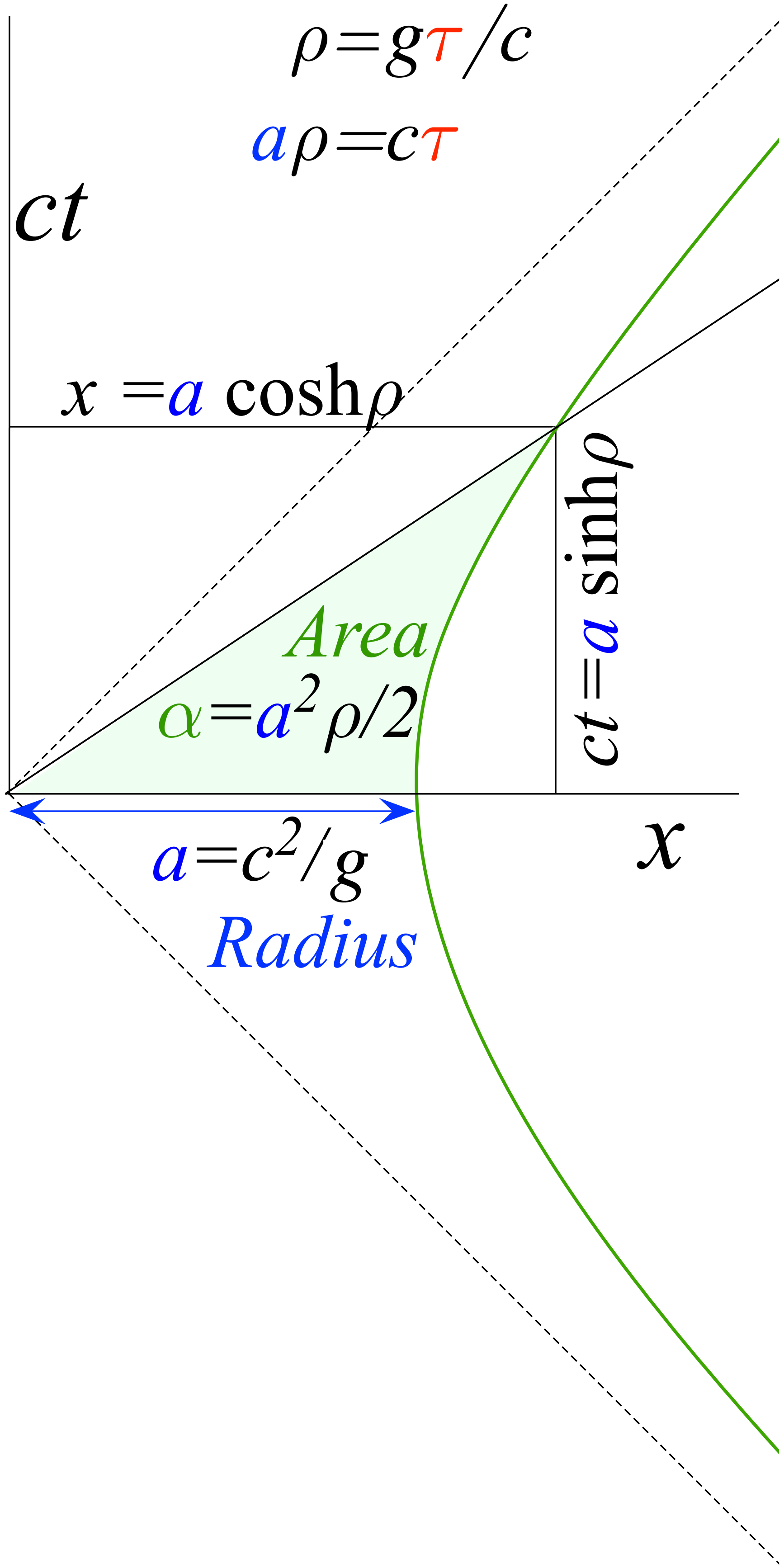


$$\alpha(Area) = \frac{x\cdot ct}{2} - \int_0^{x_1} ct\cdot dx = \frac{x\cdot ct}{2} - \int_0^{\rho_1} ct\cdot\frac{dx}{d\rho}d\rho$$

(a) Constant acceleration g

Rapidity ρ vs proper time τ



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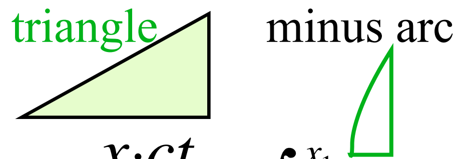

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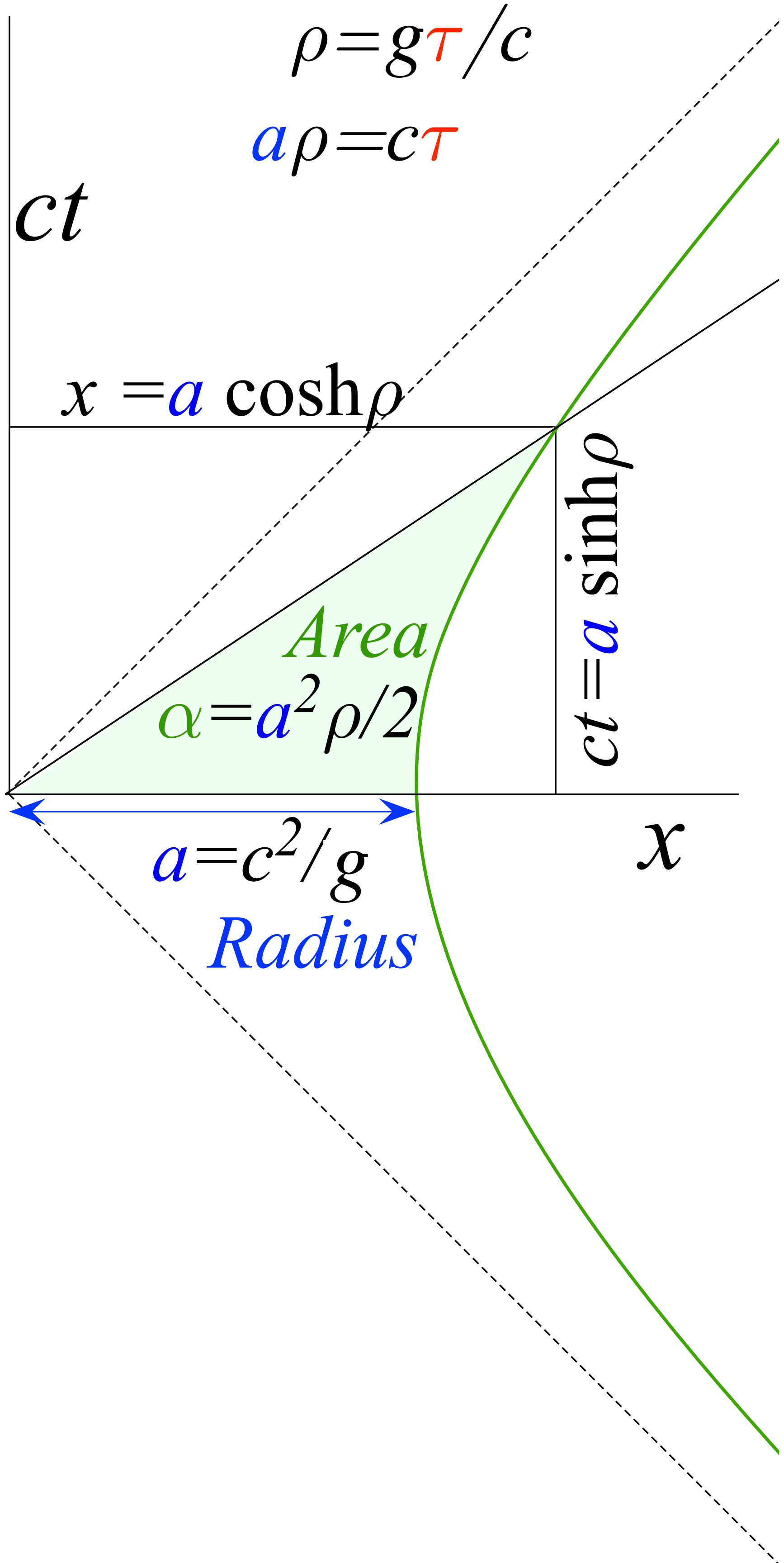
$$\alpha(\rho_1) = a^2\frac{\cosh\rho_1\cdot\sinh\rho_1}{2} - \int_0^{\rho_1}(a\sinh\rho)^2 d\rho = \frac{a^2}{2}\rho_1 = \frac{a^2}{2}\frac{g\tau_1}{c}$$

(a) Constant acceleration g

Rapidity ρ vs proper time τ



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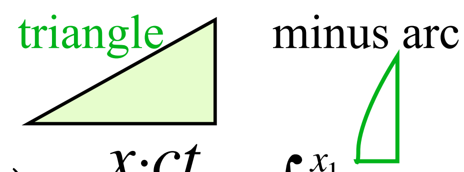

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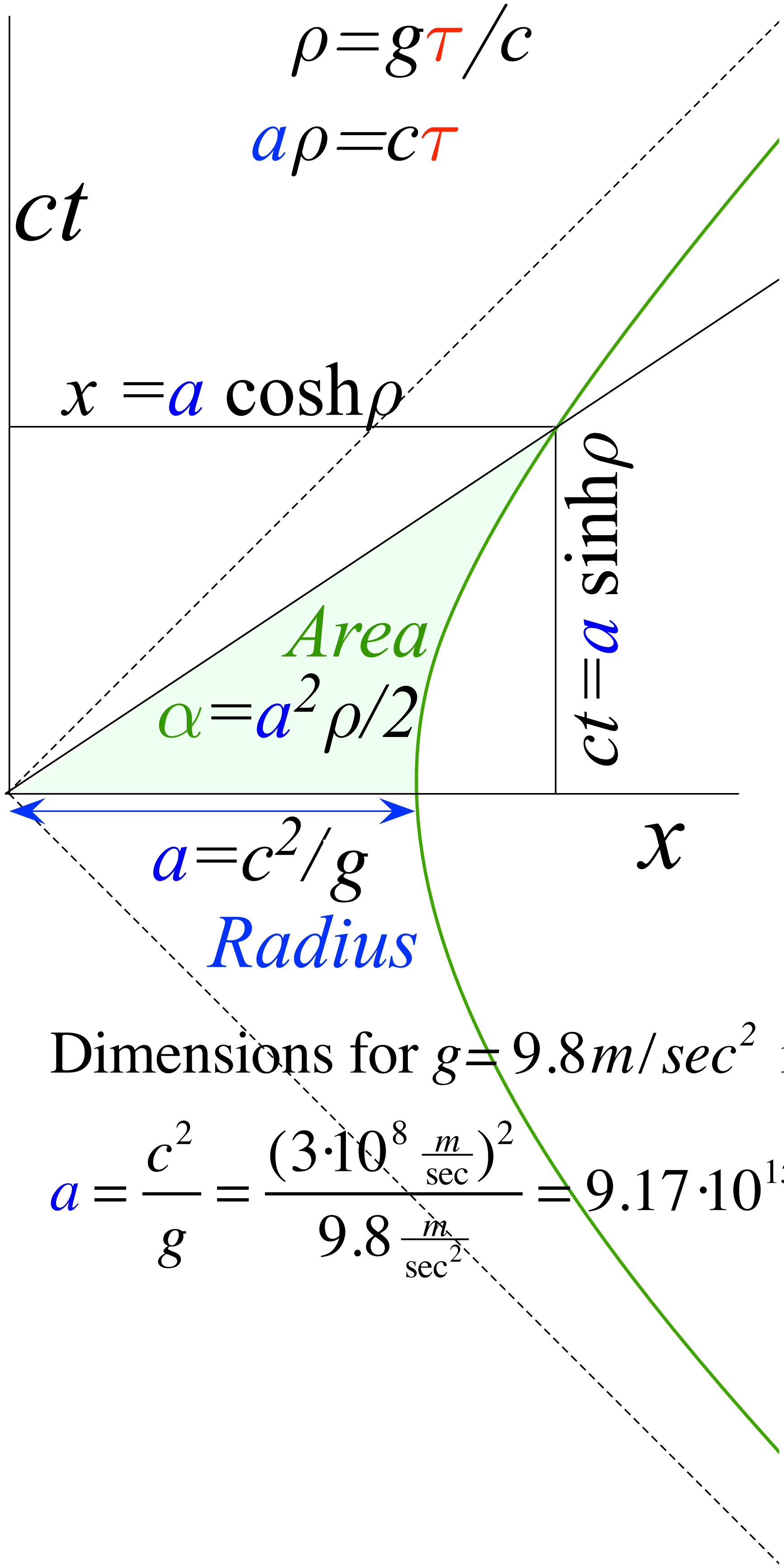
$$a\rho_1 = c\tau_1$$

(a) Constant acceleration g

Rapidity ρ vs proper time τ



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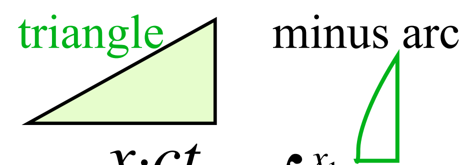

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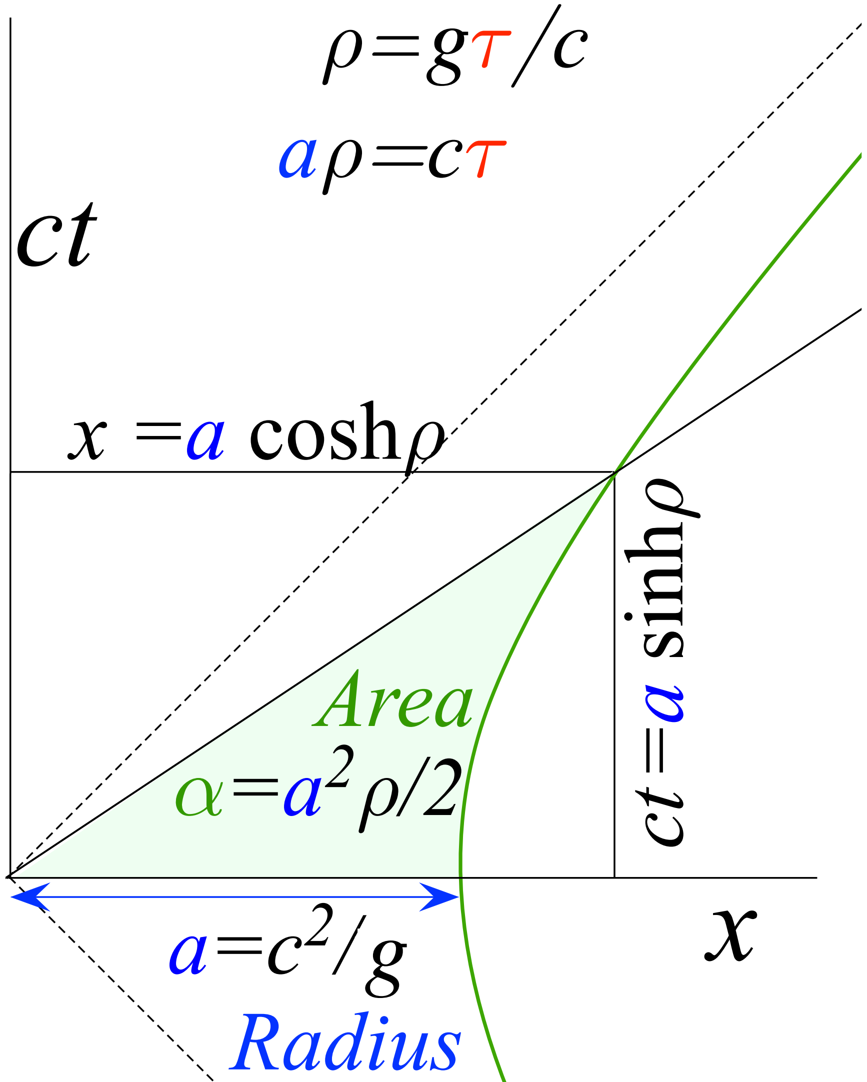
$$a\rho_1 = c\tau_1$$

Dimensions for $g = 9.8 m/sec^2$ in unit of 1 *lite yr* $= c(3.15\cdot 10^7 \text{ sec}) = 9.44\cdot 10^{15} m$

$$a = \frac{c^2}{g} = \frac{(3\cdot 10^8 \frac{m}{\sec})^2}{9.8\frac{m}{\sec^2}} = 9.17\cdot 10^{15} m = 0.97 \textit{lite yr}$$

(a) Constant acceleration g

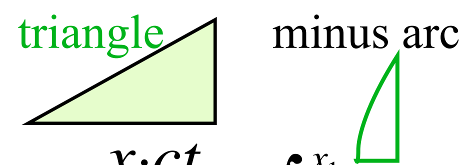
Rapidity ρ vs proper time τ



for: $\rho c = g\tau$ or: $\rho = \frac{g\tau}{c}$

$$ct = c\int \cosh\left(\frac{g\tau}{c}\right)d\tau = \frac{c^2}{g}\sinh\left(\frac{g\tau}{c}\right) = a\sinh\rho$$

$$x = c\int \sinh\left(\frac{g\tau}{c}\right)d\tau = \frac{c^2}{g}\cosh\left(\frac{g\tau}{c}\right) = a\cosh\rho$$



$$\alpha(Area) = \frac{x\cdot ct}{2} - \int_0^{x_1} ct\cdot dx = \frac{x\cdot ct}{2} - \int_0^{\rho_1} ct\cdot\frac{dx}{d\rho}d\rho$$

$$\alpha(\rho_1) = a^2\frac{\cosh\rho_1\cdot\sinh\rho_1}{2} - \int_0^{\rho_1}(a\sinh\rho)^2 d\rho = \frac{a^2}{2}\rho_1 = \frac{a^2}{2}\frac{g\tau_1}{c}$$

$$a\rho_1 = c\tau_1$$

Dimensions for $g = 9.8\,m/sec^2$ in unit of 1 $lite\ yr = c(3.15\cdot 10^7\ \mathrm{sec}) = 9.44\cdot 10^{15}\,m$

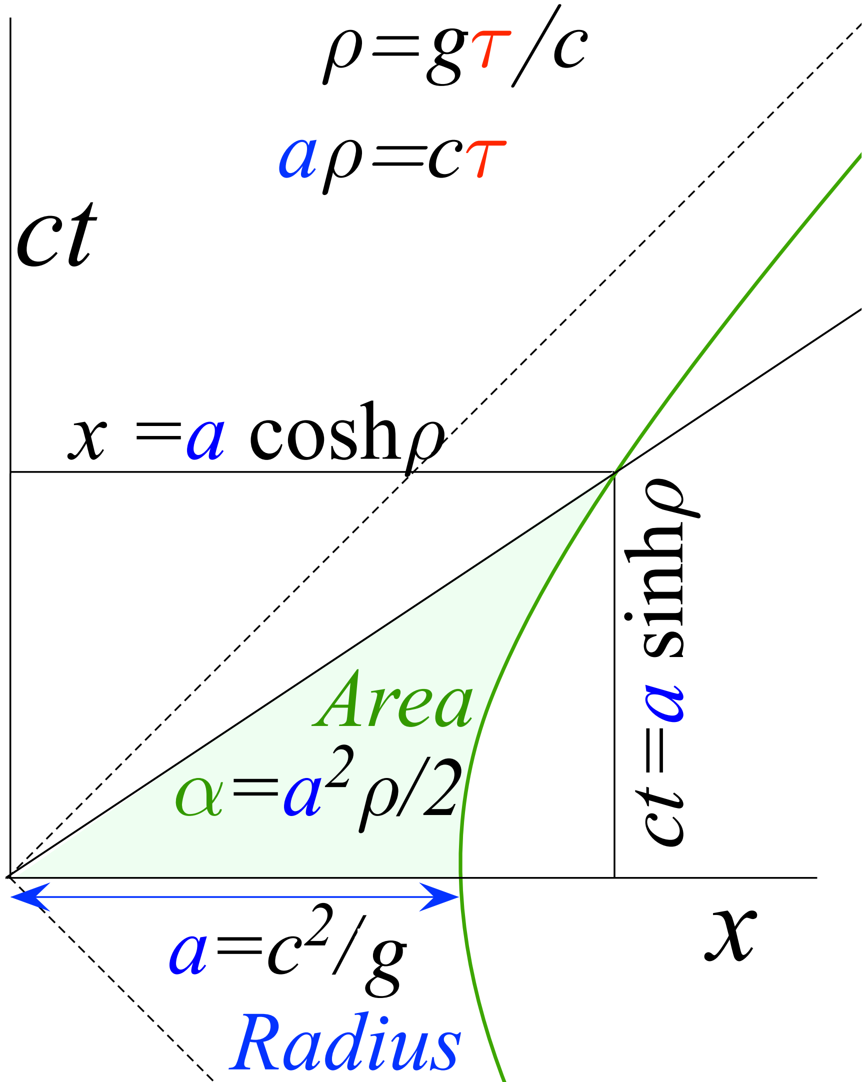
$$a = \frac{c^2}{g} = \frac{(3\cdot 10^8\,\frac{m}{\mathrm{sec}})^2}{9.8\,\frac{m}{\mathrm{sec}^2}} = 9.17\cdot 10^{15}\,m = 0.97\,lite\ yr$$

Rapidity for $c\tau_1 = 1yr = 3.15\cdot 10^7\mathrm{sec}$ is $\rho_1 = \frac{c\tau_1}{a} = \frac{3\cdot 10^8(3.15\cdot 10^7)}{9.17\cdot 10^{15}} = 1.03$

Then $x = a\cosh\rho_1 = 0.97(\cosh 1.03) = 1.53\,liteyr$

(a) Constant acceleration g

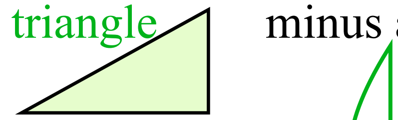
Rapidity ρ *vs proper time* τ



for: $\rho c = g\tau$ or: $\rho = \frac{g\tau}{c}$

$$ct = c\int \cosh\left(\frac{g\tau}{c}\right)d\tau = \frac{c^2}{g}\sinh\left(\frac{g\tau}{c}\right) = a\sinh\rho$$

$$x = c\int \sinh\left(\frac{g\tau}{c}\right)d\tau = \frac{c^2}{g}\cosh\left(\frac{g\tau}{c}\right) = a\cosh\rho$$



$$\alpha(Area) = \frac{x\cdot ct}{2} - \int_0^{x_1} ct\cdot dx = \frac{x\cdot ct}{2} - \int_0^{\rho_1} ct\cdot\frac{dx}{d\rho}d\rho$$

$$\alpha(\rho_1) = a^2\frac{\cosh\rho_1\cdot\sinh\rho_1}{2} - \int_0^{\rho_1}(a\sinh\rho)^2 d\rho = \frac{a^2}{2}\rho_1 = \frac{a^2}{2}\frac{g\tau_1}{c}$$

$$a\rho_1 = c\tau_1$$

Dimensions for $g = 9.8\,m/sec^2$ in units of 1 *lite yr* $= c(3.15\cdot 10^7\,\text{sec}) = 9.44\cdot 10^{15}\,m$

$$a = \frac{c^2}{g} = \frac{(3\cdot 10^8\,\frac{m}{\text{sec}})^2}{9.8\,\frac{m}{\text{sec}^2}} = 9.17\cdot 10^{15}\,m = 0.97\,lite\ yr$$

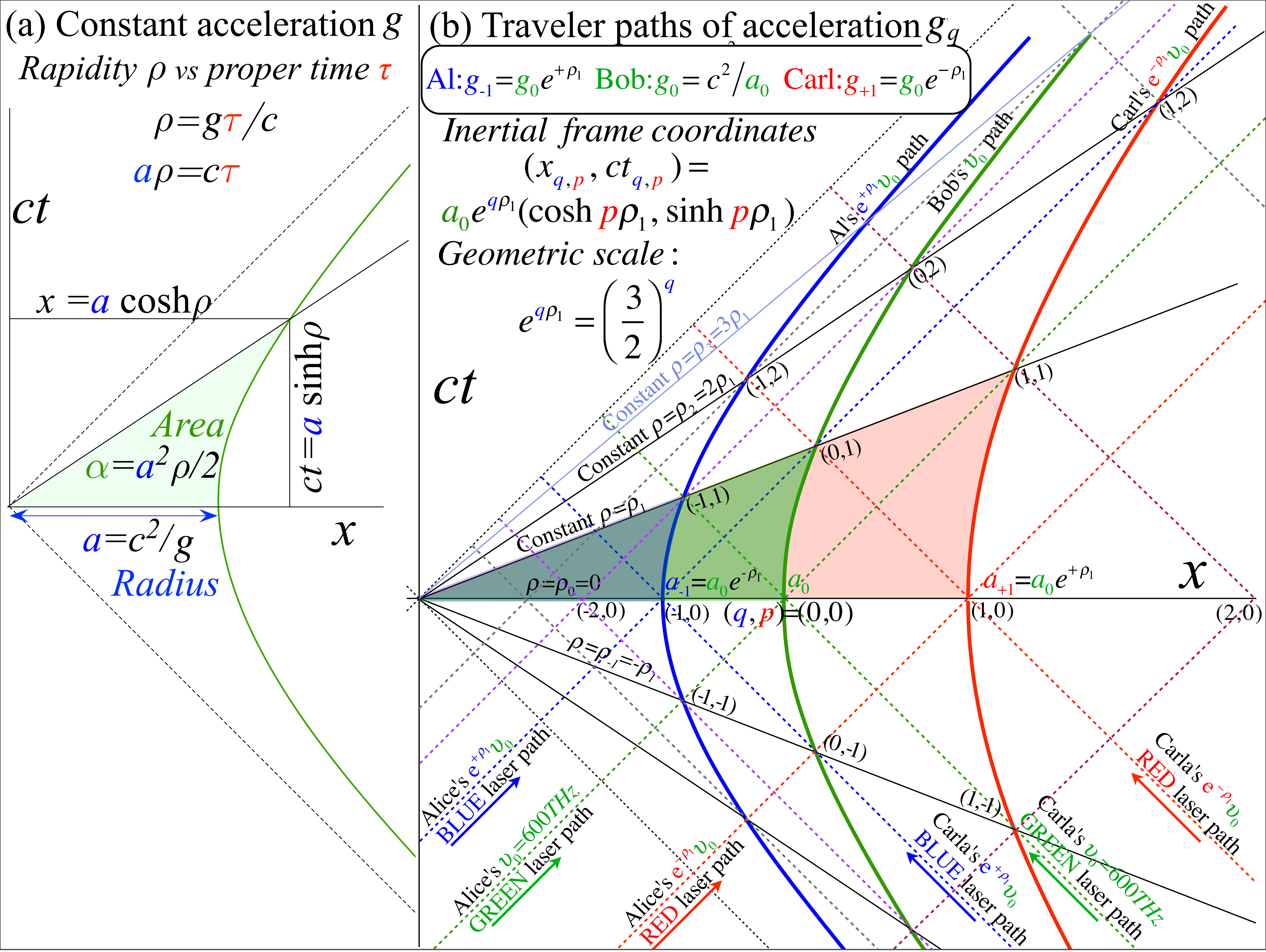
Rapidity for $c\tau_1 = 1yr = 3.15\cdot 10^7\,\text{sec}$ is $\rho_1 = \frac{c\tau_1}{a} = \frac{3\cdot 10^8(3.15\cdot 10^7)}{9.17\cdot 10^{15}} = 1.03$

Then $x = a\cosh\rho_1 = 0.97(\cosh 1.03) = 1.53\,liteyr$

Rapidity for $c\tau_1 = 21yr = 6.62\cdot 10^8\,\text{sec}$ is $\rho_1 = \frac{c\tau_1}{a} = \frac{3\cdot 10^8(6.62\cdot 10^8)}{9.17\cdot 10^{15}} = 21.63$

A long way to go to get a beer!

Then $x = a\cosh\rho_1 = 0.97(\cosh 21.63) = 1.201\cdot 10^9\,liteyr$



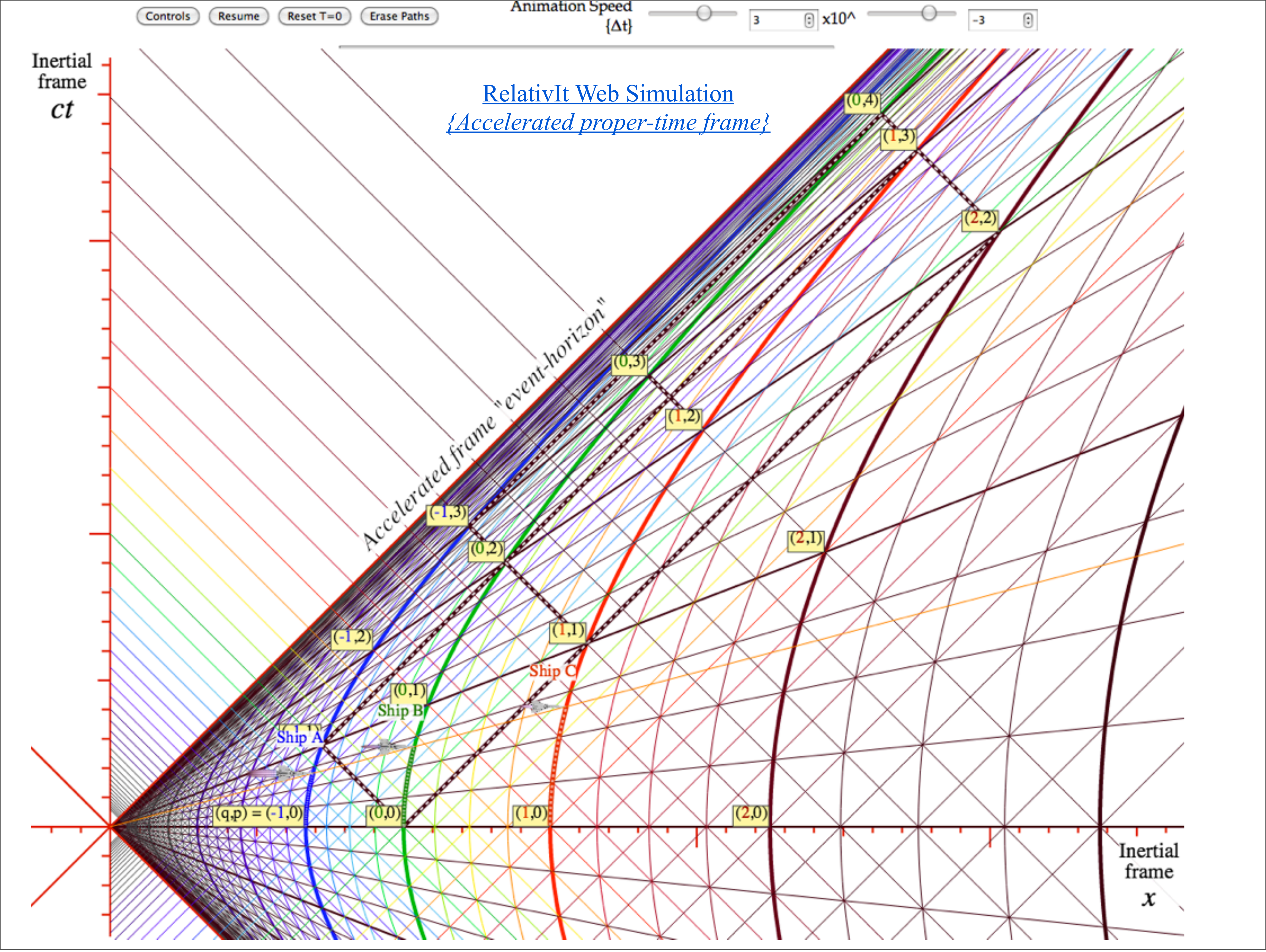
(a) Constant acceleration g
Rapidity ρ vs proper time τ
ρ=gτ/c
aρ=cτ
ct
x =a coshρ
ct=a sinhρ
Area
α=a²ρ/2
a=c²/g
Radius
x
(b) Traveler paths of acceleration g_q
Al: g_{-1}=g_0 e^{+ρ_1} Bob: g_0 = c²/a_0 Carl: g_{+1}=g_0 e^{-ρ_1}
Inertial frame coordinates
(x_{q,p}, ct_{q,p}) =
a_0 e^{qρ_1}(cosh pρ_1, sinh pρ_1)
Geometric scale:
e^{qρ_1} = (3/2)^q
ct
x
Constant ρ=ρ_3=3ρ_1
Constant ρ=ρ_2=2ρ_1
Constant ρ=ρ_1
ρ=ρ_0=0
ρ=ρ_{-1}=-ρ_1
a_{-1}=a_0 e^{-ρ_1}
a_0
a_{+1}=a_0 e^{+ρ_1}
(q,p)=(0,0)
(-2,0)
(-1,0)
(1,0)
(2,0)
(-1,1)
(0,1)
(1,1)
(-1,2)
(0,2)
(1,2)
(-1,-1)
(0,-1)
(1,-1)
Al's e^{+ρ_1}υ_0 path
Bob's υ_0 path
Carl's e^{-ρ_1}υ_0 path
Alice's e^{+ρ_1}υ_0 BLUE laser path
Alice's υ_0=600THz GREEN laser path
Alice's e^{-ρ_1}υ_0 RED laser path
Carla's e^{+ρ_1}υ_0 BLUE laser path
Carla's υ_0=600THz GREEN laser path
Carla's e^{-ρ_1}υ_0 RED laser path

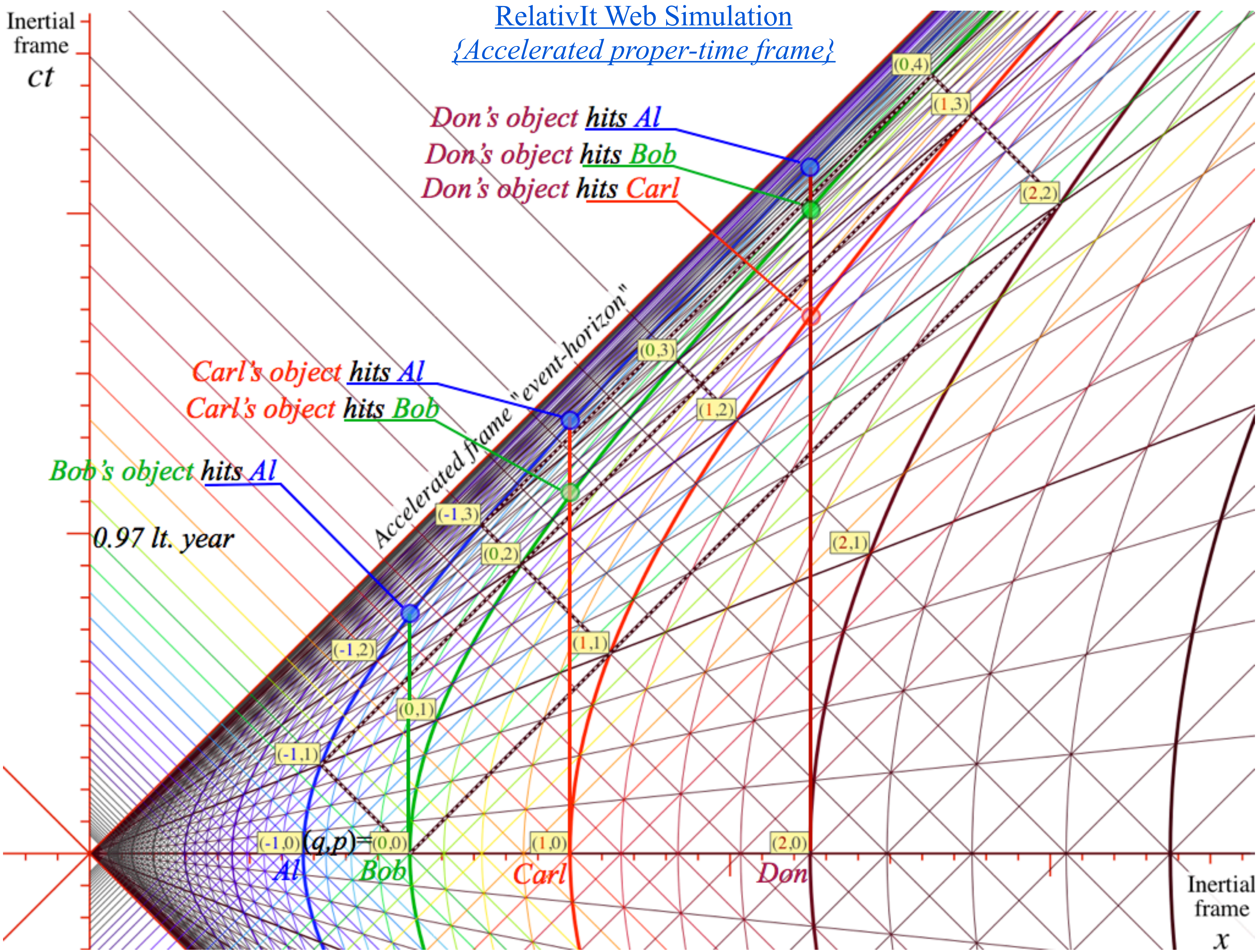
Relativity relates charge, current, and magnetic fields
Geometric derivation of magnetic constant μ_0 from electric ε_0

Lorentz-Poincare symmetry and energy-momentum spectral conservation rules
Review of 2nd-quantization "photon" number N and 1st-quantization wavenumber $\kappa = m$
Sketches of atomic and molecular spectroscopy

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Recoils shifts
Compton recoil related to rocket velocity formula
Geometric transition coordinate grids

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes g-acceleration grid
Analysis of constant-g grid compared to zero-g Minkowsi frame
Animation of mechanics and metrology of constant-g grid


Controls
Resume
Reset T=0
Erase Paths
Animation Speed
{Δt}
3
x10^
-3
RelativIt Web Simulation
{Accelerated proper-time frame}
Inertial frame
ct
Accelerated frame "event-horizon"
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(1,1)
(-1,2)
Ship C
(0,1)
Ship B
Ship A
(q,p) = (-1,0)
(0,0)
(1,0)
(2,0)
Inertial frame
x


RelativIt Web Simulation
{Accelerated proper-time frame}
Inertial frame
ct
Don's object hits Al
Don's object hits Bob
Don's object hits Carl
Carl's object hits Al
Carl's object hits Bob
Bob's object hits Al
Accelerated frame "event-horizon"
0.97 lt. year
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(-1,2)
(1,1)
(0,1)
(-1,1)
(-1,0)
(q,p)=(0,0)
(1,0)
(2,0)
Al
Bob
Carl
Don
Inertial frame
x

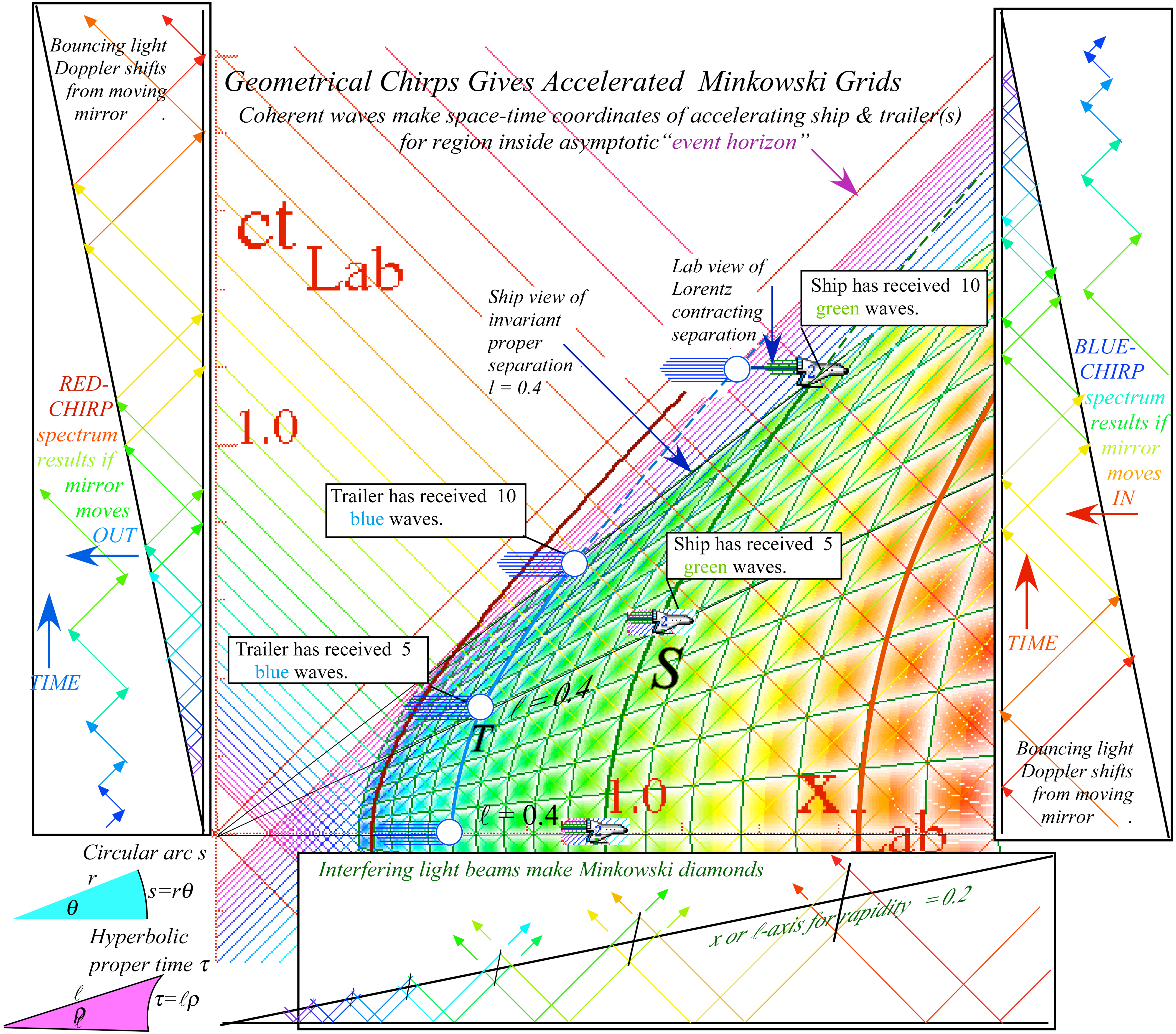


Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light

Relativity relates charge, current, and magnetic fields

Geometric derivation of magnetic constant μ_0 from electric ε_0

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Xtra stuff: *Some numerology: Which is bigger...H-atom or an electron? What' spin?*

Space-Space waves gone mad

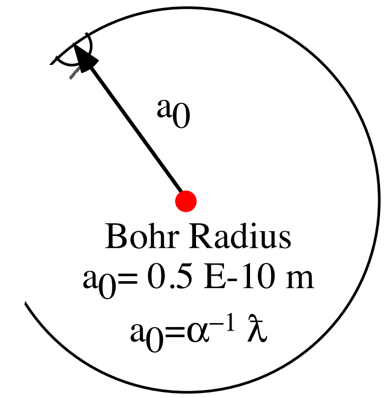

Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant α = 1/137.

Bohr model has electron orbiting at radius *r* so centrifugal force balances Coulomb attraction to the opposite charged proton.

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad \text{or:} \qquad \frac{m_e v^2 r^2}{r} = \frac{e^2}{4\pi\varepsilon_0} \qquad \text{or:} \qquad r = \frac{4\pi\varepsilon_0 m_e v^2 r^2}{e^2} = \frac{4\pi\varepsilon_0 (m_e v r)^2}{m_e e^2} = \frac{4\pi\varepsilon_0 \ell^2}{m_e e^2}$$

Bohr hypothesis: orbital momentum ℓ is a multiple N of $\hbar$ or

$$\ell = m_e v r = N\hbar \qquad (N = 1, 2, \ldots).$$

This gives the *atomic Bohr radius a_0 =0.05nm*

$$r = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} N^2 \left(= r_{Bohr} = 5.28 \times 10^{-11}\ m. = 0.528\ \ddot{A} \text{ for } N=1 \right)$$

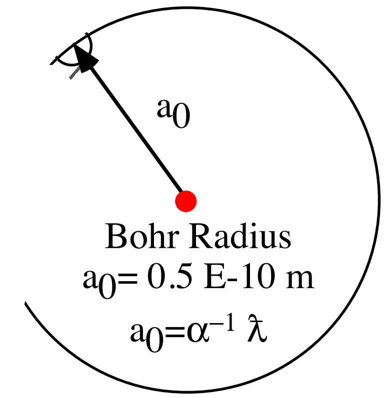

Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant $\alpha = 1/137$.

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$$r = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} N^2 \left(= r_{Bohr} = 5.28 \cdot 10^{-11}\ m. = 0.528\ \ddot{A} \text{ for } N=1 \right)$$

It also implies rear-relativistic electron orbit speed *v* that is fraction *1/N* of *0.073c*.

$$\frac{v}{c} = \frac{\ell}{m_e r c} = \frac{N\hbar}{m_e r_{Bohr} c} = \frac{N\hbar}{m_e c} \frac{m_e e^2}{4\pi\varepsilon_0 \hbar^2 N^2} = \frac{1}{N} \frac{e^2}{4\pi\varepsilon_0 \hbar c} \quad \left(= 7.29 \cdot 10^{-3} = \frac{1}{137.} \text{ for } N=1 \right)$$

The *dimensionless* ratio $\alpha = e^2/(4\pi\varepsilon_0 \hbar c) = 1/137.036$ is called the *fine-structure constant* α.

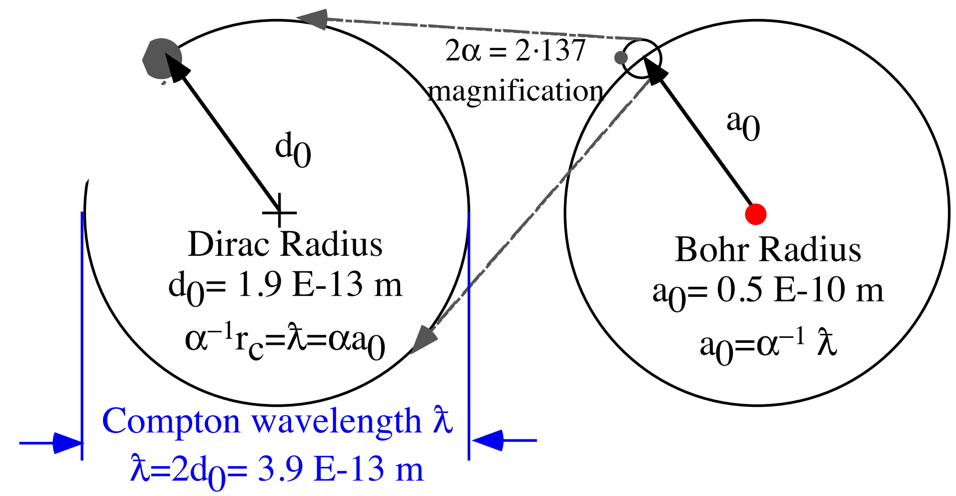


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant α = 1/137.

Bohr model has electron orbiting at radius r so centrifugal force balances Coulomb attraction to the opposite charged proton.

$$\frac{m_e v^2}{r}=\frac{e^2}{4\pi\varepsilon_0 r^2} \qquad \text{or:} \qquad \frac{m_e v^2 r^2}{r}=\frac{e^2}{4\pi\varepsilon_0} \qquad \text{or:} \qquad r=\frac{4\pi\varepsilon_0 m_e v^2 r^2}{e^2}=\frac{4\pi\varepsilon_0 (m_e vr)^2}{m_e e^2}=\frac{4\pi\varepsilon_0 \ell^2}{m_e e^2}$$

Bohr hypothesis: orbital momentum ℓ is a multiple N of $\hbar$ or

$$\ell = m_e\, v\, r = N\hbar \qquad (N = 1, 2,...).$$

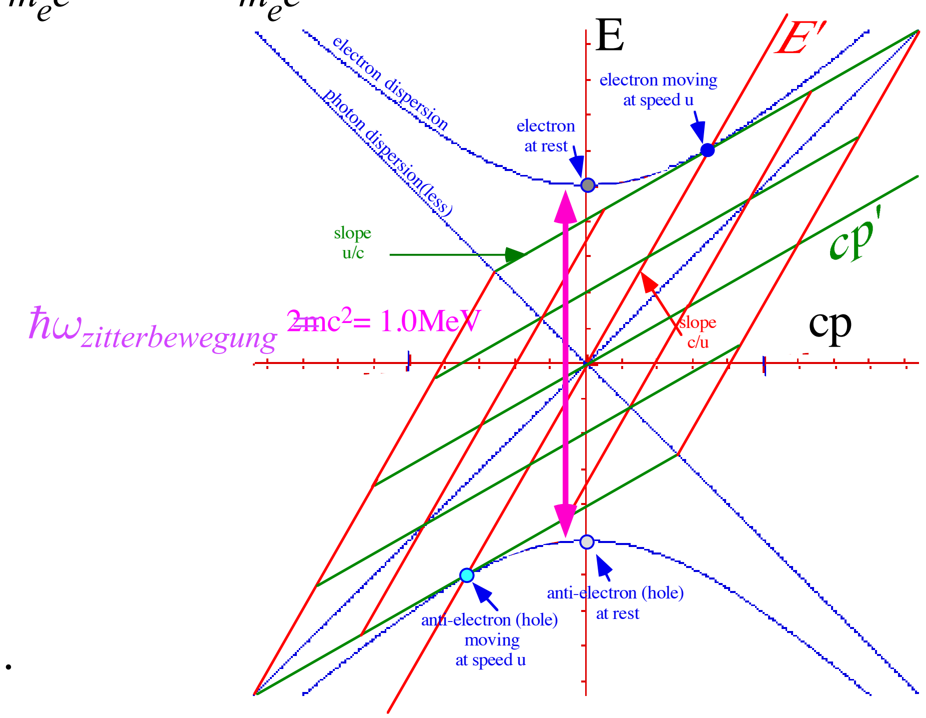
This gives the *atomic Bohr radius* a_0 *=0.05nm*

$$r=\frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}N^2\left(=r_{Bohr}=5.28\cdot 10^{-11}\ m. =0.528\ \ddot{A}\ \text{for}\ N{=}1\right)$$

It also implies rear-relativistic electron orbit speed v that is fraction $1/N$ of $0.073c$.

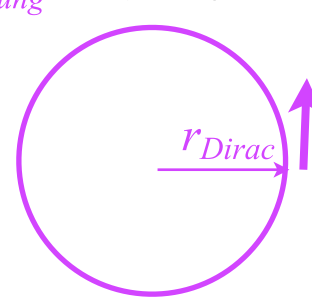
$$\frac{v}{c}=\frac{\ell}{m_e rc}=\frac{N\hbar}{m_e r_{Bohr} c}=\frac{N\hbar}{m_e c}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2 N^2}=\frac{1}{N}\frac{e^2}{4\pi\varepsilon_0\hbar c}\quad\left(=7.29\cdot 10^{-3}=\frac{1}{137.}\ \text{for}\ N{=}1\right)$$

The <u>*dimensionless*</u> ratio $\alpha = e^2/(4\pi\varepsilon_0\hbar c)=1/137.036$ is called the *fine-structure constant* α.



Now, some *numerology* of Dirac's electron radius involving *zwitterbewegung* where $\omega_{zitterbewegung} = 2mc^2/\hbar = 1.56\cdot 10^{21}$ *(radian)Hz*

$\omega_{zitterbewegung}\, r = c$ or $r_{Dirac} = c/\omega_{zitterbewegung} = \hbar/2mc = 1.93\cdot 10^{-13}\ m$ relates to the *Compton wavelength* $ƛ=\hbar/mc=3.8616\cdot 10^{-13}$ *m*



(Some crazy "thing" going c around a circle!)

Reduced Compton wavelength: $2\pi\, ƛ = h/mc = 2.4263\cdot 10^{-12}$ *or Compton "circumference"*

$2.4263102175 \pm 33\times 10^{-12}$ m

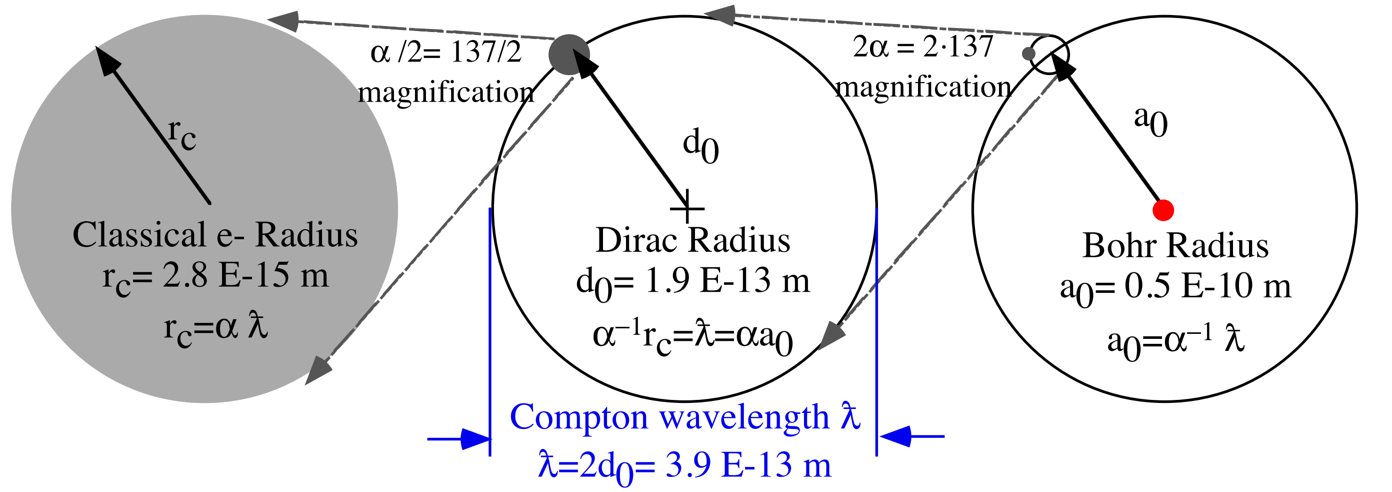


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant α = 1/137.

The classical radius of the electron defined by setting its electrostatic PE to $m_e c^2$:

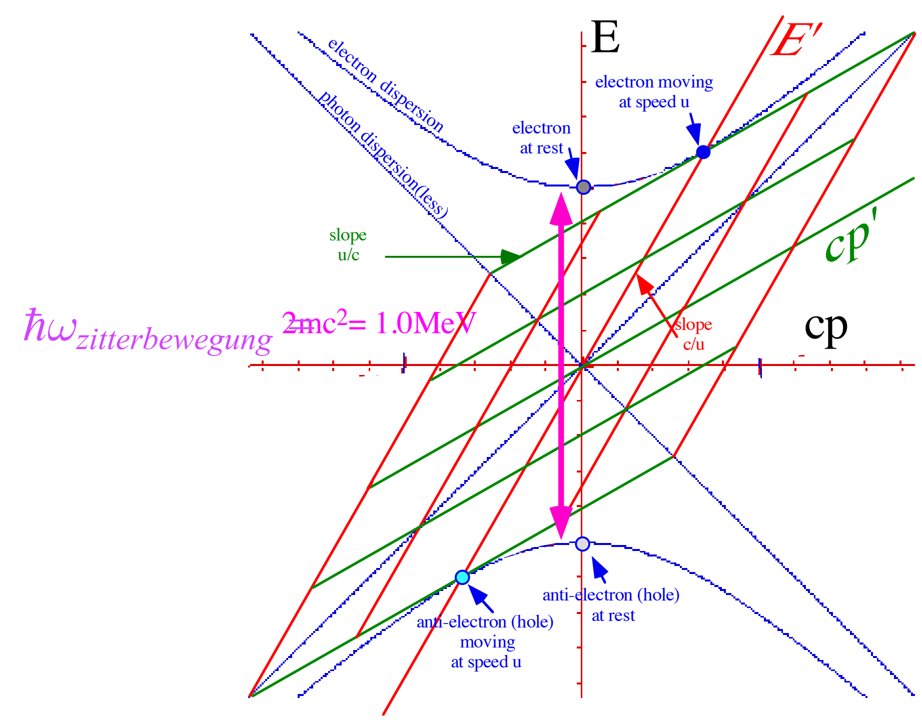
$$e^2/(4\pi e_0\, r_{classical}) = m_e c^2 \quad \text{or} \quad r_{classical} = e^2/(4\pi e_0\, m_e c^2) = 2.8\cdot 10^{-15}\ m.$$

Another fine-structure ratio to r_{Bohr}.

$$\frac{r_{Classical}}{r_{Bohr}} = \frac{e^2\ /4\pi\varepsilon_0 m_e c^2}{4\pi\varepsilon_0 \hbar^2\ /\ m_e e^2} = \left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)^2 = \left(\frac{1}{137.}\right)^2$$

As a final numerological exercise, find angular momentum $\ell = m_e vr$ of fictitious "*zitterbewegung*" orbit inside the electron.
With $v=c$ and $r = r_{Dirac}$ the following is obtained.

$$\ell = m_e c\, r_{Dirac} = m_e c\hbar/(2m_e c)$$
$$= \hbar/2$$



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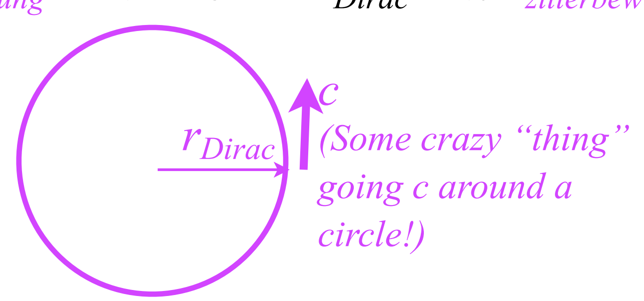
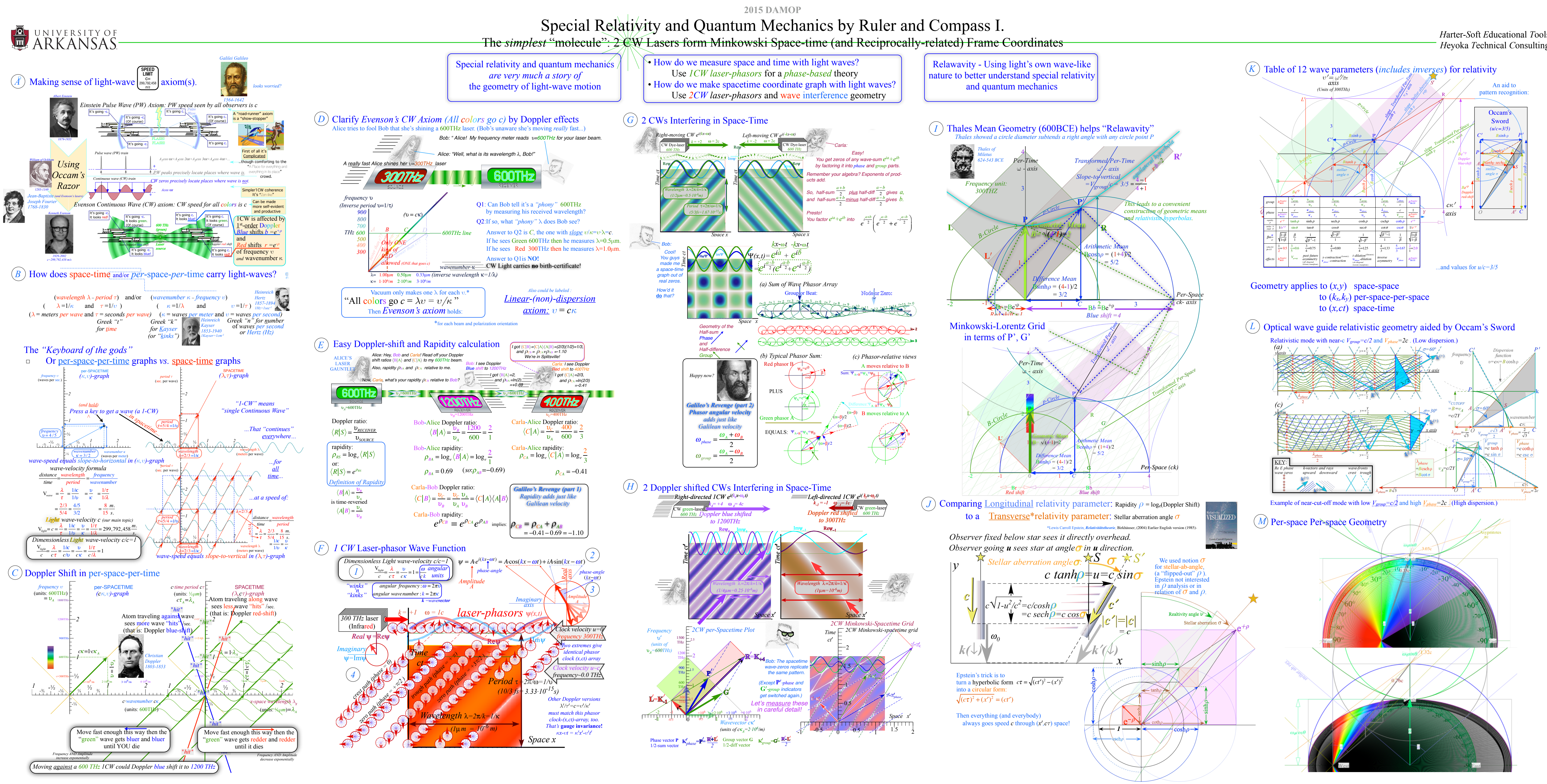
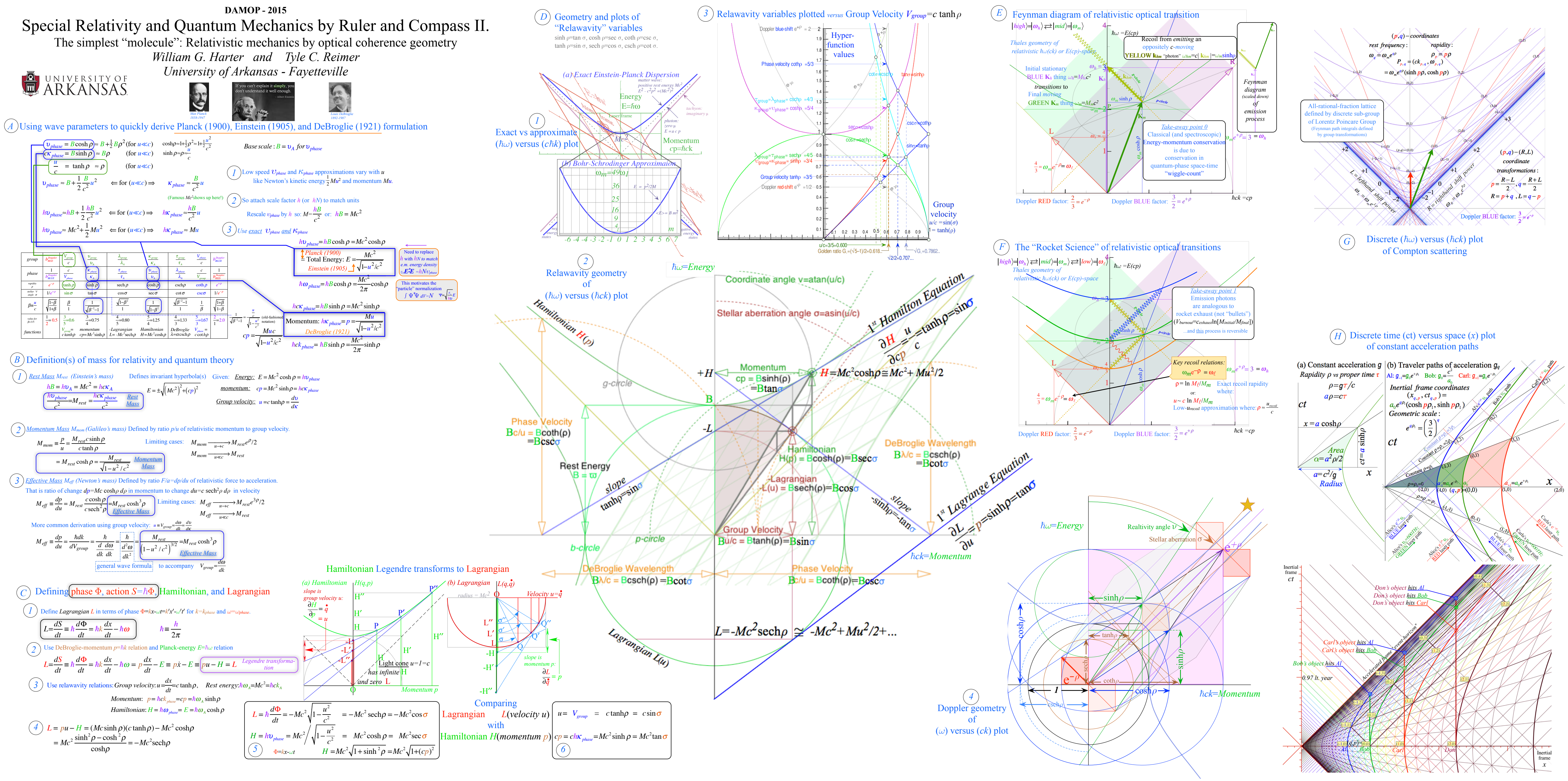




Link to pdf version of *Part I* online

Note: When printed at their optimal resolution, each poster is *7 feet across!*



[Link to pdf version of *Part II* online]

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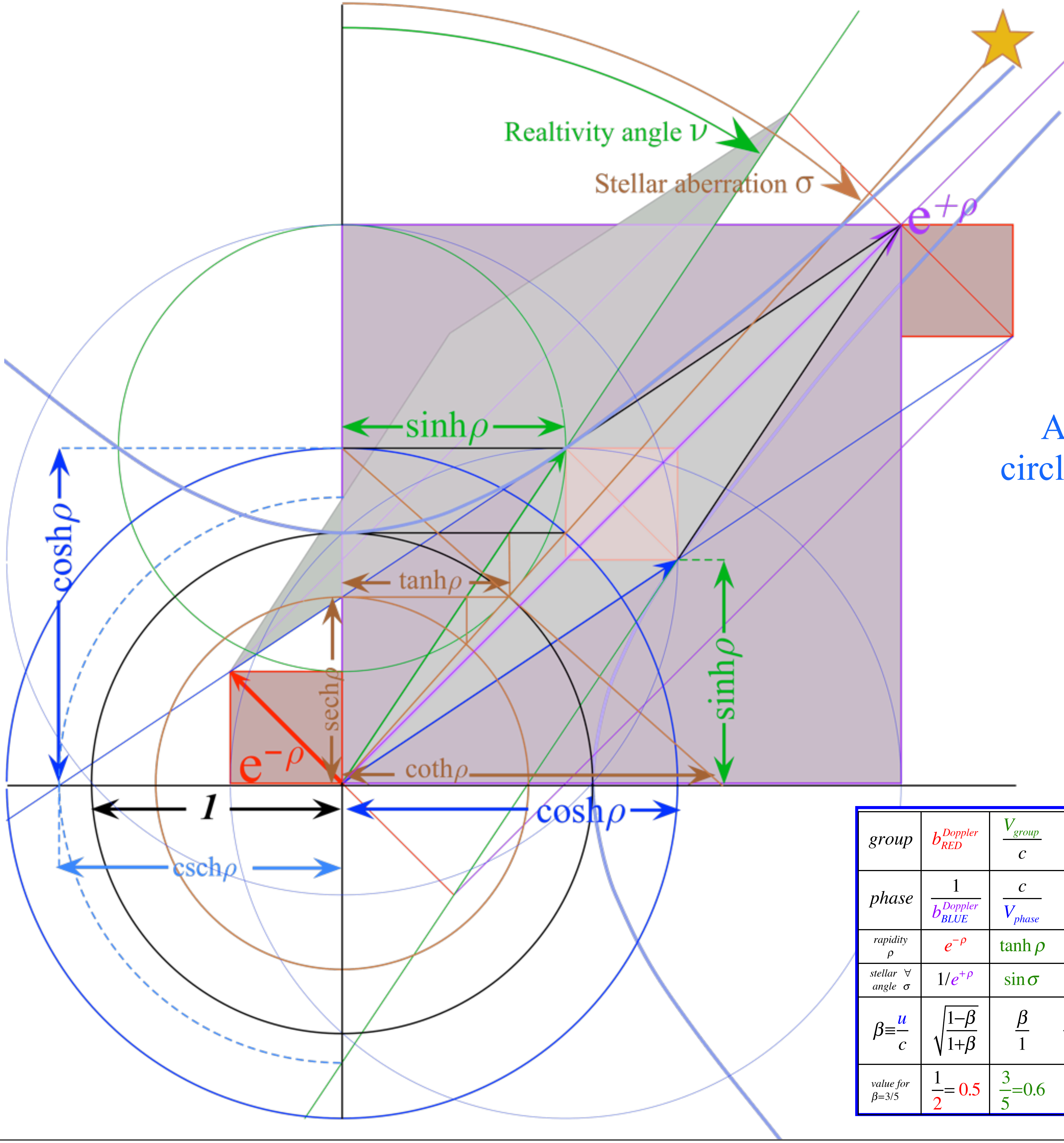


A more compact
circle-based geometry

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

from CMWith a BANG! Lecture 31
Thur. 12.10.2015

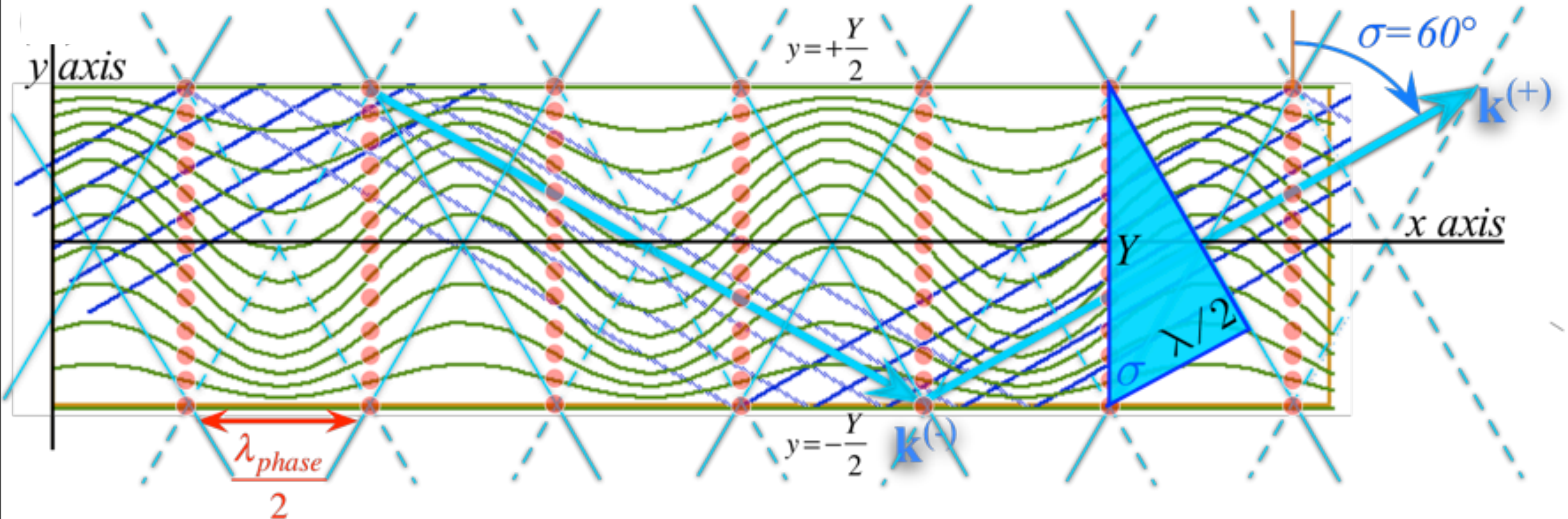
Xtra stuff: *Some numerology: Which is bigger...H-atom or an electron? What' spin?*
Space-Space waves gone mad

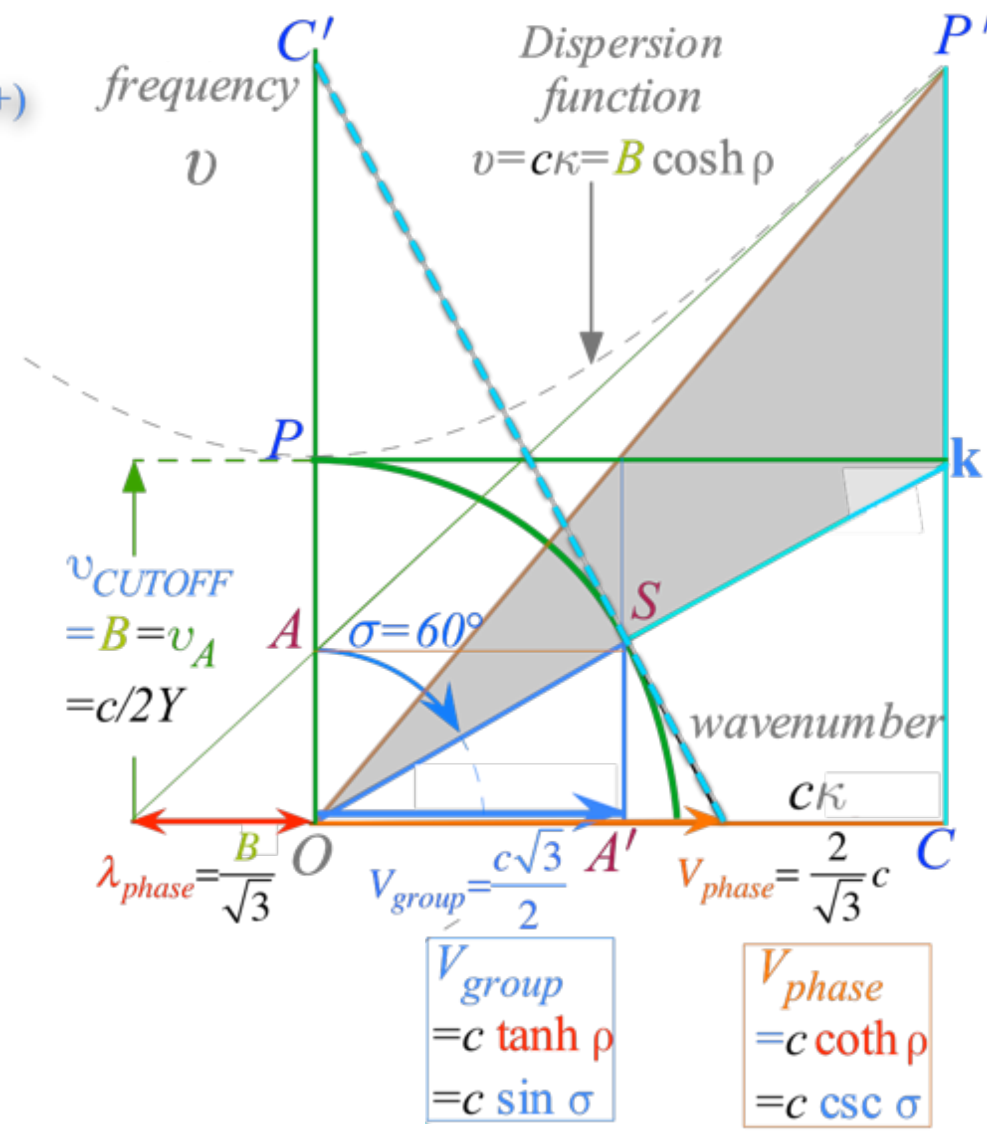
Applications to optical waveguide, spherical waves, and accelerator radiation

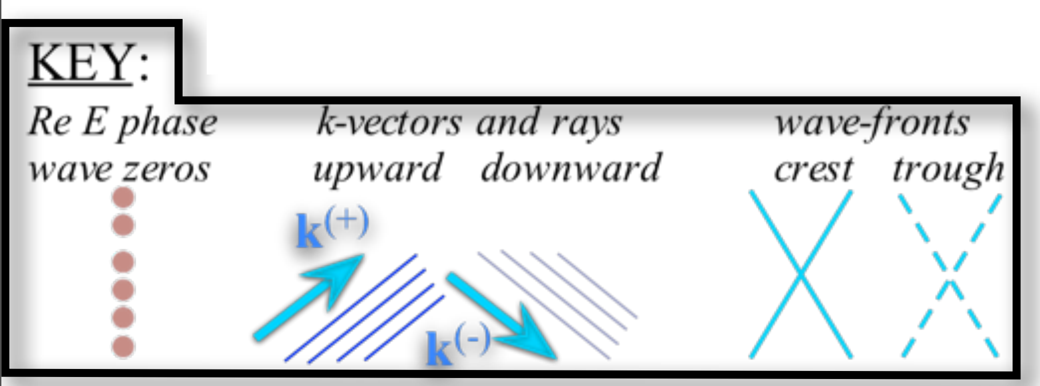
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c\sqrt{3}/2$ and $V_{phase}=2/\sqrt{3}c$. (Low dispersion.)







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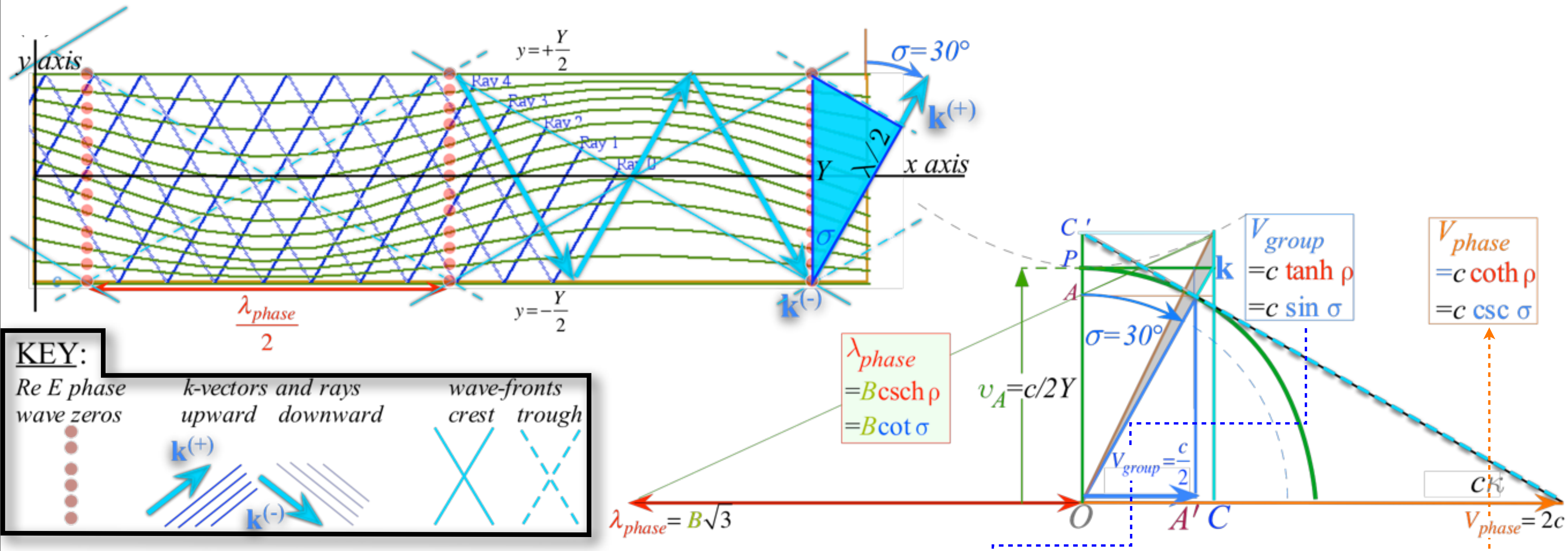
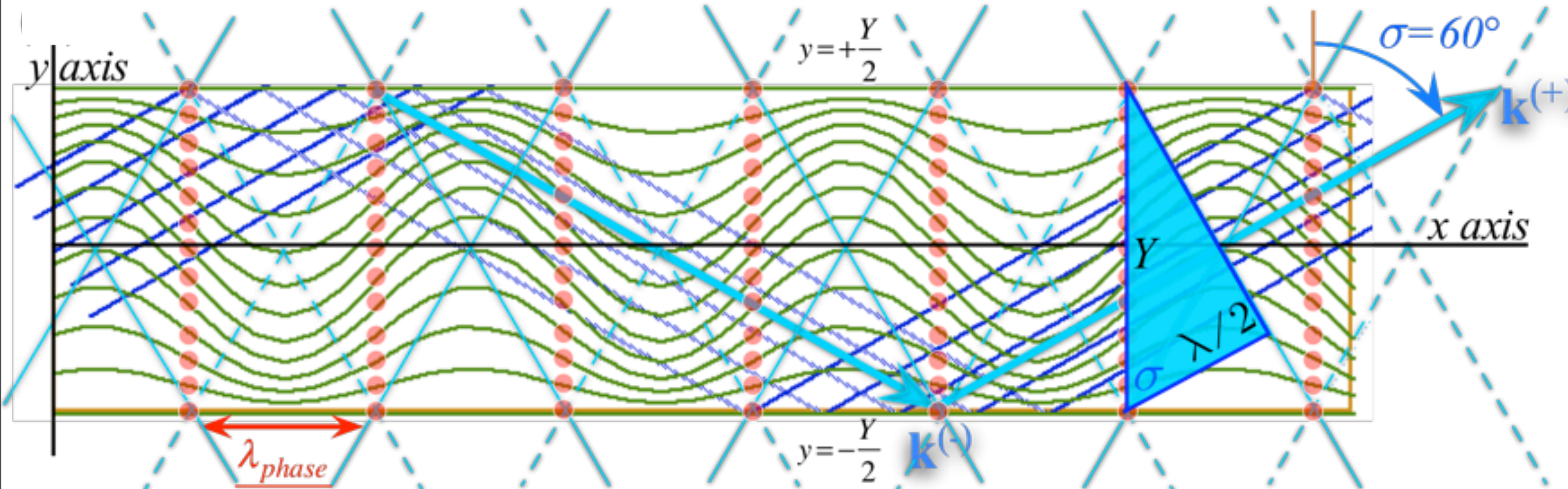


Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

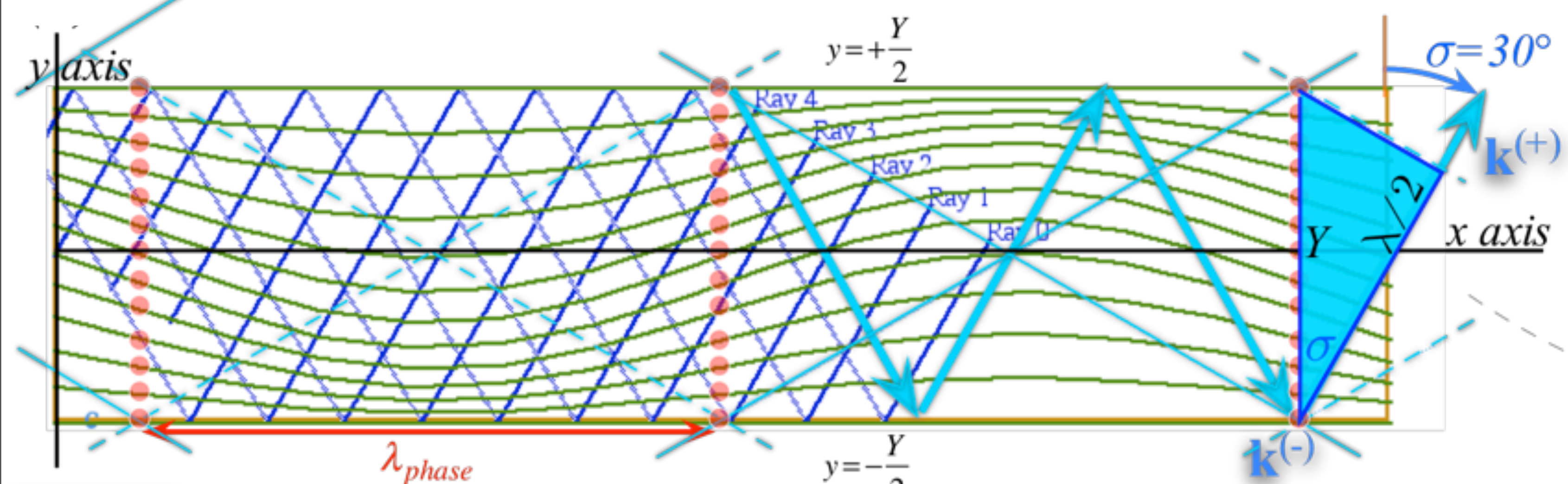
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

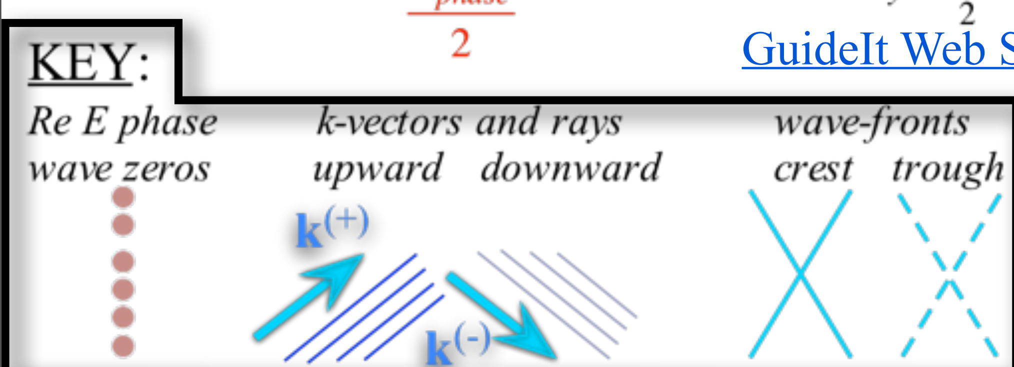
Relativistic mode with near-c $V_{group}=c\sqrt{3}/2$ and $V_{phase}=2/\sqrt{3}c$. (Low dispersion.)

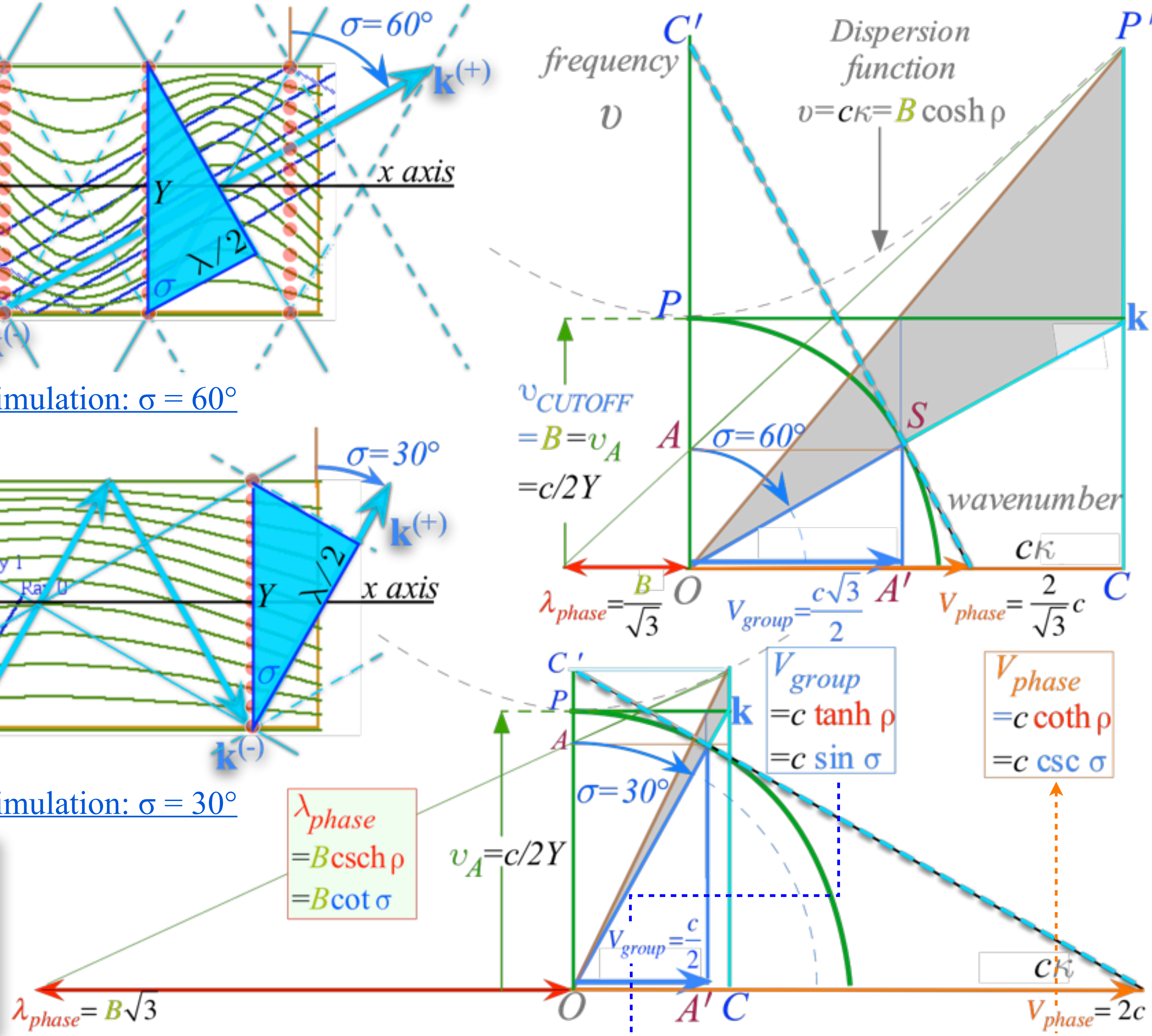


GuideIt Web Simulation: σ = 60°



GuideIt Web Simulation: σ = 30°





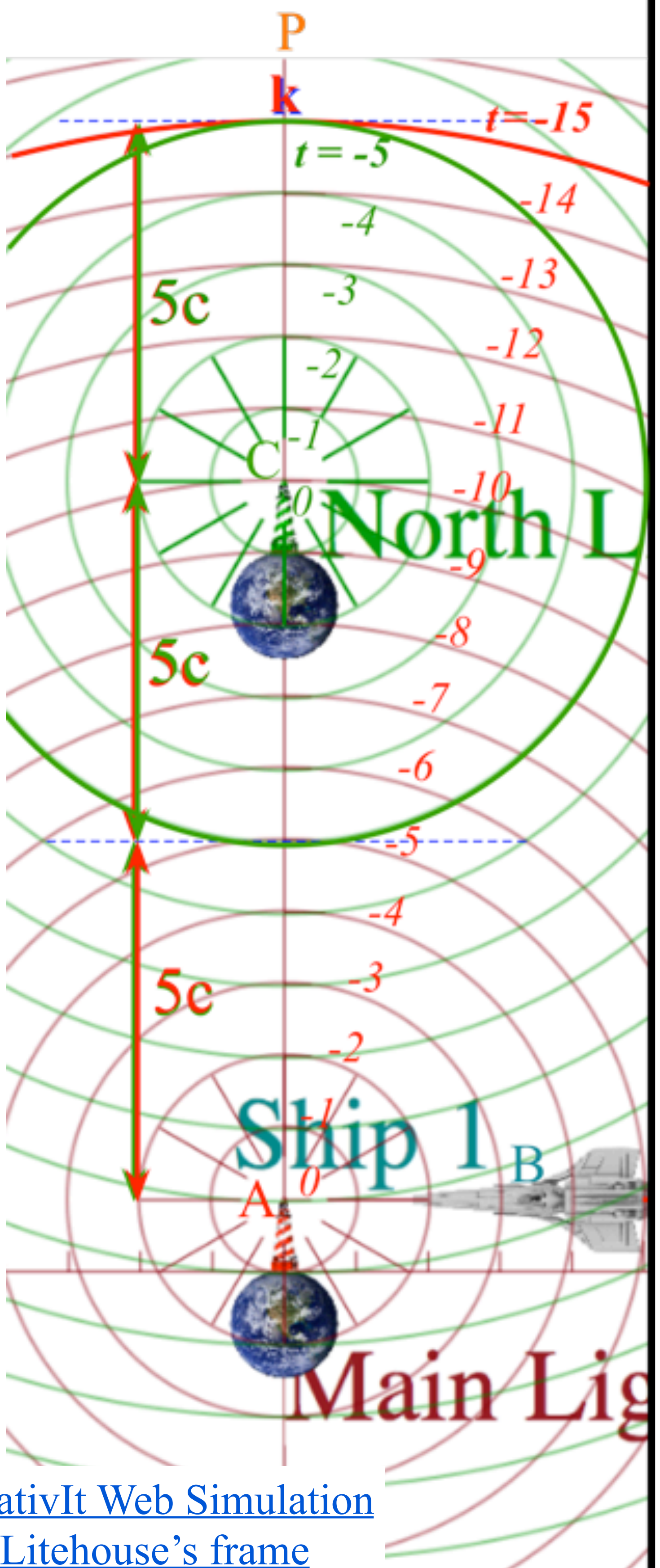
Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

Spherical wave relativistic geometry

Also, aided by Occam's Sword

(a) Spherical wave pair

In Alice-Carla frame

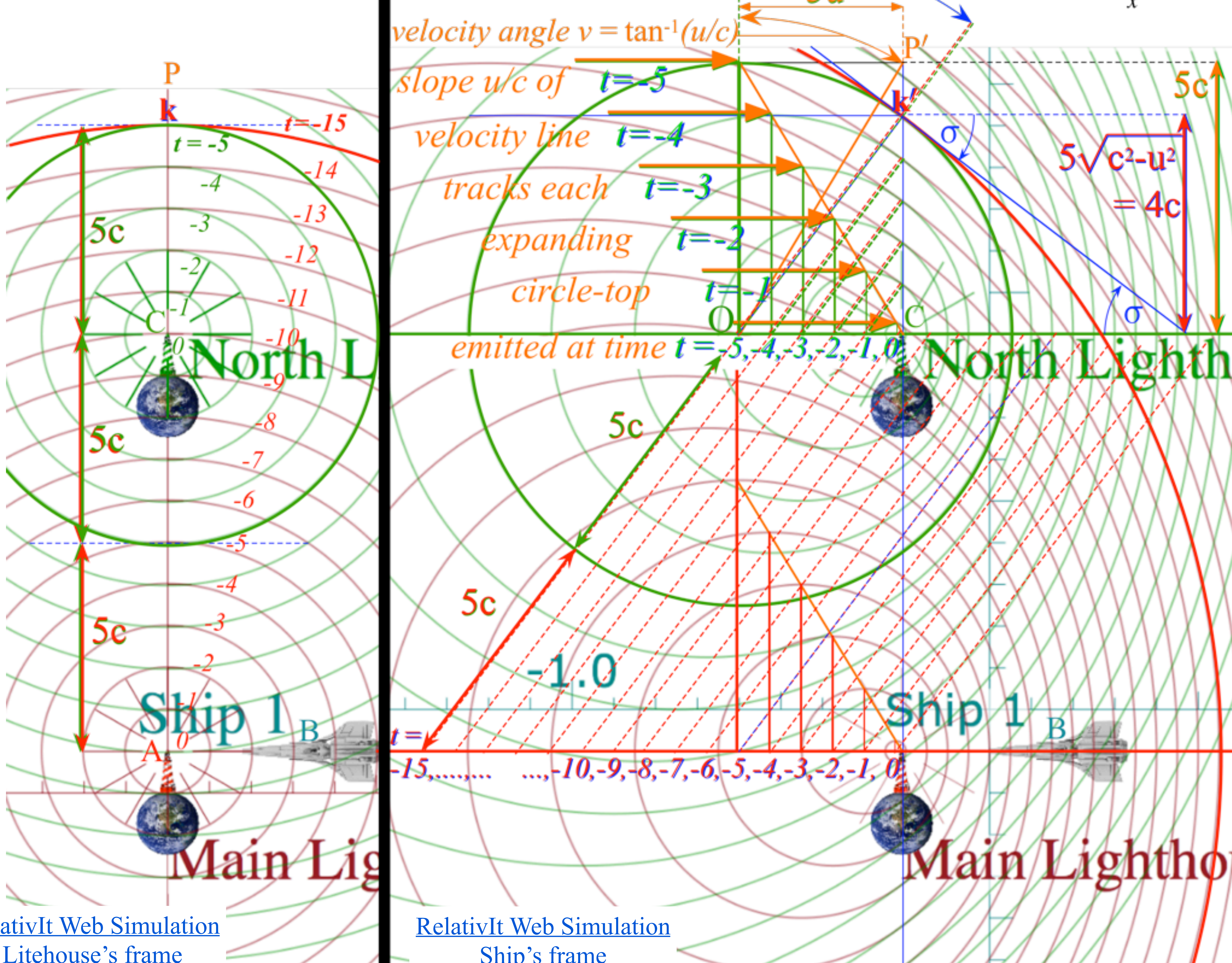


[RelativIt Web Simulation Litehouse's frame]

(a) Spherical wave pair
In Alice-Carla frame

(b) Spherical wave pair
In Bob's frame: $u_x/c = -3/5$

Occam Sword geometry in (x,y) space-space



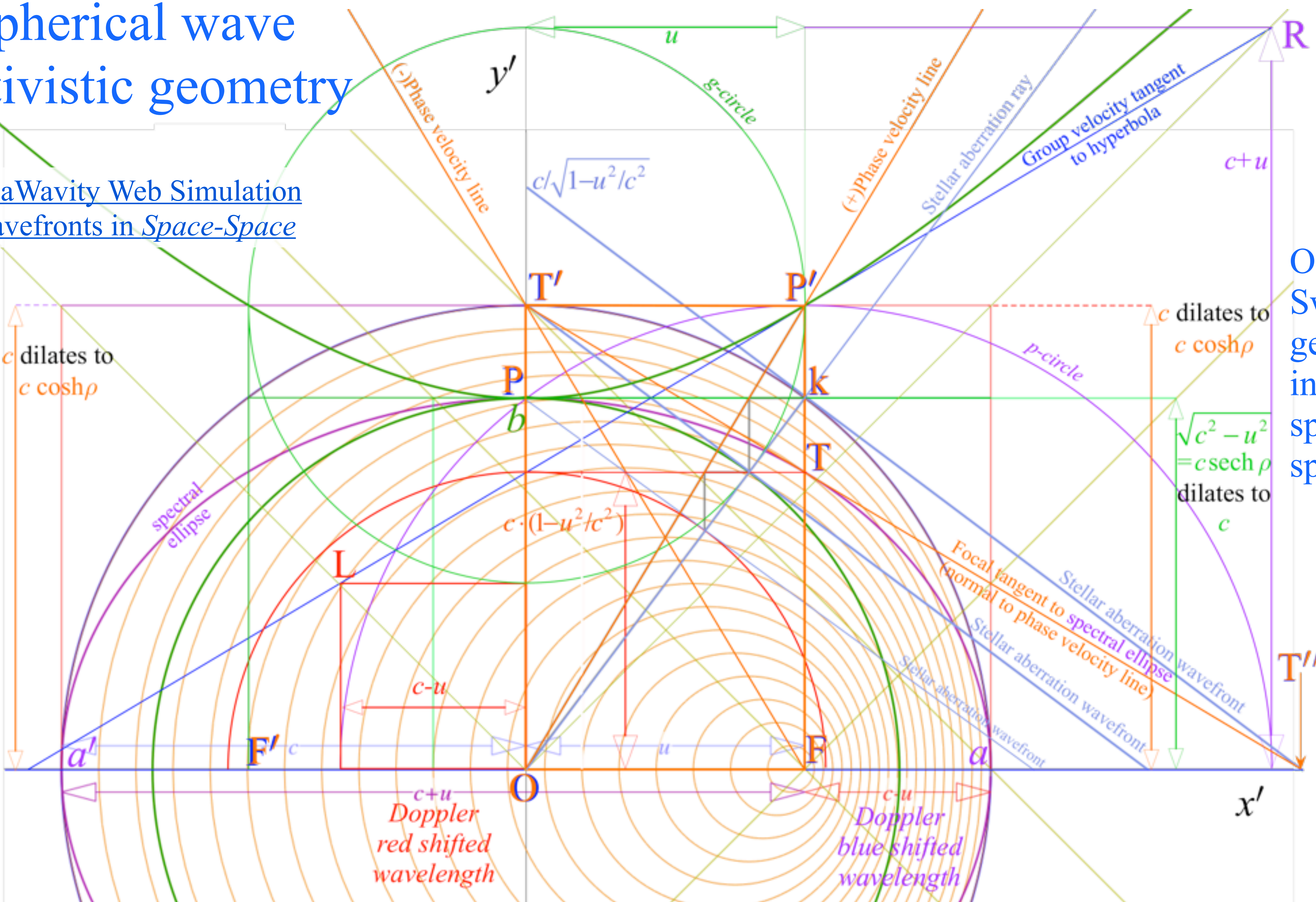
RelativIt Web Simulation
Litehouse's frame

RelativIt Web Simulation
Ship's frame

Spherical wave relativistic geometry

[RelaWavity Web Simulation Wavefronts in *Space-Space*]

Occam Sword geometry in (x,y) space-space



Doppler Red λ=c+u

dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse major radius a=OFa=c

dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor:

$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse focal length **FO**= $u = c\tanh\rho$

dilates to: $u\cosh\rho = c\sinh\rho$

ellipse latus radius **FT**=$c(1-u^2/c^2)$

dilates to: $c(1-u^2/c^2)\cosh\rho$

$= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

Doppler Blue λ=$c-u$

dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

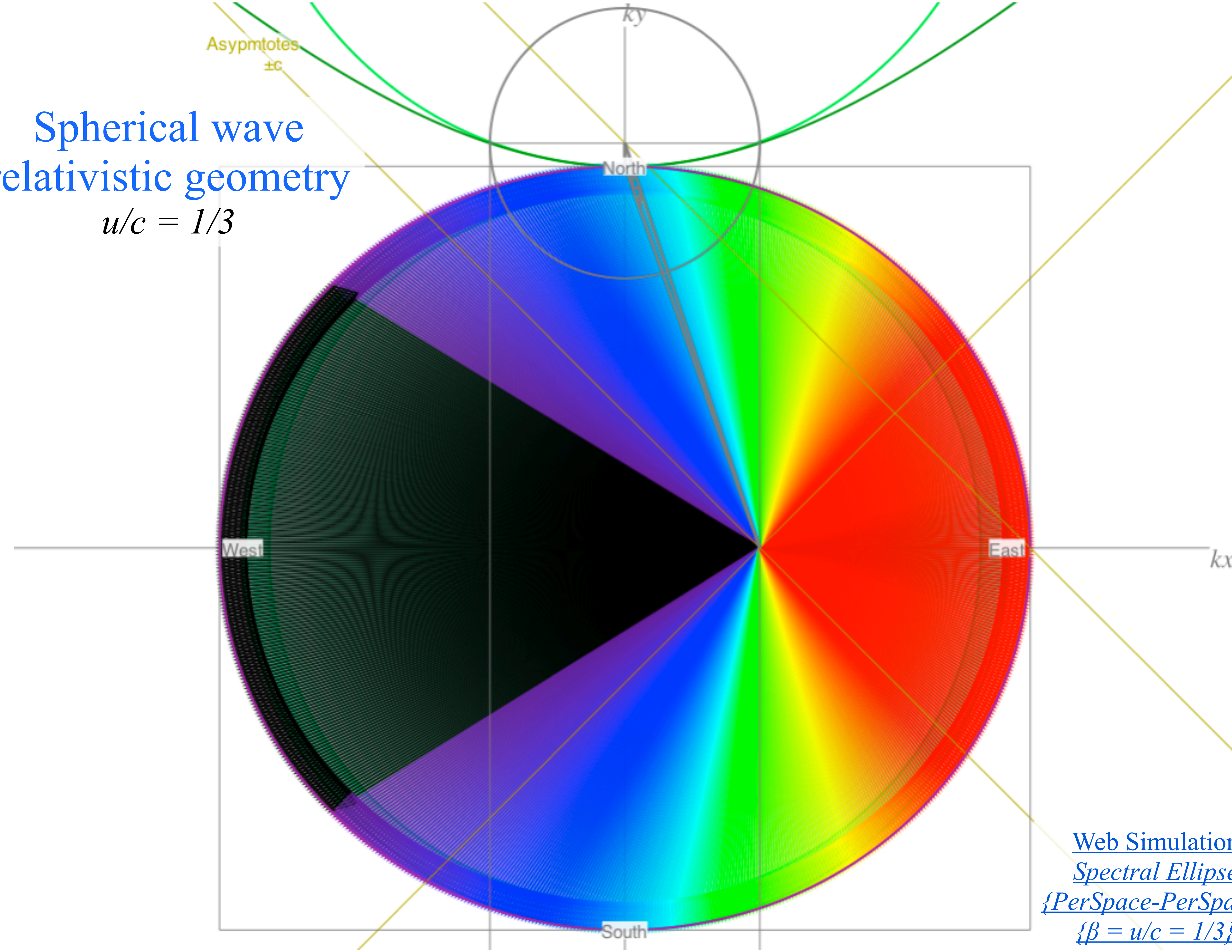
Base height **FTk**=$\sqrt{c^2-u^2}$

dilates to: $\sqrt{c^2-u^2}\cosh\rho = c$

(equal to ellipse minor radius b)

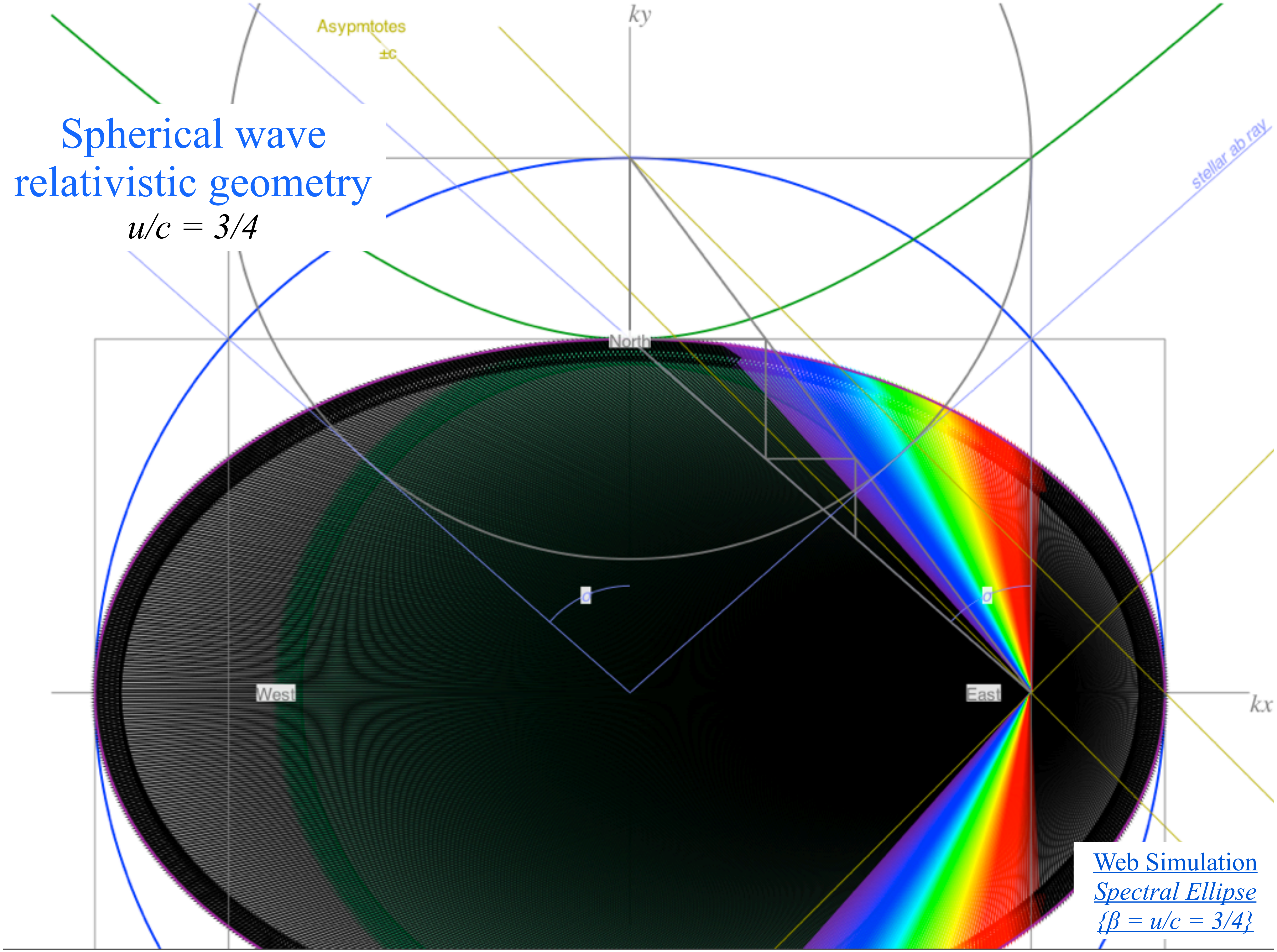
Spherical wave relativistic geometry

u/c = 1/3



[Web Simulation](#)
Spectral Ellipse
{PerSpace-PerSpace}
{β = u/c = 1/3}

Spherical wave relativistic geometry
u/c = 3/4



[Web Simulation
Spectral Ellipse
{β = u/c = 3/4}]