

Lecture 32 *Relativity*-Dynamics

Thursday 5.05.2016

Relativity: Spectroscopy, transitions, and acceleration

(Unit 3 p.45-61 - 4.26.16)

Relativity relates charge, current, and magnetic fields

Geometric derivation of magnetic constant μ_0 from electric ϵ_0

Lorentz-Poincare symmetry and energy-momentum spectral conservation rules

Review of 2nd-quantization “photon” number N and 1st-quantization wavenumber $\kappa = m$

Sketches of atomic and molecular spectroscopy

Relativistic optical transitions and Compton recoil formulae

Feynman diagram geometry

Recoils shifts

Compton recoil related to rocket velocity formula

Geometric transition coordinate grids

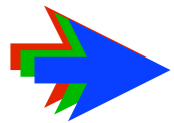
Relativity in accelerated frames

Laser up-tuning by Alice and down-tuning by Carla makes g -acceleration grid

Analysis of constant- g grid compared to zero- g Minkowski frame

Animation of mechanics and metrology of constant- g grid

Xtra stuff: *Some numerology: Which is bigger...H-atom or an electron? What's spin?*
Space-Space waves gone mad



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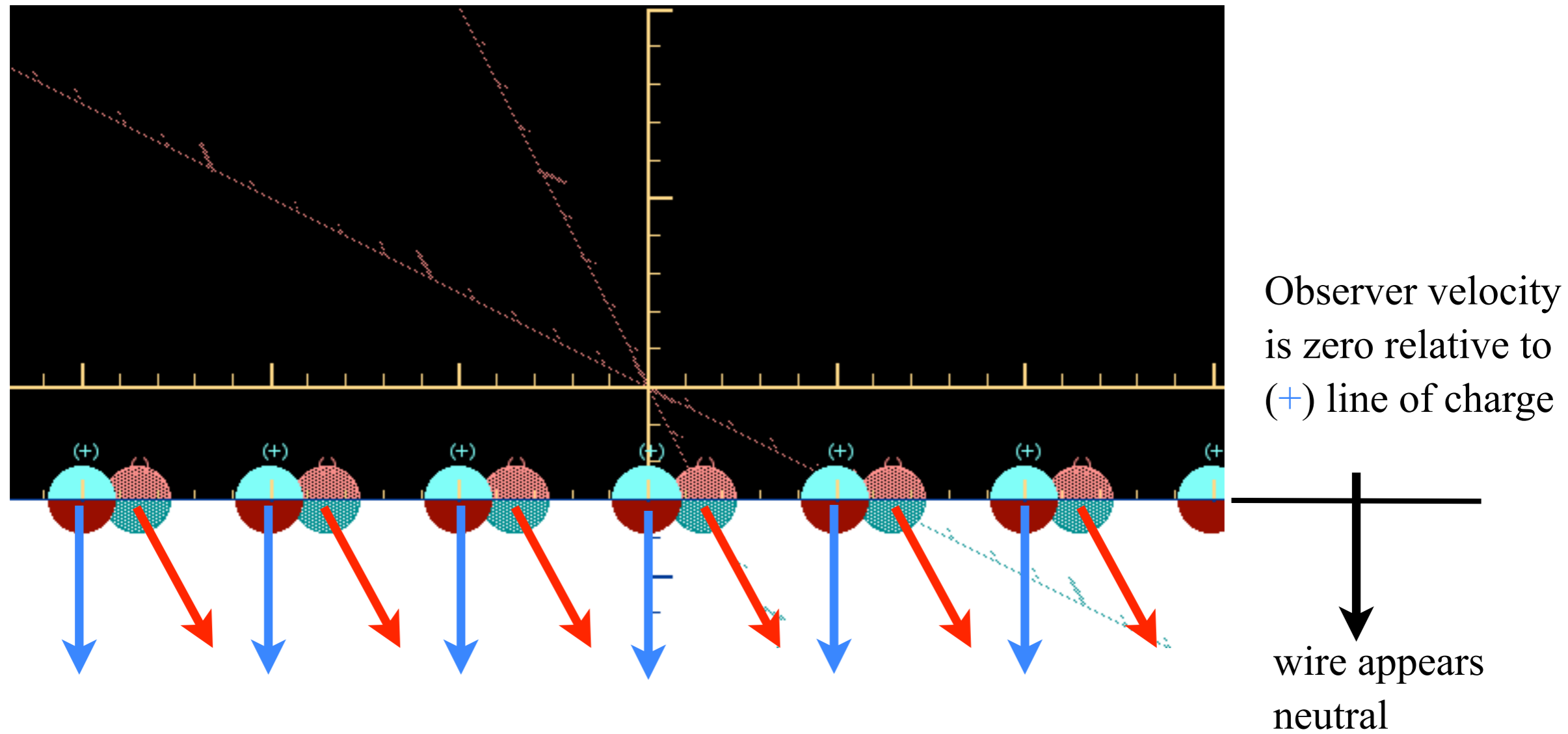
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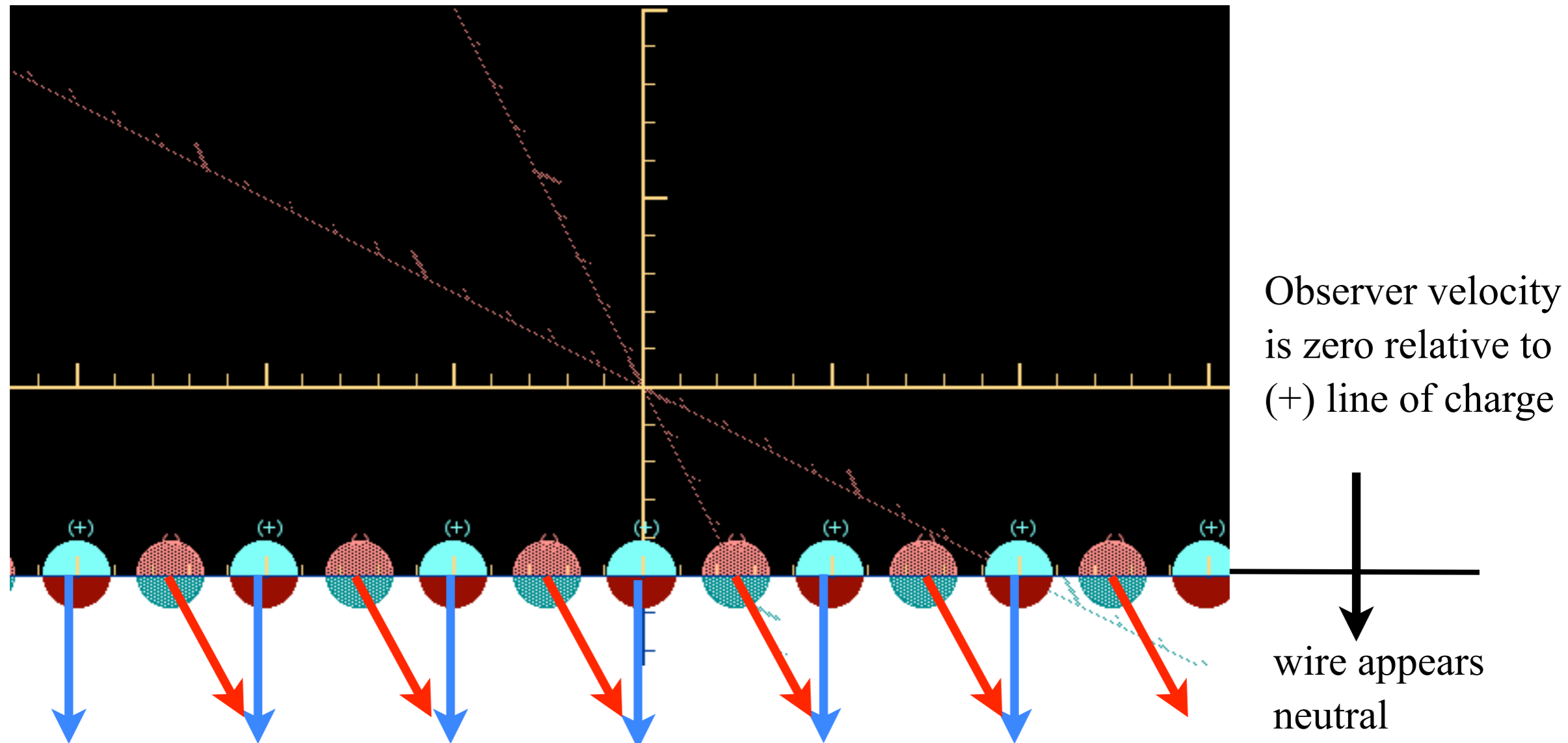
Relativistic effects on charge, current, and Maxwell Fields



(+) Charge fixed (-) Charge moving to right (*Negative current density*)

(+) Charge density is Equal to the (-) Charge density

Relativistic effects on charge, current, and Maxwell Fields



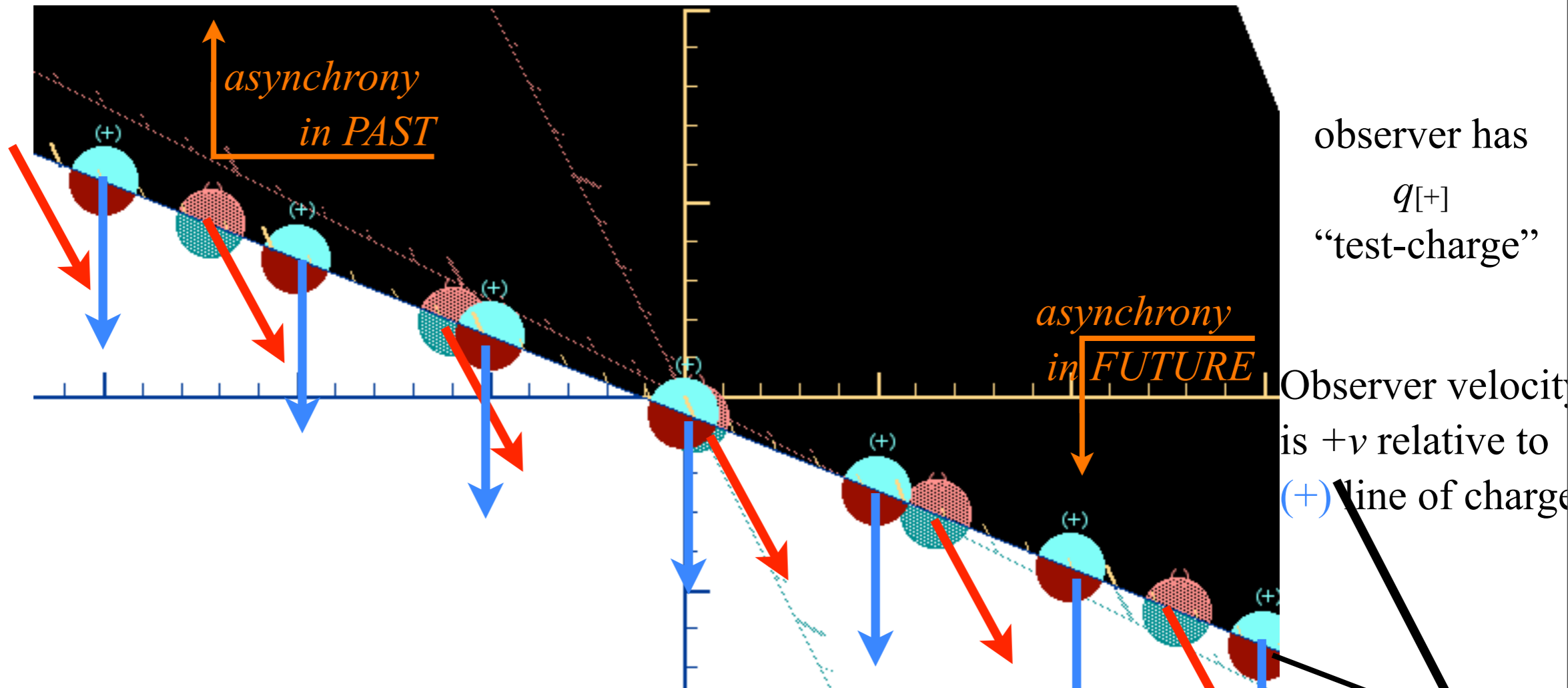
- (+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{j}(x,t)$)
- (+) Charge density is Equal to the (-) Charge density (*Zero* $\rho(x,t)=0$)

Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz *asynchrony*

Asynchrony due to off-diagonal $\sinh \rho$ (a 1st-order effect)

in Lorentz transform:
$$\begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \sim \begin{pmatrix} 1 & v/c \\ v/c & 1 \end{pmatrix}$$



(+) Charge fixed (-) Charge moving to right (Negative current density $\vec{j}(x,t)$)
 (+) Charge density is Greater than (-) Charge density (Positive $\rho(x,t) > 0$)

observer has $q_{[+]}$ "test-charge"

Observer velocity is $+v$ relative to (+) line of charge

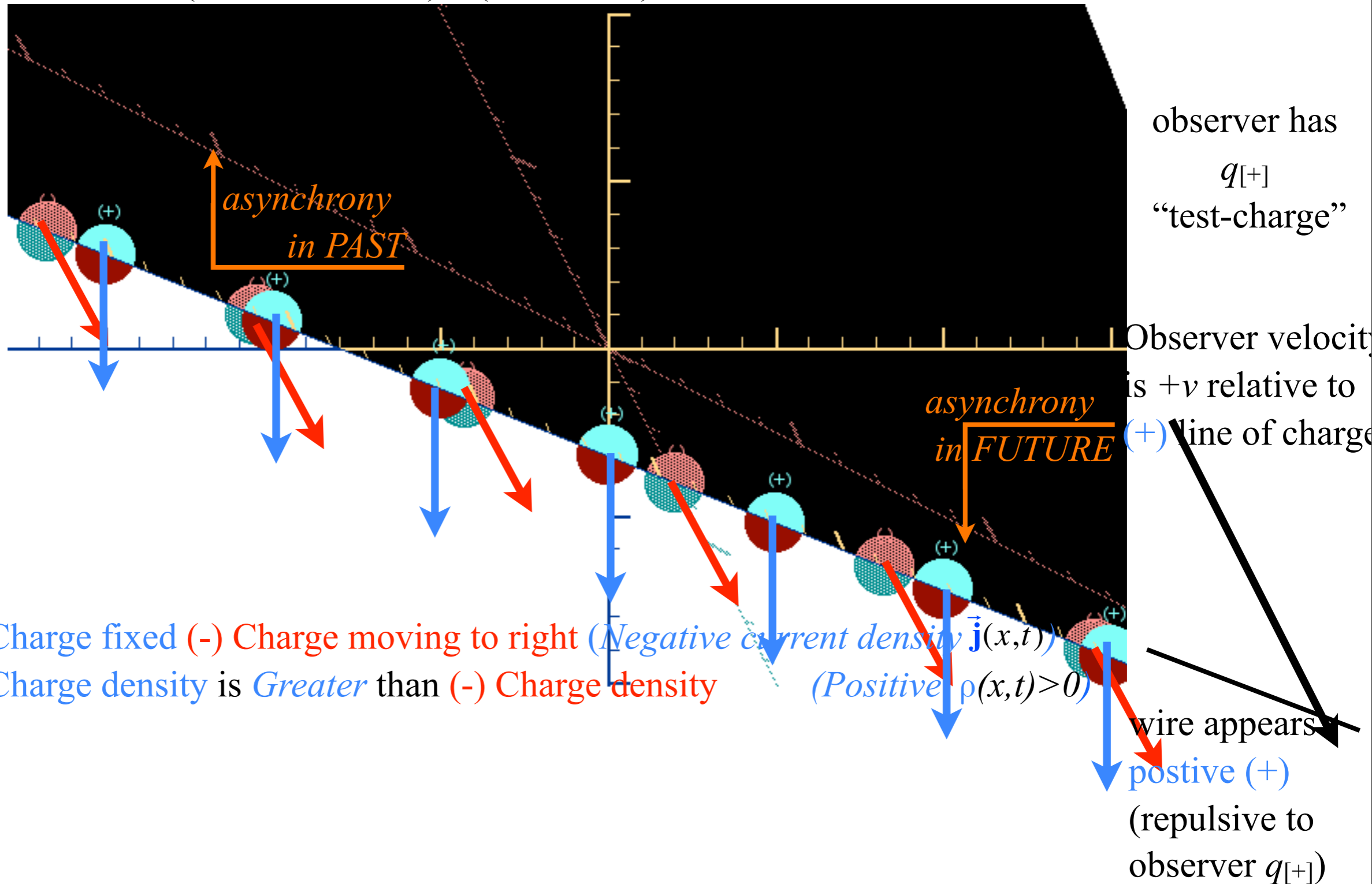
wire appears positive (+) (repulsive to observer $q_{[+]}$)

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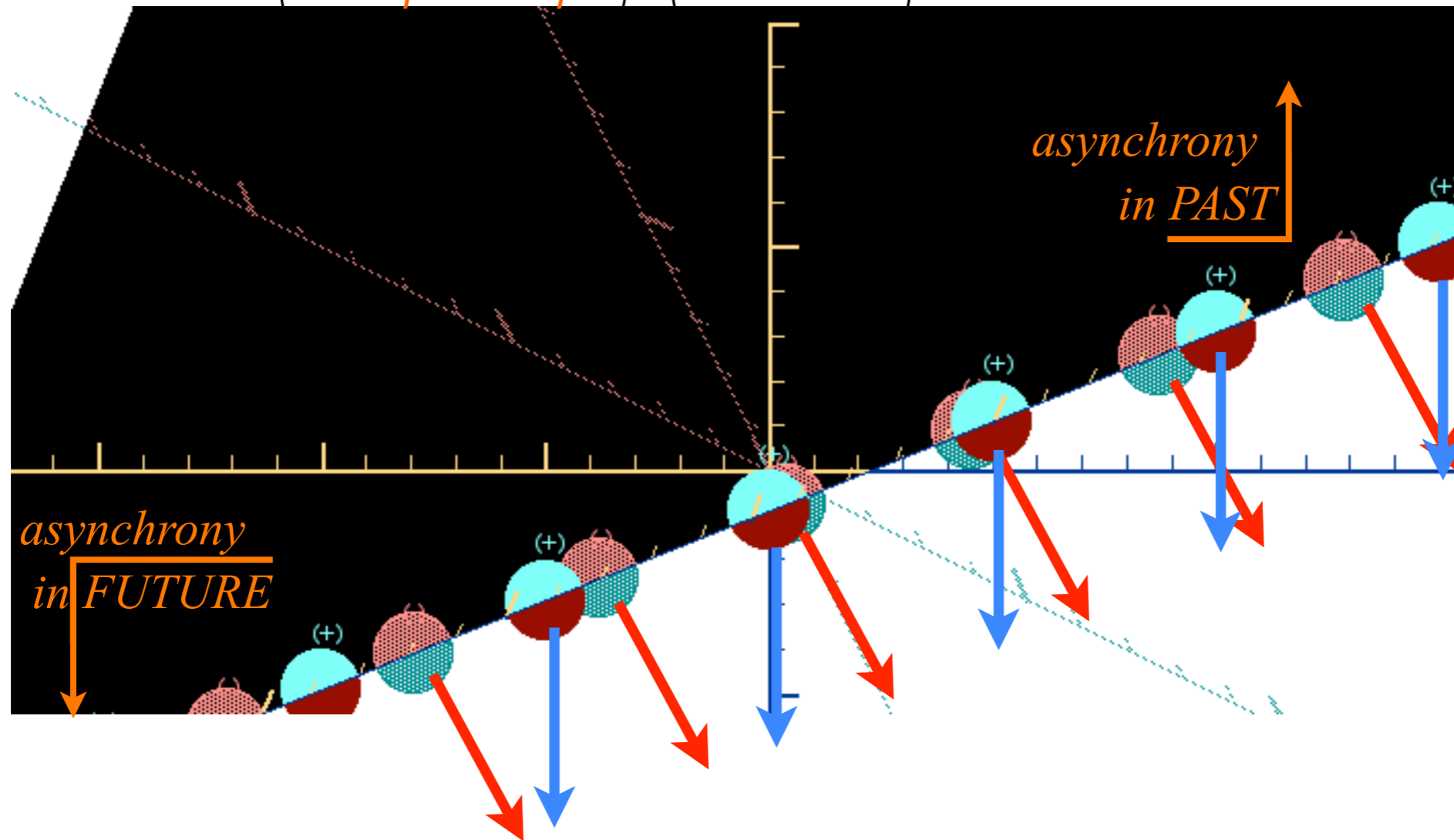
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observer has

$q_{[+]}$

“test-charge”

Observer velocity is $-v$ relative to (+) line of charge



wire appears **negative (-)** (attractive to observer $q_{[+]}$)

(+) Charge fixed (-) Charge moving to right (Negative current density $\vec{j}(x,t)$)

(+) Charge density is *Less* than (-) Charge density (Negative $\rho(x,t) < 0$)

Relativistic effects on charge, current, and Maxwell Fields

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Asynchrony due to off-diagonal $\sinh \rho$ (a 1st-order effect)

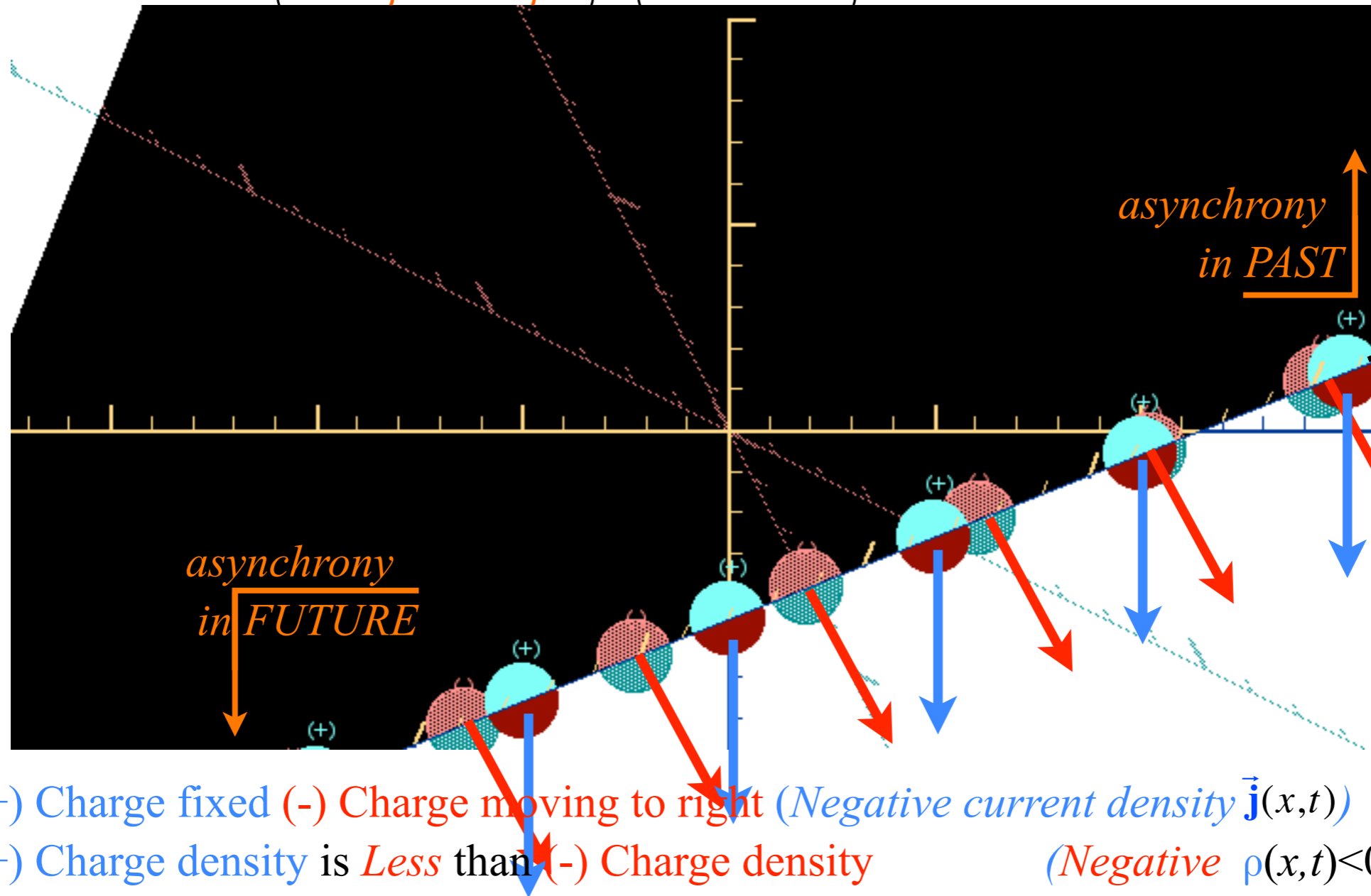
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observer has

$q_{[+]}$

“test-charge”

Observer velocity is $-v$ relative to $(+)$ line of charge

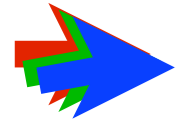


asynchrony in PAST

asynchrony in FUTURE

wire appears **negative (-)** (attractive to observer $q_{[+]}$)

- (+) Charge fixed (-) Charge moving to right (*Negative current density* $\vec{j}(x,t)$)
- (+) Charge density is *Less* than (-) Charge density (*Negative* $\rho(x,t) < 0$)



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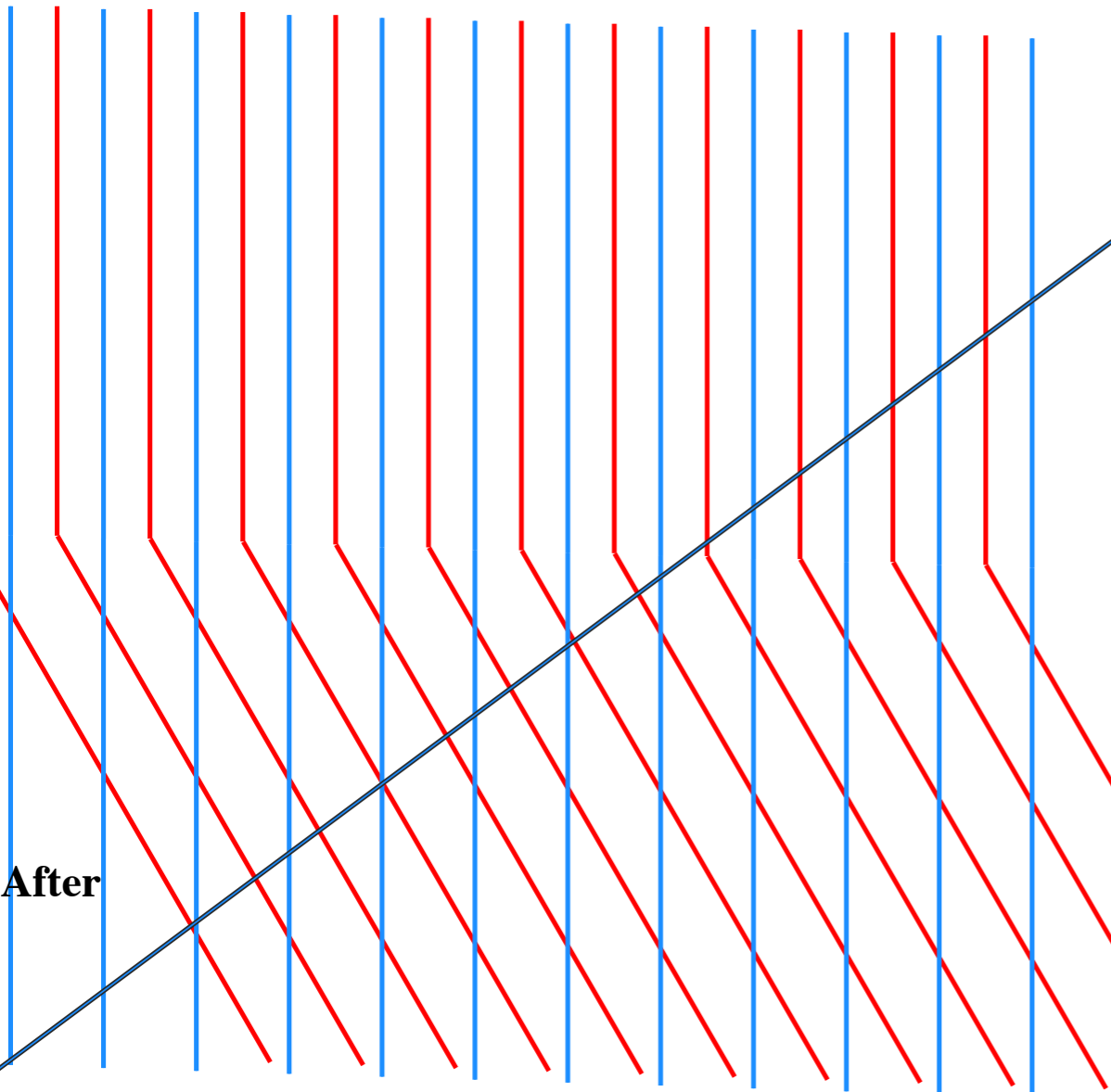
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Animation of mechanics and metrology of constant- g grid

Simple 1st-order relativistic geometry of magnetism

Before



After

If Black is moving to Left

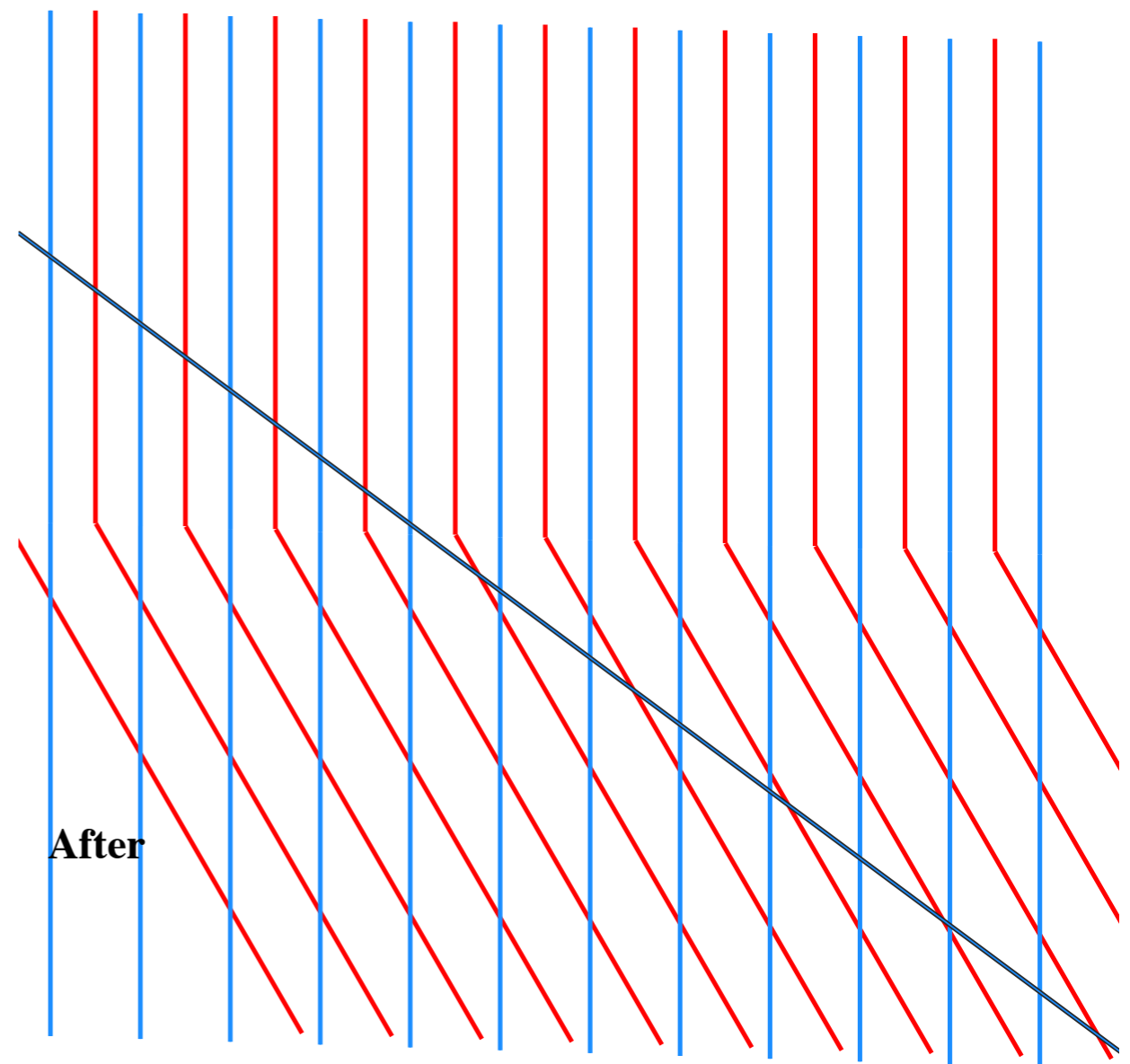
Before red starts moving to right

Black sees same number of red and blue

After red starts moving to right

Black sees more red than blue

Before



After

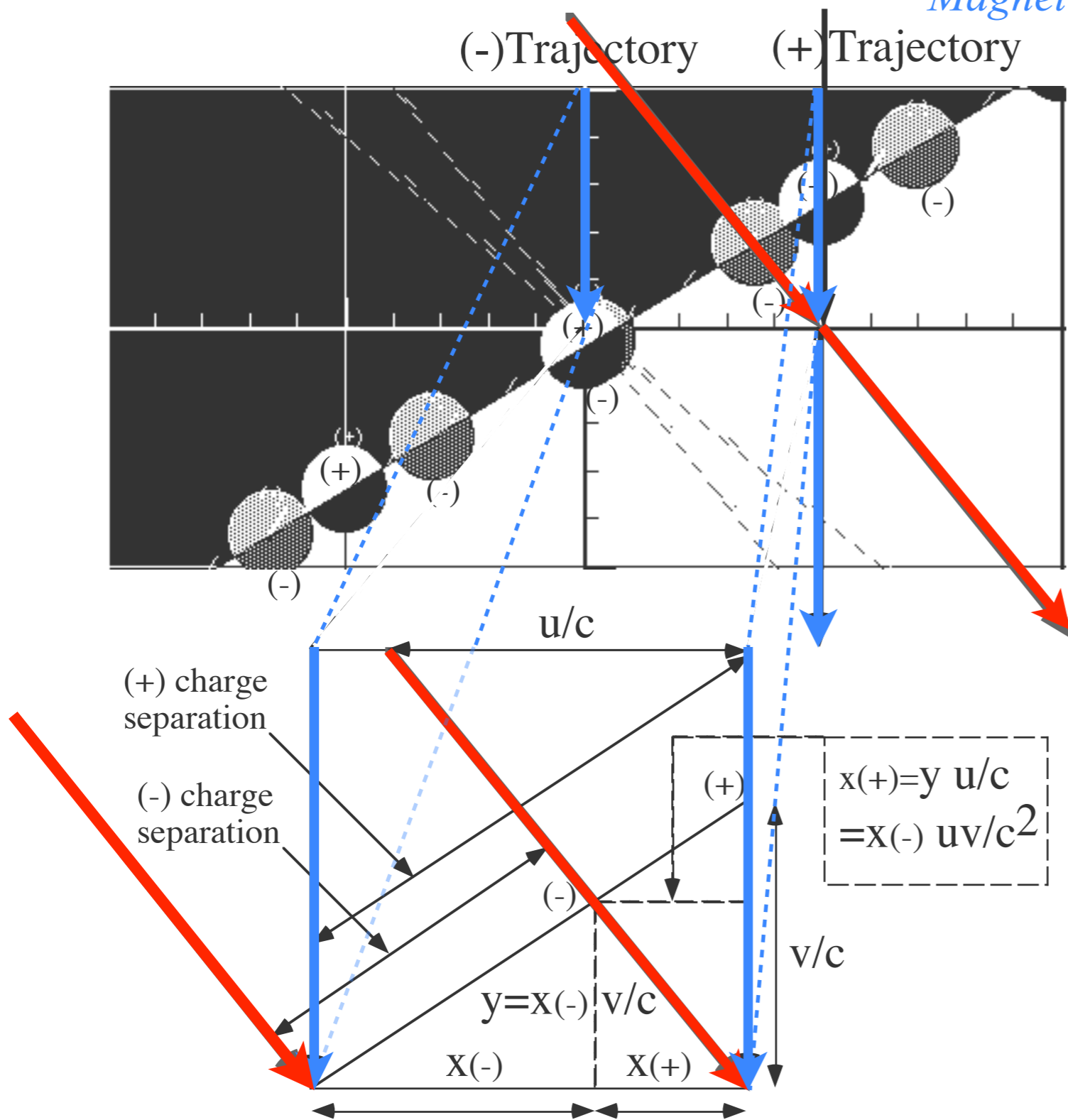
If Black is moving to Right

Before red starts moving to right

Black sees same number of red and blue

After red starts moving to right

Black sees more blue than red

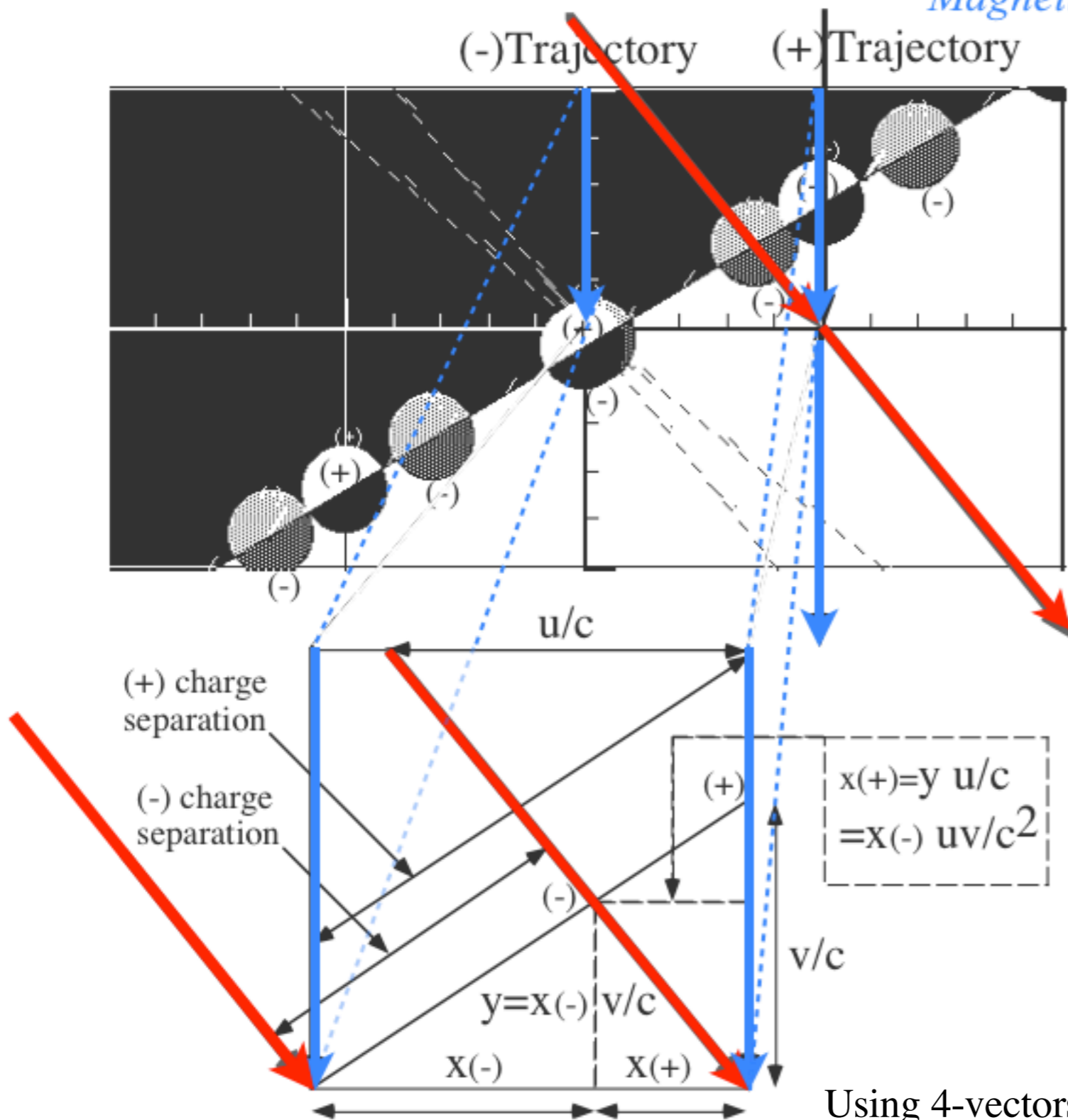


$$\frac{\rho(-)}{\rho(+)} = \frac{(+)\text{ charge separation}}{(-)\text{ charge separation}} = \frac{x(+)+x(-)}{x(-)}$$

$$\frac{\rho(-)}{\rho(+)} = \frac{x(+)}{x(-)} + 1 = \frac{uv}{c^2} + 1$$

$$\rho(+)-\rho(-) = \rho(+)\left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2}\rho(+)$$

Unit square: $(u/c) / 1 = x(+)/y$
 $(v/c) / 1 = y/x(-)$



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Unit square: $(u/c) / 1 = x(+)/y$
 $(v/c) / 1 = y/x(-)$

Using 4-vectors to EL Transform (charge-current) = $(c\rho, \mathbf{j})$

$$\begin{pmatrix} c\rho' \\ j_{x'} \\ j_{y'} \\ j_{z'} \end{pmatrix} = \begin{pmatrix} \cosh \rho & \sinh \rho & \cdot & \cdot \\ \sinh \rho & \cosh \rho & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{pmatrix} c\rho \\ j_x \\ j_y \\ j_z \end{pmatrix}$$

Magnetic B-field is relativistic $\sinh\rho$ 1st order-effect

The electric force field \mathbf{E} of a charged line varies inversely with radius. The Gauss formula for force in mks units :

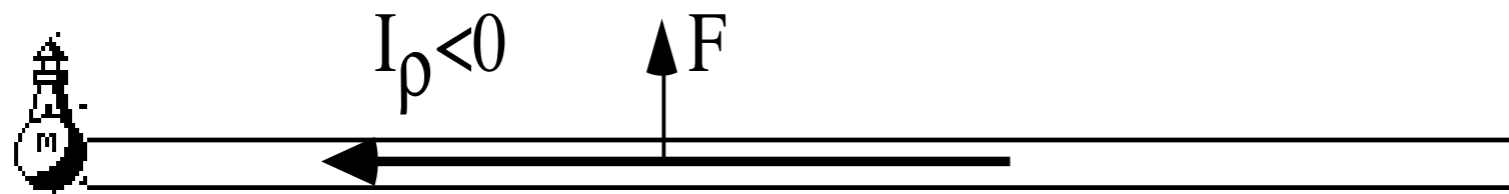
$$F = qE = q \left[\frac{1}{4\pi\epsilon_0} \frac{2\rho}{r} \right], \quad \text{where: } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{N \cdot m^2}{\text{Coul.}}$$

$$F = qE = q \left[\frac{1}{4\pi\epsilon_0} \frac{2}{r} \left(-\frac{uv}{c^2} \rho(+)\right) \right] = -\frac{2qv\rho(+)}{4\pi\epsilon_0 c^2 r} = -2 \times 10^{-7} \frac{I_q I_\rho}{r}$$

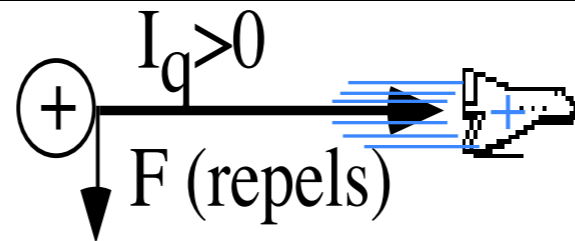
$$1/4\pi\epsilon_0 = 9 \cdot 10^9$$

$$c^2 = 9 \cdot 10^{16}$$

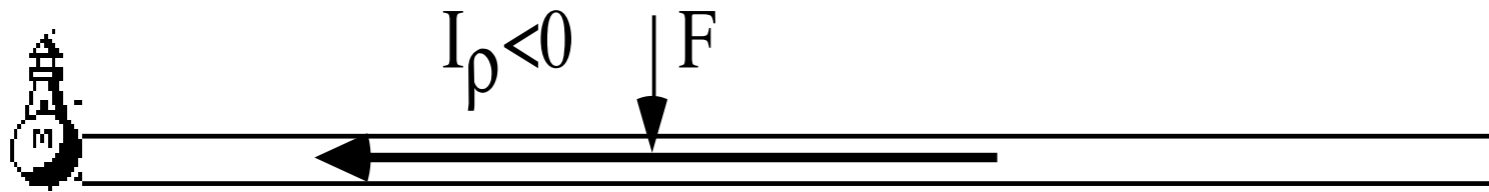
$$1/(4\pi\epsilon_0 c^2) = 10^{-7}$$



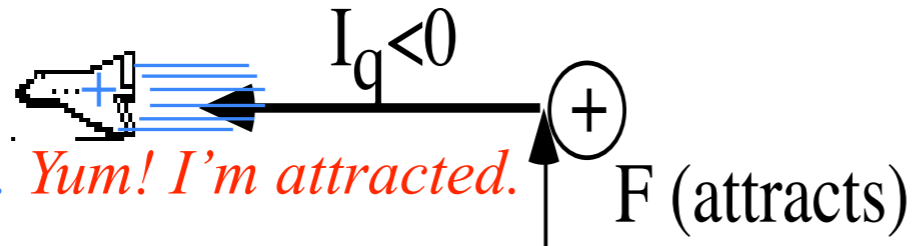
Right moving ship holding (+)-charge



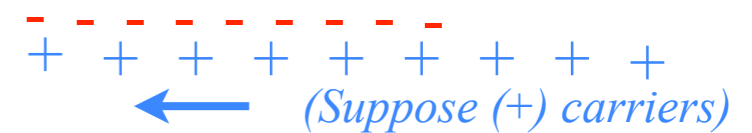
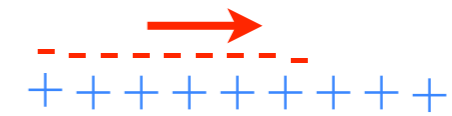
I see excess (+) charge up there. Yuk! I'm repelled.



I see excess (-) charge up there. Yum! I'm attracted.

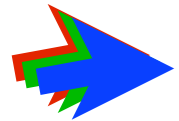


Left moving ship holding (+)-charge



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Lorentz-Poincare symmetry and $(E, c\mathbf{P})$ spectral conservation rules

Evenson axiom has symmetry based on isotropy of space-time and invariance to translation operation $\mathbf{T}(\vec{\delta}, \tau)$ having plane wave eigenfunctions $\langle x, t | \psi_{\mathbf{k}, \omega} \rangle$ and roots-of-unity eigenvalues.

$$\mathbf{T}(\vec{\delta}, \tau) | \psi_{\mathbf{k}, \omega} \rangle = e^{i(\mathbf{k} \cdot \vec{\delta} - \omega \cdot \tau)} | \psi_{\mathbf{k}, \omega} \rangle \quad (\text{eigen-ket relation}) \qquad \langle \psi_{\mathbf{k}, \omega} | \mathbf{T}^\dagger(\vec{\delta}, \tau) = \langle \psi_{\mathbf{k}, \omega} | e^{-i(\mathbf{k} \cdot \vec{\delta} - \omega \cdot \tau)} \quad (\text{eigen-bra relation})$$

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This also applies to N -particle product states $\Psi_{\mathbf{K}, \Omega} = \psi_{\mathbf{k}_1, \omega_1} \psi_{\mathbf{k}_2, \omega_2} \cdots \psi_{\mathbf{k}_N, \omega_N}$ where exponents add (k, ω) -values of each constituent in $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 + \dots$ and $\Omega = \omega_1 + \omega_2 + \dots$ so $\mathbf{T}(\vec{\delta}, \tau)$ -eigenvalues have the form $e^{i(\mathbf{K} \cdot \vec{\delta} - \Omega \cdot \tau)}$.

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Exponents must cancel for all (δ, τ)

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This requires that $\langle \Psi'_{\mathbf{K}', \Omega'} | \mathbf{U} | \Psi_{\mathbf{K}, \Omega} \rangle = 0$ unless: $\mathbf{K}' = \mathbf{K}$ and: $\Omega' = \Omega$.

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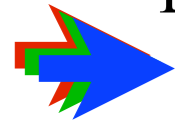
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That's TOTAL momentum ($\mathbf{P} = \hbar \mathbf{K}$) and energy ($E = \hbar \Omega$) conservation!

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Laser up-tuning by Alice and down-tuning by Carla makes g -acceleration grid

Analysis of constant- g grid compared to zero- g Minkowski frame

Animation of mechanics and metrology of constant- g grid

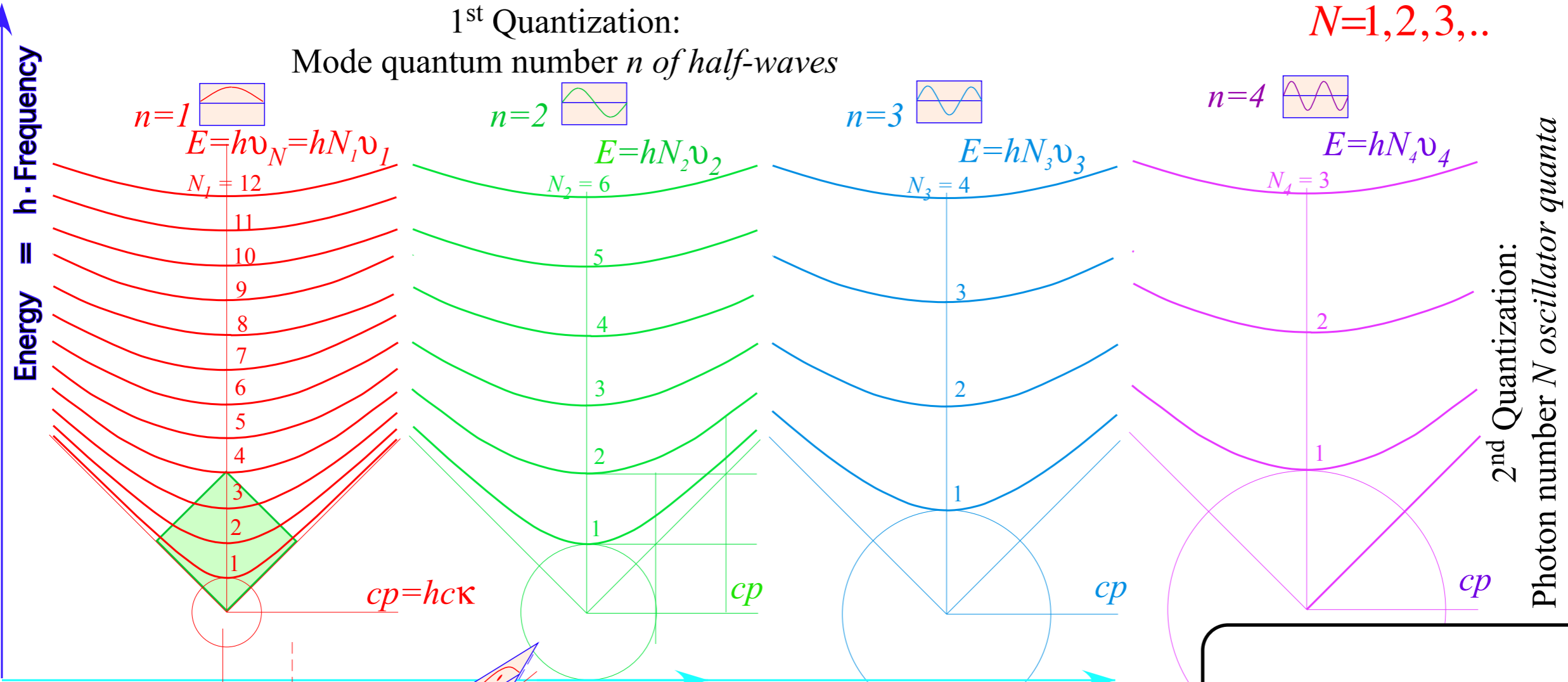
2nd Quantization: $h\nu$ implies $hN\nu$

$(h\nu_{phase} = E = h\nu_A \cosh \rho)$ implies $(hN\nu_{phase} = E_N = hN\nu_A \cosh \rho)$ with quantum numbers

$N=1,2,3,..$

1st Quantization:

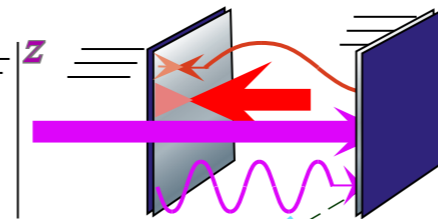
Mode quantum number n of half-waves



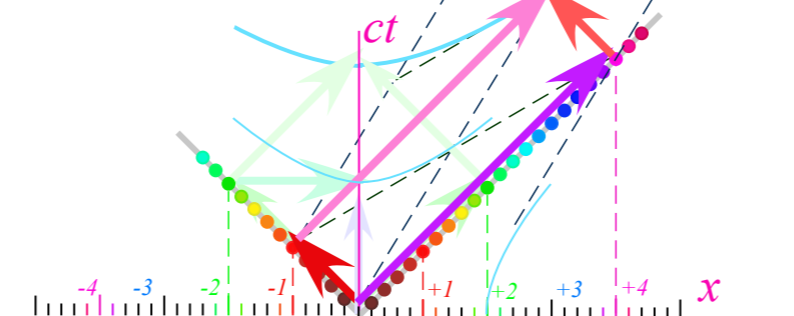
$c \cdot \text{Momentum} = hc \cdot \text{Wavenumber}$

Boosted wave mode

Boosted cavity wave has invariant mode number n photon number N_n



Lorentz contracted cavity length $L=3.2$
Proper length $l=4.0$

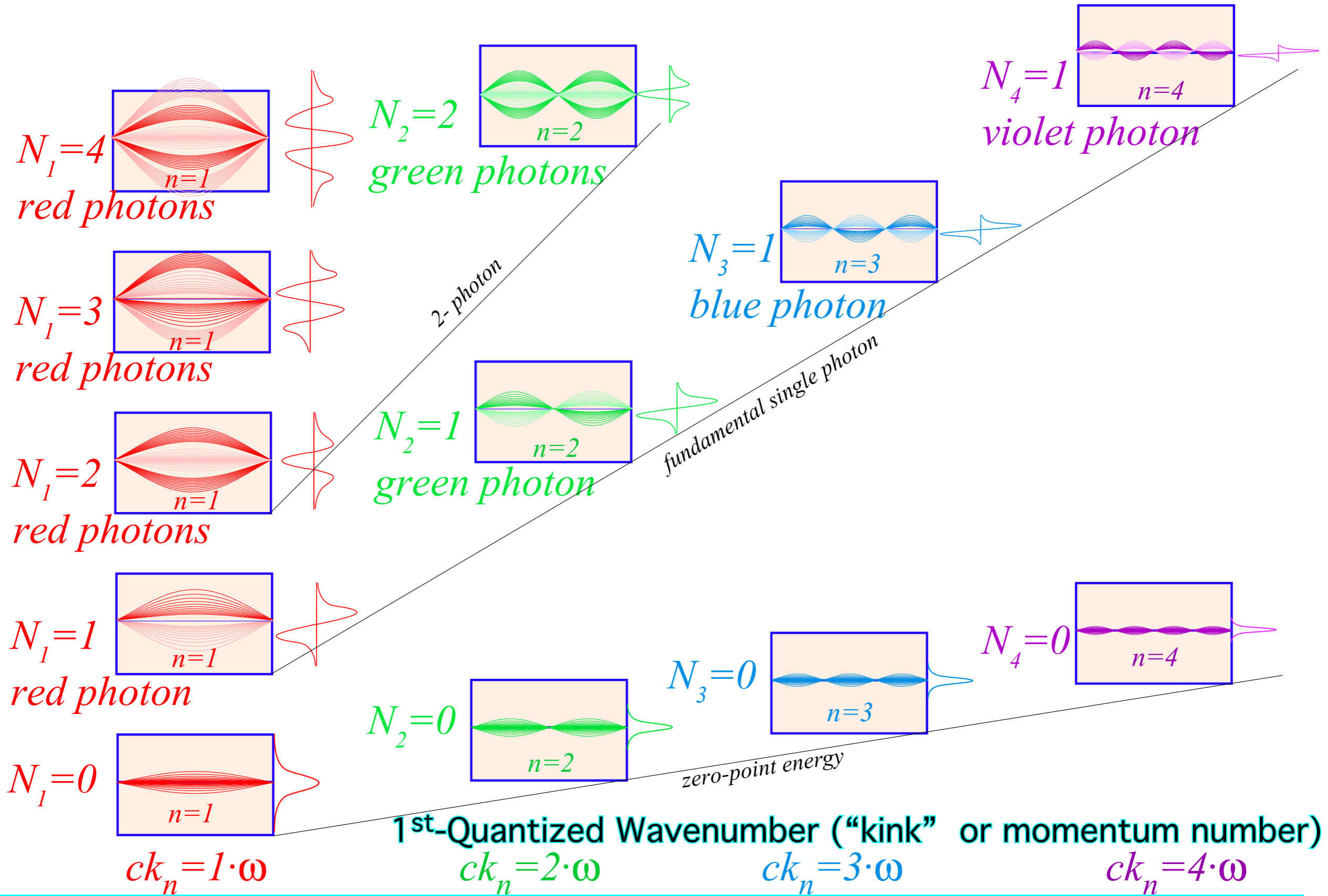


Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2nd Quantization: $h\nu$ implies $hN\nu$

$$(h\nu_{phase} = E = h\nu_A \cosh \rho) \text{ implies } (hN\nu_{phase} = E_N = hN\nu_A \cosh \rho \quad (N=1,2,\dots))$$

2nd-Quantized Amplitude ("photon" number)

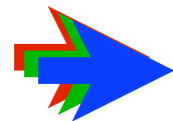


Relativity relates charge, current, and magnetic fields

Geometric derivation of magnetic constant μ_0 from electric ϵ_0

Lorentz-Poincare symmetry and energy-momentum spectral conservation rules

Review of 2nd-quantization “photon” number N and 1st-quantization wavenumber $\kappa = m$



Sketches of atomic and molecular spectroscopy

Relativistic optical transitions and Compton recoil formulae

Feynman diagram geometry

Recoils shifts

Compton recoil related to rocket velocity formula

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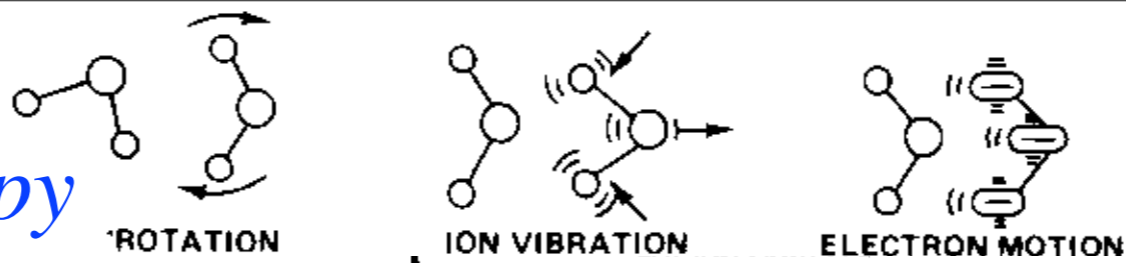
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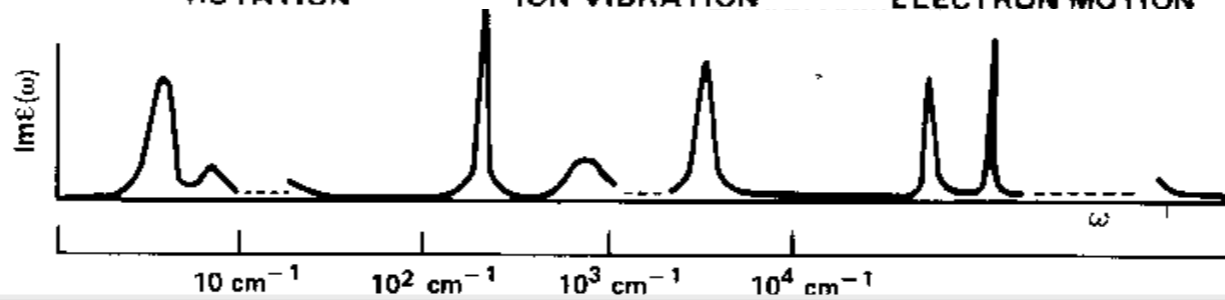
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A sketch of modern molecular spectroscopy



From Fig. 6.5.5.
Principles of Symmetry, Dynamics, and Spectroscopy
W. G. Harter, Wiley Interscience, NY (1993)



The frequency hierarchy

Radio-frequency Microwave to far-infrared Infrared Near-infrared to visible to ultraviolet to X-ray

fine structure

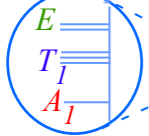
rotational spectra

vibrational spectra

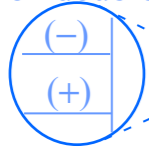
electronic spectra

Other types of spectral splitting

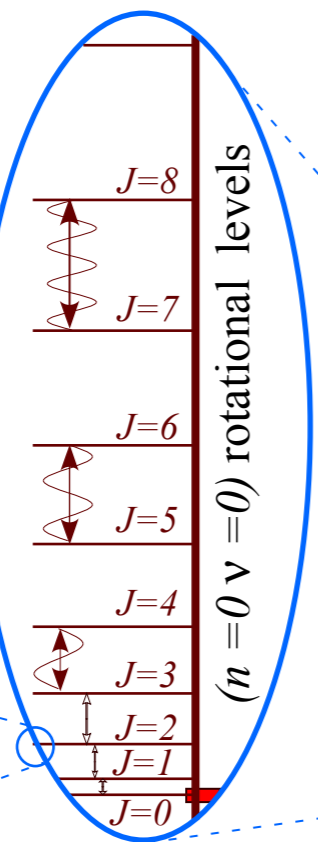
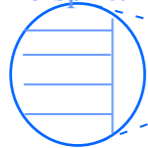
CF₄ and SF₆
J-tunneling
superfine splitting



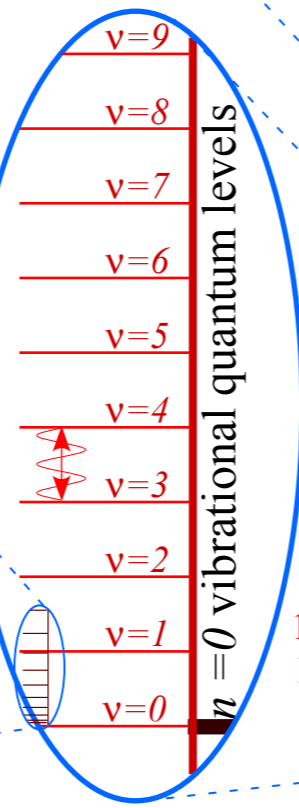
Ammonia NH₃
inversion doublet



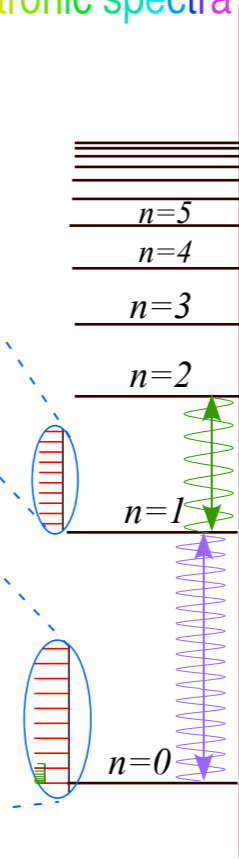
Nuclear spin
hyperfine splitting



CO₂
MICROWAVE
 $B_0(1/\lambda)=0.2\text{cm}^{-1}$
 $\lambda=5\text{cm}$
 $\nu=60\text{MHz}$



CO₂ laser
INFRARED
 $\nu=30\text{THz}$
 $\lambda=10\mu\text{m}$
 $1/\lambda=1000\text{cm}^{-1}$
 $E_{eV}=0.124\text{eV}$



electronic quantum levels

Typical
VISIBLE
 $\nu=600\text{THz}$
 $1/\lambda=2\cdot 10^6\text{m}^{-1}$
 $=2\cdot 10^4\text{cm}^{-1}$
 $\lambda=0.5\mu\text{m}$
 $=500\text{nm}$
 $=5000\text{A}$
 $E_{eV}=2.48\text{eV}$
or
H-Lyman α
ULTRAVIOLET
 $\nu=2.4\text{PHz}$
 $E_{Ly\alpha}=10.2\text{eV}$
 $\lambda=125\text{nm}$

rovibrational spectra

vibronic spectra

rovibronic spectra

Spectral
Quantities

Frequency ν
Hertz(sec⁻¹)
THz 10¹²s⁻¹
GHz 10⁹s⁻¹
MHz 10⁶s⁻¹
kHz 10³s⁻¹

Wavelength λ
meters(m)
fm 10⁻¹⁵m
pm 10⁻¹²m
nm 10⁻⁹m
 μm 10⁻⁶m
mm 10⁻³m
cm 10⁻²m
km 10³m
Wavenumber
per meter(m⁻¹)
cm⁻¹ 10²m⁻¹

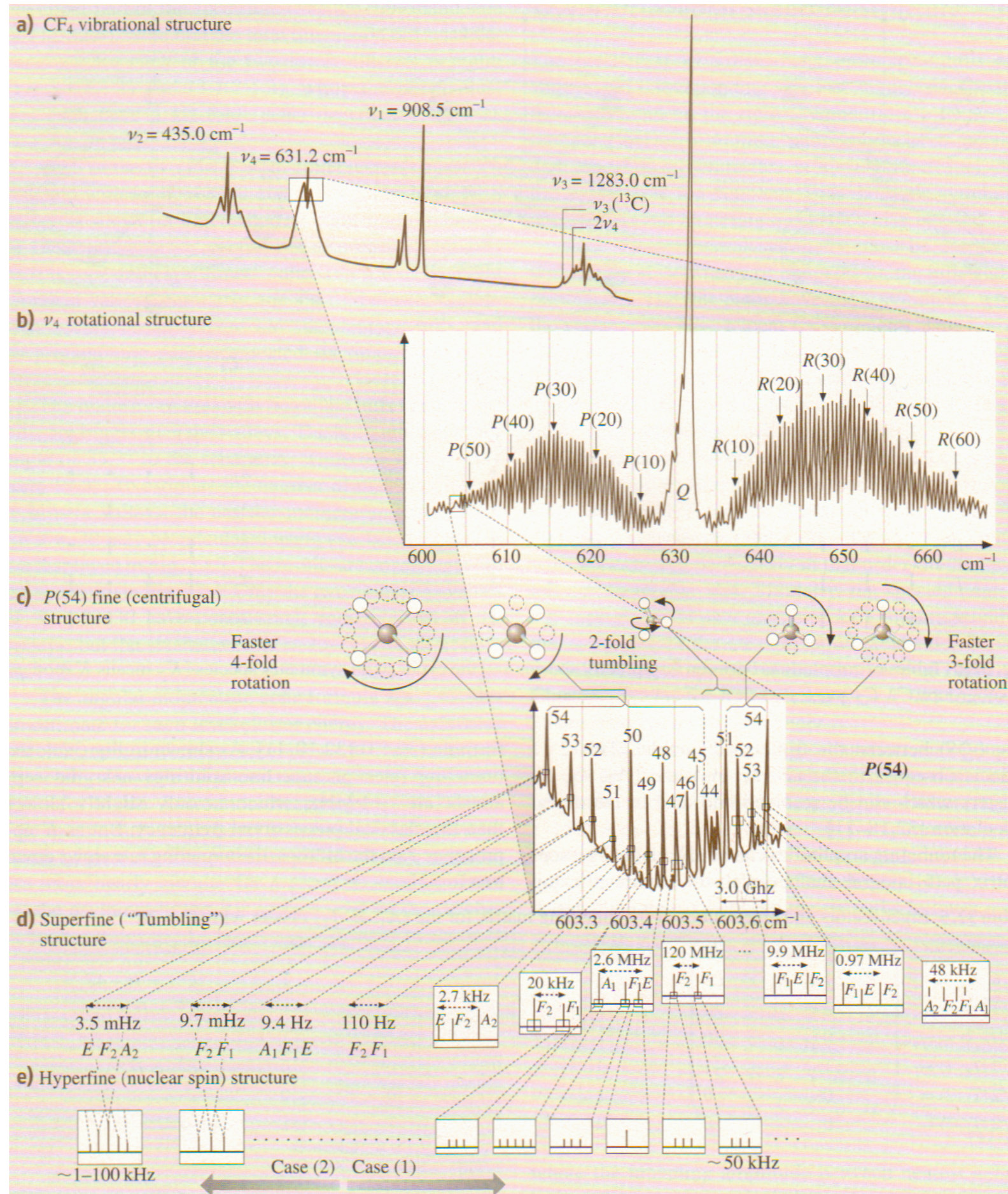
Energy $eh\nu$
electronVolts
(eV)

Example of frequency hierarchy for $16\mu\text{m}$ spectra of CF_4 (Freon-14)

W.G.Harter

Fig. 32.7

Springer Handbook of Atomic, Molecular, & Optical Physics
Gordon Drake Editor (2005)

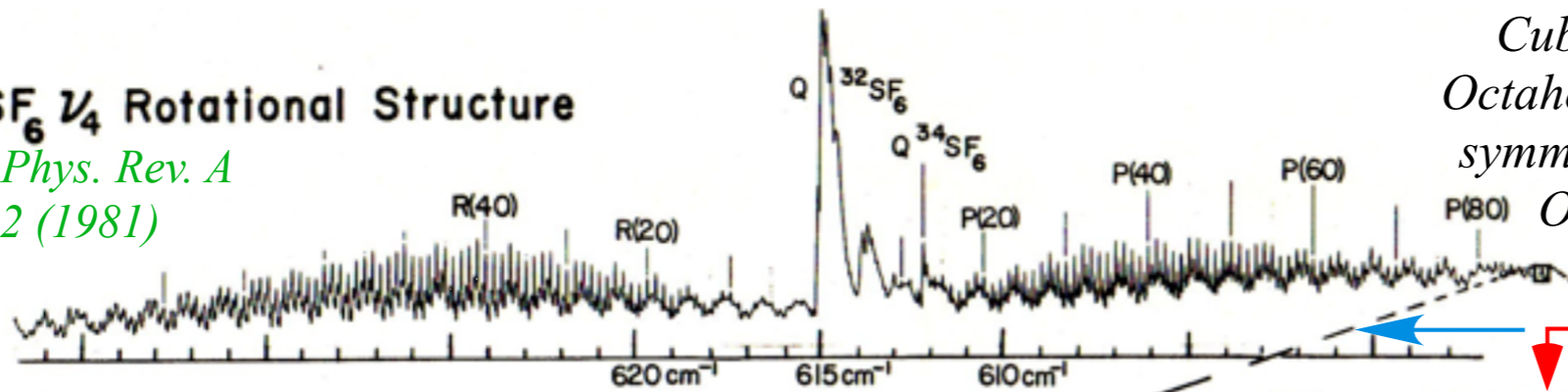


(a) SF₆ ν₄ Rotational Structure

WGH Phys. Rev. A
24, 192 (1981)

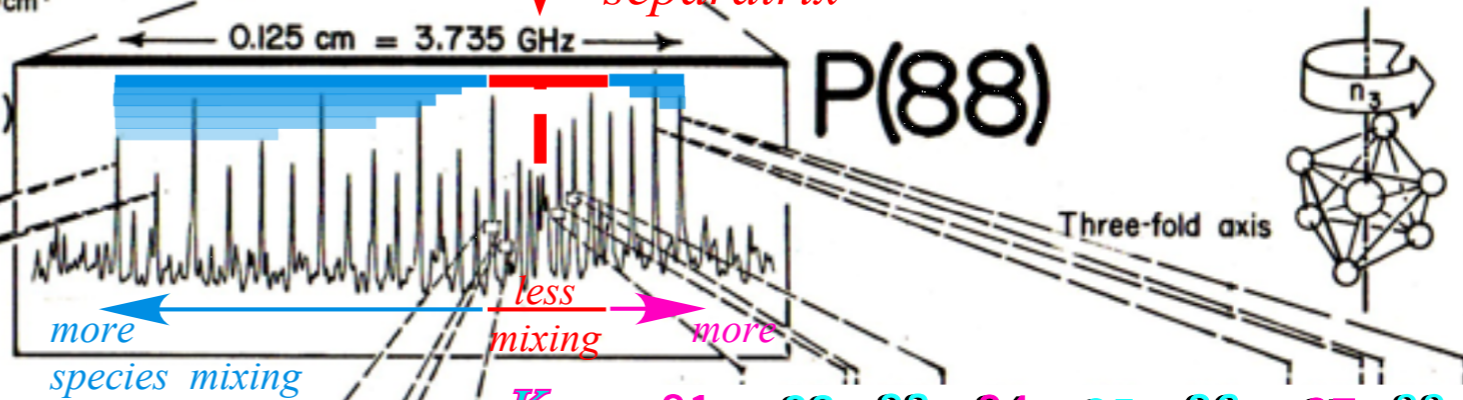
Cubic
Octahedral
symmetry

FT IR and Laser Diode Spectra
K.C. Kim, W.B. Person, D. Seitz, and B.J. Krohn
J. Mol. Spectrosc. 76, 322 (1979).



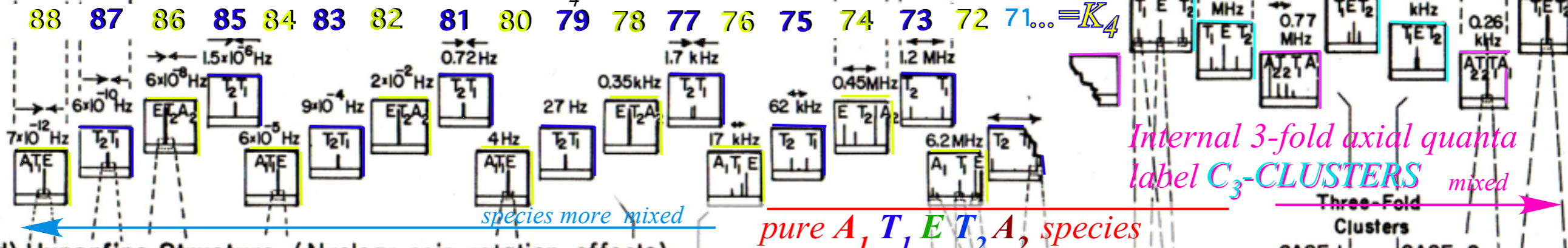
Primary AET species mixing
increases with distance from
"separatrix"

(b) P(88) Fine Structure (Rotational anisotropy effects)



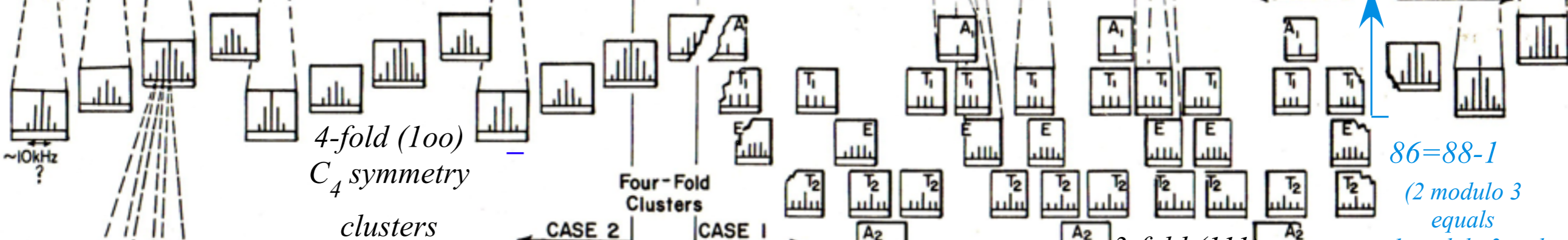
(c) Superfine Structure (Rotational axis tunneling)

4-fold (100)-clusters C₄ symmetry



(d) Hyperfine Structure (Nuclear spin-rotation effects)

4-fold (100)
C₄ symmetry
clusters



(e) Superhyperfine Structure (Spin frame correlation effects)

3-fold (111)
C₃ symmetry
clusters



86=88-1
(2 modulo 3
equals
-1 modulo 3 and
86 mod 3)

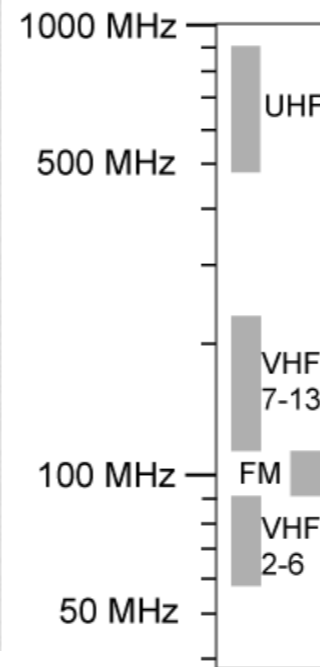
Units of frequency (Hz), wavelength (m), and energy (eV)

CLASS	FREQUENCY	WAVELENGTH	ENERGY
Y	300 EHz	1 pm	1.24 MeV
HX	30 EHz	10 pm	124 keV
SX	3 EHz	100 pm	12.4 keV
EUV	300 PHz	1 nm	1.24 keV
NUV	30 PHz	10 nm	124 eV
	3 PHz	100 nm	12.4 eV
NIR	300 THz	1 μm	1.24 eV
MIR	30 THz	10 μm	124 meV
FIR	3 THz	100 μm	12.4 meV
EHF	300 GHz	1 mm	1.24 meV
SHF	30 GHz	1 cm	124 μeV
UHF	3 GHz	1 dm	12.4 μeV
VHF	300 MHz	1 m	1.24 μeV
HF	30 MHz	10 m	124 neV
MF	3 MHz	100 m	12.4 neV
LF	300 kHz	1 km	1.24 neV
VLF	30 kHz	10 km	124 peV
VF/ULF	3 kHz	100 km	12.4 peV
SLF	300 Hz	1 Mm	1.24 peV
ELF	30 Hz	10 Mm	124 feV
	3 Hz	100 Mm	12.4 feV

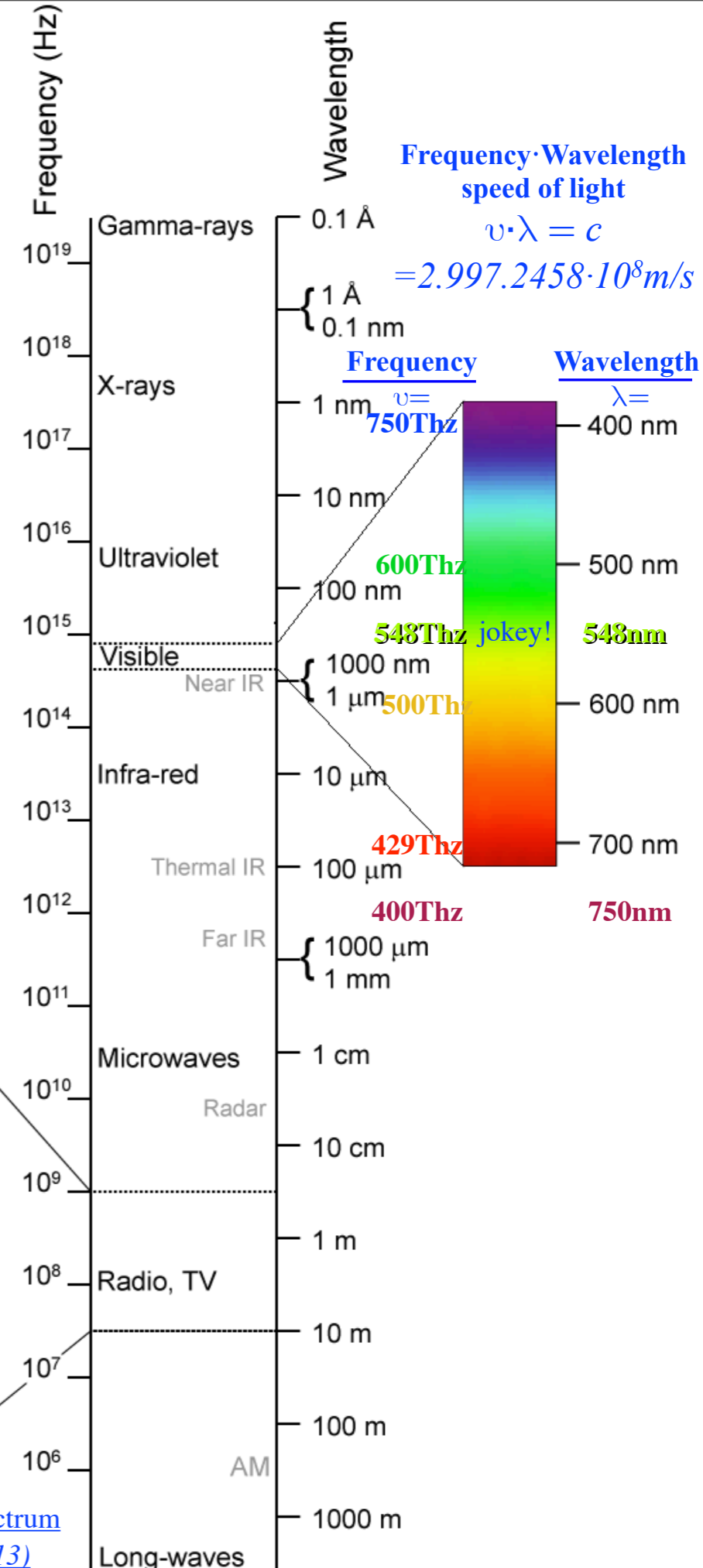
From: [Electromagnetic Spectrum](#)
[Wikipedia Commons \(2013\)](#)

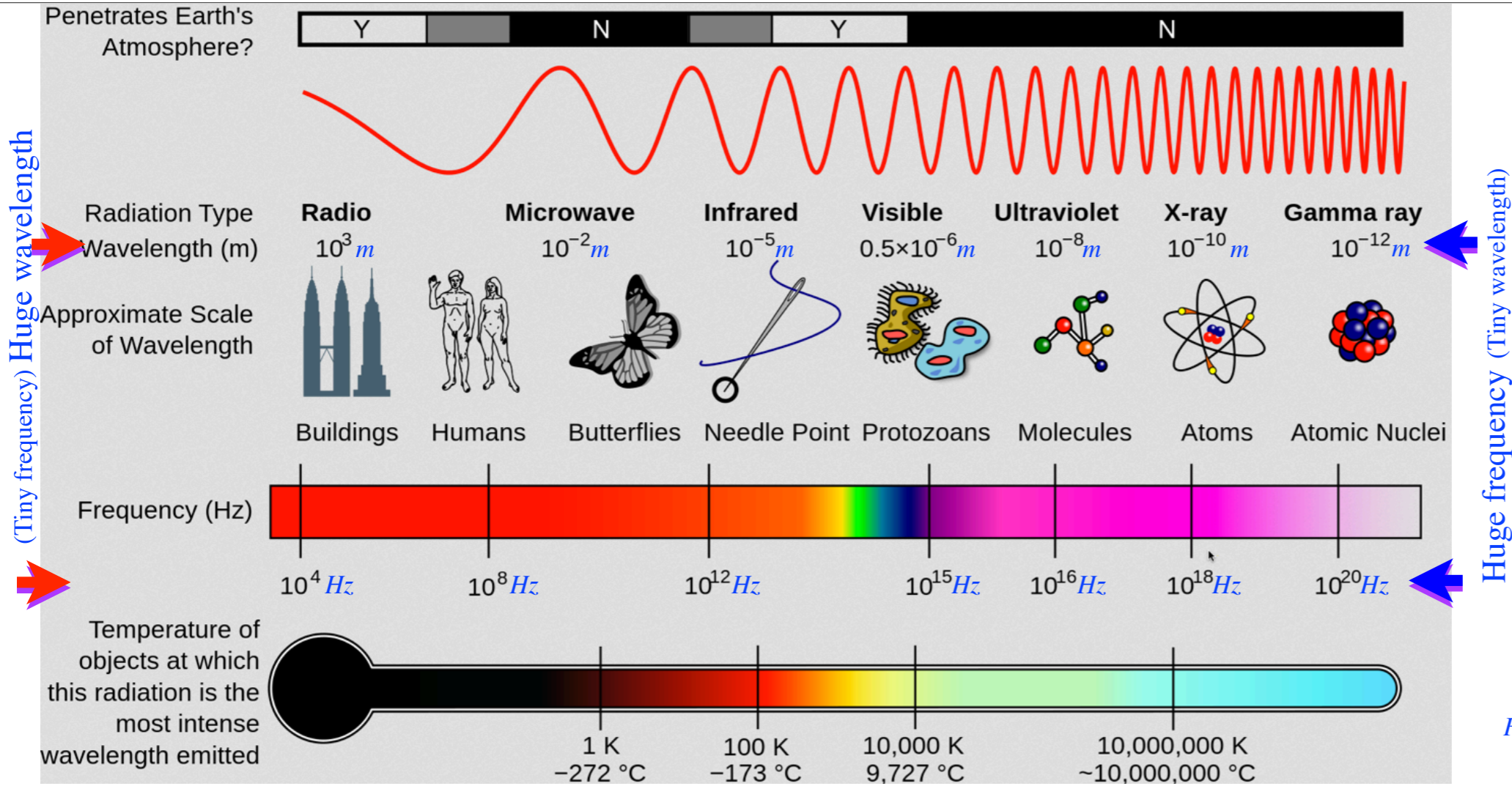
Exa: 10^{18}
 Peta: 10^{15}
 Tera: 10^{12}
 Giga: 10^9
 Mega: 10^6
 kilo: 10^3

milli: 10^{-3}
 micro: 10^{-6}
 nano: 10^{-9}
 pico: 10^{-12}
 femto: 10^{-15}
 atto: 10^{-18}

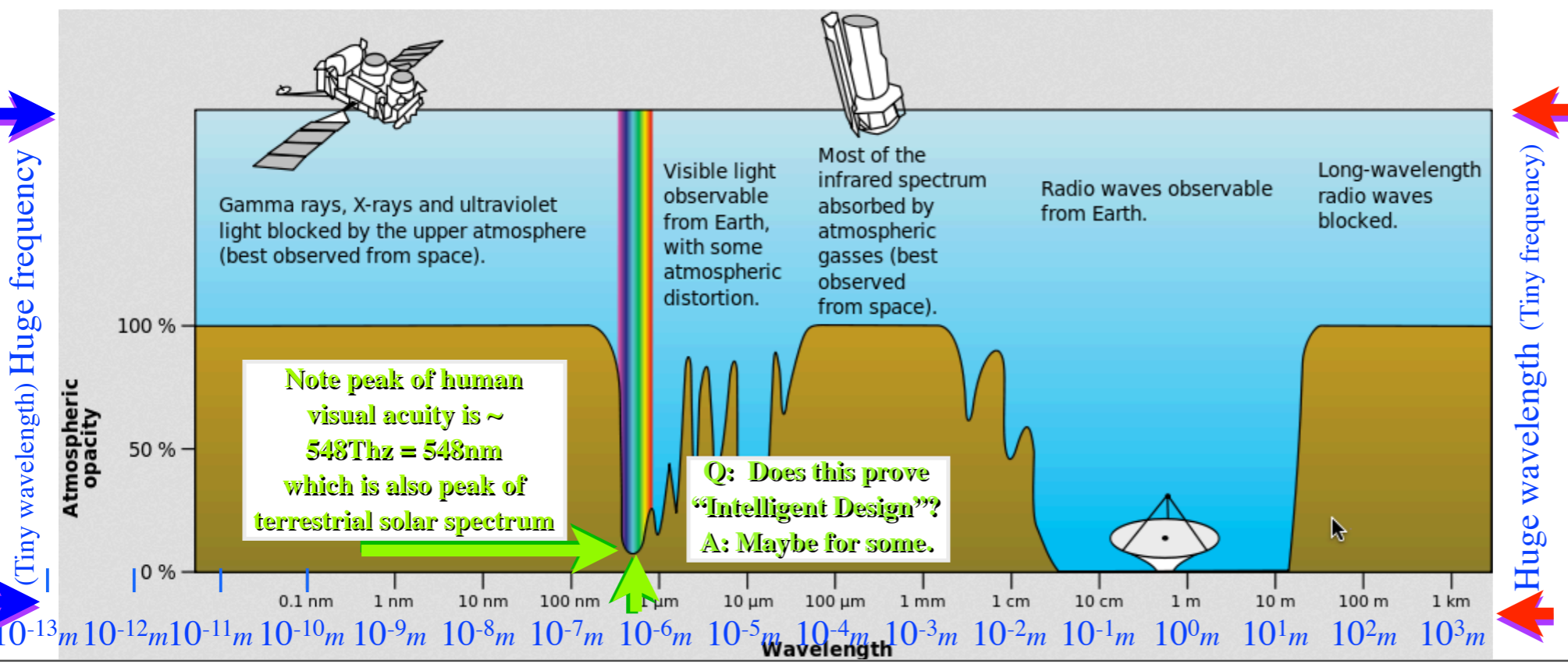


From: [Electromagnetic Spectrum](#)
[Wikipedia Commons \(2013\)](#)



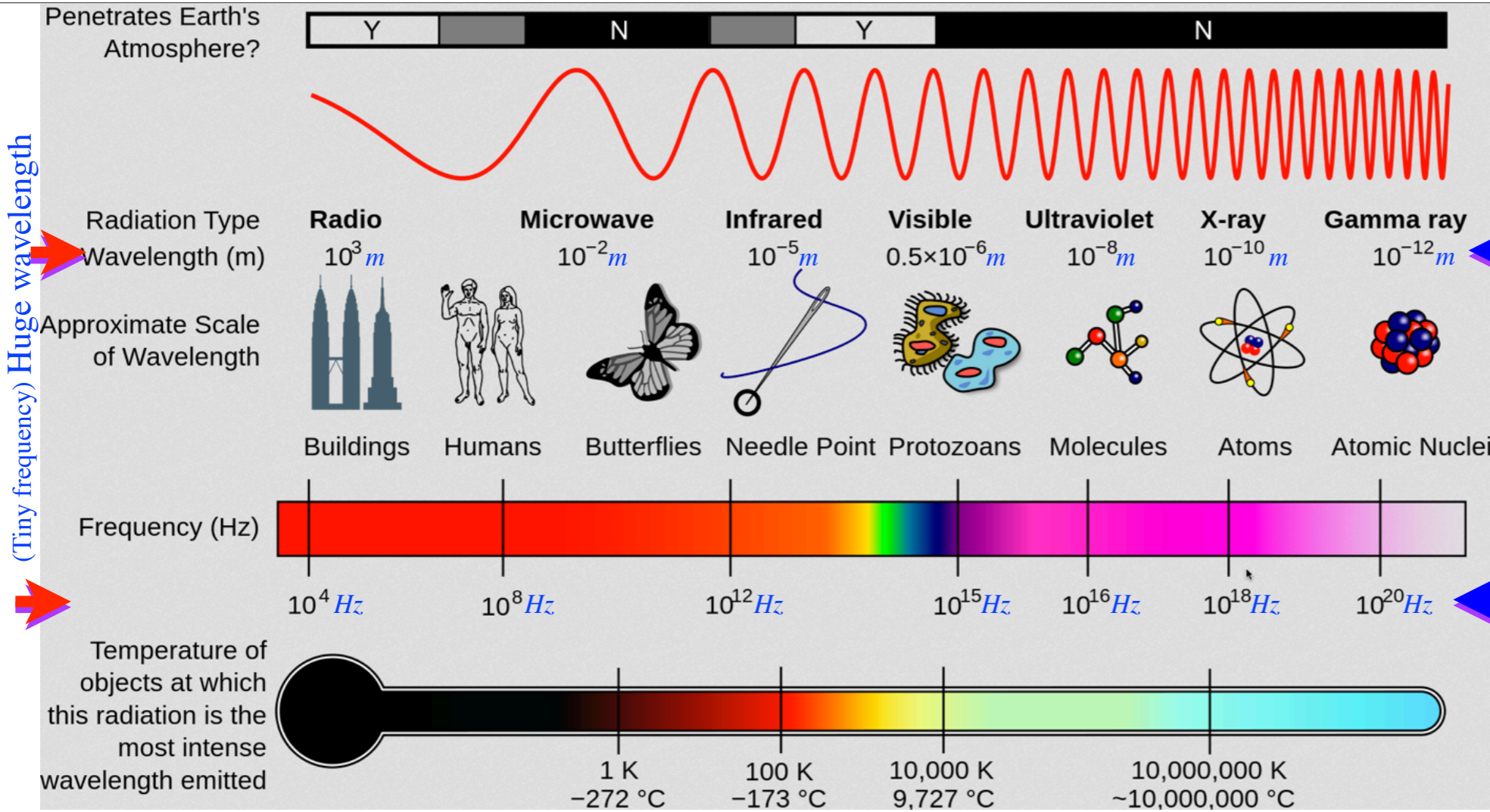


From: Electromagnetic Spectrum
Wikipedia Commons (2013)

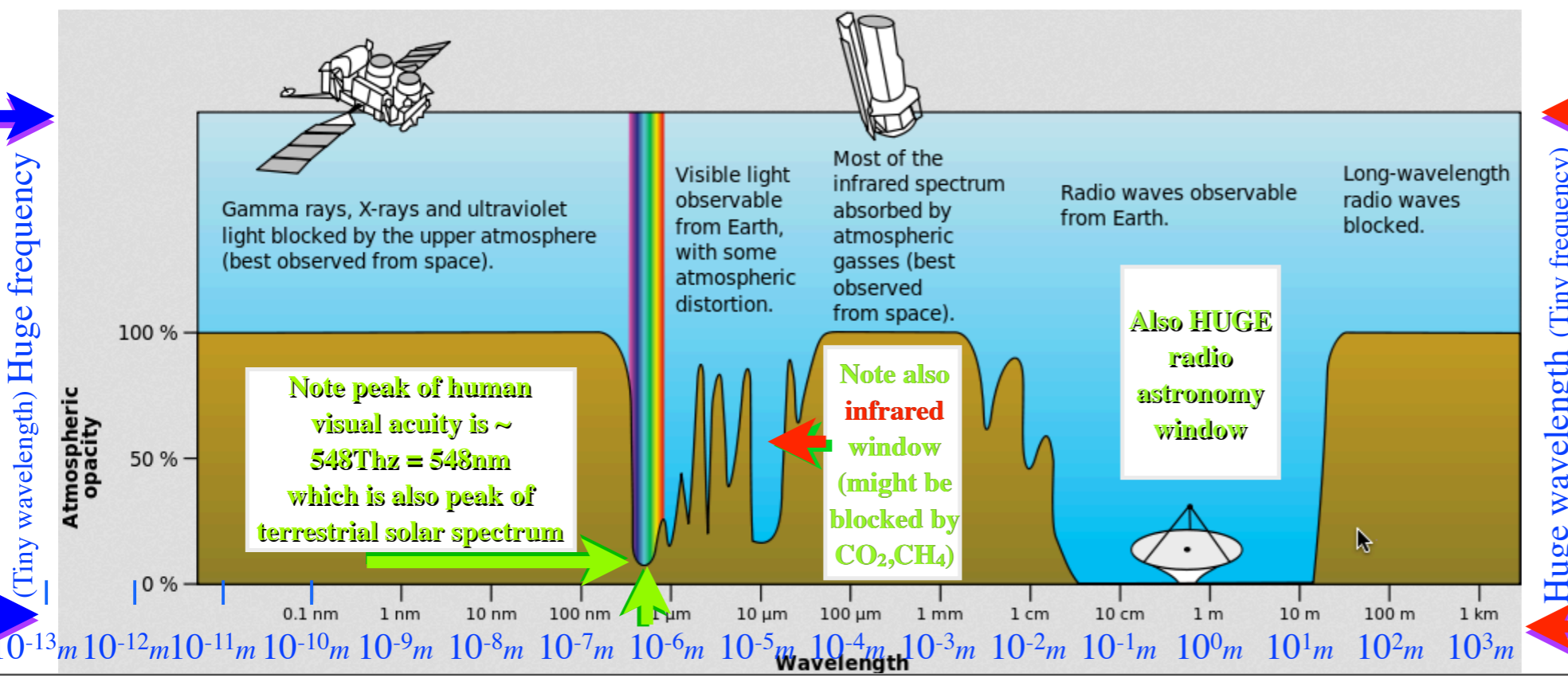


Spectral windows in Earth atmosphere

From: Electromagnetic Spectrum
Wikipedia Commons (2013)



From: Electromagnetic Spectrum
Wikipedia Commons (2013)

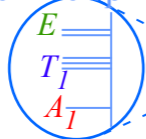


Spectral windows in Earth atmosphere

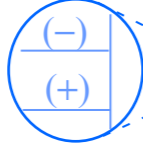
From: Electromagnetic Spectrum
Wikipedia Commons (2013)

Simple Molecular Spectra Models

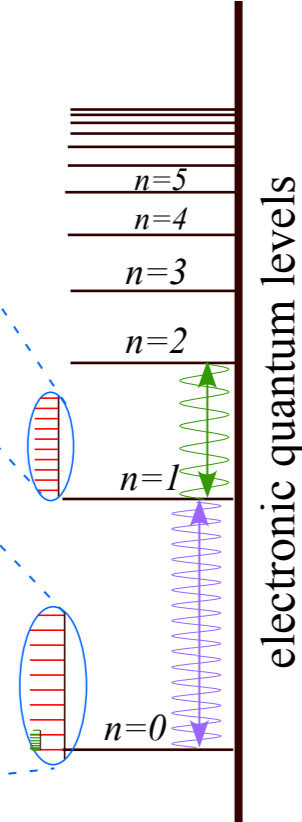
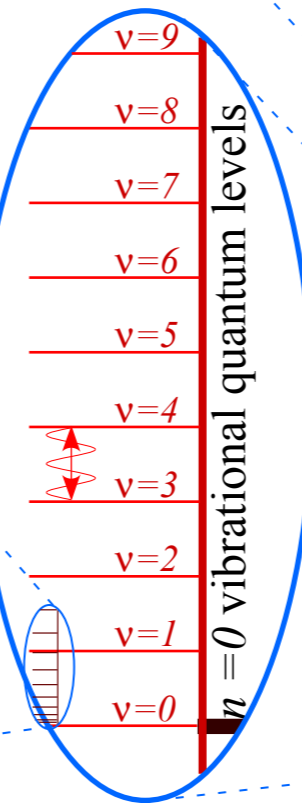
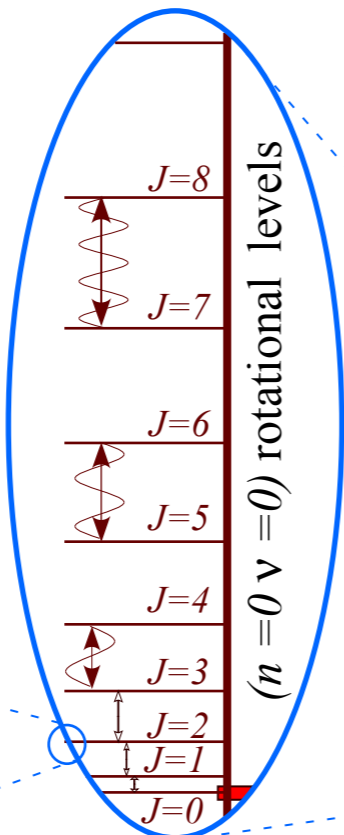
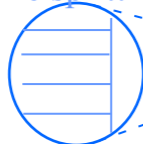
CF₄ and SF₆
J-tunneling
superfine splitting



Ammonia NH₃
inversion doublet



Nuclear spin
hyperfine splitting



fine structure

rotational spectra

vibrational spectra

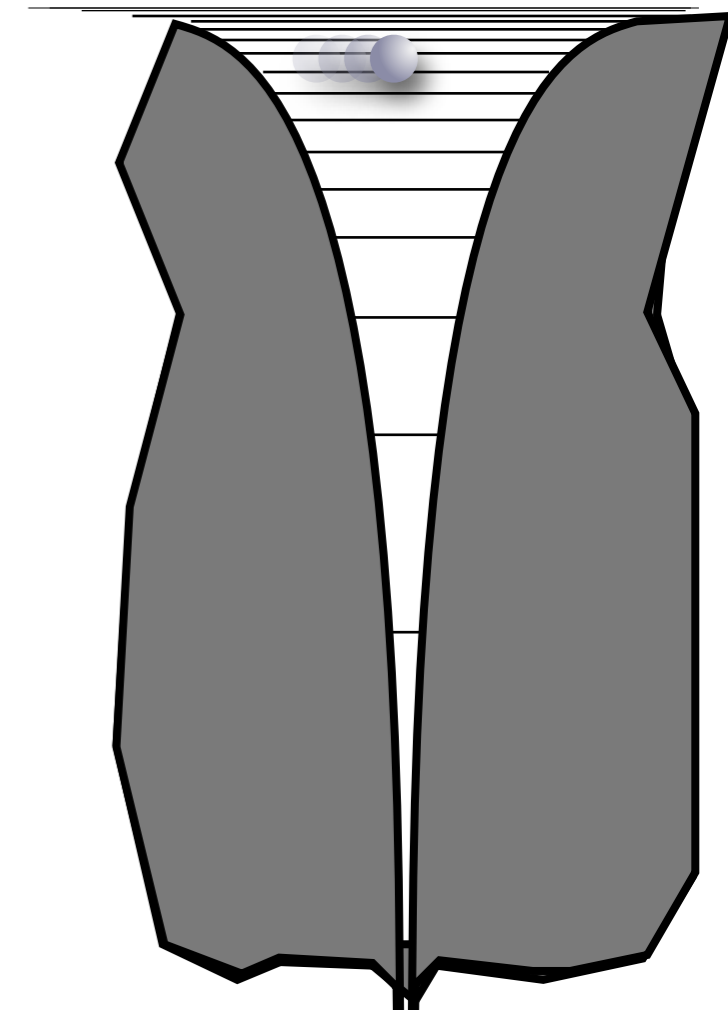
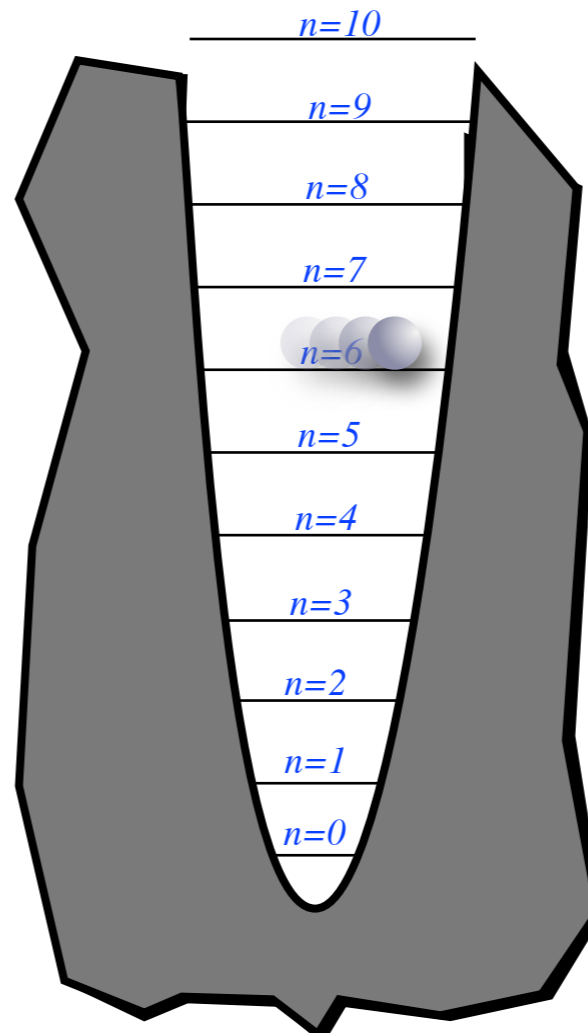
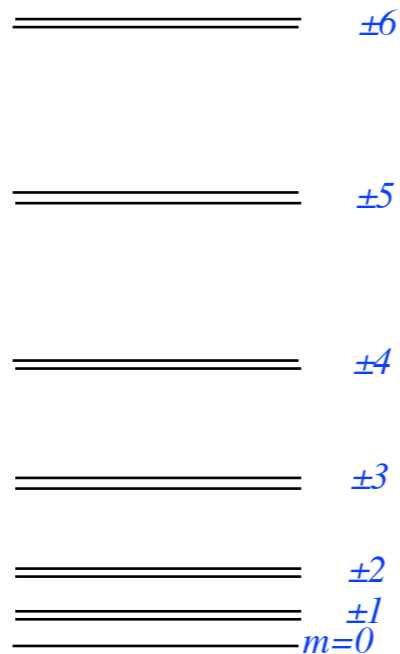
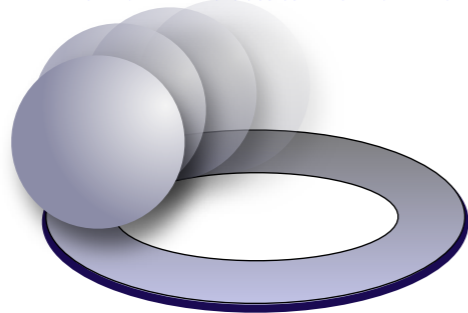
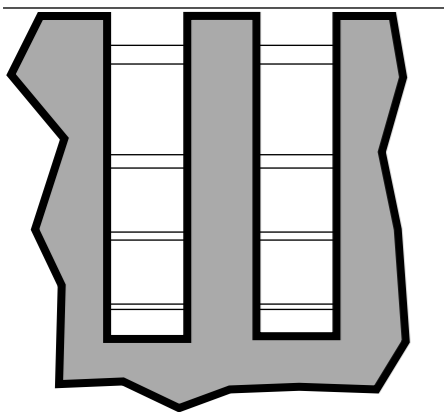
electronic spectra

2-well tunneling

Bohr mass-on-ring

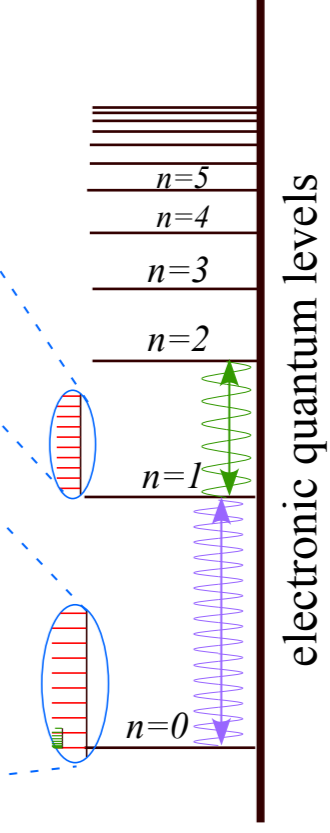
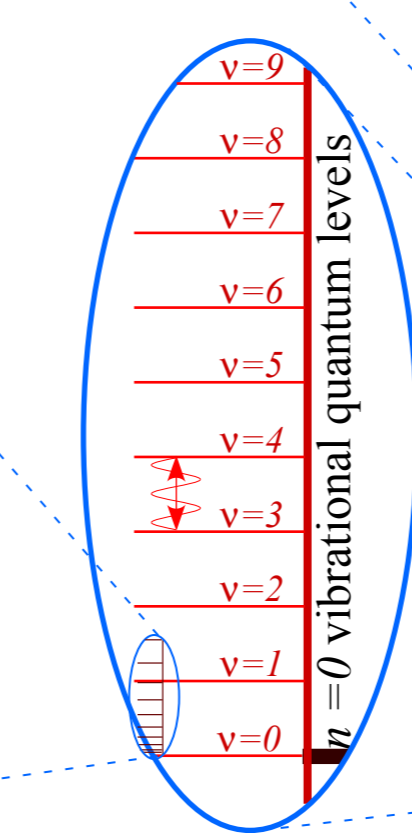
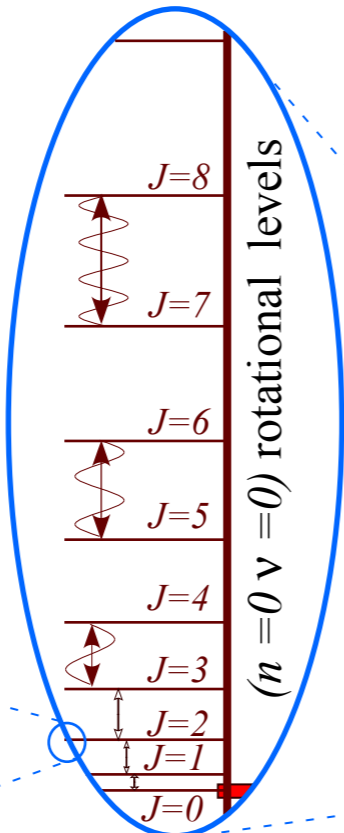
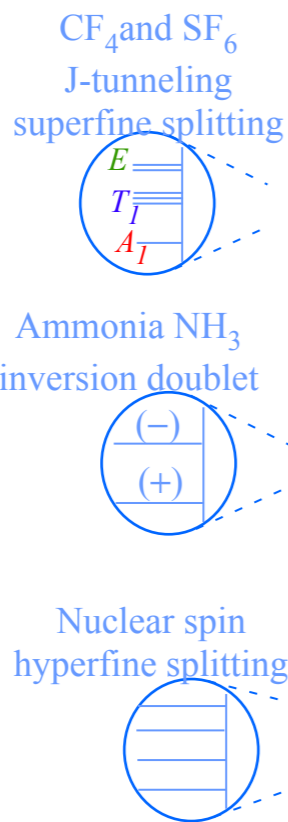
1D harmonic oscillator

Coulomb PE models



More Advanced Molecular Spectra Models

(Use symmetry group theory)



fine structure

rotational spectra

vibrational spectra

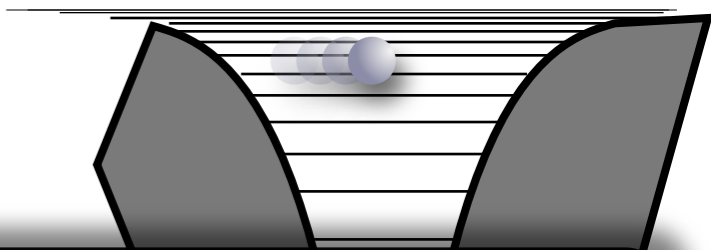
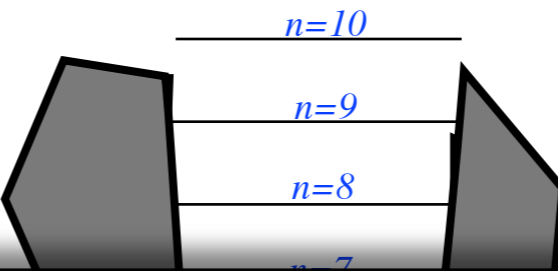
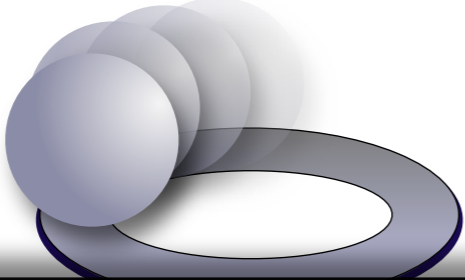
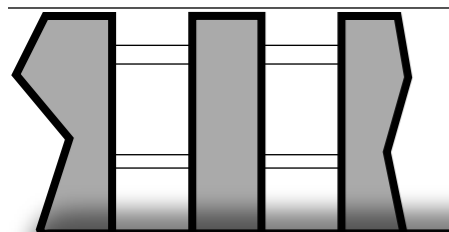
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Bohr mass-on-a-ring

1D harmonic oscillator

Coulomb PE models



2-state U(2)-spin and quasi-spin tunneling models

3D R(3)-rotor and D-function lab-body wave models

2D harmonic oscillator and U(2) 2nd quantization

*U(m)*S_n analysis of multi-electron states*



Rotational Energy Surface (RES) analysis of rovibronic tensor spectra

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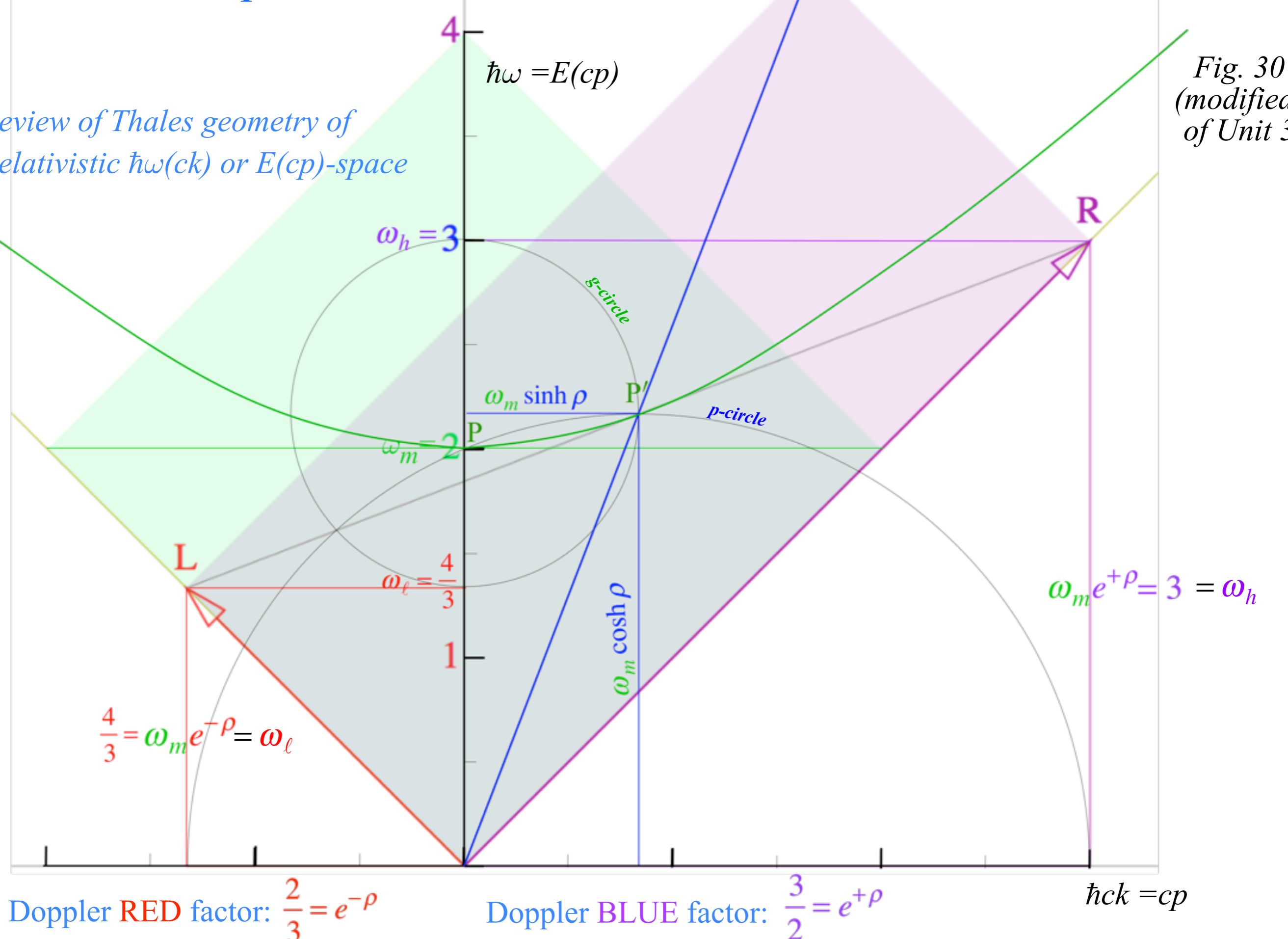
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Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftharpoons |mid\rangle = |\omega_m\rangle \rightleftharpoons |low\rangle = |\omega_l\rangle$

Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$ -space

Fig. 30 (modified) of Unit 3

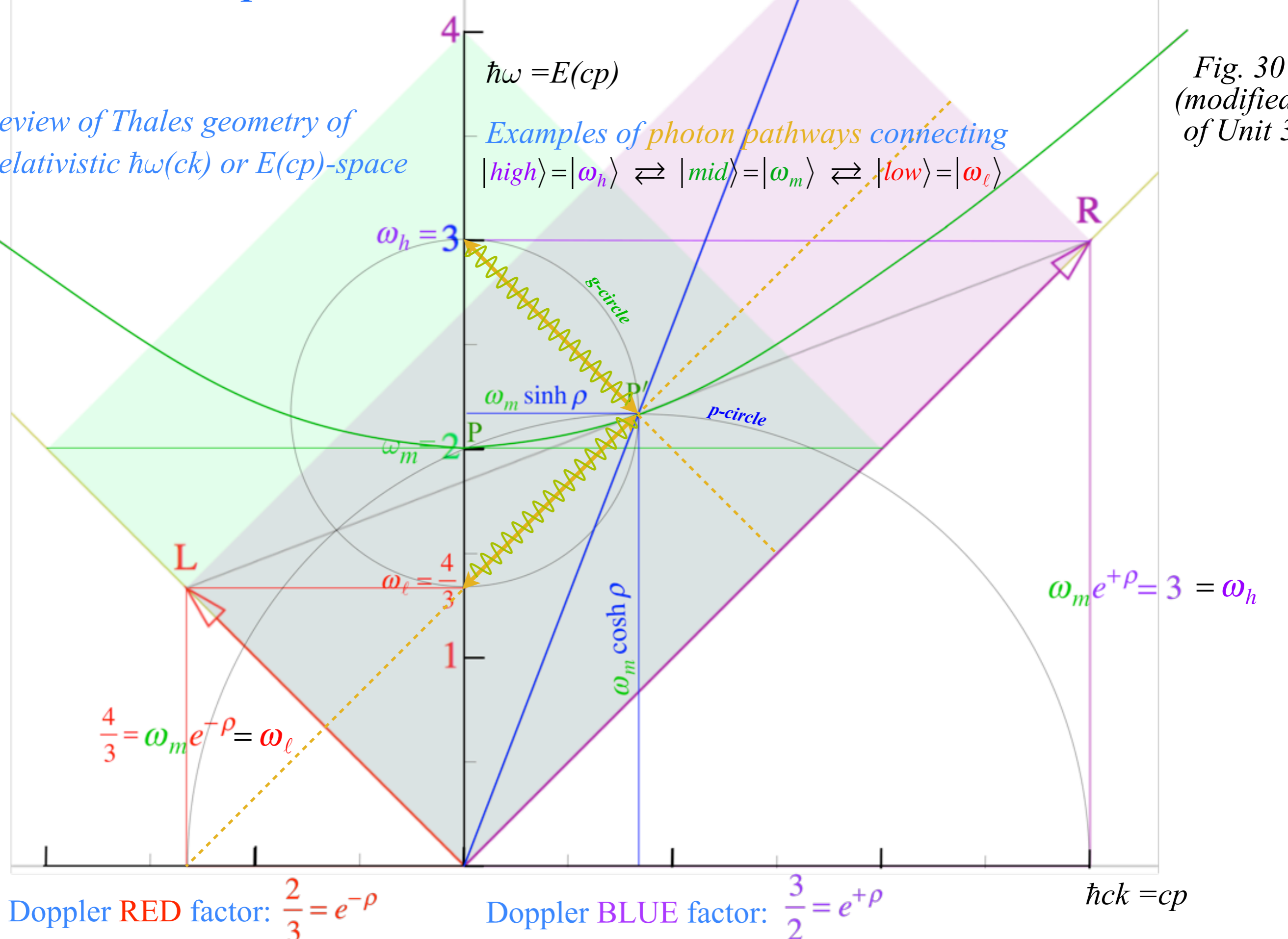


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Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$ -space

Fig. 30 (modified) of Unit 3

Examples of photon pathways connecting $|high\rangle = |\omega_h\rangle \rightleftharpoons |mid\rangle = |\omega_m\rangle \rightleftharpoons |low\rangle = |\omega_l\rangle$



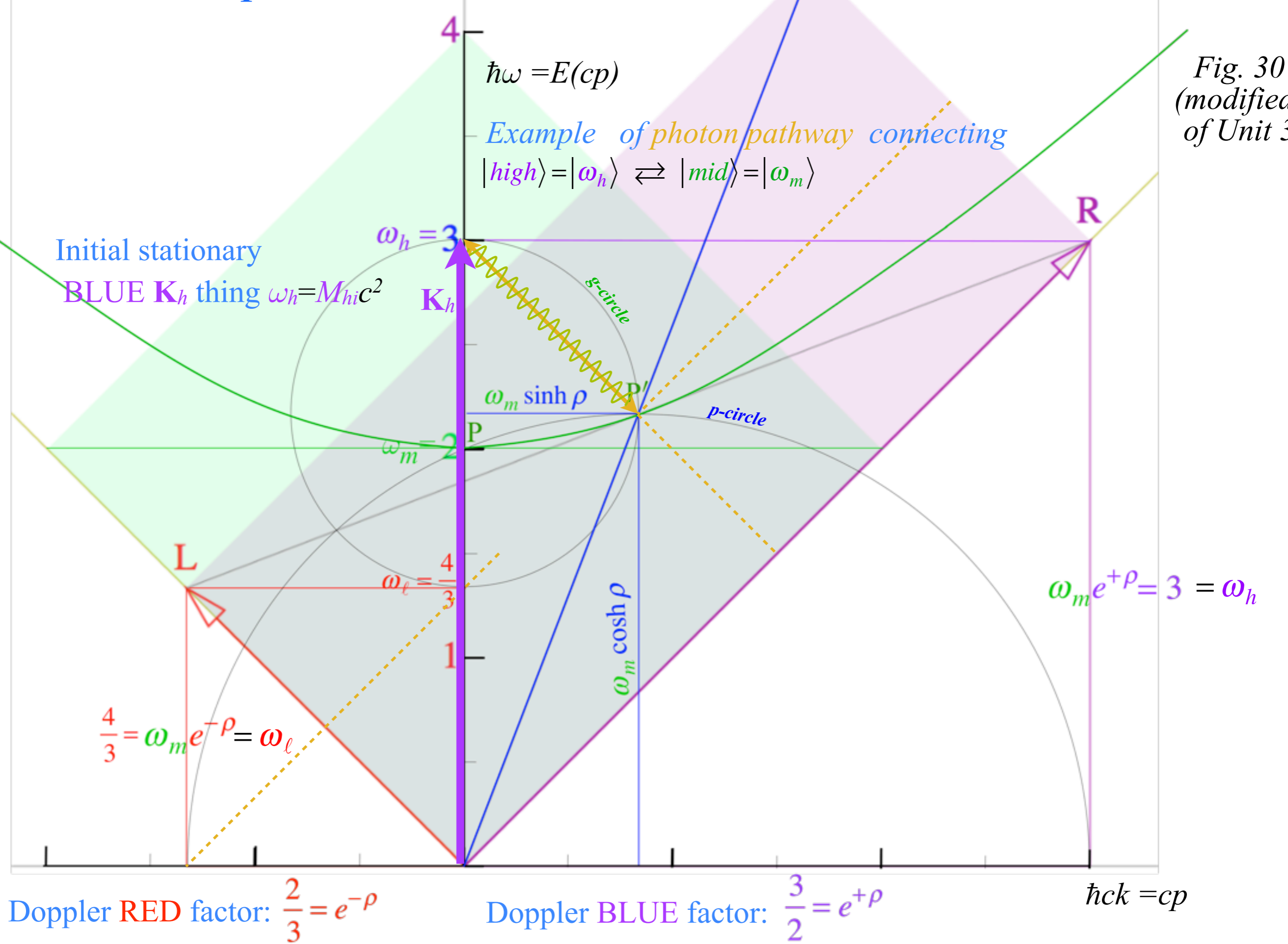
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

$\hbar ck = cp$

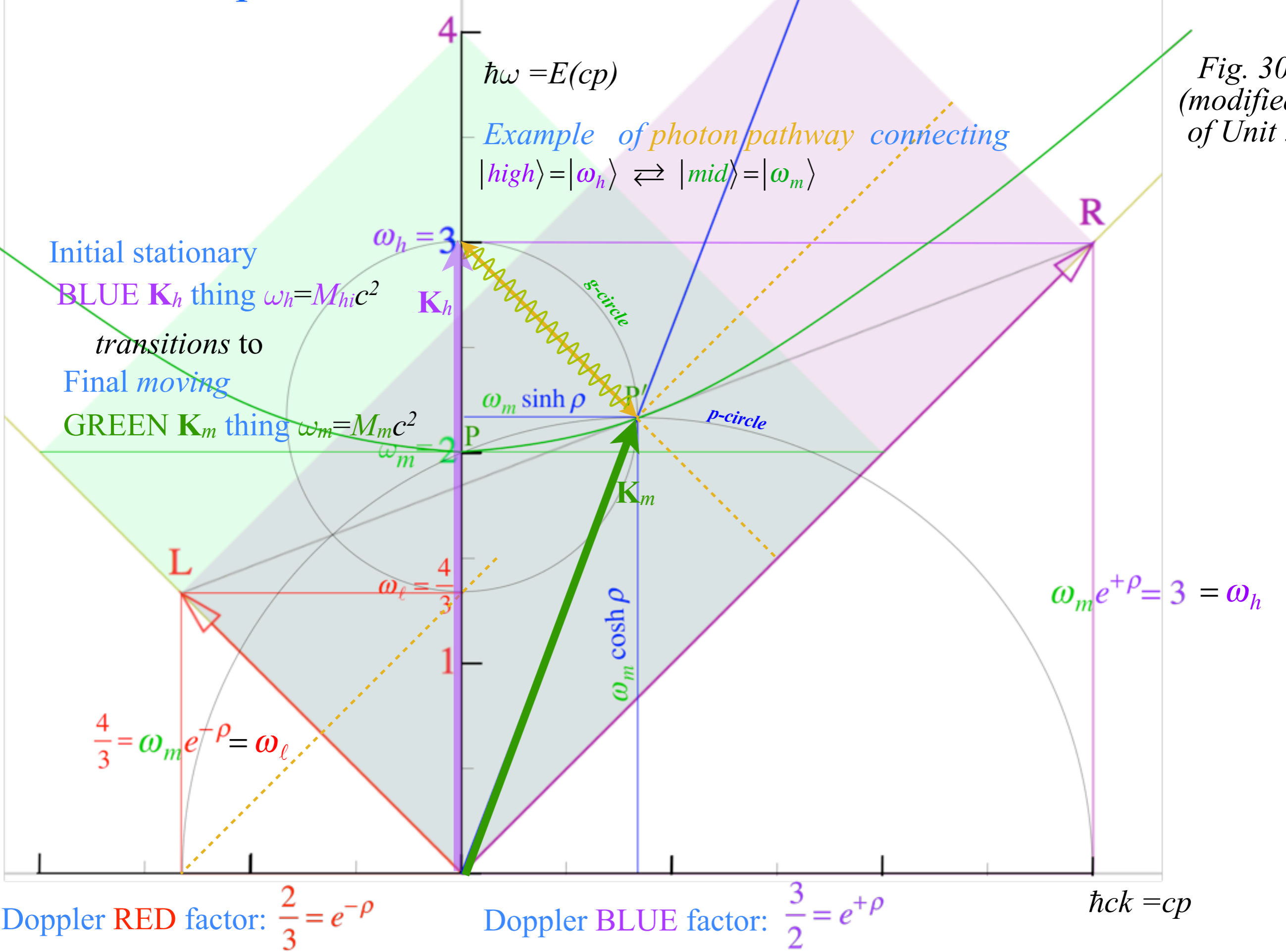
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftharpoons |mid\rangle = |\omega_m\rangle \rightleftharpoons |low\rangle = |\omega_l\rangle$

Fig. 30 (modified) of Unit 3



Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftharpoons |mid\rangle = |\omega_m\rangle \rightleftharpoons |low\rangle = |\omega_l\rangle$

Fig. 30 (modified) of Unit 3



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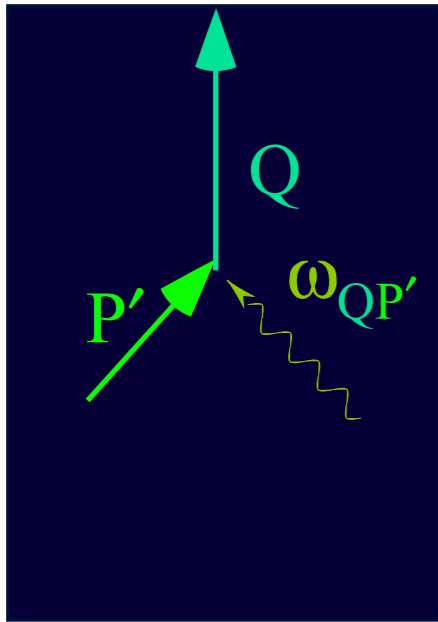
Fundamental light-matter processes:

Absorption A

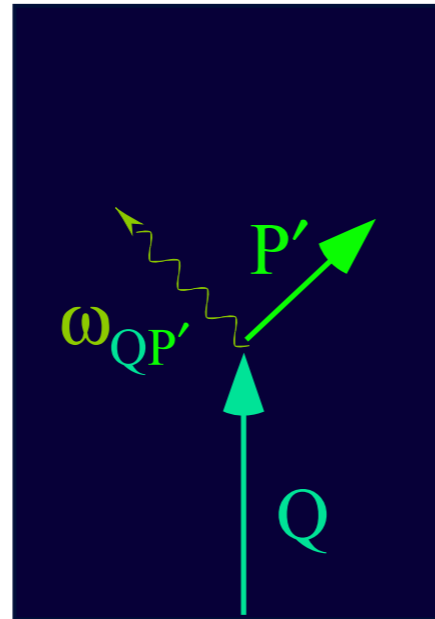
Emission E

AE Together

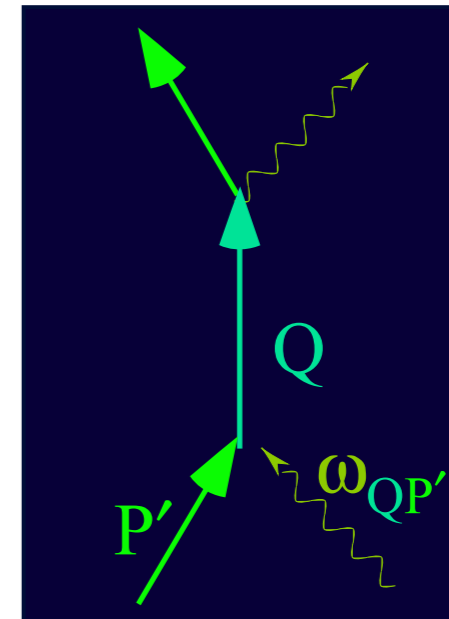
(Compton Scattering)



*1-photon
processes*



*2-photon
process*

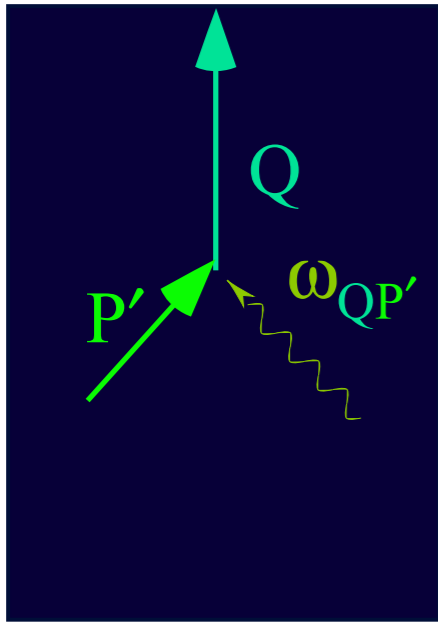


Fundamental light-matter processes:

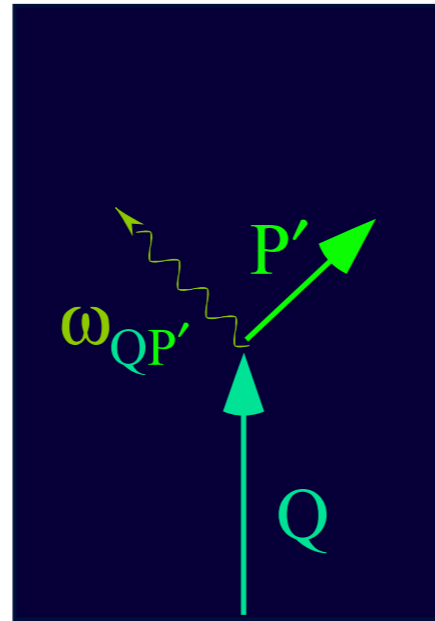
Absorption A

Emission E

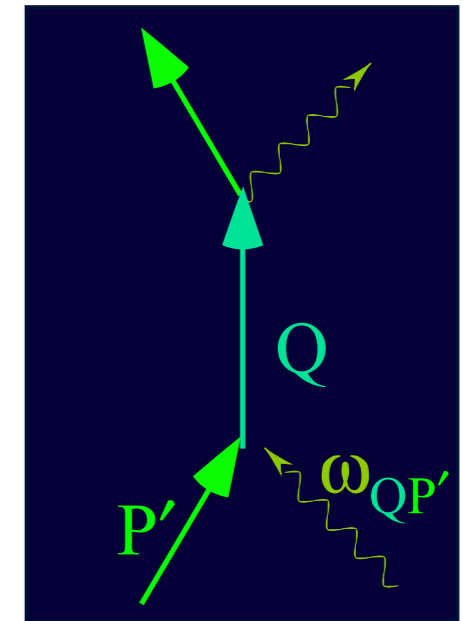
AE Together



1-photon processes



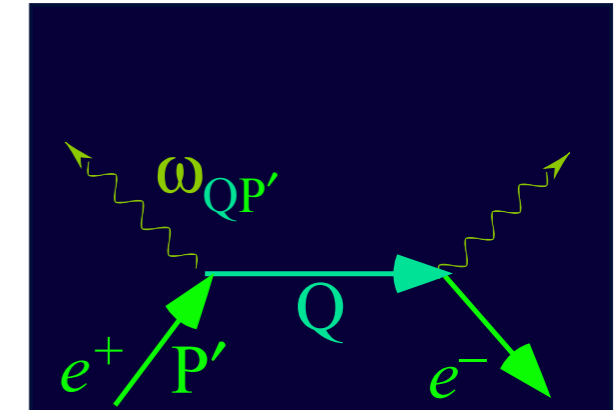
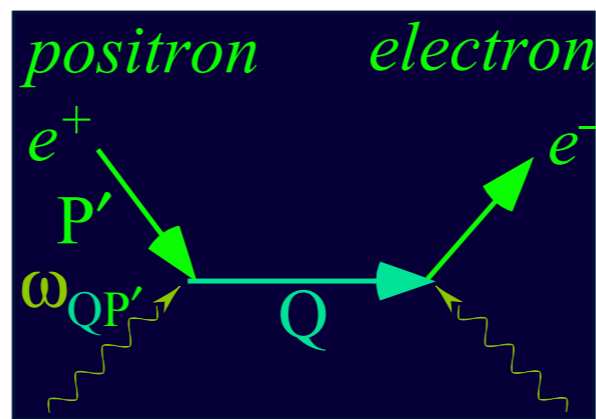
(Compton Scattering)



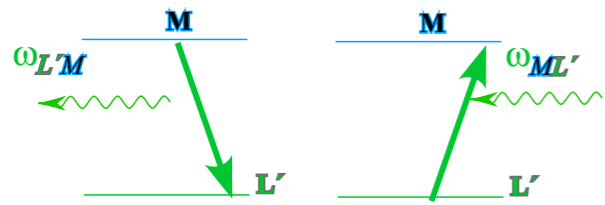
2-photon processes

“Exotic” processes: AA Together (Pair-Creation)

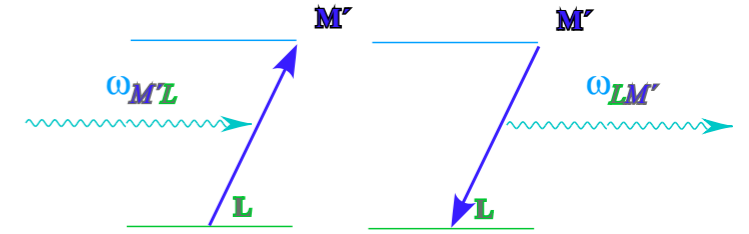
EE Together (Pair-Annihilation)



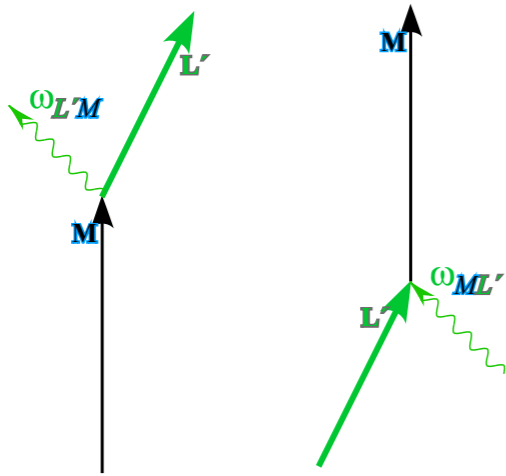
Wave geometry of 1-photon transitions and Compton recoil



Grotian 2-level diagrams

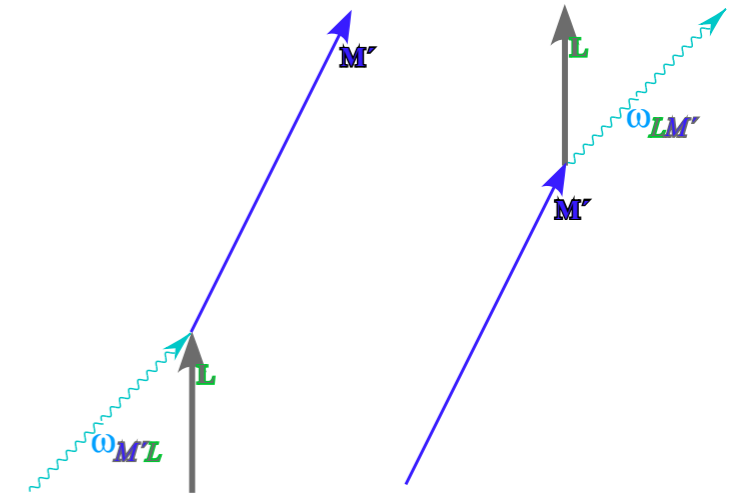


Feynman (ω, ck) diagrams
(1-photon)



M-to-L' emission

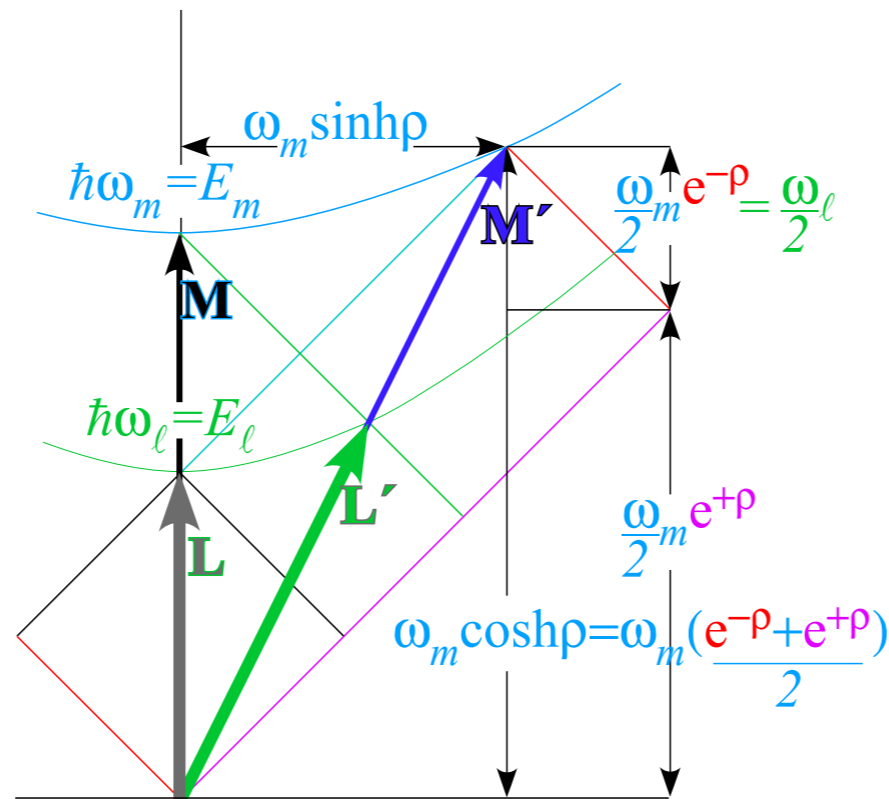
L'-to-M absorption



L-to-M' absorption

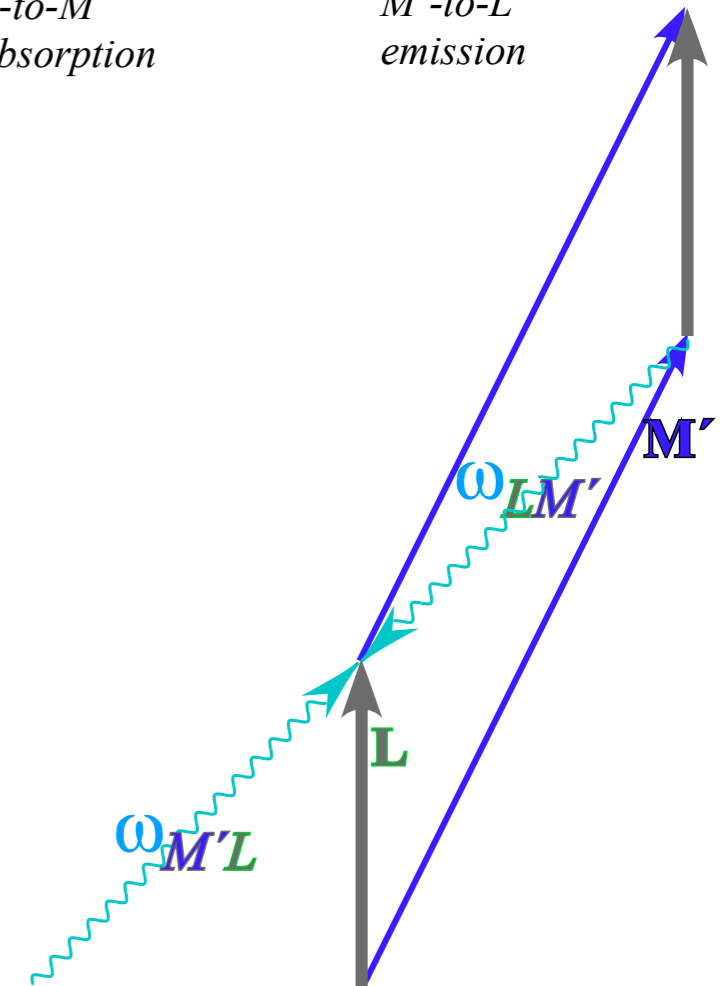
M'-to-L emission

2-Level (ω, ck) "baseball" diamonds



(ω, ck)
vector sum
(energy-
momentum
conservation)

Modified from Mod. Phys. Lect. 33

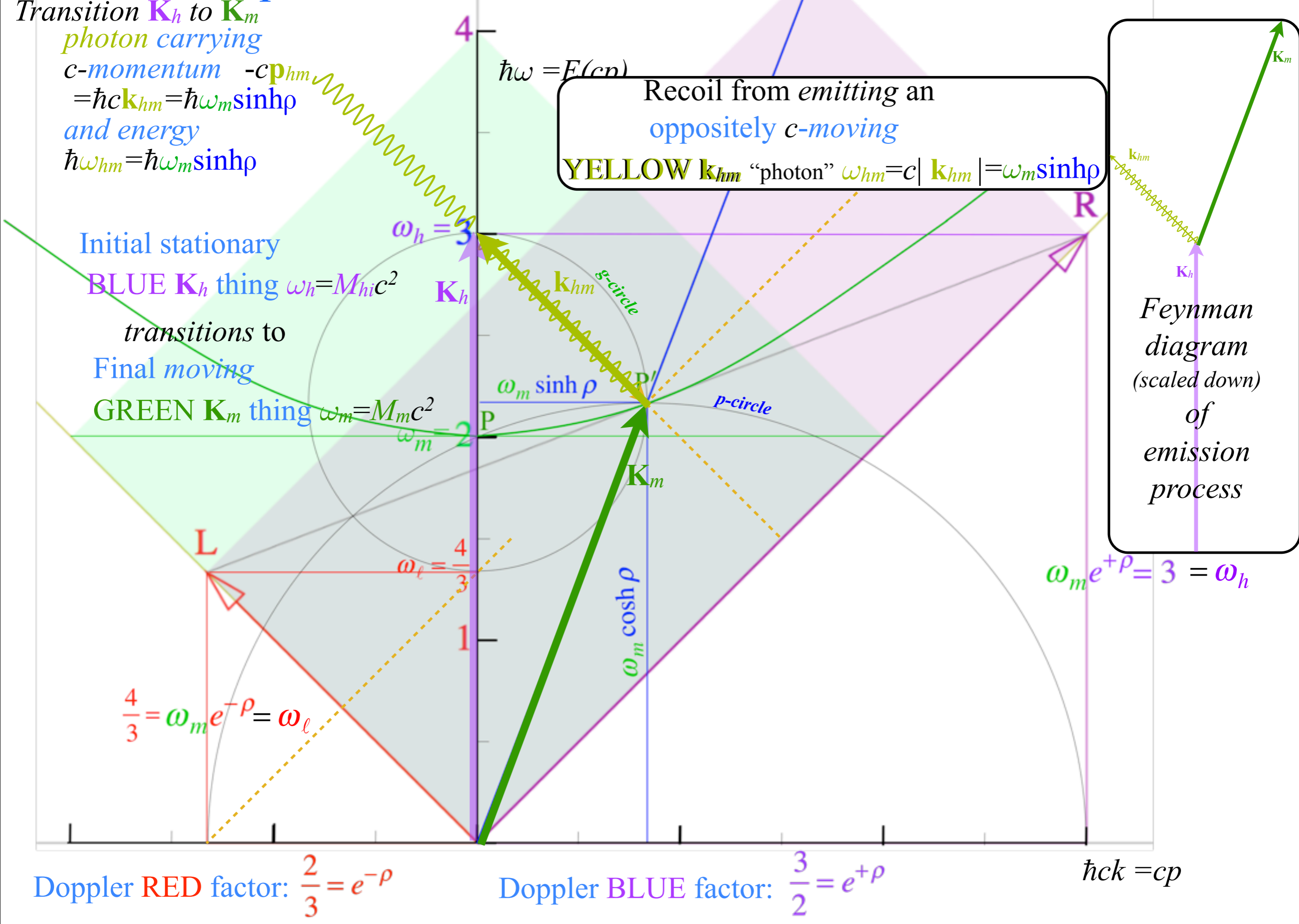
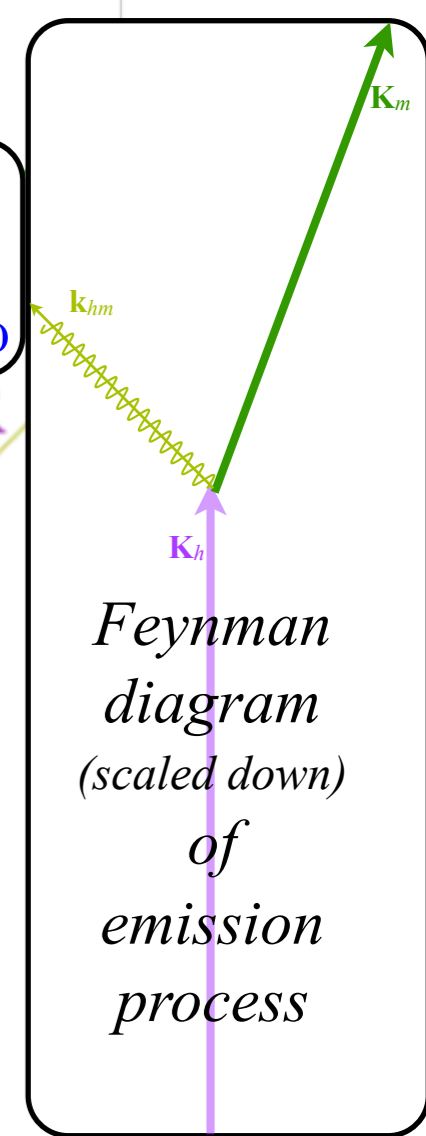


Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftharpoons |mid\rangle = |\omega_m\rangle \rightleftharpoons |low\rangle = |\omega_l\rangle$

Transition \mathbf{K}_h to \mathbf{K}_m
 photon carrying
 c-momentum $-c\mathbf{p}_{hm}$
 $= \hbar c \mathbf{k}_{hm} = \hbar \omega_m \sinh \rho$
 and energy
 $\hbar \omega_{hm} = \hbar \omega_m \sinh \rho$

Recoil from emitting an
 oppositely c-moving
YELLOW \mathbf{k}_{hm} "photon" $\omega_{hm} = c |\mathbf{k}_{hm}| = \omega_m \sinh \rho$

Initial stationary
BLUE \mathbf{K}_h thing $\omega_h = M_h c^2$
 transitions to
 Final moving
GREEN \mathbf{K}_m thing $\omega_m = M_m c^2$

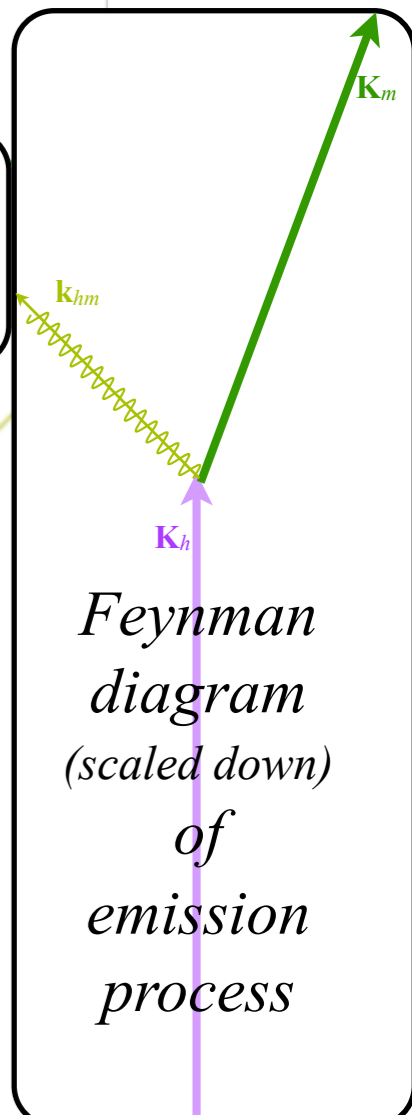


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Recoil from emitting an
 oppositely c-moving
 YELLOW \mathbf{k}_{hm} "photon" $\omega_{hm} = c |\mathbf{k}_{hm}| = \omega_m \sinh \rho$



Classical (and spectroscopic)
 Energy-momentum conservation
 is due to
 conservation in
 quantum-phase space-time
 "wiggle-count"

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

$\hbar ck = cp$

Transition \mathbf{K}_h to \mathbf{K}_m

photon carrying
 c-momentum $-c\mathbf{p}_{hm}$
 $=\hbar c\mathbf{k}_{hm}=\hbar\omega_m\sinh\rho$
 and energy
 $\hbar\omega_{hm}=\hbar\omega_m\sinh\rho$

$|high\rangle = |\omega_h\rangle$

$|mid\rangle = |\omega_m\rangle$

$|low\rangle = |\omega_l\rangle$ L

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

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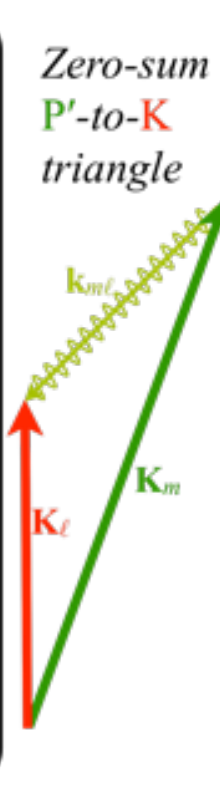
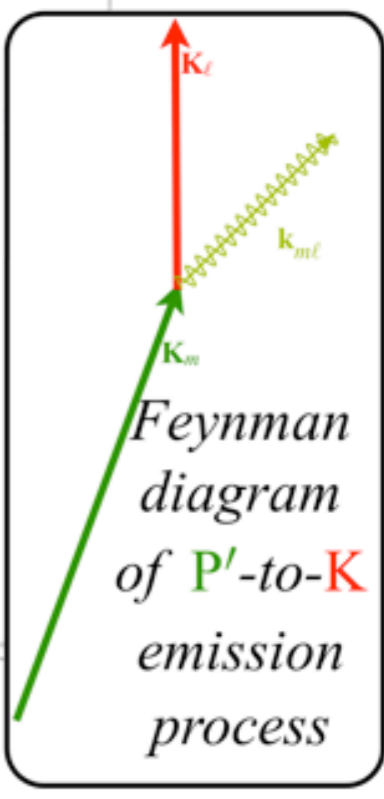
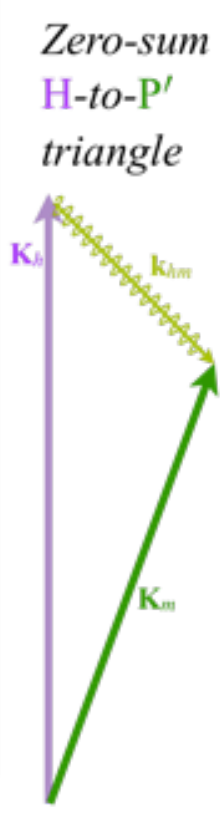
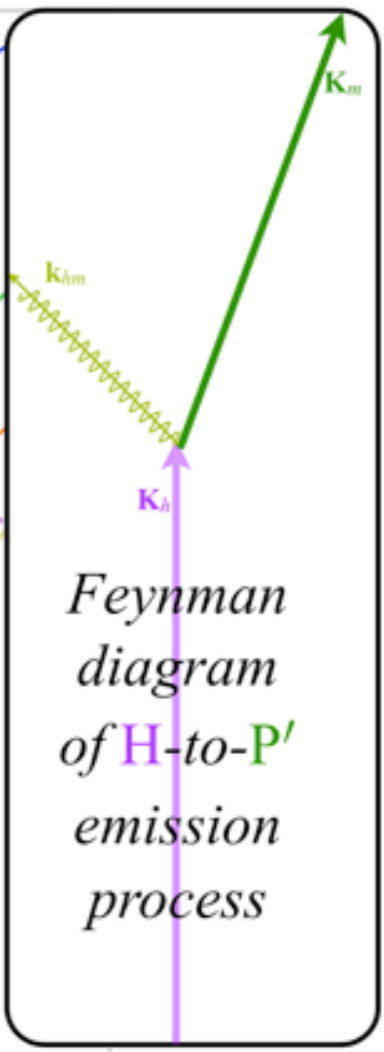
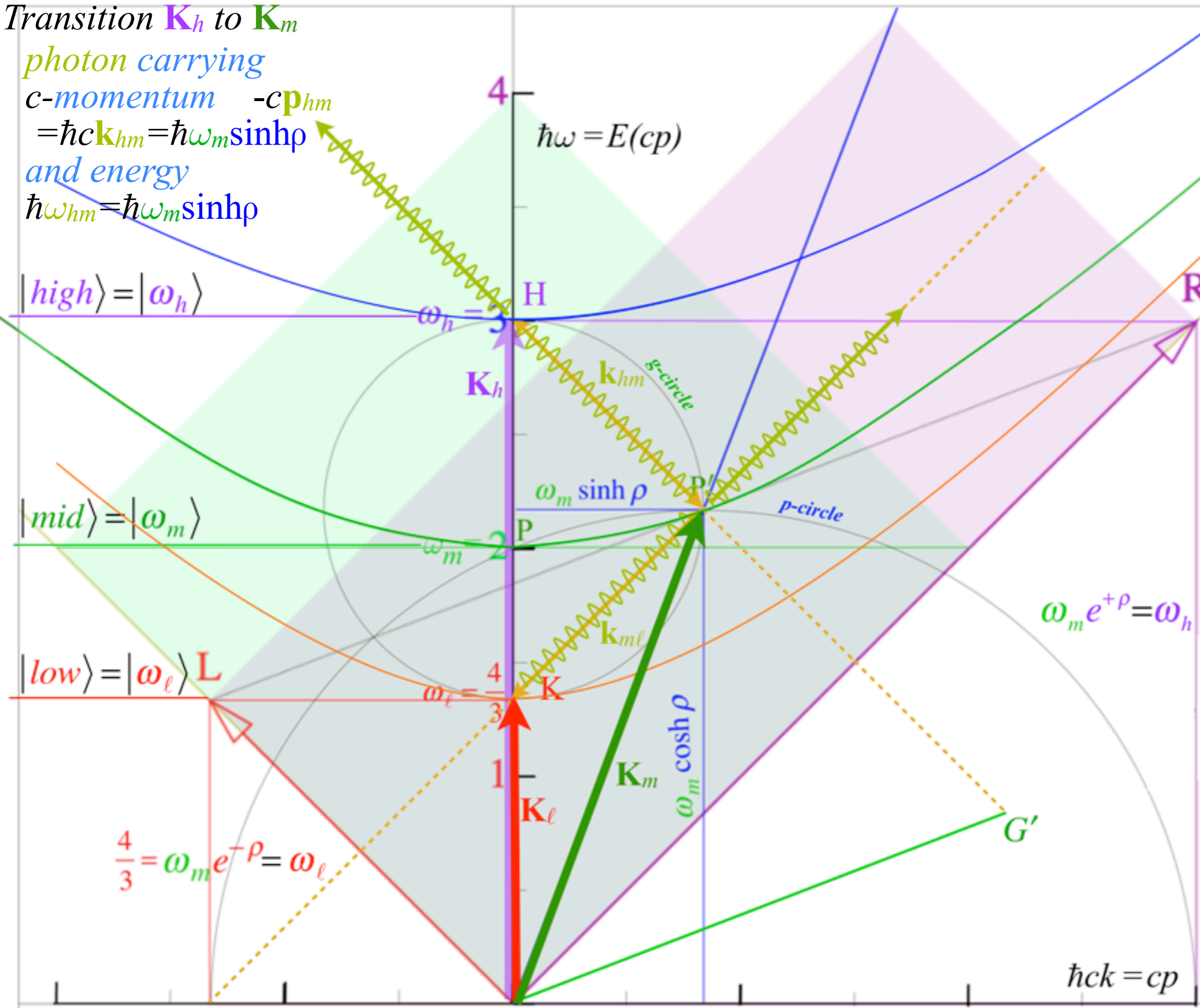


Fig. 31 (modified) of Unit 3

Transition \mathbf{K}_m to \mathbf{K}_ℓ

Transition \mathbf{K}_h to \mathbf{K}_m

photon carrying
c-momentum $-c\mathbf{p}_{hm}$
 $=\hbar c\mathbf{k}_{hm}=\hbar\omega_m\sinh\rho$
and energy
 $\hbar\omega_{hm}=\hbar\omega_m\sinh\rho$

photon carrying
c-momentum $+c\mathbf{p}_{ml}=c\mathbf{p}_{hm}$
 $=\hbar c\mathbf{k}_{ml}=\hbar\omega_m\sinh\rho$
and energy
 $\hbar\omega_{ml}=\hbar\omega_m\sinh\rho$

$|high\rangle = |\omega_h\rangle$

$|mid\rangle = |\omega_m\rangle$

$|low\rangle = |\omega_\ell\rangle$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

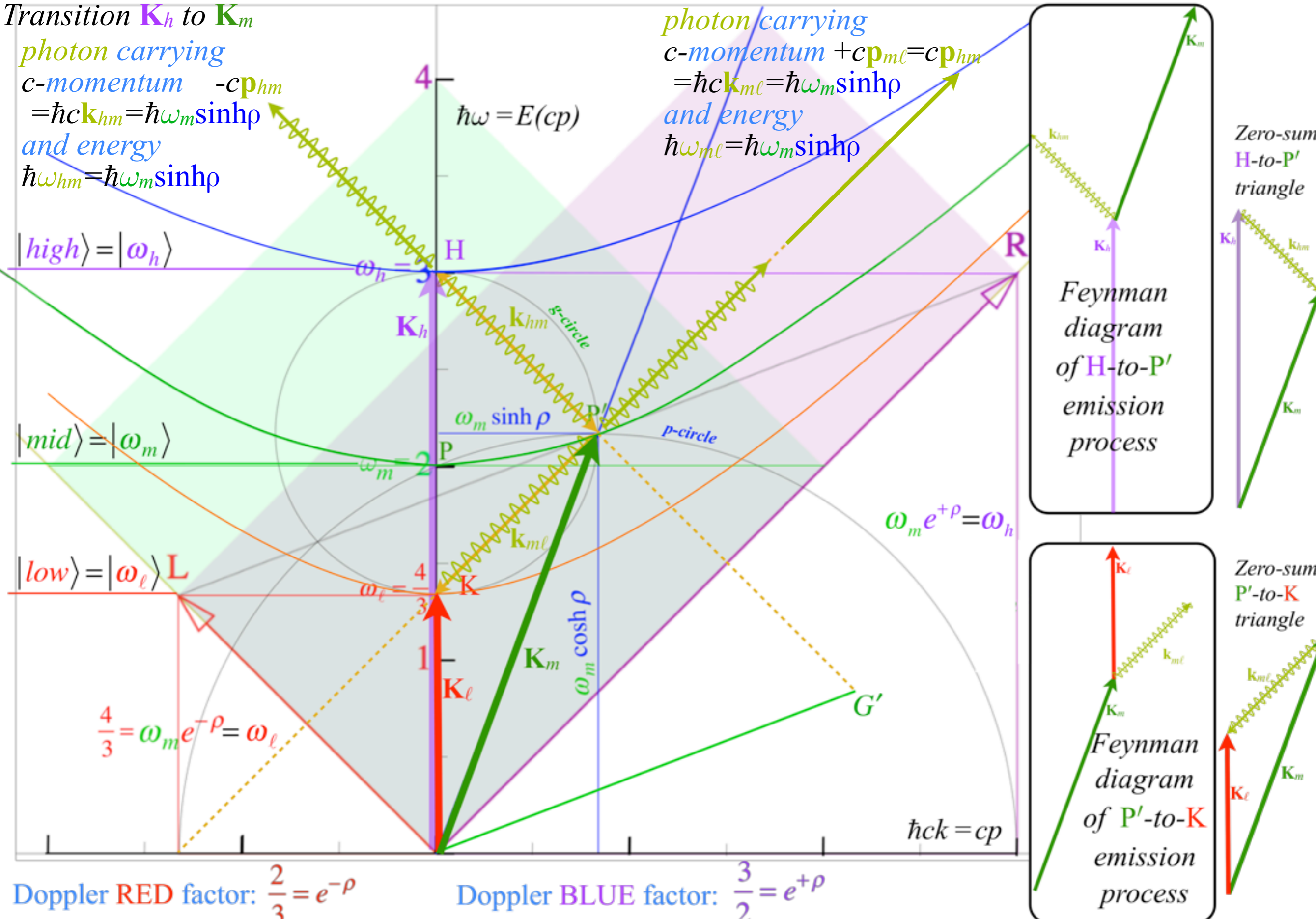


Fig. 31 (modified) of Unit 3

Relativity relates charge, current, and magnetic fields

Geometric derivation of magnetic constant μ_0 from electric ϵ_0

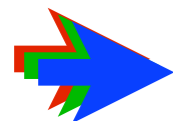
Lorentz-Poincare symmetry and energy-momentum spectral conservation rules

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Relativistic optical transitions and Compton recoil formulae

Feynman diagram geometry



Recoils shifts

Compton recoil related to rocket velocity formula

Geometric transition coordinate grids

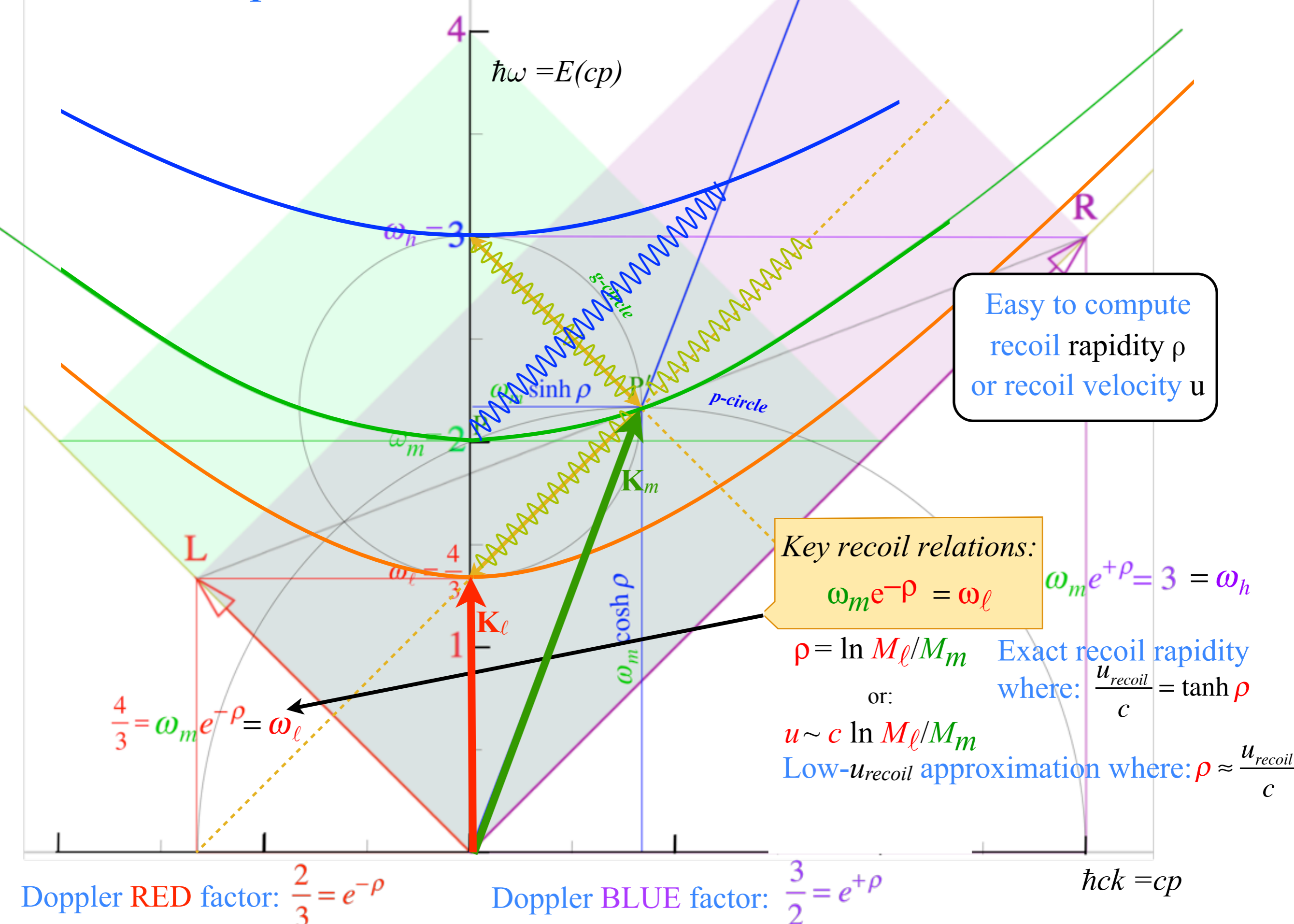
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2-3 transition long by δ

Recoil upshift = δ

$$\delta = \omega_h \cosh \rho - \omega_h \approx \frac{\omega_h}{2} \rho^2$$

$$KE_{recoil} \approx \frac{\hbar \omega_h}{2} \rho^2 = \frac{M_h}{2} u^2$$

3-2 transition short by δ

Recoil downshift = δ

$$\delta = \omega_m \cosh \rho - \omega_m \approx \frac{\omega_m}{2} \rho^2$$

$$KE_{recoil} \approx \frac{\hbar \omega_m}{2} \rho^2 = \frac{M_m}{2} u^2$$

Easy to compute recoil rapidity ρ or recoil velocity u

Key recoil relations:

$$\omega_m e^{-\rho} = \omega_l \quad \omega_m e^{+\rho} = 3 = \omega_h$$

$\rho = \ln M_l / M_m$ Exact recoil rapidity where: $\frac{u_{recoil}}{c} = \tanh \rho$

or:
 $u \sim c \ln M_l / M_m$
 Low- u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

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$\hbar ck = cp$

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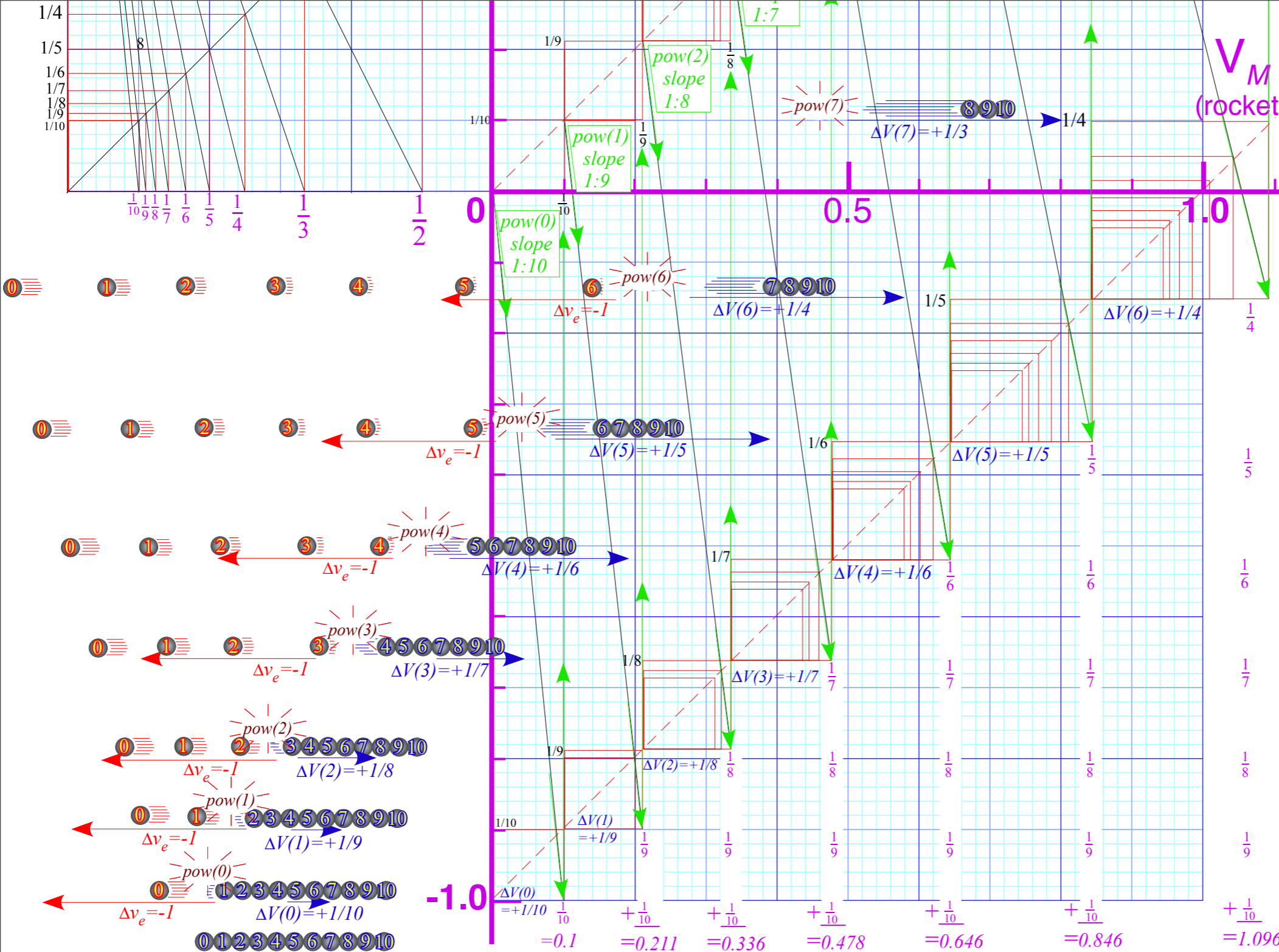
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Unit 1
Fig. 7.8a-b

Rocket Science!
from
Lecture 10
p.85



- | | | |
|---|---|---|
| 0 th : $V(0) = 1/10 = 0.1$ | 1 st : $V(1) = 1/10 + 1/9 = 0.211$ | 2 nd : $V(2) = 1/10 + 1/9 + 1/8 = 0.336$ |
| 3 rd : $V(3) = V(2) + 1/7 = 0.478$ | 4 th : $V(4) = V(3) + 1/6 = 0.646$ | 5 th : $V(5) = V(4) + 1/5 = 0.846$ |
| 6 th : $V(6) = V(5) + 1/4 = 1.096$ | 7 th : $V(7) = V(6) + 1/3 = 1.429$ | 8 th : $V(8) = V(7) + 1/2 = 1.929$ |

v_e known as
"Specific Impulse"

By calculus: $M \cdot \Delta V = -v_e \cdot \Delta M$ or: $dV = -v_e \frac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV = -v_e \int_{M_{IN}}^{M_{FIN}} \frac{dM}{M}$

The Rocket Equation: $V_{FIN} - V_{IN} = -v_e [\ln M_{FIN} - \ln M_{IN}] = v_e \left[\ln \frac{M_{IN}}{M_{FIN}} \right]$

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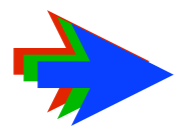
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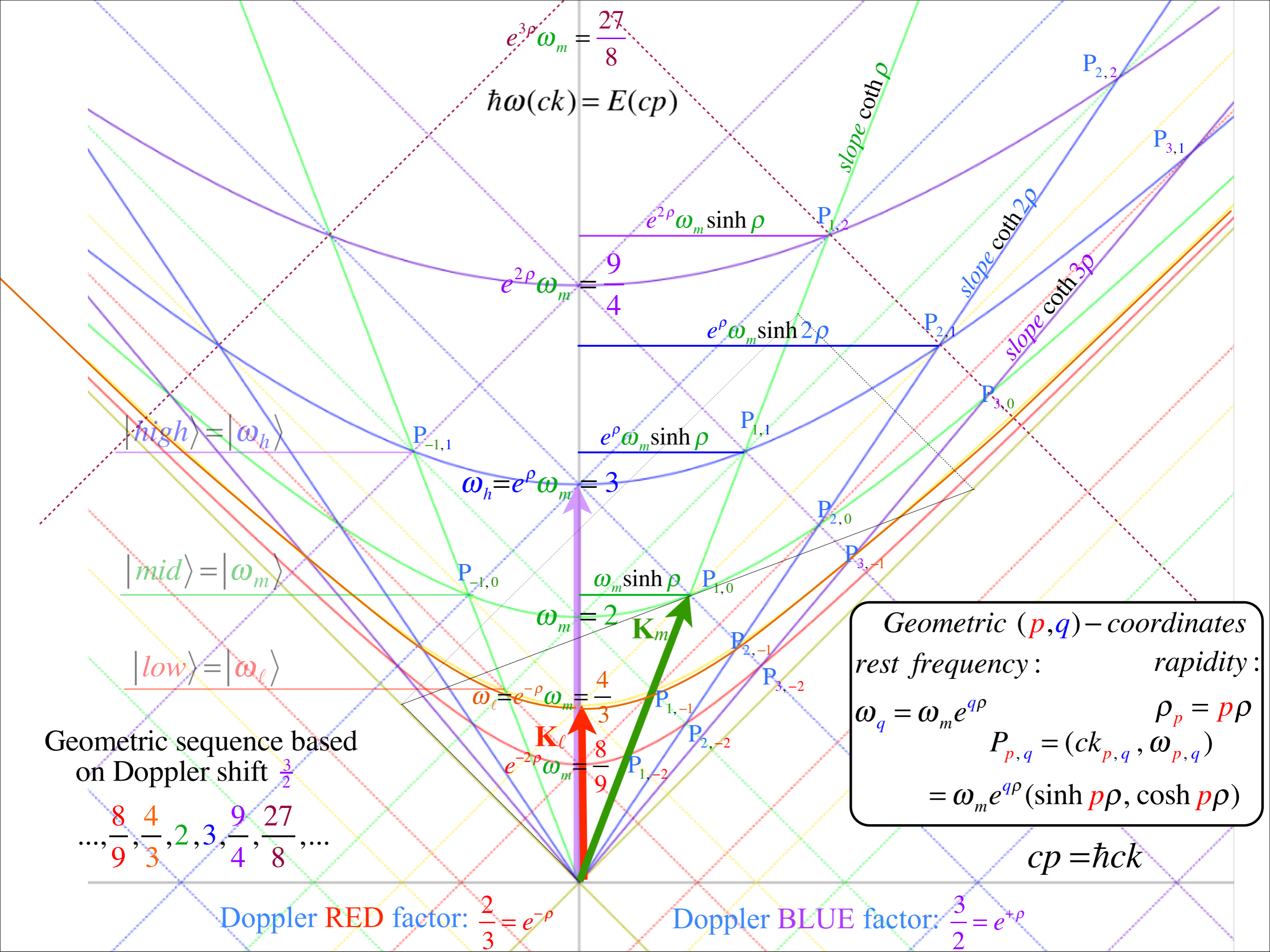
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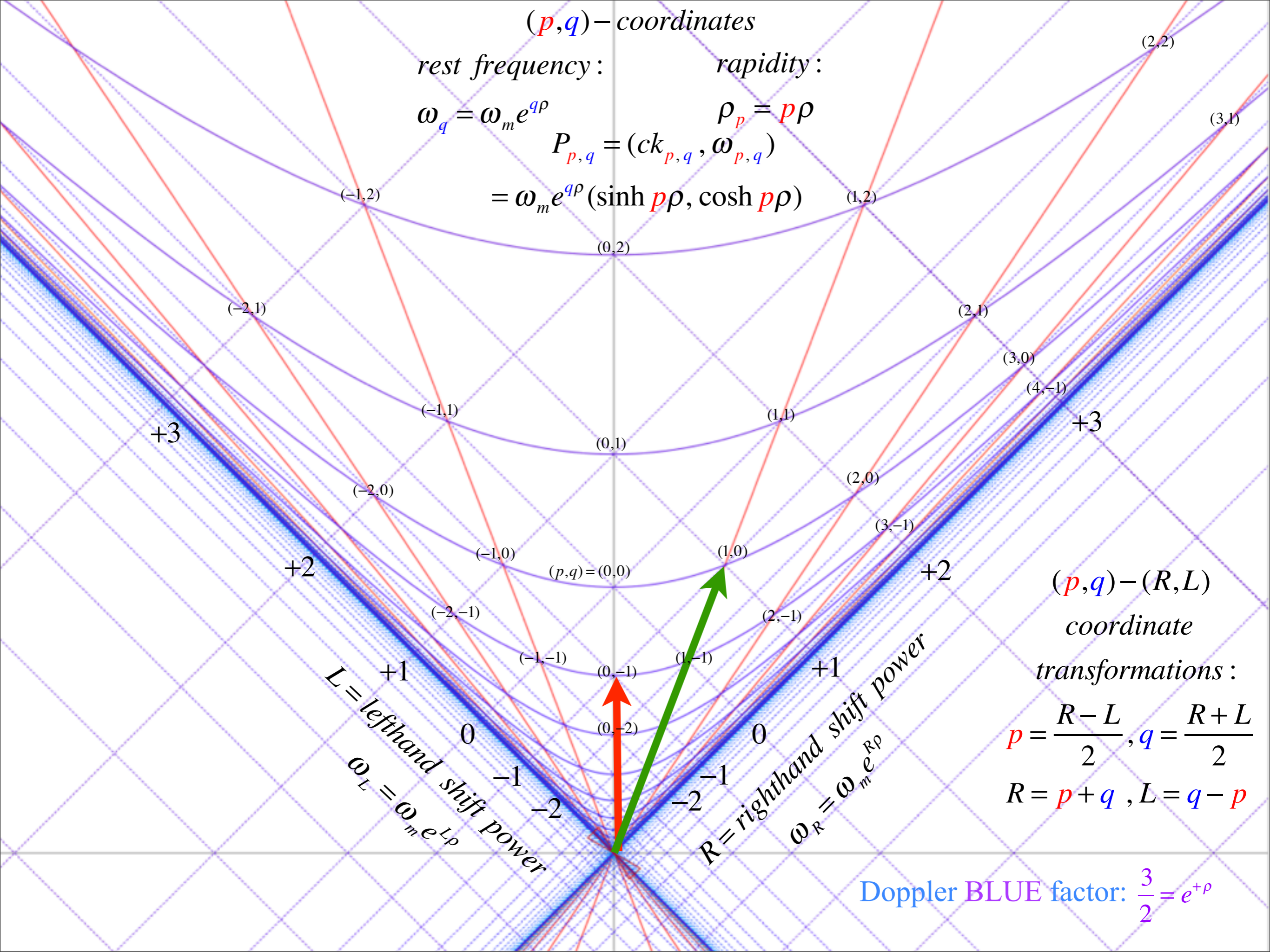
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(p, q) -coordinates

rest frequency:

rapidity:

$$\omega_q = \omega_m e^{q\rho}$$

$$\rho_p = p\rho$$

$$P_{p,q} = (ck_{p,q}, \omega_{p,q})$$

$$= \omega_m e^{q\rho} (\sinh p\rho, \cosh p\rho)$$

All-rational-fraction lattice
defined by discrete sub-group
of Lorentz Poincare Group
(Feynman path integrals defined
by group transformations)

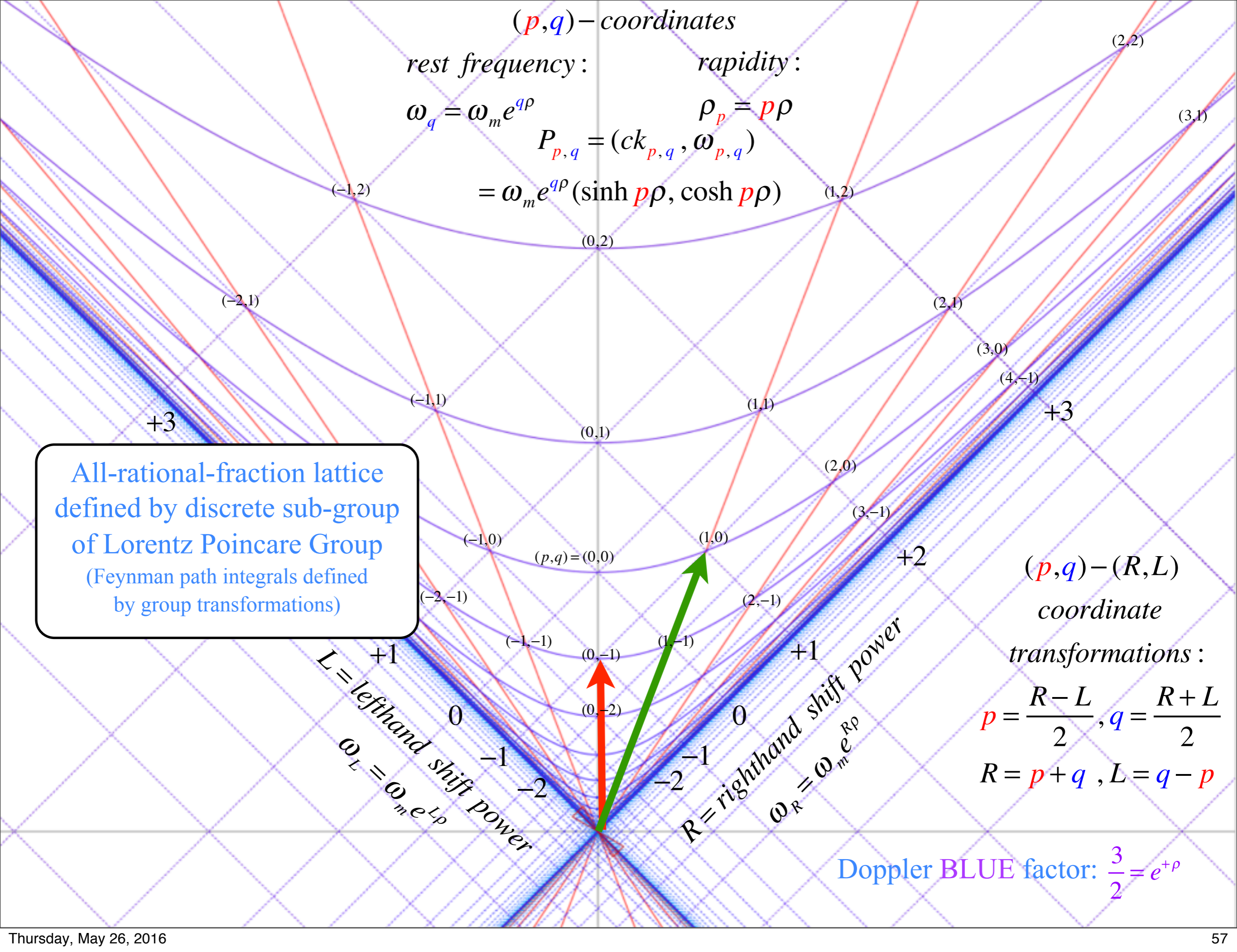
(p, q) - (R, L)
coordinate

transformations:

$$p = \frac{R-L}{2}, q = \frac{R+L}{2}$$

$$R = p + q, L = q - p$$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$



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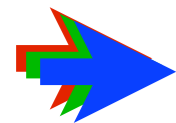
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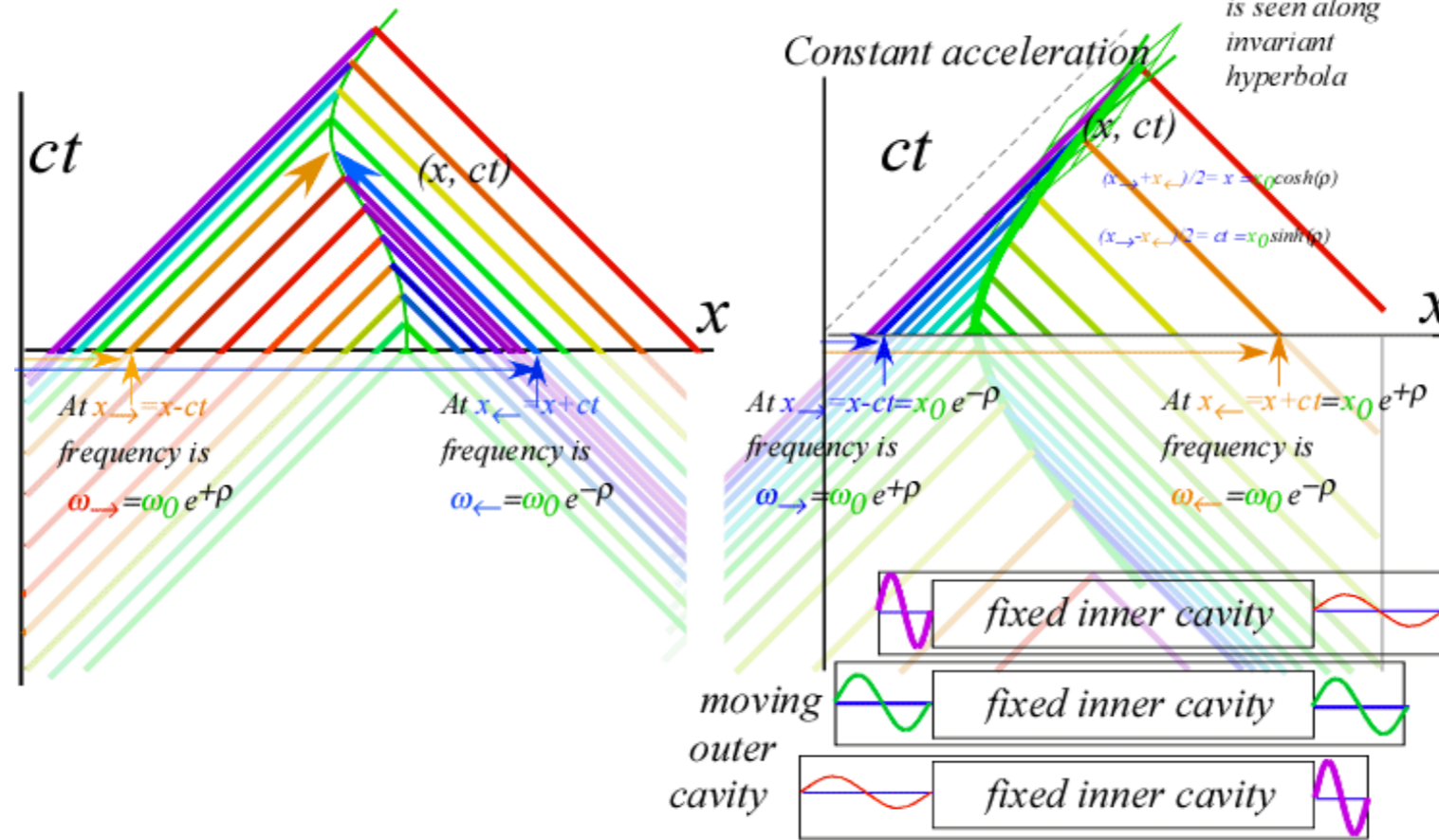
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Einstein Elevators Made by Chirped 2-CW Light

Varying Acceleration by Chirping



Wave frames of **varying** acceleration

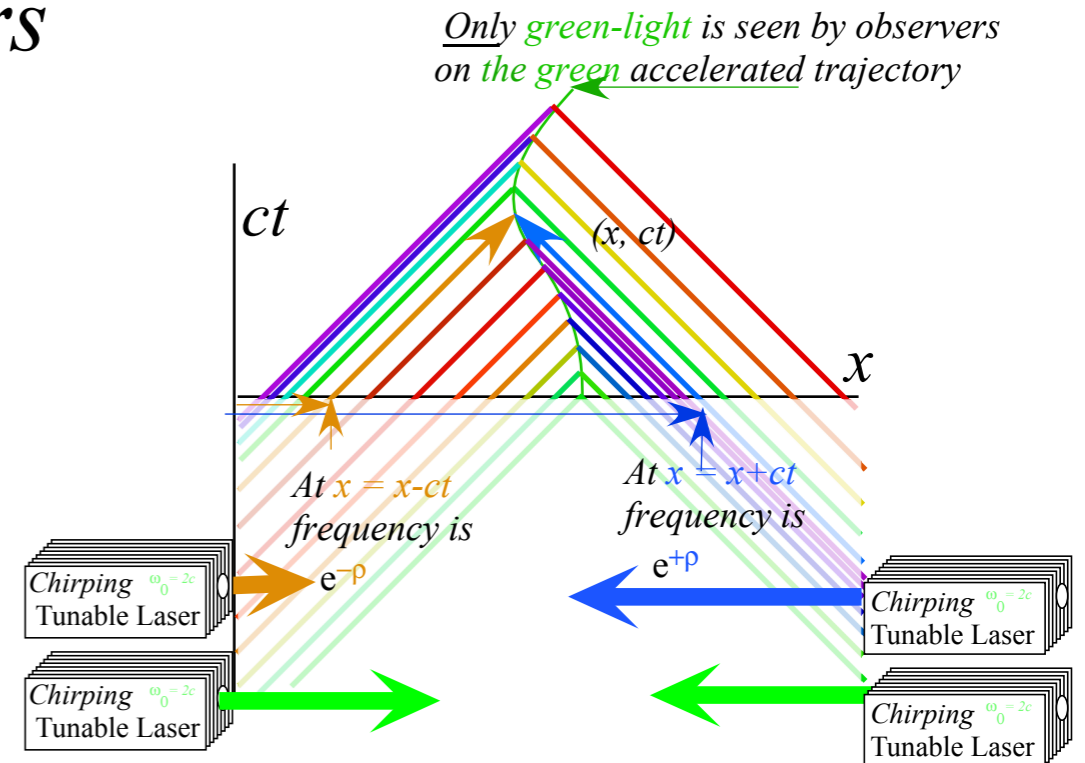
Acceleration by chirping laser pairs

Varying acceleration (General case)

Varying local acceleration $\rho = \rho(\tau)$ Lab time dt vs proper time $d\tau$

$$u = \frac{dx}{dt} = c \tanh \rho(\tau)$$

$$dt = d\tau \cosh \rho(\tau) = \frac{d\tau}{\sqrt{1 - u^2/c^2}}$$



Previous examples involved constant velocity

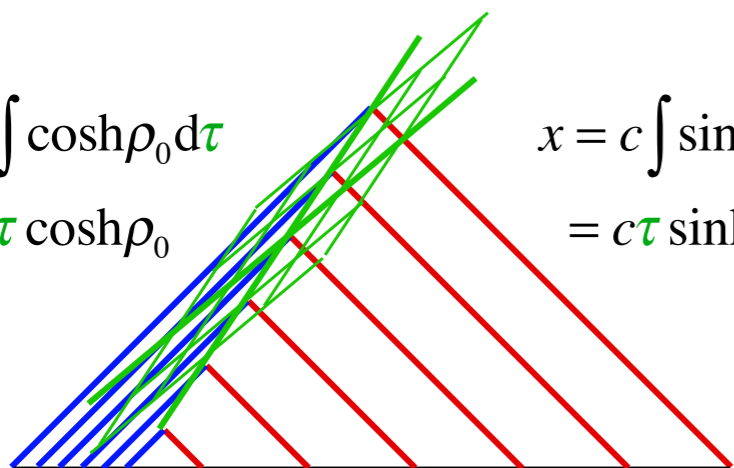
Constant velocity $\rho = \rho_0$ "Lorentz-Transformation"

$$ct = c \int \cosh \rho_0 d\tau$$

$$= c\tau \cosh \rho_0$$

$$x = c \int \sinh \rho_0 d\tau$$

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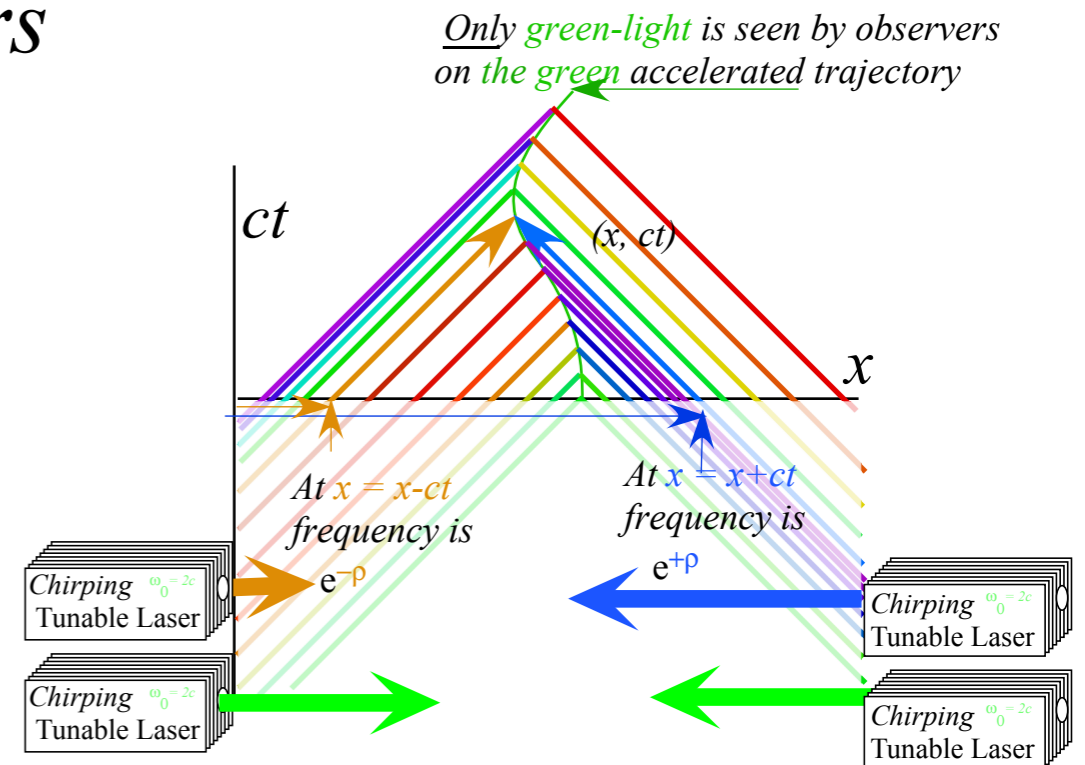
$$dt = d\tau \cosh \rho(\tau) = \frac{d\tau}{\sqrt{1 - u^2/c^2}}$$

$$\frac{dt}{d\tau} = \cosh \rho(\tau)$$

$$\frac{dx}{d\tau} = \frac{dx}{dt} \frac{dt}{d\tau} = c \tanh \rho(\tau) \cosh \rho(\tau)$$

$$ct = c \int \cosh \rho(\tau) d\tau$$

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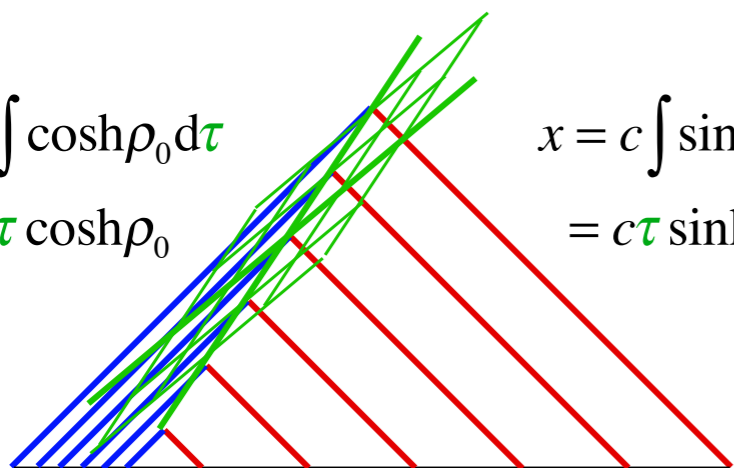


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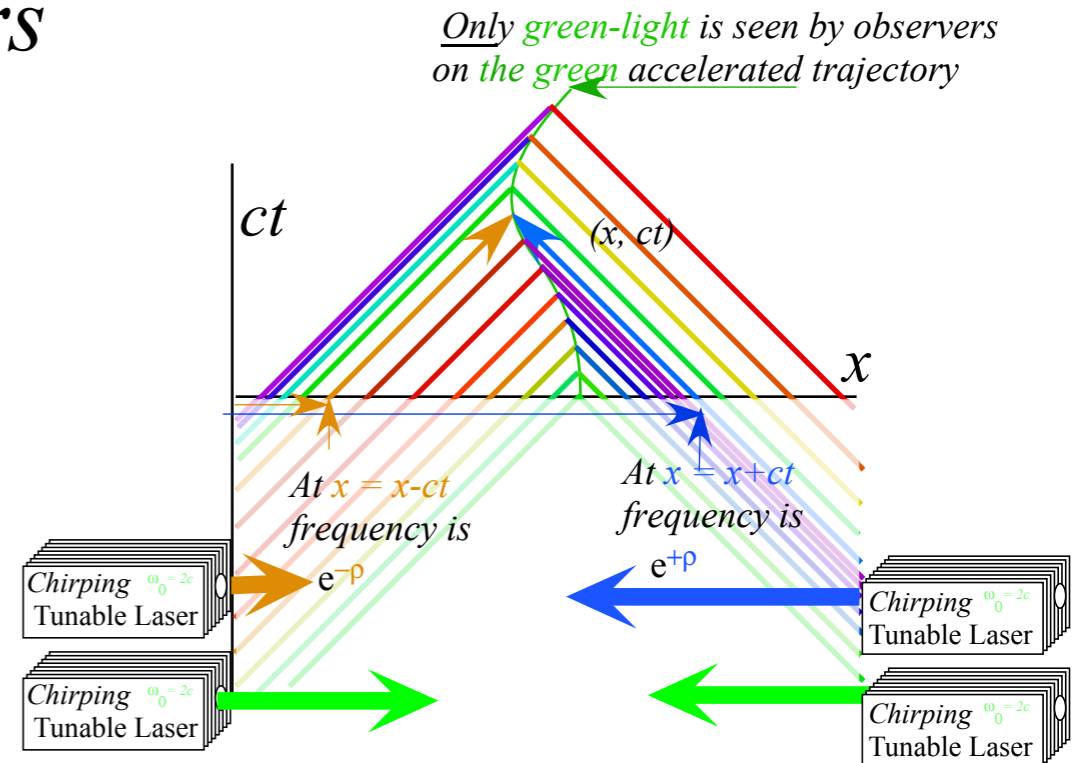
$$ct = c \int \cosh \rho(\tau) d\tau$$

$$x = c \int \sinh \rho(\tau) d\tau$$

Constant local acceleration $\rho = \frac{g\tau}{c}$ "Einstein-Elevator"

$$ct = c \int \cosh \frac{g\tau}{c} d\tau = \frac{c^2}{g} \sinh \frac{g\tau}{c}$$

$$x = c \int \sinh \frac{g\tau}{c} d\tau = \frac{c^2}{g} \cosh \frac{g\tau}{c}$$

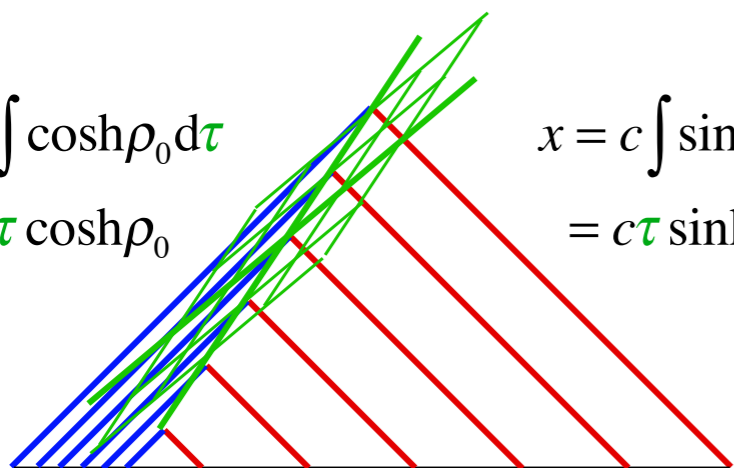


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$$\frac{dt}{d\tau} = \cosh \rho(\tau) \quad \frac{dx}{d\tau} = \frac{dx}{dt} \frac{dt}{d\tau} = c \tanh \rho(\tau) \cosh \rho(\tau)$$

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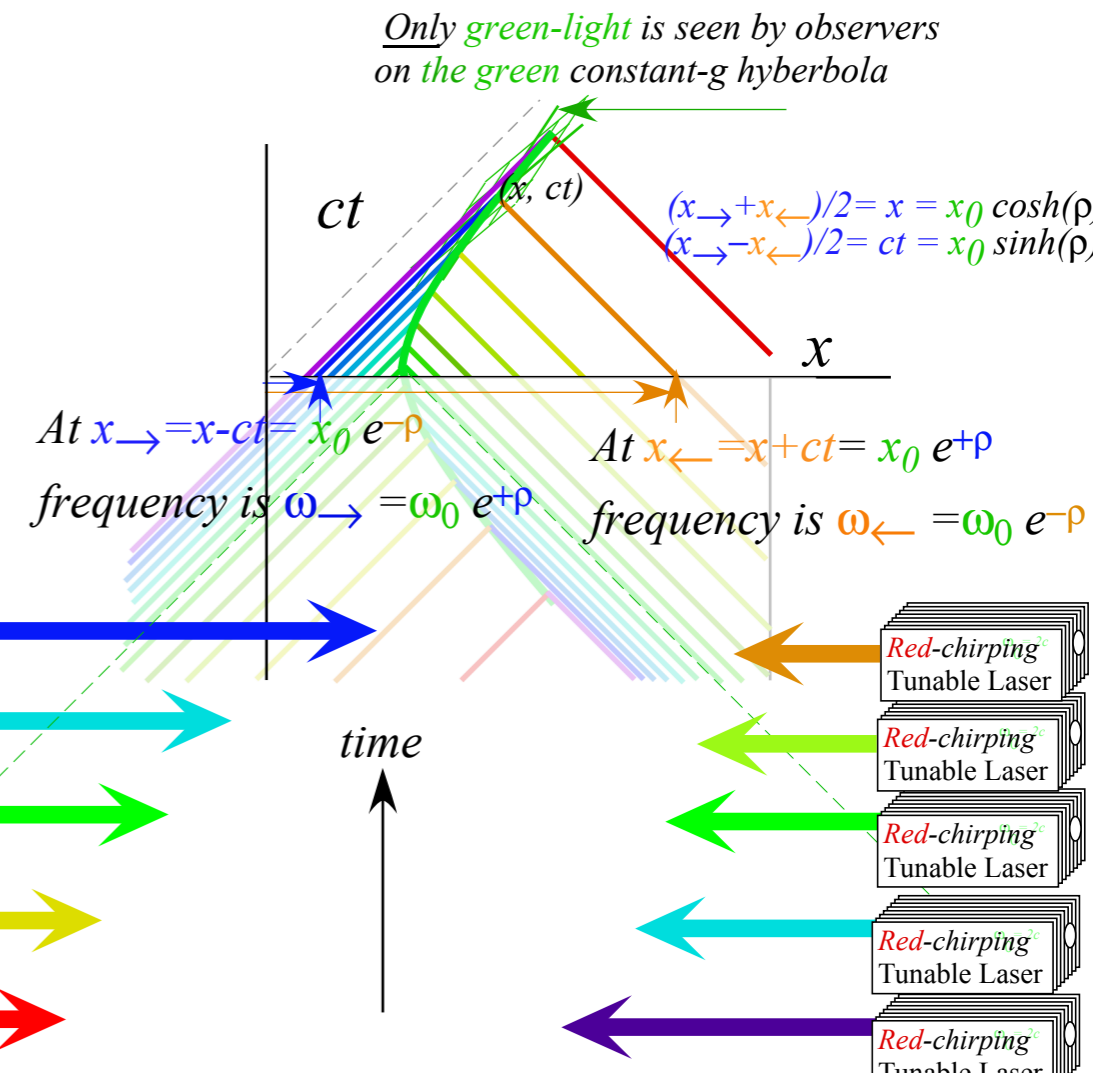
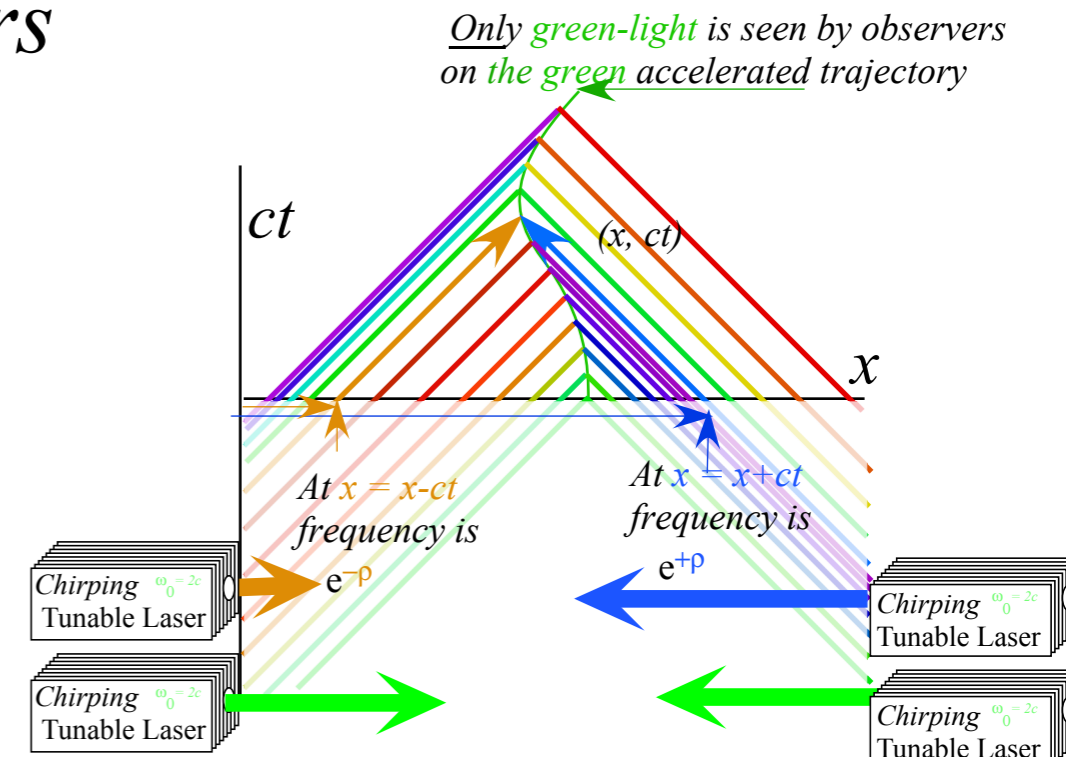
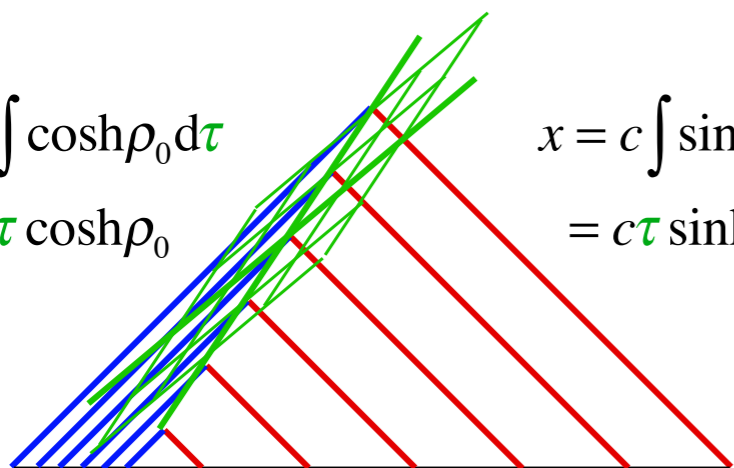


Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

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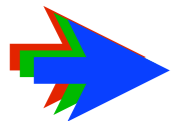
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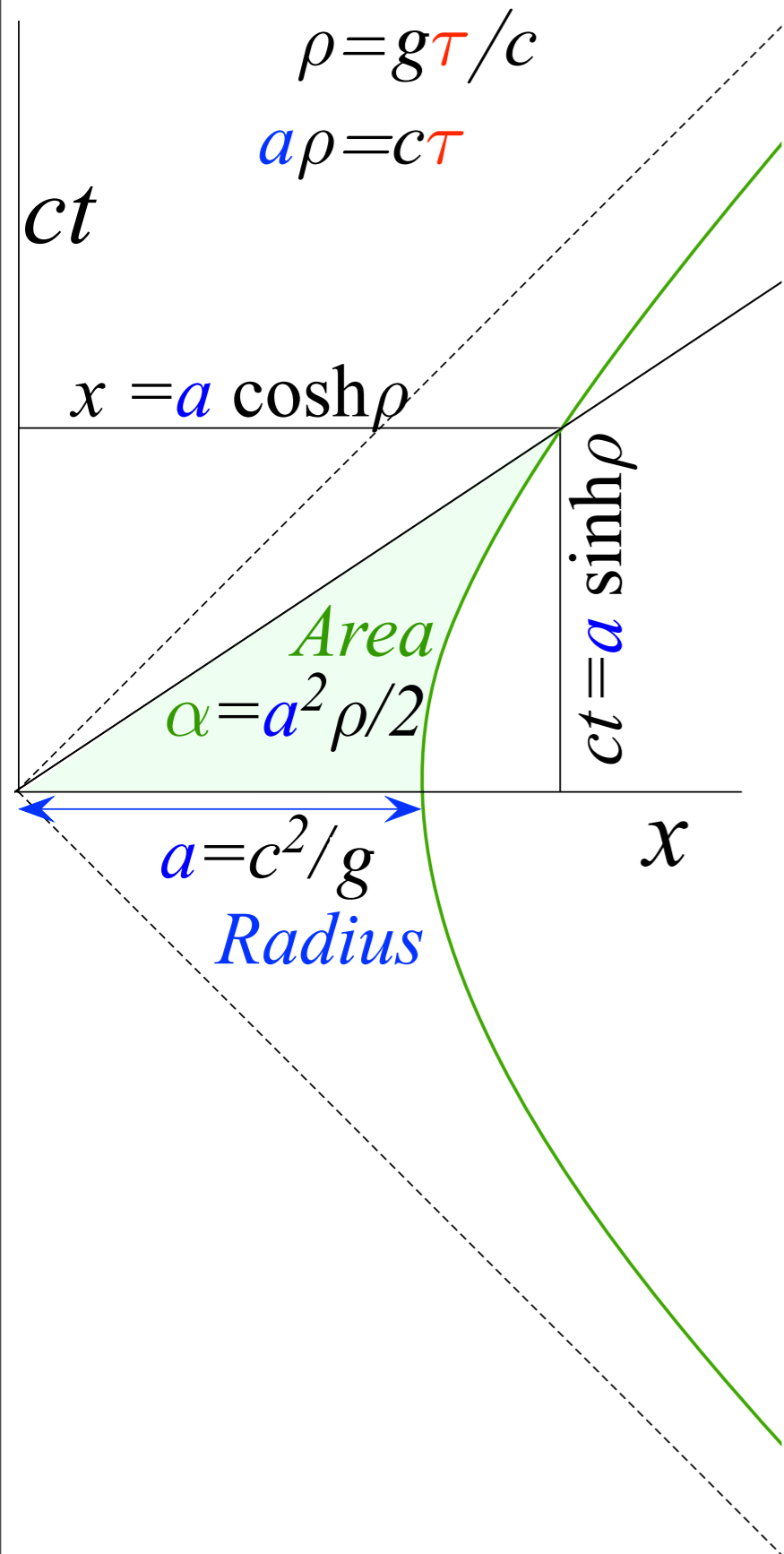
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(a) Constant acceleration g

Rapidity ρ vs proper time τ



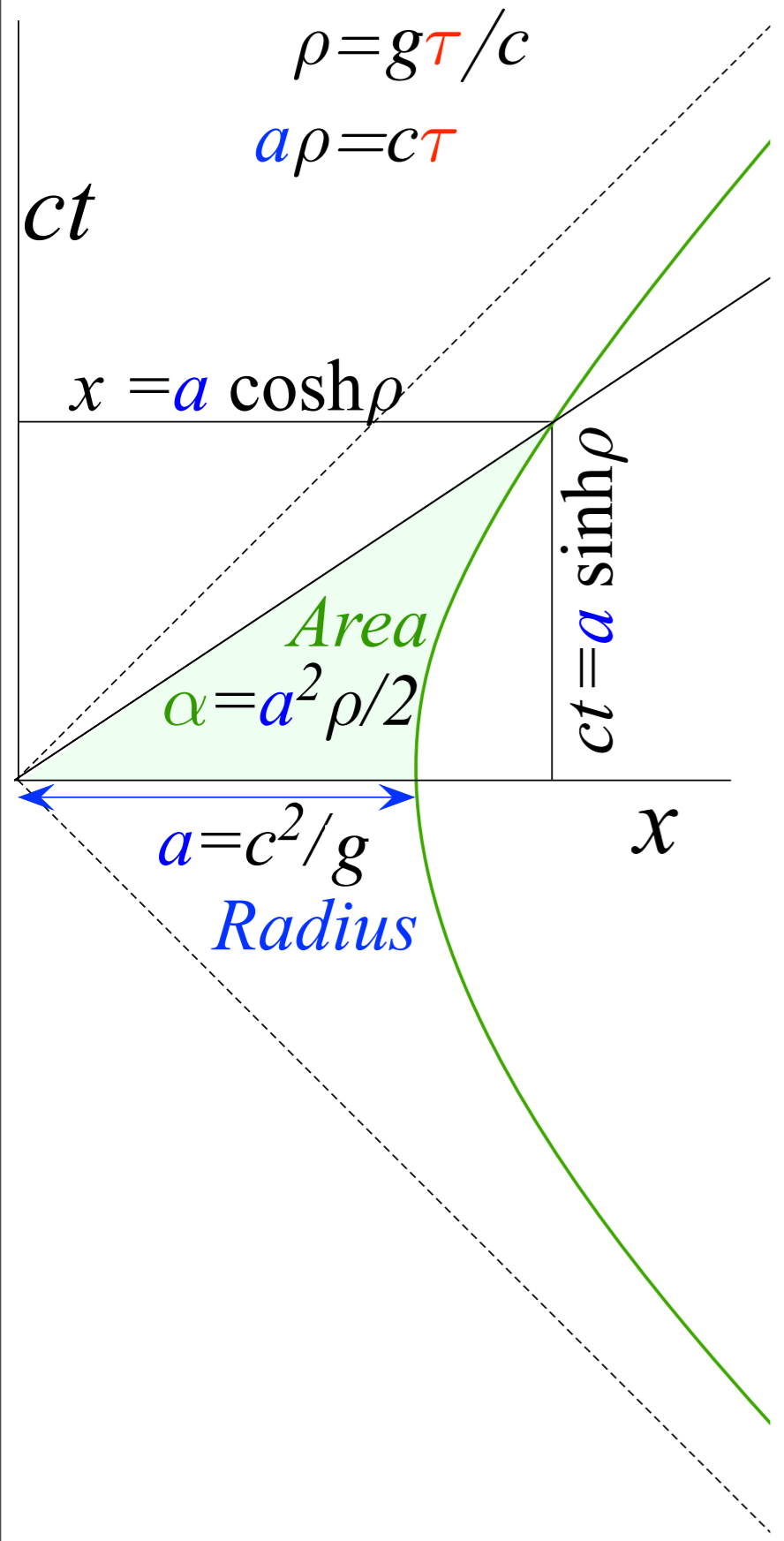
for: $\rho c = g\tau$ or: $\rho = \frac{g\tau}{c}$

$$ct = c \int \cosh\left(\frac{g\tau}{c}\right) d\tau = \frac{c^2}{g} \sinh\left(\frac{g\tau}{c}\right) = a \sinh \rho$$

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(a) Constant acceleration g

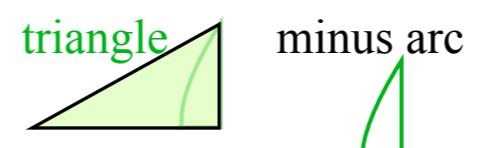
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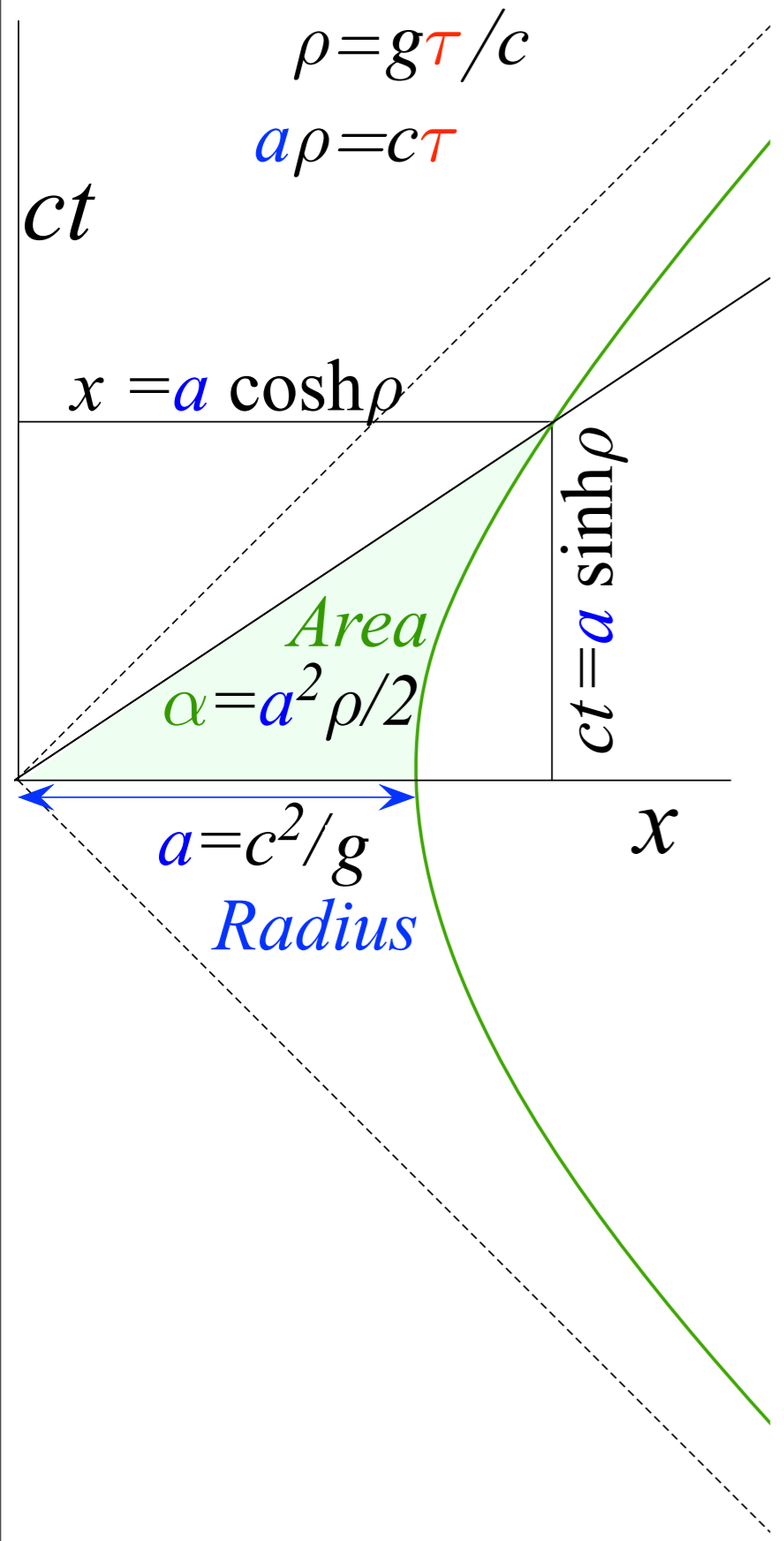
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$$\alpha(\text{Area}) = \frac{x \cdot ct}{2} - \int_0^{x_1} ct \cdot dx = \frac{x \cdot ct}{2} - \int_0^{\rho_1} ct \cdot \frac{dx}{d\rho} d\rho$$

(a) Constant acceleration g

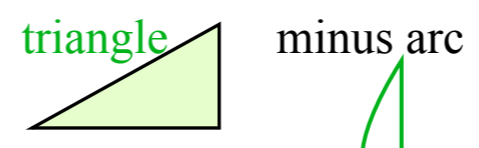
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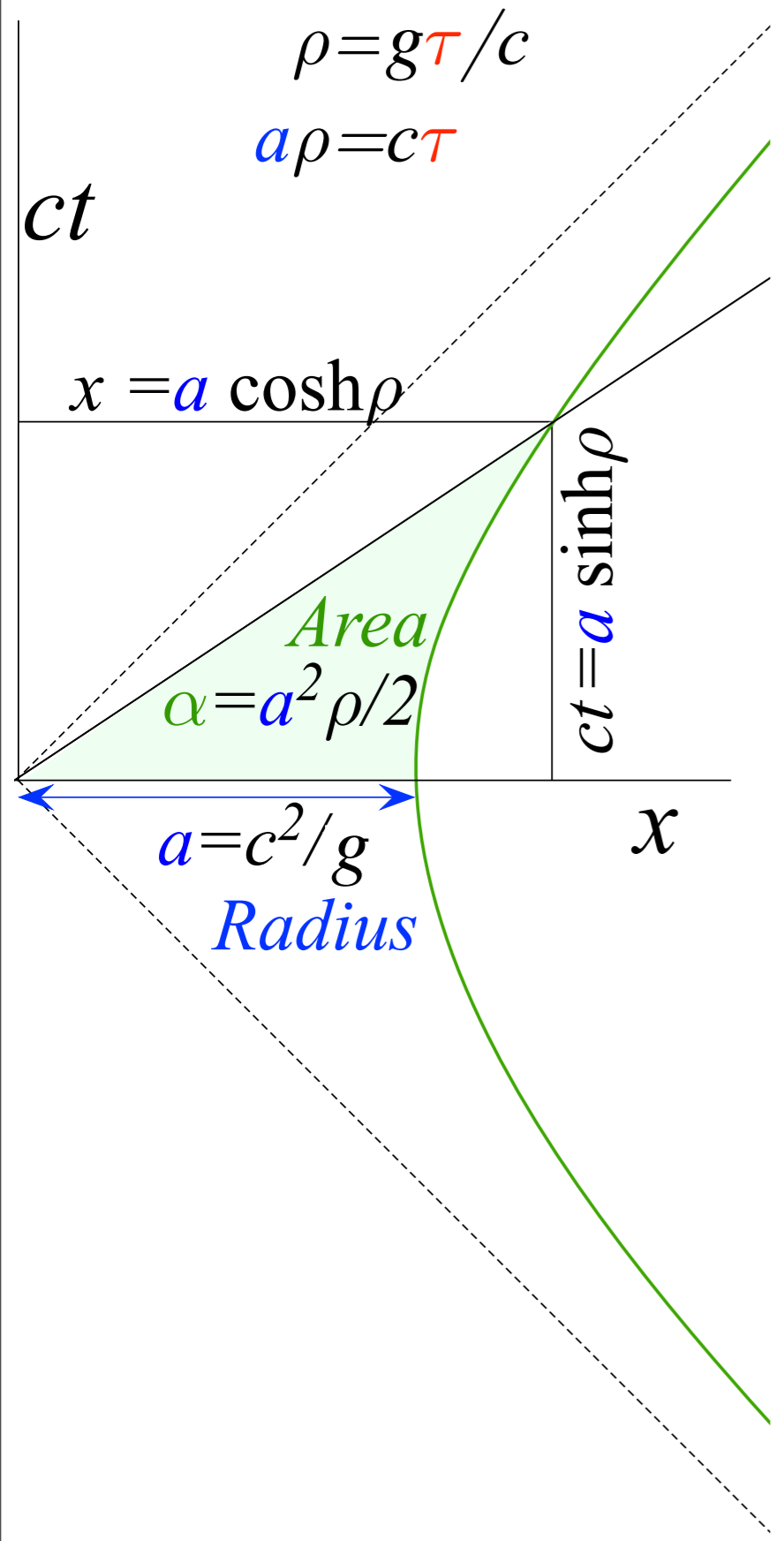


$$\alpha(\text{Area}) = \frac{x \cdot ct}{2} - \int_0^{x_1} ct \cdot dx = \frac{x \cdot ct}{2} - \int_0^{\rho_1} ct \cdot \frac{dx}{d\rho} d\rho$$

$$\alpha(\rho_1) = a^2 \frac{\cosh \rho_1 \cdot \sinh \rho_1}{2} - \int_0^{\rho_1} (a \sinh \rho)^2 d\rho = \frac{a^2}{2} \rho_1 = \frac{a^2}{2} \frac{g\tau_1}{c}$$

(a) Constant acceleration g

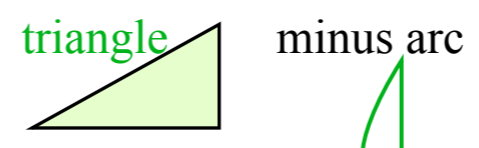
Rapidity ρ vs proper time τ



for: $\rho c = g\tau$ or: $\rho = \frac{g\tau}{c}$

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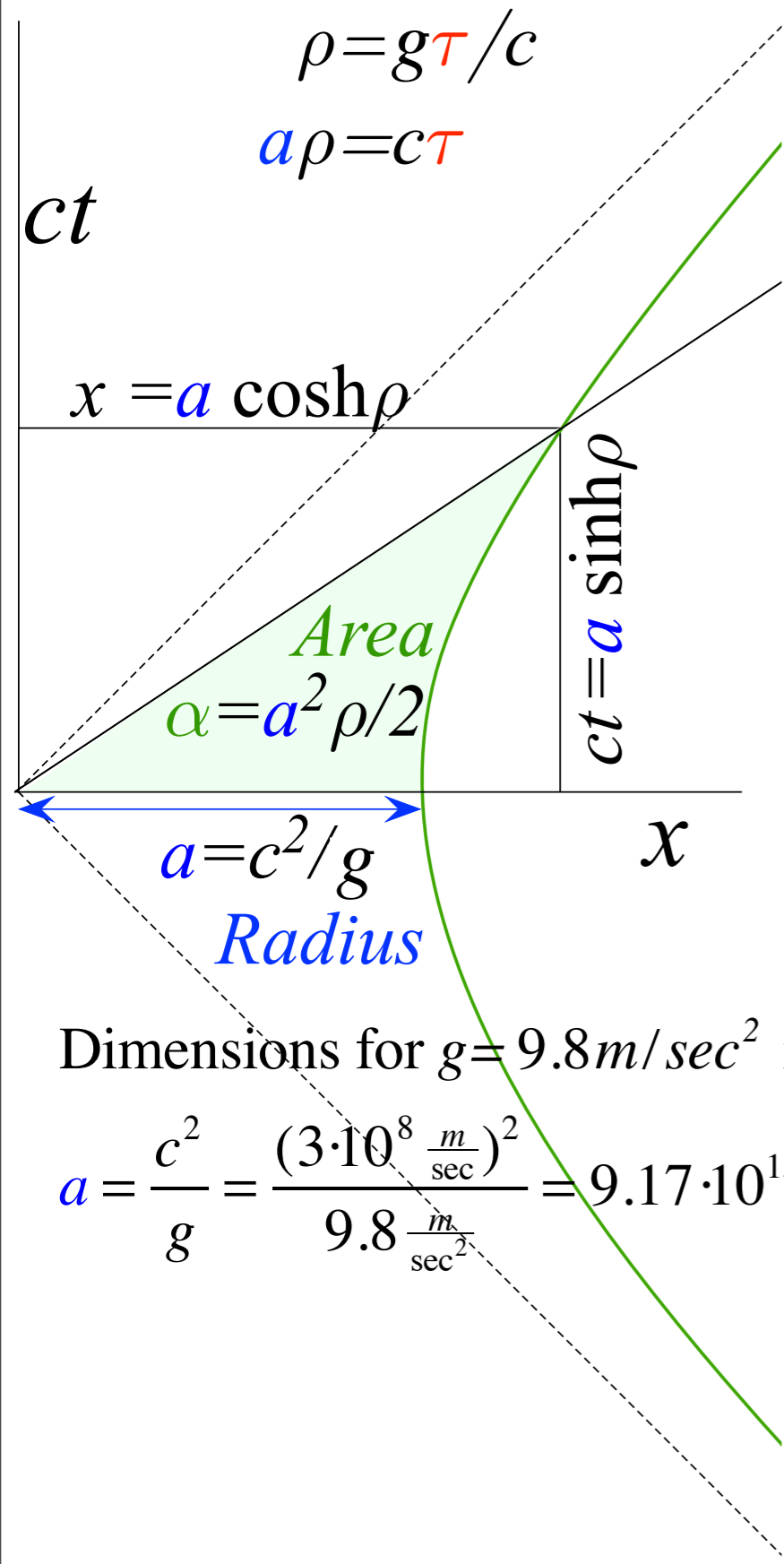
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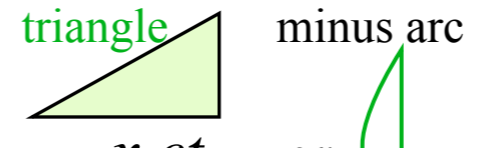
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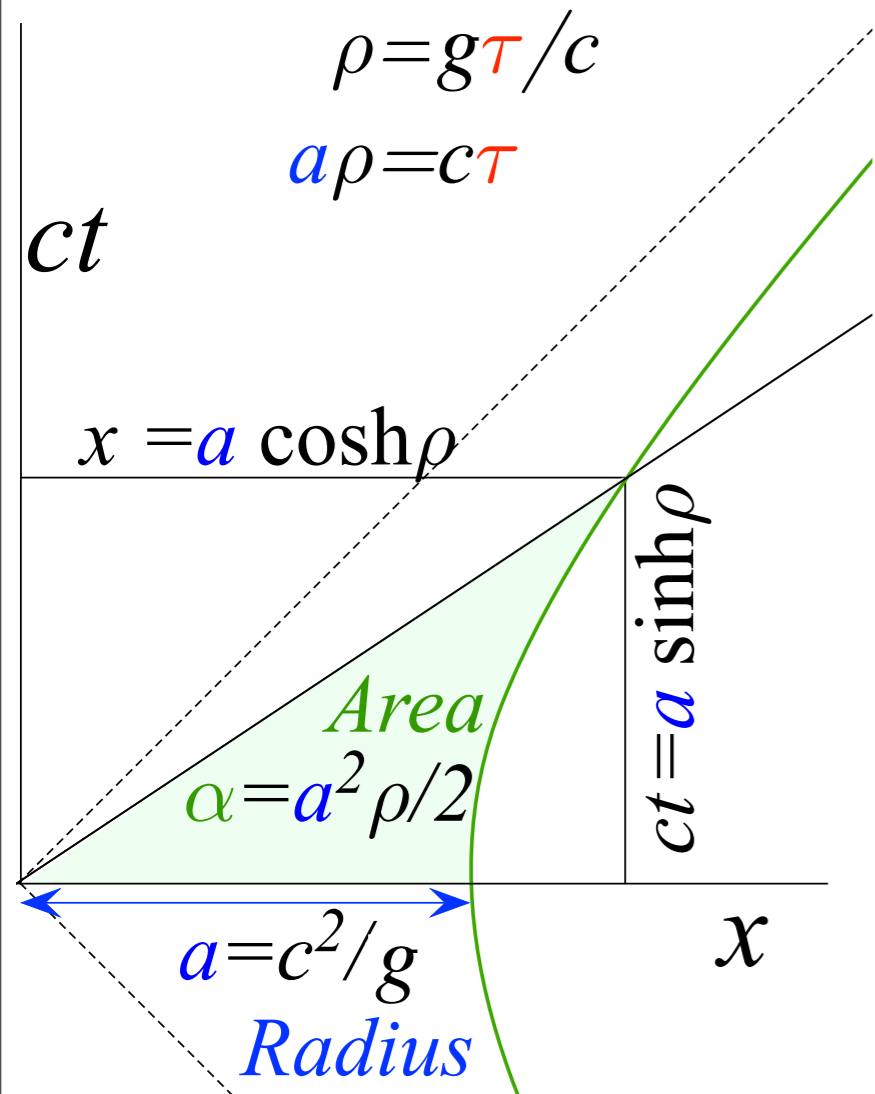
$$a\rho_1 = c\tau_1$$

Dimensions for $g = 9.8 \text{ m/sec}^2$ in unit of 1 lite yr = $c(3.15 \cdot 10^7 \text{ sec}) = 9.44 \cdot 10^{15} \text{ m}$

$$a = \frac{c^2}{g} = \frac{(3 \cdot 10^8 \frac{\text{m}}{\text{sec}})^2}{9.8 \frac{\text{m}}{\text{sec}^2}} = 9.17 \cdot 10^{15} \text{ m} = 0.97 \text{ lite yr}$$

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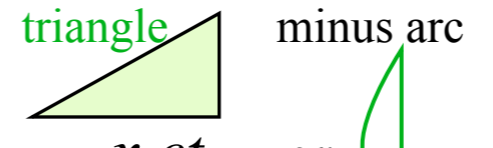
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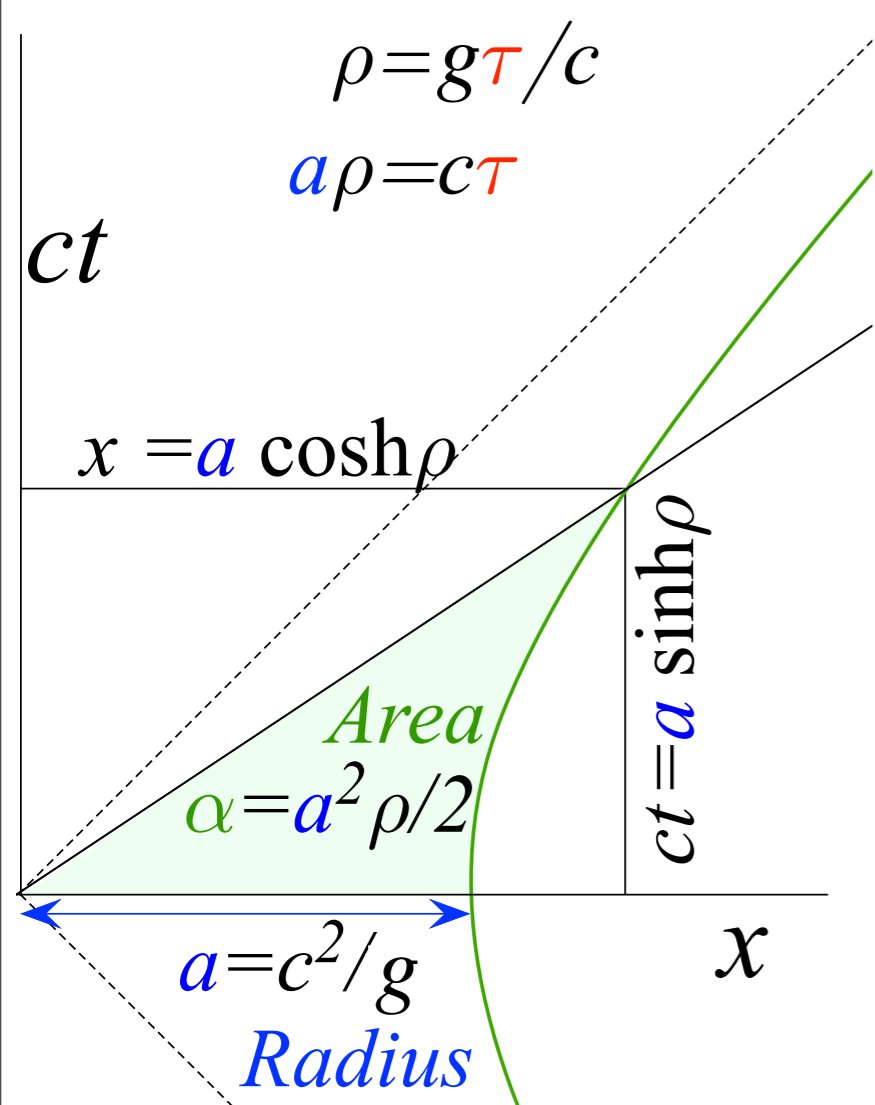
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Then $x = a \cosh \rho_1 = 0.97 (\cosh 1.03) = 1.53 \text{ lite yr}$

(a) Constant acceleration g

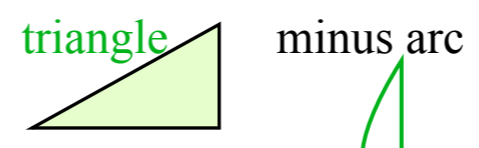
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Then $x = a \cosh \rho_1 = 0.97 (\cosh 1.03) = 1.53 \text{ lite yr}$

Rapidity for $c\tau_1 = 21 \text{ yr} = 6.62 \cdot 10^8 \text{ sec}$ is $\rho_1 = \frac{c\tau_1}{a} = \frac{3 \cdot 10^8 (6.62 \cdot 10^8)}{9.17 \cdot 10^{15}} = 21.63$

Then $x = a \cosh \rho_1 = 0.97 (\cosh 21.63) = 1.201 \cdot 10^9 \text{ lite yr}$

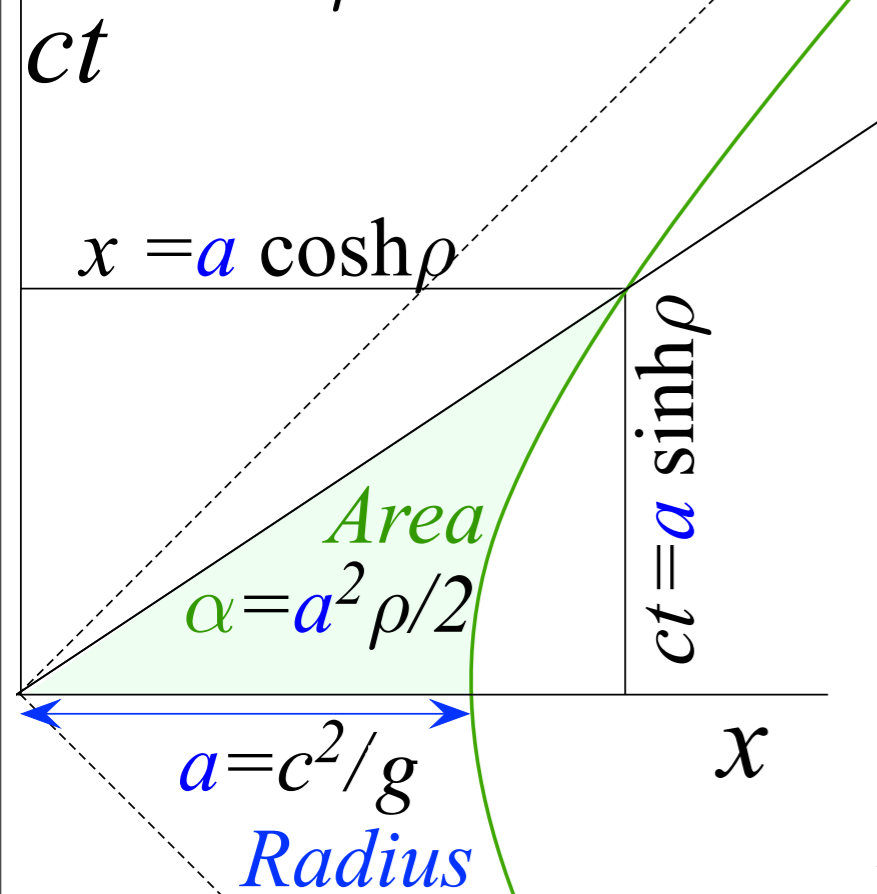
A long way to go to get a beer!

(a) Constant acceleration g

Rapidity ρ vs proper time τ

$$\rho = g\tau / c$$

$$a\rho = c\tau$$



(b) Traveler paths of acceleration g_q

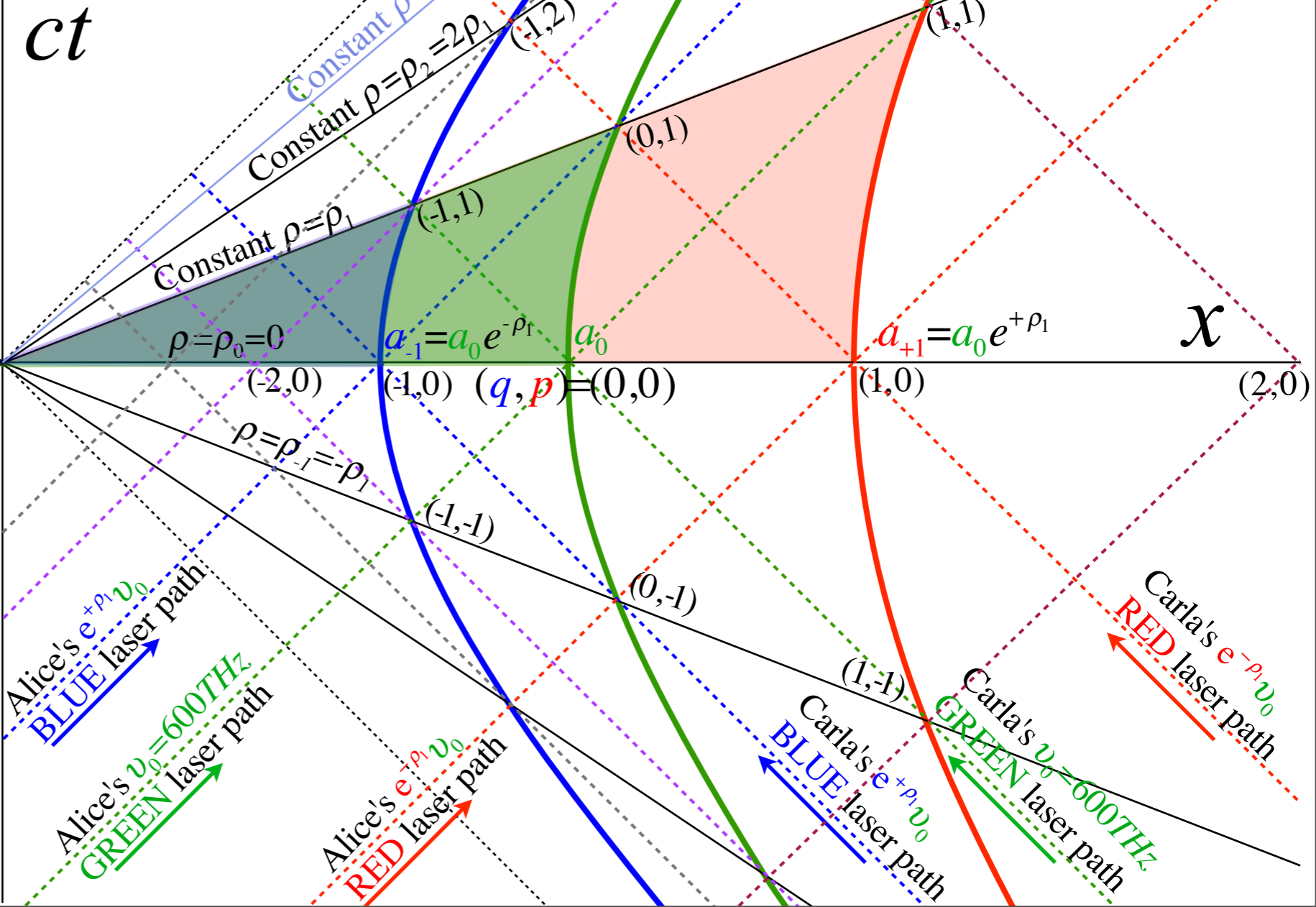
Al: $g_{-1} = g_0 e^{+\rho_1}$ Bob: $g_0 = c^2/a_0$ Carl: $g_{+1} = g_0 e^{-\rho_1}$

Inertial frame coordinates

$$(x_{q,p}, ct_{q,p}) = a_0 e^{q\rho_1} (\cosh p\rho_1, \sinh p\rho_1)$$

Geometric scale:

$$e^{q\rho_1} = \left(\frac{3}{2}\right)^q$$



Relativity relates charge, current, and magnetic fields

Geometric derivation of magnetic constant μ_0 from electric ϵ_0

Lorentz-Poincare symmetry and energy-momentum spectral conservation rules

Review of 2nd-quantization “photon” number N and 1st-quantization wavenumber $\kappa = m$

Sketches of atomic and molecular spectroscopy

Relativistic optical transitions and Compton recoil formulae

Feynman diagram geometry

Recoils shifts

Compton recoil related to rocket velocity formula

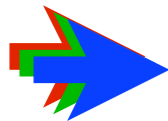
Geometric transition coordinate grids

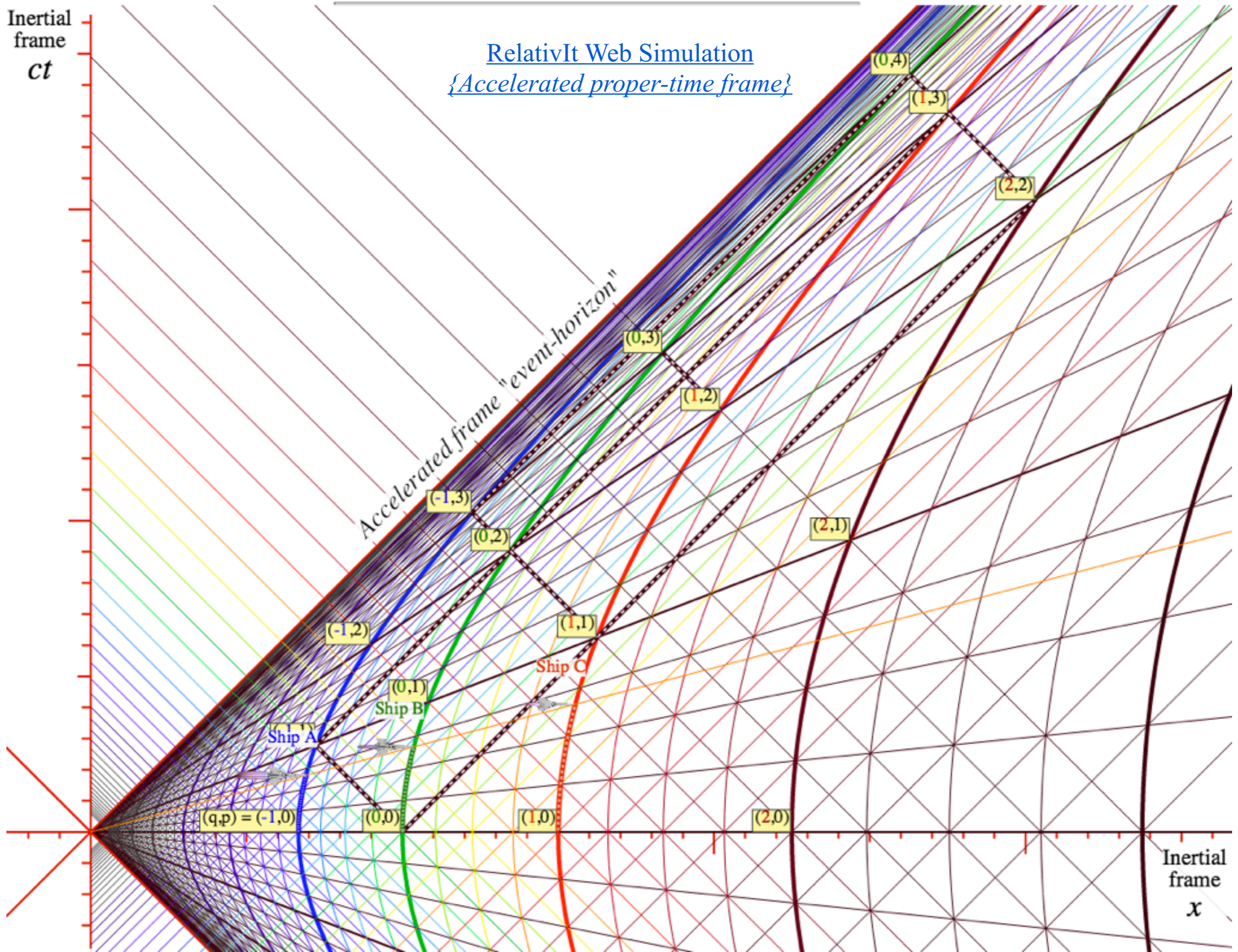
Relativity in accelerated frames

Laser up-tuning by Alice and down-tuning by Carla makes g -acceleration grid

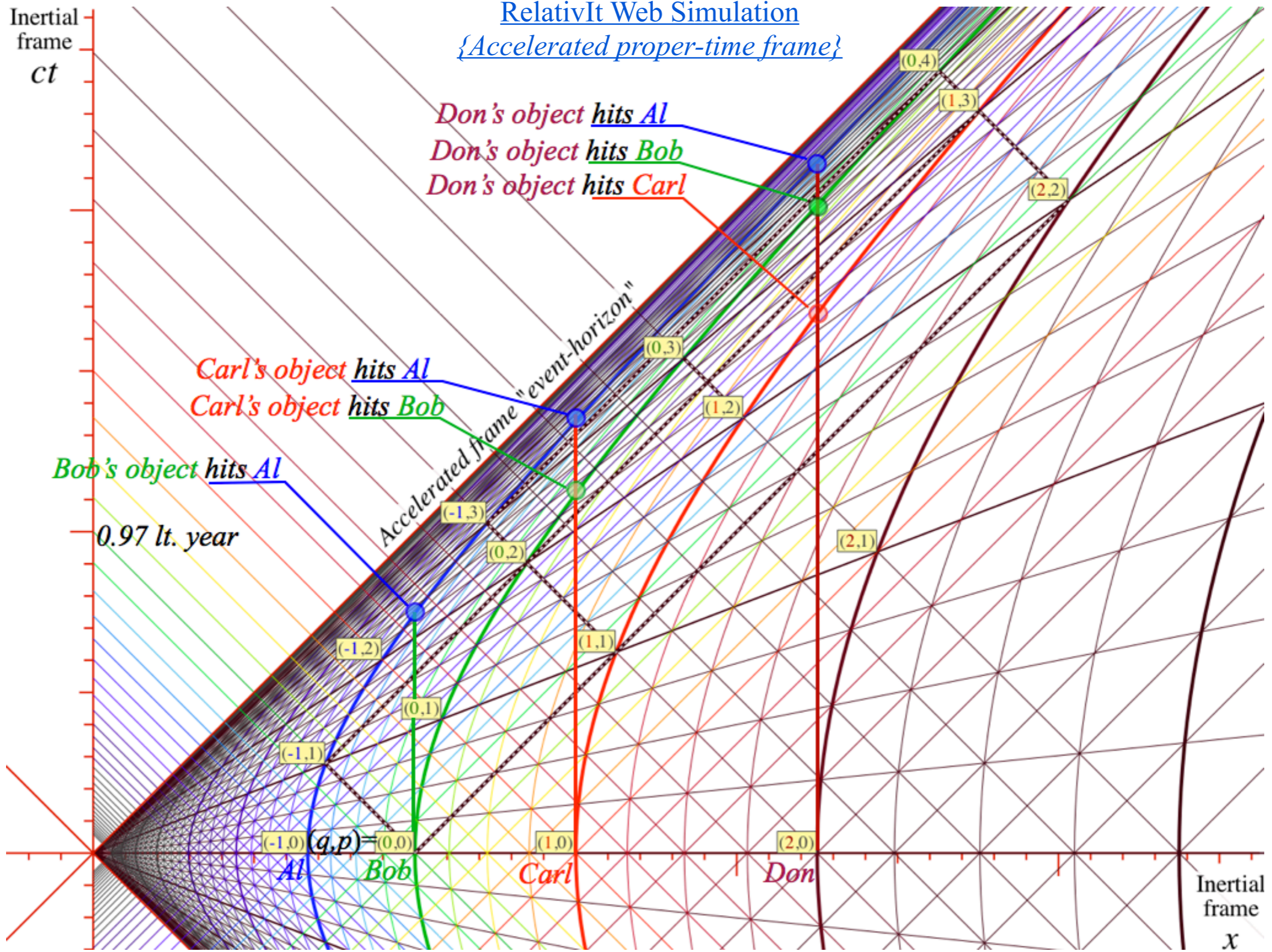
Analysis of constant- g grid compared to zero- g Minkowski frame

Animation of mechanics and metrology of constant- g grid





RelativIt Web Simulation
 {Accelerated proper-time frame}



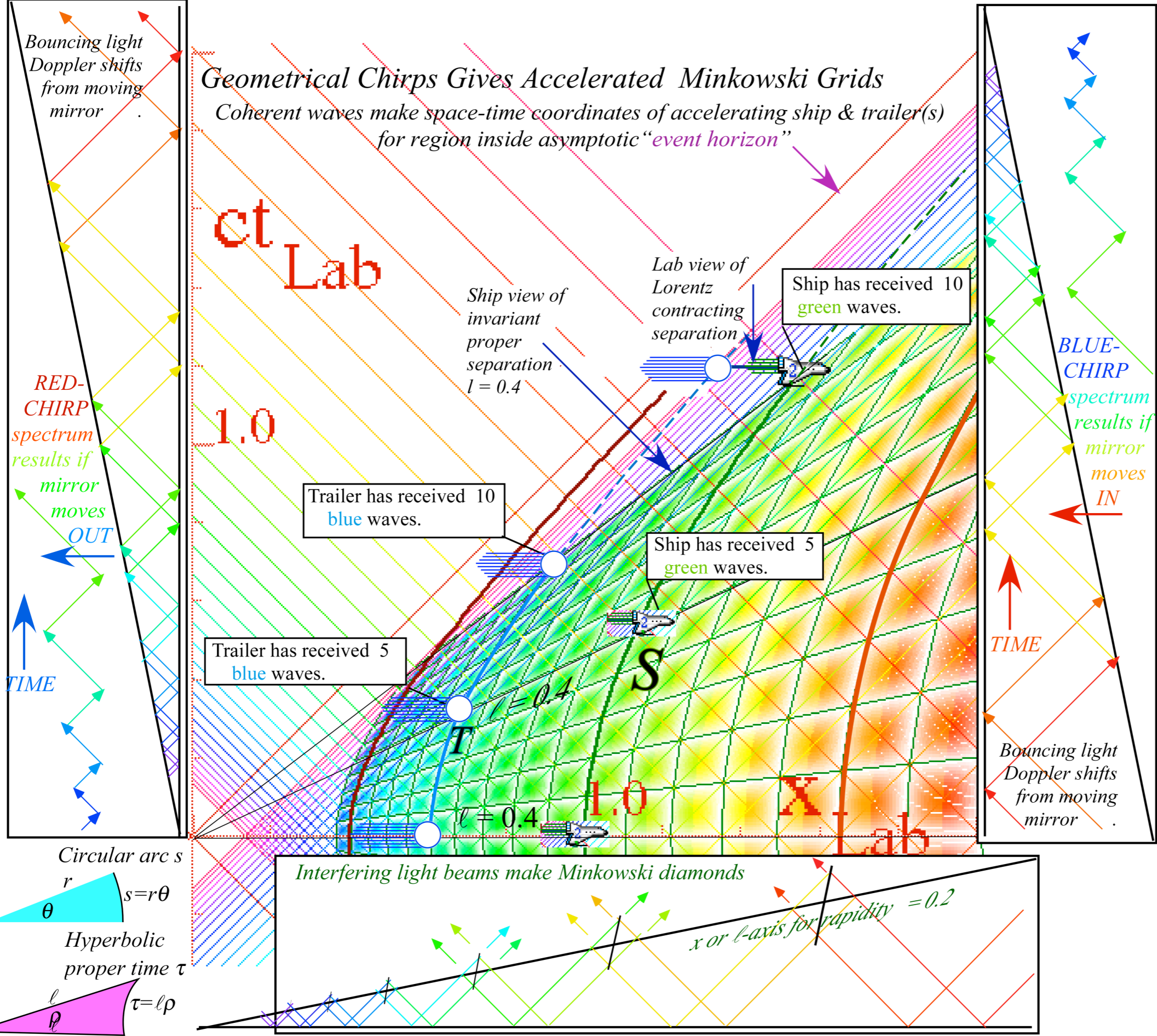


Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light

Relativity relates charge, current, and magnetic fields

Geometric derivation of magnetic constant μ_0 from electric ϵ_0

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Animation of mechanics and metrology of constant- g grid

 Xtra stuff: *Some numerology: Which is bigger...H-atom or an electron? What's spin?*
Space-Space waves gone mad

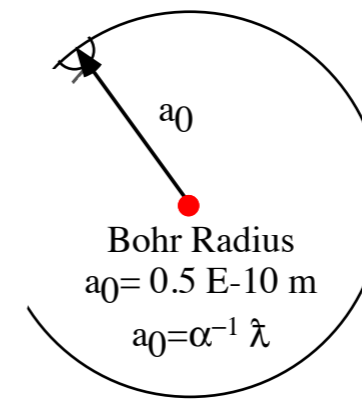


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant $\alpha = 1/137$.

Bohr model has electron orbiting at radius r so centrifugal force balances Coulomb attraction to the opposite charged proton.

$$\frac{m_e v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \quad \text{or:} \quad \frac{m_e v^2 r^2}{r} = \frac{e^2}{4\pi\epsilon_0} \quad \text{or:} \quad r = \frac{4\pi\epsilon_0 m_e v^2 r^2}{e^2} = \frac{4\pi\epsilon_0 (m_e v r)^2}{m_e e^2} = \frac{4\pi\epsilon_0 \ell^2}{m_e e^2}$$

Bohr hypothesis: orbital momentum ℓ is a multiple N of \hbar or

$$\ell = m_e v r = N \hbar \quad (N = 1, 2, \dots).$$

This gives the *atomic Bohr radius* $a_0 = 0.05 \text{ nm}$

$$r = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} N^2 \left(= r_{Bohr} = 5.28 \cdot 10^{-11} \text{ m.} = 0.528 \text{ \AA} \text{ for } N=1 \right)$$

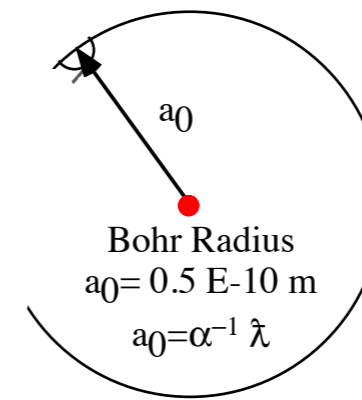


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It also implies rear-relativistic electron orbit speed v that is fraction $1/N$ of $0.073c$.

$$\frac{v}{c} = \frac{\ell}{m_e r c} = \frac{N \hbar}{m_e r_{Bohr} c} = \frac{N \hbar}{m_e c} \frac{m_e e^2}{4\pi\epsilon_0 \hbar^2 N^2} = \frac{1}{N} \frac{e^2}{4\pi\epsilon_0 \hbar c} \left(= 7.29 \cdot 10^{-3} = \frac{1}{137} \text{ for } N=1 \right)$$

The *dimensionless* ratio $\alpha = e^2 / (4\pi\epsilon_0 \hbar c) = 1/137.036$ is called the *fine-structure constant* α .

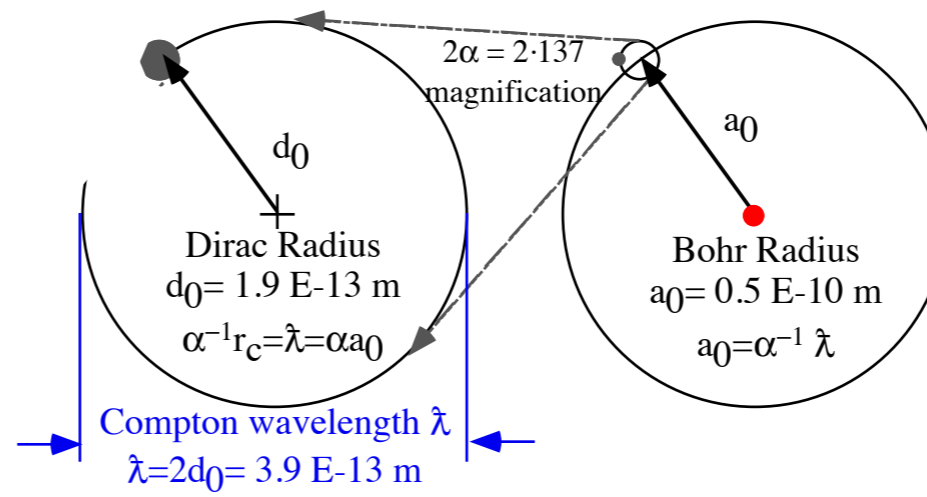


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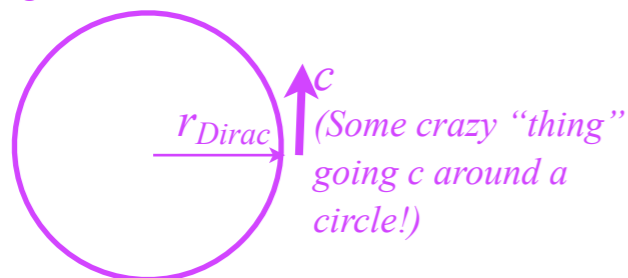
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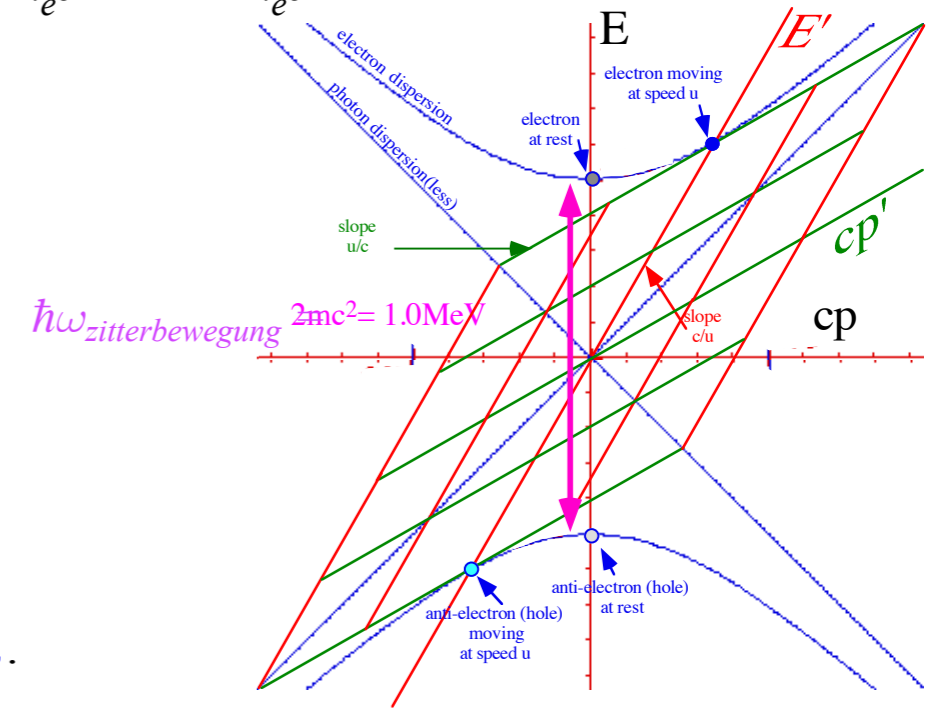
Now, some *numerology* of Dirac's electron radius involving *zitterbewegung* where $\omega_{zitterbewegung} = 2mc^2/\hbar = 1.56 \cdot 10^{21} \text{ (radian)Hz}$

$\omega_{zitterbewegung} r = c$ or $r_{Dirac} = c/\omega_{zitterbewegung} = \hbar/2mc = 1.93 \cdot 10^{-13} \text{ m}$ relates to the *Compton wavelength* $\lambda = \hbar/mc = 3.8616 \cdot 10^{-13} \text{ m}$



Reduced Compton wavelength: $2\pi \lambda = h/mc = 2.4263 \cdot 10^{-12}$
or Compton "circumference"

$$2.4263102175 \pm 33 \times 10^{-12} \text{ m}$$



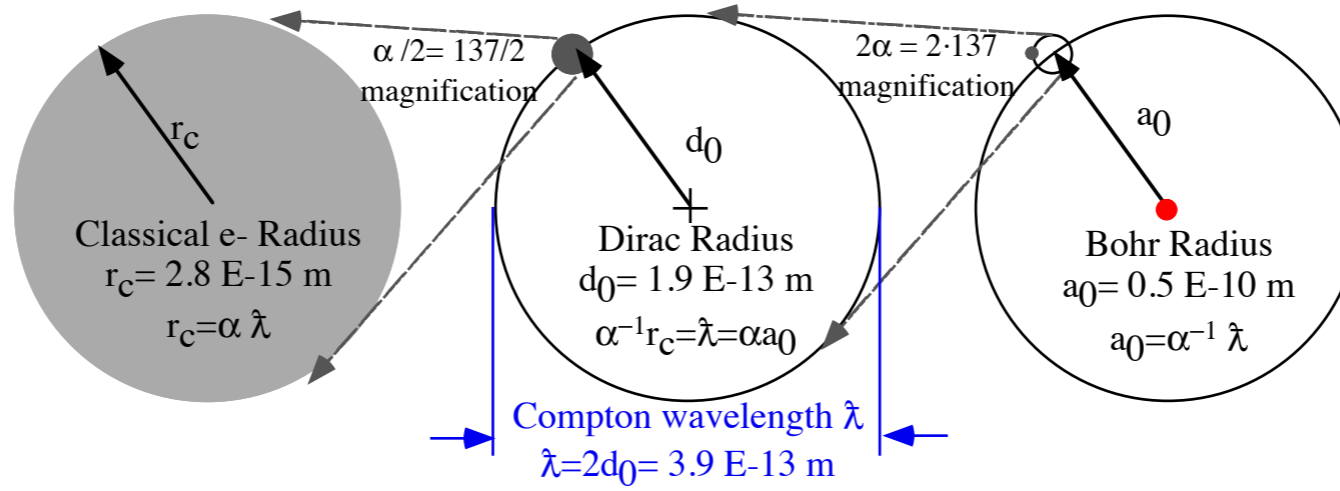


Fig.8A.2 Various electron radii and their relative sizes related by fine-structure constant $\alpha = 1/137$.

The classical radius of the electron defined by setting its electrostatic PE to $m_e c^2$:

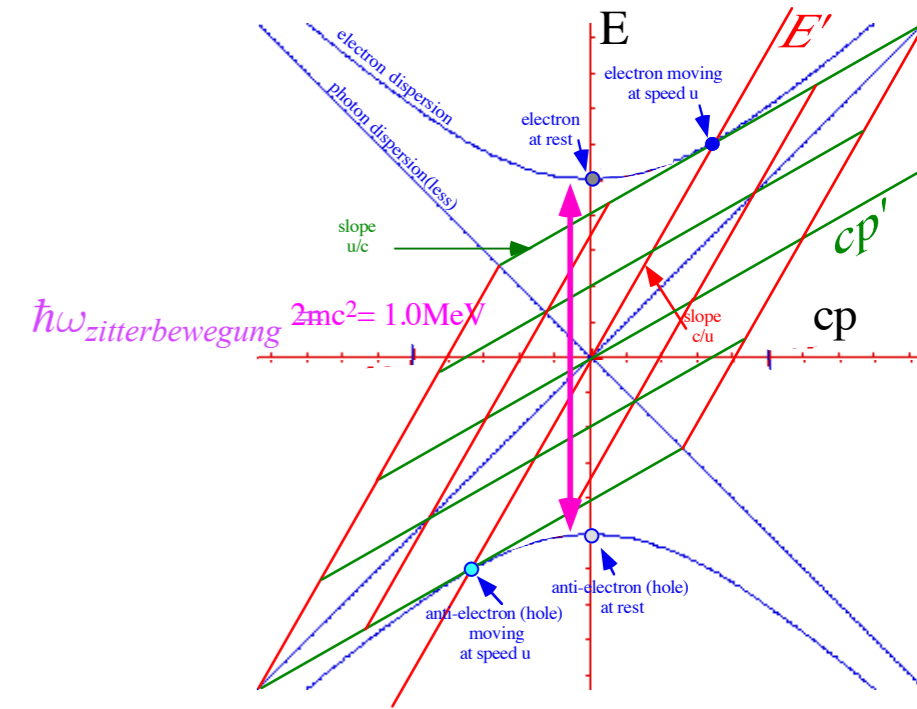
$$e^2 / (4\pi\epsilon_0 r_{classical}) = m_e c^2 \quad \text{or} \quad r_{classical} = e^2 / (4\pi\epsilon_0 m_e c^2) = 2.8 \cdot 10^{-15} \text{ m.}$$

Another fine-structure ratio to r_{Bohr} .

$$\frac{r_{Classical}}{r_{Bohr}} = \frac{e^2 / 4\pi\epsilon_0 m_e c^2}{4\pi\epsilon_0 \hbar^2 / m_e e^2} = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c} \right)^2 = \left(\frac{1}{137.} \right)^2$$

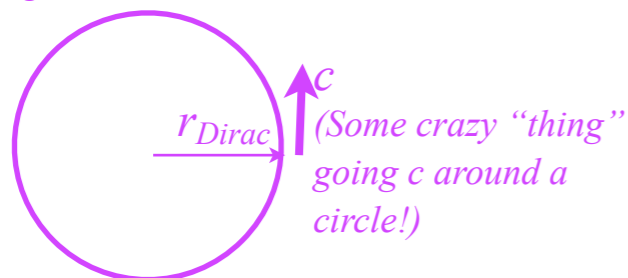
As a final numerological exercise, find angular momentum $\ell = m_e v r$ of fictitious "zitterbewegung" orbit inside the electron. With $v=c$ and $r = r_{Dirac}$ the following is obtained.

$$\begin{aligned} \ell &= m_e c r_{Dirac} = m_e c \hbar / (2m_e c) \\ &= \hbar / 2 \end{aligned}$$



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Special Relativity and Quantum Mechanics by Ruler and Compass I.

The simplest "molecule": 2 CW Lasers form Minkowski Space-time (and Reciprocally-related) Frame Coordinates

A Making sense of light-wave axiom(s).

Using Occam's Razor

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

1CW is affected by 1st-order Doppler and Red shifts $c = v/c'$ of frequency ν and wavenumber k

B How does space-time and/or per-space-per-time carry light-waves?

$(\text{wavelength } \lambda = \text{period } \tau)$ and/or $(\text{wavenumber } k = \text{frequency } \nu)$

$(\lambda = 1/k \text{ and } \tau = 1/\nu)$ and $(k = 1/\lambda \text{ and } \nu = 1/\tau)$

$(\lambda = \text{meters per wave and } \tau = \text{seconds per wave})$ and $(k = \text{waves per meter and } \nu = \text{waves per second})$

Greek "k" for wavenumber for Kayser (or "kinks")

The "Keyboard of the gods" Or per-space-per-time graphs vs. space-time graphs

wave-speed equals slope-to-horizontal in (c, \nu)-graph

wave-speed equals slope-to-vertical in (\lambda, \tau)-graph

Dimensionless Light wave-velocity $c/c = 1$

C Doppler Shift in per-space-per-time

Atom traveling along wave sees less wave "hits"/sec. (that is: Doppler red-shift)

Atom traveling against wave sees more wave "hits"/sec. (that is: Doppler blue-shift)

Moving against a 600 THz 1CW could Doppler blue shift it to 1200 THz

D Clarify Evenson's CW Axiom (All colors go c) by Doppler effects

Special relativity and quantum mechanics are very much a story of the geometry of light-wave motion

How do we measure space and time with light waves?
Use **1CW laser-phasers** for a **phase-based** theory
How do we make spacetime coordinate graph with light waves?
Use **2CW laser-phasers** and **wave interference** geometry

Relativity - Using light's own wave-like nature to better understand special relativity and quantum mechanics

E Easy Doppler-shift and Rapidity calculation

Doppler ratio: $(R/S) = \frac{v_{\text{observer}}}{v_{\text{source}}}$

rapidity: $\rho_{RS} = \log_e(R/S)$

Bob-Alice Doppler ratio: $(R/A) = \frac{v_B}{v_A} = \frac{1200}{600} = 2$

Bob-Alice rapidity: $\rho_{BA} = \log_e(2)$

Carla-Alice Doppler ratio: $(C/A) = \frac{v_C}{v_A} = \frac{400}{600} = \frac{2}{3}$

Carla-Alice rapidity: $\rho_{CA} = \log_e(2/3)$

Carla-Bob Doppler ratio: $(C/B) = \frac{v_C}{v_B} = \frac{400}{1200} = \frac{1}{3}$

Carla-Bob rapidity: $\rho_{CB} = \log_e(1/3)$

Galileo's Revenge (part 1) Rapidity adds just like Galilean velocity

$\rho_{CB} = \rho_{CA} + \rho_{AB} = -0.41 - 0.69 = -1.10$

F 1 CW Laser-phaser Wave Function

Dimensionless Light wave-velocity $c/c = 1$

Wave function: $\psi = A e^{i(kx - \omega t)}$

Wavelength $\lambda = 2\pi/k = 1/k$

Period $T = 2\pi/\omega = 1/\nu$

Other Doppler versions $\lambda'/\lambda = c'/c$ must match this phasor clock/clock-array, too. That's gauge invariance! $c'c = c'^2/c^2$

G 2 CWs Interfering in Space-Time

Geometry of the High-Speed Phase and Half-Difference Group

Typical Phasor Sum

Phasor-relative views

H 2 Doppler shifted CWs Interfering in Space-Time

Right-directed 1CW $e^{i(kx - \omega t)}$

Left-directed 1CW $e^{i(kx + \omega t)}$

2CW per-Spacetime Plot

2CW Minkowski-Spacetime grid

I Thales Mean Geometry (600BCE) helps "Relativity"

Thales showed a circle diameter subtends a right angle with any circle point P

Minkowski-Lorentz Grid in terms of P', G'

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e(\text{Doppler Shift})$ to a Transverse*relativity parameter: Stellar aberration angle σ

Observer fixed below star sees it directly overhead. Observer going u sees star at angle σ in u direction.

Stellar aberration angle σ

Epstein's trick is to turn a hyperbolic form $ct = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(ct')^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed c through (x', ct') space!

J Table of 12 wave parameters (includes inverses) for relativity

Geometry applies to (x, y) space-space to (k_x, k_y) per-space-per-space to (x, ct) space-time

Optical wave guide relativistic geometry aided by Occam's Sword

Relativistic mode with near- c $V_{\text{group}} = c/2$ and $V_{\text{phase}} = 2c$ (Low dispersion)

Example of near-cut-off mode with low $V_{\text{group}} = c/3$ and high $V_{\text{phase}} = 2c$ (High dispersion)

Per-space Per-space Geometry

Per-space Per-space Geometry

Stellar aberration angle σ

Reality angle ν

Stellar aberration σ

Per-space Per-space Geometry

Stellar aberration angle σ

Reality angle ν

Stellar aberration σ

[Link to pdf version of Part I online](#)

Note: When printed at their optimal resolution, each poster is 7 feet across!

DAMOP - 2015
Special Relativity and Quantum Mechanics by Ruler and Compass II.
 The simplest "molecule": Relativistic mechanics by optical coherence geometry

William G. Harter and Tyle C. Reimer
 University of Arkansas - Fayetteville



A Using wave parameters to quickly derive Planck (1900), Einstein (1905), and DeBroglie (1921) formulation

$v_{phase} = B \cosh \rho = B + \frac{1}{2} B \rho^2$ (for $u \ll c$) $\cosh \rho = 1 + \frac{1}{2} \rho^2 + \dots$
 $u_{phase} = B \sinh \rho = B \rho$ (for $u \ll c$) $\sinh \rho = \rho + \frac{1}{6} \rho^3 + \dots$
 $\frac{u}{c} = \tanh \rho = \rho$ (for $u \ll c$)
 $v_{phase} = B + \frac{1}{2} \frac{B}{c^2} u^2$ \Leftrightarrow (for $u \ll c$) $\Rightarrow K_{phase} = \frac{B}{c^2} u$
 $h v_{phase} = h B + \frac{1}{2} \frac{h B}{c^2} u^2$ \Leftrightarrow (for $u \ll c$) $\Rightarrow h K_{phase} = \frac{h B}{c^2} u$
 $h v_{phase} = M c^2 + \frac{1}{2} M u^2$ \Leftrightarrow (for $u \ll c$) $\Rightarrow h K_{phase} = M u$

Base scale: $B = v_A$ for v_{phase}

1 Low speed v_{phase} and K_{phase} approximations vary with u like Newton's kinetic energy $\frac{1}{2} M u^2$ and momentum $M u$.
2 So attach scale factor h (or \hbar) to match units. Rescale K_{phase} by h so: $M = \frac{h B}{c^2}$ or: $h B = M c^2$
3 Use exact v_{phase} and K_{phase}

$h v_{phase} = h B \cosh \rho = M c^2 \cosh \rho$
 \uparrow Planck (1900) Total Energy: $E = \frac{M c^2}{\sqrt{1-u^2/c^2}}$
 \uparrow Einstein (1905) \downarrow DeBroglie (1921)
 $h \omega_{phase} = h B \cosh \rho = M c^2 \cosh \rho$
 $h c k_{phase} = h B \sinh \rho = M c^2 \sinh \rho$
 Momentum: $h c p_{phase} = p = \frac{M u}{\sqrt{1-u^2/c^2}}$
 \uparrow DeBroglie (1921) \downarrow DeBroglie (1921)
 $h c k_{phase} = h B \sinh \rho = M c^2 \sinh \rho$

group	$\frac{h v_{phase}}{h c}$	$\frac{h c k_{phase}}{h c}$	$\frac{h v_{phase}}{h c}$	$\frac{h c k_{phase}}{h c}$	$\frac{h v_{phase}}{h c}$	$\frac{h c k_{phase}}{h c}$	$\frac{h v_{phase}}{h c}$	$\frac{h c k_{phase}}{h c}$	$\frac{h v_{phase}}{h c}$	$\frac{h c k_{phase}}{h c}$
rest	1	0	1	0	1	0	1	0	1	0
low	$1 + \frac{1}{2} \frac{u^2}{c^2}$	$\frac{u}{c}$	$1 + \frac{1}{2} \frac{u^2}{c^2}$	$\frac{u}{c}$	$1 + \frac{1}{2} \frac{u^2}{c^2}$	$\frac{u}{c}$	$1 + \frac{1}{2} \frac{u^2}{c^2}$	$\frac{u}{c}$	$1 + \frac{1}{2} \frac{u^2}{c^2}$	$\frac{u}{c}$
high	$\cosh \rho$	$\sinh \rho$	$\cosh \rho$	$\sinh \rho$	$\cosh \rho$	$\sinh \rho$	$\cosh \rho$	$\sinh \rho$	$\cosh \rho$	$\sinh \rho$

B Definition(s) of mass for relativity and quantum theory

1 Rest Mass M_{rest} (Einstein's mass) Defines invariant hyperbola(s) Given: Energy: $E = M c^2 \cosh \rho = h v_{phase}$
 Momentum: $h c p = M c^2 \sinh \rho = h c k_{phase}$
 $h B = h v_A = M c^2 = h c k_A$
 $E = \pm \sqrt{(M c^2)^2 + (c p)^2}$ momentum: $c p = M c^2 \sinh \rho = h c k_{phase}$
 Group velocity: $u = c \tanh \rho = \frac{d v_{phase}}{d k}$

2 Momentum Mass M_{mom} (Galileo's mass) Defined by ratio p/u of relativistic momentum to group velocity.

$M_{mom} = \frac{p}{u} = \frac{M_{rest} \cosh \rho}{c \tanh \rho}$ Limiting cases: $M_{mom} \xrightarrow{u \rightarrow 0} M_{rest} e^{\rho/2}$
 $M_{mom} \xrightarrow{u \rightarrow c} M_{rest}$
 $M_{mom} \cosh \rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} = \frac{\text{Momentum}}{\text{Mass}}$

3 Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = M c \cosh \rho d\rho$ in momentum to change $du = c \operatorname{sech}^3 \rho d\rho$ in velocity
 $M_{eff} = \frac{dp}{du} = M_{rest} \frac{c \cosh \rho}{c \operatorname{sech}^3 \rho} = M_{rest} \cosh^4 \rho$ Limiting cases: $M_{eff} \xrightarrow{u \rightarrow 0} M_{rest} e^{2\rho}$
 $M_{eff} \xrightarrow{u \rightarrow c} M_{rest}$
 More common derivation using group velocity: $u = v_{phase} \frac{d v_{phase}}{d k}$

$M_{eff} = \frac{d p}{d u} = \frac{h c k}{d v_{group}} = \frac{h}{\frac{d v_{phase}}{d k}} = \frac{h}{\frac{d}{d k} \left(\frac{M_{rest} c \cosh \rho}{\sqrt{1-u^2/c^2}} \right)}$
 $M_{eff} = \frac{d p}{d u} = \frac{h c k}{d v_{group}} = \frac{h}{\frac{d v_{phase}}{d k}} = \frac{h}{\frac{d}{d k} \left(\frac{M_{rest} c \cosh \rho}{\sqrt{1-u^2/c^2}} \right)}$
 general wave formula: to accompany $v_{group} = \frac{d v_{phase}}{d k}$

C Defining phase Φ , action $S = \hbar \Phi$, Hamiltonian, and Lagrangian

1 Define Lagrangian L in terms of phase $\Phi = \int \mathbf{k} \cdot d\mathbf{x} - \int \omega dt = \int \mathbf{k} \cdot d\mathbf{x} - \int \omega dt$ for $k = k_{phase}$ and $\omega = \omega_{phase}$.
 $L = \frac{dS}{dt} \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$ $\hbar \equiv \frac{h}{2\pi}$

2 Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar \omega$ relation
 $L = \frac{dS}{dt} \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega = \frac{dx}{dt} p - E \equiv p u - E \equiv p u - H = L$ Legendre transformation

3 Use relativity relations: Group velocity: $u = \frac{d v_{phase}}{d k} = c \tanh \rho$. Rest energy: $\omega_0 = M c^2 = \hbar c k_A$
 Momentum: $p = \hbar c k_{phase} = M c^2 \sinh \rho$
 Hamiltonian: $H = \hbar \omega_{phase} = E = \hbar \omega_0 \cosh \rho$

4 $L = p u - H = (M c \sinh \rho)(c \tanh \rho) - M c^2 \cosh \rho$
 $= M c^2 \frac{\sinh^2 \rho - \cosh^2 \rho}{\cosh \rho} = -M c^2 \operatorname{sech} \rho$

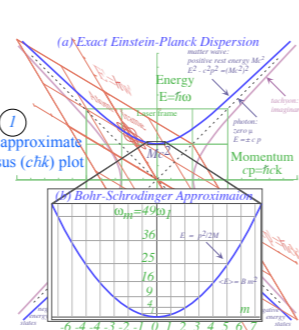
(a) Hamiltonian $H(q,p)$
 slope is group velocity $u = \frac{d v_{phase}}{d k}$
 $H = M c^2 \cosh \rho$
 $H' = M c^2 \sinh \rho = p$
 $H'' = M c^2 \cosh \rho = H$
 has infinite H' and zero L
 Momentum p

(b) Lagrangian $L(q,\dot{q})$
 radius = $M c^2$
 $L = -M c^2 \operatorname{sech} \rho$
 $L' = M c^2 \tanh \rho = u$
 $L'' = M c^2 \operatorname{sech}^3 \rho = \frac{1}{\cosh^3 \rho}$
 slope is momentum $p = \frac{\partial L}{\partial \dot{q}}$
 Velocity $u = \dot{q}$

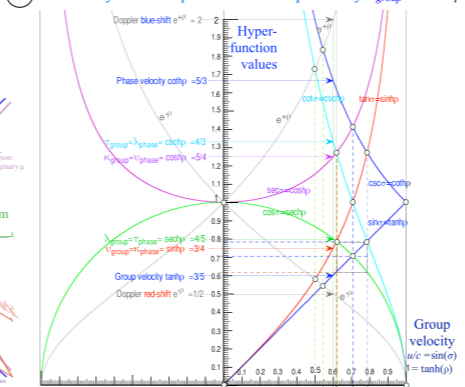
Comparing Lagrangian $L(\text{velocity } u)$ with Hamiltonian $H(\text{momentum } p)$
 $L = -M c^2 \operatorname{sech} \rho = -M c^2 \cos \sigma$
 $H = M c^2 \cosh \rho = M c^2 \sec \sigma$
 $H = M c^2 \sqrt{1 + \sinh^2 \rho} = M c^2 \sqrt{1 + (c p)^2}$
 $L = -M c^2 \sqrt{1 - \frac{u^2}{c^2}} = -M c^2 \operatorname{sech} \rho = -M c^2 \cos \sigma$
 $H = M c^2 \sqrt{1 + \frac{p^2}{M^2 c^2}} = M c^2 \cosh \rho = M c^2 \sec \sigma$
 $\Phi = \int \mathbf{k} \cdot d\mathbf{x} - \omega t$

D Geometry and plots of "Relativity" variables

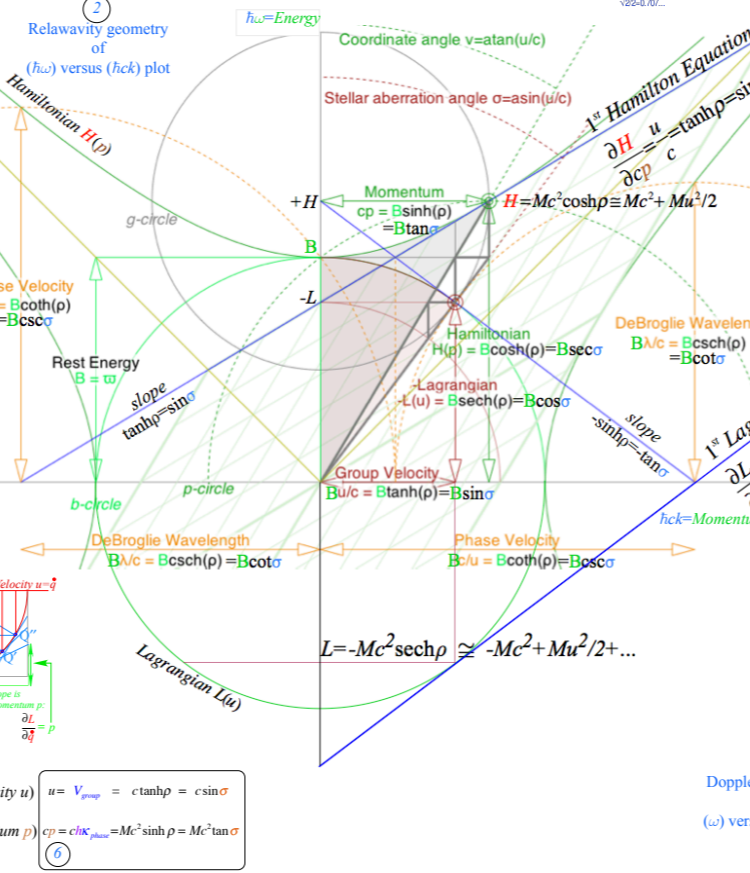
$\sinh \rho = \tan \sigma$, $\cosh \rho = \sec \sigma$, $\coth \rho = \csc \sigma$,
 $\tanh \rho = \sin \sigma$, $\operatorname{sech} \rho = \cos \sigma$, $\operatorname{csch} \rho = \cot \sigma$.



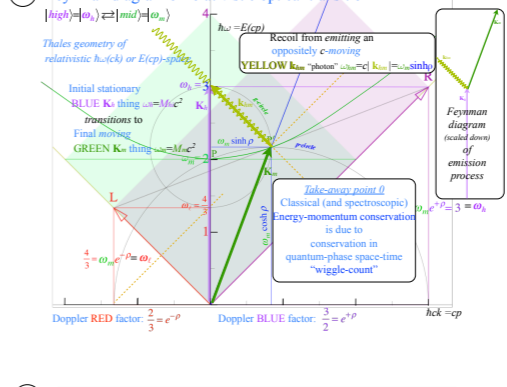
3 Relativity variables plotted versus Group Velocity $V_{group} = c \tanh \rho$



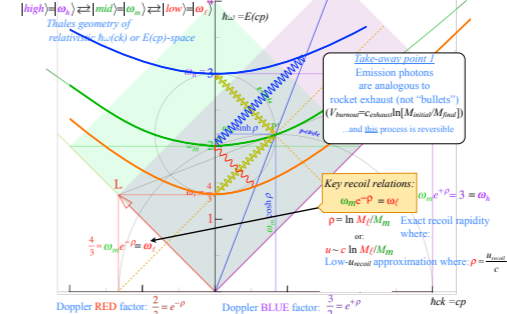
2 Relativity geometry of $(h\omega)$ versus (hck) plot



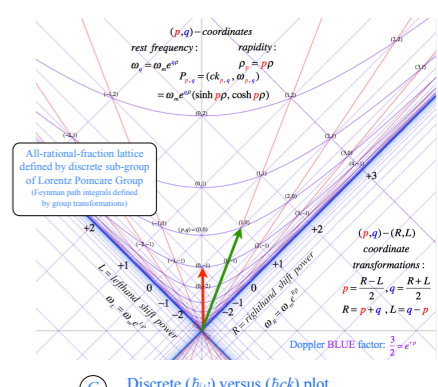
E Feynman diagram of relativistic optical transition



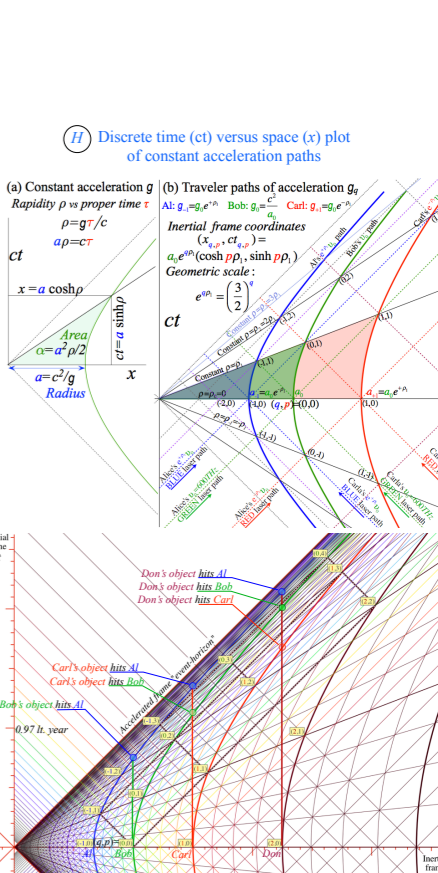
F The "Rocket Science" of relativistic optical transitions



G Discrete $(h\omega)$ versus (hck) plot of Compton scattering

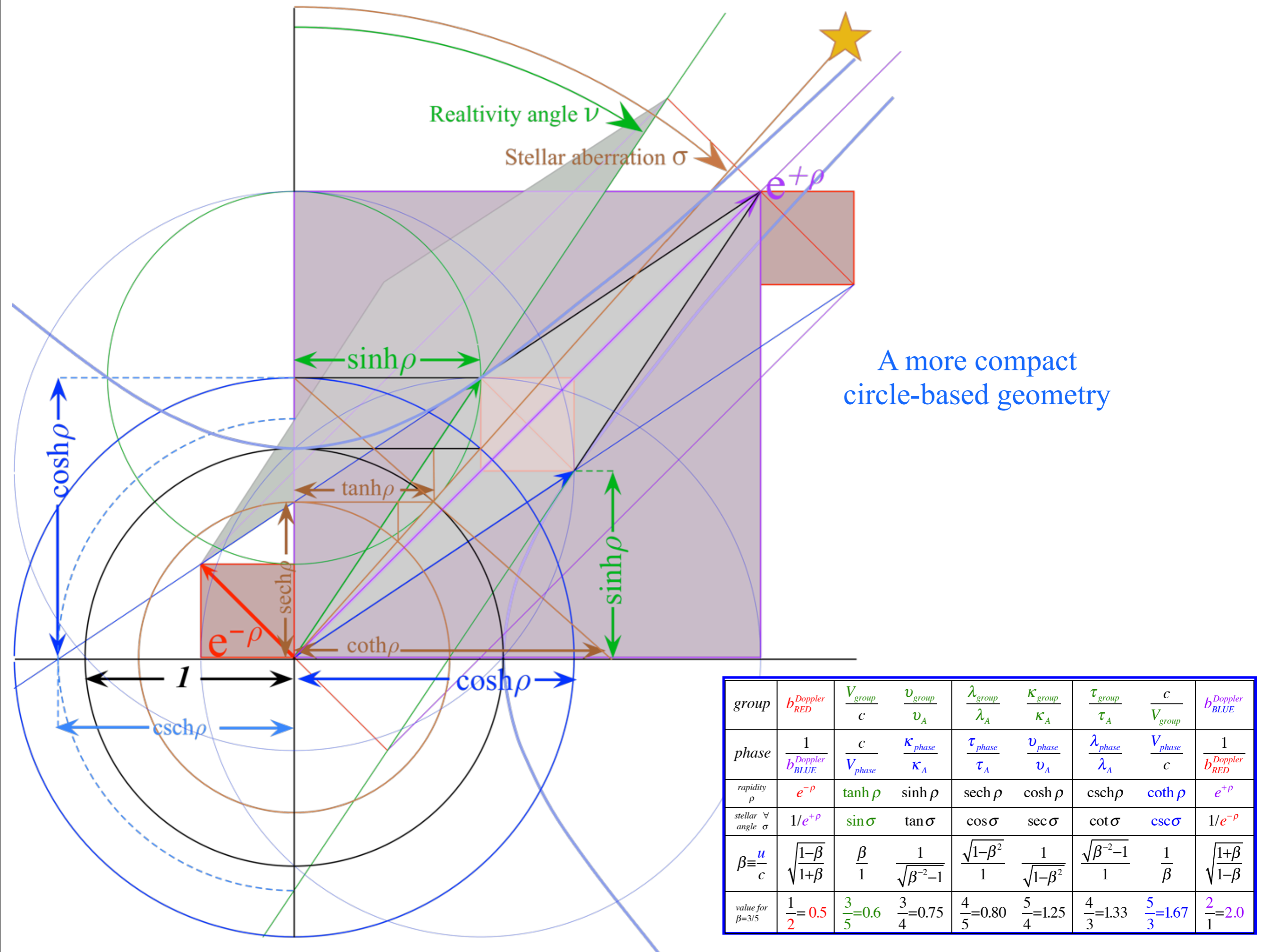


H Discrete time (ct) versus space (x) plot of constant acceleration paths



[Link to pdf version of Part II online](#)

Note: When printed at their optimal resolution, each poster is 7 feet across!



A more compact circle-based geometry

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{v_{group}}{v_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{v_{phase}}{v_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh \rho$	$\sinh \rho$	$\operatorname{sech} \rho$	$\cosh \rho$	$\operatorname{csch} \rho$	$\operatorname{coth} \rho$	$e^{+\rho}$
stellar ∇ angle σ	$1/e^{+\rho}$	$\sin \sigma$	$\tan \sigma$	$\cos \sigma$	$\sec \sigma$	$\cot \sigma$	$\csc \sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^2-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^2-1}}{1}$	$\frac{1}{\beta}$	$\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

from CMWith a BANG! Lecture 31
Thur. 12.10.2015

Xtra stuff: *Some numerology: Which is bigger...H-atom or an electron? What's spin?*



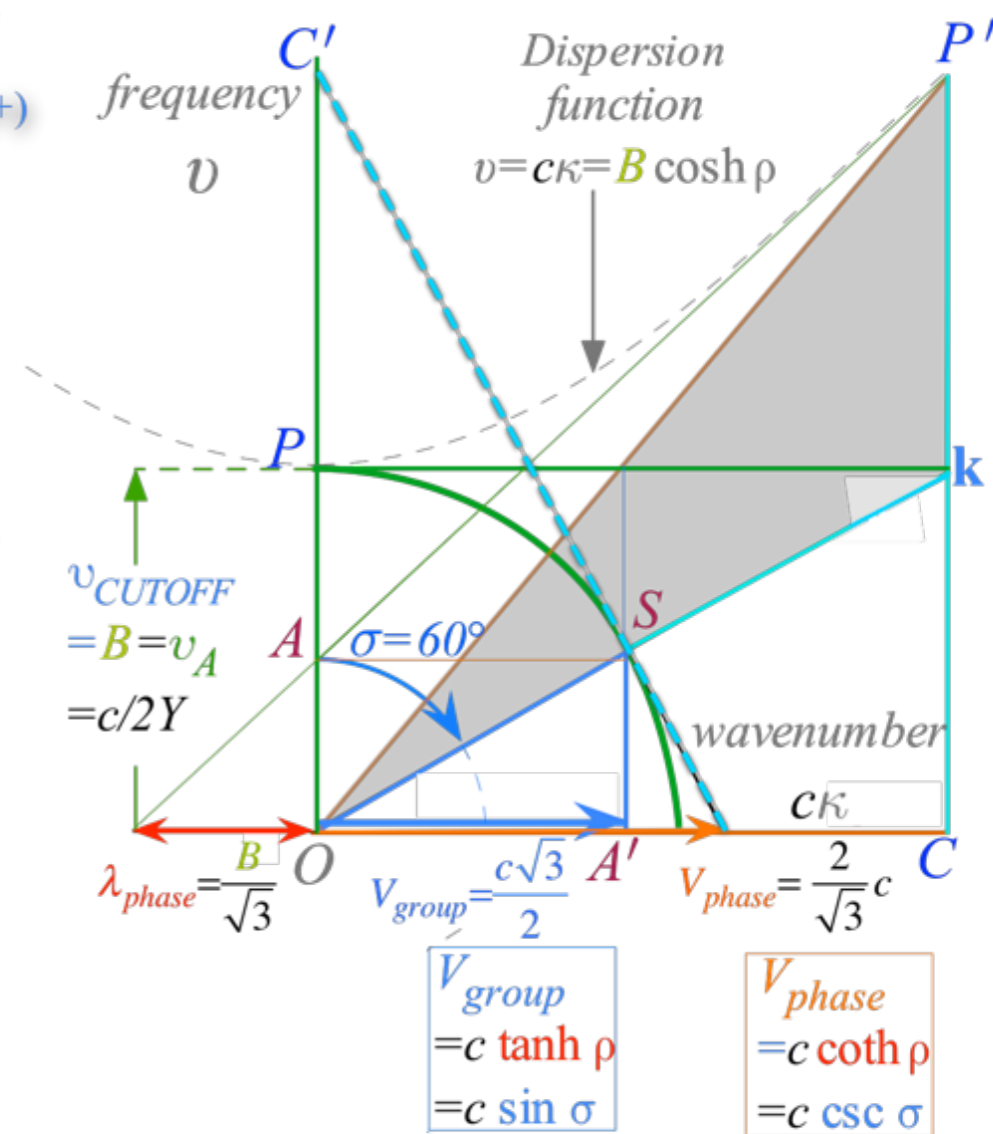
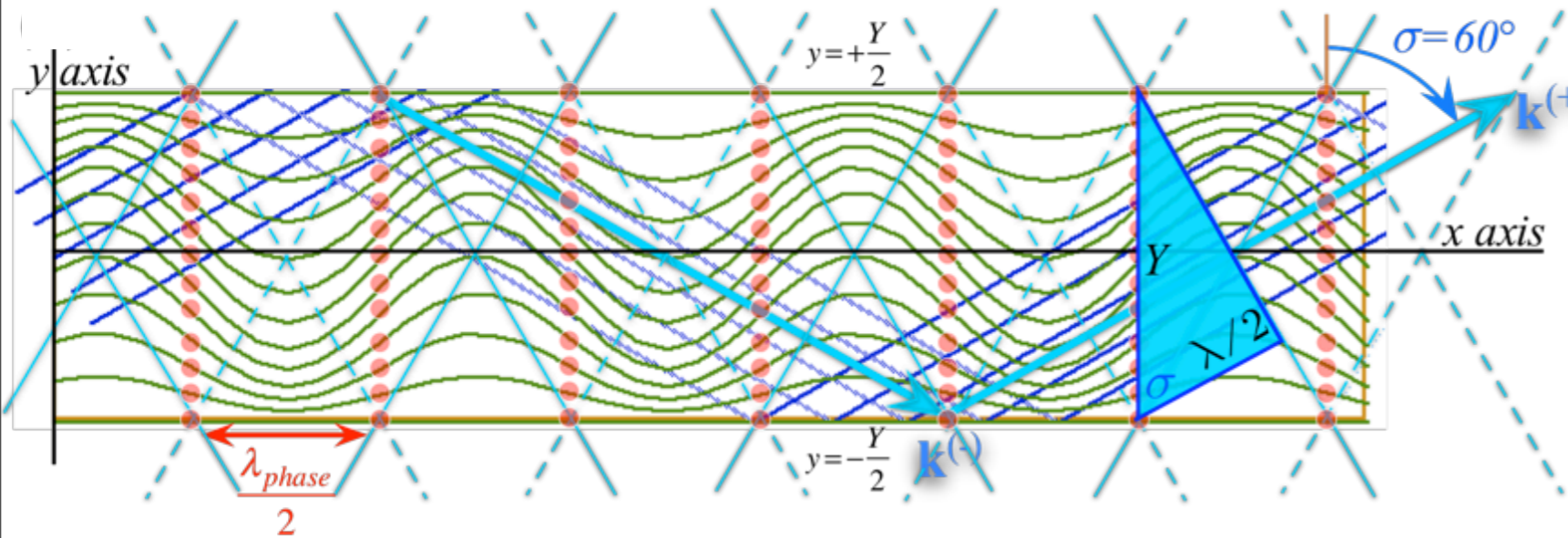
Space-Space waves gone mad

➔ Applications to optical waveguide, spherical waves, and accelerator radiation

Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
 to (k_x, k_y) per-space-per-space
 to (x, ct) space-time

Relativistic mode with near-c $V_{group} = c\sqrt{3}/2$ and $V_{phase} = 2/\sqrt{3}c$. (Low dispersion.)

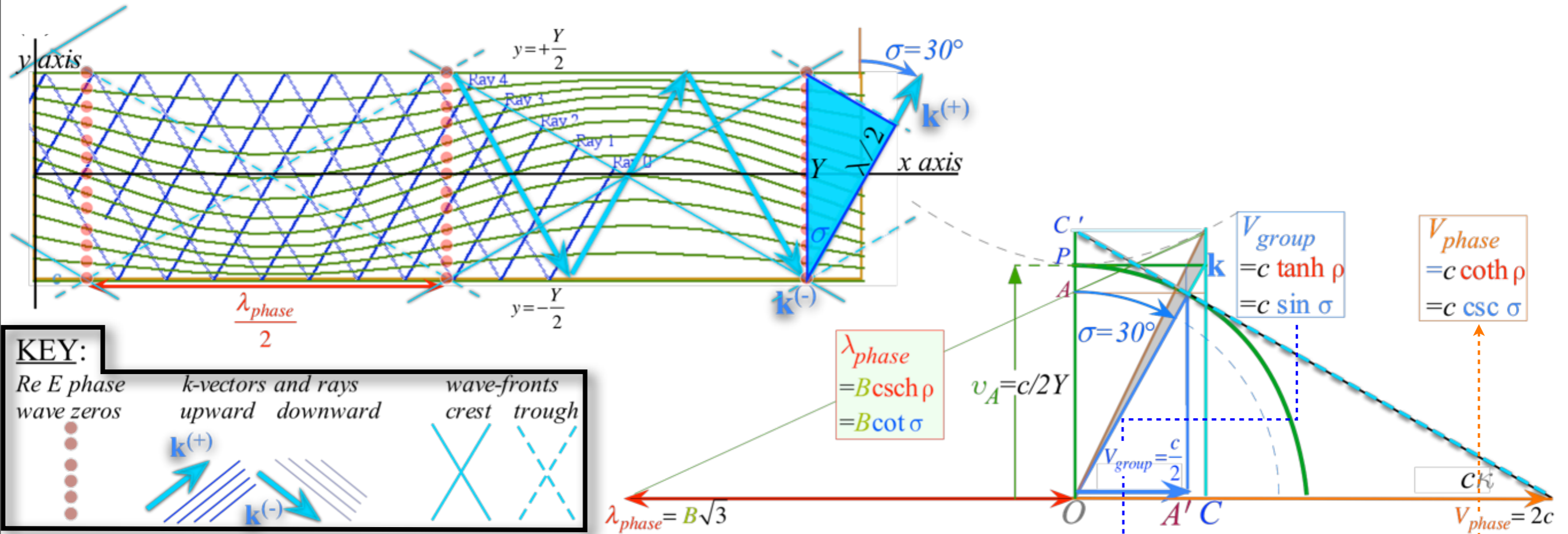


KEY:

Re E phase wave zeros	k-vectors and rays upward downward	wave-fronts crest trough

Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
 to (k_x, k_y) per-space-per-space
 to (x, ct) space-time

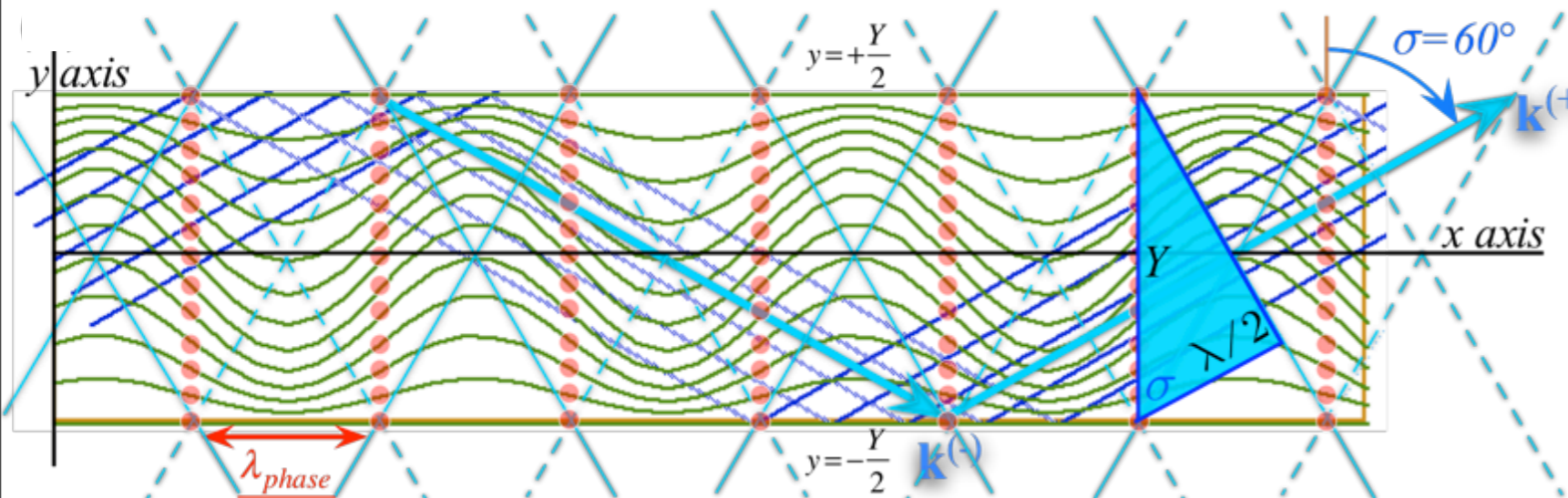


Example of near-cut-off mode with low $V_{group} = c/2$ and high $V_{phase} = 2c$. (High dispersion.)

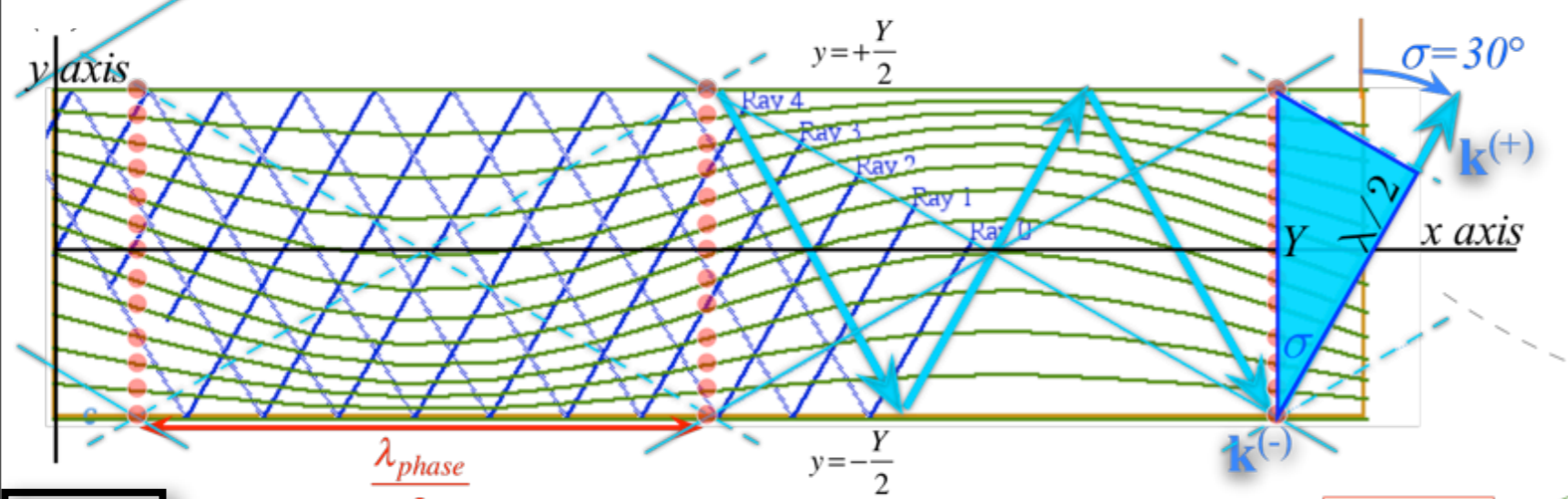
Optical wave guide relativistic geometry aided by Occam's Sword

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Relativistic mode with near-c $V_{group} = c\sqrt{3}/2$ and $V_{phase} = 2/\sqrt{3}c$. (Low dispersion.)



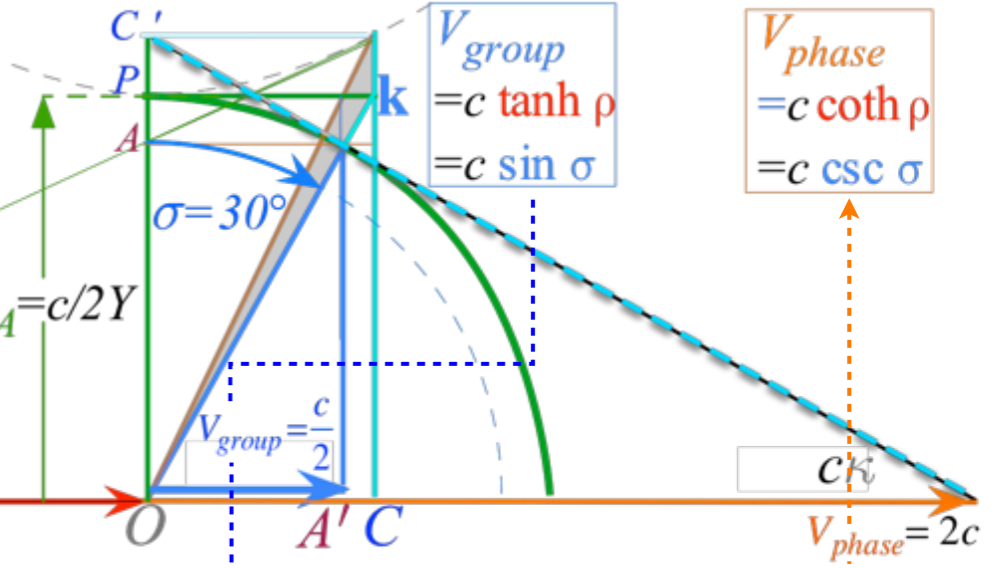
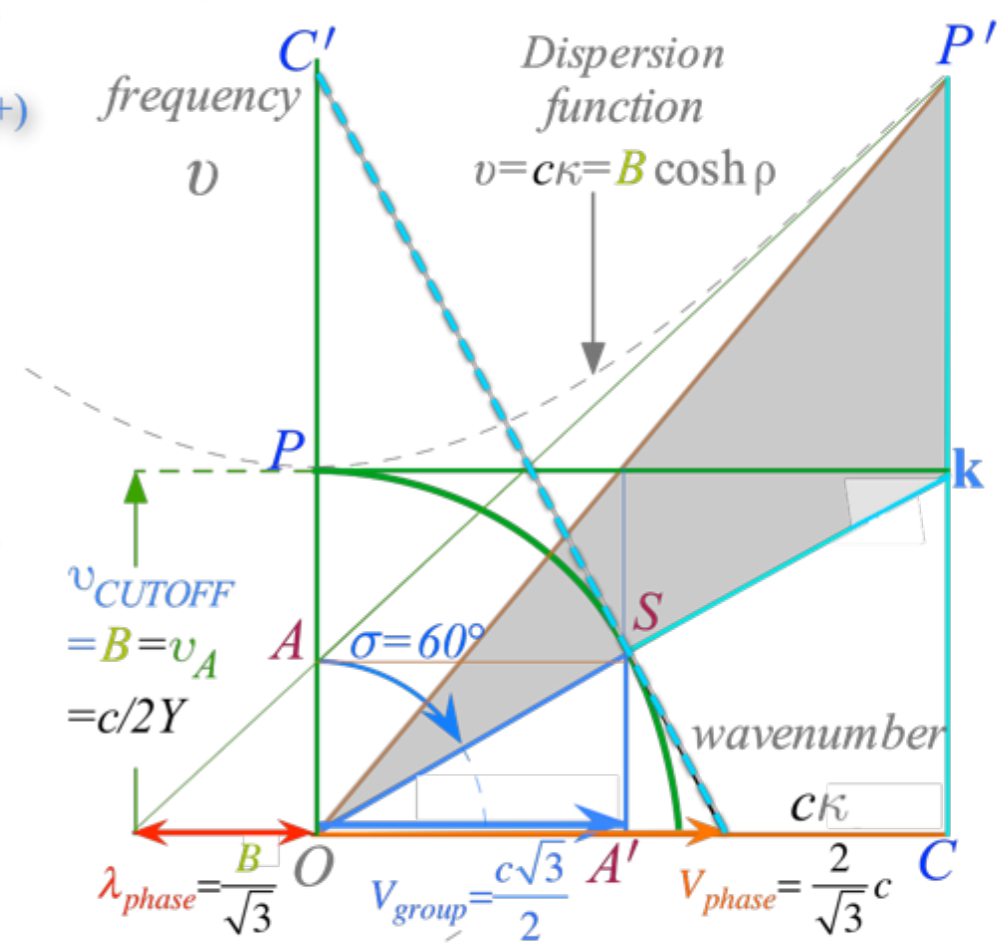
GuideIt Web Simulation: $\sigma = 60^\circ$



GuideIt Web Simulation: $\sigma = 30^\circ$

KEY:

Re E phase wave zeros	k-vectors and rays upward downward	wave-fronts crest trough



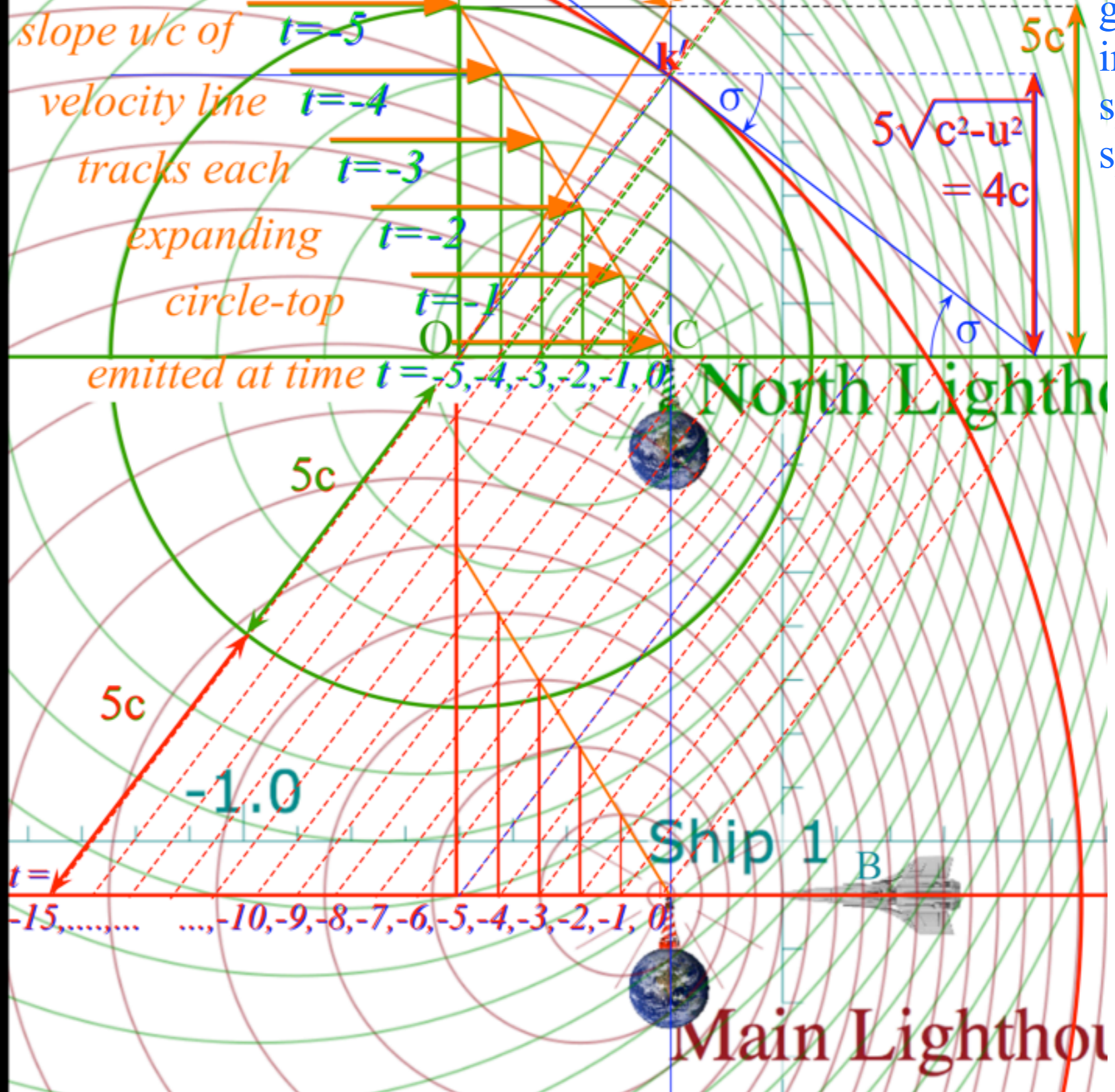
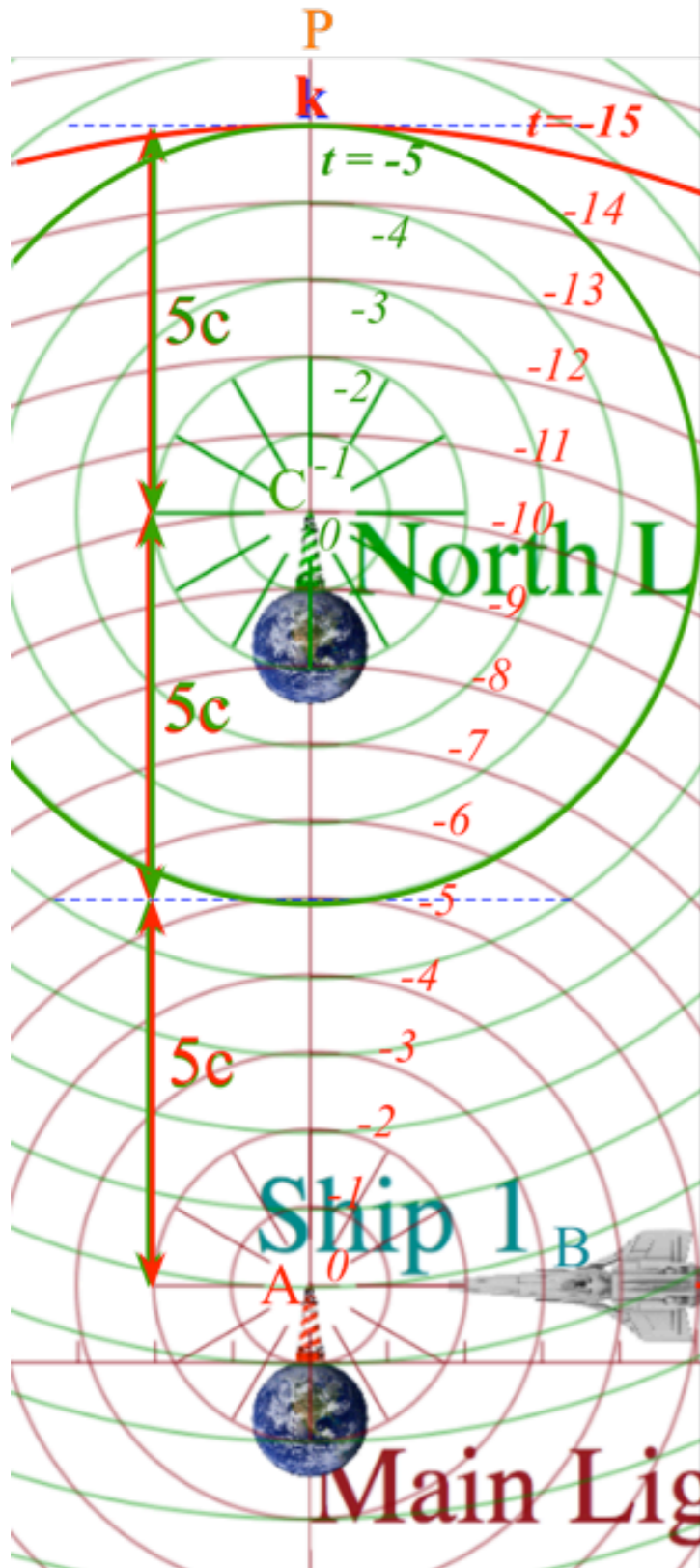
Example of near-cut-off mode with low $V_{group} = c/2$ and high $V_{phase} = 2c$. (High dispersion.)

(a) Spherical wave pair
In Alice-Carla frame

stellar angle $\sigma = \sin^{-1}(u/c)$
velocity angle $v = \tan^{-1}(u/c)$
slope u/c of $t=-5$
velocity line $t=-4$
tracks each $t=-3$
expanding $t=-2$
circle-top $t=-1$
emitted at time $t=-5, -4, -3, -2, -1, 0$

(b) Spherical wave pair
In Bob's frame: $u_x/c = -3/5$

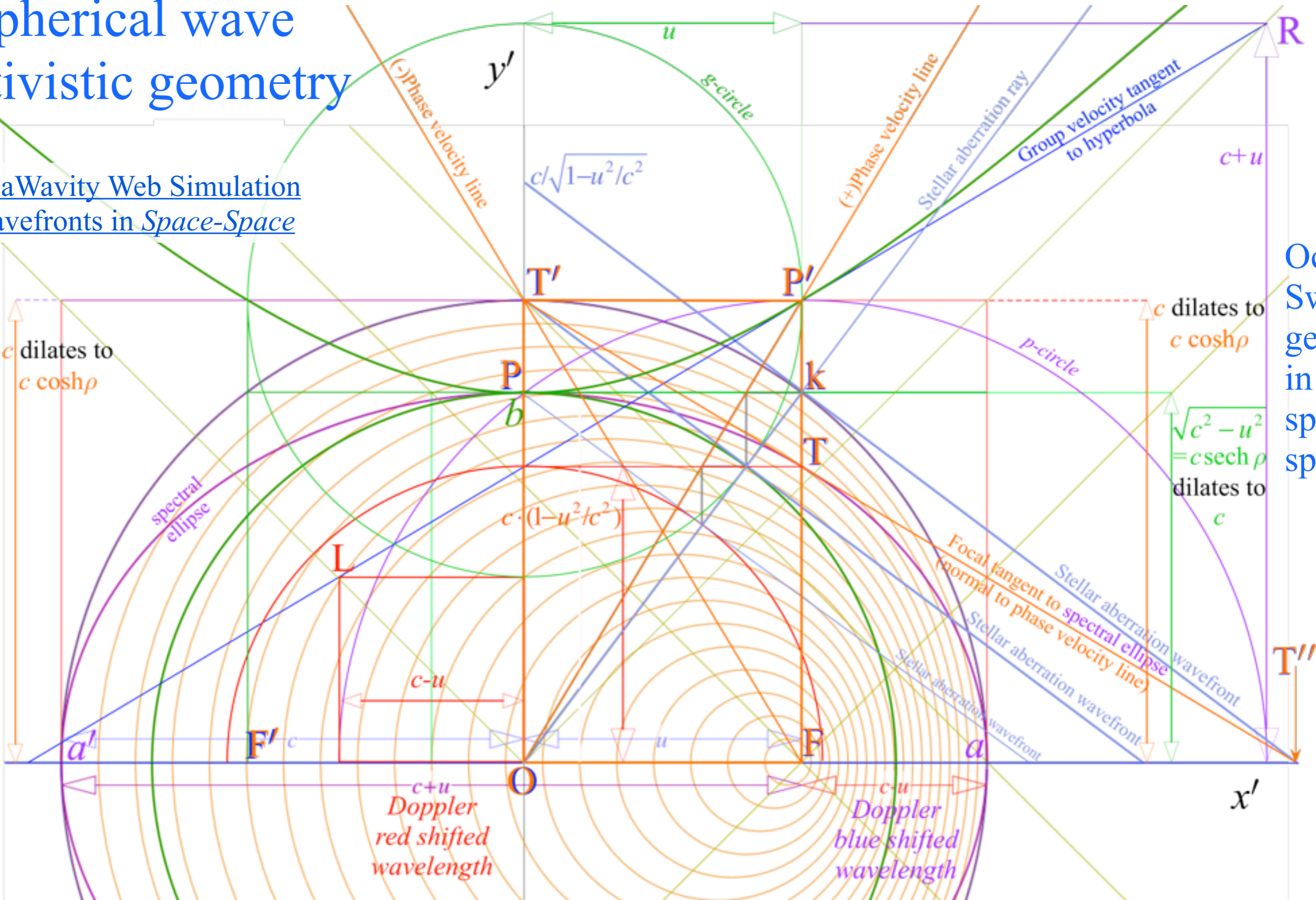
Occam
Sword
geometry
in (x,y)
space-
space



Spherical wave relativistic geometry

RelaWavity Web Simulation
Wavefronts in Space-Space

Occam
Sword
geometry
in (x,y)
space-space

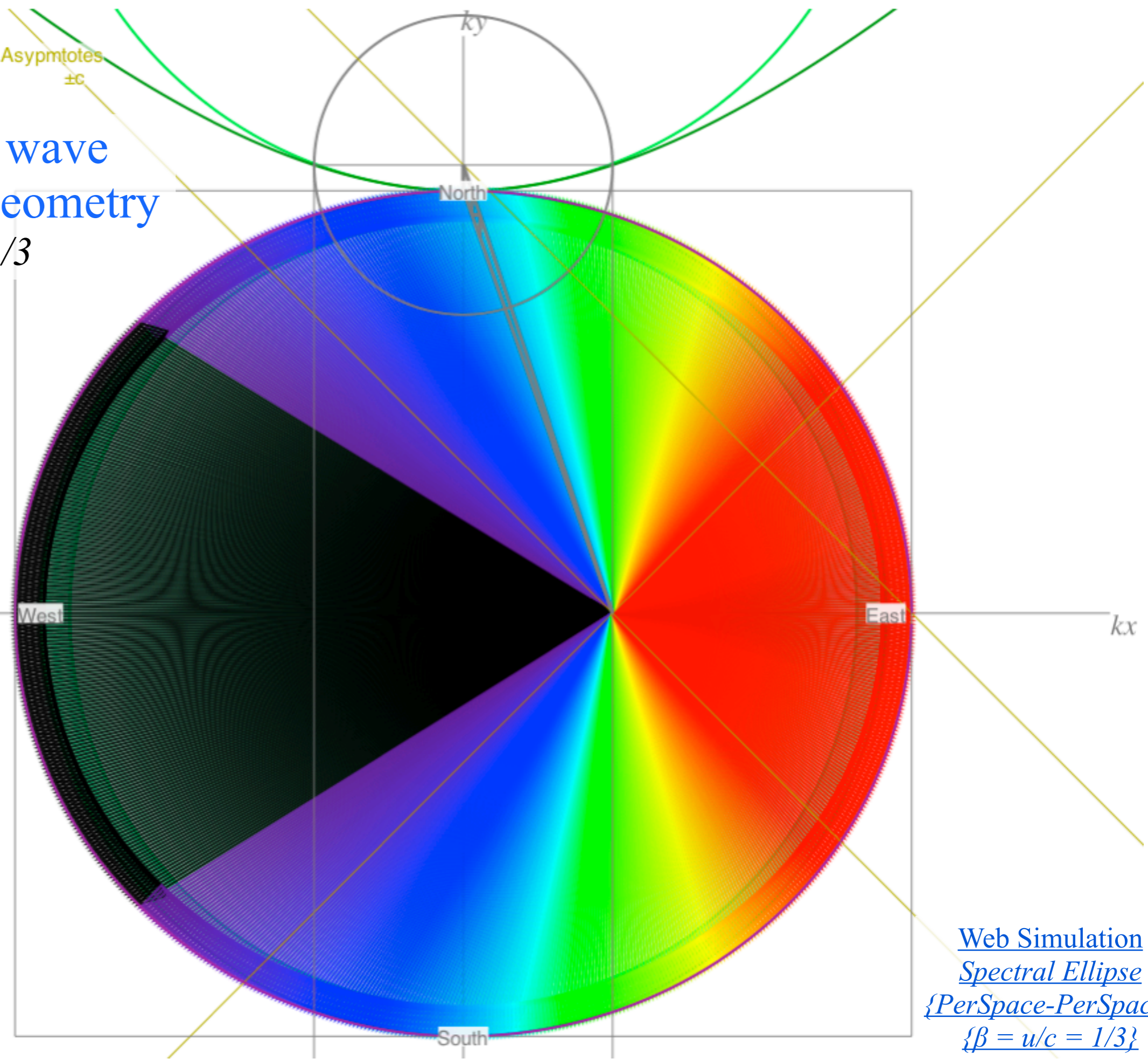


<p>Doppler Red $\lambda=c+u$ dilates to: $(c+u) \cosh \rho = c \sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$</p>	<p>ellipse focal length $FO = u = c \tanh \rho$ dilates to: $u \cosh \rho = c \sinh \rho$</p>	<p>Doppler Blue $\lambda=c-u$ dilates to: $(c-u) \cosh \rho = c \sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$</p>
<p>ellipse major radius $a=OFa=c$ dilates to: $c \cosh \rho = c/\sqrt{1-u^2/c^2}$</p>	<p>ellipse latus radius $FT=c(1-u^2/c^2)$ dilates to: $c(1-u^2/c^2) \cosh \rho = c\sqrt{1-u^2/c^2} = c \operatorname{sech} \rho$</p>	<p>Base height $FTk = \sqrt{c^2 - u^2}$ dilates to: $\sqrt{c^2 - u^2} \cosh \rho = c$ (equal to ellipse minor radius b)</p>

Applications of Einstein dilation factor:
 $\gamma = \cosh \rho = 1/\sqrt{1-u^2/c^2}$

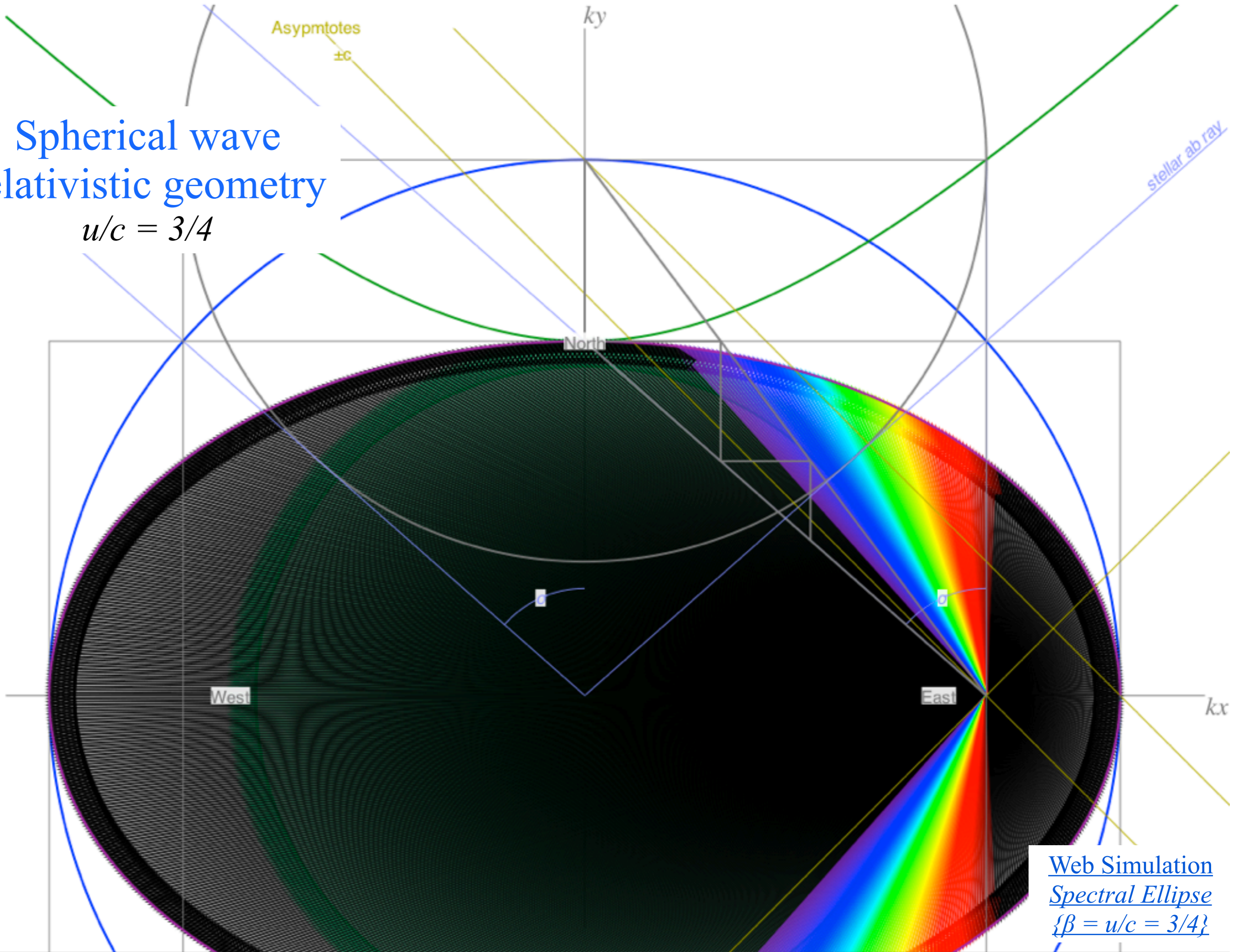
Spherical wave relativistic geometry

$$u/c = 1/3$$



[Web Simulation](#)
[Spectral Ellipse](#)
[{PerSpace-PerSpace}](#)
[{ \$\beta = u/c = 1/3\$ }](#)

Spherical wave
relativistic geometry
 $u/c = 3/4$



Web Simulation
Spectral Ellipse
 $\{\beta = u/c = 3/4\}$