

Lectures 3-4
Tue-Thur., 1.(26,28).2016

Analysis of 1D 2-Body Collisions (Ch. 2 to Ch. 4 of Unit 1)

Review of elastic Kinetic Energy ellipse geometry

The X2 Superball pen launcher

Perfectly elastic “ka-bong” velocity amplification effects (Faux-Flubber)

Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(s)

Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots

Integration of (V_1, V_2) data to space-space plots (y_1, y_2) Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on

Review of elastic Kinetic Energy ellipse geometry

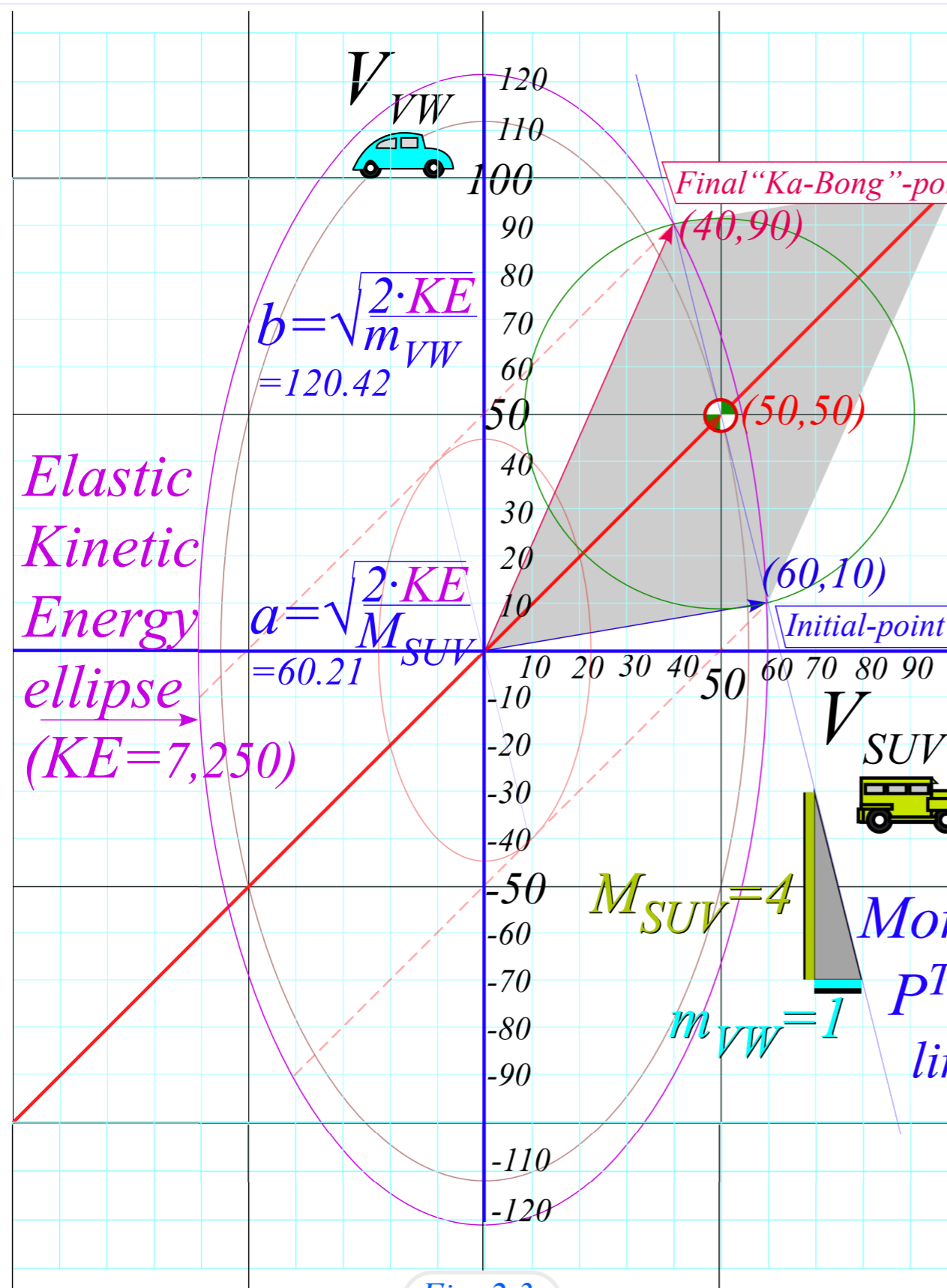


Fig. 2.3a
(Unit 1)

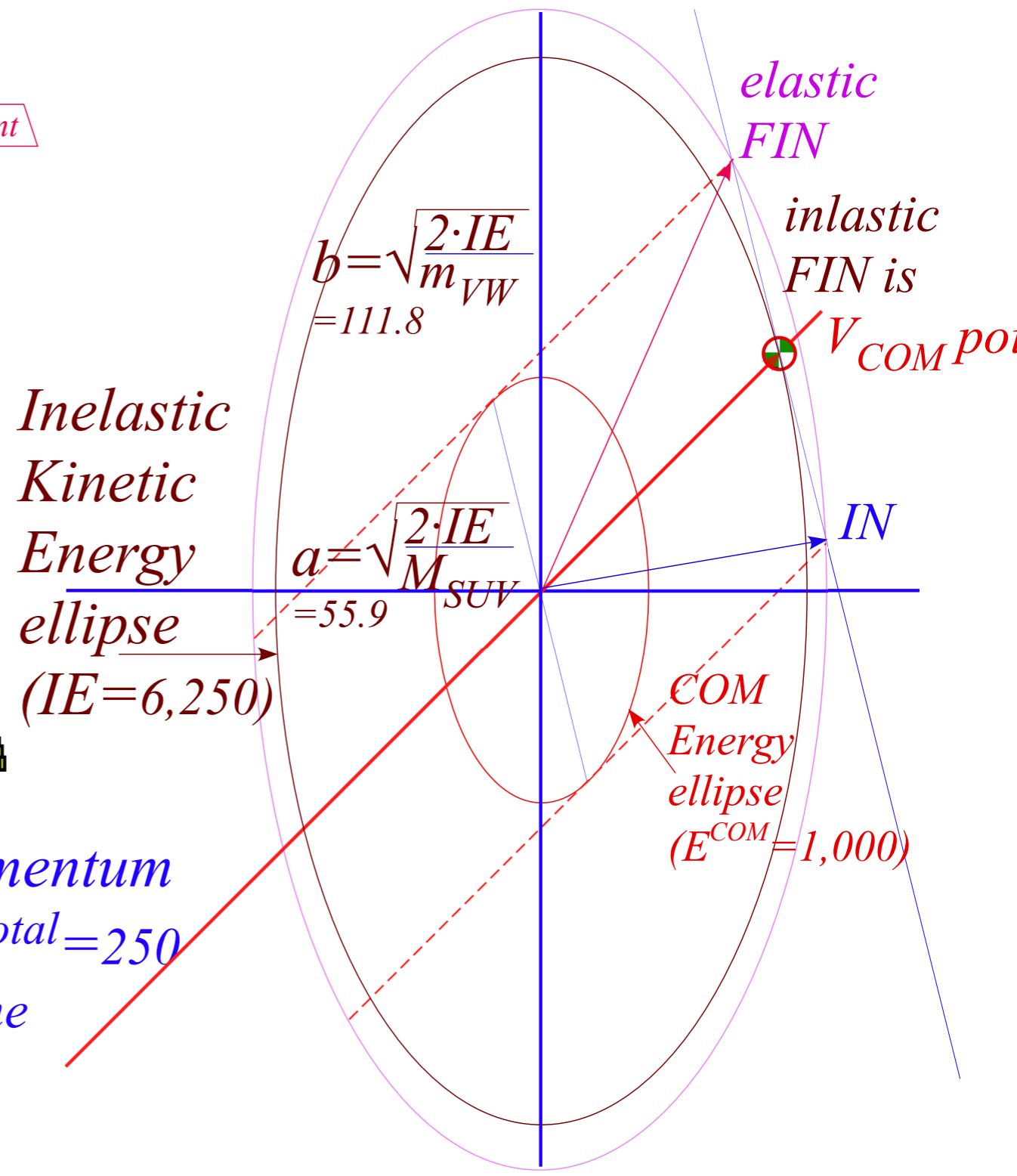


Fig. 2.3b
(Unit 1)

The X-2 Pen launcher and Superball Collision Simulator*

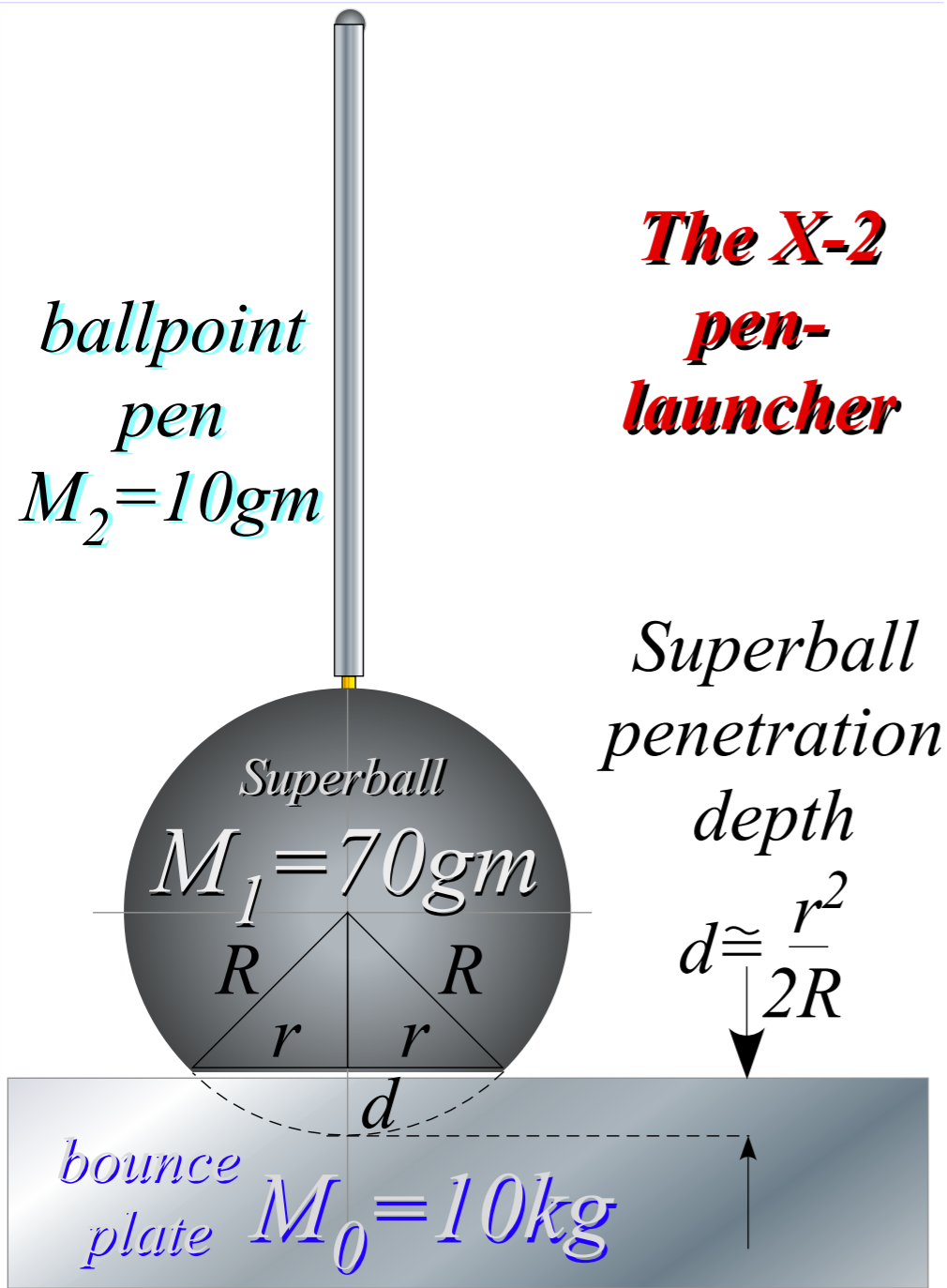
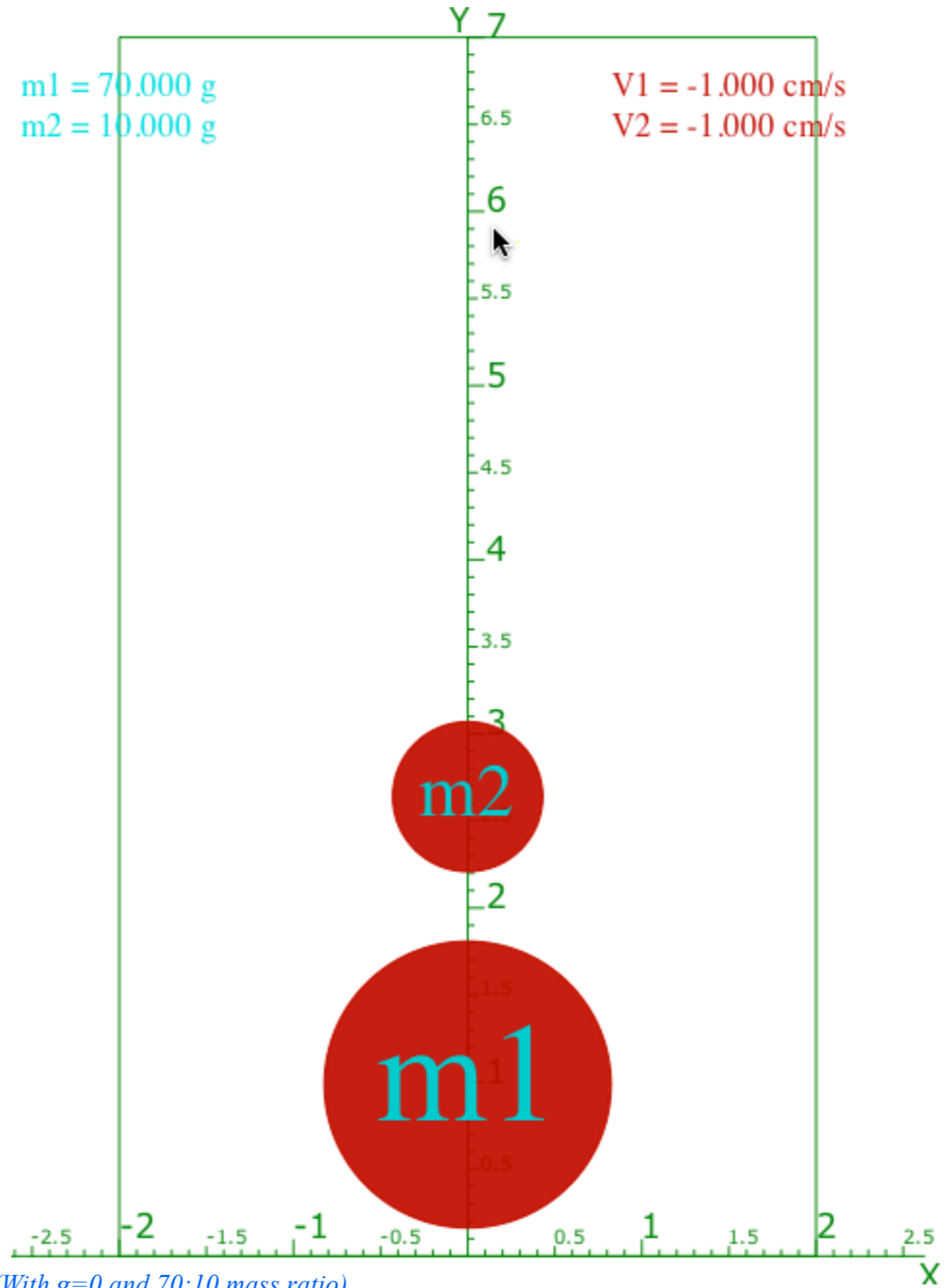


Fig. 3.1
(Unit 1)



(With $g=0$ and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator <http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

The X-2 Pen launcher and Superball Collision Simulator*

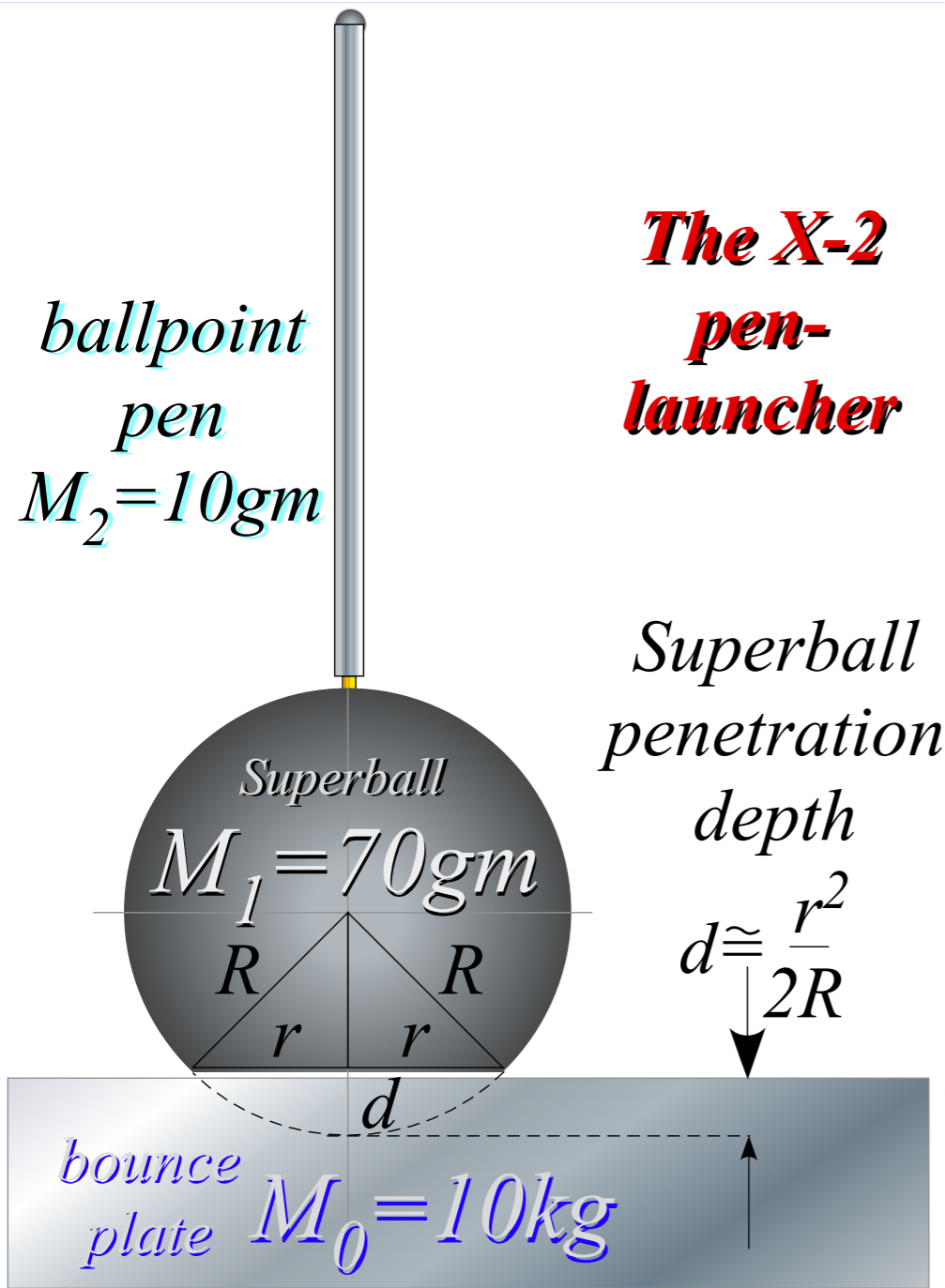


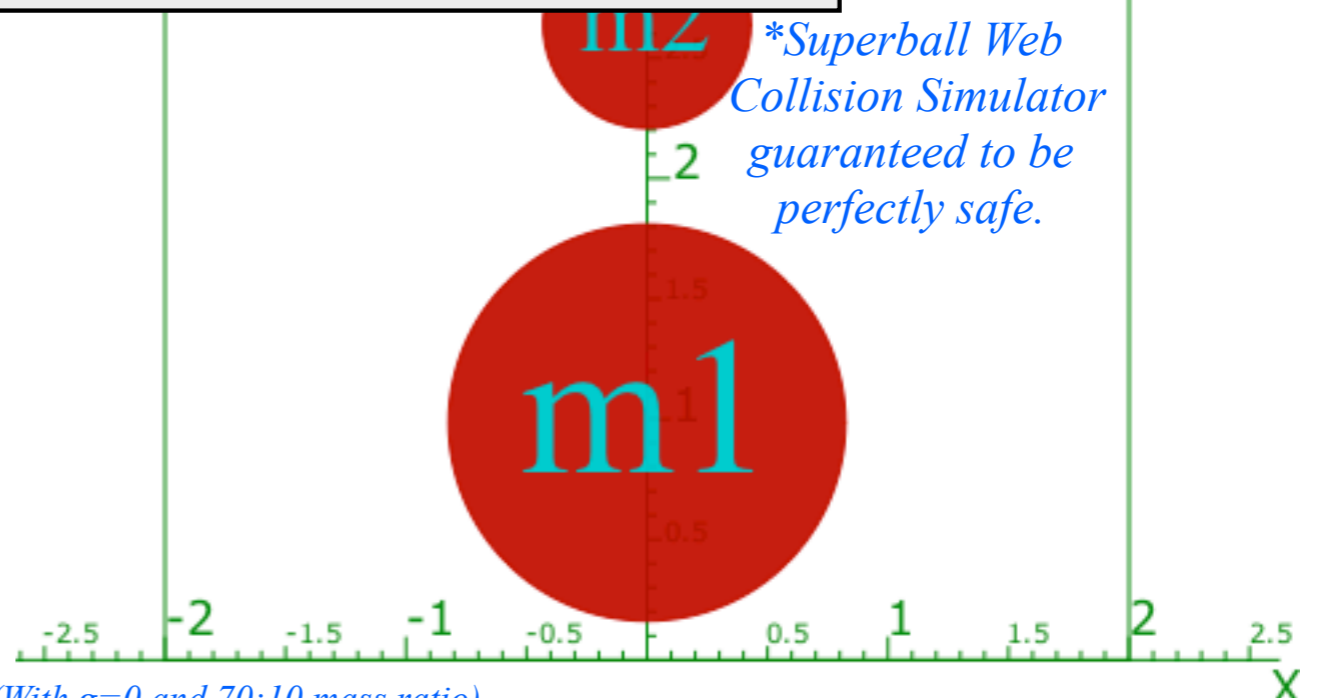
Fig. 3.1
(Unit 1)

Caution: Product Liability Disclaimer

This ballpoint pen could be hazardous to your health! The experiments which are the subject of this discussion are both spectacular and potentially dangerous, and care to protect one's eyes should be taken. The simplest experiment involves sticking a ball point pen into a superball or other hard rubber ball and dropping the two onto a hard floor. If done correctly the pen will eject the ball with such force it may stick in the ceiling of the room. Obviously you want to be careful with this weapon. And, this goes doubly and triply for the more advanced models that may be developed in the course of studying this stuff. It is recommended that experimenters wear safety glasses when doing these experiments with pens. (We could just say don't use pens, but that's an easy way to do this experiment and probably the way most people will go about it.) Some of the tangential experiments associated with this development are less hazardous. To measure the potential force function of a ball one may simply paint the ball and measure the spot size as a function of drop height h .

The sagittal approximation $d = r^2 / 2R$ allows one to quickly convert spot radius r to penetration depth x for a superball of radius R as shown in the figure. Equating this to Mgh gives the ball potential energy function $V(x)$.

$V_1 = -1.000 \text{ cm/s}$
 $V_2 = -1.000 \text{ cm/s}$



(With $g=0$ and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

ballpoint pen
 $M_2 = 10\text{gm}$

The X-2 pen-launcher

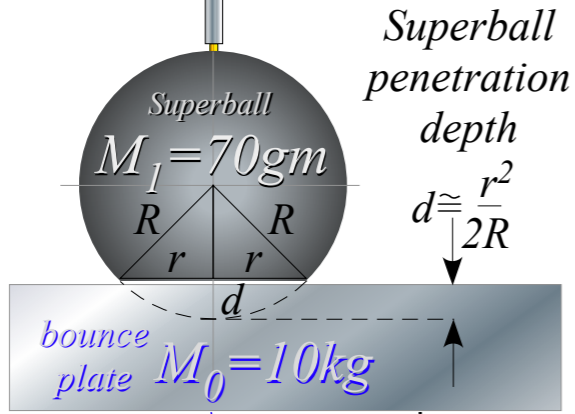


Fig. 3.3
 (Unit 1)

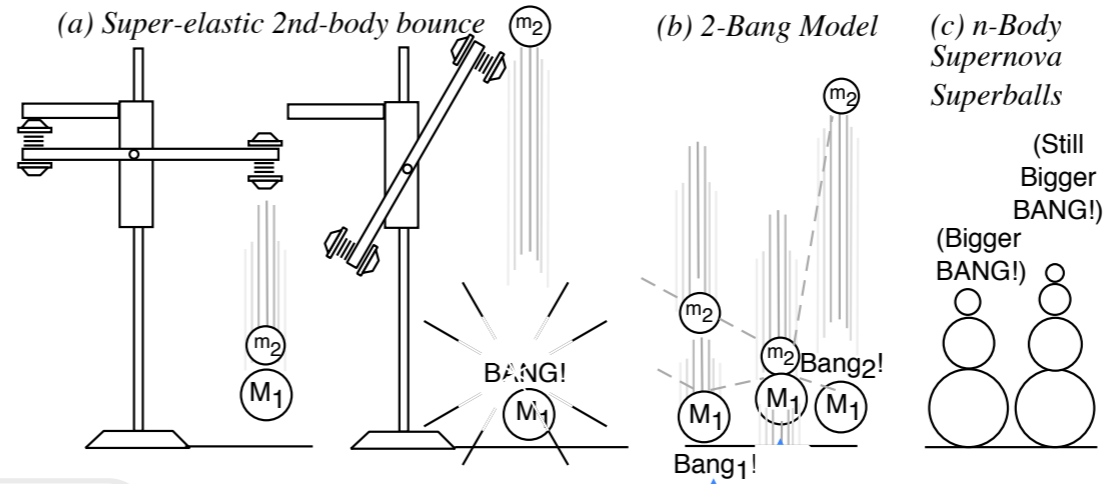
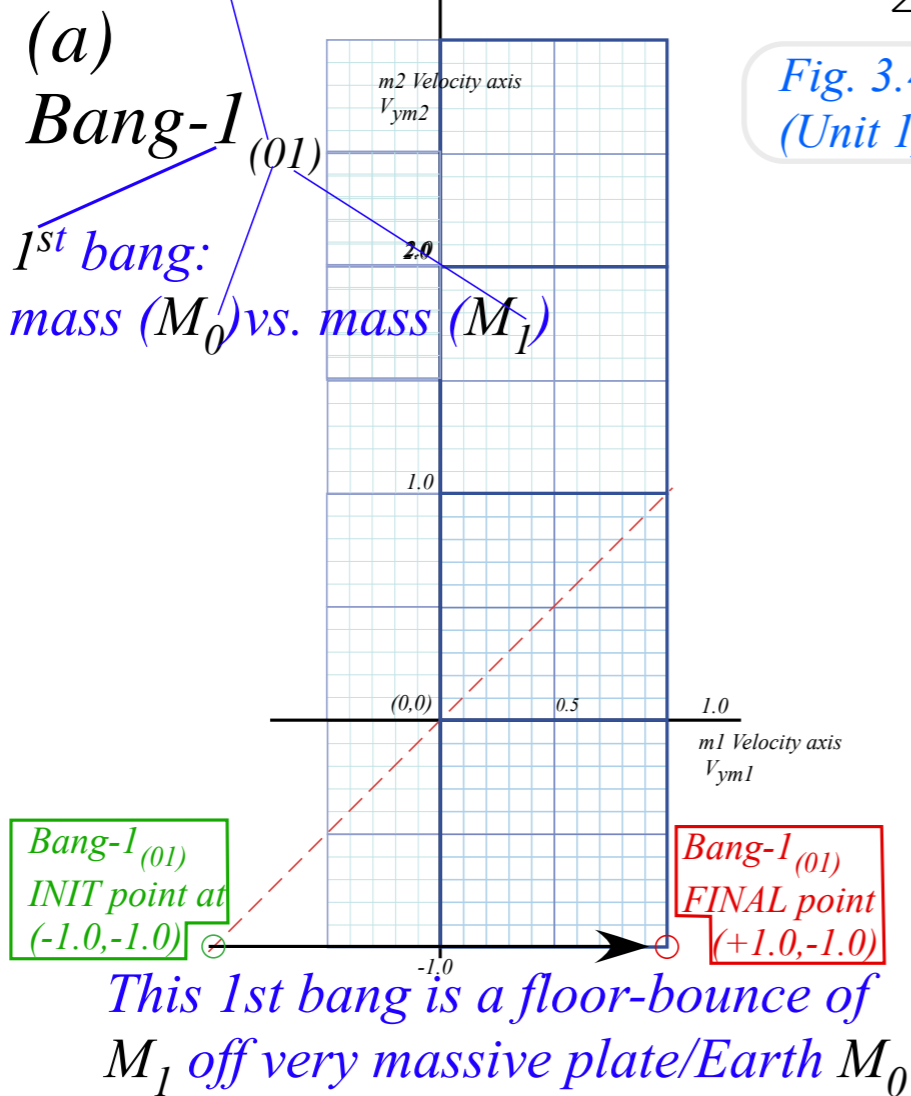


Fig. 3.4
 (Unit 1)



1st bang:
 M_1 off floor

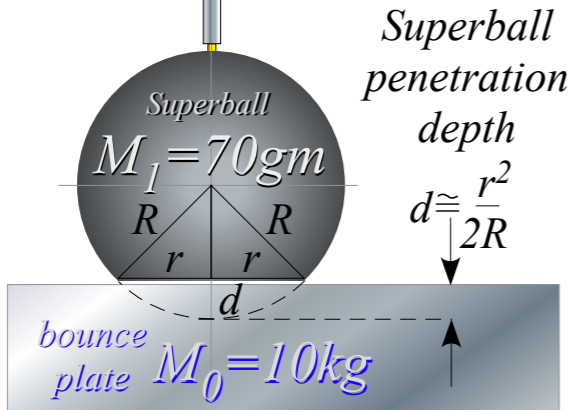
(With $g=0$ and 70:10 mass ratio)

*Launch Generic Superball Collision Web Simulator

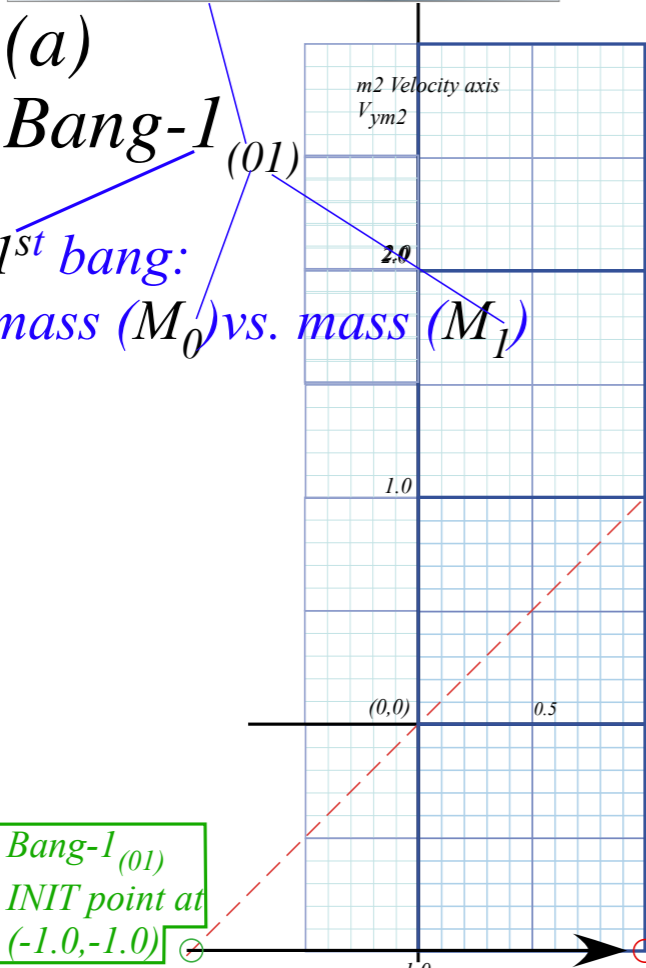
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ballpoint pen
 $M_2 = 10\text{gm}$

The X-2 pen-launcher



Superball penetration depth
 $d \approx \frac{r^2}{2R}$



This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Fig. 3.3 (Unit 1)

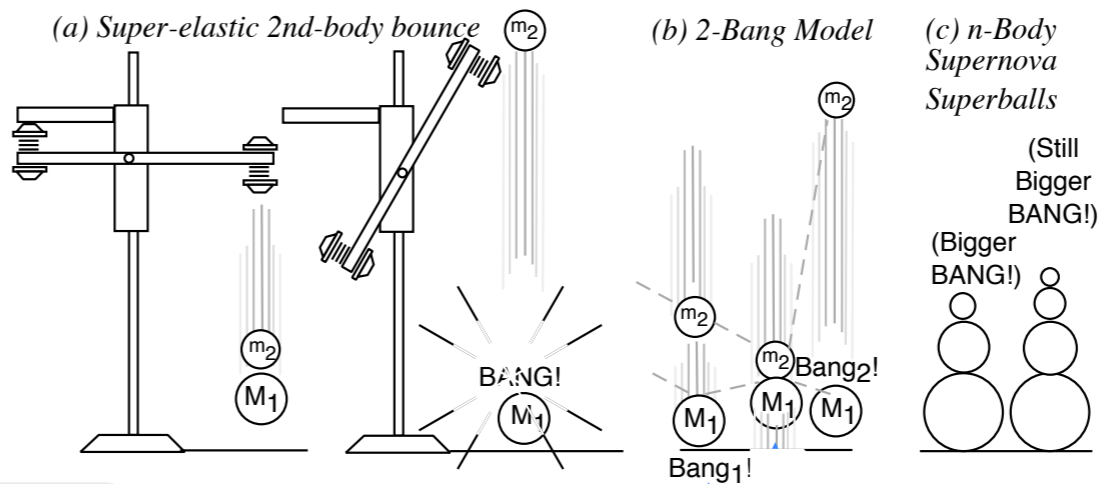
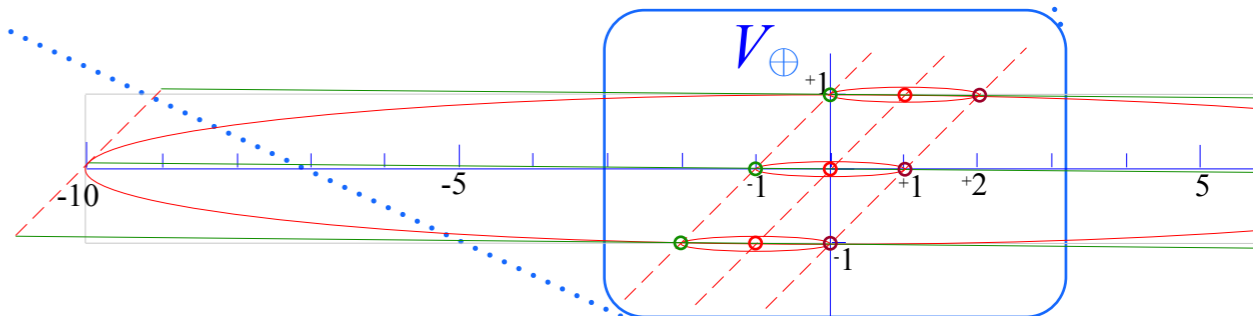
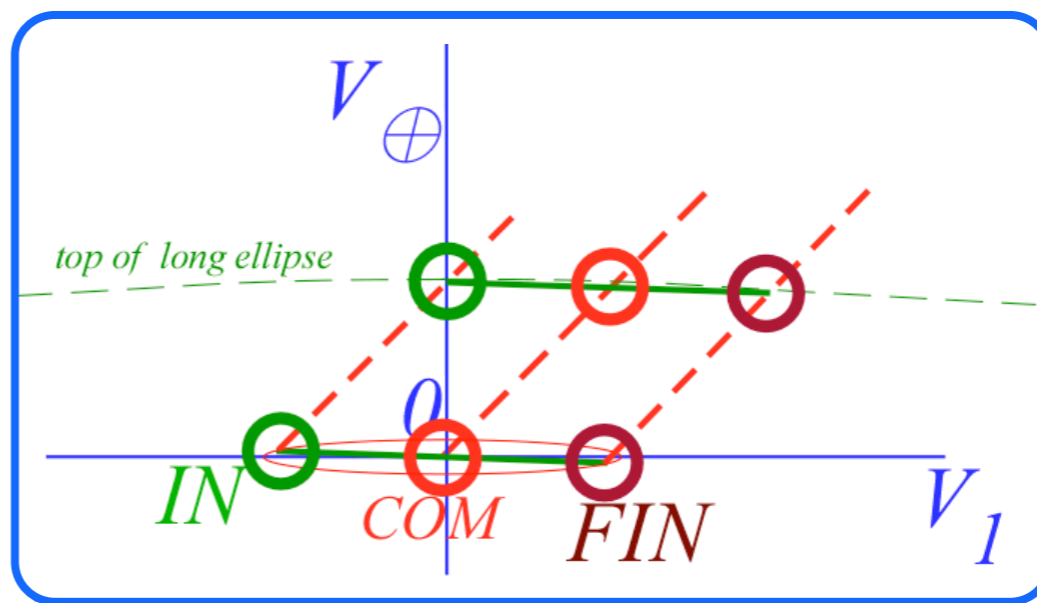


Fig. 3.4 (Unit 1)

1st bang: M_1 off floor



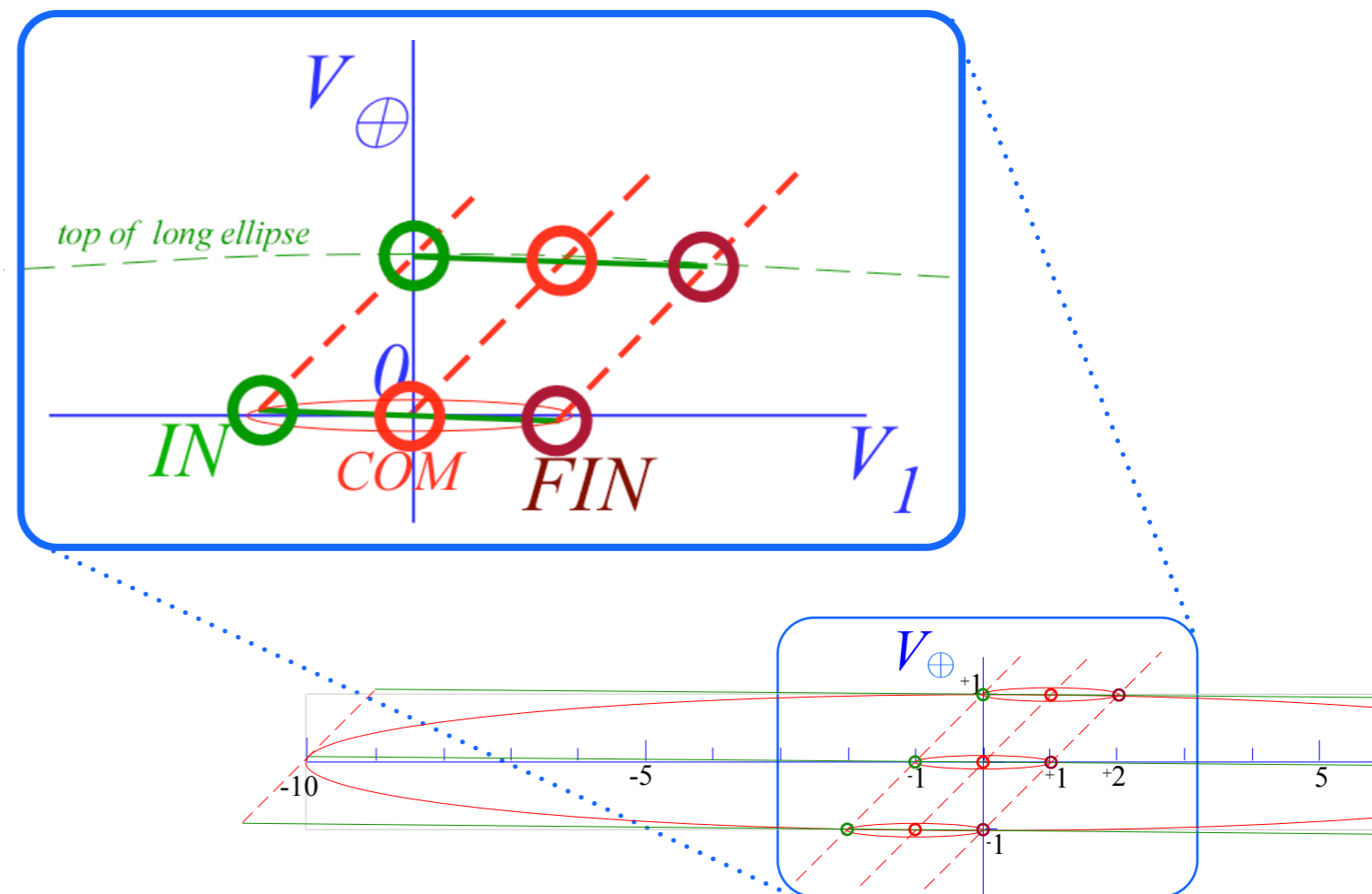
(With $g=0$ and 70:10 mass ratio)

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

(With g and 70:35 mass ratio)

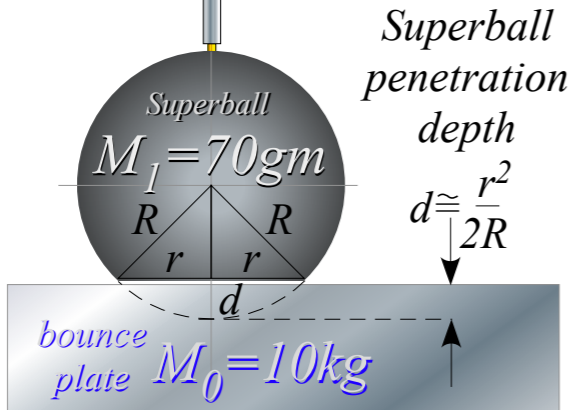
<http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html>

(a) 1st bang of M_1 off
 floor plate $M_{\oplus} = 100 M_1$ along
 (V_1, V_{\oplus}) -momentum line of slope
 $-M_1/M_{\oplus} = -1/100$
 from IN-end to COM to FIN-end
 of ($a/b = \sqrt{M_{\oplus}}/\sqrt{M_1} = 10$) ellipse



ballpoint pen
 $M_2=10gm$

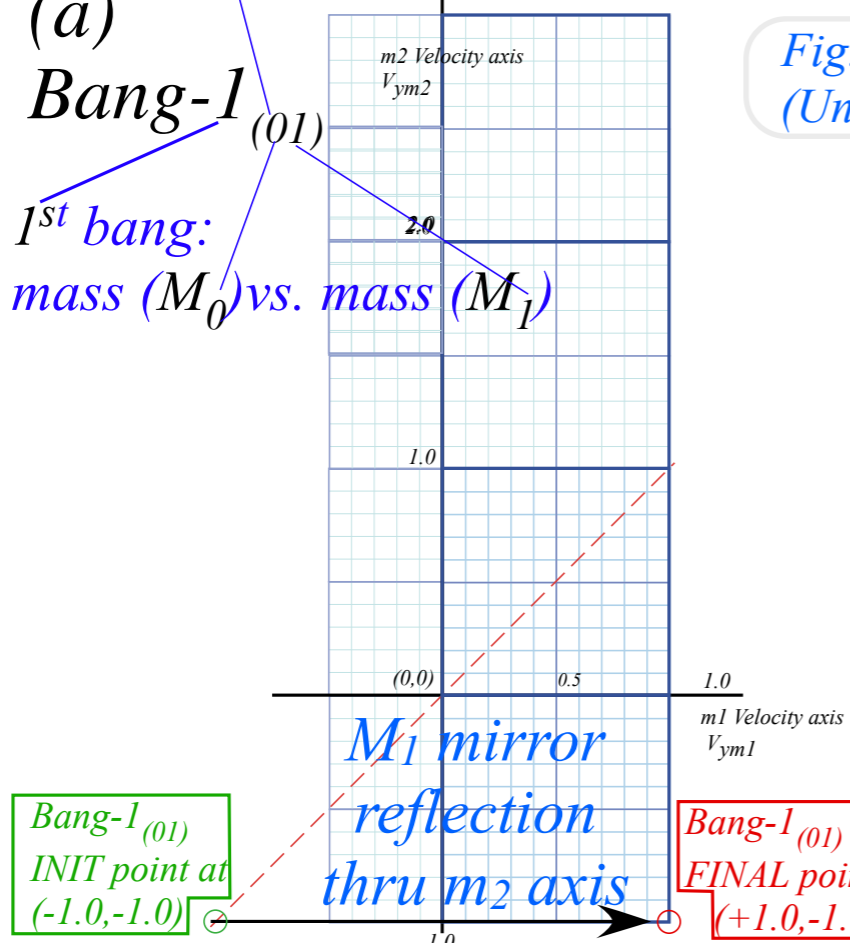
The X-2 pen-launcher



(a)

Bang-1
(01)

1st bang:
mass (M_0) vs. mass (M_1)



This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Fig. 3.3
(Unit 1)

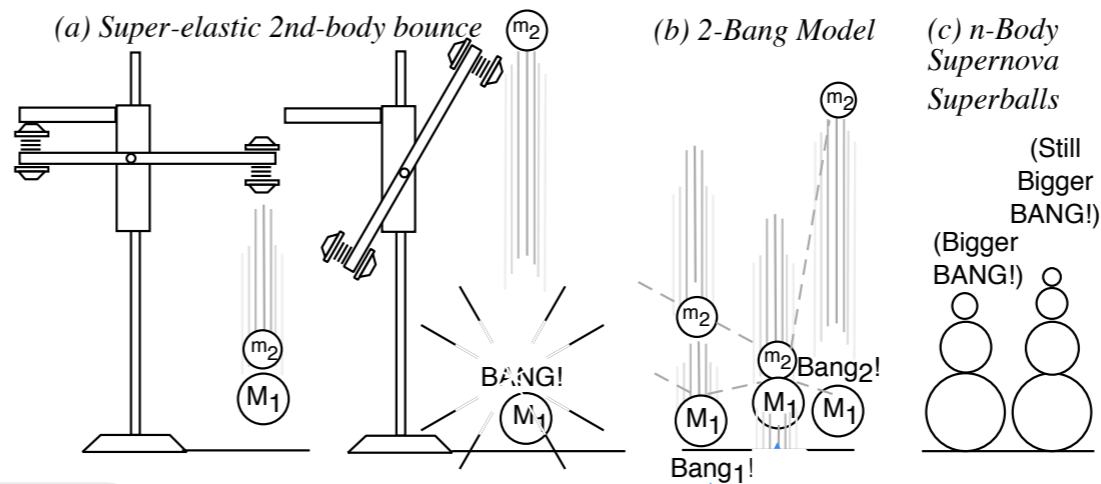


Fig. 3.4
(Unit 1)

1st bang:
 M_1 off floor

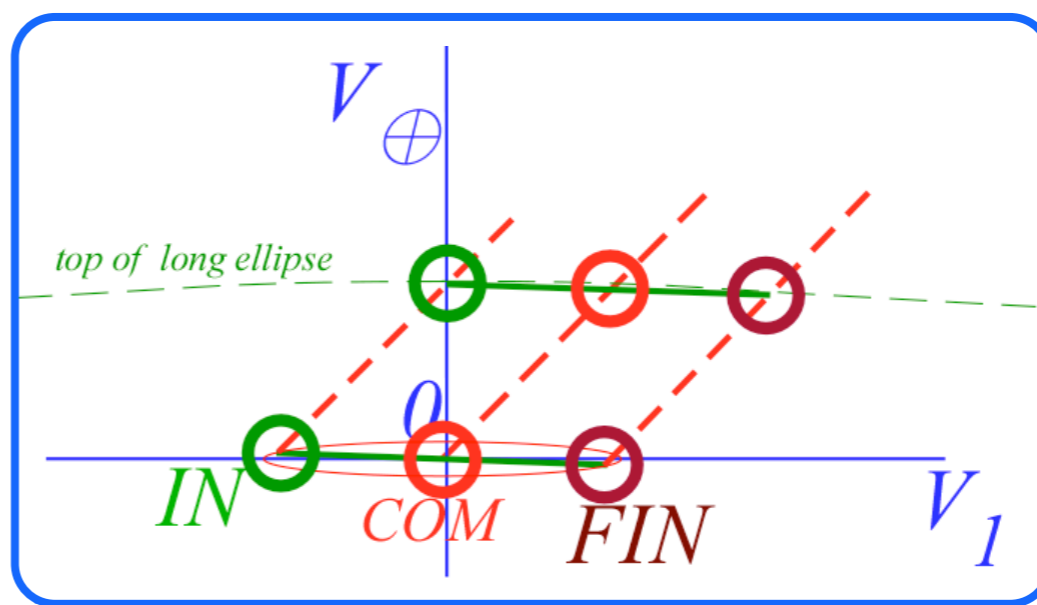
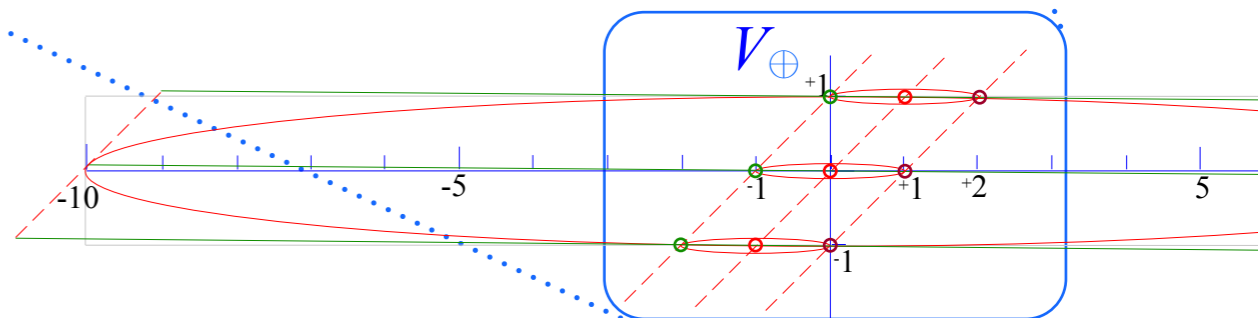


Fig. 3.2a
(Unit 1)



(With $g=0$ and 70:10 mass ratio)

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

ballpoint pen
pen
 $M_2=10gm$

The X-2 pen-launcher

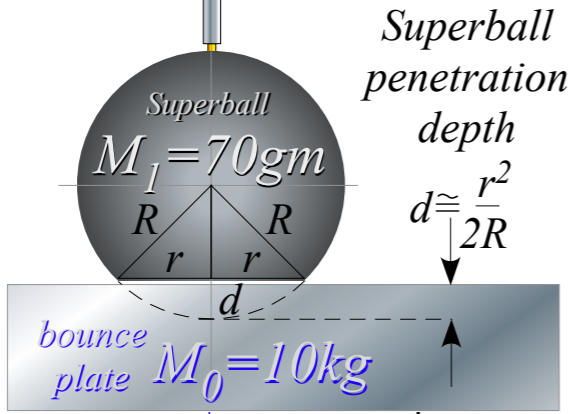


Fig. 3.3
(Unit 1)

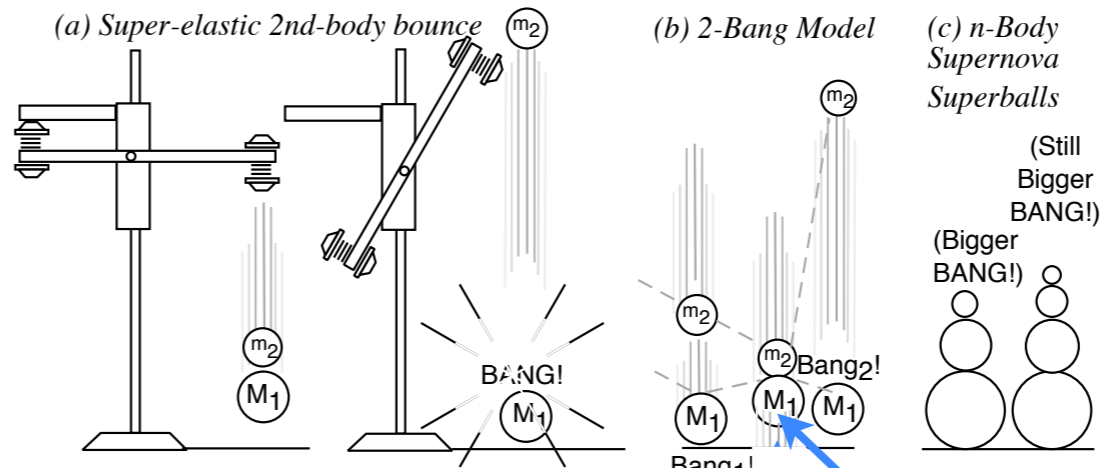
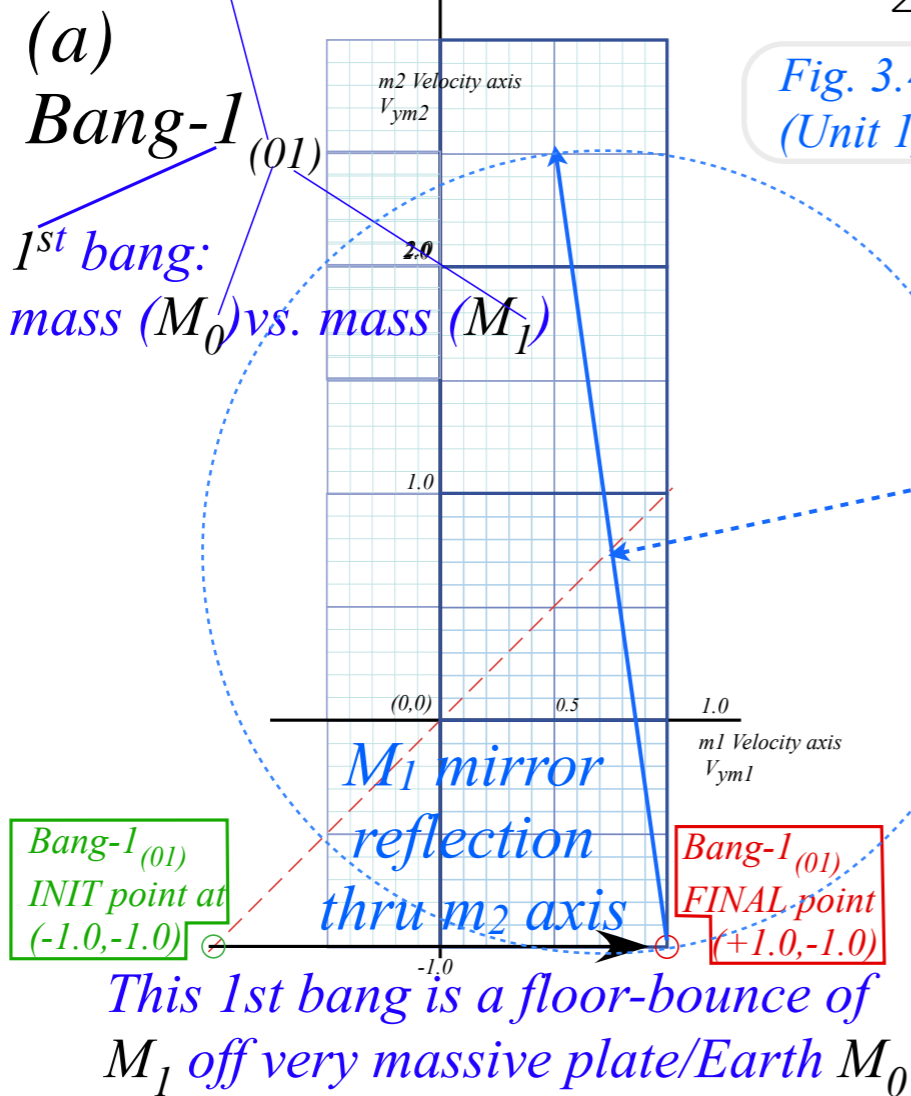


Fig. 3.4
(Unit 1)



1st bang: M_1 off floor

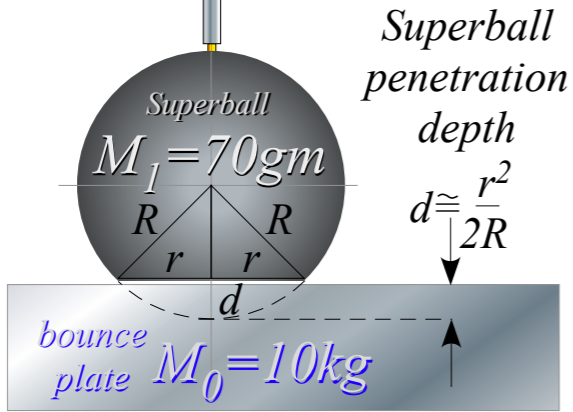
2nd bang: m_2 off M_1

(With $g=0$ and 70:10 mass ratio)
<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

(With g and 70:35 mass ratio)
<http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html>

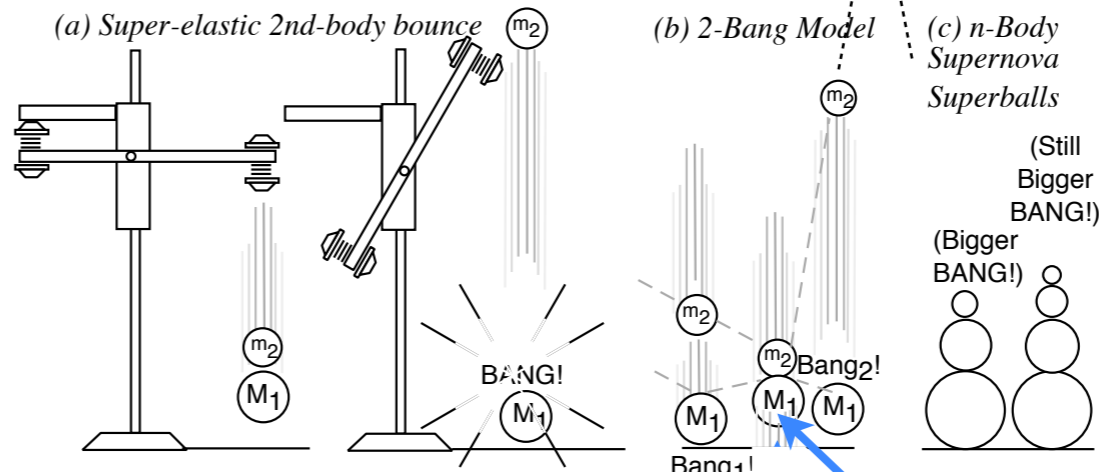
ballpoint pen
 $M_2 = 10\text{gm}$

The X-2 pen-launcher



Superball penetration depth
 $d \approx \frac{r^2}{2R}$

Fig. 3.3 (Unit 1)



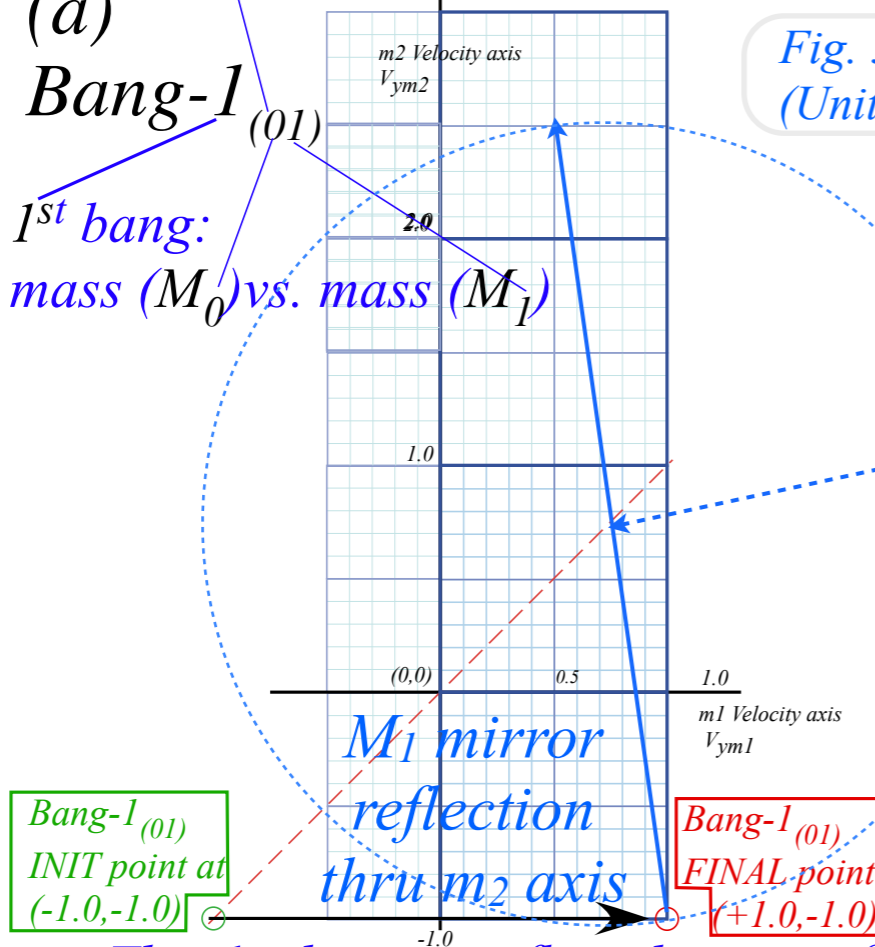
3rd bang:
 m_2 off ceiling

(a)

Bang-1 (01)

1st bang:
 mass (M_0) vs. mass (M_1)

Fig. 3.4 (Unit 1)



Bang-1 (01)
 INIT point at
 (-1.0, -1.0)

Bang-1 (01)
 FINAL point
 (+1.0, -1.0)

This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

1st bang:
 M_1 off floor

2nd bang:
 m_2 off M_1

(With $g=0$ and 70:10 mass ratio)
<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

(With g and 70:35 mass ratio)
<http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html>

Geometry of X2 launcher bouncing in box

 *Independent Bounce Model (IBM)*

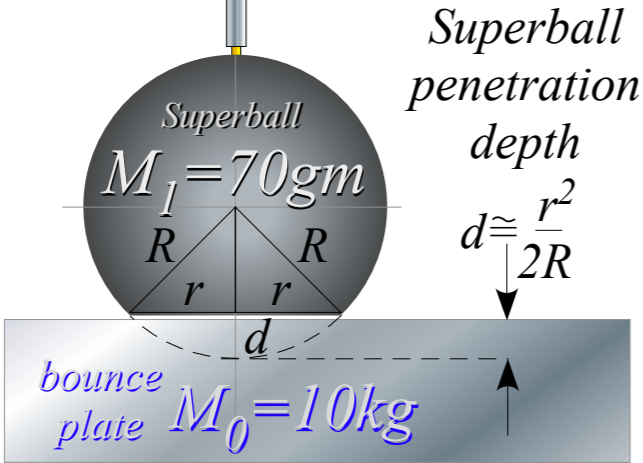
Geometric optimization and range-of-motion calculation(t)

Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots

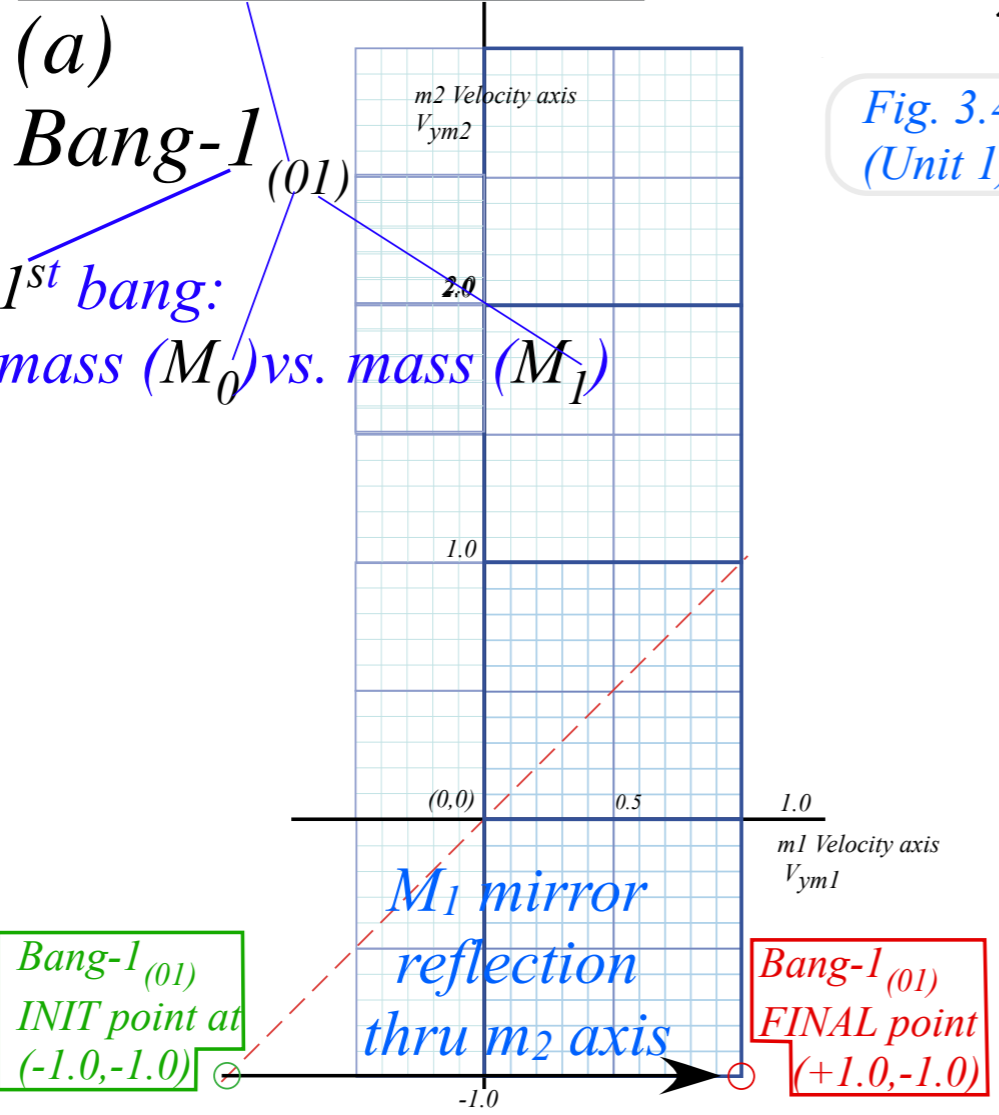
Integration of (V_1, V_2) data to space-space plots (y_1, y_2)

ballpoint pen
 $M_2=10gm$

The X-2 pen-launcher



Superball penetration depth
 $d \approx \frac{r^2}{2R}$



This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

Fig. 3.3 (Unit 1)

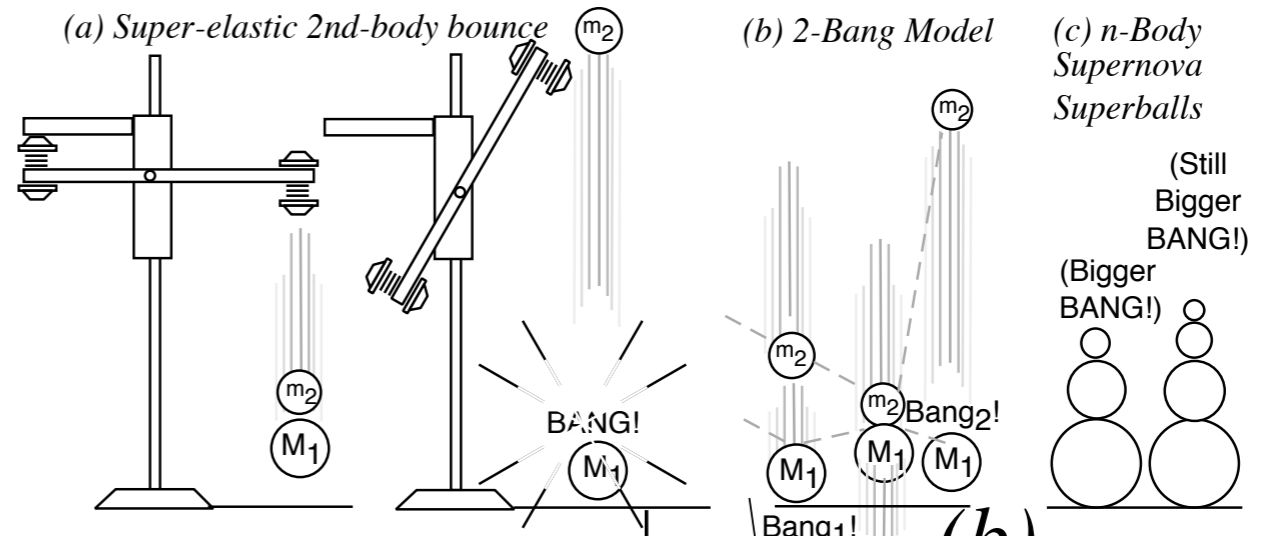
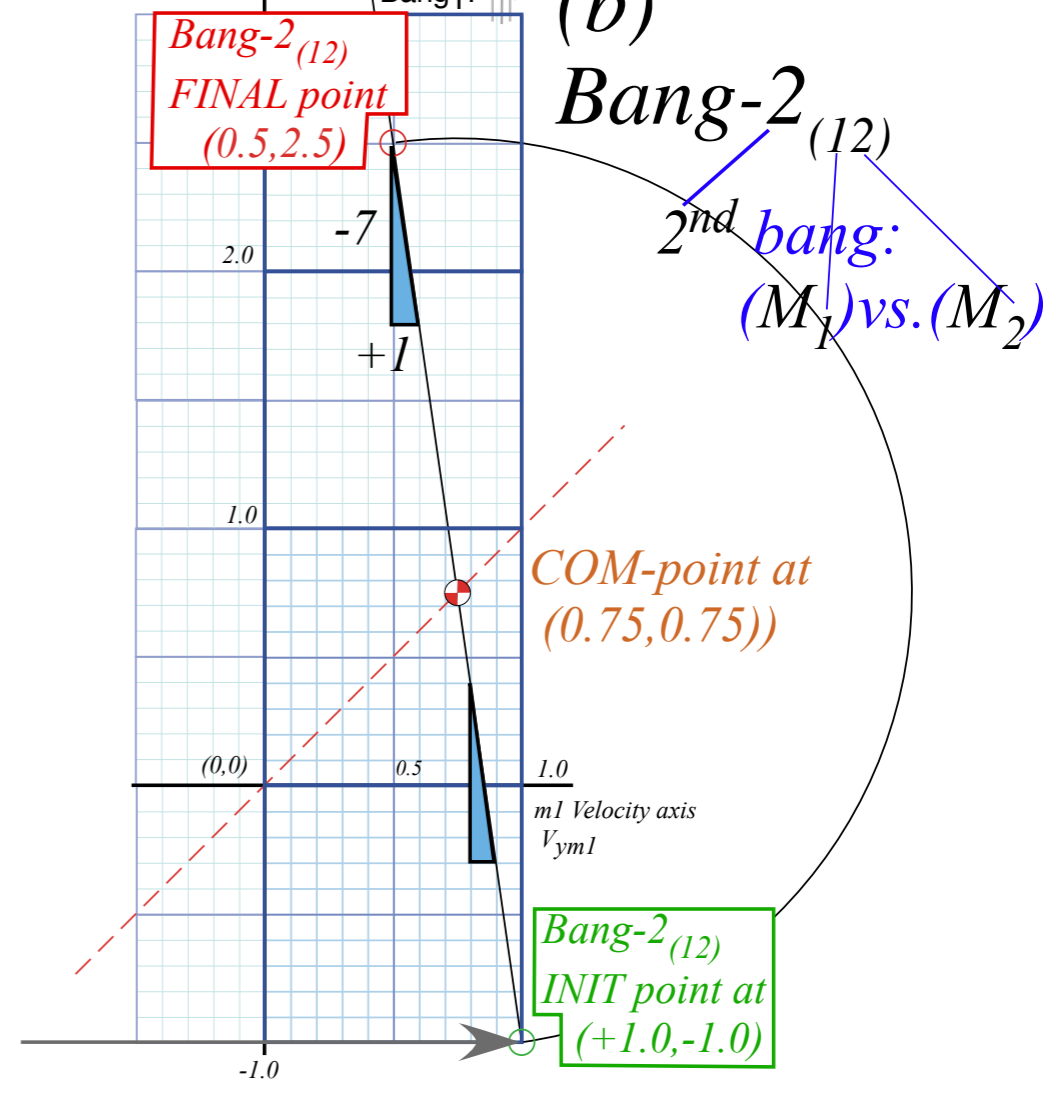


Fig. 3.4 (Unit 1)



Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

 *Geometric optimization and range-of-motion calculation(s)*

Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots

Integration of (V_1, V_2) data to space-space plots (y_1, y_2)

ballpoint pen
pen
 $M_2=10\text{gm}$

The X-2 pen-launcher

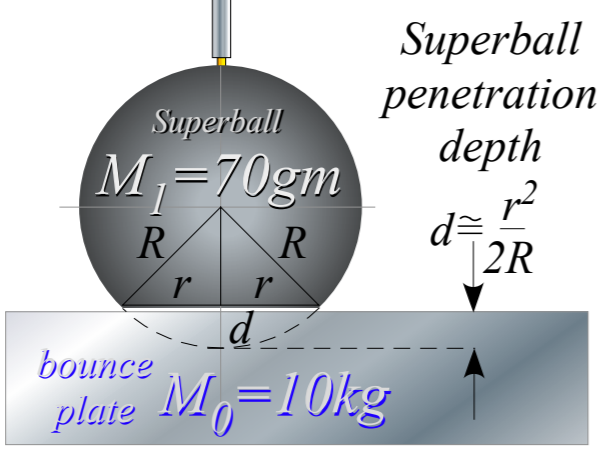


Fig. 3.3 (Unit 1)

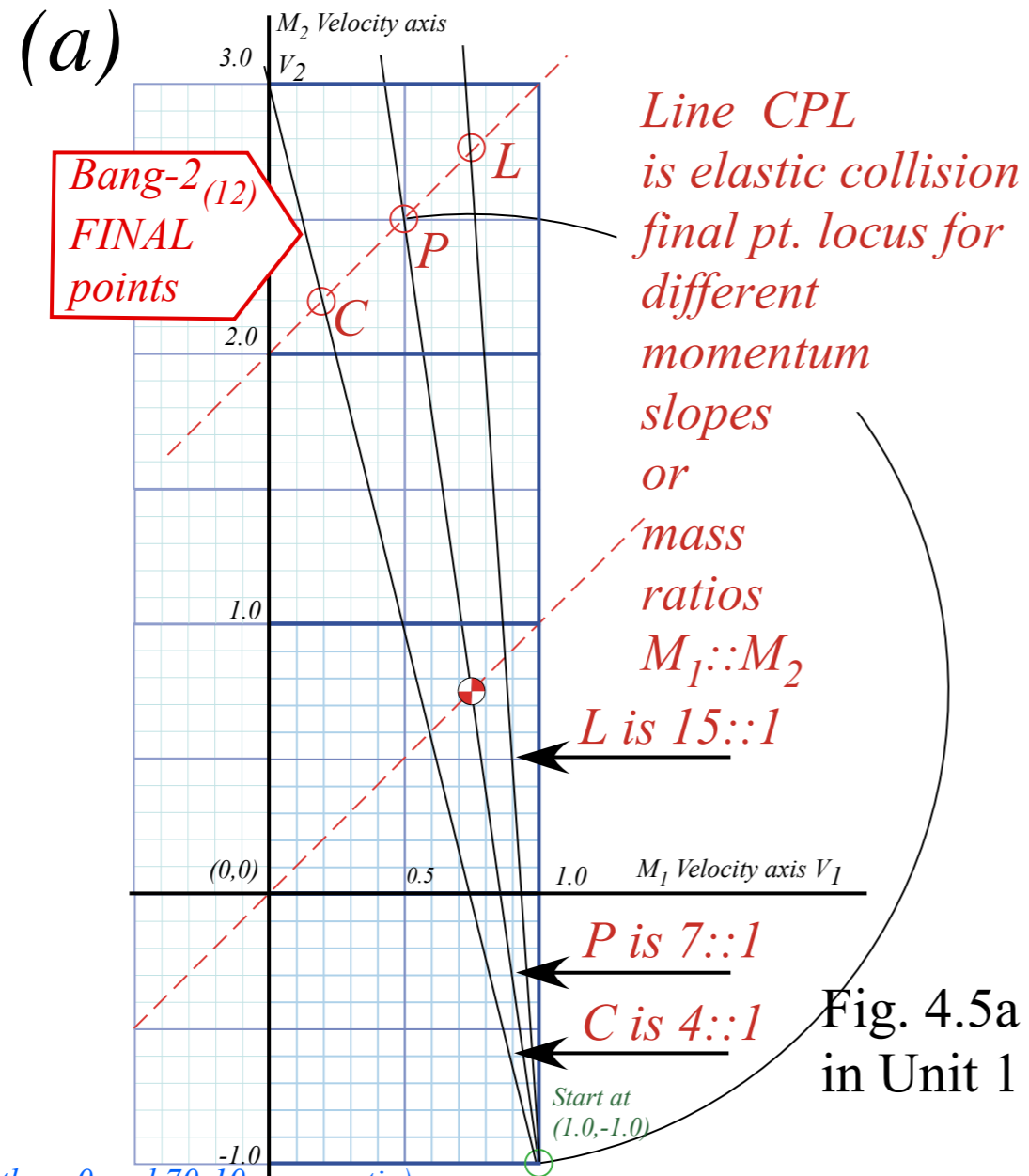
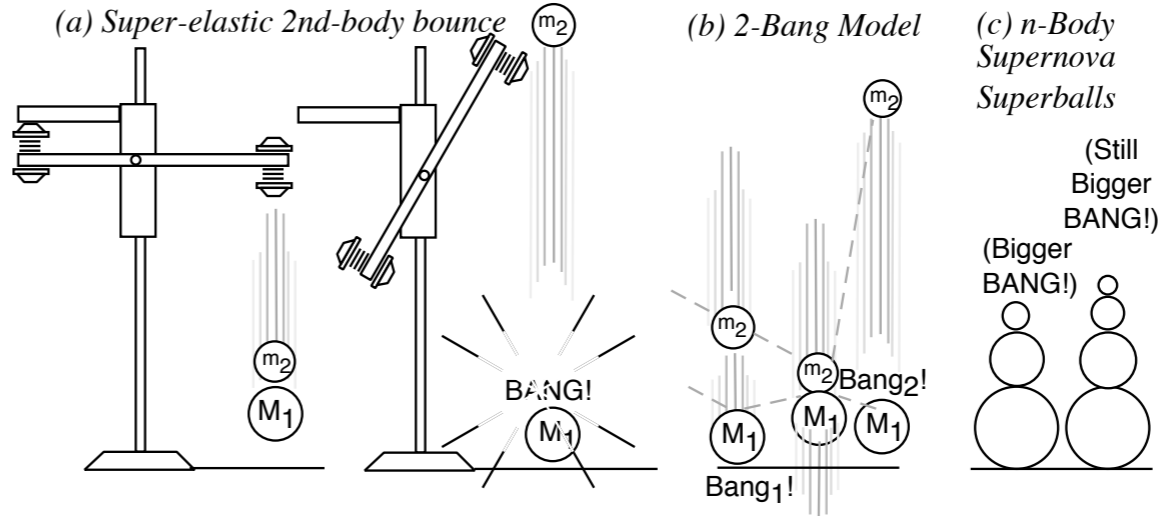


Fig. 4.5a in Unit 1

(With $g=0$ and 70:10 mass ratio)
<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

(With g and 70:35 mass ratio)
<http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html>

ballpoint pen
 $M_2=10\text{gm}$

The X-2 pen-launcher

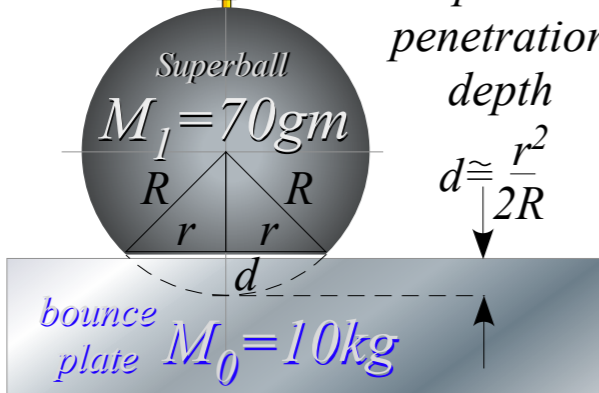


Fig. 3.3 (Unit 1)

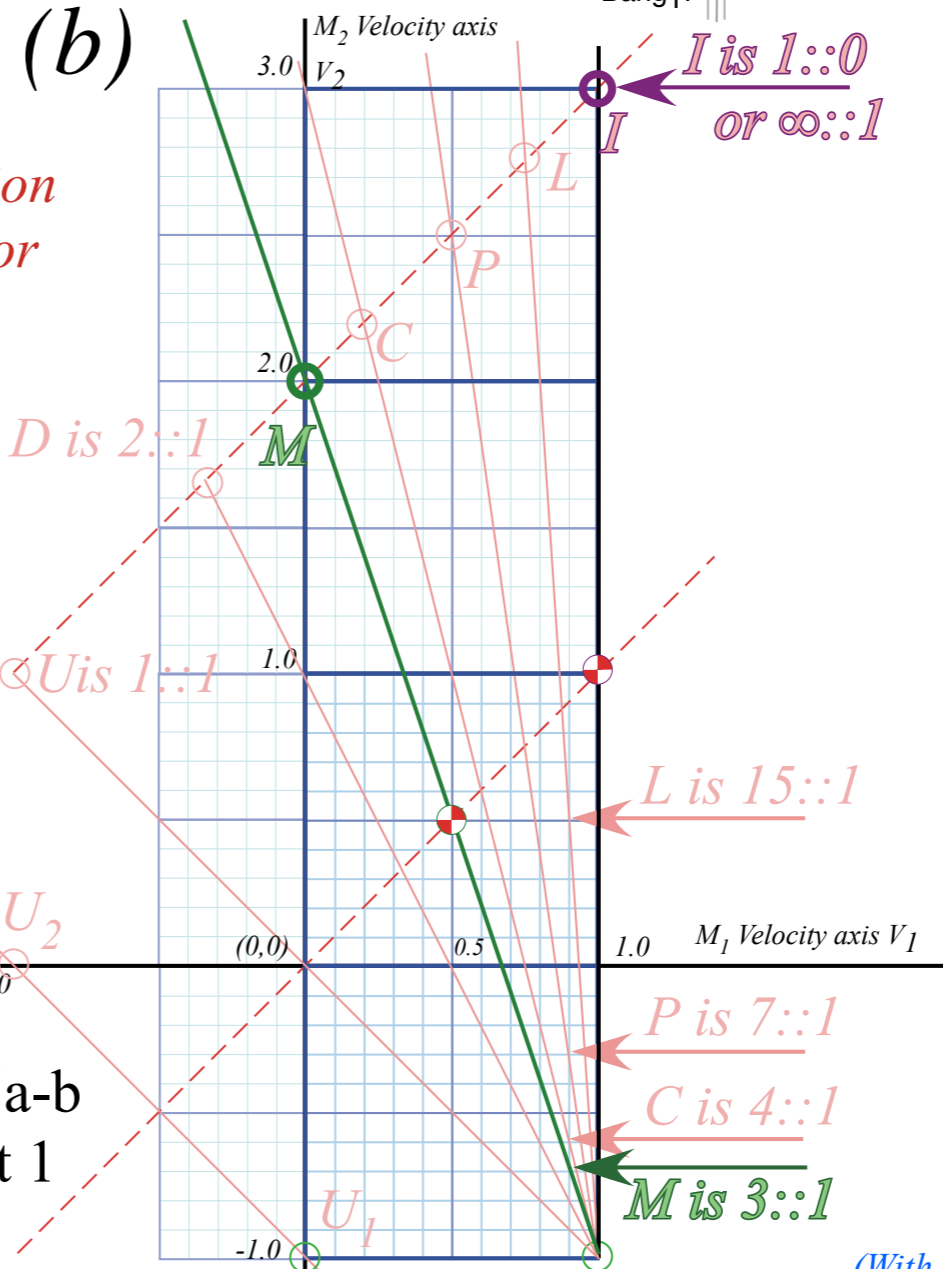
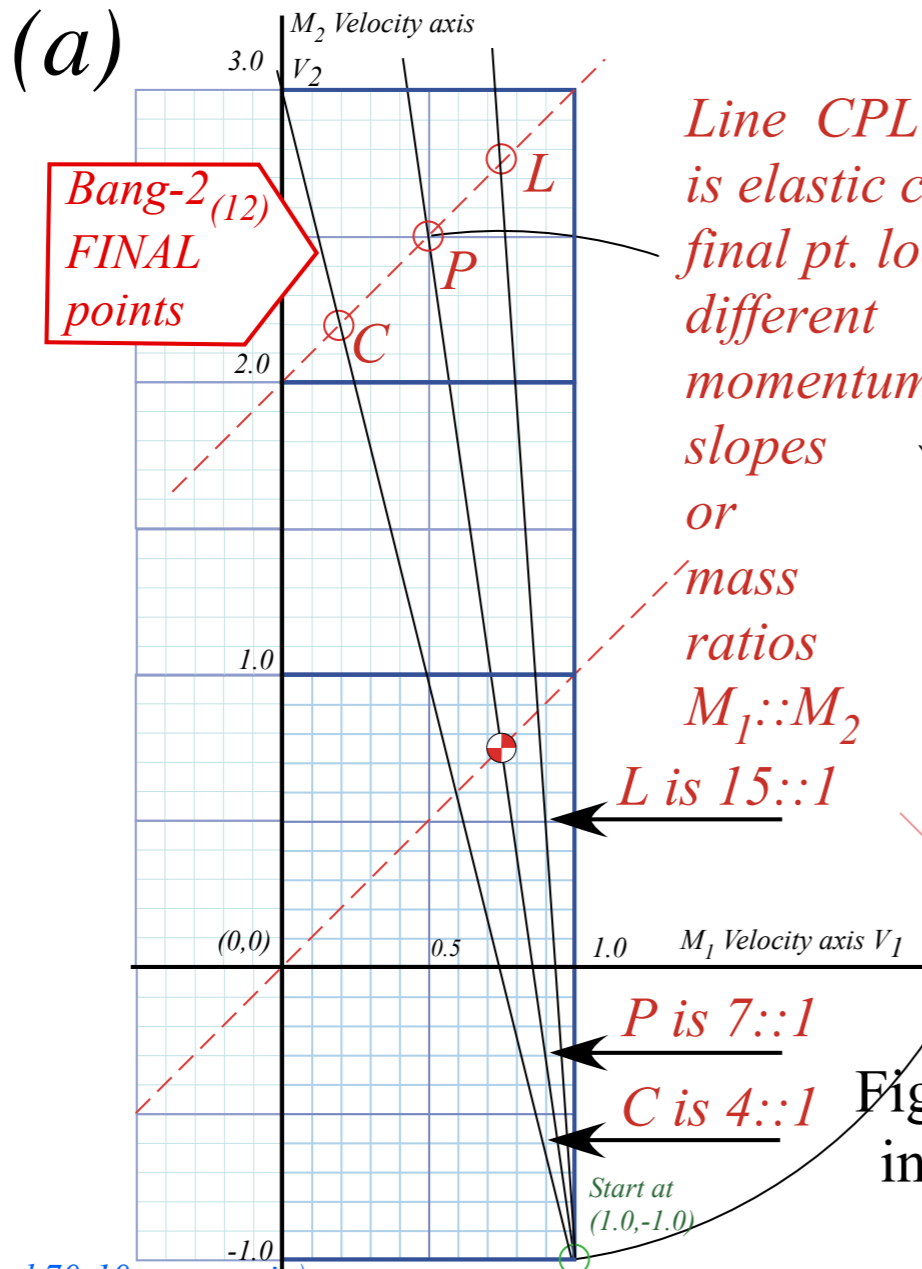
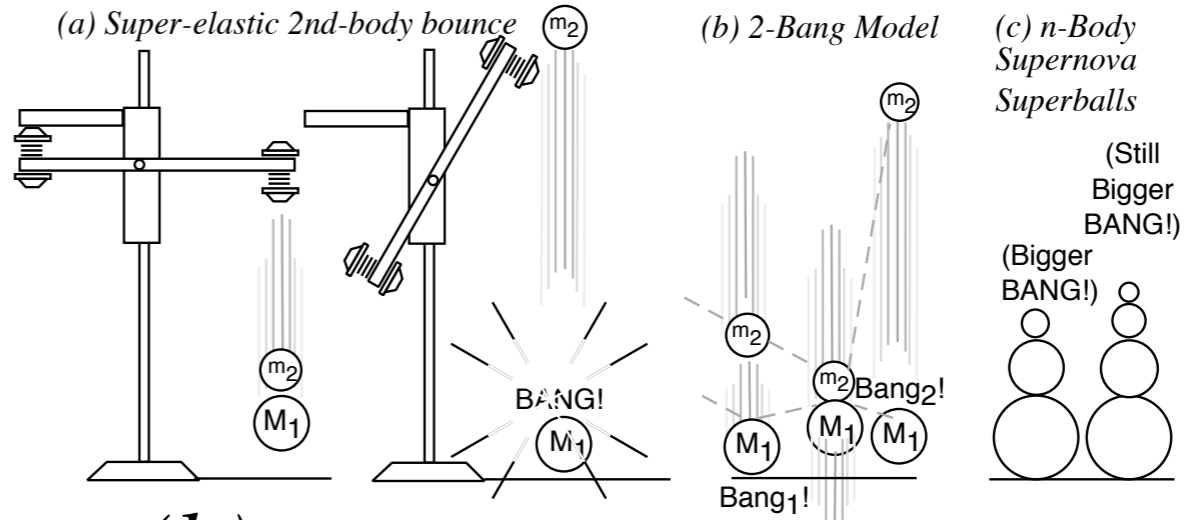


Fig. 4.5a-b in Unit 1

(With $g=0$ and 70:10 mass ratio)

<http://www.uark.edu/ua/modphys/markup/BounceItWeb.html?scenario=1007>

(With g and 70:35 mass ratio)

<http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html>

Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

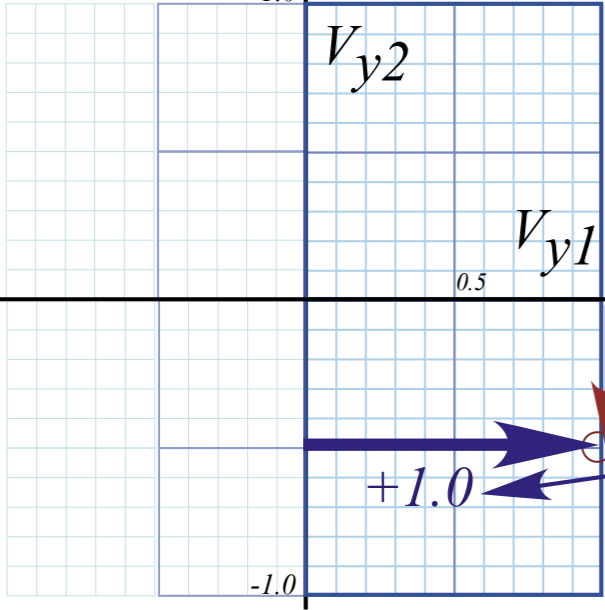
Geometric optimization and range-of-motion calculation(s)

 *Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots*

Integration of (V_1, V_2) data to space-space plots (y_1, y_2) Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$

Geometric "Integration" (Converting Velocity data to Spacetime)

Velocity V_{y2} vs. V_{y1} Plot



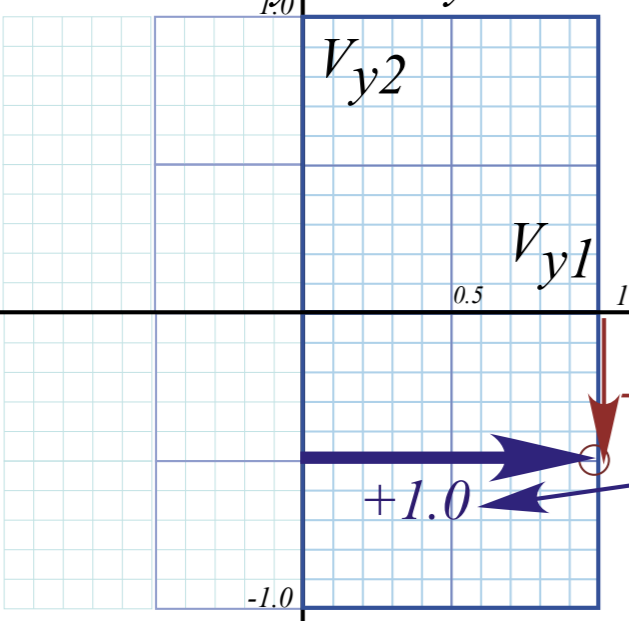
$V_{y2} = -0.5$
means M_2 is
somewhere
on some path of slope -0.5

$V_{y1} = +1.0$
means M_1 is
somewhere
on some path of slope $+1.0$

Position y vs. Time t Plot

Geometric "Integration" (Converting Velocity data to Spacetime)

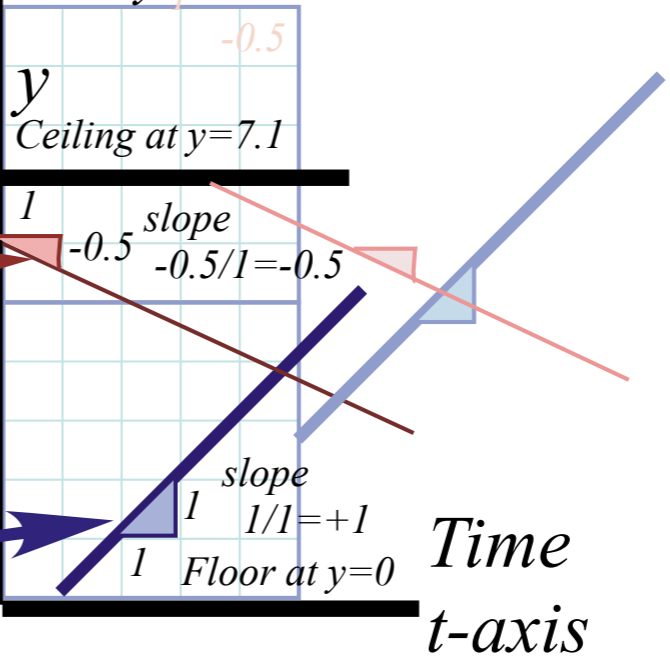
Velocity V_{y2} vs. V_{y1} Plot



$V_{y2} = -0.5$
means M_2 is
somewhere
on some path of slope -0.5

$V_{y1} = +1.0$
means M_1 is
somewhere
on some path of slope $+1.0$

Position y vs. Time t Plot

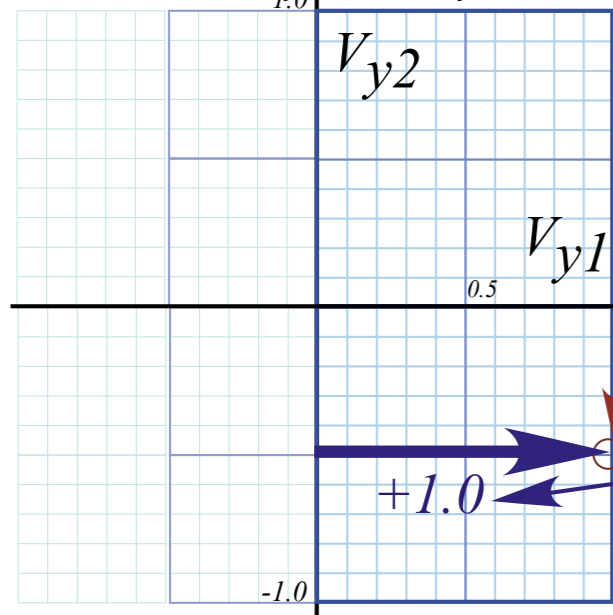


Height
 y -axis

Time
 t -axis

Geometric "Integration" (Converting Velocity data to Spacetime)

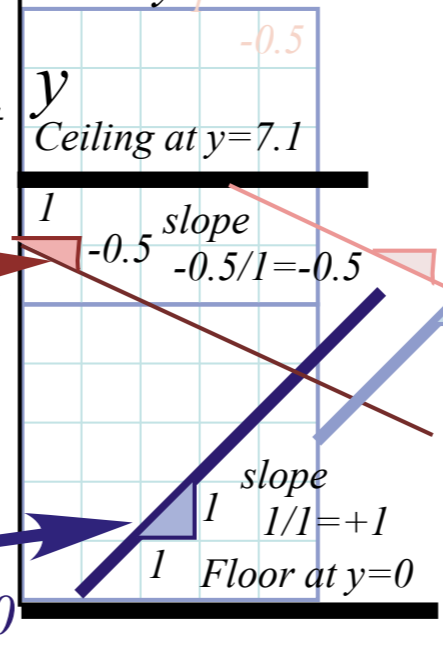
Velocity V_{y2} vs. V_{y1} Plot



$V_{y2} = -0.5$
means M_2 is
somewhere
on some path of slope -0.5

$V_{y1} = +1.0$
means M_1 is
somewhere
on some path of slope $+1.0$

Position y vs. Time t Plot



Until you specify
initial conditions $y_0(t_0)$...
...you don't know what
 v_y -line to use

Geometric "Integration" (Converting Velocity data to Spacetime)

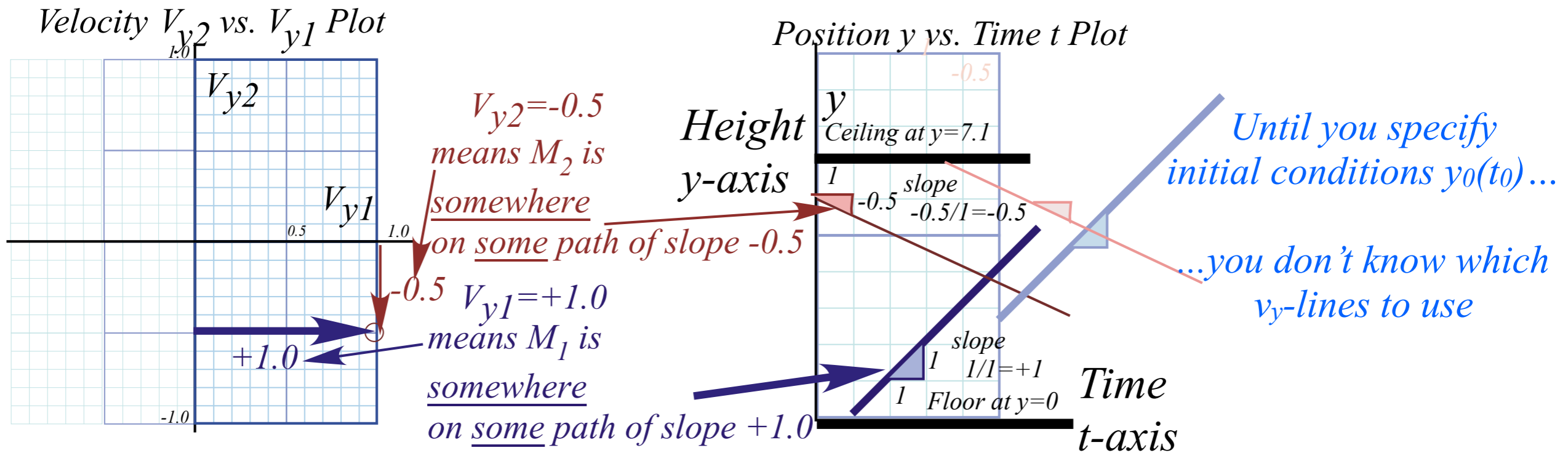
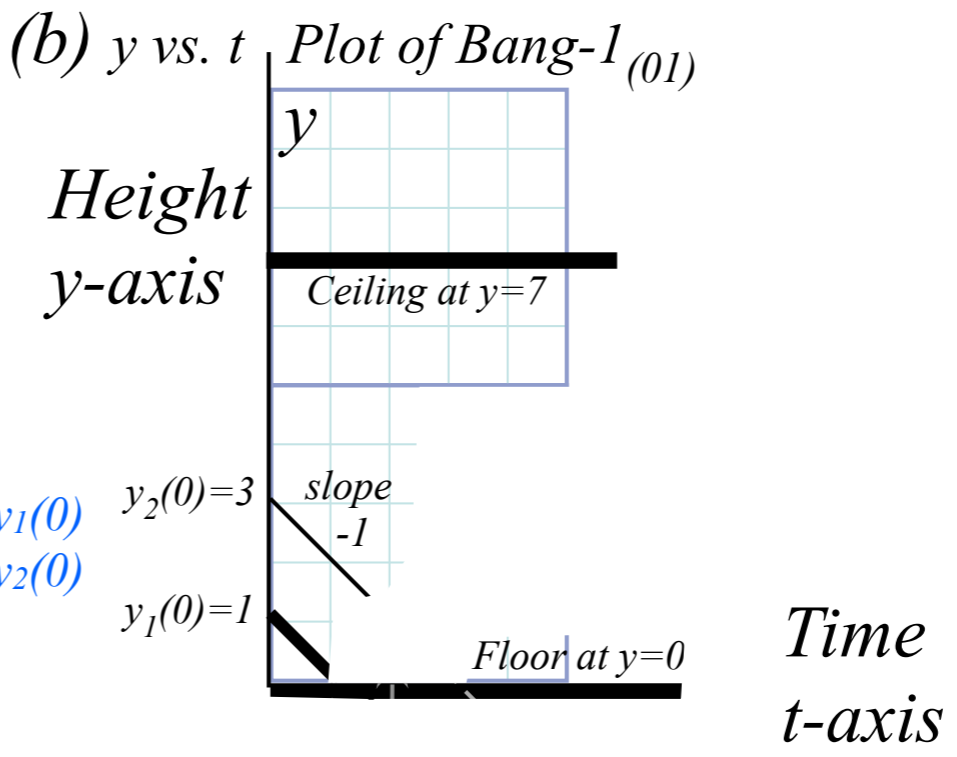
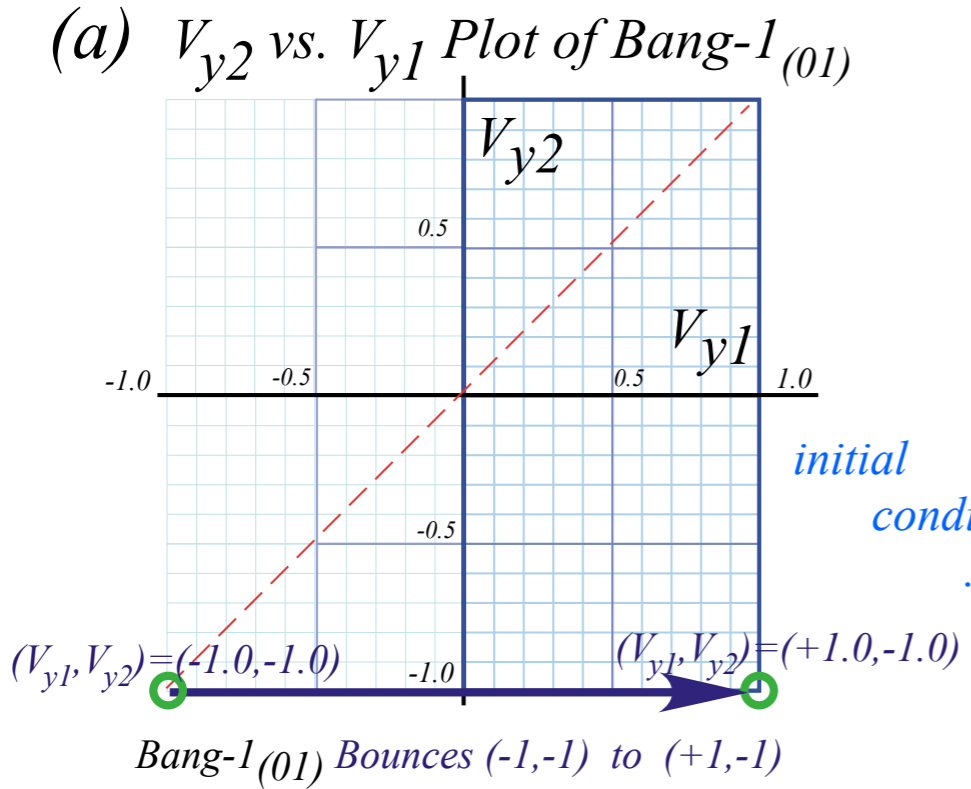


Fig. 3.6 (Unit 1)



Geometric "Integration" (Converting Velocity data to Spacetime)

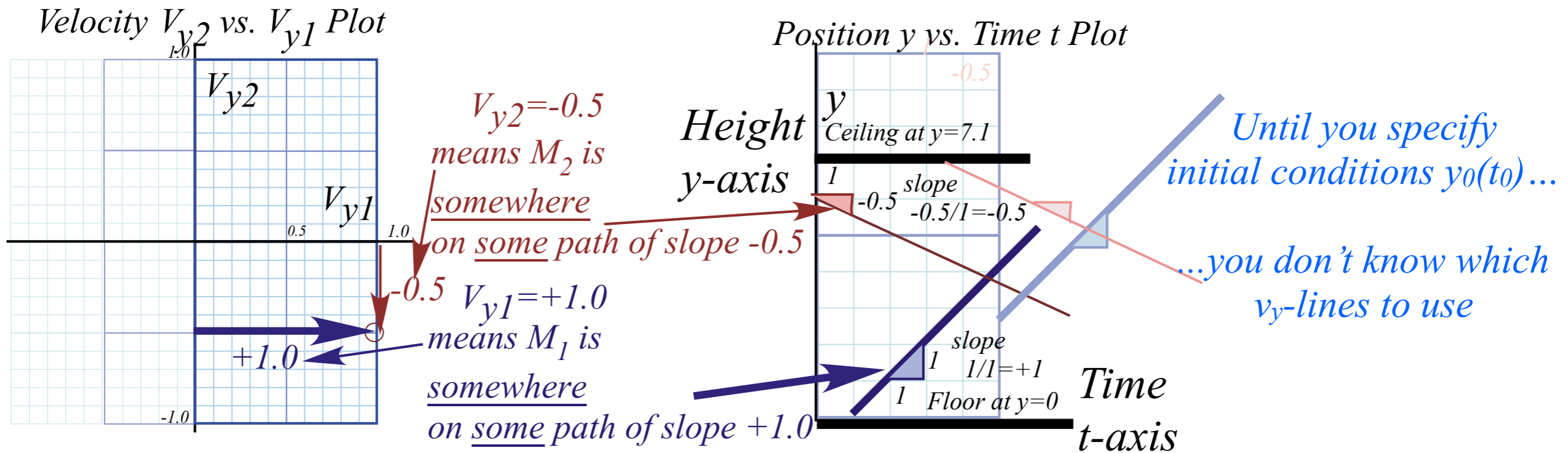
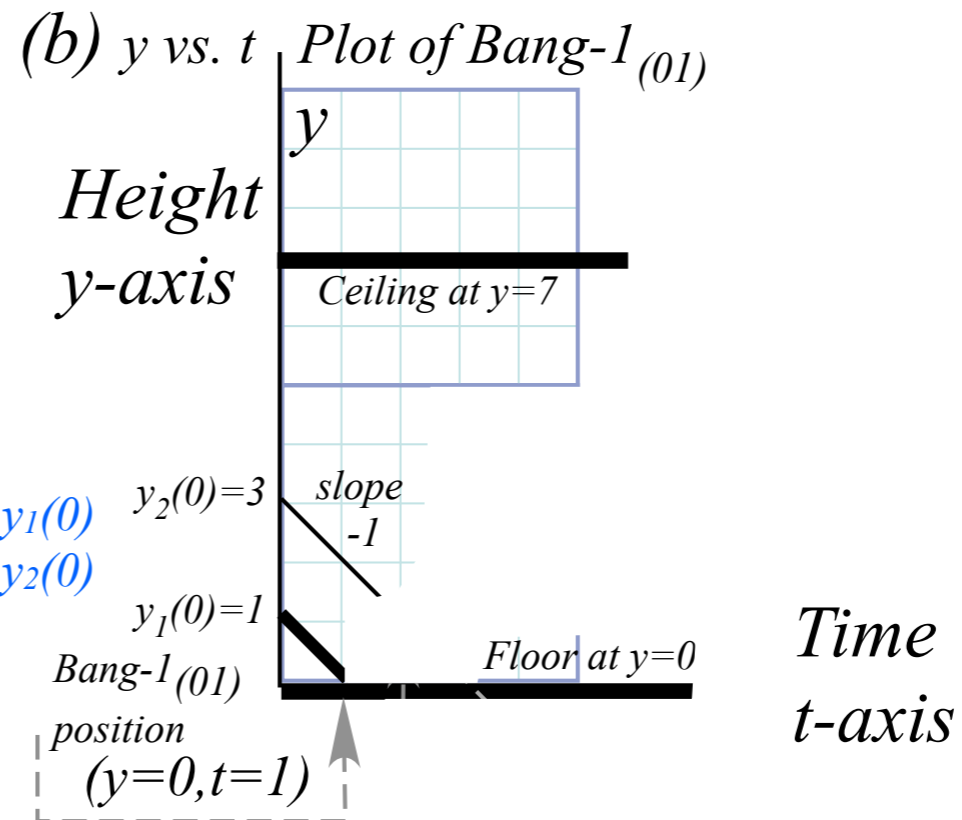
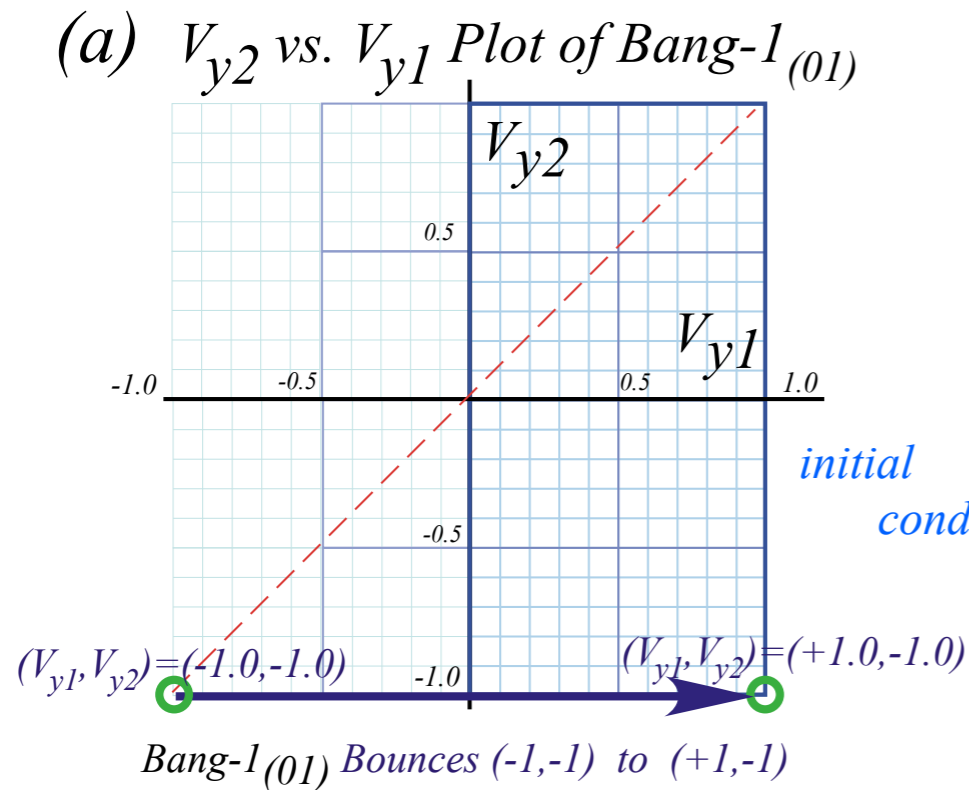


Fig. 3.6
(Unit 1)



Geometric "Integration" (Converting Velocity data to Spacetime)

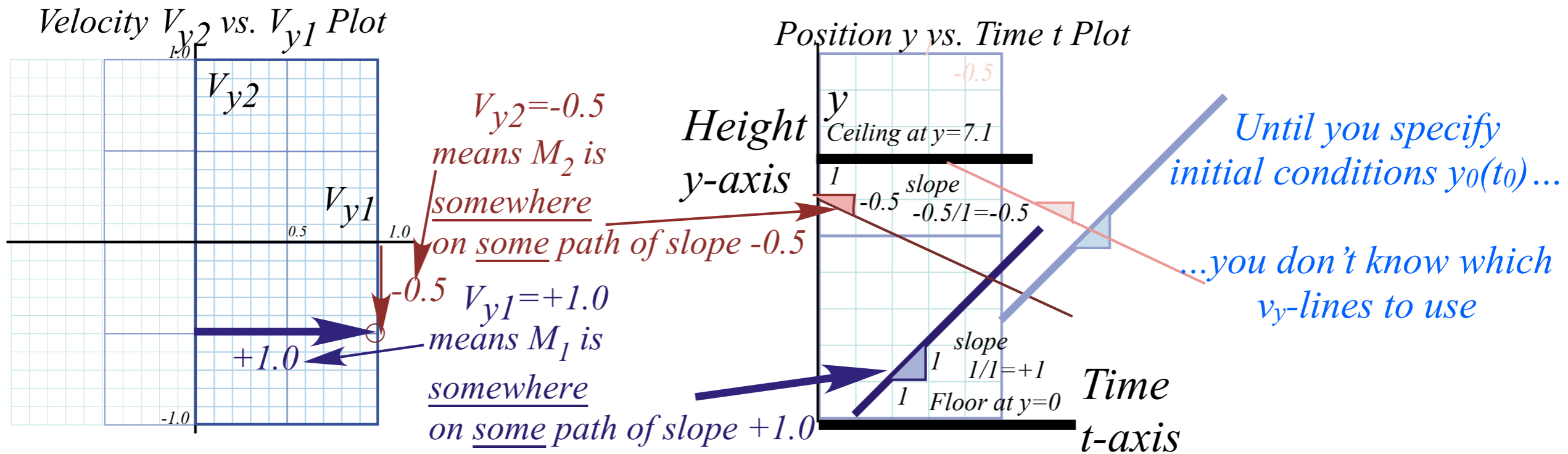
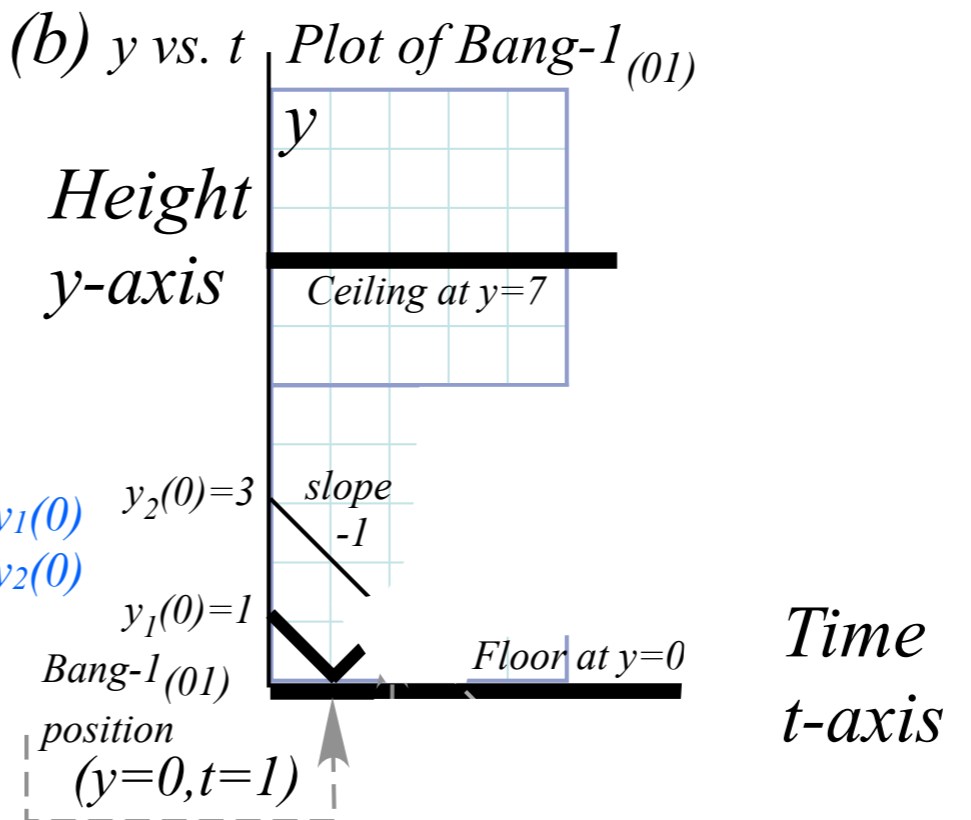
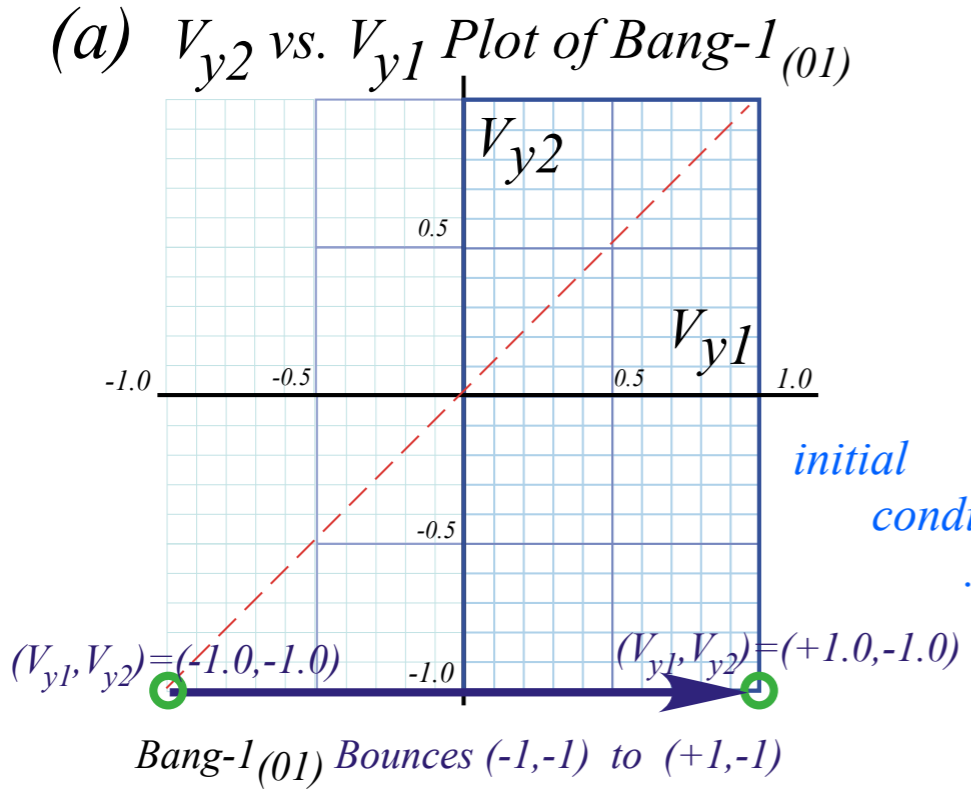


Fig. 3.6 (Unit 1)



Geometric "Integration" (Converting Velocity data to Spacetime)

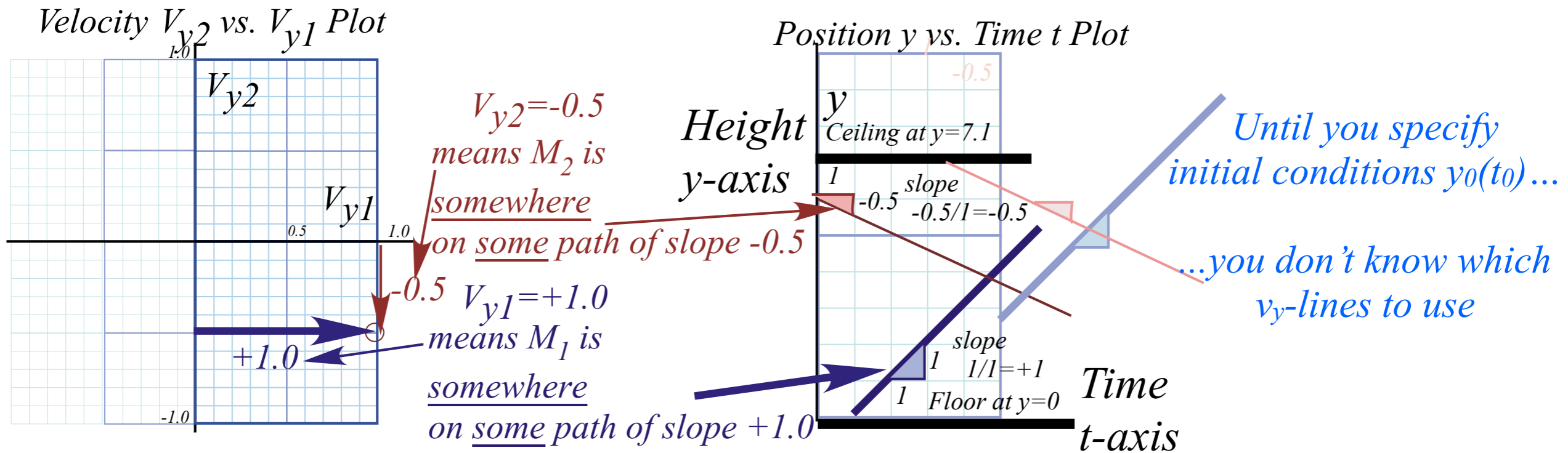
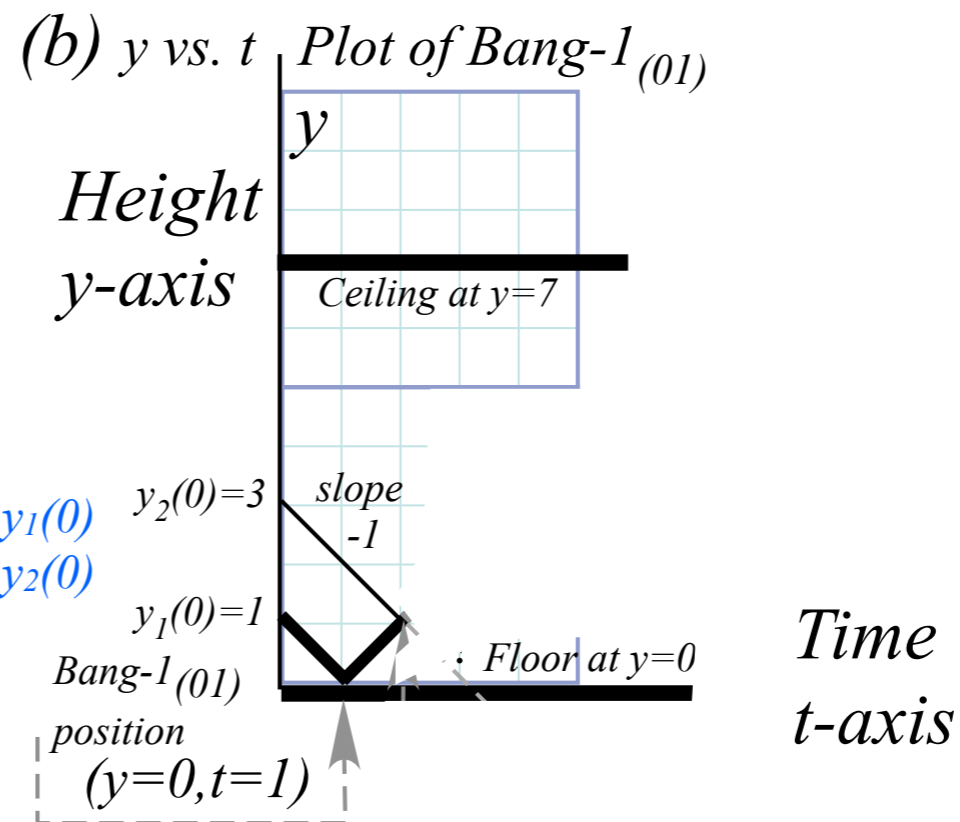
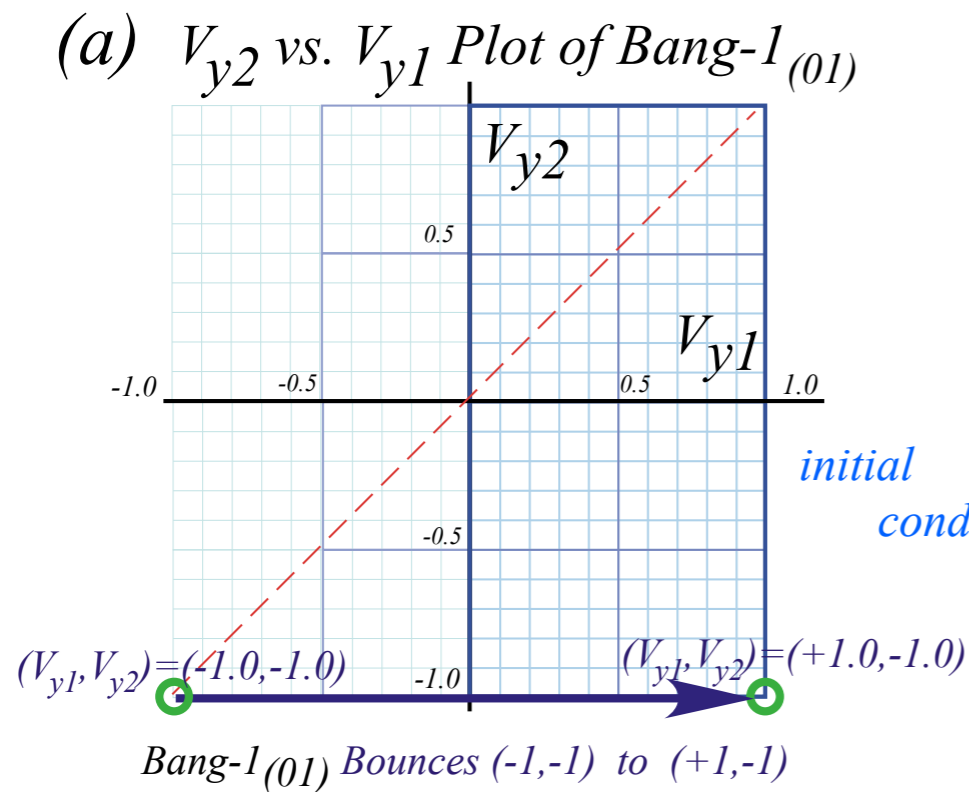


Fig. 3.6 (Unit 1)



Geometric "Integration" (Converting Velocity data to Spacetime)

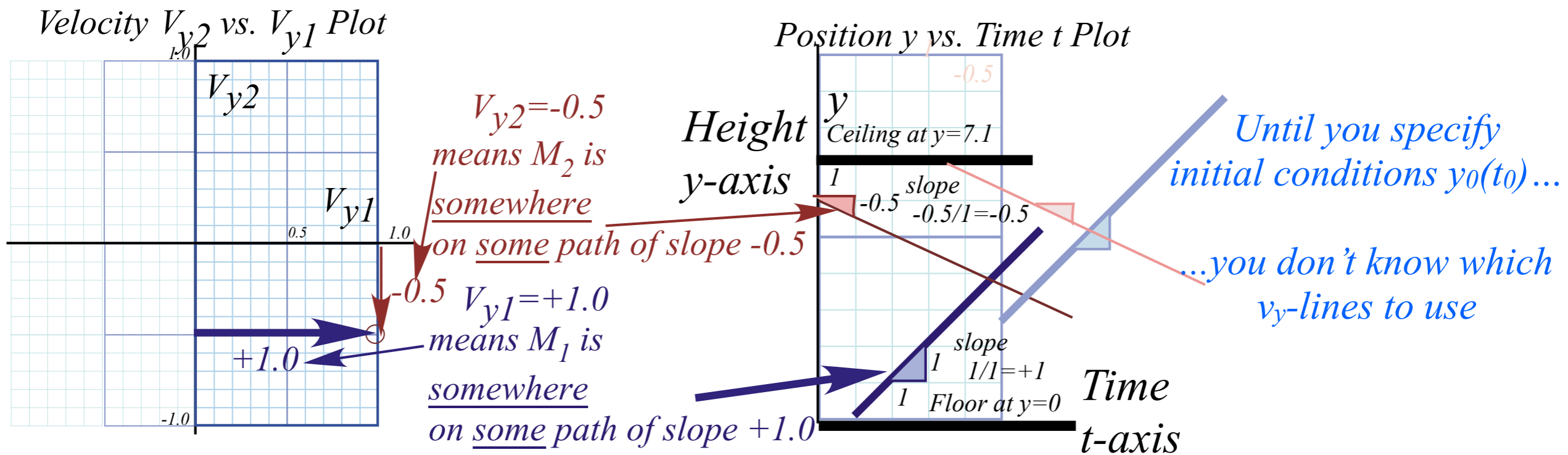
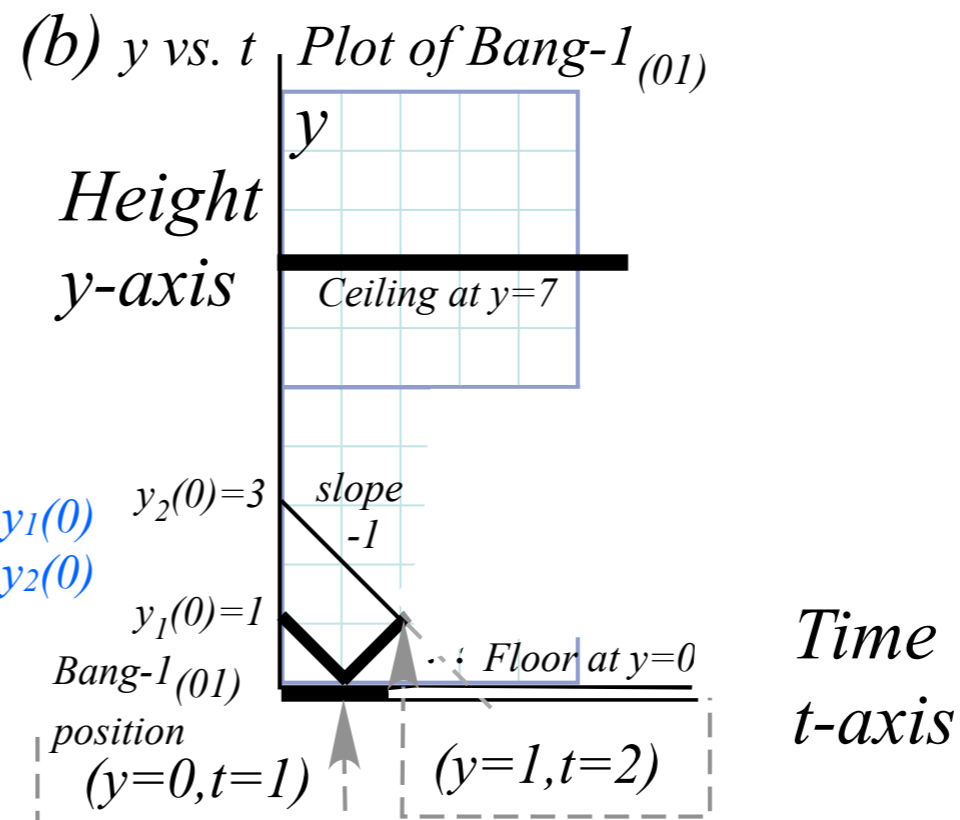
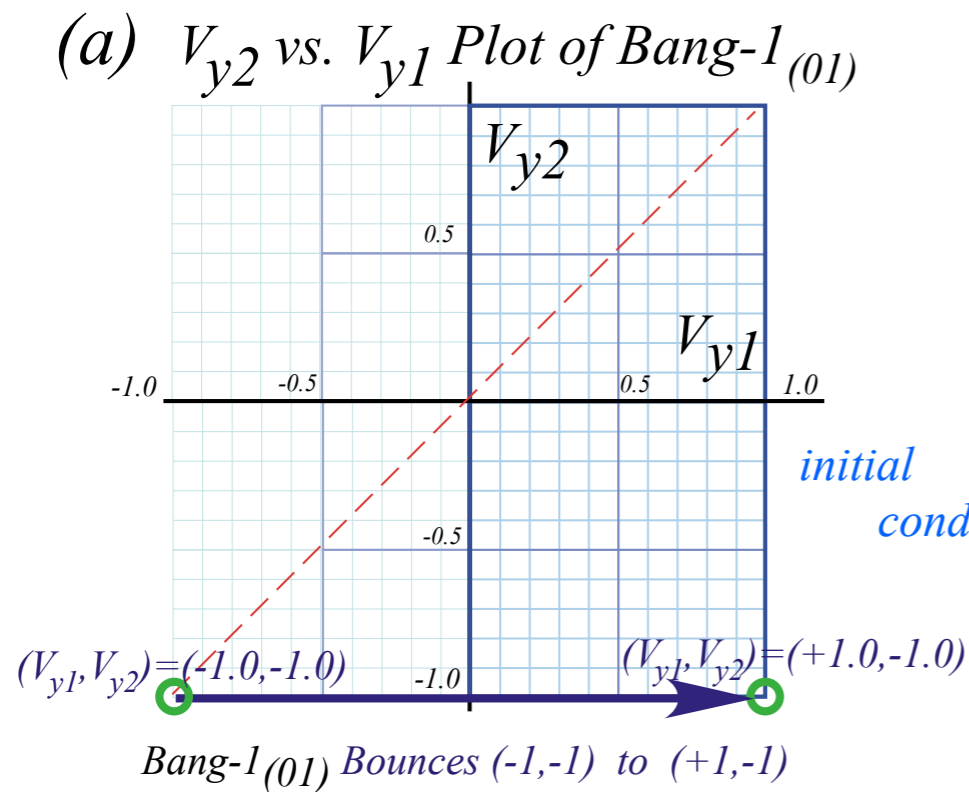


Fig. 3.6 (Unit 1)



Geometric "Integration" (Converting Velocity data to Spacetime)

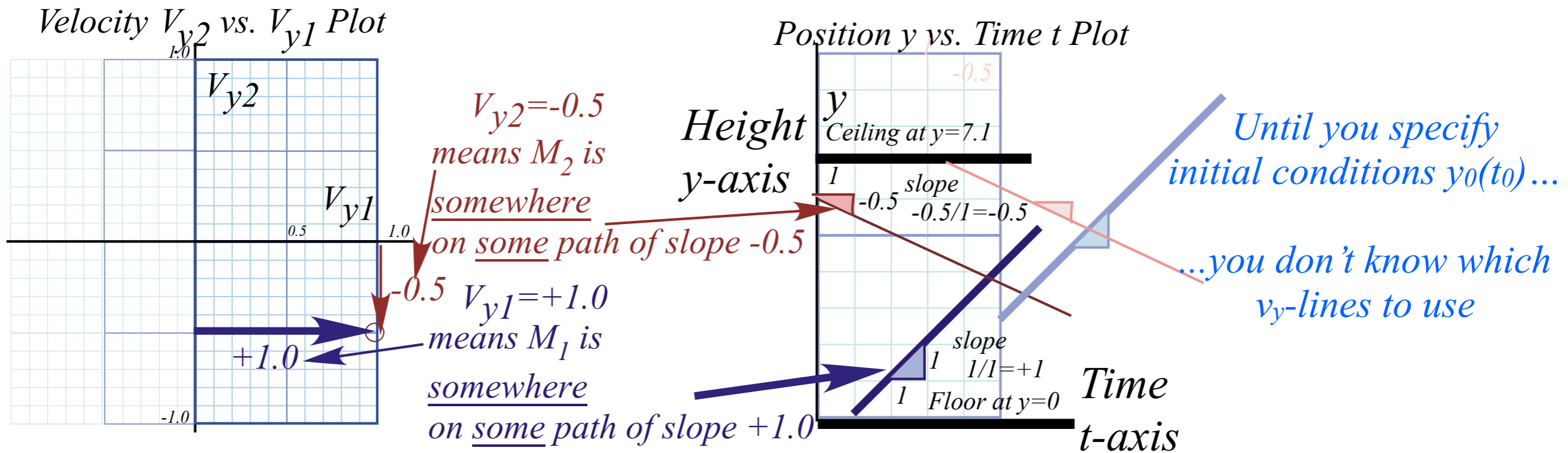
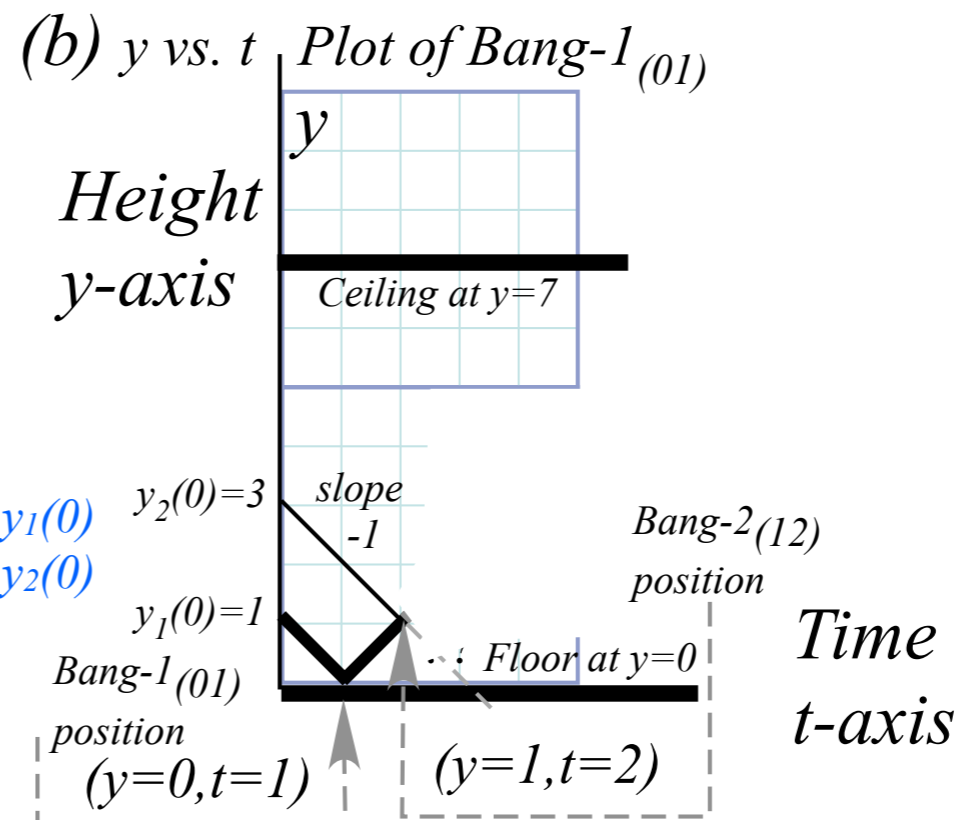
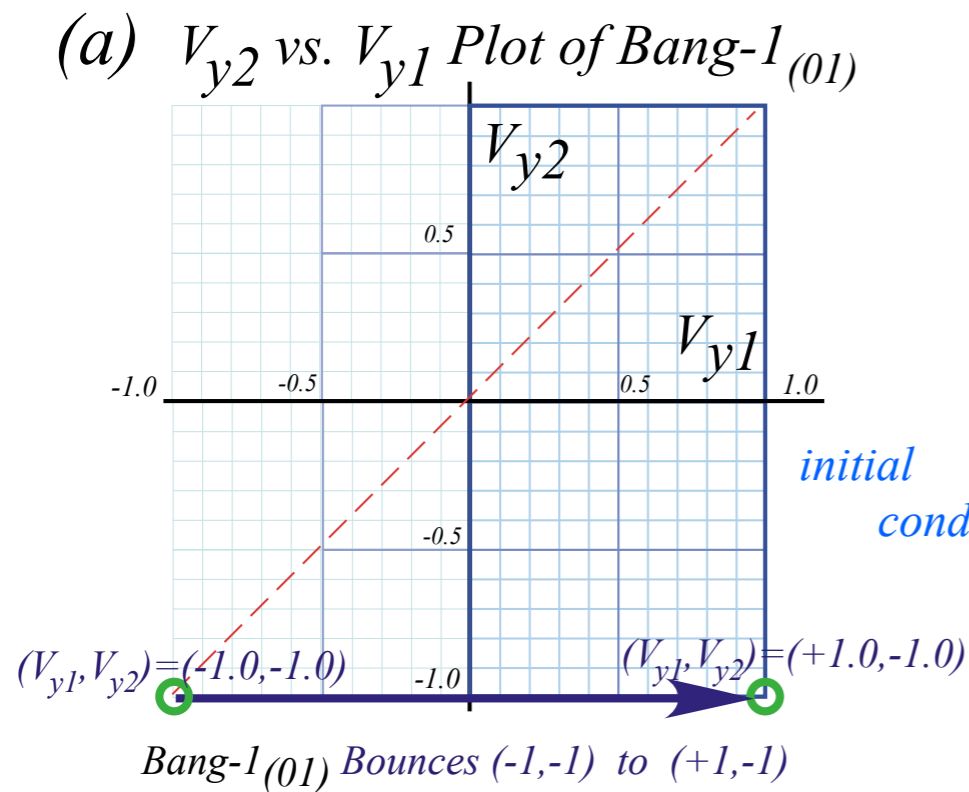
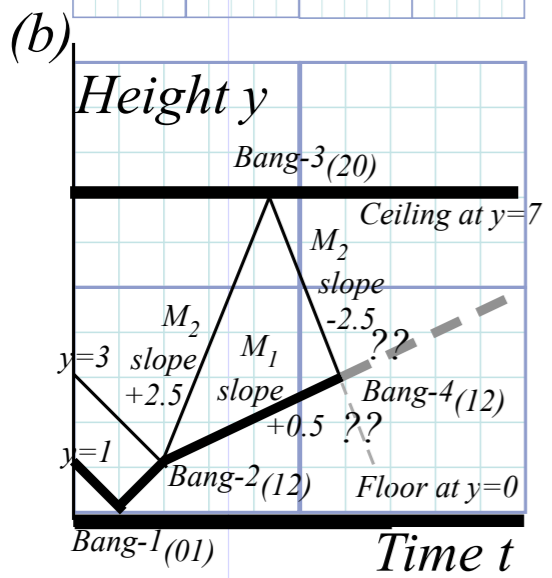
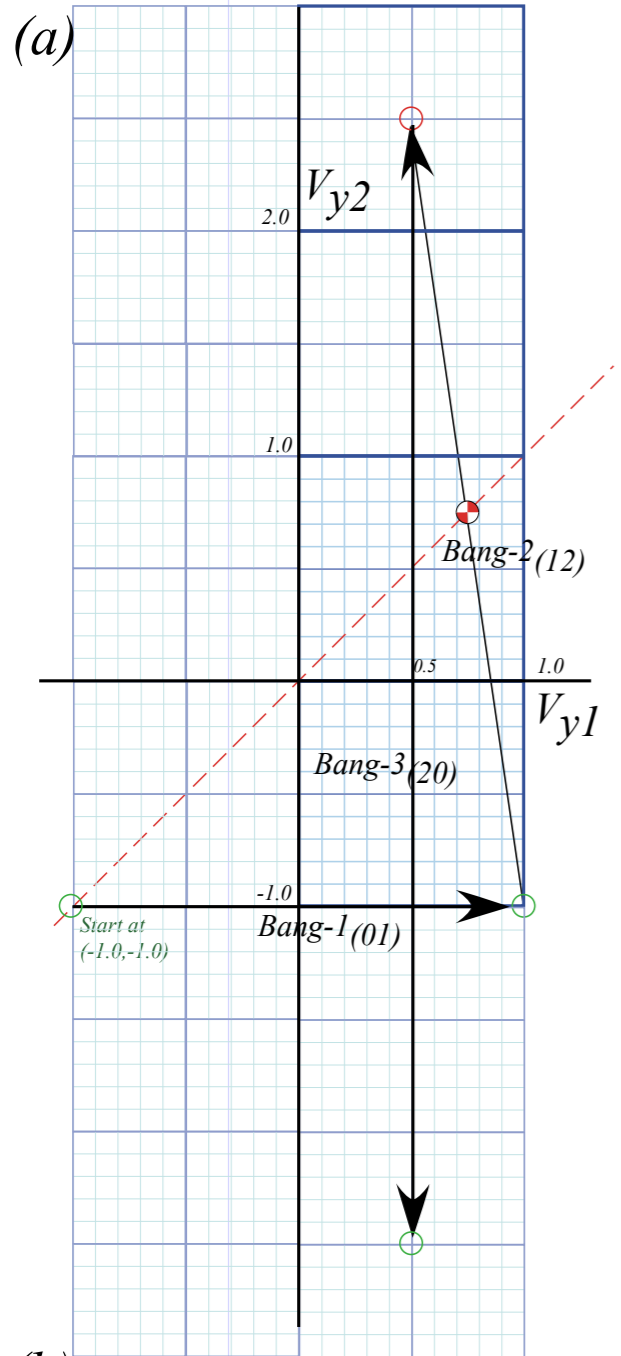


Fig. 3.6
(Unit 1)



Geometric "Integration" (Converting Velocity data to Spacetime)



Geometric "Integration" (Converting Velocity data to Spacetime)

Kinetic Energy Ellipse

$$KE = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 = \frac{1}{2} + \frac{7}{2} = 4$$

$$1 = \frac{V_1^2}{2KE / M_1} + \frac{V_2^2}{2KE / M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

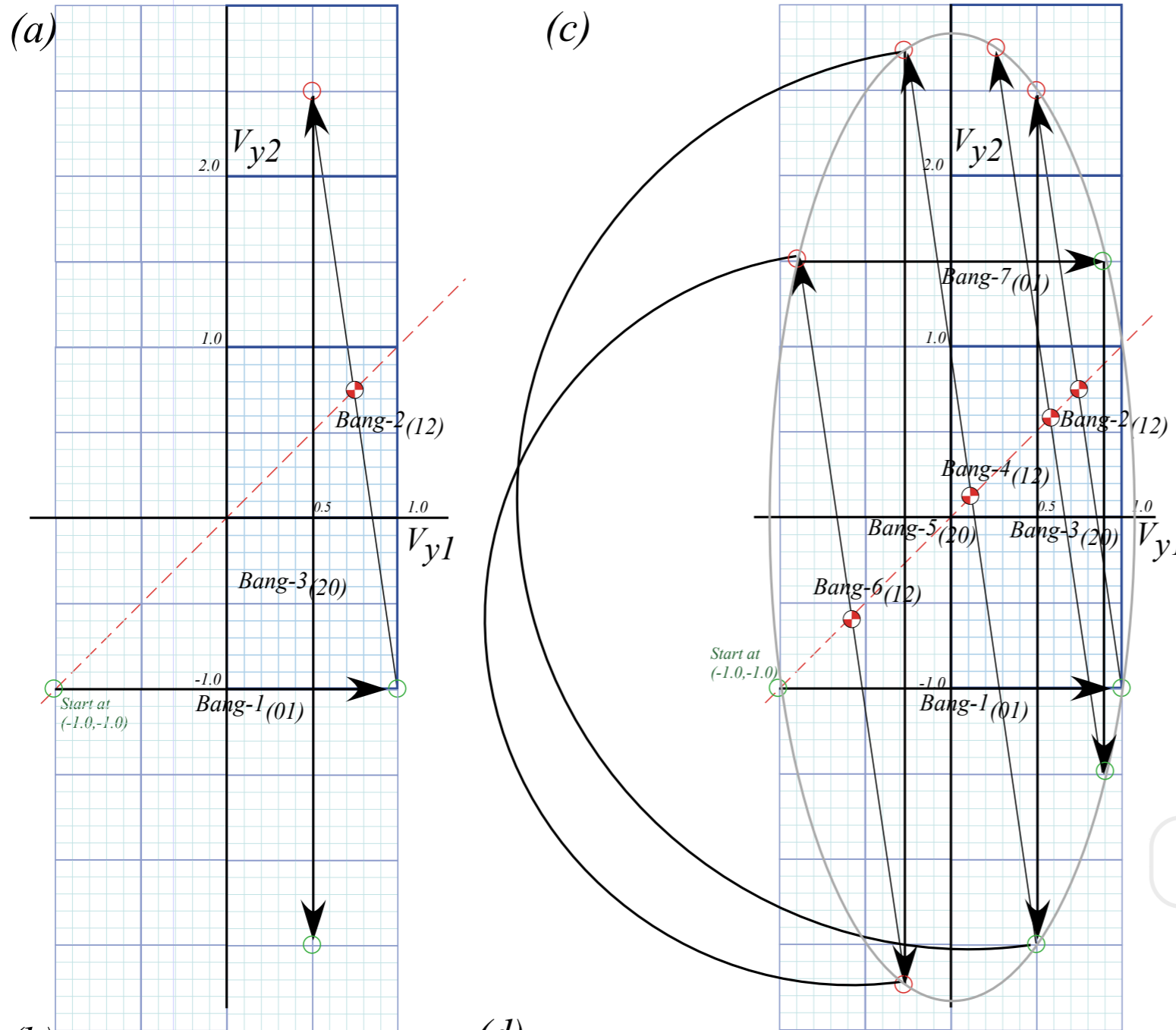


Fig. 3.7
(Unit 1)

Geometric "Integration" (Converting Velocity data to Spacetime)

Kinetic Energy Ellipse

$$KE = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE / M_1} + \frac{V_2^2}{2KE / M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

Ellipse radius 2

$$a_1 = \sqrt{2KE / M_1}$$

$$a_2 = \sqrt{2KE / M_2}$$

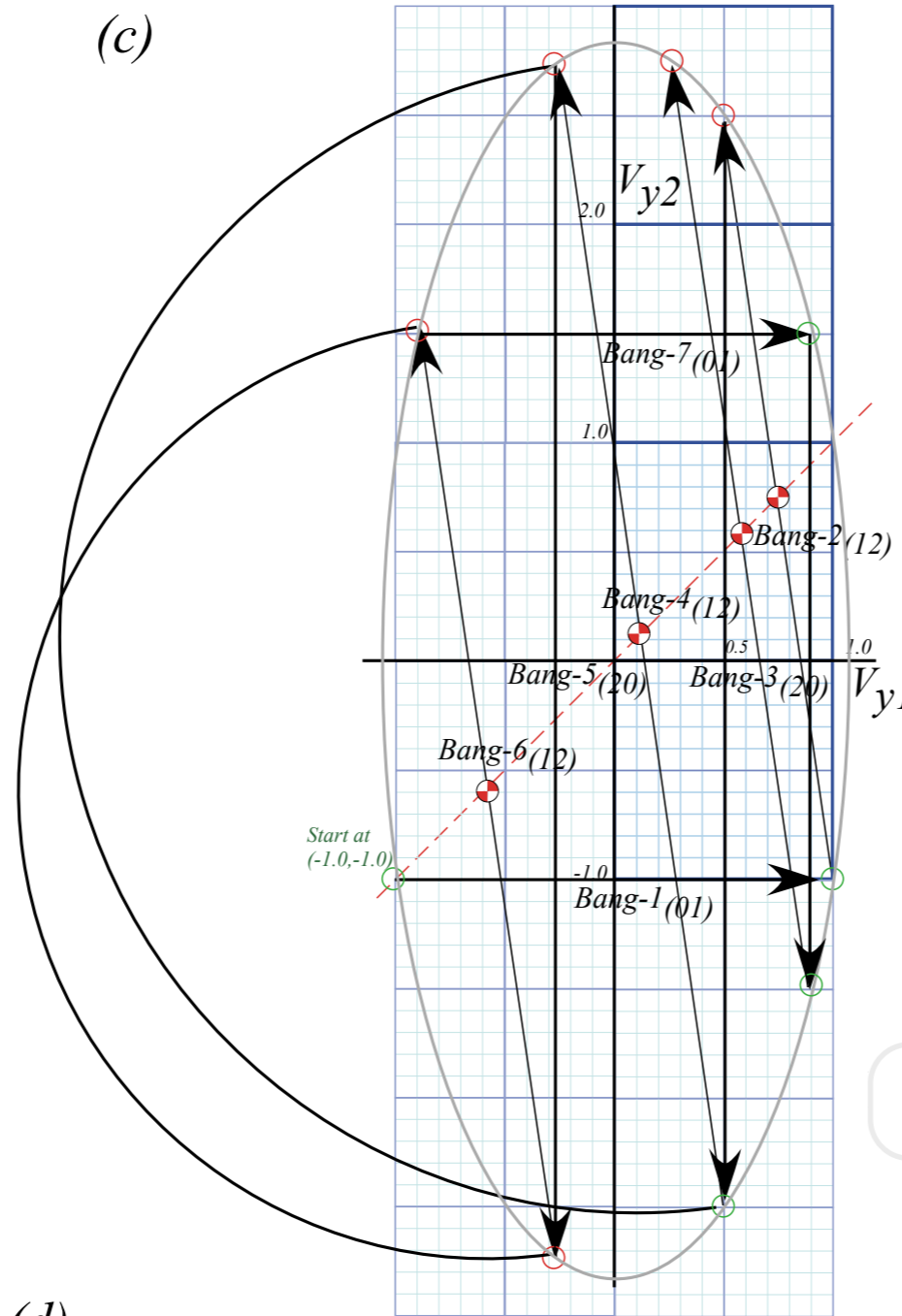
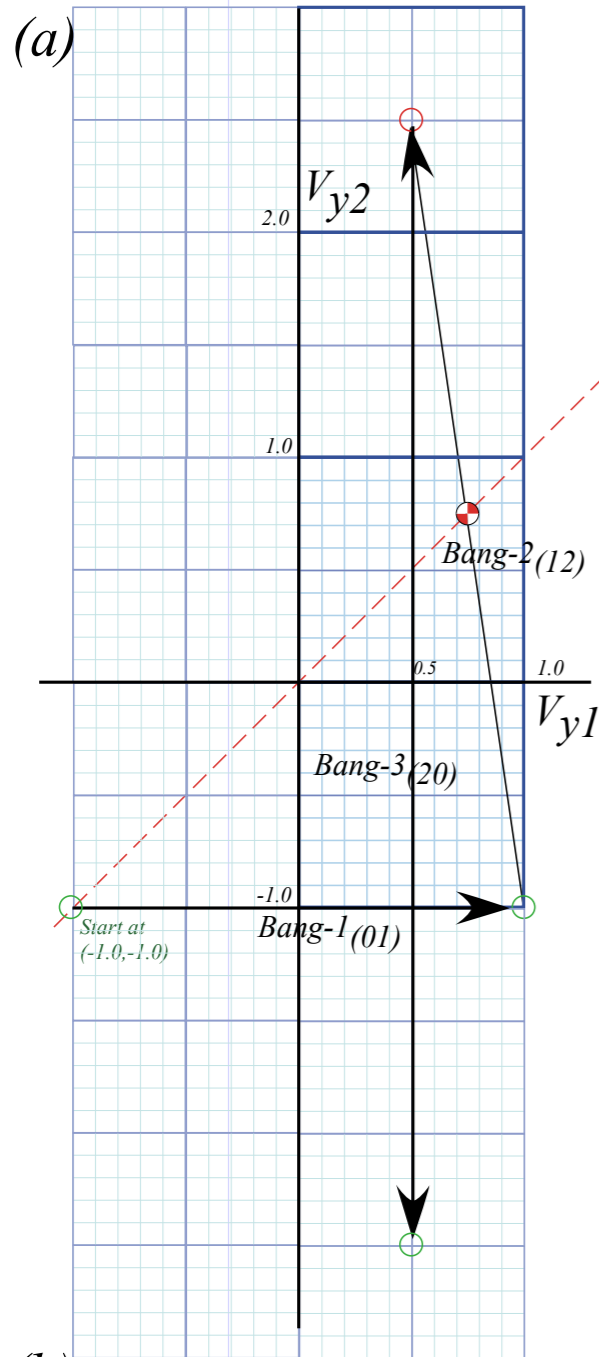
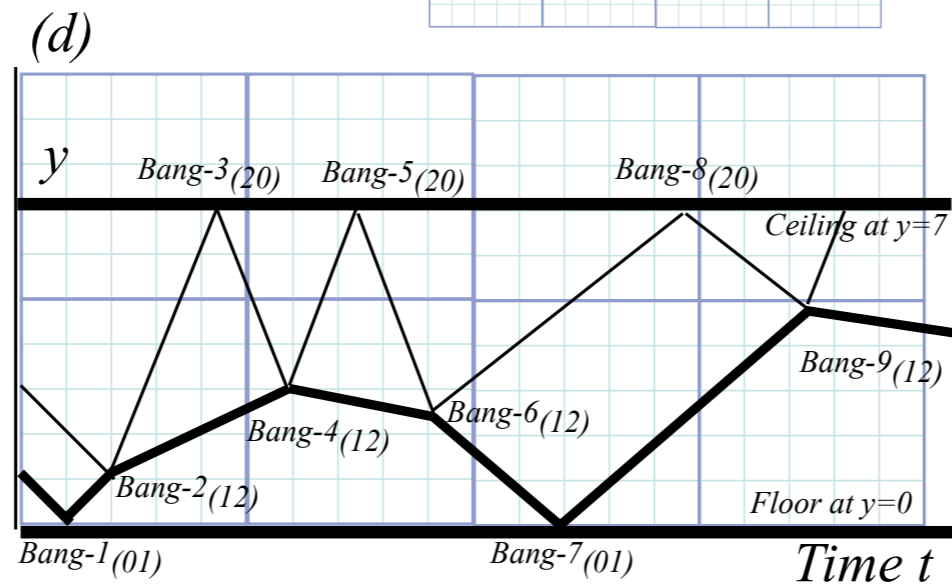
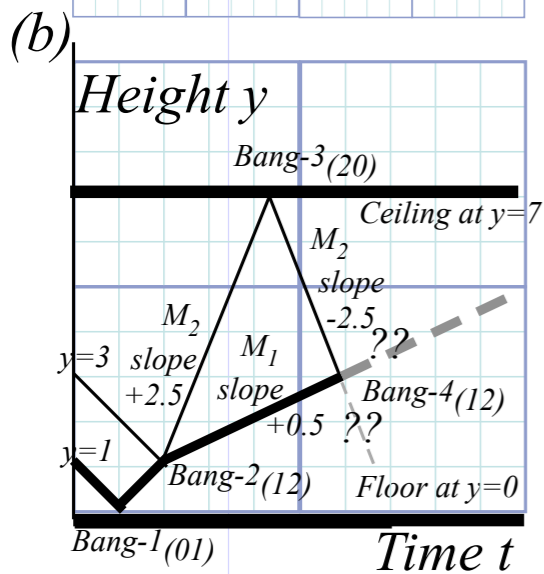


Fig. 3.7
(Unit 1)



Geometric "Integration" (Converting Velocity data to Spacetime)

Kinetic Energy Ellipse

$$KE = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2 = \frac{7}{2} + \frac{1}{2} = 4$$

$$1 = \frac{V_1^2}{2KE / M_1} + \frac{V_2^2}{2KE / M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE / M_1}$$

$$= \sqrt{2KE / 7}$$

$$= \sqrt{8/7}$$

$$= 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE / M_2}$$

$$= \sqrt{2KE / 1}$$

$$= \sqrt{8/1}$$

$$= 2.83$$

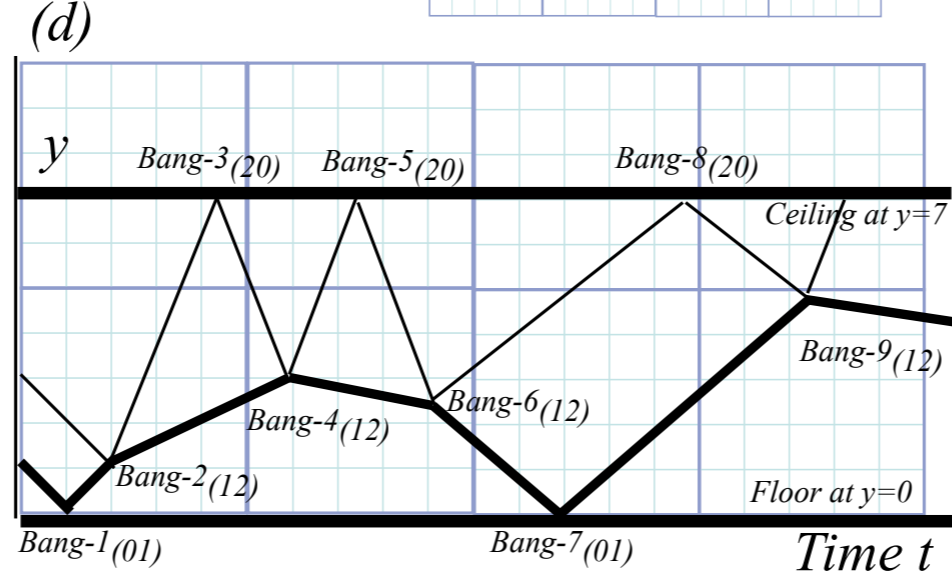
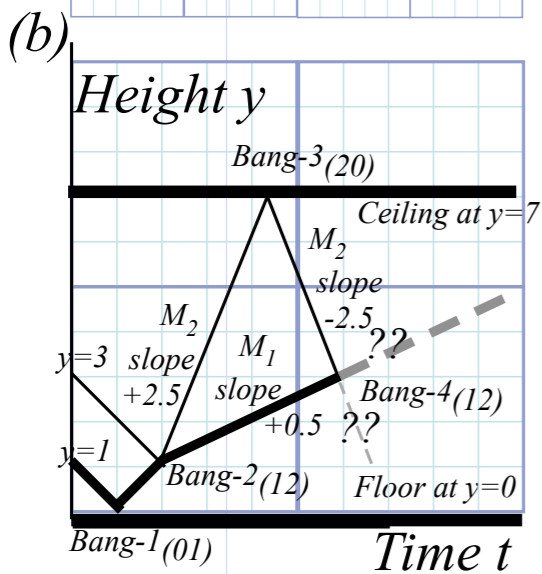
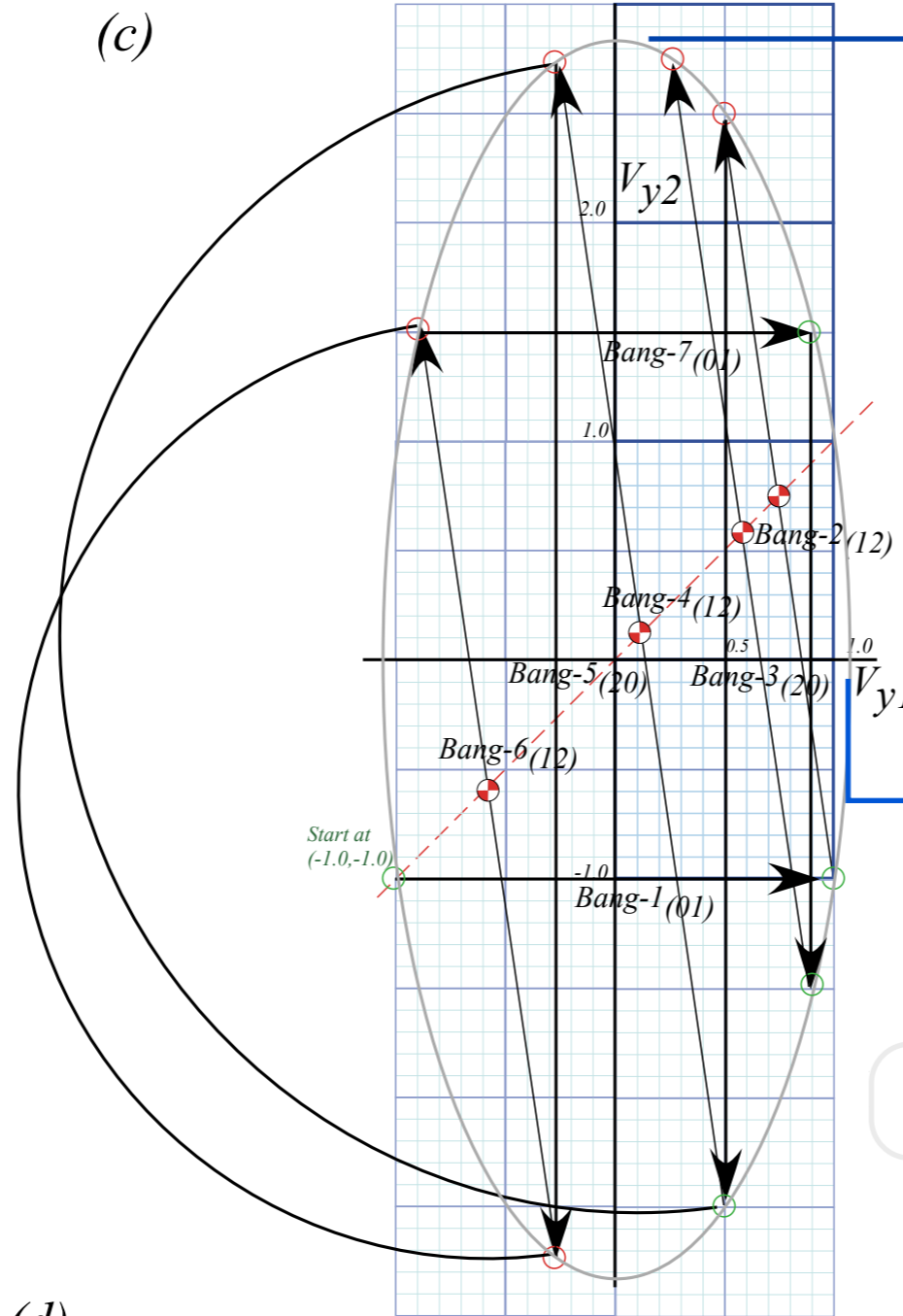
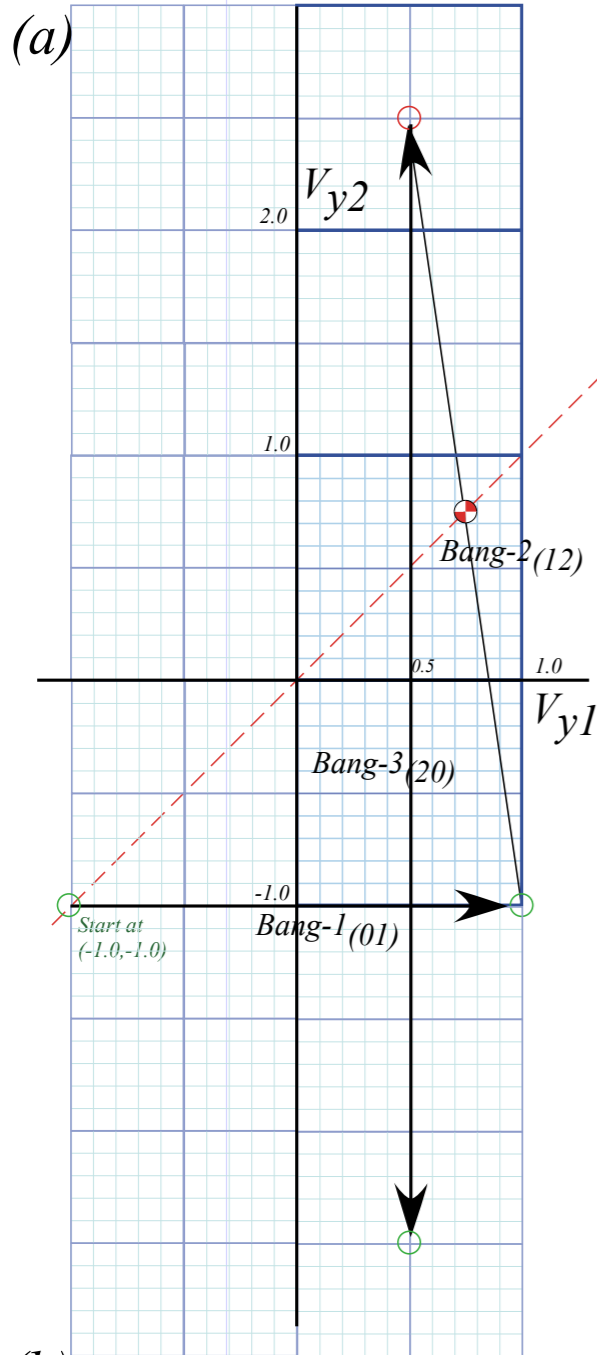


Fig. 3.7
(Unit 1)

Geometric "Integration" (Converting Velocity data to Spacetime)

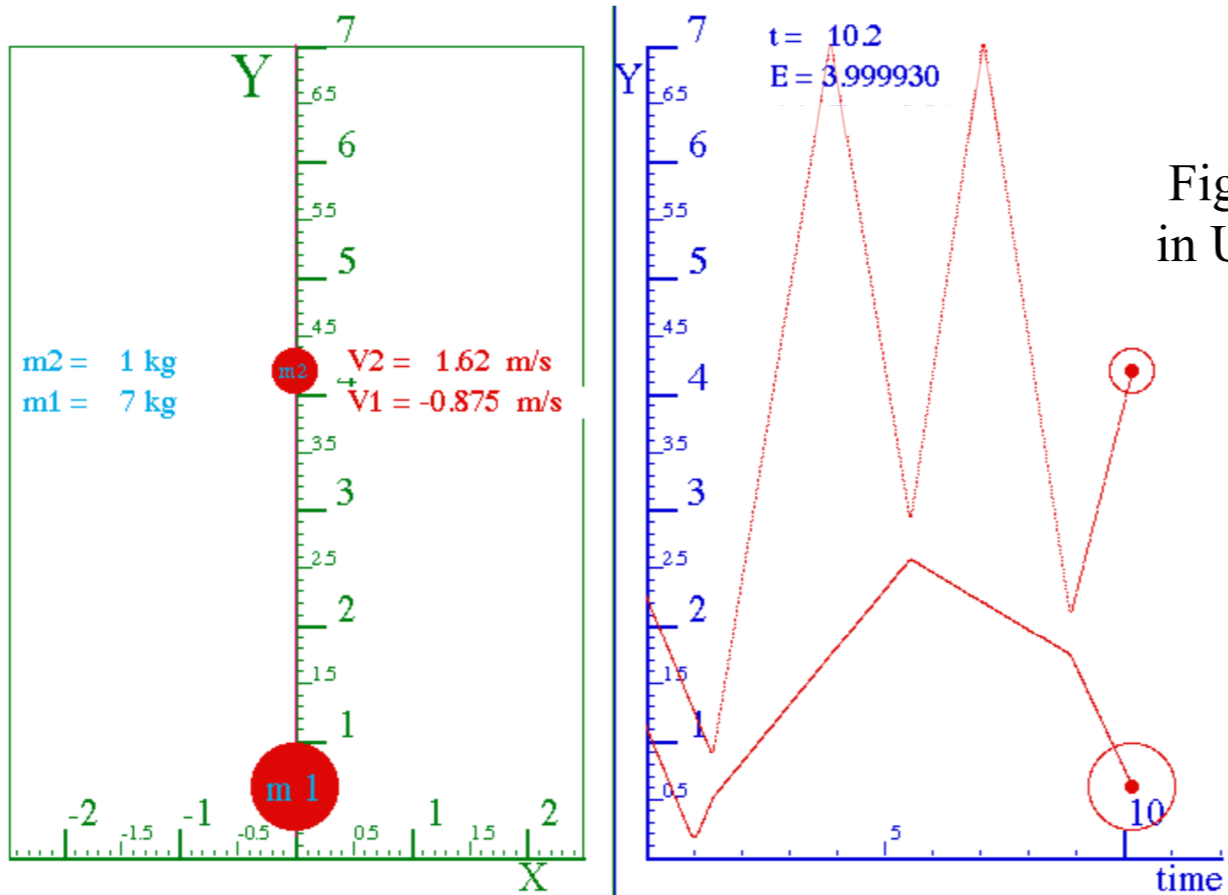


Fig. 4.8
in Unit 1

*BounceIt Superball Collision Web Simulator:
 $M_1=70, M_2=10$ with Newtonian time plot*

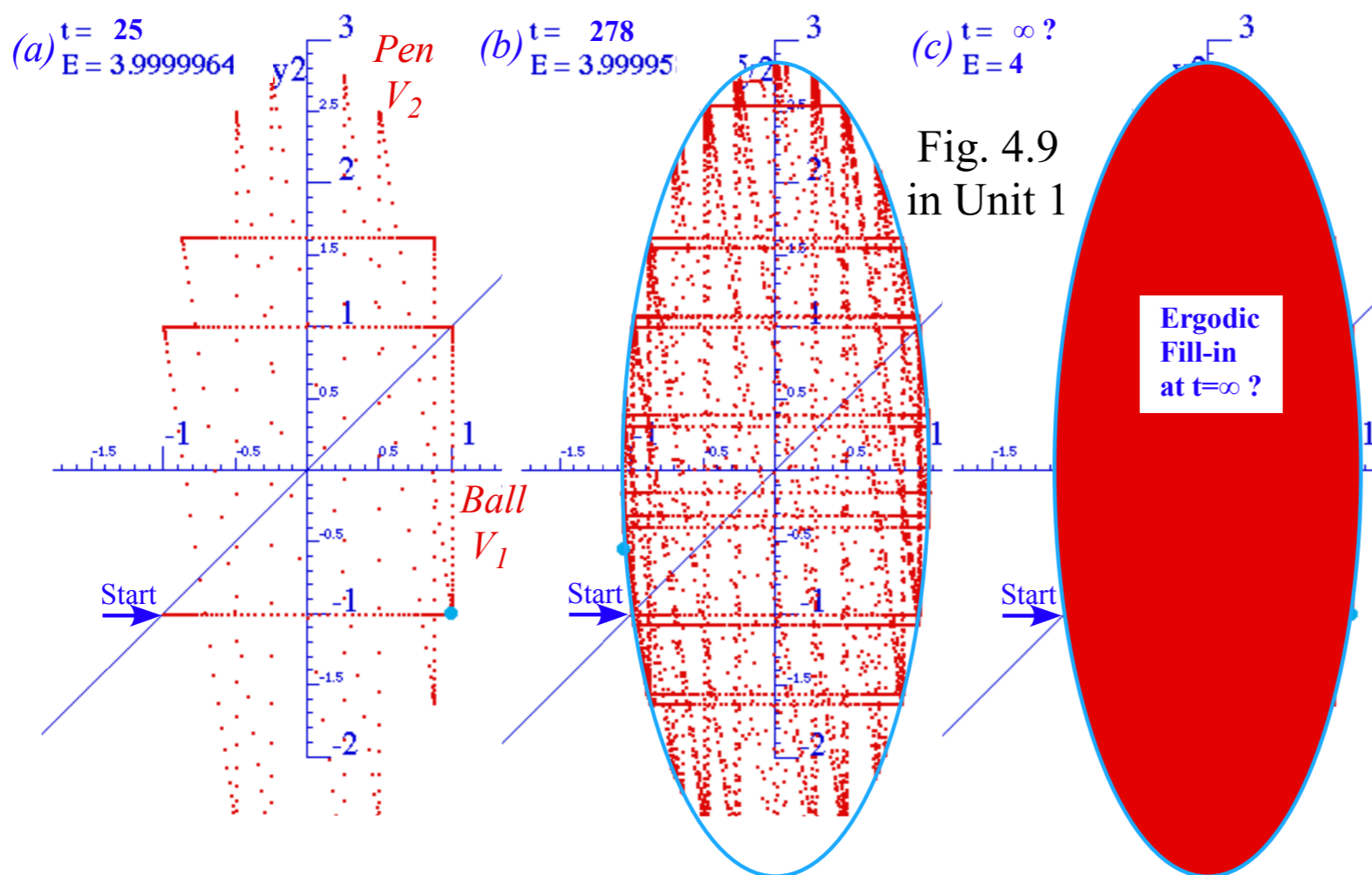


Fig. 4.9
in Unit 1

*BounceIt Superball Collision Web Simulator:
 $M_1=70, M_2=10$ with V_2 vs V_1 plot*

Geometry of X2 launcher bouncing in box

Independent Bounce Model (IBM)

Geometric optimization and range-of-motion calculation(t)

Integration of (V_1, V_2) data to space-time plots $(y_1(t), t)$ and $(y_2(t), t)$ plots

 *Integration of (V_1, V_2) data to space-space plots (y_1, y_2) Examples $(M_1=7, M_2=1)$ and $(M_1=49, M_2=1)$*

Geometric "Integration" (Converting Velocity data to Space-space trajectory)

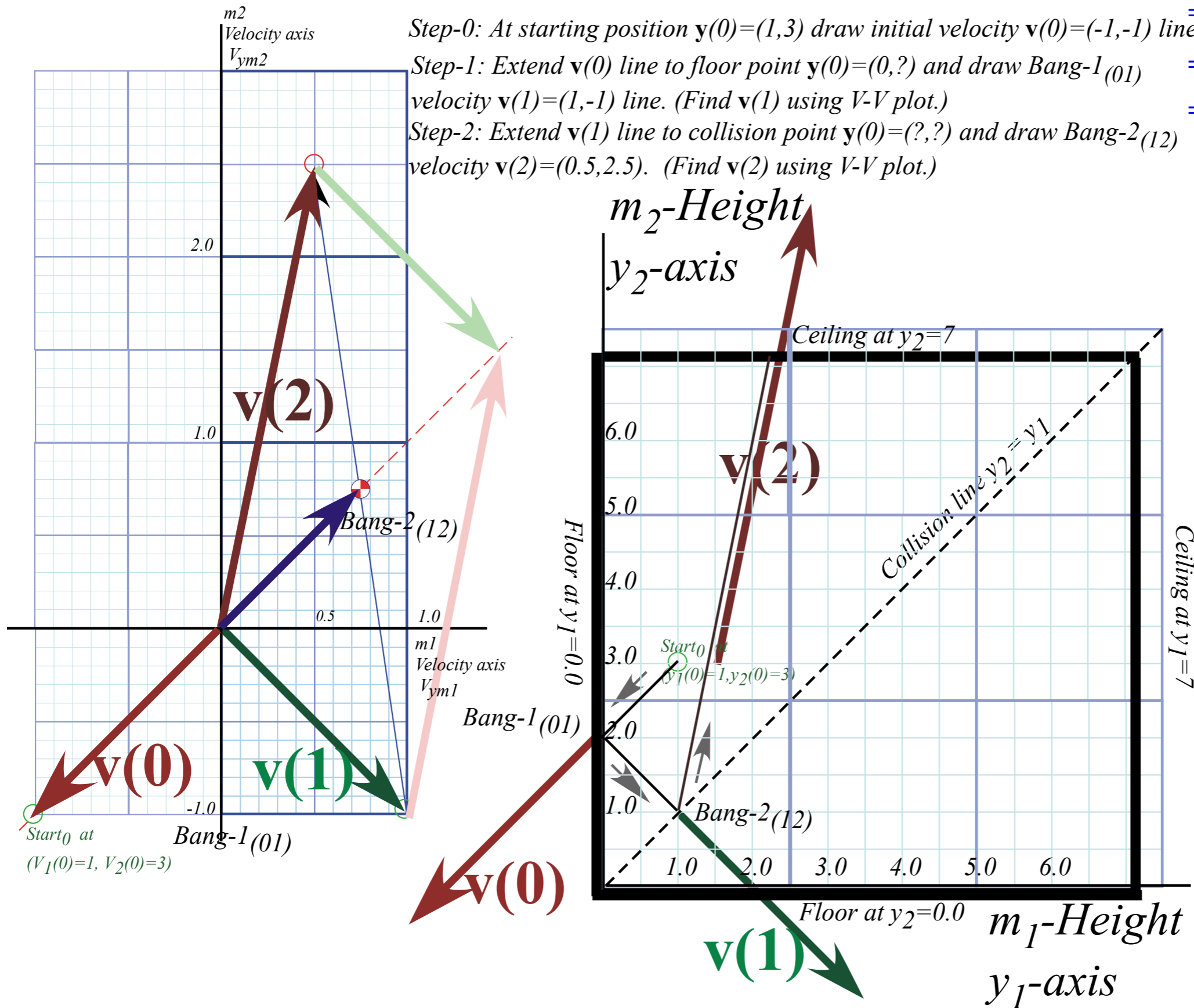
Fig. 3.8
(Unit 1)

<u>Ellipse radius 1</u>	<u>Ellipse radius 2</u>
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/M_1}$
$= \sqrt{2KE/7}$	$= \sqrt{2KE/1}$
$= \sqrt{8/7}$	$= \sqrt{8/1}$
$= 1.07$	$= 2.83$

Step-0: At starting position $\mathbf{y}(0)=(1,3)$ draw initial velocity $\mathbf{v}(0)=(-1,-1)$ line.

Step-1: Extend $\mathbf{v}(0)$ line to floor point $\mathbf{y}(0)=(0,?)$ and draw Bang-1(01) velocity $\mathbf{v}(1)=(1,-1)$ line. (Find $\mathbf{v}(1)$ using V-V plot.)

Step-2: Extend $\mathbf{v}(1)$ line to collision point $\mathbf{y}(0)=(?,?)$ and draw Bang-2(12) velocity $\mathbf{v}(2)=(0.5,2.5)$. (Find $\mathbf{v}(2)$ using V-V plot.)

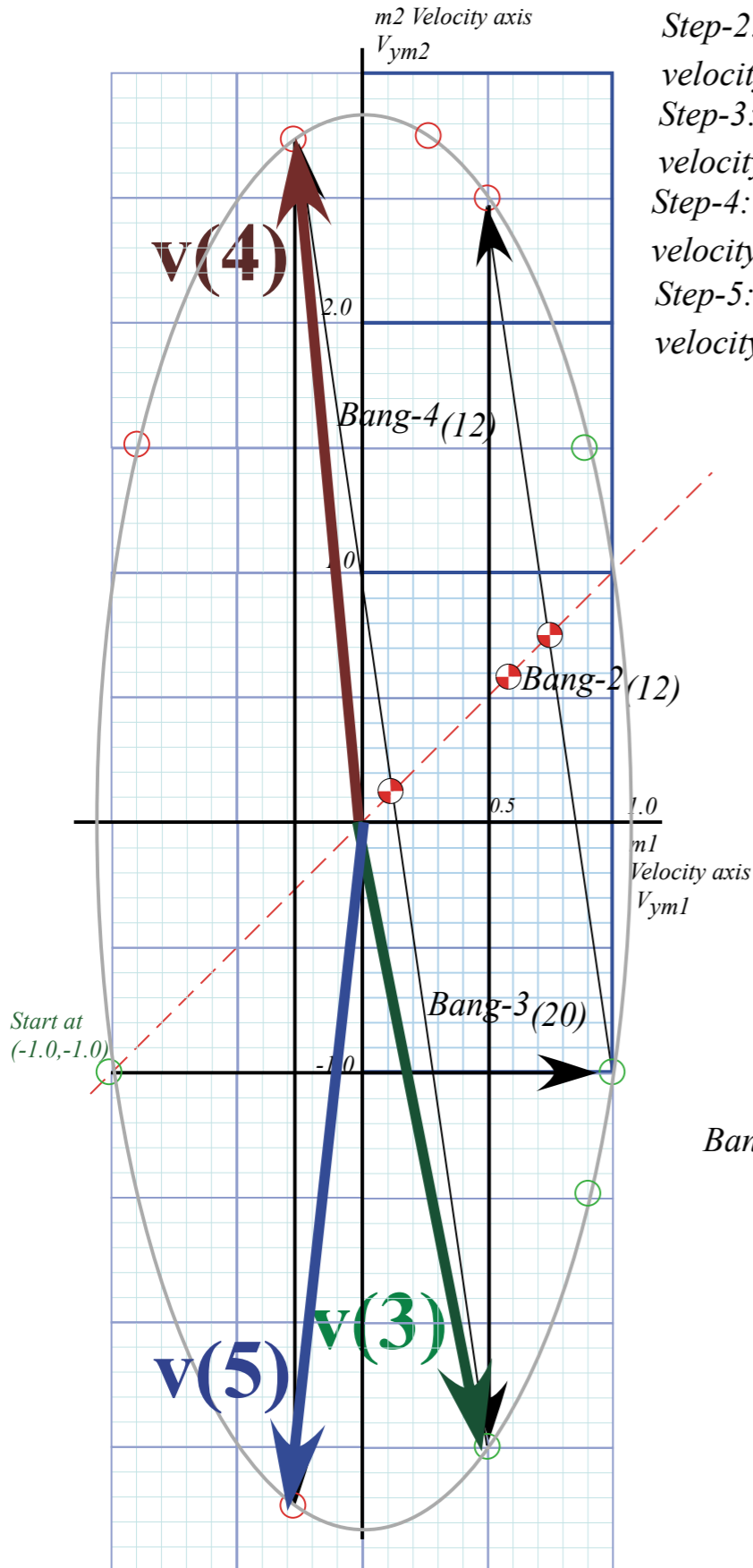


Geometric "Integration" (Converting Velocity data to Space-space trajectory)

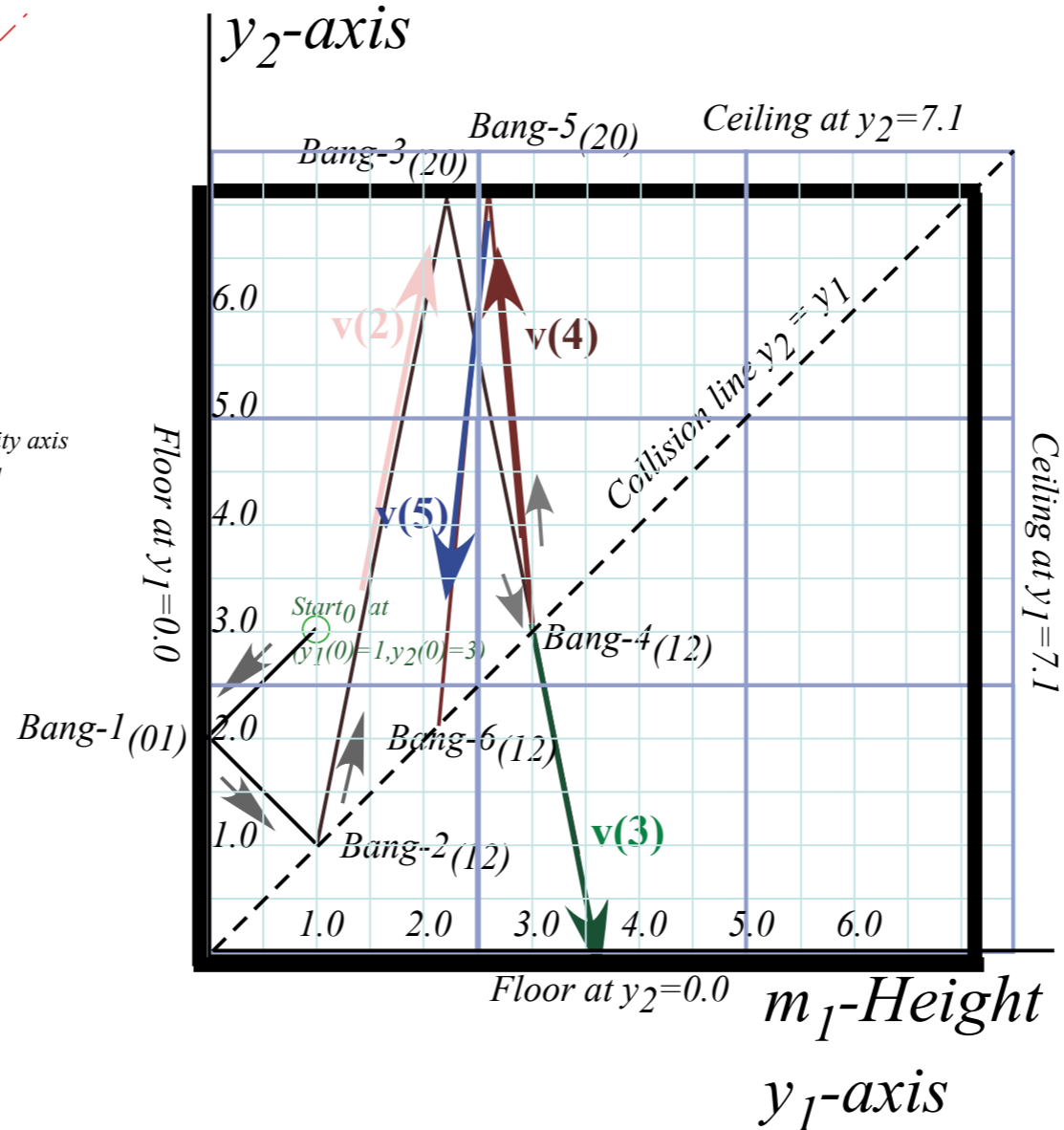
Fig. 3.9
(Unit 1)

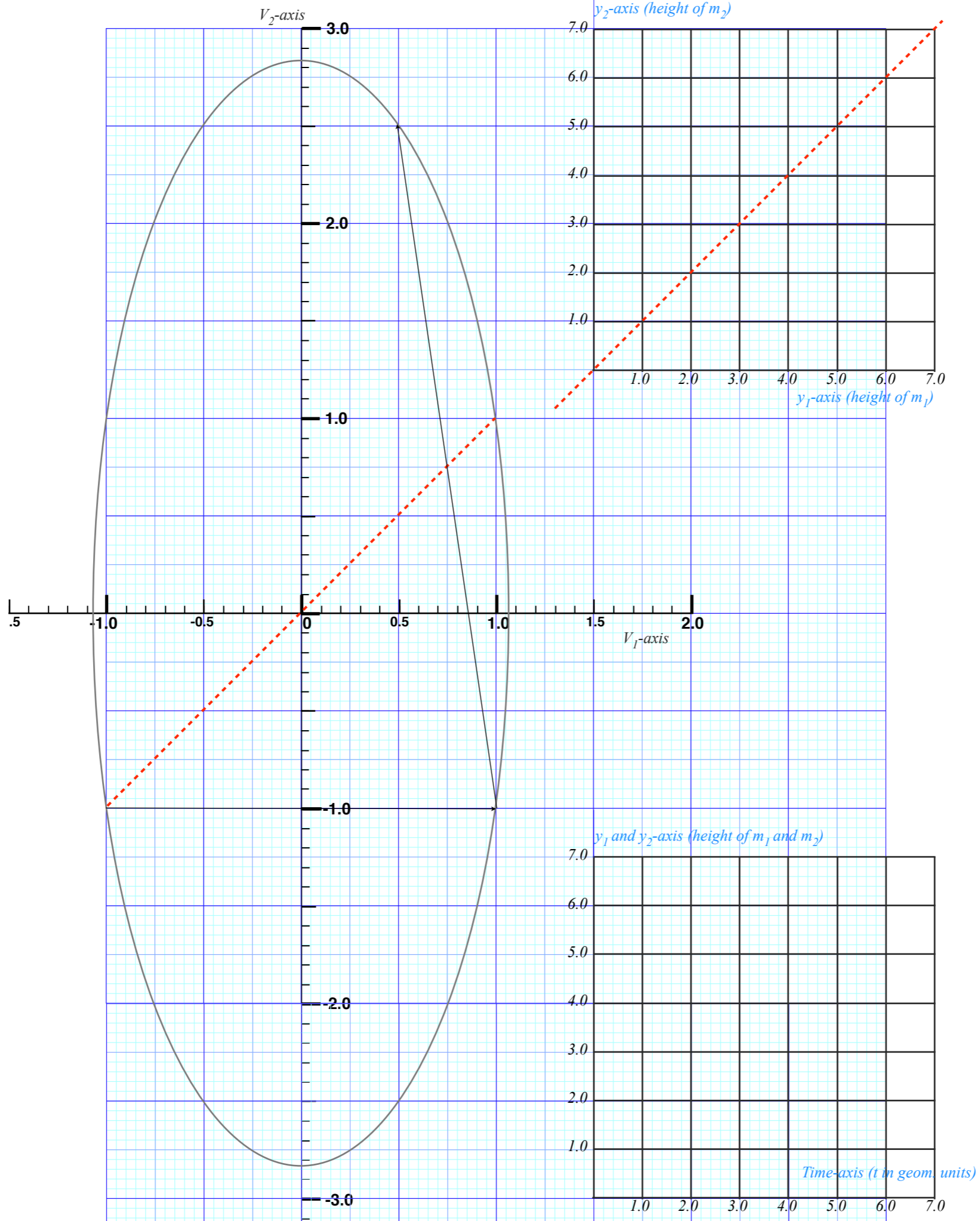
<u>Ellipse radius 1</u>	<u>Ellipse radius 2</u>
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/M_1}$
$= \sqrt{2KE/7}$	$= \sqrt{2KE/1}$
$= \sqrt{8/7}$	$= \sqrt{8/1}$
$= 1.07$	$= 2.83$

Step-2: Extend $v(2)$ line to ceiling point $y(3)=(?, 7.1)$ and draw Bang-3(20) velocity $v(3)=(1, -1)$ line. (Find $v(3)$ using V-V plot.)
 Step-3: Extend $v(3)$ line to collision point $y(4)=(?, ?)$ and draw Bang-4(12) velocity $v(4)=(0.5, 2.5)$. (Find $v(4)$ using V-V plot.)
 Step-4: Extend $v(4)$ line to ceiling point $y(4)=(?, 7.1)$ and draw Bang-5(20) velocity $v(5)=(1, -1)$ line. (Find $v(5)$ using V-V plot.)
 Step-5: Extend $v(5)$ line to collision point $y(6)=(?, ?)$ and draw Bang-6(12) velocity $v(6)=(0.5, 2.5)$. (Find $v(6)$ using V-V plot.)



m_2 -Height





<u>Ellipse radius 1</u>	<u>Ellipse radius 2</u>
$a_1 = \sqrt{2KE/M_1}$	$a_2 = \sqrt{2KE/M_1}$
$= \sqrt{2KE/7}$	$= \sqrt{2KE/1}$
$= \sqrt{8/7}$	$= \sqrt{8/1}$
$= 1.07$	$= 2.83$

Example with masses: $m_1=49$ and $m_2=1$

Geometric “Integration” (Converting Velocity data to Space-time trajectory)

Example with masses: $m_1=49$ and $m_2=1$

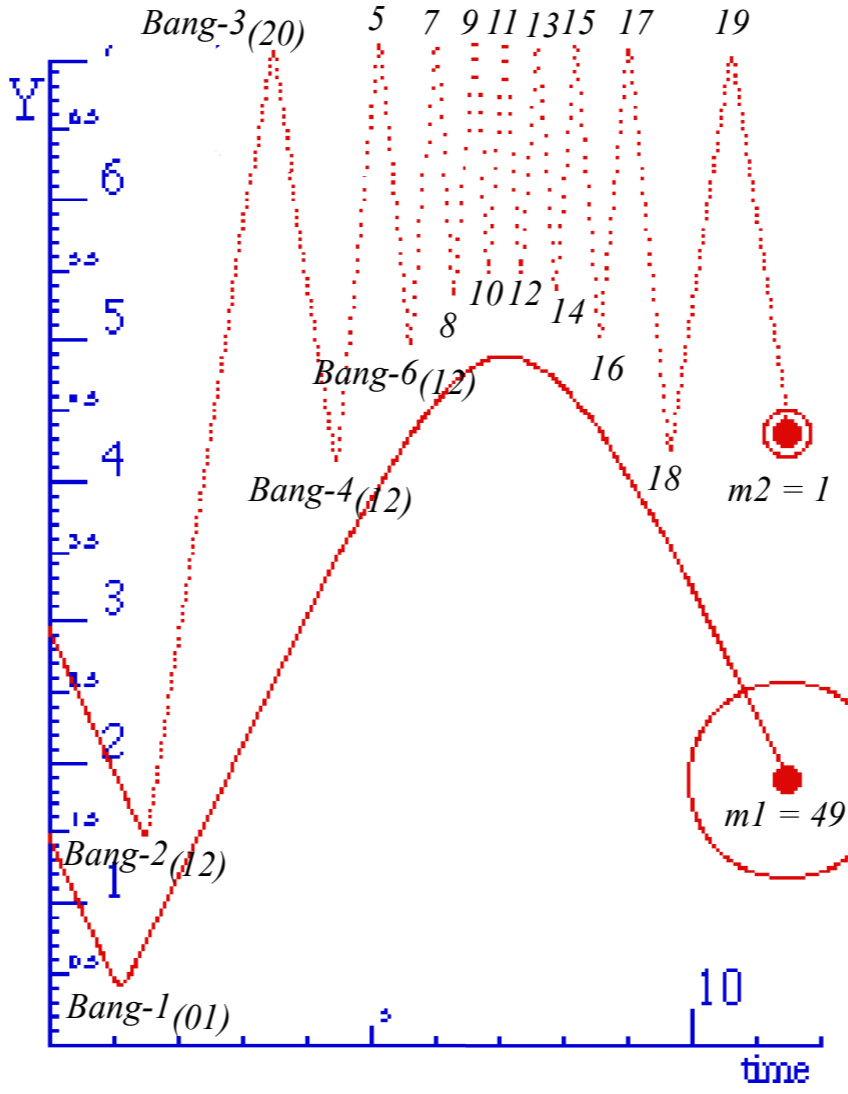
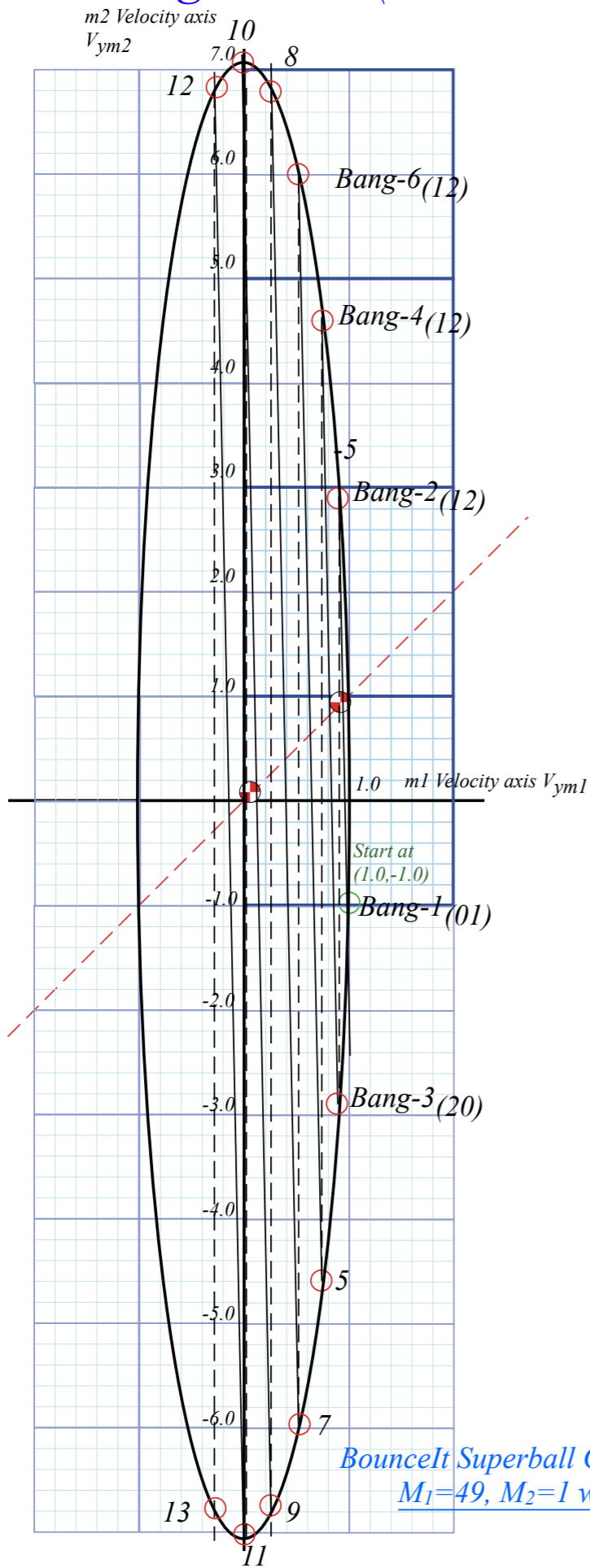


Fig. 4.1 (Unit 1) BounceIt Superball Collision Web Simulator: $M_1=49, M_2=1$ with Newtonian time plot

BounceIt Superball Collision Web Simulator: $M_1=49, M_2=1$ with V_2 vs V_1 plot

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



BounceIt Superball Collision Web Simulator:
 $M_1=49, M_2=1$ with V_2 vs V_1 plot

Example with masses: $m_1=49$ and $m_2=1$

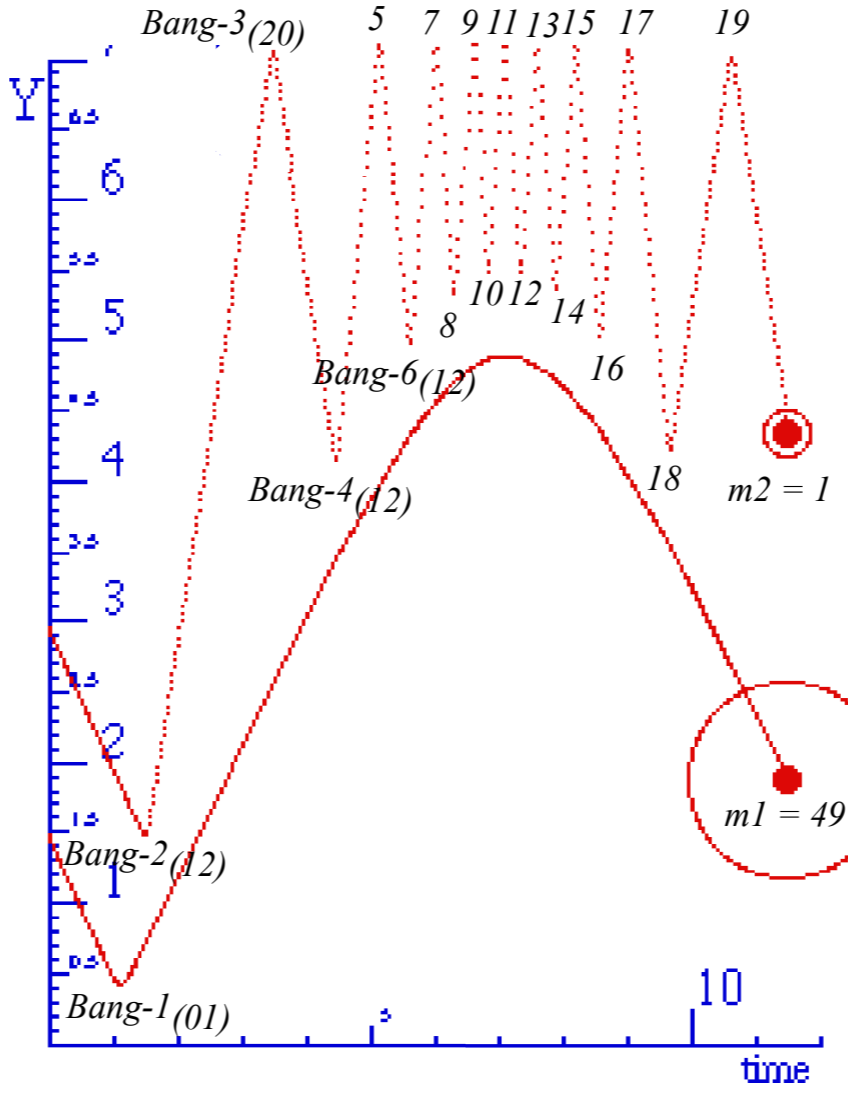
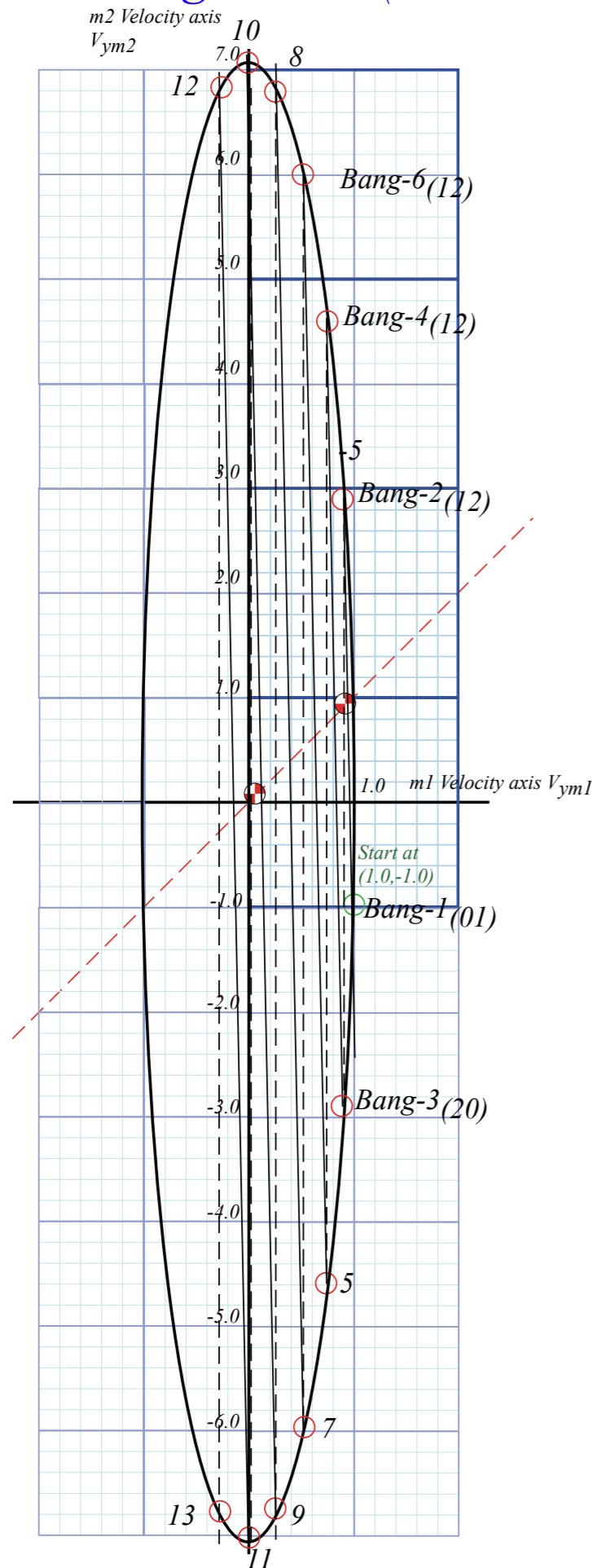


Fig. 4.1
(Unit 1)

BounceIt Superball Collision Web Simulator:
 $M_1=49, M_2=1$ with Newtonian time plot

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: $m_1=49$ and $m_2=1$

Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{V_1^2}{2KE/m_1} + \frac{V_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$\begin{aligned} a_1 &= \sqrt{2KE/M_1} \\ &= \sqrt{2KE/49} \\ &= \sqrt{50/49} \\ &= 1.01 \end{aligned}$$

Ellipse radius 2

$$\begin{aligned} a_2 &= \sqrt{2KE/m_2} \\ &= \sqrt{2KE/1} \\ &= \sqrt{50/1} \\ &= 7.07 \end{aligned}$$

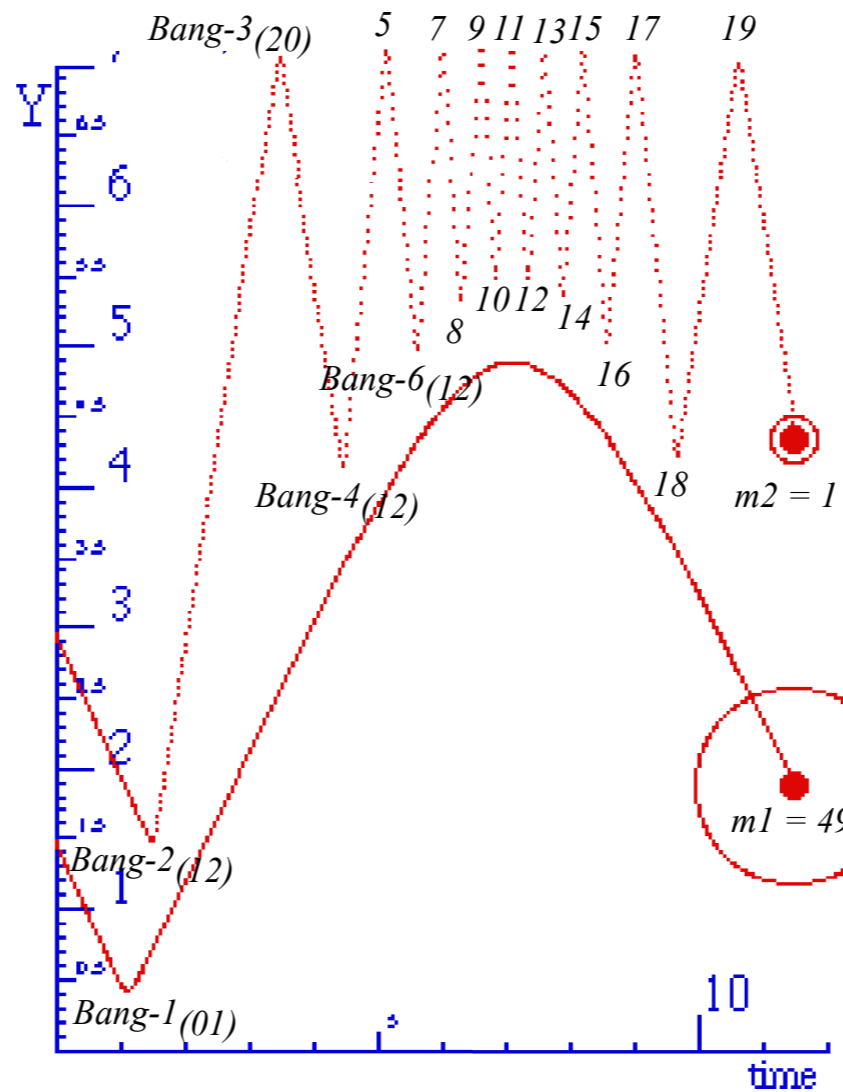


Fig. 4.1
(Unit 1)

Multiple collisions calculated by matrix operator products

 *Matrix or tensor algebra of 1-D 2-body collisions*

*“Mass-bang” matrix **M**, “Floor-bang” matrix **F**, “Ceiling-bang” matrix **C**.*

*Geometry and algebra of “ellipse-Rotation” group product: **R= C•M***

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give:
$$V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives v^{FIN} in terms of v^{IN} ...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} =$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives v^{FIN} in terms of v^{IN} ...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives v^{FIN} in terms of v^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Quiz question about linear solution

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$ gives just *one* solution to *quadratic* collision equations.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives \mathbf{v}^{FIN} in terms of \mathbf{v}^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Quiz question about linear solution

Linear formula $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$ gives just *one* solution to quadratic collision equations.

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}$$

Q: What is the *second* solution and to what simple process would it correspond?

[Example with friction](#)

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

→ *“Mass-bang” matrix **M**, “Floor-bang” matrix **F**, “Ceiling-bang” matrix **C**.*

*Geometry and algebra of “ellipse-Rotation” group product: **R= C•M***

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives v^{FIN} in terms of v^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives v^{FIN} in terms of v^{IN} ...

Finally as a matrix operation: $\mathbf{v}^{FIN} = \mathbf{M} \cdot \mathbf{v}^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

Ceiling-bang \mathbf{C} of m_2 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives v^{FIN} in terms of v^{IN} ...

Finally as a matrix operation: $v^{FIN} = \mathbf{M} \cdot v^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang \mathbf{M} of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang \mathbf{C} of m_2 :

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives v^{FIN} in terms of v^{IN} ...

Finally as a matrix operation: $v^{FIN} = \mathbf{M} \cdot v^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang \mathbf{M} of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang \mathbf{C} of m_2 :

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ and $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Multiple collisions calculated by matrix operator products

Matrix or tensor algebra of 1-D 2-body collisions

*“Mass-bang” matrix **M**, “Floor-bang” matrix **F**, “Ceiling-bang” matrix **C**.*

*Geometry and algebra of “ellipse-Rotation” group product: **R= C•M***



Multiple Collisions by Matrix Operator Products

T-Symmetry & Momentum Axioms give: $V^{COM} = \frac{V^{FIN} + V^{IN}}{2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Gives v^{FIN} in terms of v^{IN} ...

Finally as a matrix operation: $v^{FIN} = \mathbf{M} \cdot v^{IN}$...

$$\begin{pmatrix} v_1^{FIN} \\ v_2^{FIN} \end{pmatrix} = \begin{pmatrix} 2V^{COM} - v_1^{IN} \\ 2V^{COM} - v_2^{IN} \end{pmatrix} = \begin{pmatrix} 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_1^{IN} \\ 2 \frac{m_1 v_1^{IN} + m_2 v_2^{IN}}{m_1 + m_2} - v_2^{IN} \end{pmatrix} = \frac{\begin{pmatrix} m_1 v_1^{IN} - m_2 v_1^{IN} + 2m_2 v_2^{IN} \\ 2m_1 v_1^{IN} + m_2 v_2^{IN} - m_1 v_2^{IN} \end{pmatrix}}{m_1 + m_2} = \frac{\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1^{IN} \\ v_2^{IN} \end{pmatrix}}{m_1 + m_2}$$

Matrix operations include...

Floor-bang \mathbf{F} of m_1 :

$$\mathbf{F} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mass-bang \mathbf{M} of m_1 and m_2 :

$$\mathbf{M} = \begin{pmatrix} \frac{m_1 - m_2}{m_1 + m_2} & \frac{2m_2}{m_1 + m_2} \\ \frac{2m_1}{m_1 + m_2} & \frac{m_2 - m_1}{m_1 + m_2} \end{pmatrix}$$

Ceiling-bang \mathbf{C} of m_2 :

$$\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let: $m_1=49$ and $m_2=1$

$$\mathbf{M} = \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix}$$

Define "ellipse-Rotation" \mathbf{R} as group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} = \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix}$

$$\begin{aligned}
 |FIN^9\rangle &= \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL } (0))}
 \end{aligned}$$

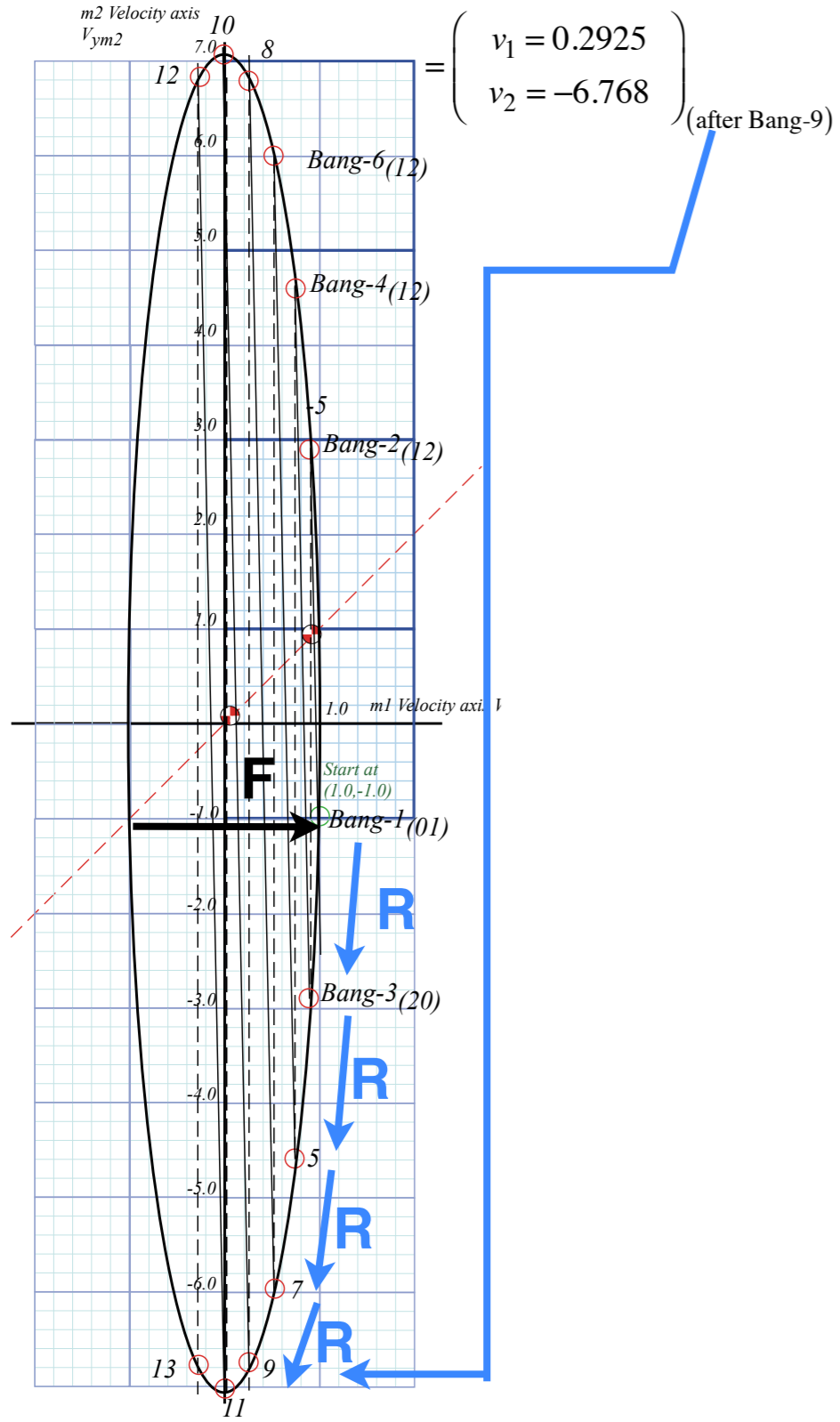
$$\begin{aligned}
 \begin{pmatrix} |FIN^9\rangle \\ v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \underbrace{\mathbf{C} \cdot \mathbf{M}}_{\mathbf{R}} \cdot \underbrace{\mathbf{C} \cdot \mathbf{M}}_{\mathbf{R}} \cdot \underbrace{\mathbf{C} \cdot \mathbf{M}}_{\mathbf{R}} \cdot \underbrace{\mathbf{C} \cdot \mathbf{M}}_{\mathbf{R}} \cdot \mathbf{F} \begin{pmatrix} |IN^0\rangle \\ v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix}_{(\text{INITIAL } (0))} \\
 \begin{pmatrix} |FIN^9\rangle \\ v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} \mathbf{R} \\ \mathbf{R} \\ \mathbf{R} \\ \mathbf{R} \end{pmatrix} \cdot \mathbf{F} |IN^0\rangle \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix}_{(\text{after Bang-1})}
 \end{aligned}$$

“ellipse-Rotation” group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

$$\begin{aligned}
\left\langle \begin{matrix} |FIN^9\rangle \\ v_1^{FIN-9} \\ v_2^{FIN-9} \end{matrix} \right\rangle &= \underbrace{\mathbf{C} \cdot \mathbf{M}}_{\mathbf{R}} \cdot \underbrace{\mathbf{C} \cdot \mathbf{M}}_{\mathbf{R}} \cdot \underbrace{\mathbf{C} \cdot \mathbf{M}}_{\mathbf{R}} \cdot \underbrace{\mathbf{C} \cdot \mathbf{M}}_{\mathbf{R}} \cdot \mathbf{F} \left\langle \begin{matrix} |IN^0\rangle \\ v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{matrix} \right\rangle_{(\text{INITIAL } (0))} \\
\left\langle \begin{matrix} |FIN^9\rangle \\ v_1^{FIN-9} \\ v_2^{FIN-9} \end{matrix} \right\rangle &= \left(\begin{matrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{matrix} \right) \cdot \left(\begin{matrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{matrix} \right) \cdot \left(\begin{matrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{matrix} \right) \cdot \left(\begin{matrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{matrix} \right) \cdot \left(\begin{matrix} v_1 = 1 \\ v_2 = -1 \end{matrix} \right)_{(\text{after Bang-1})} \\
&= \left(\begin{matrix} v_1 = 0.2925 \\ v_2 = -6.768 \end{matrix} \right)_{(\text{after Bang-9})}
\end{aligned}$$

“ellipse-Rotation” group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

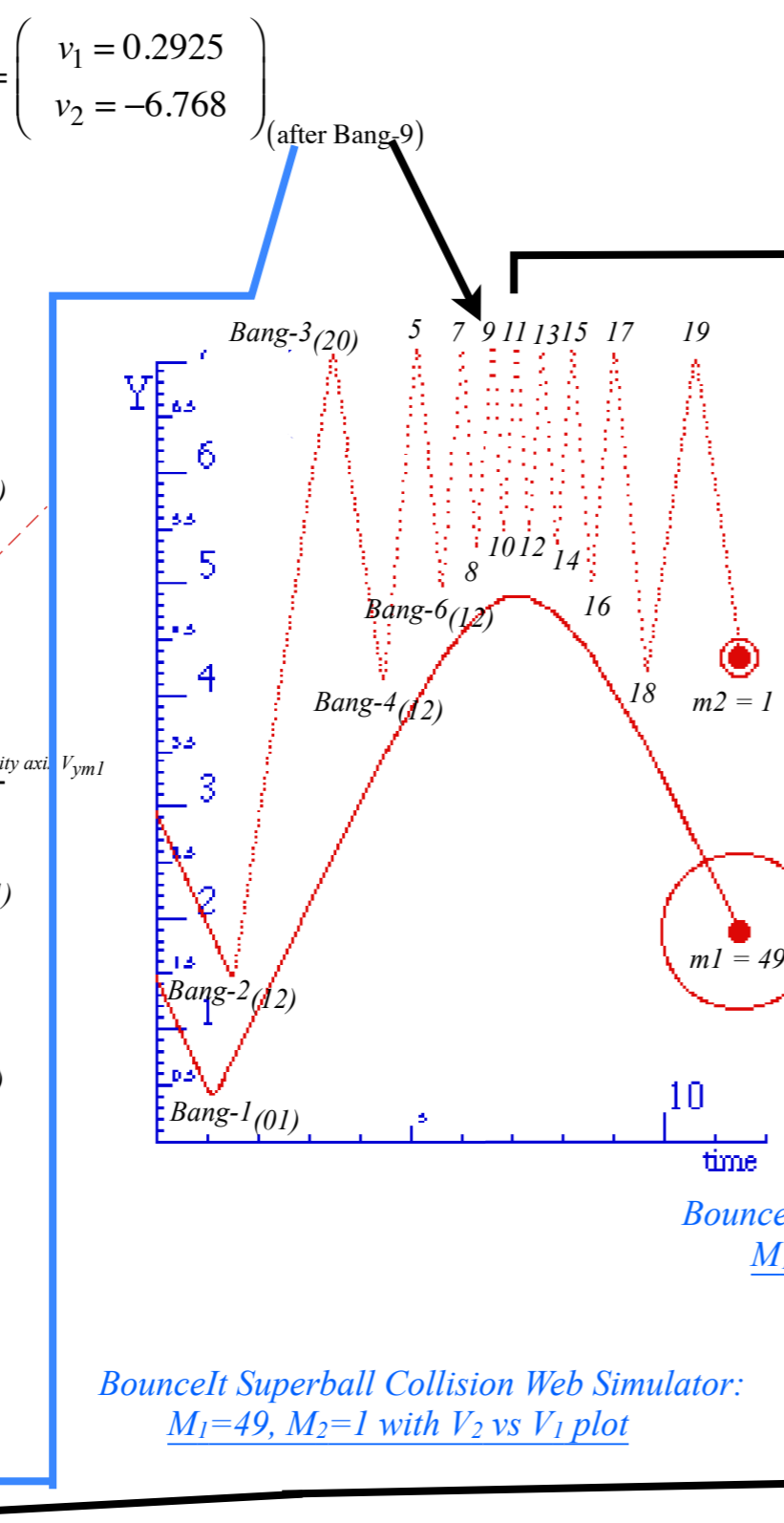
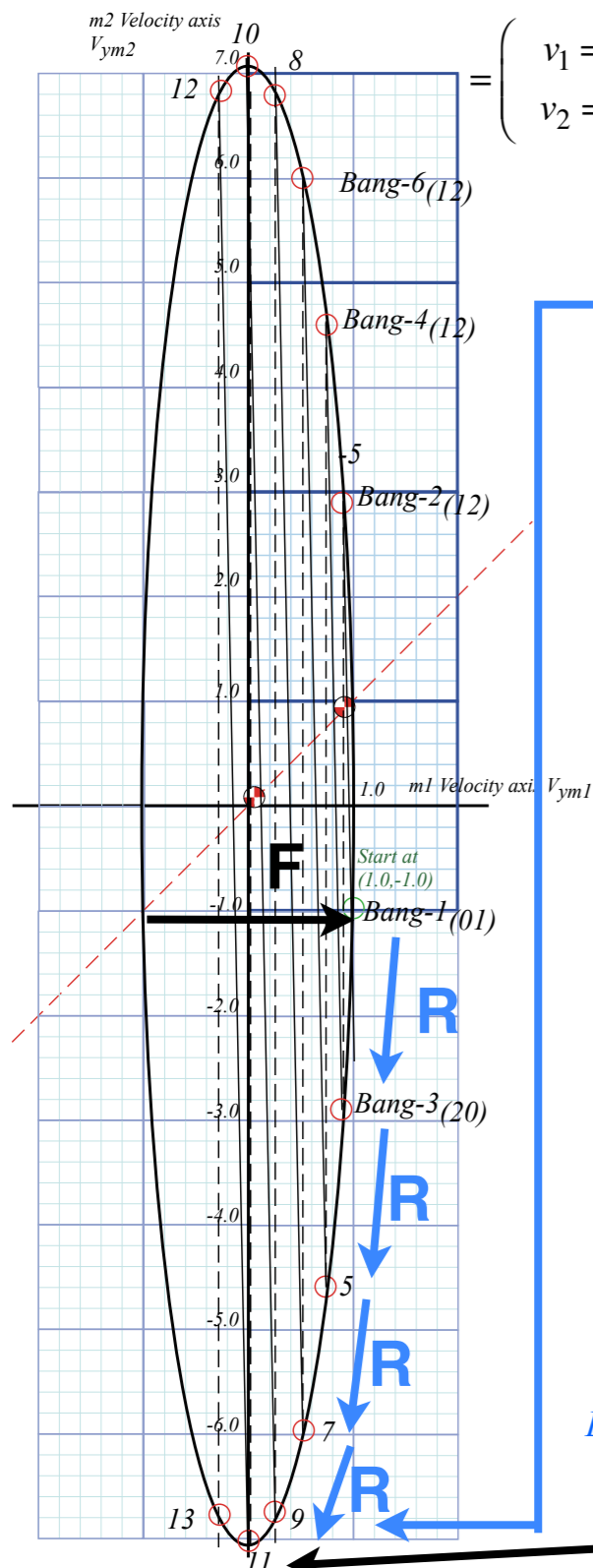
$$\begin{aligned}
 |FIN^9\rangle &= \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} |IN^0\rangle \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix} \quad (\text{INITIAL } (0)) \\
 |FIN^9\rangle &= \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} |IN^0\rangle \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix} \quad (\text{after Bang-1})
 \end{aligned}$$



“ellipse-Rotation” group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

Fig. 4.1a-b (revised)

$$\begin{aligned}
 |FIN^9\rangle &= \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{C} \cdot \mathbf{M} \cdot \mathbf{F} |IN^0\rangle \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ 1.96 & -0.96 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} v_1^{IN} = -1 \\ v_2^{IN} = -1 \end{pmatrix} \quad (\text{INITIAL (0)}) \\
 |FIN^9\rangle &= \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{F} |IN^0\rangle \\
 \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1 = 1 \\ v_2 = -1 \end{pmatrix} \quad (\text{after Bang-1})
 \end{aligned}$$



“ellipse-Rotation” group product: $\mathbf{R} = \mathbf{C} \cdot \mathbf{M}$

$$\begin{aligned}
 \begin{pmatrix} v_1^{FIN-11} \\ v_2^{FIN-11} \end{pmatrix} &= \begin{pmatrix} 0.96 & 0.04 \\ -1.96 & 0.96 \end{pmatrix} \cdot \begin{pmatrix} v_1^{FIN-9} \\ v_2^{FIN-9} \end{pmatrix} \\
 &= \begin{pmatrix} v_1 = 0.0100 \\ v_2 = -7.071 \end{pmatrix} \quad (\text{after Bang-11})
 \end{aligned}$$

<<Under Construction>>
Matrix Collision Web Simulator:
 $M_1=49, M_2=1$ V_2 vs V_1 plot

BounceIt Superball Collision Web Simulator:
 $M_1=49, M_2=1$ with Newtonian time plot

BounceIt Superball Collision Web Simulator:
 $M_1=49, M_2=1$ with V_2 vs V_1 plot

Fig. 4.1a-b
(revised)

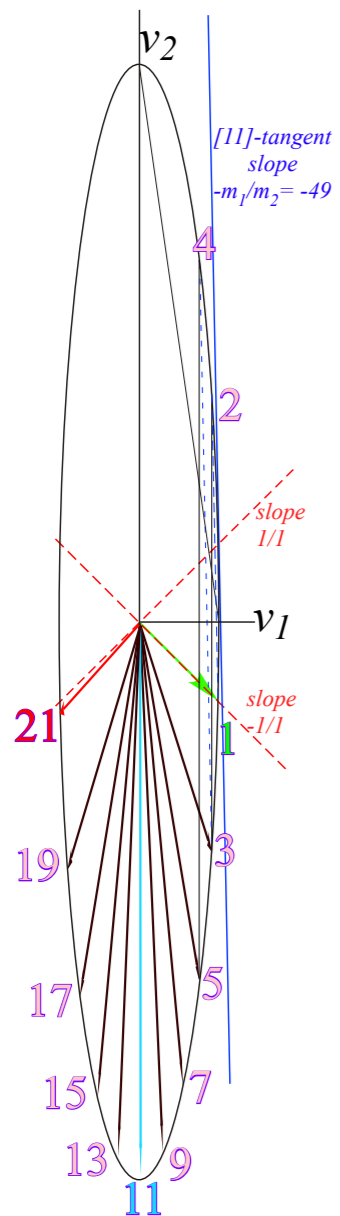
Ellipse rescaling-geometry and reflection-symmetry analysis

 *Rescaling KE ellipse to circle*

How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics in Ch. 12

Ellipse rescaling geometry and reflection symmetry analysis

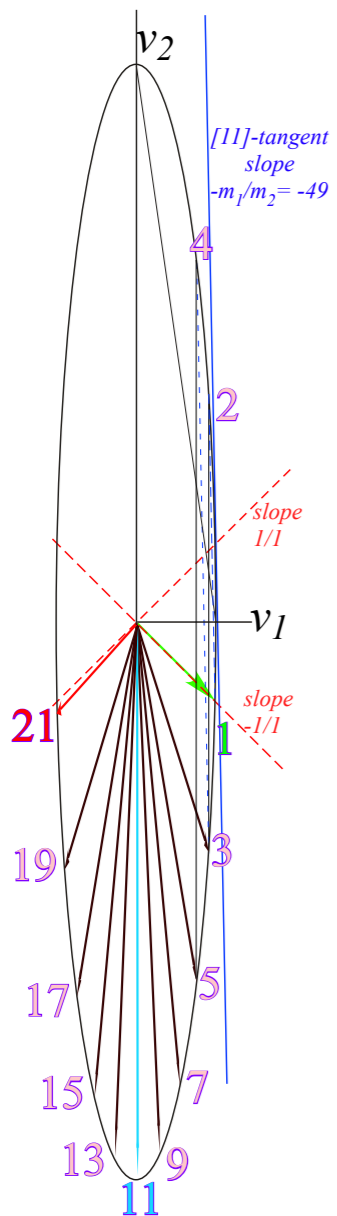
Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$



Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN1} \\ v_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} V_1^{FIN1} / \sqrt{m_1} \\ V_2^{FIN1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

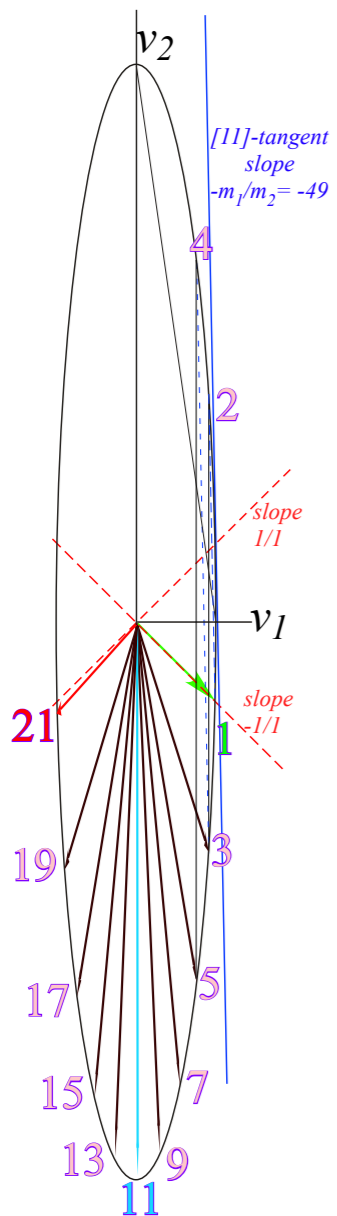


Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN1} \\ v_2^{FIN1} \end{pmatrix} = \frac{I}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} V_1^{FIN1} / \sqrt{m_1} \\ V_2^{FIN1} / \sqrt{m_2} \end{pmatrix} = \frac{I}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

$$\text{or:} \quad \begin{pmatrix} V_1^{FIN1} \\ V_2^{FIN1} \end{pmatrix} = \frac{I}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \cdot \vec{V}, \quad \text{or:} \quad \begin{pmatrix} V_1^{FIN2} \\ V_2^{FIN2} \end{pmatrix} = \frac{I}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \cdot \mathbf{M} \cdot \vec{V}$$



Ellipse rescaling geometry and reflection symmetry analysis

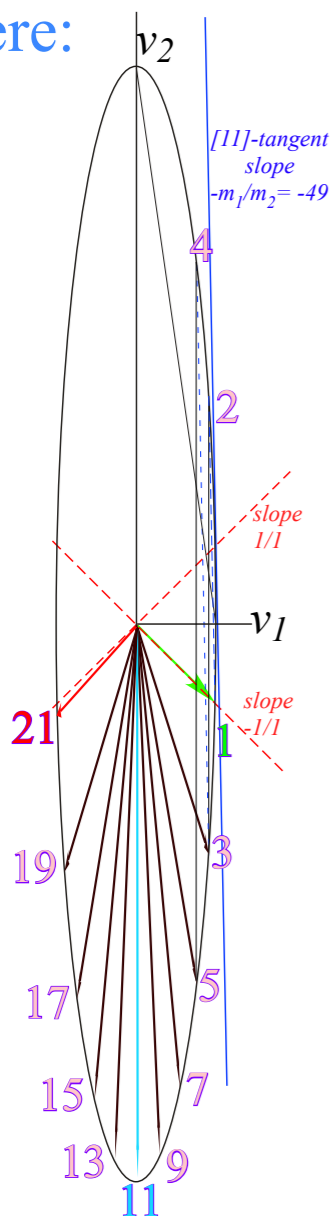
Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN1} \\ v_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} V_1^{FIN1} / \sqrt{m_1} \\ V_2^{FIN1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

$$\text{or: } \begin{pmatrix} V_1^{FIN1} \\ V_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \cdot \vec{V}, \quad \text{or: } \begin{pmatrix} V_1^{FIN2} \\ V_2^{FIN2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \cdot \mathbf{M} \cdot \vec{V}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$



Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN1} \\ v_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} V_1^{FIN1} / \sqrt{m_1} \\ V_2^{FIN1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

$$\text{or: } \begin{pmatrix} V_1^{FIN1} \\ V_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \cdot \vec{V}, \quad \text{or: } \begin{pmatrix} V_1^{FIN2} \\ V_2^{FIN2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \cdot \mathbf{M} \cdot \vec{V}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \frac{m_1 - m_2}{m_1 + m_2}$ and: $\sin\theta \equiv \frac{2\sqrt{m_1 m_2}}{m_1 + m_2}$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$

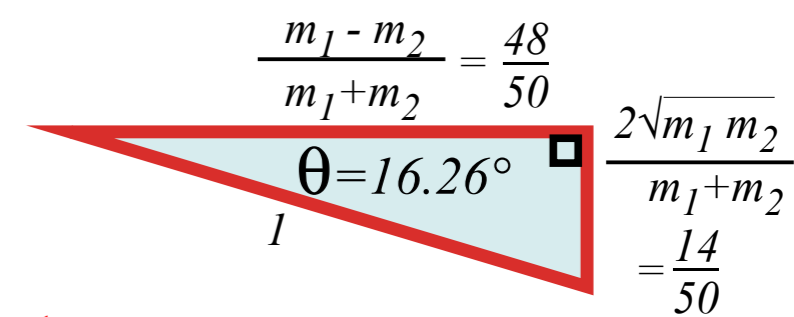
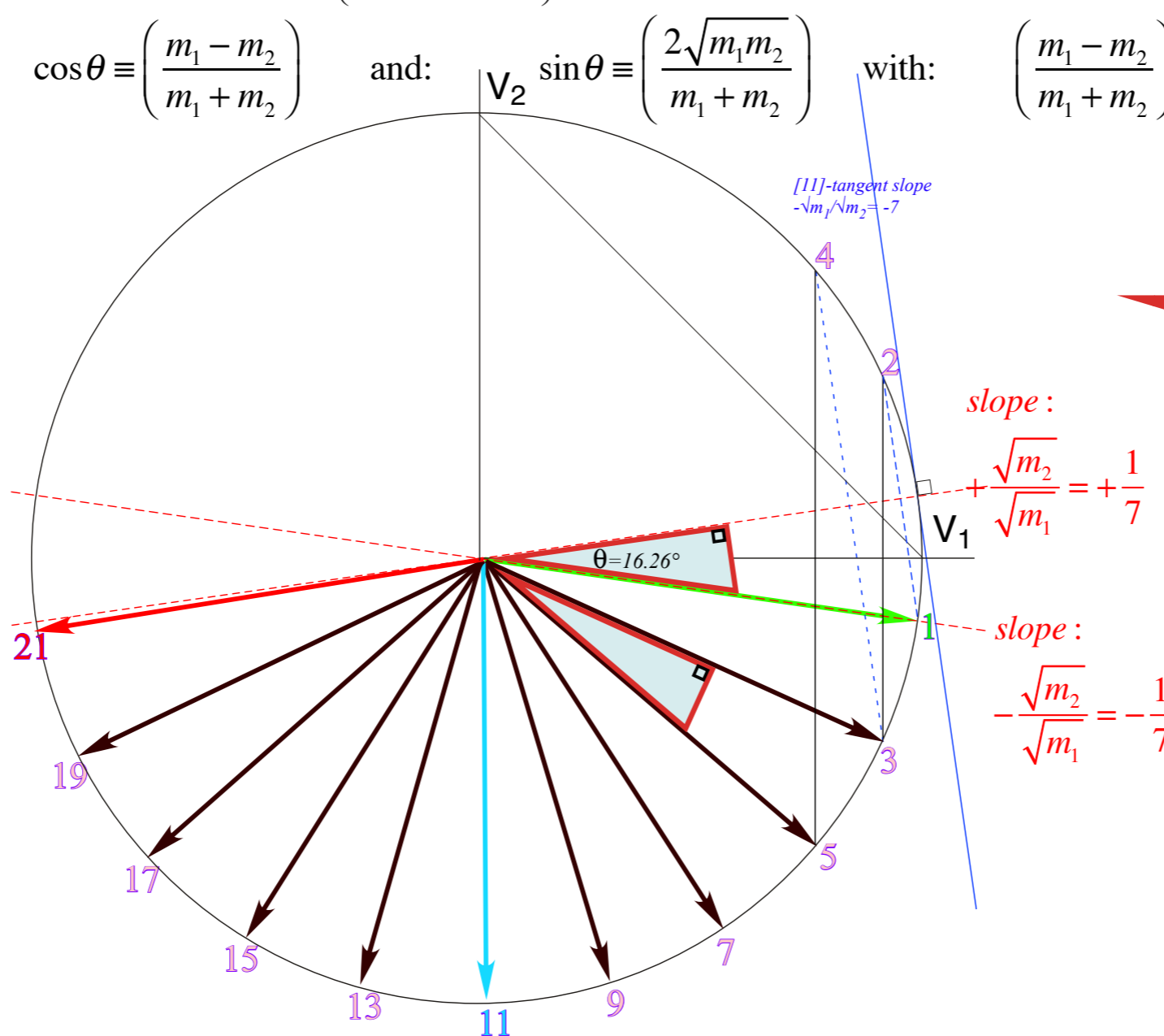
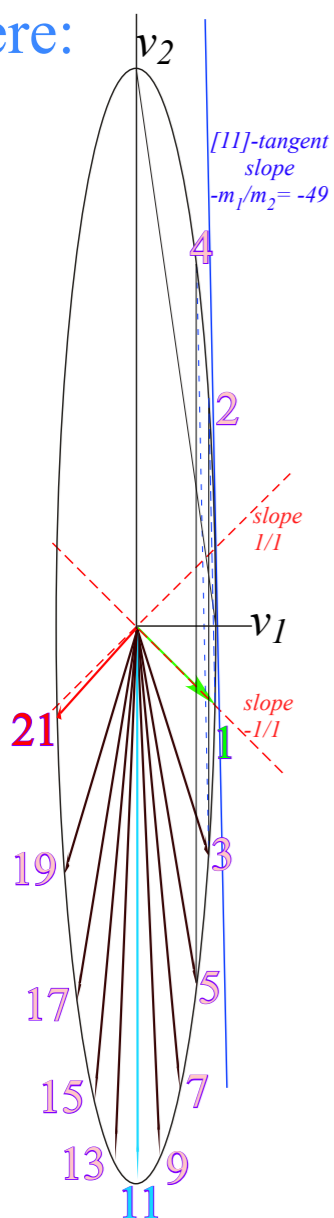


Fig. 4.2a-c
(revised)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_2}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

$$\begin{pmatrix} v_1^{FIN1} \\ v_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad \text{becomes:} \quad \begin{pmatrix} V_1^{FIN1} / \sqrt{m_1} \\ V_2^{FIN1} / \sqrt{m_2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 / \sqrt{m_1} \\ V_2 / \sqrt{m_2} \end{pmatrix}$$

$$\text{or: } \begin{pmatrix} V_1^{FIN1} \\ V_2^{FIN1} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ 2\sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{M} \cdot \vec{V}, \quad \text{or: } \begin{pmatrix} V_1^{FIN2} \\ V_2^{FIN2} \end{pmatrix} = \frac{1}{M} \begin{pmatrix} m_1 - m_2 & 2\sqrt{m_1 m_2} \\ -2\sqrt{m_1 m_2} & m_1 - m_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \mathbf{C} \cdot \mathbf{M} \cdot \vec{V}$$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations* $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

where: $\cos\theta \equiv \frac{m_1 - m_2}{m_1 + m_2}$ and: $\sin\theta \equiv \frac{2\sqrt{m_1 m_2}}{m_1 + m_2}$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$

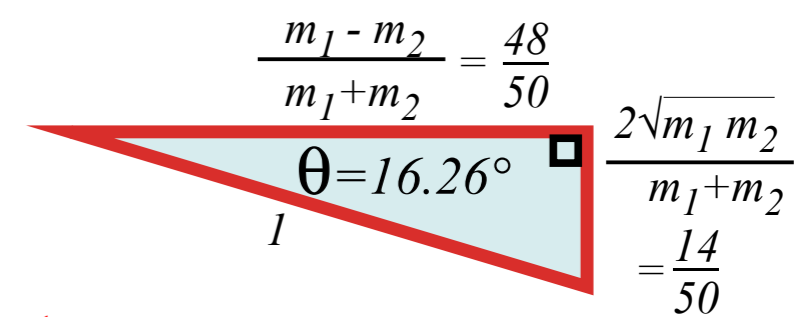
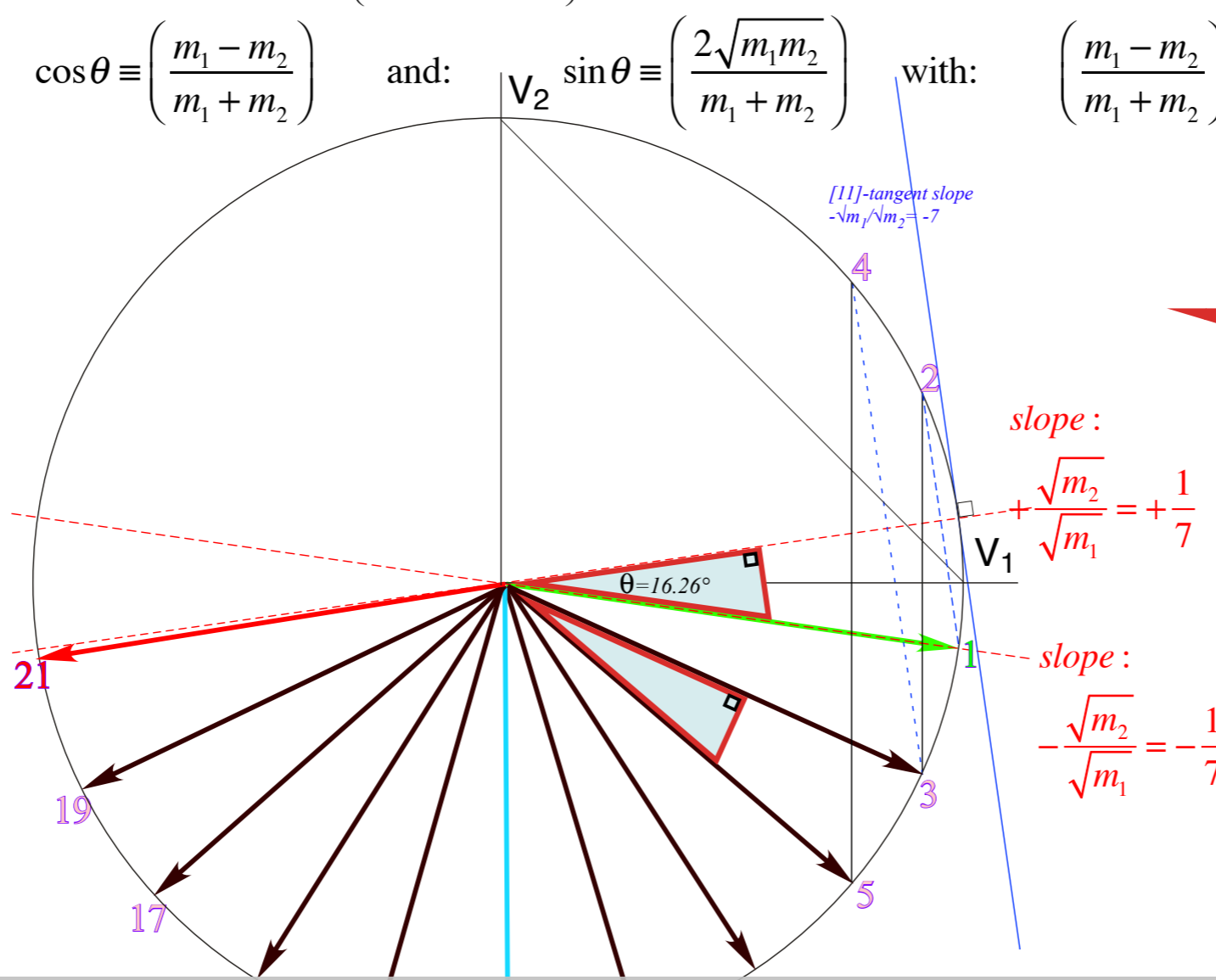
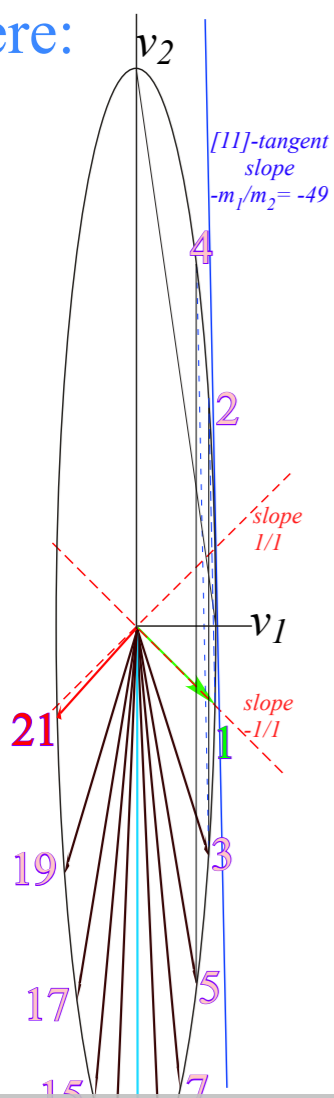


Fig. 4.2a-c
(revised)

Note: If $m_1 \cdot m_2$ is perfect-square, then θ -triangle is rational ($3^2 + 4^2 = 5^2$, etc.)

Ellipse rescaling geometry and reflection symmetry analysis

Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations*

where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2} \right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2} \right)^2 = 1$

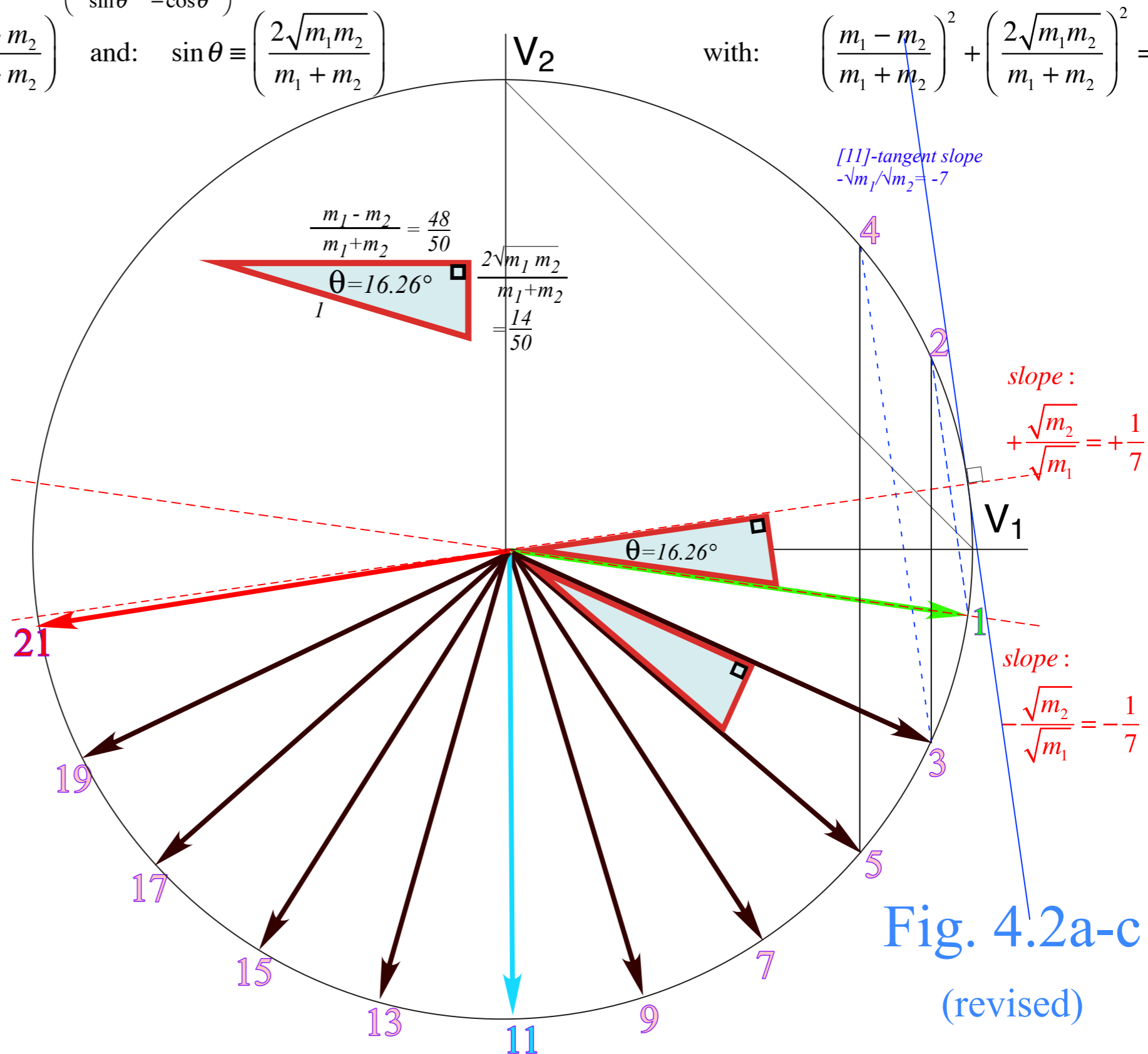
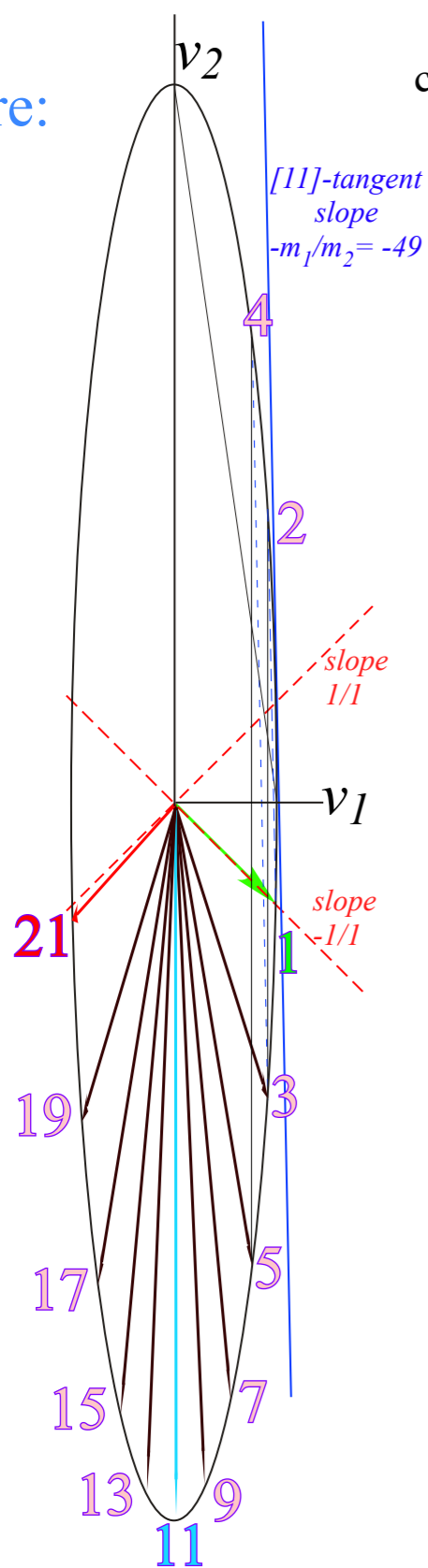


Fig. 4.2a-c
(revised)

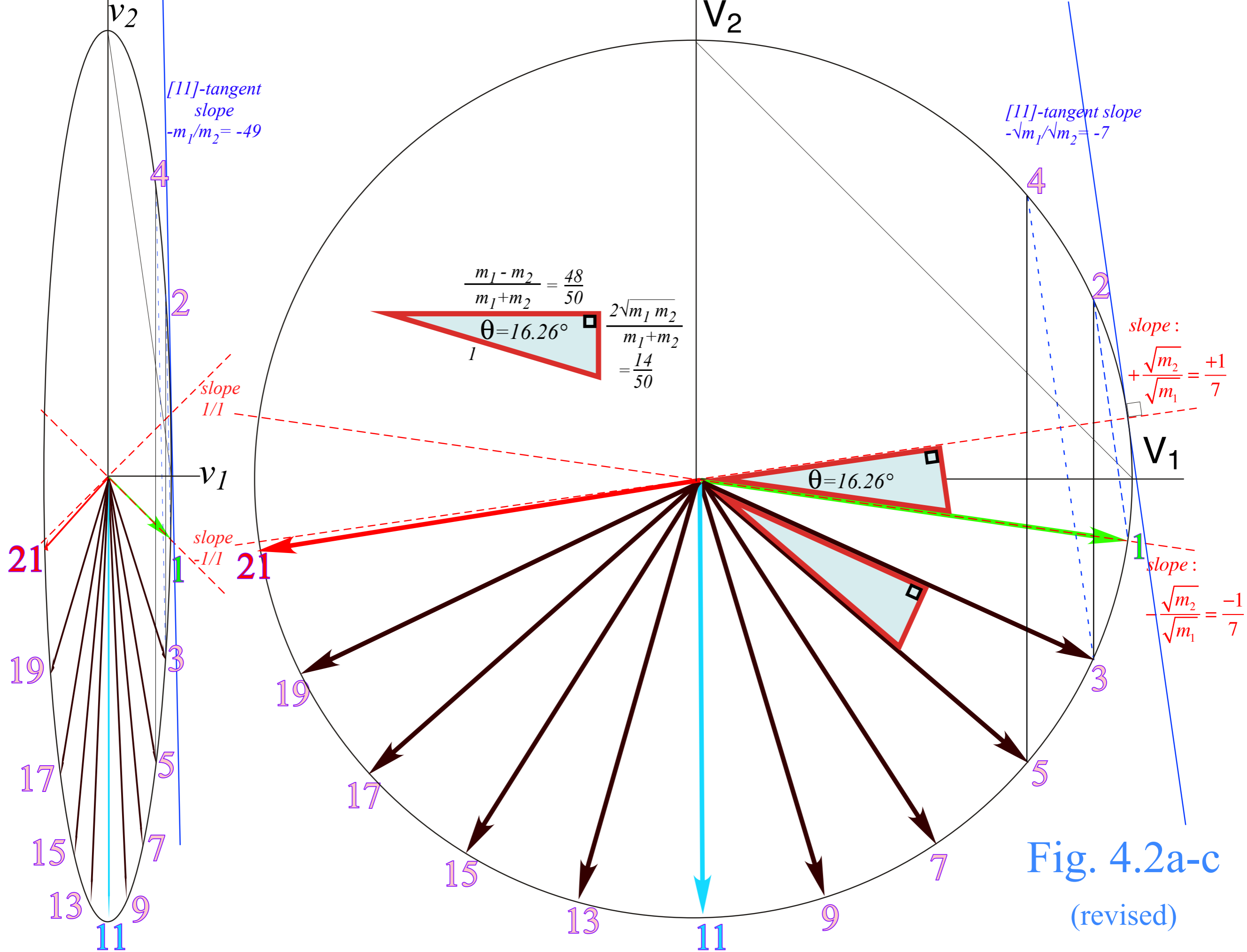


Fig. 4.2a-c
(revised)

Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

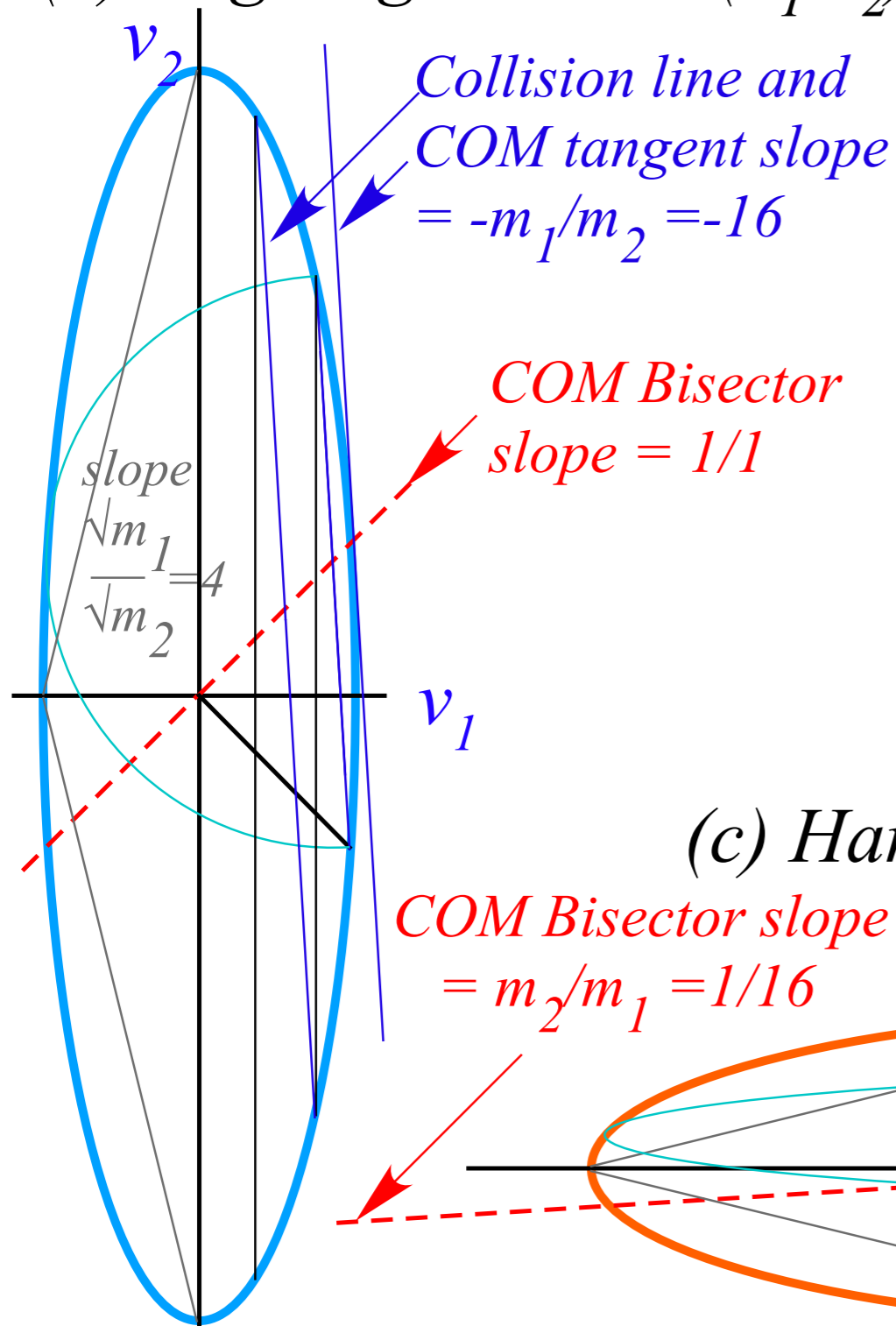
 *How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on*

What ellipse rescaling leads to...(in Ch. 9-12)

Fig. 10.1?
(Unit 1)

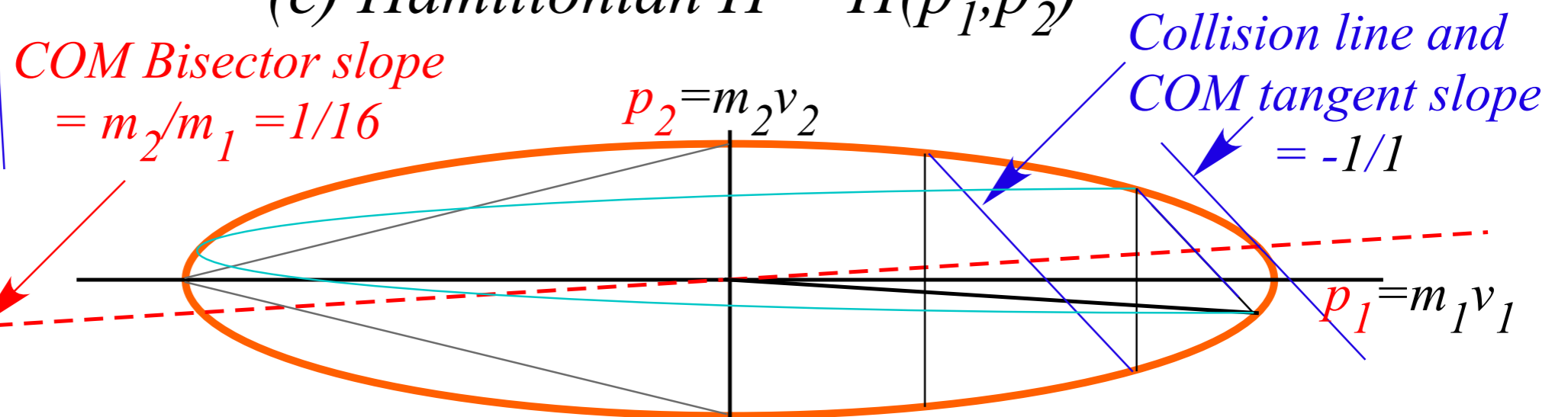
How this relates to *Lagrangian*, and *Hamiltonian* mechanics in Ch. 10

(a) Lagrangian $L = L(v_1, v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$
 velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

(c) Hamiltonian $H = H(p_1, p_2)$

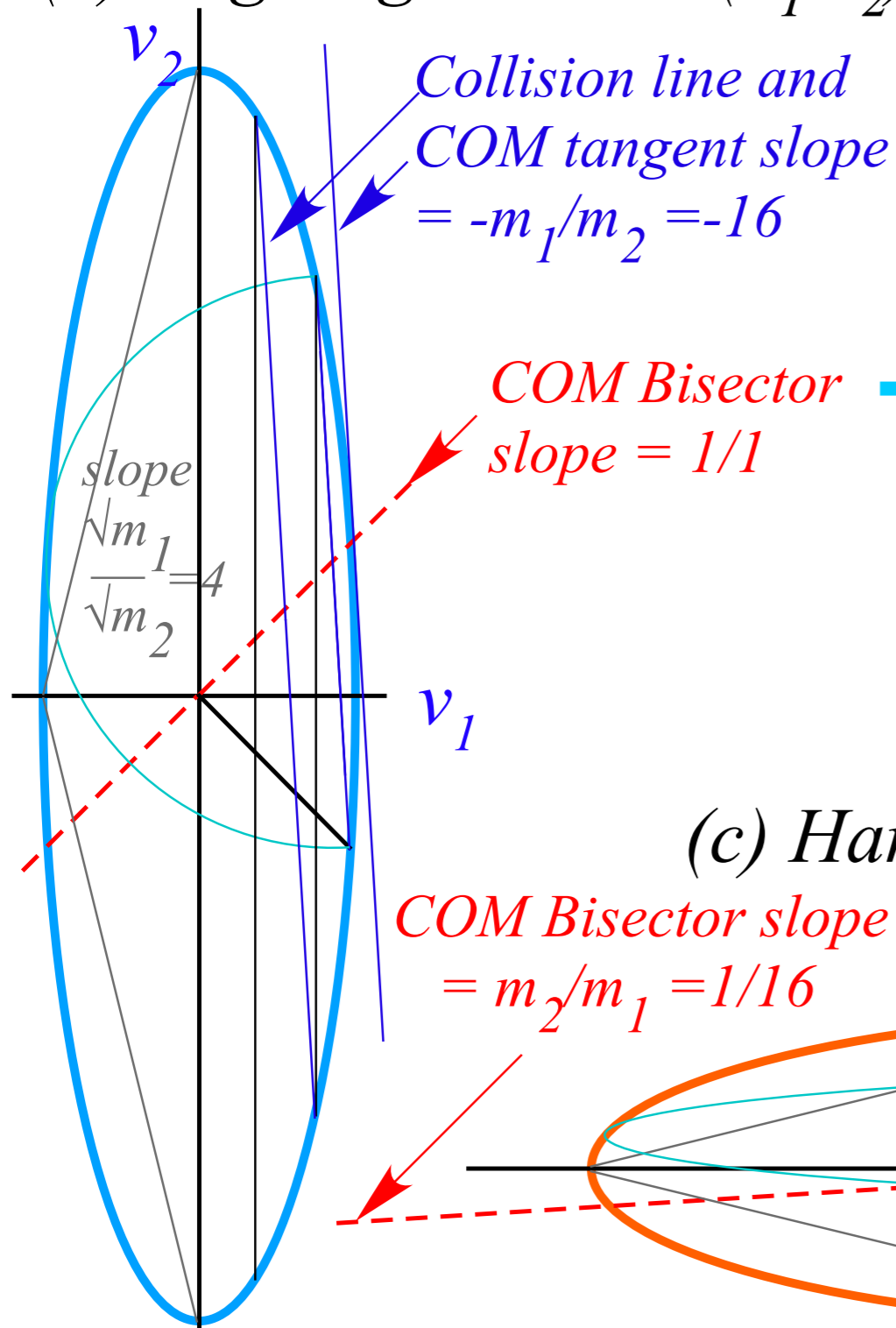


What ellipse rescaling leads to...(in Ch. 9-12)

Fig. 10.1?
(Unit 1)

How this relates to *Lagrangian*, and *Hamiltonian* mechanics in Ch. 10

(a) Lagrangian $L = L(v_1, v_2)$



velocity v_1 rescaled to *momentum*: $p_1 = m_1 v_1$
 velocity v_2 rescaled to *momentum*: $p_2 = m_2 v_2$

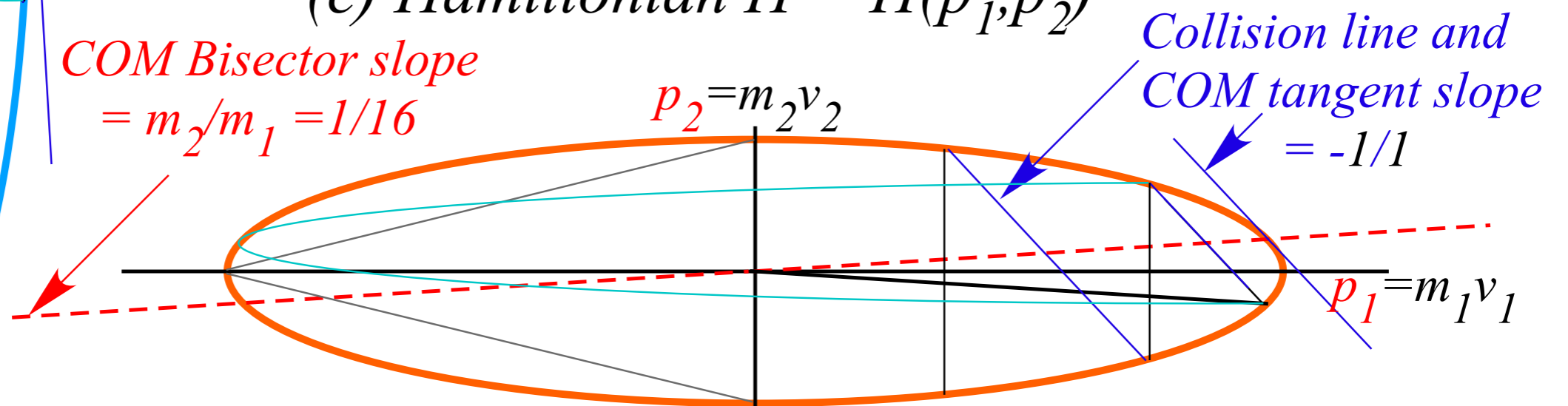
Lagrangian $L(v_1, v_2) = KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

rescaled to

Hamiltonian $H(p_1, p_2) = KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$

(c) Hamiltonian $H = H(p_1, p_2)$

COM Bisector slope
 $= m_2/m_1 = 1/16$

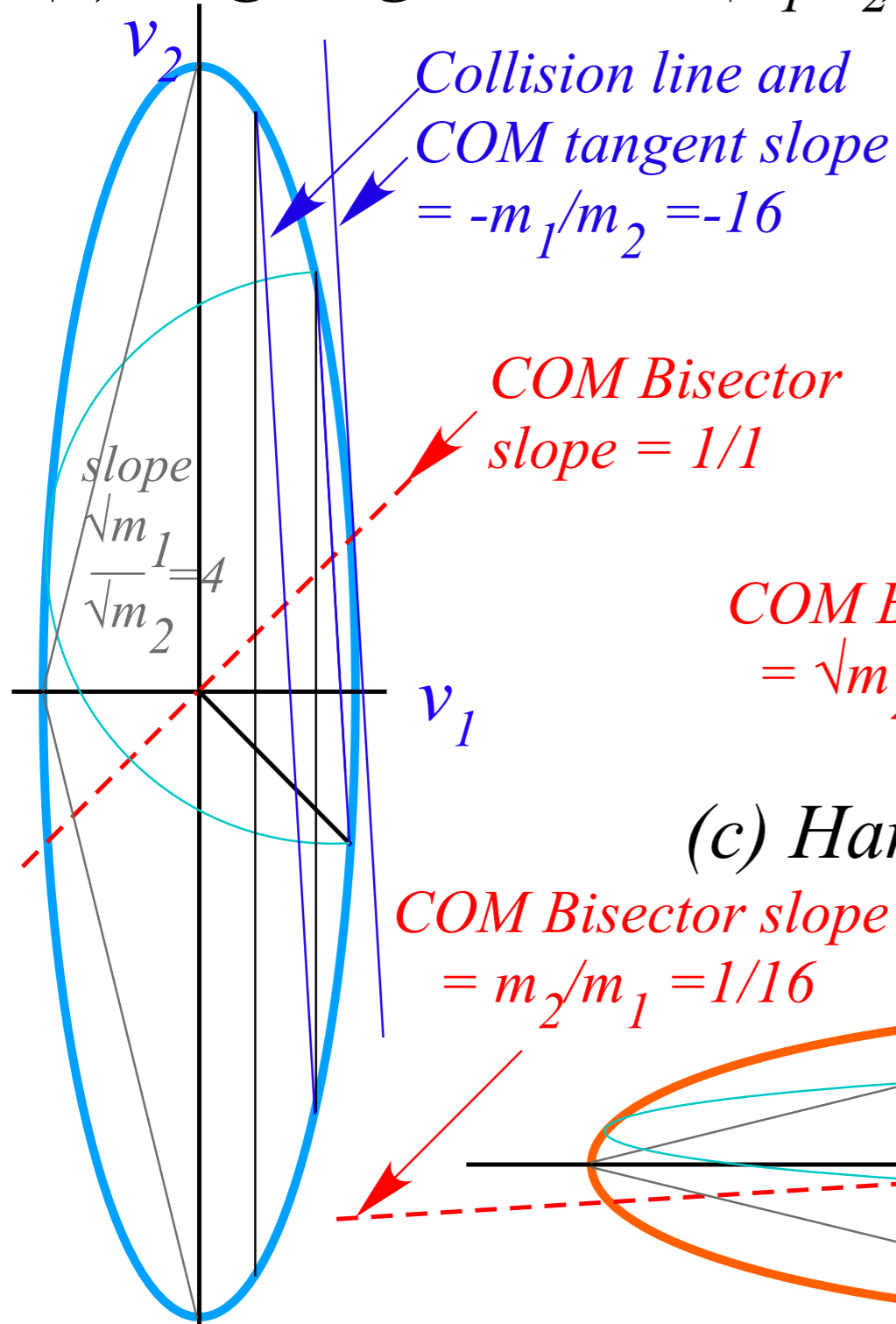


What ellipse rescaling leads to...(in Ch. 9-12)

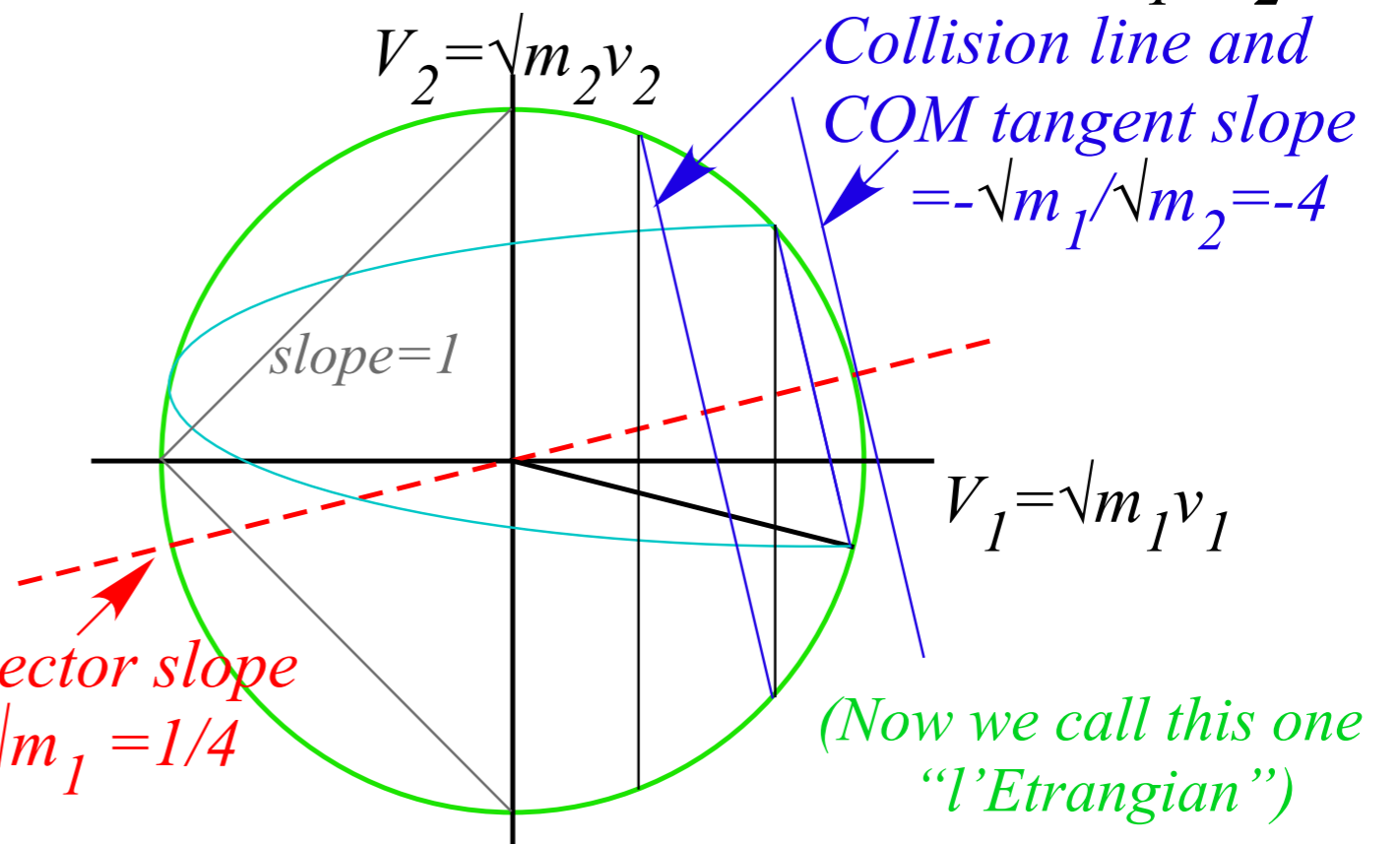
Fig. 10.1?
(Unit 1)

How this relates to *Lagrangian*, *l'Etrangian*, and *Hamiltonian* mechanics in Ch. 10

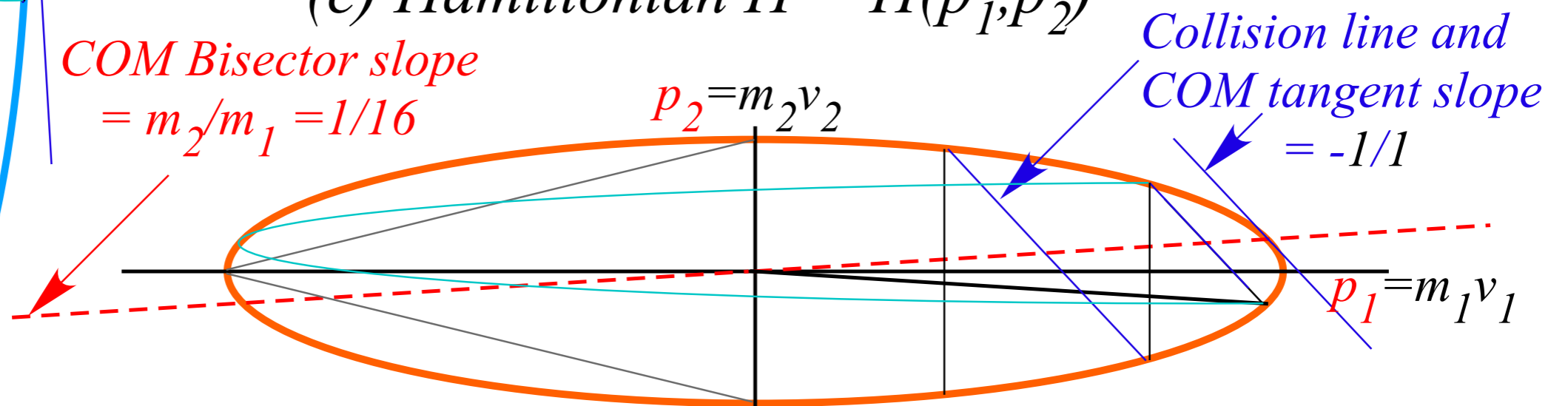
(a) Lagrangian $L = L(v_1, v_2)$



(b) Estrangian $E = E(V_1, V_2)$



(c) Hamiltonian $H = H(p_1, p_2)$



Ellipse rescaling-geometry and reflection-symmetry analysis

Rescaling KE ellipse to circle

*How this relates to Lagrangian, l'Etrangian, and Hamiltonian mechanics later on
(Preview of next Lecture)*

 *Reflections in the clothing store: “It’s all done with mirrors!”*

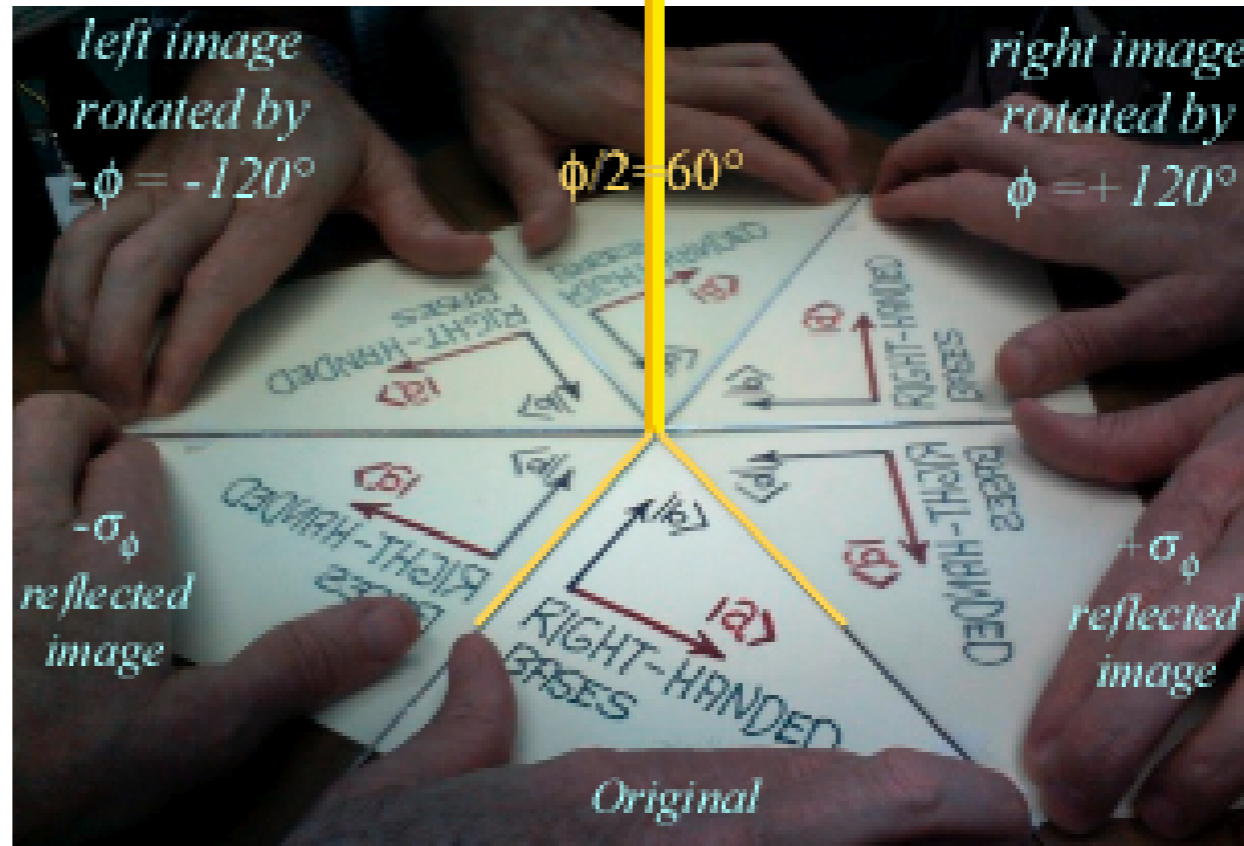
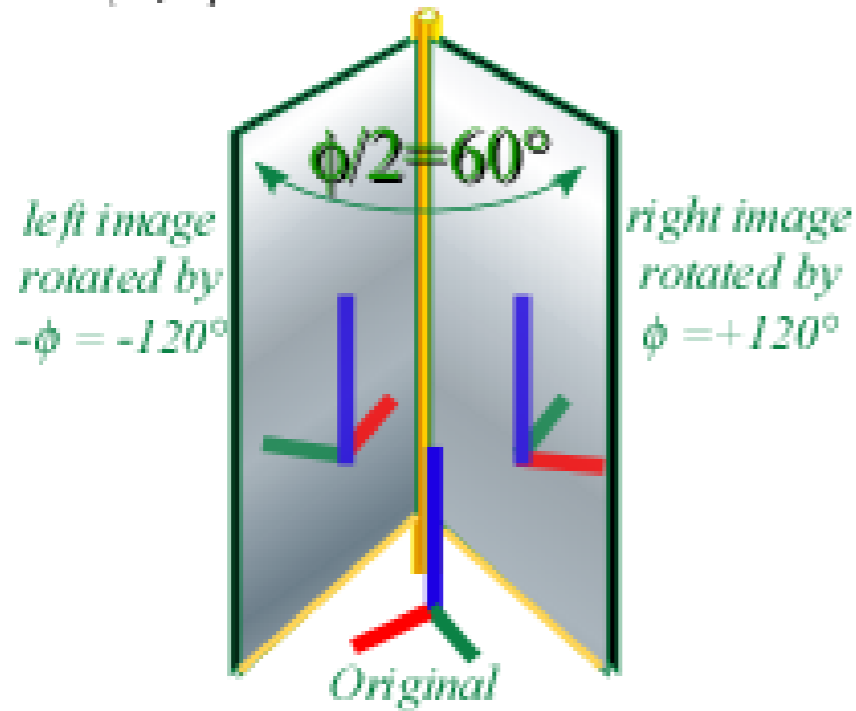
Introducing hexagonal symmetry $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Group multiplication and product table

Classical collision paths with $D_6 \sim C_{6v}$ (Resulting from $m_1/m_2=3$)

Reflections in clothing store mirrors

(a) $\phi = \pm 120^\circ$ rotations



(b) $\phi = \pm 180^\circ$ rotations

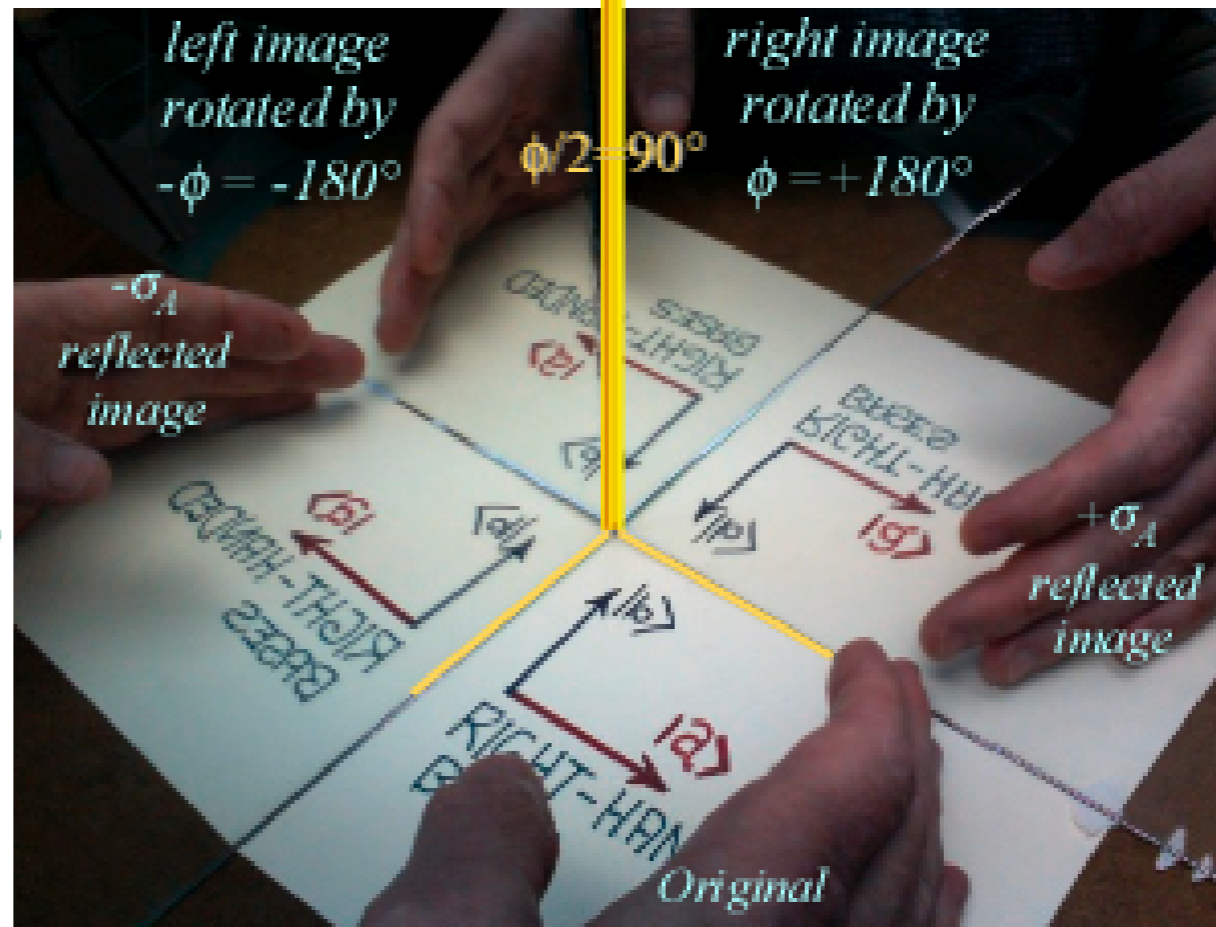
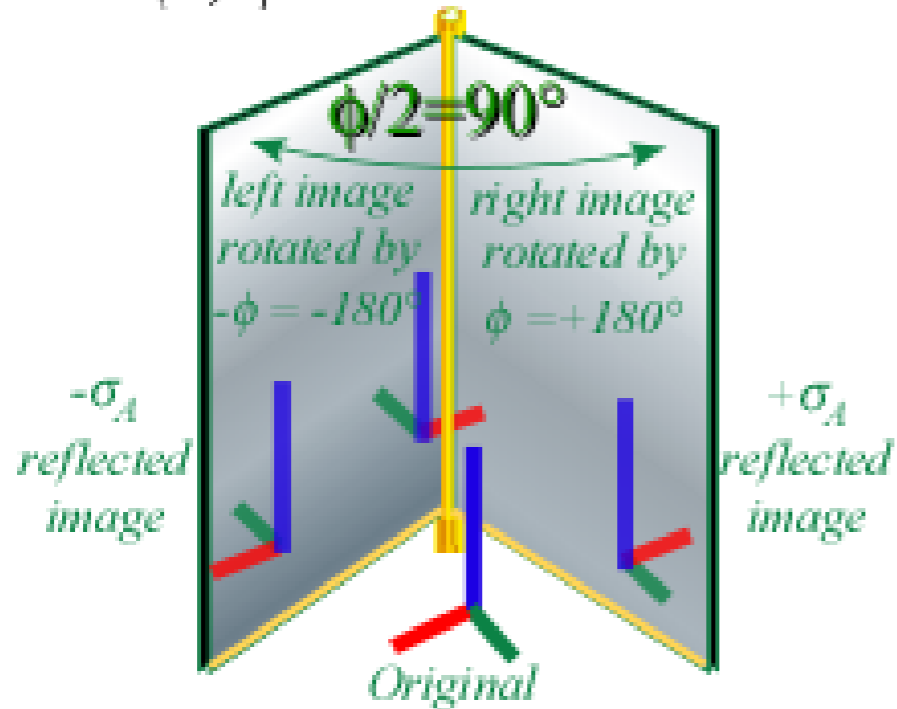
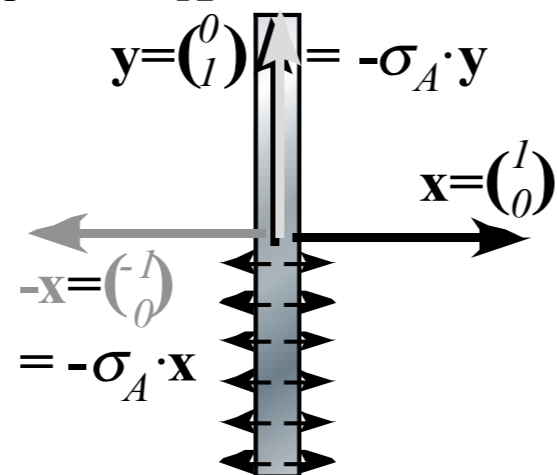
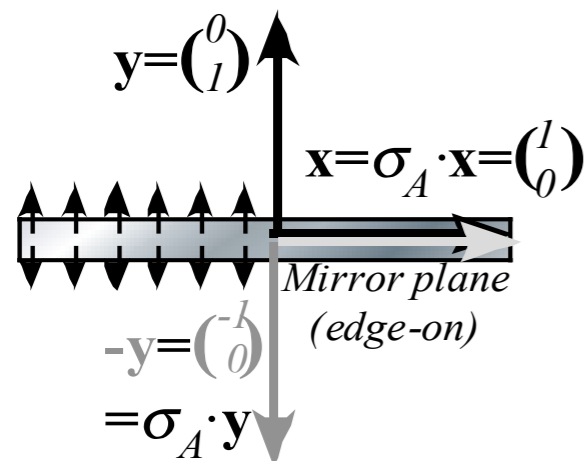


Fig. 5.4a-b

Symmetry: It's all done with mirrors!

(a) Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$



(b) Reflections $\sigma_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $-\sigma_B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

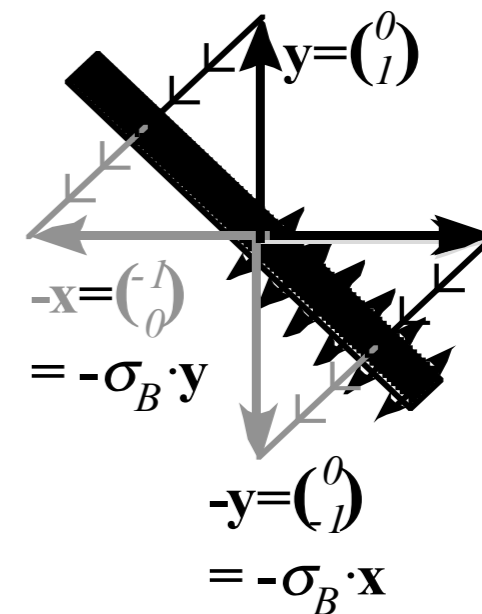
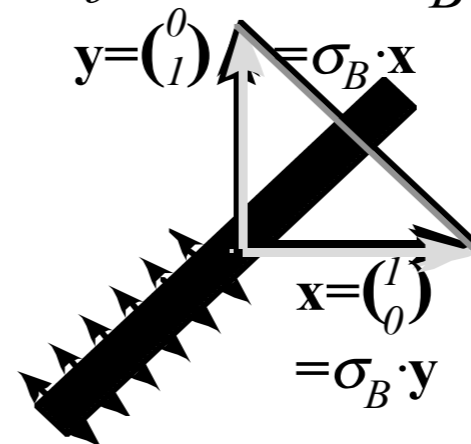
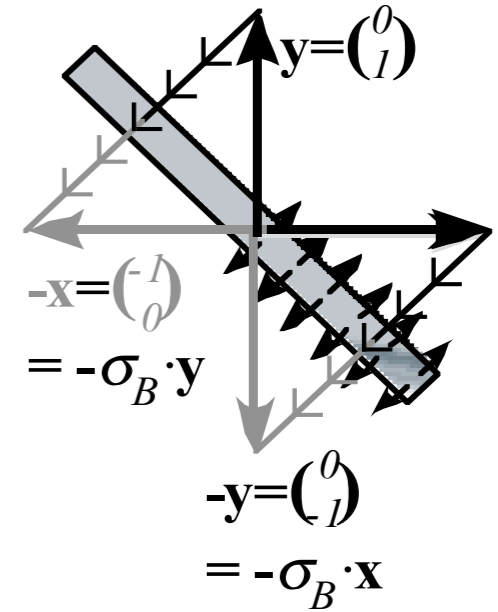
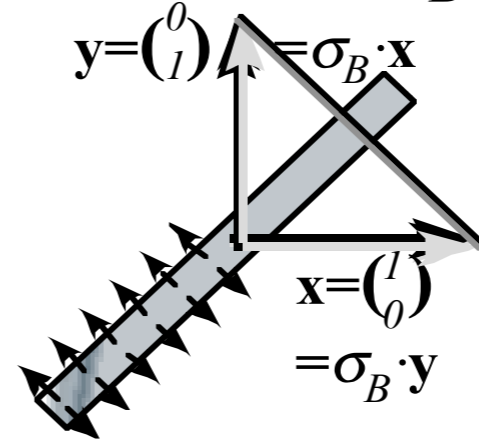
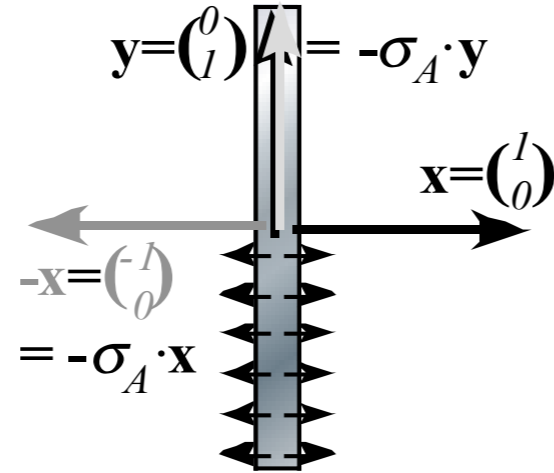
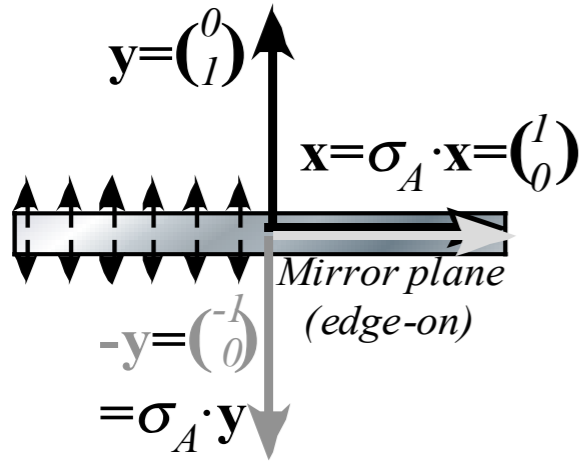


Fig. 4.3
(Unit 1)

Symmetry: It's all done with mirrors!

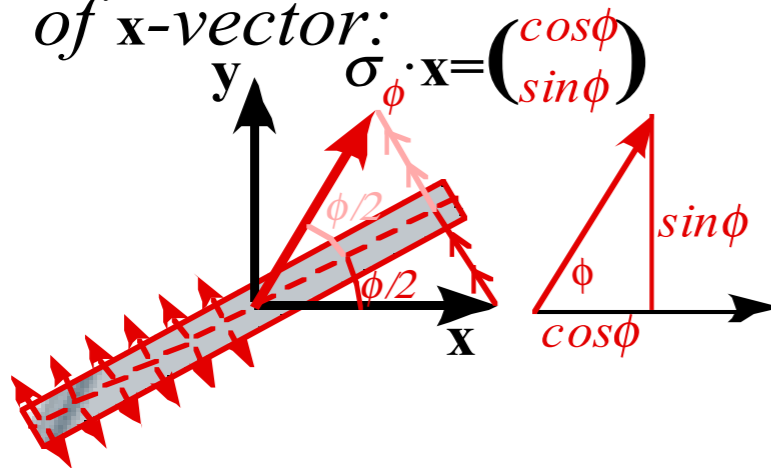
(a) Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) Reflections $\sigma_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $-\sigma_B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$



(c) σ_ϕ reflection $\begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{pmatrix}$

of x -vector:



...of y -vector:

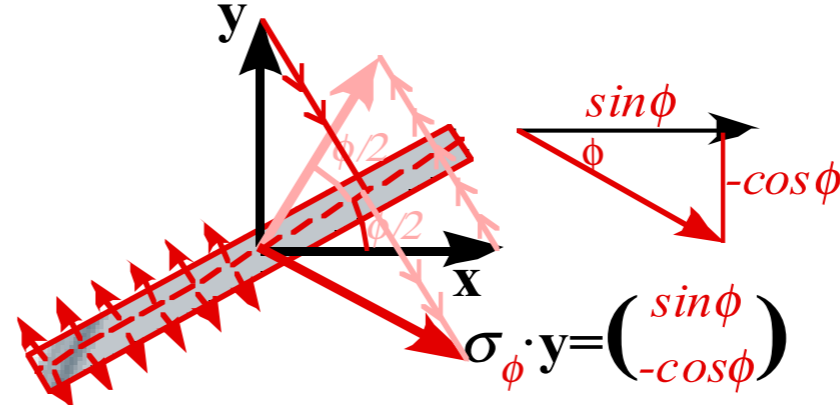
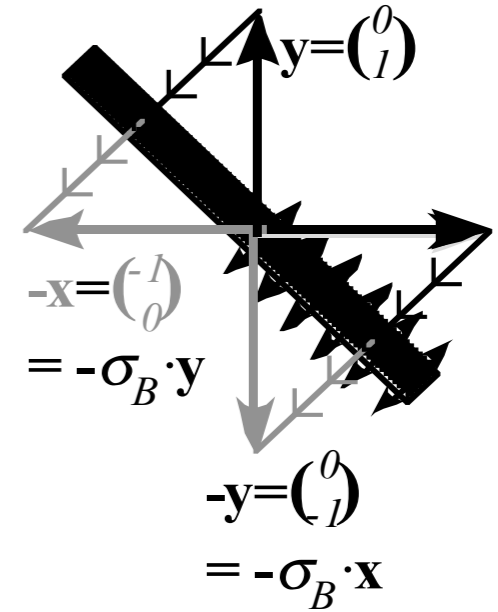
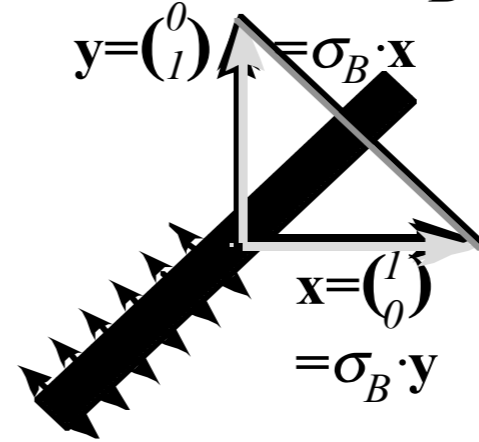
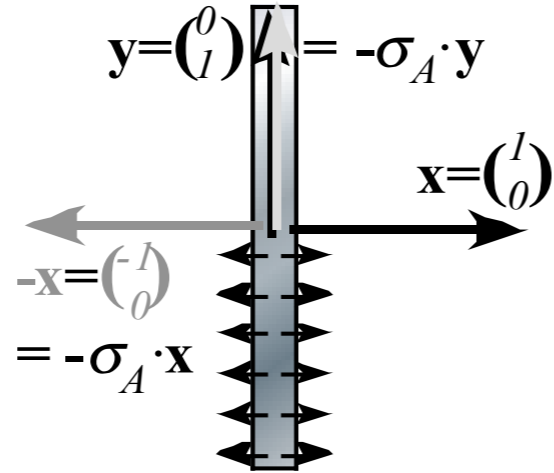
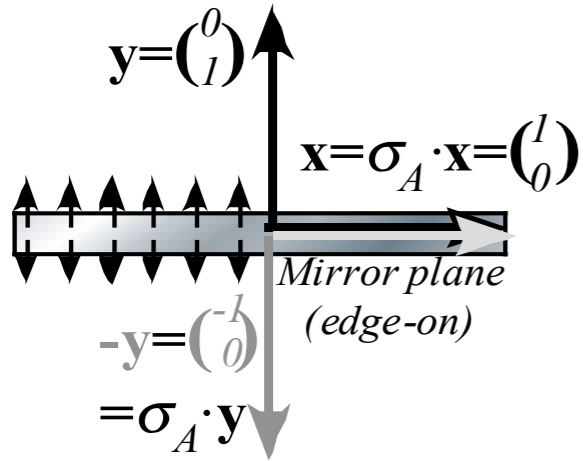


Fig. 4.3
(Unit 1)

Symmetry: It's all done with mirrors!

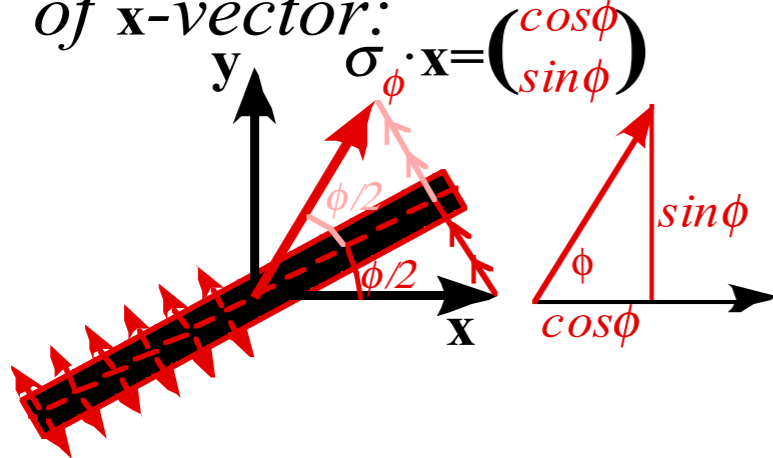
(a) Reflections $\sigma_A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $-\sigma_A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) Reflections $\sigma_B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $-\sigma_B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$



(c) σ_ϕ reflection $\begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{pmatrix}$

of \mathbf{x} -vector:



...of \mathbf{y} -vector:

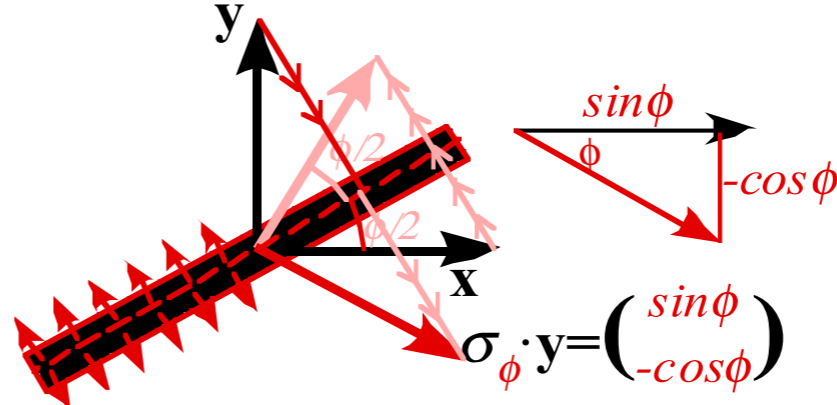
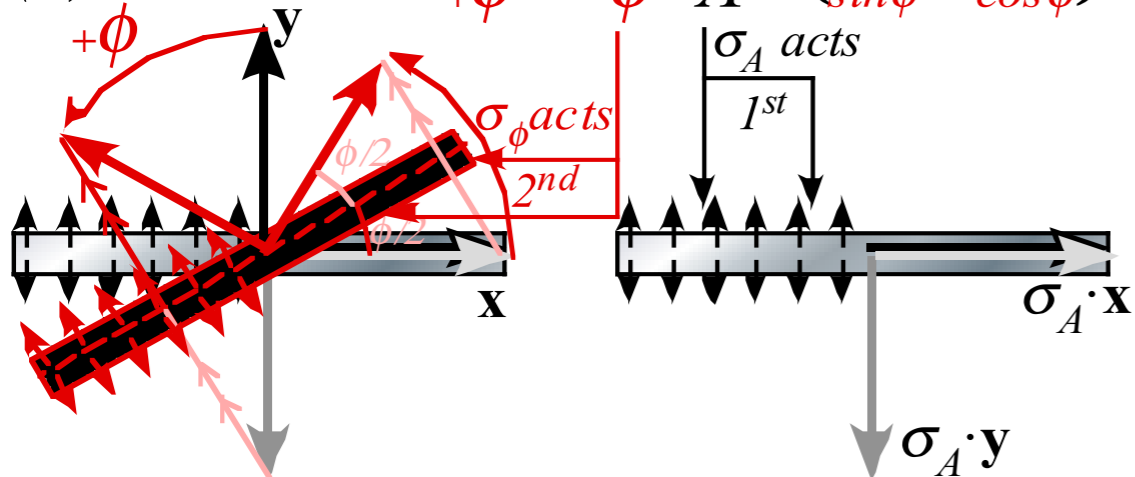


Fig. 4.3
(Unit 1)

(d) Rotation: $R_{+\phi} = \sigma_\phi \sigma_A = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$



(e) Rotation: $R_{-\phi} = \sigma_A \sigma_\phi = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$

