Lecture 22.

Relativity of wave-optics and Lorentz-Minkowski coordinates I.

(Ch. 2 of Unit 2)

1. Optical wave coordinates and frames
   - Old-fashioned vs. New-fashioned spacetime frames
   - Dueling lasers make lab frame space-time grid (CW or PW)
     - Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames

2. Applying Occam's razor to relativity axioms
   - Einstein PW Axioms versus Evenson CW Axioms
     - CW light clearly shows Doppler shifts
     - Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.
       - Is dispersion linear?... does astronomy work?... how about spectroscopy?
     - Is Doppler a geometric factor or arithmetic sum?
       - Introducing rapidity $\rho = \ln b$.
     - That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

3. Spectral theory of Einstein-Lorentz relativity
   - Applying Doppler Shifts to per-space-time (ck,ω) graph
   - CW Minkowski space-time coordinates (x,ct) and PW grids ← Lecture 22 ended (about) here
     - Relating Doppler Shifts b or $r=1/b$ to velocity u/c or rapidity ρ
     - Lorentz transformation

1. Optical wave coordinates and frames

→ *Old-fashioned vs. New-fashioned spacetime frames*

Dueling lasers make lab frame space-time grid (CW or PW)

Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames

• *Optical wave coordinate manifolds and frames*

Shining some light on light using complex phasor analysis

Old-fashioned meter-stick-clock frames

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)

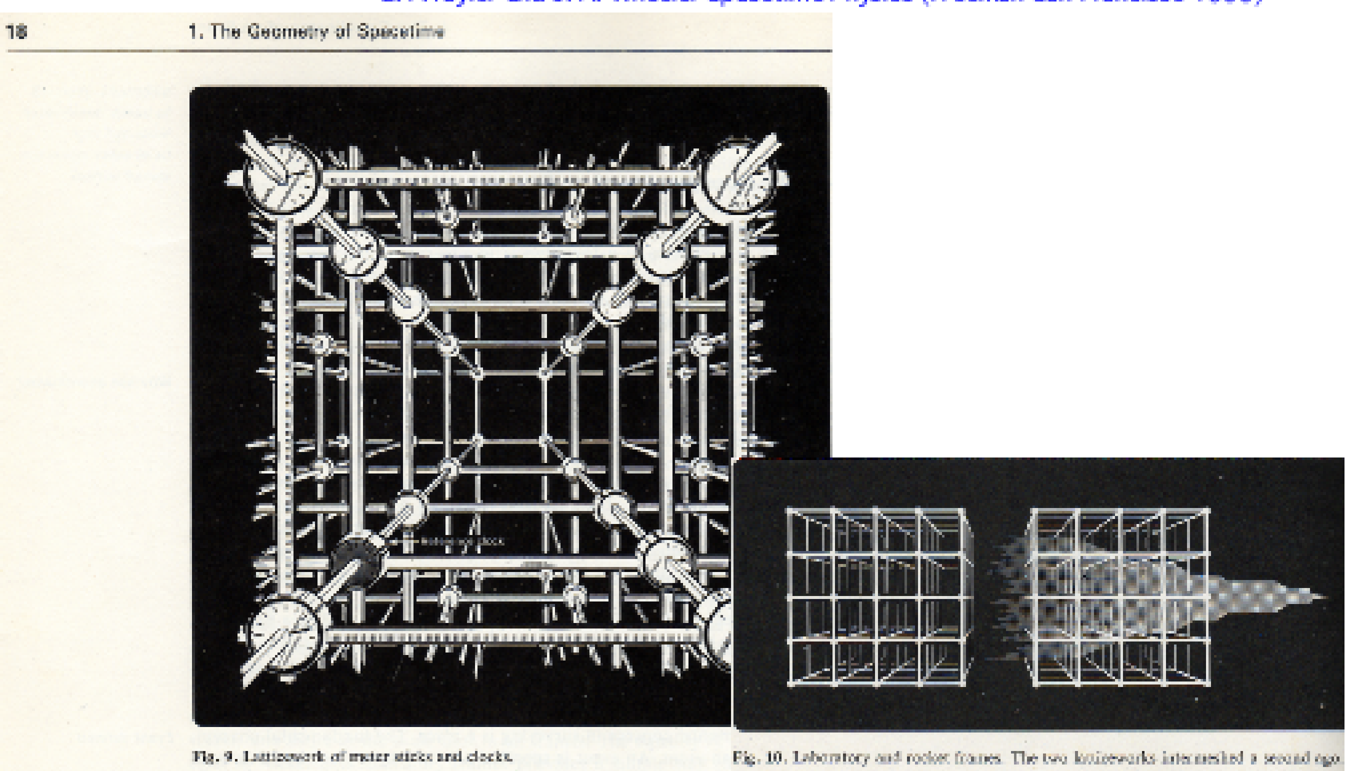


Fig. 9. Latticework of meter sticks and clocks.

Fig. 10. Laboratory and rocket frames. The two latticeworks intermeshed a second ago

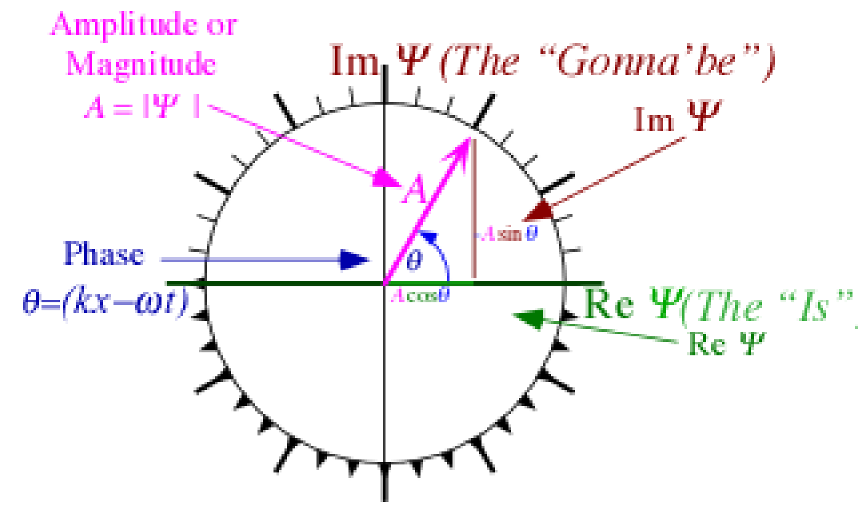
New-fashioned laser clocks & meter sticks

Complex Phasor Clocks : Tesla's AC "phasor"

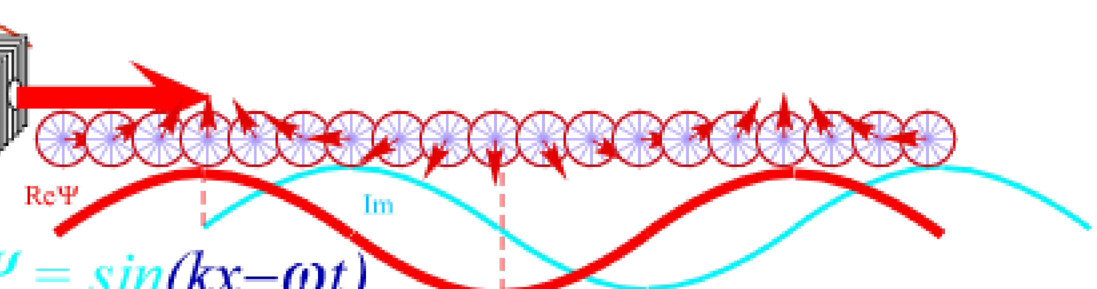
Quantum Phasor Clock

$$\Psi = Ae^{i(kx-\omega t)}$$
$$= A\cos(kx-\omega t)$$
$$+ i\,A\sin(kx-\omega t)$$

Phasor clocks turn clockwise in time for positive ω



300THz Laser plane wave $\langle x,t\,|\,k,\omega\rangle = Ae^{i(kx - wt)}$



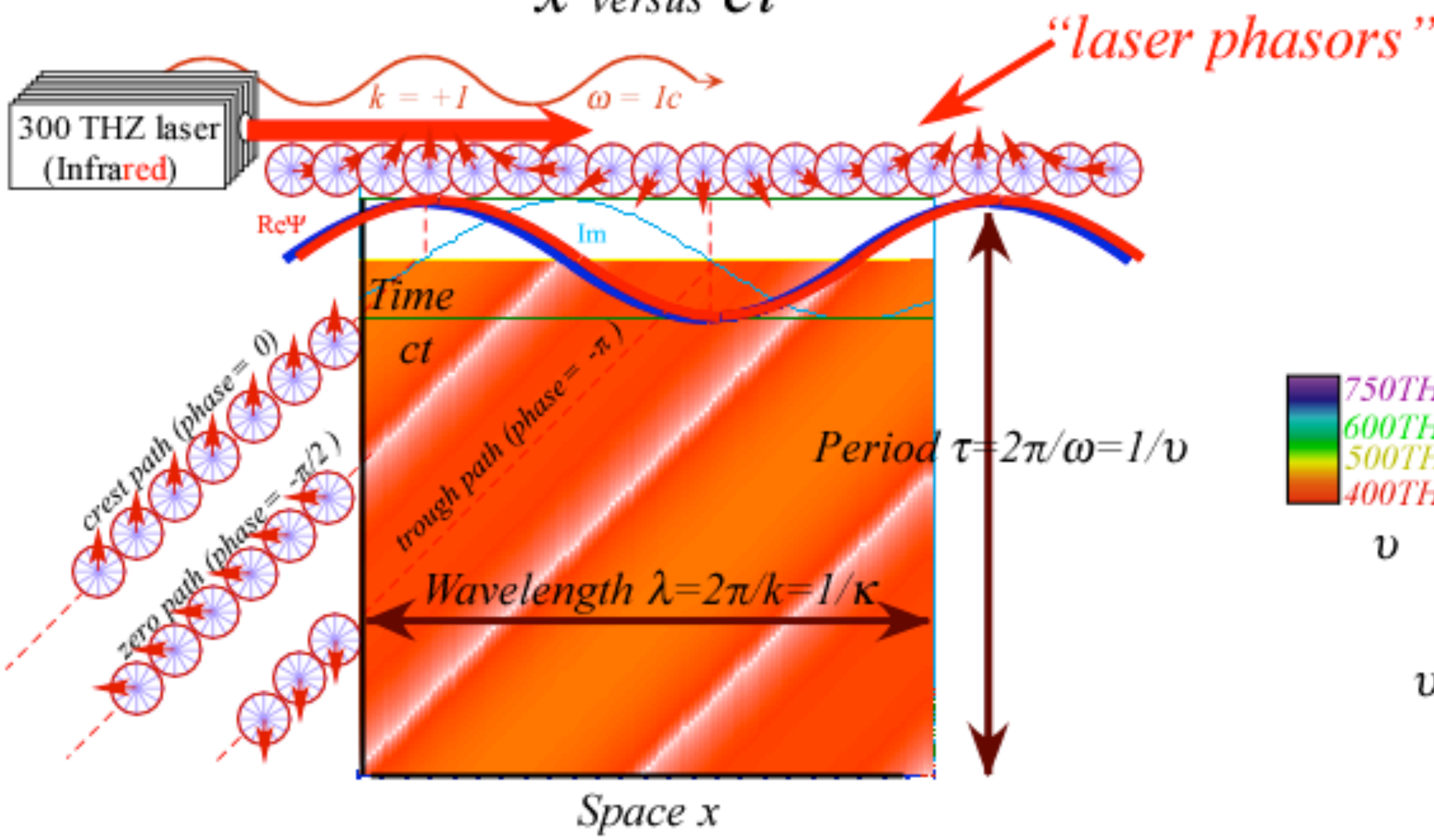
$Re\,\Psi = \cos(kx-\omega t)$

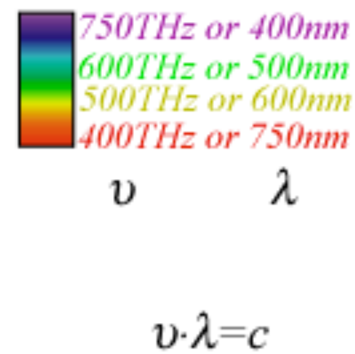
$Im\,\Psi = \sin(kx-\omega t)$

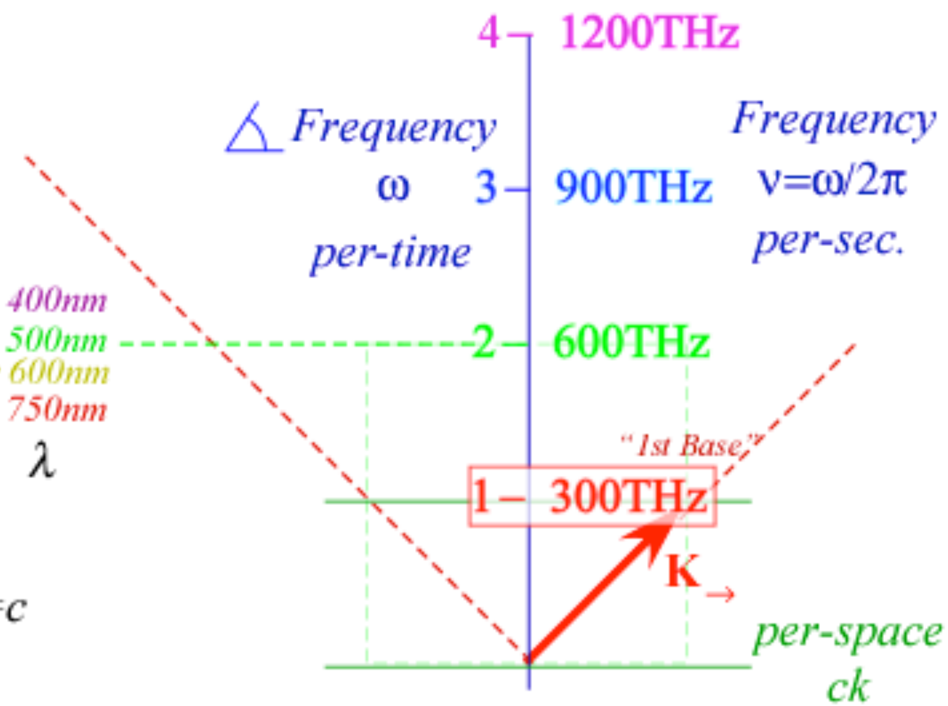
New-fashioned laser clocks & meter sticks (contd.)
Dual views:

(1.) Spacetime
x versus ct

(2.) Per-Spacetime
ω versus ck







Single plane-wave meter-stick-clocks are too fast (can't catch'em)

(...But at least this view is constant)

Interfering wave pairs needed to make rest frame coordinates...

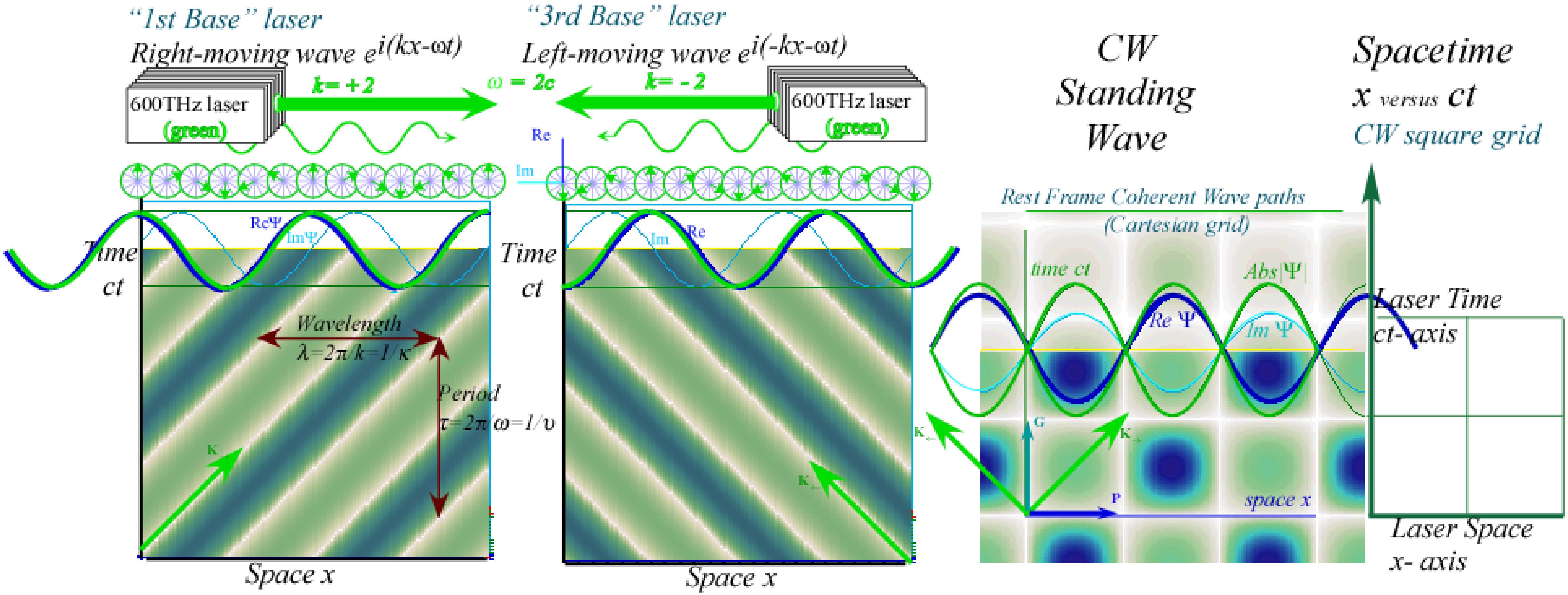
1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames

➡ *Dueling lasers make lab frame space-time grid (CW or PW)*

Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames

Zeros of head-on CW sum gives (x,ct)-grid



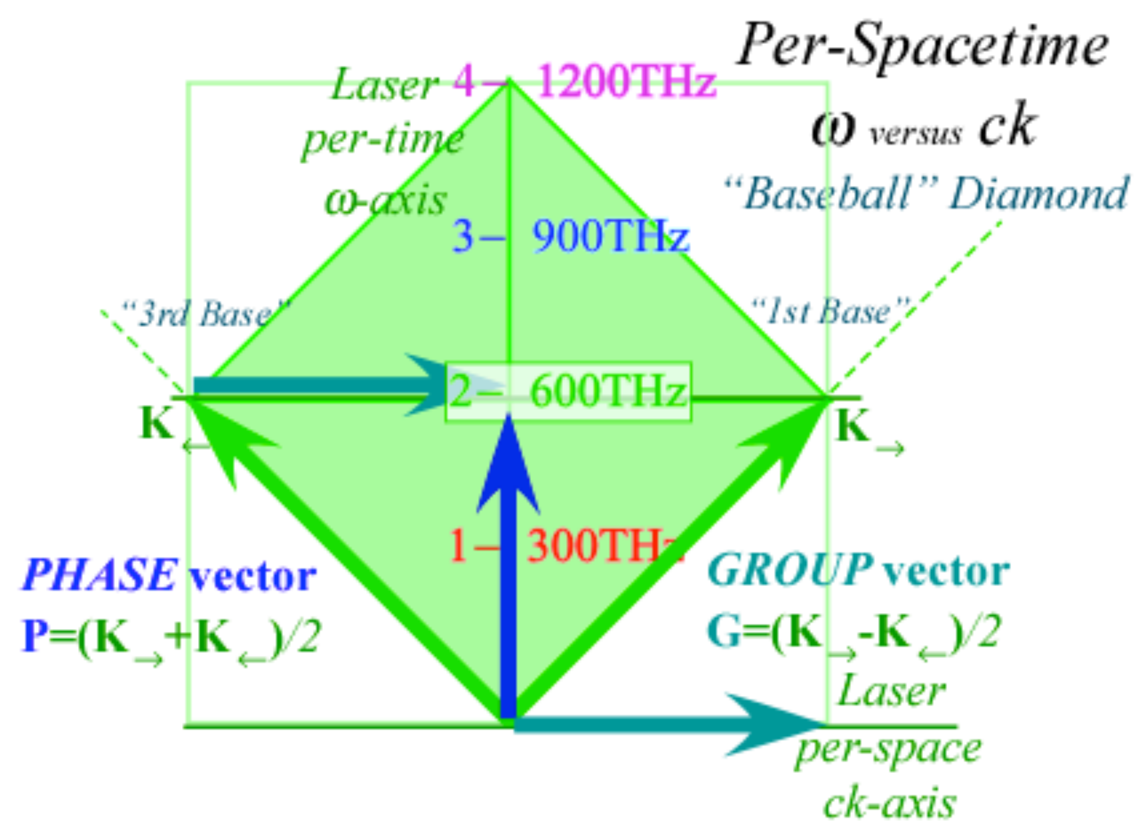
Find zeros by factoring sum:

$$\Psi = \overbrace{e^{ia}}^{(kx-\omega t)} + \overbrace{e^{ib}}^{(-kx-\omega t)}$$

$$= \underbrace{e^{i(a+b)/2}}_{\text{Phase factor}} \underbrace{\left(e^{i(a-b)/2} + e^{-i(a-b)/2}\right)}_{\text{Group factor}}$$

Phase factor: $\exp i\left(\frac{a+b}{2}\right) = e^{-i\omega t}$

Group factor: $2\cos\left(\frac{a-b}{2}\right) = 2\cos(kx)$



1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames

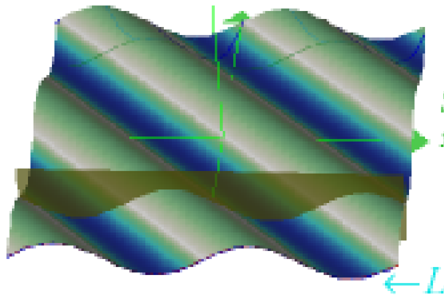
Dueling lasers make lab frame space-time grid (CW or PW)

→ *Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames*

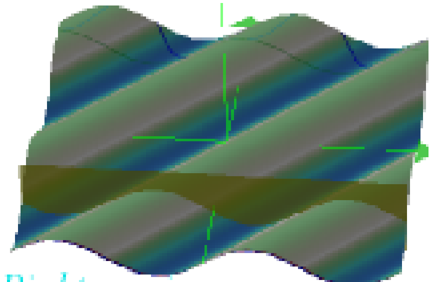
Newton's "Fits" in Optical Interference

Newton complained that light waves have *"fits"* (what we now know as wave *interference* or *resonance*.)
Examples of interference are head-on collision of two *Continuous Waves (2-CW)* or two *Pulse Waves (PW)*

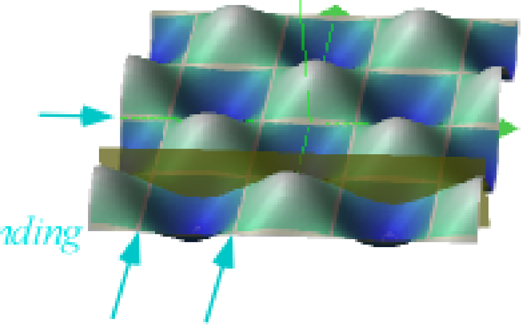
Continuous Wave (CW) Addition



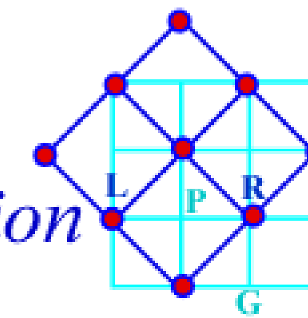
PLUS



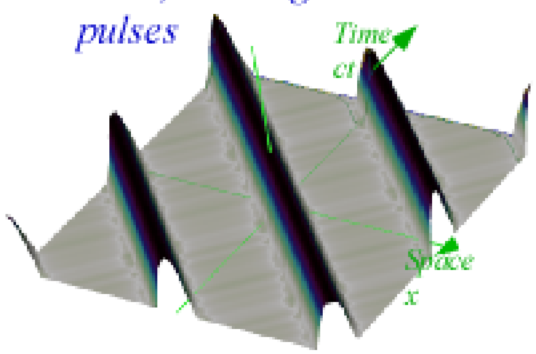
EQUALS



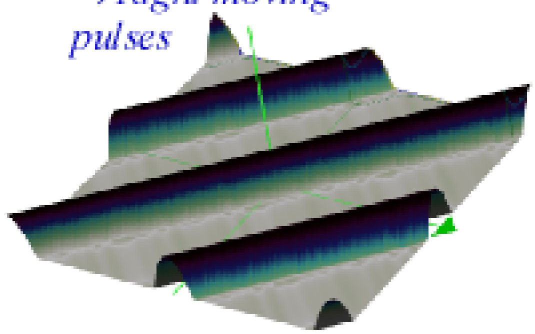
Sharp zeros trace
square grid
(Peaks are diffuse)



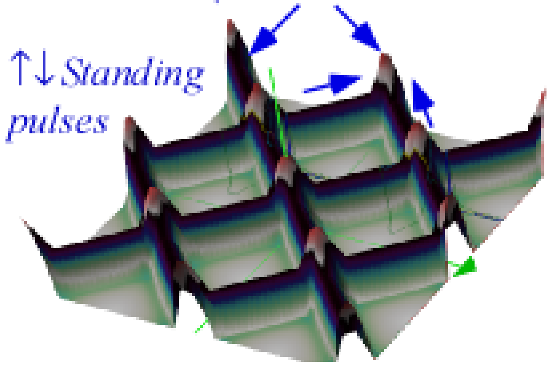
Pulse Wave (PW) Addition



PLUS



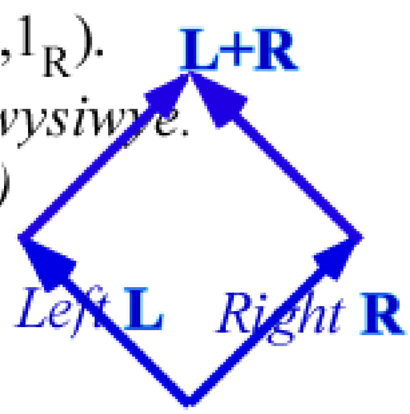
EQUALS



(Zeros are diffuse)
Sharp peaks trace
diamond grid

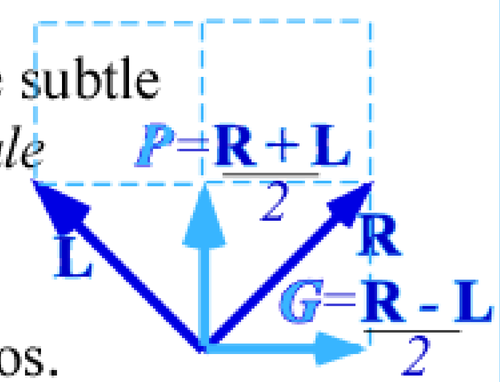
Pulse Wave (PW) sum compared with

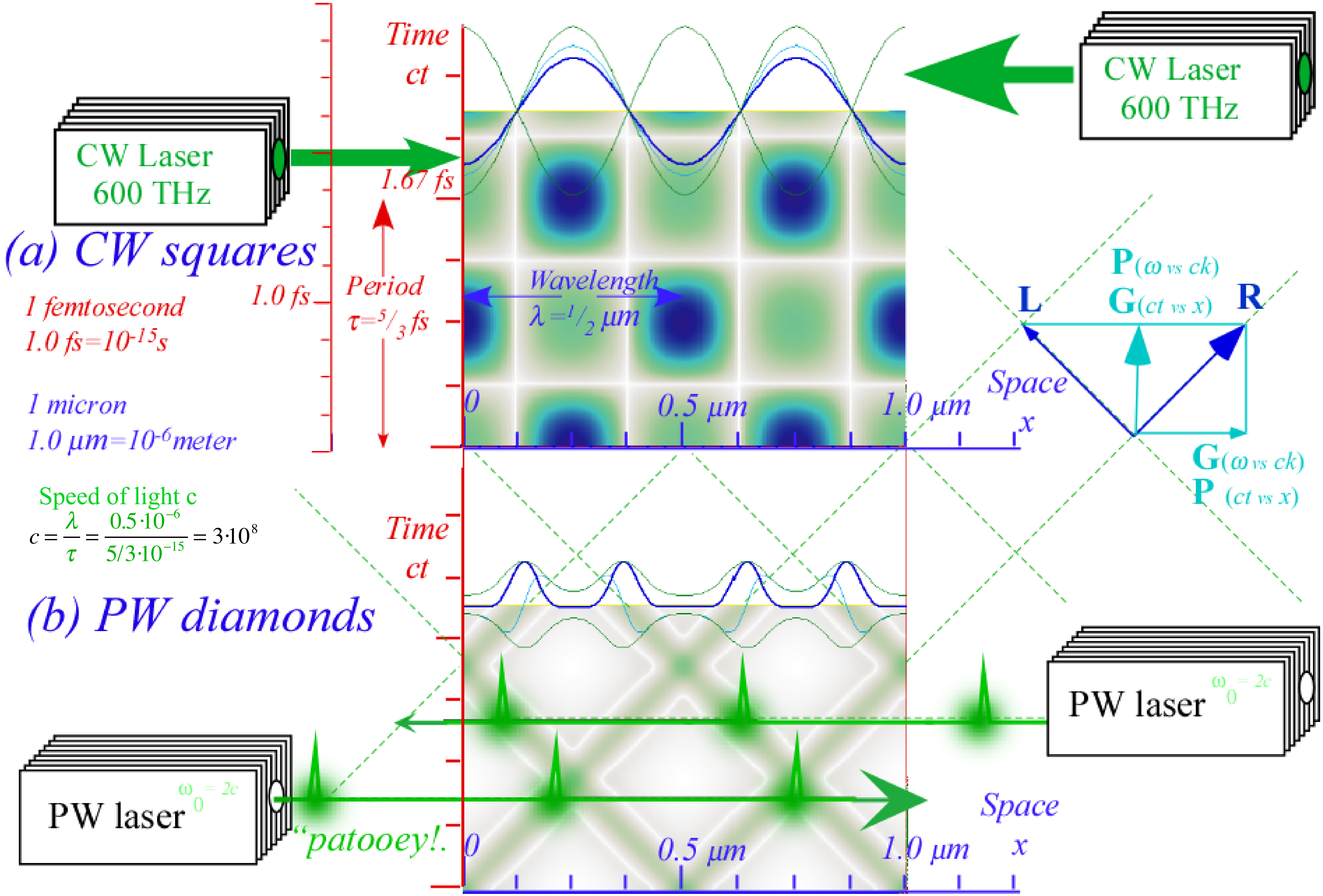
- *PW* waves are OFF (0) or ON (1)
- *PW* sum is Boolean $(0_L,0_R),(0_L,1_R),(1_L,0_R),(1_L,1_R)$.
- *PW* time *peak-diamond* paths are *wysiwye*.
  (What you see is what you expect!)



Continuous Wave (CW) sum

- *CW* waves range continously from -1 to +1
- *CW* sum is more subtle and nuanced *interference*.
- *CW* time *zero-square* paths are subtle results of the *half-sum* ***P****-rule* and the *half-difference* ***G****-rule* of *phase* ***P*** and *group* ***G*** zeros.





CW Laser
600 THz
CW Laser
600 THz
Time
ct
1.67 fs
(a) CW squares
1.0 fs
Period
τ=5/3 fs
Wavelength
λ=1/2 μm
1 femtosecond
1.0 fs=10-15s
1 micron
1.0 μm=10-6meter
0 0.5 μm 1.0 μm
Space
x
P(ω vs ck)
G(ct vs x)
L
R
G(ω vs ck)
P (ct vs x)
Speed of light c
c = λ/τ = 0.5·10-6 / 5/3·10-15 = 3·108
Time
ct
(b) PW diamonds
PW laser
PW laser
"patooey!.
0 0.5 μm 1.0 μm
Space
x

2. Applying Occam's razor to relativity axioms

→ *Einstein PW Axioms versus Evenson CW Axioms*

CW light clearly shows Doppler shifts

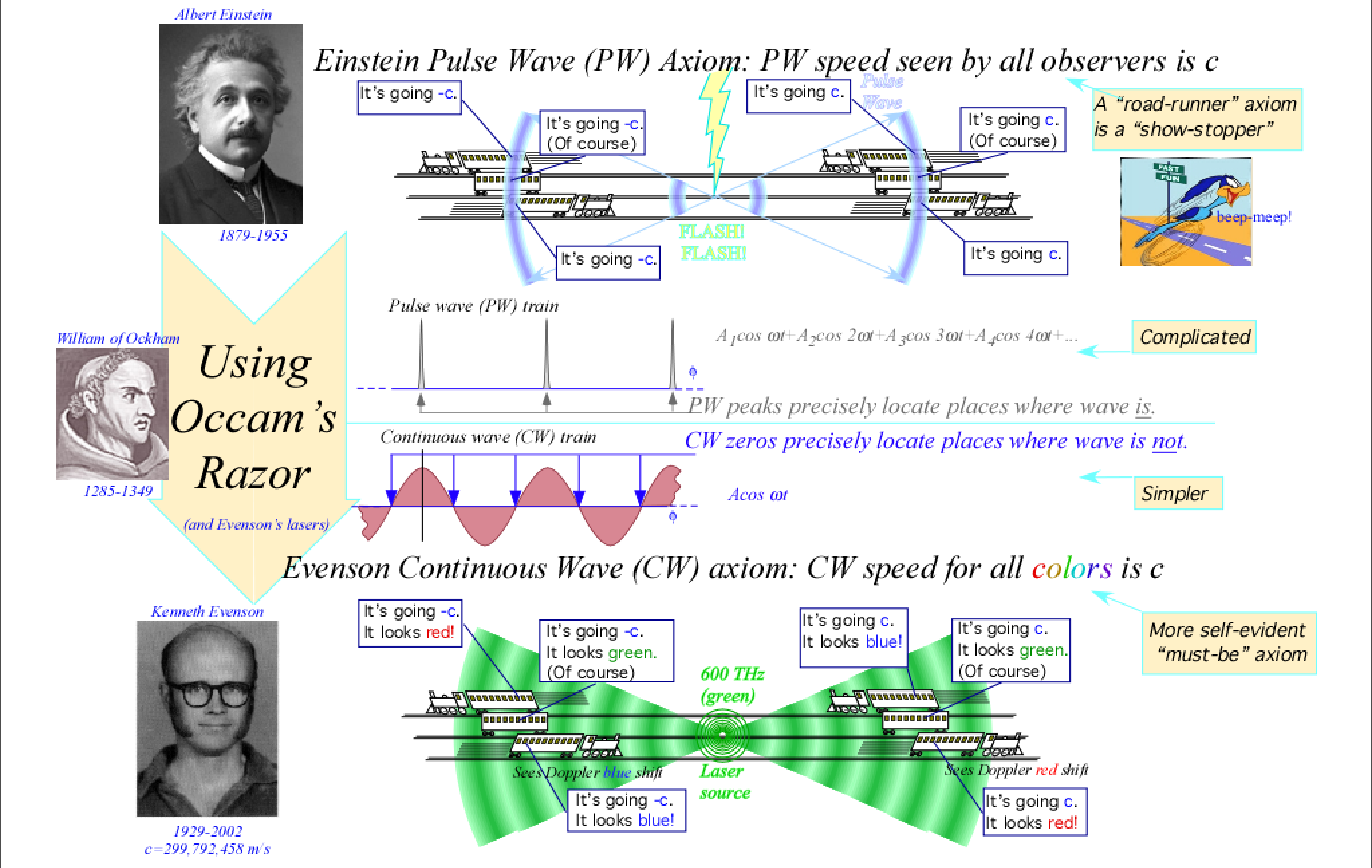
Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear?... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)


Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c
Albert Einstein
1879-1955
It's going -c.
It's going -c. (Of course)
It's going -c.
It's going c.
It's going c. (Of course)
It's going c.
Pulse Wave
FLASH! FLASH!
A "road-runner" axiom is a "show-stopper"
beep-meep!
Pulse wave (PW) train
$A_1\cos\omega t+A_2\cos 2\omega t+A_3\cos 3\omega t+A_4\cos 4\omega t+\ldots$
Complicated
PW peaks precisely locate places where wave is.
Continuous wave (CW) train
CW zeros precisely locate places where wave is not.
$A\cos\omega t$
Simpler
William of Ockham
1285-1349
Using Occam's Razor
(and Evenson's lasers)
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c
More self-evident "must-be" axiom
Kenneth Evenson
1929-2002
c=299,792,458 m/s
It's going -c. It looks red!
It's going -c. It looks green. (Of course)
It's going -c. It looks blue!
Sees Doppler blue shift
600 THz (green)
Laser source
It's going c. It looks blue!
It's going c. It looks green. (Of course)
It's going c. It looks red!
Sees Doppler red shift

2. Applying Occam's razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms

→ *CW light clearly shows Doppler shifts*

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear? ... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old ***Time-Reversal*** *meta-Axiom (that is so-oo-o neglected!)*

Doppler Shifts in Spacetime

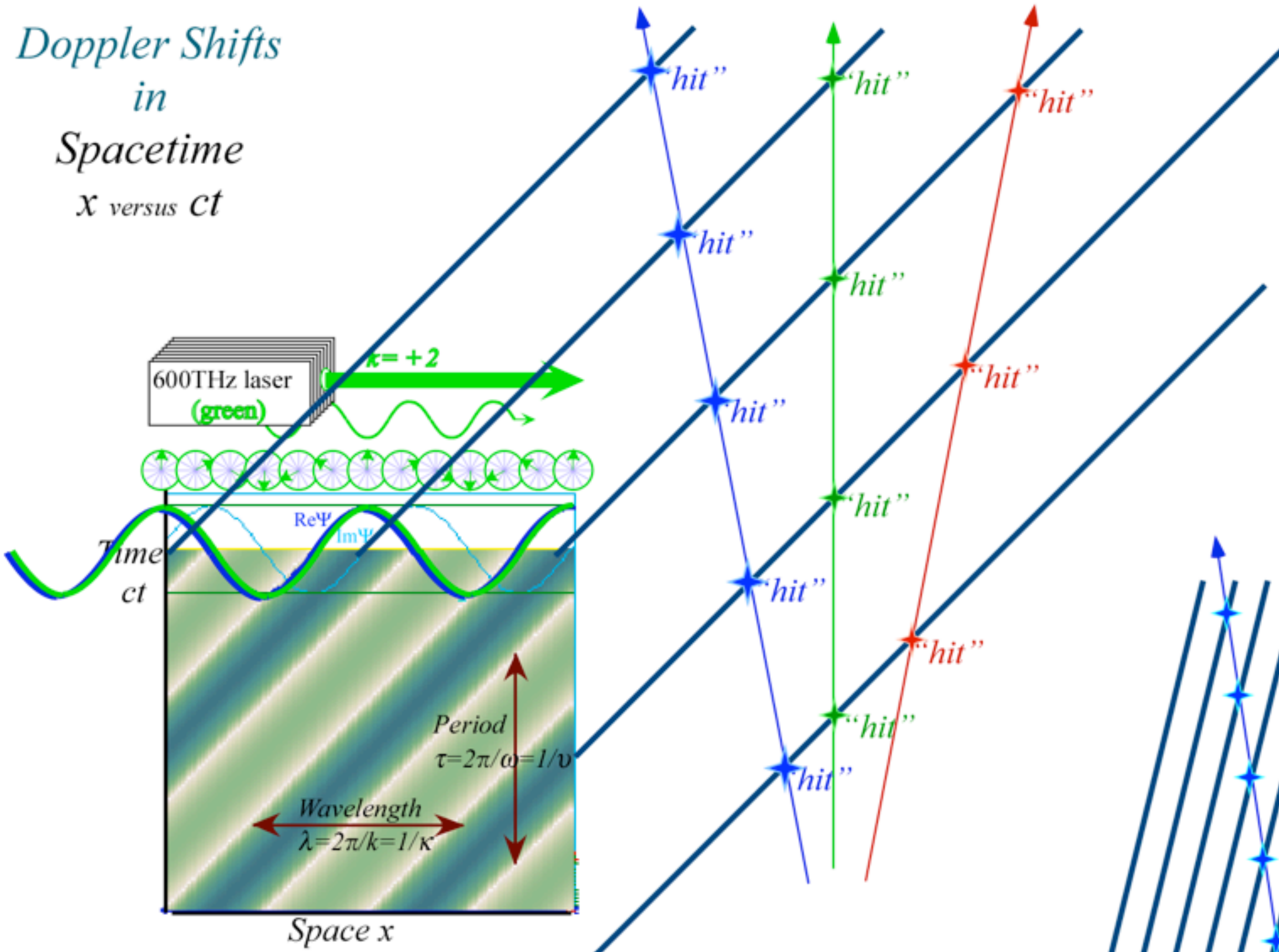
x versus *ct*

Doppler Blueshift

More "hits" per sec. if moving toward laser source

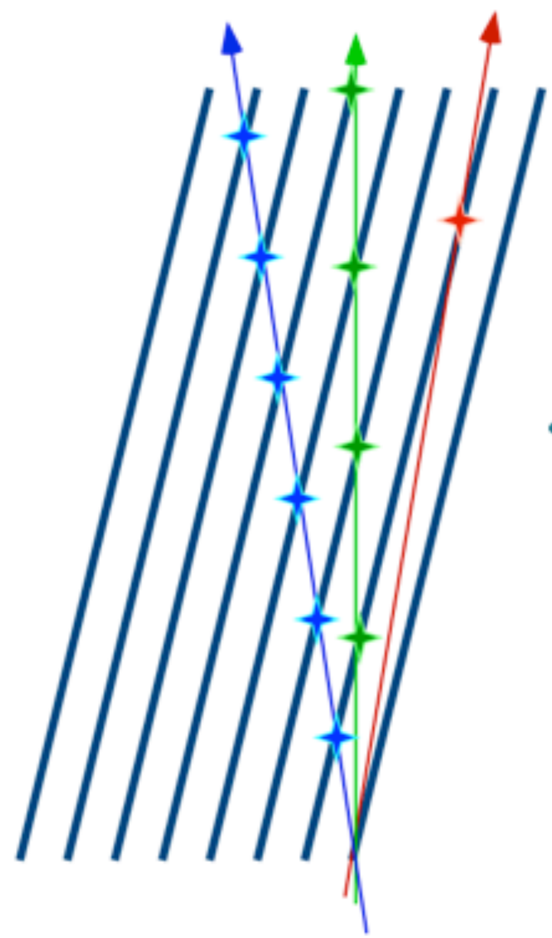
Doppler Redshift

Fewer "hits" per sec. if moving away from laser source



Doppler's picture needs revision for light whose period and wavelength both shift.

Why?



...So that all colors go the same speed!

$$\upsilon \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c$$

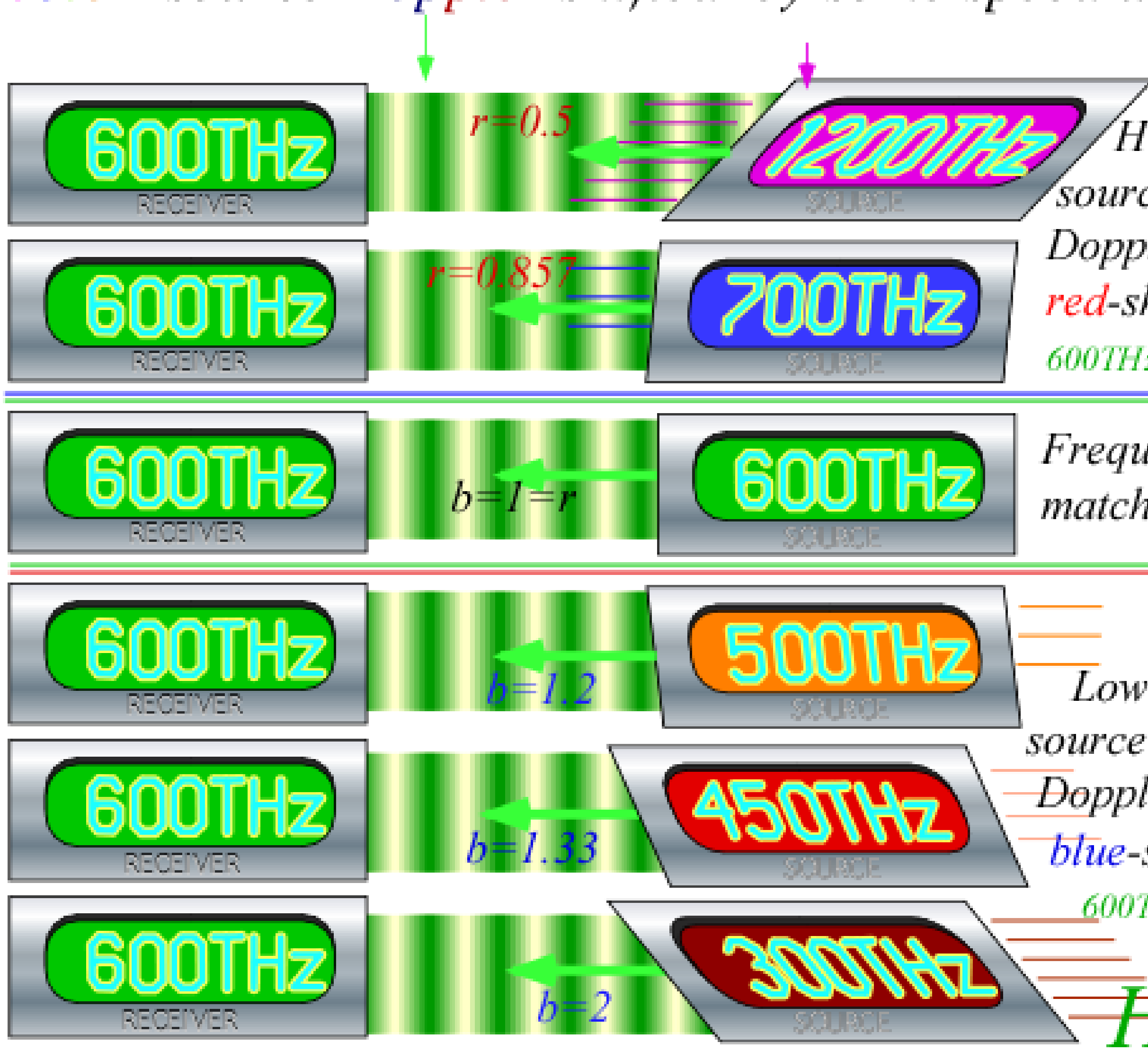
$$\upsilon \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c \quad \text{etc.}$$

$$\upsilon \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c$$

CW Axiom ("All colors go c.") based on Doppler effects

Showing that Green is Green is Green...(and all the same speed)...

Any color (like 600THz green) may be made by any other color source Doppler shifted by some speed u (less than c)

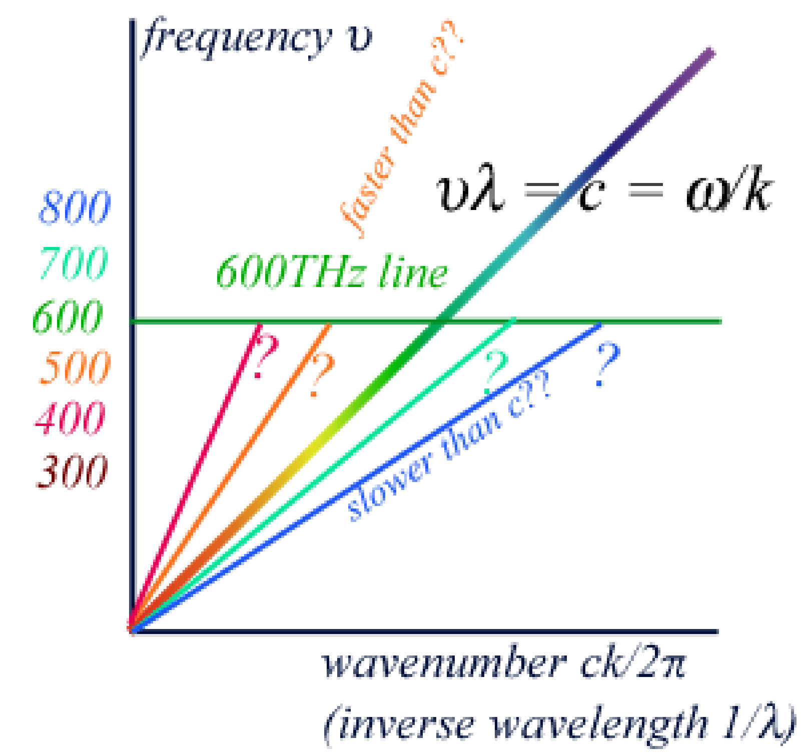


Higher frequency υ source recedes so its Doppler factor (0<r<1) red-shifts υ to match a 600THz-tuned receiver.

Frequency υ=600THz matches receiver.

Lower frequency υ source approaches so its Doppler factor (1<b<∞) blue-shifts υ to match a 600THz-tuned receiver.

How many ways can you make 600THz green?



How many kinds of green exist?
(It's either 1 or ∞.)

2. Applying Occam's razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms

CW light clearly shows Doppler shifts

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear?... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

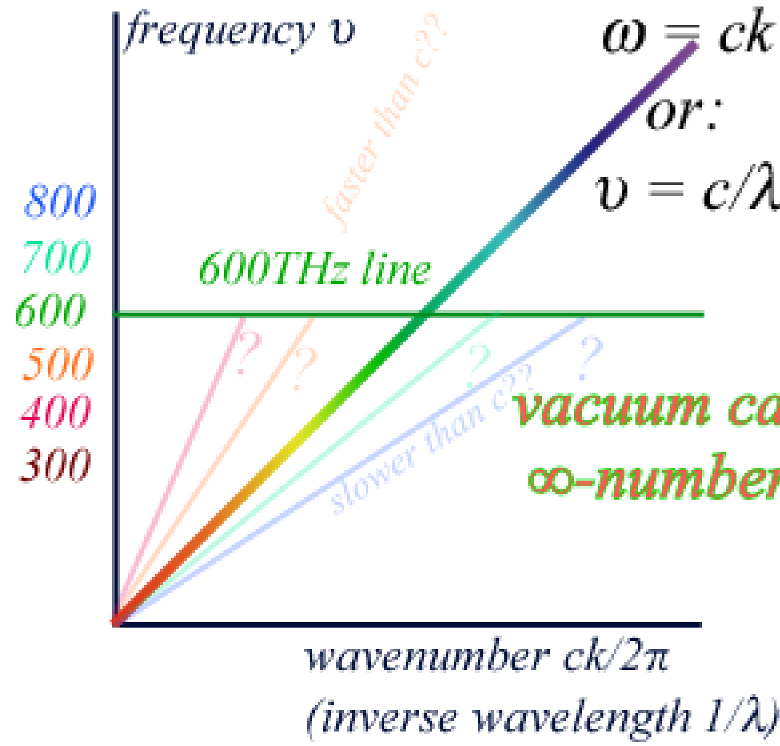
That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

Linear dispersion: $\omega = ck$

Linear dispersion means NO dispersion

Einstein PW is corollary of Evenson CW



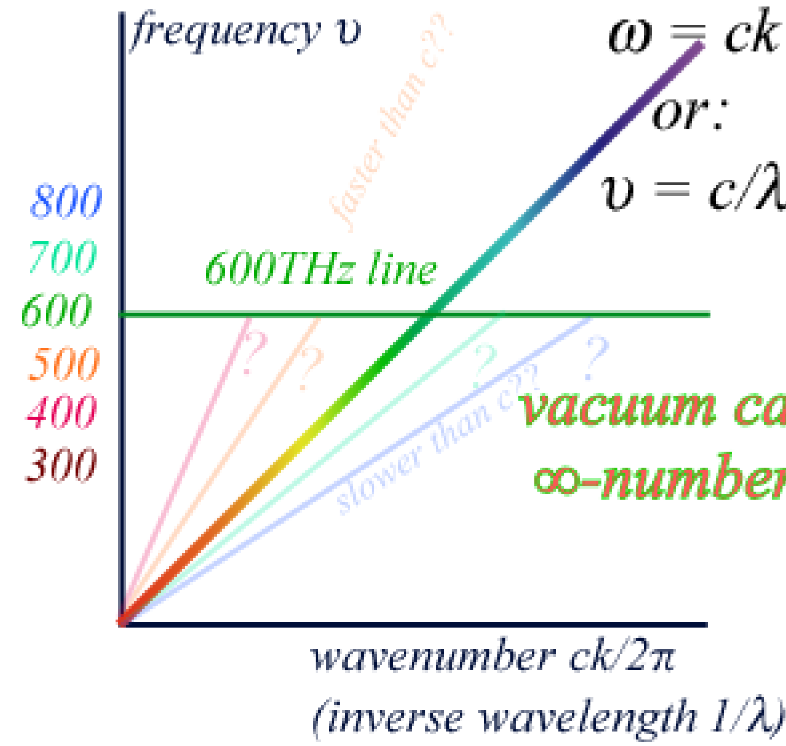
vacuum can't support an ∞-number of "other speeds"

Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

Linear dispersion: $\omega = ck$

Linear dispersion means NO dispersion

Einstein PW is corollary of Evenson CW



vacuum can't support an ∞-number of "other speeds"

What if blue were to travel 0.001% slower than red from a galaxy 9 billion light years away? (..and show up 10^5 years late)

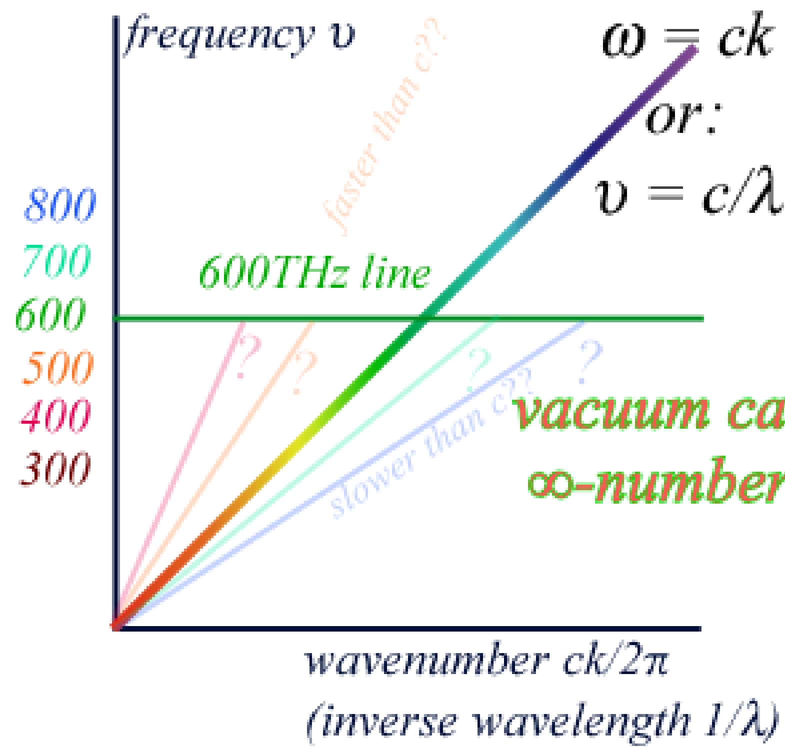
That would mean Good-Bye Hubble Astronomy!

Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

Linear dispersion: $\omega = ck$

Linear dispersion means NO dispersion

Einstein PW is corollary of Evenson CW

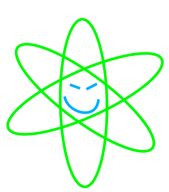


vacuum can't support an ∞-number of "other speeds"

What if blue were to travel 0.001% slower than red from a galaxy 9 billion light years away? (..and show up 10^5 years late)

That would mean Good-Bye Hubble Astronomy!

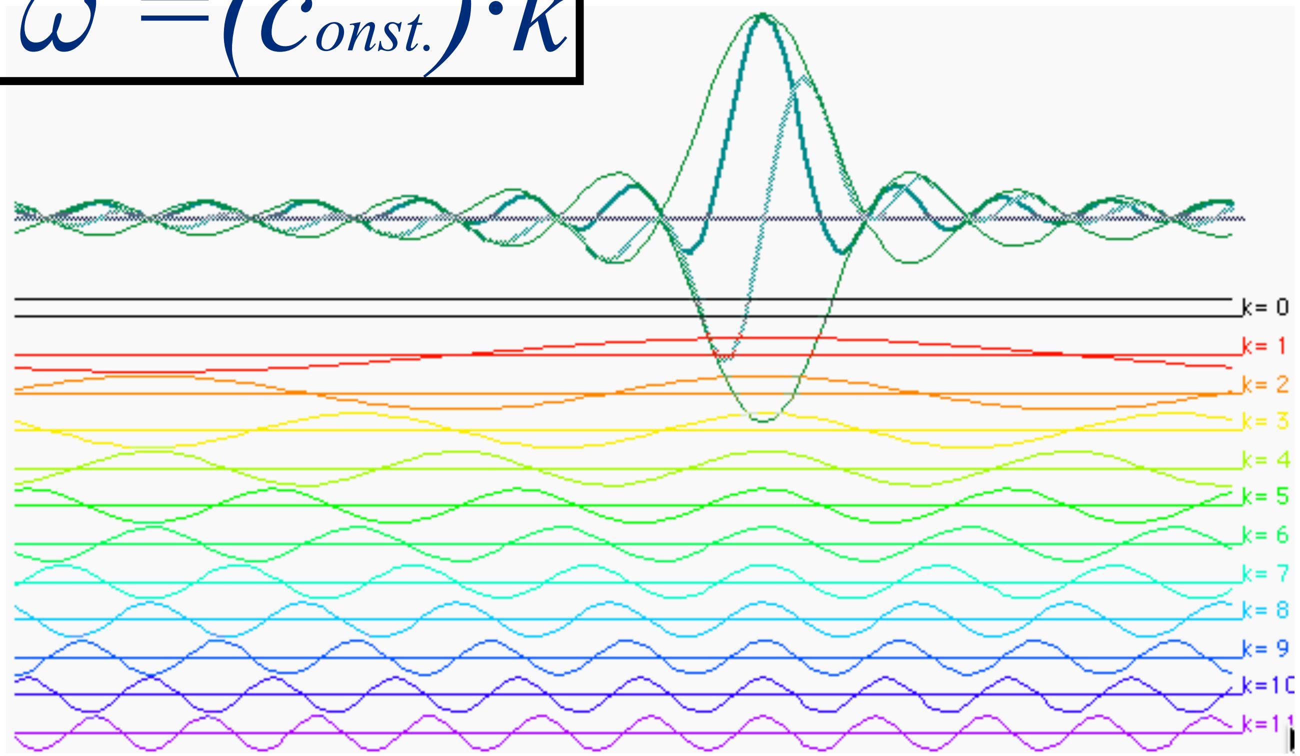
What if ν=600THz green excited an Ar atom but NOT a λ=0.500μm optical cavity? (or vice-versa?)

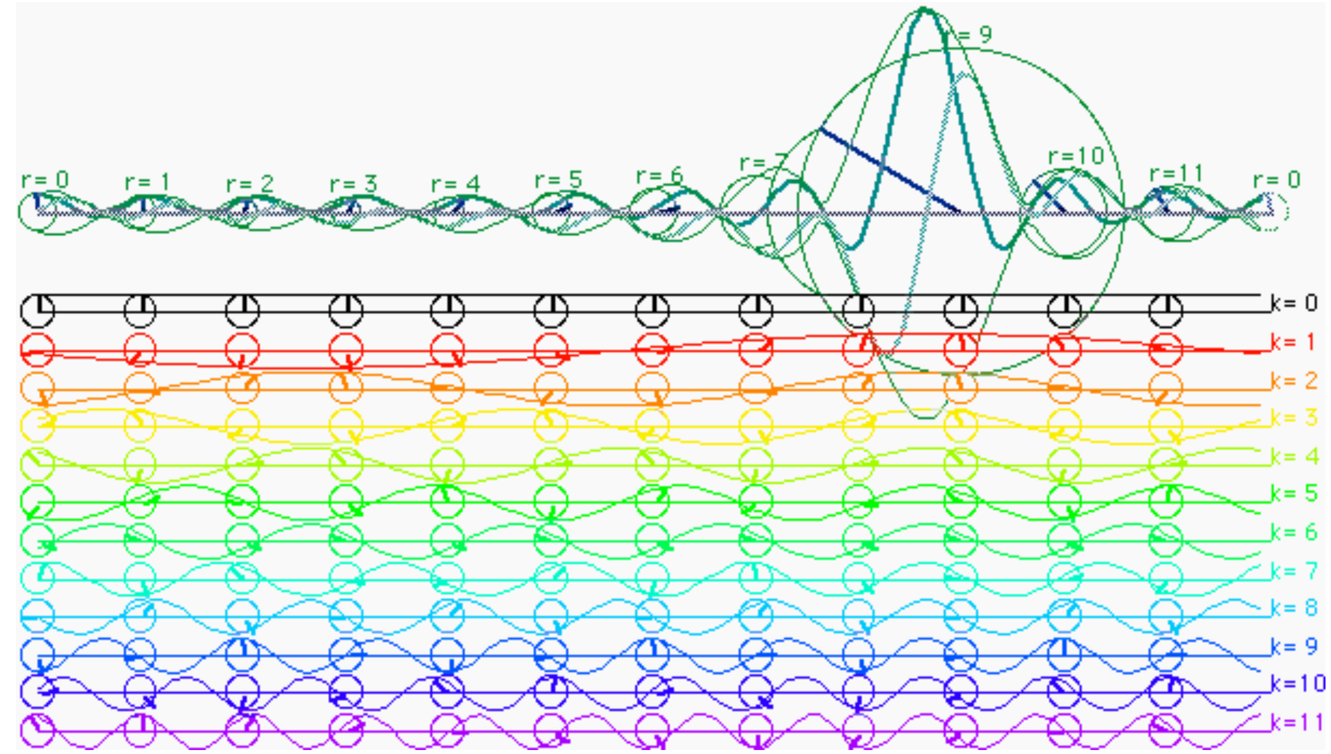


That would mean Good-Bye Light Amplification by Stimulated Emission of Radiation .

Linear Dispersion (means NO dispersion) has all colors (Fourier components) march in "lockstep"

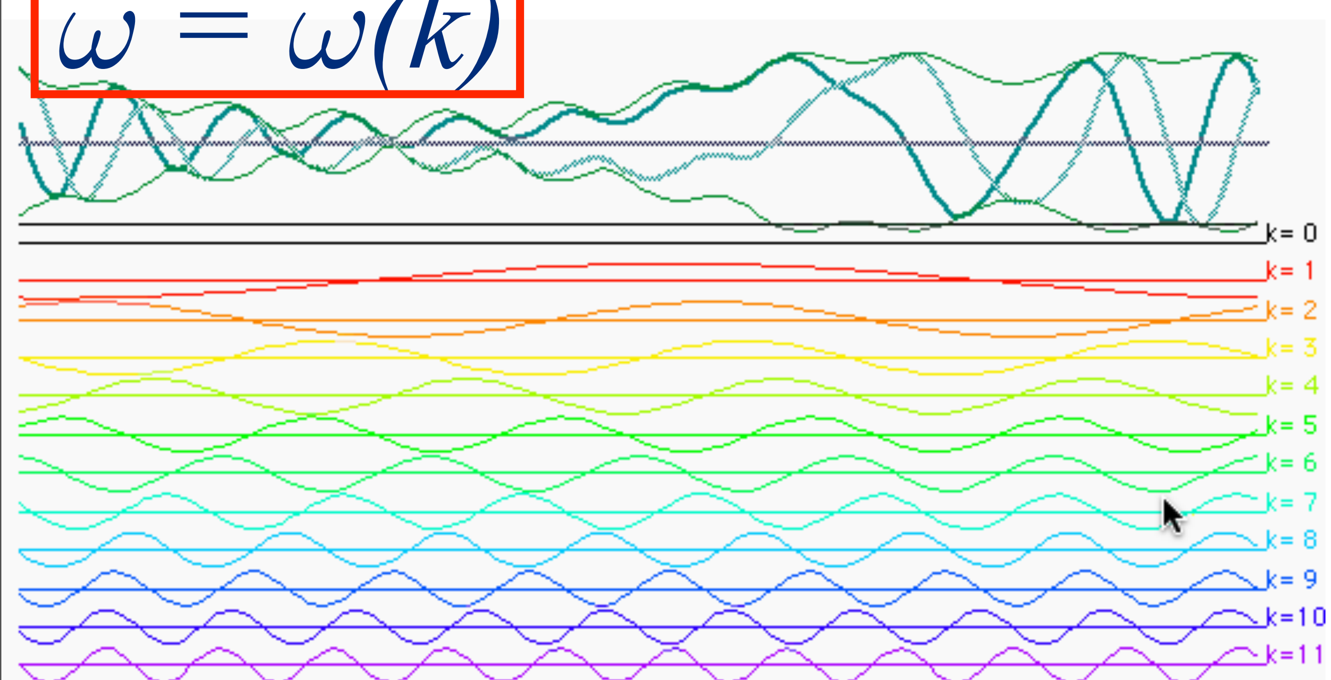
$$\omega = (c_{onst.}) \cdot k$$

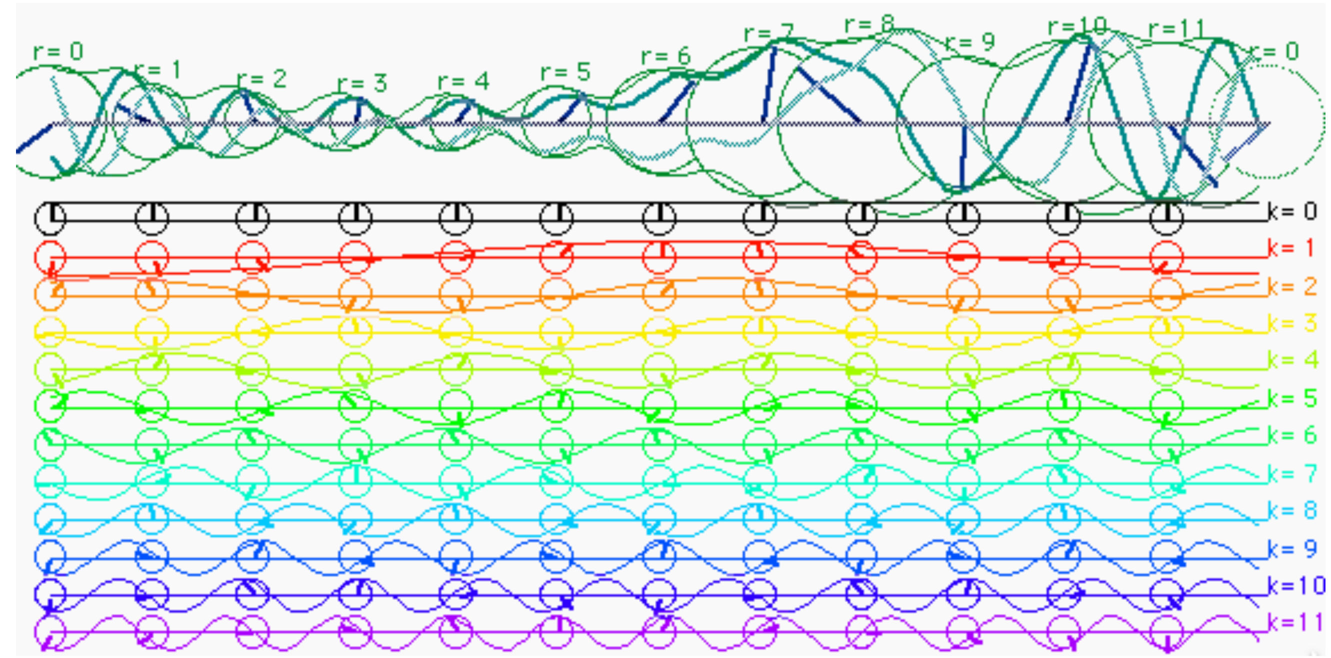




NON-linear Dispersion (has dispersion) so different colors (Fourier components) go different speeds

$$\omega = \omega(k)$$





See animation: www.uark.edu/ua/pirelli/php/train_PW_Occum_Evenson.php

2. Applying Occam's razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms

CW light clearly shows Doppler shifts

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear? ... does astronomy work?... how about spectroscopy?

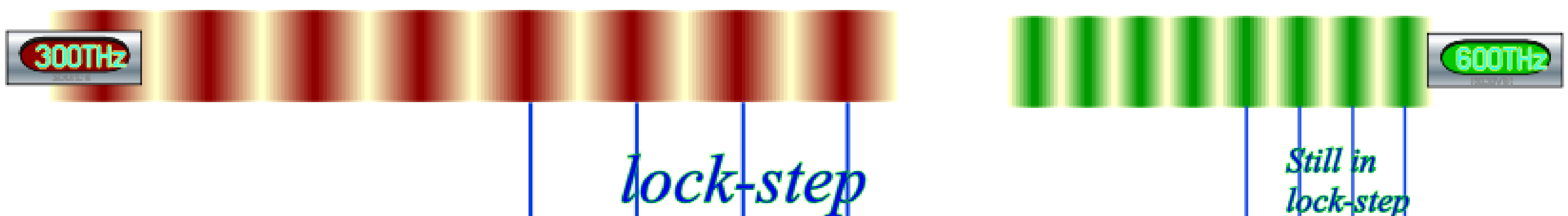
Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

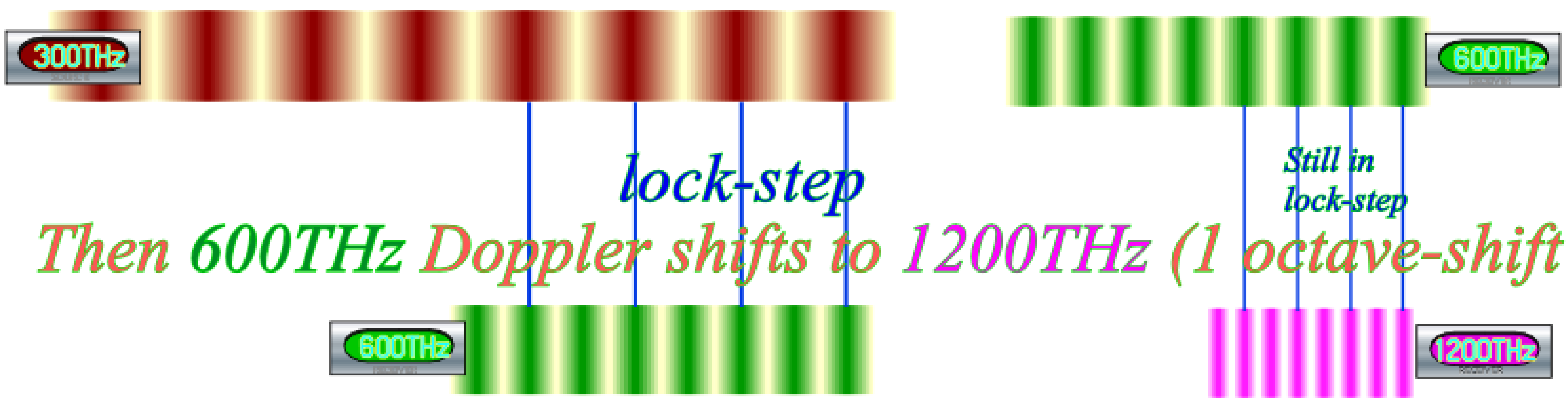
If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)



Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)

If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)



Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)

Doppler shifts maintain frequency ratios (not differences)

1-D Doppler shifts {red $= e^{-\rho}$ ...blue $= e^{+\rho}$} *form a Lie Group*

3-D Doppler shifts are hypercomplex elements of Lorentz Group

2. Applying Occam's razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms

CW light clearly shows Doppler shifts

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear?... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = ln\ b$.

That old ***Time-Reversal*** *meta-Axiom (that is so-oo-o neglected!)*

Frequency blue shift b when Source-Receiver interval is >>CLOSING<<

Frequency red shift r when Source-Receiver interval is <<OPENING>>

$$\frac{\upsilon_{IN}}{\upsilon_{OUT}} = \frac{\upsilon_{Receiver}}{\upsilon_{Source}} = b = e^{+|\rho|} > 1$$

$$\frac{\upsilon_{Receiver}}{\upsilon_{Source}} = r = e^{-|\rho|} < 1$$

Defining ***Rapidity*** ρ *as logarithm of Doppler*

$\rho = ln(b \text{ or } r)$

Frequency blue shift b when Source-Receiver interval is >>CLOSING<<

Frequency red shift r when Source-Receiver interval is <<OPENING>>

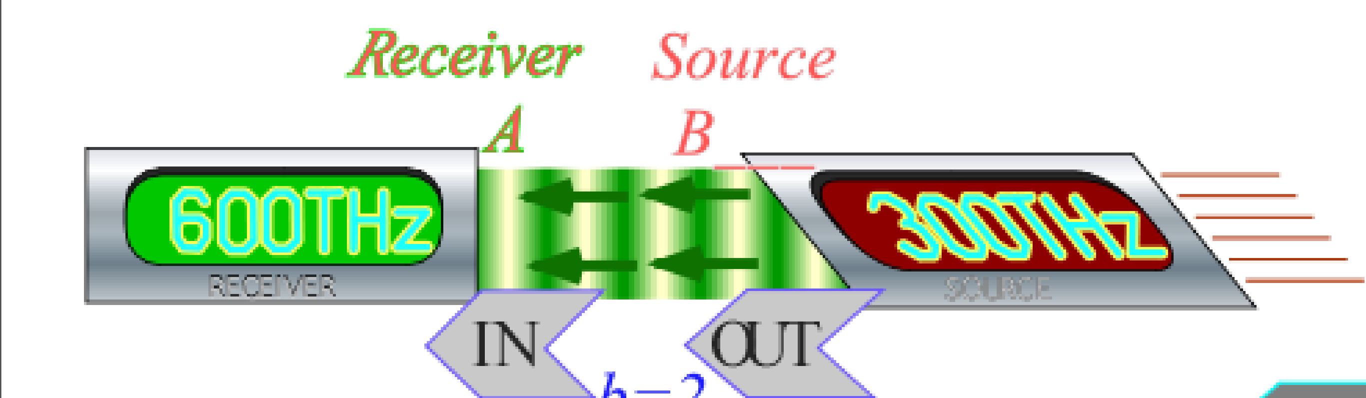
$$\frac{\upsilon_{IN}}{\upsilon_{OUT}} = \frac{\upsilon_{Receiver}}{\upsilon_{Source}} = b = e^{+|\rho|} > 1$$

$$\frac{\upsilon_{Receiver}}{\upsilon_{Source}} = r = e^{-|\rho|} < 1$$

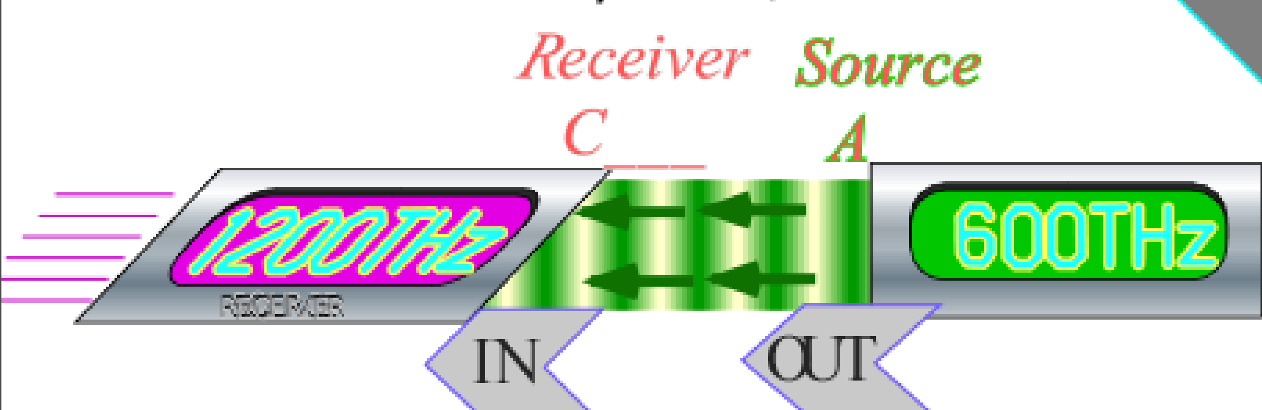
*Defining **Rapidity** ρ as logarithm of Doppler*

$\rho = ln(b \ \text{ or } r)$

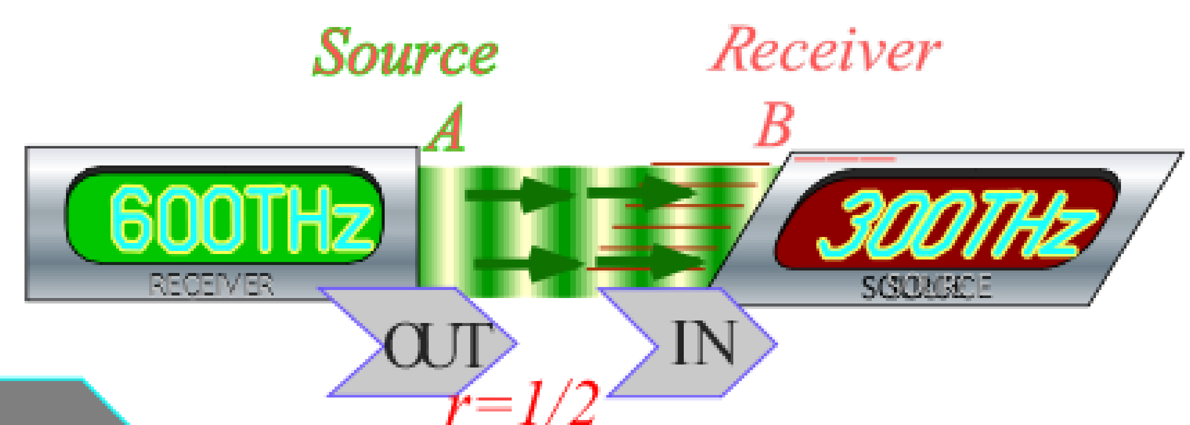
Examples:



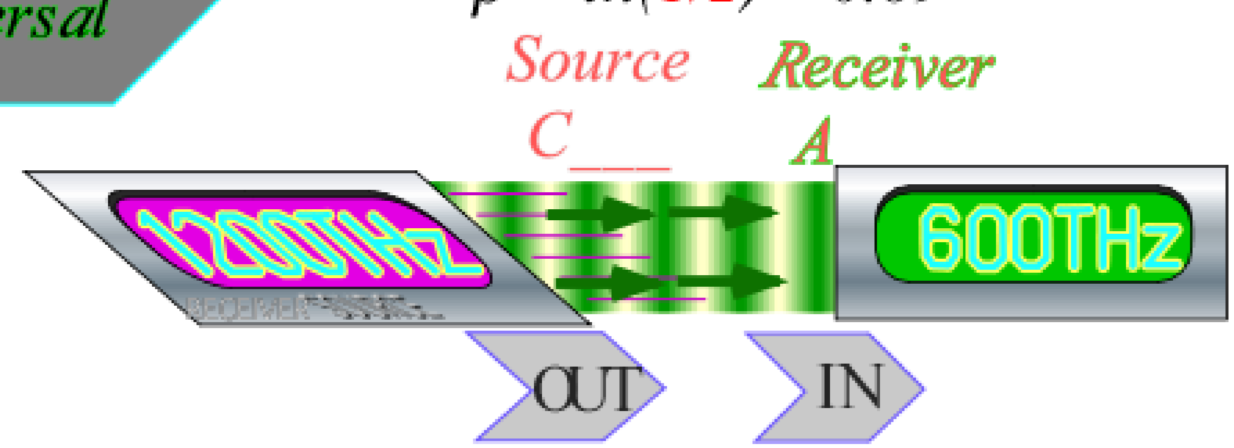
$\rho = ln(2) = 0.69$



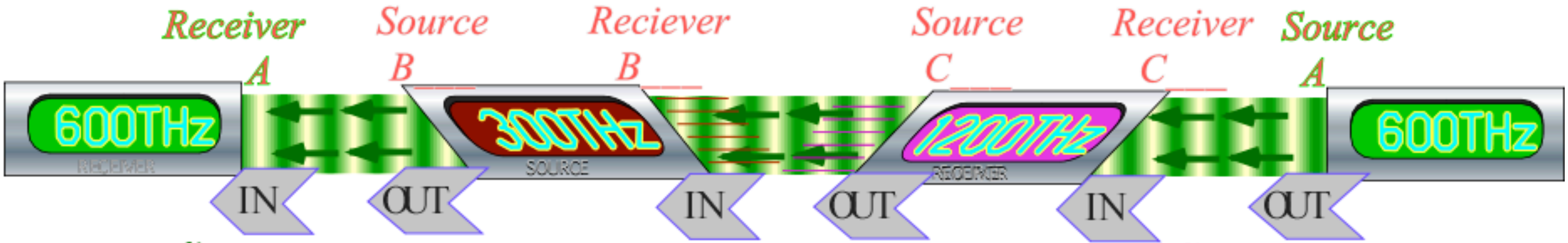
Examples:



$\rho = ln(1/2) = -0.69$



Each Doppler shift $\frac{\upsilon_A}{\upsilon_B}$ maps to a Lorentz transformation T_{AB}



$\frac{\upsilon_A}{\upsilon_B}=b_{AB}=e^{\rho_{AB}}=2$

$\rho_{AB} = \ln(2)=0.69$

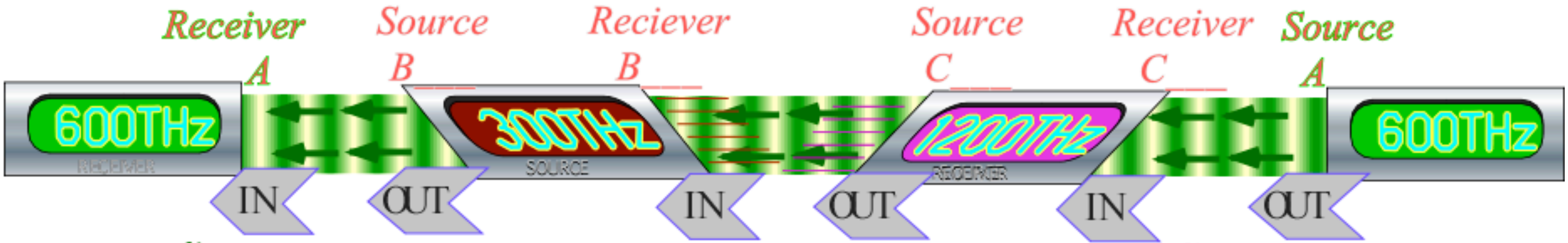
$\frac{\upsilon_B}{\upsilon_C}=b_{BC}=e^{\rho_{BC}}=1/4$

$\rho_{BC}= \ln(1/4)=-1.38$

$\frac{\upsilon_C}{\upsilon_A}=b_{CA}=e^{\rho_{CA}}=2$

$\rho_{CA}= \ln(2)=0.69$

Each Doppler shift $\frac{\upsilon_A}{\upsilon_B}$ maps to a Lorentz transformation T_{AB}



$\frac{\upsilon_A}{\upsilon_B} = b_{AB} = e^{\rho_{AB}} = 2$

$\rho_{AB} = \ln(2) = 0.69$

$\frac{\upsilon_B}{\upsilon_C} = b_{BC} = e^{\rho_{BC}} = 1/4$

$\rho_{BC} = \ln(1/4) = -1.38$

$\frac{\upsilon_C}{\upsilon_A} = b_{CA} = e^{\rho_{CA}} = 2$

$\rho_{CA} = \ln(2) = 0.69$

Group product is represented:
(by IN-OUT "nematodes")

$T_{AB} \cdot T_{BC} = T_{CA}$

$\frac{\upsilon_A}{\upsilon_B} \frac{\upsilon_B}{\upsilon_C} = \frac{\upsilon_A}{\upsilon_C}$

$e^{\rho_{AB}} e^{\rho_{BC}} = e^{\rho_{AC}} = e^{(\rho_{AB} + \rho_{BC})}$

...and rapidity ρ_{AB} is a Galilean (arithmetic) parameter

To be shown: $\rho_{AB} = \operatorname{atanh}(u_{AB}/c)$ approaches (u_{AB}/c) for: $\rho_{AB} << 1$

2. Applying Occam's razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms

CW light clearly shows Doppler shifts

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear?... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

Inverse to Lorentz transformation T_{AB} is T_{BA}

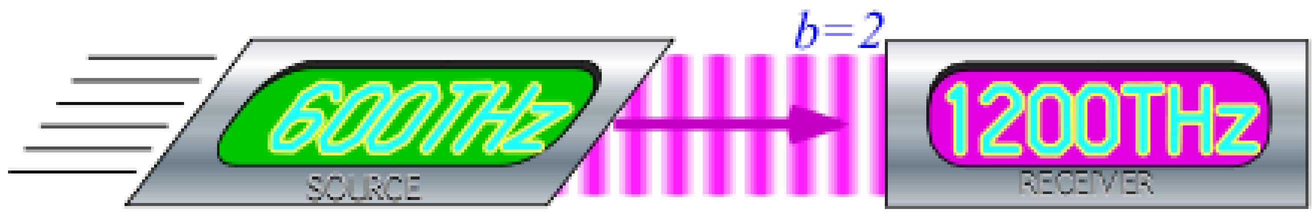
..just as the arithmetic inverse of $\frac{\upsilon_A}{\upsilon_B}$ is $\frac{\upsilon_B}{\upsilon_A}$

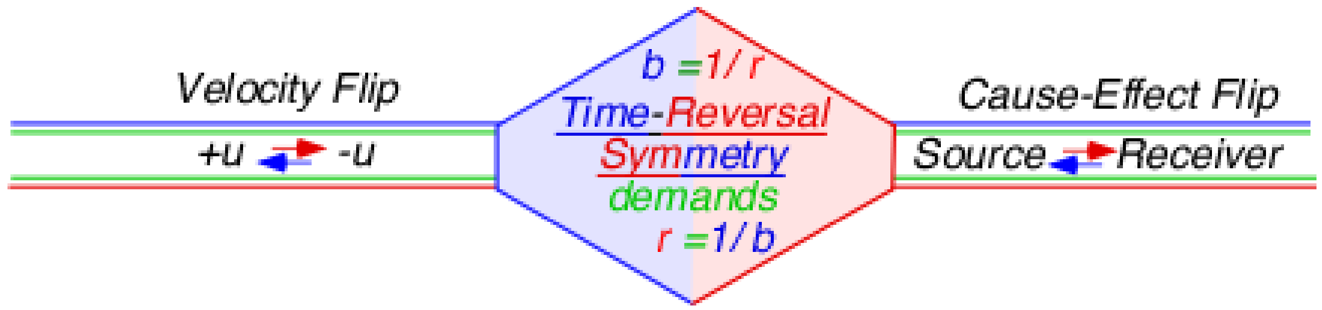
..just as the arithmetic inver... of $e^{\rho_{AB}}$ is $e^{\rho_{BA}} = e^{-\rho_{AB}}$

..just as the arithmetic inver... of ρ_{AB} is $\rho_{BA} = -\rho_{AB}$

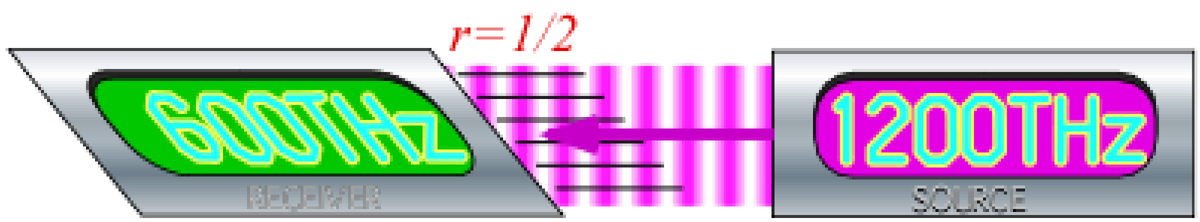
Detailed time reversal symmetry implies $r=1/b$.

Approaching source (600THz green)





Receding receiver sees Doppler red-shift of 1200THz source to 600THz (600THz)=r·(1200THz) with r=1/2



See animation: www.uark.edu/ua/pirelli/php/time_rev_sym.php

3. Spectral theory of Einstein-Lorentz relativity

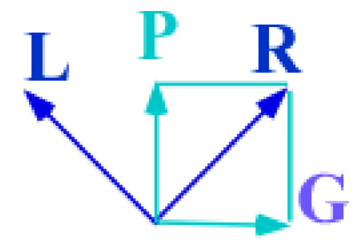
- Applying Doppler Shifts to per-space-time (ck,ω) graph
  - CW Minkowski space-time coordinates (x,ct) and PW grids
    - Relating Doppler Shifts b or $r=1/b$ to velocity u/c or rapidity ρ
      - Lorentz transformation

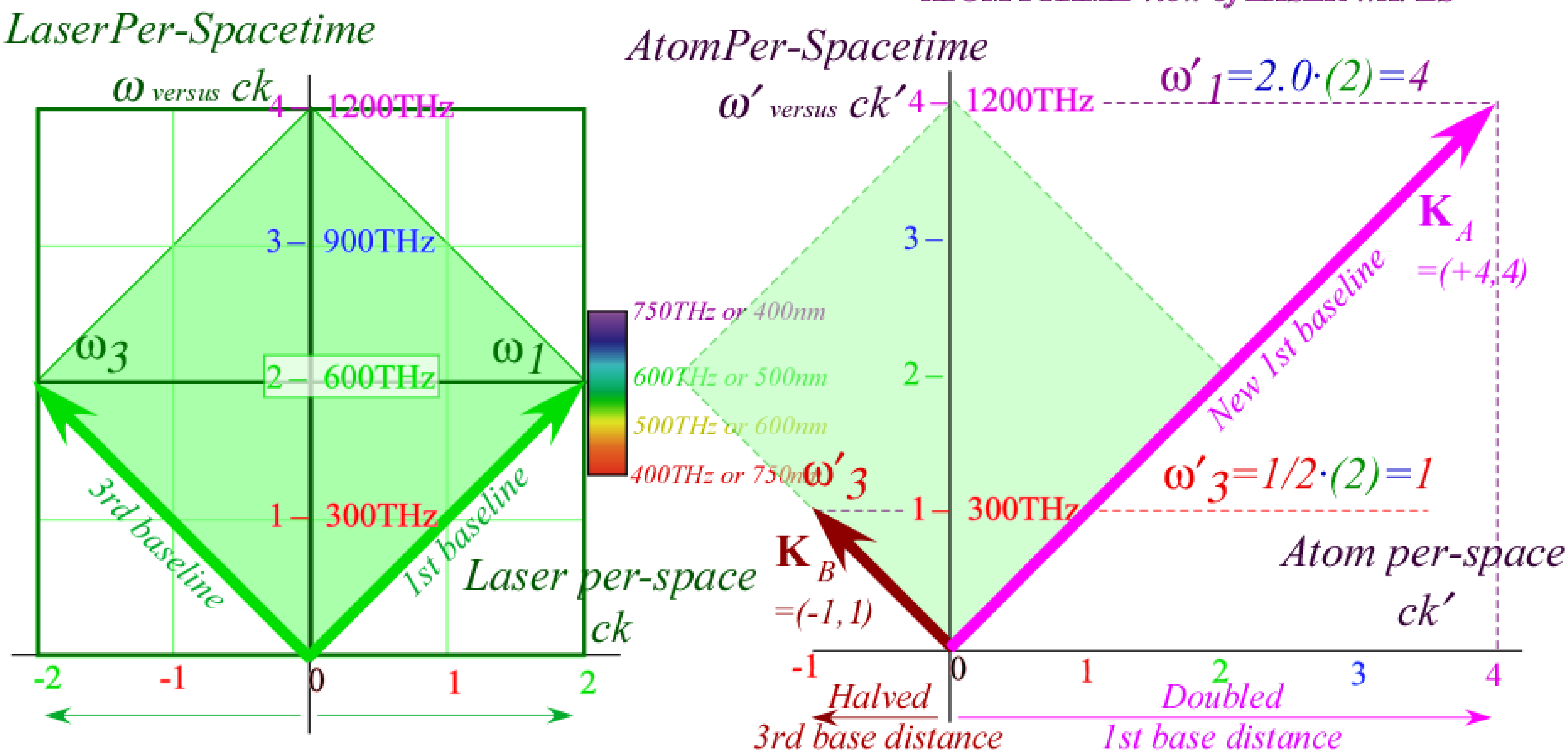
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$





3. Spectral theory of Einstein-Lorentz relativity

Applying Doppler Shifts to per-space-time (ck,ω) graph

CW Minkowski space-time coordinates (x,ct) and PW grids

Relating Doppler Shifts b or $r=1/b$ to velocity u/c or rapidity ρ

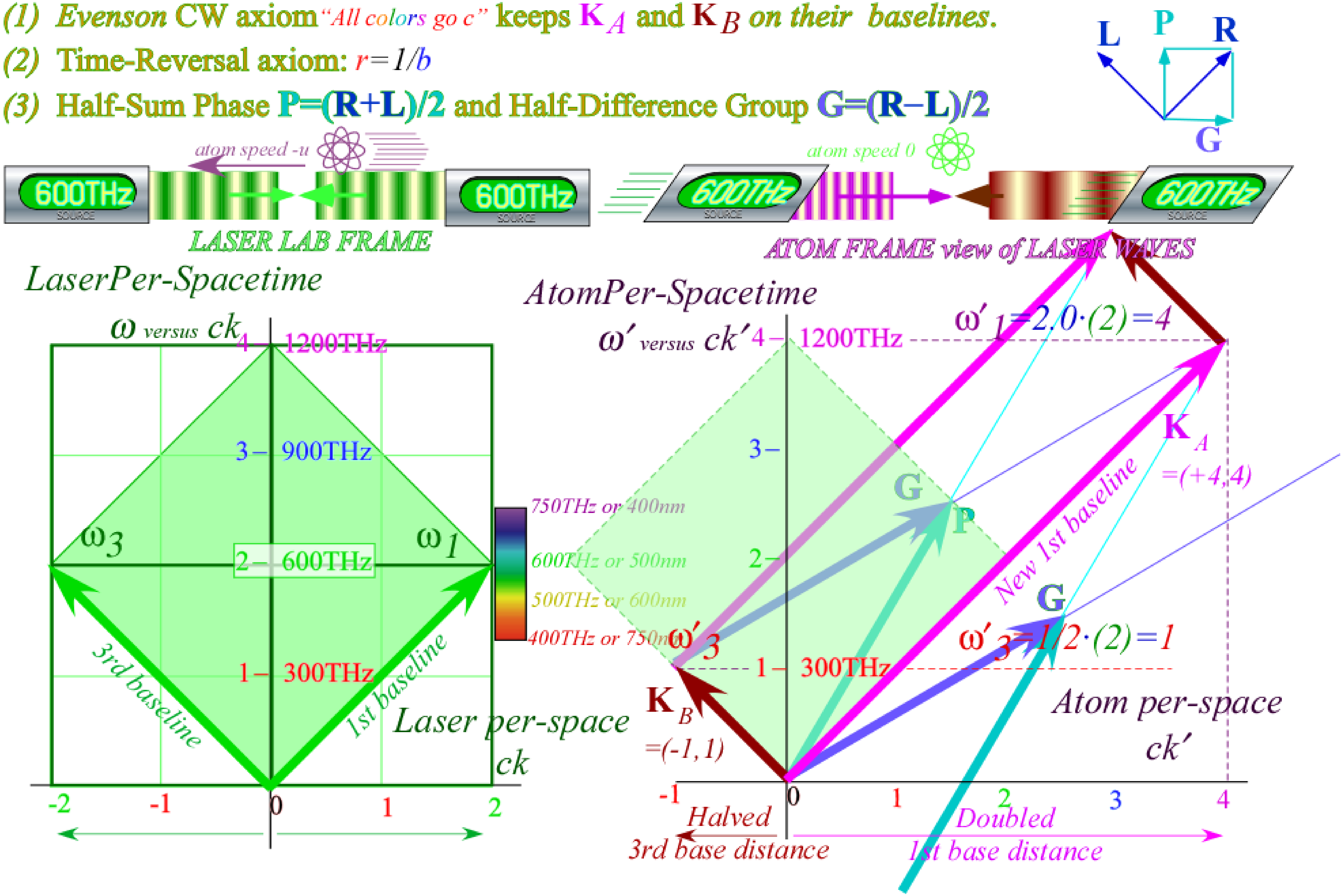
Lorentz transformation

Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom *"All colors go c"* keeps K_A and K_B *on their baselines.*

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase **P=(R+L)/2** and Half-Difference Group **G=(R−L)/2**

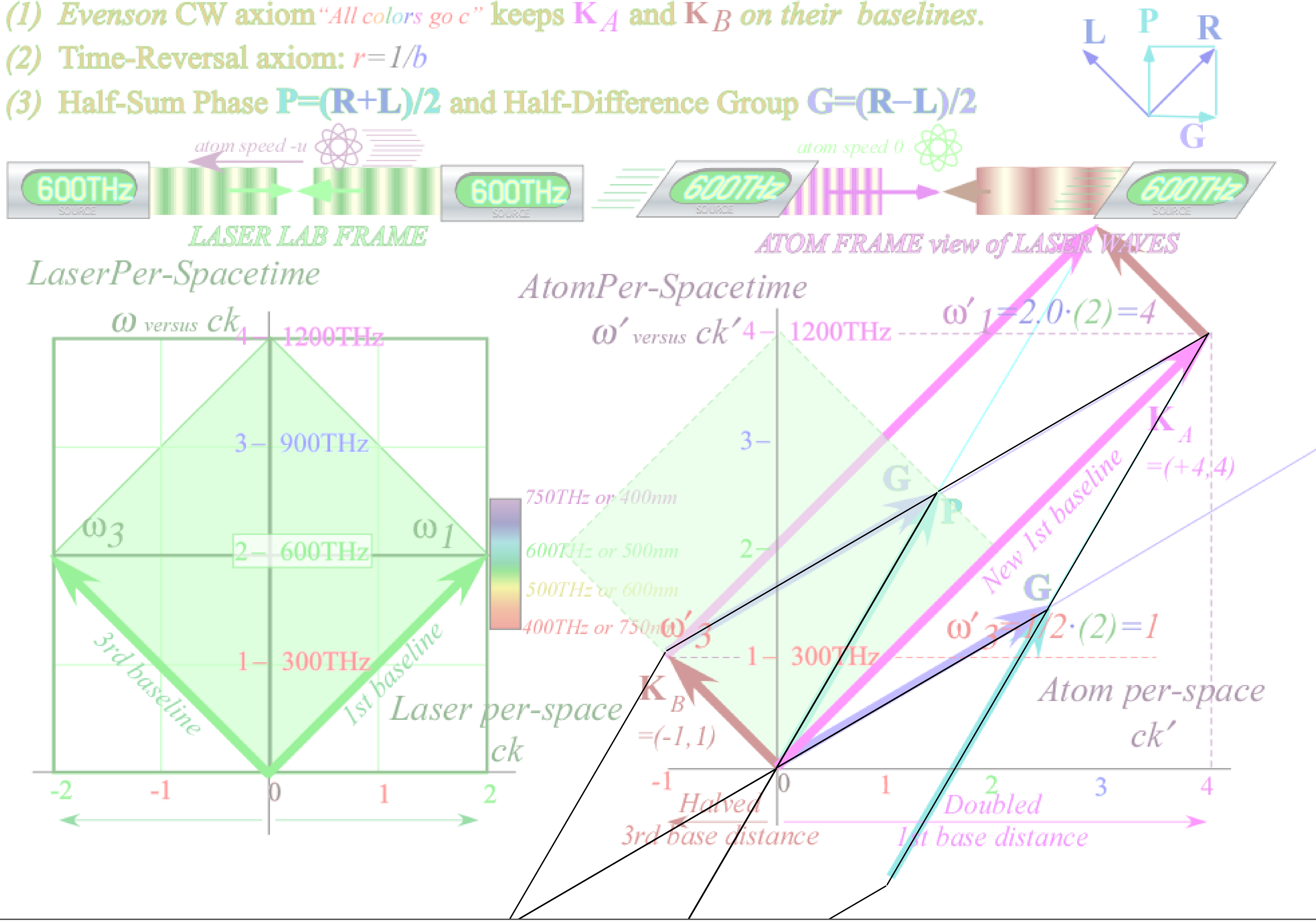


Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$

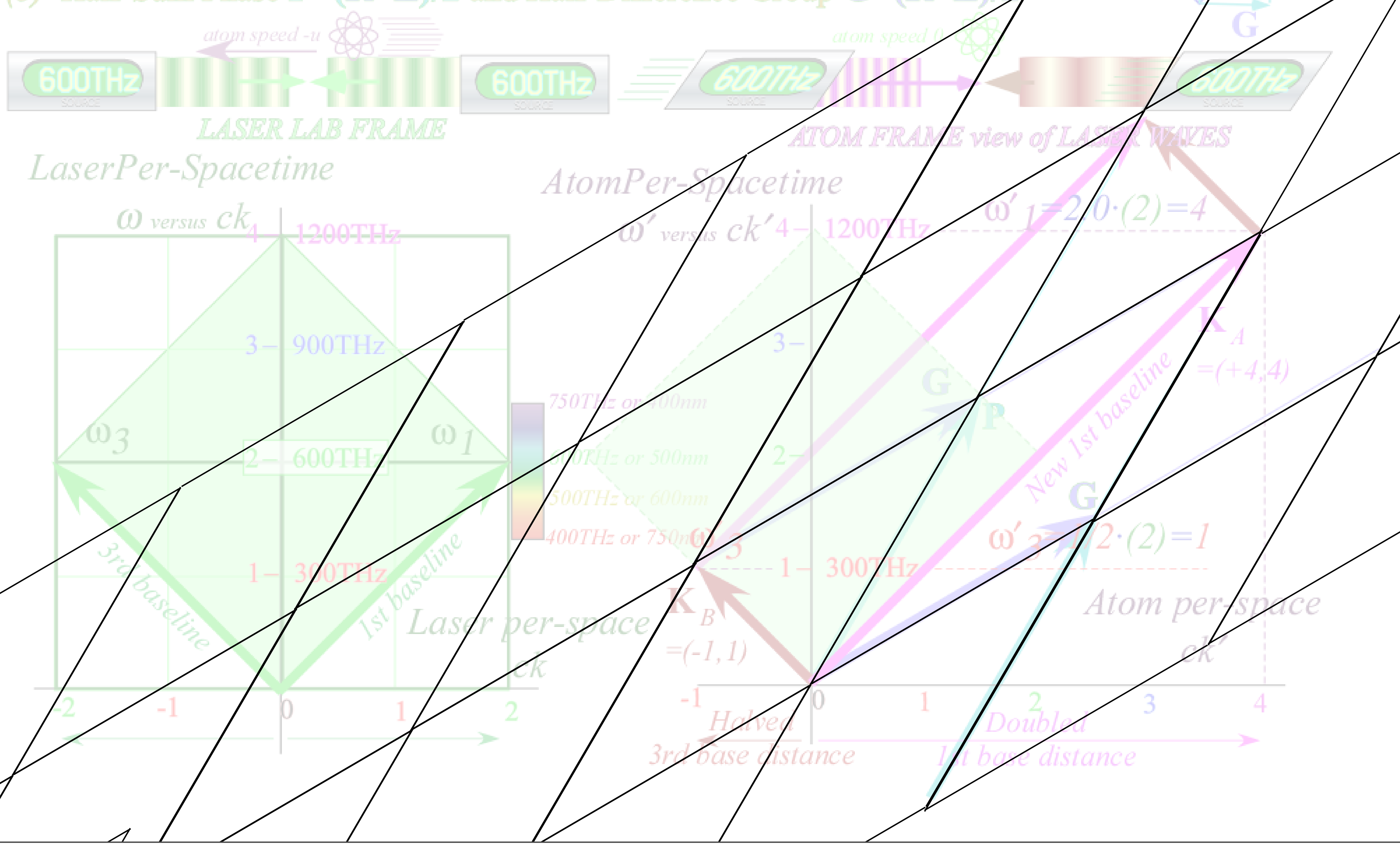


Deriving Spacetime and per-spacetime coordinate geometry by:

(1) **Evenson CW axiom** *"All colors go c"* **keeps** K_A **and** K_B ***on their baselines.***

(2) **Time-Reversal axiom:** $r=1/b$

(3) **Half-Sum Phase P=(R+L)/2 and Half-Difference Group G=(R−L)/2**

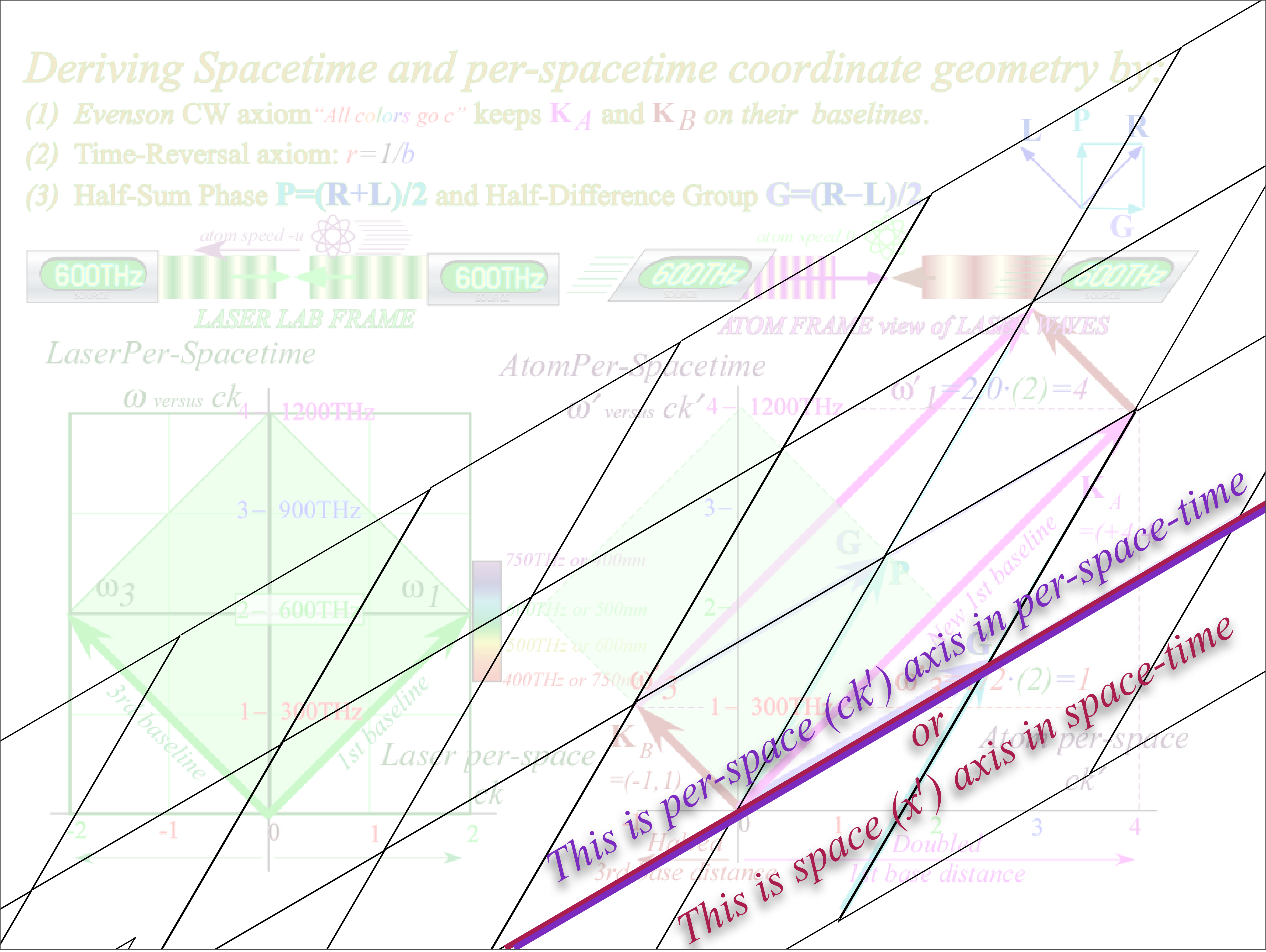


Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase P=(R+L)/2 and Half-Difference Group G=(R−L)/2

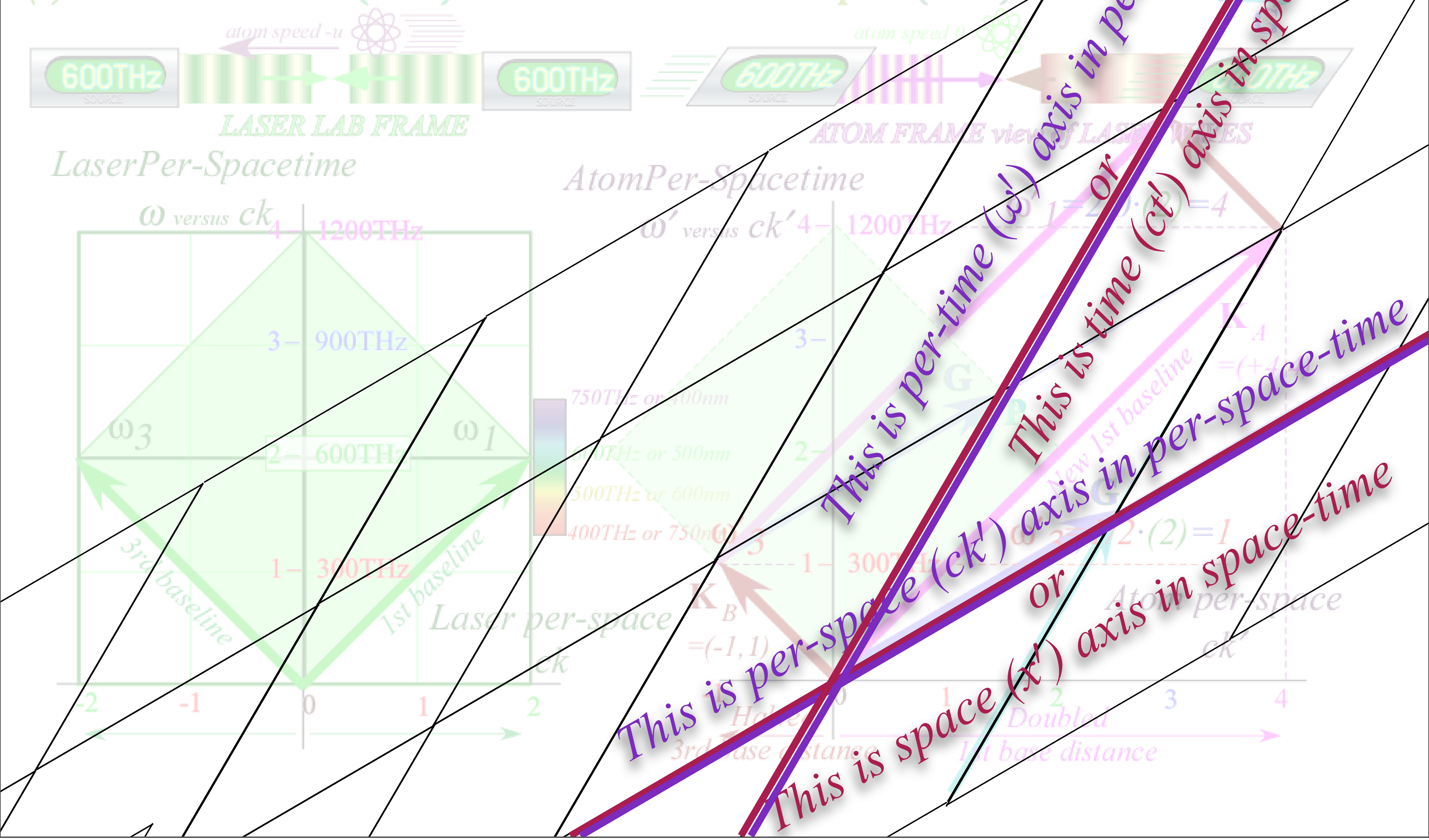


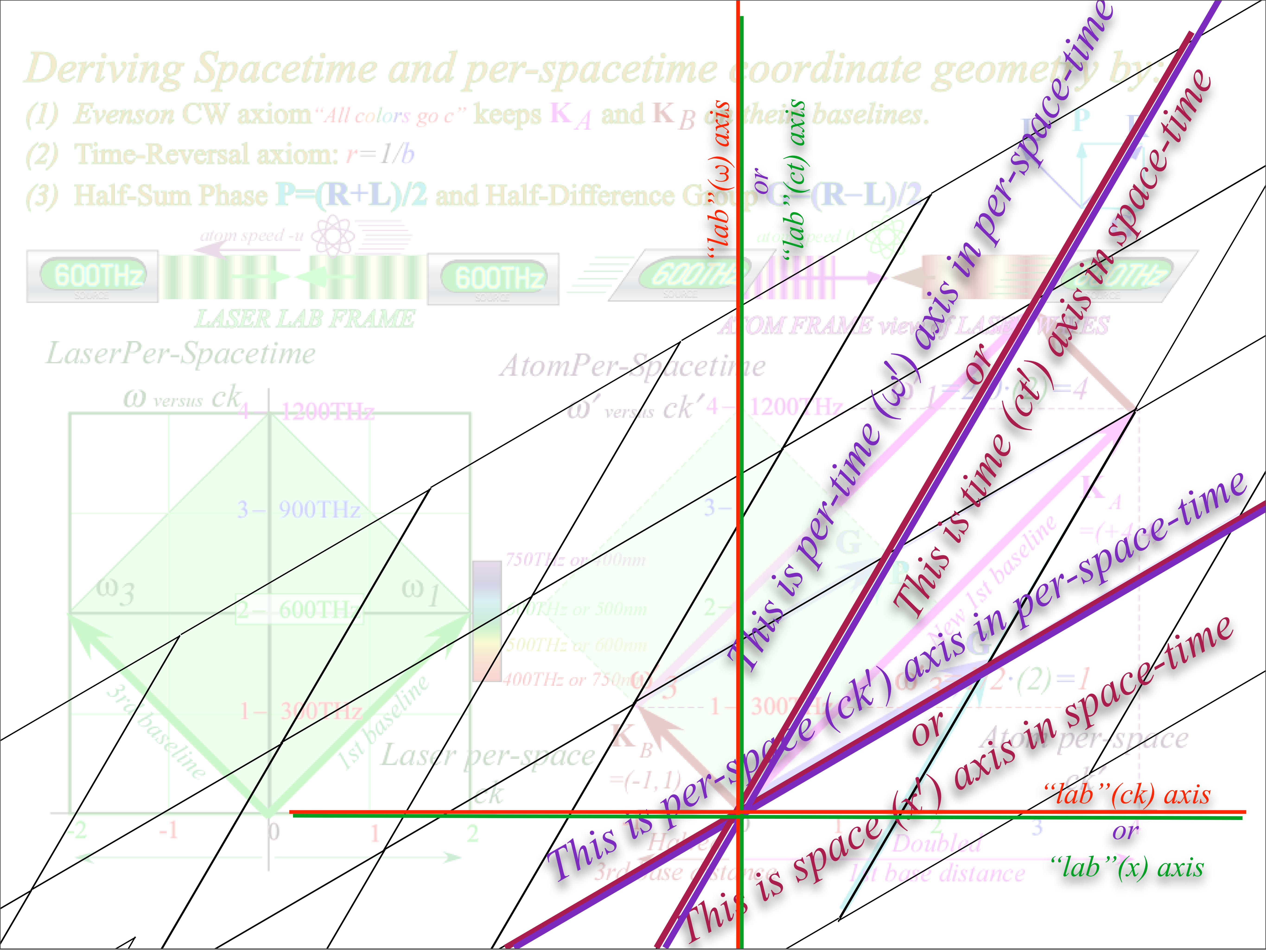
Deriving Spacetime and per-spacetime coordinate geometry by:

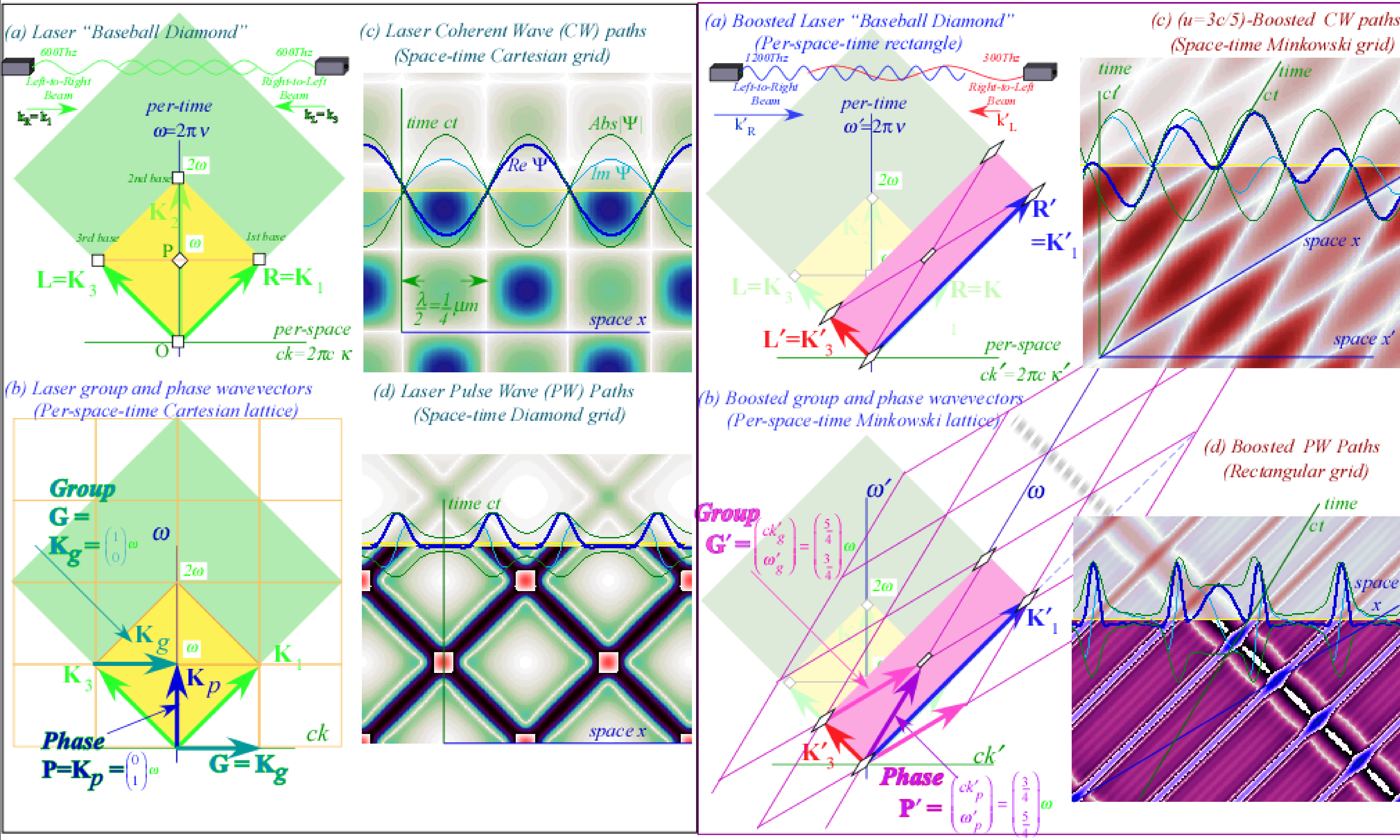
(1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$




Deriving Spacetime and per-spacetime coordinate geometry by:
(1) Evenson CW axiom "All colors go c" keeps K A and K B on their baselines.
(2) Time-Reversal axiom: r=1/b
(3) Half-Sum Phase P=(R+L)/2 and Half-Difference Group G=(R−L)/2
"lab"(ω) axis
or
"lab"(ct) axis
This is per-time (ω′) axis in per-space-time
or
This is time (ct′) axis in space-time
This is per-space (ck′) axis in per-space-time
or
This is space (x′) axis in space-time
"lab"(ck) axis
or
"lab"(x) axis
LASER LAB FRAME
ATOM FRAME view of LASER WAVES
LaserPer-Spacetime
AtomPer-Spacetime
ω versus ck
ω′ versus ck′
1200THz
900THz
600THz
300THz
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
3rd baseline
1st baseline
New 1st baseline
Laser per-space
Atom per-space
Doubled
1st base distance
3rd base distance
=(-1,1)



Laser lab views

atom speed $-u=-\frac{3}{5}c$

Atom views (sees lab going $+u=\frac{3}{5}c$)

Lecture 22 ended (about) here