Lecture 32.

Relativity of interfering and galloping waves: SWR and SWQ IV.

(Ch. 4-6 of Unit 2   4.17.12)

Relativistic effects on charge, current, and Maxwell Fields

Review of Lecture 31

Current density changes by Lorentz asynchrony

Magnetic B-field is relativistic effect

Lecture 31 ended here

Relating photons to Maxwell energy density and Poynting flux

Field Energy $=|\mathsf{E}|^2\varepsilon_0$

$1/4\pi\varepsilon_0=9\cdot 10^9$

Relativistic variation and invariance of frequency (ω,k) and amplitudes

How probability ψ-waves and flux ψ-waves evolved

*Properties of amplitude ψ*ψ-squares*

More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta

The Ship-Barn-and-Butler saga of confused causality

(More about galloping)

Review of Lecture 30

1st Quantization: Quantizing phase variables ω and k

Understanding how quantum transitions require "mixed-up" states

Closed cavity vs Ring cavity

Quantized ω and *k* *Counting wave kink numbers*

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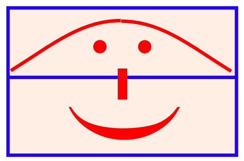
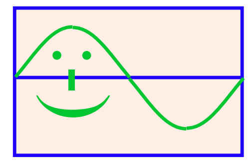
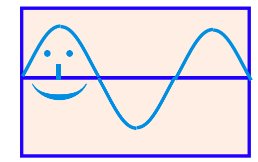
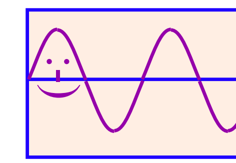

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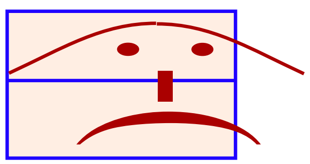

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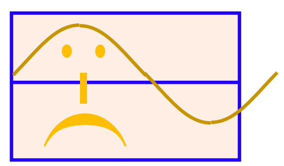

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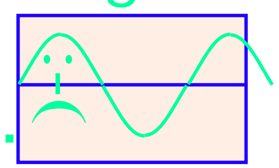

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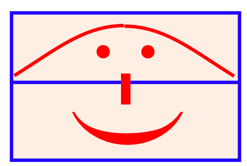

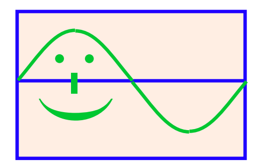

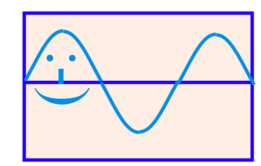

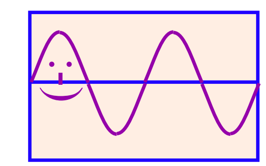

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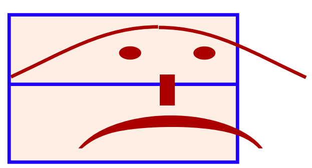

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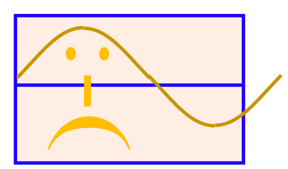

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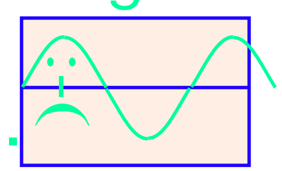

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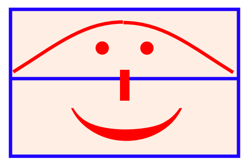
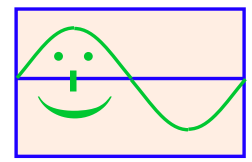
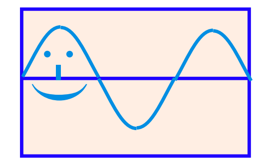
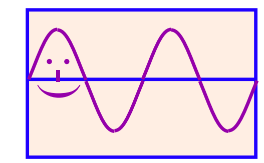

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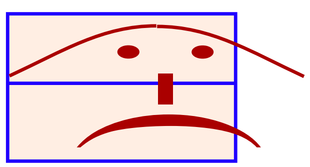

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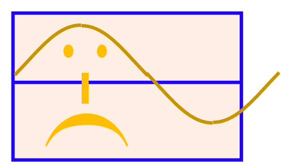

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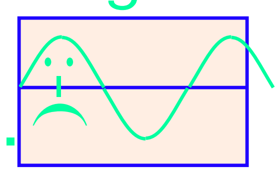

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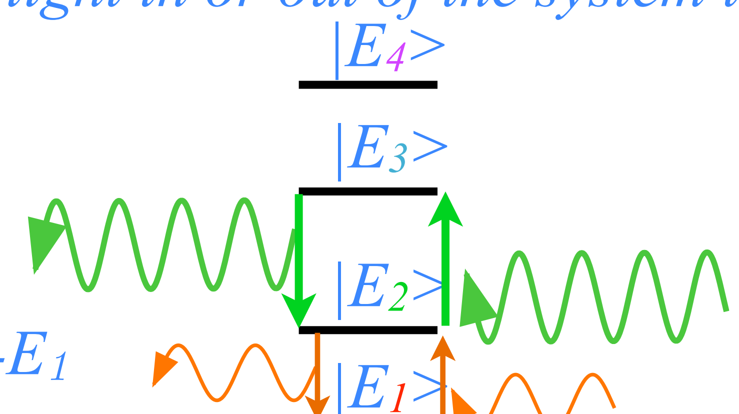
That's the only way you get any light in or out of the system to "see" it.

frequency $\hbar\omega_{32} = E_3 - E_2$

frequency $\hbar\omega_{21} = E_2 - E_1$



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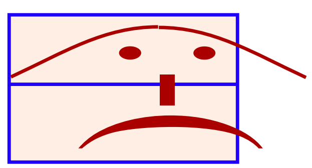

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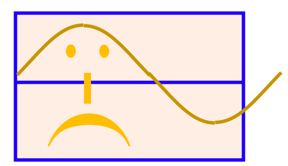

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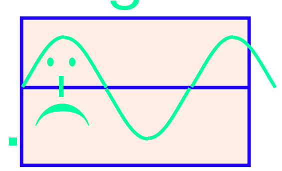

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frequency $\omega_{32} = (E_3 - E_2)/\hbar$

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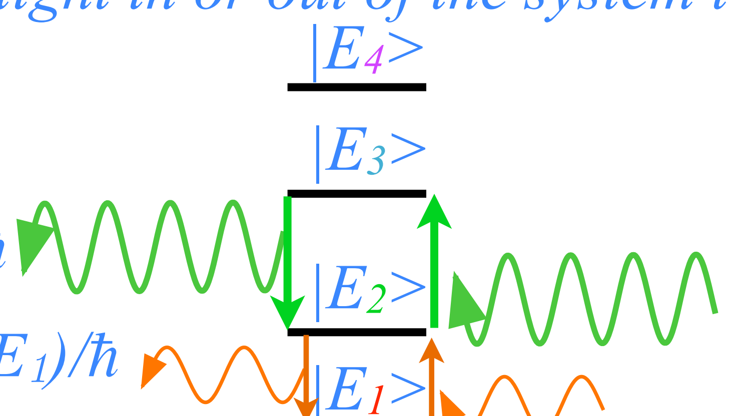

These eigenstates are the only ways the system can "play dead"… … " sleep with the fishes"…

Quantized ω and *k* *Counting wave kink numbers*

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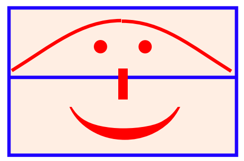
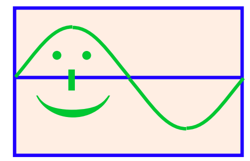
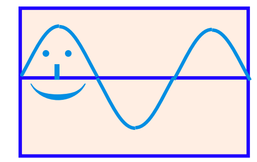
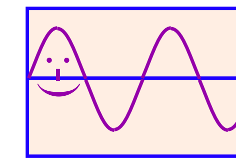

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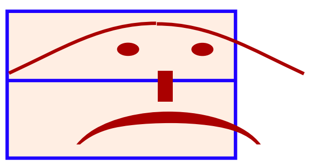

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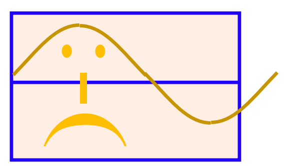

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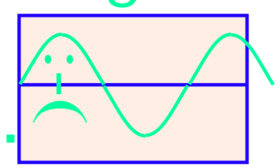

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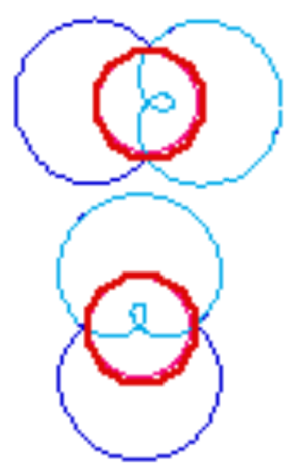
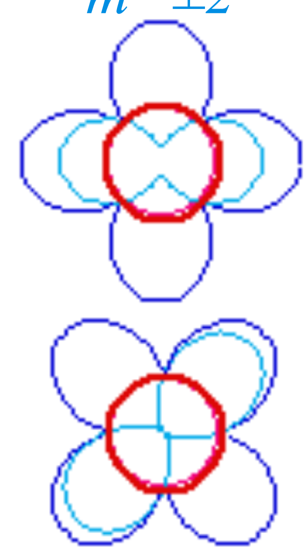
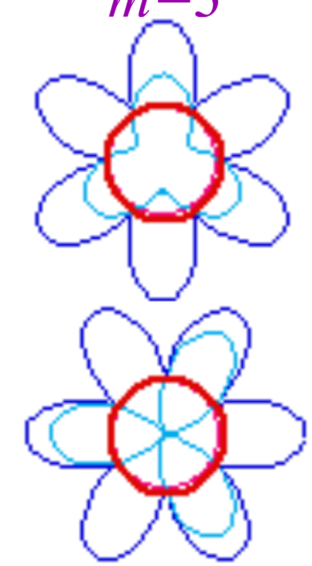
Rings tolerate a *zero* (kinkless) quantum wave but require ±*integral wave number.*

OK ring quantum numbers: *m=0* *m=±1* *m=±2* *m=3*

(± integral number of wavelengths)


Bohr's models of *atomic spectra (1913-1923)* are beginnings of *quantum wave mechanics* built on *Planck-Einstein (1900-1905)* relation *E=hυ*. *DeBroglie* relation *p=h/λ* comes around *1923*.

2nd Quantization: Quantizing amplitudes ("photons","vibrons", and "what-ever-ons")

Introducing coherent states (What lasers use)

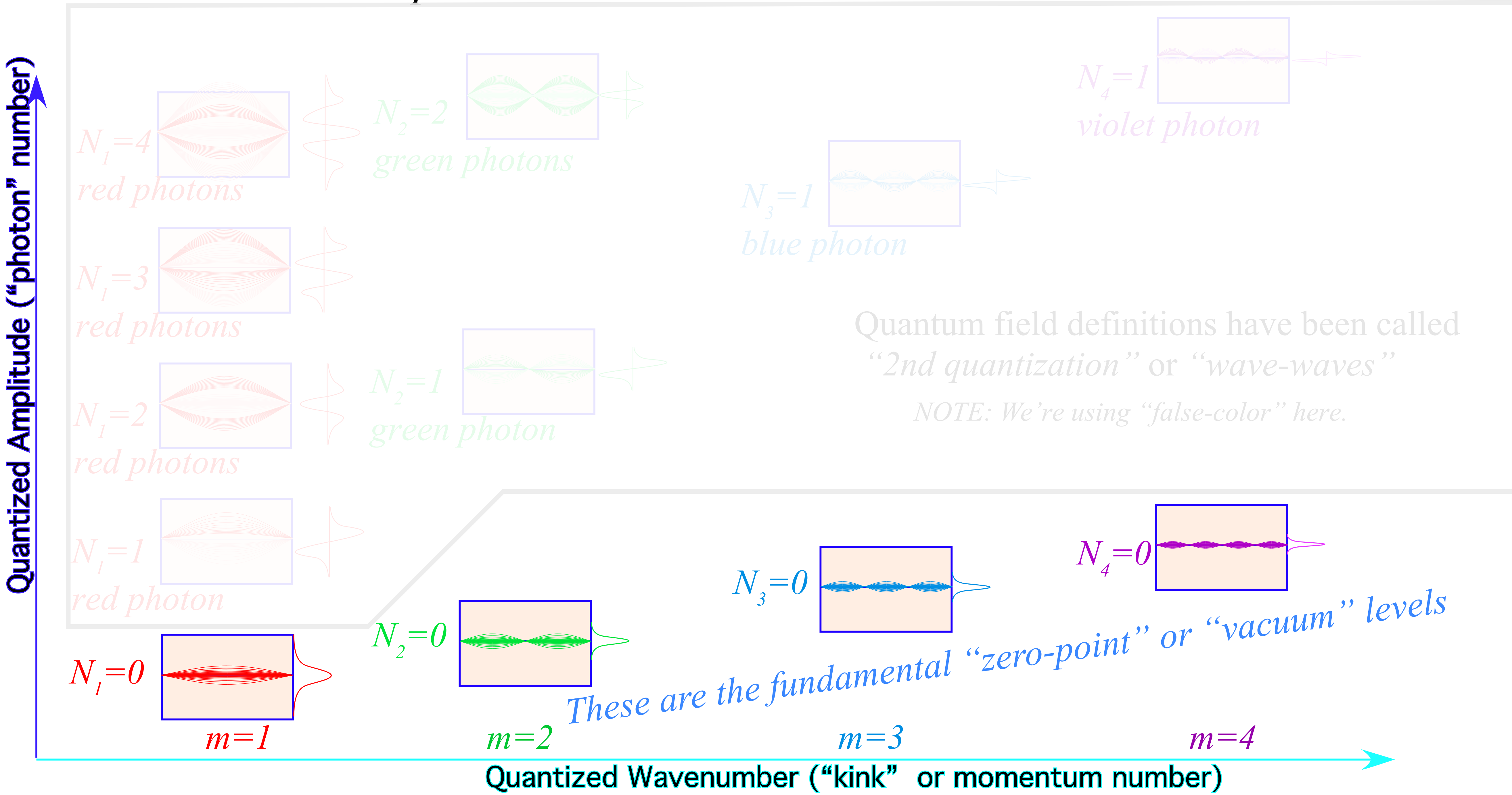
Analogy with (ω,k) wave packets

Wave coordinates need coherence

Quantized *Amplitude* *Counting "photon" number*

Planck's relation $E=Nh\upsilon$ began as a tenative axiom to explain low-T light. Then he tried to disavow it! Einstein picked it up in his 1905 paper. Since then its use has grown enormously and continues to amaze, amuse (or bewilder) all who study it.

A current view is that it represents the *quantization* of optical field *amplitude*. We picture this below as *N-photon* wave states for each box-mode of *m* wave kinks.



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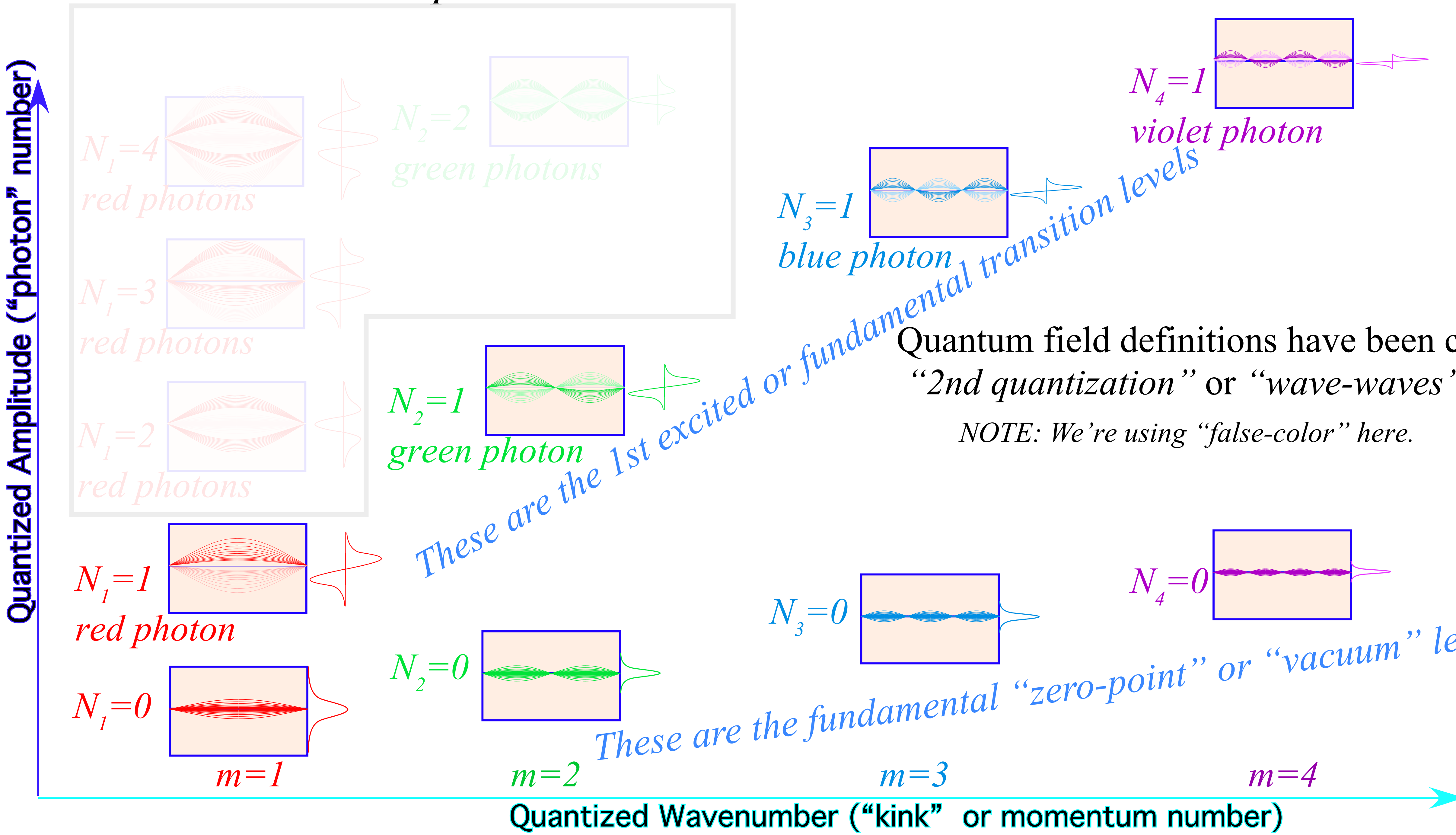

Quantum field definitions have been called *"2nd quantization"* or *"wave-waves"*

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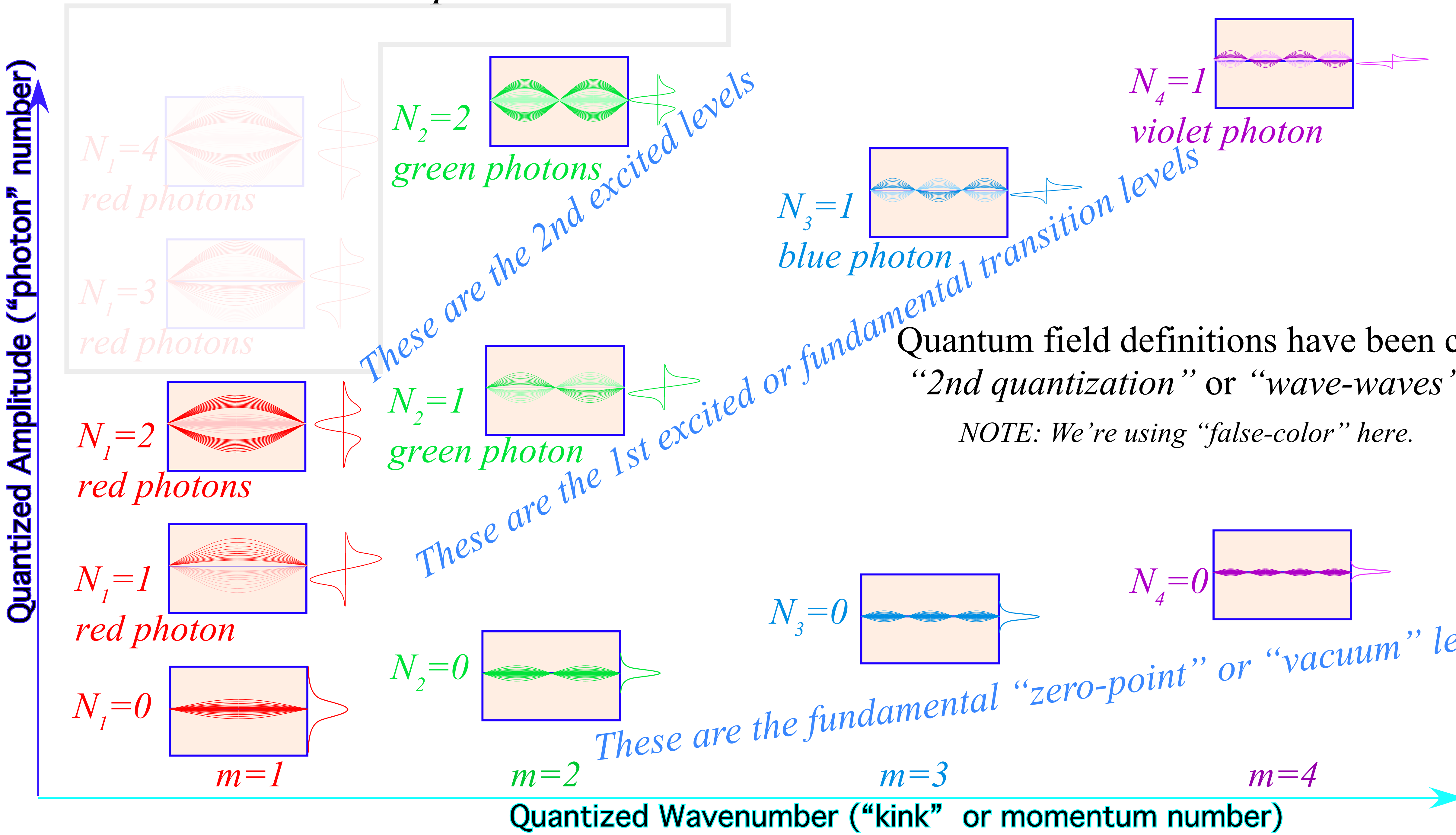

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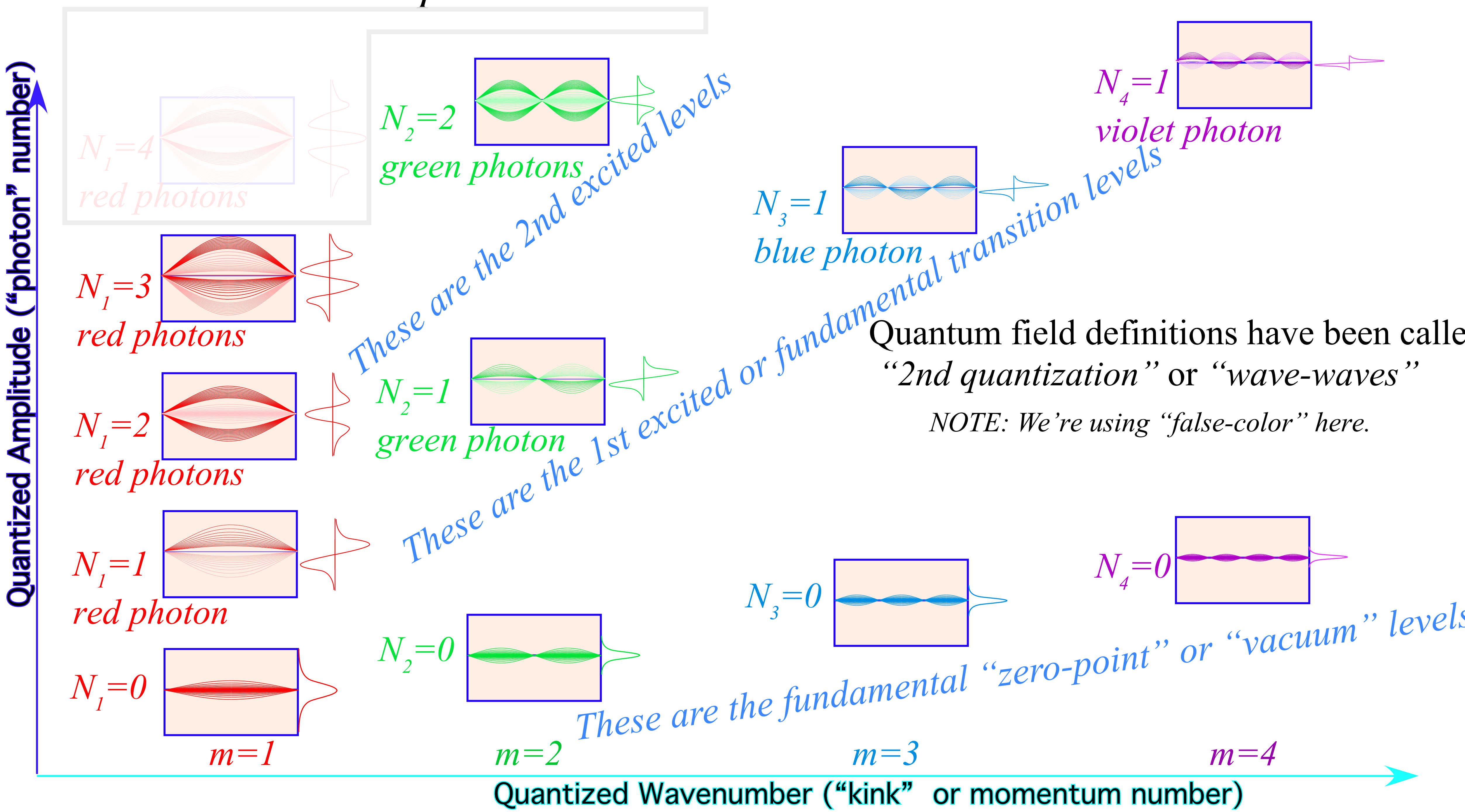

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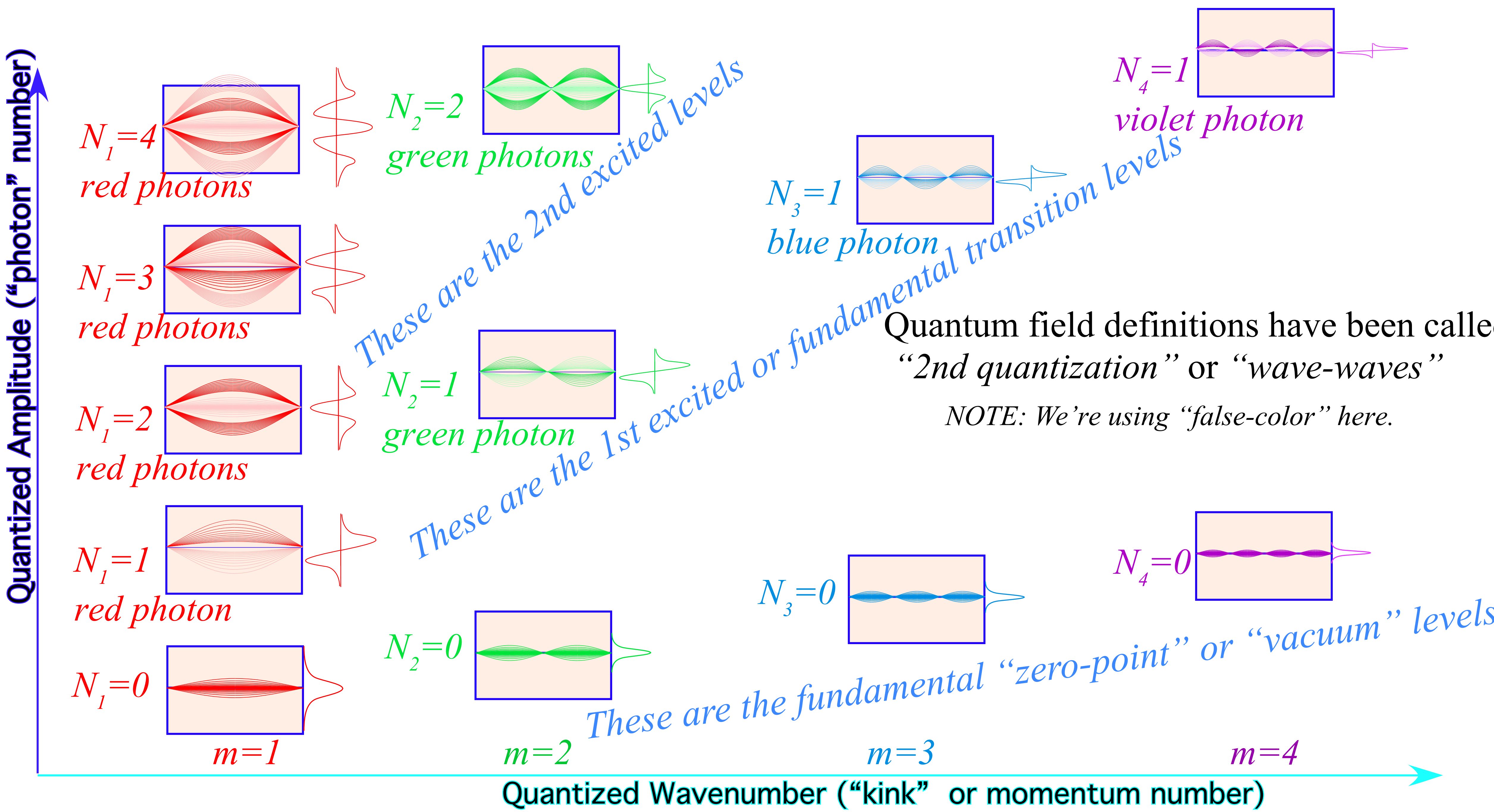

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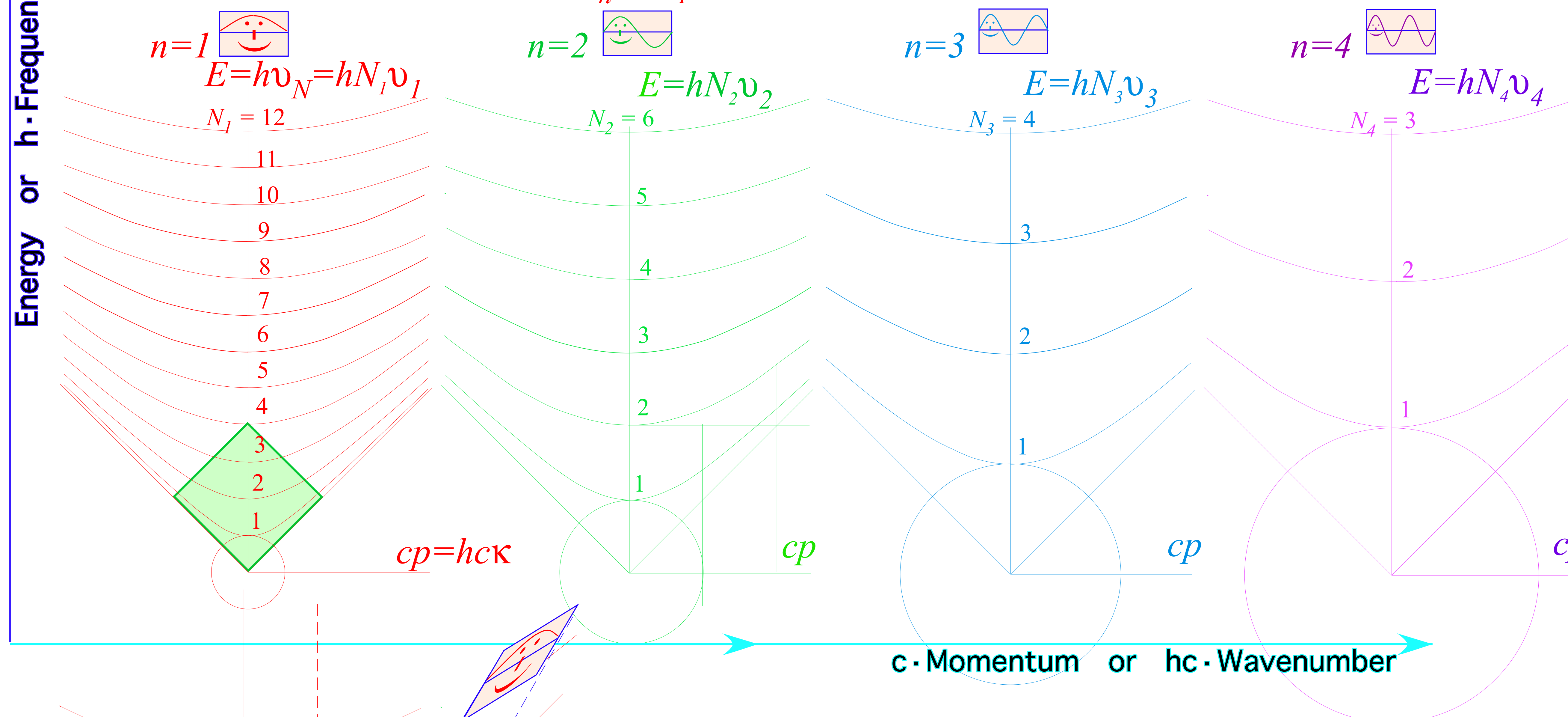
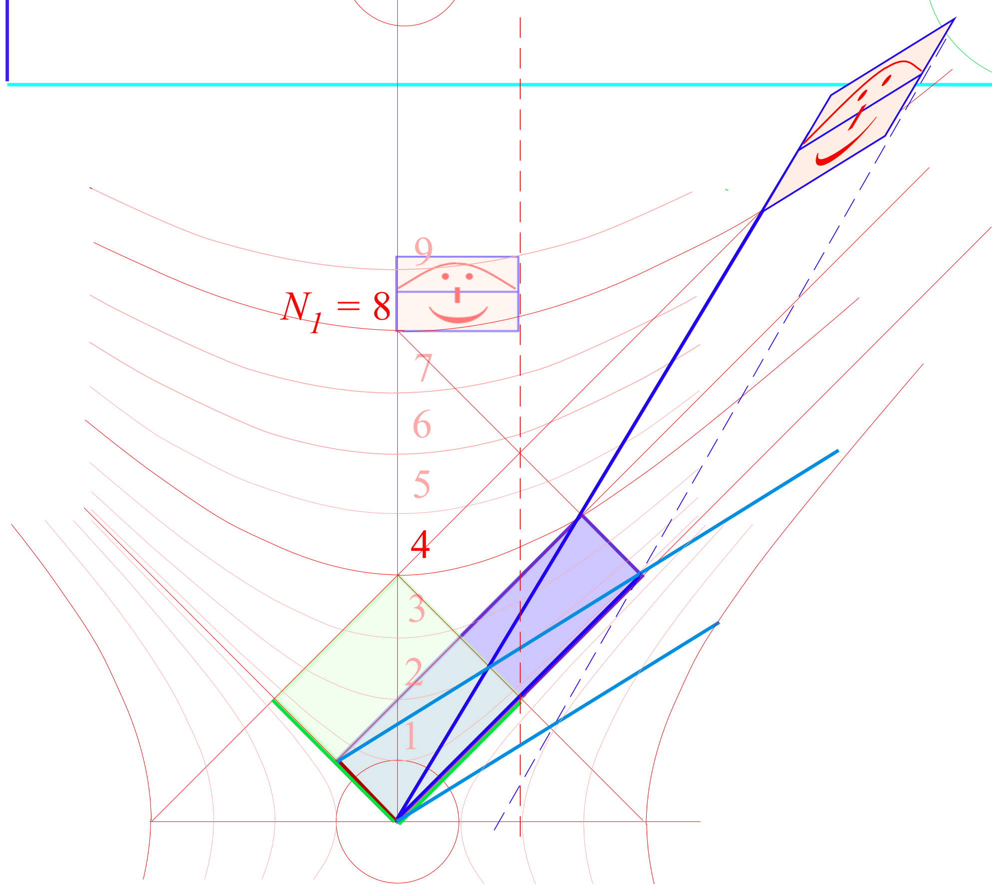
Quantum numbers *N* of field or *n, m,..* of modes are *invariants* and *not* changed by boosting velocity.
Each mode fundamental frequency $\upsilon_n = n\upsilon_1$ and its *N*-photon multiples belong to invariant hyperbolas.





Boosted observers see distorted frequencies and lengths, but will agree on the *numbers n* and *N* of mode *nodes* and *photons*.

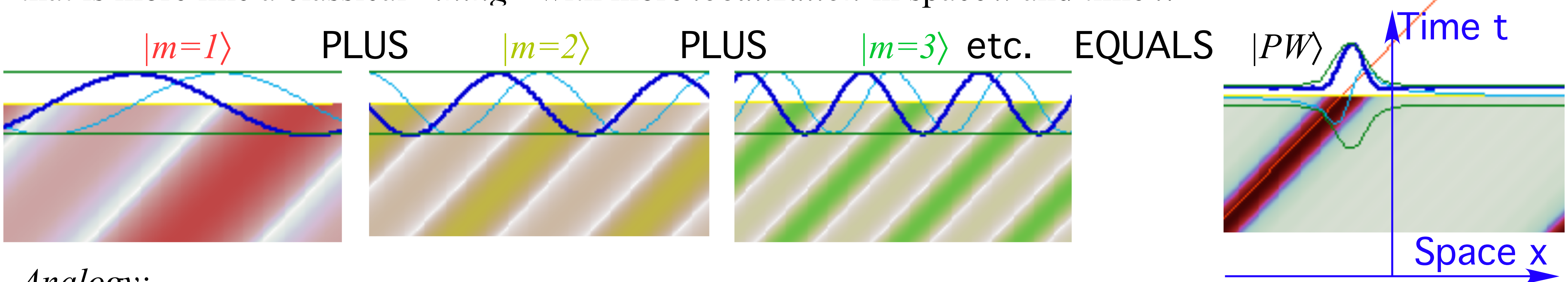
This is how light waves can “fake” some of the properties of classical “things” such as *invariance* or *object permanence*.

It takes at least *TWO CW*’s to achieve such invariance. One CW is not enough and cannot have non-zero invariant *N* . Invariance is an *interference* effect that needs at least *two-to-tango!*

www.uark.edu/ua/pirelli/php/quantized_2.php

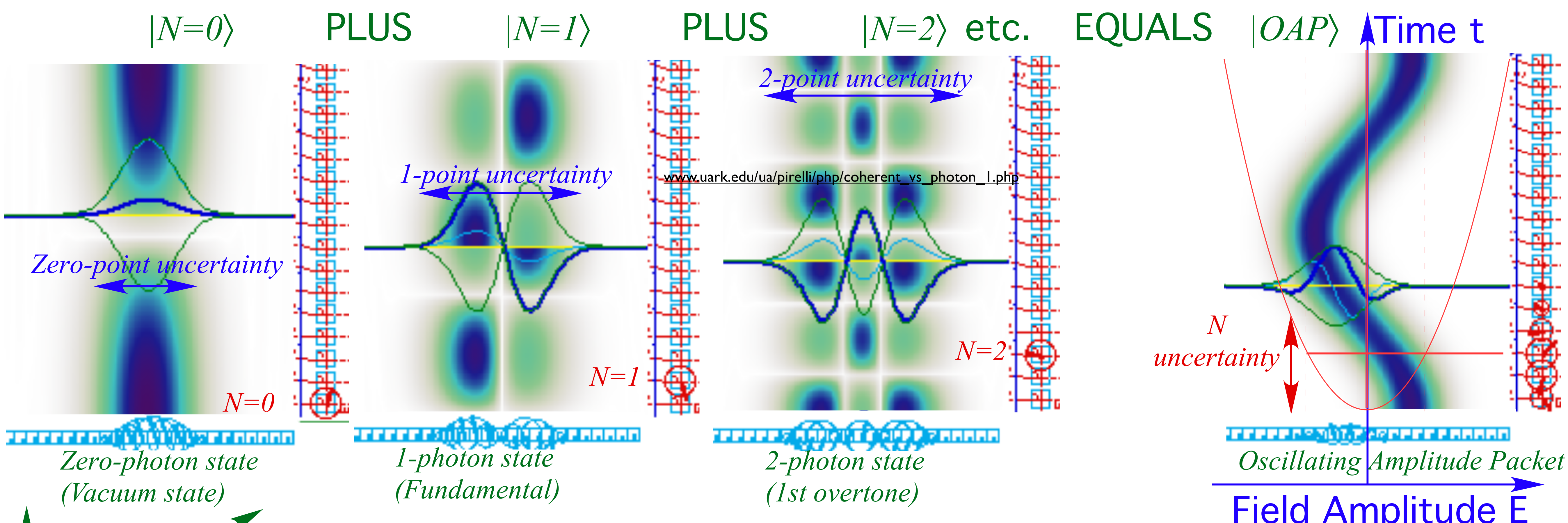
Coherent States: Oscillator Amplitude Packets *analogous to* *Wave Packets*

We saw how adding *CW*'s *(Continuous Waves m=1,2,3...)* can make *PW (Pulse Wave)* or *WP (Wave Packet)* that is more like a classical *"thing"* with more *localization* in space *x* and time *t*.



Analogy:

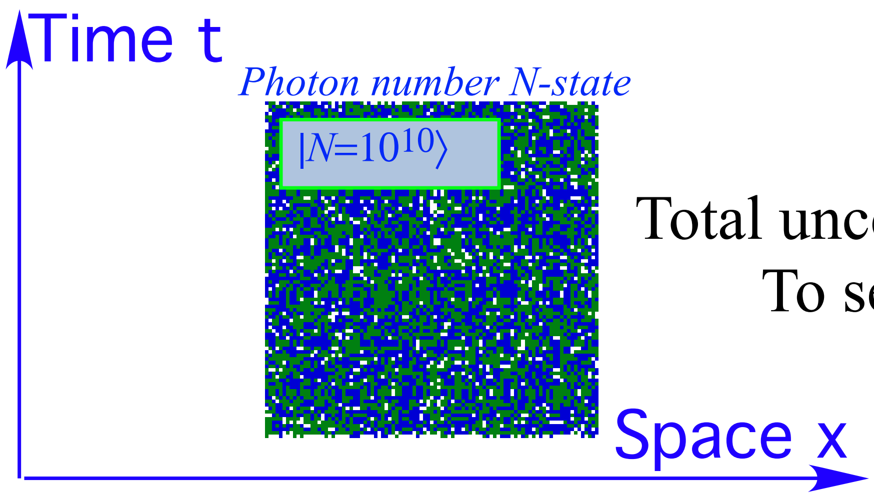
Adding *photons (Quantized amplitude N=0,1,2...)* can make a *CS (Coherent State)* or *OAP (Oscillator Amplitude Packet)* that is more like a *classical wave oscillation* with more *localization* in field amplitude.



Pure photon states have localized (certain) *N* but *de*localized (*un*certain) amplitude and phase.
OAP states have *de*localized (*un*certain) *N* but more localized (certain) amplitude and phase.

Coherent States(contd.) *Spacetime wave grid is impossible without coherent states*

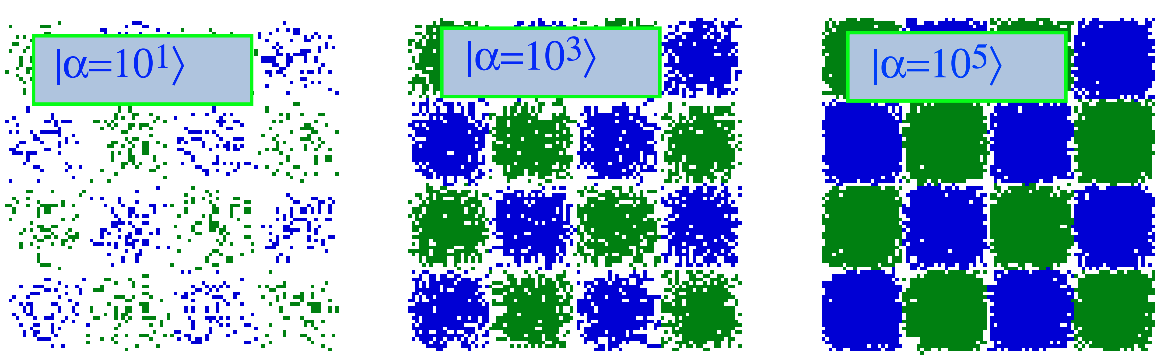
Pure photon number N-states would make useless spacetime coordinates



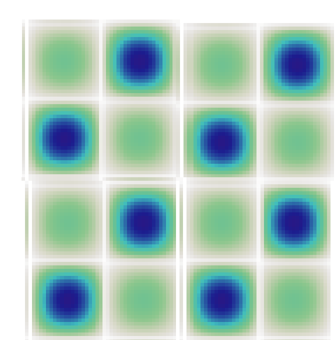
Total uncertainty of amplitude and phase makes the count pattern a wash.
To see grids *some N-uncertainty is necessary!*

Coherent-α-states are defined by continuous amplitude-packet parameter α whose square is average photon number $\bar{N}=|\alpha|^2$. Coherent-states make better spacetime coordinates for larger $\bar{N}=|\alpha|^2$.

Quantum field coherent α-states



Classical limit



$\bar{N}=100$ $\Delta N=10$ | $\bar{N}=10^6$ $\Delta N=10^3$ | $\bar{N}=10^{10}$ $\Delta N=10^5$

Coherent-state uncertainty in photon number (and mass) varies with amplitude parameter $\Delta N \sim \alpha \sim \sqrt{N}$ so a coherent state with $\bar{N}=|\alpha|^2=10^6$ only has a 1-in-1000 uncertainty $\Delta N \sim \alpha \sim \sqrt{N}=1000$.

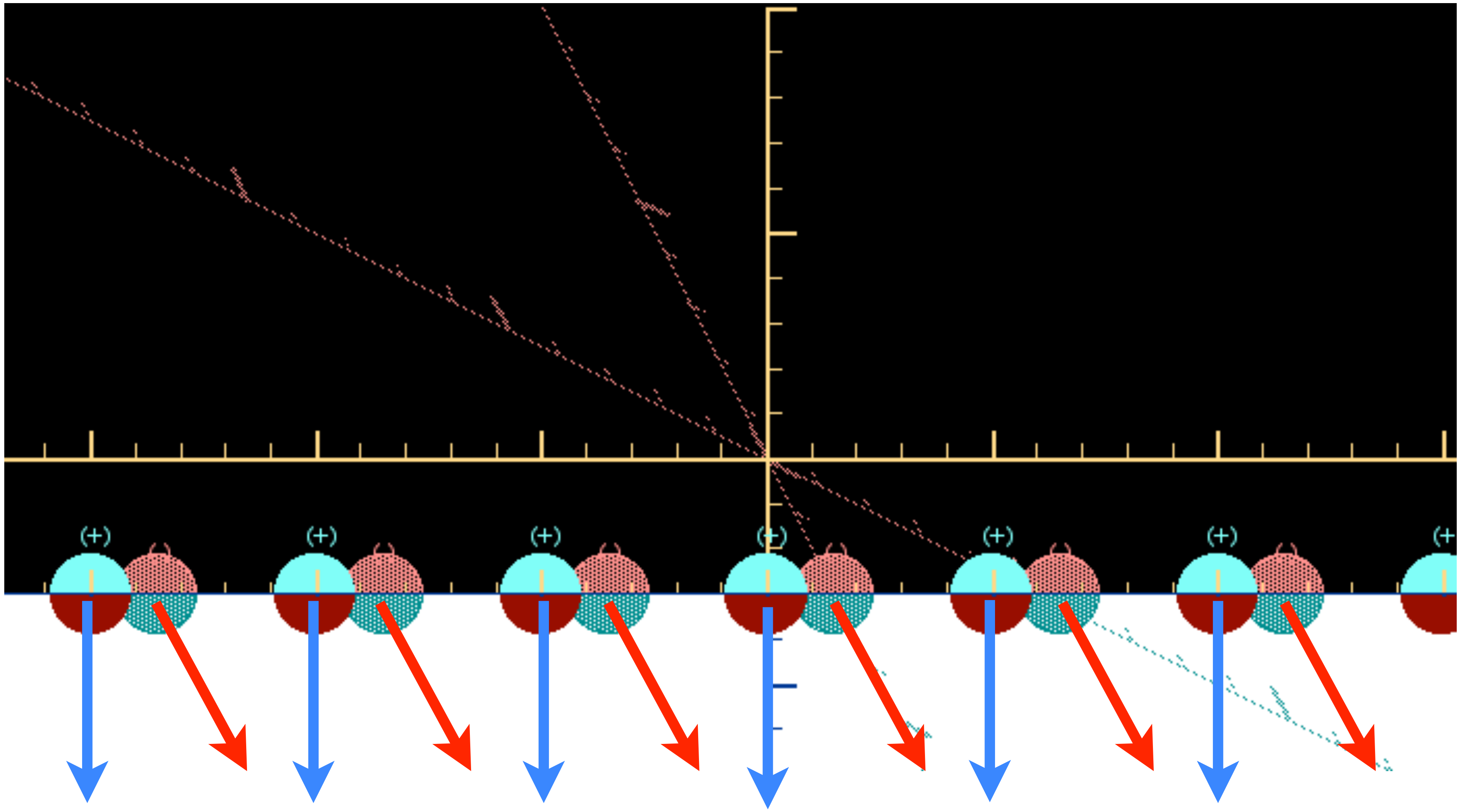
Relativistic effects on charge, current, and Maxwell Fields

➡ *Current density changes by Lorentz asynchrony*

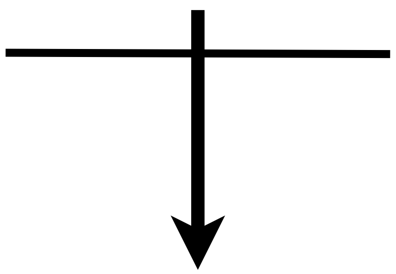
Magnetic B-field is relativistic effect

Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony



Observer velocity is zero relative to (+) line of charge



wire appears neutral

(+) Charge fixed (-) Charge moving to left (*Negative current density)*

(+) Charge density is Equal to the (-) Charge density

Relativistic effects on charge, current, and Maxwell Fields

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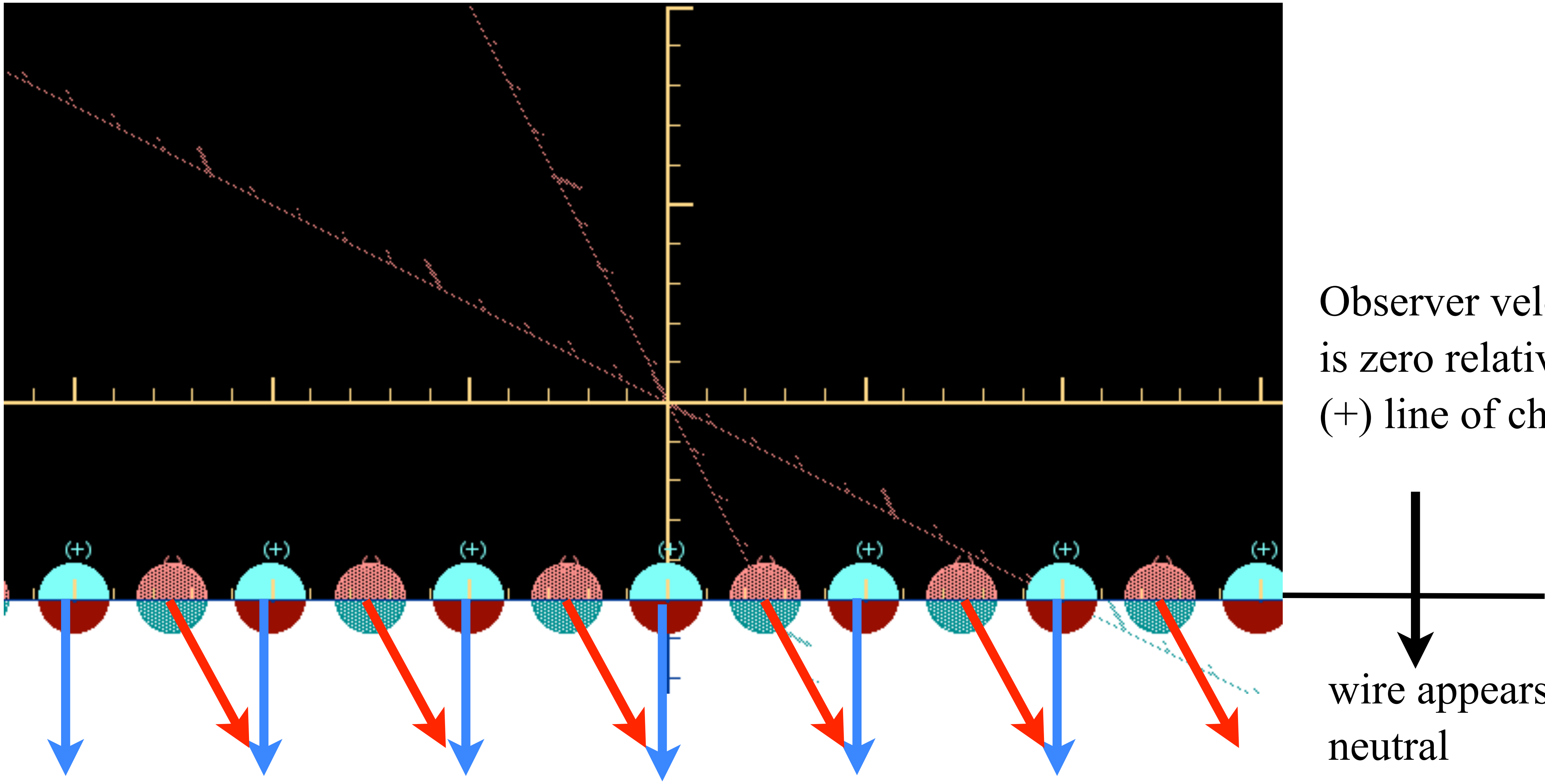

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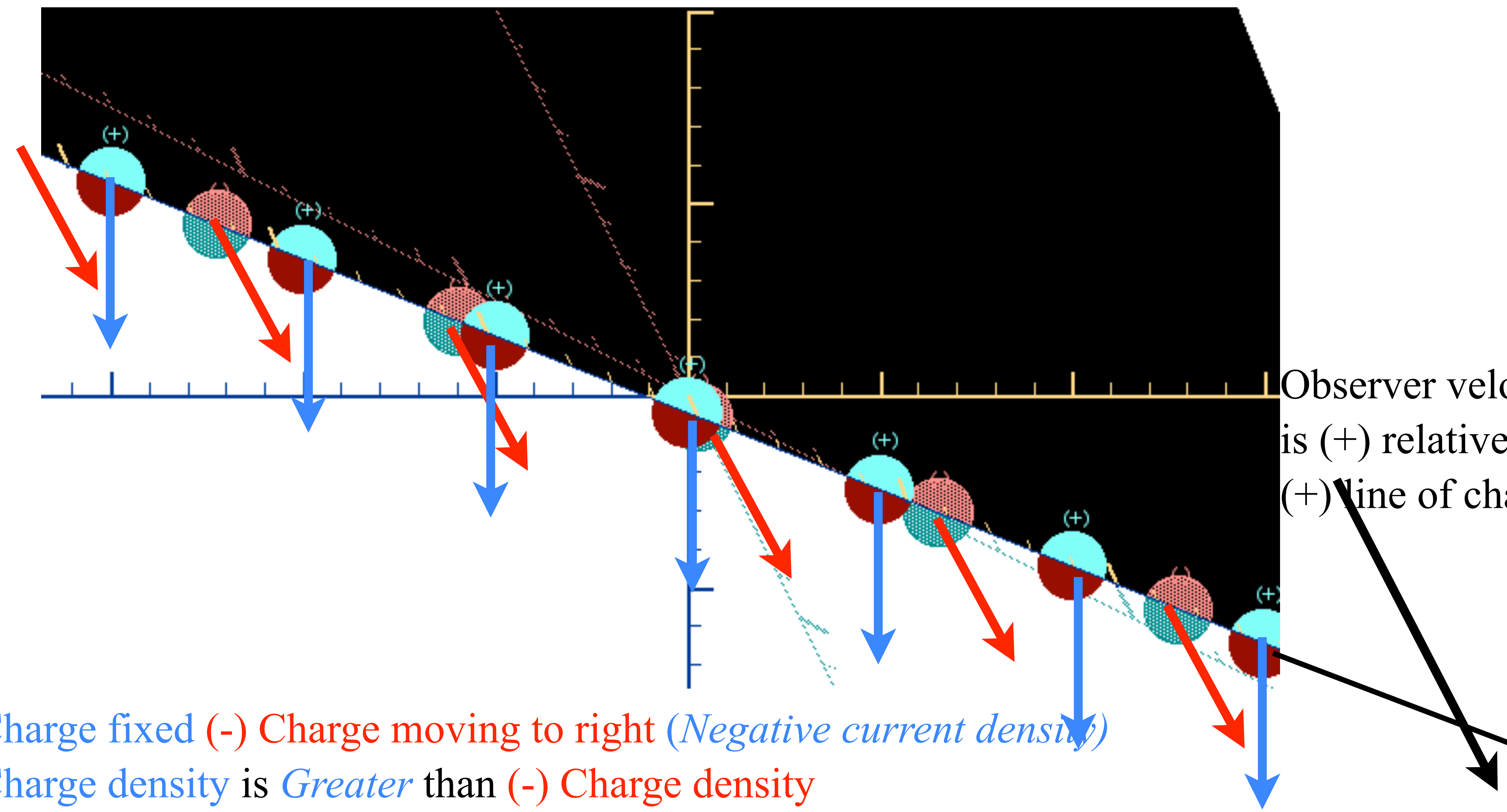


Observer velocity is (+) relative to (+) line of charge

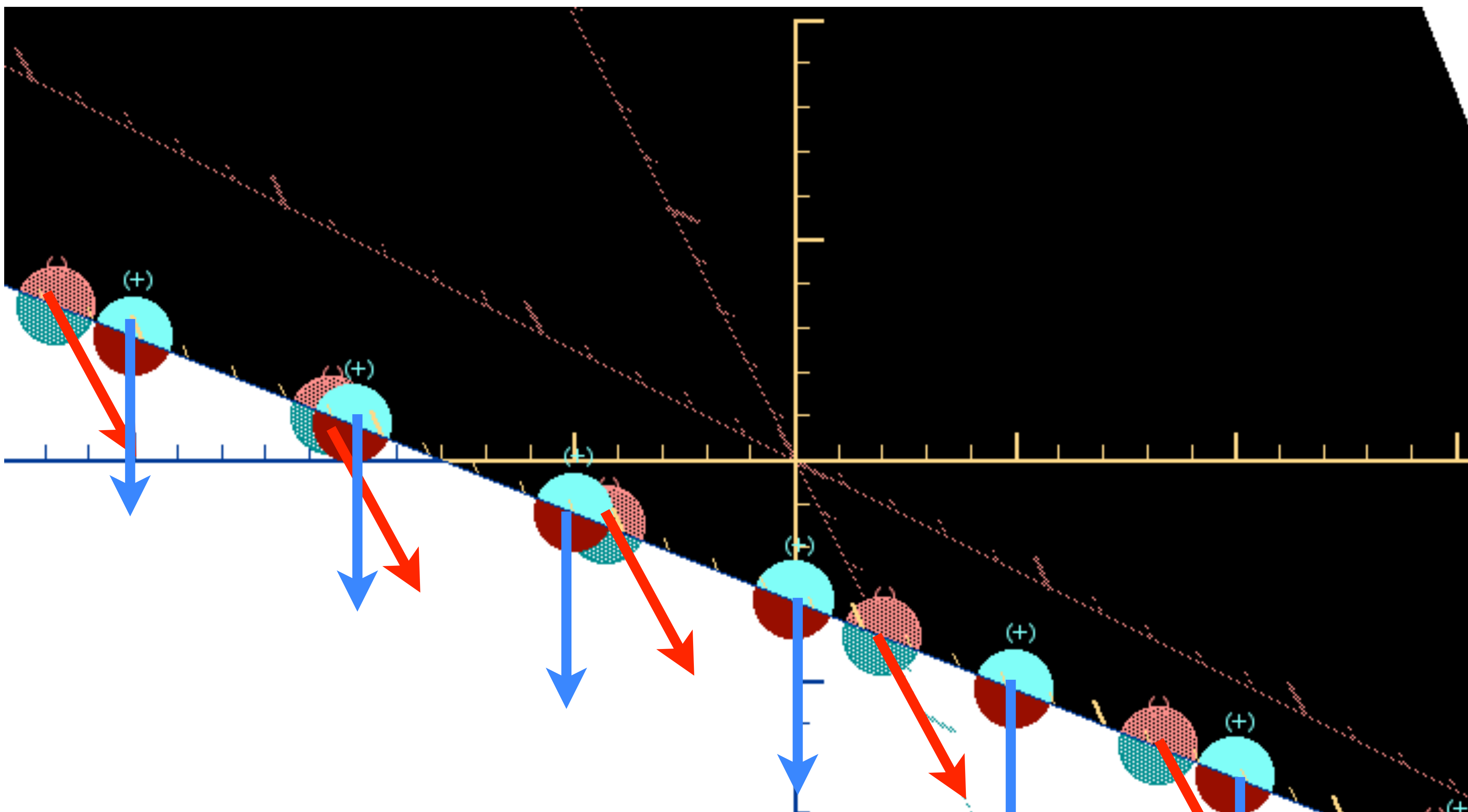
(+) Charge fixed (-) Charge moving to right (*Negative current density*)

(+) Charge density is *Greater* than (-) Charge density

wire appears postive (+) (repulsive to +)

Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony



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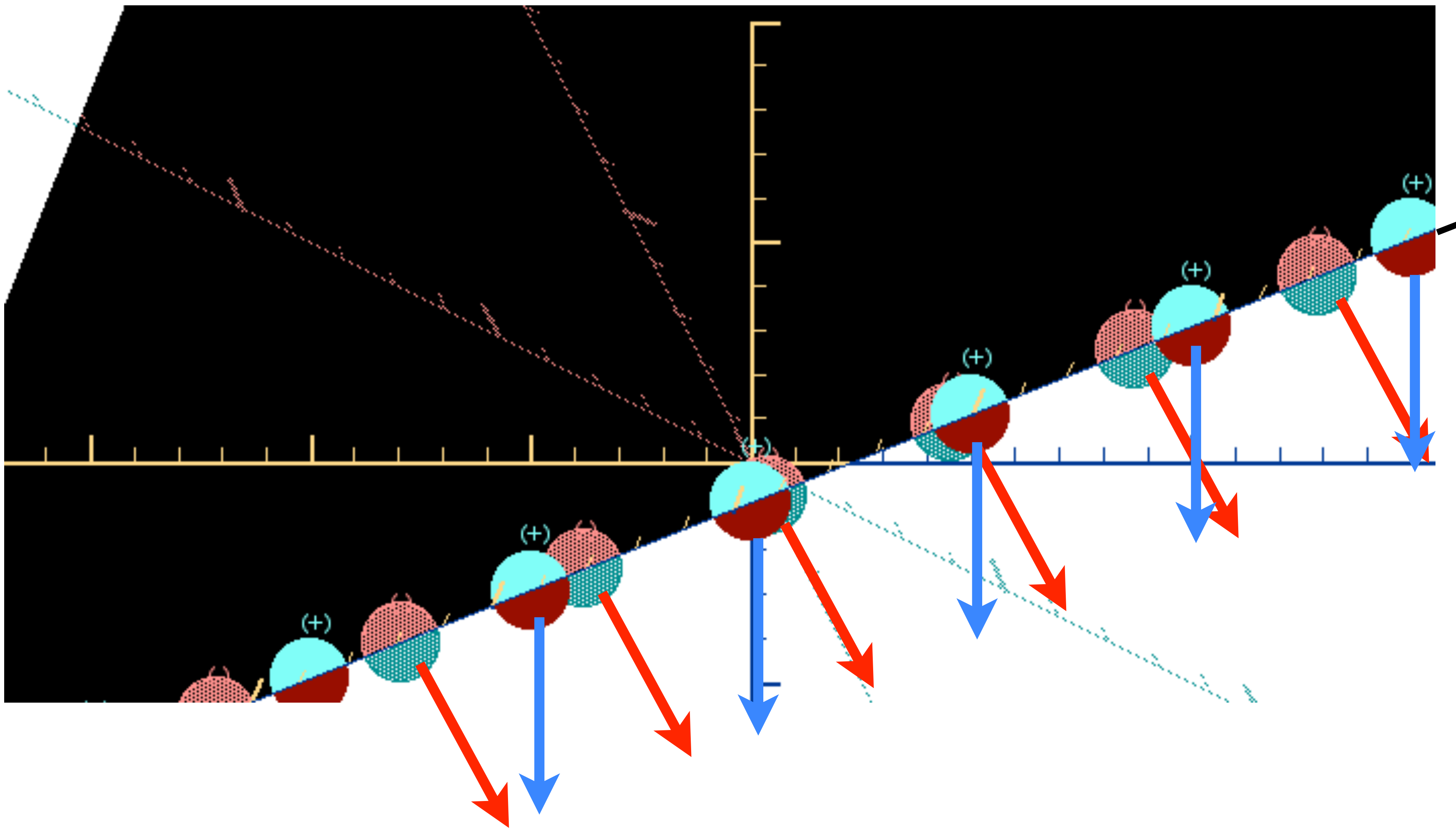

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wire appears negative (-) (attractive to +)

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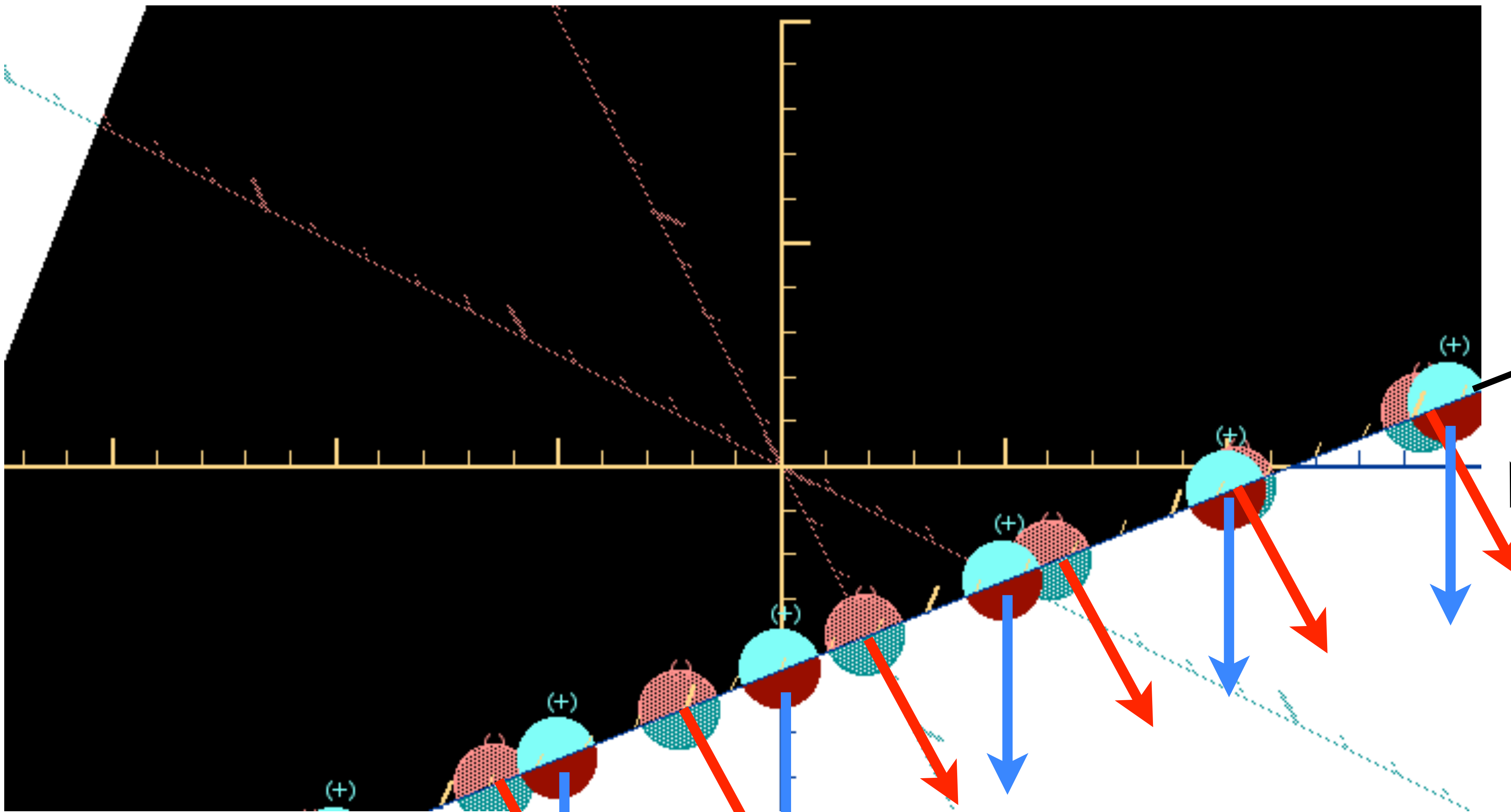
Relativistic effects on charge, current, and Maxwell Fields

Current density changes by Lorentz asynchrony



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Relativistic effects on charge, current, and Maxwell Fields

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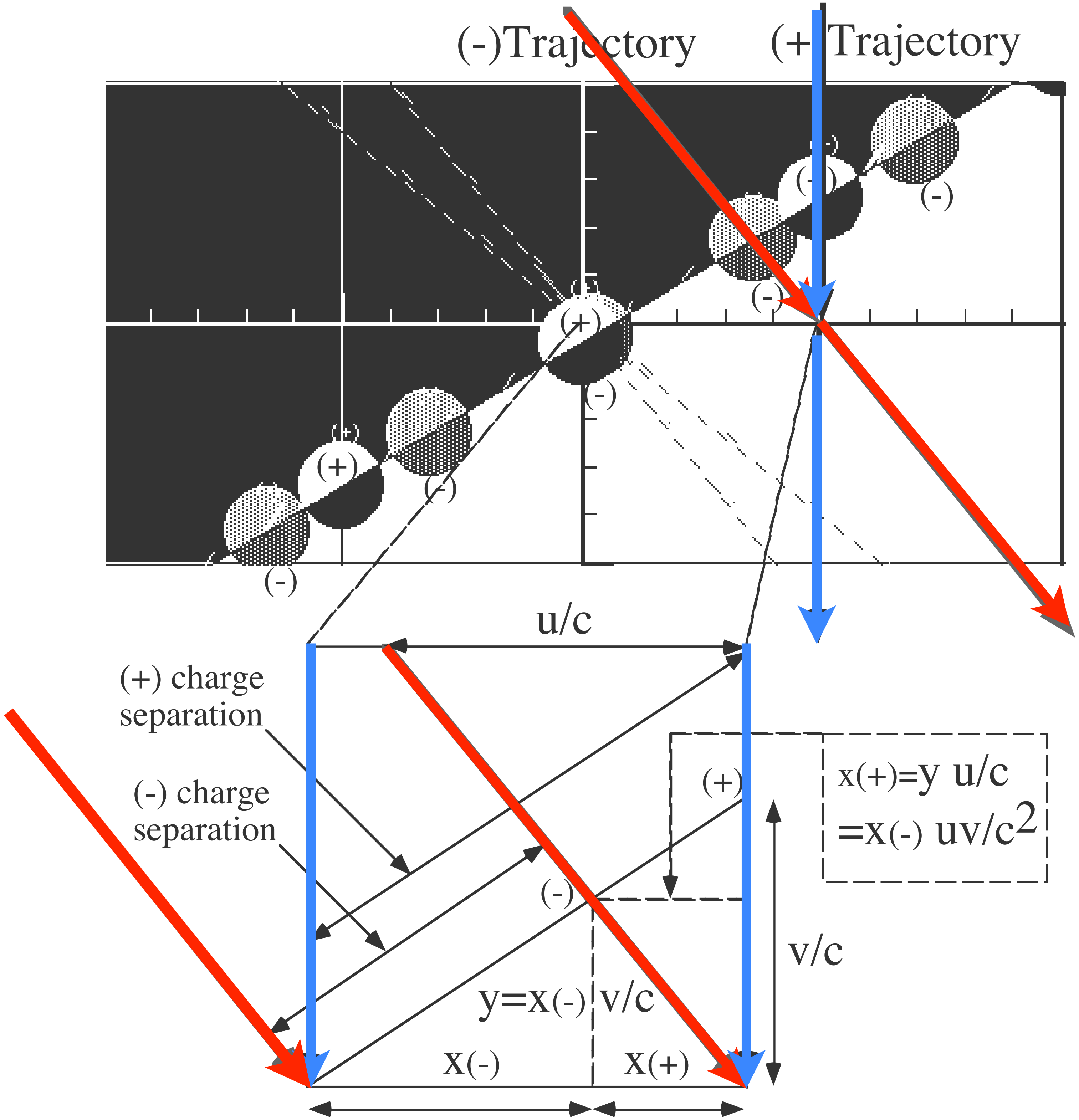
➡ *Magnetic B-field is relativistic effect*



$$\frac{\rho(-)}{\rho(+)} = \frac{(+)\text{ charge separation}}{(-)\text{ charge separation}} = \frac{x(+)+x(-)}{x(-)}$$

$$\frac{\rho(-)}{\rho(+)} = \frac{x(+)}{x(-)} + 1 = \frac{uv}{c^2} + 1$$

$$\rho(+) - \rho(-) = \rho(+)\left(1 - \frac{\rho(-)}{\rho(+)}\right) = -\frac{uv}{c^2}\rho(+)$$

Unit square: (u/c) /1 = x(+)/y

(v/c) /1 = y/x(-)

Magnetic B-field is relativistic effect!

The electric force field **E** of a charged line varies inversely with radius. The Gauss formula for force in mks units :

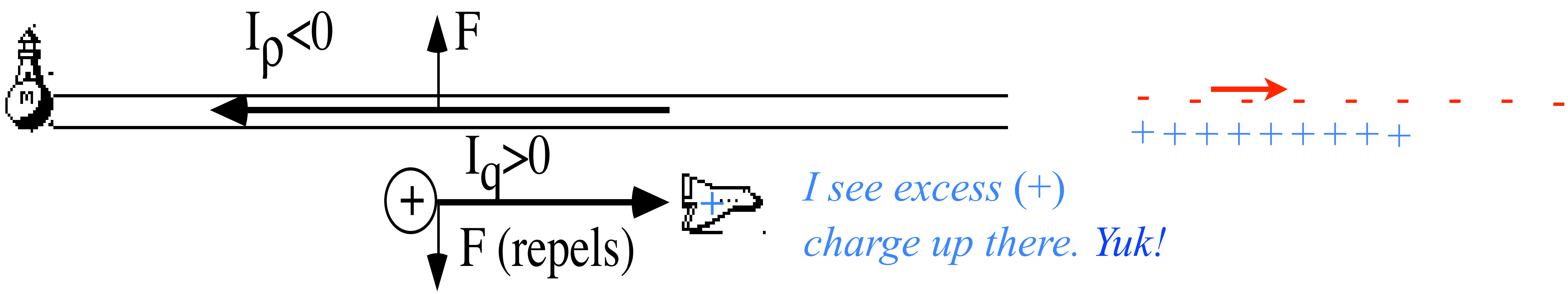
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2\rho}{r}\right] \quad , \quad \text{where:} \quad \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\frac{N\cdot m^2}{Coul.}$$

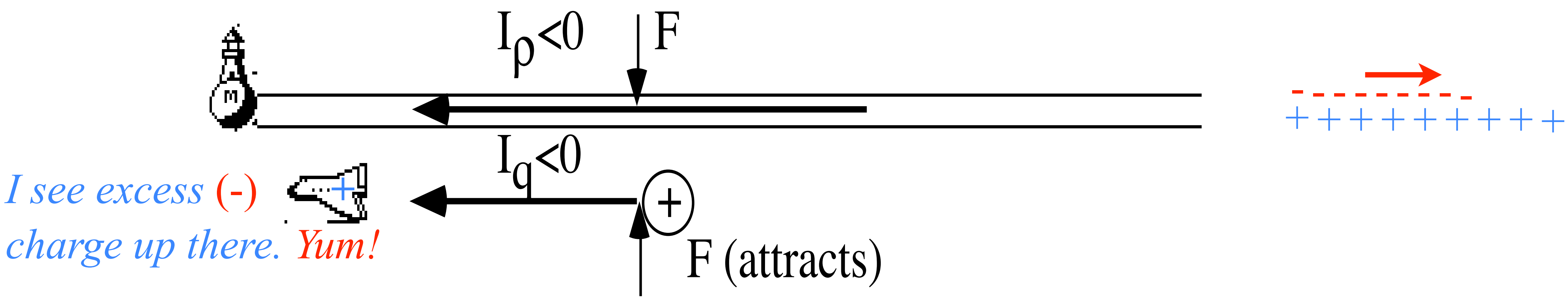
$$F = qE = q\left[\frac{1}{4\pi\varepsilon_0}\frac{2}{r}\left(-\frac{uv}{c^2}\rho(+)\right)\right] = -\frac{2\,qv\,\rho(+)u}{4\pi\varepsilon_0 c^2\, r} = -2\times10^{-7}\,\frac{I_q\, I_\rho}{r}$$

$1/4\pi\varepsilon_0 = 9\cdot10^9$

$c^2 = 9\cdot10^{-16}$

$1/(4\pi\varepsilon_0 c^2) = 10^{-7}$





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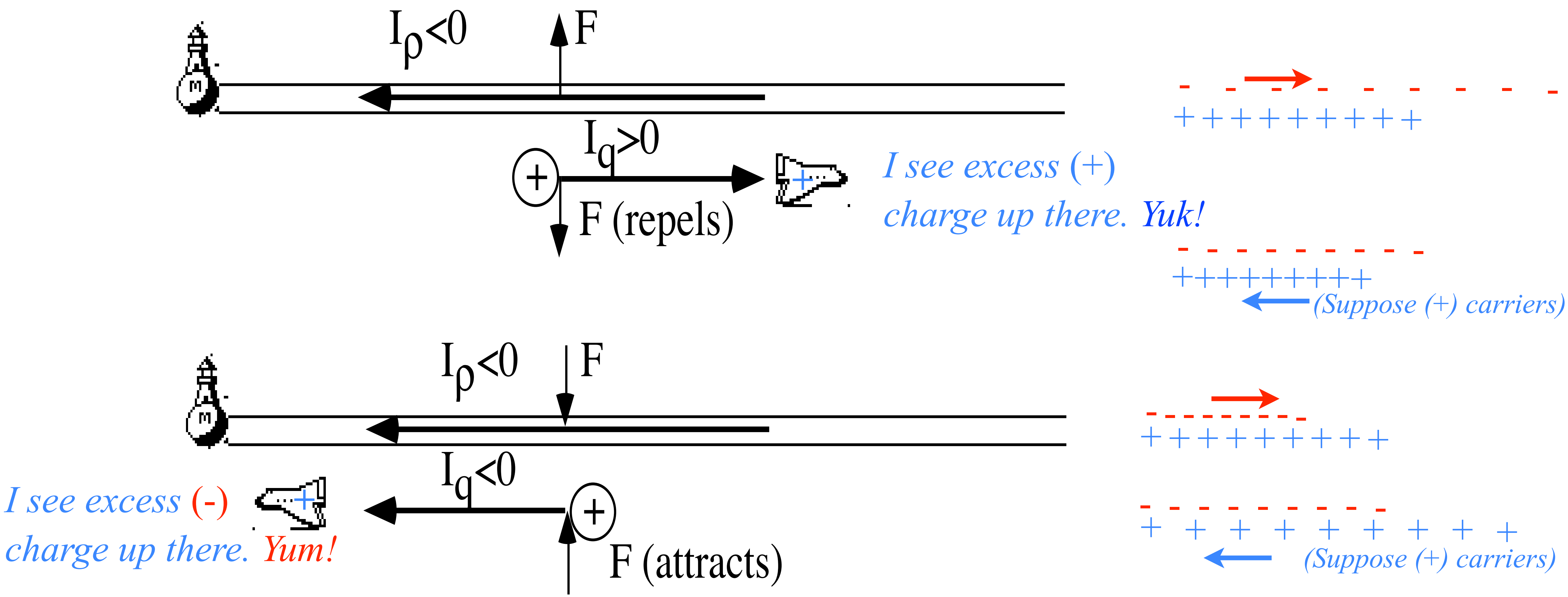


➔ *Relating photons to Maxwell energy density and Poynting flux*

Relativistic variation and invariance of frequency (ω,k) and amplitudes

How probability ψ-waves and flux ψ-waves evolved

*Properties of amplitude ψ*ψ-squares*

More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta

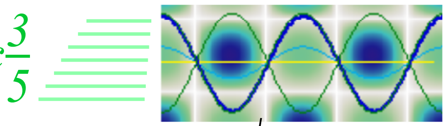
Light Energy and Flux *2-CW vs 1-CW-light*

What if head-on CW's υ_A*=1200THz* and υ_B*=300THz* pair-up in a *2-CW*-light beam?

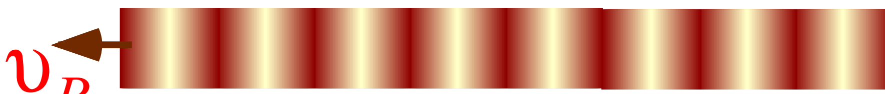
υ_A

(Ultraviolet 1200THz)

Group velocity $u=c\frac{\upsilon_A-\upsilon_B}{\upsilon_A+\upsilon_B}=c\frac{3}{5}$



Green 600THz$=\upsilon_0=\sqrt{(\upsilon_A\,\upsilon_B)}$



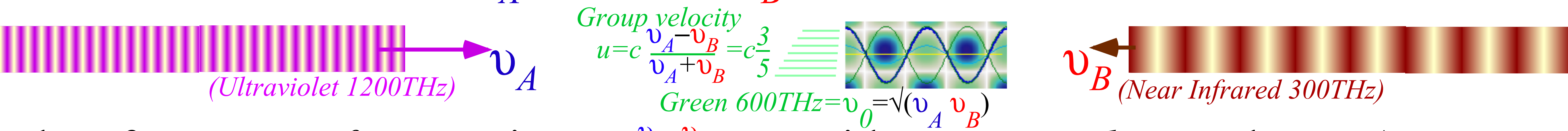
υ_B *(Near Infrared 300THz)*

They form a *rest frame* going $u=c\frac{\upsilon_A-\upsilon_B}{\upsilon_A+\upsilon_B}=3c/5$ with a *mean* or *base* color $\upsilon_0=\sqrt{(\upsilon_A\,\upsilon_B)}$

($\upsilon_0=B=600THz$ is *green* here. Neither has this singly.)

Light Energy and Flux *2-CW vs 1-CW-light*

What if head-on CW's $\upsilon_{A=1200THz}$ and $\upsilon_{B=300THz}$ pair-up in a *2-CW*-light beam?



They form a *rest frame* going $u=c\frac{\upsilon_A-\upsilon_B}{\upsilon_A+\upsilon_B}=3c/5$ with a *mean* or *base* color $\upsilon_0=\sqrt{(\upsilon_A\ \upsilon_B)}$ ($\upsilon_0=B=600THz$ is *green* here. Neither has this singly.) *All* observers agree on υ_0 since *all* shift-products $b\upsilon_A\ r\upsilon_B$ equal $(\upsilon_0)^2$ due to Doppler-time-symmetry ($b=1/r$). Single *CW*'s get *invariant* properties if they pair-up. The υ_A-υ_B pairing above makes a number $\bar{N}$ of *invariant mass quanta* $M_1=h\upsilon_0/c^2=4.42\cdot 10^{-36}kg$ where the number $\bar{N}$ is invariant, too. $\bar{N}$ is Planck's *photon number* for the cavity rest energy $E=\bar{N}h\upsilon_0$.

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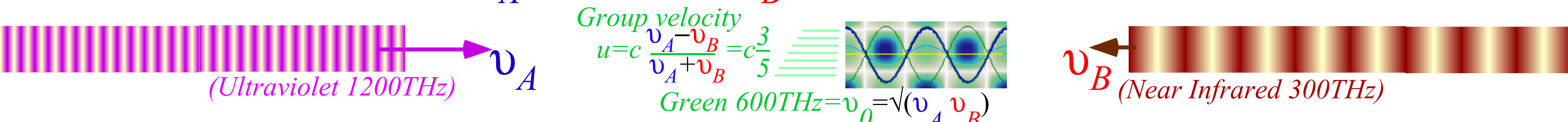

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Relating Planck's E to Maxwell's Density U=E/V

Maxwell field energy *E*, a product of mean-square electric field $\langle \mathsf{E}^2\rangle$, volume of cavity *V*, and constant $\varepsilon_0=8.854\cdot 10^{-12}C^2/N\cdot m^2$, approximates Planck's energy $\bar{N}h\upsilon_0$.

$E=\langle \mathsf{E}^2\rangle V\varepsilon_0=\bar{N}h\upsilon_0$ *Maxwell-Planck Energy* $U=\langle \mathsf{E}^2\rangle\varepsilon_0=\bar{N}h\upsilon_0/V$ *Maxwell-Planck Density*

Field Energy $=|\mathsf{E}|^2\varepsilon_0$ $1/4\pi\varepsilon_0=9\cdot 10^9$

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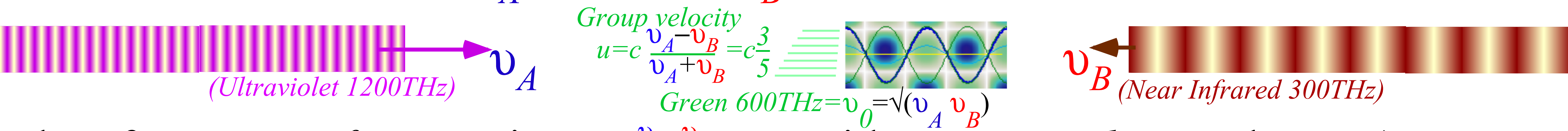

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$E=\langle \mathsf{E}^2\rangle V\varepsilon_0=\bar{N}h\upsilon_0$ *Maxwell-Planck Energy* $U=\langle \mathsf{E}^2\rangle\varepsilon_0=\bar{N}h\upsilon_0/V$ *Maxwell-Planck Density*

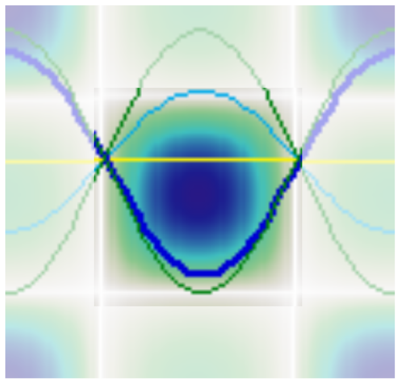
Example: Let a $\frac{1}{4}\mu m$-cube cavity *(Half-wave at 600Thz)* have $\bar{N}=10^{10}$ photons in volume $V=(\frac{1}{4}10^{-6}m)^3$.

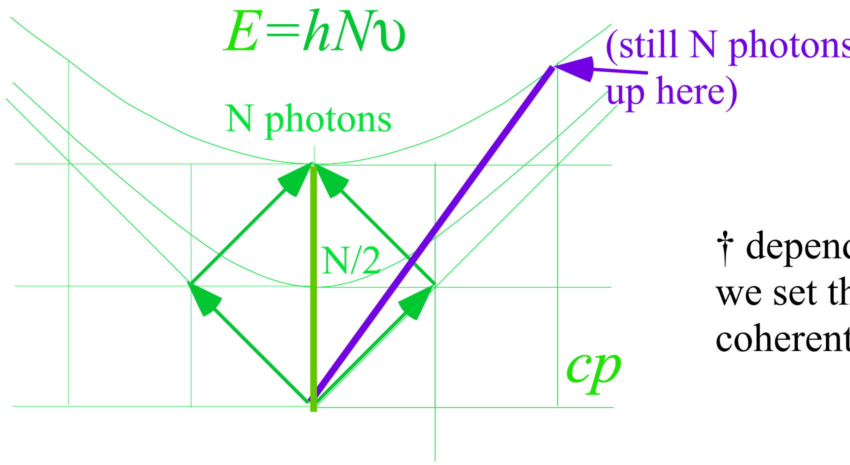
Energy per photon: $h\upsilon_0=4\cdot 10^{-19}J=2.5\ eV$ Energy of $\bar{N}$ photons: $\bar{N}h\upsilon_0=4\cdot 10^{-9}J=25GeV$

E-field per photon: $\mathsf{E}_1=\sqrt{(h\upsilon_0/V\varepsilon_0)}=7.6\cdot 10^3 V/m$ E-field of $\bar{N}$ photons: $\mathsf{E}_N=7.6\cdot 10^{13} V/m$

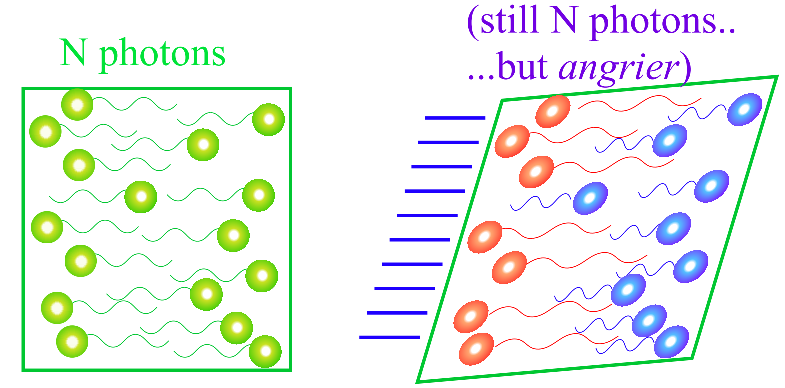
Energy and Flux (contd) *2-CW- vs 1-CW-light*

Planck $E=Nh\upsilon$ relation allows us to interpret our N-quantized *2-CW* mode as a box or *cavity* of $N_{\text{(more-or-less†)}}$ photons where N *is invariant* to speed u of box.





† depends on how we set the mode's coherent state.



Energy and Flux (contd) *2-CW- vs 1-CW-light*

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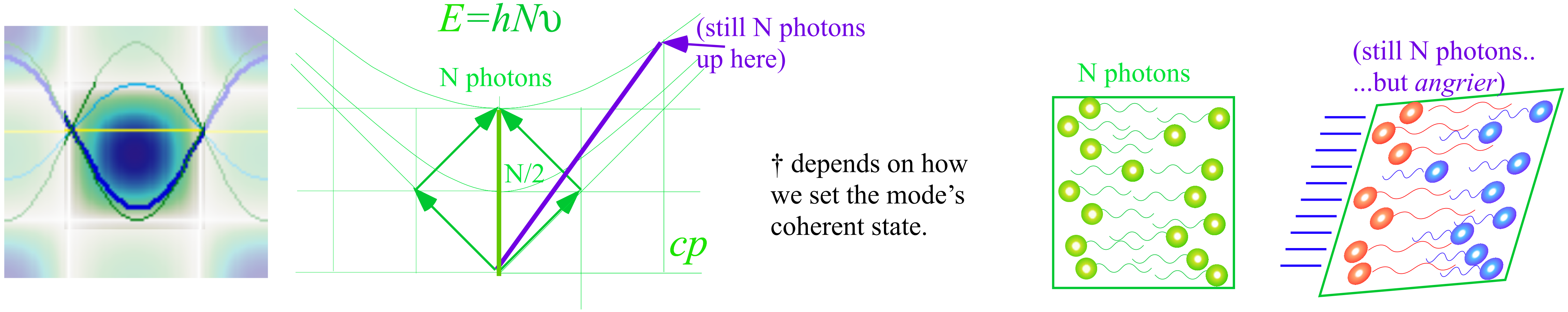

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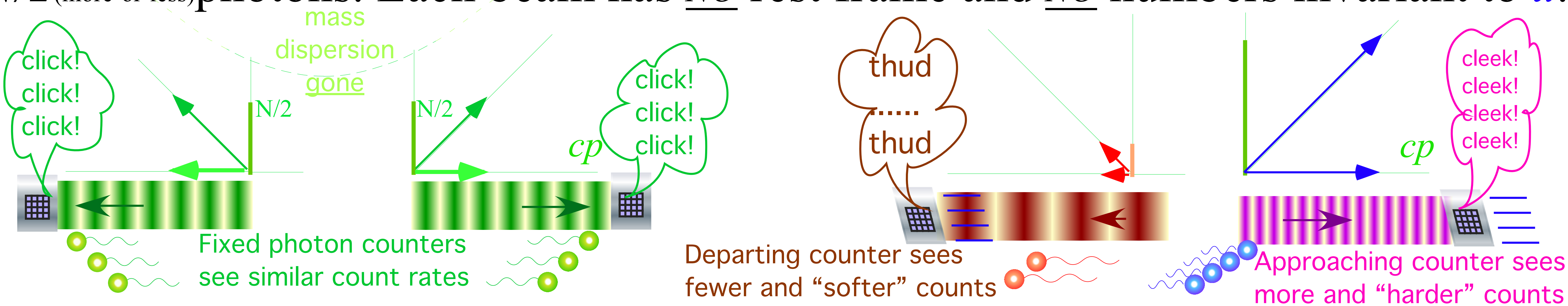
If we open the box our *2-CW* mode "divorces" into two separate *1-CW* beams of *N/2*(more-or-less) photons. Each beam has NO rest frame and NO numbers invariant to *u*.



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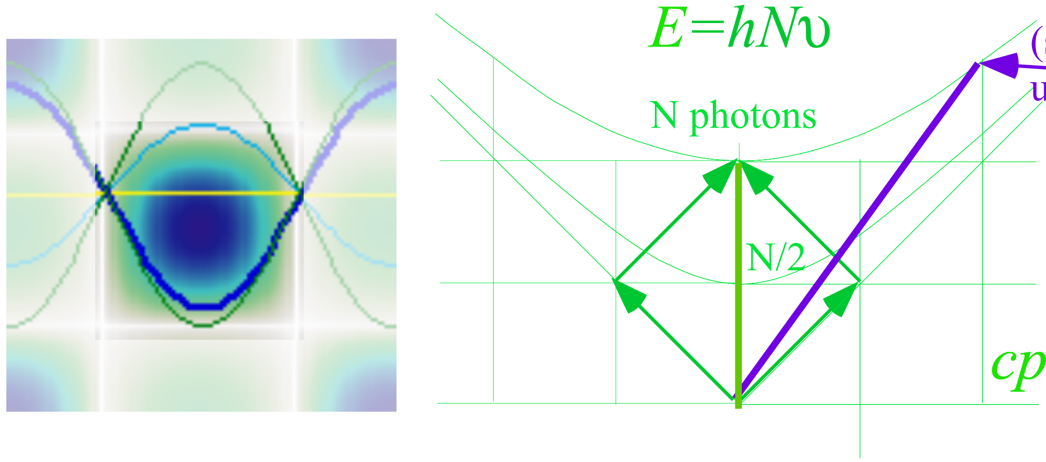

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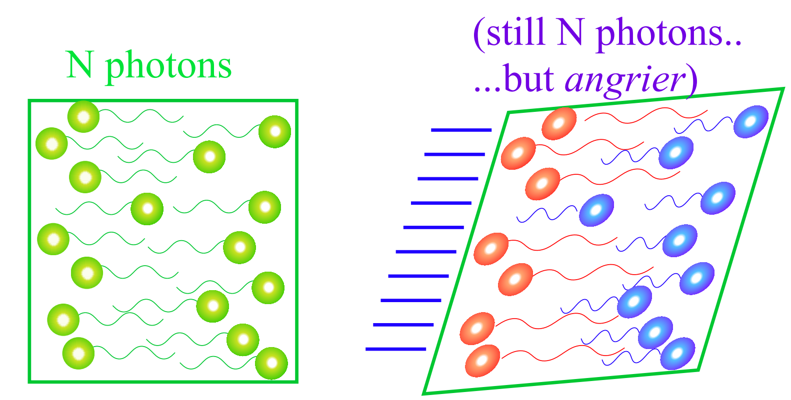

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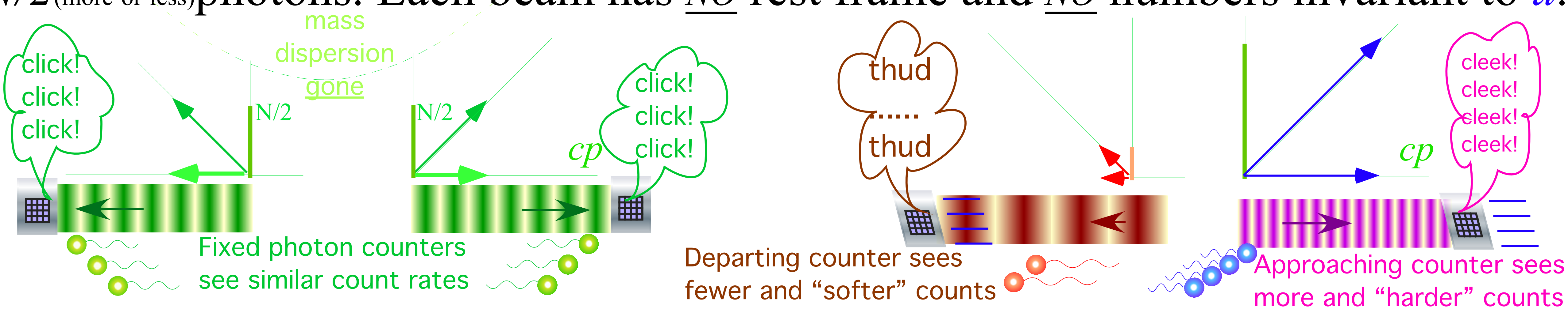


Relating Poynting's Intensity S=cU to Planck's Flux

Poynting intensity *S* is a product of $c=2.99792458m/s$ and density *U*. It approximates Planck's energy $E=Nh\upsilon$ times *c* and divided by cavity volume *V*.

$$S=cU=(Nc/V)h\upsilon = n\, h\upsilon \quad \textit{Poynting-Planck Flux} \quad \textit{(Watts per square meter)}$$

The *photon-count rate* is $n=Nc/V$ *(per square meter per second)* and $h\upsilon$ is energy *(per count)*.

Relating photons to Maxwell energy density and Poynting flux

- ➔ *Relativistic variation and invariance of frequency (ω,k) and amplitudes*
- *How probability ψ-waves and flux ψ-waves evolved*
    - *Properties of amplitude ψ*ψ-squares*
- *More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta*

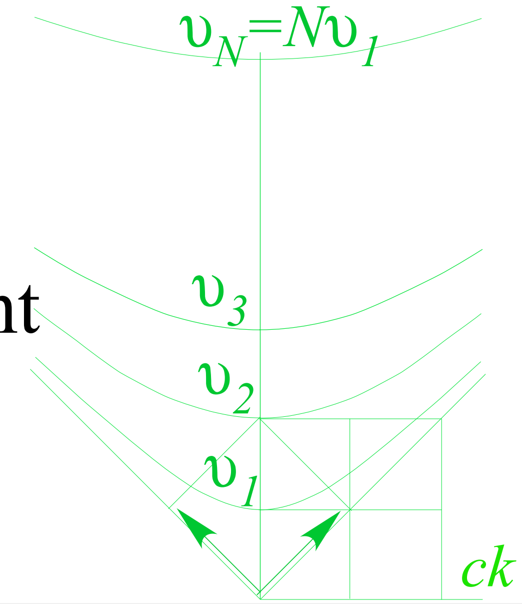
Frequency and Amplitude Variance 2-CW-light vs 1-CW-light
2-CW modes have invariance

Maxwell-Planck energy E is *photon number* $N(m^{-3})$ times *2-CW-frequency* υ_1.

Invariant to ρ — Each is ρ-invariant

$$E=\langle U\rangle\cdot V=\varepsilon_0\langle \mathsf{E}^2\rangle\cdot V=\varepsilon_0\langle \mathsf{E}_{2\text{-}CW}{}^*\mathsf{E}_{2\text{-}CW}\rangle\cdot V=hN\upsilon_1=h\upsilon_N$$

Photon number N and rest-frame frequencies $\upsilon_1 \ldots \upsilon_N$ are invariant to rapidity ρ and occupy *(ω,ck)-hyperbolas* in per-spacetime.



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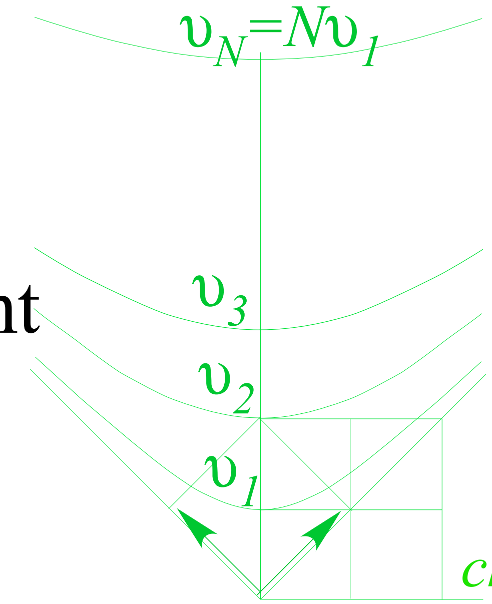


Photon number N and rest-frame frequencies $\upsilon_1 \ldots \upsilon_N$ are invariant to rapidity ρ and occupy *(ω,ck)-hyperbolas* in per-spacetime.

1-CW beams lack invariance (have *"variance"* ala′ Doppler)

Planck-Poynting flux S is *count rate* $n=Nc/V(m^{-2}s^{-1})$ times *1-CW-frequency* $\upsilon_{\leftarrow}$ or $\upsilon_{\rightarrow}$.

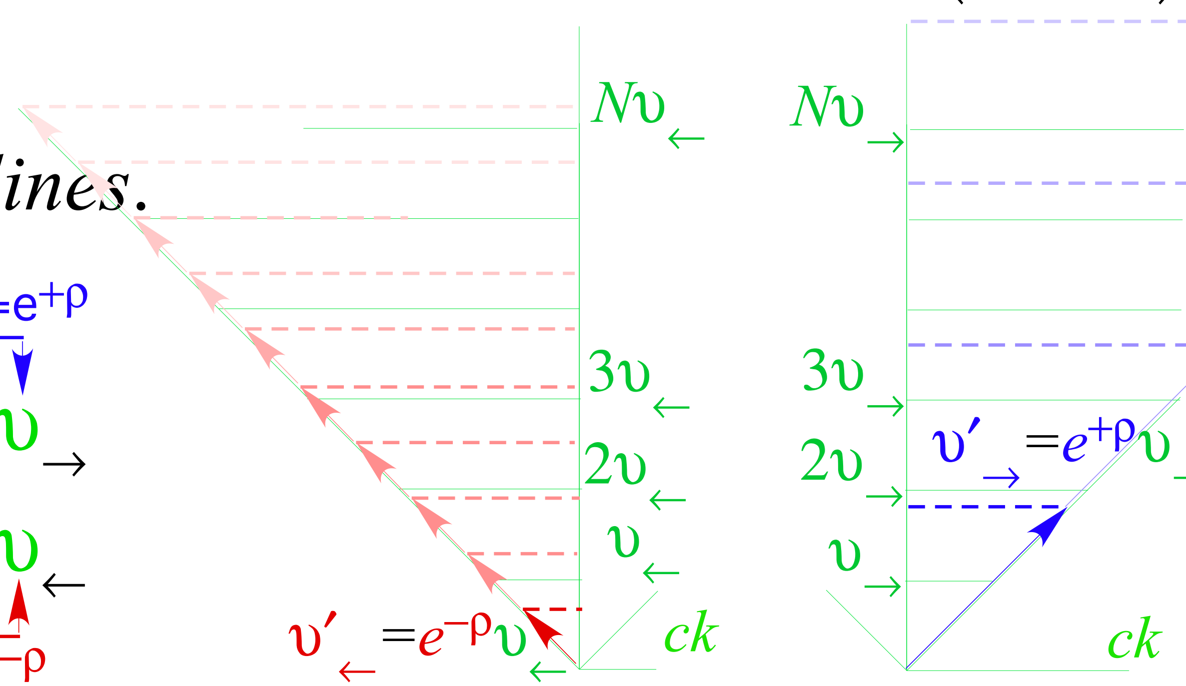
Count rate n and frequency υ Doppler shift by $b=e^{\pm\rho}$ factors and occupy *(ω=±ck)-baselines*.

Shifts by $b=e^{+2\rho}$ — Each blue shifts by $b=e^{+\rho}$

$$S_{\rightarrow}=cU_{\rightarrow}=c\varepsilon_0\langle \mathsf{E}^2\rangle=c\varepsilon_0\langle \mathsf{E}^{\rightarrow}_{1\text{-}CW}{}^*\mathsf{E}^{\rightarrow}_{1\text{-}CW}\rangle=hn_{\rightarrow}\upsilon_{\rightarrow}$$

$$S_{\leftarrow}=cU_{\leftarrow}=c\varepsilon_0\langle \mathsf{E}^2\rangle=c\varepsilon_0\langle \mathsf{E}^{\leftarrow}_{1\text{-}CW}{}^*\mathsf{E}^{\leftarrow}_{1\text{-}CW}\rangle=hn_{\leftarrow}\upsilon_{\leftarrow}$$

Shifts by $r=e^{-2\rho}$ — Each red shifts by $r=e^{-\rho}$



Note: $\mathsf{E}^{\leftrightarrow}_{1\text{-}CW}\sqrt{(c\varepsilon_0/h)}=\sqrt{(n_{\leftrightarrow}\upsilon_{\leftrightarrow})}$ is geometric mean of *amplitude frequency* $n_{\leftrightarrow}$ and *phase frequency* $\upsilon_{\leftrightarrow}$.

Important result below:

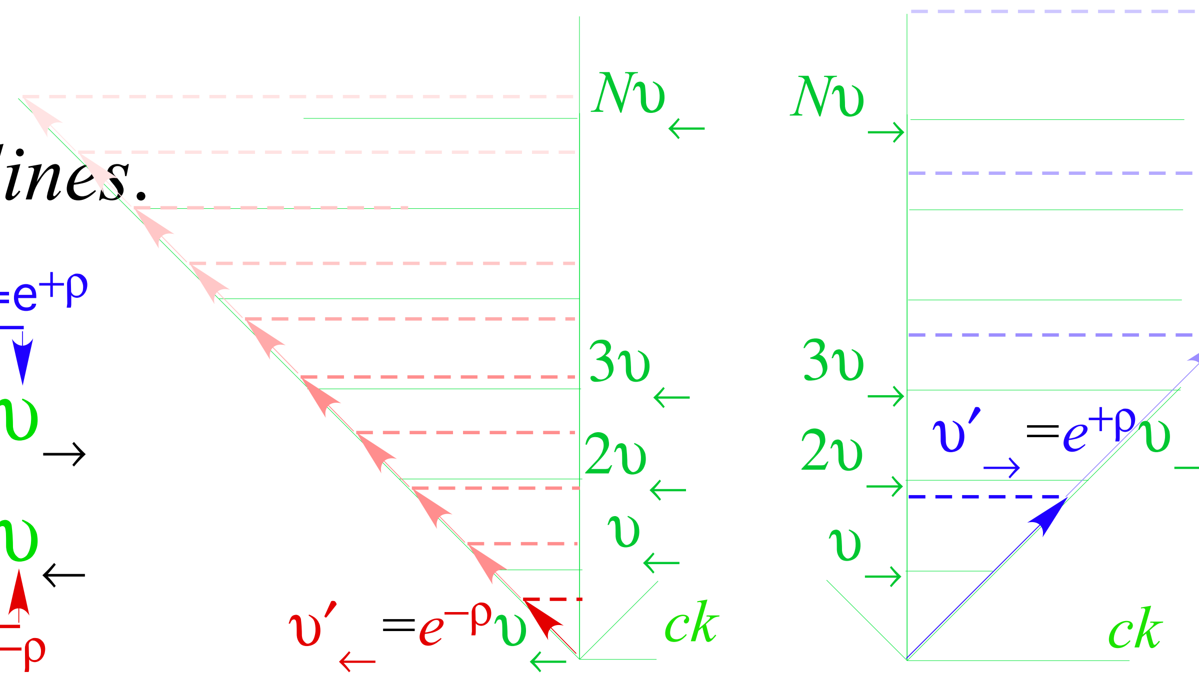
Amplitudes of 1-CW "exponentiate" just like frequency, and intensity does at twice the rate

(A double-double whammy!)

1-CW beams lack invariance (have *"variance"* ala′ Doppler)

Planck-Poynting flux S is *count rate* $n = Nc/V(m^{-2}s^{-1})$ times *1-CW-frequency* $\upsilon_{\leftarrow}$ or $\upsilon_{\rightarrow}$.

Count rate n and frequency υ Doppler shift by $b=e^{\pm\rho}$ factors and occupy *($\omega=\pm ck$)-baselines*.



Shifts by $b=e^{+2\rho}$ — Each blue shifts by $b=e^{+\rho}$

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Shifts by $r=e^{-2\rho}$ — Each red shifts by $r=e^{-\rho}$

Note: $\mathrm{E}^{\leftrightarrow}_{1\text{-CW}}\sqrt{(c\varepsilon_0/h)}=\sqrt{(n_{\leftrightarrow}\upsilon_{\leftrightarrow})}$ is geometric mean of *amplitude frequency* $n_{\leftrightarrow}$ and *phase frequency* $\upsilon_{\leftrightarrow}$.

Relating photons to Maxwell energy density and Poynting flux

Relativistic variation and invariance of frequency (ω,k) and amplitudes

➡ *How probability ψ-waves and flux ψ-waves evolved*

*Properties of amplitude ψ*ψ-squares*

More on unmatched amplitudes AND unmatched frequencies AND unmatched quanta

How Probability Amplitudes ψ *or* ψ *Come About* *(An optical view)*

Maxwell-Planck-Poynting flux $S = cU = c\varepsilon_0 |E|^2 = c\varepsilon_0 E^* E = nh\upsilon$ has count rate $n = Nc/V (m^{-2}s^{-1})$

If each E-field amplitude factor is scaled by a factor $\sqrt{\frac{c\varepsilon_0}{h\upsilon}} = \sqrt{\frac{\varepsilon_0}{h\kappa}}$ the result is a *flux probability amplitude* $\psi = E\sqrt{\frac{c\varepsilon_0}{h\upsilon}}$ whose square equals flux count rate $n (m^{-2}s^{-1})$.

$$\psi^* \psi = n \quad (m^{-2}s^{-1})$$

A *fixed probability amplitude* $\psi = E\sqrt{\frac{\varepsilon_0}{h\upsilon}}$ has square equal to N/V *(particles per volume)*.

$$\psi^* \psi = N/V \quad (m^{-3})$$

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$$\psi^*\psi=N/V \quad (m^{-3})$$

Here's how to answer Planck's worry about photons

Q: How can classical oscillator energy (Amplitude)²(frequency)² jive with linear Planck law S=nhυ?

A: Let amplitude ψ or ψ contain inverse square root of frequency: $\psi=E\sqrt{\frac{c\varepsilon_0}{h\upsilon}}$ *the "quantum amplitude"*

$$Energy\sim|A|^2\upsilon^2 \text{ where vector potential } \mathbf{A} \text{ defines electric field: } \mathbf{E}=\frac{\partial\mathbf{A}}{\partial t}=i\omega\mathbf{A}=2\pi i\upsilon\mathbf{A}$$

$$Energy\sim|A|^2\upsilon^2=\left|A\sqrt{\upsilon}\right|^2\upsilon=\left|\frac{E}{2\pi\upsilon}\sqrt{\upsilon}\right|^2\upsilon=\left|\frac{E}{2\pi\sqrt{\upsilon}}\right|^2\upsilon\sim\left|E\sqrt{\frac{c\varepsilon_0}{h\upsilon}}\right|^2=nh\upsilon$$

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Probability Waves ψ*(x,t)* *(More optical views)*

Optical E-field amplitudes like $\mathsf{E}(x,t) = \mathsf{E}_0 e^{i(kx-\omega t)}$ vary with space x and time t. So do scaled $\psi(x,t)$ ampliudes whose sum-Σ (integral-∫) over cells ΔV (or dV) must be particle number N. For 1-particle systems $(N=1)$ this is the *unit norm* rule.

$$\Sigma_j \psi(x_j,t)^* \psi(x_j,t) \Delta V_j = N \qquad \text{or:} \qquad \int \psi(x,t)^* \psi(x,t) dV = N$$

How Probability Amplitudes ψ *or* ψ *Come About* *(An optical view)*

Maxwell-Planck-Poynting flux $S=cU=c\varepsilon_0|\mathsf{E}|^2=c\varepsilon_0\mathsf{E}^*\mathsf{E}=nh\upsilon$ has count rate $n=Nc/V(m^{-2}s^{-1})$
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Probability Waves $\psi(x,t)$ *(More optical views)*

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$$\Sigma_j\psi(x_j,t)^*\psi(x_j,t)\Delta V_j=N \qquad \text{or:} \qquad \int\psi(x,t)^*\psi(x,t)dV=N$$

Born interpreted $\psi(x,t)^*\psi(x,t)$ as *probable expectation* of particle count. Schrodinger objected to the *probability wave* interpretation that is now accepted and called the Schrodinger theory. A relativistic wave view lends merit to his objections.

Doppler Transformation of 2-CW Modes

Doppler shift of *opposite-k 1-CW* beams. As derived before phases are invariant: $(k'x'-\omega't'=kx-\omega t)$

E-*wave:* $E=E_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)}+E_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)}$

Ψ-*wave:* $\Psi=\psi_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)}+\psi_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)}$

blue shift

$$E'_{\rightarrow}=b\,E_{\rightarrow}=e^{+\rho}E_{\rightarrow}$$

red shift

$$E'_{\leftarrow}=r\,E_{\leftarrow}=e^{-\rho}E_{\leftarrow}$$

$$\psi=E\sqrt{\frac{\varepsilon_0}{h\upsilon}}$$

scaled blue shift

$$\psi'_{\rightarrow}=\sqrt{b}\;\psi_{\rightarrow}=e^{+\rho/2}\psi_{\rightarrow}$$

scaled red shift

$$\psi'_{\leftarrow}=\sqrt{r}\;\psi_{\leftarrow}=e^{-\rho/2}\psi_{\leftarrow}$$

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scaled blue shift

$$\psi'_{\rightarrow}=\sqrt{b}\,\psi_{\rightarrow}=e^{+\rho/2}\psi_{\rightarrow}$$

scaled red shift

$$\psi'_{\leftarrow}=\sqrt{r}\,\psi_{\leftarrow}=e^{-\rho/2}\psi_{\leftarrow}$$

Parameters related to *relative velocity* u:

$$\beta=u/c=\tanh\rho=\frac{\sinh\rho}{\cosh\rho}=\frac{e^{+\rho}-e^{-\rho}}{e^{+\rho}+e^{-\rho}}=\frac{b^2-1}{b^2+1}$$

$$b^2=\frac{1+\beta}{1-\beta}=\frac{1+\tanh\rho}{1-\tanh\rho}$$

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Ψ-wave: $\Psi=\psi_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)}+\psi_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)}$

scaled blue shift

$$\psi'_{\rightarrow}=\sqrt{b}\;\psi_{\rightarrow}=e^{+\rho/2}\psi_{\rightarrow}$$

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Transformation of *SWR* (or *SWQ*) and u_{GROUP} (or u_{PHASE}) is a *non-linear* transformation

$$SWR'=\frac{E'_{\rightarrow}-E'_{\leftarrow}}{E'_{\rightarrow}+E'_{\leftarrow}}=\frac{b^2E_{\rightarrow}-E_{\leftarrow}}{b^2E_{\rightarrow}+E_{\leftarrow}}=\frac{(1+\beta)E_{\rightarrow}-(1-\beta)E_{\leftarrow}}{(1+\beta)E_{\rightarrow}+(1-\beta)E_{\leftarrow}}=\frac{(E_{\rightarrow}-E_{\leftarrow})+\beta(E_{\rightarrow}+E_{\leftarrow})}{(E_{\rightarrow}+E_{\leftarrow})+\beta(E_{\rightarrow}-E_{\leftarrow})}=\frac{SWR+\beta}{1+\beta\cdot SWR}$$

SWR (or *SWQ*) *Transformation*

$$SWR'=\frac{SWR+\beta}{1+SWR\cdot\beta}=\frac{SWR+u/c}{1+SWR\cdot u/c}$$

u_{GROUP} (or u_{PHASE}) *Transformation*

$$u'_{GROUP}/c=\frac{u_{GROUP}/c+\beta}{1+u_{GROUP}\cdot\beta/c}=\frac{(u_{GROUP}+u)/c}{1+u_{GROUP}\cdot u/c^2}$$

Doppler Transformation of 2-CW Modes

Doppler shift of *opposite-k 1-CW* beams.

As derived before phases are invariant: $(k'x'-\omega't'=kx-\omega t)$

E-*wave:* $E=E_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)}+E_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)}$

blue shift: $E'_{\rightarrow}=b\,E_{\rightarrow}=e^{+\rho}E_{\rightarrow}$

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Ψ-*wave:* $\Psi=\psi_{\rightarrow}e^{i(k_{\rightarrow}x-\omega_{\rightarrow}t)}+\psi_{\leftarrow}e^{i(k_{\leftarrow}x-\omega_{\leftarrow}t)}$

$\psi=E\sqrt{\frac{\varepsilon_0}{h\upsilon}}$

scaled blue shift: $\psi'_{\rightarrow}=\sqrt{b}\;\psi_{\rightarrow}=e^{+\rho/2}\psi_{\rightarrow}$

scaled red shift: $\psi'_{\leftarrow}=\sqrt{r}\;\psi_{\leftarrow}=e^{-\rho/2}\psi_{\leftarrow}$

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SWR (or *SWQ*) *Transformation*

$$SWR'=\frac{SWR+\beta}{1+SWR\cdot\beta}=\frac{SWR+u/c}{1+SWR\cdot u/c}$$

u_{GROUP} (or u_{PHASE}) *Transformation*

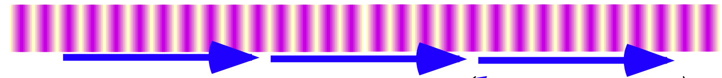
$$u'_{GROUP}/c=\frac{u_{GROUP}/c+\beta}{1+u_{GROUP}\cdot\beta/c}=\frac{(u_{GROUP}+u)/c}{1+u_{GROUP}\cdot u/c^2}$$

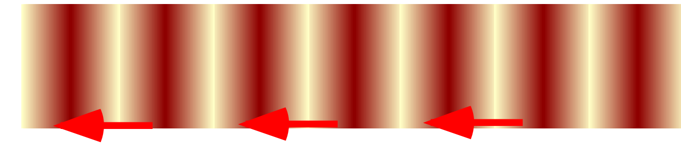
Both are restatements of hyperbolic trig identity: $\tanh(a+b)=\frac{\tanh(a)+\tanh(b)}{1+\tanh(a)\cdot\tanh(b)}$

last term is ignorable if both a and b are small

Velocity addition is *non-linear* but rapidity addition is always *linear*: $\rho_{a+b}=\rho_a+\rho_b$

Unequal amplitudes and Unequal frequencies



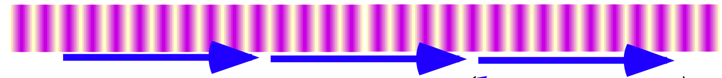


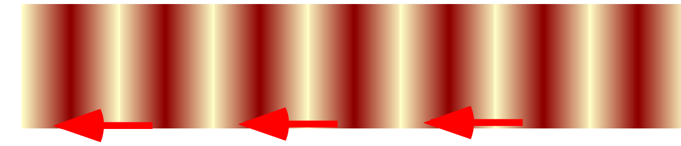
Suppose a general 2-CW Ψ-*wave:* $\Psi = \psi_{\rightarrow} e^{i(k_{\rightarrow} x - \omega_{\rightarrow} t)} + \psi_{\leftarrow} e^{i(k_{\leftarrow} x - \omega_{\leftarrow} t)}$

where probable count is $N_{\rightarrow} = |\psi_{\rightarrow}|^2$ for *right* and $N_{\leftarrow} = |\psi_{\leftarrow}|^2$ for *left*-going beams.

www.uark.edu/ua/pirelli/php/amplitude_probability_7.php

Unequal amplitudes and Unequal frequencies





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Amplitudes ($\psi_{\rightarrow}=\sqrt{\frac{\varepsilon_0}{\hbar\omega_{\rightarrow}}}E_{\rightarrow}$,$\psi_{\leftarrow}=\sqrt{\frac{\varepsilon_0}{\hbar\omega_{\leftarrow}}}E_{\leftarrow}$) of frequencies ($\omega_{\rightarrow}=ck_{\rightarrow}$,$\omega_{\leftarrow}=ck_{\leftarrow}$) determine

$$\textit{probable momentum-flux } \langle p\rangle=\langle\hbar k\rangle=\overbrace{|\psi_{\rightarrow}|^2}^{\text{right count } N_{\rightarrow}}\hbar k_{\rightarrow}-\overbrace{|\psi_{\leftarrow}|^2}^{\text{left count } N_{\leftarrow}}\hbar k_{\leftarrow}$$

$$=\frac{\varepsilon_0}{\hbar\omega_{\rightarrow}}|E_{\rightarrow}|^2\hbar k_{\rightarrow}-\frac{\varepsilon_0}{\hbar\omega_{\leftarrow}}|E_{\leftarrow}|^2\hbar k_{\leftarrow}=\frac{\varepsilon_0}{c}\left(|E_{\rightarrow}|^2-|E_{\leftarrow}|^2\right)$$

$$\textit{probable energy-flux } \langle E\rangle=\langle\hbar\omega\rangle=|\psi_{\rightarrow}|^2\hbar\omega_{\rightarrow}+|\psi_{\leftarrow}|^2\hbar\omega_{\leftarrow}$$

$$=\frac{\varepsilon_0}{\hbar\omega_{\rightarrow}}|E_{\rightarrow}|^2\hbar\omega_{\rightarrow}+\frac{\varepsilon_0}{\hbar\omega_{\leftarrow}}|E_{\leftarrow}|^2\hbar\omega_{\leftarrow}=\varepsilon_0\left(|E_{\rightarrow}|^2+|E_{\leftarrow}|^2\right)$$

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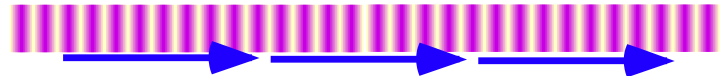
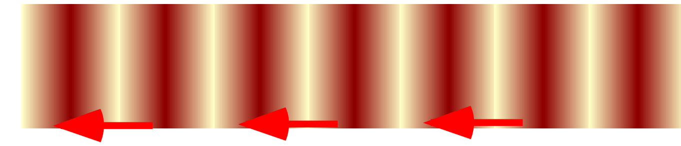
Unequal amplitudes and Unequal frequencies





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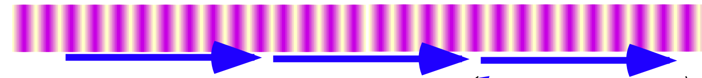
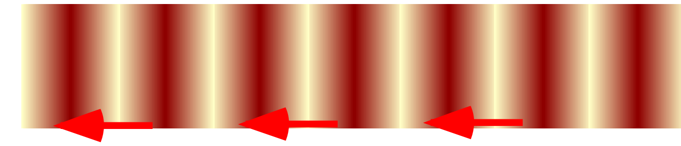
$$\textit{Invariant hyperbola } \langle E\rangle^2-c^2\langle p\rangle^2=4\varepsilon_0|E_{\rightarrow}|^2\varepsilon_0|E_{\leftarrow}|^2=\hbar^2\omega_{\rightarrow}\omega_{\leftarrow}4N_{\rightarrow}N_{\leftarrow}=(\hbar\bar{\omega}\bar{N})^2=(2\varepsilon_0\bar{E}^2)^2$$

Unequal amplitudes and Unequal frequencies





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$$\textit{probable energy-flux } \langle E\rangle=\langle\hbar\omega\rangle=|\psi_{\rightarrow}|^2\hbar\omega_{\rightarrow}+|\psi_{\leftarrow}|^2\hbar\omega_{\leftarrow}$$

$$=\frac{\varepsilon_0}{\hbar\omega_{\rightarrow}}|E_{\rightarrow}|^2\hbar\omega_{\rightarrow}+\frac{\varepsilon_0}{\hbar\omega_{\leftarrow}}|E_{\leftarrow}|^2\hbar\omega_{\leftarrow}=\varepsilon_0(|E_{\rightarrow}|^2+|E_{\leftarrow}|^2)$$

Invariant hyperbola $\langle E\rangle^2-c^2\langle p\rangle^2=4\varepsilon_0|E_{\rightarrow}|^2\varepsilon_0|E_{\leftarrow}|^2=\hbar^2\omega_{\rightarrow}\omega_{\leftarrow}4N_{\rightarrow}N_{\leftarrow}=(\hbar\bar{\omega}\bar{N})^2=(2\varepsilon_0\bar{E}^2)^2$

In Center-of-Momentum (COM) frame

$[E'_{\rightarrow}=\bar{E}=E'_{\leftarrow}]$ speed is $u_{COM}=c\frac{E_{\rightarrow}-E_{\leftarrow}}{E_{\rightarrow}+E_{\leftarrow}}$

Mean amplitude

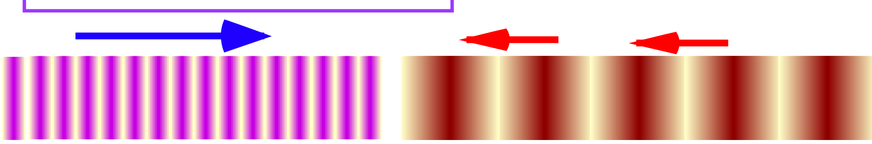
$\bar{E}=\sqrt{E_{\rightarrow}E_{\leftarrow}}$

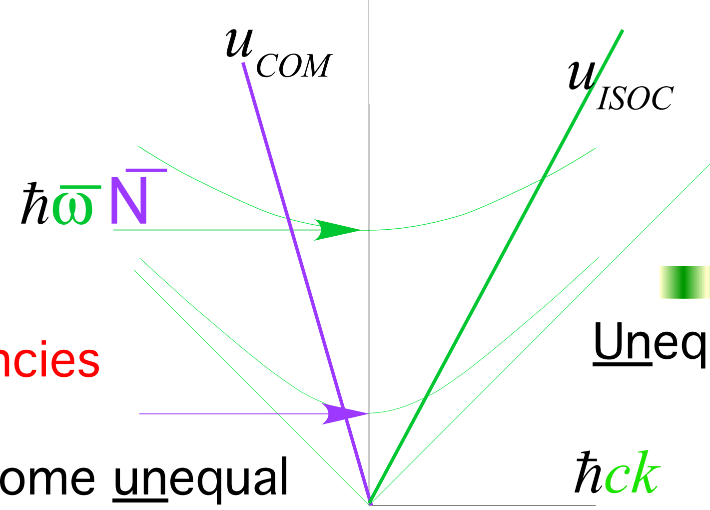
In *Isochromatic (ISOC) frame*

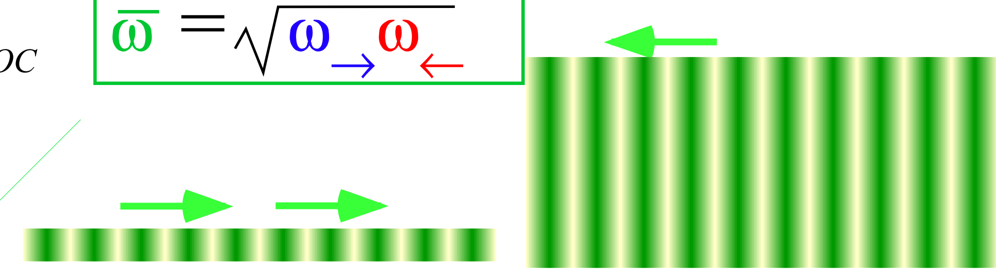
$[\omega'_{\rightarrow}=\bar{\omega}=\omega'_{\leftarrow}]$ speed is $u_{ISO}=c\frac{\omega_{\leftarrow}-\omega_{\rightarrow}}{\omega_{\leftarrow}+\omega_{\rightarrow}}$

Mean color

$\bar{\omega}=\sqrt{\omega_{\rightarrow}\omega_{\leftarrow}}$







Equal amplitudes but Unequal frequencies

Unequal amplitudes but Equal frequencies

Mean count

$\bar{N}=\sqrt{4N_{\rightarrow}N_{\leftarrow}}$

Hyberbola drops as $E_{\rightarrow}$ and $E_{\leftarrow}$ become unequal

www.uark.edu/ua/pirelli/php/amplitude_probability_7.php

The Ship-Barn-and-Butler saga of confused causality

(More about galloping)

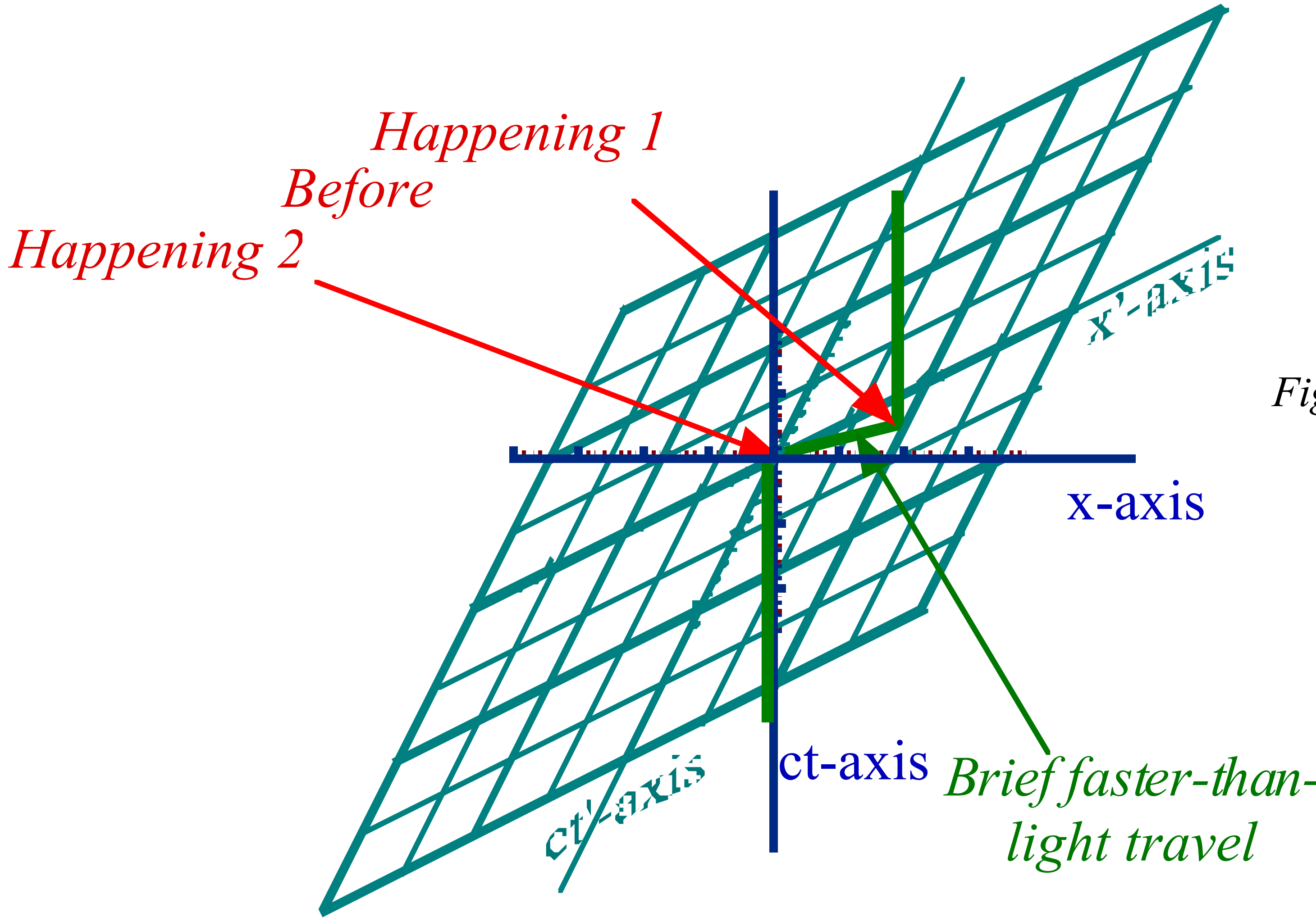


Fig. 2.B.10 Lighthouse plot of two Happenings

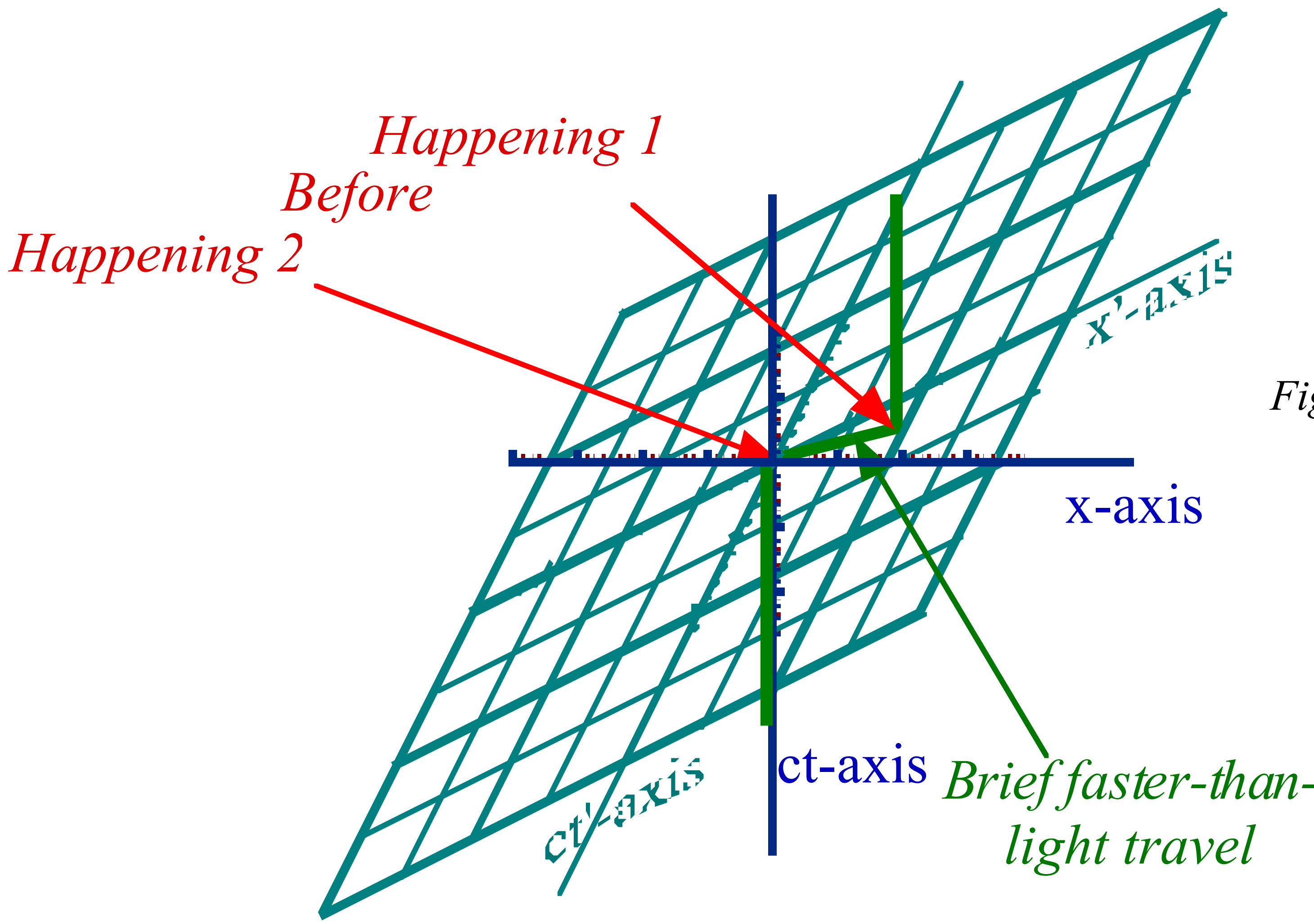


Fig. 2.B.10 Lighthouse plot of two Happenings

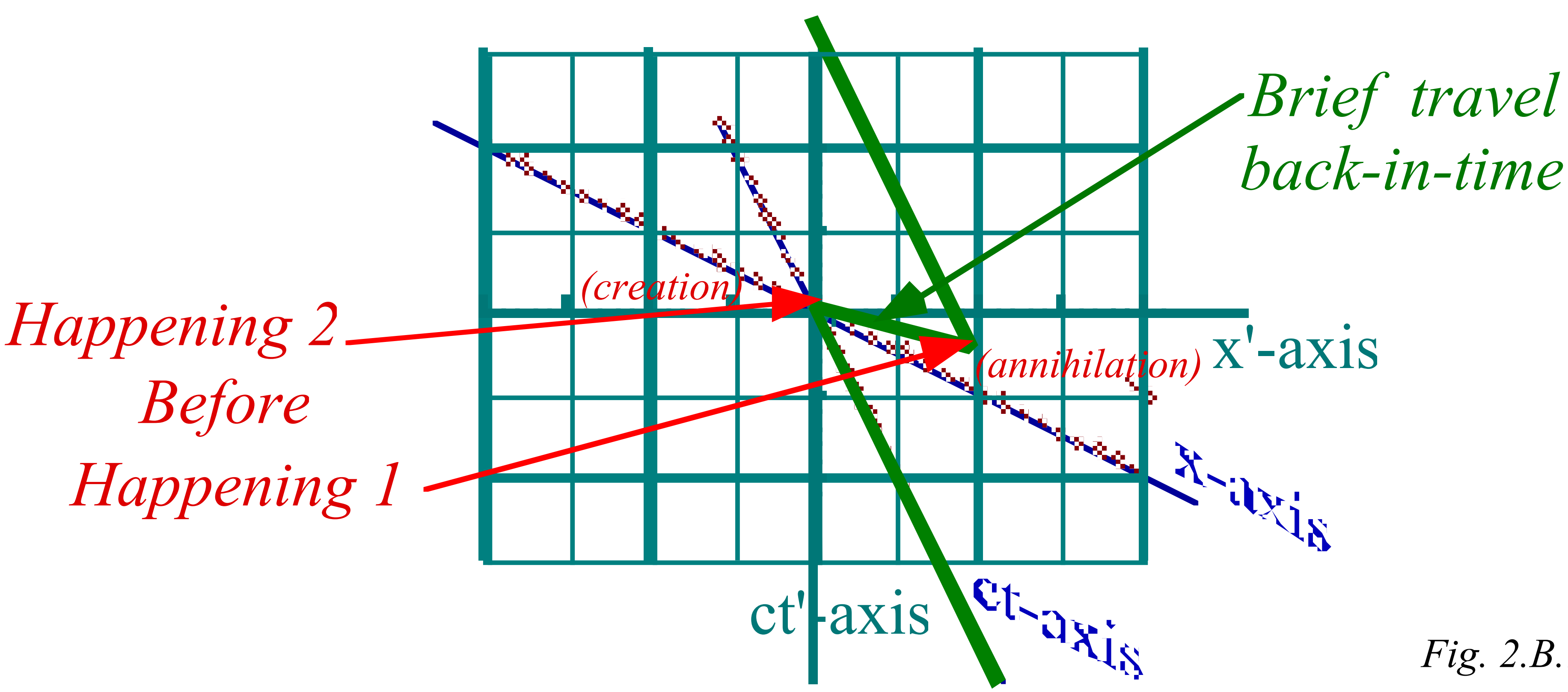


Fig. 2.B.11 Ship plot of two Happenings

Waves that go back in time - The Feynman-Wheeler Switchback

Minkowski Zero-Grids are Spacetime *Switchbacks* for $-u_{GROUP}<SWR<0$

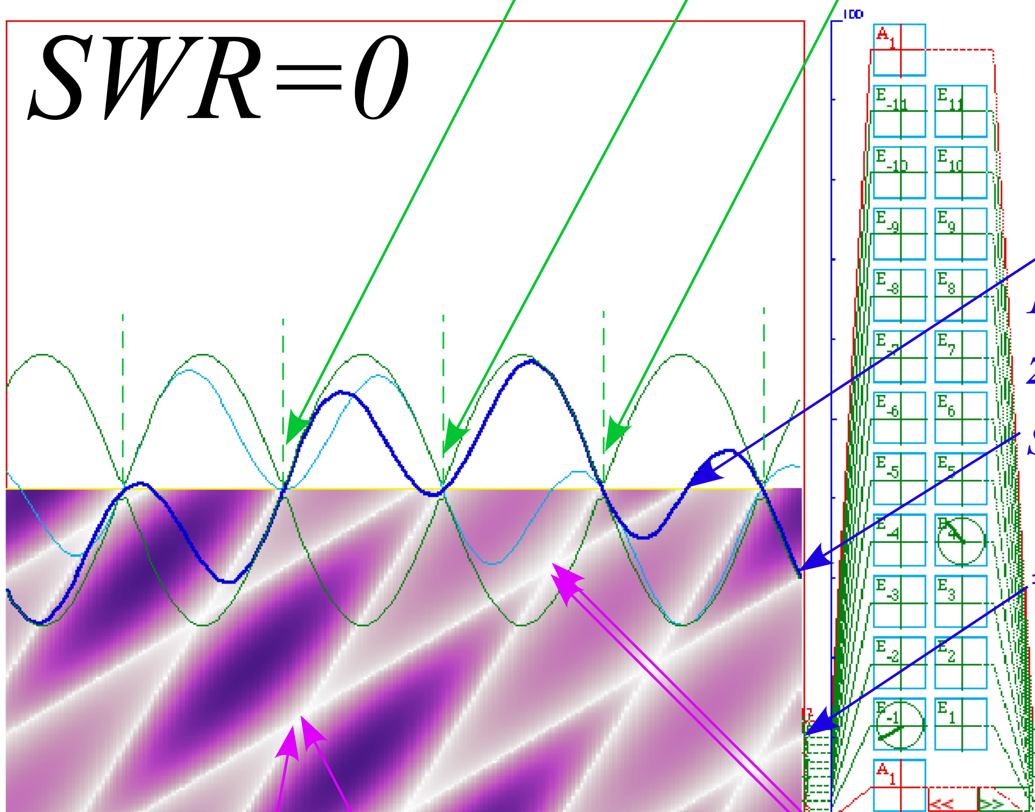
$\omega_{\rightarrow}=4c$	$\omega_{\leftarrow}=1c$
$k_{\rightarrow}=4,$	$k_{\leftarrow}=-1$
$u_{GROUP}=c3/5$	$u_{PHASE}=c5/3$

Group zero speed limit $\dfrac{u_{GROUP}+SWR}{1+u_{GROUP}\cdot\frac{SWR}{c^2}}=5c/11$

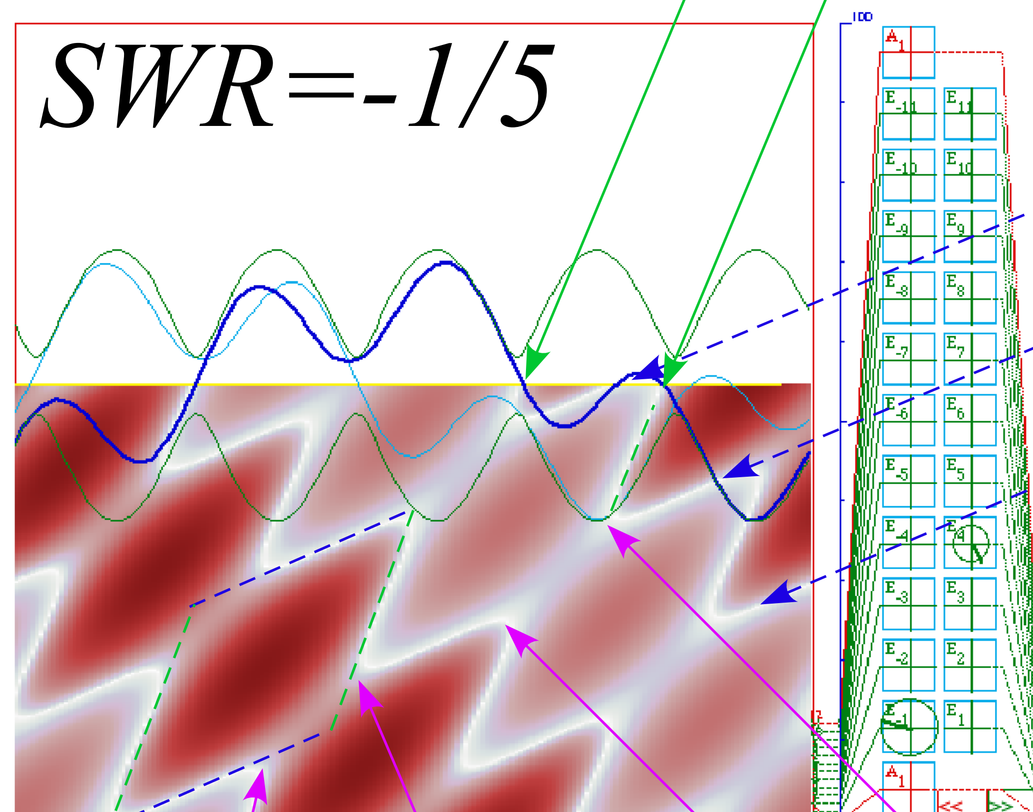
$$\frac{\frac{3}{5}+\frac{-1}{5}}{1+\frac{3}{5}\frac{-1}{5}}=\frac{\frac{2}{5}}{\frac{22}{25}}=\frac{5}{11}$$

Group-zero speed $u_{GROUP}=c3/5$

Phase "anti-zero" going "back-in-time"



Phase zero speed $u_{PHASE}=c5/3$



Phase zero speed limit $\dfrac{u_{PHASE}+SWR}{1+u_{PHASE}\cdot\frac{SWR}{c^2}}=11c/5$

$$\frac{\frac{5}{3}+\frac{-1}{5}}{1+\frac{5}{3}\frac{-1}{5}}=\frac{\frac{22}{15}}{\frac{10}{25}}=\frac{11}{5}$$

$E_{\leftarrow}=0.5$, $E_{\rightarrow}=0.5$

$E_{\leftarrow}=0.6$, $E_{\rightarrow}=0.4$

Wave zero-anti-zero annihilation *and* creation occur *together* at *the same* spacetime point for $SWR=0$

Wave zero-anti-zero annihilation and creation occur *separately* at *different* spacetime points for $-u_{GROUP}<SWR<0$