Lecture 33.
Relativity of quantum collisions and energy momentum transfer

(Ch. 7 of Unit 2 4.19.12)

Fundamental 1-and2-photon processes and their Feynman diagrams
- *(A) Absorption, (E) Emission, and (AE together) Compton scattering*
- *"Exotic" processes (AA) pair-creation, (EE) Pair-annihilation*

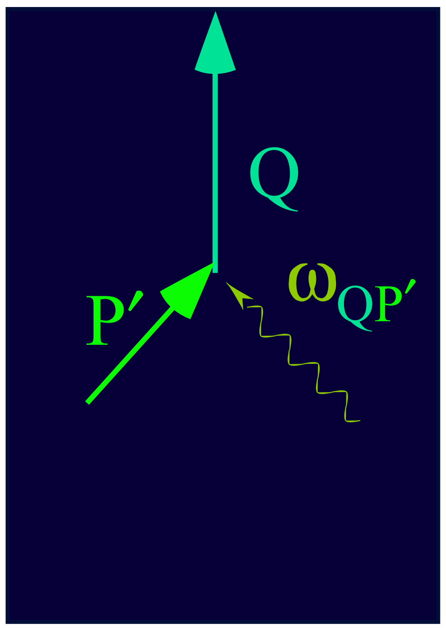
Wave geometry of 1-photon transitions and atomic recoil
- *1-Photon emission and recoil*
  - *Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams*
    - *"Baseball diamond" geometric formulas (Rocket-science of Photon emission)*
  - *Feynman's Father's question: "Where is photon before it comes out?*
    - *An answer: that gives recoil frequency down-shift δ*
- *1-Photon absorption and recoil*
  - *Similar diagrams and analysis gives recoil frequency up-shift δ*

Wave geometry of 2-photon transitions and Compton scattering
- *2-Photon emission and recoil*
  - *Grotian 3-level diagrams vs. Feynman (ω,ck) diagrams*
    - *"Photon diamond" geometric formulas*
    - *Geometric frequency shifting*

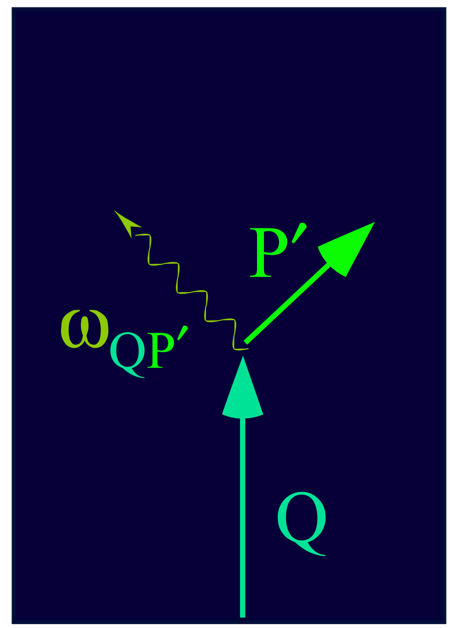
Fundamental light-matter processes:

Absorption A | *Emission E* | *AE Together*

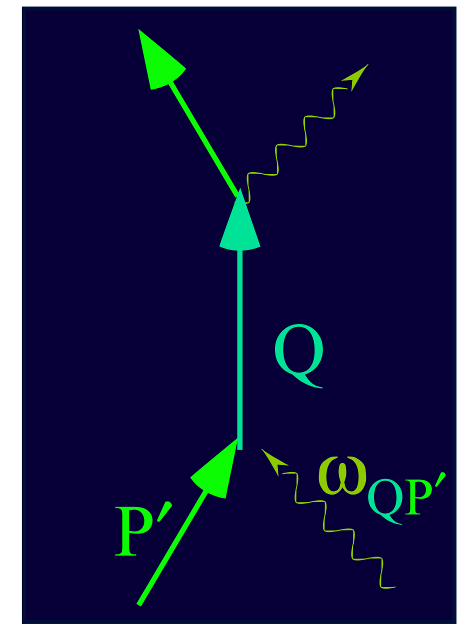
(Compton Scattering)



1-photon processes

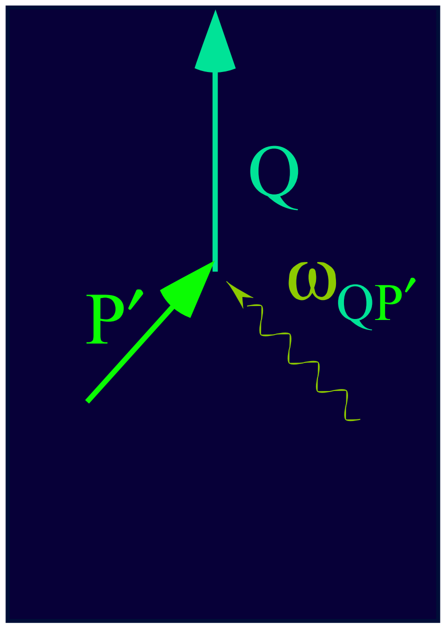


2-photon process



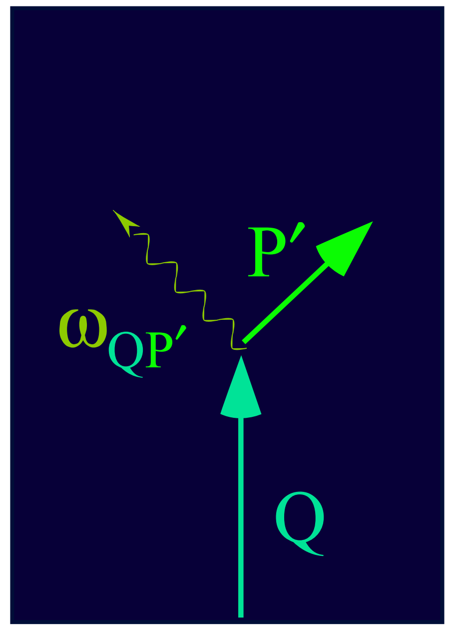
Fundamental light-matter processes:

Absorption A

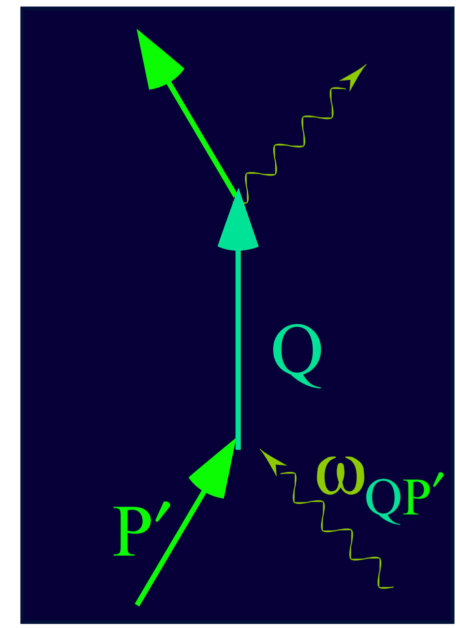


1-photon processes

Emission E

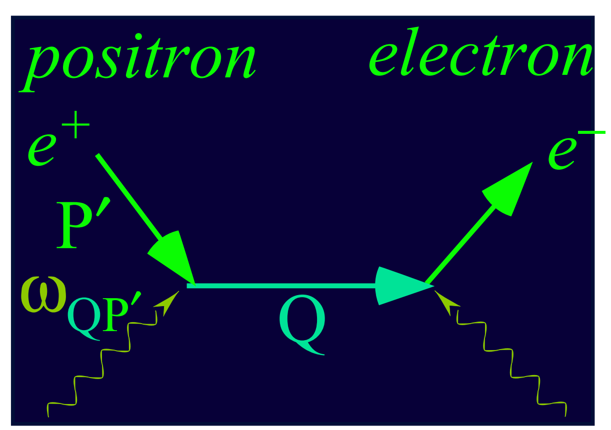


AE Together (Compton Scattering)

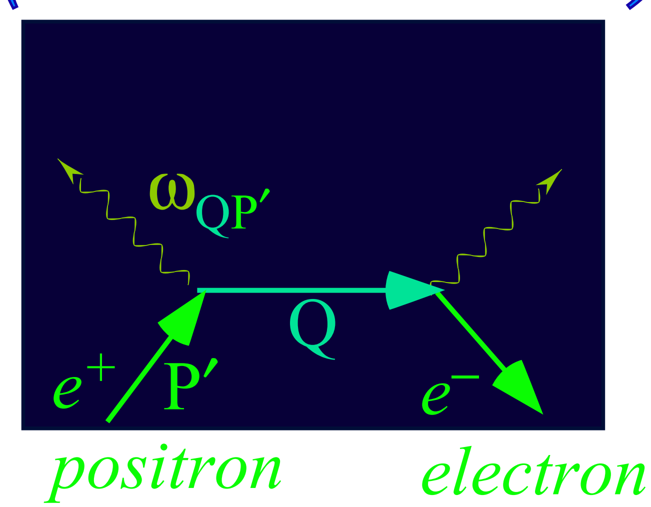


2-photon processes

"Exotic" processes: AA Together (Pair-Creation)



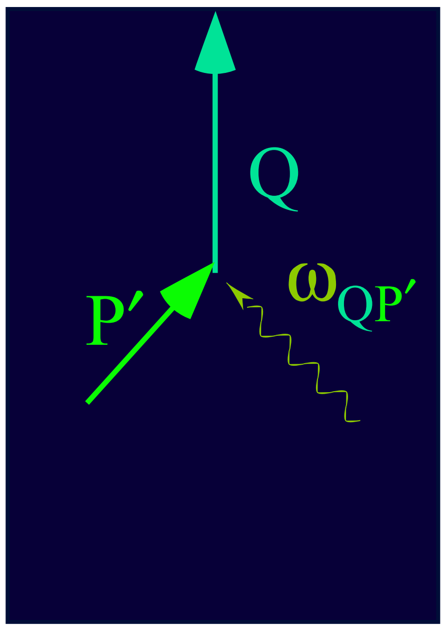
EE Together (Pair-Annhilation)



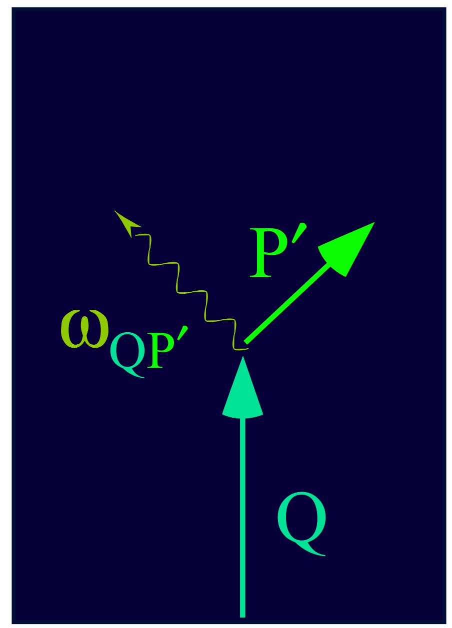
Fundamental light-matter processes:

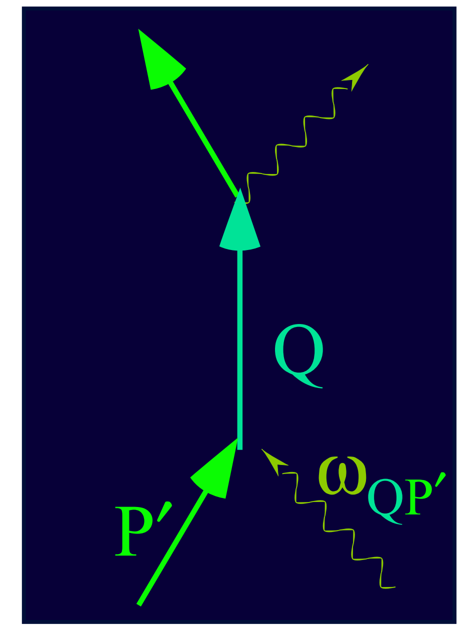
Absorption A | *Emission E* | *AE Together*

(Compton Scattering)



1-photon processes





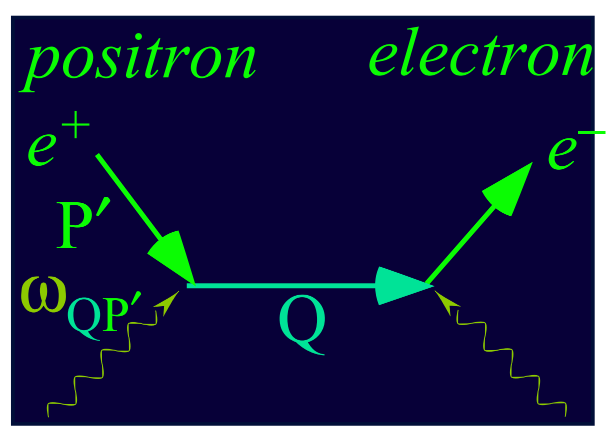
2-photon processes

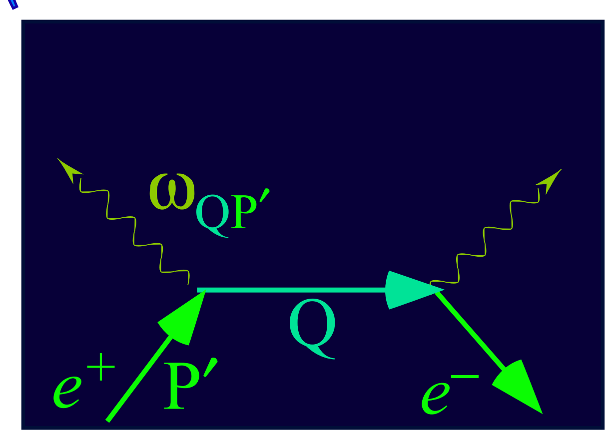
"Exotic" processes: AA Together | *EE Together*

(Pair-Creation) | *(Pair-Annhilation)*

N-photon processes

2-CW approach to QED



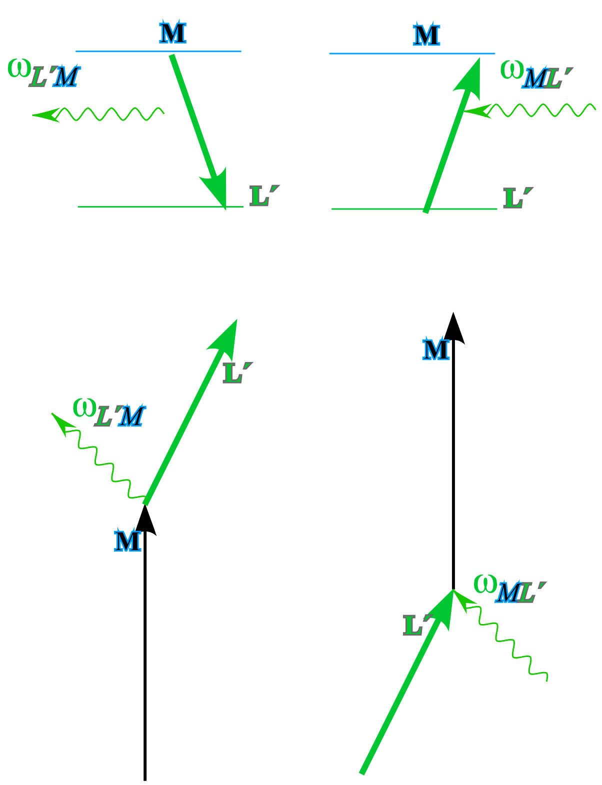


positron *electron*

Hyper-complex geometry of optical interference...

...formulation of relativistic QM

Wave geometry of 1-photon transitions and Compton recoil

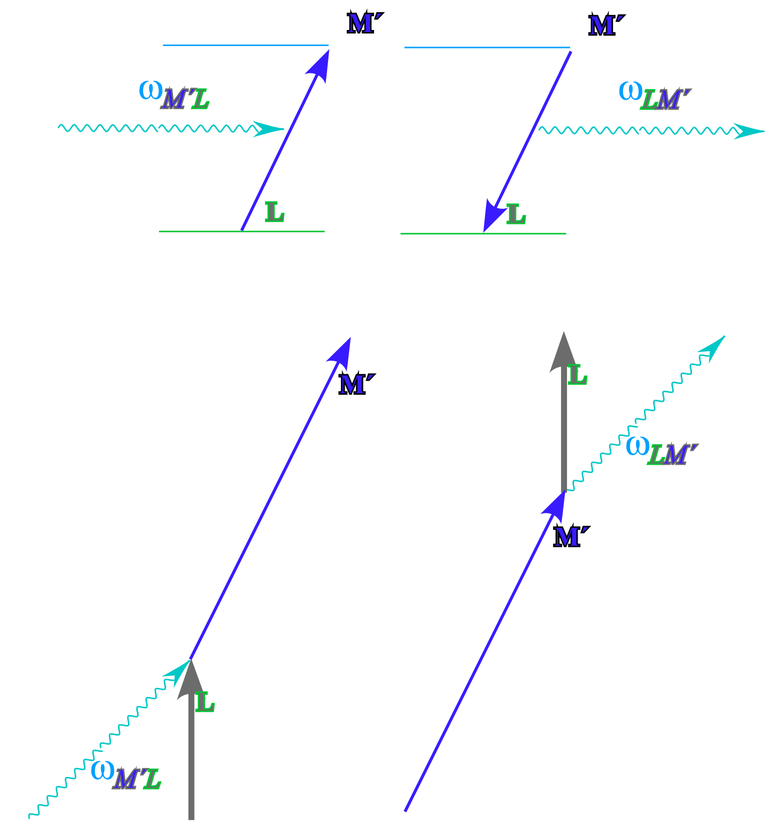


M-to-L′ emission

L′-to-M absorption

Grotian 2-level diagrams

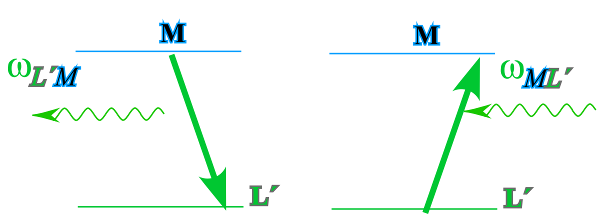
Feynman (ω,ck) diagrams (1-photon)



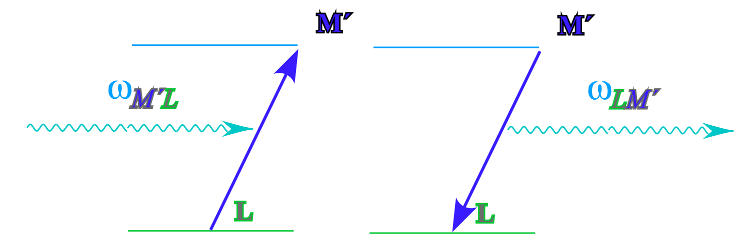
L-to-M′ absorption

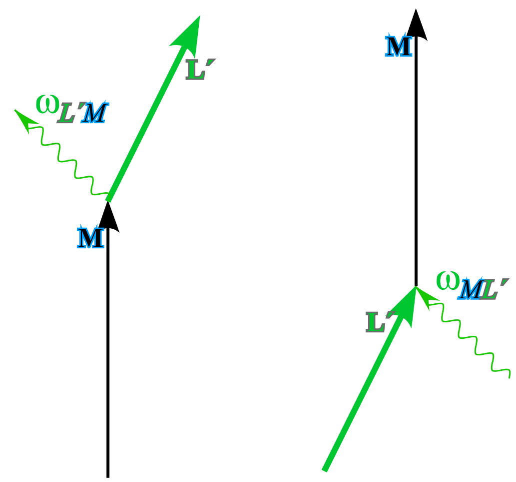
M′-to-L emission

Wave geometry of 1-photon transitions and Compton recoil

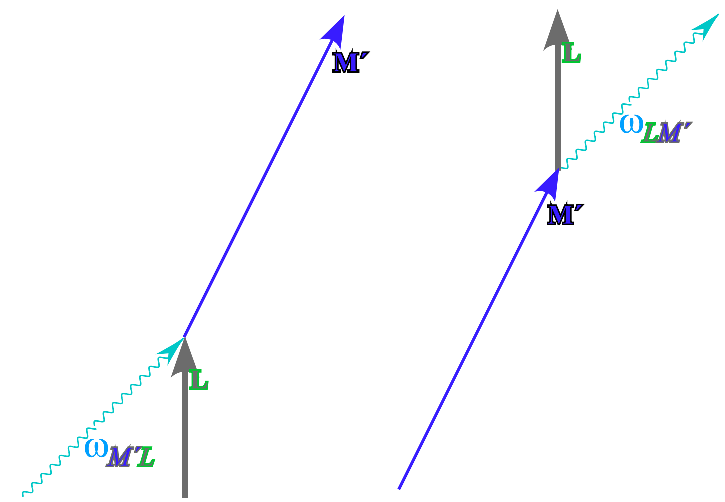


Grotian 2-level diagrams





Feynman (ω,ck) diagrams (1-photon)



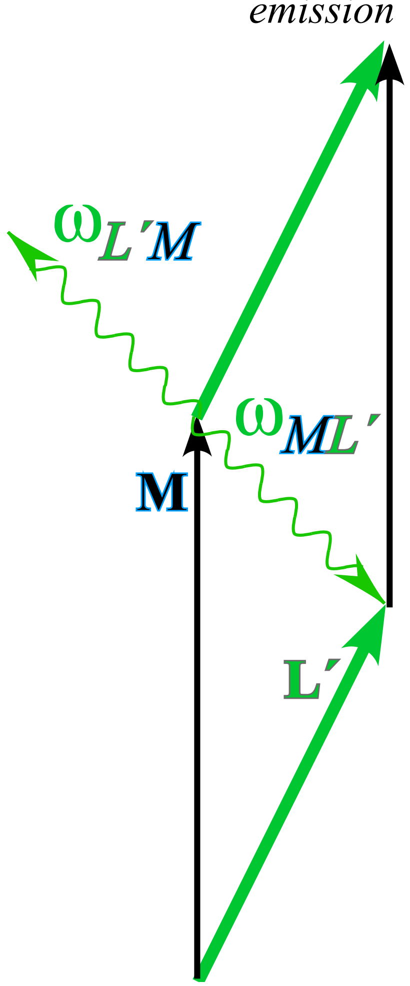
M-to-L′ emission

L′-to-M absorption

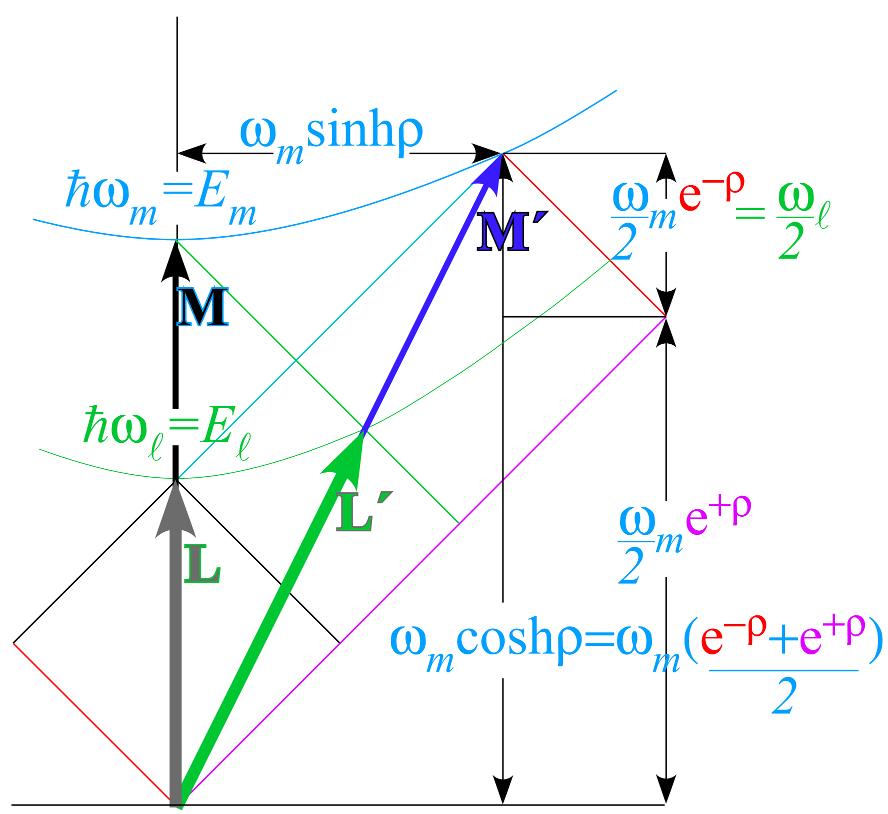
L-to-M′ absorption

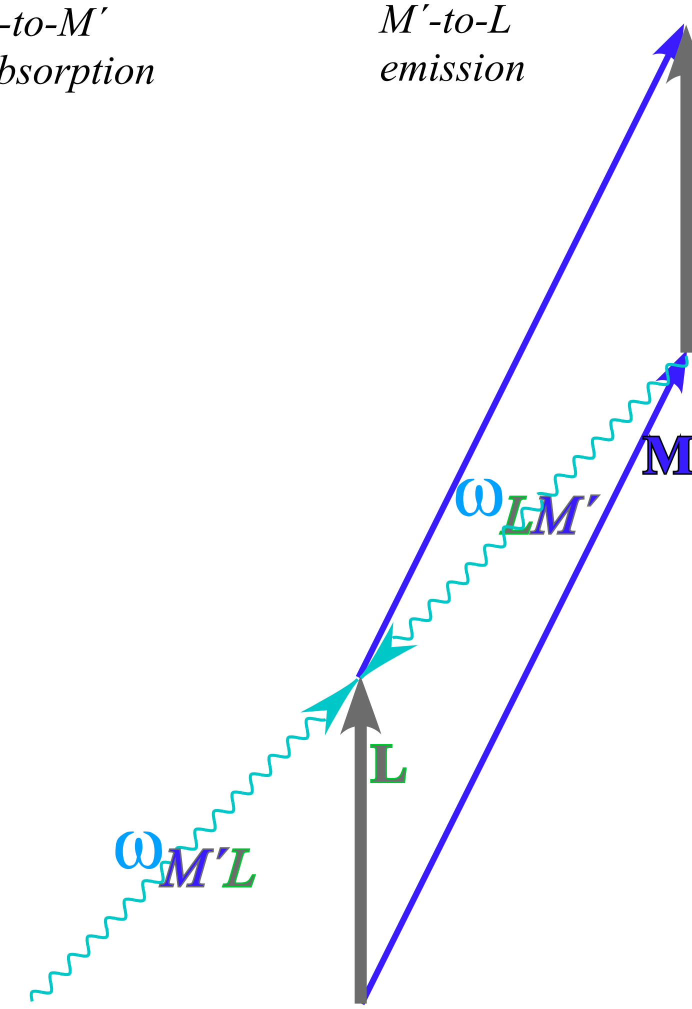
M′-to-L emission

2-Level (ω,ck) "baseball" diamonds



(ω,ck) vector sum (energy-momentum conservation)





Wave geometry of 1-photon transitions and atomic recoil

1-Photon emission and recoil

Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams

➡ *"Baseball diamond"geometric formulas (Rocket-science of Photon emission)*

Feynman's Father's question: "Where is photon before it comes out?

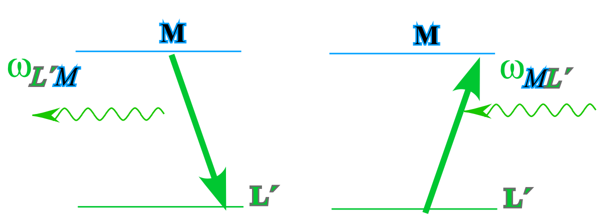
An answer: that gives recoil frequency down-shift δ

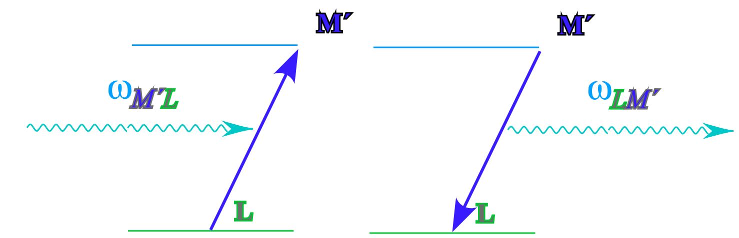
1-Photon absorption and recoil

Similar diagrams and analysis gives recoil frequency up-shift δ

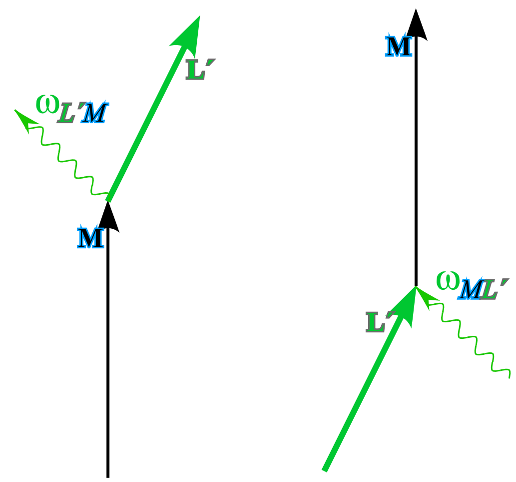
Wave geometry of 1-photon transitions and Compton recoil

Grotian 2-level diagrams

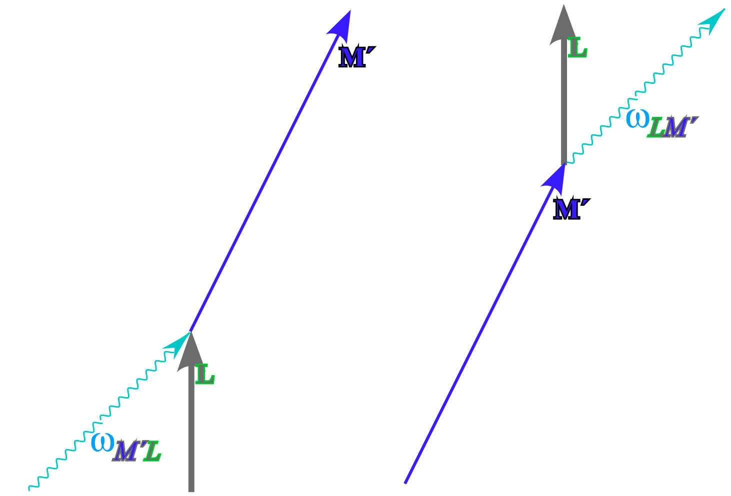




Feynman (ω,ck) diagrams (1-photon)

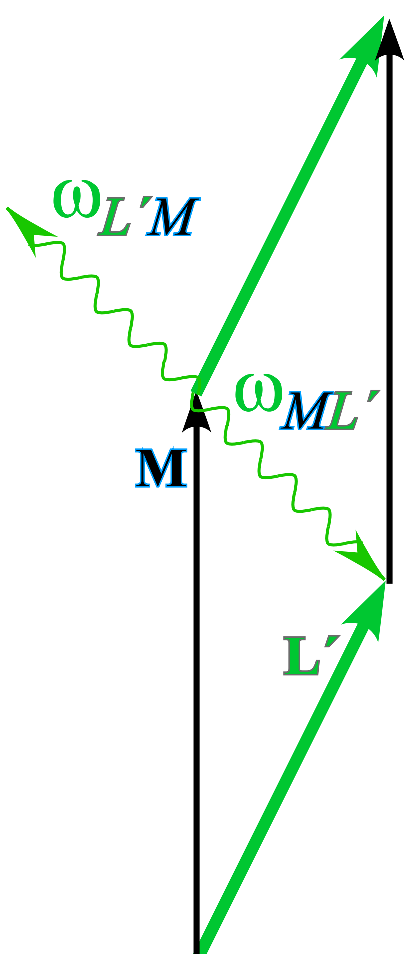


M-to-L′ emission *L′-to-M absorption*

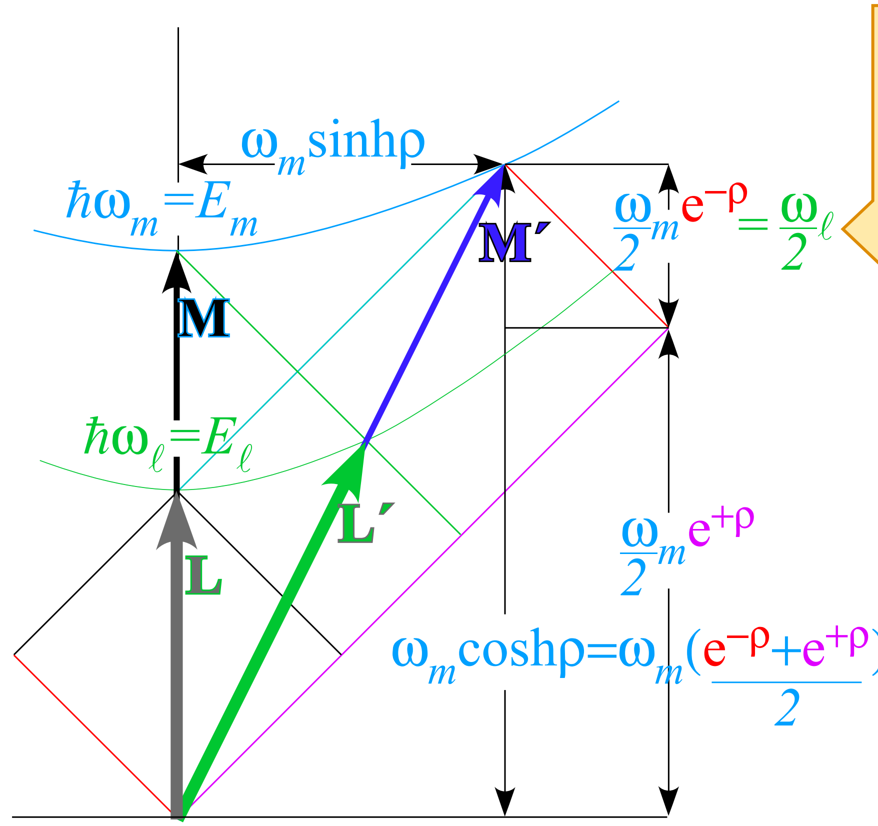


L-to-M′ absorption *M′-to-L emission*

2-Level (ω,ck) "baseball" diamonds



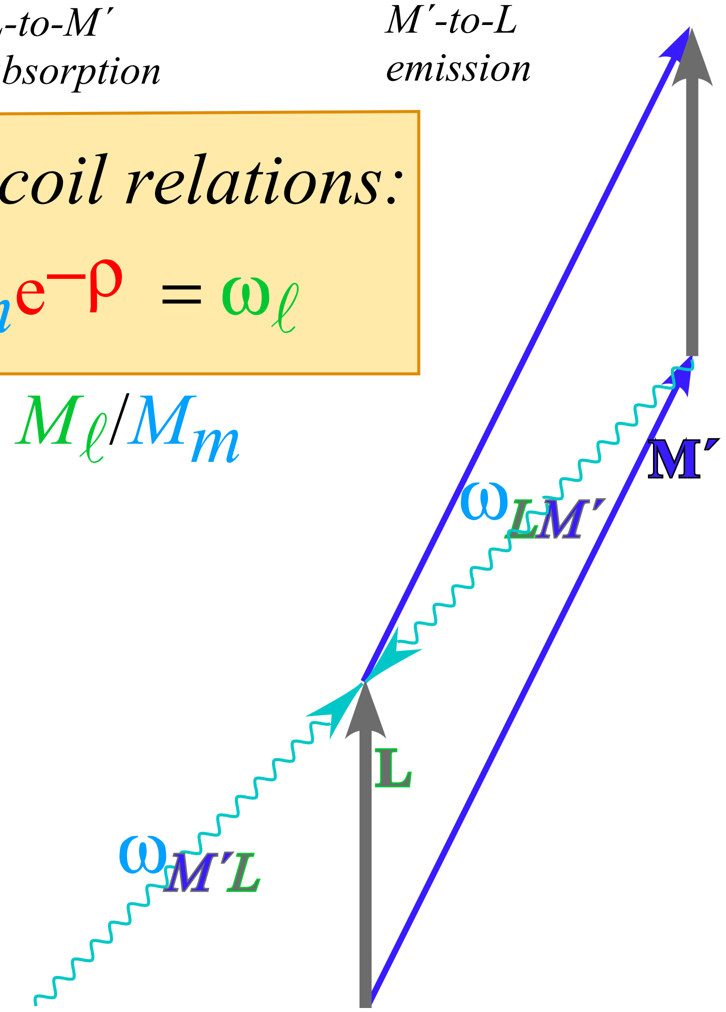
(ω,ck) vector sum (energy-momentum conservation)



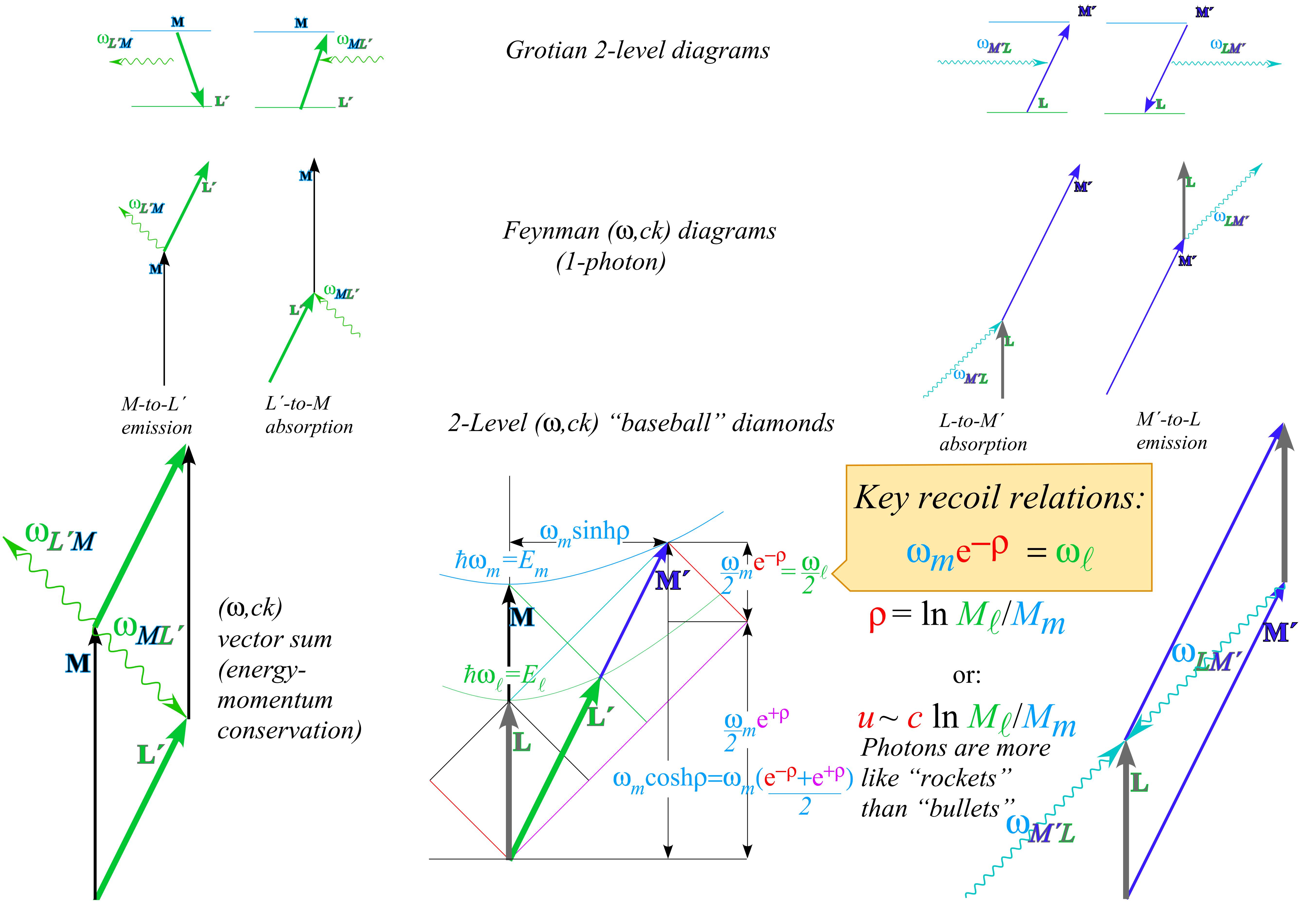
Key recoil relations:

$$\omega_m e^{-\rho} = \omega_\ell$$

$$\rho = \ln M_\ell / M_m$$



Wave geometry of 1-photon transitions and Compton recoil



Wave geometry of 1-photon transitions and atomic recoil

1-Photon emission and recoil

Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams

"Baseball diamond"geometric formulas (Rocket-science of Photon emission)

➡ *Feynman's Father's question: "Where is photon before it comes out?*

An answer: that gives recoil frequency down-shift δ

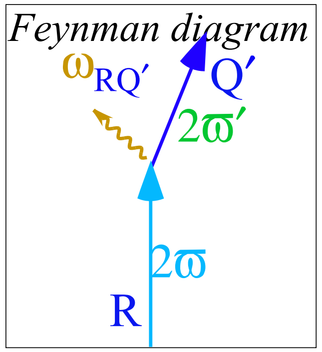
1-Photon absorption and recoil

Similar diagrams and analysis gives recoil frequency up-shift δ

Photon Recoil Effects (Answers for Feynman's father using geometric means)

R→Q′ *Photo-Emission Process*

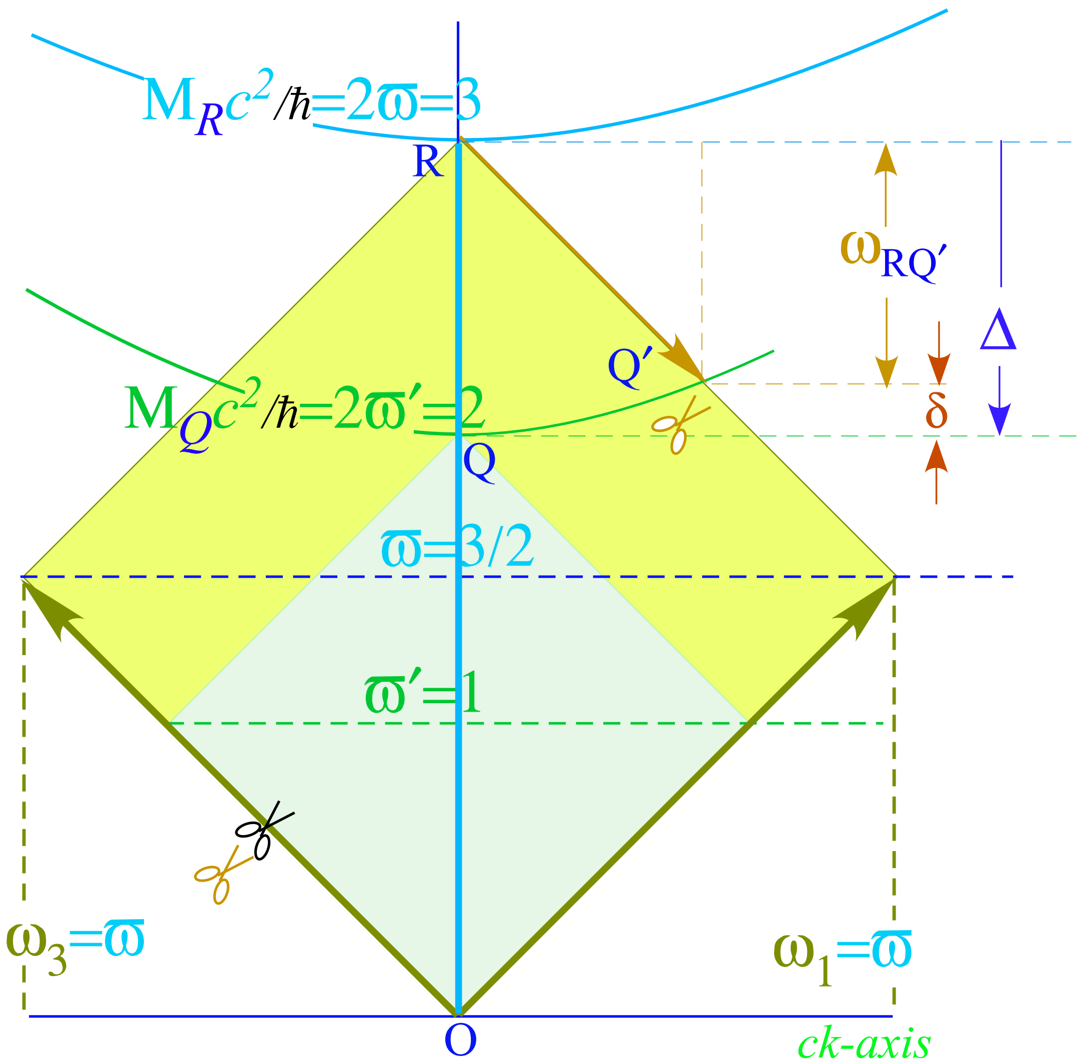
Feynman tells of his father's query, *"Where's the photon before an atom emits it?"* and how he didn't answer it (despite a high-$ MIT education). Here is a short answer that uses the *geometric mean* in a "baseball" diamond model of excited atom R. Back-emitted photon $\omega_{RQ'}$ is *cut out (✂)* of 3rd-baseline but 1st stays put.

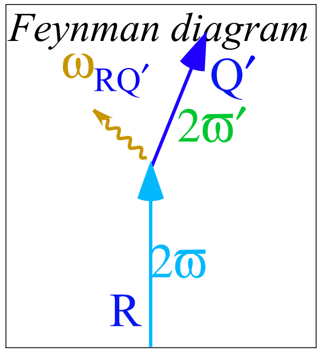


Photon Recoil Effects (Answers for Feynman's father using geometric means)

R→Q′ *Photo-Emission Process*

Feynman tells of his father's query, *"Where's the photon before an atom emits it?"* and how he didn't answer it (despite a high-$ MIT education). Here is a short answer that uses the *geometric mean* in a "baseball" diamond model of excited atom R. Back-emitted photon $\omega_{RQ'}$ is *cut out* (✂) of 3rd-baseline but 1st stays put.

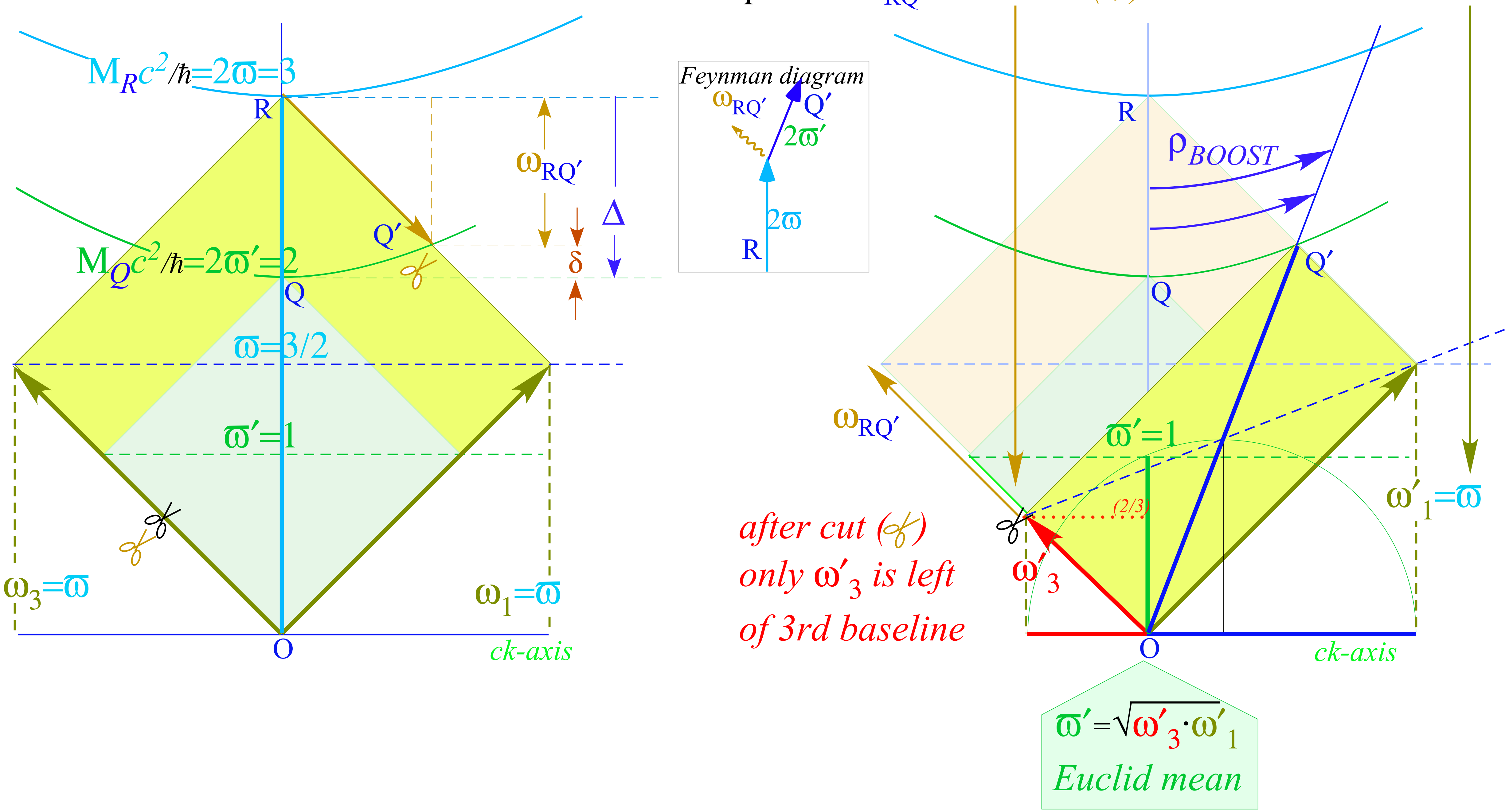




Photon Recoil Effects (Answers for Feynman's father using geometric means)

R→Q′ *Photo-Emission Process*

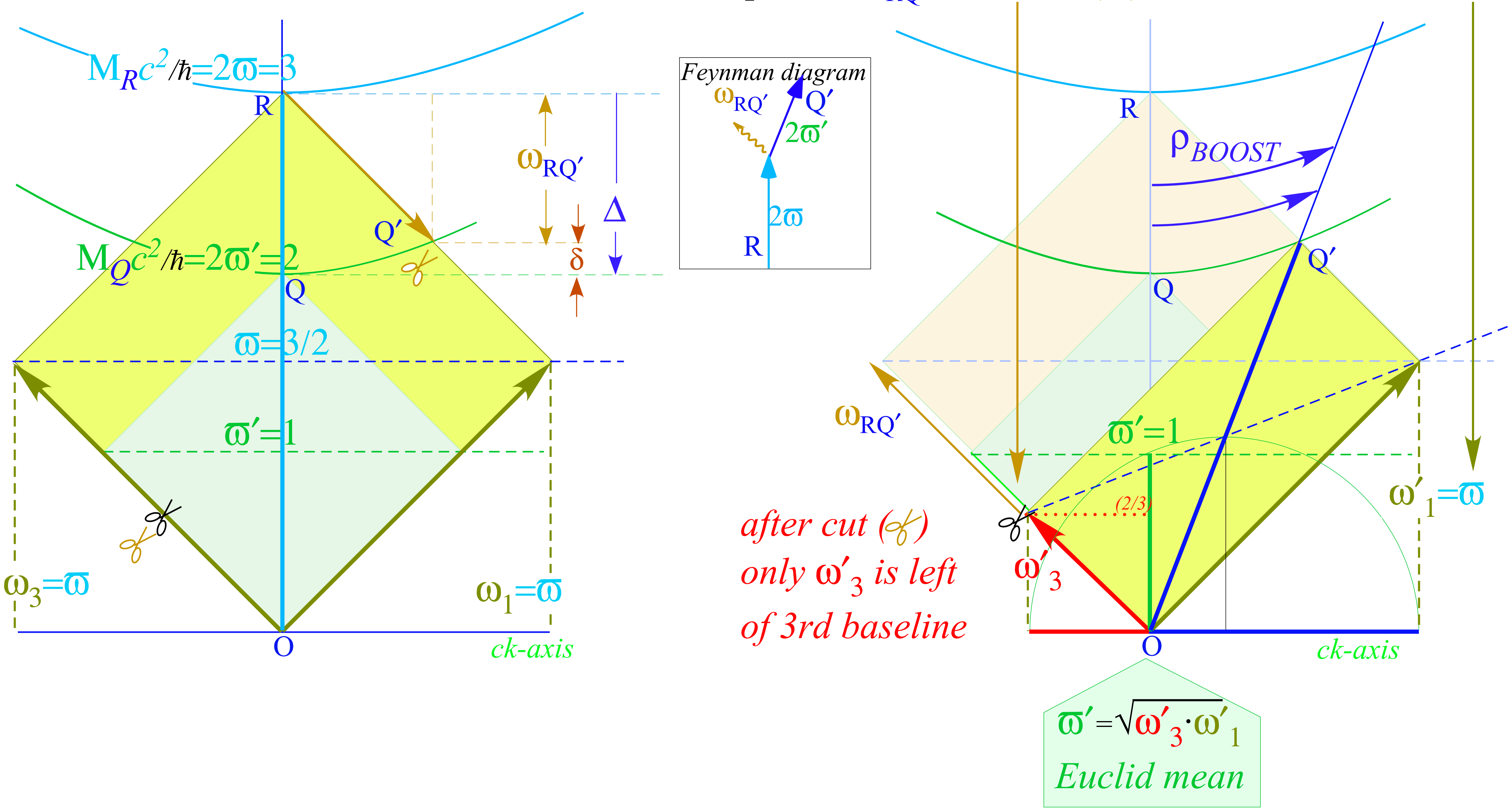
Feynman tells of his father's query, *"Where's the photon before an atom emits it?"* and how he didn't answer it (despite a high-$ MIT education). Here is a short answer that uses the *geometric mean* in a "baseball" diamond model of excited atom R. Back-emitted photon $\omega_{RQ'}$ is *cut out (✂)* of 3rd-baseline but 1st stays put.



Photon Recoil Effects (Answers for Feynman's father using geometric means)

R→Q′ *Photo-Emission Process*

Feynman tells of his father's query, *"Where's the photon before an atom emits it?"* and how he didn't answer it (despite a high-$ MIT education). Here is a short answer that uses the *geometric mean* in a "baseball" diamond model of excited atom R. Back-emitted photon $\omega_{RQ'}$ is *cut out* (✂) of 3rd-baseline but 1st stays put.



Atom recoils from R down to Q′ as it *loses mass* $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$.

Atom 3rd-baseline ϖ shrinks by *Feynman's-father-factor* $ff = \frac{\omega'_3}{\varpi}$.

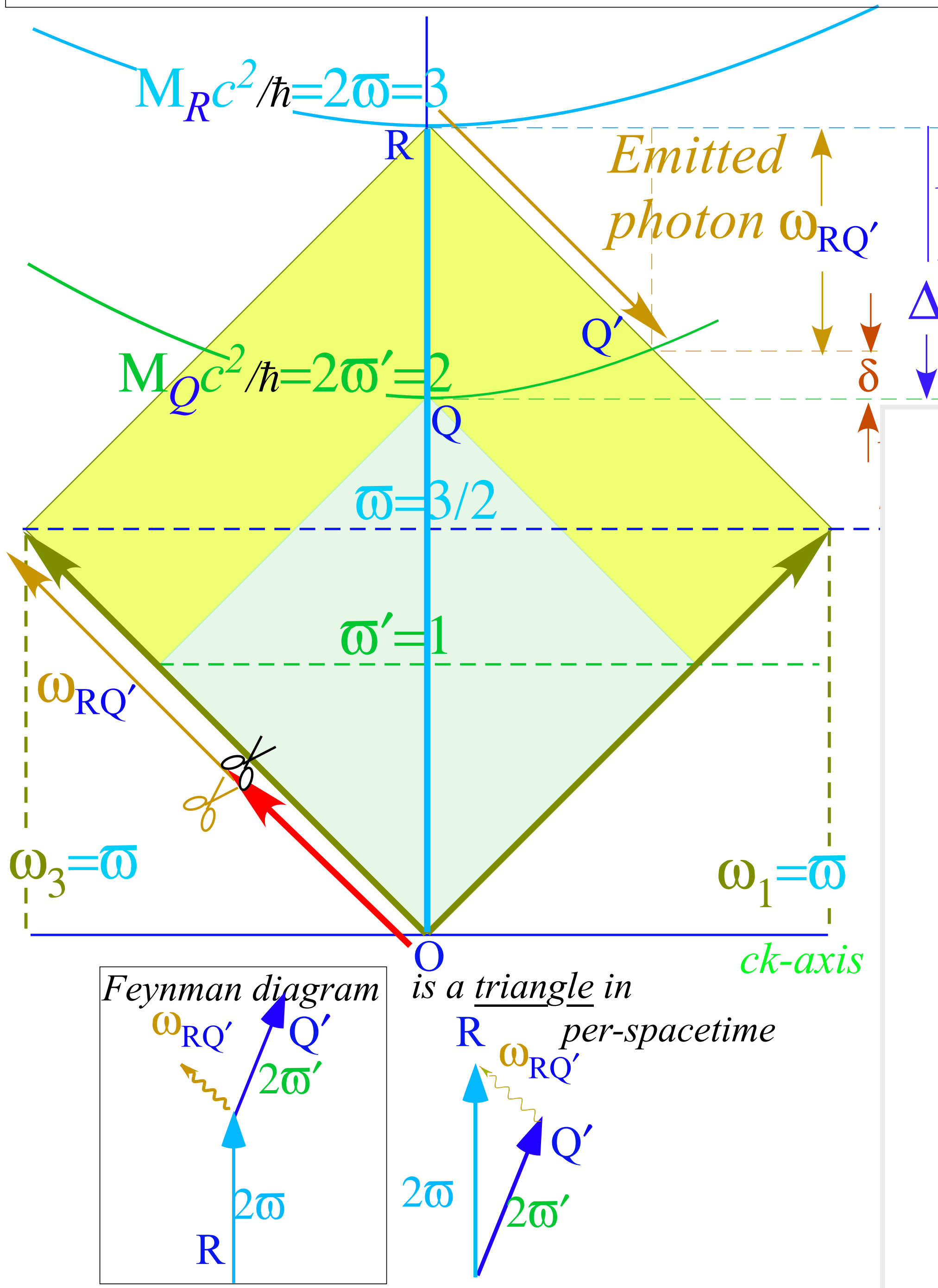
(Here we let: ff=(2/3)/(3/2)=4/9)

R→Q′ *Photo-Emission Process(contd.)*

A short answer that uses the *geometric mean* in a "baseball" diamond model of excited atom R.

Back-emitted photon $\omega_{RQ'}$ is *cut out* (✂) of 3rd-baseline but 1st stays put.

Atom recoils from R down to Q′ as it *loses mass* $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$.

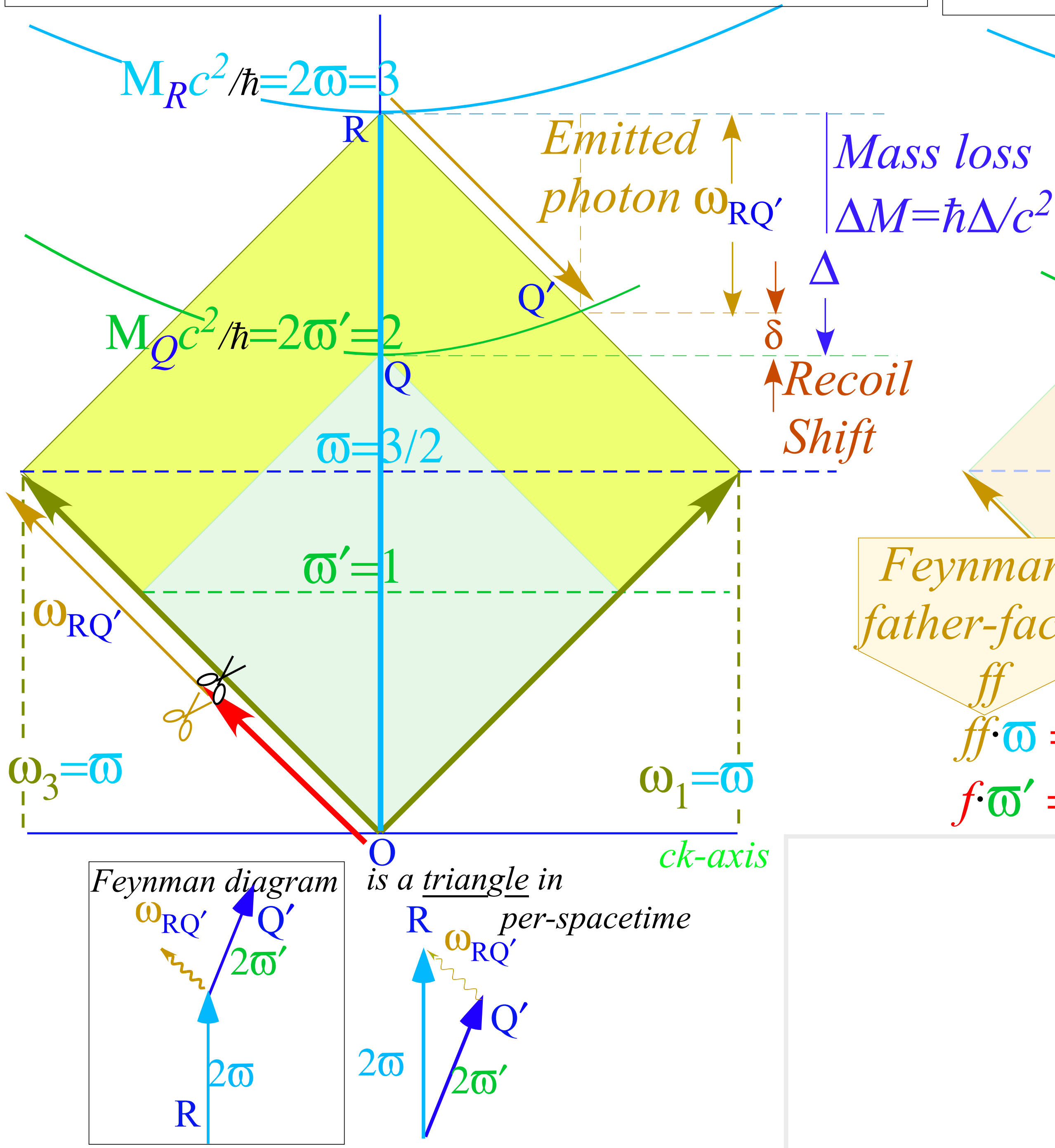


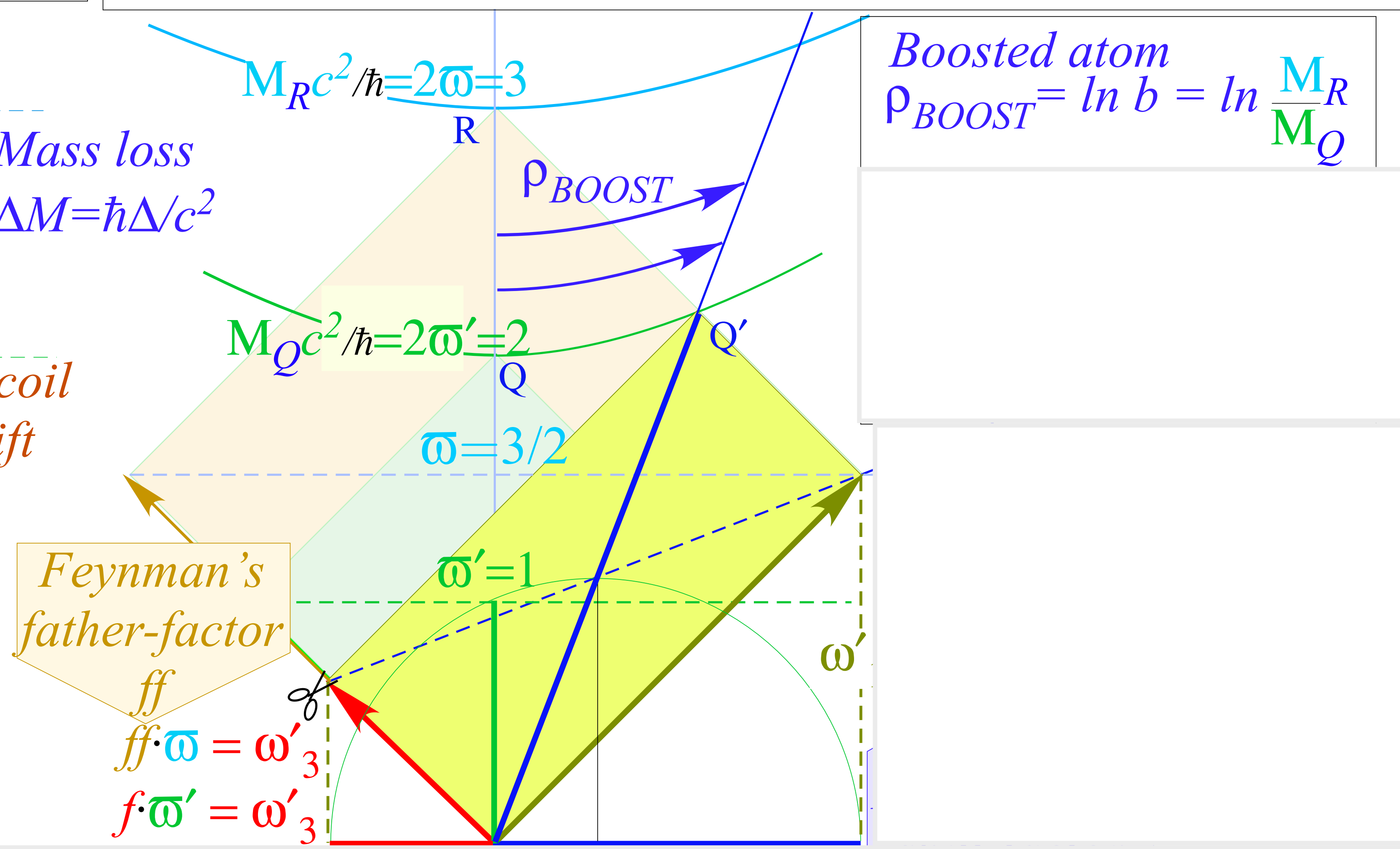
R→Q′ *Photo-Emission Process(contd.)*

A short answer that uses the *geometric mean* in a "baseball" diamond model of excited atom R.

Back-emitted photon $\omega_{RQ'}$ is *cut out* (✂) of 3rd-baseline but 1st stays put.

Atom recoils from R down to Q′ as it *loses mass* $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$.	Atom 3rd-baseline ϖ shrinks by *Feynman's-father-factor* $ff = \frac{\omega'_3}{\varpi}$. *ff* is square f^2 of Doppler *red-shift factor* $f = \frac{\varpi'}{\varpi} = \frac{M_Q}{M_R} = e^{-\rho_{BOOST}}$.



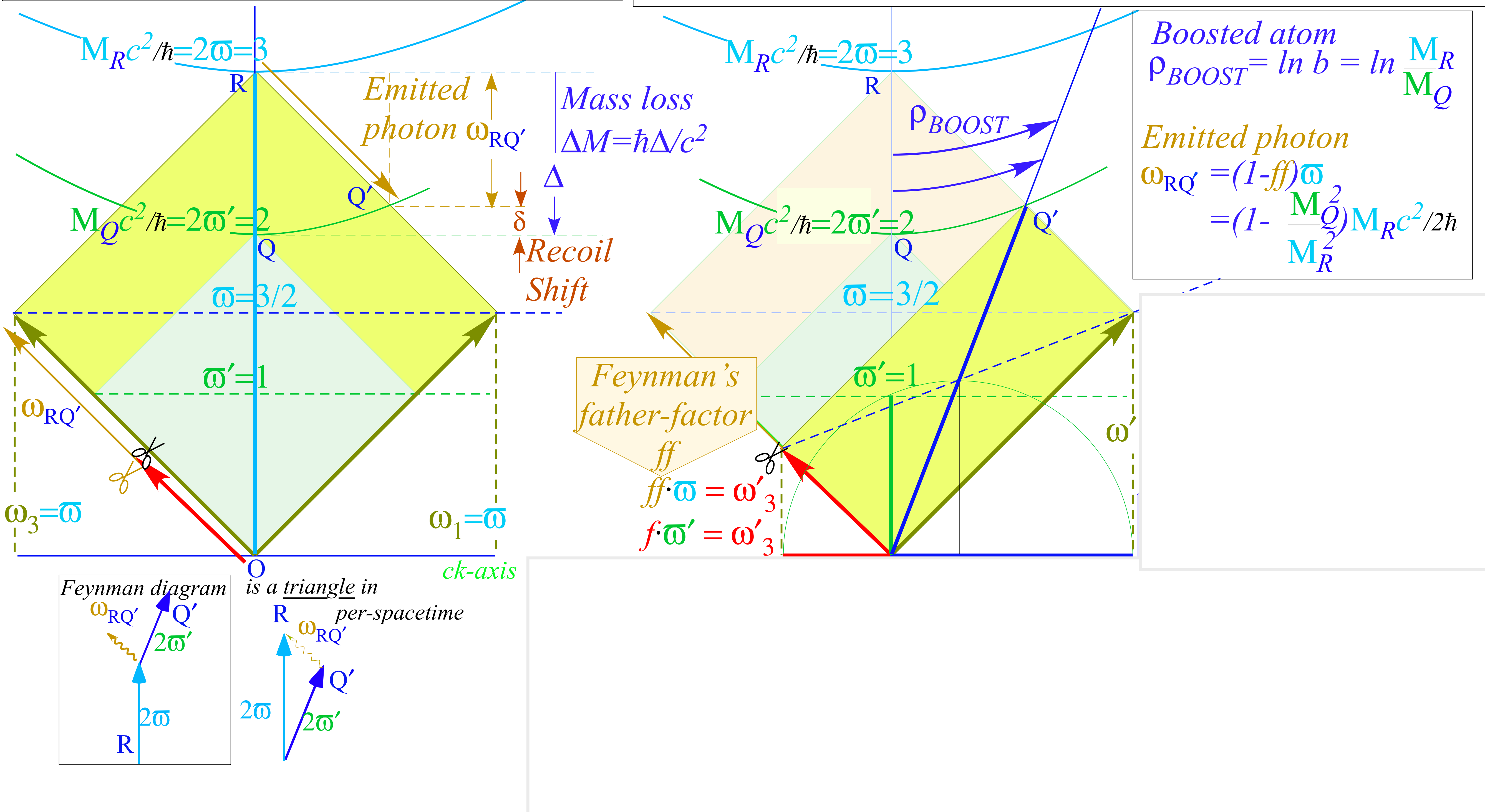


$R \to Q'$ *Photo-Emission Process(contd.)*

A short answer that uses the *geometric mean* in a “baseball” diamond model of excited atom R.

Back-emitted photon $\omega_{RQ'}$ is *cut out (✂)* of 3rd-baseline but 1st stays put.

Atom recoils from R down to Q′ as it *loses mass* $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$.	Atom 3rd-baseline ϖ shrinks by *Feynman's-father-factor* $ff = \frac{\omega'_3}{\varpi}$. *ff* is square f^2 of Doppler *red-shift factor* $f = \frac{\varpi'}{\varpi} = \frac{M_Q}{M_R} = e^{-\rho_{BOOST}}$.

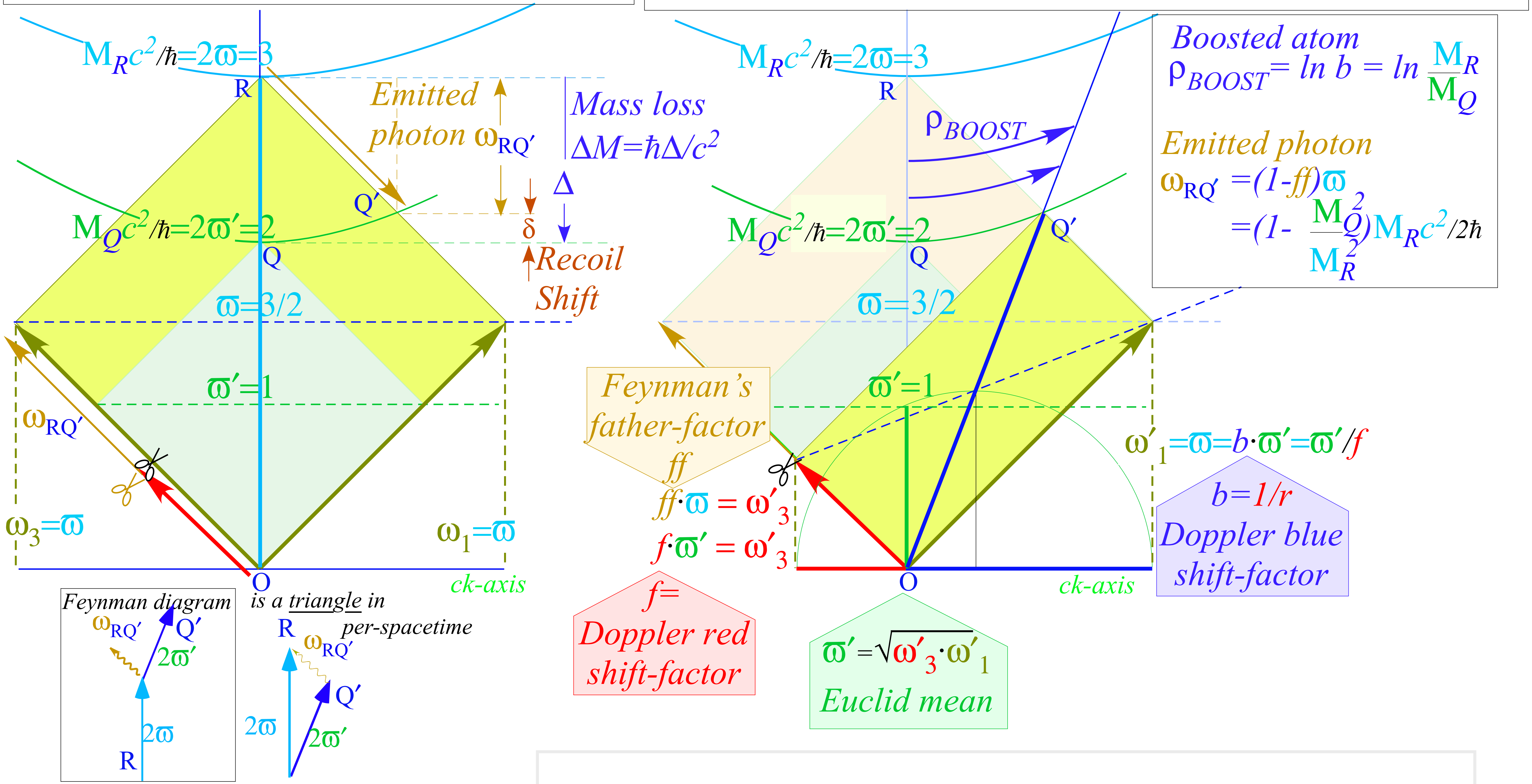


$R \to Q'$ *Photo-Emission Process(contd.)*

A short answer that uses the *geometric mean* in a “baseball” diamond model of excited atom R.

Back-emitted photon $\omega_{RQ'}$ is *cut out* (✂) of 3[rd]-baseline but 1[st] stays put.

Atom recoils from R down to Q′ as it *loses mass* $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$.	Atom 3[rd]-baseline ϖ shrinks by *Feynman's-father-factor* $ff = \frac{\omega'_3}{\varpi}$. *ff* is square f^2 of Doppler *red-shift factor* $f = \frac{\varpi'}{\varpi} = \frac{M_Q}{M_R} = e^{-\rho_{BOOST}}$.



Wave geometry of 1-photon transitions and atomic recoil

1-Photon emission and recoil

Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams

"Baseball diamond"geometric formulas (Rocket-science of Photon emission)

Feynman's Father's question: "Where is photon before it comes out?

➡ *An answer: that gives recoil frequency down-shift δ*

1-Photon absorption and recoil

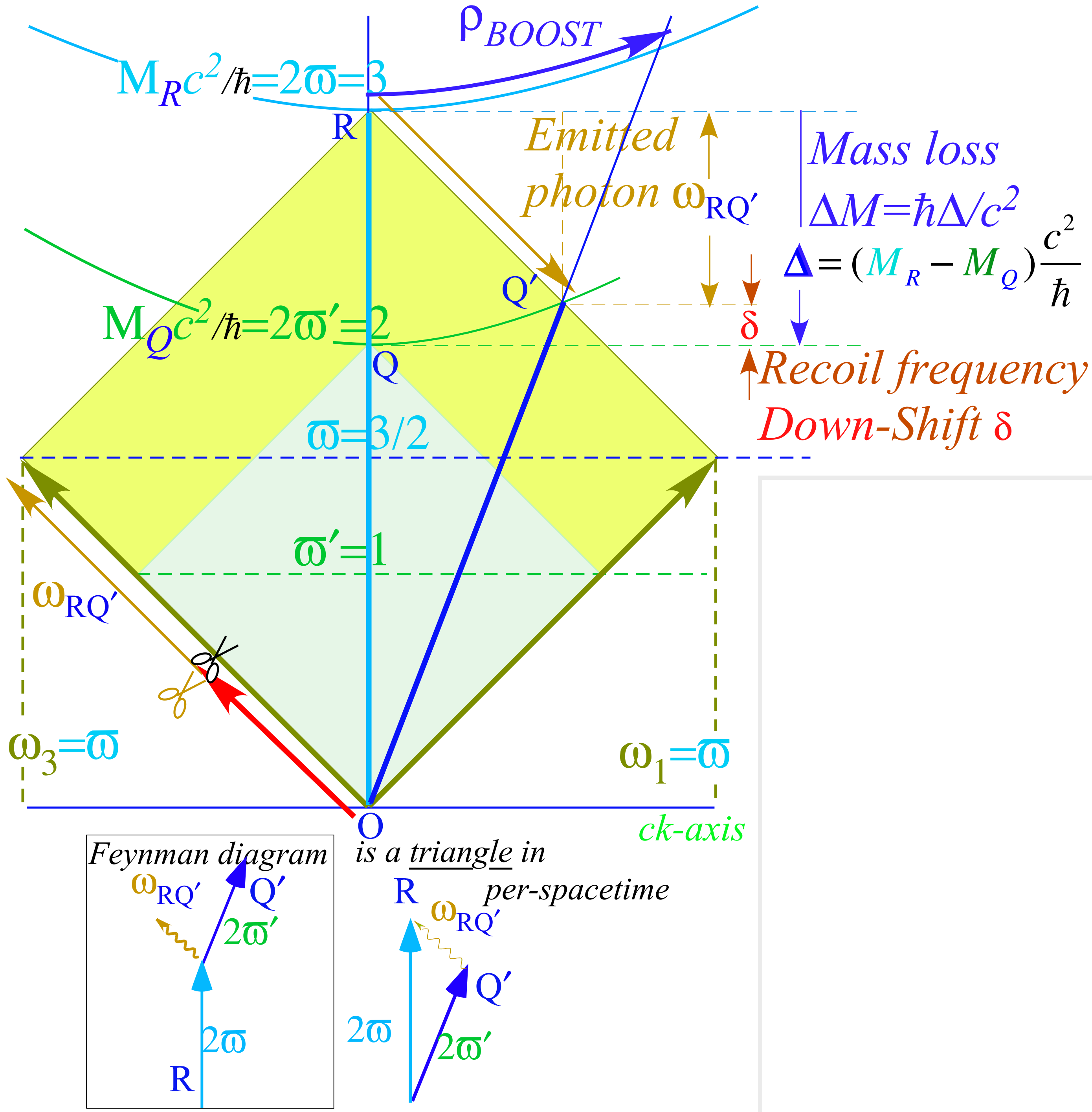
Similar diagrams and analysis gives recoil frequency up-shift δ

R→Q′ *Photo-Emission Process (contd.)*

Atoms lose only *tiny tiny* fractions of mass M_R to photons $\omega_{RQ'}$. (But, suppose a γ-emitter loses *1/3* M_R as in the case here.)

Here are the numbers for this case and approximations for atoms that only lose *tiny tiny* mass. ($\hbar\omega_{RQ'} << M_R c^2$)

$M_R c^2/2\hbar = \varpi = 3/2$ *excited rest mass*	$f = \frac{M_Q}{M_R} = 2/3$	*Doppler red shift factor*	$b = 3/2$
$M_Q c^2/2\hbar = \varpi' = 1$ *ground rest mass*	$ff = f \cdot f = 4/9$	*Feynman's-father factor*	$b^2 = 9/4$



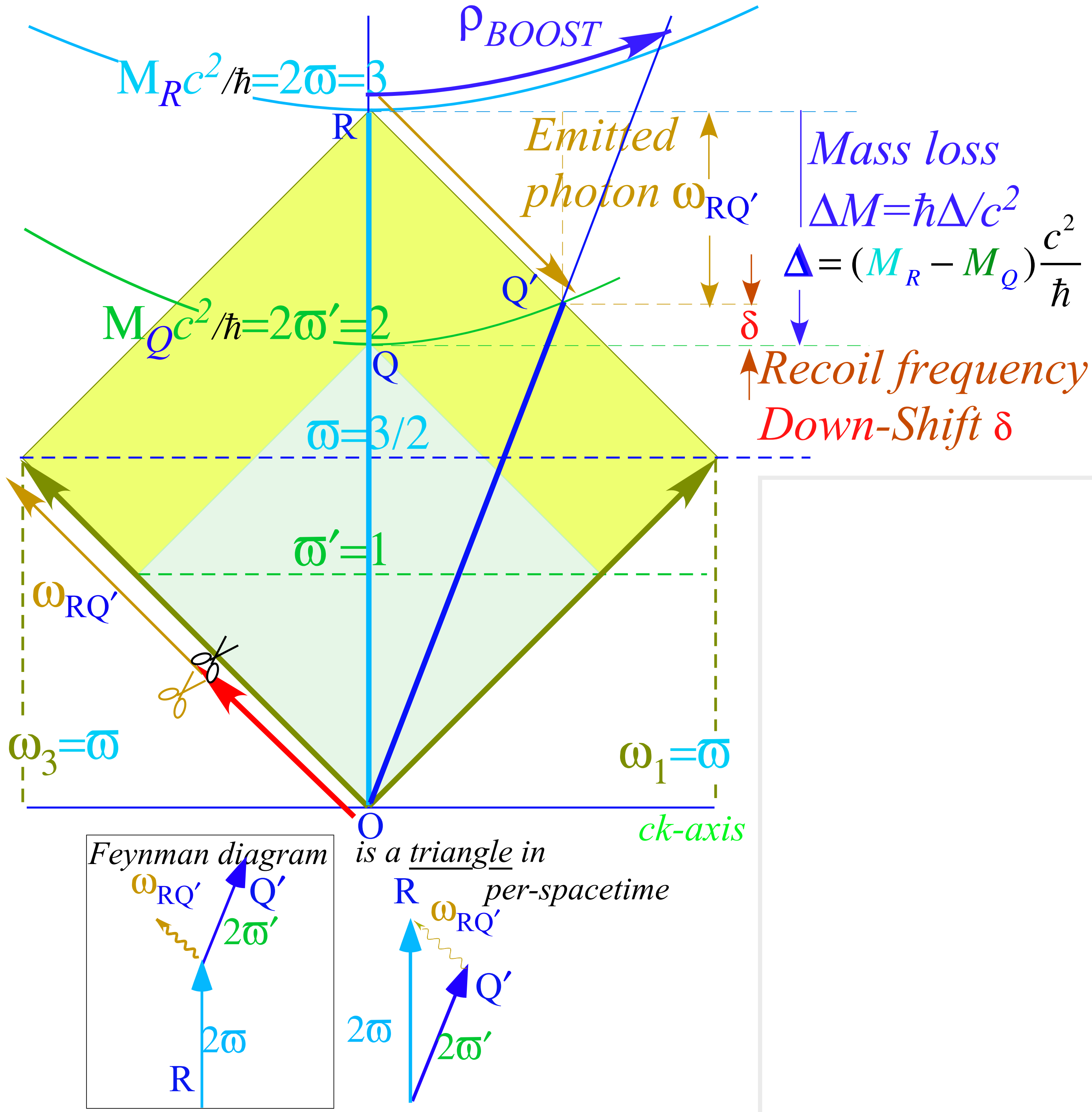
R→Q′ *Photo-Emission Process (contd.)*

Atoms lose only *tiny tiny* fractions of mass M_R to photons $\omega_{RQ'}$. (But, suppose a γ-emitter loses *1/3* M_R as in the case here.)

Here are the numbers for this case and approximations for atoms that only lose *tiny tiny* mass. ($\hbar\omega_{RQ'} << M_R c^2$)

$M_R c^2/2\hbar = \varpi = 3/2$ *excited rest mass* $\qquad f = \frac{M_Q}{M_R} = 2/3$ *Doppler red shift factor* $\qquad b = 3/2$

$M_Q c^2/2\hbar = \varpi' = 1$ *ground rest mass* $\qquad ff = f \cdot f = 4/9$ *Feynman's-father factor* $\qquad b^2 = 9/4$



Boosted atom

$\rho_{BOOST} = \ln b = \ln \frac{M_R}{M_Q}$

Emitted photon

$\omega_{RQ'} = (1 - ff)\varpi$

$= (1 - \frac{M_Q^2}{M_R^2}) M_R c^2/2\hbar$

$u_{BOOST} = c\frac{b^2 - 1}{b^2 + 1} = c\ 5/13$

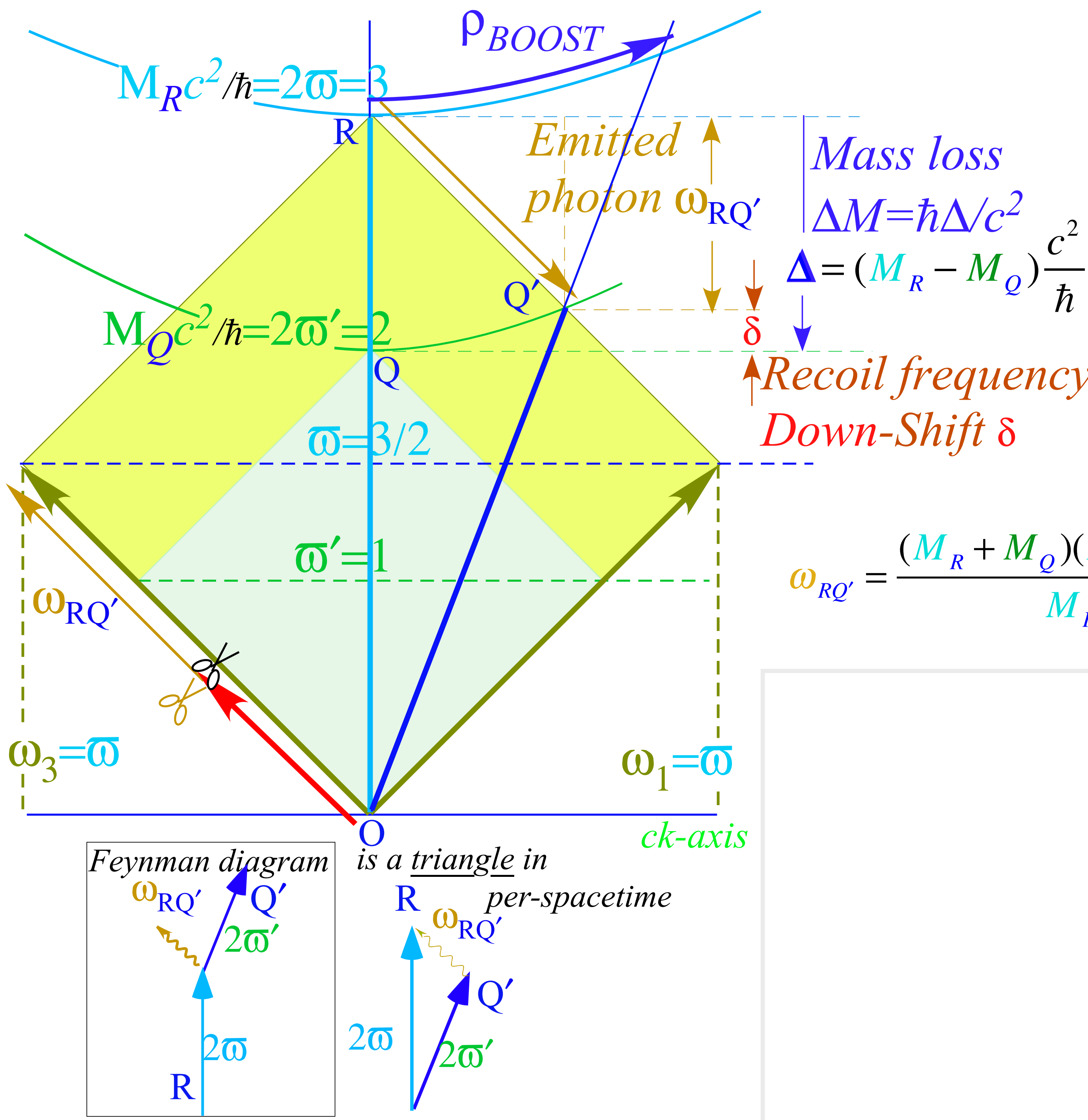
$\omega_{RQ'} = (1 - 4/9)3/2 = 5/6$

R→Q′ *Photo-Emission Process (contd.)*

Atoms lose only *tiny* *tiny* fractions of mass M_R to photons $\omega_{RQ'}$. (But, suppose a γ-emitter loses *1/3* M_R as in the case here.)

Here are the numbers for this case and approximations for atoms that only lose *tiny* *tiny* mass. ($\hbar\omega_{RQ'} << M_R c^2$)

$M_R c^2/2\hbar = \varpi = 3/2$ *excited rest mass*	$f = \frac{M_Q}{M_R} = 2/3$	*Doppler red shift factor*	$b = 3/2$
$M_Q c^2/2\hbar = \varpi' = 1$ *ground rest mass*	$ff = f \cdot f = 4/9$	*Feynman's-father factor*	$b^2 = 9/4$



Boosted atom
$\rho_{BOOST} = \ln b = \ln \frac{M_R}{M_Q}$

Emitted photon
$\omega_{RQ'} = (1-ff)\varpi$
$= \left(1 - \frac{M_Q^2}{M_R^2}\right) M_R c^2/2\hbar$

$u_{BOOST} = c\frac{b^2 - 1}{b^2 + 1} = c\ 5/13$

$\omega_{RQ'} = (1-4/9)3/2 = 5/6$

$$\omega_{RQ'} = \frac{(M_R + M_Q)(M_R - M_Q)}{M_R}\frac{c^2}{2\hbar} = \frac{(2M_Q + \hbar\Delta/c^2)\Delta}{M_Q + \hbar\Delta/c^2}\frac{1}{2} = \frac{(M_Q\Delta + \hbar\Delta^2/2c^2)}{M_Q + \hbar\Delta/c^2}$$

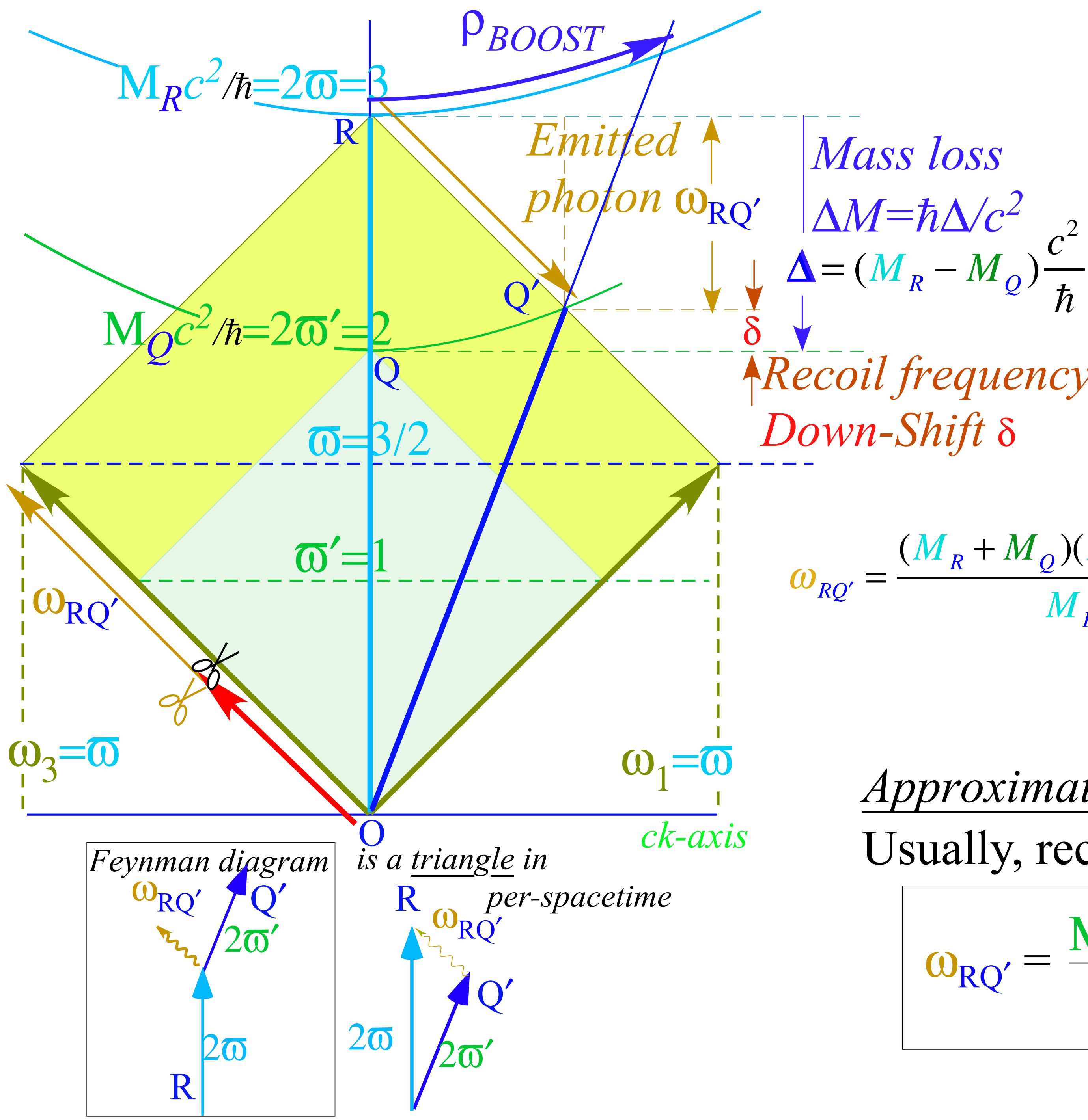
R→Q′ *Photo-Emission Process (contd.)*

Atoms lose only *tiny* *tiny* fractions of mass M_R to photons $\omega_{RQ'}$. (But, suppose a γ-emitter loses *1/3* M_R as in the case here.)

Here are the numbers for this case and approximations for atoms that only lose *tiny* *tiny* mass. ($\hbar\omega_{RQ'} \ll M_R c^2$)

$M_R c^2/2\hbar = \varpi = 3/2$ *excited rest mass* $\quad f = \dfrac{M_Q}{M_R} = 2/3$ *Doppler red shift factor* $\quad b = 3/2$

$M_Q c^2/2\hbar = \varpi' = 1$ *ground rest mass* $\quad ff = f \cdot f = 4/9$ *Feynman's-father factor* $\quad b^2 = 9/4$



Boosted atom

$$\rho_{BOOST} = \ln b = \ln \frac{M_R}{M_Q}$$

Emitted photon

$$\omega_{RQ'} = (1-ff)\varpi = \left(1 - \frac{M_Q^2}{M_R^2}\right) M_R c^2/2\hbar$$

$$u_{BOOST} = c\frac{b^2-1}{b^2+1} = c\ 5/13$$

$$\omega_{RQ'} = (1-4/9)3/2 = 5/6$$

$$\omega_{RQ'} = \frac{(M_R+M_Q)(M_R-M_Q)}{M_R}\frac{c^2}{2\hbar} = \frac{(2M_Q+\hbar\Delta/c^2)\Delta}{M_Q+\hbar\Delta/c^2}\frac{1}{2} = \frac{(M_Q\Delta+\hbar\Delta^2/2c^2)}{M_Q+\hbar\Delta/c^2}$$

Approximations for "soft-photon" atomic transitions

Usually, recoil shift δ and mass-loss $\Delta M = \hbar\Delta/c^2$ are neglible.

$$\omega_{RQ'} = \frac{M_Q c^2\Delta/\hbar + \Delta^2/2}{M_Q c^2/\hbar + \Delta} \sim \Delta - \frac{\hbar\Delta}{M_Q c^2}$$

Recoil frequency Down-Shift δ

Wave geometry of 1-photon transitions and atomic recoil

1-Photon emission and recoil

Grotian 2-level diagrams vs. Feynman (ω,ck) diagrams

"Baseball diamond" geometric formulas (Rocket-science of Photon emission)

Feynman's Father's question: "Where is photon before it comes out?

An answer: that gives recoil frequency down-shift δ

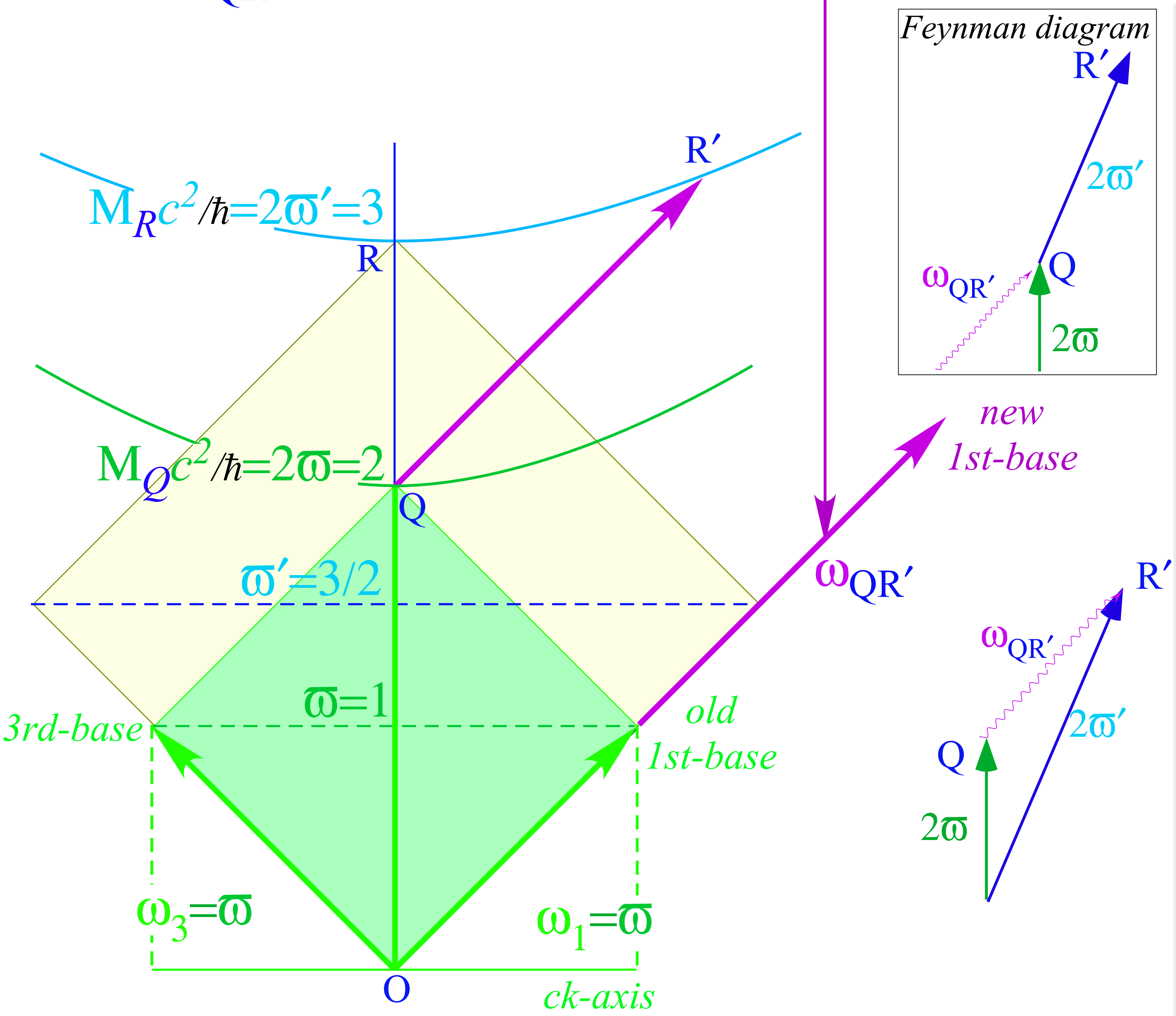
1-Photon absorption and recoil

➡ *Similar diagrams and analysis gives recoil frequency up-shift δ*

Photon Recoil Effects (contd.)

Q→R′ *Photo-Absorption Process*

Photon $\omega_{QR'}$ is *pasted* onto the 1st-baseline but 3rd-base stays put.

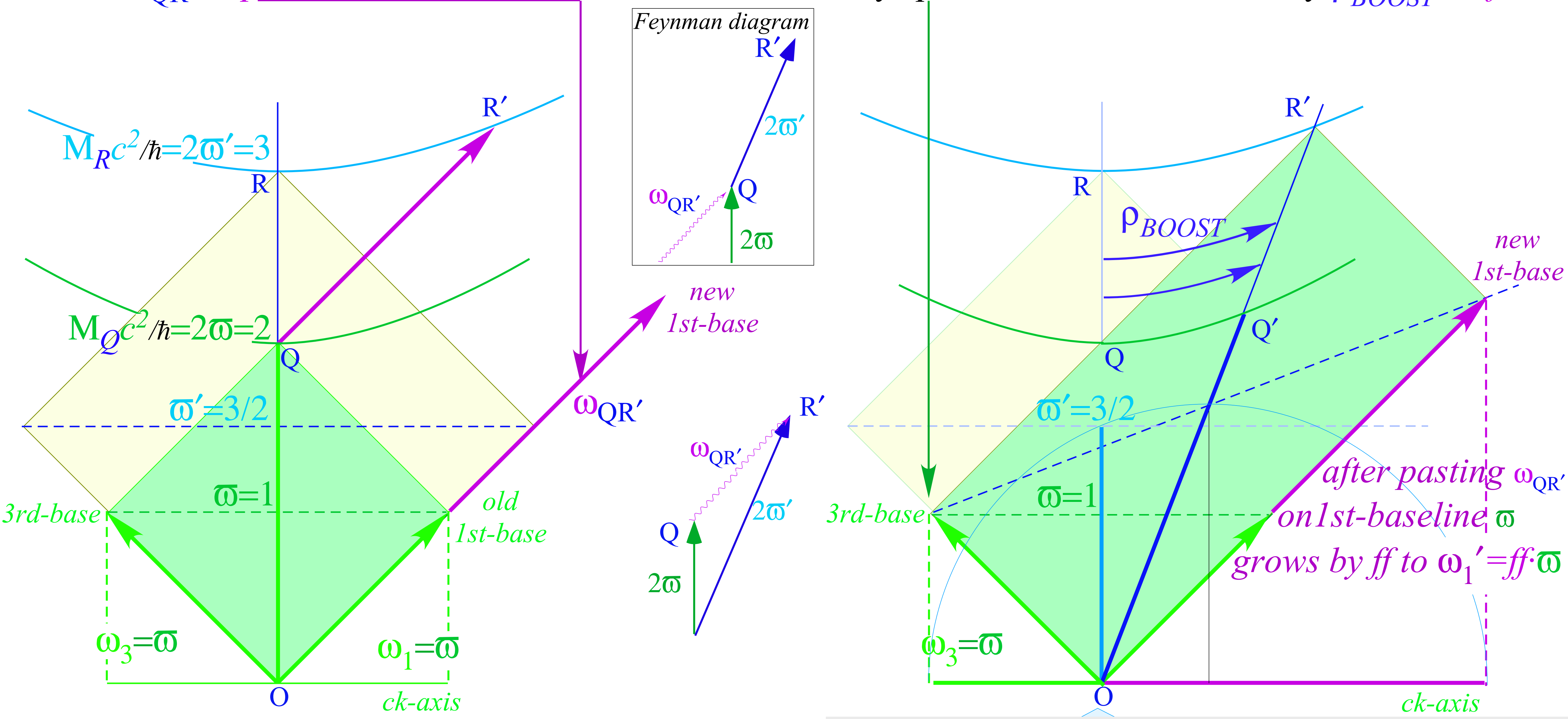


Atom accelerates from Q up to R′ as it *gains mass* $\Delta M = M_R - M_Q$ or phase frequency $\Delta = 1$.

Photon Recoil Effects (contd.)

Q→R′ *Photo-Absorption Process*

Photon $\omega_{QR'}$ is *pasted* onto the 1st-baseline but 3rd-base stays put. This boosts the atom by ρ_{BOOST}=ln *f*.

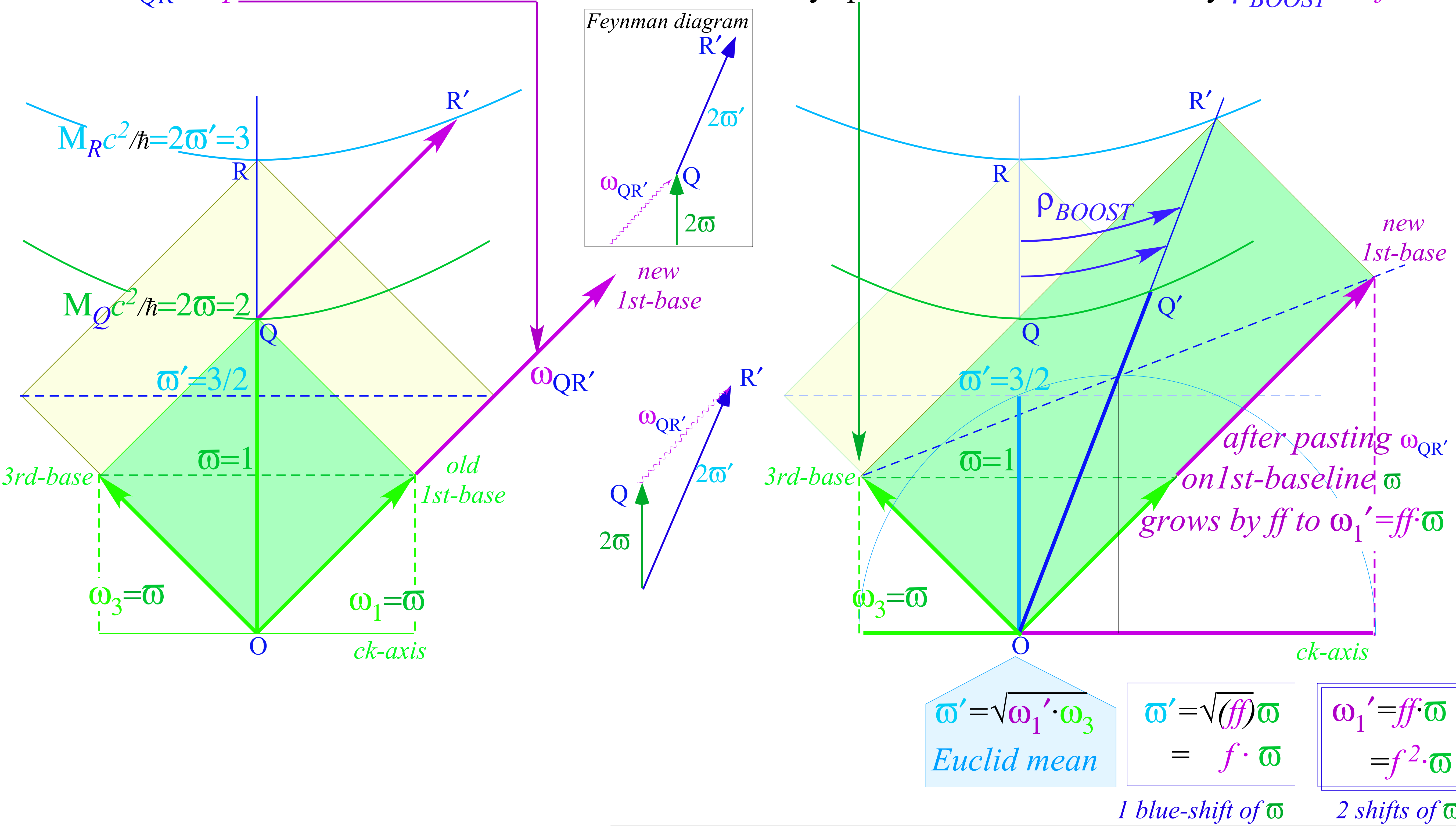


Atom accelerates from Q up to R′ as it *gains mass* ΔM=M_R -M_Q or phase frequency Δ=*1*.

Photon Recoil Effects (contd.)

Q→R′ *Photo-Absorption Process*

Photon $\omega_{QR'}$ is *pasted* onto the 1st-baseline but 3rd-base stays put. This boosts the atom by $\rho_{BOOST}=\ln f$.

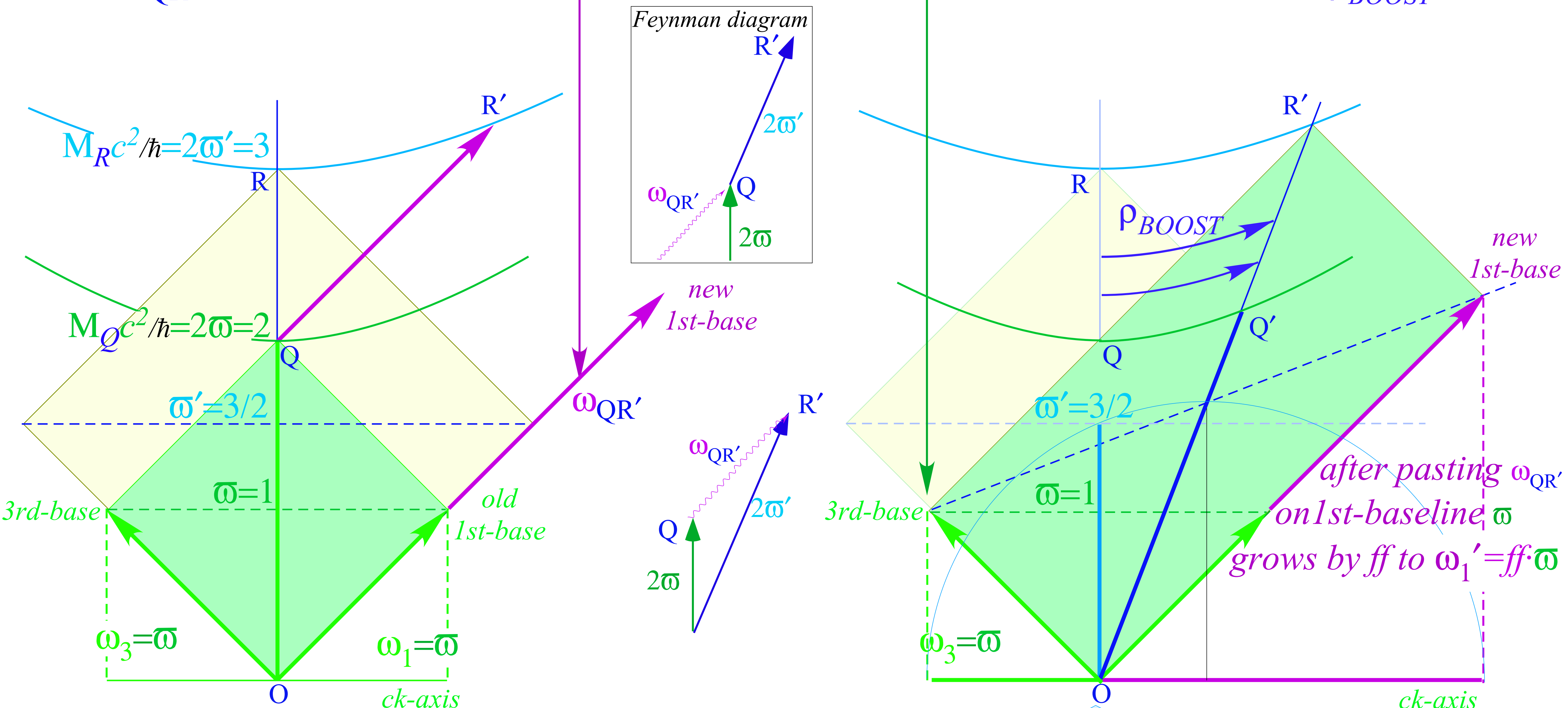


Atom accelerates from Q up to R′ as it *gains mass* $\Delta M=M_R-M_Q$ or phase frequency $\Delta=1$.

Photon Recoil Effects (contd.)

Q→R′ *Photo-Absorption Process*

Photon $\omega_{QR'}$ is *pasted* onto the 1st-baseline but 3rd-base stays put. This boosts the atom by $\rho_{BOOST}=\ln f$.



$\varpi'=\sqrt{\omega_1'\cdot\omega_3}$
Euclid mean

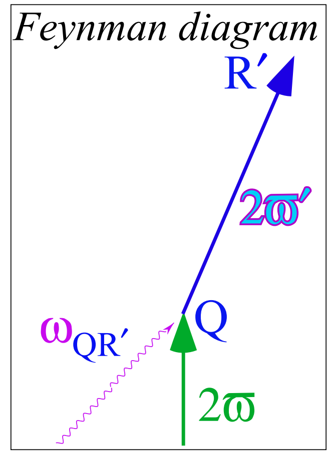
$\varpi'=\sqrt{(ff)}\varpi$
$= f\cdot\varpi$
1 blue-shift of ϖ

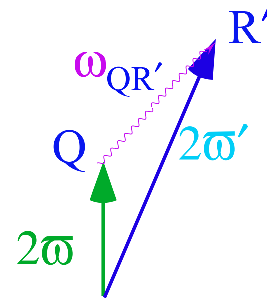
$\omega_1'=ff\cdot\varpi$
$=f^2\cdot\varpi$
2 shifts of ϖ

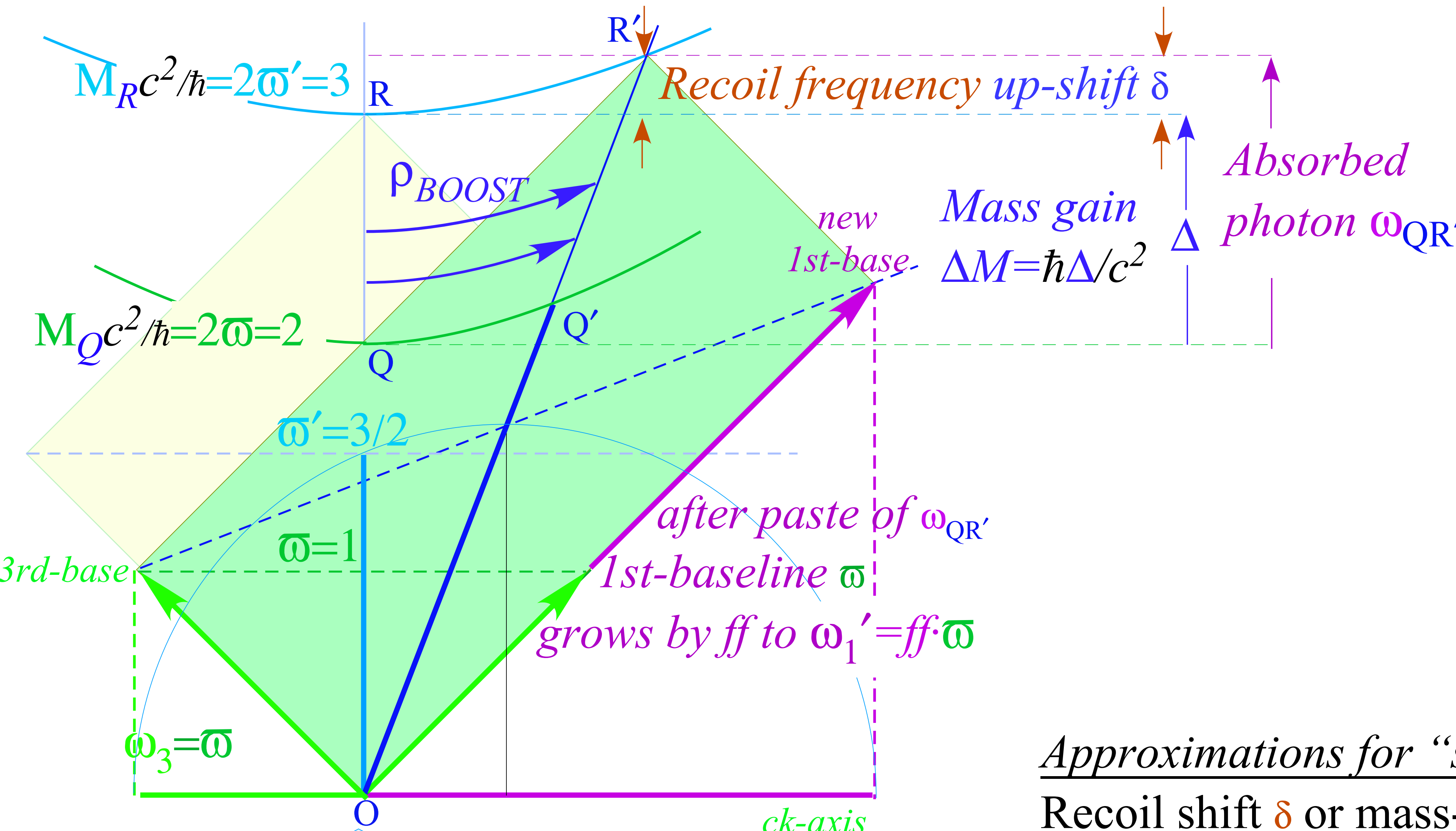
Atom accelerates from Q up to R′ as it *gains mass* $\Delta M=M_R - M_Q$ or phase frequency $\Delta=1$.

Atom 1st-baseline ϖ grows by *Feynman's-father-factor* $ff=\frac{\omega_1'}{\varpi}$. *Doppler blue shift* factor is $b=f=\sqrt{ff}$ that relates ϖ, ϖ′, and ω_1'.

$Q \rightarrow R'$ *Photo-Absorption Process(contd.)*







Boosted atom

$$\rho_{BOOST} = \ln f = \ln \frac{M_R}{M_Q} \qquad f=3/2$$

Absorbed photon

$$\omega_{QR'} = (ff-1)\varpi \qquad \varpi=1$$

$$= \left(\frac{M_R^2}{M_Q^2} - 1\right) M_Q c^2/2\hbar$$

$$u_{BOOST} = c\frac{f^2-1}{f^2+1} = c\ 5/13$$

$$\omega_{QR'} = (9/4-1)1 = 5/4$$

$\varpi' = \sqrt{\omega_1' \cdot \omega_3}$

Euclid mean

$\varpi' = \sqrt{(ff)}\varpi$

$= f \cdot \varpi$

1 blue-shift of ϖ

$\omega_1' = ff \cdot \varpi$

$= f^2 \cdot \varpi$

2 blue-shifts of ϖ

Approximations for "soft-photon" atomic transitions

Recoil shift δ or mass-gain $\Delta M = \hbar\Delta/c^2$ assumed small.

$$\omega_{QR'} = \frac{M_Q c^2 \Delta/\hbar + \Delta^2/2}{M_Q c^2/\hbar} \sim \Delta + \frac{\hbar\Delta^2}{2M_Q c^2}$$

Recoil frequency up-shift δ

Wave geometry of 2-photon transitions and Compton scattering

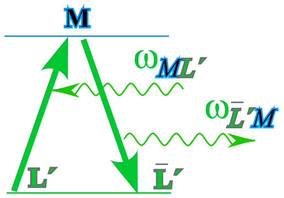
➡ *2-Photon emission and recoil*

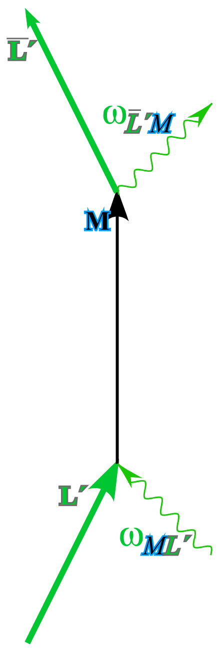
Grotian 3-level diagrams vs. Feynman (ω,ck) diagrams

"Photon diamond"geometric formulas

Geometric frequency shifting and "Compton stairs"

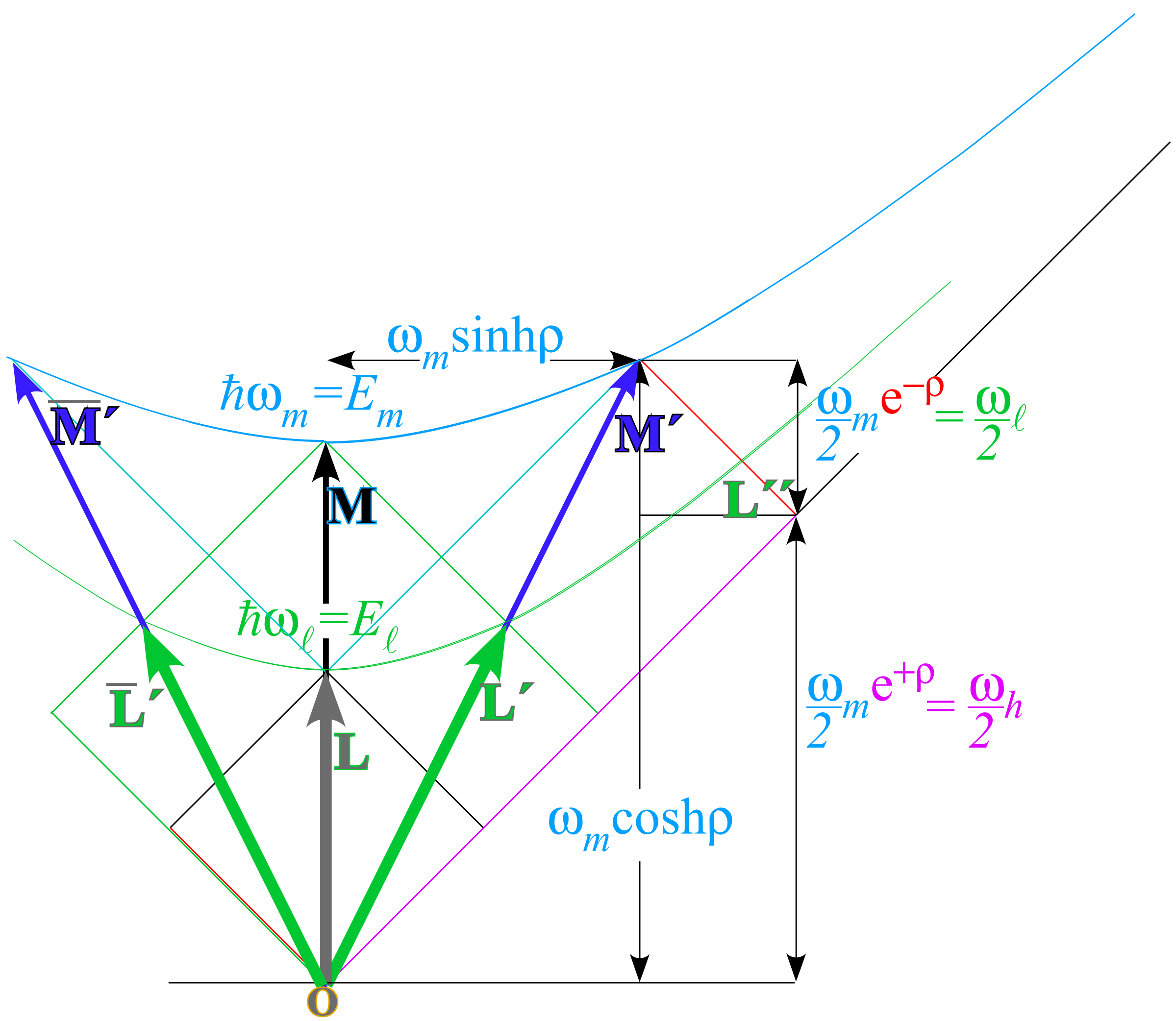
Wave geometry of 2-photon transitions and Compton scattering



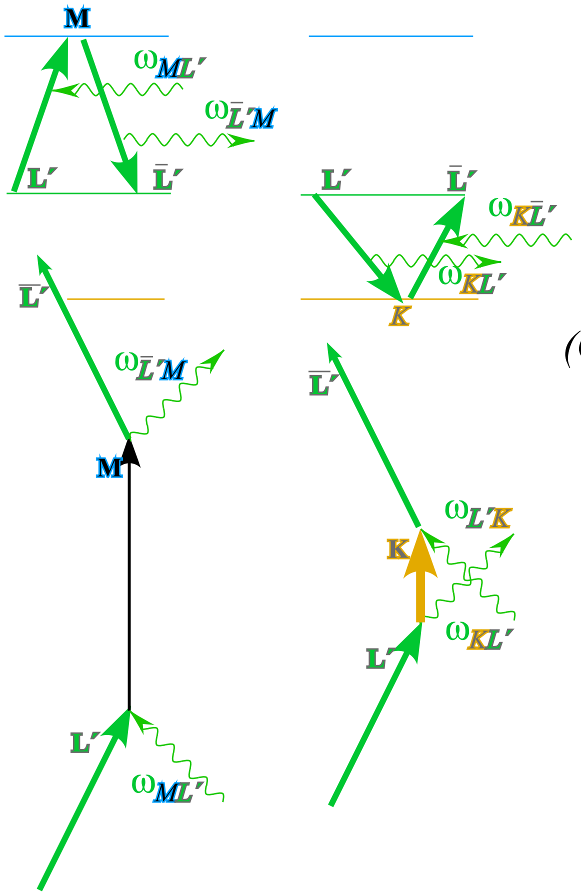


Compton scattering (Center of Momentum view)

Geometric 2-Level (ck,ω) diagram

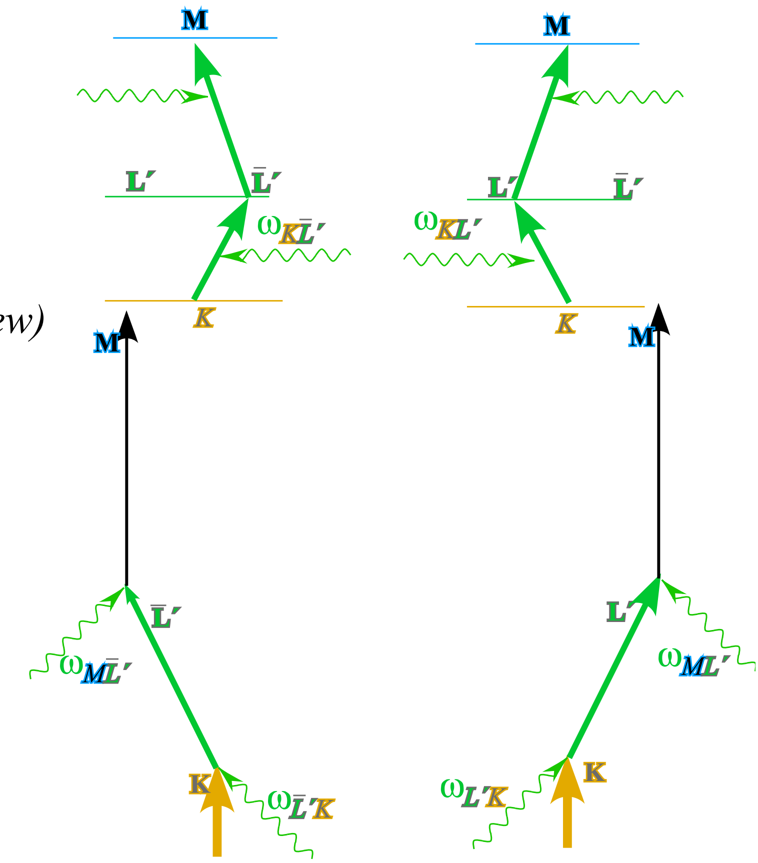


Wave geometry of 2-photon transitions and Compton scattering

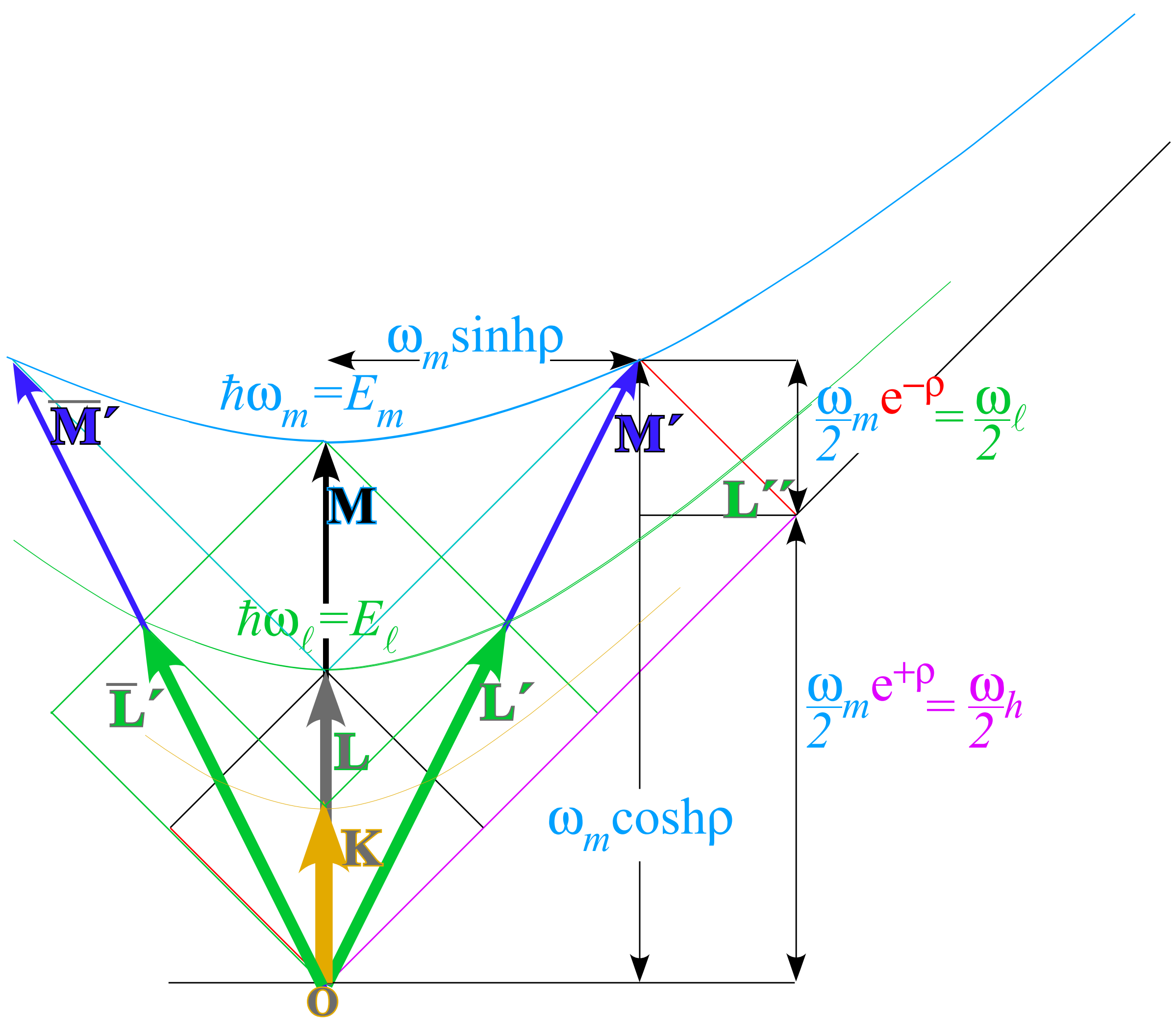


Compton scattering (Center of Momentum view)

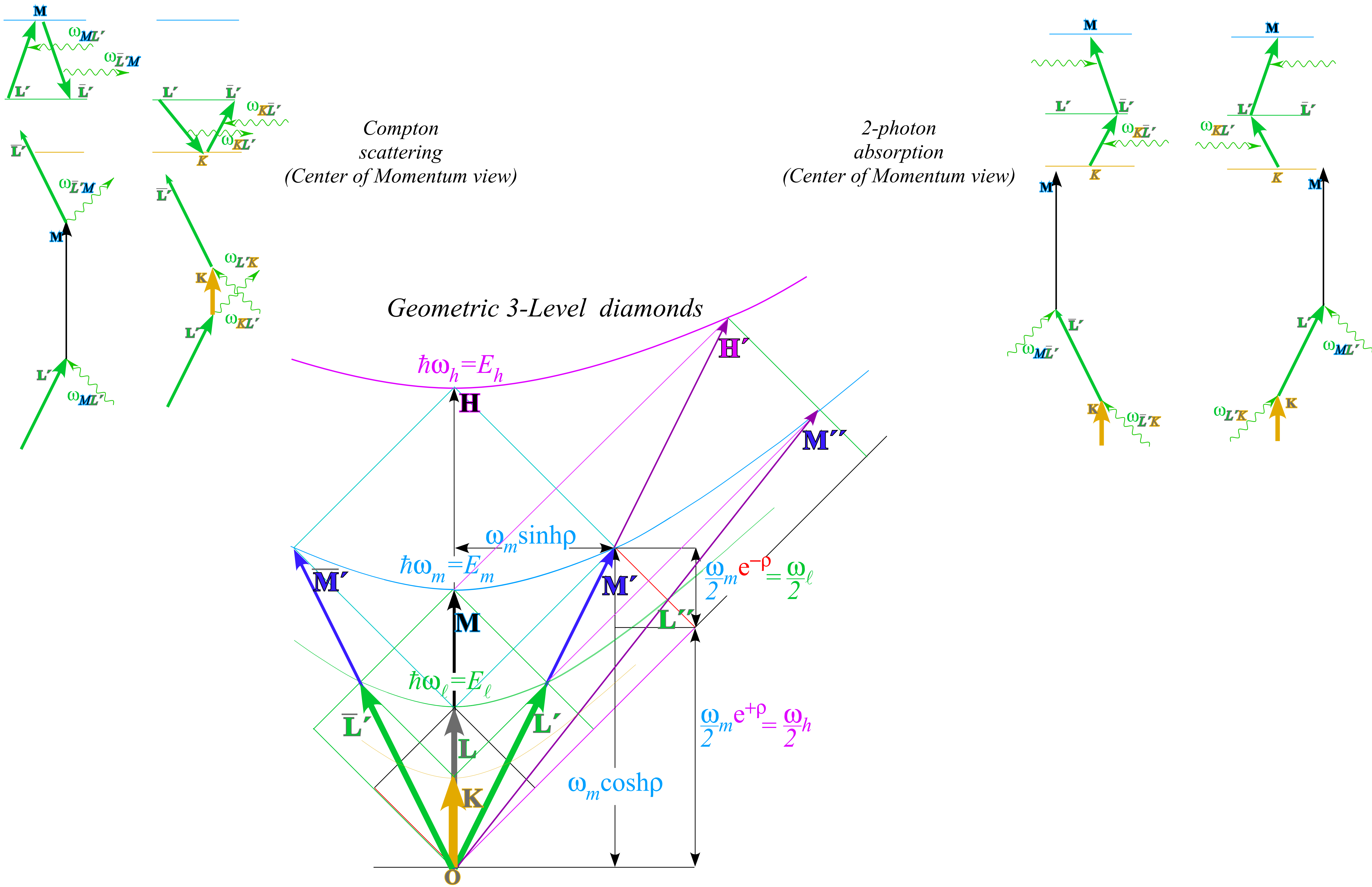
2-photon absorption (Center of Momentum view)



Geometric 3-Level diamonds



Wave geometry of 2-photon transitions and Compton scattering



Wave geometry of 2-photon transitions and Compton scattering

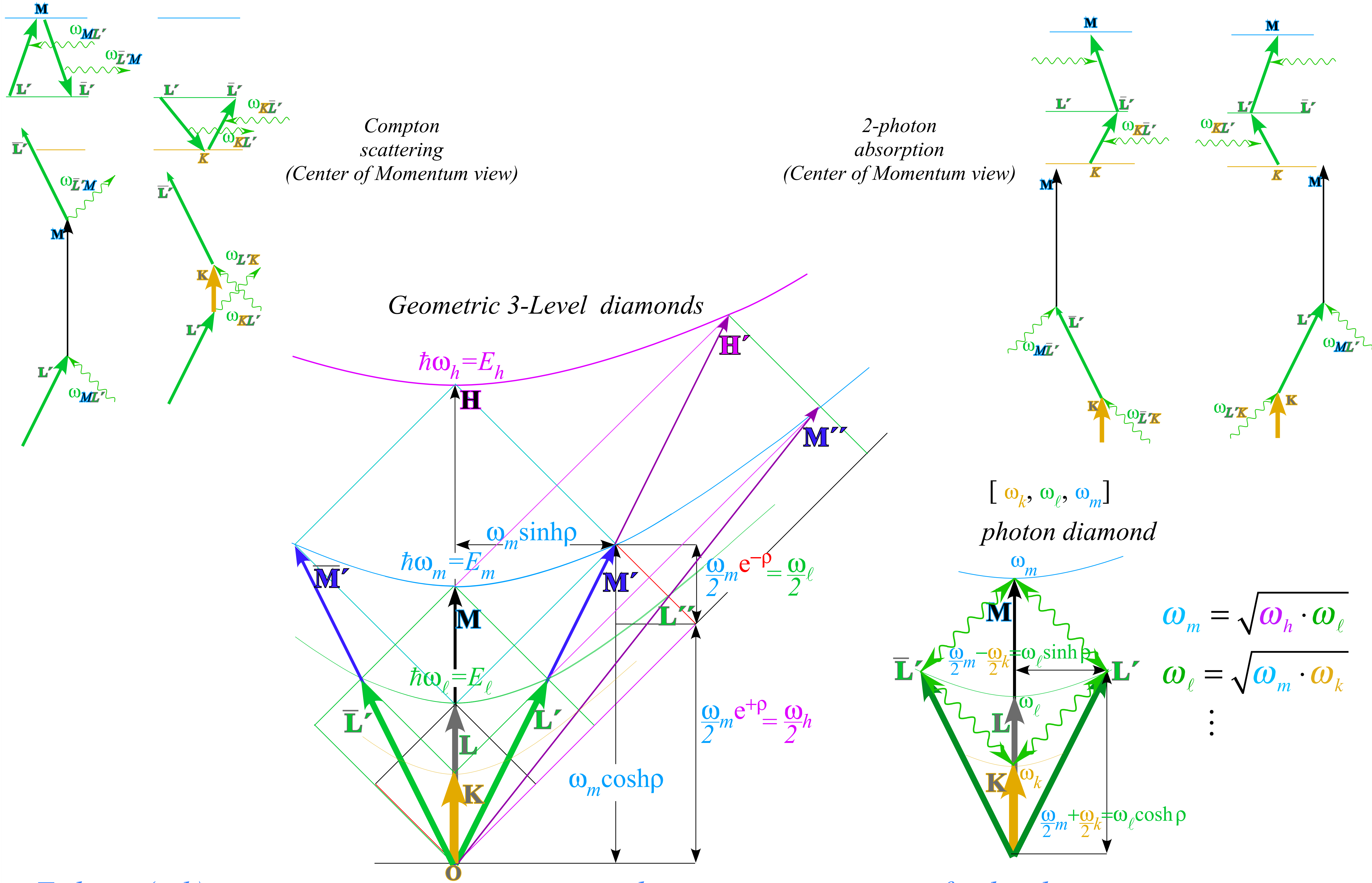
2-Photon emission and recoil

Grotian 3-level diagrams vs. Feynman (ω,ck) diagrams

➔ *"Photon diamond" geometric formulas*

Geometric frequency shifting and "Compton stairs"

Wave geometry of 2-photon transitions and Compton scattering



To have (ω,k) conservation in transitions must have geometric series of ω-levels: ω_k, ω_l, ω_m, ω_h,...

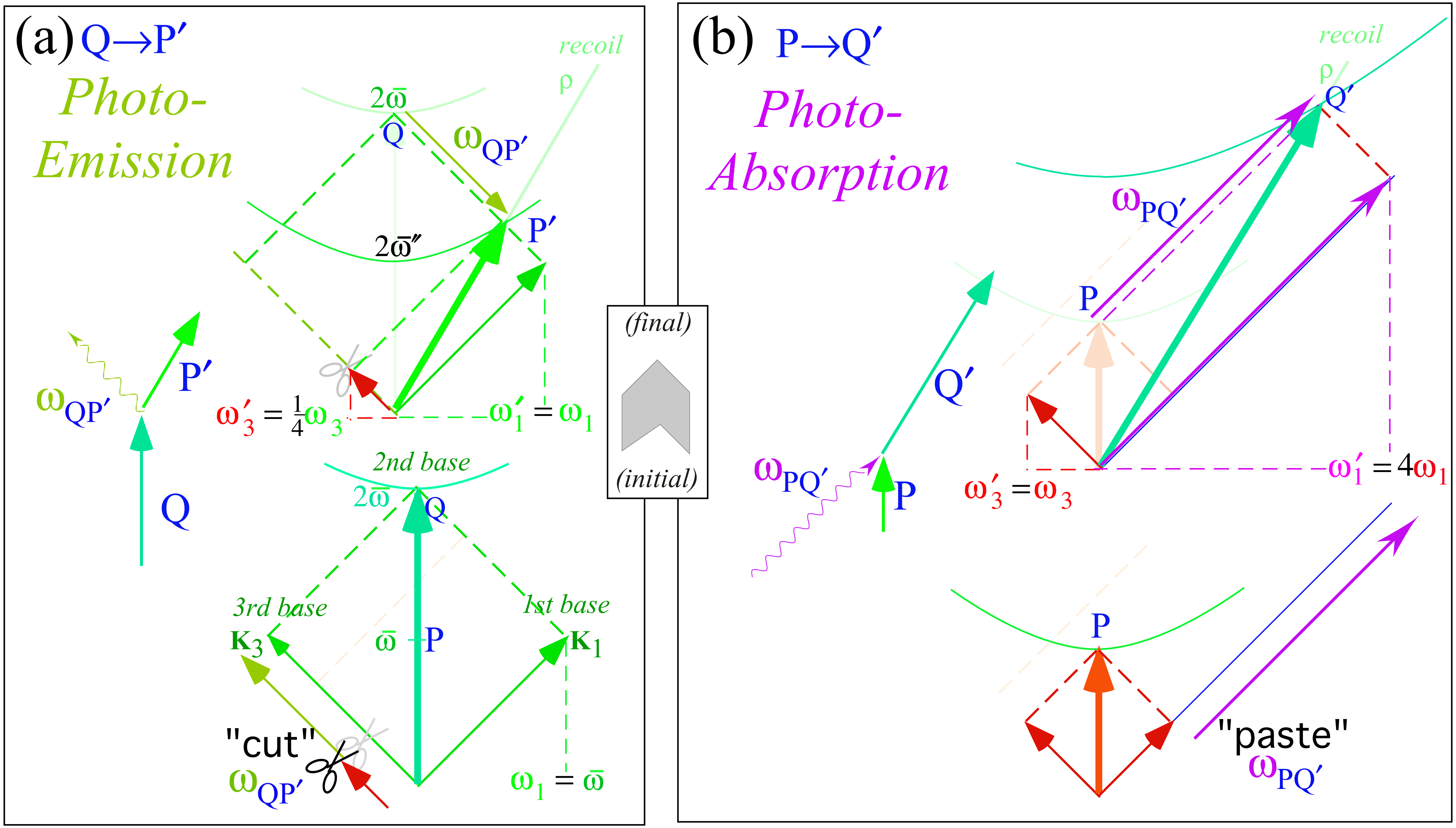
Wave geometry of 2-photon transitions and Compton scattering

2-Photon emission and recoil

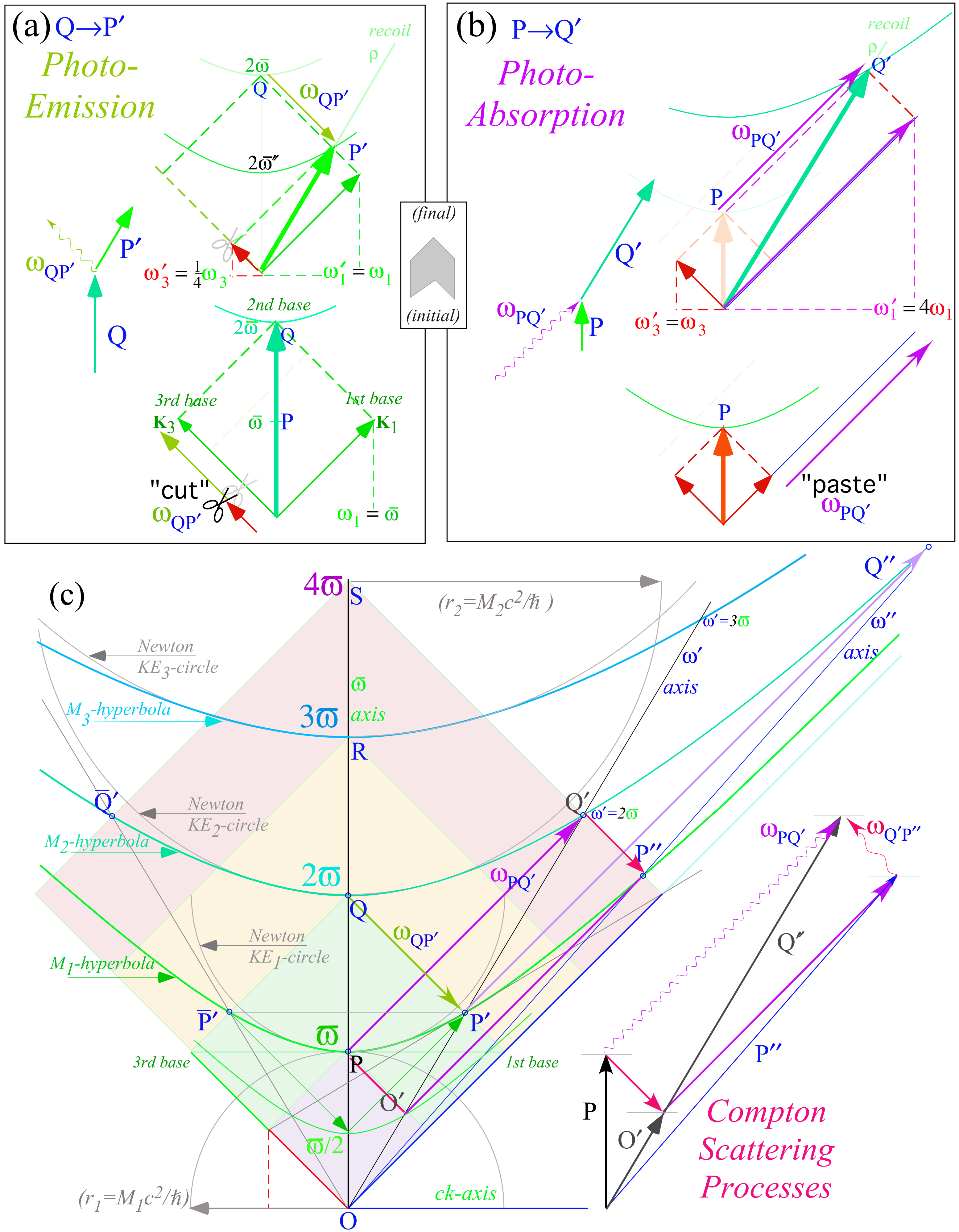
Grotian 3-level diagrams vs. Feynman (ω,ck) diagrams

➔ *"Photon diamond"geometric formulas (Examples with geometric ratio of 2)*

Geometric frequency shifting and "Compton stairs"

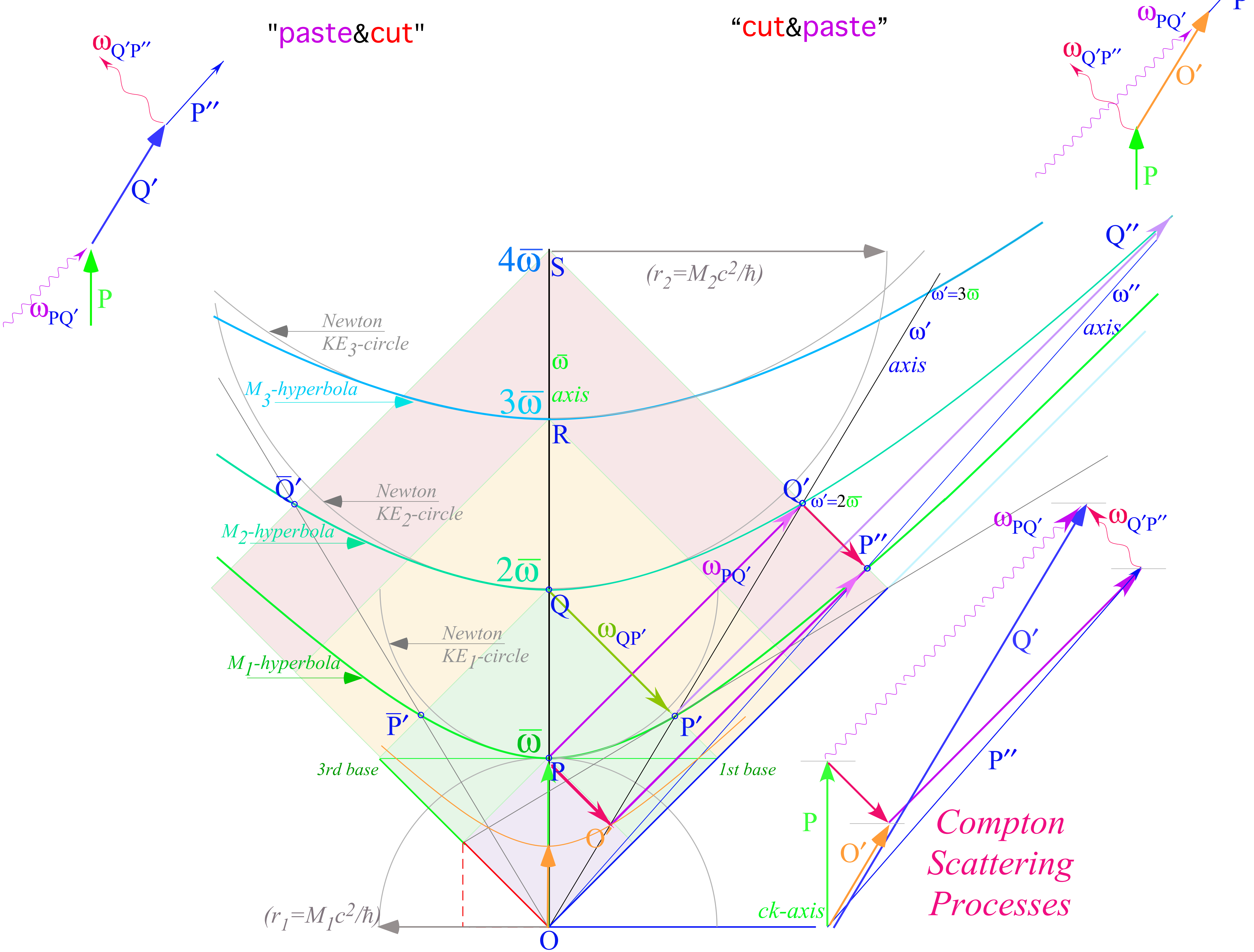


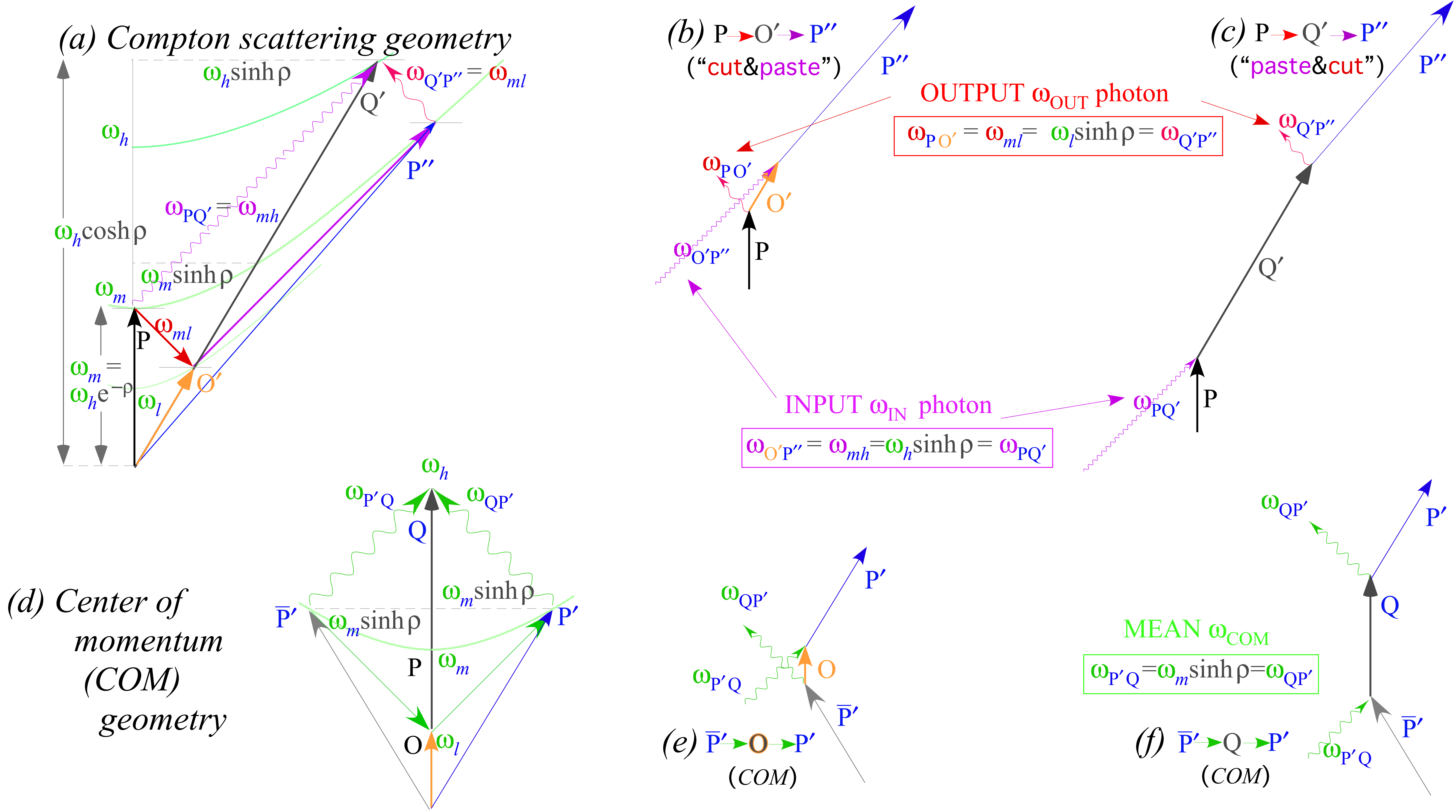
Feynman-father-factor of ff=4 *Doppler factor of f=2 gives familar recoil* *u/c= 3/5*



(a) Q→P′
Photo-Emission
(b) P→Q′
Photo-Absorption
(final)
(initial)
"cut"
"paste"
2nd base
3rd base
1st base
recoil
(c)
Newton KE3-circle
M3-hyperbola
Newton KE2-circle
M2-hyperbola
Newton KE1-circle
M1-hyperbola
ck-axis
Compton Scattering Processes

P→Q′ →P″ *Compton-Scattering Process*   P→O′ →P″ *Inverse Compton-Process*




(a) Compton scattering geometry
ω_h sinh ρ
ω_Q′P″ = ω_ml
Q′
ω_h
P″
ω_PQ′ = ω_mh
ω_h cosh ρ
ω_m sinh ρ
ω_m
ω_ml
P
ω_m =
ω_h e^−ρ
ω_l
O′
(b) P→O′→P″
("cut&paste")
P″
OUTPUT ω_OUT photon
ω_PO′ = ω_ml = ω_l sinh ρ = ω_Q′P″
ω_PO′
O′
ω_O′P″
P
INPUT ω_IN photon
ω_O′P″ = ω_mh = ω_h sinh ρ = ω_PQ′
(c) P→Q′→P″
("paste&cut")
P″
ω_Q′P″
Q′
ω_PQ′
P
(d) Center of momentum (COM) geometry
ω_h
ω_P′Q
ω_QP′
Q
ω_m sinh ρ
P̄′
ω_m sinh ρ
P′
P
ω_m
O
ω_l
ω_QP′
P′
ω_P′Q
O
P̄′
(e) P̄′→O→P′
(COM)
MEAN ω_COM
ω_P′Q = ω_m sinh ρ = ω_QP′
ω_QP′
P′
Q
P̄′
ω_P′Q
(f) P̄′→Q→P′
(COM)

Wave geometry of 2-photon transitions and Compton scattering

2-Photon emission and recoil

Grotian 3-level diagrams vs. Feynman (ω,ck) diagrams

"Photon diamond" geometric formulas (Examples with geometric ratio of 2)

➔ *Geometric frequency shifting and "Compton stairs"*

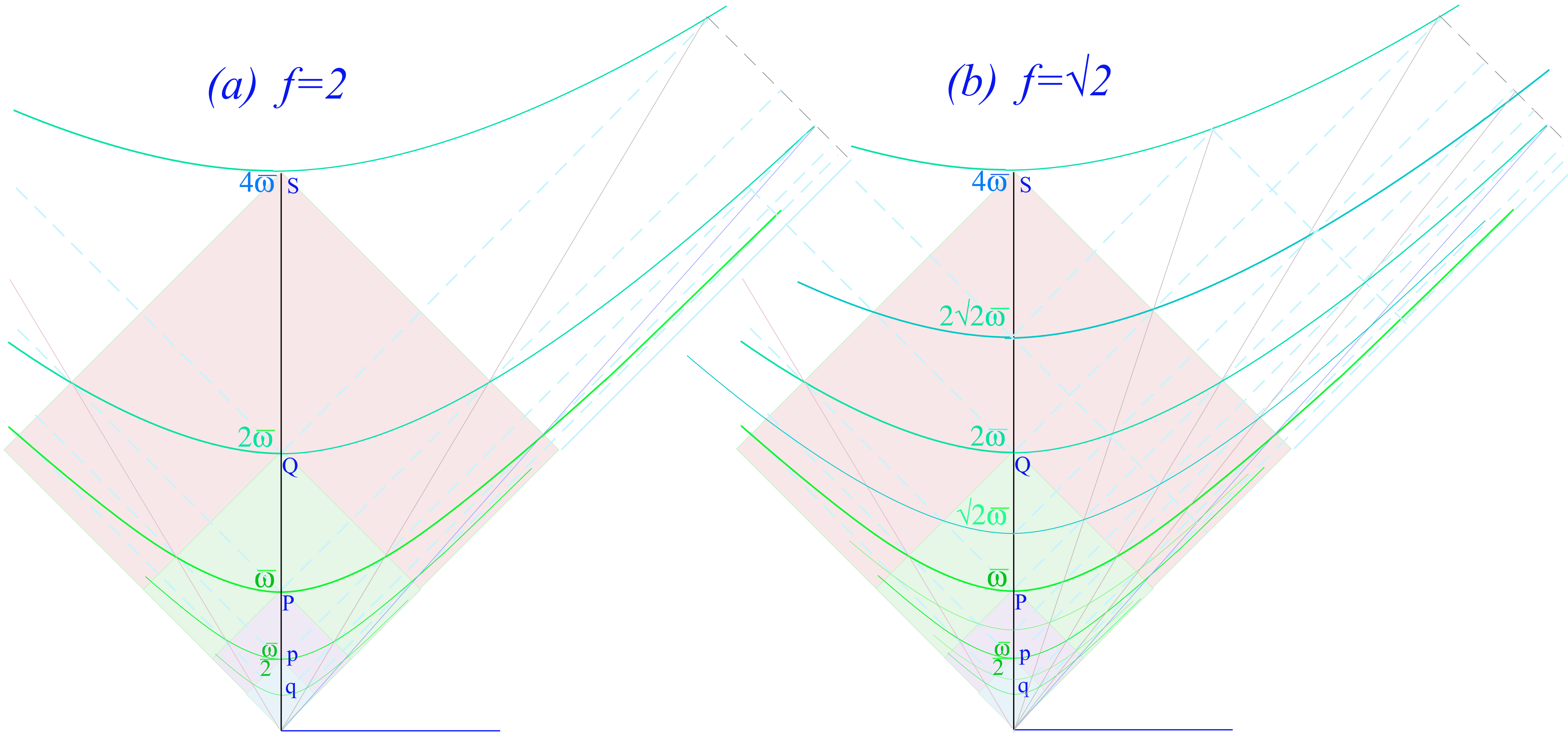


Fig. 7.7 Compton nets are congruent Compton staircases of transitions. (a) f=2:1 (b) f= $\sqrt{2}$ *:1*

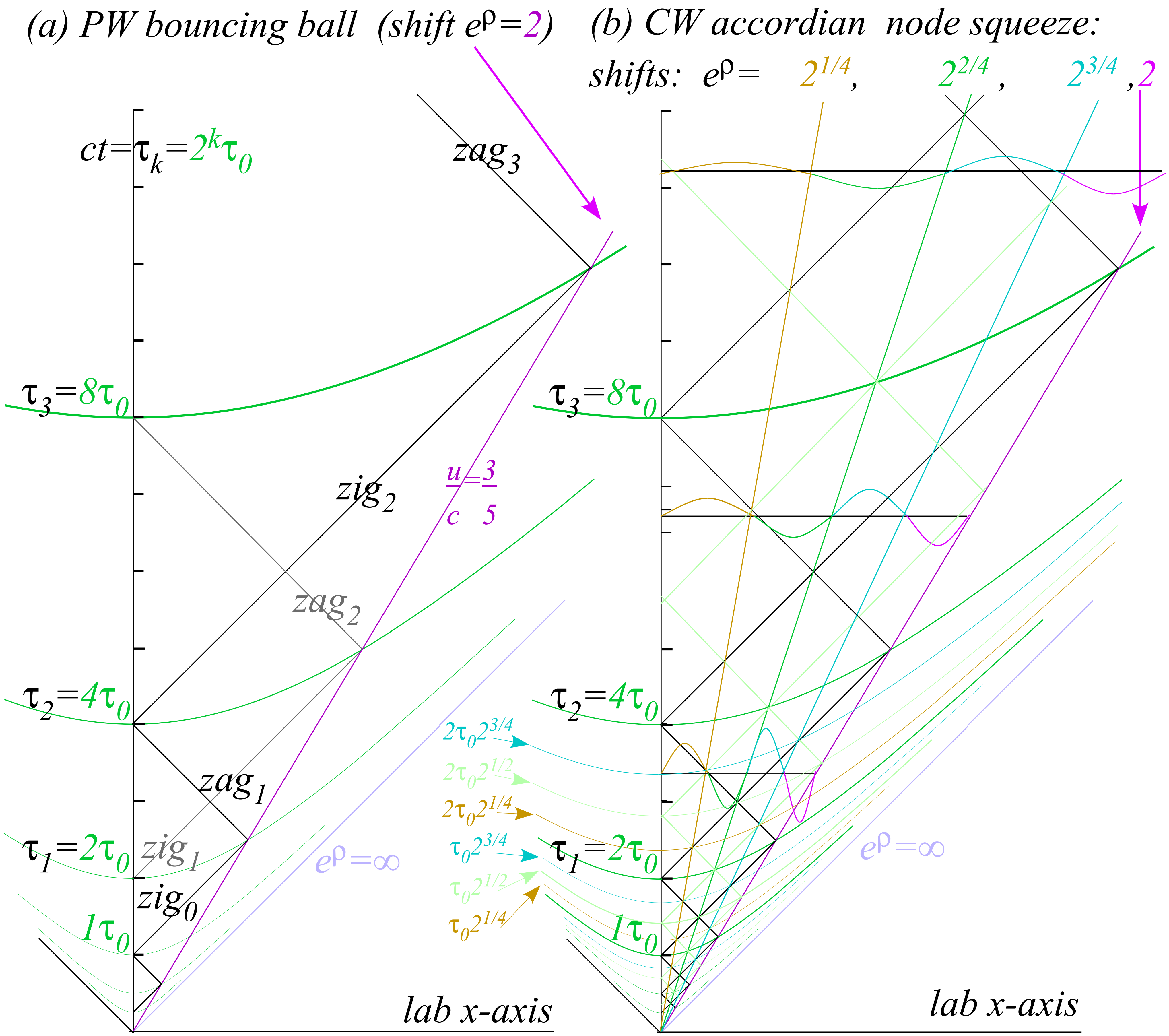
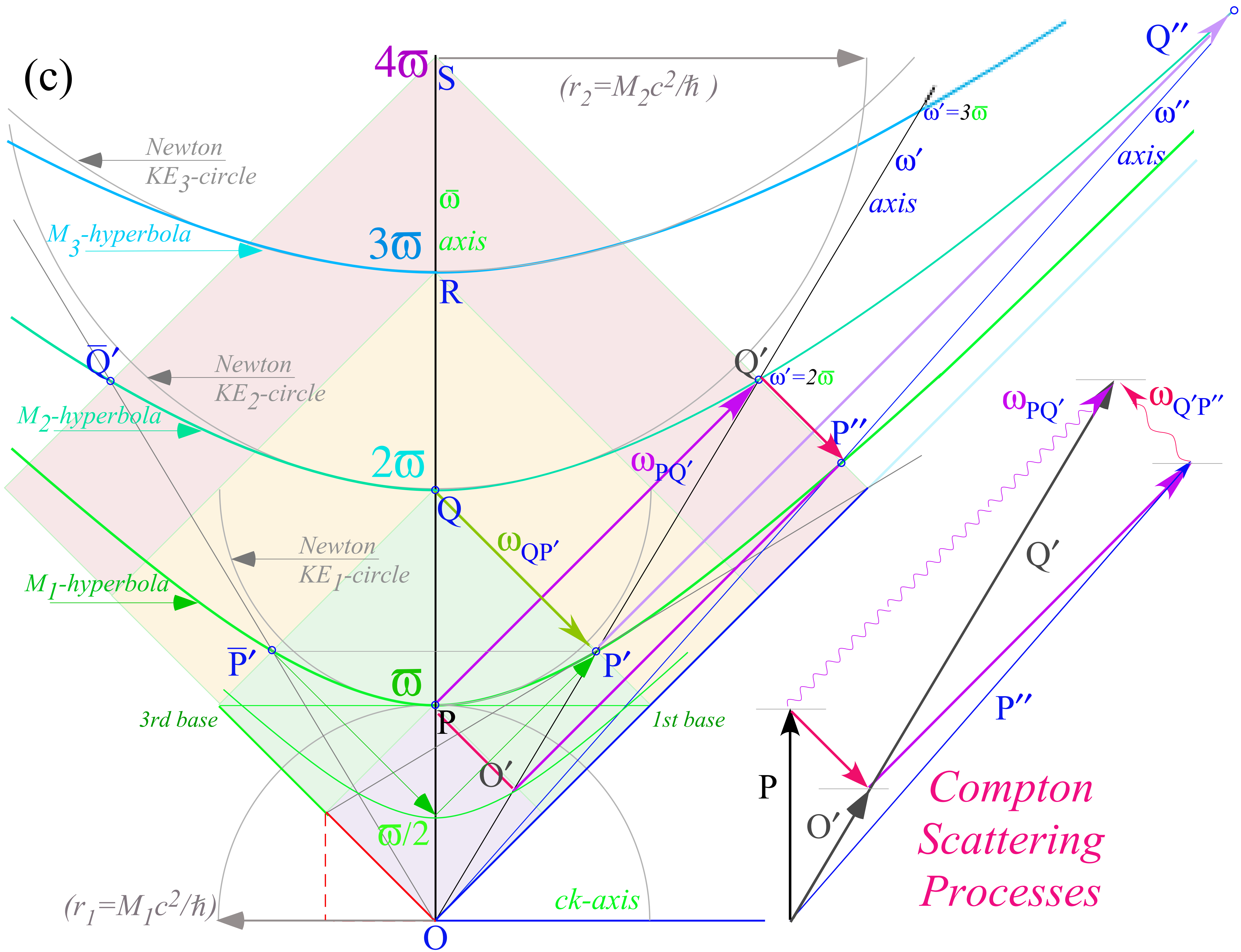


Fig. 7.8 Space-time nets (a) PW zigzag paths bounce. (b) CW nodes squeeze like an accordian.


(c)
4ϖ
S
$(r_2=M_2c^2/\hbar)$
Newton
KE_3-circle
ϖ
axis
M_3-hyperbola
3ϖ
R
ω′=3ϖ
ω′
axis
Q″
ω″
axis
Q̄′
Newton
KE_2-circle
M_2-hyperbola
2ϖ
Q
Q′
ω′=2ϖ
P″
$\omega_{PQ'}$
$\omega_{QP'}$
$\omega_{PQ'}$
$\omega_{Q'P''}$
Newton
KE_1-circle
M_1-hyperbola
P̄′
P′
ϖ
P
3rd base
1st base
O′
ϖ/2
Q′
P″
P
O′
$(r_1=M_1c^2/\hbar)$
ck-axis
O
Compton
Scattering
Processes