Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation
Learning about sin! *and* cos *and...*

➔ Derivation of relativistic quantum mechanics
What's the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2$

$\sinh\rho \approx \rho$

At low speeds:

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\boxed{\frac{\kappa_{phase}}{\kappa_A}}$	$\frac{\tau_{phase}}{\tau_A}$	$\boxed{\frac{\upsilon_{phase}}{\upsilon_A}}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation - Relativistic *Terms (Expanded Table)*

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \qquad \text{(for } u \ll c)$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

group	$b_{RED}^{Doppler}$	$\boxed{\frac{V_{group}}{c}}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\boxed{\frac{\kappa_{phase}}{\kappa_A}}$	$\frac{\tau_{phase}}{\tau_A}$	$\boxed{\frac{\upsilon_{phase}}{\upsilon_A}}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\boxed{\tanh\rho}$	$\boxed{\sinh\rho}$	$\operatorname{sech}\rho$	$\boxed{\cosh\rho}$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \quad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \quad \text{(for } u \ll c)$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c)$$

group	$b_{RED}^{Doppler}$	$\boxed{\frac{V_{group}}{c}}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\boxed{\frac{\kappa_{phase}}{\kappa_A}}$	$\frac{\tau_{phase}}{\tau_A}$	$\boxed{\frac{\upsilon_{phase}}{\upsilon_A}}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\boxed{\tanh\rho}$	$\boxed{\sinh\rho}$	$\text{sech}\,\rho$	$\boxed{\cosh\rho}$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c\text{)}$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{(for } u \ll c\text{)}$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad \text{(for } u \ll c\text{)}$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \qquad \Leftarrow \text{for } (u \ll c) \Rightarrow \qquad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton’s kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \quad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \quad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

So attach scale factor h to match units.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{(for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

So attach scale factor h to match units.

Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b^{Doppler}_{RED}$	$\boxed{\frac{V_{group}}{c}}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\boxed{\frac{\kappa_{phase}}{\kappa_A}}$	$\frac{\tau_{phase}}{\tau_A}$	$\boxed{\frac{\upsilon_{phase}}{\upsilon_A}}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\boxed{\tanh\rho}$	$\boxed{\sinh\rho}$	$\text{sech}\,\rho$	$\boxed{\cosh\rho}$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \quad (\text{for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c\text{)}$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c\text{)}$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \qquad \text{(for } u \ll c\text{)}$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

group	$b_{RED}^{Doppler}$	$\boxed{\frac{V_{group}}{c}}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\boxed{\frac{\kappa_{phase}}{\kappa_A}}$	$\frac{\tau_{phase}}{\tau_A}$	$\boxed{\frac{\upsilon_{phase}}{\upsilon_A}}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\boxed{\tanh\rho}$	$\boxed{\sinh\rho}$	$\operatorname{sech}\rho$	$\boxed{\cosh\rho}$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \quad (\text{for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? Cheap trick??

group	$b_{RED}^{Doppler}$	$\boxed{\frac{V_{group}}{c}}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\boxed{\frac{\kappa_{phase}}{\kappa_A}}$	$\frac{\tau_{phase}}{\tau_A}$	$\boxed{\frac{\upsilon_{phase}}{\upsilon_A}}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\boxed{\tanh\rho}$	$\boxed{\sinh\rho}$	$\operatorname{sech}\rho$	$\boxed{\cosh\rho}$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? Cheap trick??

…Try exact υ_{phase} *…*

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation - Relativistic *Terms*
(Expanded Table)

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \quad (\text{for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *...*

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

(old-fashioned notation)

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \quad (\text{for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *...*

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

(old-fashioned notation)

group	$b^{Doppler}_{RED}$	$\boxed{\frac{V_{group}}{c}}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\boxed{\frac{\kappa_{phase}}{\kappa_A}}$	$\frac{\tau_{phase}}{\tau_A}$	$\boxed{\frac{\upsilon_{phase}}{\upsilon_A}}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\boxed{\tanh\rho}$	$\boxed{\sinh\rho}$	$\mathrm{sech}\,\rho$	$\boxed{\cosh\rho}$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\boxed{\frac{1}{\sqrt{1-\beta^2}}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Max Planck 1858-1947

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

Max Planck
1858-1947

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h (or hN) to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? Cheap trick??

...Try <u>exact</u> υ_{phase} ...

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \bullet \mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$$

Einstein (1905)

group	$b^{Doppler}_{RED}$	$\boxed{\dfrac{V_{group}}{c}}$	$\dfrac{\upsilon_{group}}{\upsilon_A}$	$\dfrac{\lambda_{group}}{\lambda_A}$	$\dfrac{\kappa_{group}}{\kappa_A}$	$\dfrac{\tau_{group}}{\tau_A}$	$\dfrac{V_{phas}}{c}$	
phase	$\dfrac{1}{b^{Doppler}_{BLUE}}$	$\dfrac{c}{V_{phase}}$	$\boxed{\dfrac{\kappa_{phase}}{\kappa_A}}$	$\dfrac{\tau_{phase}}{\tau_A}$	$\boxed{\dfrac{\upsilon_{phase}}{\upsilon_A}}$	$\dfrac{\lambda_{phase}}{\lambda_A}$	$\dfrac{c}{V_{group}}$	$\dfrac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\boxed{\tanh\rho}$	$\boxed{\sinh\rho}$	$\text{sech}\,\rho$	$\boxed{\cosh\rho}$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \dfrac{u}{c}$	$\sqrt{\dfrac{1-\beta}{1+\beta}}$	$\dfrac{\beta}{1}$	$\dfrac{1}{\sqrt{\beta^{-2}-1}}$	$\dfrac{\sqrt{1-\beta^2}}{1}$	$\boxed{\dfrac{1}{\sqrt{1-\beta^2}}}$	$\dfrac{\sqrt{\beta^{-2}-1}}{1}$	$\dfrac{1}{\beta}$	$\sqrt{\dfrac{1+\beta}{1-\beta}}$
value for β=3/5	$\dfrac{1}{2}=0.5$	$\dfrac{3}{5}=0.6$	$\dfrac{3}{4}=0.75$	$\dfrac{4}{5}=0.80$	$\dfrac{5}{4}=1.25$	$\dfrac{4}{3}=1.33$	$\dfrac{5}{3}=1.67$	$\dfrac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B+\tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \quad (\text{for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1+\tfrac{1}{2}\rho^2 \approx 1+\tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M=\dfrac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *...*

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \boldsymbol{E}^* \boldsymbol{\cdot} \boldsymbol{E} = hN\upsilon_{phase}$

This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N \qquad \Psi = \sqrt{\dfrac{\varepsilon_0}{h\upsilon}}E$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phas}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$			
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

For more visit the Pirelli Challenge Site
Quantized amplitude

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B+\tfrac{1}{2}B\rho^2 \text{ (for } u\ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u\ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \qquad \text{(for } u\ll c)$$

$$\cosh\rho\approx 1+\tfrac{1}{2}\rho^2\approx 1+\tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho\approx\rho\approx\frac{u}{c}$$

Max Planck
1858-1947

$B=\upsilon_A$

$B=\upsilon_A=c\kappa_A$

At low speeds:

$$\upsilon_{phase}\approx B+\frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u\ll c) \Rightarrow \quad \kappa_{phase}\approx\frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M=\frac{hB}{c^2}$ or: $hB=Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase}\approx hB+\frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u\ll c) \Rightarrow \quad h\kappa_{phase}\approx\frac{hB}{c^2}u$$

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase}\approx Mc^2+\frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u\ll c) \Rightarrow \quad h\kappa_{phase}\approx Mu$$

Lucky coincidences?? Cheap trick??

...Try <u>exact</u> υ_{phase} ...

$$h\upsilon_{phase}=hB\cosh\rho=Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E=\frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

Need to replace ~~h~~ with hN *to match e.m. energy density*
$\varepsilon_0 \mathbf{E}^*\!\cdot\!\mathbf{E} = hN\upsilon_{phase}$

This motivates the "particle" normalization
$\int \Psi^*\Psi\, dV=N \qquad \Psi=\sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

Big worry: Is not oscillator energy quadratic in frequency υ?

HO energy$=\frac{1}{2}A^2\upsilon^2$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$			
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B+\tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \quad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \quad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1+\tfrac{1}{2}\rho^2 \approx 1+\tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M=\frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h *(or* hN*)* to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try ***exact*** υ_{phase} *...*

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

Need to replace ~~h~~ h with hN *to match e.m. energy density* $\varepsilon_0\mathbf{E}^*\!\cdot\mathbf{E} = hN\upsilon_{phase}$

This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phas}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$			
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Big worry: Is not oscillator energy quadratic in frequency υ?

HO energy$=\frac{1}{2}A^2\upsilon^2$

Resolution and dirty secret: $\mathbf{E}$, N, and υ_{phase} are all frequencies!

So $\varepsilon_0\mathbf{E}\cdot\mathbf{E} = hN\upsilon_{phase}$ *is quadratic in frequency*

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B+\tfrac{1}{2}B\rho^2 \text{ (for } u\ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u\ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \qquad \text{(for } u\ll c)$$

$$\cosh\rho \approx 1+\tfrac{1}{2}\rho^2 \approx 1+\tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B+\frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u\ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M=\dfrac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB+\frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u\ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx Mc^2+\frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u\ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} and κ_{phase}...

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0\mathbf{E}^*\cdot\mathbf{E} = hN\upsilon_{phase}$

group	$b_{RED}^{Doppler}$	$\boxed{\frac{V_{group}}{c}}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\boxed{\frac{\kappa_{phase}}{\kappa_A}}$	$\frac{\tau_{phase}}{\tau_A}$	$\boxed{\frac{\upsilon_{phase}}{\upsilon_A}}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\boxed{\tanh\rho}$	$\boxed{\sinh\rho}$	$\text{sech}\,\rho$	$\boxed{\cosh\rho}$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\boxed{\frac{1}{\sqrt{1-\beta^2}}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} and κ_{phase}...

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \bullet \mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1 - u^2/c^2}}$$

Einstein (1905)

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$cp = \frac{Mcu}{\sqrt{1 - u^2/c^2}}$$

group	$b_{RED}^{Doppler}$	$\dfrac{V_{group}}{c}$	$\dfrac{\upsilon_{group}}{\upsilon_A}$	$\dfrac{\lambda_{group}}{\lambda_A}$	$\dfrac{\kappa_{group}}{\kappa_A}$	$\dfrac{\tau_{group}}{\tau_A}$	$\dfrac{V_{phase}}{c}$	
phase	$\dfrac{1}{b_{BLUE}^{Doppler}}$	$\dfrac{c}{V_{phase}}$	$\dfrac{\kappa_{phase}}{\kappa_A}$	$\dfrac{\tau_{phase}}{\tau_A}$	$\dfrac{\upsilon_{phase}}{\upsilon_A}$	$\dfrac{\lambda_{phase}}{\lambda_A}$	$\dfrac{c}{V_{group}}$	$\dfrac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \dfrac{u}{c}$	$\sqrt{\dfrac{1-\beta}{1+\beta}}$	$\dfrac{\beta}{1}$	$\dfrac{1}{\sqrt{\beta^{-2}-1}}$	$\dfrac{\sqrt{1-\beta^2}}{1}$	$\dfrac{1}{\sqrt{1-\beta^2}}$	$\dfrac{\sqrt{\beta^{-2}-1}}{1}$	$\dfrac{1}{\beta}$	$\sqrt{\dfrac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2} = 0.5$	$\frac{3}{5} = 0.6$	$\frac{3}{4} = 0.75$	$\frac{4}{5} = 0.80$	$\frac{5}{4} = 1.25$	$\frac{4}{3} = 1.33$	$\frac{5}{3} = 1.67$	$\frac{2}{1} = 2.0$

$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1 - \frac{u^2}{c^2}}}$$ (old-fashioned notation)

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho} \approx \rho \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$


Max Planck
1858-1947


Louis DeBroglie
1892-1987

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Natural wave conspiracy
~~Lucky coincidences~~?? *Expensive* *~~Cheap~~ trick??*

...Try exact υ_{phase} *and* κ_{phase}*...*

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}} \quad \text{(old-fashioned notation)}$$

$$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$$

$$\boxed{\text{Momentum: } h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}}$$

DeBroglie (1921)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\boxed{\upsilon_{phase} = B\cosh\rho} \approx B + \tfrac{1}{2}B\rho^2 \text{(for } u \ll c)$$

$$\boxed{c\kappa_{phase} = B\sinh\rho} \approx B\rho \qquad \text{(for } u \ll c)$$

$$\boxed{\frac{u}{c} = \tanh\rho \approx \rho} \qquad \text{(for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

Max Planck
1858-1947

Louis DeBroglie
1892-1987

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h (or hN) to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Natural wave conspiracy ~~Lucky coincidences??~~ Expensive ~~Cheap~~ trick??

...Try exact υ_{phase} *and* κ_{phase}*...*

Need to replace h with hN to match e.m. energy density $\varepsilon_0 \mathbf{E}^*\bullet\mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N \quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$			
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

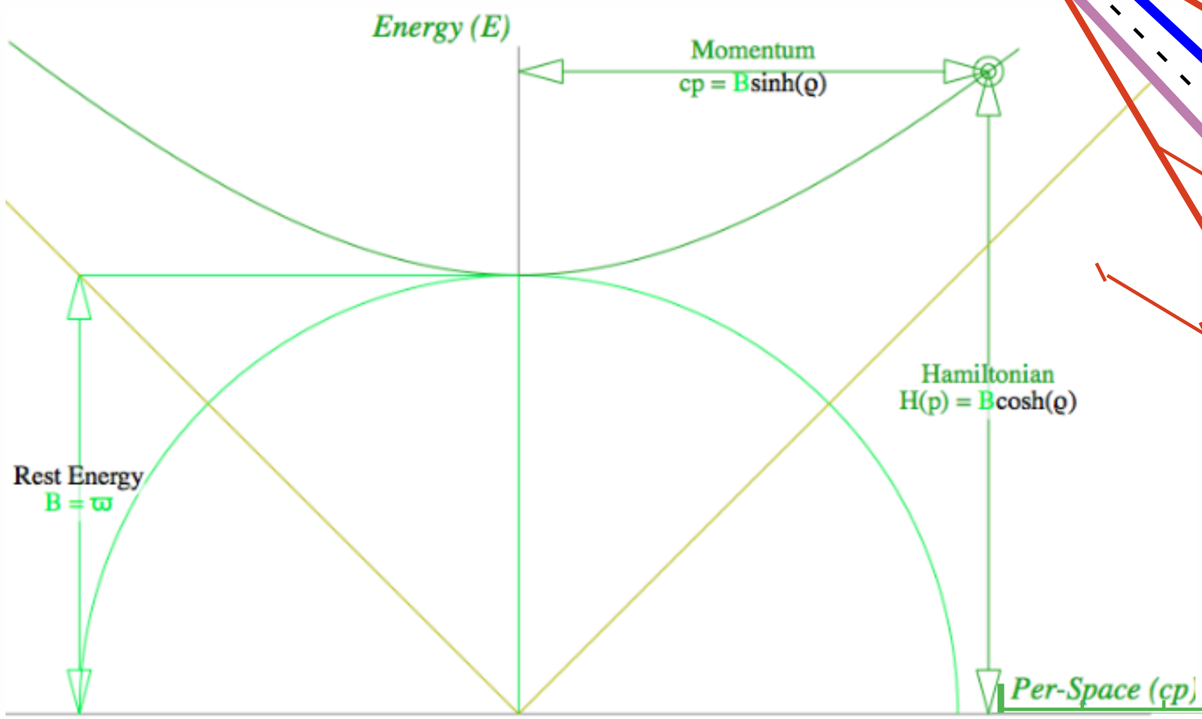
$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$$ (old-fashioned notation)

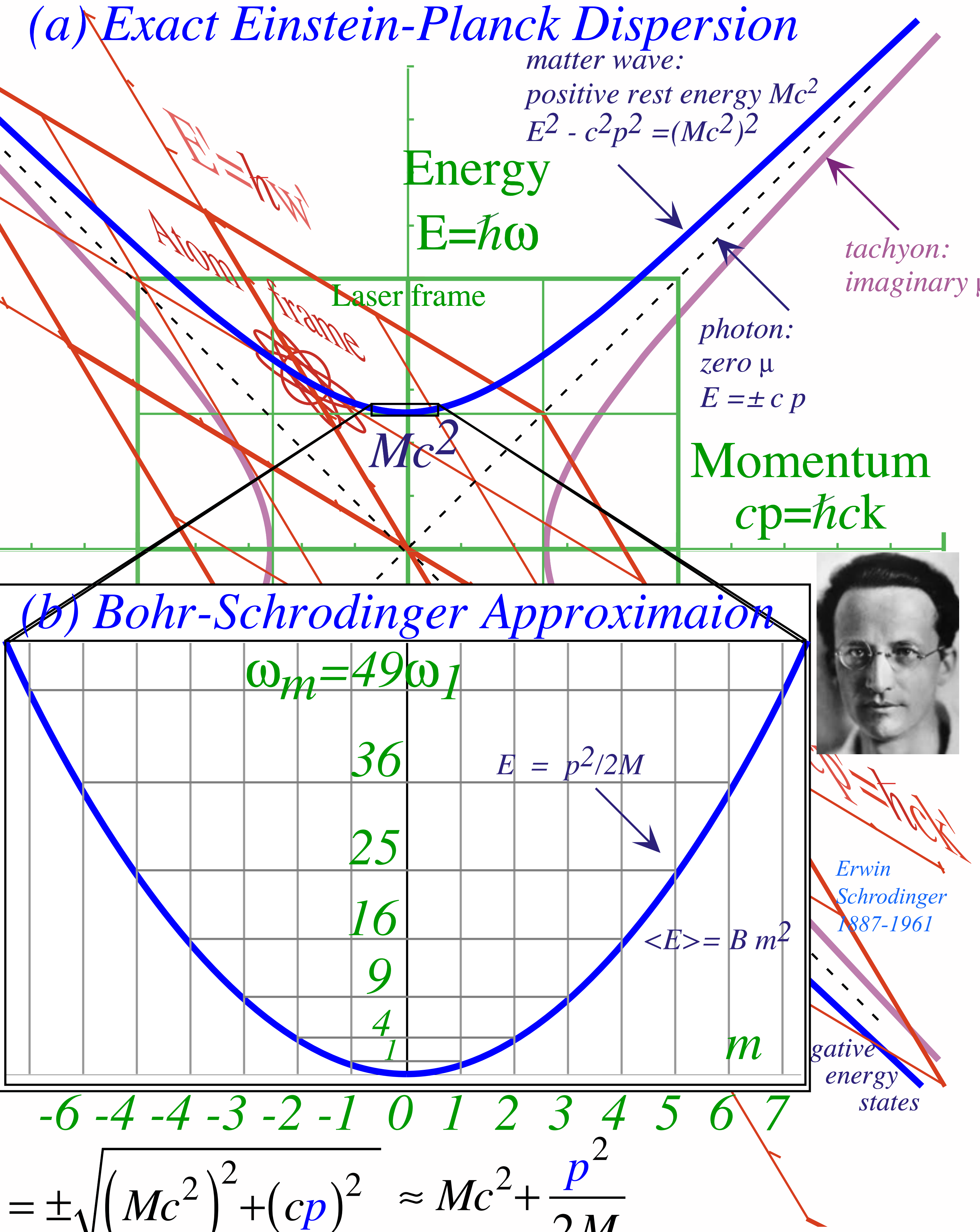
$$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$$

$$\text{Momentum: } h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$$

DeBroglie (1921)

Using (some) wave coordinates for relativistic quantum theory





Erwin Schrodinger 1887-1961

Neils Bohr 1885-1962

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

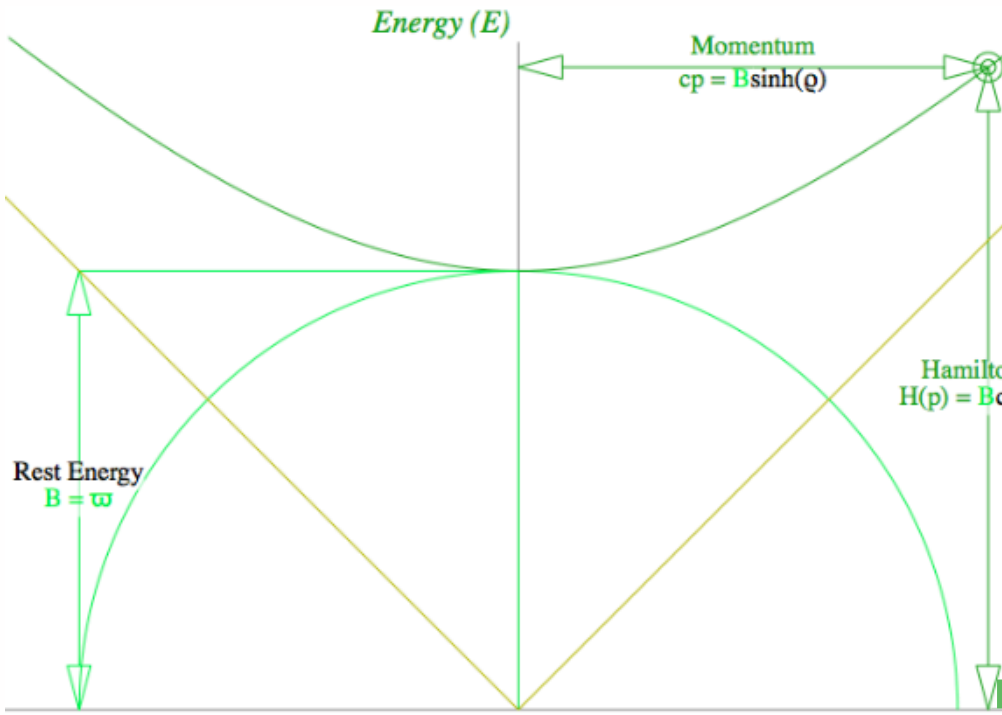
$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

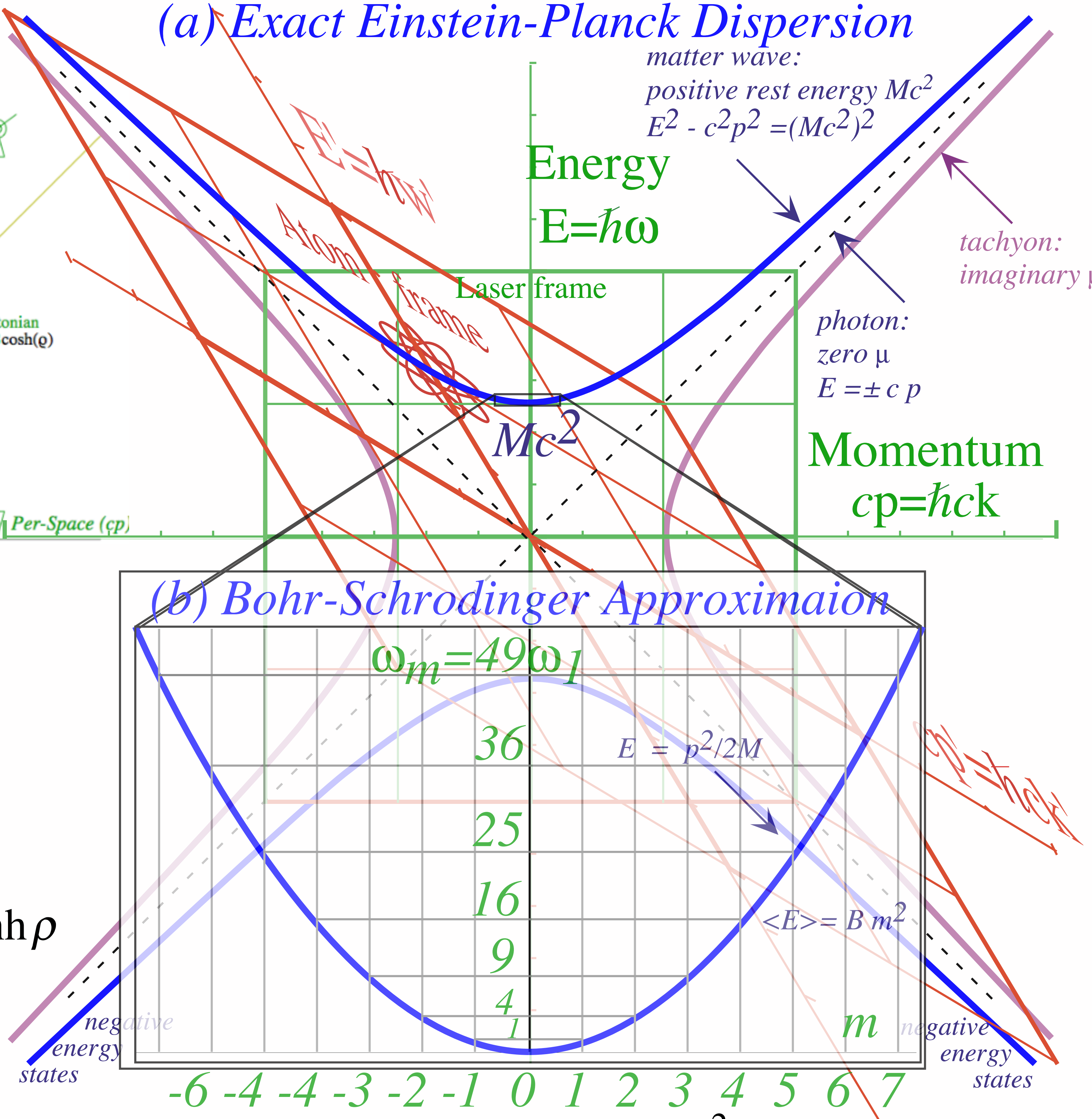
Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2+(cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

Using (some) wave coordinates for relativistic quantum theory




(a) Exact Einstein-Planck Dispersion

(b) Bohr-Schrodinger Approximaion

Mass (resting)

$$hB = h\upsilon_A = \boxed{Mc^2} = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = \boxed{h\upsilon_A}\cosh\rho$$

Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = \boxed{h\upsilon_A}\sinh\rho$$

Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2\cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2+(cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

low speed approximation

Relawavity variable tables

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b^{Doppler}_{RED}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*$^{(Lorentz)}$ τ_{phase}*-contraction*	*t-dilation*$^{(Einstein)}$ υ_{phase}*-dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b^{Doppler}_{BLUE}$

Relativistic quantum mechanics variable tables

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{\beta}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{1-\beta^2}}{\beta}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
functions		$V_{group}=$ $c\tanh\rho$	*momentum* $cp=Mc^2\sinh\rho$	*-Lagrangian* $L=-Mc^2\operatorname{sech}\rho$	*Hamiltonian* $H=Mc^2\cosh\rho$	*DeBroglie* $\lambda=\alpha\operatorname{csch}\rho$	$V_{phase}=$ $c\cot h\rho$	

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
- ➔ What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

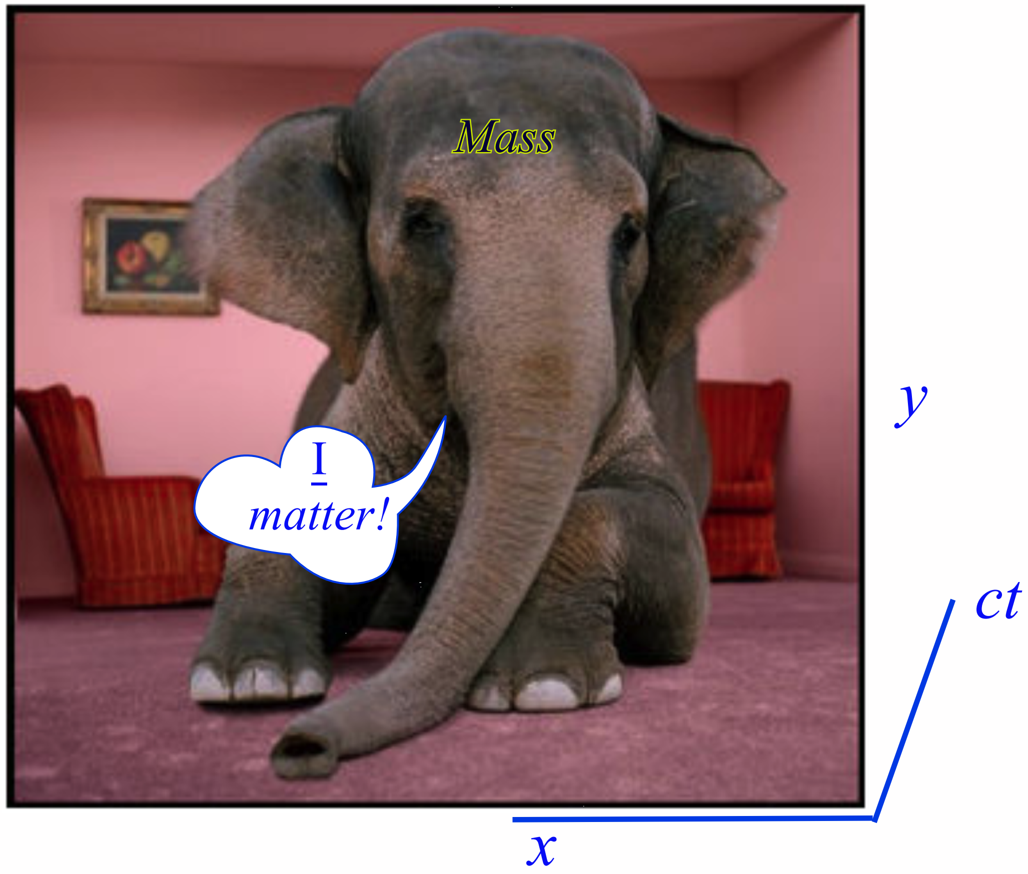
Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

- *What's the matter with Mass?*



Shining some light on the elephant in the spacetime room

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest} \qquad \textit{Rest Mass}}$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest}}$$

Rest Mass

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c\sinh\rho}{c\tanh\rho}$$

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)* Defines invariant hyperbola(s)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest}}$$

Rest Mass

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c\sinh\rho}{c\tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}}$$

Momentum Mass

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest}}$$ *Rest Mass*

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + \left(cp\right)^2}$$

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c\sinh\rho}{c\tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}}$$ *Momentum Mass*

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

Defines invariant hyperbola(s)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest}}$$

Rest Mass

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c \sinh\rho}{c \tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1 - u^2 / c^2}}}$$

Momentum Mass

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} *(Newton's mass)* Defined by ratio *F/a=dp/du* of relativistic force to acceleration.

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \dfrac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest}}$$

Rest Mass

Momentum Mass M_{mom} (Galileo's mass) Defined by ratio p/u of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c\sinh\rho}{c\tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}}$$

Momentum Mass

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\mathrm{sech}^2\rho\, d\rho$ in velocity

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest} \qquad \textit{Rest Mass}}$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \qquad \textit{Momentum Mass}}$$

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} *(Newton's mass)* Defined by ratio *F/a=dp/du* of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\text{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\text{sech}^2\rho} = M_{rest}\cosh^3\rho$$

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} (Einstein's mass) Defines invariant hyperbola(s)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest} \qquad \text{Rest Mass}}$$

Momentum Mass M_{mom} (Galileo's mass) Defined by ratio p/u of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \qquad \text{Momentum Mass}}$$

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} (Newton's mass) Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\mathrm{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho}\boxed{= M_{rest}\cosh^3\rho \quad \text{Effective Mass}}$$

Limiting cases:

$$M_{eff} \xrightarrow[u\to c]{} M_{rest}e^{3\rho}/2$$

$$M_{eff} \xrightarrow[u\ll c]{} M_{rest}$$

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} (Einstein's mass) Defines invariant hyperbola(s)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest} \quad \text{Rest Mass}}$$

Momentum Mass M_{mom} (Galileo's mass) Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c\sinh\rho}{c\tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \text{Momentum Mass}}$$

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} (Newton's mass) Defined by ratio *F/a=dp/du* of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\mathrm{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho} \boxed{= M_{rest}\cosh^3\rho \quad \text{Effective Mass}}$$

Limiting cases:

$$M_{eff} \xrightarrow[u\to c]{} M_{rest}e^{3\rho}/2$$

$$M_{eff} \xrightarrow[u\ll c]{} M_{rest}$$

More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}}$$

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

Group velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest} \qquad \text{Rest Mass}}$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

Momentum Mass M_{mom} (Galileo's mass) Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c\sinh\rho}{c\tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \qquad \text{Momentum Mass}}$$

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}\, e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} (Newton's mass) Defined by ratio *F/a=dp/du* of relativistic force to acceleration.

That is ratio of change $dp = Mc\cosh\rho\, d\rho$ in momentum to change $du = c\,\text{sech}^2\rho\, d\rho$ in velocity

$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\text{sech}^2\rho} \boxed{= M_{rest}\cosh^3\rho \quad \text{Effective Mass}}$$

Limiting cases:

$$M_{eff} \xrightarrow[u\to c]{} M_{rest}\, e^{3\rho}/2$$

$$M_{eff} \xrightarrow[u\ll c]{} M_{rest}$$

More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \boxed{\frac{\hbar}{\frac{d^2\omega}{dk^2}}} = \boxed{\frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}} = M_{rest}\cosh^3\rho \quad \text{Effective Mass}}$$

general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Definition(s) of mass for relativity/quantum

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\boxed{\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2}}$$ *Rest Mass*

Momentum Mass M_{mom} *(Galileo's mass)* Defined by p/u

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

$$\boxed{= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}}$$ *Momentum Mass*

Effective Mass M_{eff} *(Newton's mass)* Defined by $F/a=dp/du$

That is ratio of $dp=Mc\cosh\rho\, d\rho$ to change $du=c\,\mathrm{sech}^2\rho\, d\rho$ in velocity

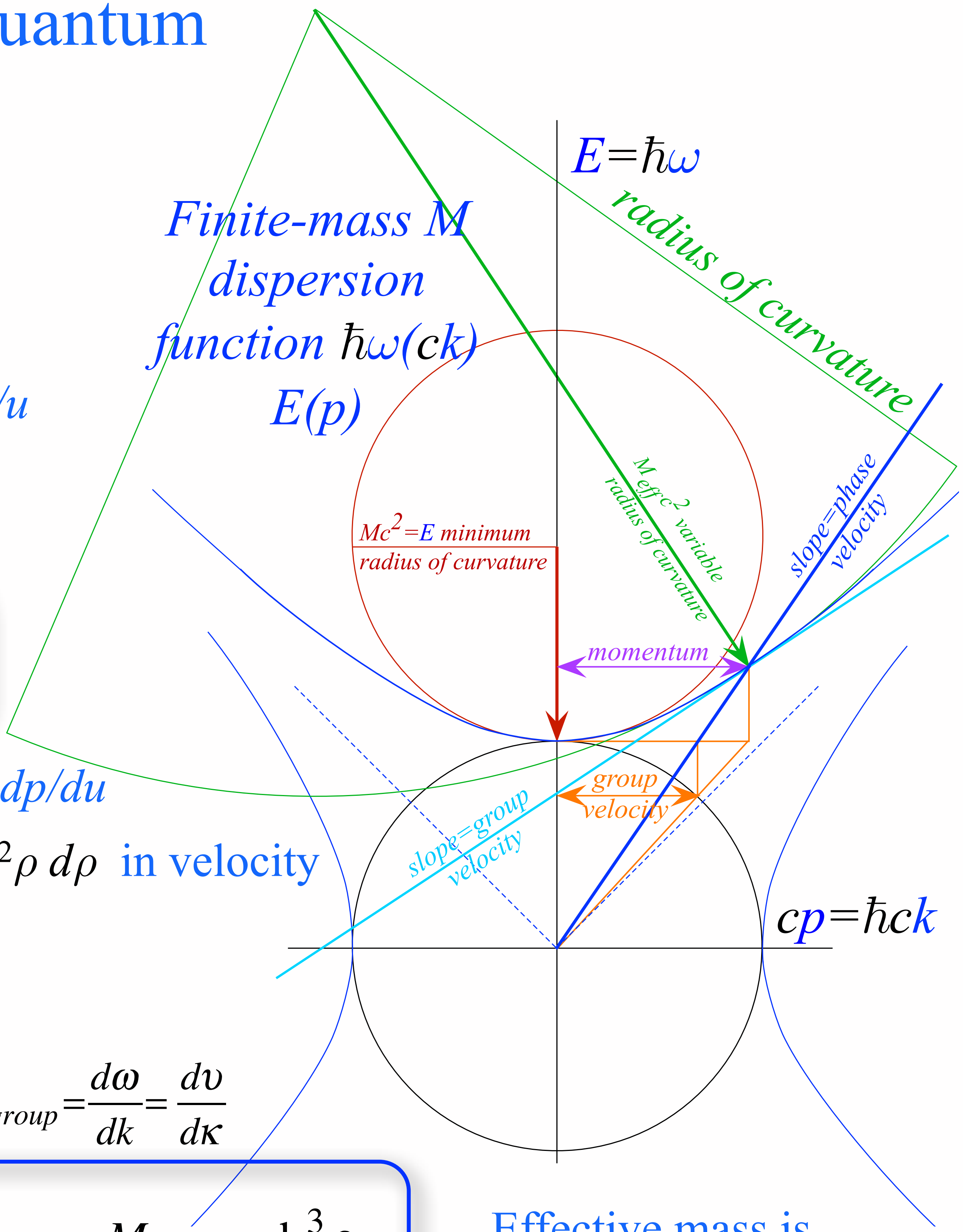
$$M_{eff} \equiv \frac{dp}{du} = M_{rest}\frac{c\cosh\rho}{c\,\mathrm{sech}^2\rho} \boxed{= M_{rest}\cosh^3\rho}$$ *Effective Mass*

More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \boxed{\frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}} = M_{rest}\cosh^3\rho}$$ *Effective Mass*

general wave formula

to accompany $V_{group} = \frac{d\omega}{dk}$



Effective mass is proportional to the *radius of curvature of* $\omega(k)$ *dispersion.*

Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

Rest Mass $(a)\gamma$-rest mass: $M^{\gamma}_{rest} = 0,$

Momentum Mass $(b)\gamma$-momentum mass: $M^{\gamma}_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\upsilon}{c^2},$

Effective Mass $(c)\gamma$-effective mass: $M^{\gamma}_{eff} = \infty.$

Newton complained about *his* "corpuscles" of light having *"fits"* (going *crazy*).

(All *this* would be evidence of *triple Schizophrenia*.)

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2\cdot 10^{-51})kg\cdot s = 4.5\cdot 10^{-36}\,kg \quad \text{(for: } \upsilon\text{=600THz)}$$

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of e^{ρ}=2 spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- ➔ Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar ck_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$$
$$\hbar ck_{phase} = cp = \hbar\omega_A \sinh\rho$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

$$p = \hbar k = Mc \sinh\rho$$

$$E = \hbar\omega = Mc^2 \cosh\rho$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar c k_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$$
$$\hbar c k_{phase} = cp = \hbar\omega_A \sinh\rho$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic $\boxed{\text{action } S}$ and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E$$

$$\hbar \equiv \frac{h}{2\pi}$$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A\cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A\sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A = Mc^2 = \hbar ck_A$$
$$\hbar\omega_{phase} = E = \hbar\omega_A\cosh\rho$$
$$\hbar ck_{phase} = cp = \hbar\omega_A\sinh\rho$$

Relativistic $\boxed{\text{action } S}$ and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega' t'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv \boxed{pu - H = L \qquad \textit{Legendre transformation}}$$

$$p = \hbar k = Mc\sinh\rho$$

$$E = \hbar\omega = Mc^2\cosh\rho = H$$

Prior wave relations

$$\boxed{\begin{aligned} h\upsilon_A &= Mc^2 = hc\kappa_A \\ h\upsilon_{phase} &= E = h\upsilon_A\cosh\rho \\ hc\kappa_{phase} &= cp = h\upsilon_A\sinh\rho \end{aligned}}$$

←linear Hz format

angular phasor→ format

$$\boxed{\begin{aligned} \hbar\omega_A &= Mc^2 = \hbar ck_A \\ \hbar\omega_{phase} &= E = \hbar\omega_A\cosh\rho \\ \hbar ck_{phase} &= cp = \hbar\omega_A\sinh\rho \end{aligned}}$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic $\boxed{\text{action } S}$ and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv \boxed{pu - H = L \quad \textit{Legendre transformation}}$$

Use *Group velocity* $: u=\dfrac{dx}{dt}=c\tanh\rho$

$$p=\hbar k = Mc\sinh\rho \qquad\qquad E=\hbar\omega = Mc^2\cosh\rho = H$$

$$\boxed{\begin{array}{l} h\upsilon_A = Mc^2 = hc\kappa_A \\ h\upsilon_{phase} = E = h\upsilon_A\cosh\rho \\ hc\kappa_{phase} = cp = h\upsilon_A\sinh\rho \end{array}}$$

Prior wave relations

←linear Hz format

angular phasor→ format

$$\boxed{\begin{array}{l} \hbar\omega_A = Mc^2 = \hbar ck_A \\ \hbar\omega_{phase} = E = \hbar\omega_A\cosh\rho \\ \hbar ck_{phase} = cp = \hbar\omega_A\sinh\rho \end{array}}$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic $\boxed{\text{action } S}$ and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv \boxed{pu - H = L \quad \textit{Legendre transformation}}$$

Use *Group velocity*: $u=\frac{dx}{dt}=c\tanh\rho$

$$p=\hbar k=Mc\sinh\rho \qquad\qquad E=\hbar\omega= Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

Prior wave relations

$$\boxed{\begin{array}{l} h\upsilon_A=Mc^2=hc\kappa_A \\ h\upsilon_{phase}= E = h\upsilon_A\cosh\rho \\ hc\kappa_{phase}=cp = h\upsilon_A\sinh\rho \end{array}}$$

←linear Hz format

angular phasor→ format

$$\boxed{\begin{array}{l} \hbar\omega_A=Mc^2=\hbar ck_A \\ \hbar\omega_{phase}= E = \hbar\omega_A\cosh\rho \\ \hbar ck_{phase}=cp = \hbar\omega_A\sinh\rho \end{array}}$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic $\boxed{\text{action } S}$ and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv \boxed{pu - H = L \qquad \textit{Legendre transformation}}$$

Use Group velocity: $u=\frac{dx}{dt}=c\tanh\rho$

$$p = \hbar k = Mc\sinh\rho \qquad\qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$$= Mc^2\frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

$$L \text{ is}: Mc^2\frac{-1}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

Prior wave relations

$$\boxed{\begin{aligned} h\upsilon_A &= Mc^2 = hc\kappa_A \\ h\upsilon_{phase} &= E = h\upsilon_A\cosh\rho \\ hc\kappa_{phase} &= cp = h\upsilon_A\sinh\rho \end{aligned}}$$

←linear Hz format

angular phasor→ format

$$\boxed{\begin{aligned} \hbar\omega_A &= Mc^2 = \hbar ck_A \\ \hbar\omega_{phase} &= E = \hbar\omega_A\cosh\rho \\ \hbar ck_{phase} &= cp = \hbar\omega_A\sinh\rho \end{aligned}}$$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv \boxed{pu - H = L \quad \text{Legendre transformation}}$$

Use Group velocity : $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc \sinh\rho \qquad\qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

Note: $Mcu = Mc^2 \tanh\rho$

$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

Prior wave relations

←linear Hz format	angular phasor→ format
$h\upsilon_A = Mc^2 = hc\kappa_A$	$\hbar\omega_A = Mc^2 = \hbar ck_A$
$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$	$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$
$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$	$\hbar ck_{phase} = cp = \hbar\omega_A \sinh\rho$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv \boxed{pu - H = L \qquad \textit{Legendre transformation}}$$

Use *Group velocity* : $u=\frac{dx}{dt}=c\tanh\rho$

$$p = \hbar k = Mc\sinh\rho \qquad\qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$$= Mc^2\frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

Note: $Mcu = Mc^2\tanh\rho$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho$$

with *Hamiltonian H=E*

$$H = \hbar\omega = Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho$$

Prior wave relations

$h\upsilon_A=Mc^2=hc\kappa_A$

$h\upsilon_{phase}= E = h\upsilon_A\cosh\rho$

$hc\kappa_{phase}=cp = h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor format→

$\hbar\omega_A=Mc^2=\hbar ck_A$

$\hbar\omega_{phase}= E = \hbar\omega_A\cosh\rho$

$\hbar ck_{phase}=cp = \hbar\omega_A\sinh\rho$

$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian* L using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega' t'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$L\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv\boxed{pu-H=L}$$

Legendre transformation

Use *Group velocity* : $u=\frac{dx}{dt}=c\tanh\rho$

$$p=\hbar k=Mc\sinh\rho \qquad E=\hbar\omega=Mc^2\cosh\rho=H$$

$$L=pu-H=(Mc\sinh\rho)(c\tanh\rho)-Mc^2\cosh\rho$$

$$=Mc^2\frac{\sinh^2\rho-\cosh^2\rho}{\cosh\rho}=-Mc^2\operatorname{sech}\rho$$

Note: $Mcu=Mc^2\tanh\rho$

Also: $cp=Mc^2\sinh\rho$

Compare *Lagrangian L*

$$L=\hbar\dot{\Phi}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

$\hbar=h/2\pi$

Prior wave relations

$h\upsilon_A=Mc^2=hc\kappa_A$

$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$

$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor format→

$\hbar\omega_A=Mc^2=\hbar ck_A$

$\hbar\omega_{phase}=E=\hbar\omega_A\cosh\rho$

$\hbar ck_{phase}=cp=\hbar\omega_A\sinh\rho$

$$\hbar\equiv\frac{h}{2\pi}$$

Relativistic $\boxed{\text{action } S}$ and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$\boxed{\frac{dS}{dt}=L}\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv\boxed{pu-H=L\quad\textit{Legendre transformation}}$$

Use *Group velocity* : $u=\frac{dx}{dt}=c\tanh\rho$

$=c\sin\sigma$

$$p=\hbar k=Mc\sinh\rho \qquad E=\hbar\omega=Mc^2\cosh\rho=H$$

$$L=pu-H=(Mc\sinh\rho)(c\tanh\rho)-Mc^2\cosh\rho$$

$$=Mc^2\frac{\sinh^2\rho-\cosh^2\rho}{\cosh\rho}=-Mc^2\operatorname{sech}\rho$$

Note: $Mcu=Mc^2\tanh\rho$

$=Mc^2\sin\sigma$

Also: $cp=Mc^2\sinh\rho$

$=\hbar ck=Mc^2\tan\sigma$

Compare *Lagrangian L*

$$\boxed{\dot{S}=L=\hbar\dot{\Phi}}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho=-Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S=\hbar\Phi$

Including stellar angle σ

Prior wave relations

$h\upsilon_A=Mc^2=hc\kappa_A$

$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$

$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$

←linear Hz format

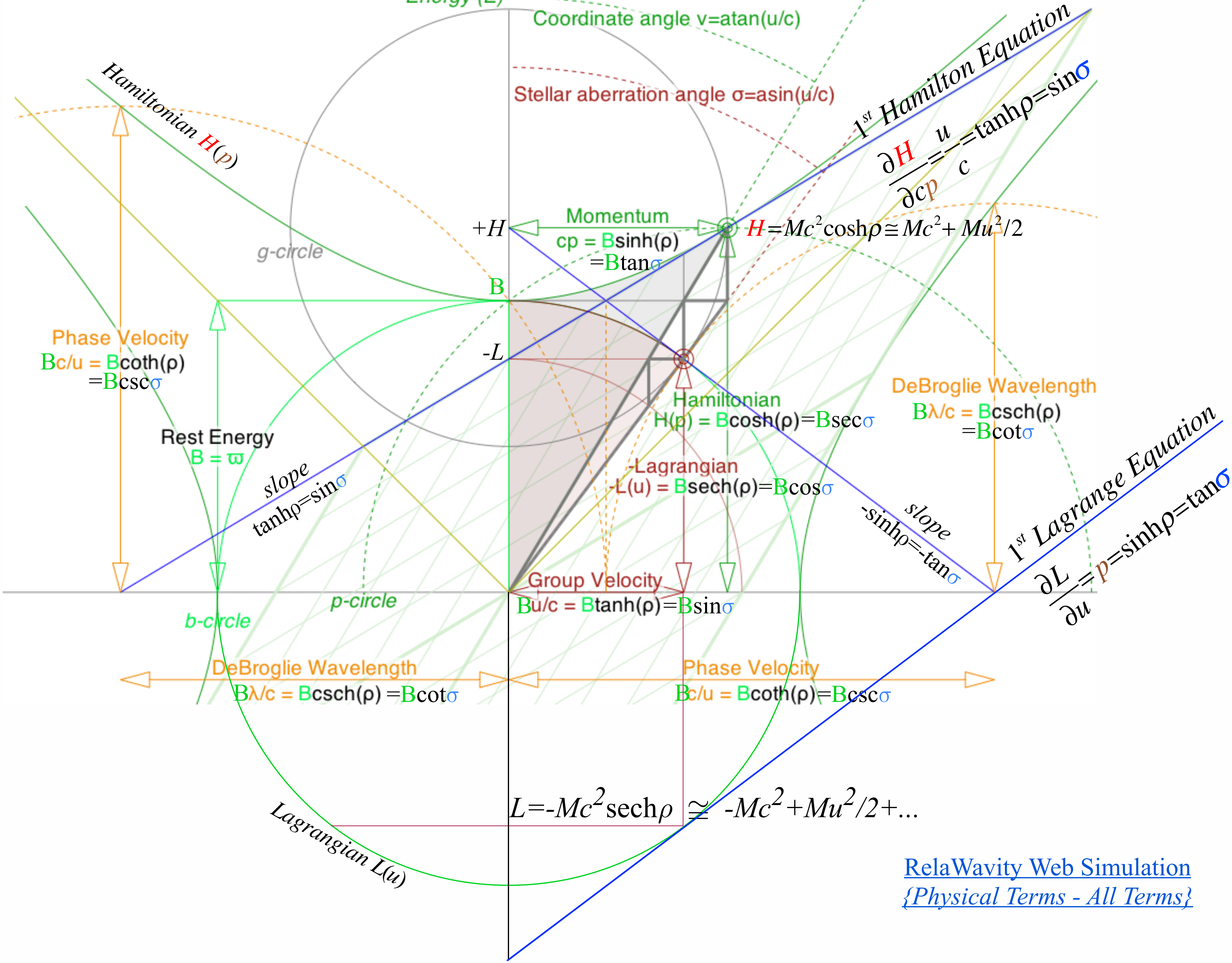
angular phasor format→

$\hbar\omega_A=Mc^2=\hbar ck_A$

$\hbar\omega_{phase}=E=\hbar\omega_A\cosh\rho$

$\hbar ck_{phase}=cp=\hbar\omega_A\sinh\rho$

$$\hbar\equiv\frac{h}{2\pi}$$



RelaWavity Web Simulation

{Physical Terms - All Terms}

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - ➔ Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

Relativistic $\boxed{\text{action } S}$ and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega' t'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$\boxed{\frac{dS}{dt}=L}\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv\boxed{pu-H=L \qquad \textit{Legendre transformation}}$$

Compare *Lagrangian L*

$$\boxed{\dot{S}=L=\hbar\dot{\Phi}}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho=-Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S=\hbar\Phi$

Prior wave relations

$$h\upsilon_A=Mc^2=hc\kappa_A$$
$$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$$
$$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A=Mc^2=\hbar ck_A$$
$$\hbar\omega_{phase}=E=\hbar\omega_A\cosh\rho$$
$$\hbar ck_{phase}=cp=\hbar\omega_A\sinh\rho$$

$$\hbar\equiv\frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega' t'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$\boxed{\frac{dS}{dt}=L}\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv\boxed{pu-H=L}$$

Legendre transformation

$$\boxed{dS\equiv L\,dt\equiv\hbar\,d\Phi}=\hbar k\,dx-\hbar\omega\,dt=p\,dx-H\,dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

$$\boxed{\dot{S}=L=\hbar\dot{\Phi}}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho=-Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S=\hbar\Phi$

Prior wave relations

$$h\upsilon_A=Mc^2=hc\kappa_A$$
$$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$$
$$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$$

←linear Hz format

angular phasor→ format

$$\hbar\omega_A=Mc^2=\hbar ck_A$$
$$\hbar\omega_{phase}=E=\hbar\omega_A\cosh\rho$$
$$\hbar ck_{phase}=cp=\hbar\omega_A\sinh\rho$$

$$\hbar\equiv\frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi=kx-\omega t=k'x'-\omega't'$ for wave of $k=k_{phase}$ and $\omega=\omega_{phase}$.

Use DeBroglie-momentum $p=\hbar k$ relation and Planck-energy $E=\hbar\omega$ relation to define *Hamiltonian* $H=E$

$$\boxed{\frac{dS}{dt}=L}\equiv\hbar\frac{d\Phi}{dt}=\hbar k\frac{dx}{dt}-\hbar\omega=p\frac{dx}{dt}-E\equiv p\dot{x}-E\equiv\boxed{pu-H=L}$$

Legendre transformation

Use *Group velocity* : $u=\frac{dx}{dt}=c\tanh\rho$

$$\boxed{dS\equiv Ldt\equiv\hbar\, d\Phi}=\hbar k\,dx-\hbar\omega\,dt=p\,dx-H\,dt$$

Poincare Invariant action differential

$$\frac{\partial S}{\partial x}=p\qquad\qquad\frac{\partial S}{\partial t}=-H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

$$\boxed{\dot{S}=L=\hbar\dot{\Phi}}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho=-Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H=\hbar\omega=Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S=\hbar\Phi$

Prior wave relations

$h\upsilon_A=Mc^2=hc\kappa_A$
$h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$
$hc\kappa_{phase}=cp=h\upsilon_A\sinh\rho$

←linear Hz format

angular phasor format→

$\hbar\omega_A=Mc^2=\hbar ck_A$
$\hbar\omega_{phase}=E=\hbar\omega_A\cosh\rho$
$\hbar ck_{phase}=cp=\hbar\omega_A\sinh\rho$

$$\hbar\equiv\frac{h}{2\pi}$$

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

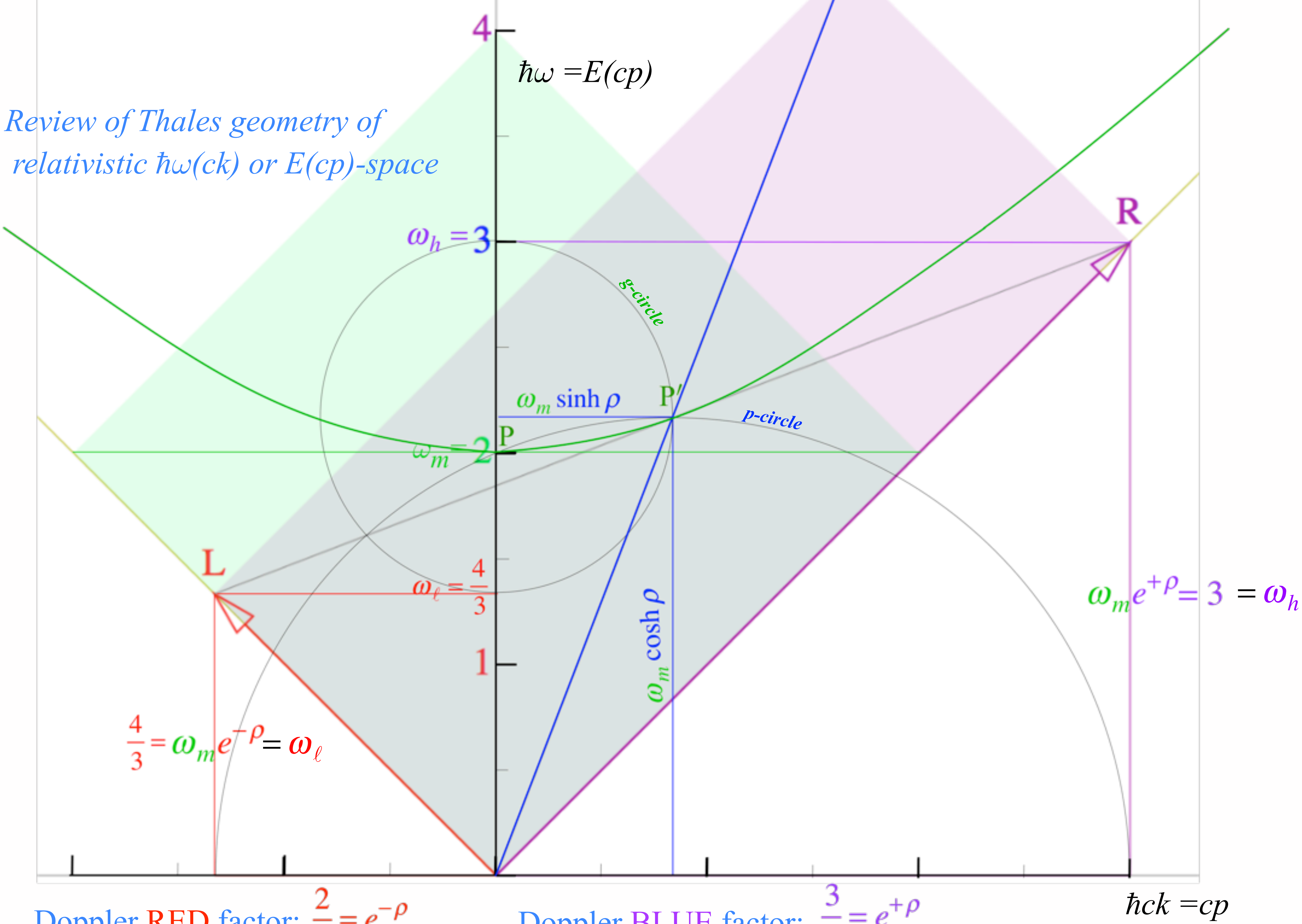
➔ Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

Relativistic optical transitions

$$|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

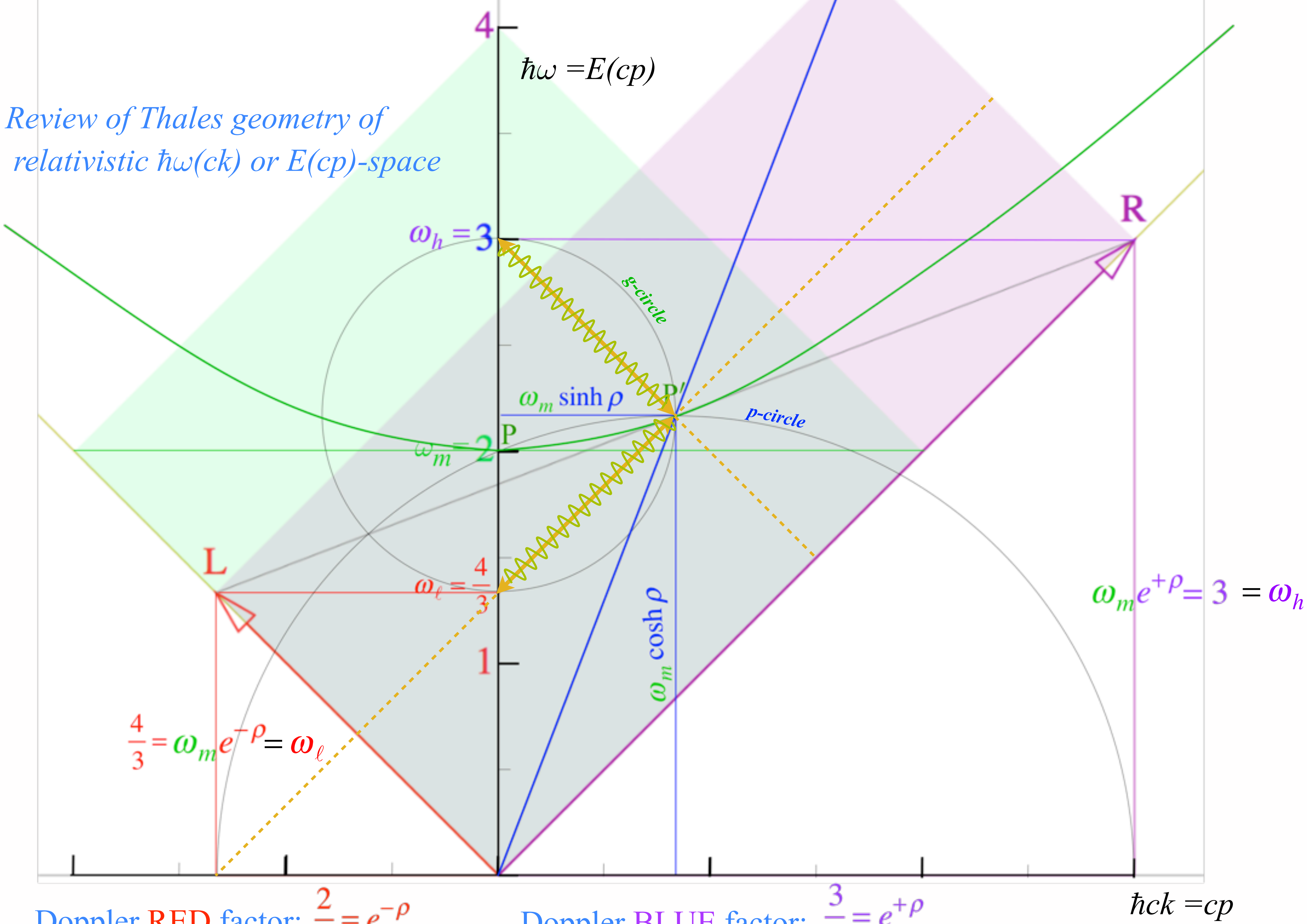


Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

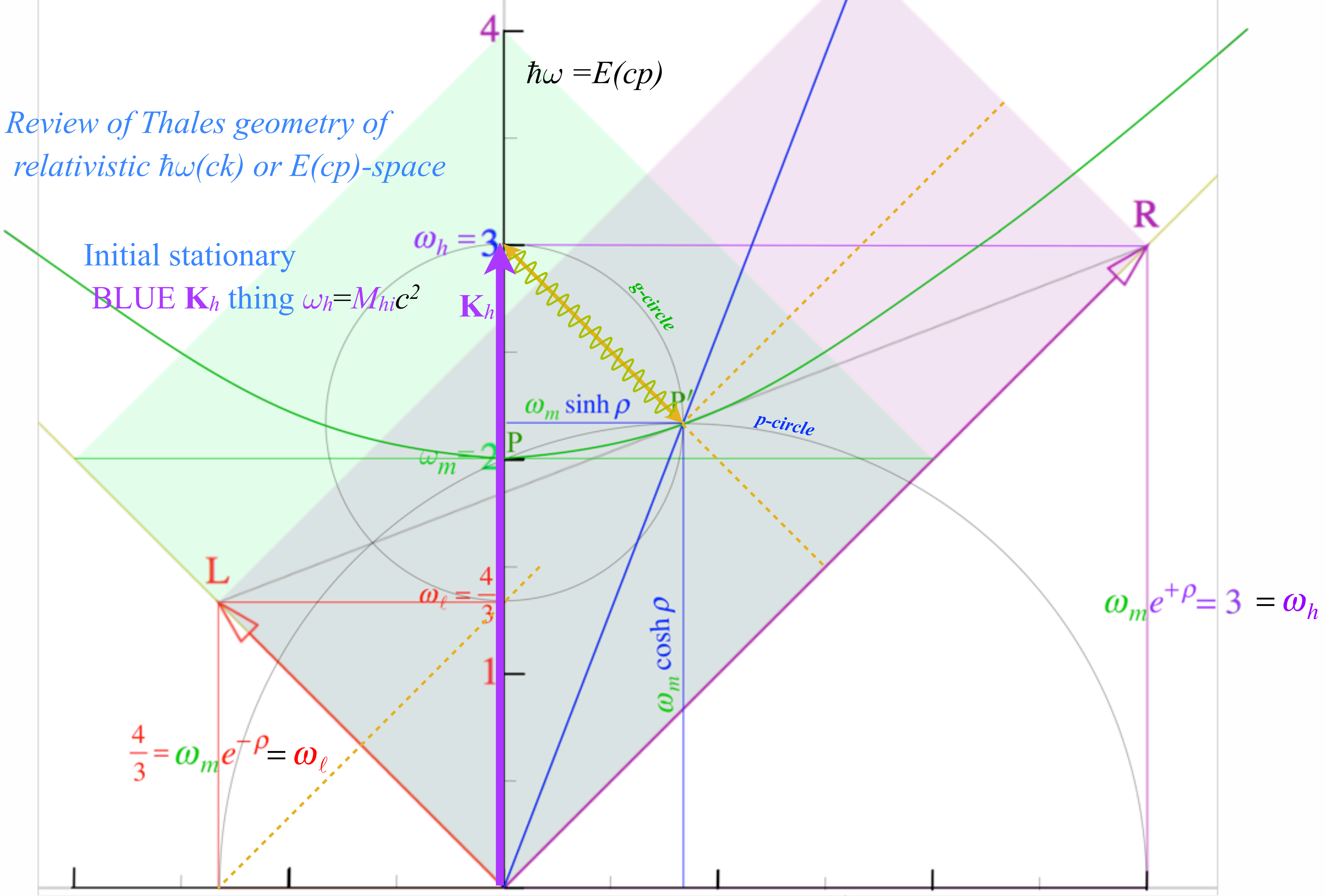


Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



Initial stationary
BLUE $\mathbf{K}_h$ thing $\omega_h = M_{hi}c^2$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

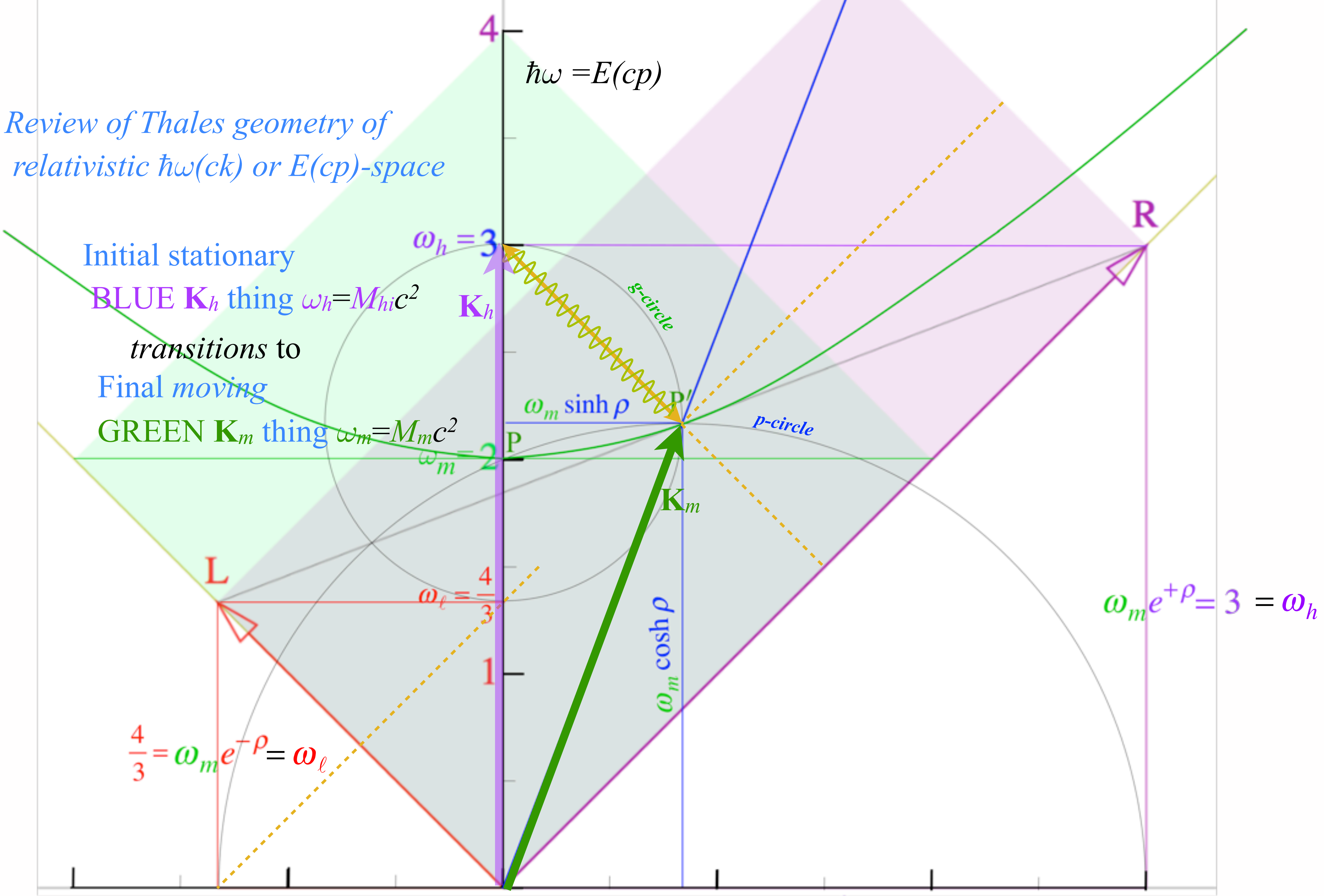
Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

Initial stationary BLUE $\mathbf{K}_h$ thing $\omega_h=M_{hi}c^2$

transitions to

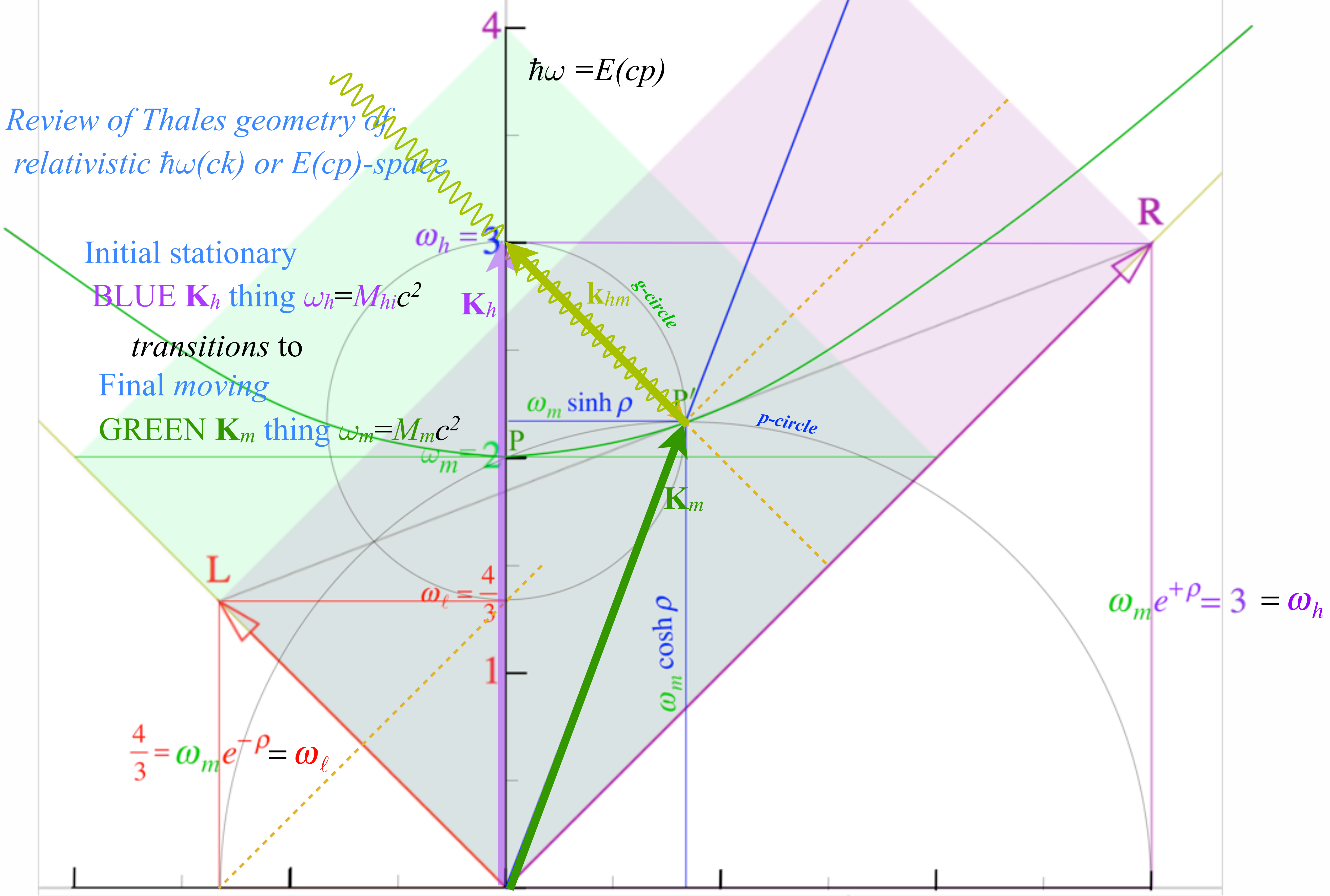
Final *moving* GREEN $\mathbf{K}_m$ thing $\omega_m=M_mc^2$



Doppler RED factor: $\frac{2}{3}=e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2}=e^{+\rho}$

Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$



Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

Initial stationary

BLUE $\mathbf{K}_h$ thing $\omega_h = M_{hi}c^2$

transitions to

Final *moving*

GREEN $\mathbf{K}_m$ thing $\omega_m = M_m c^2$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

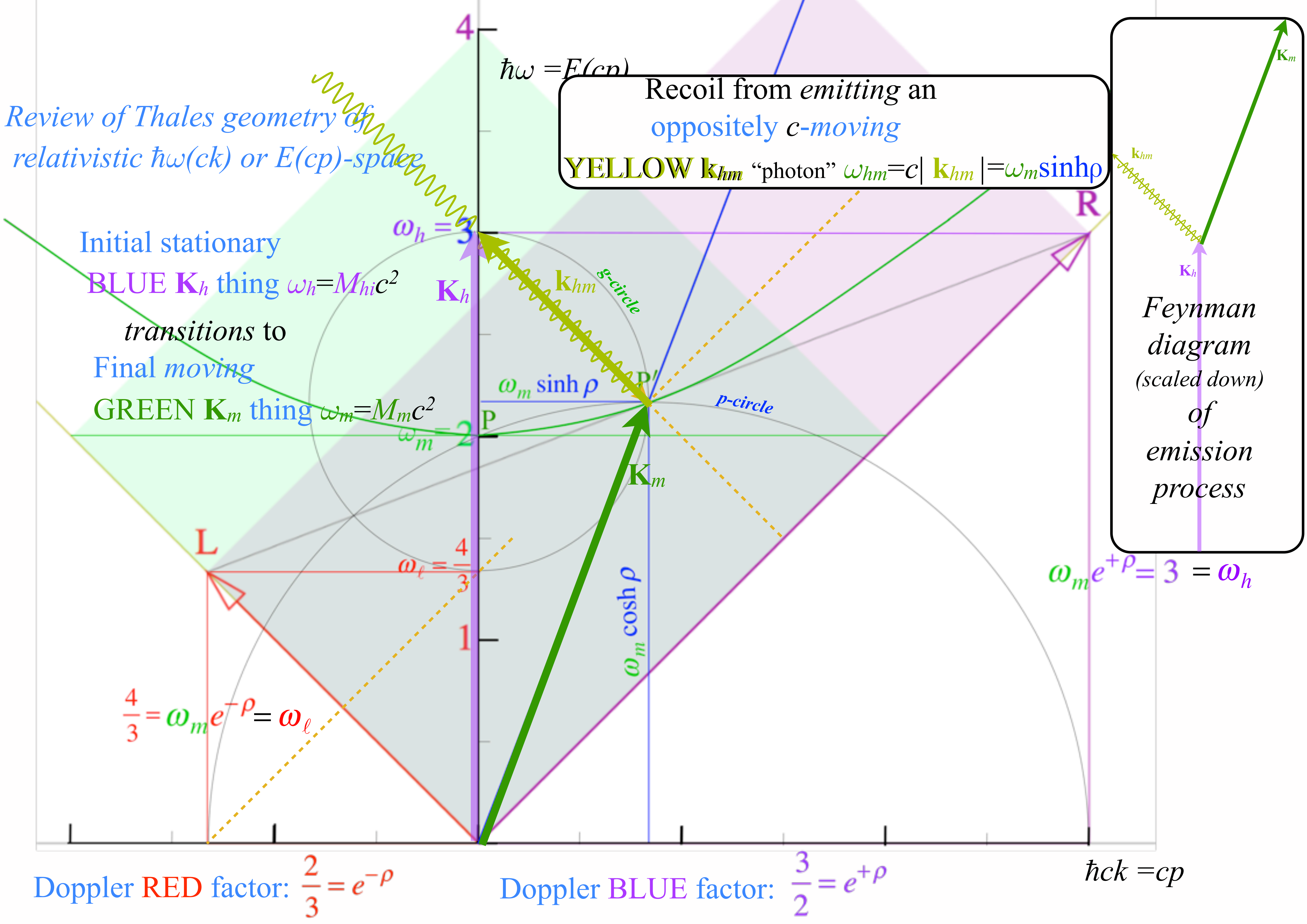
Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
What's the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
➔ Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$



Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

Recoil from *emitting* an oppositely *c-moving* YELLOW $\mathbf{k}_{hm}$ "photon" $\omega_{hm}=c|\mathbf{k}_{hm}|=\omega_m\sinh\rho$

Initial stationary BLUE $\mathbf{K}_h$ thing $\omega_h=M_{hi}c^2$ *transitions* to Final *moving* GREEN $\mathbf{K}_m$ thing $\omega_m=M_mc^2$

Feynman diagram (scaled down) of emission process

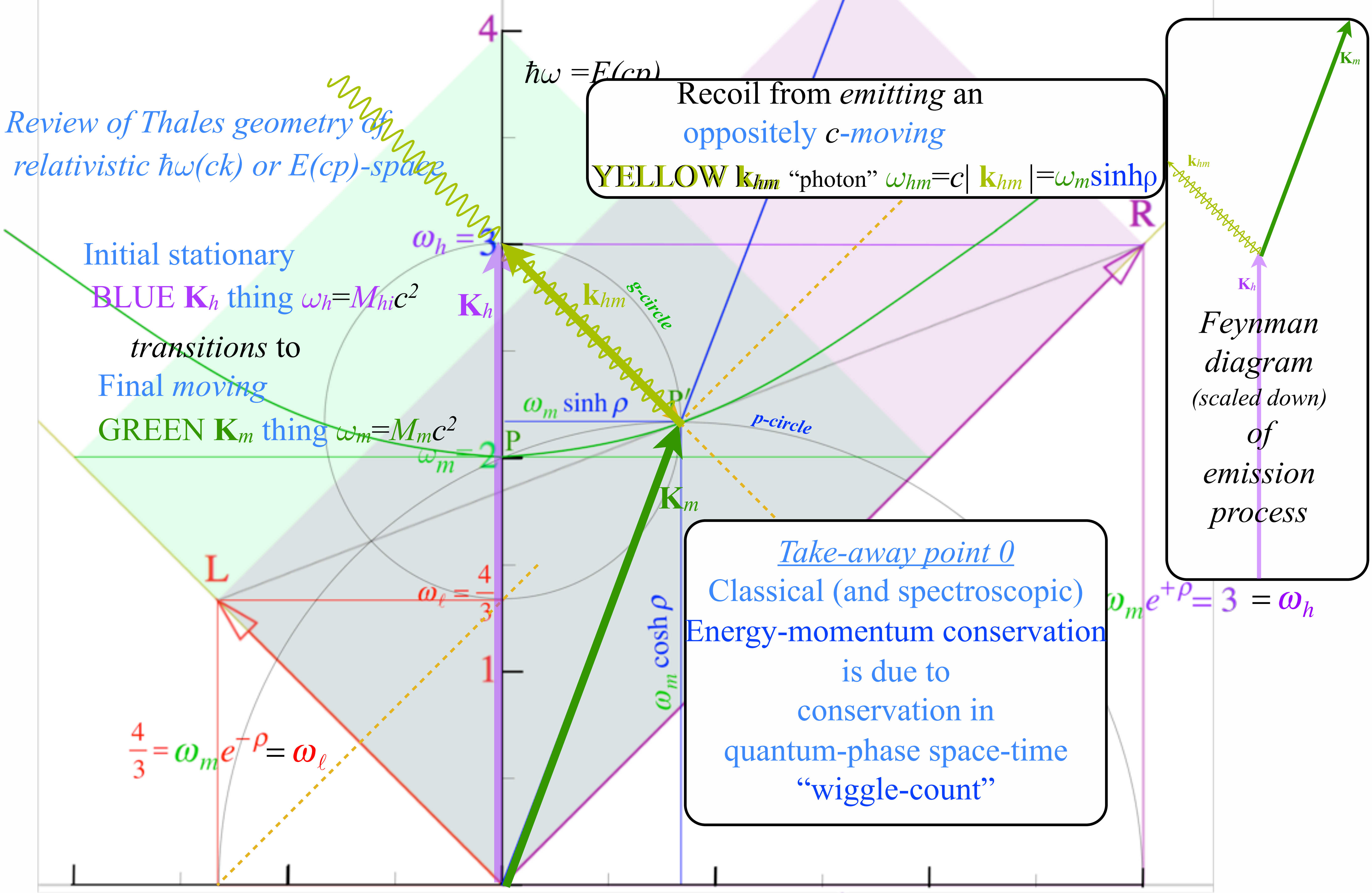
$\omega_m e^{+\rho} = 3 = \omega_h$

$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$



Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

Initial stationary
BLUE $\mathbf{K}_h$ thing $\omega_h=M_{hi}c^2$
transitions to
Final *moving*
GREEN $\mathbf{K}_m$ thing $\omega_m=M_m c^2$

Recoil from *emitting* an
oppositely *c-moving*
YELLOW $\mathbf{k}_{hm}$ "photon" $\omega_{hm}=c|\mathbf{k}_{hm}|=\omega_m \sinh\rho$

Take-away point 0
Classical (and spectroscopic)
Energy-momentum conservation
is due to
conservation in
quantum-phase space-time
"wiggle-count"

Feynman diagram
(scaled down)
of
emission
process

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation

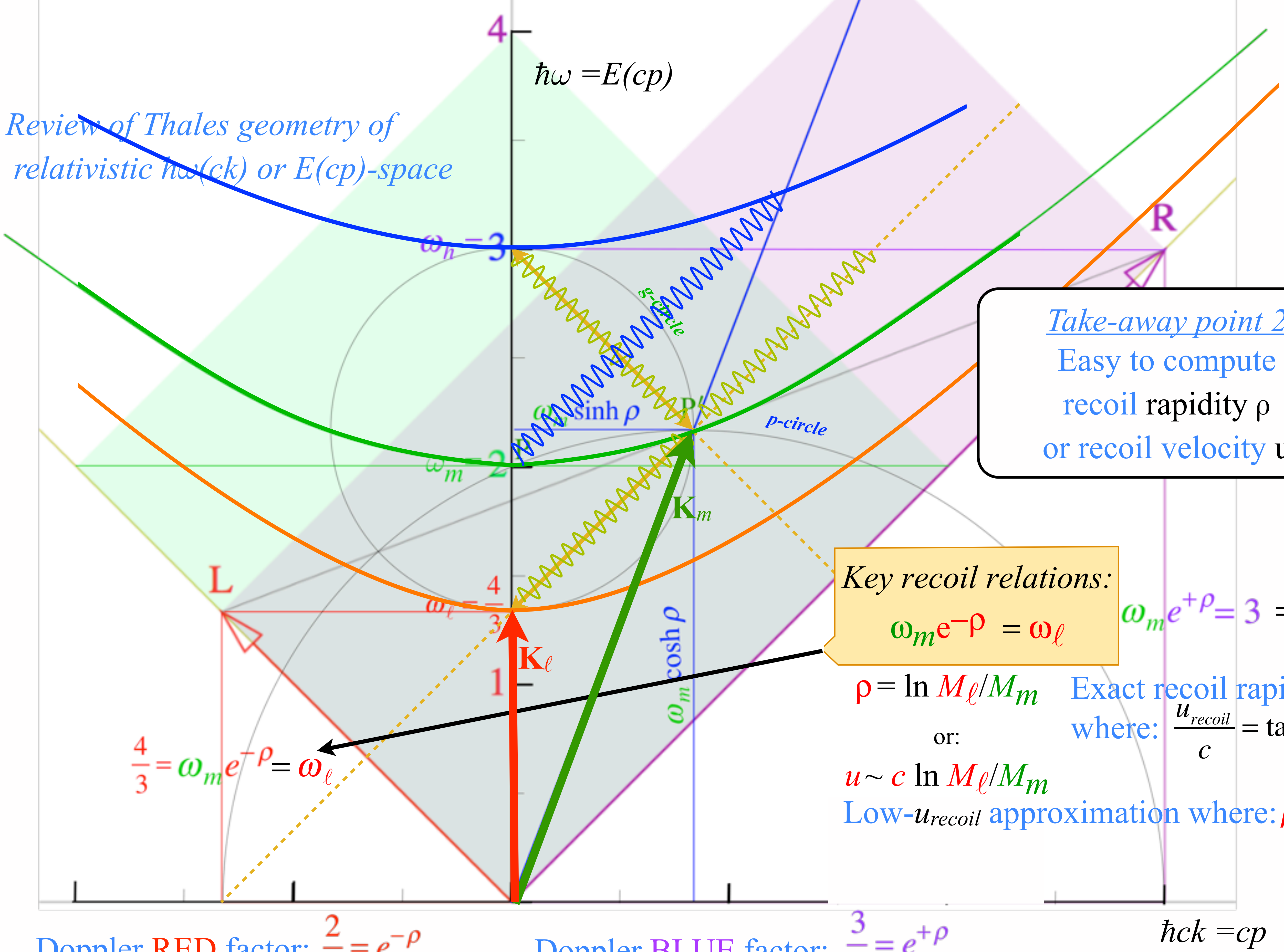
Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - ➔ Compton recoil related to rocket velocity formula
  - Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



Take-away point 2
Easy to compute
recoil rapidity ρ
or recoil velocity u

Key recoil relations:

$$\omega_m e^{-\rho} = \omega_\ell$$

$$\omega_m e^{+\rho} = 3 = \omega_h$$

$$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$$

$\rho = \ln M_\ell / M_m$

or:

$u \sim c \ln M_\ell / M_m$

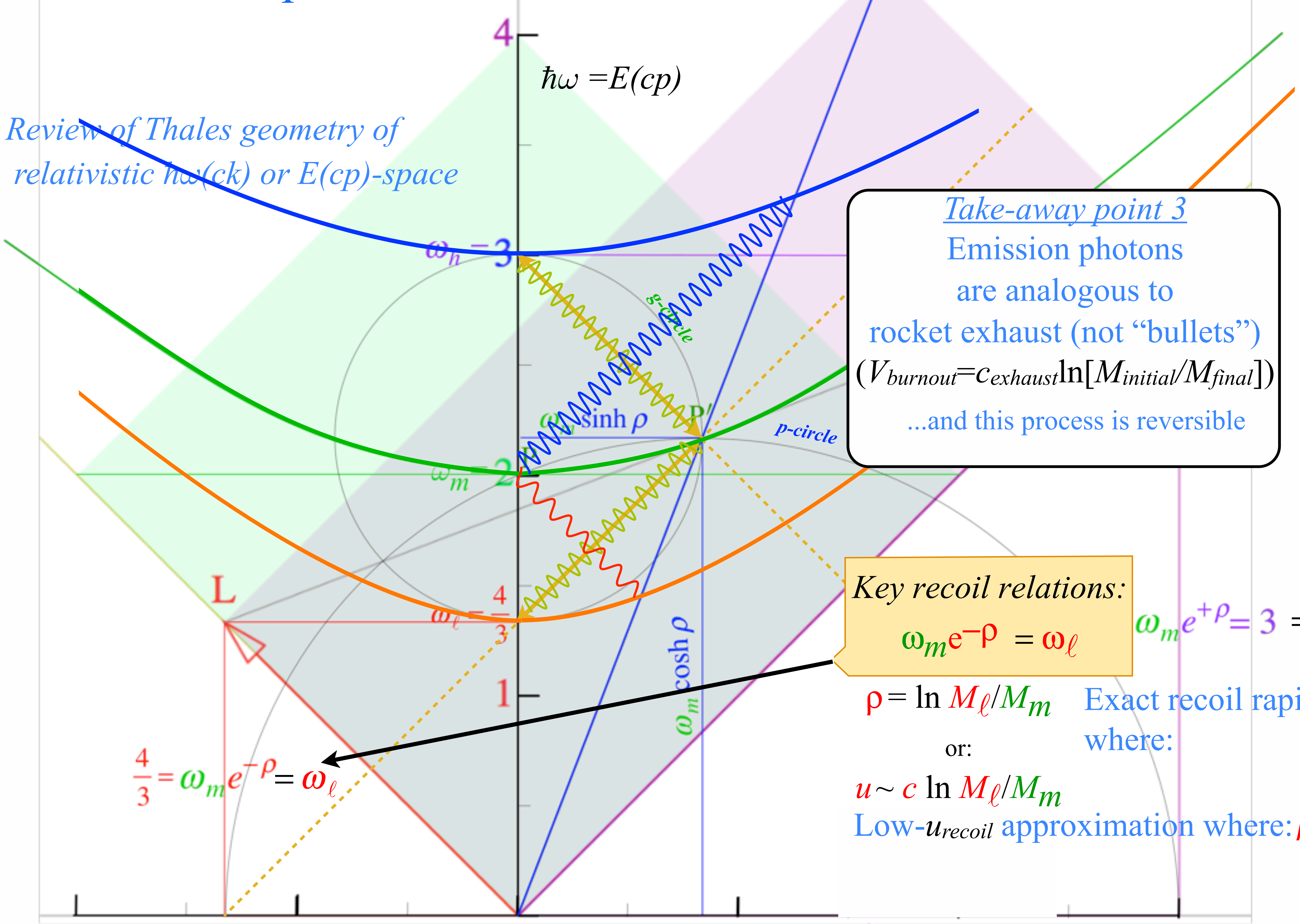
Exact recoil rapidity where: $\frac{u_{recoil}}{c} = \tanh\rho$

Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$



Review of Thales geometry of relativistic ħω(ck) or E(cp)-space

Take-away point 3

Emission photons are analogous to rocket exhaust (not "bullets")

$(V_{burnout}=c_{exhaust}\ln[M_{initial}/M_{final}])$

...and this process is reversible

Key recoil relations:

$\omega_m e^{-\rho} = \omega_\ell$

$\omega_m e^{+\rho} = 3 = \omega_h$

$\rho = \ln M_\ell / M_m$

Exact recoil rapidity where:

or:

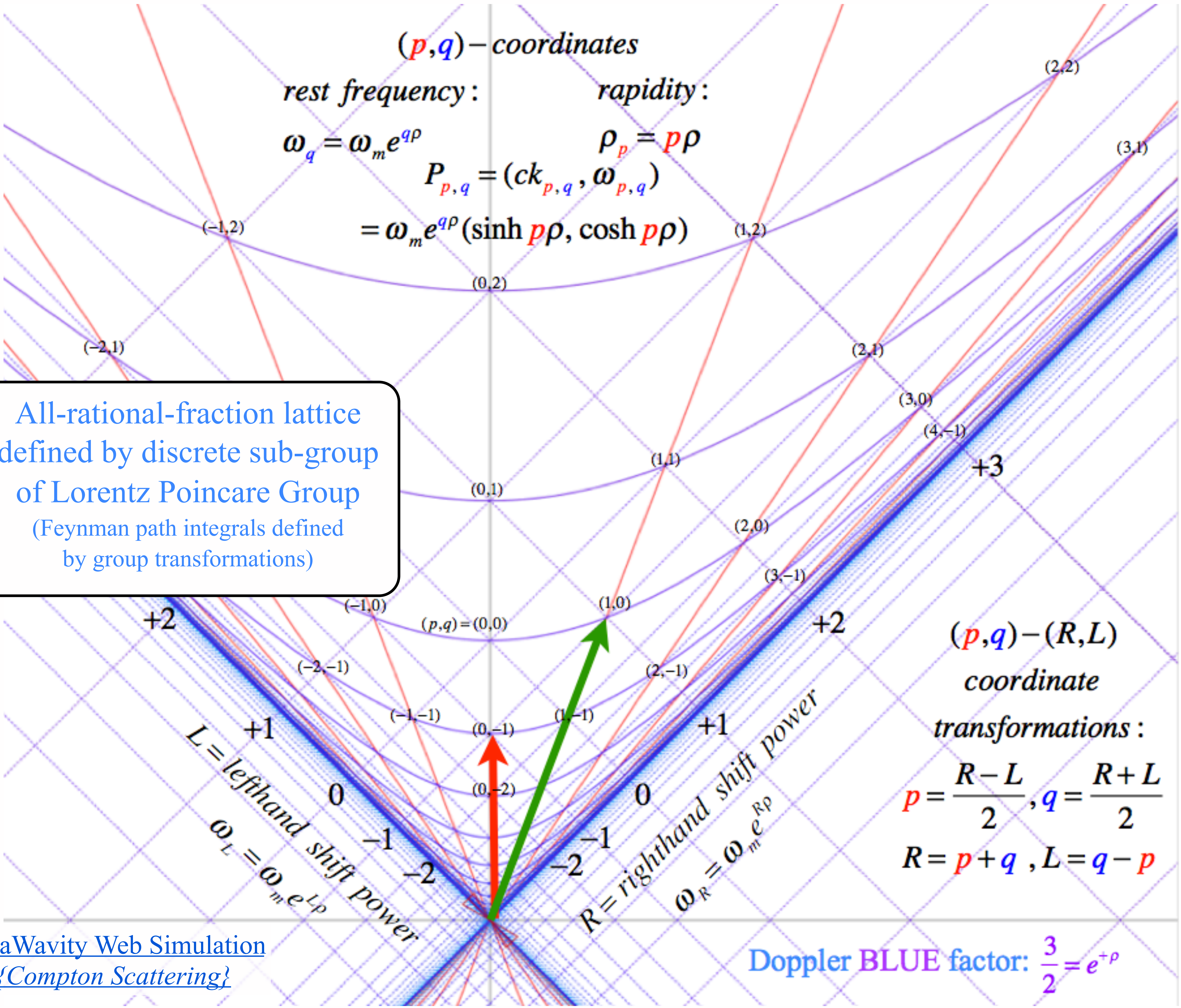
$u \sim c \ln M_\ell / M_m$

Low-u_{recoil} approximation where: $\rho \approx \frac{u_{recoil}}{c}$

$\frac{4}{3} = \omega_m e^{-\rho} = \omega_\ell$

Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$



$(p,q) -$ *coordinates*

rest frequency: $\omega_q = \omega_m e^{q\rho}$

rapidity: $\rho_p = p\rho$

$$P_{p,q} = (ck_{p,q}, \omega_{p,q})$$
$$= \omega_m e^{q\rho}(\sinh p\rho, \cosh p\rho)$$

All-rational-fraction lattice defined by discrete sub-group of Lorentz Poincare Group

(Feynman path integrals defined by group transformations)

$L =$ *lefthand shift power*

$\omega_L = \omega_m e^{L\rho}$

$R =$ *righthand shift power*

$\omega_R = \omega_m e^{R\rho}$

$(p,q) - (R,L)$ *coordinate transformations:*

$$p = \frac{R-L}{2}, q = \frac{R+L}{2}$$
$$R = p + q, L = q - p$$

RelaWavity Web Simulation
{Compton Scattering}

Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
What's the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
➔ Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

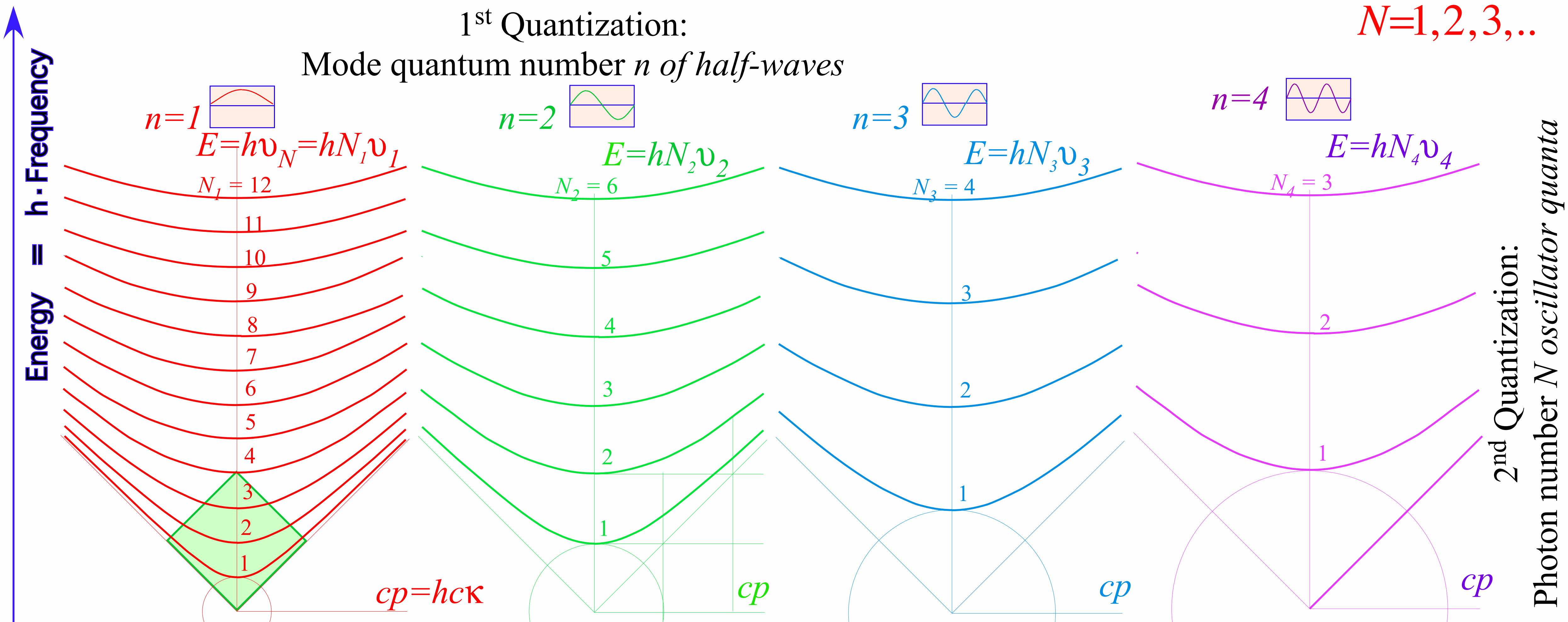
Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

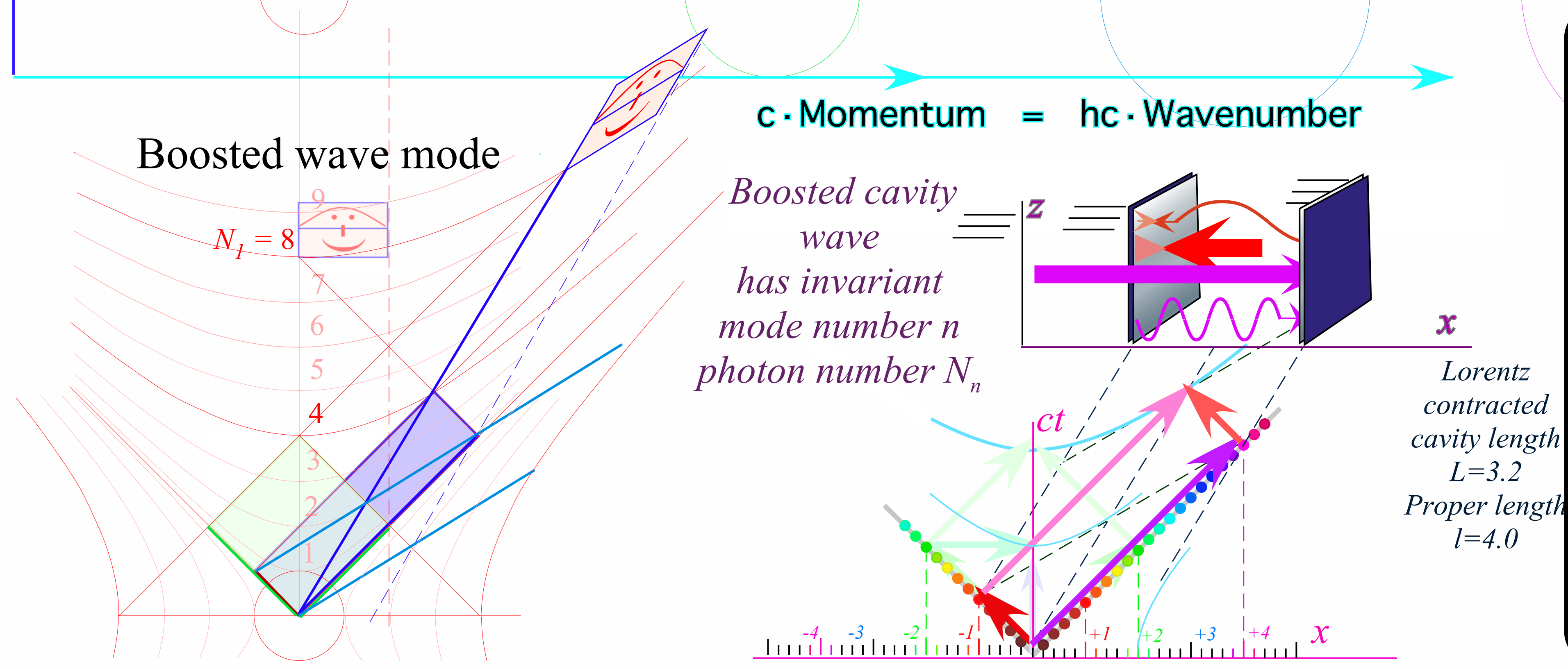
2nd Quantization:

$h\upsilon$ *is actually* $hN\upsilon$

($h\upsilon_{phase}=E=h\upsilon_A\cosh\rho$) *is actually* ($hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho$ with *quantum* numbers)

N=1,2,3,..





Boosted cavity wave has invariant mode number n photon number N_n

Lorentz contracted cavity length L=3.2 Proper length l=4.0

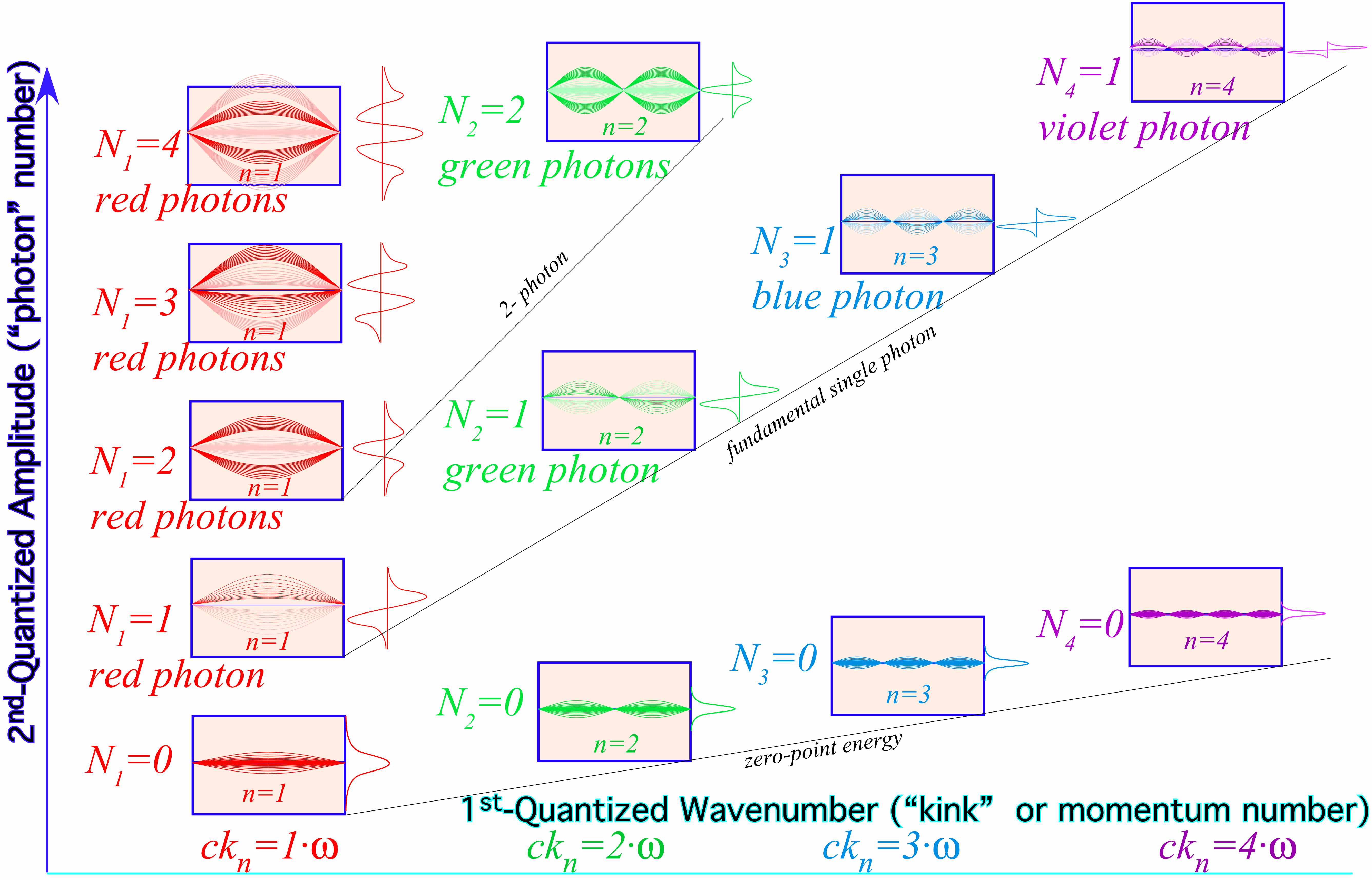
Take-away point 4

Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2nd Quantization:

hυ is actually hNυ

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ *is actually* $(h N\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho \quad (N=1,2,..))$



Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation
    - *Learning about* sin! *and* cos *and...*

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
- ➔ Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

Acceleration by chirping laser pairs

Varying acceleration (General case)

From Lect. 35 ModPhys (2012)

Varying local acceleration $\rho = \rho(\tau)$

$$u = \frac{dx}{dt} = c\tanh(\tau)$$

$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

$$ct = c\int \cosh\rho(\tau)\,d\tau \qquad x = c\int \sinh\rho(\tau)\,d\tau$$

Constant local acceleration $\rho = \frac{g\tau}{c}$ *"Einstein Elevator"*

$$ct = c\int \cosh\frac{g\tau}{c}\,d\tau \qquad x = c\int \sinh\frac{g\tau}{c}\,d\tau$$

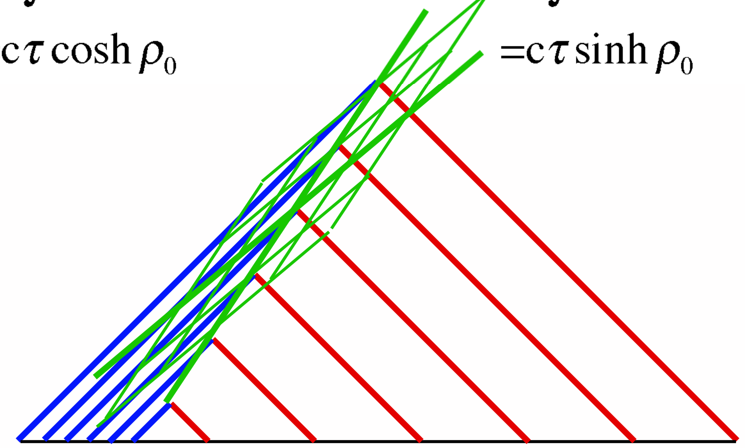
$$= \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$

Previous examples involved constant velocity

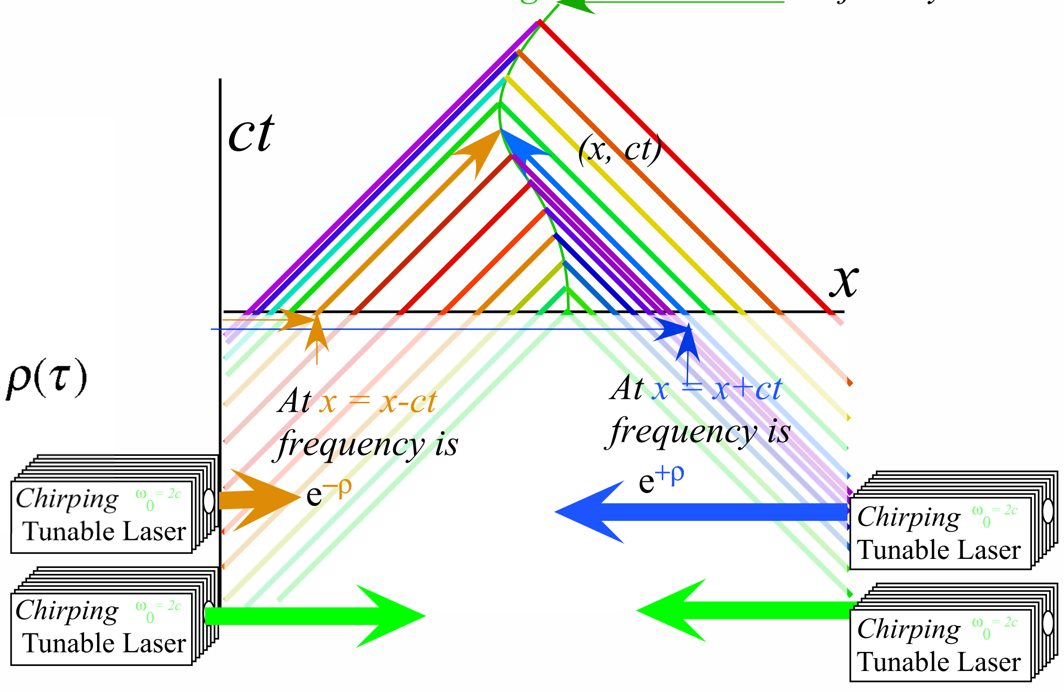
Constant velocity $\rho = \rho_0 = const.$ *"Lorentztransformation"*

$$ct = c\int \cosh\rho_0\,d\tau \qquad x = c\int \sinh\rho_0\,d\tau$$

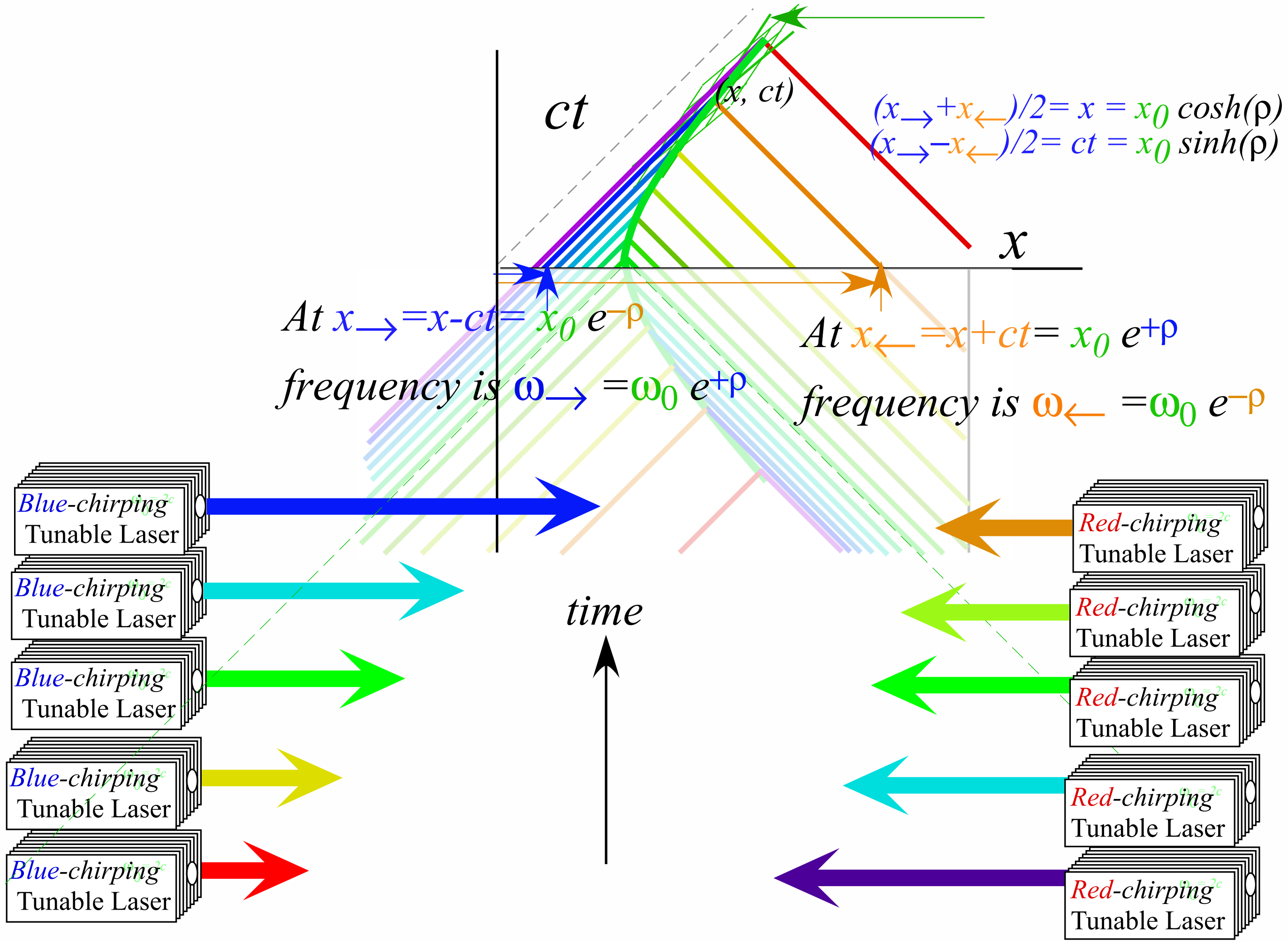
$$= c\tau\cosh\rho_0 \qquad = c\tau\sinh\rho_0$$



Only green-light is seen by observers on the green accelerated trajectory



Only green-light is seen by observers on the green constant-g hyberbola



$(x_{\rightarrow}+x_{\leftarrow})/2 = x = x_0\cosh(\rho)$

$(x_{\rightarrow}-x_{\leftarrow})/2 = ct = x_0\sinh(\rho)$

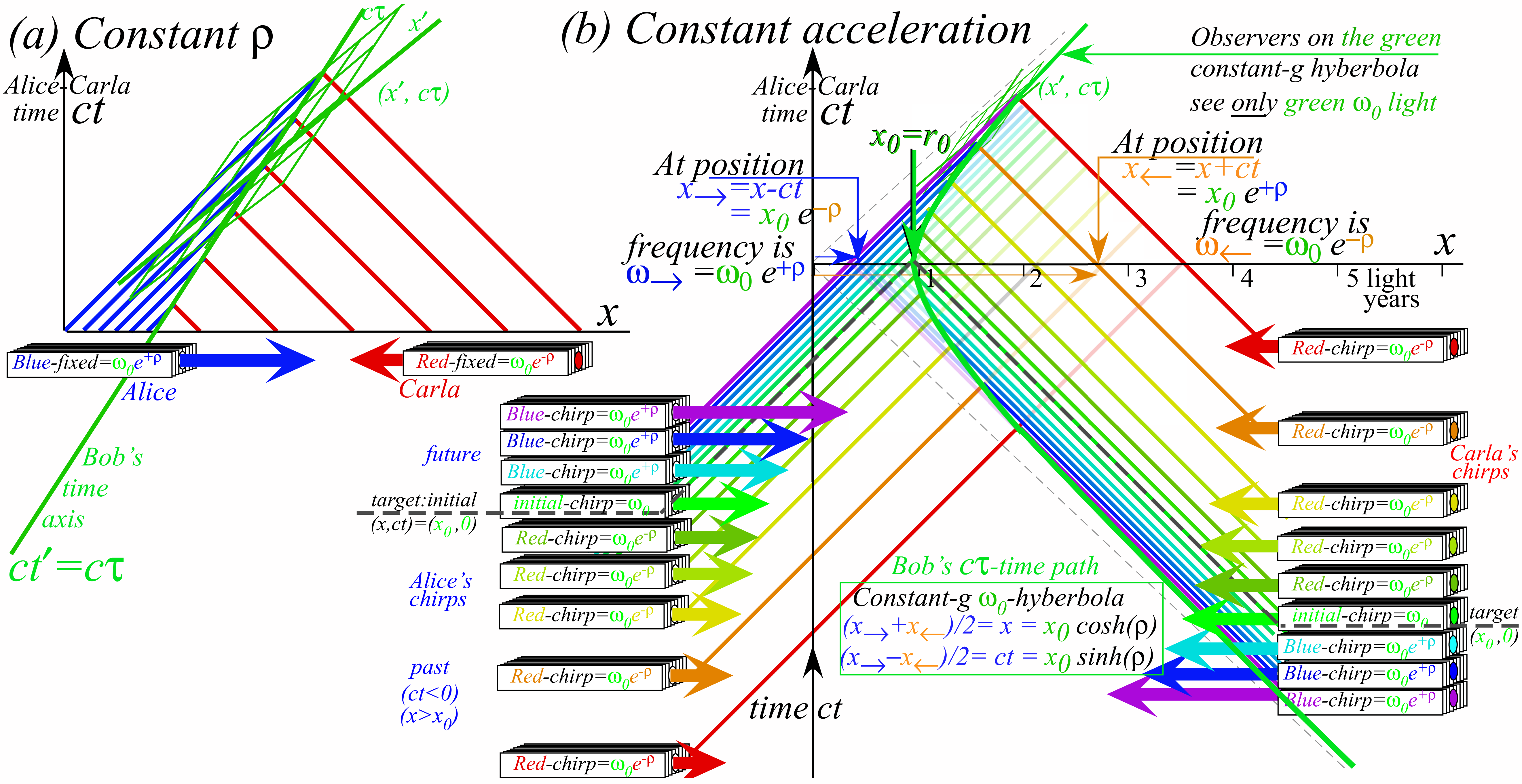
At $x_{\rightarrow} = x-ct = x_0\,e^{-\rho}$

frequency is $\omega_{\rightarrow} = \omega_0\,e^{+\rho}$

At $x_{\leftarrow} = x+ct = x_0\,e^{+\rho}$

frequency is $\omega_{\leftarrow} = \omega_0\,e^{-\rho}$

Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

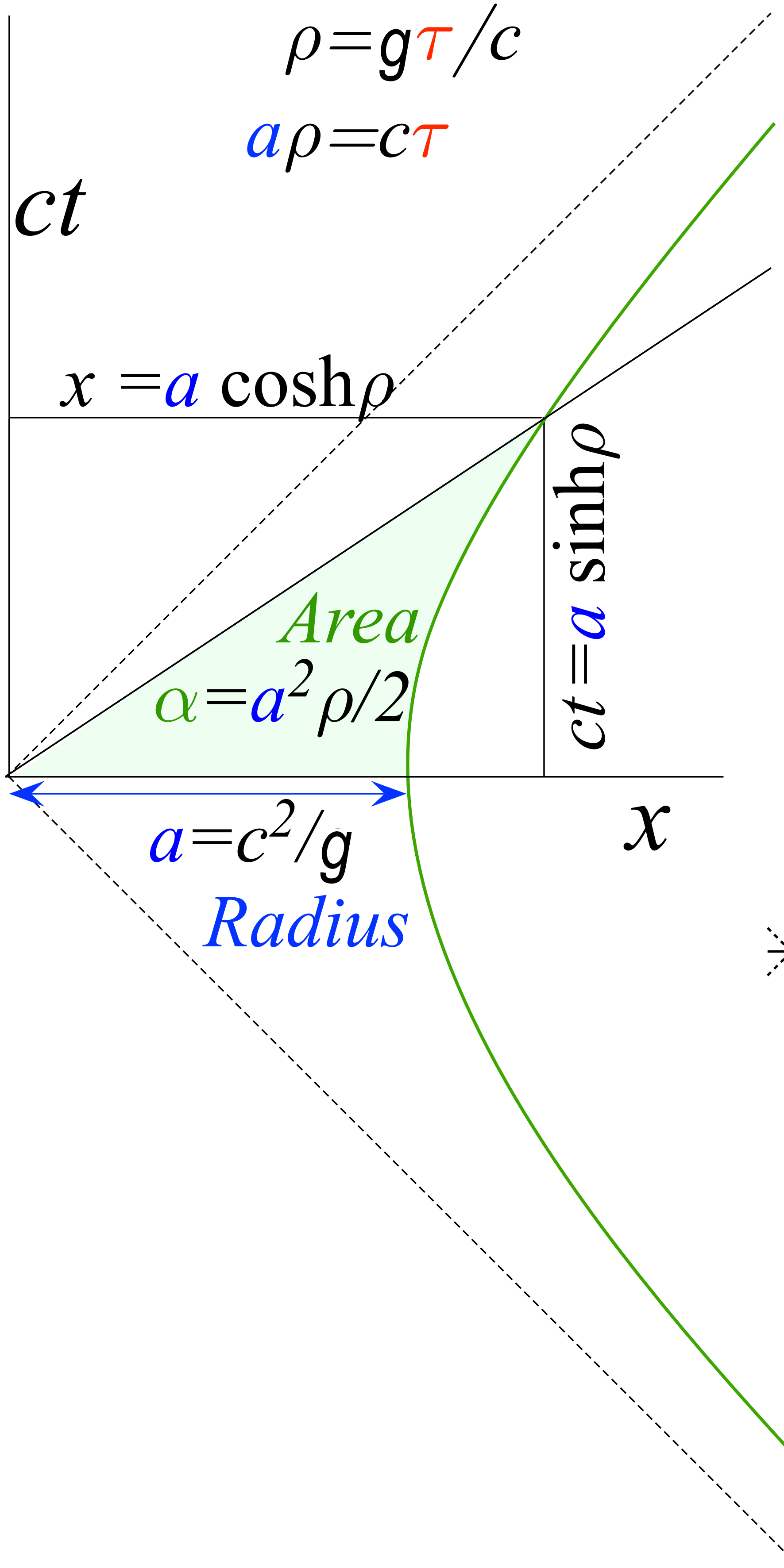

(a) Constant ρ
Alice-Carla time ct
cτ
x′
(x′, cτ)
x
Blue-fixed=$\omega_0 e^{+\rho}$
Alice
Red-fixed=$\omega_0 e^{-\rho}$
Carla
Bob's time axis
$ct'=c\tau$
(b) Constant acceleration
Observers on the green constant-g hyberbola see only green ω_0 light
Alice-Carla time ct
(x′, cτ)
$x_0=r_0$
At position $x_{\rightarrow}=x-ct = x_0 e^{-\rho}$
frequency is $\omega_{\rightarrow}=\omega_0 e^{+\rho}$
At position $x_{\leftarrow}=x+ct = x_0 e^{+\rho}$
frequency is $\omega_{\leftarrow}=\omega_0 e^{-\rho}$
x
1
2
3
4
5 light years
Blue-chirp=$\omega_0 e^{+\rho}$
Blue-chirp=$\omega_0 e^{+\rho}$
Blue-chirp=$\omega_0 e^{+\rho}$
future
target:initial (x,ct)=(x_0,0)
initial-chirp=ω_0
Red-chirp=$\omega_0 e^{-\rho}$
Red-chirp=$\omega_0 e^{-\rho}$
Alice's chirps
Red-chirp=$\omega_0 e^{-\rho}$
past (ct<0) (x>x_0)
Red-chirp=$\omega_0 e^{-\rho}$
Red-chirp=$\omega_0 e^{-\rho}$
time ct
Red-chirp=$\omega_0 e^{-\rho}$
Red-chirp=$\omega_0 e^{-\rho}$
Carla's chirps
Red-chirp=$\omega_0 e^{-\rho}$
Red-chirp=$\omega_0 e^{-\rho}$
Red-chirp=$\omega_0 e^{-\rho}$
initial-chirp=ω_0
target (x_0,0)
Blue-chirp=$\omega_0 e^{+\rho}$
Blue-chirp=$\omega_0 e^{+\rho}$
Blue-chirp=$\omega_0 e^{+\rho}$
Bob's cτ-time path
Constant-g ω_0-hyberbola
$(x_{\rightarrow}+x_{\leftarrow})/2 = x = x_0 \cosh(\rho)$
$(x_{\rightarrow}-x_{\leftarrow})/2 = ct = x_0 \sinh(\rho)$

(a) Constant acceleration g

Rapidity ρ vs proper time τ

$$\rho = g\tau/c$$

$$a\rho = c\tau$$



(b) Traveler paths of acceleration g_q

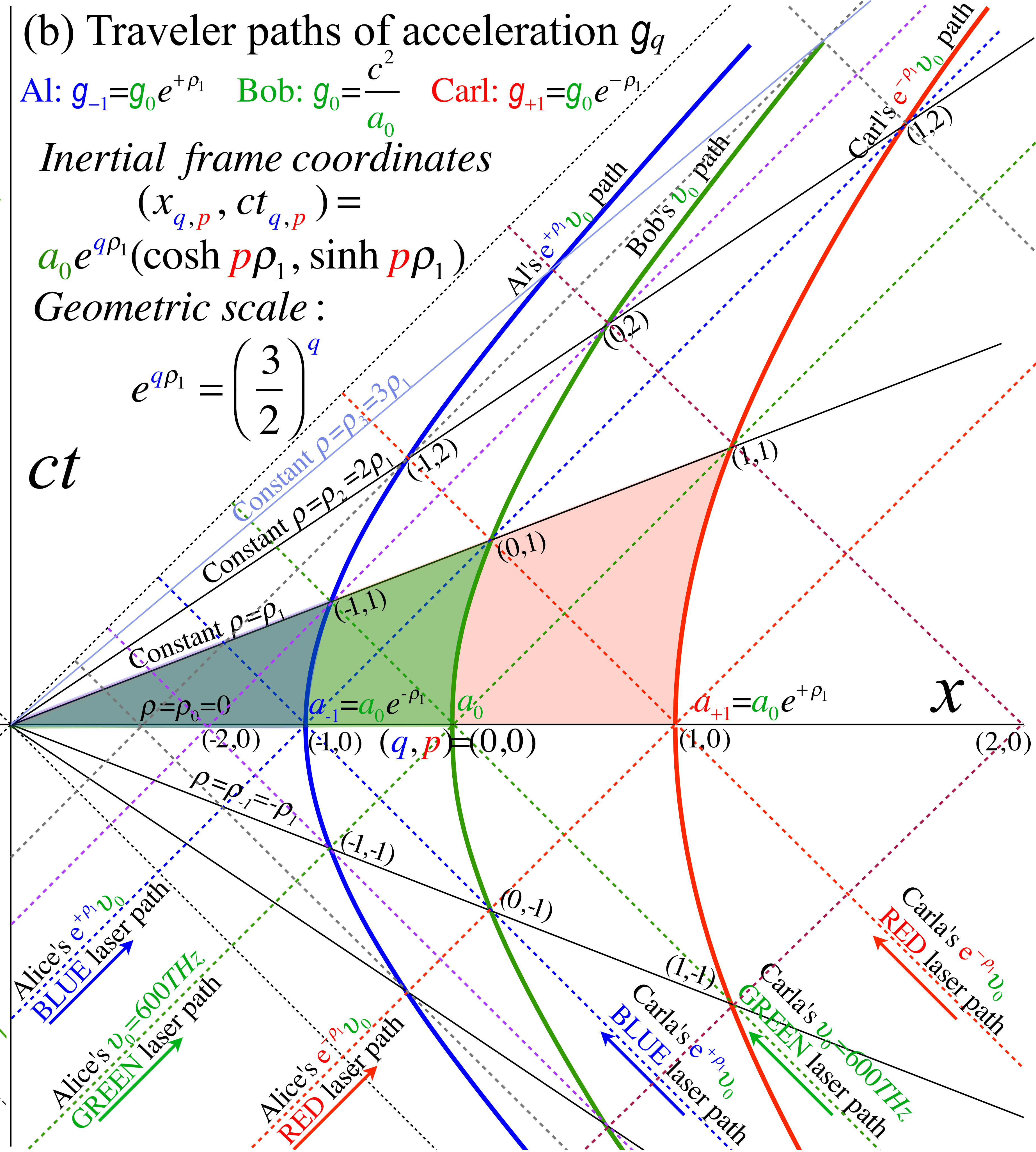
Al: $g_{-1} = g_0 e^{+\rho_1}$ Bob: $g_0 = \frac{c^2}{a_0}$ Carl: $g_{+1} = g_0 e^{-\rho_1}$

Inertial frame coordinates

$$(x_{q,p}, ct_{q,p}) = a_0 e^{q\rho_1}(\cosh p\rho_1, \sinh p\rho_1)$$

Geometric scale:

$$e^{q\rho_1} = \left(\frac{3}{2}\right)^q$$



Lecture 31
Thur. 12.10.2015

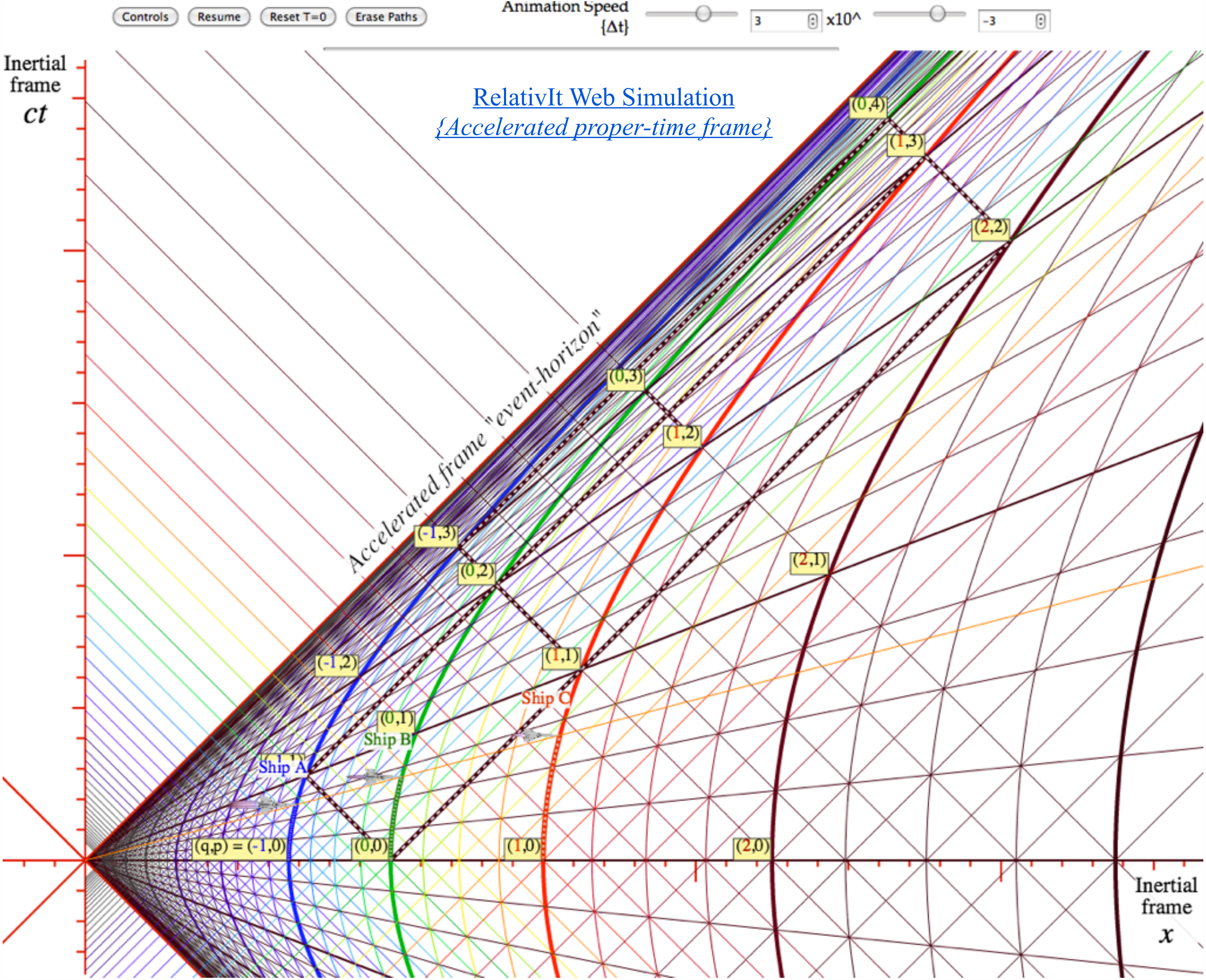
Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation
    - *Learning about* sin! *and* cos *and...*

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - ➔ Animation of mechanics and metrology of constant-g grid

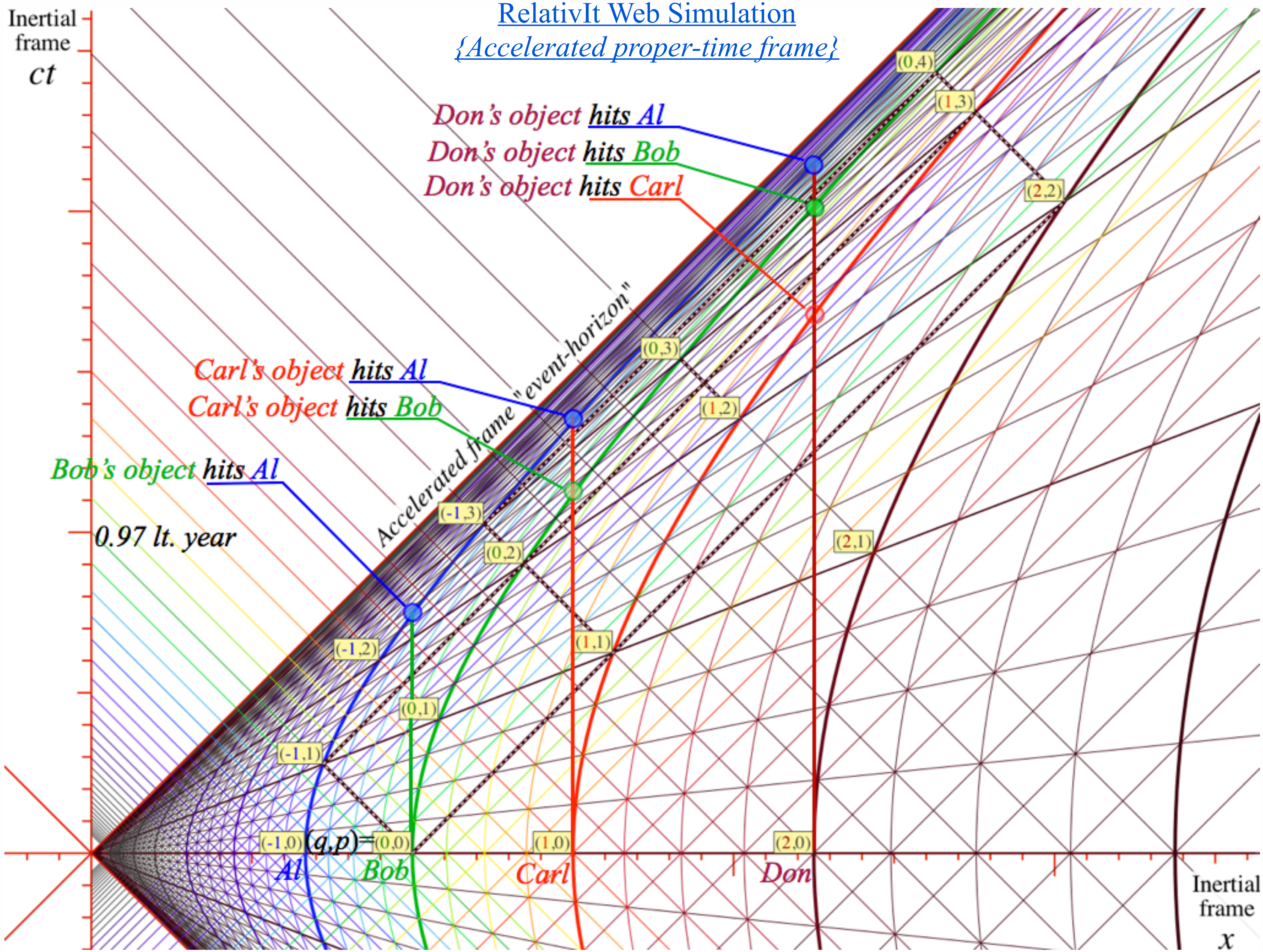


RelativIt Web Simulation

{Accelerated proper-time frame}

RelativIt Web Simulation

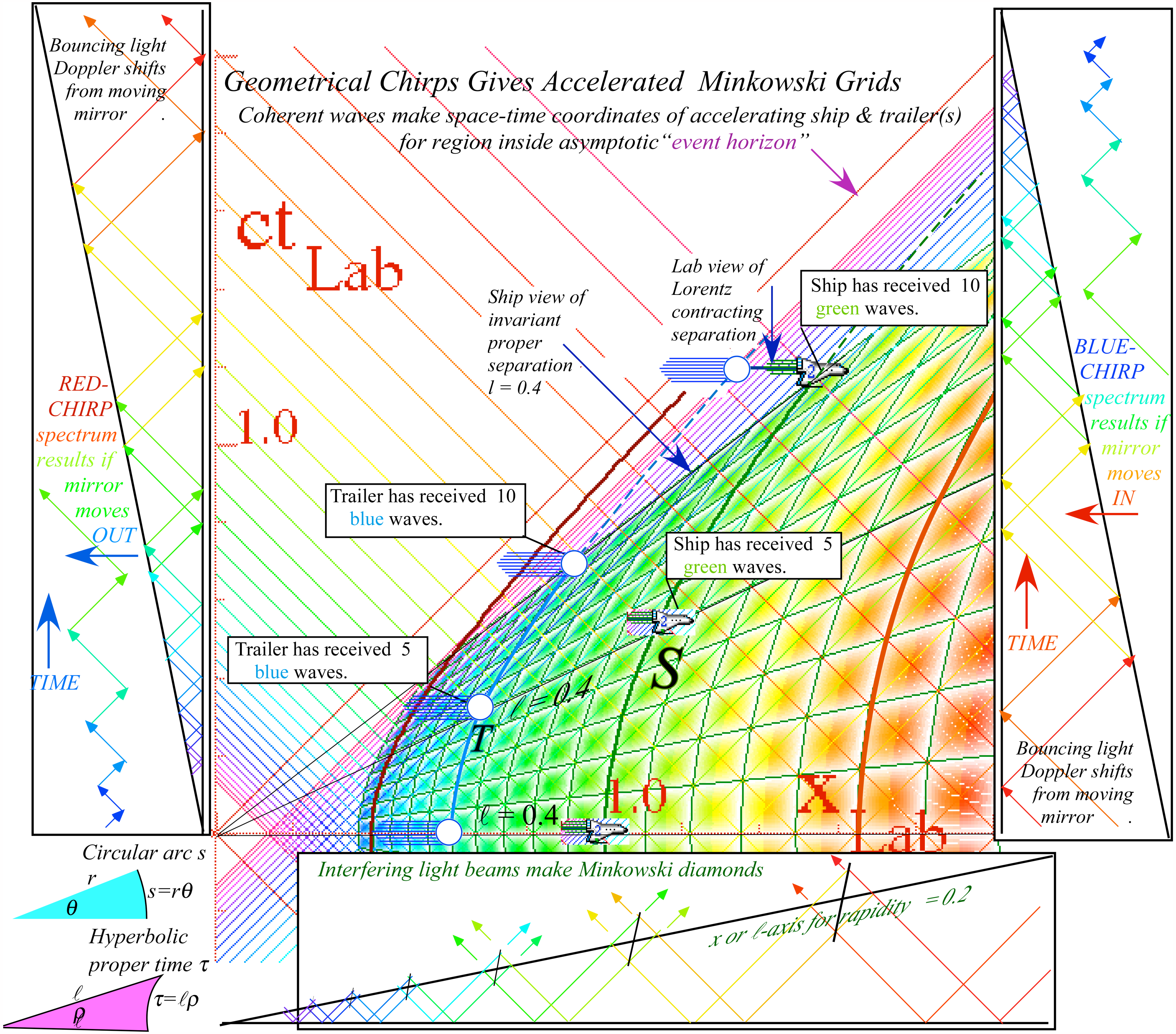
{Accelerated proper-time frame}



From Lect. 35
ModPhys (2012)

Geometrical Chirps Gives Accelerated Minkowski Grids

Coherent waves make space-time coordinates of accelerating ship & trailer(s) for region inside asymptotic "event horizon"



Bouncing light Doppler shifts from moving mirror

RED-CHIRP spectrum results if mirror moves OUT

TIME

Ship view of invariant proper separation $l = 0.4$

Lab view of Lorentz contracting separation

Ship has received 10 green waves.

Trailer has received 10 blue waves.

Ship has received 5 green waves.

Trailer has received 5 blue waves.

BLUE-CHIRP spectrum results if mirror moves IN

TIME

Bouncing light Doppler shifts from moving mirror

Circular arc s

r θ $s=r\theta$

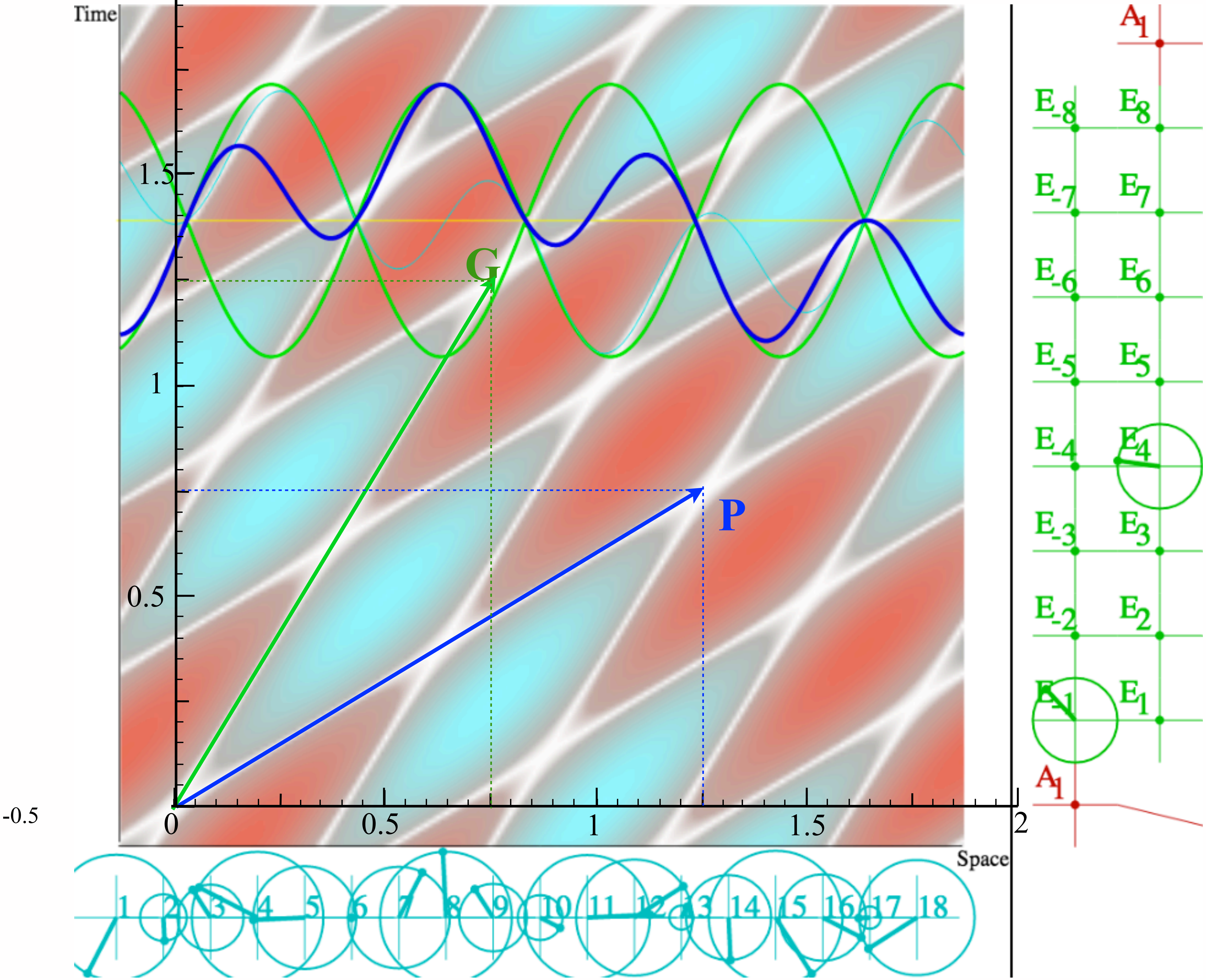
Hyperbolic proper time τ

ℓ ρ $\tau=\ell\rho$

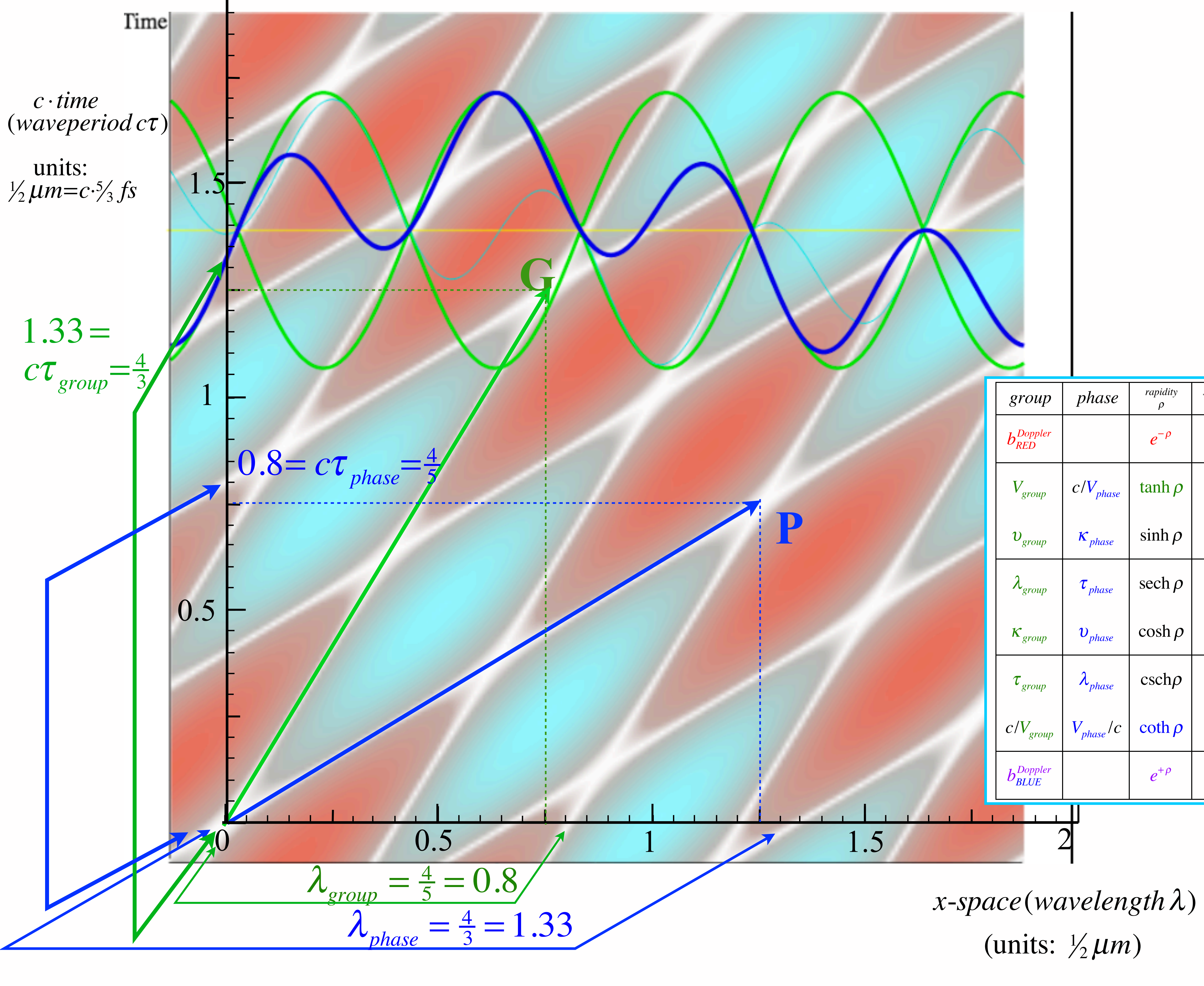
Interfering light beams make Minkowski diamonds

x or ℓ-axis for rapidity $=0.2$

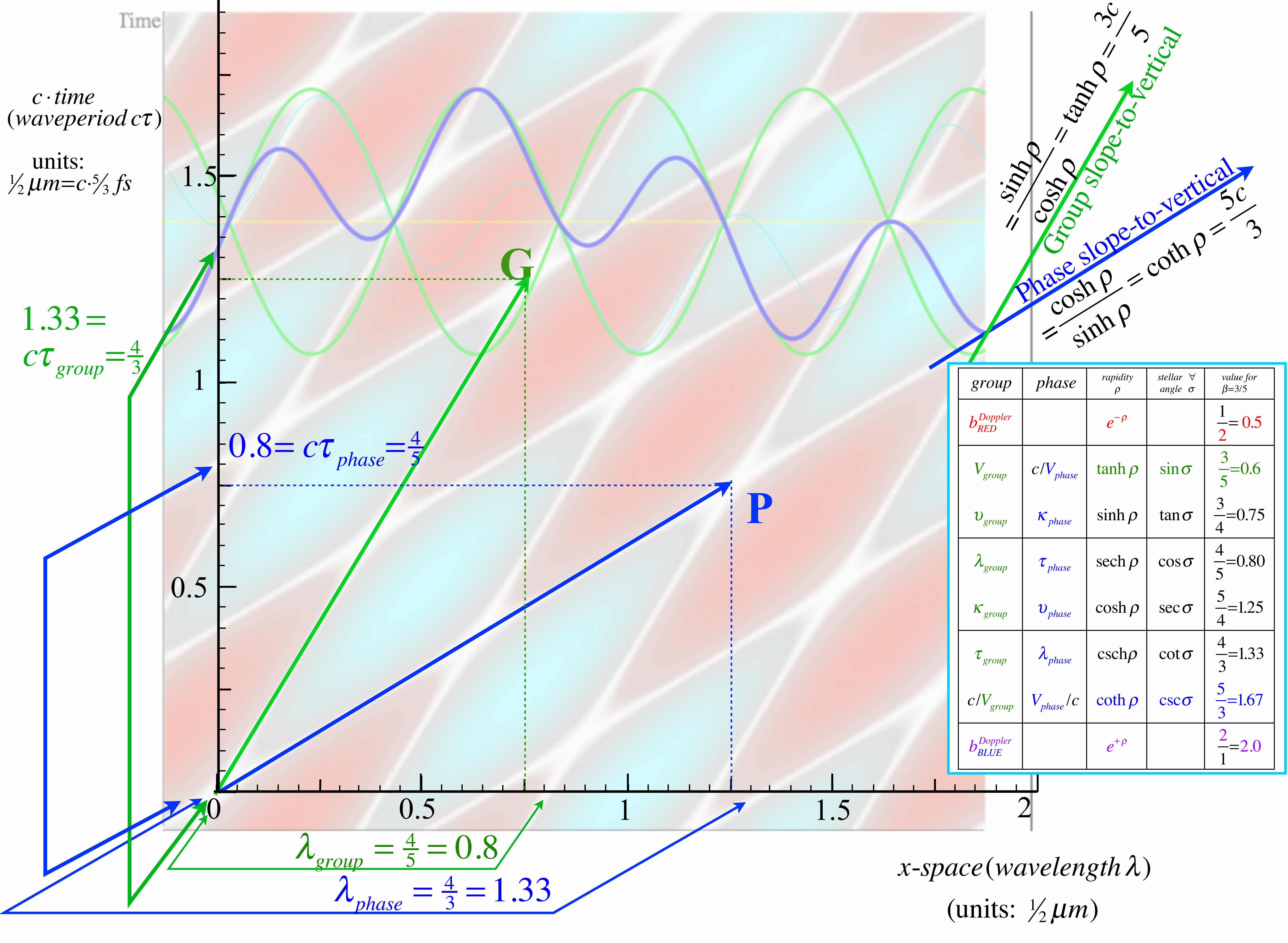
Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light



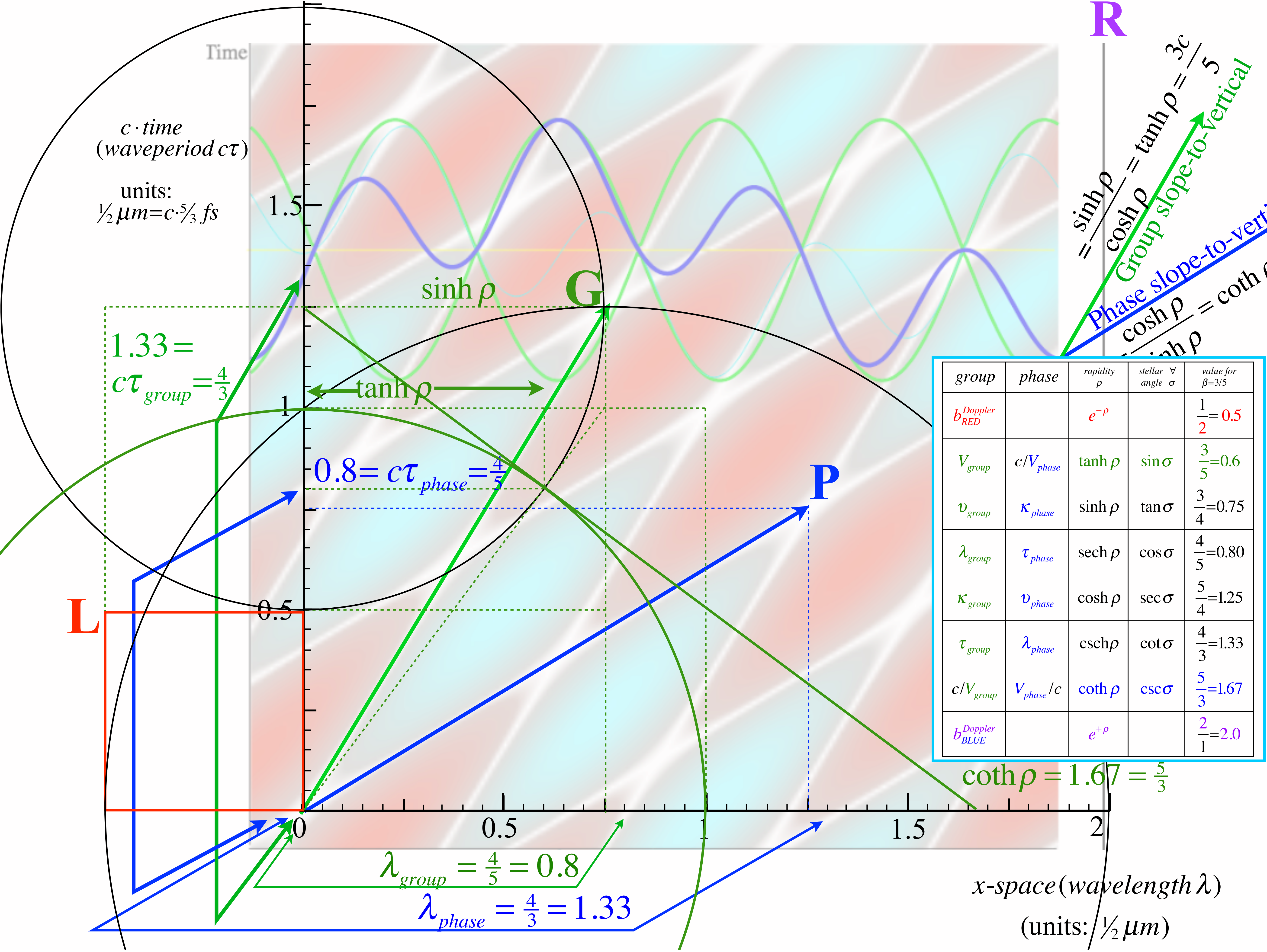
Time
1.5
1
0.5
-0.5
0
0.5
1
1.5
2
Space
G
P
A1
E-8
E8
E-7
E7
E-6
E6
E-5
E5
E-4
E4
E-3
E3
E-2
E2
E-1
E1
A1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18



group	*phase*	*rapidity* ρ	*stellar* ∀ *angle* σ	*value for* β=3/5
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2}$= 0.5
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}$=0.6
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}$=0.75
λ_{group}	τ_{phase}	$\operatorname{sech}\rho$	$\cos\sigma$	$\frac{4}{5}$=0.80
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}$=1.25
τ_{group}	λ_{phase}	$\operatorname{csch}\rho$	$\cot\sigma$	$\frac{4}{3}$=1.33
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}$=1.67
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}$=2.0

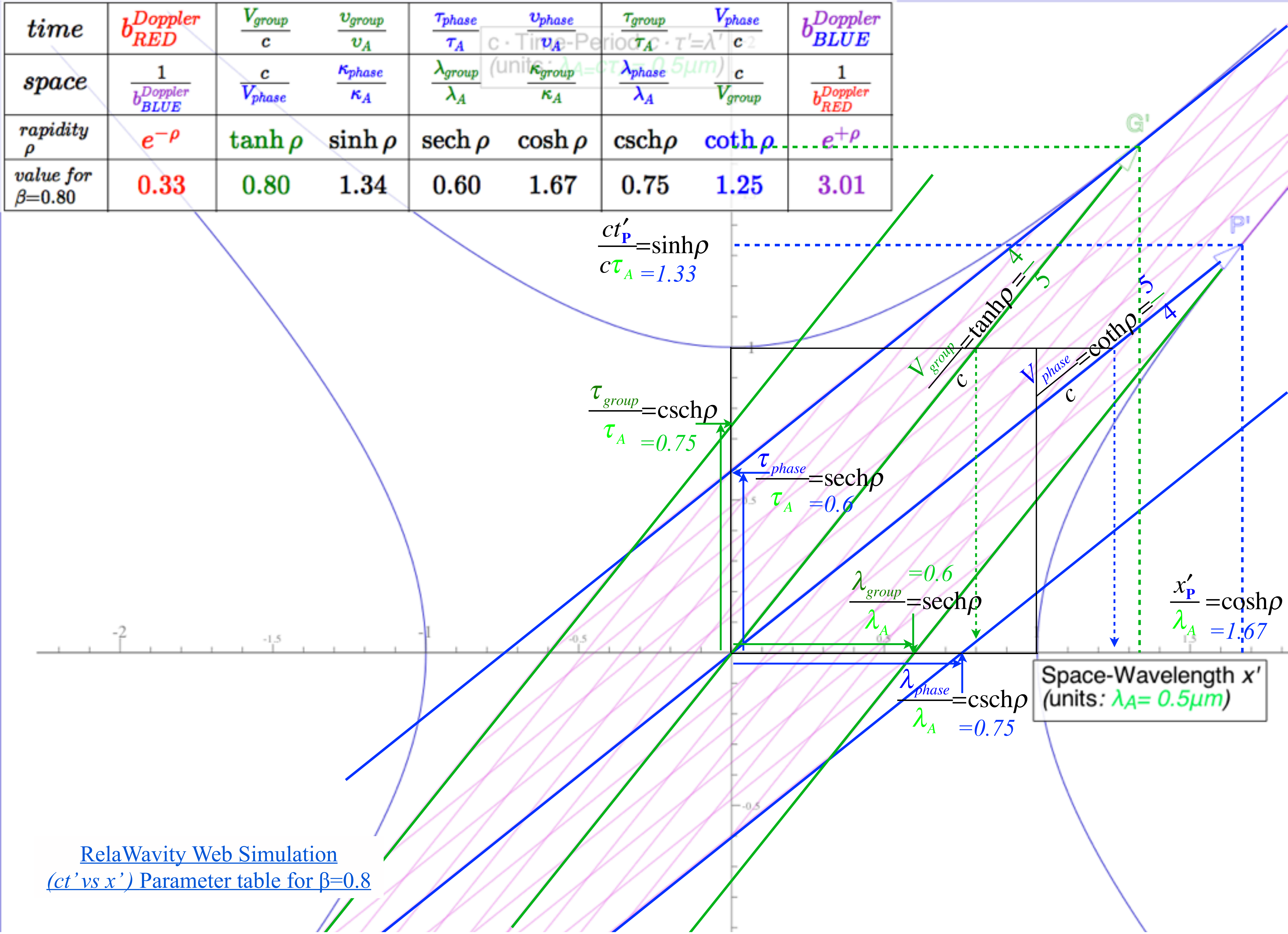


group	*phase*	*rapidity* ρ	*stellar* ∀ *angle* σ	*value for* β=3/5
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2}$= 0.5
V_{group} υ_{group}	c/V_{phase} κ_{phase}	$\tanh\rho$ $\sinh\rho$	$\sin\sigma$ $\tan\sigma$	$\frac{3}{5}$=0.6 $\frac{3}{4}$=0.75
λ_{group} κ_{group}	τ_{phase} υ_{phase}	$\text{sech}\,\rho$ $\cosh\rho$	$\cos\sigma$ $\sec\sigma$	$\frac{4}{5}$=0.80 $\frac{5}{4}$=1.25
τ_{group} c/V_{group}	λ_{phase} V_{phase}/c	$\text{csch}\rho$ $\coth\rho$	$\cot\sigma$ $\csc\sigma$	$\frac{4}{3}$=1.33 $\frac{5}{3}$=1.67
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}$=2.0

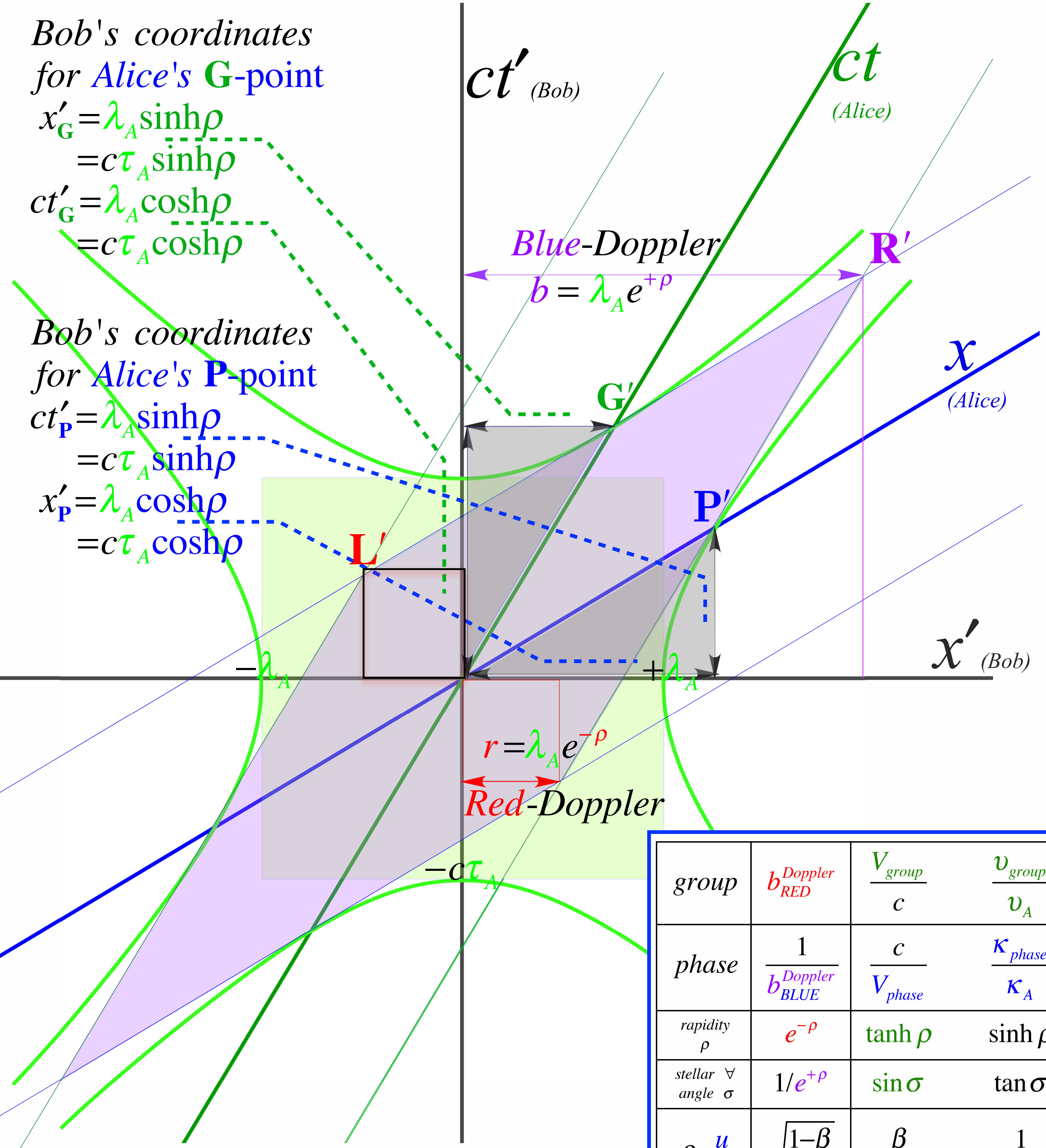


group	phase	rapidity ρ	stellar ∀ angle σ	value for $\beta=3/5$
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2}=0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\mathrm{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}=1.25$
τ_{group}	λ_{phase}	$\mathrm{csch}\rho$	$\cot\sigma$	$\frac{4}{3}=1.33$
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}=1.67$
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}=2.0$

time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=0.80	0.33	0.80	1.34	0.60	1.67	0.75	1.25	3.01



RelaWavity Web Simulation
(ct' vs x') Parameter table for β=0.8



Bob's coordinates for Alice's **G**-point

$x'_{\mathbf{G}}=\lambda_A\sinh\rho$

$=c\tau_A\sinh\rho$

$ct'_{\mathbf{G}}=\lambda_A\cosh\rho$

$=c\tau_A\cosh\rho$

Bob's coordinates for Alice's **P**-point

$ct'_{\mathbf{P}}=\lambda_A\sinh\rho$

$=c\tau_A\sinh\rho$

$x'_{\mathbf{P}}=\lambda_A\cosh\rho$

$=c\tau_A\cosh\rho$

Space-time parameters

$\lambda_{phase}=\lambda_A\operatorname{csch}\rho$

$\lambda_{group}=\lambda_A\operatorname{sech}\rho$

$c\tau_{phase}=c\tau_A\operatorname{sech}\rho$

$c\tau_{group}=c\tau_A\operatorname{csch}\rho$

Per-space-time parameters

$c\kappa_{phase}=c\kappa_A\sinh\rho$

$c\kappa_{group}=c\kappa_A\cosh\rho$

$\upsilon_{phase}=\upsilon_A\cosh\rho$

$\upsilon_{group}=\upsilon_A\sinh\rho$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b^{Doppler}_{RED}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*$^{(Lorentz)}$ τ_{phase}*-contraction*	*t-dilation*$^{(Einstein)}$ υ_{phase}*-dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b^{Doppler}_{BLUE}$

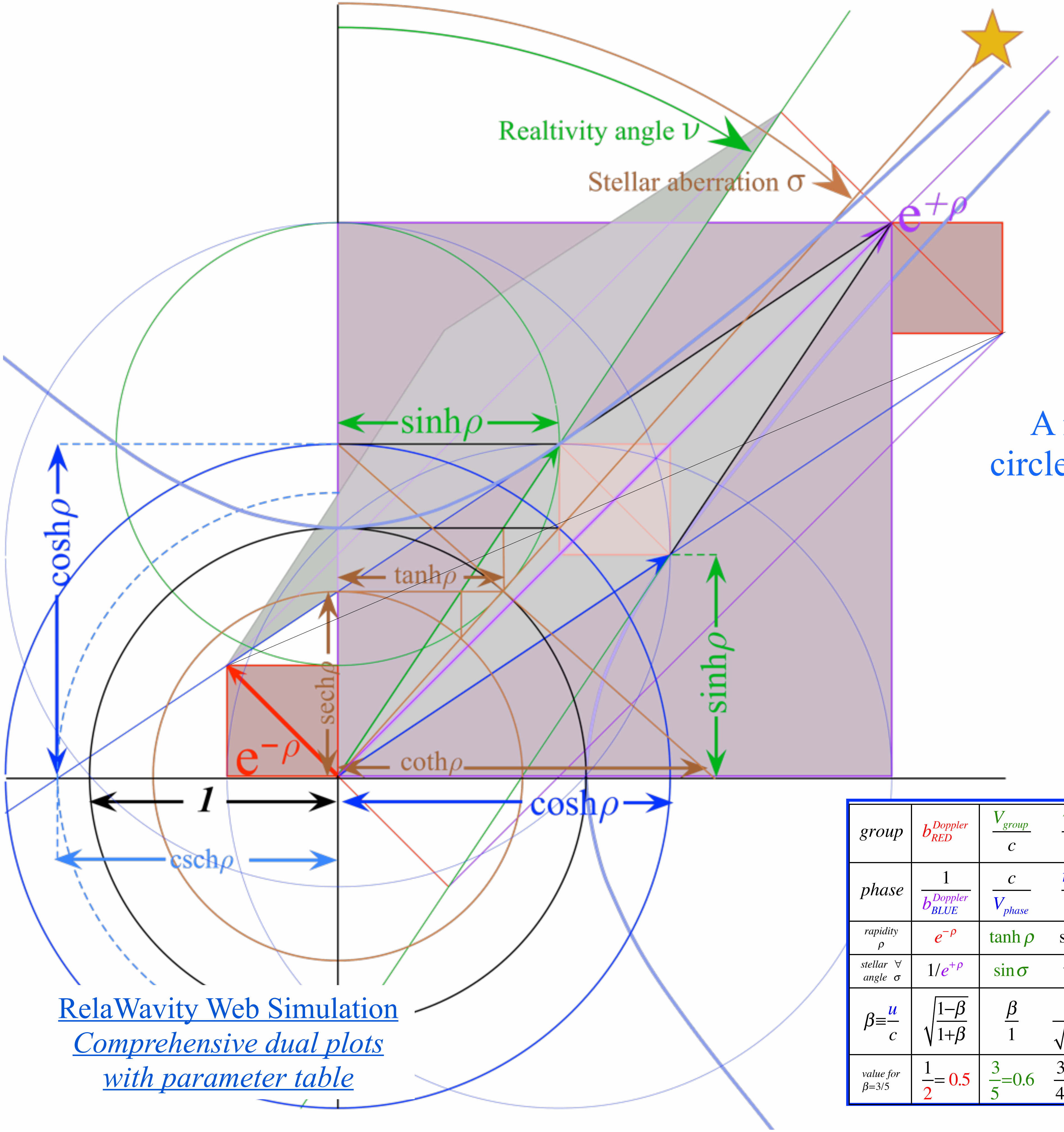
RelaWavity Web Simulation

Comprehensive dual plots

with parameter table

RelaWavity Web Simulation

(ct' vs x') with parameter table



A more compact circle-based geometry

RelaWavity Web Simulation
Comprehensive dual plots
with parameter table

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$