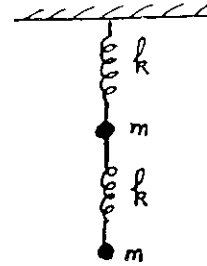


## SECTION A:

### MECHANICS

- A1. Two identical masses  $m$  are suspended from two identical springs of harmonic constant  $k$ , as shown. Find

1. the characteristic frequencies;
2. the equations of motion in normal coordinates;
3. the equations of motion in original coordinates.



- A2. A mass  $m$  moves in a circular orbit of radius  $r_0$  under the influence of a central force whose potential is  $-km/r^n$ . Show that the circular orbit is stable under small oscillations (that is, the mass will oscillate about the circular orbit) if  $n < 2$ .

- A3. Particles 1, 2, 3, and 4 with  $m_1 = m_3 = m$  and  $m_2 = m_4 = 3m$  are connected by massless rods and moving along the inside circle with radius  $a$ . Particles 1 and 4 are located at the midpoint of the radius, and  $\theta_0$  is fixed. The circle is vertical. Hence, there is gravitations force acting on all 4 particles.

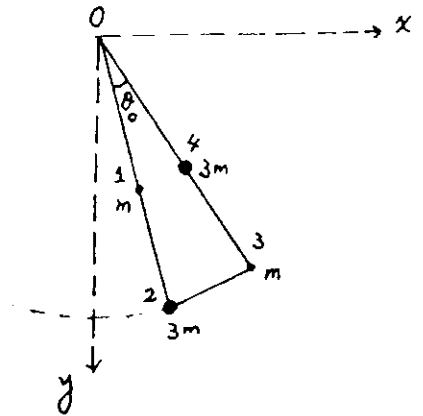
You need to choose the general coordinates for describing the motion of the system.

Find the **equation of motion** (no need to find the solutions) via,

- (1) Newton's 2<sup>nd</sup> law;
- (2) Lagrange's Eqs.;
- (3) Hamilton's Eqs.;
- (4) Poisson brackets.

Consider the limiting case:  $\theta_0$  is very small and the oscillation is around the vertical direction.

What is the period of the oscillation of the system?



SECTION B:

E & M

- B1. A point charge  $q$  is placed in a semi-infinite dielectric with a dielectric constant  $\epsilon_2$  at a distance  $d$  away from a plane interface which separates the first medium from another semi-infinite dielectric with a dielectric constant  $\epsilon_1$ . Find the electric potential in both regions.
- B2. The infinite plane  $z = 0$  is kept at zero potential except for a disk of radius  $a$  centered on the  $z$ -axis. The disk is maintained at a constant potential  $V$ . It is required to determine the potential in the  $z > 0$  region.
- (a) Write down (do not derive) the Green's function for the region  $z > 0$ .
  - (b) Write down an integral expression for the potential for the region  $z > 0$ . Do not carry out any integration but show all limits of integration and the integrand explicitly.
  - (c) Calculate the form of the potential far from the disk and evaluate the leading term for  $z > 0$ .
- B3. Two plane parallel conducting plates of width  $b$ , height  $a$  are separated by a small distance  $d$ . The plates are immersed vertically in a fluid of dielectric constant  $K = \epsilon/\epsilon_0$  and mass density  $\rho_m$ . Calculate the height to which the fluid rises between the plates when the plates are connected to a battery that maintains a constant potential  $V$  across them. List what approximations have been made in arriving at your result.

## SECTION C:

### STATISTICAL MECHANICS & THERMODYNAMICS

C1. Consider a system of  $N$  magnetic dipoles each of magnitude  $\mu$  and spin  $J = \frac{1}{2}$  placed in a magnetic field  $H$ . Derive an expansion for entropy, magnetization, and heat capacity for this system. Plot them as a function of temperature.

C2. Experimentally one finds that for a rubber band

$$\left. \frac{\partial J}{\partial L} \right|_T = \frac{\alpha T}{L_0} \left( 1 + 2 \left( \frac{L_0}{L} \right)^3 \right)$$

and

$$\left. \frac{\partial J}{\partial T} \right|_L = \frac{\alpha L}{L_0} \left( 1 - \left( \frac{L_0}{L} \right)^3 \right)$$

where  $J$  is the tension,  $\alpha = 1.0 \times 10^3 \text{ dyn/K}$ , and  $L_0 = 0.5 \text{ m}$  is the length of the band with no tension.

- (a) Compute  $(\partial L / \partial T)_J$  and discuss its physical meaning.
  - (b) Find the equation of state and show that  $dJ$  is an exact differential.
  - (c) Assume the heat capacity  $C_L = 1.0 \text{ joules/K}$ . Find the work necessary to stretch the band reversibly and adiabatically to a length of  $lm$ . What is the change in temperature.
- C3. Consider a gas contained in volume  $V$  at temperature  $T$ . The gas is composed of  $N$  distinguishable particles of zero rest mass, so that energy  $E$  and momentum  $p$  of the particles are related by  $E = pc$ . The number of single-particle energy states in the range  $p$  to  $(p + dp)$  is  $4\pi V p^2 dp / h^3$ . Find the equation of state and the internal energy of the gas and compare with an ordinary gas.

## SECTION D:

### MODERN PHYSICS

- D1. In 1950, two groups at the Berkeley synchrocyclotron simultaneously discovered evidence for the existence of the neutral pion decaying into two gamma rays,  $\pi^0 \rightarrow 2\gamma$ .
- (a) Calculate the opening angle,  $\theta$ , in the lab frame between the two gamma rays in terms of the rest mass,  $m_0$ , of the moving pion and the energies  $E_1$  and  $E_2$  of the two photons.
  - (b) What is the minimum opening angle,  $\theta_{\min}$ , in the lab frame in terms of  $m_0$  and the initial pion energy  $E_\pi$ ?
  - (c) Why is  $\theta_{\min} \neq 0$ ?
- D2. A particle of mass  $m$  moves in a force field given by  $\vec{F} = -kr^\alpha \hat{r}$  where  $k$  and  $\alpha$  are constants ( $\alpha \neq -1$ ). Using the Bohr model find the radii of the allowed orbits and the energy levels. Check your answers using the following two special cases:
- (a)  $\alpha = 1, k = m\omega^2$  (3 - D simple harmonic oscillator)
  - (b)  $\alpha = -2, k = Ze^2$  (hydrogenic atom)
- D3. The ground state electronic configuration of  $^{87}\text{Rb}$  is  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 5s^1$  and its nuclear spin is  $I = 3/2$ .
- (a) Draw a diagram showing energy levels (including fine and hyperfine structure splittings) in the ground and the first electronically excited states. Label each level by its spectroscopic designation and appropriate quantum numbers. State the selection rules for electric dipole transition, and draw all allowed transitions by vertical lines on the diagram.
  - (b) Now consider the atom to be placed in a weak external magnetic field. For convenience just consider an  $F = 1$  level of the excited state and the  $F = 2$  level of the ground state. In a separate diagram show all the splittings of these levels and identify each level by its quantum number. Draw vertical lines to indicate allowed transitions and indicate the polarization of the emitted radiation corresponding to each transition.

- D4. We wish to produce  $^{64}\text{Cu}$  by bombarding natural copper (69%  $^{63}\text{Cu}$ , 31%  $^{65}\text{Cu}$ ) with low-energy neutrons. The relevant cross-section are

$$\sigma_{63} = 4.5 \text{ barns for } ^{63}\text{Cu} + n \rightarrow ^{64}\text{Cu}$$

$$\sigma_{65} = 2.3 \text{ barns for } ^{65}\text{Cu} + n \rightarrow ^{66}\text{Cu}$$

If the copper target is  $1.0 \text{ cm}^2$  in area and  $0.01 \text{ cm}$  thick (this sample can be considered as a thin sample) and neutron flux is  $10^{12}/\text{cm}^2 \cdot \text{s}$ , find the production rate of  $^{64}\text{Cu}$  and activity (decay rate  $\times$  number of atoms) for the  $^{64}\text{Cu}$  after 12.8 hours.

(1 barn =  $10^{-24} \text{ cm}^2$ , half-life of  $^{64}\text{Cu}$  = 12.8 hours, number of copper atoms per unit volume =  $0.83 \times 10^{23}/\text{cm}^3$ ).

- E3. A particle moves in the following potential:  $U(x) = U_0/(x-a)^2 (x+a)^2/a^4$ .
- (a) If this particle has a total energy equal to  $U_0/2$ , indicate the classically allowed region on a sketch of  $U(x)$  versus  $x$ .
  - (b) What is the functional form of the kinetic energy for this particle?
  - (c) On separate figures, sketch your “guesses” for the shapes of the wave functions for both the ground state and the first excited state (assume that total energy is less than  $U_0$  for both).
  - (d) For your wave functions “guess” the expected values of the particle’s positions and the most probable values of the particle’s positions.
  - (e) If the particle is a nitrogen atom ( $Z = 7$ ) and  $a = 1 \text{ \AA}$ , estimate an average kinetic energy for the ground state.