Equilibrium Unemployment and Excess Capacity in Steady-state and Growth Cycles

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This paper is addressed to the interpretation of the average or trend rate of unemployment. Many contemporary macroeconomists provide a "nominal" interpretation of that rate: the trend or "natural" rate of unemployment is understood to be that rate which is consistent with a stable rate of inflation. Another, distinctly minority, strand of thought gives a "real" interpretation of unemployment: the trend rate is understood to be that rate which is consistent with a stable distribution of income between capital and labour. The present contribution is of the latter variety.

I. STEADY STATE AND THE DYNAMICS OF THE REAL WAGE AND REAL PRICE

The seminal work in this vein is R. M. Goodwin's (1967) paper, "A Growth Cycle". Although his paper focused on the cyclical behaviour of employment, profit and wage rates, we may briefly consider the steady state implicit in his type of model. The steady-state profit and wage rates, $r^*$ and $w^*$, are given by the solution to the Harrod equation, as in most brands of growth theory:

$$s\{r^*(w^*)\} = g^*$$

where $s$ is the average propensity to save, $v$ is the capital-output ratio, and $g^*$ is the exogenous natural rate of growth. Abstracting from technical progress, $r(w)$ describes a stable wage-profit frontier. Typically, $s$ and $1/v$ are taken to be non-decreasing functions of $r$, with one or the other or both being strictly increasing. The steady-state profit and wage rates equilibrate the growth rate of capital, $s/v$, with the growth rate of labour, providing for a constant rate of employment.

The other steady-state condition is

$$w = h(n^*) = 0, \quad h' > 0$$

where the dot denotes a time derivative and $n$ is the employment rate (one minus the unemployment rate). Equation (2) posits a "law of supply and demand" relating changes in the real wage to the state of the labour market. It is, in other words, a "real" Phillips curve. Steady state requires a constant real wage (and, hence, profit rate), so (2) solves for the steady-state employment rate, $n^*$.

The Goodwin formulation specifies $h(1) > 0$, so $n^* < 1$. This immediately raises the question of why it should be supposed that $h(1) > 0$. To some this might suggest that labour has some non-competitive bargaining power. This would in turn imply that steady-state unemployment ($n^* < 1$) could only be
due to such imperfections, i.e. that slack in the labour market is required only to counterbalance labour's non-competitive bargaining power.\(^4\)

However, such an interpretation need not hold; for the real wage is, by definition, the relative price between labour and output. In other words, (2) is not only the real-wage dynamic, it is also the real-price dynamic (i.e. price in wage units). The problem with the specification in (2) is that the only market explicitly considered is the labour market: the state of the product market is not considered (or else it is held constant). If the state of the product market is to be considered, the natural analogue of (2) would be

\[
(3) \quad \dot{w} = h(n, u), \quad h_1 > 0, h_2 < 0
\]

where \(u\) represents the state of the product market. We take \(u\) to be the ratio of quantity demanded to notional short-run supply, or utilization rate. It should be noted that auctioneer rules or "laws of supply and demand" such as (2) or (3) have not yet been rigorously grounded in general theory.\(^5\) Still, as heuristics go, symmetry would certainly suggest that (3) is preferable to (2) and our analysis proceeds from there.\(^6\)

Given (3), then, we may now analyse what combinations of \(n\) and \(u\) are admissible for steady state; i.e. for stationary wage/profit configurations, \(h(n^*, u^*) = 0\). It is immediately obvious that the steady-state levels of unemployment and excess capacity are positively associated. Unemployment and excess capacity may be considered to counterbalance each other, the former tending to keep labour from raising real wages, the latter tending to keep firms from raising real prices. That is the main point of this paper.

We may note in passing that conventional notions of competitive price dynamics would suggest that \(h(1, 1) = 0\). (This is the obvious generalization of the objection to Goodwin, noted above, that (2) should be characterized by \(h(1) = 0\).) Even in this case, however, we would find unemployment in steady state if and only if there is excess capacity in steady state, i.e. \(n^* \geq 1\) as \(u^* \geq 1\).

One of the virtues of (3), which allows us to bring excess capacity into the distribution dynamics, is that it allows us to treat increasing returns. Increasing returns is one of the explanations of excess capacity that is most compatible with steady state.\(^7\)

Cost-minimizing firms meeting any given level of demand will, under increasing returns, operate at a point where the marginal products of labour and capital stand in equal ratios to the real wage and profit rate, but taken individually the marginal products exceed the factor prices. The short-run notional supply, of course, is given by that output at which the marginal product of labour equals the real wage. The cost-minimizing firm will satisfy any given demand with a capital stock which implies a larger notional short-run supply. This is illustrated by a familiar Viner-Wong type of diagram, in Figure 1. Following the non-Walrasian views of Hahn and Negishi, we suppose that the representative firm perceives (or "conjectures", to use Hahn's term) a demand curve that is kinked at real price \(p\) and quantity \(Q^d\). The marginal revenue equals short- and long-run marginal cost at quantity \(Q^d\). The cost-minimizing firm installs a capacity that implies \(p = LAC = SAC > LMC = SMC\), so short-run notional supply, \(Q^s\), exceeds \(Q^d\). Therefore, under
increasing returns, \( u^* = Q^d/Q^s < 1 \). That is, under increasing returns there is excess capacity, which according to (3) would drive down real prices unless there were sufficient unemployment to keep real wages from rising.

II. NON-STEADY-STATE BEHAVIOUR

In this section, we consider behaviour of such a system out of steady state. In particular, it will be shown that, if the elasticity of substitution is less than unity and if the system is cyclical, it will accurately reproduce the cyclical relationships among the capacity utilization, profit and employment rates observed in the United States, but not those involving productivity and output.

Utilization and profit rates typically lead the employment rate over the cycle. For example, the US Commerce Department's Bureau of Economic Analysis reports that, over the period 1948–1970, peaks and troughs of capacity utilization in manufacturing lead those of the employment rate, on average, by 4.8 months, or 8 per cent of the average business cycle. The cyclical timing of the profit rate is not analysed, but the closely related profit share in domestic income of non-financial corporations leads the employment rate, on average, by 9.1 months, or 15 per cent of the average business cycle.8

Before turning to models of the cycle, a word of interpretation may be in order. We have in mind "equilibrium" cycles, in the sense of Hahn's "conjectural equilibrium". At each point in time all agents are satisfied with the price and quantity chosen, subject to the conjectured constraint that his demand curve is kinked at that price and quantity. Such conjectures, however, move over time. In particular, the prices at which agents perceive a kink respond to the state of unemployment and excess capacity according to (2) or (3) in the models below.9 The models also trace out the quantities associated with
the kink, namely those quantities that would constitute "rational expectations."

**The Goodwin model**

Turning now to models of the cycle, we consider first the original Goodwin model. Non-steady-state behaviour here is described by two equations. First, real-wage movements are governed by a simple real Phillips curve, such as (2) above. The second equation dynamizes (1) to describe movement of the employment rate, n. We make use of the identity \( n = \frac{L}{L_s} = \frac{K}{kL_s} \), where \( L \) is employment, \( L_s \) is labour supply, \( K \) is capital, and \( k \) is the capital–labour ratio. In Goodwin's original model, \( k \) is fixed (leaving aside technical progress), so

\[
\frac{\dot{n}}{n} = g_K - g_L
\]

where \( g_K \) is the growth rate of capital and \( g_L \) is the growth rate of labour supply. We take \( g_L \) as exogenous and recall that \( g_K = \frac{s(r(w))}{v(r(w))} \). In particular, \( dg_K/dr > 0 \), so \( dg_K/dw < 0 \).
The system (2) and (4) can be illustrated in the phase diagram of Figure 2(a) and the wave diagram of Figure 2(b). The bars over the variables denote deviations from steady-state values. In Figure 2(a), the \( \bar{n} \) axis denotes points where \( n = 0 \), since \( w = 0 \). Similarly, the horizontal axis, which can represent either \( \bar{r} \) or \( -\bar{w} \), denotes points where \( r = \bar{w} = 0 \), since \( \bar{n} = 0 \).

The most commonly noted feature of this model is that the orbits are closed.10 We are more interested, however, in the following two points. First, the profit rate does lead the employment rate in this model, as it does in the data, but perhaps by too long a margin. The lead is on the order of a quarter of a cycle, rather than the 15 per cent suggested above. The second point of concern, of course, is that the model exhibits no excess capacity, owing to the assumption of constant returns, fixed coefficients.

**Variable coefficients under constant returns**

In leading up to our model of the cycle under increasing returns to scale, consider first a variable coefficients model under constant returns. Here, we still have full capacity utilization over the cycle (assuming that factor prices are correctly foreseen and that there are no adjustment costs). Thus (2) is still relevant, even if (3) holds, since \( u = 1 \) at all times.

Equation (4), however, must be recast as

\[
\frac{\dot{n}}{n} = \frac{g_k - g_L}{k} - \frac{\dot{k}}{k} = g_k - g_L - \frac{1}{k} \frac{dk}{dw} \bar{w} = g_k - g_L - \frac{1}{k} \frac{dk}{dw} h(n).
\]

We consider the linear approximation of (2), (5) around the steady state:

\[
(2') \quad \frac{\dot{w}}{h'} = \bar{n}
\]

\[
(5') \quad \frac{\dot{n}}{n} = n \frac{dg_k}{dw} \bar{w} - n \frac{dk}{k} \frac{dw}{h'} \bar{n}.
\]

This system is stable, provided that \( \frac{dk}{dw} > 0 \), as would be the case under neoclassical technology. Thus, capital–labour substitution dampens Goodwin cycles. Indeed, if \( \frac{dk}{dw} \) is large enough, the system converges monotonically.

The oscillatory case is illustrated by Figures 3(a) and (b). The line of \( n \)-stationarity in Figure 3(a) can be derived from (5'). It is described in the space of \( \bar{n} \) and \( -\bar{w} \) by \( \bar{n}/-\bar{w} = -k (dg_k/dw)/h'(dk/dw) \). Comparing with Figure 2(a), we see that the line of \( n \)-stationarity is rotated clockwise from the \( \bar{n} \)-axis. This has the effect of bringing the turning points of \( \bar{n} \) closer to those of \( \bar{r} \), as seen by comparing Figures 3(b) and 2(b). This brings the model into closer accord with the data in this respect.

**Variable coefficients under increasing returns**

Still, the model has no place yet for excess capacity. Desai has introduced capacity utilization into the model by positing a relationship between the capacity utilization rate and the employment rate. The model developed below also introduces the capacity utilization rate, but there are two key differences from Desai. First, our approach derives the utilization rate from cost-minimization under increasing returns. This will tie the utilization rate directly to factor prices rather than to the employment rate. We can then derive the cyclical relationship between the employment rate and the utilization rate.
We will find that the relationship will be close, but not coincident, as posited by Desai. Rather, under conditions to be specified, the utilization rate will slightly lead the employment rate, as, in fact, the data show.

The second key difference from Desai is our symmetric real-wage dynamic (actually, a slight modification of (3) to be introduced), rather than his version of (2), with money illusion, as discussed in note 3 above.

To model increasing returns, we must make some special assumptions. First, we assume that the number of firms is constant, perhaps owing to government anti-merger policy. This permits us to use the "representative firm" construction.

Second, we must narrow down the class of production functions, \( Q = G(K, L) \), to be considered. We continue to abstract from technical progress. Levhari and Sheshinski (1972) have shown (alternatively, see Arrow and Kurz, 1970, pp. 22–5) that a production function is amenable to steady growth if it can be expressed as

\[
Q = G(K, L) = F(K, H(K, L^\delta)),
\]

where \( F \) and \( H \) are concave and linearly homogeneous. We confine ourselves here to the case \( Q = F(K, L^\delta) \), \( \delta > 1 \). It will be useful to express the production
function in intensive form:

\[ Q = L^\delta F(K/L^\delta, 1) = L^\delta f(\kappa), \quad f' > 0, f'' < 0 \]

where \( \kappa = K/L^\delta \). By analogy with labour-augmenting technical progress, we may think of \( L^\delta \) as a measure of labour in efficiency units, so each worker has \( L^{\delta-1} \) efficiency units. Thus, \( \kappa \) can be thought of as the capital–labour ratio measured in efficiency units. Similarly, \( f(\kappa) \) is output per efficiency unit. We may define \( \omega = w/L^{\delta-1} \) as the wage per efficiency unit.

Third, we suppose that each firm faces quantitative demand constraints as well as parametric prices of inputs and output. In meeting its demand, each firm minimizes costs at each point in time (arguably the most restrictive assumption).\(^{11}\) The paths traced out should be interpreted as equilibrium paths, characterized by fulfilled expectations of quantities and prices.

Formally, at all times the firm chooses the capital–labour ratio to satisfy \( r/\omega = G_K/G_L \), where, by product exhaustion, \( r = \{f(\kappa) - \omega\}/\kappa \). Under these conditions it can be shown that \( \kappa, v = f(\kappa) \), and \( r \) are functions of \( \omega \). In particular, it can be shown that \( d\kappa/d\omega > 0, dv/d\omega > 0, \) and \( dr/d\omega < 0 \). We can also express the capacity utilization rate as a function of \( \omega \). It is shown in the Appendix that \( du/d\omega \geq 0 \) as the elasticity of substitution between capital and labour is greater or less than unity.

Fourth, we modify the real-wage dynamic (3) to account for scale effects. We suppose that the dependent variable is not wage per natural unit of labour, but instead wage per efficiency unit. That is, we recast (3) as

\[ \dot{\omega} = h_1(\nu, u(\omega)), \quad h_1 > 0, h_2 < 0. \]

The second equation of our two-variable dynamic system dynamizes (1) and describes movement of the employment rate, \( n \). We make use of the identity \( n = L/L^\delta = K/L^\delta \kappa L^{\delta-1} \). Then

\[ \frac{\dot{n}}{du(\omega)} = n \left\{ g_K - g_L - \frac{\kappa}{\kappa} (\delta - 1) \frac{L}{L} \right\} \]

\[ = n \left\{ g_K - g_L - \frac{1}{\kappa} \frac{d\kappa}{d\omega} \omega - (\delta - 1) \left( \frac{\dot{n}}{n} + g_L \right) \right\} \]

\[ = \frac{n}{\delta} \left( g_K - \delta g_L - \frac{1}{\kappa} \frac{d\kappa}{d\omega} \omega \right). \]

We continue to assume that \( dg_K/\omega > 0 \), so \( dg_K/d\omega < 0 \). Note that the stationary solution of (8) is equivalent to the Harrod equation (1), with \( g^* = \delta g_L \).

We consider the linear approximation of (7), (8) around the steady state. Using bars to denote deviations from steady-state values, we have

\[ \dot{\omega} = h_2 \frac{du}{d\omega} \bar{\omega} + h_1 \bar{n} \]

\[ \dot{n} = n \left( \frac{dg_K}{\delta d\omega} \frac{\omega}{\delta k} \frac{\delta}{d\omega} \right) \bar{\omega} - nh_1 \frac{d\kappa}{\delta d\omega} \bar{n}. \]
The linearized system is stable or unstable as \( h_2(du/d\omega) \leq (nh_1/\delta \kappa)(d\kappa/d\omega) \).\(^{12}\) As shown in the Appendix, \( du/d\omega \leq 0 \) as technology is inelastic or elastic. Thus, for inelastic technology it is possible for the system to be unstable. The system is oscillatory or monotonically convergent as

\[
\left( h_2 \frac{du}{d\omega} - \frac{nh_1}{\delta \kappa} \frac{d\kappa}{d\omega} \right)^2 \geq -4nh_1 \frac{dg_K}{d\omega}.
\]

We consider more closely the behaviour of the system in the cyclical case where the elasticity of substitution is less than unity, and, hence, the utilization rate is an inverse function of \( \omega \). The system’s phase diagram can be drawn with \( \bar{n} \) on one axis and any of \( \bar{u}, \bar{r}, \) or \( -\bar{\omega} \) on the other, as in Figure 4(a). The lines of stationarity can be derived from (7') and (8'). For example, stationarity of \( u, r, \) and \( \omega \) is given by (7') and is described in the space of \( \bar{n} \) and \(-\bar{\omega}\) by \( \bar{n}/\bar{\omega} = h_2(du/d\omega)/h_1 \). Alternatively, it could be described in the space of \( \bar{n} \)
and \( \bar{u} \) or of \( \bar{n} \) and \( \bar{r} \) by
\[
\bar{u}/\bar{n} = -h_2/h_1 \quad \text{or} \quad \bar{n}/\bar{r} = -h_2(du/d\omega)/h_1(dr/d\omega),
\]
respectively, where use is made of the approximations \( \bar{u} \approx (du/d\omega)\bar{\omega} \) and \( \bar{r} \approx (dr/d\omega)\bar{\omega} \). Stationarity of \( n \) is described by similar constructions derived from (8'). It can be verified that the stationarity lines are positively sloped, with the \( n \)-line being steeper, as depicted.

Comparing Figures 3(a) and 4(a), we see that the increasing returns model with the symmetric real-wage dynamic rotates the \( r \)-stationarity line counterclockwise towards the \( n \)-stationarity line. Thus \( n \) and \( r \) are brought yet closer into phase, as shown in Figures 3(b) and 4(b). More importantly, the utilization rate varies over the cycle and slightly leads the employment rate, as it does in the US economy.\(^{13}\)

It is difficult to tell a simple story to go along with the sequence depicted in Figure 4 without doing violence to the feedbacks captured in the equations. One version, however, might read as follows. At point A, sustained high employment has begun to raise wages and reduce the profit rate, leading to capital–labour substitution. Under inelastic technology, the substitution of capital for labour raises capacity relative to output, and brings down utilization rates. Employment keeps rising for a short time, until point B, because, although the profit rate is falling, it is still high, so saving and investment are high. After point B, the profit rate and investment have fallen enough to begin reducing the employment rate. Employment, though falling, is still high, so the profit rate keeps falling, which continues to bring down the utilization rate. By the time we get to point C, employment has fallen enough that the profit rate starts to rise, leading to labour–capital substitution, and a turnaround in utilization. Employment continues to fall until point D, since profits are still low, though rising. Finally, after point D, profits have risen enough that employment starts increasing.

Although this model accurately replicates the lead of utilization and profit rates over employment rates, it does not track other variables that well. In particular, it can be shown that output relative to trend and labour productivity relative to trend lag employment in this model, but not in the US economy.\(^{14}\)

III. SUMMARY

This paper has extended the Goodwin model of unemployment to accommodate increasing returns and the excess capacity associated with it. It was suggested that real wage (real price) movements should depend on excess capacity as well as unemployment, with unemployment tending to reduce the real wage (raise the real price) and excess capacity tending to raise the real wage (reduce the real price). As a result, in steady state, when the real wage (real price) is stationary, unemployment and excess capacity are positively associated and counterbalance each other. In particular, if increasing returns leads to steady-state excess capacity, then there is reason to expect steady-state unemployment.

The cyclical behaviour of this model replicates certain aspects of US experience somewhat better than the original Goodwin model. In our model, if the elasticity of substitution is less than unity and if the solution is oscillatory, then the profit rate and utilization rate lead the employment rate by short margins.
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APPENDIX

Consider the production function

\[(A1)\]

\[Q = G(K, L) = \frac{F(K/L^\delta)}{L^\delta}, \quad \delta > 1\]

where \(K = K/L^\delta\), capital per efficiency unit. The first and second derivatives of \(G\) can be expressed as

\[(A2)\]

\[
G_K = f' \\
G_L = \delta L^{\delta-1}(f - \kappa f') \\
G_{KK} = \delta L^{\delta-1} f'' \\
G_{LL} = \delta L^{\delta-2}[(\delta - 1)(f - \kappa f') + \delta \kappa^2 f''] \\
G_{KL} = -\delta L^{\delta-1} \kappa f''
\]

Positive marginal products imply \(f' > 0\) and \(f - \kappa f' > 0\). We restrict ourselves to CES functions with diminishing marginal productivity of labour, the latter implying \(f'' < 0\), so \(G_{KK} < 0\) and \(G_{KL} > 0\).

The Allen elasticity of substitution is defined by

\[(A3)\]

\[-\sigma = -\frac{G_K G_L (KG_K + LG_L)}{KL (G_{KK} G_L^2 + G_{KL} G_K^2)} = \frac{-f'(f - \kappa f')\{\delta(f - \kappa f') + \kappa f'}{\delta f^2 f'' + (\delta - 1)(f - \kappa f')\kappa f'^2}
\]

using (2). Re-expressing for use below, we have

\[(A4)\]

\[f'' = \frac{-(\delta - 1)(f - \kappa f') f'^2 - f''(f - \kappa f')\{\delta(f - \kappa f') + \kappa f'}{\delta f^2}.
\]

Output is exhausted between profits and wages. That is,

\[(A5)\]

\[f = r\kappa + \omega
\]

where \(f\) is output per efficiency unit, \(r\) is the profit rate, and \(\omega = w/L^{\delta-1}\) is the wage per efficiency unit.

Firms take the profit rate and the wage rate, \(w\), as the costs of capital and labour respectively. Minimizing costs, they choose \(\kappa\) to satisfy \(r/w = G_K/G_L\), or, using (A2),

\[(A6)\]

\[r\delta (f - \kappa f') = \omega f'.
\]

We use (A5) and (A6) to verify \(dk/d\omega > 0\), \(dv/d\omega > 0\), and \(dr/d\omega < 0\). Solving (A5) and (A6), we express \(\kappa(\omega)\) in inverse form:

\[(A7)\]

\[\omega = \frac{\delta f(f - \kappa f')}{\delta (f - \kappa f') + \kappa f'}.
\]

Differentiating (A7), we find

\[(A8)\]

\[
\frac{dk}{d\omega} \propto \frac{d\omega}{dk} \propto \{\delta(f - \kappa f') + \kappa f'}\{f''(f - \kappa f') - \kappa f''\} - (f - \kappa f')\{f'' - (\delta - 1)\kappa f''\} = -\kappa f'^2 f'' + (\delta - 1)f''(f - \kappa f')^2 > 0
\]

where we use '\(\propto\)' to denote sign equality.

Verifying the sign of \(dv/d\omega\), we use \(v = \kappa/f(\kappa)\) to derive

\[(A9)\]

\[\frac{dv}{d\omega} \propto \frac{dv}{dk} = \frac{f - \kappa f'}{f^2} > 0
\]
Turning to \( r \), we use (A5) and (A6) to express

\[
\begin{align*}
\text{(A10)} \quad r &= \frac{ff'}{\delta(f - \kappa f') + \kappa f'} \\
\text{and}
\end{align*}
\]

\[
\begin{align*}
\text{(A11)} \quad \frac{dr}{d\omega} &\sim \frac{dr}{d\kappa} \sim \{\delta(f - \kappa f') + \kappa f'}\{ff'' + f'^2\} - ff'\{f' - (\delta - 1)\kappa f''\} \\
&= \delta f^2 f'' + (\delta - 1)(f - \kappa f')f'^2.
\end{align*}
\]

Multiplying through by \( \kappa^2/f^2 \), we find

\[
\begin{align*}
\text{(A12)} \quad \frac{dr}{d\omega} &\sim \kappa^2 f'' + (\delta - 1)(f - \kappa f')\left(\frac{\kappa f'}{f}\right)^2 < \delta \kappa^2 f'' + (\delta - 1)(f - \kappa f') < 0
\end{align*}
\]

where the first inequality follows from \( \kappa f'/f < 1 \), and the second from (A2), given that \( G_{LL} < 0 \).

We now consider capacity utilization, the main focus of this Appendix. Define \( L_c \) as the capacity complement of labour, i.e. that amount of labour which would bring the marginal product of labour down to the real wage, and thus produce \( Q_s \), as depicted in Figure 1. Define \( \kappa_c = K/L_c^{\delta-1} \) and denote \( f(K_c) \) by \( f_c \) and \( f'(K_c) \) by \( f'_c \). Unsubscripted notation will continue to denote functions evaluated at the cost-minimizing capital–effective labour ratio, \( \kappa = K/L^\delta \), described by (A5) and (A6).

Formally, \( L_c \) is implicitly defined by

\[
\begin{align*}
\text{(A13)} \quad w &= G_L(K_c, L_c) = \delta L_c^{\delta-1}(f_c - \kappa_c f'_c)
\end{align*}
\]

or

\[
\begin{align*}
\text{(A14)} \quad \omega &= \frac{w}{L_c^{\delta-1}} = \delta \left(\frac{L_c}{L}\right)^{\delta-1}(f_c - \kappa_c f'_c) = \delta \left(\frac{\kappa_c}{\kappa}\right)^{\delta-1/\delta}(f_c - \kappa_c f'_c).
\end{align*}
\]

Equating (A14) with (A7) allows us to express \( \kappa_c \) as an implicit function of \( \kappa \).

\[
\begin{align*}
\text{(A15)} \quad f(f - \kappa f')\kappa^{(\delta-1)/\delta} &= \{\delta(f - \kappa f') + \kappa f'(f_c - \kappa_c f'_c)\} \kappa^{(\delta-1)/\delta}
\end{align*}
\]

It will be useful below to have an expression for \( d\kappa_c/d\kappa \). Differentiating (A15), we find

\[
\begin{align*}
\text{(A16)} \quad \frac{d\kappa_c}{d\kappa} &= \left[ -\delta \kappa_c \kappa ((f - \kappa f') f'' - \kappa f''') \kappa^{(\delta-1)/\delta} \\
& \quad + (\delta - 1) \kappa_c (f_c - \kappa_c f'_c) \delta(f - \kappa f' + \kappa f') \kappa^{(\delta-1)/\delta} \\
& \quad + \delta \kappa_c (f_c - \kappa_c f'_c) (f' - (\delta - 1) \kappa f'') \kappa^{(\delta-1)/\delta}/(\delta - 1) \kappa f(f - \kappa f') \kappa^{(\delta-1)/\delta} \\
& \quad + \delta \kappa_c^2 f'' \delta(f - \kappa f') + \kappa f' \kappa^{(\delta-1)/\delta}\right].
\end{align*}
\]

Multiplying through numerator and denominator by \( f(f - \kappa f') \) and using (A15), we can factor out terms in \( \kappa^{(\delta-1)/\delta} \). After some rearrangement, we find \( d\kappa_c/d\kappa = N/D \), where, to simplify notation, we take \( \theta = (f - \kappa f')/f \) and \( \theta_c = (f_c - \kappa_c f'_c)/f_c \):

\[
\begin{align*}
\text{(A17)} \quad N &= \delta \kappa^2 \kappa_c \theta_c f'' - \delta \kappa f' \kappa_c \theta_c \{\delta \theta + (1 - \theta)\} + (\delta - 1) \kappa_c \theta_c \theta_c \{\delta \theta + (1 - \theta)\} \\
& \quad + \delta \kappa_c f(1 - \theta) \theta_c \theta_c \\
D &= \kappa f(\delta \theta + (1 - \theta))\{\delta - 1\} \theta_c + \delta \kappa_c^2 f''/f_c.
\end{align*}
\]

We now turn to our main concern, the utilization rate.

\[
\begin{align*}
\text{(A18)} \quad u &= \frac{Q^d}{Q^d} = G(K, L) \frac{L_f^\delta}{L_c^\delta} = \frac{\kappa_c f}{\kappa_c f_c}.
\end{align*}
\]

Differentiating, we have

\[
\begin{align*}
\text{(A19)} \quad \frac{du}{d\omega} \sim \frac{du}{d\kappa} \sim (\kappa_c f_c) \frac{d\kappa_c}{d\kappa} - \kappa_c \theta_c = (\kappa_c f_c) \frac{N}{D} - \kappa_c \theta_c.
\end{align*}
\]
Examining the expression for $D$ in (A17), one finds that the last bracketed term is $1/f$, times the bracketed term in the expression for $G_{LL}$ in (2). That is, $G_{LL} < 0$ implies $D < 0$. Therefore, multiplying through (A19) by $-D/\kappa_kf$, we have

$$\frac{du}{d\omega} \propto \theta^2(\delta \theta + (1 - \theta))(\delta - 1)\theta_c + \frac{\kappa^2 f''/f_c'}{f} - \delta \kappa^2 \theta_c f''/f$$

Using (A4) for $f''$ and $f'_c$, multiplying through by $\sigma/\theta_c(\delta \theta + (1 - \theta))$, and performing many simplifications, we end up with

$$\frac{du}{d\omega} \propto (\sigma - 1)(\delta - 1)\theta_c(1 - \theta_c) + (\theta_c - \theta).$$

If it can be shown that $\theta_c \geq \theta$ as $\sigma \geq 1$, that will prove $du/d\omega \equiv 0$ as $\sigma \equiv 1$, as stated in the text.

To prove this, note that the share of output going to labour is given by $\omega/f$, or, using (A7), $\delta \theta/(\delta \theta + (1 - \theta))$. If the wage-profit ratio were to fall to that level which would make $\kappa_c$ the cost-minimizing configuration for firms, then the share of labour would be $\delta \theta_c/(\delta \theta + (1 - \theta_c))$. This hypothetical share is greater or less than the actual share as $\sigma \equiv 1$. But simple algebra shows that

$$\frac{\delta \theta_c}{\delta \theta + (1 - \theta)} \geq \frac{\delta \theta}{\delta \theta + (1 - \theta)} \quad \text{as } \theta_c \equiv \theta.$$

This proves the result, $du/d\omega \equiv 0$ as $\sigma \equiv 1$.

NOTES

1 Goodwin takes $v$ as a constant, but $s = rv$, which increases in $r$. His steady-state profit and wage rates, therefore, are given by the particularly simple form of (1), $r^*(w^*) = g^*$.

2 Goodwin’s equation was specified in terms of the logarithmic derivative of the real wage, but this difference is inconsequential.

3 Desai (1973) considers a Goodwin-type model with money illusion. In his model, (2) is replaced by two equations: a Phillips curve in nominal wages, with only a partial pass-through of price inflation (Goodwin’s version, (2), has a full pass-through); and a price adjustment equation, by which firms always try to reduce the gap between a fixed target mark-up and the actual mark-up.

These modifications have the following implications for the steady state: the steady-state wage and profit rate are still given by (1), independent of the degree of money illusion. This means that the steady-state share of wages, and hence the steady-state actual mark-up, are also fully governed by (1) without reference to the Phillips and price adjustment equations. However, the steady-state employment rate, $n^* (\nu^* \text{ in Desai’s notation})$, is affected by the degree of money illusion and the target mark-up. To see this, first note that there is nothing in the model to bring the target mark-up into line with the steady-state actual mark-up. The gap between the two determines the steady-state rate of inflation (or deflation). The higher is the steady-state rate of inflation, the higher is the steady-state employment rate consistent with the steady-state share of wages. This is an immediate consequence of money illusion, and the more money illusion there is, the greater is the steady-state employment rate associated with any positive level of inflation.

Desai also shows that money illusion affects the stability properties of the system. Goodwin’s system, with no money illusion, produces closed orbits, but money illusion stabilizes the system, producing convergent cycles.

4 This would be a serious problem if, as Goodwin maintains, his model is aimed at formalizing the Marxian theory of the “reserve army”. Marx’s doctrine of the reserve army is embedded in a volume largely devoted to detailing how weak labour is vis-à-vis capital, not how strong it would be without unemployment.

5 See Arrow (1959).

6 The specification (3) is implicit in the work of others. For example, Arrow and Hahn’s (1971) assumption A.11.1 takes wage and price dynamics, as measured in fictional units of account, to be governed separately by the state of the labour and product markets respectively.
If so, then (3) follows. Alternatively, their assumption A.11.2 takes movements in money wages and prices to be so governed, as do most conventional macro-models. Once again, (3) follows. Solow and Stiglitz modify such expressions for money wage and price dynamics by including a partial pass-through of price and wage increases, respectively. As they show, (3) follows here, too. The specification (3) receives some econometric support from US data (see Costrell, 1978, pp. 47–56). The symmetry argument underlying (3) can also be applied to questions of stagflation and the real balance effect (see Costrell, 1982).

Another common explanation of persistent excess capacity is derived from the cyclical variability of demand. Of course, strictly speaking, one would find no such variability in steady state, but it may be helpful in understanding the level of the trend around which we observe cycles.

Table 2 of the Handbook of Cyclical Indicators (US Department of Commerce, 1977) gives the average lead or lag of the turns of various series relative to the reference cycle. The employment rate is series 90, the ratio of civilian employment to total population of working age. The capacity utilization rate is series 82, the Federal Reserve Board’s measure. The profit share is series 81, the ratio of profits (after taxes) with inventory valuation and capital consumption adjustments to total corporate domestic income. (According to Lovell, quarterly data on this variable show a 0.86 correlation with interest plus profits relative to the capital stock, the concept that is, perhaps, closest to the profit rate of the models under consideration.) The average length of the business cycle over the period 1948–1970 (60.2 months) was calculated from Appendix E of the February 1982 issue of Business Conditions Digest.

Note that this interpretation differs from the usual, overly rigid, assertion that the kink price is the most recent price. This was Sweezy’s (1939) original argument, which led to Stigler’s (1947) criticism and hence to the eclipse of kinked demand theory until Hahn’s and Negishi’s revival.

For an elegant proof provided by Arrow, see Costrell (1978, pp. 212–213).

The introduction of adjustment costs might improve the model’s depiction of the relationship between the profit rate and the utilization rate. Without adjustment costs, we shall see that the two move together, whereas the data suggest that the utilization rate lags the profit rate, as mentioned above. Adjustment costs, however, may interpose a lag between factor proportions and factor prices, which in the present model would mean a lag between the utilization rate and the profit rate.

The special case δ = 1 is the constant returns case discussed above, where w = ω, k = κ, and u = 1, independent of ω. Thus, du/dω = 0 and the stability condition is satisfied, as stated above.

There are, of course, other reasons for the observed relationship between the utilization rate and the employment rate, such as general and specific labour hoarding (see Costrell, 1981–1982).

See Costrell (1978, pp. 235–238). As noted there, to model the cyclical pattern of productivity it may be necessary to argue that the employment rate (or its rate of change) enters the production function.

REFERENCES


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